Statistical Science

Chapter 2.5 Ratio Scale Units

ReCap (Ch 1)
Quantities (Ch2)
2.1 Five part definition
2.2 Types of measurement scale
2.3 Data collection, recording, and error checking
2.4 Graphical and tabular display of data
Critique of graphs and tables
2.5 Ratio scale units
2.5.1 Ratio scale units in biology
2.5.2 The utility of ratio scale units
2.5.3 Base units, derived units, and standard multiples
2.5.4 Unconventional Units
2.5.5 Unfamiliar units
2.6 Dimensions

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Not here last time?
     Course Outline
     Name on roster
Questionnaire results
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Discussion of Cards Lab:
  Anybody come up with "wrong"
rule that works?
      In 1997 mutually exclusive
pairs introduced ("test" cards),
before going to multiple working
hypotheses ("crucial" cards).
   Ask for discussion of this,
comparison of "test" and
"crucial."
     In 1998 crucial cards only.
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on chalk board

Recap Chapter 1
Quantitative reasoning: Example of scallops, which combined stats and models
Biological reasoning will take the lead.
Stats will be a tool, rather than defining the way we think about biological problems.
Models (equations) are ideas about the relation of quantities
Statistical analyses are based on models

Recap Chapter 2
Quantities: 5 components Name
Symbol
Procedural Statement (write out and review: Could I use this recipe?)
Set of numbers collected into a vector
Units on a defined measurement scale
Measurements made on four types of scale: nominal, ordinal, interval, ratio
Data collection, recording, and error checking
Graphical and tabular display of fully defined quantities

Today: Use of ratio scale units in quantitative reasoning (Schneider 2009 Ch4)
Begin with definition of base units, derived units, and standard multiples
Then explore unconventional units and their utility in science.

Wrap-up:
Poor practice in use of units results in poor science.
Ratio scale units are useful in reasoning about quantities.
Distinguish derived from base units, then define standard multiples.
Unconventional units are useful in sciences.

2.5.1 Ratio Scale Units (Schneider 2009 *Quantitative Ecology* Chapter 4)

Measurement is the hallmark if science and engineering. There are strong traditions of sound practice in the use of ratio scale measurement units in physiology, chemistry, physics, and biochemistry. Unfortunately, practice is less sound in ecology, the health related sciences, and statistical applications in biology.

Ratio scale units are distinguished from measurements on other types of scales by just that--ratios can be taken and interpreted. Ratios of measurements on nominal, ordinal, or interval scales cannot be used.

Nominal scale. Cannot compute the ratio. presence/absence = ?
    Units scored as having an attribute can be gathered together to compute
    a valid ratio: p = % with attribute.

Ordinal scale. Can compute the ratio, but what does it mean ?
    Rank3 / Rank1 ≠ 3 times higher than Rank 1

Interval scale. Can compute the ratio, but not interpretable.
    For example. 50°F = 10° C, 41° F = 5°C
        50°F / 41° F = 1.2
        10°C / 5° C = 2 ≠ 1.2
    But difference in temperature is on ratio scale, hence ratios can be computed.
        10°C rise = 18° F rise, 5°C rise = 9° F rise
        10°C rise / 5° C rise = 2
        18°F rise / 9° F rise = 2

It is of note that in science, nominal scale measurements are of considerable utility and that such units are readily converted to ratio scale measurements. Nominal scale measurements have a high prevalence in biology because of the utility categories and mutually exclusive scoring of the units of life (live/dead, female/male, presence/absence of gene, behavior, species, *etc*). When living units are scored on a nominal scale, it is natural and informative to collect scores as proportions, which are on a ratio scale.

Usage of units in some areas of science is at present cavalier and sloppy, to the detriment of the science. Units do not appear in the procedural statements found in methods sections of research reports and theses. Units are omitted when reporting results in the form of summary statistics, parameter estimates, and graphs. This is serious, because the results cannot be checked or reproduced by another investigator. The time and effort put into theses and research reports were wasted, if the results are irreproducible.

