

732A96/TDDE15 Advanced Machine Learning

Reinforcement Learning

Jose M. Peña
IDA, Linköping University, Sweden

Lectures 9: REINFORCE and Deep Q-Learning Algorithms

Contents

- ▶ REINFORCE
- ▶ Neural Networks
- ▶ Example: Grid Worlds
- ▶ Deep Q-Learning
- ▶ Example: CartPole

Literature

- ▶ Main source
 - ▶ Sutton, R. S. and Barto, A. G. *Reinforcement Learning: An Introduction*. The MIT Press, 2018. Chapters 13 and 16.
 - ▶ Mnih, V. et al. Playing Atari with Deep Reinforcement Learning. *NIPS Deep Learning Workshop*, 2013.
- ▶ Additional source
 - ▶ Russel, S. and Norvig, P. *Artificial Intelligence: A Modern Approach*. Pearson, 2010. Chapters 16, 17 and 21.
 - ▶ Mnih, V. et al. Human-level Control through Deep Reinforcement Learning. *Nature* 518, 529–533, 2015.

REINFORCE

- ▶ Consider learning a **parameterized policy** $\pi(a|s, \theta)$ to select actions without consulting state or action values.
- ▶ Advantages:
 - ▶ The policy function may be simpler to learn than the state and action value functions.
 - ▶ Possibility to inject prior knowledge through the parameterization chosen.
 - ▶ Possibility to model deterministic and stochastic policies. The latter may be interesting in problems with imperfect knowledge, e.g. bluffing in poker. Learning is also easier as the gradient is smoother.
- ▶ Choose the parameter values that maximize the following function for **episodic** tasks: $J(\theta) = v_{\pi}(S_0)$ where S_0 is the initial state. It can also be adapted to continuing tasks.
- ▶ Stochastic **gradient ascent**: $\theta^{i+1} \leftarrow \theta^i + \alpha \nabla_{\theta^i} J(\theta^i)$.

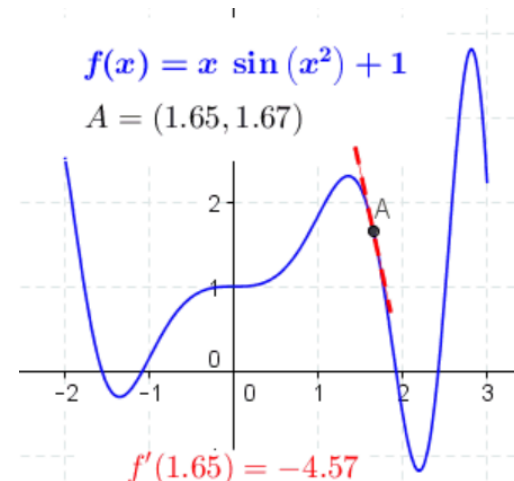
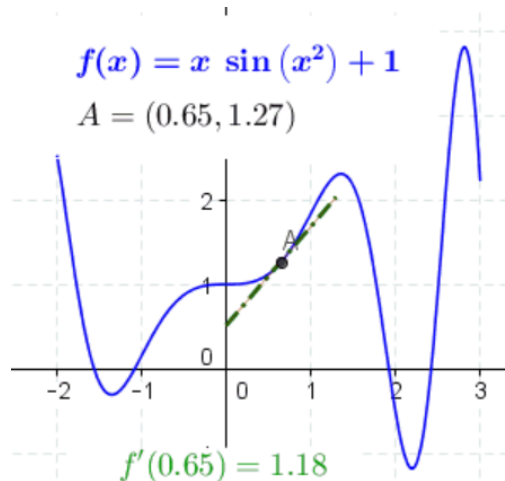
Parameters



