OBSERVATORIO

ASTRONÓMICO

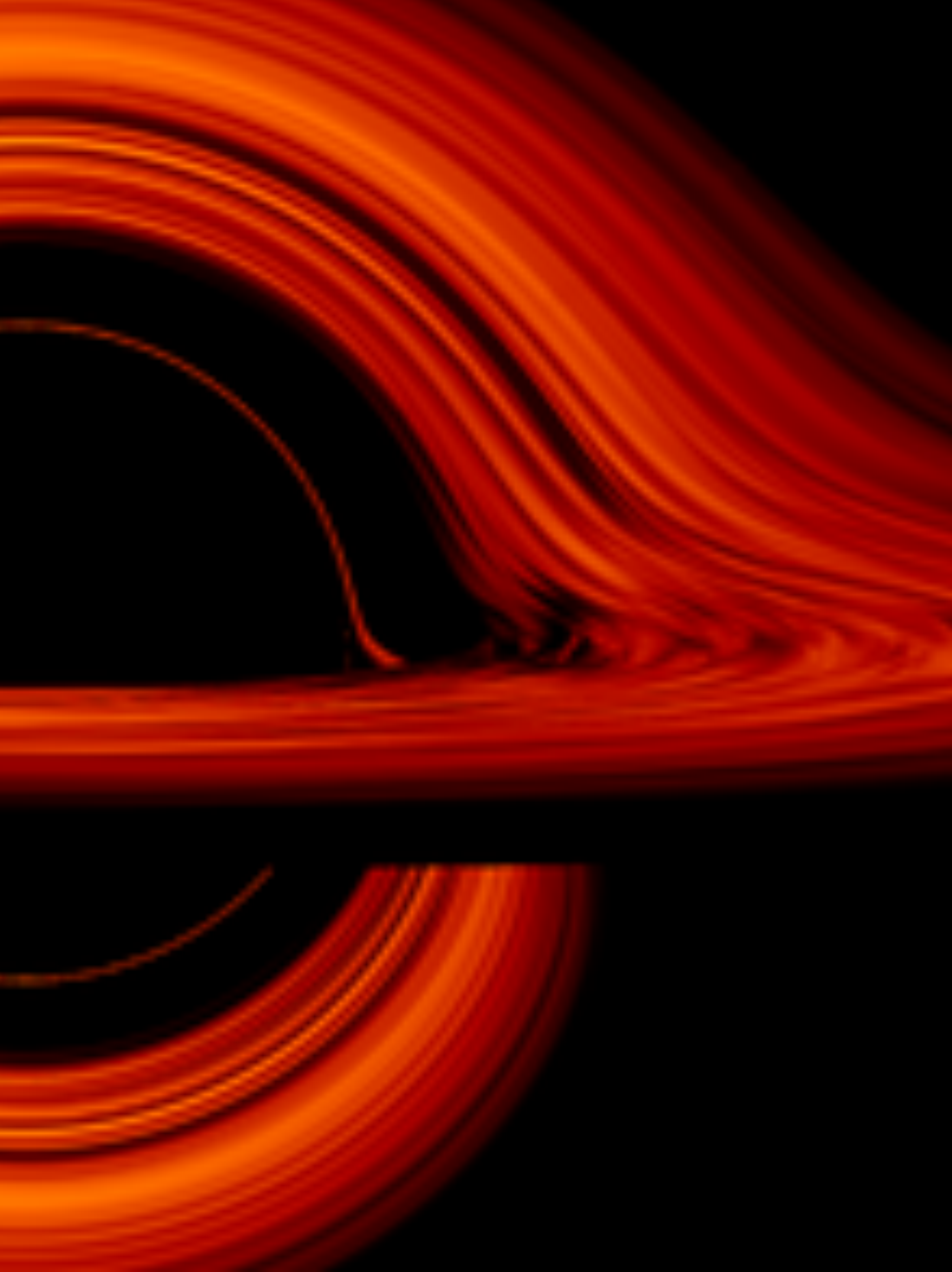
NACIONAL DE COLOMBIA

COMPUTATIONAL ASTROPHYSICS

Observatorio
Astronómico
Nacional

Extended Seminar
“A code to generate the image of a black hole”

