A Small Intro to Harmonic Oscillators

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$$\sum \vec{F} = m\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t}$$

There are multiple formulas that can be used to describe the motion of a harmonic oscillator. The most important to be aware of is that usually, a harmonic oscillator follows some given motion.

start with newton's second law.

$$\sum \vec{F} = m\vec{a}$$

Adjust for spring force.

(and other forces if they apply, doesn't matter in this example but you get my point)

$$-kx = m\ddot{x}$$

$$0 = m\ddot{x} + kx$$

Now we have it in a form where we can do some really useful things to find the oscillations.

divide by m, note that we've put k over m for the convenience of a particular formula

$$0 = \ddot{x} + \frac{k}{m}x$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Where ω_0 is the natural frequency of a simple harmonic oscillator, with a spring constant k and mass m.

This leaves us with an interesting differential equation.

$$0 = \ddot{x} + \omega_0^2 x$$

note: this equation represents the position as a function of time. If this isn't clear, remember that $\ddot{x}$ is simply the acceleration, which is also a function of time.

So how can we solve this? One way is to use the convenient $e^{i\omega t}$ who's derivative is very similar.

$$\frac{d^2}{dt^2} e^{i\omega t} = -\omega^2 e^{i\omega t}$$

Leaving us with an equation to properly model the motion in terms of natural frequency!

note: don't confuse natural frequency ω_0 with the actual oscillating frequency ω which is the frequency that the system is actually oscillating at.

So let's look at our new equation.

$$0 = \ddot{x} + \omega_0^2 x$$

$$f(x) = e^{i\omega t}$$

$$0 = -\omega^2 e^{i\omega t} + \omega_0^2 e^{i\omega t}$$

$$0 = e^{i\omega t}(-\omega^2 + \omega_0^2)$$

In order for this equality to hold it must be the case that