732A90: Lab 2

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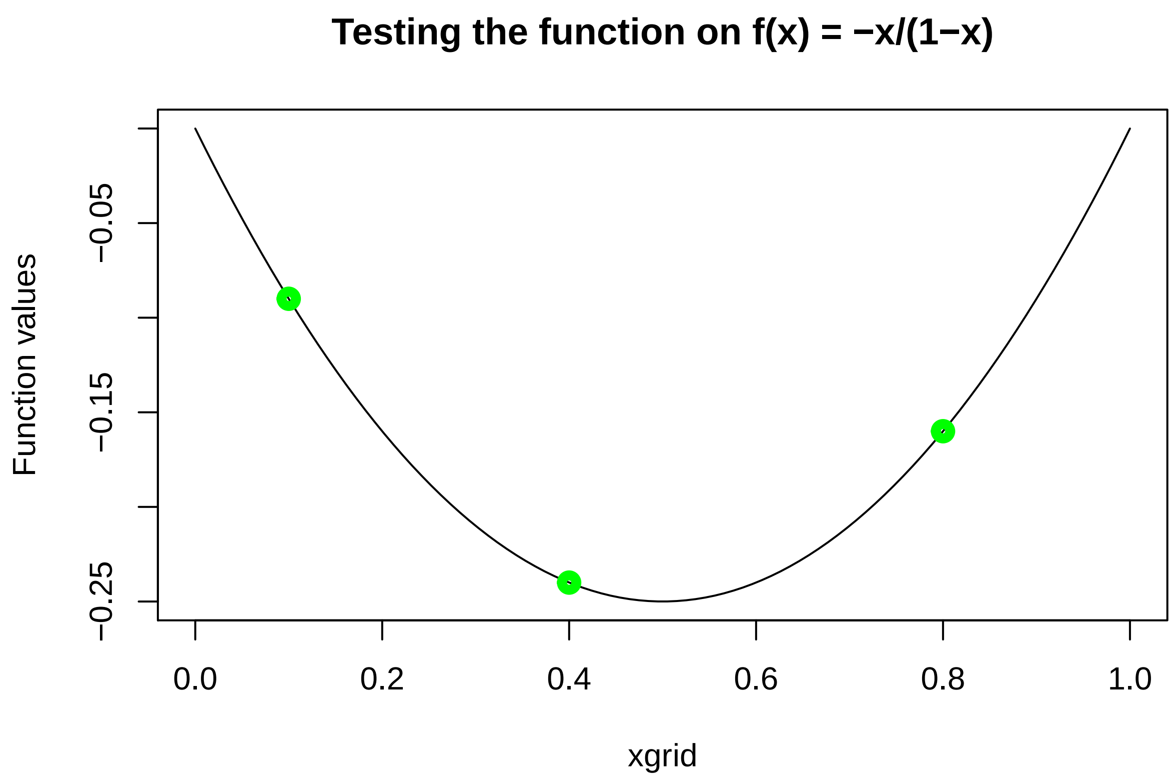
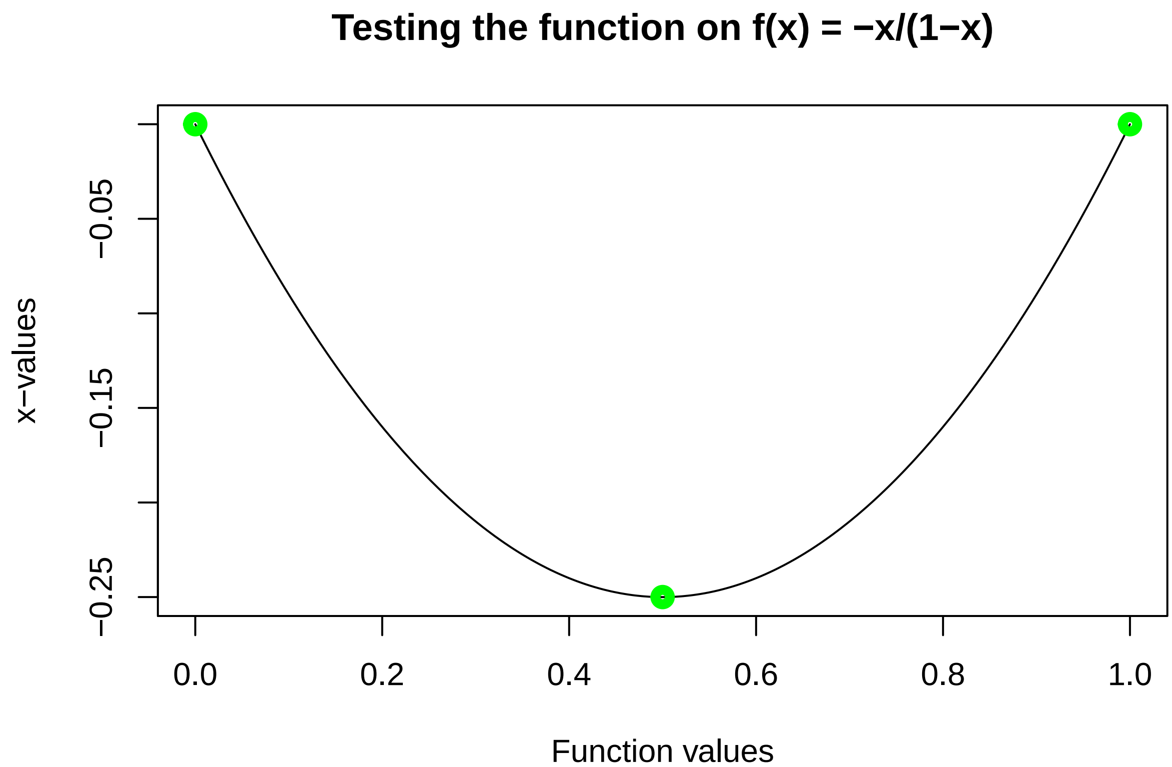
Question 1: Optimizing parameters