Eduard Larrañaga
Observatorio Astronómico Nacional



Contents

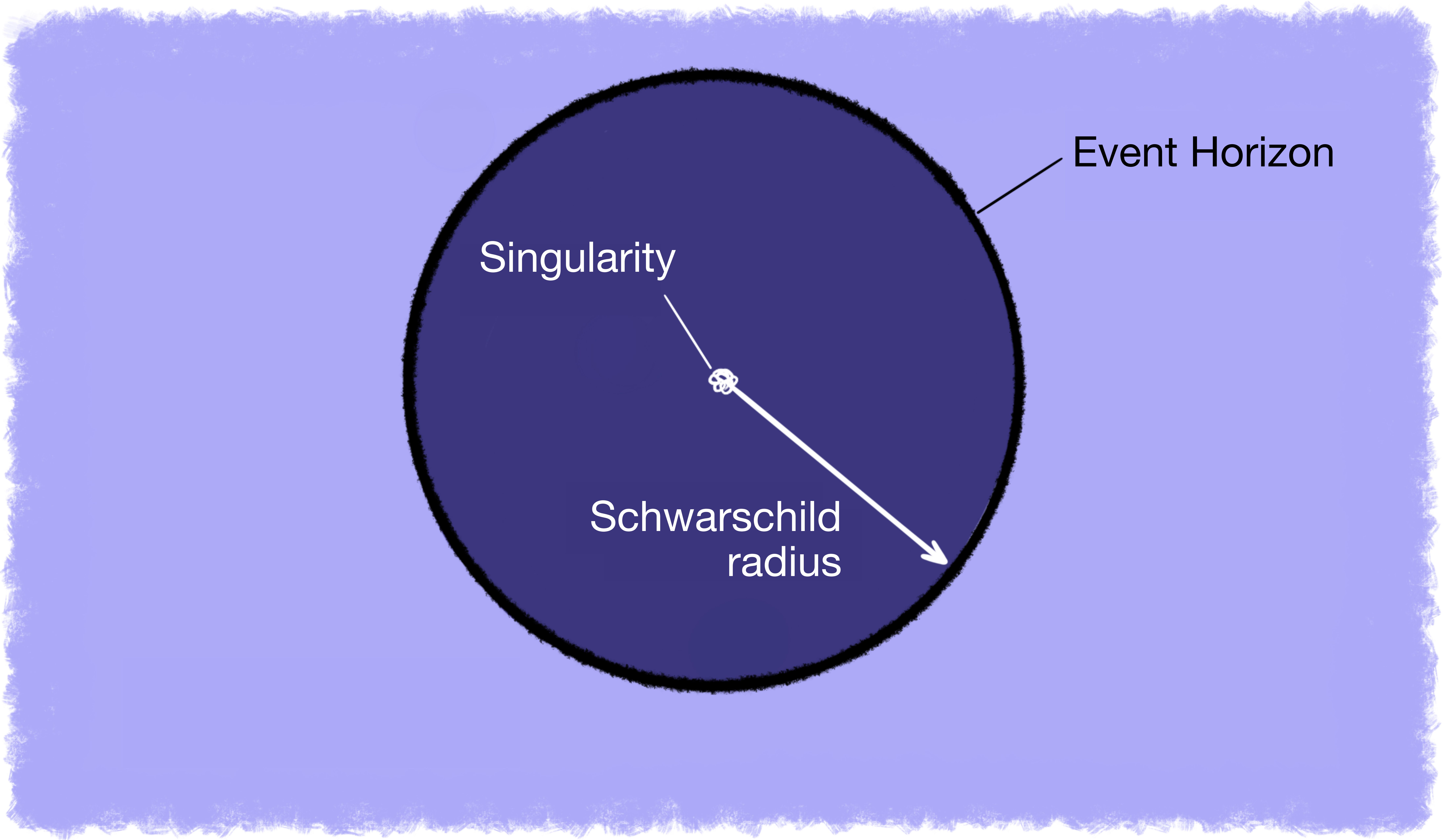
Session 01

Motion of a Photon in a Schwarzschild Background

The Schwarzschild Spacetime

The Schwarzschild spacetime

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$


Event Horizon
Singularity
Schwarschild
radius

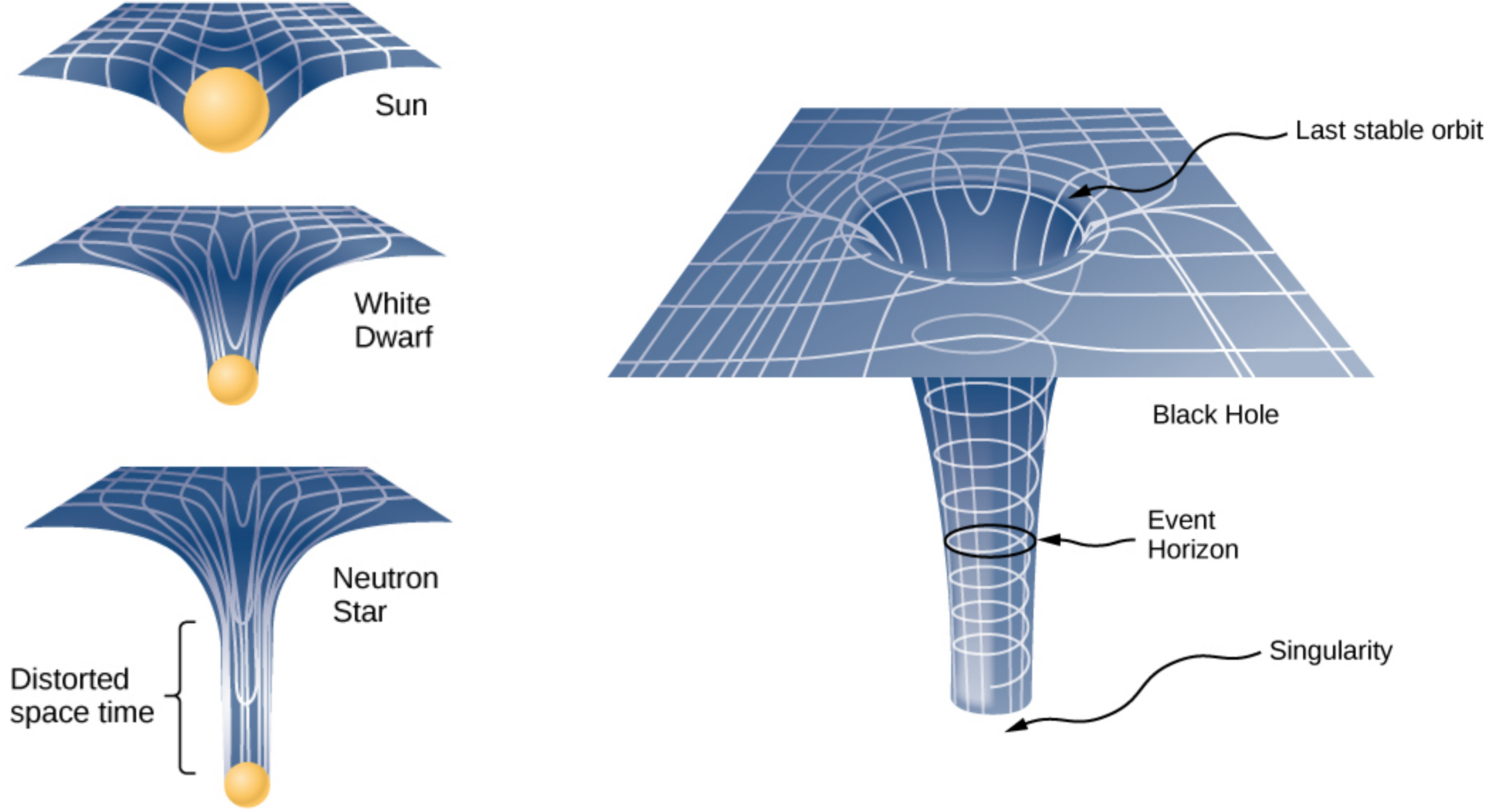


Image Credit: OpenStax University Physics at: https://commons.wikimedia.org/wiki/File:CNX_UPhysics_13_07_spacetime.png

