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Permutation Tests

A Practical Guide to Resampling Methods for Testing Hypotheses

Second Edition

With 14 Figures



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