UNIT 1 VECTORS – 1

Stucture

1.0 INTRODUCTION

In this unit, we shall study vectors. A vectors is a quantity having both magnitude and direction, such as displacement, velocity, force etc. A scalar is a quantity having magnitude only but no direction, such as mass, length, time etc. A vector is represented by a directed line segment. A directed line segment is a portion of a straight line, where the two end-points are distinguished as initial and terminal.

Scalars are represented using single real numbers as complex numbers only. A vector in plane is represented using two real numbers. This is done by considering a rectangular coordinate system by which every point in place is associated with a pair of numbers (x,y). Then the vector whose initial point is origin and terminal pont is (x,y) is $x\,\hat{\imath} + y\,\hat{\jmath}$. This is component form of a vector. Similarly, component form of a vector in space is $x\,\hat{\imath} + y\,\hat{\jmath} + z\hat{k}$. We shall discuss it in detail in this Unit. Further, we shall learn to add and subtract vectors and to multiply a vector by a scalar. There are many applications of vectors in geometry. We shall prove the section formula for vectors and solve many problems in geometry using vectors.

1.1 OBJECTIVES

After studying this unit, you will be able to :

- define the terms scalar and vector;
- define position vector of a point;
- find direction cosines and direction ratios of a vector;
- find sum and difference of two vectors;
- multiply a vector by a scalar;
- write a vector in component form; and
- use selection formula in geometrical problems.

1.2 VECTORS AND SCALARS

Many quantities in geometry and physics, such as area, volume, mass, temperature, speed etc. are characterized by a single real number called magnitude only. Such quantities are called **scalars.** There are, however, other physical quantities such as displacement, force, velocity, acceleration etc. which cannot be characterized by a single real number only. We require both magnitude and direction to specify them completely. Such quantities are called **vectors.**

Definition : A quantity having both magnitude and direction is called a vector. Graphically, a vector is represented by a directed line segement. A directed line segment with initial point A and terminal point B is denoted by $\overrightarrow{AB}$.

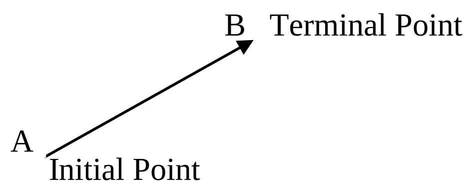


Figure 1

We usually denote vectors by lower case, bold face letters $\bar{a}, \bar{b}$, etc. or by letters with arrows above them such as $\bar{a}, \bar{b}$,etc.

If $\vec{a}$ is a vector represented by directed line segment $\overrightarrow{AB}$, then magnitude of $\vec{a}$ is the length of $\overrightarrow{AB}$ and is dentoed by $|\vec{a}|$or $\left|\overrightarrow{AB}\right|$.

Two vectors are said to be equal if they have same magnitude and the same direction.

A vector with zero magnitude (i.e., when initial point and terminal point coincide) is called a **zero vector** or **null vector.** A vector whose magnitude is unity (i.e., 1 unit) is called a **unit vector.**

Position Vector and Direction Cosines

The position vector $\vec{r}$ of any point P with respect to the origin of reference O is a vector $\overrightarrow{OP}$. Recall that a point P in space is uniquely determined by three coordinates. If P has coordinates (x,y,z) and O is the origin of the rectangular coordinates system, then the magnitude of the position vector $\overrightarrow{OP}$ is given by

$$|\vec{r}| = \left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$$

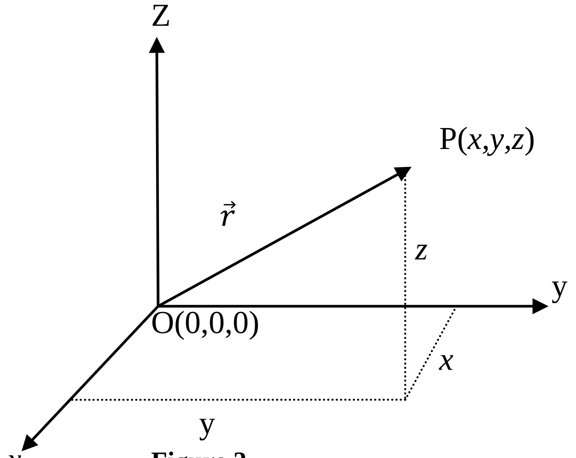


Figure 2

Suppose $\overrightarrow{OP}$ makes angles α, β and γ with the positive directions of x, y and z – axes respectively. These angles are called **direction angles** of $\overrightarrow{OP}$ $(or\ \vec{r}\ )$. The cosines of these angles i.e., $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are called **direction cosines** of the vector $\vec{r}$ and usually denoted by l, m and n respectively.

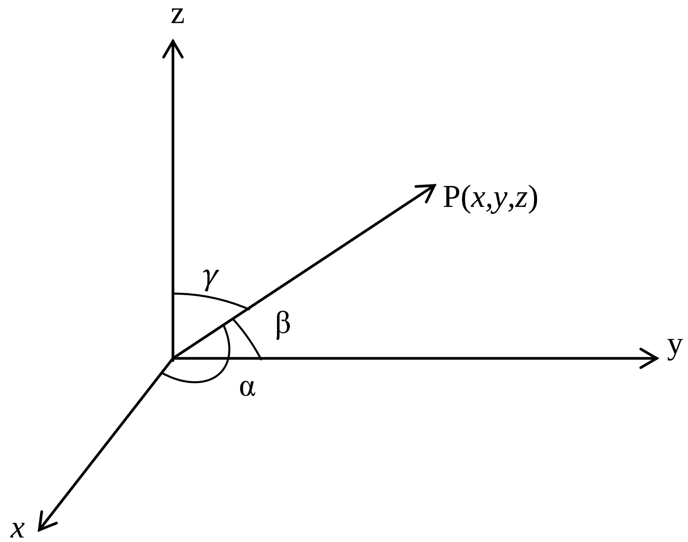


Figure 3

It must be noted that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

i.e., $l^2 + m^2 + n^2 = 1$

In fact, note that $\cos\alpha = \frac{x}{r}$ (here, $r = |\vec{r}|$). Similarly, $\cos\beta = \frac{y}{r}$ and $\cos\gamma = \frac{z}{r}$.

Thus, the coordinates of the point P may also be expressed as (lr, mr, nr). The numbers lr, mr and nr, proportional to the direction cosines are called **direction ratios** of vector $\vec{r}$, and denoted by a, b and c respectively.

Example 1 : Find the magnitude and direction angles of the position vector of the point P(1,2,−2).

