CS314 Homework 1

Sample solution

Spring 2014

1. (a) $01 \Rightarrow \epsilon$, $10 \Rightarrow \epsilon$
   or
   $01 \Rightarrow T$, $10 \Rightarrow T$, $T\# \Rightarrow \#$, $T1 \Rightarrow 1$, $1T \Rightarrow 1$, $T0 \Rightarrow 0$, $0T \Rightarrow 0$

   (b) You may have multiple options to which rule to apply, but you can only apply one rule at a time. \$0110# we have two options \$0110# $\Rightarrow$ \$10# using rule $01 \Rightarrow \epsilon$ *or* \$0110# $\Rightarrow$ \$01# using rule $10 \Rightarrow \epsilon$

   (c) i. \$0110# use rule $01 \Rightarrow \epsilon$
   \$10# use rule $10 \Rightarrow \epsilon$
   \$# no rules
   or
   \$0110# use rule $10 \Rightarrow \epsilon$
   \$01# use rule $01 \Rightarrow \epsilon$
   \$# no rules

   ii. \$00010# use rule $10 \Rightarrow \epsilon$
   \$000# no rules
   or
   \$00010# use rule $01 \Rightarrow \epsilon$
   \$000# no rules

2. $(+ \mid - \mid \epsilon)$ $(0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)^+$
   $(\epsilon \mid . \; (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)^+$
   $(\epsilon \mid E \; (\epsilon \mid - \;) \; (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)^+))$

   A more readable version

   $(+ \mid - \mid \epsilon)$ digit$^+$ $(\epsilon \mid .$ digit$^+$ $(\epsilon \mid E \; (\epsilon \mid -)$ digit$^+))$

3. (a) All binary strings, including the empty string.

   (b) All binary strings of length ≥ 3 with 0 as the third to last digit.

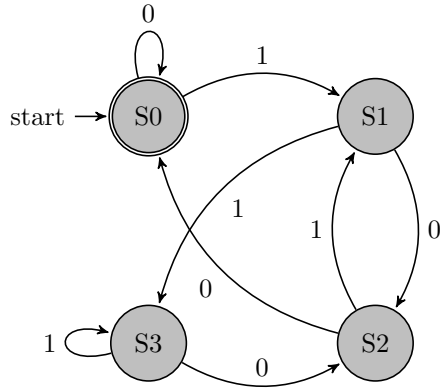
   (c) All binary strings of even length, including the emtpy string, where the # of zeroes is even and the # of ones is even.

4. (a) i. No b's **directly** following any c's: $(ca^+ \mid a \mid b)^*c^*$ *or* $b^*(a^+b^* \mid c)^*$

   ii. No b's following any c's: $(a \mid b)^*(a \mid c)^*$

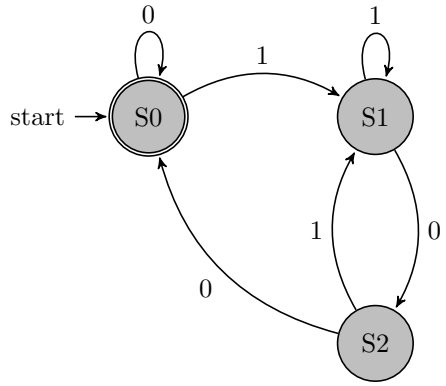
   (b) $c^*(\epsilon \mid b \mid a \mid ac^*a \mid ac^*ac^*a \mid bc^*a \mid bc^*ac^*a \mid bc^*ac^*ac^*a \mid ac^*b \mid ac^*bc^*a \mid ac^*bc^*ac^*a \mid ac^*ac^*b \mid ac^*ac^*bc^*a \mid ac^*ac^*ac^*b)c^*$
   or
   $c^*(b \mid \epsilon)c^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*$ | */*no b or b in first position*/*
   $c^*(a \mid \epsilon)^*c^*bc^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*$ | */*b in second position*/*
   $c^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*bc^*(a \mid \epsilon)^*c^*$ | */*b in third position*/*
   $c^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*(a \mid \epsilon)^*c^*bc^*$ */*b in last position*/*

5. (a) i. <S, s, F, T>
S = {S0, S1, S2 , S3}
s = S0
F = {S0}
T = {S0 x 0 ⇒ S0,
S0 x 1 ⇒ S1,
S1 x 0 ⇒ S2,
S1 x 1 ⇒ S3,
S2 x 0 ⇒ S0,
S2 x 1 ⇒ S1,
S3 x 0 ⇒ S2,
S3 x 1 ⇒ S30}



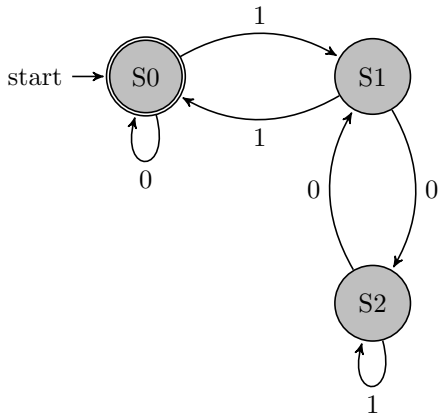
S0 represents mod4 = 0, S1 is mod4=1, and S2 is mod4=2, and S3 is mod4=3

ii. <S, s, F, T>
S = {S0, S1, S2}
s = S0
F = {S0}
T = {S0 x 0 ⇒ S0,
S0 x 1 ⇒ S1,
S1 x 0 ⇒ S2,
S1 x 1 ⇒ S1,
S2 x 0 ⇒ S0,
S2 x 1 ⇒ S1}



Divisible for 4 means that the last two digits should be 00 or nothing. S0 encodes that the last two digits were nothing (empty string) or two 0's. S1 encodes that the last digit was a 1. S2 encodes that the last two digits were a 1 followed by a 0.

(b) <S, s, F, T>
S = {S0, S1, S2}
s = S0
F = {S0}
T = {S0 x 0 ⇒ S0,
S0 x 1 ⇒ S1,
S1 x 0 ⇒ S2,
S1 x 1 ⇒ S0,
S2 x 0 ⇒ S1,
S2 x 1 ⇒ S2}



S0 represents mod3 = 0, S1 is mod3=1, and S2 is mod3=2