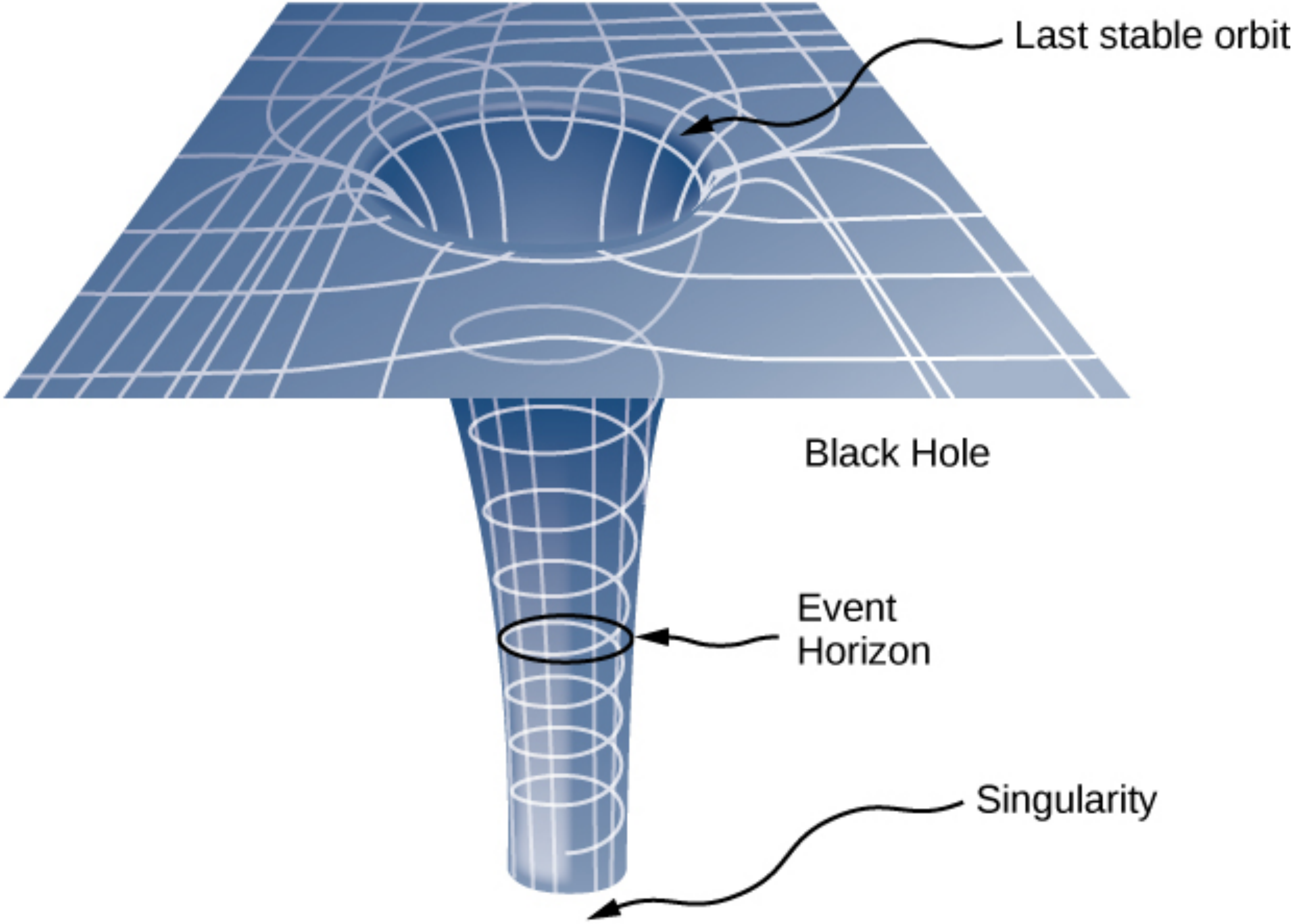


Image Credit: OpenStax University Physics at: https://commons.wikimedia.org/wiki/File:CNX_UPhysics_13_07_spacetime.png

Image Credit: NASA, ESA, STScI, Joseph Olmsted at: https://commons.wikimedia.org/wiki/File:Gravitational_lensing_by_a_black_hole.jpg

The Schwarzschild spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$g_{\mu\nu} = \begin{bmatrix} -\left(1 - \frac{2M}{r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2M}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}$$

Motion of a Photon in Schwarzschild Spacetime

Equations of Motion

$$x^\mu = [t, r, \theta, \phi]$$
$$k_\mu = [k_t, k_r, k_\theta, k_\phi]$$

Constants of Motion: $k_t = \varepsilon$, $k_\phi = \ell$

$$\dot{t} = \frac{\varepsilon}{\left(1 - \frac{2M}{r}\right)}$$

$$\dot{r} = \left(1 - \frac{2M}{r}\right) k_r$$

$$\dot{\theta} = \frac{k_\theta}{r^2}$$

$$\dot{\phi} = \frac{\ell}{r^2 \sin^2\theta}$$

$$\dot{k}_t = \dot{\varepsilon} = 0$$

$$\dot{k}_r = -\frac{M}{(r-2M)^2}\varepsilon^2 - \frac{M}{r^2}k_r^2 + \frac{k_\theta^2}{r^3} + \frac{\ell^2}{r^3 \sin^2\theta}$$

$$\dot{k}_\theta = \frac{\cos\theta}{r^2 \sin^3\theta}\ell$$

$$\dot{k}_\phi = \dot{\ell} = 0$$

Equations of Motion

$$\dot{t} = \frac{\varepsilon}{\left(1 - \frac{2M}{r}\right)}$$

$$\dot{r} = \left(1 - \frac{2M}{r}\right) k_r$$

$$\dot{\theta} = \frac{k_\theta}{r^2}$$

$$\dot{\phi} = \frac{\ell}{r^2 \sin^2 \theta}$$

$$\dot{k}_t = \dot{\varepsilon} = 0$$

$$\dot{k}_r = -\frac{M}{(r-2M)^2}\varepsilon^2 - \frac{M}{r^2}k_r^2 + \frac{k_\theta^2}{r^3} + \frac{\ell^2}{r^3 \sin^2 \theta}$$

