# Basic concepts about signals & systems (Part II)

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# Outline

1. Introduction
2. Basic signals and operations
3. The Fourier Transform
4. Linear Time Invariant Systems
5. Filters and resonators
6. The source-filter model

# Linear Time Invariant systems

- Linearity
  - Linear systems fulfill the superposition principle:
    - 1) Additive property
      - $T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$
    - 2) Scaling property
      - $T\{a\, x_1[n]\} = a\, T\{x_1[n]\} = a\, y[n]$
    - Expressing both at the same time:

$$T\{ax_1[n] + bx_2[n]\} = a\, T\{x_1[n]\} + b\, T\{x_2[n]\}$$

$$x(t) \longrightarrow \boxed{T()} \longrightarrow y(t)=T(x(t))$$

# Linear Time Invariant systems

- Linearity
  - In a more general way:

    if $x[n] = \Sigma_k\, a_k\, x_k[n] \Rightarrow y[n] = \Sigma_k\, a_k\, y_k[n]$

    where $y_k[n]$ is the response of the system to the input signal $x_k[n]$

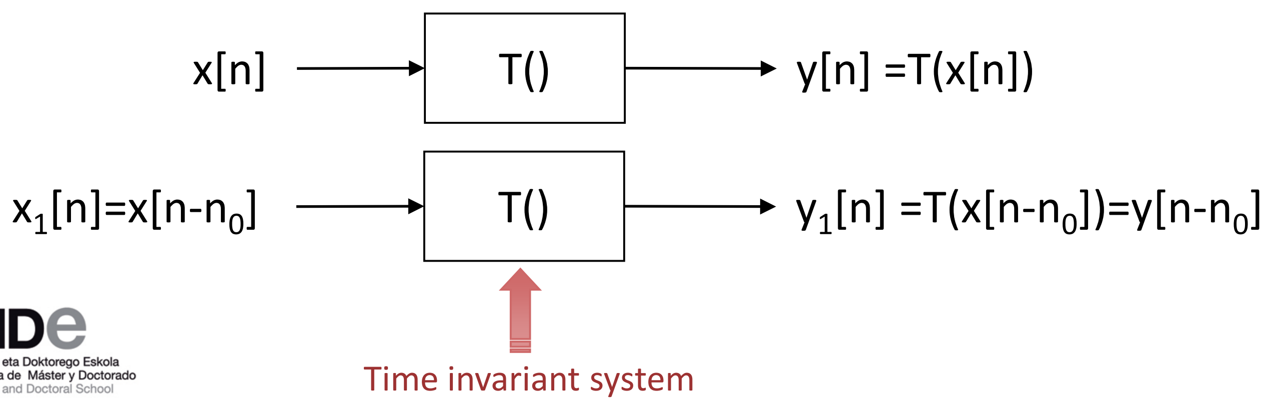
$$\Sigma_k\, a_k\, x_k[n] \longrightarrow \boxed{T()} \longrightarrow y[n] = T(x[n]) = \Sigma_k\, a_k\, y_k[n]$$

Linear system

# Linear Time Invariant systems

- Time invariance
  - A system is time invariant if a delay in the input signal signal produces the same delay in the output signal
    - If $x[n]$ produces $y[n]$ as output, the system is time invariant if for every $n_0$

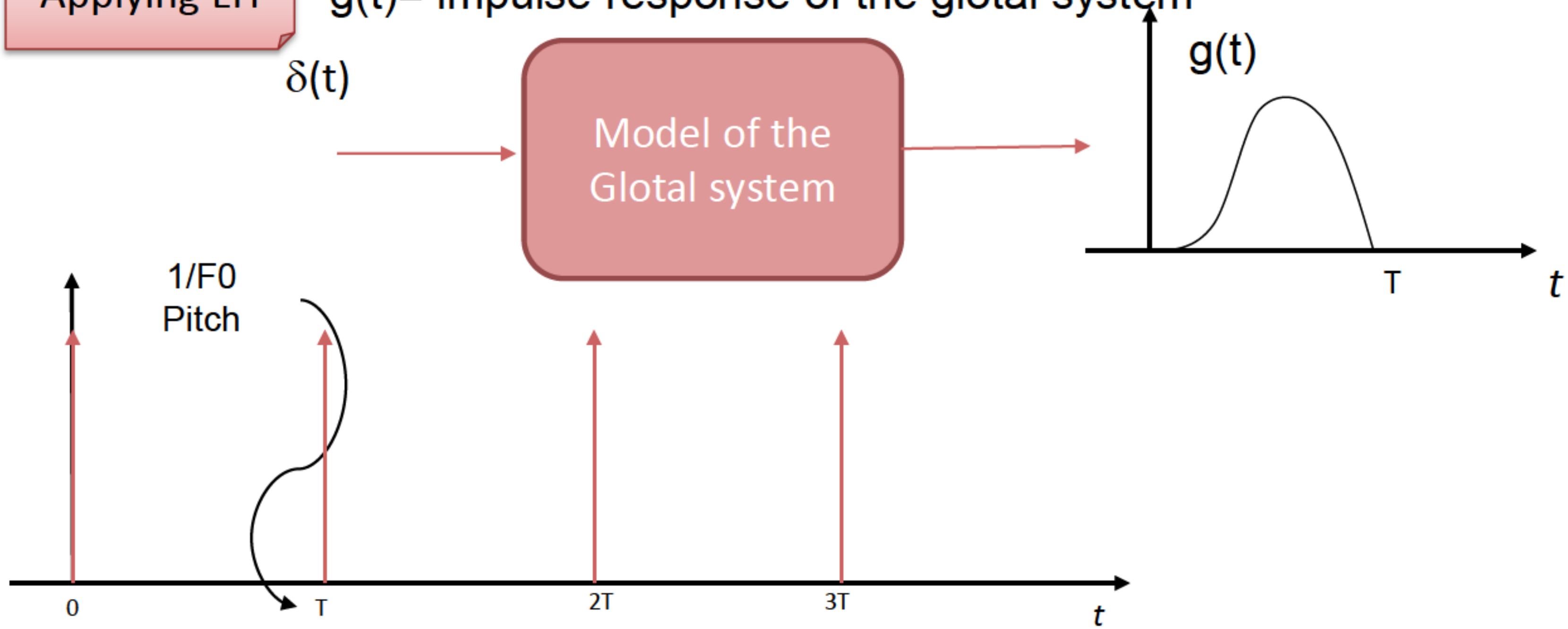
      $x_1\,[n] = x\,[n\text{-}n_0]$ produces $y_1[n] = y\,[n\text{-}n_0]$