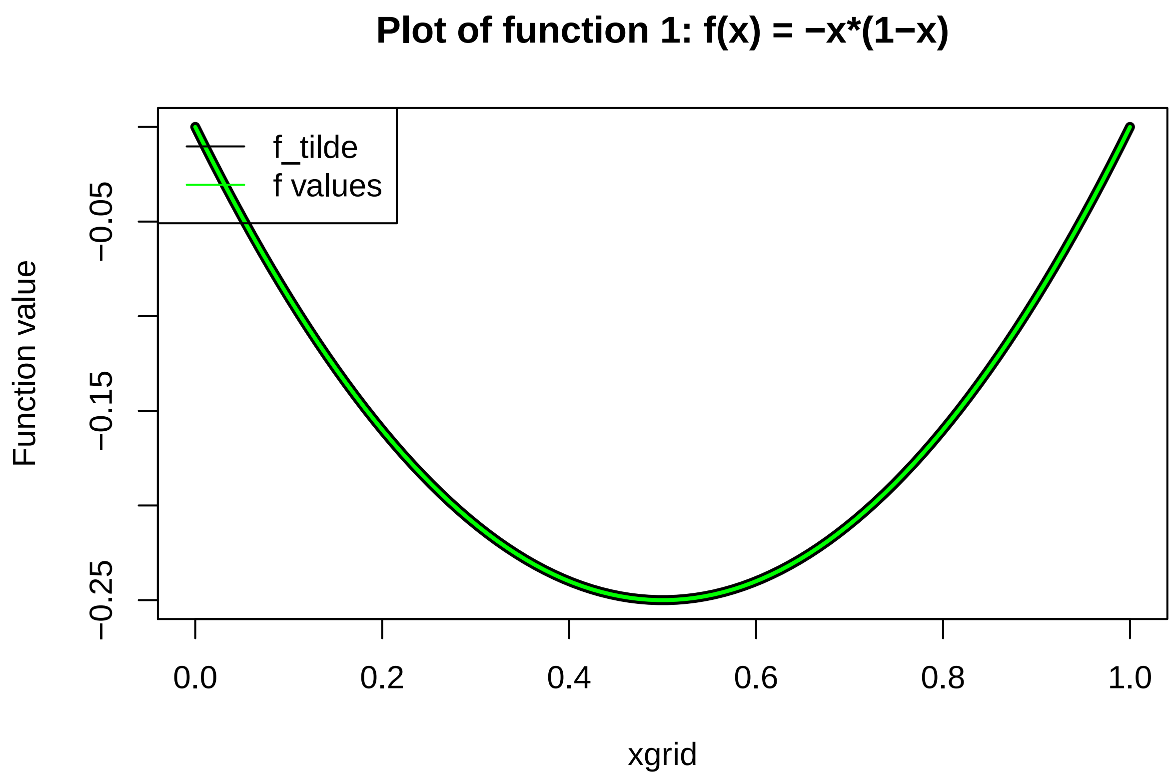
1:

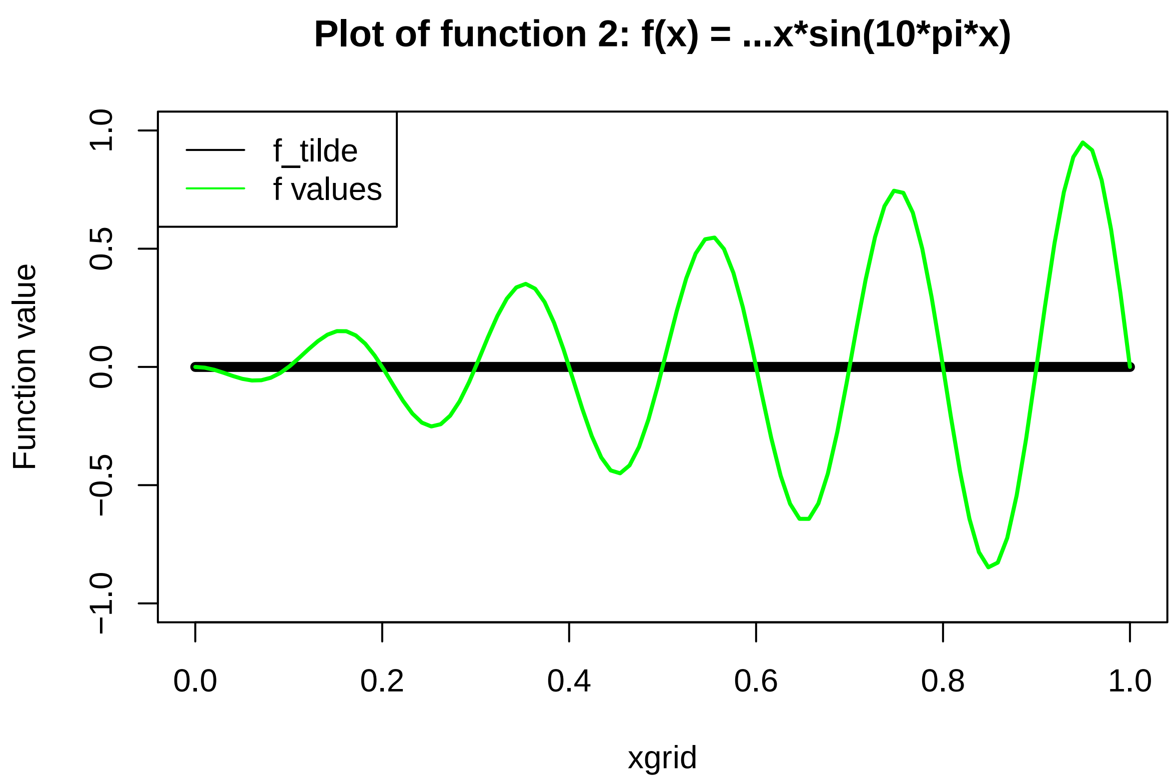


2:


Testing the function on f(x) = −x/(1−x)
x−values
−0.05
−0.15
−0.25
0.0
0.2
0.4
0.6
0.8
1.0
Function values

3:


Plot of function 1: f(x) = −x*(1−x)
f_tilde
f values
Function value
−0.05
−0.15
−0.25
0.0
0.2
0.4
0.6
0.8
1.0
xgrid


Plot of function 2: f(x) = ...x*sin(10*pi*x)
f_tilde
f values
Function value
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
xgrid

The first function get a very good fit, the approximated function basially fits on top of the real function. For the second function on the other hand, the approximation does not fit well. This is because the fit only can do quadratic functions, whereas the sin-function needs a more complex approximation in order to produce a good fit. The best fit for the second function is a straight line across the x-axis, since the real function is above and under it an equal amount.

