Basic concepts about signals & systems

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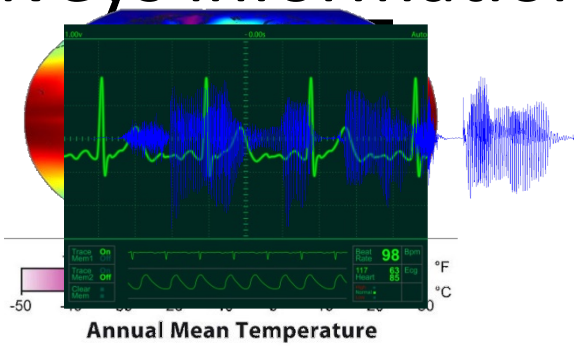
Universidad del País Vasco
Euskal Herriko Unibertsitatea
MDe
Master eta Doktorego Eskola
Escuela de Máster y Doctorado
Master and Doctoral School

Outline

1. Introduction
2. Basic signals and operations with signals
3. The Fourier Transform
4. Linear Time Invariant Systems
5. Filters and resonators
6. The source-filter model

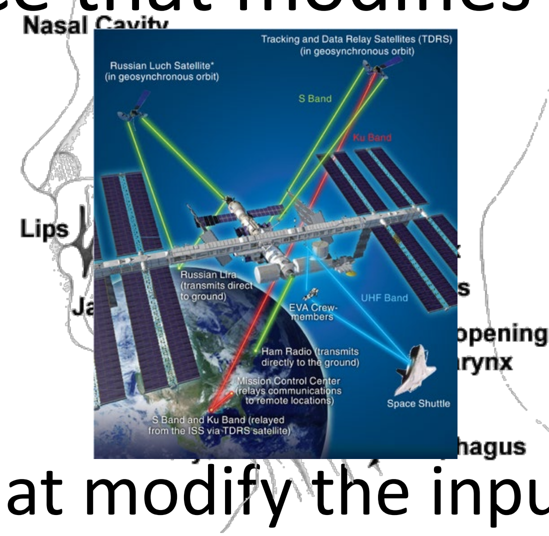
Introduction: what's a signal?

- Signal: magnitude that conveys information
  - Examples:
    - Speech signal
    - Image or photograph
    - Data about blood pressure, temperature… of a patient
    - Data about temperature, humidity, atmospheric pressure
  - Formally they are represented as a function of one or more variables



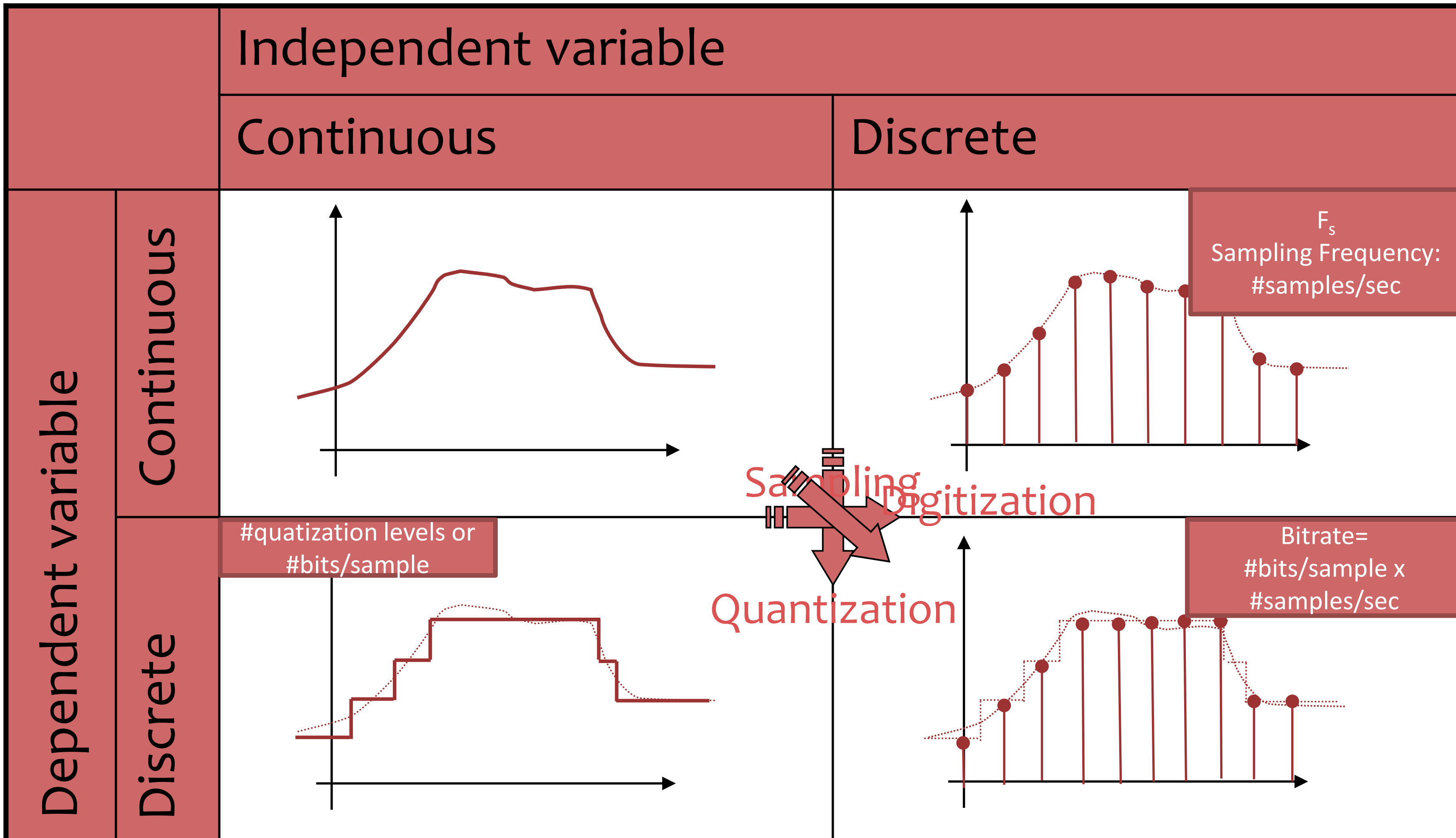
Introduction: what’s a system?