2.5.1 Ratio Scale Units

Two factors contribute to prevalence of poor usage.
One is inappropriate systems of units. The mechanical system of units (mass length time) used in physics leaves out the other sciences. For example, tree density has units of stems per unit area, but in a mechanical system only area is recognized as a unit, while stems are not. Chemistry has developed an appropriate set of units (moles of atoms, ions, and molecules). There is no reason that sciences such as biology cannot be similarly successful in developing appropriate units.

Another contributing factor is that units are not central to mathematics and are not taught in statistical courses for scientists. Units have a sound mathematical basis (Abelian sets). But in the interests of generality, units are omitted from statistics courses, or from the occasional science course that features mathematical reasoning.

2.5.2 The Utility of Ratio Scale Units

Ratio scale units have several useful properties. One is that they guide accurate computation. An example is the calculation of energy flow through populations. Computing the energy flow through a system such as salt marsh (Teal 1962) or a freshwater spring (Odum 1957) typically requires a table of conversion factors because there are several units of energy occur in the literature--Joules, gram-calories, kilogram-calories, British thermal units. If we are interested in rates of energy transfer even more units exist–kilocalories per day, liters of oxygen per hour, ergs, Watts, and horsepower-hours.

Ratio scale units guide the interpretation of symbolic notation used in equations. The abstract and forbidding strangeness of the symbol $e^{-\dot{D}\cdot t}$ is overcome by giving it a name then undertaking calculations with scaled values. The symbol $e^{-\dot{D}\cdot t}$ stands for the percent of the population remaining after suffering a death rate of $\dot{D}$. A fuller sense of the meaning of this percentage comes from substituting scaled values such as t = 2 days and $\dot{D}$ = 0.01 day^{-1}, instead of just numbers. Box 2.1 shows a series of calculations. Far more is learned by taking out a calculator right now to do the calculations, than by just examining Box 2.1.

instantaneous mortality $\dot{D}$	time t	% remaining $e^{-\dot{D}\cdot t}$
0.01 year^{-1}	1 year	99%
0.1 year^{-1}	1 year	90%
0.2 year^{-1}	1 year	82%
0.2 year^{-1}	2 years	67%
0.4 year^{-1}	1 year	67%

Box 2.1 Interpretation of the symbol $e^{-\dot{D}t}$ via calculation with scaled values

2.5.2 The Utility of Ratio Scale Units

Ratio scale units have another useful quality, which is that they can be combined to make new units via multiplication and division. Multiplication of a unit by itself is equivalent to changing the exponent of the units. Think of sweeping sticks at right angles to make areas, or measuring velocity as the frequency (in units of time^{-1}) that a unit of distance is covered. This operation of changing exponents will become important in working with fractal objects, such as the convoluted structure of the stream beds inhabited by fish, or the surface of the lung.

Yet another useful quality is that ratio scale units permit analysis at multiple scales of space and time. Pattern and process evident at one space or time scale differ from those evident at other scales. For example, hourly food intake varies during the according to predictable patterns, one pattern for nocturnal animals, another for crepuscular animals. Monthly food intake varies according to seasonal patterns, especially in animals that hibernate. Over the course of a lifetime, food intake as the animals grows larger.

We can visualize multiscale analysis as the operation of zooming in toward greater detail, or conversely, expanding the scale to reveal larger scale pattern and process. To represent this in formal terms, so that calculations can be made, we need units that can be doubled (or halved) repeatedly. Ratio scale units can be reduced by factors two, ten, or any other base. This increases the resolution, allowing us to zoom in on detail, an operation that we can represent mathematically by dividing units repeatedly to finer scales of resolution. Or conversely we can zoom back to pull in larger scale pattern. This is represented mathematically by expanding the range and the resolution. These operations, which are natural with ratio scale units, cannot be carried out on ordinal or interval types of measurement scales.

Because ratio scale units have these useful properties, some people have taken the position that only ratio scale units are valid (e.g., Campbell 1942). This narrow view does not stand up to logical analysis (Stevens 1975, Luce and Narens 1987). Thus in defining a biological quantity it is more important to provide a clear statement of the type of units, than to provide no definition because a ratio scale unit is not applicable.

> Another look at 2.5.2
>
> Ecologists often use transformations before undertaking inferential statistical analysis of data. Comment on what is lost by taking the square root of data on tree numbers per 1 m^2 area.