Solution : Let $\vec{r} = \overrightarrow{\mathrm{OP}}$. We have

$$r = |\vec{r}| = \left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{1^2 + 2^2 + (-2)^2} = 3$$

$$\text{and} \quad \cos\alpha = \frac{x}{r} = \frac{1}{3}$$

$$\text{and} \quad \cos\beta = \frac{y}{r} = \frac{2}{3}$$

$$\cos \Upsilon = \frac{z}{r} = \frac{-2}{3}.$$

$$\text{Hence} \quad \alpha = \cos^{-1}\frac{1}{3} \approx 70^0 30'$$

$$\beta = \cos^{-1}\frac{2}{3} \approx 48^0 10'$$

$$\gamma = \cos^{-1}\left(\frac{2}{3}\right) \approx 131^0 50'$$

Thus, the vector $\vec{r}$ forms acute antles with the *x*-axis and *y*-axis, and an obtuse angle with the *z*-axis.

Coinitial, Collinear and Coplanar Vectors

Two or more vector having the same initial points are called **coinitial Vectors.** Two vectors are said to be **collinear** if they are parallel to the same line, irrespective of their magnitude and directions. A set of vectors is said to be c**oplanar** if they lie on the same plane, or the planes in which the different vectors lie are all parallel to the same plane.

Addition of Vectors

Let $\vec{a}$ and $\vec{\mathrm{b}}$ be two vectors. We position them so that the initial point of $\vec{\mathrm{b}}$ is the terminal point of $\vec{a}$. Then the vector extending from the intial point of $\vec{a}$ to the terminal point of $\vec{\mathrm{b}}$ is defined as the **sum** of $\vec{a}$ and $\vec{\mathrm{b}}$ and is denoted as $\vec{a} + \vec{\mathrm{b}}$.

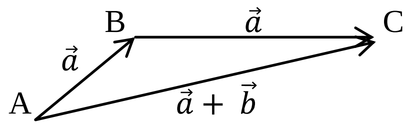


Figure 4

In the Figure 4 $\quad \vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{BC}$ and

$\vec{a} + \vec{b} = \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

This is called **triangle law of addition of** vectors.

If the vectors $\vec{a}$ and $\vec{\mathrm{b}}$ are represented by the two adjacent sides of a parallelogram, then their sum $\vec{a} + \vec{\mathrm{b}}$ is the vector represented by the diagonal of the parallelogram through their common point. **This is called parallelogram law of vector addition.**

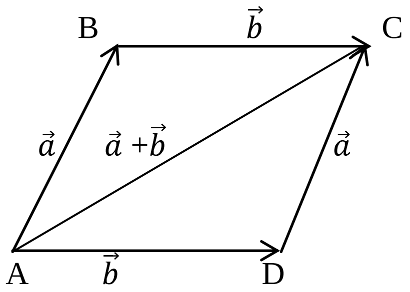


Figure 5

In above figure 5, by triangle law

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

But $\overrightarrow{BC}=\overrightarrow{AD}$

$\therefore\quad \overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$

which is the paralellogram law.

Properties of Vector Addition

1. Addition of vectors is commulative i.e.,
   $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
   for any vectors $\vec{a}$ and $\vec{b}$.

Proof : Let $\overrightarrow{\mathrm{OA}}=\vec{a}$ and $\overrightarrow{\mathrm{AB}}=\vec{b}$. We have $\vec{a}+\vec{b}=\overrightarrow{\mathrm{OB}}$.

Completing the parallelogram OABC having OA and OB as adjacent sides (see Figure 6), we have

$\overrightarrow{OC}=\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{CB}=\overrightarrow{OA}=\vec{a}$ = $\overrightarrow{AB}$=$\vec{a}$ and $\overrightarrow{CB}=\overrightarrow{OA}=\vec{a}$

So, we have

$\overrightarrow{OC}=\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{CB}=\overrightarrow{OA}=\vec{a}$

Here, $\vec{a}+\vec{b}=\vec{b}+\vec{a}$

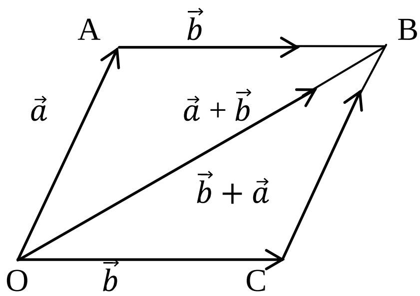


Figure 6

2. Addition of vectors is associative, i.e.,

   $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors.

Proof : Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be represented by $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ respectively.

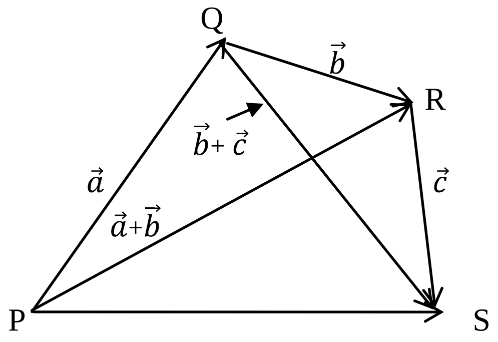


Figure 7

Then $\vec{a}+\vec{b}=\overrightarrow{PQ}+\overrightarrow{QR}=\overrightarrow{PR}$

$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$.

Again, $\vec{b}+\vec{c}=\overrightarrow{QR}+\overrightarrow{RS}=\overrightarrow{QS}$

$\Rightarrow \quad \vec{a}+(\vec{b}+\vec{c})=\overrightarrow{PQ}+\overrightarrow{QS}=\overrightarrow{PS}$

Hence $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

3. If $\vec{a}$ is any vector, and $\vec{0}$ is the zero vector then $\vec{a}+\vec{0}=\vec{a}$.
   Here, zero vector $\vec{0}$ is the additive identity for vector addition.

Difference of Vectors

Let $\vec{a}$ be a vector represented by $\overrightarrow{AB}$. Then ***negative*** of $\vec{a}$, is denoted by $-\vec{a}$ is defined by the vector $\overrightarrow{BA}$. So, $-\vec{a}$ is a vector having same magnitude as $\vec{a}$ but direction opposite to that of $\vec{a}$. It is also clear that

$\vec{a}+(-\vec{a})=\overrightarrow{AB}+\overrightarrow{BA}=\overrightarrow{AA}=\vec{0}$

If $\vec{a}$ and $\vec{b}$ are two vectors, then their difference is defined by

$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$.

Geometrically, let $\vec{a}=\overrightarrow{AB}$ and $\vec{b}=\overrightarrow{BC}$ and construct $\overrightarrow{BC'}$, so that its magnitude is same as the vector $\overrightarrow{BC}$, but the direction opposite to $\overrightarrow{BC}$ *i.e.*, $\overrightarrow{BC'}=-\overrightarrow{BC}$, as shown in figure below (Figure 8).

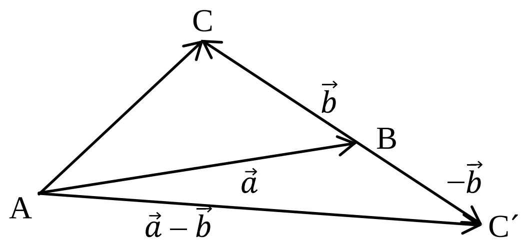


Figure 8

Then, by triangle law

$$\overrightarrow{AC}' = \overrightarrow{AB} + \overrightarrow{BC}' = \vec{a} + (-\vec{b}) = \vec{a} - \vec{b}.$$

The vector $\overrightarrow{AC}'$ represents difference of $\vec{a}$ and $\vec{b}$.