- System: any process or device that modifies a signal
  - Examples:
    - Speech production system
    - Communication system
  - Represented by transforms that modify the input signal to produce the output signal



$$x(t) \longrightarrow \boxed{T()} \longrightarrow y(t)=T(x(t))$$

Analog vs. digital signals



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Basic signals: Periodic signals

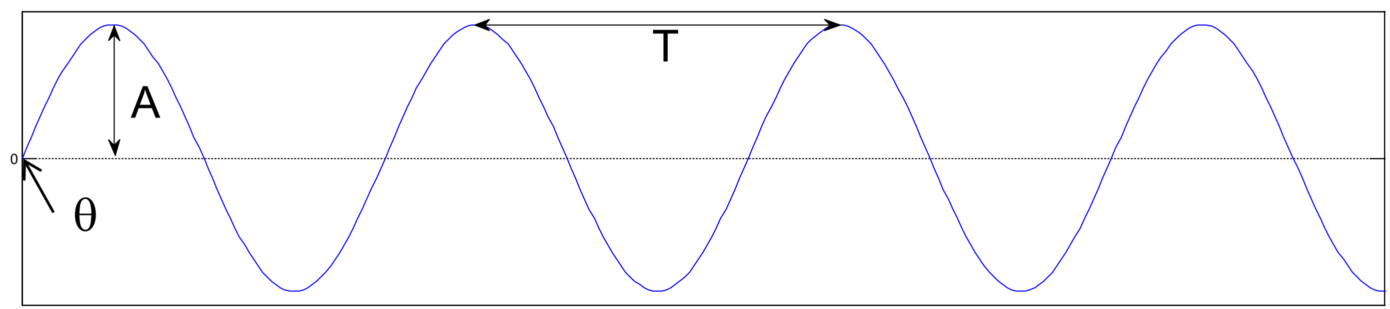
- The simplest vibration in nature is the sinusoid

$$s(t) = A\sin(2\pi f_0 t + \theta)$$

A: amplitude

θ: phase

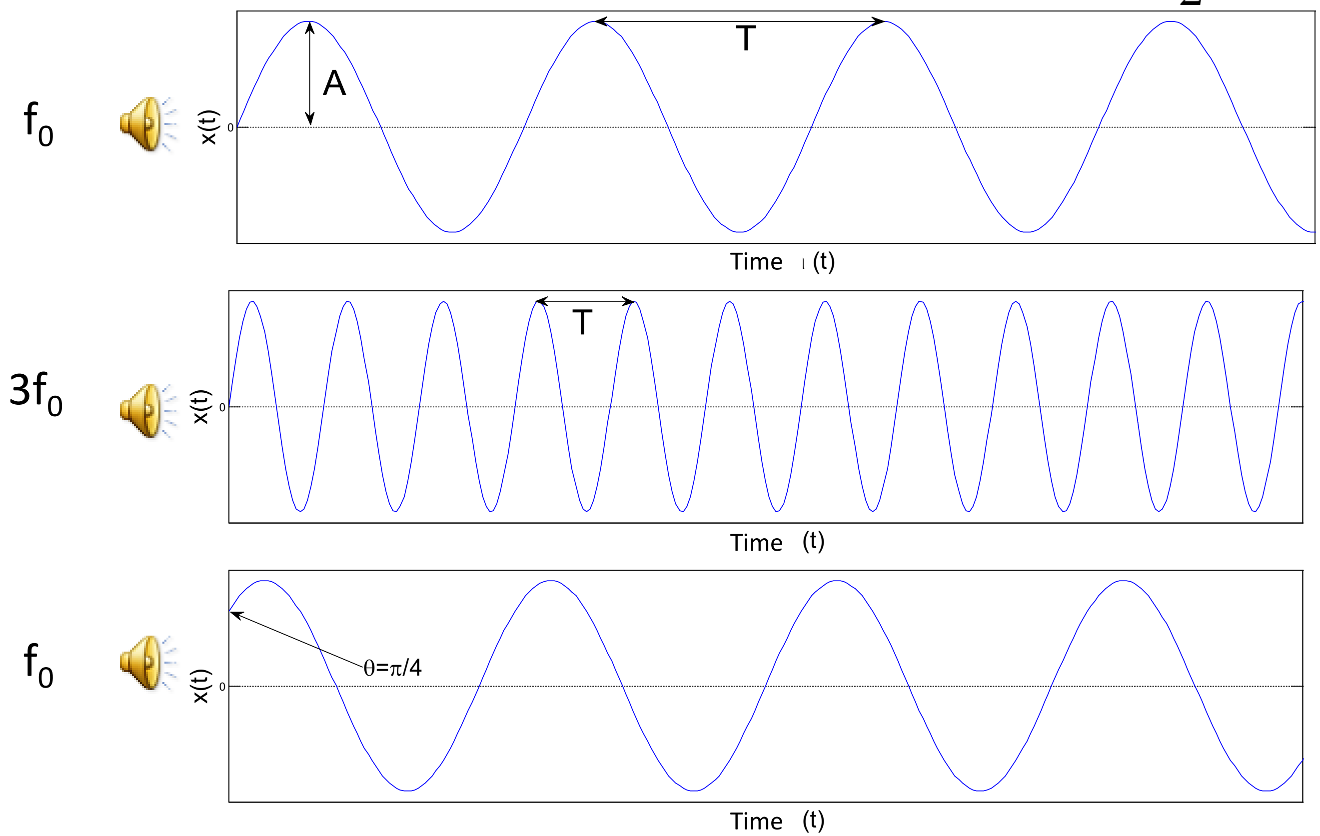
f_0 (Hz): frequency
$T=1/f_0$: period



Time (t)

Basic signals: Periodic signals

$$s(t) = A sin(2\pi f_0 t + \theta) = A cos(2\pi f_0 t + \theta - \frac{\pi}{2})$$



For simplicity, we will ignore the phase of the sinusoides: i.e. we will treat equally sinus (*sin*) and cosinus (*cos*)