Question 2: Maximizing likelihood

2:

```
## MLE mu:  1.275528
```

```
## MLE sigma:  2.005976
```

```
## log-likelihood:  -211.5069
```

3:

Bad idea to maximize likelihood! This is because we want to prevent overflow. Without log, we would try to multiply 100 relatively large number which could cause overflow. With log, we sum these numbers instead which gives us a much smaller value.

4:

Here follows a table with optimal parameters, best log-likelihood value, and iteration counts for all the cases:

```
##                           mu    sigma log-likelihood count: fn count: gr conv
## CG                  1.275528 2.005977      -211.5069       274        43    0
## CG with gradients   1.275528 2.005976      -211.5069        53        17    0
## BFGS                1.275528 2.005977      -211.5069        37        15    0
## BFGS with gradient 1.275528 2.005977      -211.5069        38        15    0
```

Here follows a table with gradients for all the cases:

```
##                      mu gradient sigma gradient
## CG                 -2.907985e-06  -1.966048e-06
## CG with gradients   2.463027e-07  -1.927738e-08
## BFGS                1.902664e-06  -2.438016e-05
## BFGS with gradient  1.366082e-06  -3.826563e-06
```

From the results we can that in all of the cases, the algorithm converged (given **0** as convergence value). This is further confirmed by seeing that the gradients of optimal parameters are (close to) zero. When using CG we get a great advantage from using gradients, whereas when using BFGS there is no advantage in using gradients. This means that the "finite-difference approximation" does the job as well as the gradient does at finding new points.

The optimal parameters were also found by the maximum likelihood estimators using their analytical expressions!

The recommended setting would be **BFGS** without gradient. It gets the same number of iterations as with gradient, making it simple to use.