Multiplication of a Vector by a Scalar

The product of vector $\vec{a}$ by a scalar, λ denoted by $\lambda\vec{a}$ is defined as a vector whose length is $|\lambda|$ times that of $\vec{a}$ and whose direction is the same or opposite as that of $\vec{a}$ according as λ is positive or negative.

If $\lambda = -1$ then $\lambda\vec{a} = (-1)\,\vec{a} = -\vec{a}$, which is negative of vector $\vec{a}$. Also, note that $\vec{a} + (-1)\vec{b} = \vec{a} - \vec{b}$ and $-\lambda\vec{a} = \lambda\,(-\vec{a})$.

Let $\vec{a}$ be a non zero vector. If we take $\lambda = \frac{1}{|\vec{a}|}$, then $\lambda\,\vec{a}$ is a unit vector take because

$$|\lambda\,\vec{a}| = |\lambda||\,\vec{a}| = \frac{1}{|\,\vec{a}|}|\,\vec{a}| = 1.$$

This vector $\lambda\vec{a}$ is denoted by $\hat{a}$, and is the unit vector in the direction of $\vec{a}$.

Thus, $\hat{a} = \frac{1}{|\vec{a}|}$.

Let $\vec{a}$ be a vector and λ_1 and λ_2 are scalars. Then it is easy to prove that

$$\lambda_1(\lambda_2\,\vec{a}\,) = (\lambda_1\lambda_2)\vec{a}$$

Distributive Property : Let $\vec{a}$ and $\vec{b}$ be vectors and α and β be any scalars. Then

(i) $(\vec{a} + \beta)\vec{a} = \alpha\vec{a} + \beta\vec{a}$

(ii) $\alpha\,(\vec{a} + \vec{b}) = \alpha\vec{a} + \alpha\vec{b}.$

Proof of (i)

Let $\overrightarrow{AB}$ be vector $\vec{a}$ as shown in figure below (Figure 9)

$$A \xrightarrow{\quad\vec{a}\quad} B \xrightarrow{\alpha\vec{a} = \overrightarrow{AC}} C \xrightarrow{\beta\vec{a} = \overrightarrow{CD}} D$$

Figure 9

First assume that α and β are positive. Then $\alpha\vec{a}$ is given by $\overrightarrow{AC}$ which is in same direction as $\overrightarrow{AB}$ such that $|\overrightarrow{AC}| = \alpha\,|\overrightarrow{AB}|$. To consider $\beta\vec{a}$, We choose the initial point C. If $\overrightarrow{CD} = \beta\vec{a}$, then by triangle law of vector addition,

$$\alpha\vec{a} + \beta\vec{a} = \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$$

Now, $|\overrightarrow{AD}| = AC + CD = \alpha\,|\vec{a}| + \beta\,|\vec{a}|$
$= (\alpha + \beta)\,|\vec{a}|$

Also, the direction of $\overrightarrow{AD}$ is the same as that of $\vec{a}$. Thus, $\overrightarrow{AD} = (\alpha + \beta)\,\vec{a}$ by definition of scalar multiple of a vector. Hence, we have

$$(\alpha + \beta)\,\vec{a} = \alpha\,\vec{a} + \beta\,\vec{a}.$$

Also, it is clear that the property holds when one or both of α, β are negative since $-\alpha\vec{a} = \alpha\vec{a}$.

Proof of (ii)

As shown in figure (Fig. 10) below, Let $\vec{a}$ & $\vec{b}$ be represented by $\overline{AB}$ and $\overline{BC}$ respectively. Then $\overrightarrow{AC} = \vec{a} + \vec{b}$.

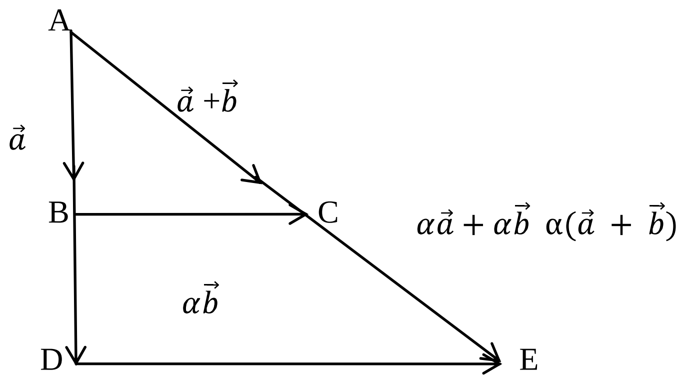


Figure 10

We assume α to be positive. Take a point D on AB such that AD = α AB and point E on AC such that AE = α AC. Join D and E.

But, $\overrightarrow{OP_1} = \overrightarrow{OQ} + \overrightarrow{QP_1}$

Now, Δ ABC ~Δ ADE (by construction)

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \alpha$$

and DE is parallel to BC.

$\Rightarrow DE = \alpha\,\text{BC}$

i.e., $\overrightarrow{DE} = \alpha\ \overrightarrow{BC} = \alpha\,\vec{\text{b}}$

Now from ΔADE,

$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$

i.e., $\alpha(\vec{a} + \vec{b}) = \alpha\vec{a} + \alpha\vec{b}.$

Check Your Progress – 1

1. Classify the following quantities as vectors or scalars.

(a) distance	(b) force
(c) Velocity	(d) workdone
(e) temperature	(f) length
(g) Speed	(h) acceleration

2. Prove that $l^2 + m^2 + n^2 = 1$, where *l*, *m* and *n* are direction cosines of a vector.
3. Prove that in a triangle ABC,

   $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$
4. Find the magnitude and direction of the position vector of point P(1, –1, $\sqrt{2}$)
5. If the position vector of the point P (*x*, 0,3) has magnitude 5, find the value of *x*.

1.3 COMPONENT OF A VECTOR

Consider the points A(1,0,0) B(0,1,0) and C(0,0,1) on *x*, *y* and *z–axes* respectively. Then, the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are unit vectors because and $\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR} = \vec{b} - \vec{r}$

The vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are called **units vectors along the *x-axis, y, axis* and *z–axis*** respectively and are denoted by $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively.

Suppose P (*x*, *y* , *z*) is a point in space and consider the position vector $\overrightarrow{OP}$ shown in following figure (Figure 11). Let P_1 be the foot of the perpendicular from P on the *xy* plane.