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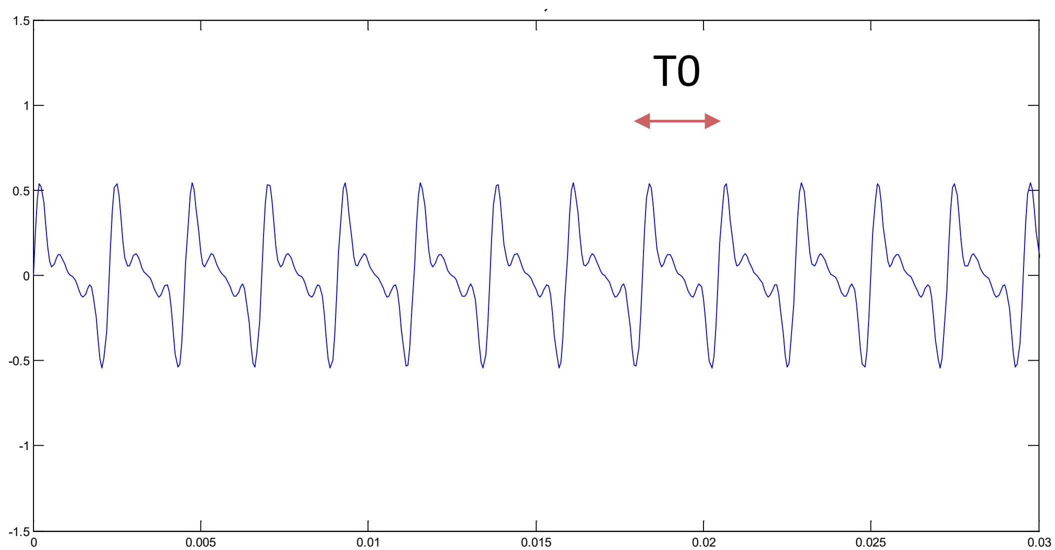
Basic signals: Periodic signals

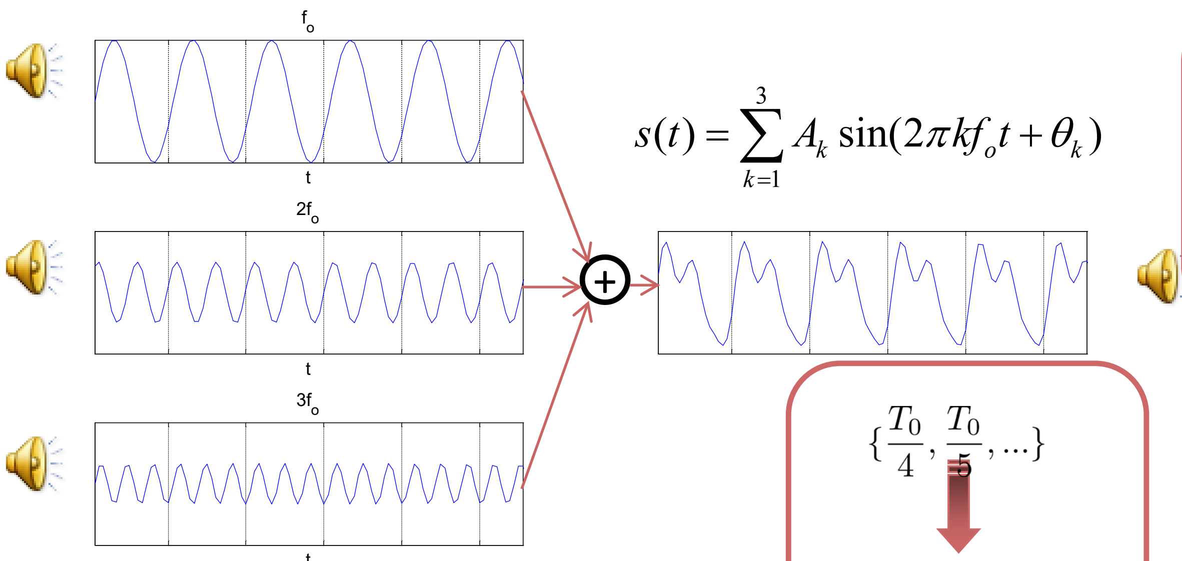
- A periodic signal is a signal that repeats its values in regular intervals or **periods**



- The minimum repetition interval is the ***fundamental period*** (measured in seconds s)
- The inverse of the fundamental period is the ***fundamental frequency***, it expresses the number of cicles (or periods) by second, and its unit is the Herz (Hz)

Basic signals: Periodic signals

If we combine sinusoids with periods multiples one to each other, a periodic signal will result, with a period the **least common multiple** of the individual periods:



$$s(t)=\sum_{k=1}^{3} A_k \sin(2\pi k f_o t+\theta_k)$$

$$\{T_0, \frac{T_0}{2}, \frac{T_0}{3}, \frac{T_0}{4}, \frac{T_0}{5}, \ldots\}$$

$\{T_0\}$

$$\{\frac{T_0}{4}, \frac{T_0}{5}, \ldots\}$$

$\{T_0\}$

$$\{\frac{T_0}{2}, \frac{T_0}{3}, \frac{T_0}{4}, \frac{T_0}{5}, \ldots\}$$

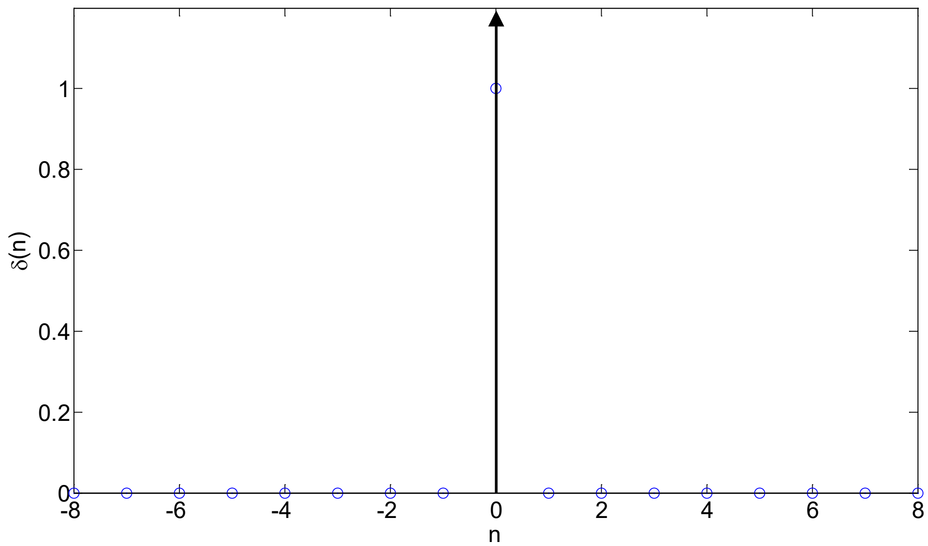
$\{T_0\}$

The **least common multiple** of the frequency components is perceived as the 'tone' of the signal *(Missing fundamental effect)*

