Relativistic Changes in Physics

David Awad

The Lorentz transformations relate the coordinates of a *single event* in one inertial system to the coordinates of the *same event* in another inertial system.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Since $v^2/c^2 \leq 1$, γ is greater than or equal to one.

Variable	prime	inertial
x	$x' = \gamma(x - vt)$	$x = \gamma(x' + vt')$
y	$y' = y$	$y = y'$
z	$z' = z$	$z = z'$
t	$t' = \gamma\left(t - \frac{xv}{c^2}\right)$	$t = \gamma\left(t' - \frac{x'v}{c^2}\right)$

Length Contraction

$$L = \frac{L_0}{\gamma(v)} = L_0\sqrt{1 - v^2/c^2}$$

Time Dilation

$$t' = L_0\gamma(v) = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}$$

Relativistic Addition of Velocity.

Let's imagine the speed of a space ship from the reference frame of another. That would be modeled with the following relation. And this is done simply by differentiating the Lorentz transformations for x over y.

$$\frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma(dt - \frac{vdx}{c^2})} = \frac{\frac{dx}{dt} - v}{1 - \frac{v\frac{dx}{dt}}{c^2}}$$

Following the math we end up at the following relationship.

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Applying this transformation to a spacecraft traveling at 0.8c which fires a projectile that it observes to be moving at 0.7c with respect to it. We would expect the classical results to be 0.15c. However we actually obtain a velocity of 1.5c/1.56 = 0.96c rather than the 1.5c which seems to be the common sense answer.

Let's imagine the speed of the death star from the reference frame of the enterprise. The enterprise is moving west at a speed of $0.9c$. Entirely possible. Now imagine another vessel, the death star going in the opposite direction at the same speed. If I were an observer on the death star, I would see the enterprise moving away at a speed of $1.8c$. Well surely that can't be. That would be faster than light.

Relativistic Doppler Effects

observer	model
For a moving observer	$v_0{}' = v_0(1 + v/w)$
For a moving source	$v_0' = v_0\left(\frac{1}{1 - v/w}\right)$

$$v_D = v_0\left(\sqrt{\frac{1 + v/c}{1 - v/c}}\right)$$

ν_D is the frequency in the observer's rest frame and v is the relative speed of source and observer. As we expect, there is no mention of motion relative to a medium. The relativistic result plays no favorites with the classical results; it disagrees with both but treats them.

Relativistic Momentum

The energy and momentum of a particle of rest mass m moving at velocity v:

$$p = \gamma m_0 v$$

$$E = \gamma mc^2$$

Let's say it's moving close to the speed of light, it's mass may not necessarily stay m. For a particle moving at arbitrary speed u, the mass is represented as.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

As for kinetic energy:

$$KE = mc^2 - m_0c^2$$

The energy and momentum of a *photon* are related by $E = pc$.

And in general, for particles of non-zero rest mass, $E^2 = (mc^2)^2 + (pc)^2$

How energy and momentum are conserved in collisions or decays.