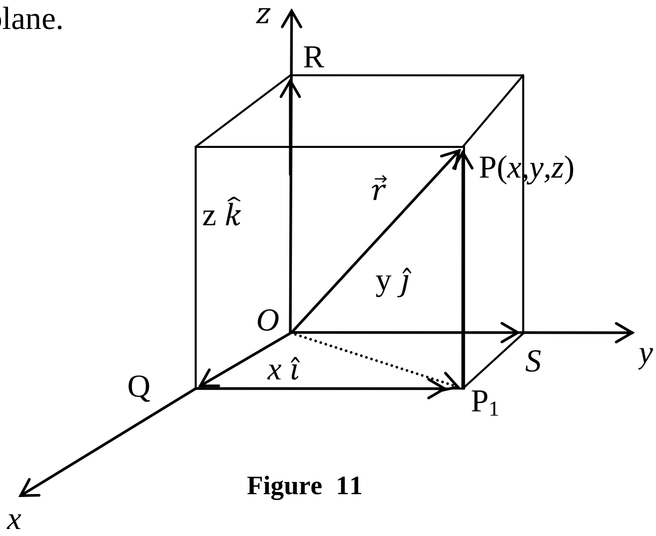


Figure 11

Since $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z-axes* respectively and *P* has coordinates (*x*,*y*, *z*), therefore

$\overrightarrow{OQ} = x\hat{i}$, $\overrightarrow{OS} = y\hat{j}$ and $\overrightarrow{OR} = z\hat{k}$.

So, $\overrightarrow{QP_1} = \overrightarrow{OS} = y\hat{j}$ and $\overrightarrow{P_1P} = \overrightarrow{OR} = z\hat{k}$.

Now, $\overrightarrow{OP} = \overrightarrow{OP_1} + \overrightarrow{P_1P}$

But, $\overrightarrow{OP_1} = \overrightarrow{OQ} + \overrightarrow{QP_1}$

$\therefore \quad \overrightarrow{OP} = \overrightarrow{OP_1} + \overrightarrow{OQ} + \overrightarrow{QP_1}$

$or, \quad \overrightarrow{OP}(or\ \vec{r}) = y\hat{j} + z\hat{k}$

This form of any vector is called **component form.** Here, x, y and z are called scalar components of $\vec{r}$ and $x\hat{\imath}$, $y\hat{\jmath}$, $z\hat{k}$ and called vector components of $\vec{r}$.

Also note that length of vector $\vec{r} = x\hat{\imath}, + y\hat{\jmath} + z\hat{k}$ is

$|\vec{r}\,| = |\,\overrightarrow{\mathrm{OP}}| = \sqrt{x^2 + y^2 + z^2}$.

Let $\vec{a} = a_1\hat{\imath}, + a_2\,\hat{\jmath} + a_3\hat{k}$ and

$\vec{b} = b_1\hat{\imath}, + b_2\,\hat{\jmath} + b_3\hat{k}$ be component form of two vectors. Then

1. The vectors $\vec{a}$ and $\vec{b}$ are equal if and only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$,
2. The sum of vectors $\vec{a}$ and $\vec{b}$ is given by
   $\vec{a} + \vec{b} = (a_1 + b_1)\,\hat{\imath}, + (a_2 + b_2)\,\hat{\jmath} + (a_3 + b_3)\,\hat{k}$
3. The difference of vectors $\vec{a}$ and $\vec{b}$ is given by
   $\vec{a} - \vec{b} = (a_1 - b_1)\,\hat{\imath}, + (a_2 - b_2)\,\hat{\jmath} + (a_3 - b_3)\,\hat{k}$
4. The multiplication of vector $\vec{a}$ by any scalar λ is given by
   $\lambda\,\vec{a} = (\lambda a_1)\,\hat{\imath} + (\lambda a_2)\,\hat{\jmath} + (\lambda a_3)\,\hat{k}$

We may observe that vectors $\vec{a}$ and $\lambda\,\vec{a}$ are always collinear, whatever be the value of λ. In fact, two vectors $\vec{a}$ and $\vec{b}$are collinear if and only if there exists a non zero scalar λ such that $\vec{b} = \lambda\,\vec{a}$.

If $\vec{a} = a_1\,\hat{\imath} + a_2)\,\hat{\jmath} + a_3\,\hat{k}$ and

$\vec{b} = b_1\,\hat{\imath} + b_2)\,\hat{\jmath} + b_3\,\hat{k},$

then the two vectors are collinear if and only if

$(b_1\hat{\imath} + b_2)\hat{\jmath} + b_3\,\hat{k} = \lambda\,(a_1\,\hat{\imath} + a_2\,\hat{\jmath} + a_3\,\hat{k})$

$\Leftrightarrow \quad b_1 = \lambda\,a_1,\ b_2 = \lambda\,a_2\,,\ b_3 = \lambda\,a_3$

$$\Leftrightarrow \quad \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda.$$

Example 2 : Let $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k}$ and

$\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

Evaluate

(i) $2\,\vec{a} + 3\,\vec{b}$ (ii) $\vec{a} - 2\,\vec{b}$

Solution : (i) $2\,\vec{a} = 2(2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k}\,) = 4\hat{\imath} + 6\,\hat{\jmath} - 2\,\hat{k}$

$3\vec{b} = 3(\hat{\imath} + 2\,\hat{\jmath} + 3\hat{k}\,) = 3\hat{\imath} + 6\,\hat{\jmath} + 9\,\hat{k}$

$\therefore \quad 2\vec{a} + 3\,\vec{b} = 7\hat{\imath} + 12\,\hat{\jmath} + 7\,\hat{k}$

(ii) $\vec{a} - 2\,\vec{b} = (2\hat{\imath} + 3\,\hat{\jmath} - \hat{k}\,) - 2(\hat{\imath} + 2\,\hat{\jmath} + 3\hat{k}\,)$
$= (2\hat{\imath} + 3\,\hat{\jmath} - \hat{k}\,) - (2\hat{\imath} + 4\,\hat{\jmath} + 6\hat{k}\,)$
$= 0\hat{i} - \hat{j} - 7\hat{k}$

$$\hat{a} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{5}(3\hat{i} - 4\hat{k}) = \left(\frac{3}{5}\right)\hat{i} - \left(\frac{4}{5}\right)\hat{k}$$

Example 3 : Find a unit vector in the direction of the vector

$\vec{a} = 3\hat{\imath} - 4\hat{k}$

Solution : $\vec{a} = 3\hat{\imath} - 4\hat{k} = 3\hat{\imath} + 0\hat{\jmath} - 4\hat{k}$

$\therefore |\vec{a}| = \sqrt{3^2 + 0^2 + (-4)^2} = \sqrt{25} = 5.$

So, unit vector in the direction of $\vec{a}$ is

$$\hat{a} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{5}(3\hat{i} - 4\hat{k}) = \left(\frac{3}{5}\right)\hat{i} - \left(\frac{4}{5}\right)\hat{k}$$

Example 4 : Find a unit vector in the direction of $(\vec{a} - \vec{b})$ where

$\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and
$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}.$

Solution : Here, $\vec{a} - \vec{b} = (-\hat{\imath} + \hat{\jmath} + \hat{k}) - (2\hat{\imath} + \hat{\jmath} - 3\,\hat{k})$
$= -3\,\hat{\imath} + 4\hat{k}$

and $|\vec{a} - \vec{b}\,| = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5.$

$\therefore$ unit vector in the direction of $\vec{a} - \vec{b}$

$$= \frac{1}{|\vec{a} - \vec{b}|}\left(\vec{a} - \vec{b}\right)$$

$$= \frac{1}{5}\left(-3\,\hat{\imath} + 4\hat{k}\right) = \frac{-3}{5}\hat{\imath} + \frac{4}{5}\hat{k}$$

Example 5 : Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\,\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath}$
Find a vector in the direction of $\vec{a} + \vec{b}$ that has magnitude 7 units.

