TDDE07: Bayesian Learning
Solutions on paper:

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Exam 2020-06-04 davbj395

TDDE07: Bayesian Learning

1] Binomial distribution

$x \mid N,\theta \sim Bin(N,\theta)$, $N=50$

a) Prior: $\theta \sim U(0,1)$

Posterior distribution:

$$p(\theta \mid x,N) \propto p(x \mid \theta,N) \cdot p(\theta \mid N) = \binom{N}{x}\theta^{x} \cdot (1-\theta)^{N-x} \cdot \frac{1}{1-0} \propto \binom{N}{x}\theta^{x}(1-\theta)^{(N-x)}$$

We can recognice α as $x+1$ and β as $N-x+1 = 51-x$ for the Beta distribution. $p(\theta \mid x,N) \sim Beta(x+1, 51-x)$

2] d)

Laplace Approximation: Taylor approximation of the log-likelihood using parameters obtained by optimization. After parameters are obtained, one can make approximated draws from the posterior

Simulation: i) Make a draw of β (5 of them) from our prior. ii) Use these β in the likelihood to get an estimated value for $\tilde{y}$.

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(2) 3] $p(\beta | y_i) \propto p(y_i | \beta) \cdot p(\beta)$

4] 20 fish: 16 female & 4 male $x=1$: female

a) Prior: $\theta \sim Beta(2,2)$, $x_i \sim Bern(\theta)$

Predictive distribution:

$$p(x_{n+1} | x_{1:n}) = \int p(x_{n+1} | \theta, x_{1:n}) \cdot \underbrace{p(\theta | x_{1:n})}_{\sim B(\alpha+s, \beta+f)} d\theta =$$

$$= \int p(x_{n+1} | \theta) \cdot p(\theta) \, d\theta = \text{, due to no time series}$$

$$= \int \theta^{x} (1-\theta)^{1-x} \cdot \frac{\Gamma(2+16+2+4)}{\Gamma(2+16) \cdot \Gamma(2+4)} \cdot \theta^{2+16-1} \cdot (1-\theta)^{2+4-1} \, d\theta =$$

$$= \frac{\Gamma(24)}{\Gamma(18) \cdot \Gamma(6)} \cdot \int \theta^{x+17} (1-\theta)^{6-x} \, d\theta =$$

$$= \frac{\Gamma(24)}{\Gamma(18)\Gamma(6)} \cdot \frac{\Gamma(x+18)\Gamma(7-x)}{\Gamma(x+18+7-x)} \cdot \underbrace{\int \frac{\Gamma(x+18)\Gamma(7-x)}{\Gamma(x+18)\Gamma(7-x)} \theta^{x+17} (1-\theta)^{6-x} d\theta}_{=1} =$$

$$= \frac{\Gamma(24) \cdot \Gamma(x+18) \cdot \Gamma(7-x)}{\Gamma(18)\Gamma(6) \cdot \Gamma(25)} = 24 \cdot \Gamma(24) = \frac{\Gamma(x+18)\Gamma(7-x)}{\Gamma(18)\Gamma(6) \cdot 24} =$$

$$= \left| x=1, \text{female} \right| = \frac{\overbrace{\Gamma(19)}^{18 \cdot \Gamma(18) =} \cdot \Gamma(6)}{\Gamma(18) \cdot \Gamma(6) \cdot 24} = \frac{18}{24} = \frac{3}{4}$$

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4b) Predictive distribution of the length

$y_i \sim N(\mu_{ML}, \sigma^2_{ML} = 2)$

Normal model, uniform prior:

$p(\tilde{y}|y_{1:n}) = \int p(\tilde{y}|\theta) \cdot p(\theta|y_{1:n})\, d\theta$

$p(\tilde{y}|\theta) \sim N(\mu_{ML}, \sigma^2_{ML})$, $p(\theta|y_{1:n}) \sim N(\bar{y}, \frac{\sigma^2_{ML}}{n})$

Since likelihood & posterior are independant:

$E(\tilde{y}|y_{1:n}) = \bar{y}$, $V(\tilde{y}|y_{1:n}) = \left(\sigma^2_{ML} + \frac{\sigma^2_{ML}}{n}\right)$

Hence: $p(\tilde{y}|y_{1:n}) \sim N(\bar{y}, \sigma^2_{ML}(1+\frac{1}{n})) = N(12, \cdot 2(1+\frac{1}{20}) =$

$= N(12, (\frac{21}{10})^2)$

4c)

$$p(Fem|l,w) = \frac{\overset{\text{from b)}}{P(l,w|Fem)} \cdot \overset{\text{from a)}}{P(Fem)}}{P(l,w)} \propto$$

$\propto P(l|Fem) \cdot p(w|Fem) \cdot P(Fem)$