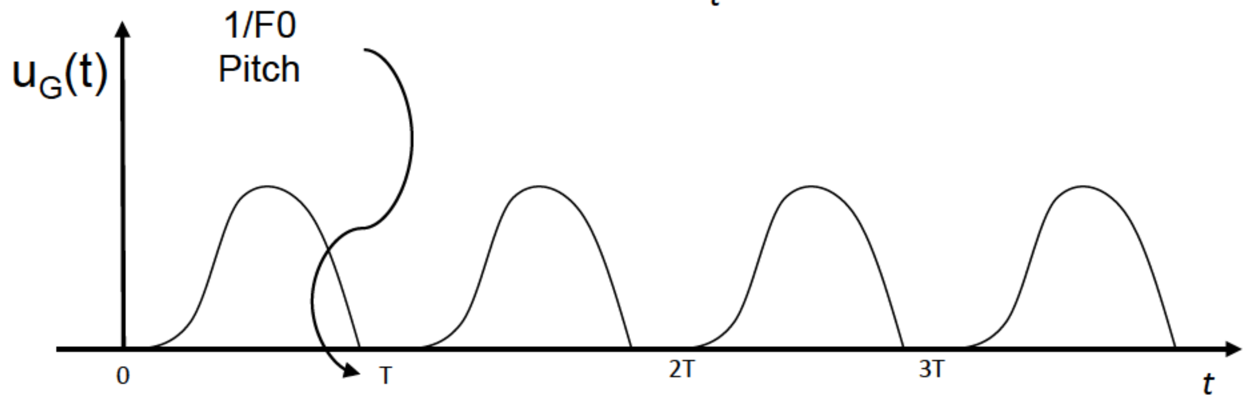


# Linear Time Invariant systems

Applying LTI

g(t)= impulse response of the glotal system



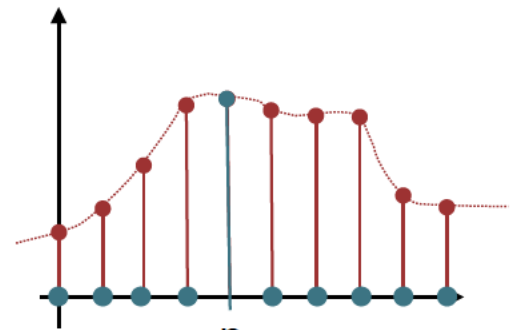



# Linear Time Invariant systems

- A signal as a combination of unit impulses (discrete signals):
  - Sampling

    $x[n]\delta[n]=x[0]\delta[n]$

    $x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$




  - Expressing any signal as a combination of unit impulses

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Ch.1 id_demo

# Linear Time Invariant systems

- A signal as a combination of impulses (continuous case):
  - Sampling

$$x(t)\delta(t)=x(0)\delta(t) \qquad x[n]\delta[n]=x[0]\delta[n]$$

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0) \qquad x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$$

  - Expressing any signal as a combination of unit impulses

$$x(t)=\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \qquad x[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Ch.1 id_demo

# Linear Time Invariant systems

- If the system is **linear and invariant:**
  - It is completely characterized by its **impulse response**
    - h[n] response to δ[n]

$$\delta[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]=h[n]$$

    - If  we have any input signal x[n]

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]=x[n]*h[n]$$

$$y[n] = T(x[n]) = T\left(\sum_{k=\infty}^{+\infty} x[k]\,\delta[n-k]\right) \{\text{Linearity}\} = \sum_{k=\infty}^{+\infty} T(x[k]\,\delta[n-k])$$

$$\{\text{Linearity}\} = \sum_{k=\infty}^{+\infty} x[k]\,T(\delta[n-k])\,\{\text{Invariance}\} = \sum_{k=\infty}^{+\infty} x[k]\,h[n-k]$$

$$\boxed{\boldsymbol{y[n] = x[n] * h[n]} \longleftarrow \textit{\textbf{Convolution}}\text{ operation}}$$

# Linear Time Invariant systems

- If the system is **linear and invariant:**
  - It is completely characterized by its **impulse response**
    - h(t) response to δ(t)

$$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)=h(t)$$

    - If we have any input signal x(t)

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)=x(t)*h(t)$$

$$y(t)=T(x(t))=T\left(\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right)\overset{\text{Linear system}}{=}\int_{-\infty}^{\infty} x(\tau)T\left(\delta(t-\tau)\right)d\tau \overset{\text{Invariant system}}{=}$$

$$=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \boxed{x(t)*h(t) \leftarrow \textbf{\textit{Convolution}} \text{ operation}}$$

# Linear Time Invariant systems

- If we know the response of a system (h(t) or h[n]) to an impulse signal ($\delta(t)$ or $\delta[n]$), then:

*we can obtain the response of the system (y(t) or y[n]) through the* ***convolution*** *of the input with the impulse response:*

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]=x[n]*h[n]$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)=x(t)*h(t)$$

# Convolution equation

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Continuous time convolution equation

1º invert

2º move to τ=t

3º Multiply both signals

4º Calculate the product area

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Discrete time convolution equation

Same procedure as in continuous time

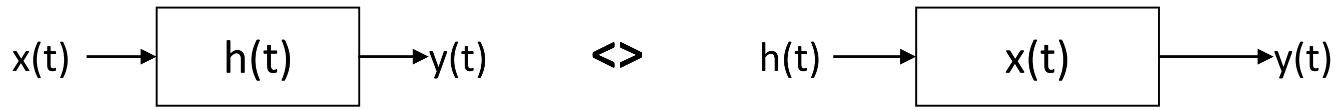
## Examples:

*https://lpsa.swarthmore.edu/Convolution/CI.html*

# Linear Time Invariant systems: properties of convolution

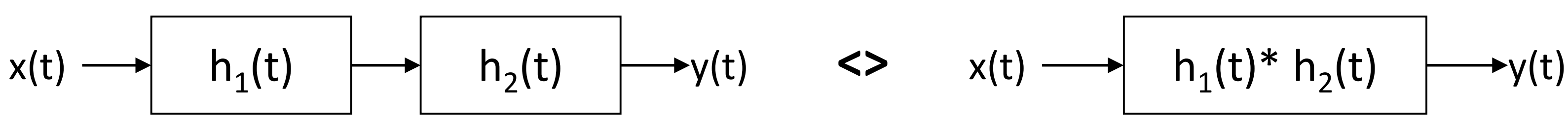
- Commutative

$$x(t) * h(t) = h(t) * x(t)$$




- Associative: systems connected in series

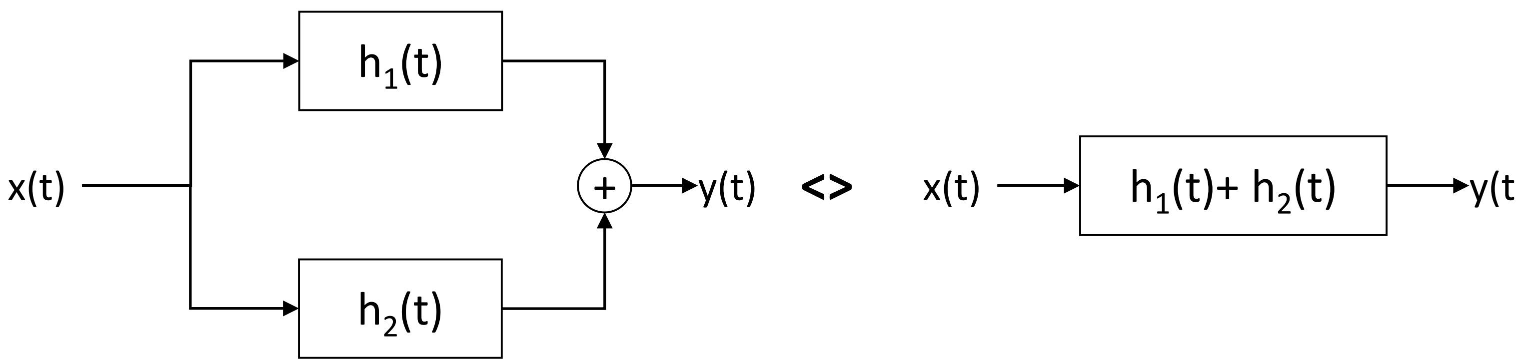
$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$