Finding Q-table

→ Making a policy

Learning rate



REINFORCE

- ▶ Theorem:

$$\nabla_{\theta} J(\theta) = \nabla_{\theta} v_{\pi}(S_0) \propto E_{\pi} \left[\sum_a q_{\pi}(S_t, a) \nabla_{\theta} \pi(a|S_t, \theta) \right]$$

and thus

$$\begin{aligned} \nabla_{\theta} J(\theta) &\propto E_{\pi} \left[\sum_a \pi(a|S_t, \theta) q_{\pi}(S_t, a) \frac{\nabla_{\theta} \pi(a|S_t, \theta)}{\pi(a|S_t, \theta)} \right] = E_{\pi} \left[q_{\pi}(S_t, A_t) \frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \right] \\ &= E_{\pi} \left[G_t \frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \right] = E_{\pi} [G_t \nabla_{\theta} \ln \pi(A_t|S_t, \theta)]. \end{aligned}$$

- ▶ All this results in the REINFORCE algorithm, which asymptotically converges to a local optimum of $J(\theta)$.

REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s, \theta)$

Algorithm parameter: step size $\alpha > 0$

Initialize policy parameter $\theta \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):

 Generate an episode $S_0, A_0, R_1, \dots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \theta)$

 Loop for each step of the episode $t = 0, 1, \dots, T-1$:

$$\begin{aligned} G &\leftarrow \sum_{k=t+1}^T \gamma^{k-t-1} R_k \\ \theta &\leftarrow \theta + \alpha \gamma^t G \nabla \ln \pi(A_t|S_t, \theta) \end{aligned} \tag{G_t}$$

$$E[f(x)] = \sum_x f(x) \cdot p(x=x)$$

$$E[E[f(x)]] = E[f(x)] \quad ???$$

$$\frac{\partial \ln f(x)}{\partial x} = \frac{1}{f(x)} \cdot f'(x)$$

$$E[x] = \sum_x x \cdot p(x=x) \approx \frac{x_1 + x_2 + \dots + x_n}{n} = \bar{x} \rightarrow E[x]$$

REINFORCE

- ▶ REINFORCE has one major disadvantage: It updates the policy only **at the end of each episode**, which makes it not suitable for incremental online learning and, typically, implies slow convergence and high variance.
- ▶ A particularly interesting advantage of REINFORCE is that updating the policy parameters may change the policy for every action-state pair, and not only for the pairs visited, i.e. **generalization**.
- ▶ Another interesting feature of REINFORCE is that it can be applied to **continuous state and action spaces** by choosing an appropriate parametric policy model, e.g. $\mathcal{N}(\mu s, \sigma)$:

$$\pi(a|s, \theta) = \pi(a|s, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(a - \mu s)^2}{2\sigma^2}\right)$$

which implies that

$$\nabla_{\mu} \ln \pi(a|s, \mu, \sigma) = \frac{a - \mu s}{\sigma^2} s$$

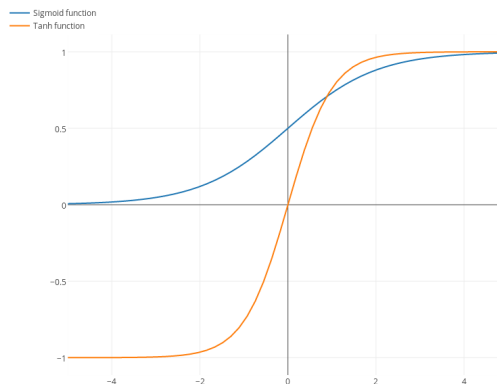
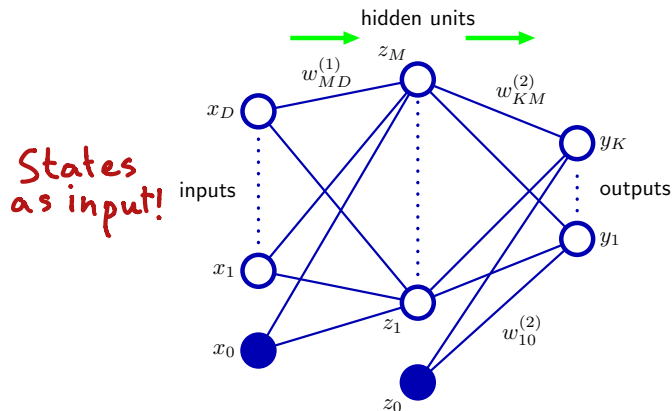
and

$$\nabla_{\sigma} \ln \pi(a|s, \mu, \sigma) = \left(\frac{(a - \mu s)^2}{\sigma^2} - 1\right) s$$

and, thus, the REINFORCE update has a simple form.