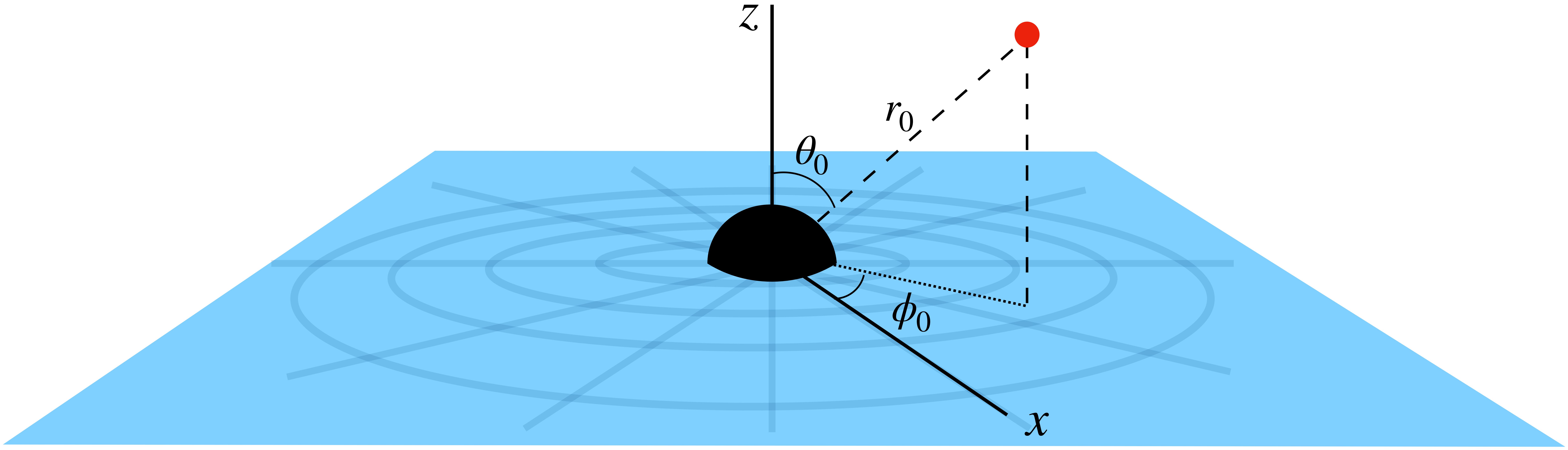
$$\dot{k}_\theta = \frac{\cos \theta}{r^2 \sin^3 \theta}\ell$$

$$\dot{k}_\phi = \dot{\ell} = 0$$

The Initial Conditions

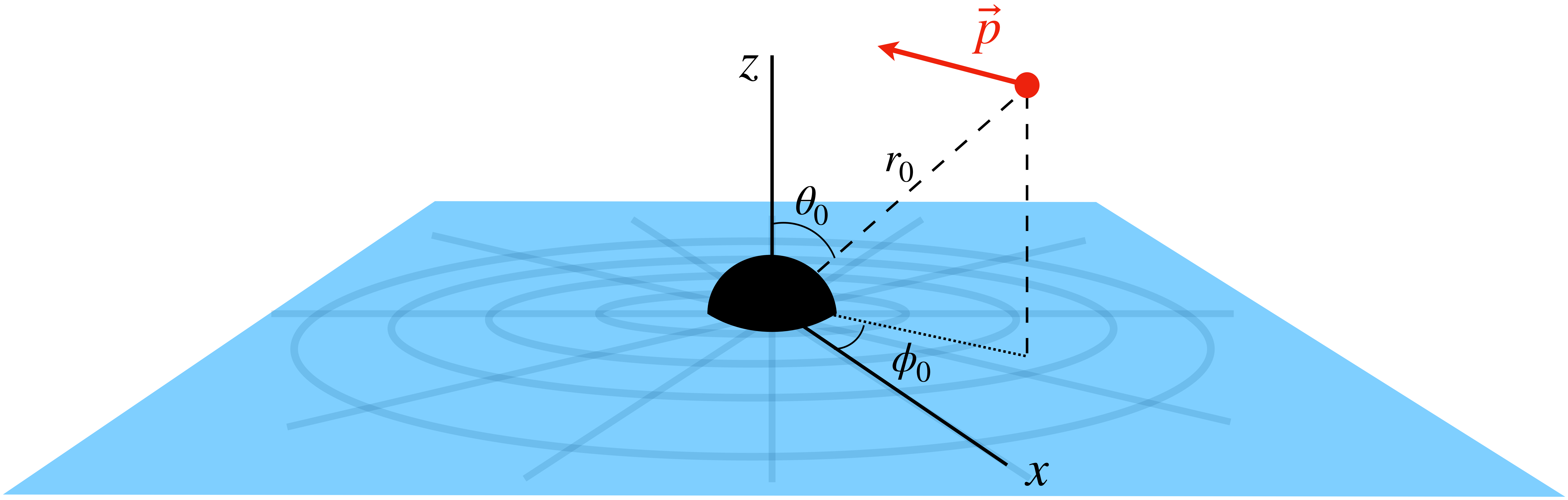
Initial Conditions

$$[0, r_0, \theta_0, \phi_0]$$



Initial Conditions

$$\left[(k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0\right] \longleftarrow [k_0^t, k_0^r, k_0^\theta, k_0^\phi]$$



Initial Conditions

$$\left[(k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0\right] \longleftarrow [k_0^t, k_0^r, k_0^\theta, k_0^\phi]$$