Solution : Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) + (-i + j)$

$= 3\hat{j} + 3\hat{k} = \vec{c}$

The unit vector in the direction of $\vec{c} = \vec{a} + \vec{b}$ is

$$\vec{c} = \frac{1}{|\vec{c}|}\vec{c} = \frac{1}{\sqrt{18}}\,(3\hat{\jmath} + 3\hat{k})$$

$$= \frac{3}{3\sqrt{2}}\,\hat{\jmath} + \frac{3}{3\sqrt{2}}\hat{k} = \frac{1}{\sqrt{2}}\hat{\jmath} + \frac{3}{\sqrt{2}}\hat{k}$$

Therefore, the vector having magnitude equal to 7 and in the direction of $\vec{c}$ is

$$= 7\left(\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right) = \frac{7}{\sqrt{2}}\hat{j} + \frac{7}{\sqrt{2}}\hat{k} = 7\hat{c}.$$

Example 6 : If $\vec{a}$ and $\vec{b}$ are position vectors of the points (1, –1) and (–2, *m*) respectively, then find the value of *m* for which $\vec{a}$ and $\vec{b}$ are collinear.

Solution : Here $\vec{a} = \hat{\imath} - \hat{\jmath}$

$$\vec{b} = -2\hat{i} + m\hat{j}$$

$\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} = \vec{b}$ where λ is a real number

i.e., $\hat{i} - \hat{j} = \lambda(-2\hat{i} - m\hat{j}) = -2\lambda + m\lambda j$

Comparing the component on both sides, we get

$$1 = -2\lambda \Rightarrow \lambda = \frac{-1}{2} \text{ also, } \lambda m = -1 \Rightarrow \frac{-1}{2}m = -1 \Rightarrow m = 2.$$

Check Your Progress – 2

1. Let $\vec{a} = 4\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{b} = -2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$.

   Find the component form and magnitude of the following vectors:

   (a) $\vec{a} + \vec{b}$ (b) $\vec{a} - \vec{b}$

   (b) $\frac{5}{2}\vec{a}$ (d) $2\vec{a} + 3\vec{b}$

2. Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$.

   Find

   (a) $|\vec{a} - \vec{b}|(\vec{a} + \vec{b})$ (b) $|2\vec{a} - 3\vec{b}|$

3. Find a unit vector in the direction of $\vec{a} + \vec{b}$ where $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ *and* $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$.

4. Write the direction ratio's of the vector $\vec{r} = 2\hat{i} = \hat{j} - \hat{k}$ and hence calculate its direction cosines.

5. Show that vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} + 6\hat{j} - 2\hat{k}$ are collinear.

1.4 SECTION FORMULA

In this section, we shall discuss section formula and its applications. Before that let us find component form of a vector joining two points.

Vector Joining Two Points

Let P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) *be* any two points. The position vectors of P and Q are

$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and

$\overrightarrow{OQ} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

The vector joining P and Q is $\overrightarrow{PQ}$

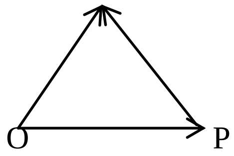


Figure 12

By triangle law, we have

$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$\therefore \quad \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$

$= (x_2 - x_2)\hat{i} + (y_2 - y_2)\hat{k} + (z_2 - z_2)\hat{k}$

The magnitude of Vector $\overrightarrow{PQ}$ is given by

$$\overrightarrow{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example 7 : Find the vector $\overrightarrow{PQ}$ where P is the point (5, 7, –1) and Q is the point (2, 9, −2)

Solution : $\overrightarrow{PQ}$= (– 2 – 5)$\hat{\imath}$ + (9–7) $\hat{\jmath}$ + (– 2 –1) $\hat{k}$ = – 7 $\hat{\imath}$ + 2 $\hat{\jmath}$ –3 $\hat{k}$.

Section Formula : To find the positon vector of the point which divides the line joining two given points in a given ratio.

Let P and Q be two points with position vectors : $\vec{a}$ and $\vec{b}$ respectively. Let O be the origin of reference so that

$\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$

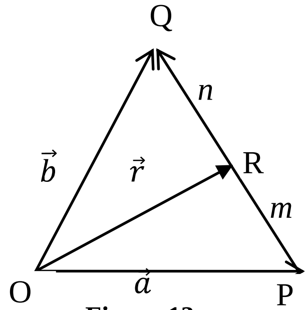


Figure 13

Let R be a point which divides PQ in the ratio *m : n.*

Let : $\overrightarrow{QR} = \vec{r}$.

Now, $\dfrac{PR}{RQ} = \dfrac{m}{n}$ i.e., $nPR = m\,RQ$ which gives the vector equality

$n\overrightarrow{PR} = m\overrightarrow{RQ}$

From above figure, (Figure 13) we have

$= \dfrac{1}{2}(1+t)\vec{a}.$

$\therefore \quad \overrightarrow{\mathrm{PQ}} = \overrightarrow{SR}$

Therefore,

$n(\vec{r} - \vec{a}) = m(\vec{b} - \vec{r})$

$$\Rightarrow \vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

Hence, the position vector of the point R which dividies P and Q in the ratio of $m : n$ internally is given by

$$\overrightarrow{OR} = \frac{m\,\vec{b} + n\,\vec{a}}{m + n}.$$

Corollary : If R is the midpoint of PQ, then $m = n$. Therefore, the position vector of the midpoint of the joint of two points with position vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{r} = \frac{1}{2}\left(\vec{a} + \vec{b}\right)$$

Remark : If R divides the line segment PQ in the ratio $m : n$ externally, then the position vector of R is given by

$$\overrightarrow{OR} = \frac{m\,\vec{b} - n\vec{a}}{m+n}.$$

Example 8 : Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ in the ratio 2:1

(i) internally (ii) externally

Solution :

(i) Position vector of R which divides PQ in the ratio 2:1 internally is

$$\overrightarrow{OR} = \frac{2\left(\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) + 1\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right)}{2+1}$$

$$= \frac{4\hat{\imath} + 5\hat{j} + \hat{k}}{3} = \frac{4}{3}\hat{\imath} + \frac{5}{3}\hat{j} + \frac{1}{3}\hat{k}$$

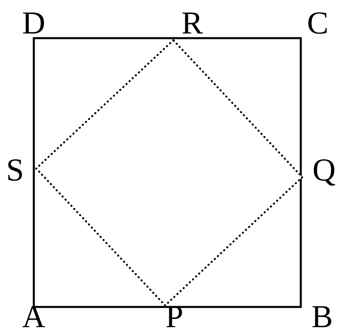
(ii) Position vector of R which divides PQ in the ratio 2:1 externally is

$$\overrightarrow{OR} = \frac{2(\hat{\imath} + 2\hat{j} + \hat{k}) - (2\hat{\imath} + \hat{j} - \hat{k})}{2-1}$$

$$= 3\hat{j} + 3\hat{k}$$

Example 9 : If the mid-points of the consecutive sides of a quadrilateral are joined, then show by using vectors that they form a parallelogram.