Include all code for this report

```
knitr::opts_chunk$set(echo = TRUE, warning=FALSE, message=FALSE)
# Include packages here
xgrid = seq(0,1, length.out = 100)

f_tilde = function(a, x){
  res = a[1] + a[2]*x + a[3]*x^2
  return(res)
}

squared_error = function(a, param){
  f_vals = f_tilde(a, param[1:3])
  error = (param[4]-f_vals[1])^2 + (param[5]-f_vals[2])^2 + (param[6]-f_vals[3])^2
  return(error)
}

interpolate = function(x_start, func){
  a = c(0,0,0)
  param = c(x_start, func(x_start)) # contains x_vals and y_vals
  opti = optim(par = a, fn = squared_error, param = param)
  return(opti)
}

# Testing the created functions
x_start = c(0.1, 0.4, 0.8)
test_func = function(x){
  return(-x*(1-x))
}
opti = interpolate(x_start, test_func)

plot(x = xgrid, y = f_tilde(opti$par, xgrid), main = "Testing the function on f(x) = -x/(1-x)", ylab ="I
lines(x = x_start, y = test_func(x_start), col = 'green', type='p', lwd = '5')

approx_func = function(nint, func){
  xgrid = seq(0,1, length.out = nint)

## SECOND APPROACH ##
#  res = c()
#  for (i in 1:nint){
#    start = 0+1/100*(i-1)
#    end = start+1/100
#    #cat("start: ", start, " end: ", end, "\n")
#    opti = interpolate(c(start, (start+end)/2, end), func)
#    val = f_tilde(opti$par, c(start, (start+end)/2))
#    res = c(res, val)
#  }
#  return(res)

## FIRST APPROACH ##
  opti = interpolate(c(0, 0.5, 1), func)
  res = f_tilde(opti$par, xgrid)
  return(res)
```

```
}

test = approx_func(1000, test_func)
plot(x = seq(0,1,length.out = 1000), y = test, type = 'l', xlab = "Function values", ylab ="x-values", m
lines(x = c(0, 0.5, 1), y = test_func(c(0, 0.5, 1)), col = 'green', type='p', lwd = '5')
f_1 = function(x){
  return(-x*(1-x))
}
f_2 = function(x){
  return(-x*sin(10*pi*x))
}
f_tilde_1 = approx_func(100, f_1)
f_tilde_2 = approx_func(100, f_2)
xgrid = seq(0,1,length.out = 100)
plot(x = xgrid, y = f_tilde_1, type = 'l', lwd=5, main="Plot of function 1: f(x) = -x*(1-x)", ylab = "Fu
lines(x = xgrid, y = f_1(xgrid), col='green', lwd=2)
legend("topleft", c("f_tilde","f values"),
       col=c("black", "green"), lty=1, cex=1)

plot(x = xgrid, y = f_tilde_2, type = 'l', lwd=5, main="Plot of function 2: f(x) = -x*sin(10*pi*x)", yla
lines(x = xgrid, y = f_2(xgrid), col='green', lwd=2)
legend("topleft", c("f_tilde","f values"),
       col=c("black", "green"), lty=1, cex=1)


load(file = 'data.rdata')

loglike = function(x, mu, sigma){
  log = sum(dnorm(x, mean = mu, sd = sigma, log = TRUE))
  return(log)
}

mle_mu = function(x){
  return(mean(x))
}
mle_sigma = function(x){
  mean = mean(x)
  a = (x-mean)^2
  res = sum(a)/length(x)
  return(sqrt(res))
}

cat("MLE mu: ", mle_mu(data), "\n")
cat("MLE sigma: ", mle_sigma(data), "\n")
cat("log-likelihood: ", loglike(data, mle_mu(data), mle_sigma(data)), "\n")
grads = function(par, x){
  n = length(x)
  mean = mean(x)
  grad_mu = n*(mean-par[1])/par[2]^2
  a = (x-par[1])^2
  grad_sigma = sum(a)/par[2]^3 - n/par[2]
  return(c(grad_mu, grad_sigma))
}
```

```r
loglike = function(par, x){
  log = sum(dnorm(x, mean = par[1], sd = par[2], log = TRUE))
  return(log)
}

# Matrix to store the answers
results = matrix(nrow = 4, ncol = 6)
rownames(results) = c("CG", "CG with gradients", "BFGS", "BFGS with gradient")
colnames(results) = c("mu", "sigma", "log-likelihood", "count: fn", "count: gr", "conv")

# Using Conjugate Gradient Method
init = c(0,1)
opti_CG = optim(par = init, fn = loglike, x=data, method=c("CG"), control=list(fnscale=-1))
results[1,] = c(opti_CG$par, opti_CG$value, opti_CG$counts, opti_CG$convergence)

# Using CG with gradients
opti_CG_g = optim(par = init, fn = loglike, gr = grads , x=data, method=c("CG"), control=list(fnscale=-
results[2,] = c(opti_CG_g$par, opti_CG_g$value, opti_CG_g$counts, opti_CG_g$convergence)

# Using BFGS
opti_BFGS = optim(par = init, fn = loglike, x=data, method=c("BFGS"), control=list(fnscale=-1))
results[3,] = c(opti_BFGS$par, opti_BFGS$value, opti_BFGS$counts, opti_BFGS$convergence)

# Using BFGS with gradients
opti_BFGS_g = optim(par = init, fn = loglike, gr = grads, x=data, method=c("BFGS"), control=list(fnscal
results[4,] = c(opti_BFGS_g$par, opti_BFGS_g$value, opti_BFGS_g$counts, opti_BFGS_g$convergence)

print(results)
cat("\n")
gradients = matrix(nrow = 4, ncol = 2)
rownames(gradients) = c("CG", "CG with gradients", "BFGS", "BFGS with gradient")
colnames(gradients) = c("mu gradient", "sigma gradient")
for (i in 1:4){
  gradients[i,] = grads(results[i,1:2], data)
}
print(gradients)
```