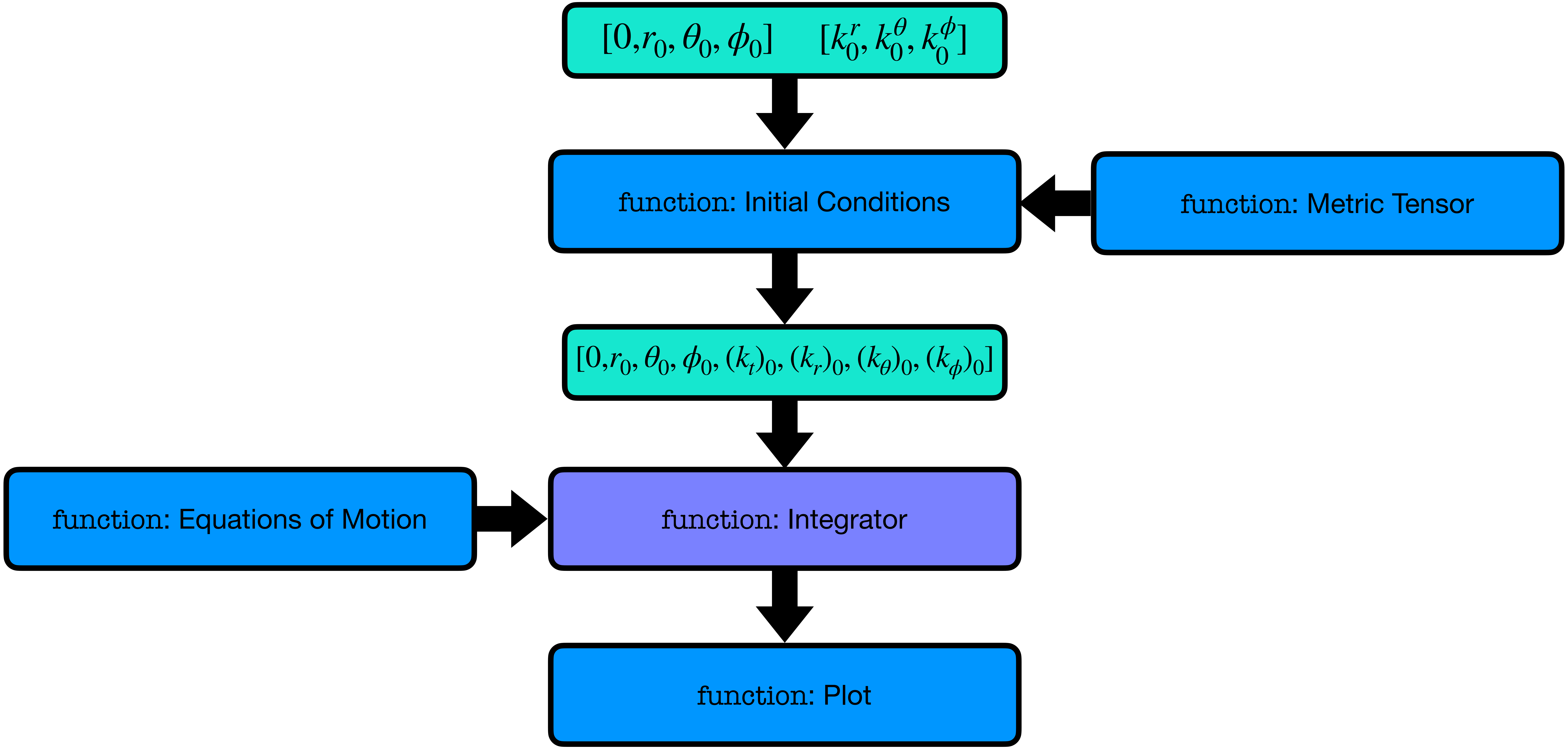
$$k^2 = g_{\mu\nu}k^\mu k^\nu = 0$$

$$k^t = \sqrt{\frac{g_{rr}(k^r)^2 + g_{\theta\theta}(k^\theta)^2 + g_{\phi\phi}(k^\phi)^2}{g_{tt}}}$$