Basic signals: The unit impulse signal

- Discrete time unit impulse or delta
  - 0 where n≠0

$$\delta[n]=\begin{cases}1 & n=0\\ 0 & rest\end{cases}$$



Basic signals: The unit impulse signal

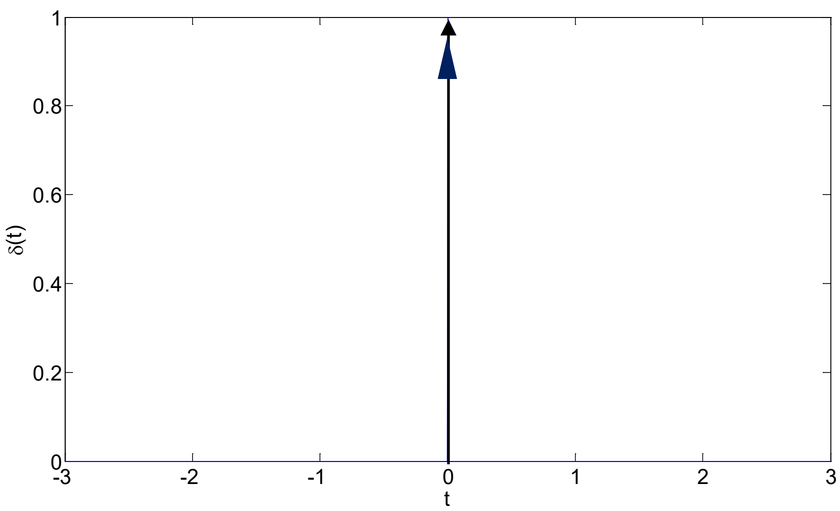
- Continuous time unit impulse or delta
  - 0 where t≠0
  - Area 1
  - Possible mathematical definition

$$\delta(t)=\begin{cases}\infty & t=0\\ 0 & rest\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)dt=1$$

$$\delta(t)=\lim_{T\to 0}\frac{1}{T}\prod\left(\frac{t}{T}\right)$$

  - Represented by an arrow in t=0



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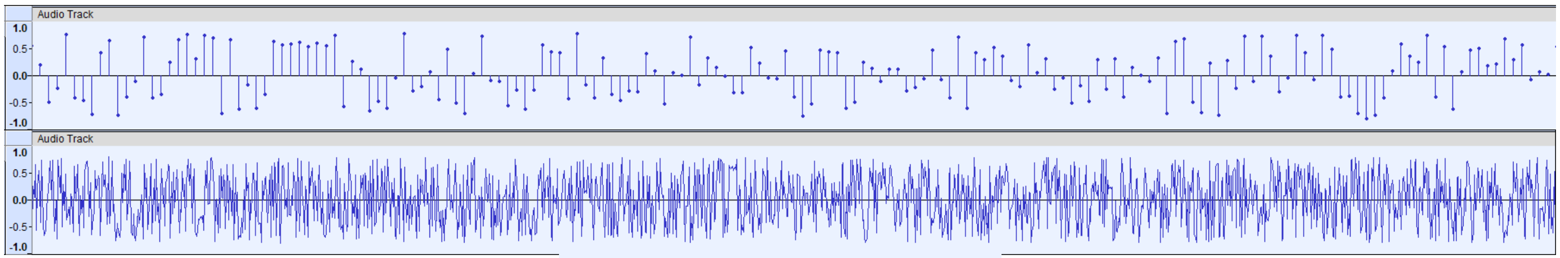
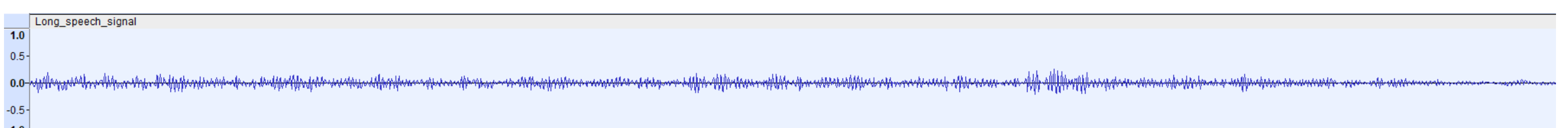
Basic signals: noise

- Noise is a signal with a random nature whose values can not be exactly determined with a formula



Segment of white gaussian noise

- It can be statistically modeled: gaussian noise is the most used model in speech.
- Noises differ also on their frequency components
- In speech modeling:
  - The speech signal for unvoiced sounds is modeled as noise
  - The glottal signal is usually taken as ‘White Gaussian Noise’ (WGN)