- ▶ However, we can easily do better than this.

Neural Networks



- ▶ Consider a multiclass classification problem, i.e. $C \in \{1, 2, \dots, K\}$.
- ▶ Activations: $a_j = \sum_{i=1}^D w_{ji}^{(1)} x_i + w_{j0}^{(1)}$ for all $j = 1, \dots, M$.
- ▶ Hidden units and activation function: $z_j = h(a_j)$ for all $j = 1, \dots, M$.
- ▶ Output activations: $a_k = \sum_{j=1}^M w_{kj}^{(2)} z_j + w_{k0}^{(2)}$ for all $k = 1, \dots, K$.
- ▶ Output : $y_k(\mathbf{x}) = a_k$ for all $k = 1, \dots, K$.
- ▶ Two-layer NN: $y_k(\mathbf{x}) = \sum_{j=1}^M w_{kj}^{(2)} h\left(\sum_{i=1}^D w_{ji}^{(1)} x_i + w_{j0}^{(1)}\right) + w_{k0}^{(2)}$.
- ▶ Class probabilities: $p(C = k|\mathbf{x}) = \frac{\exp(y_k(\mathbf{x}))}{\sum_{k=1}^K \exp(y_k(\mathbf{x}))}$.
- ▶ All the previous is generalizable to several hidden layers.

Neural Networks

- ▶ Consider some training data $\{(\mathbf{x}_n, c_n)\}$ of size N . Consider minimizing the cross-entropy or negative log-likelihood loss function:

$$L(\mathbf{w}) = \sum_{n=1}^N L_n(\mathbf{w}) = - \sum_{n=1}^N \log p(c_n | \mathbf{x}_n, \mathbf{w}).$$

- ▶ The weight space is highly multimodal and, thus, we have to resort to approximate iterative methods to minimize the previous expression.
- ▶ Stochastic gradient descent:

$$\mathbf{w}^{i+1} = \mathbf{w}^i - \alpha \nabla_{\mathbf{w}^i} L_n(\mathbf{w}^i) = \mathbf{w}^i + \alpha \nabla_{\mathbf{w}^i} \log p(c_n | \mathbf{x}_n, \mathbf{w}^i)$$

where n is chosen randomly, and $\nabla_{\mathbf{w}^i} L_n(\mathbf{w}^i)$ can be computed efficiently via **backpropagation**.