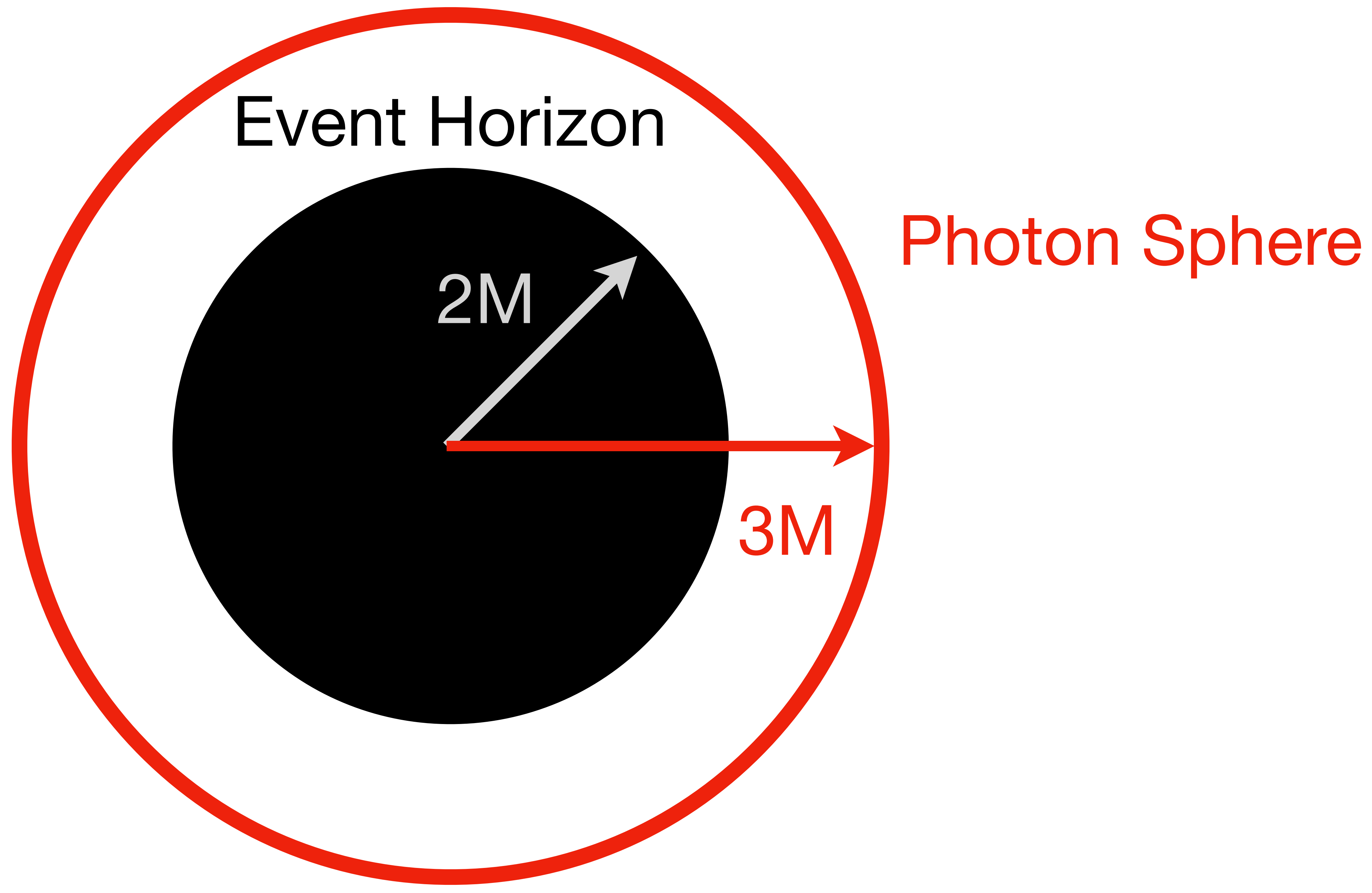
$$k_\mu = g_{\mu\nu}k^\nu$$

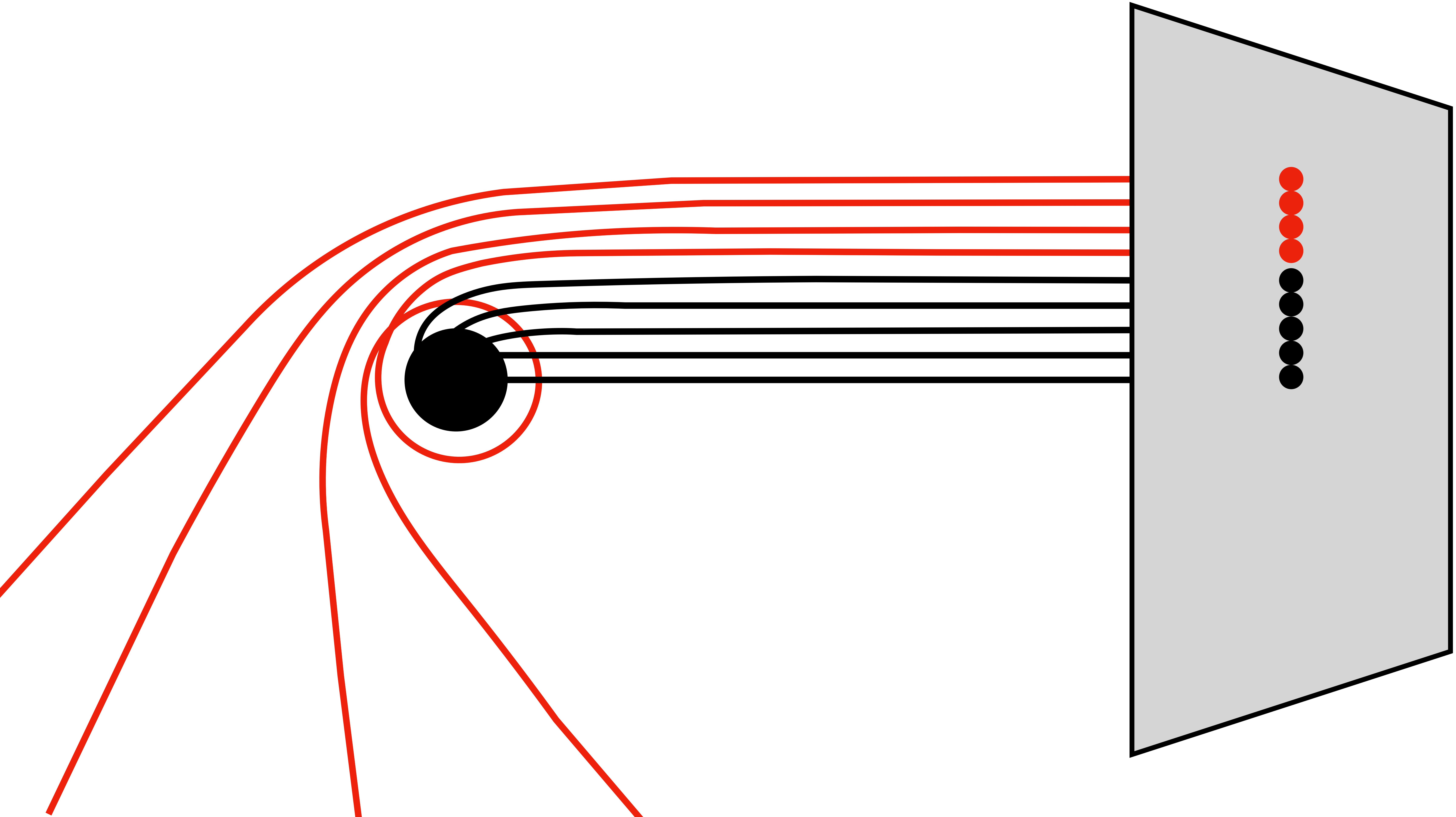
The Code

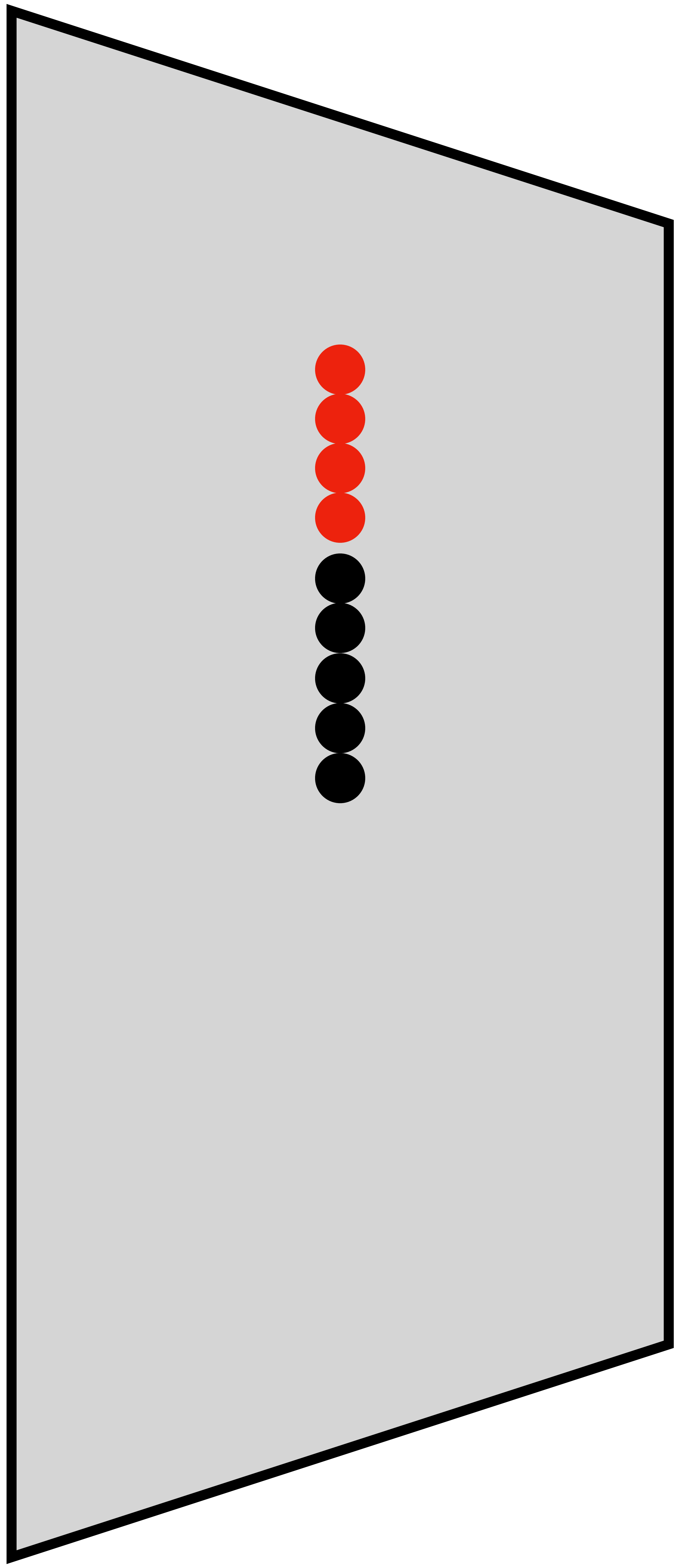
