Class Information

- Eighth homework has been posted.
- Second project extension: **Monday, April 21, 11:59pm**.
- Sakai submission will be set up soon.

Review: Dependence — Basics

Theorem

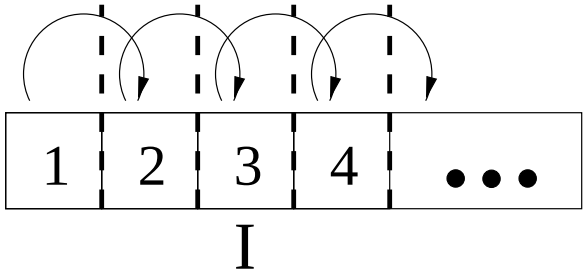
Any reordering transformation that preserves every dependence (i.e., visits first the source, and then the sink of the dependence) in a program preserves the meaning of that program.

□

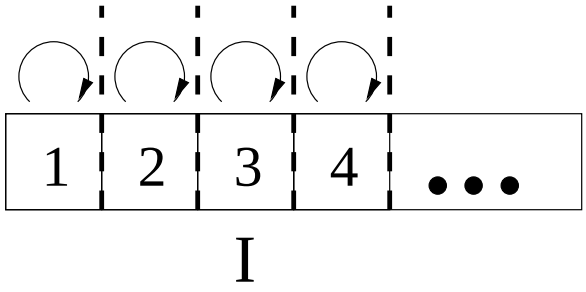
Note: Dependence starts with the notion of a sequential execution, i.e., starts with a sequential program.

Review: Dependence Analysis for Array References

```
do I = 1, 100
   A(I) =
      = A(I-1)
enddo
```




```
do I = 1, 100
   A(I) =
      = A(I)
enddo
```



A loop-independent dependence exists regardless of the loop structure. The source and sink of the dependence occur on the same loop iteration.

A loop-carried dependence is induced by the iterations of a loop. The source and sink of the dependence occur on different loop iterations.

Loop-carried dependences can inhibit parallelization and loop transformations

Dependence Testing

Given

```
do i1 = L1, U1
   ...
      do in = Ln, Un
S1       A(f1(i1, ..., in), ..., fm(i1, ..., in)) = ...
S2       ... = A(g1(i1, ..., in), ..., gm(i1, ..., in))
```

A *dependence* between statement S_1 and S_2, denoted $S_1 \delta S_2$, indicates that S_1, the *source*, must be executed before S_2, the *sink* on some iteration of the nest.

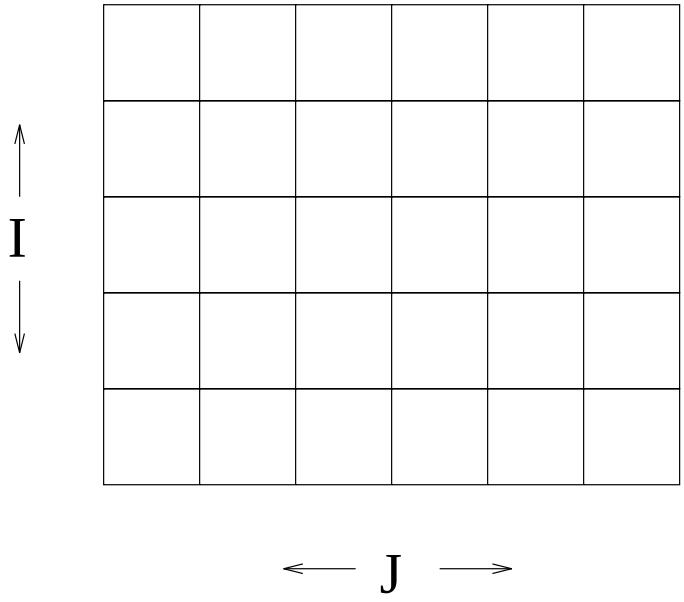
Let α & β be a vector of n integers within the ranges of the lower and upper bounds of the n loops.