# Linear Time Invariant systems: properties of convolution

- Distributive: systems connected in parallel

$$[x(t)*h_1(t)]+[x(t)*h_2(t)]=x(t)*[h_1(t)+h_2(t)]$$




- Neutral element

$x(t)*\delta(t)=x(t)$ $\quad$ $x[n]*\delta[n]=x[n]$

$x(t)*\delta(t-t_0)=x(t-t_0)$ $\quad$ $x[n]*\delta[n-n_0]=x[n-n_0]$

# Linear Time Invariant systems: properties of convolution

- FT of the convolution operation: Product!

$$x(t) * h(t) \xrightarrow{F} X(f)\cdot H(f)$$

Convolution

Product

$$\xrightarrow{F}$$

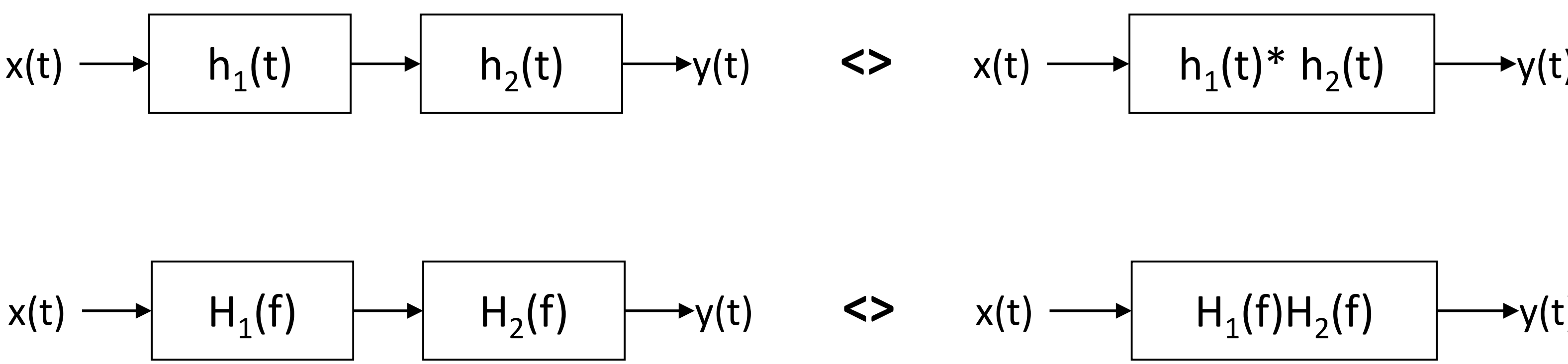
| Time | Frequency |
|---|---|
| $x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)=x(t)*h(t)$ | $X(f) \rightarrow \boxed{H(f)} \rightarrow Y(f)=X(f)H(f)$ |

Working in the frequency domain converts the convolution operation in a product of the Fourier Transforms