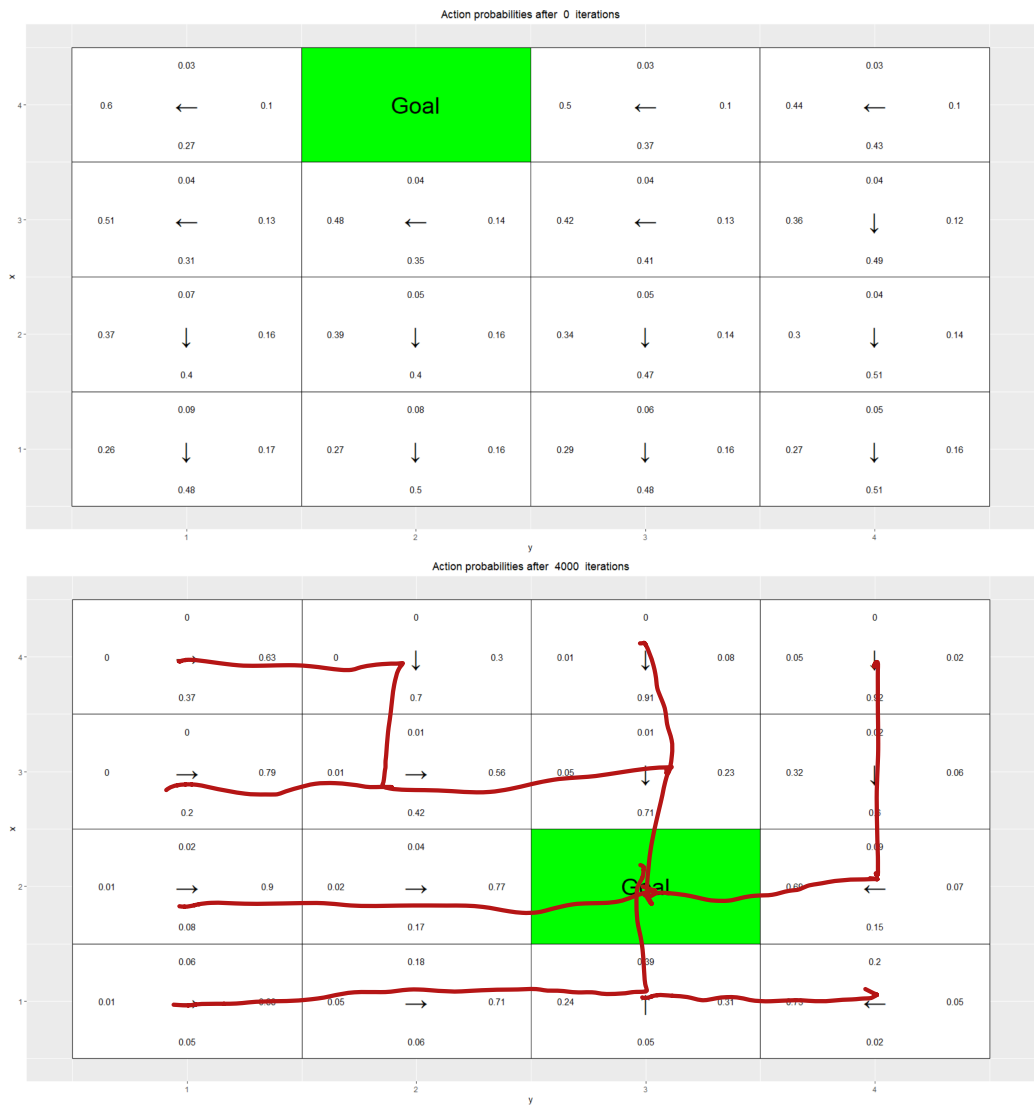
- ▶ Note the similarities between the update above and the REINFORCE update:

$$\theta^{i+1} = \theta^i + \alpha \gamma^t G \nabla_{\theta^i} \ln \pi(A_t | S_t, \theta^i).$$

Classifier from states to actions

- ▶ Therefore, REINFORCE is straightforward to implement when a NN is used to represent the parameterized policy. We only need to set the sample weights $\gamma^t G$. In other words, the policy can be seen as a classifier whose training involves certain peculiarities: Incremental online, and **delayed labels** via sample weights.

Example: Grid Worlds



Deep Q-learning

- ▶ Now, consider a K -dimensional regression problem, i.e. $\mathbf{C} = (C_1, \dots, C_K)$. Consider also some training data $\{(\mathbf{x}_n, \mathbf{c}_n)\}$ of size N . Consider minimizing the mean squared-error loss function:

$$L(\mathbf{w}) = \sum_{n=1}^N L_n(\mathbf{w}) = \sum_{n=1}^N \frac{1}{2} \|\mathbf{c}_n - \mathbf{y}(\mathbf{x}_n)\|^2 = \sum_{n=1}^N \sum_{k=1}^K \frac{1}{2} (c_{nk} - y_k(\mathbf{x}_n))^2.$$

True Prediction

- ▶ The above implies the following stochastic gradient descent update:

$$\mathbf{w}^{i+1} = \mathbf{w}^i - \alpha \nabla_{\mathbf{w}^i} L_n(\mathbf{w}^i) = \mathbf{w}^i - \alpha \sum_{k=1}^K (c_{nk} - y_k(\mathbf{x}_n)) \nabla_{\mathbf{w}^i} \mathbf{y}(\mathbf{x}_n)$$

where n is chosen randomly, and $\nabla_{\mathbf{w}^i} \mathbf{y}(\mathbf{x}_n)$ can be computed efficiently via **backpropagation**.