2.5.3 Base Units, Derived Units, and Standard Multiples

Standard units on a ratio scale are defined against a standard base so that anyone anywhere can obtain comparable results. Table 2.1 lists the seven ***SI base units*** in the International System of Units (Système Internationale, abbreviated SI). This system includes two supplementary units, one for plane angles and one for solid angles. These appear in definitions of angular velocity, acceleration, and momentum. They also appear in definitions of light flux and light exposure.

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To present following
table, ask for show of
hands of people who have
used base and
supplementary unit.
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Table 2.1

Base and supplementary units in the SI system.

Quantity	Unit	Abbreviation
Length	meter	m
Mass	kilogram	kg
Time	second	s
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd
Electrical current	ampere	A
Planar angle	radian	rad
Solid angle	steradian	sr

Combinations of the base units result in ***derived units***. Some of these derived units have names, such as Joules for units of energy. Table 2.2 lists derived units that commonly occur in science. A list of over 60 derived units can be found in Legendre and Legendre (1998, 2nd ed 2003 Elsevier). A collection of ratio scale constants and quantities used in marine ecology can be found in Mann and Lazier (1991).

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To present the following
table, pick out some
derived units and ask for
show of hands of people
who have used the derived
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Table 2.2 Units That Commonly Occur in the Natural Sciences

Quantity	Units	Equivalent Unit (Name)
Acceleration		
Angular	$rad \cdot s^{-2}$	
Linear	$m \cdot s^{-2}$	
Area	m^2	
	$10^4 \cdot m^2$	ha (hectare)
Concentration	$mol \cdot m^{-3}$	
Energy (work)	$N \cdot m$	J (Joule)
	$4185 \cdot J$	Kcal (kilocalorie)
Energy flux	$J \cdot m^{-2} \cdot s^{-1}$	
Force	$kg \cdot m \cdot s^{-2}$	N (Newton)
Frequency	s^{-1}	Hz (Hertz)
Light		
Luminance	$cd \cdot m^{-2}$	
Luminous flux	$cd \cdot sr$	lm (lumen)
Illuminance	$lm \cdot m^{-2}$	lx (lux)
	$10.764 \cdot lx$	fc (footcandle)
Photon flux (*PAR)	$1 \cdot mole \cdot m^{-2} \cdot s^{-1}$	E (Einstein)$\cdot m^{-2} \cdot s^{-1}$
Mass density	$kg \cdot m^{-2}$	
Mass flow	$kg \cdot s^{-1}$	
Mass flux	$kg \cdot m^{-2} \cdot s^{-1}$	
Power	$J \cdot s^{-1}$	W (Watt)
Pressure (stress)	$N \cdot m^{-2}$	Pa (Pascal)
Sverdrup	$10^6 \cdot m^3 \; s^{-1}$	
Surface tension	$N \cdot m^{-1}$	
Velocity		
Angular	$rad \cdot s^{-1}$	
Linear	$m \cdot s^{-1}$	
Viscosity		
Dynamic	$Pa \cdot s$	
Kinematic	$m^2 \cdot s^{-1}$	
Volume	m^3	(cubic meter)
	$10^{-3}\, m^3$	l (liter)
Volume flow rate	$m^3 \cdot s^{-1}$	
Wavelength	m	
Wavenumber	m^{-1}	

*PAR = Photosynthetically Active Radiation.

2.5.3 Base Units, Derived Units, and Standard Multiples

The divisibility of ratio scale units into smaller and yet smaller fractions allows us to define a series of standard multiples. These have standard names and abbreviations, shown in Table 4.3. These standard multiples, 10^{-1} = deci, 10^{-3} = milli, 10^{-6} = micro, produce new units from a basic unit such as the Watt, a unit of energy. The ***standard multiple units*** listed in Table 2.3 are applicable to any ratio scale unit.

Table 2.3

Standard multiples of ratio scale units. W = Watt

Name	Multiple	Abbreviation	Example
pico	10^{-12}	p	pW
nano	10^{-9}	n	nW
micro	10^{-6}	µ	µW
milli	10^{-3}	m	mW
centi	10^{-2}	c	cW
deci	10^{-1}	d	dW
	10^{0}		W
deca	10^{1}	da	daW
hecto	10^{2}	h	hW
kilo	10^{3}	k	kW
mega	10^{6}	M	MW
giga	10^{9}	G	GW

Another look at 2.5.3.