# Linear Time Invariant systems: properties of convolution

- Associative: systems connected in series

$$[x(t)*h_1(t)]*h_2(t) = x(t)*[h_1(t)*h_2(t)]$$

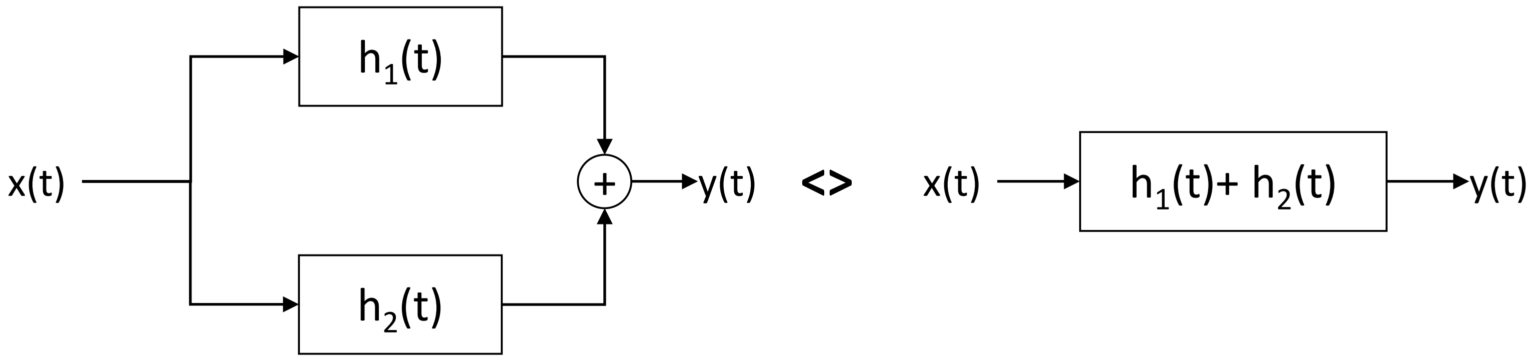


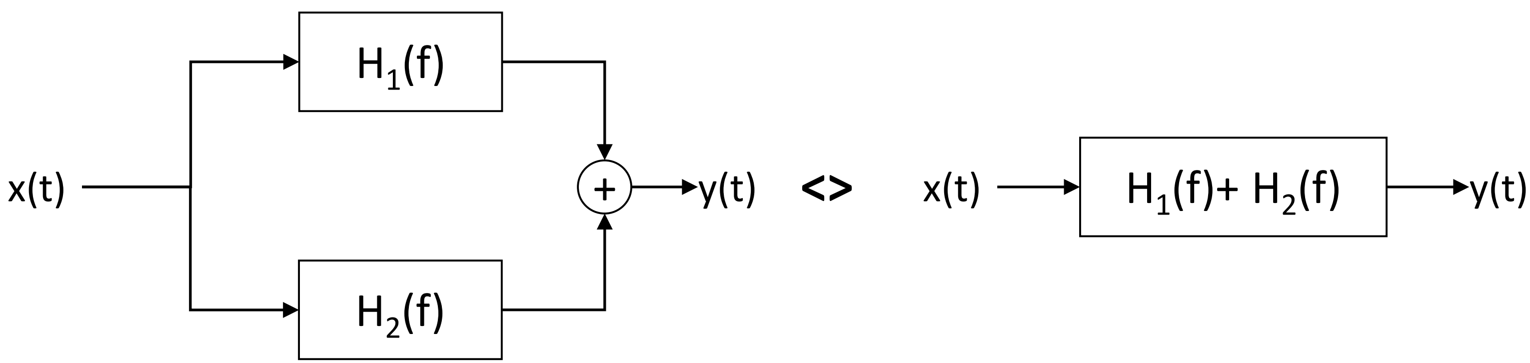

$$Y(f) = X(f)H_1(f)H_2(f)$$

# Linear Time Invariant systems: properties of convolution

- Distributive: systems connected in parallel

$$[x(t)*h_1(t)]+[x(t)*h_2(t)]=x(t)*[h_1(t)+h_2(t)]$$




$$Y(f) = X(f)\{H_1(f) + H_2(f)\}$$

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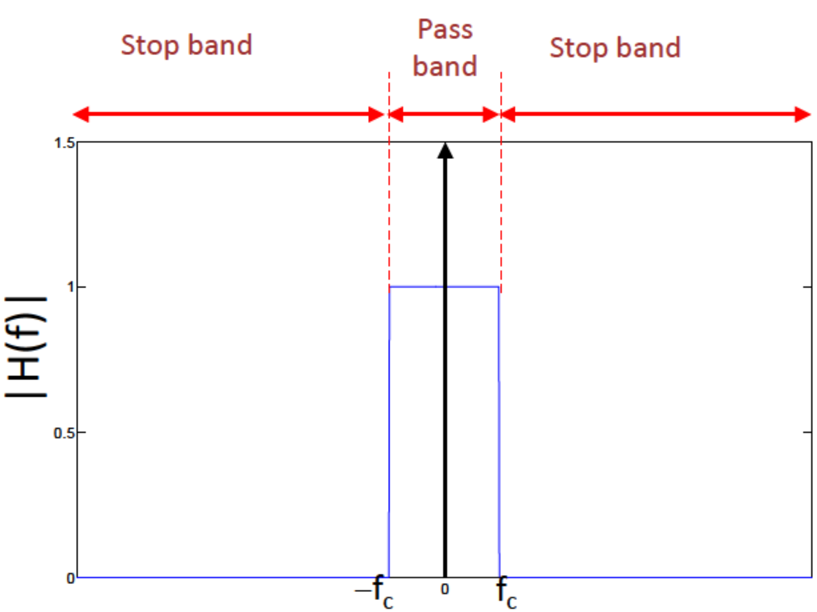
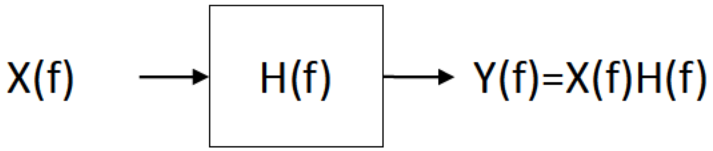
# Filters

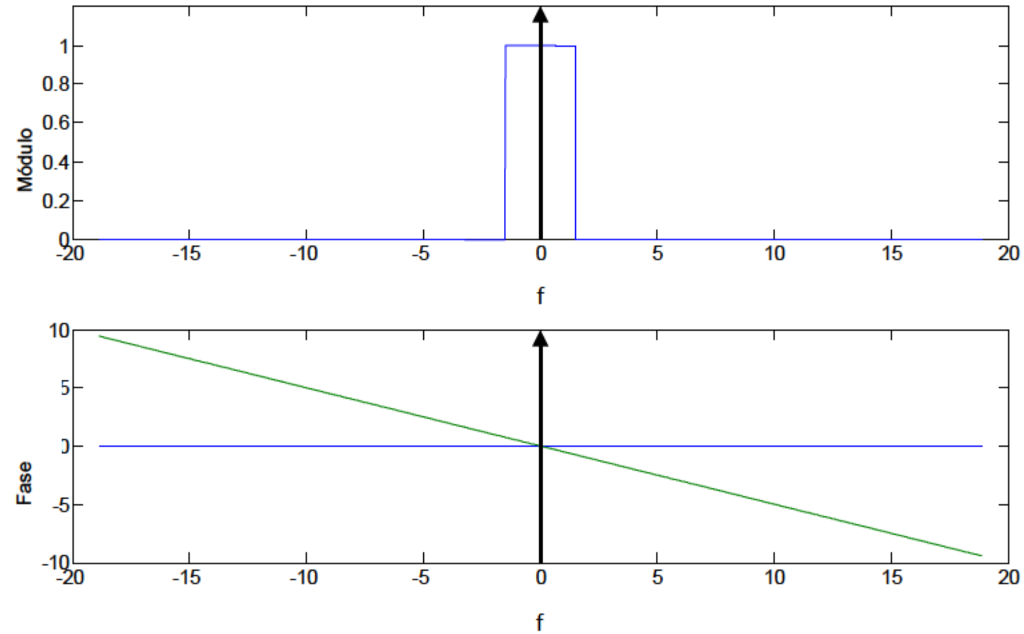
- Filters are LTI system that allows passing frequency components in a specific band (pass band) and removes the rest (stop band)



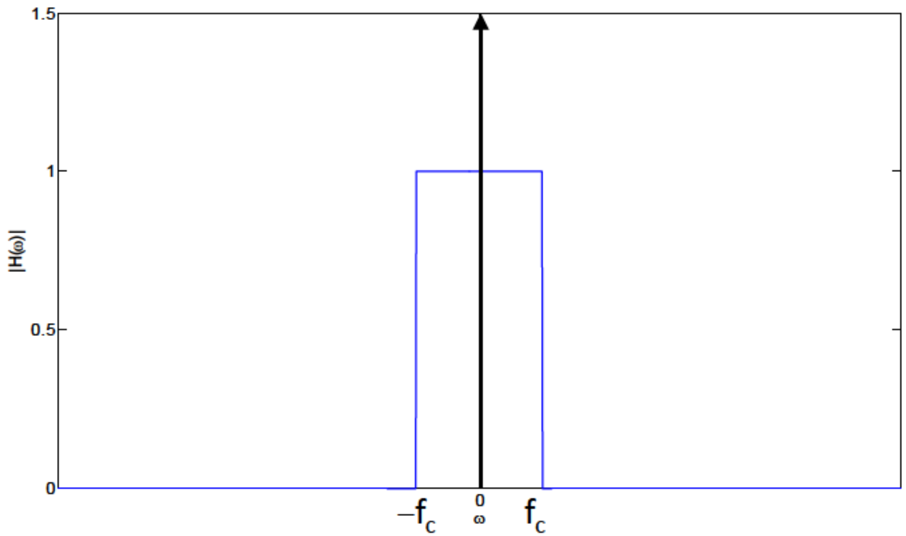



# Ideal filters

- An ideal filter does not introduce distortion
  - Module
    - 1 at pass band
    - 0 at stop band
  - Phase 0 or linear at pass band