Solution : Let $\vec{a}$, $\vec{b}$, $\vec{c}$ $\vec{d}$ be the positon vectors of the vertices A, B, C, D of the quadrilateral ABCD. Let *P*, *Q*, *R*, *S* be the mid-points of sides AB, BC, CD, DA respectively. Then the position vectors of P, Q, R and S are $\frac{1}{2}(\vec{a} + \vec{b}), \frac{1}{2}(\vec{b} + \vec{c}), \frac{1}{2}(\vec{c} + \vec{d})$ and $\frac{1}{2}(\vec{d} + \vec{a})$ respectively.



$$Now, \overrightarrow{PQ} = \overrightarrow{PQ} - \overrightarrow{PQ} = \frac{1}{2}(\vec{b} + \vec{c}) - \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}(\vec{c} - \vec{a})$$

or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ $(\because \overrightarrow{CA} = -\overrightarrow{AC})$

$\therefore$ $\overrightarrow{PQ} = \overrightarrow{SR}$

$\Rightarrow$ PQ = SR and also PQ || SR.

Since a pair of opposite sides are equal and parallel, therefore, PQRS is a parallelogram.

Example 10 : Prove that the three medians of a triangle meet at a point called the centroid of the triangle which divides each of the medians in the ratio 2:1.

Solution : Let the position vectors of the vertices A, B, C of a triangle ABC with respect to any origin O be $\vec{a}, \vec{b}, \vec{c}$. The position vectors of the mid-points D, E, F of the sides are

$$\frac{1}{2}(\vec{b} + \vec{c}), \ \frac{1}{2}(\vec{c} + \vec{a}), \ \frac{1}{2}(\vec{a} + \vec{b}), \text{respectively.}$$

Let G be the point on the median AD such that AG : GD = 2:1.

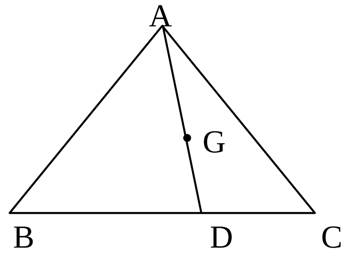


Figure 15

Then by the section formula, the position vector of G is given by

$$\overrightarrow{OG} = \frac{2\overrightarrow{OD} + \overrightarrow{OA}}{2+1} = \frac{2\left[\frac{1}{2}\ \vec{b} + \vec{c}\ \right] + \vec{a}}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

If G_1 were the point on the median BE such that $BG_1 : G_1E = 2:1$, the same argument would show that

$$\overrightarrow{OG_1} = \frac{\vec{a}\ + \vec{b} + \vec{c}}{3}$$

In other words, G and G_1 coincide. By symmetry, we conclude that all the three medians pass through the point G such that

$$\overrightarrow{OG} = \frac{\vec{a}\ + \vec{b} + \vec{c}}{3}$$

and which divides each of the medians in the ratio 2:1.

Example 11 : Prove that the straight line joining the mid-points of two non parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

Solution : Let OABC be a trapezium with parallel sides OA and CB. Take O as the origin of reference.

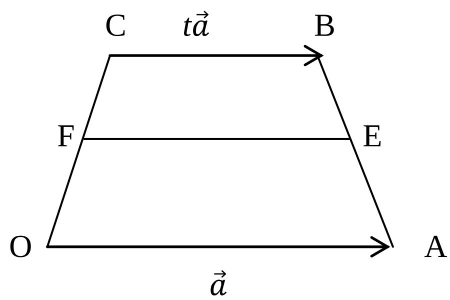


Figure 16

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{c}$, so that $\vec{a}$, $\vec{c}$ are the position vectors of the points A and C respectively.

As CB is parallel to OA, the Vector $\overrightarrow{CB}$ must be a product of the vector $\overrightarrow{OA}$ by some scalar, say, *t*.

So, $\overrightarrow{CB} = t\ \ \overrightarrow{OA} = t\vec{a}$ (1)

$\therefore$ The position vector $\overrightarrow{OB}$ of B is

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB} = c + t\vec{a}$

Since F is the mid point of OC,

$\therefore$ position vector of F $= \dfrac{\vec{o}+\vec{c}}{2} = \dfrac{1}{2}\vec{c}$

Similarly, positon vector of midpoint E of AB

$$= \frac{\vec{a} + (\vec{c} + t\vec{a})}{2} = \frac{(1+t)\vec{a} + \vec{c}}{2}$$

We have

$$\overrightarrow{FE} = \overrightarrow{OE} - \overrightarrow{OF}$$
$$= \frac{(1+t)\vec{a} + \vec{c}}{2} - \frac{\vec{c}}{2}$$

$$= \frac{1}{2}(1+t)\vec{a}.$$

$$i.e., \overrightarrow{FE} = \frac{1}{2}(1+t)\overrightarrow{OA}$$

So, $\overrightarrow{FE}$ is a scalar multiple of $\overrightarrow{OA}$

$$\Rightarrow \overrightarrow{FE} \parallel \overrightarrow{OA} \text{ and } \overrightarrow{FE} = \frac{1}{2}(1+t)\overrightarrow{OA}$$

Also, from (1) we have

CB = t OA

$\therefore$ OA + CB = (1 + t) OA = 2FE.

Example 12 : Show that the three points with position vectors
$-2\vec{a} + 3\vec{b} + 5\vec{c}, \quad \vec{a} + 2\vec{b} + 3\vec{c}, \quad 7\vec{a} - \vec{c}$
are collinear.

Solution : Let us denote the three points by A, B and C respectively.

We have

$$\overrightarrow{AB} = (\vec{a} + 2\vec{b} + 3\vec{c}) - (-2\vec{a} + 3\vec{b} + 5\vec{c})$$
$$= 3\vec{a} - \vec{b} - 2\vec{c}.$$

$$\overrightarrow{AC} = (7\vec{a} - \vec{c}) - (-2\vec{a} + 3\vec{b} + 5\vec{c})$$
$$= (9\vec{a} - 3\vec{b} - 6\vec{c}) = 3\overrightarrow{AB}$$

Thus, the vectors $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are collinear. These vectors are also cointial, therefore, the points A, B and C are collinear.