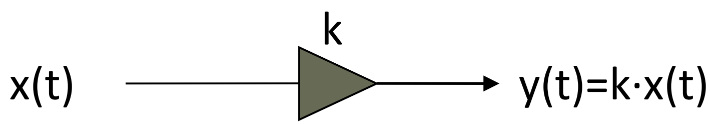


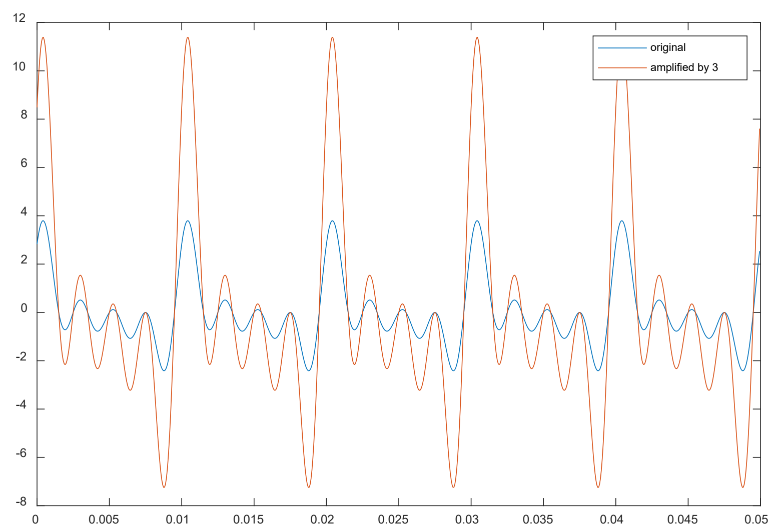
Segment of an unvoiced sound

Basic operations with signals

Amplification

$x(t) \rightarrow k\cdot x(t)$ $\qquad$ $x[n] \rightarrow k\cdot x[n]$



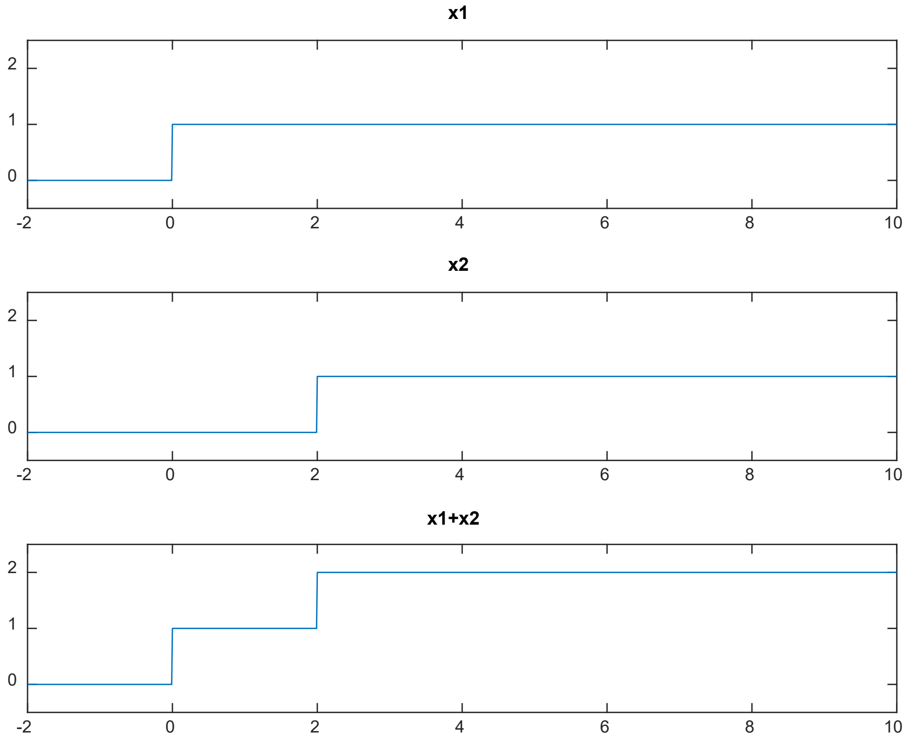


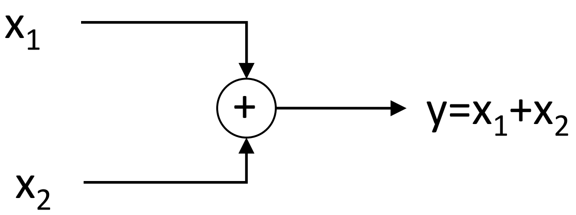
Basic operations with signals

Sum

$x_1(t),x_2(t) \Rightarrow x_1(t)+x_2(t)$

$x_1[n],x_2[n] \Rightarrow x_1[n]+x_2[n]$





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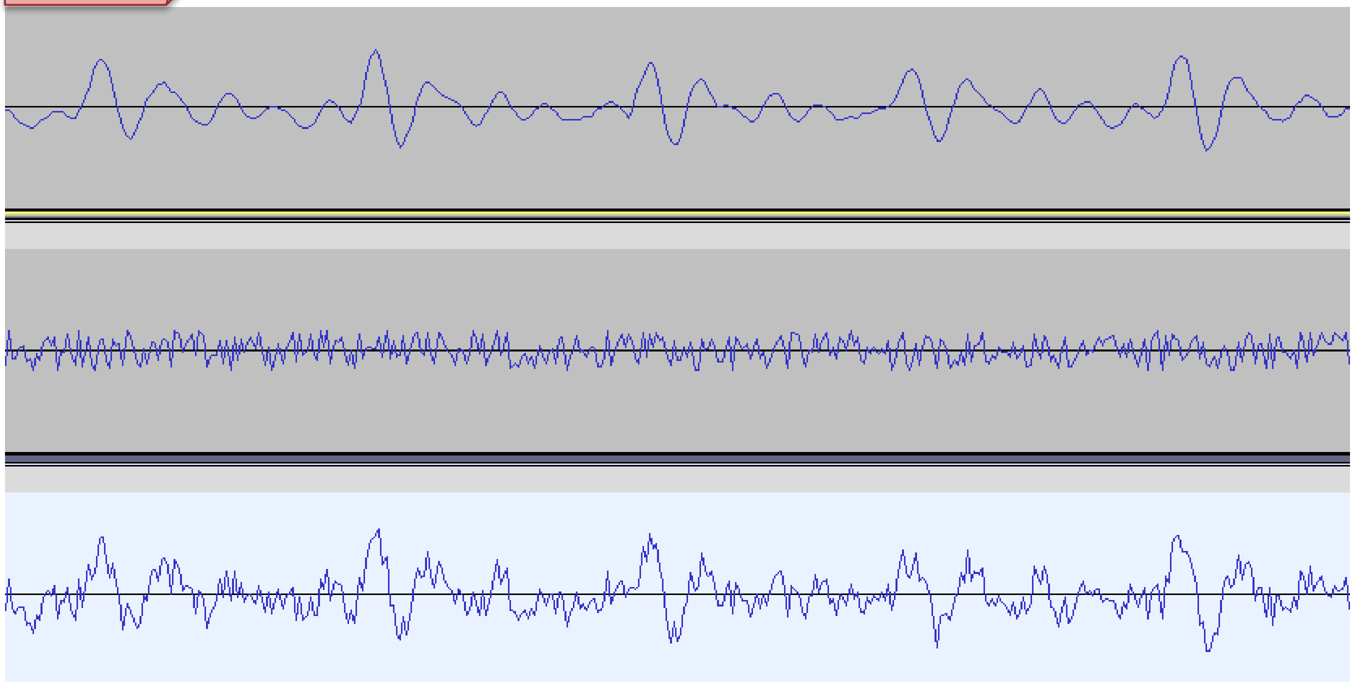
Basic operations with signals