# Basic ideal filters

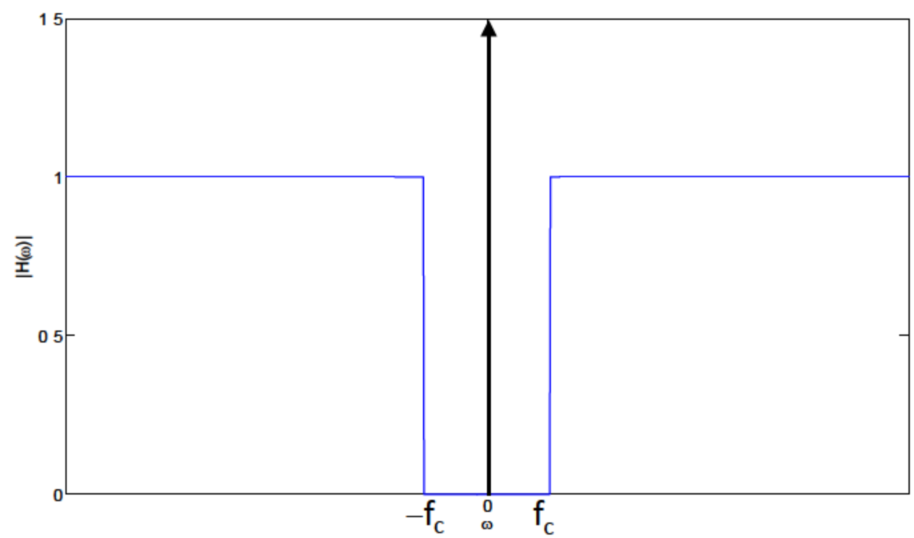



**Low-pass** filter

Allows passing all frequencies below the cut-off frequency $f_c$

**High-pass** filter

Allows passing all the frequencies above the cut-off frequency $f_c$

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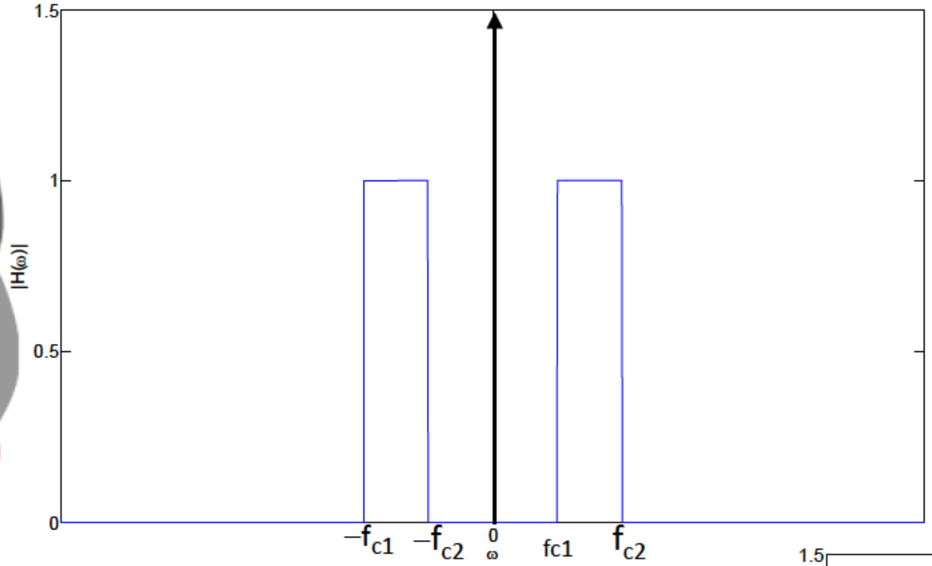
# Basic ideal filters

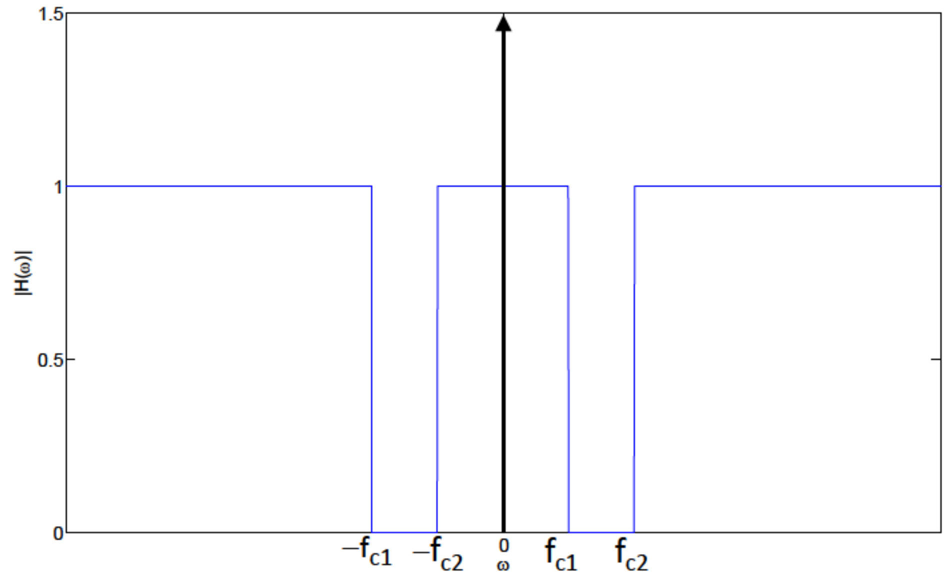



**Band-pass** filter

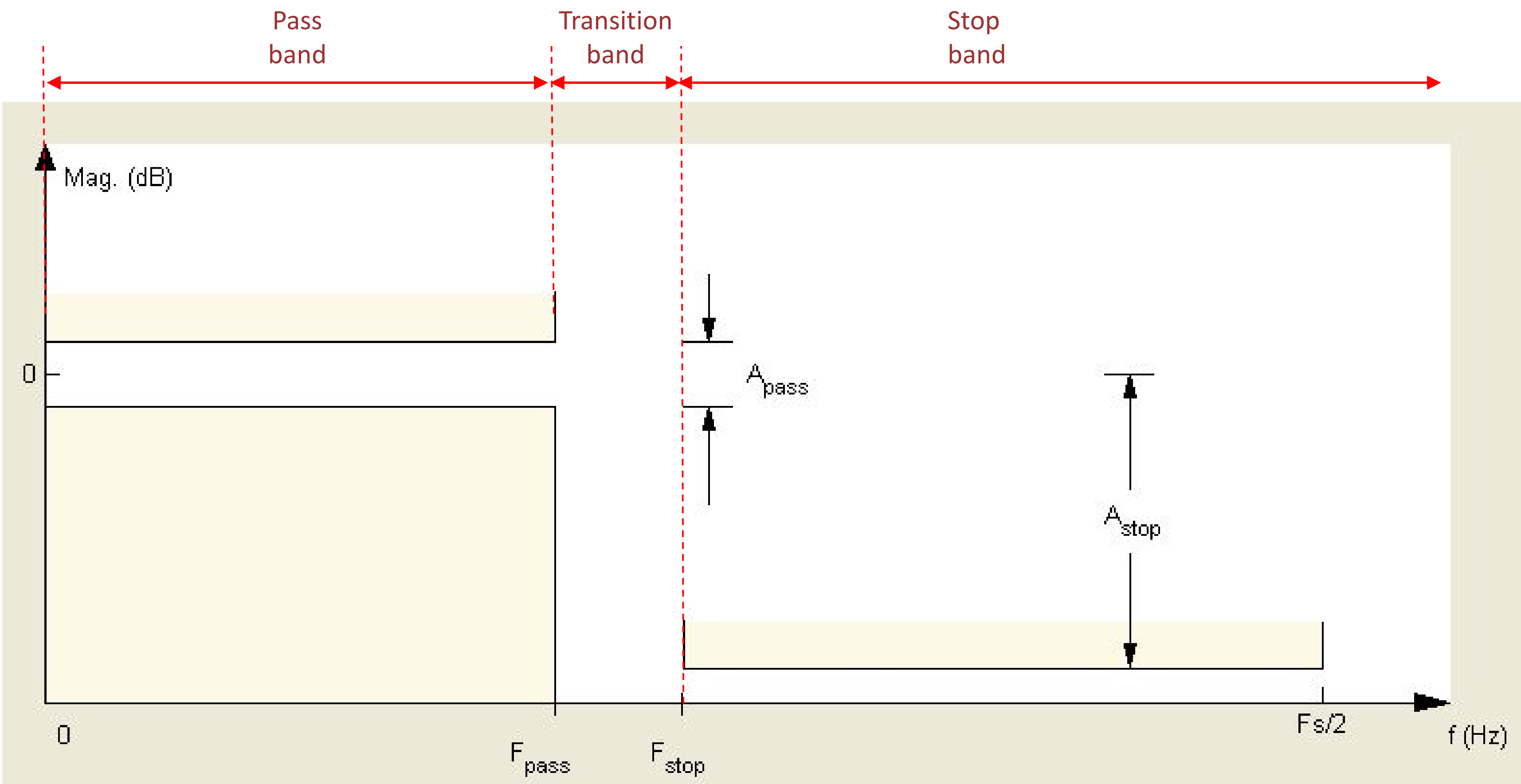
Allows passing all frequencies in the range $f_{c1}$ to $f_{c2}$

