Maths

David Björelind

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The PDF of a normal distribution is $\frac{1}{\sigma\sqrt{2\pi}} * e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$. The Likelihood then becomes

$L = (\frac{1}{2\pi\sigma^2})^{n/2} * exp(\frac{-\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2})$ after multiplying the PDF over i=1 to n.

The log-likehood is then $-\frac{n}{2}log(\frac{1}{2\pi\sigma^2}) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$ after taking the log of the likelihood. for this expression, we want to find how to pick μ and σ so that we maximize it. The derivative of the log-likelihood is calculated with regards to μ and σ and set $= 0$ to find the optimal values.

$\frac{dL}{d\mu} = \frac{2n(\bar{x}-\mu)}{2\sigma^2} = 0$

$\frac{dL}{d\sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(\bar{x}-\mu)^2 = 0$

This produces an MLE estimator for μ which is $\hat{\mu} = \bar{x}$, the sample mean. The MLE estimator for σ is $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$.