Central Force Motion

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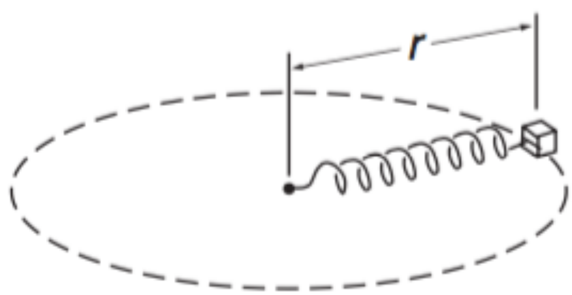
A Sample Question Question

A $2kg$ mass on a frictionless table is attached to one end of a massless spring. The other end of the spring is held by a friction- less pivot. The spring produces a force of magnitude $3r$ newtons on the mass, where r is the distance in meters from the pivot to the mass. The mass moves in a circle and has a total energy of $12J$.



a) Find the radius of the orbit and the velocity of the mass.

For motion in a circular orbit for this mass that's connected to a spring.

$$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2$$

Using sum of forces, since the only force acting on the mass is the spring force, we have $kr = \frac{mv^2}{r}$.

multiply by r

$$mv^2 = kr^2$$

substitute into E for mv^2

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = kr^2$$

$$E = 12J = kr^2$$

We're told in the problem that the force is of magnitude $3r$**, meaking** k **is 3.**

$$12J = 3r^2$$

$$r = 2m$$

For the velocity, we can simply use the relation from the sum of the forces, (sometimes called the equillibrium relation).

$$kr = \frac{mv^2}{r}$$

$$v_0 = \frac{kr^2}{m}$$

$k = 3N/m, r = 2m, m = 2kg$

$$v_0 = \sqrt{\frac{6m}{s}}$$

b) The mass is struck by a sudden sharp blow, giving it instantaneous velocity of $1m/s$ **radially outward. Show the state of the system before and**

after the blow on a sketch of the energy diagram.

Before the blow, we have a total energy such as the following:

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$$

But taking into account, this new impulse, it's extending the block radially outward. So we can simply add 1 meter per second in kinetic energy.

$$E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$$

$$\delta E = E + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}kr_i^2 + \frac{1}{2}mv_\theta^2 + \frac{1}{2}m = \frac{1}{2}k_f^2$$

And afterwards it oscillates about this new, augmented equillibrium.