**Band -stop** filter

Allows passing all the frequencies but the ones in the range $f_{c1}$ to $f_{c2}$

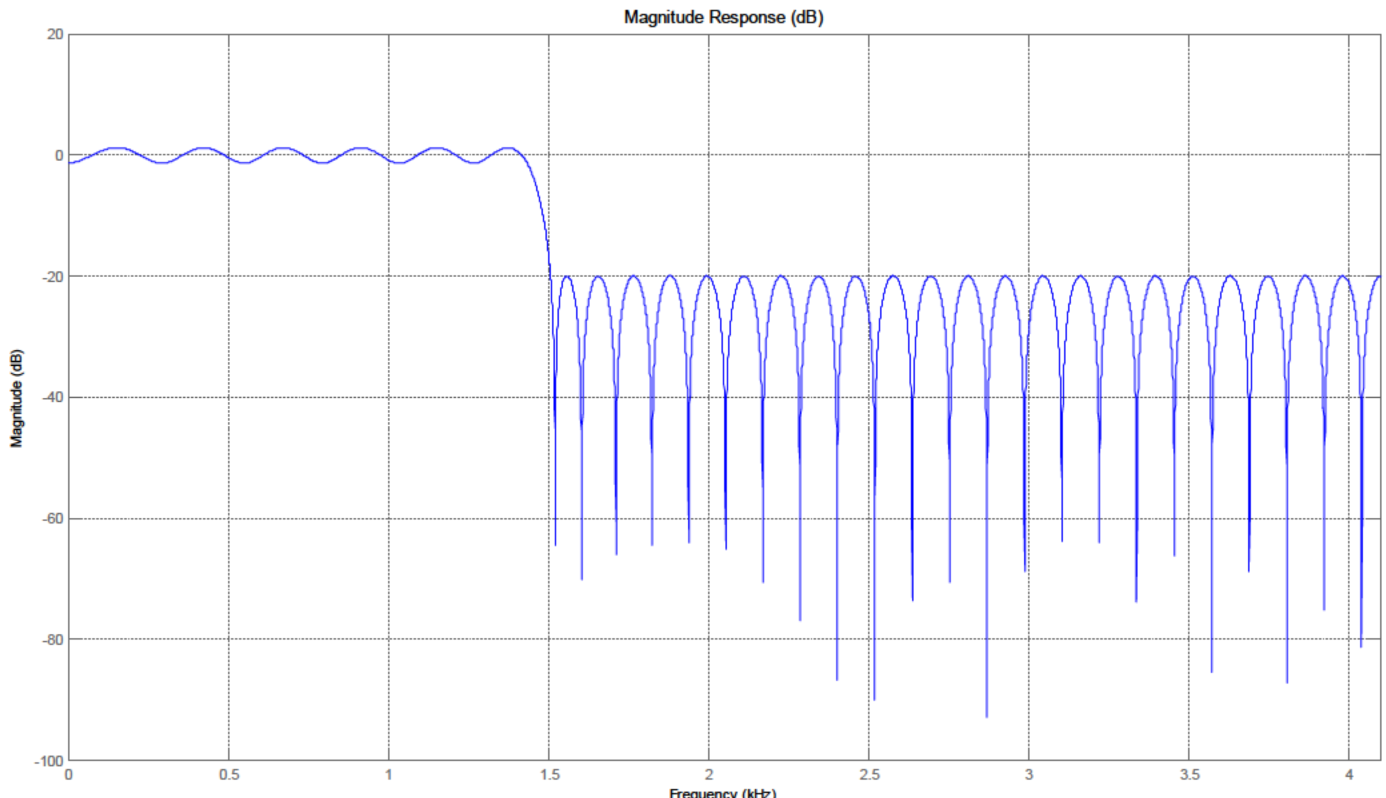


# Real filters



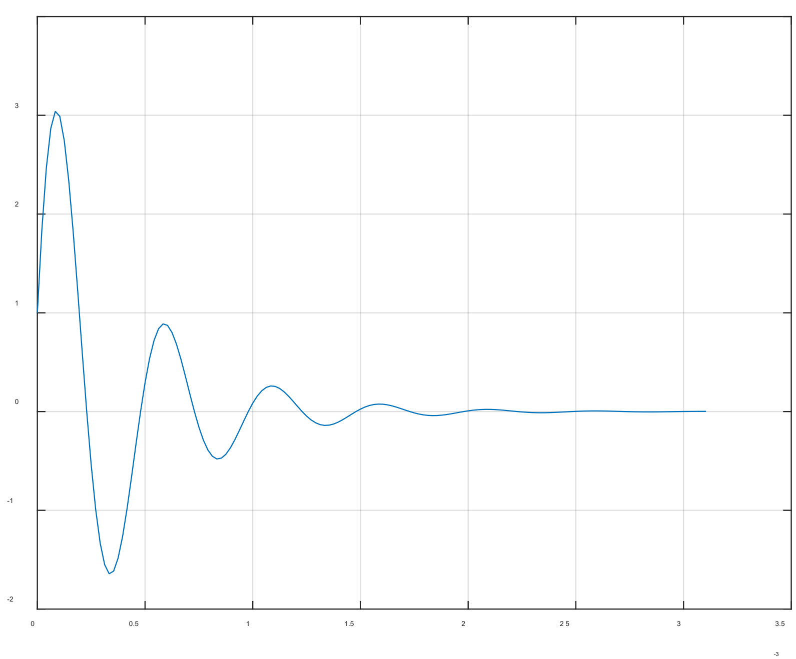
# Real filters

- Design of real filters with SPTool or FDATool in Matlab

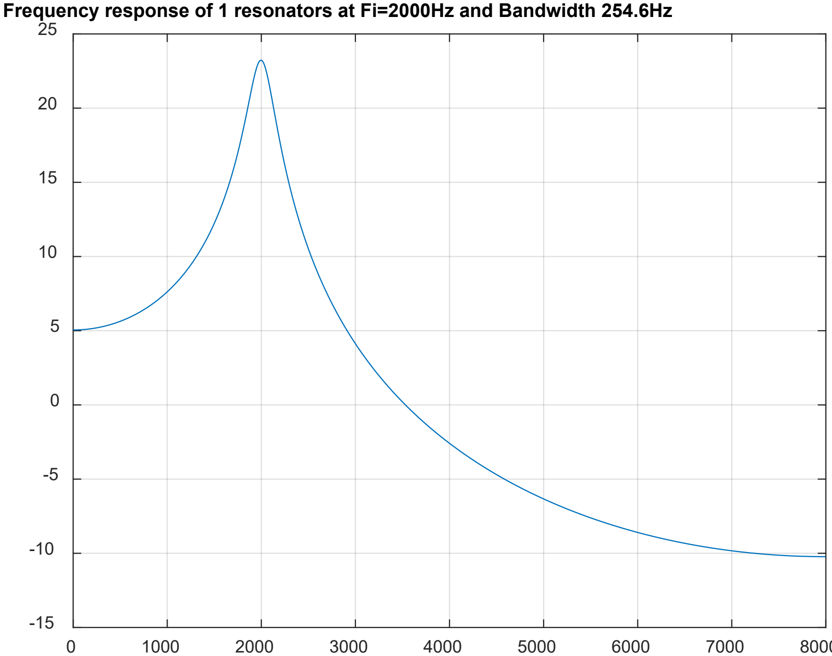


# Resonators

- A resonator is a system which favors the transfer of a certain frequency. It can be seen as a bandpass filter centered at the resonance frequency:

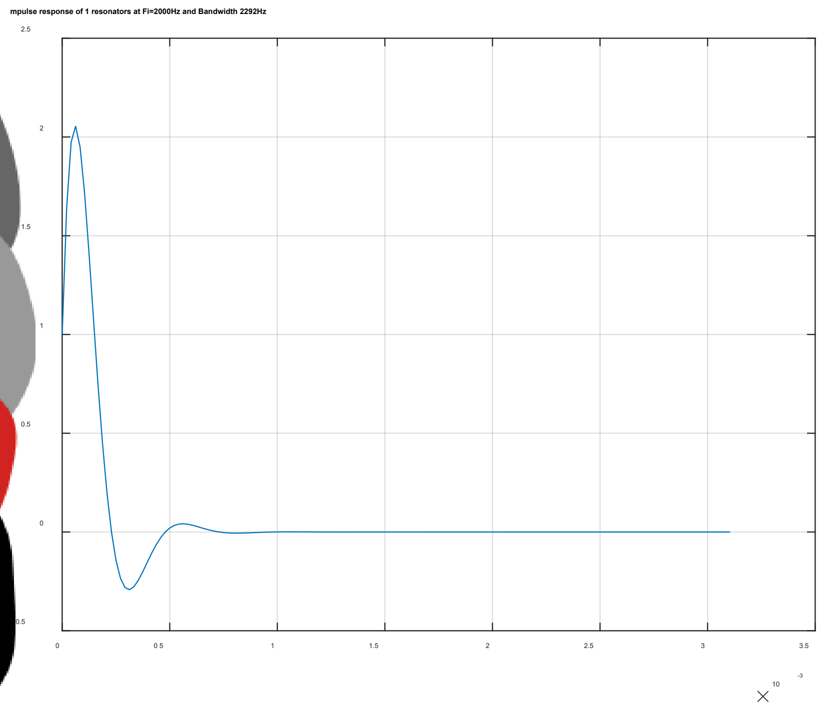


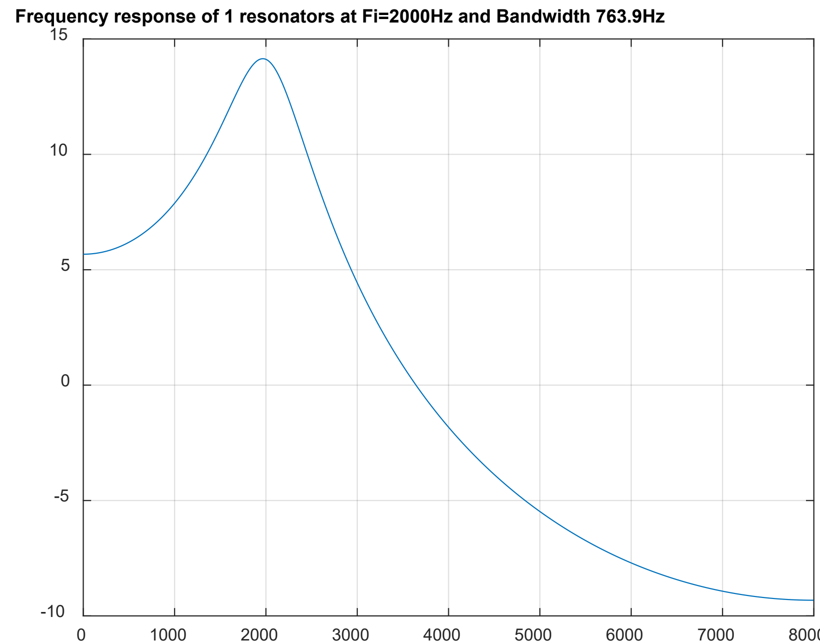

Impulse response of a 2nd order resonator




Frequency response of 2nd order resonator

# Resonators

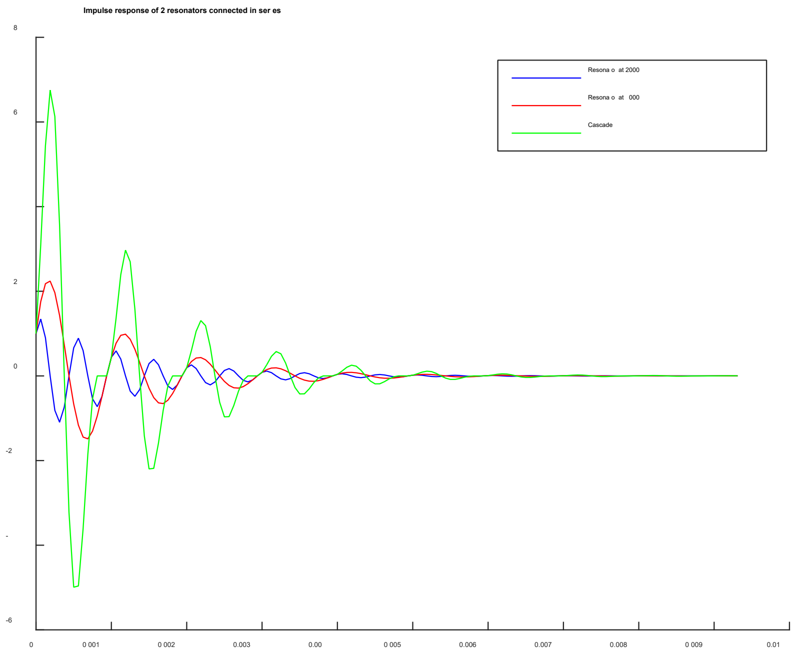


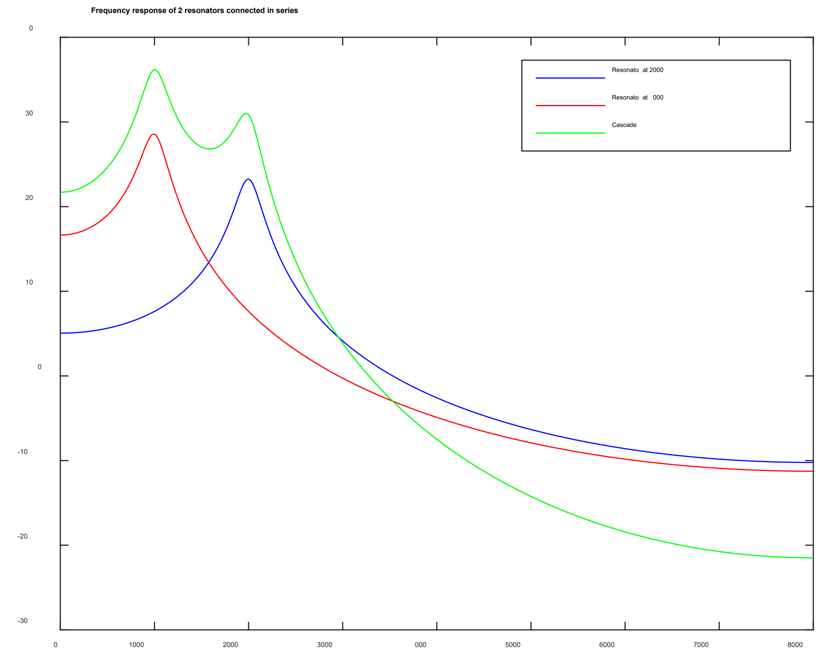



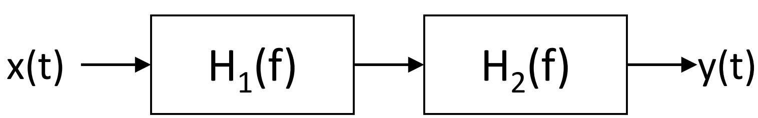

A resonator is defined by 2 parameters:

- Resonance frequency Fr
- Bandwitdth (3dB)

# Resonators connected in series

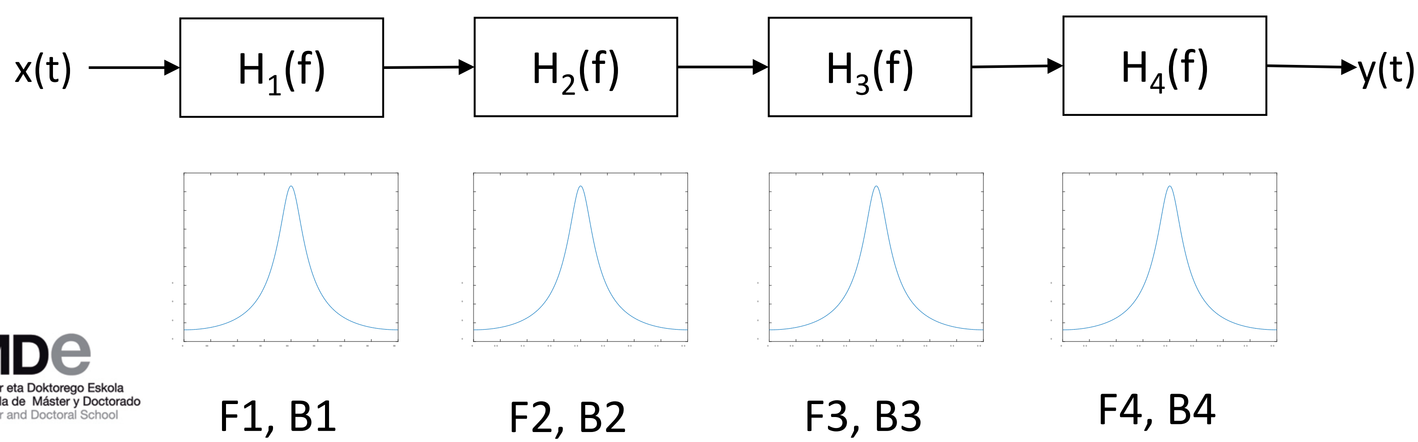


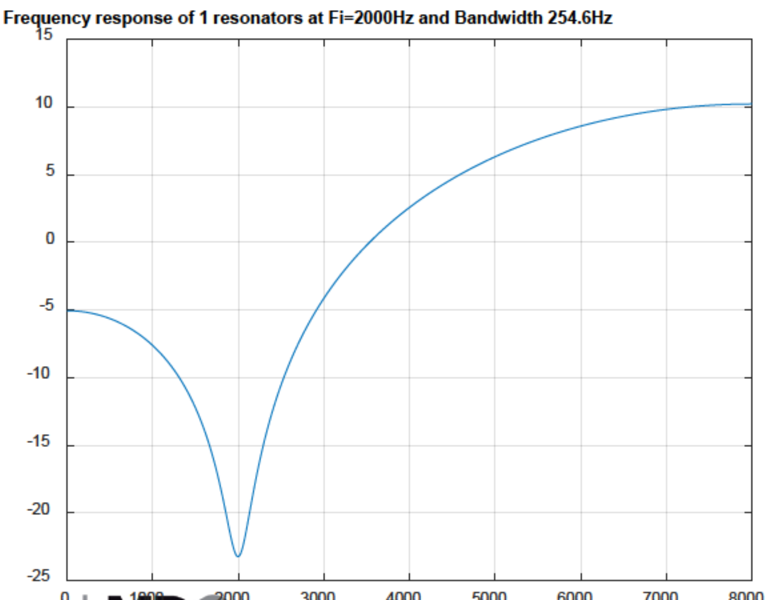
# Resonators example: Formants

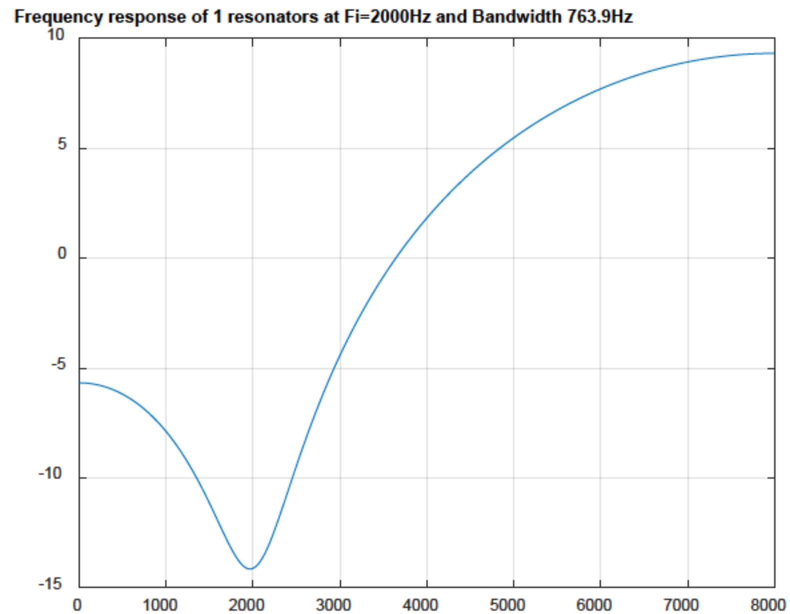
- Formants are the resonance frequencies of the oral cavity
- The resonance frequencies depend mainly on the form given to the cavity
- The oral cavity can be modeled as 4 resonators connected in cascade / parallel:



# Antiresonators

- An anti-resonator is a system which impedes the transfer of a certain frequency. It can be seen as a band-stop filter centered at the anti-resonance frequency. They are used to model ‘nasal’ zeros.



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# Recommended Lectures

- Digital Processing of Speech Signals

Lawrence Rabiner and Ronald W. Schafer

1979 (Ed.Prentice Hall)