Structure of the Code

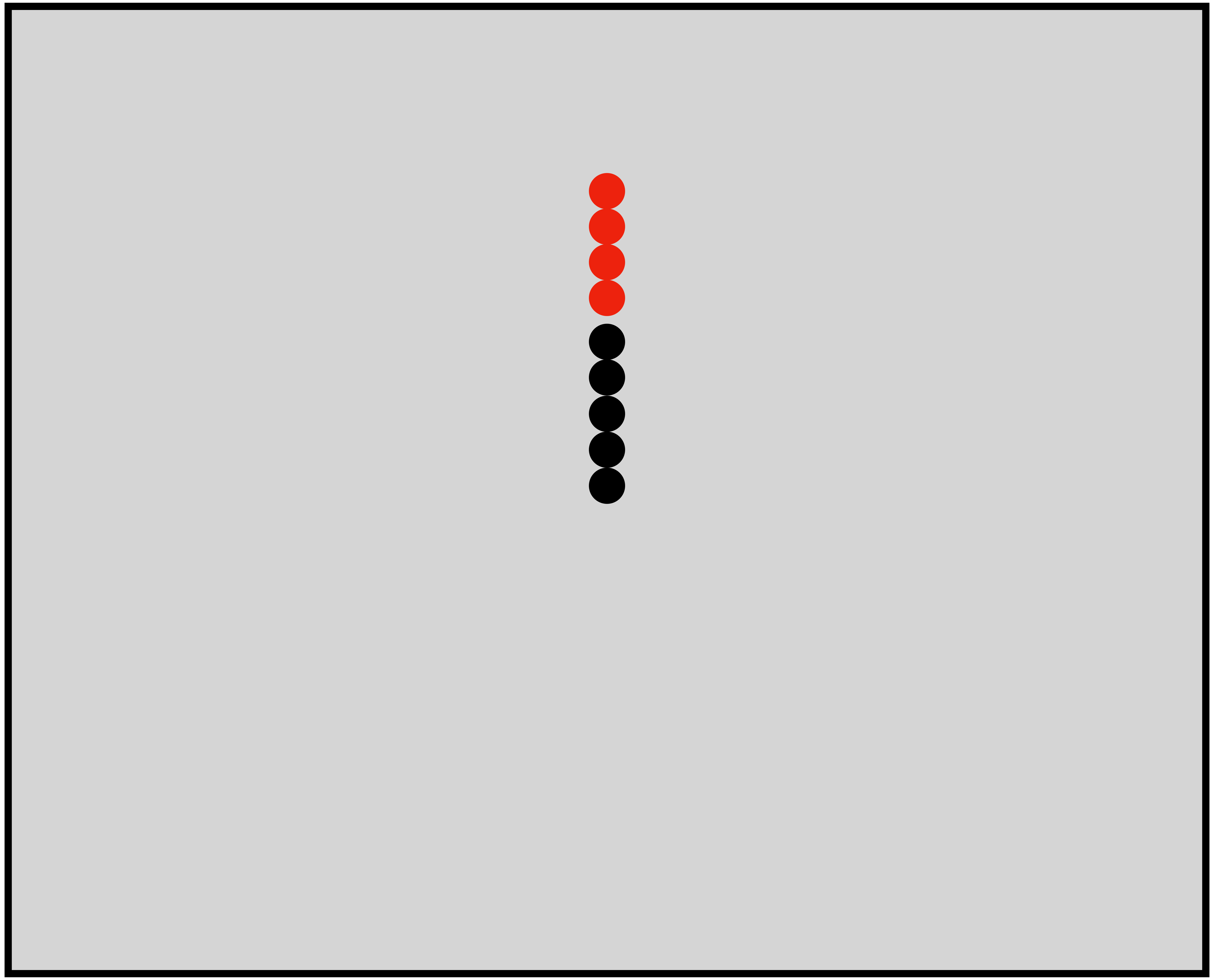


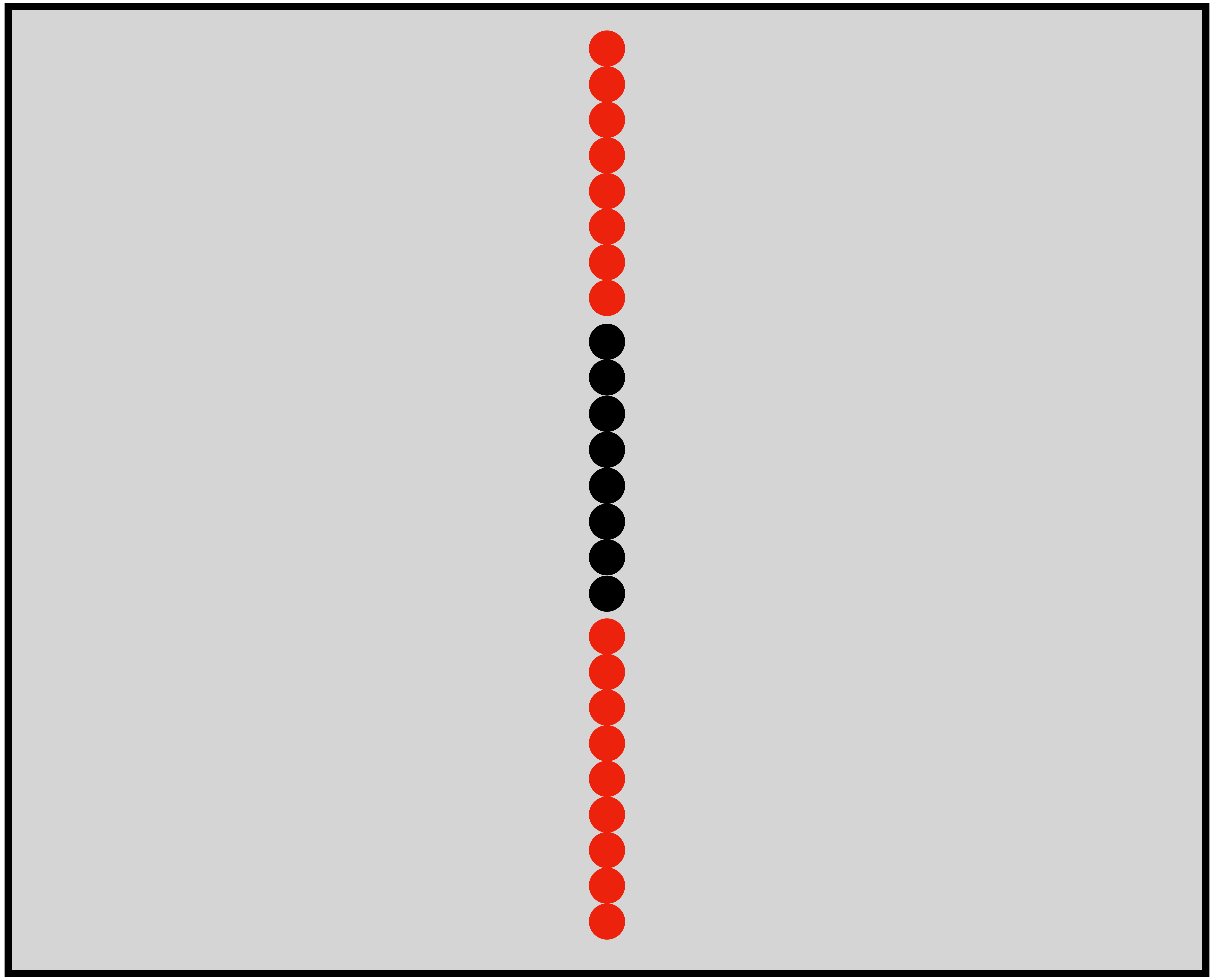
The Photon Sphere and the Shadow of a Black Hole