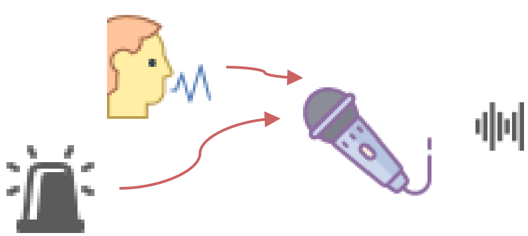
Sum



s(t) *signal*

n(t) *noise*

$s_n(t)=s(t)+n(t)$
Noisy signal



Addition occurs very frequently:

- Noise or other undesired interferences are added to the signals of interest
- Reflections of the main signal are added generating echoes and reverberation
- Signals from stereo or quadraphonic equipment are added before arriving to our ears…

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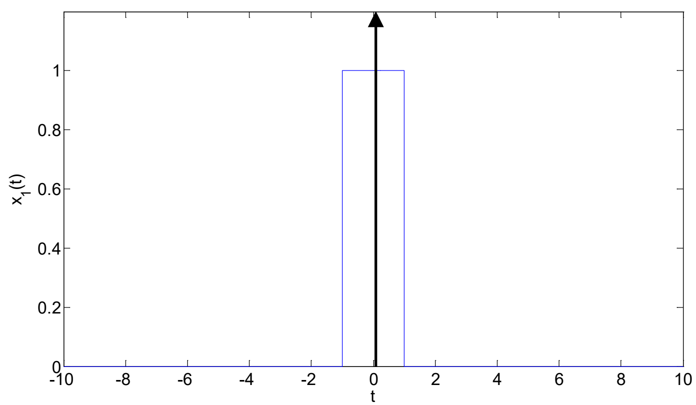
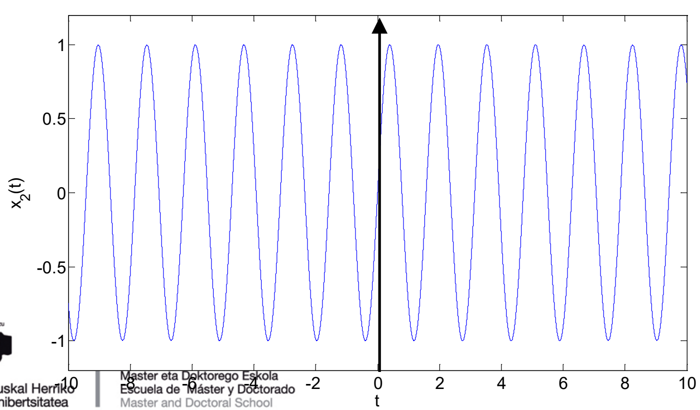
Basic operations with signals

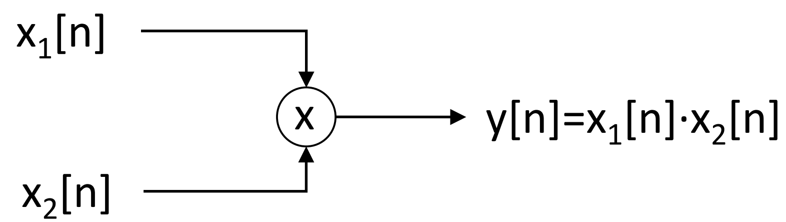
Product

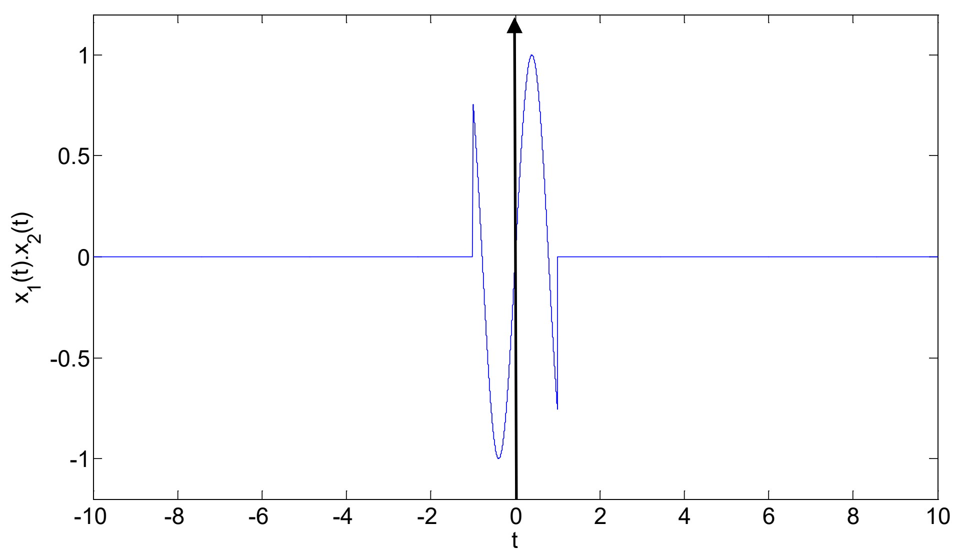
$x_1(t),x_2(t) \longrightarrow x_1(t)\cdot x_2(t)$