Check Your Progress – 3

1. Find a unit vector in the direction of vector $\overrightarrow{PQ}$ joining the points P (1, 2, 3) and Q (–1, 1, 2).

2. (i) Let P and Q be two points with position vectors $\overrightarrow{QP}= 3\vec{a} - 2\vec{b}$ and OQ= $\vec{a} + \vec{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio 2 :1. (i) internally, and (ii) externally.

3. Show that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is half of its length.

4. Show that diagonals of a quadrilateral bisect each other if and only if it is a parallelogram.

5. ABCD is a parallelogram and P is the point of intersection of its diagonals, O is the origin prove that

$$4\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$$

6. Show that the three points : A(6, –7, –1), B(2, –3, 1) and C(4, –5, 0) are collinear.

1.5 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress – 1

1. (a) Scalar (b) Vector
   (c) Vector (d) Scalar
   (e) Scalar (f) Scalar
   (g) Scalar (h) Vectors

2. If $\overrightarrow{QP} = \vec{r}$ makes angles α, β and γ with positive direction of *x, y* and *z* *axis* respectively then l = cos α, m = cos β, n = cos γ

   If P has coordinates (*x*,*y*, *z*), then $x = r\cos\alpha$, $y = r\cos\beta$ and $z = r\cos\gamma$

$$\text{where} \quad r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

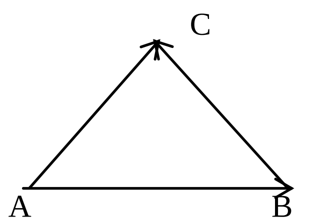
$$\text{So}, \cos\alpha = \frac{x}{r}, \cos\beta = \frac{y}{r}, \cos\gamma = \frac{z}{r},$$

$$\therefore\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$$

$$= \frac{x^2 + y^2 + z^2}{r^2} = \frac{r^2}{r^2} = 1.$$

3. From adjoining figure, we have $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ using triangle law of vector addition.

$\therefore\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \vec{0}$



or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ $\quad (\because\ \overrightarrow{CA} = -\overrightarrow{AC})$

Figure 17

4. $|\overrightarrow{OP}| = r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + (-1)^2 + \sqrt{2}^2} = \sqrt{4} = 2$

$$\cos\alpha = \frac{x}{r} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

$$\cos\beta = \frac{y}{r} = -\frac{1}{2} \Rightarrow \beta = \frac{2\pi}{3}$$

$$\cos\gamma = \frac{z}{r} = \frac{\sqrt{2}}{z} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}.$$

5. $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

$\therefore\ 5 = \sqrt{x^2 + 0^2 + 3^2}$

$25 = x^2 + 9$

$\therefore\ x^2 = 16 \qquad \Rightarrow\ x = \pm 4.$

Check Your Progress – 2

1. (a) $\vec{a} + \vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$$|\vec{a} + \vec{b}| = \sqrt{2^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$$

(b) $\vec{a} - \vec{b} = 6\hat{i} - 8\hat{j} - 2\hat{k}$

$$\left|\vec{a} - \vec{b}\right| = \sqrt{6^2 + (-8)^2 + (-2)^2} = \sqrt{104} = 2\sqrt{26}$$

(c) $\quad \frac{5}{2}\vec{a} = \frac{5}{2}(4\hat{\imath} + 3\hat{\jmath} + \hat{k}) = 10\hat{\imath} - \frac{15}{2}\hat{\jmath} + \frac{5}{2}\hat{k}$

$$\left|\frac{5}{2}\vec{a}\right| = \sqrt{100 + \frac{225}{4} + \frac{25}{4}} = \sqrt{\frac{650}{4}} = \frac{5}{2}\sqrt{26}$$

$r = 2\hat{i} + \hat{j} - \hat{k},$

$3\vec{b} = -\ 6\hat{i} + 15\hat{j} + 9\hat{k}$

$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k},$

$\therefore\ |2\vec{a} + 3\vec{b}| = \sqrt{4 + 81 + 121} = \sqrt{206}$

2. (a) $\vec{a}+\vec{b}=3\hat{i}+2\hat{j}+\hat{k}$

i.e., $\overrightarrow{OB}+\overrightarrow{OB}=\overrightarrow{OB}$ and

$\therefore\ |\vec{a}-\vec{b}| = \sqrt{(-1)^2+(-6)^2+\ 3^2}\ \ = \sqrt{46}$

$\therefore\quad |\vec{a}-\vec{b}|(\vec{a}+\vec{b}) = \sqrt{46}(3\hat{i}+2\hat{j}+\hat{k})$

(b) $2\vec{a}=2\hat{i}-4\hat{j}+4\hat{k}$

$3\vec{b}=6\hat{i}+12\hat{j}-3\hat{k}$

$\therefore\quad 2\vec{a}-3\vec{b}=-4\hat{i}-16\hat{j}+7\hat{k}$

$\therefore\quad |2\vec{a}-3\vec{b}|\ =\sqrt{(-4)^2+(-16)^2+(7)^2}=\sqrt{321}$

3. Here, $\vec{a}+\vec{b}=4\hat{i}+3\hat{j}-2\hat{k}$

$\therefore\quad |\vec{a}+\vec{b}|=\sqrt{4^2+3^2+(-2)^2}=\sqrt{29}$

$\therefore$ unit vector in the direction of $\vec{a}+\vec{b}$

$$=\frac{\vec{a}\ +\ \vec{b}}{|\vec{a}\ +\ \vec{b}|}\ =\ \frac{1}{\sqrt{29}}\ (4\hat{\imath}+3\hat{\jmath}-\ 2\hat{k})$$

4. For any vector $\vec{r}=a\hat{i}+b\hat{j}+c\hat{k},\ a,b$ and c are direction ratios and $\frac{\vec{a}}{|\vec{r}|},\frac{\vec{b}}{|\vec{r}|}$ and $\frac{\vec{c}}{|\vec{r}|}$ are the direction cosines.

Here, $r=2\hat{i}+\hat{j}-\hat{k},$ so the direction ratios are $a=2,\ \ b=1,\ \ c=-1$

Also, $|\vec{r}| = \sqrt{2^2+1^2+(-1)^2}\ \ = \sqrt{6}$

$$\therefore\ l=\ \frac{2}{\sqrt{6}},m=\ \frac{1}{\sqrt{6}},n=\ \frac{-1}{\sqrt{6}}$$

5. Let $\vec{a}=2\hat{i}+3\hat{j}=-\hat{k}$ and $\vec{b}=4\hat{i}+6\hat{j}-2\hat{k}=-2\hat{k}$

Thus clearly $\vec{b}=2(2\hat{i}+3\hat{j}-\hat{k})=2\vec{a}$

is a scalar multiple of $\vec{a}$.