Deep Q-Learning

- ▶ If the transition model were so that s and a are always followed by s' and r , then $q_*(s, a) = r + \gamma \max_{a'} q_*(s', a')$ by the Bellman optimality equation and, thus, $0 = r + \gamma \max_{a'} q_*(s', a') - q_*(s, a)$. We can try to enforce this constraint by executing π one step from s and a and, then, updating the estimate of $q_*(s, a)$ as

$$q_*(s, a) \leftarrow q_*(s, a) + \alpha(r + \gamma \max_{a'} q_*(s', a') - q_*(s, a)).$$

where $\alpha > 0$ is the learning rate.

- ▶ The reasoning above also applies to stochastic transition models, since the number of times that s and a are followed by s' and r in the sampled episodes is proportional to the transition probability.

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+$, $a \in \mathcal{A}(s)$, arbitrarily except that $Q(\text{terminal}, \cdot) = 0$

Loop for each episode:

 Initialize S

 Loop for each step of episode:

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Take action A , observe R, S'

$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$

 until S is terminal

Deep Q-Learning

- ▶ Q-learning has a major disadvantage: Representing the action value function $q(s, a)$ as a look-up table is not feasible for **large state and/or action spaces**, due to storage space and time to convergence.
- ▶ A solution is to represent it as a **parameterized function** $q(s, a|\theta)$. This brings advantages such as fewer parameters than table entries and thus easier to learn and reach convergence, less storage space, **generalization to unvisited state-action pairs**, etc.
- ▶ It also comes with the challenge of selecting an appropriate parameterized function: It has to be expressive and allow for incremental online training. A NN is typically used.

Use Neural Net to predict
Q values from states →
Predict Q values for actions

True value: $q_*(s, a) = \sum_{s', r} p(s', r|s, a)(r + \gamma \max_{a'} q_*(s', a'))$.

- For the i -th iteration (i.e., episode step), the NN is trained by minimizing the mean squared-error loss function: **Predictions**

$$L(\theta^i) = E_{s, a \sim \rho(s, a)} [(y^i - \underbrace{q(s, a|\theta^i)}_{\text{We don't know true values}})^2]$$

where $\rho(s, a)$ is the behaviour distribution induced by ϵ -greedy, and

$$y^i = E_{r, s' \sim p(r, s'|s, a)} [r + \gamma \max_{a'} q(s', a'|\theta^{i-1})|s, a].$$

Best guess

- The gradient is given by

$$\nabla_{\theta^i} L(\theta^i) = E_{s, a \sim \rho(s, a); r, s' \sim p(r, s'|s, a)} [(r + \gamma \max_{a'} q(s', a'|\theta^{i-1}) - q(s, a|\theta^i)) \nabla_{\theta^i} q(s, a|\theta^i)]$$

and stochastic gradient descent is typically considered:

$$\theta^{i+1} = \theta^i - \alpha (r + \gamma \max_{a'} q(s', a'|\theta^{i-1}) - q(s, a|\theta^i)) \nabla_{\theta^i} q(s, a|\theta^i).$$

Back propagation

- Note the similarities between the update above and the NN update:

$$\mathbf{w}^{i+1} = \mathbf{w}^i - \alpha \nabla_{\mathbf{w}^i} L_n(\mathbf{w}^i) = \mathbf{w}^i + \alpha \sum_{k=1}^K (c_{nk} - y_k(\mathbf{x}_n)) \nabla_{\mathbf{w}^i} \mathbf{y}(\mathbf{x}_n)$$

which implies that deep Q-learning is straightforward to implement when a NN is used to represent the parameterized state value function. In other words, the action value function can be seen as a regressor whose training involves certain peculiarities: Incremental online, and **delayed targets** via bootstrapping on the model from the previous iteration.