$x_1[n],x_2[n] \longrightarrow x_1[n]\cdot x_2[n]$





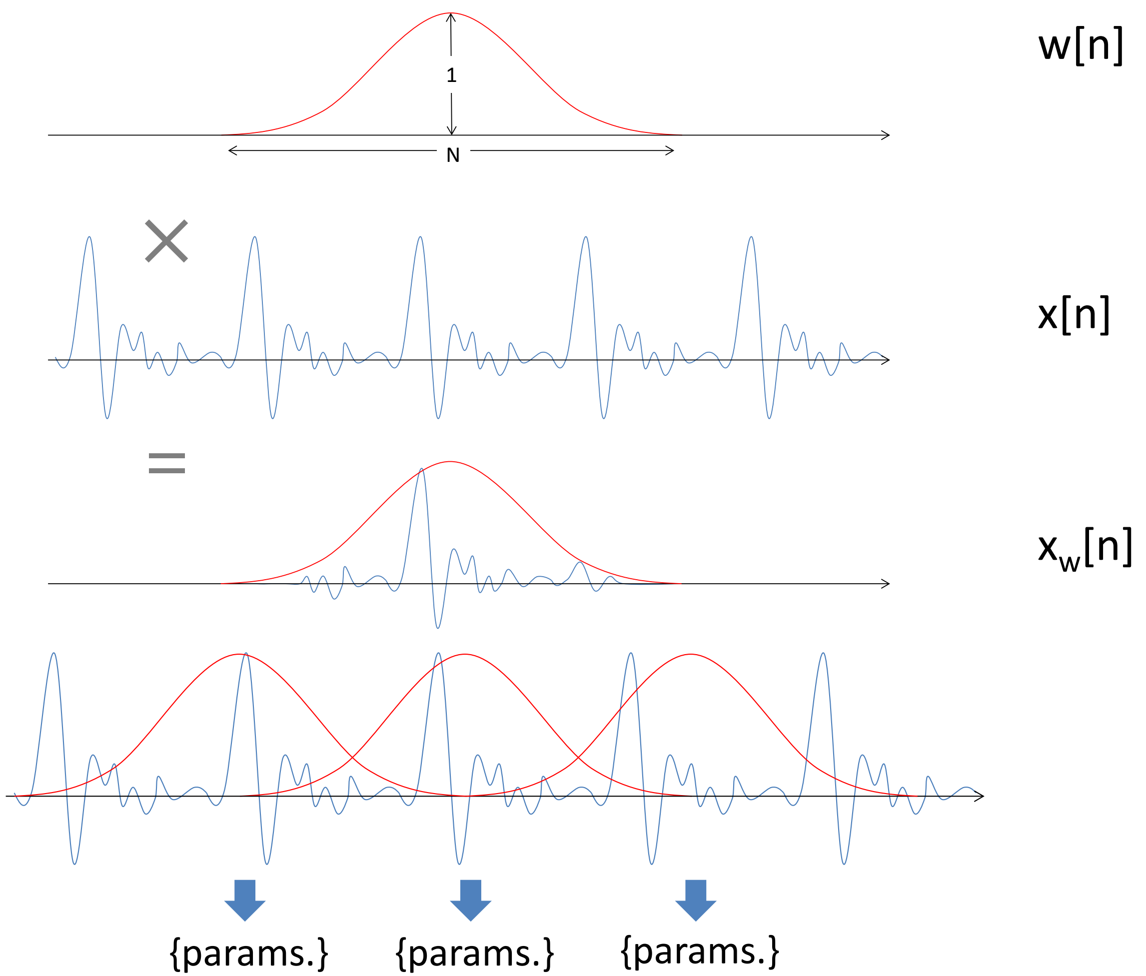




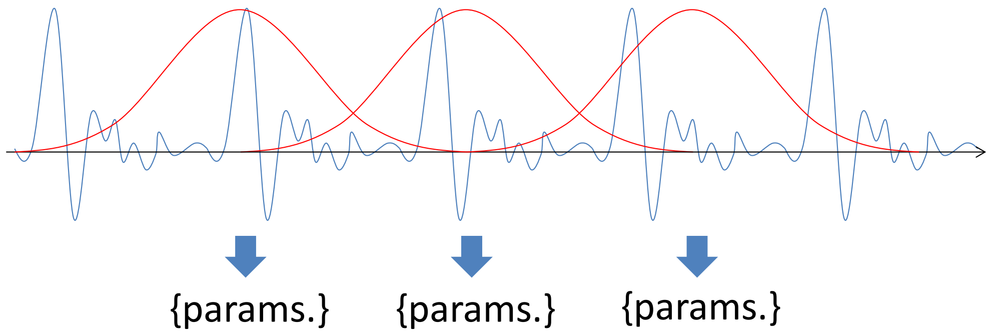
Basic operations with signals

Product

Windowing: the product of a signal and a *window* to select a short segment from the signal

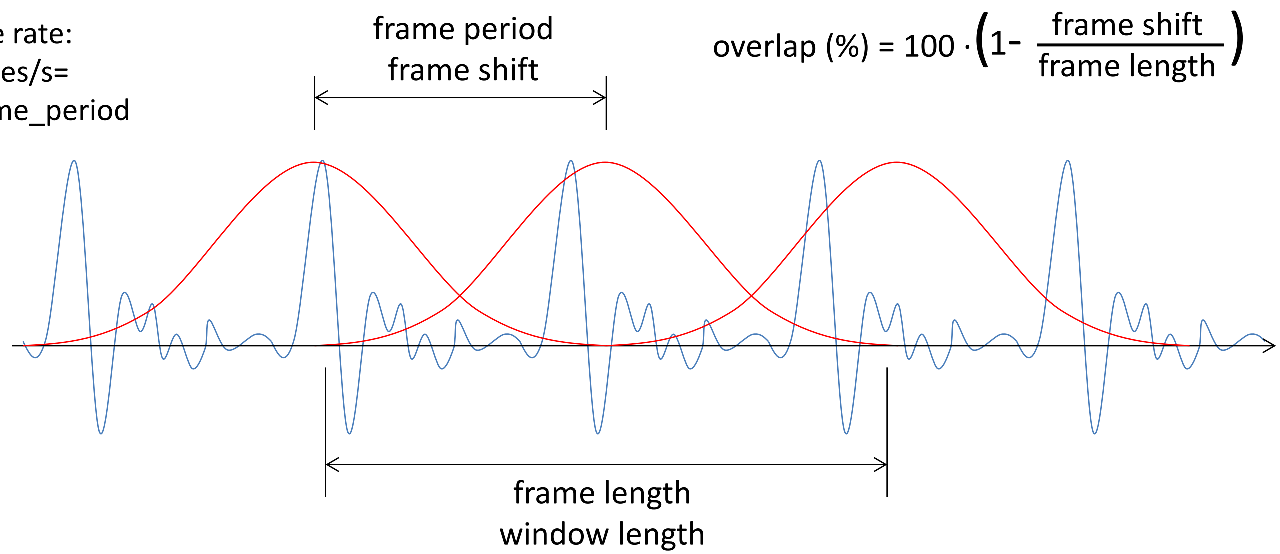


Basic operations with signals



Frame rate:
#frames/s=
1/frame_period

$$\text{overlap (\%)} = 100 \cdot \left(1 - \frac{\text{frame shift}}{\text{frame length}}\right)$$