1. Of the units in Table 2.2, how many have you used ?
2. Of the multiples in Table 2.3, how many have you used ?

2.5.4 Unconventional Units

We do not need to restrict ourselves to standard units in reasoning about quantities. Unconventional units are as valid as standard units. Galileo used a spearlength to reason about velocities and momenta. A spearlength is just as good for quantitative reasoning as the standard unit, a meter. It is in reporting measurements that we have to use standard units. Measurements must be repeatable by others, which means either using meters or using spearlengths defined relative to a meter.

The advantage of unconventional units in quantitative reasoning is that they permit a wider choice of units, leading to a direct and more immediate understanding of the relation of quantities. If our interest were in the foraging ranges of owls, we can define the range in biological terms, based on the minimum area (in standard units) required to meet daily energy requirements. Taking this area as one unit, we can then quantify the foraging area needed by a pair of owls to successfully produce 1 chick, 2 chicks, and so on, relative to the number of minimum foraging units. To phrase this as a question, if 1 owl requires a certain area to meet its own energy needs, then how many of these units will be needed by 2 owls to raise 1 or more chicks? By defining a new unit, we can address this problem with biologically meaningful units, rather than with arbitrary units.

2.5.4 **Unconventional Units** (continued)

An unconventional unit that proves useful again and again in chemistry and biology is the individual, or entity, for which a convenient symbol is the number sign #. Examples of chemical entities are atoms, molecules, and proteins. Examples of biological entities are individuals, cells, species, genes, attacks by a predator, or potential encounters. An ***entity*** is defined as a recognizable object belonging to a population of such objects. The entity is an unconventional or non-SI unit that is extremely useful in ecology and can be handled in a rigorous fashion (Stahl 1962). The conventional SI unit is the mole (Table 4.1), which is equal to $6.0225 \cdot 10^{23}$ entities. The mole is an appropriate unit for chemical entities such as atoms, ions, or molecules. It is far too large for biological populations. Even the zooplankter *Calanus finmarchicus*, one of the most abundant animals on the planet, does not amount to a picomole. The philosophical objection to using counts of objects or events as a measurement scale (Ellis 1966) can be easily met by insisting that this scale does not consist of numbers; it has units of entities (animals, genes, etc.) on a ratio scale of measurement. One distinctive feature of this unit is that we cannot halve it repeatedly in the same way that we can halve the unit of a centimeter repeatedly. But this does not prevent us from calculating expected values in fractions of entities. An example is average family size, which is expressed in fractions of individuals, even though any single family must have a discrete number of individuals.

Unconventional exponents are another source of useful units. For some problems units of temporal frequency (*e.g.*, sec^{-1}) or spatial frequency (*e.g.*, km^{-1}) are more useful than units of time or distance. Fractal units such as $m^{1.8}$ are far more informative than Euclidean lines (m^1), planes (m^2), and volumes (m^3) in describing a variety of ecological phenomena, from coastlines (Pennycuick and Kline 1986) and rivers to lungs and the circulatory system. These will be treated in more detail in Chapter 2.6.

Another look at 2.5.4.

If there are about 5 generations per century, re-express the following measurements in generation times:

- 1 solar year
- 1 lifetime of 75 years
- 5% increase per year

2.5.5 Information

Though not an SI unit, the Hartley is part of the International System of Quantities, defined by International Standard IEC 80000-13 of the International Electrotechnical Commission. It is named after Ralph Hartley.

The Hartley (symbol Hart), also called a ban, or a dit (short for decimal digit), is a logarithmic unit that measures information or entropy, based on base 10 logarithms and powers of 10. A similar measure (belonging to the same dimension) is the shannon, based on powers of 2 and base 2 logarithms. One ban or Hartley is the information content of an event if the probability of that event occurring is 1/10.

0 hartley	1:1 odds
1 hartley	10:1 odds
2 hartleys	100:1 odds
3 hartleys	1000:1 odds