Hence, $\vec{a}$ and $\vec{b}$ are collinear vectors.

1. Here, $\overrightarrow{PQ} = (-1-1)\,\hat{\imath} + (1-2)\,\hat{\imath} + (2-3)\,\hat{k}$
$= -2\,\hat{\imath} - \hat{\jmath} - \hat{k}$

$\therefore |\overrightarrow{PQ}| = \sqrt{(-2)^2 + (-1)^2 + (-1)^2} = \sqrt{6}$

$\therefore$ unit vector in the direction of $\overrightarrow{PQ}$

$$= \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{1}{\sqrt{6}}\,(-2\,\hat{\imath} - \hat{\jmath} - \hat{k}\,)$$

2. (i) $\overrightarrow{OR} = \dfrac{2\overrightarrow{OQ} + \overrightarrow{OP}}{2+1} = \dfrac{2(\vec{a} + \vec{b}) + (3\vec{a} + 2\vec{b})}{3} = \dfrac{5\vec{a}}{3}$

(ii) $\overrightarrow{OR} = \dfrac{2\overrightarrow{OQ} - \overrightarrow{OP}}{2-1} = \dfrac{2(\vec{a} + \vec{b}) - (3\vec{a} - 2\vec{b})}{3} = 3\vec{b} - \vec{a}.$

3. Let A of Δ ABC be considered as the origin of vectors (Figure 18)

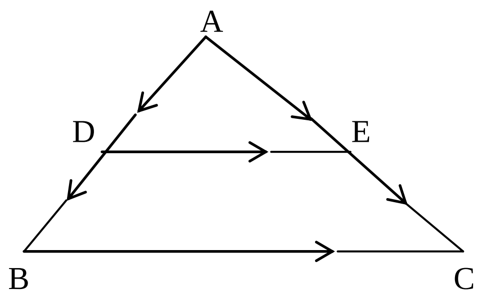


Figure 18

Let D and E be the mid-points of sides AB, CA respectively. Then

$\overrightarrow{AD} = \dfrac{1}{2}\,\overrightarrow{AB}$ and $\overrightarrow{AE} = \dfrac{1}{2}\,\overrightarrow{AC}$

Now, $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$

$= -\overrightarrow{AD} + \overrightarrow{AE}$

$= -\dfrac{1}{2}\,\overrightarrow{AB} + \dfrac{1}{2}\overrightarrow{AC}$

$= \dfrac{1}{2}\,\overrightarrow{BA} + \dfrac{1}{2}\overrightarrow{AC}$

$= \dfrac{1}{2}\,\overrightarrow{BA} + \dfrac{1}{2}\overrightarrow{AC}$

$= \dfrac{1}{2}\,\overrightarrow{BC}$

$\Rightarrow\ \overrightarrow{DE}\ ||\ \overrightarrow{BC}$ and $|\overrightarrow{DE}| = \frac{1}{2}\,|\overrightarrow{BC}|$

4. ABCD be the quadrilateral and O be the point of intersection of AC and BD (Figure 19) choose O as the origin of vectors.

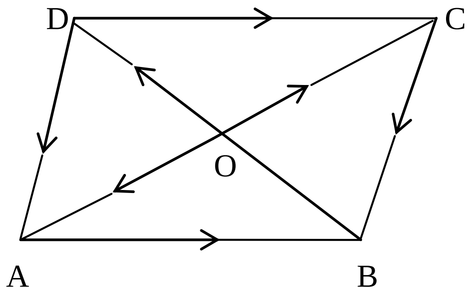


Figure 19

O is the mid-point of AC and BD

$\therefore \quad \overrightarrow{OP} = \dfrac{\overrightarrow{OA}+\overrightarrow{OD}}{2}$

i.e., $\overrightarrow{OB}+\overrightarrow{OD}=\vec{O}$ and $\overrightarrow{OC}+\overrightarrow{OA}=\vec{O}$

i.e., iff $\overrightarrow{OB} - \overrightarrow{OC} = \overrightarrow{OA}-\overrightarrow{OD}$

i.e., if $\overrightarrow{CB} = \overrightarrow{DA}$

also equivalently $\overrightarrow{OB}-\overrightarrow{OA}=\overrightarrow{OC}-\overrightarrow{OD}$

i.e., $\overrightarrow{AB}=\overrightarrow{DC}$

5. P is the mid-point of AC

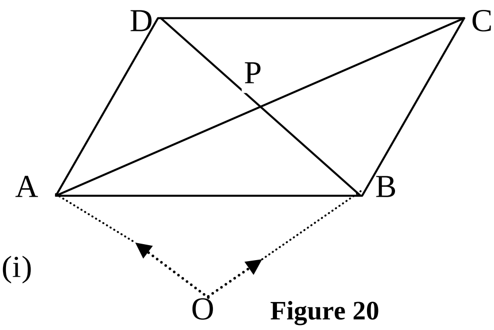


Figure 20

$\therefore \quad \overrightarrow{OP} = \dfrac{\overrightarrow{OA}+\overrightarrow{OD}}{2}$

or $2\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{OC}$.....................(i)

Again, P is also mid-point of BD

$\therefore \quad \overrightarrow{OP} = \dfrac{1}{2}\ \overrightarrow{OB}+\ \overrightarrow{OC}$ ……………..(ii)

or $2\overrightarrow{OP}=\overrightarrow{OB}+\overrightarrow{OD}$

Adding (i) and (ii) we get

or $4\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{OC}+\overrightarrow{OB}+\overrightarrow{OD}$

or $4\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{OC}+\overrightarrow{OB}+\overrightarrow{OD}$

6. Here, $\overrightarrow{AB}$ = (2 – 6) $\hat{\imath}$ + (– 3+ 7) $\hat{\jmath}$ + (1+ 1) $\hat{k}$

$= -4\hat{\imath} + 4\,\hat{\jmath} + 2\hat{k}$

and $\overrightarrow{AC}$ = (4 – 6) $\hat{\imath} + (-5+7)\hat{\jmath}$ + (0 + 1) $\hat{k}$

$= -\ 2\,\hat{\imath} + 2\hat{\jmath} + \hat{k}$

clearly, $\overrightarrow{AB} = 2\,\overrightarrow{AC}$

So, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear vectors. Since $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are also coinitial, therefore A, B and C are collinear points.

1.6 SUMMARY

In this unit, we discuss mathematical concept of vector. In **section 1.2**, first of all, the concept, as distinct from that of a scalar, is defined. Then concepts of position vector, direction cosines of a vector, coinitial vectors, collinear vectors, coplanar vectors, are defined and explained. In **section 1.3**, method of expressing a vector in 3-dimensional space in terms of standard unit vectors is discussed. In **section 1.4**, first, method of finding a vector joining two points is discussed. Then, section formula for finding position vector of the point which divides the vector joining two given points, is illustrated.

Answers/Solutions to questions/problems/exercises given in various sections of the unit are available in **section 1.5**.