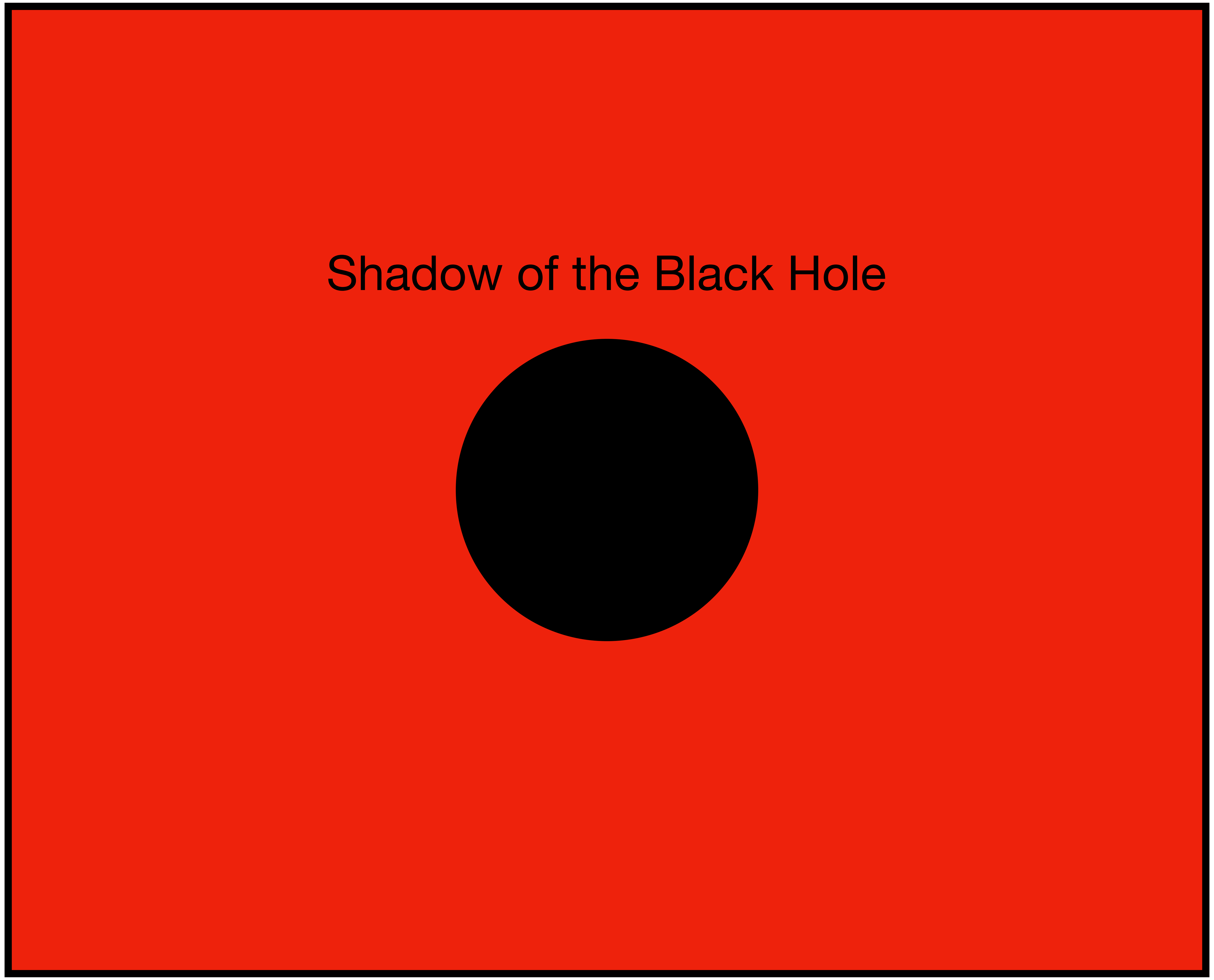

Event Horizon
2M
Photon Sphere
3M










Shadow of the Black Hole

@astronomiaOAN

AstronomiaOAN

@astronomiaOAN