Deep Q-learning with experience replay for estimating $\pi \approx \pi_*$

Algorithm parameters: small $\epsilon > 0$

Initialize $Q(S, A|\theta)$ with random weights, and initialize \mathcal{D} with N random experiences

Loop for each episode:

Initialize S

Loop for each step of episode:

Choose A from S using policy derived from Q (e.g., ϵ -greedy)

Take action A , observed R, S'

Store transition (S, A, R, S') in \mathcal{D}

Sample random minibatch of transitions (S_j, A_j, R_j, S'_j) from \mathcal{D}

$$Y_j \leftarrow \begin{cases} R_j & \text{for terminal } S'_j \\ R_j + \gamma \max_a Q(S'_j, a|\theta) & \text{for non-terminal } S'_j \end{cases}$$

Perform a gradient descent step on $(Y_j - Q(S_j, A_j|\theta))^2$

$S \leftarrow S'$

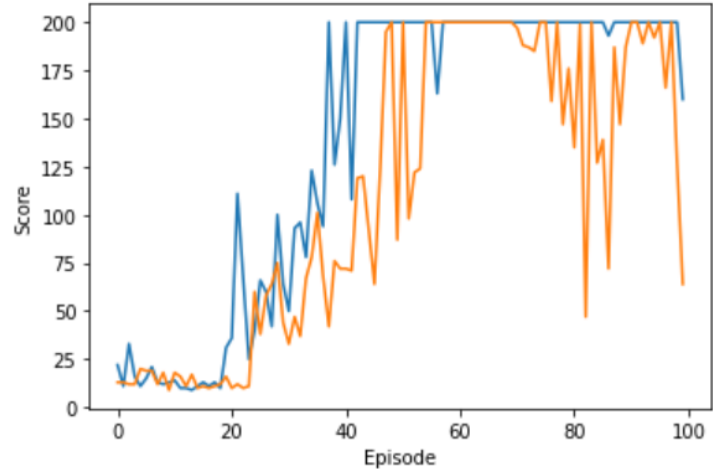
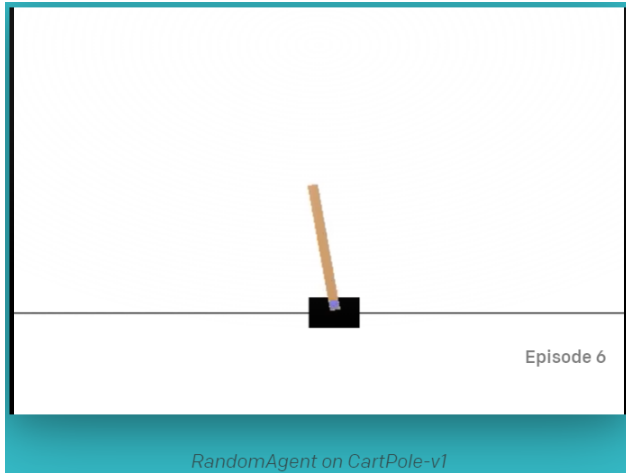
until S' is terminal

Not what the Q-table tells us

- ▶ Learning a parameterized state value function may resemble supervised learning but the learning data is **not i.i.d.** due to highly correlated states and changing agent behaviour. Experience replay alleviates these problems by smoothing the training distribution over many past behaviours.
- ▶ All action values at once: $S \rightarrow NN \rightarrow (Q(S, A_1), \dots, Q(S, A_k))$.

Example: CartPole

- With replay
- Without replay



- ▶ Open AI's gym
- ▶ PilcoLearner

Summary

- ▶ REINFORCE
- ▶ Neural Networks
- ▶ Example: Grid Worlds
- ▶ Deep Q-Learning
- ▶ Example: CartPole
- ▶ Interested in more ? Check out [AlphaGo - The Movie](#).

Thank you