# *Inverse Problem of Magnetostatics*

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## *Multipole expansion*

Biot—Savart law is given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \, d^3\mathbf{r}' \tag{1}$$

and in case of the magnetisation $\mathbf{M}$ we have

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{[\nabla \times \mathbf{M}(\mathbf{r}')] \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \, d^3\mathbf{r}' \tag{2}$$

The question I ask myself is: are any statistical properties of the magnetic field connected to the statistical properties of the magnetisation or current density distribution that produces it?

## *Popcorn model*

The popcorn model is a model of a magnetised material that consists of a large number of small magnetic dipoles. The dipoles are randomly oriented and their positions are random. The magnetisation of the material is given by the sum of all the dipoles. The magnetic field is given by the sum of the magnetic fields of all the dipoles. The magnetic field of a single dipole is given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3\mathbf{m}(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')(\mathbf{r} - \mathbf{r}') - |\mathbf{r} - \mathbf{r}'|^2 \mathbf{m}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^5} \tag{3}$$

where $\mathbf{m}(\mathbf{r}')$ is the magnetic moment of the dipole at position $\mathbf{r}'$.

## *Reconstructing magnetic field components*