Does $\exists\ \alpha \leq \beta$, s.t.

$$f_k(\alpha) = g_k(\beta) \quad \forall k,\ 1 \leq k \leq m\ ?$$

Iteration Space

```
do I = 1, 5
   do J = I, 6
      ...
   enddo
enddo
```



$1 \leq I \leq 5$

$I \leq J \leq 6$

- lexicographical (sequential) order for the above iteration space is

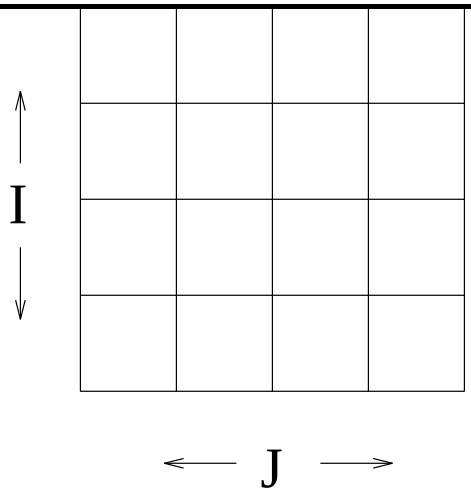
  (1,1), (1,2), . . ., (1,6)
  (2,2), (2,3), . . . (2,6)
  . . .
  (5,5), (5,6)

- given $I = (i_1, \ldots i_n)$ and $I' = (i'_1, \ldots, i'_n)$,

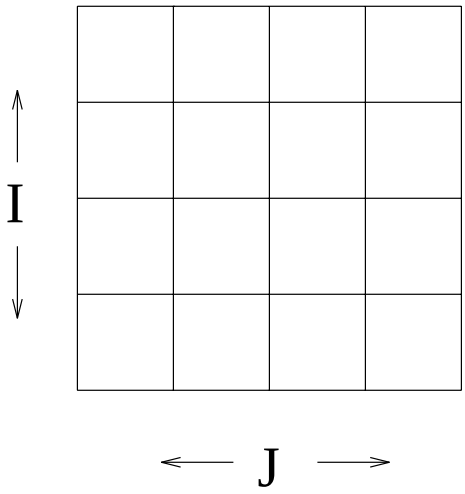
  $I \ < \ I'$ *iff*
  $(i_1, i_2, \ldots i_k) = (i'_1, i'_2, \ldots i'_k) \ \ \& \ \ i_{k+1} < i'_{k+1}$

Distance & Direction Vectors

```
 do I = 1, N
  do J = 1, N
S1 A(I,J) = A(I,J-1)
  enddo
 enddo
```




```
 do I = 1, N
  do J = 1, N
S2 A(I,J) = A(I-1,J-1)
S3 B(I,J) = B(I-1,J+1)
  enddo
 enddo
```



Distance Vector = number of iterations between accesses to the same location

Direction Vector = direction in iteration space $(=, <, >)$

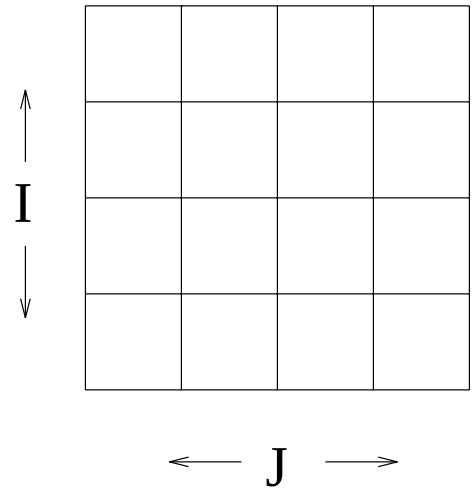
	distance vector	direction vector
$S_1 \delta S_1$	(0,1)	(=,<)
$S_2 \delta S_2$	(1,1)	(<,<)
$S_3 \delta S_3$	(1,-1)	(<,>)

Which Loops are Parallel?

```
 do I = 1, N
  do J = 1, N
S1 A(I,J) = A(I,J-1)

 do I = 1, N
  do J = 1, N
S2 A(I,J) = A(I-1,J-1)

 do I = 1, N
  do J = 1, N
S3 B(I,J) = B(I-1,J+1)
```



- a dependence $D = (d_1, \ldots, d_k)$ is *carried* at *level* i, if d_i is the first nonzero element of the distance/direction vector

- a loop l_i is *parallel*, if $\nexists$ a dependence D_j carried at level i

	distance vector	direction vector
$\forall D_j$	$d_1, \ldots, d_{i-1} > 0$	$d_1, \ldots, d_{i-1} =$ "$<$"
OR	$d_1, \ldots, d_i = 0$	$d_1, \ldots, d_i =$ "$=$"