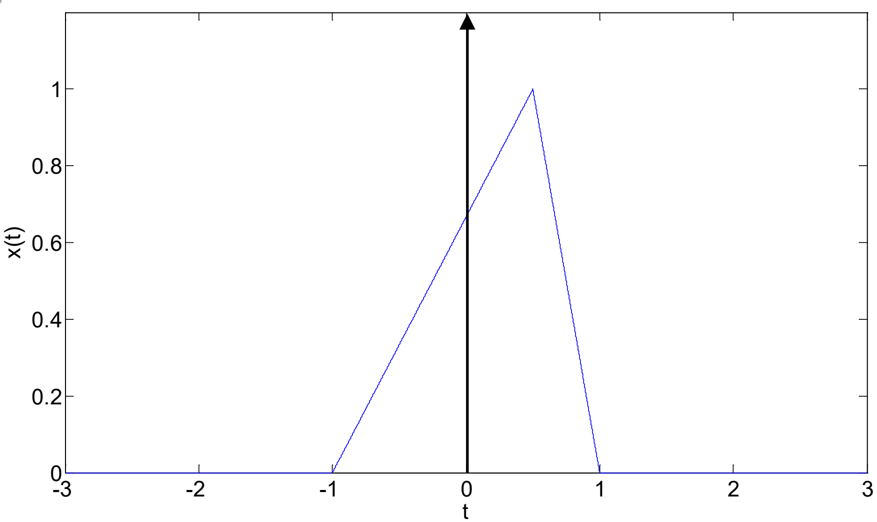
Segmental or ‘short-term’ analysis

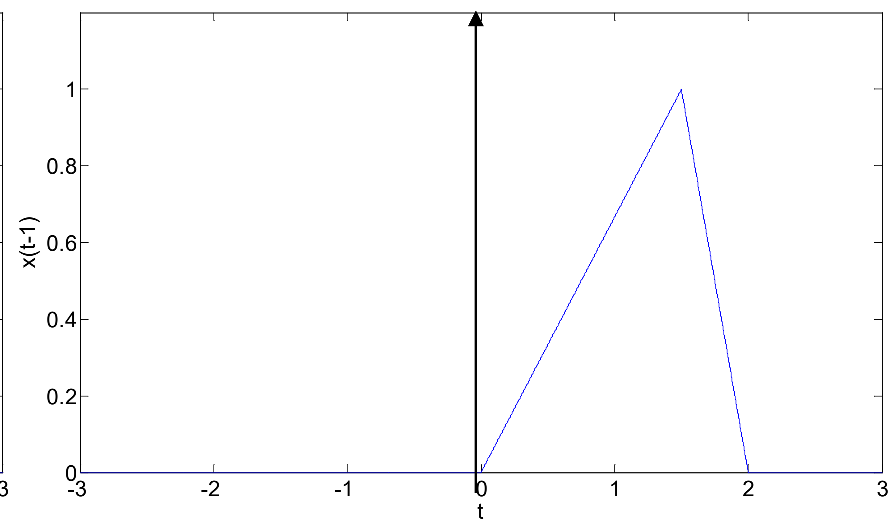
Basic operations with signals

Time-shift

$x(t) \rightarrow x(t-t_0)$ $x[n] \rightarrow x[n-n_0]$

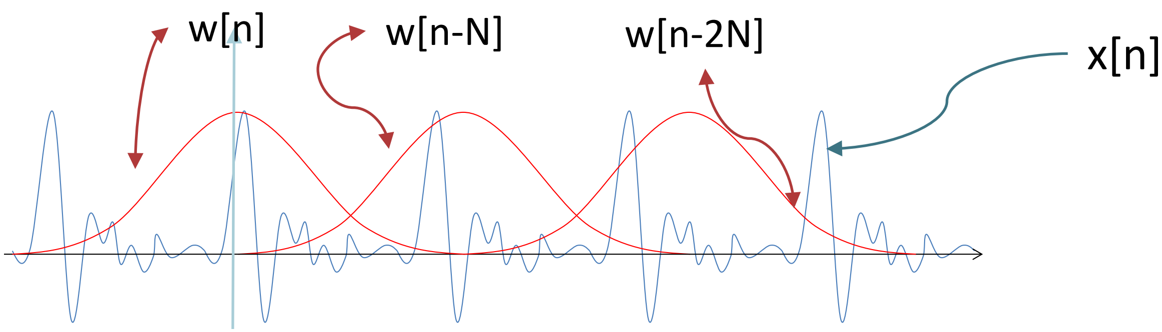
$t_0, n_0 > 0$ delays the signal $t_0, n_0 < 0$ advance the signal

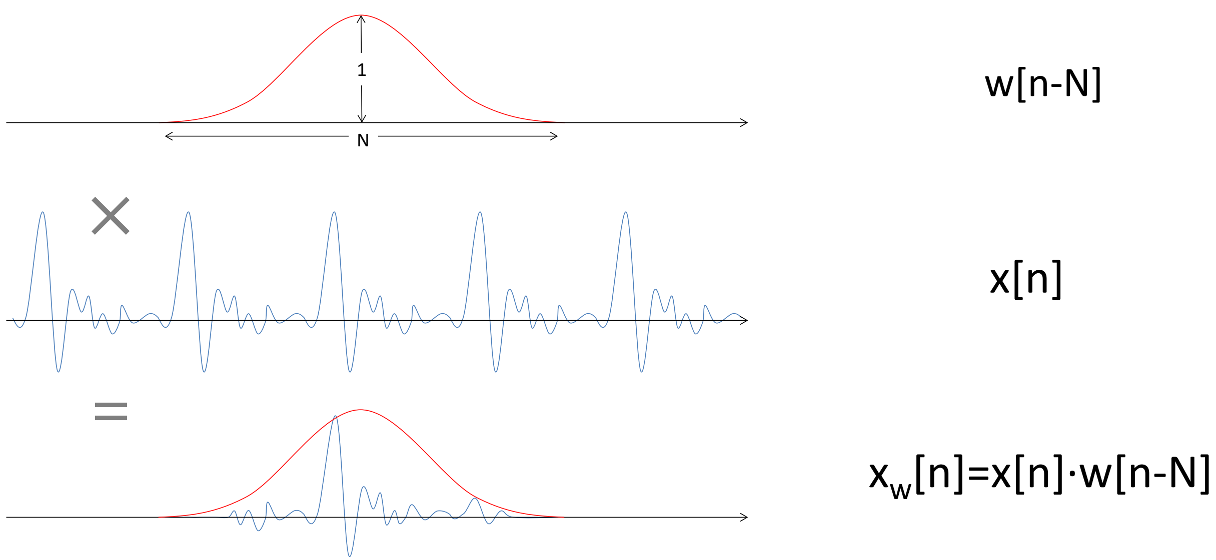




Basic operations with signals

Time-shift



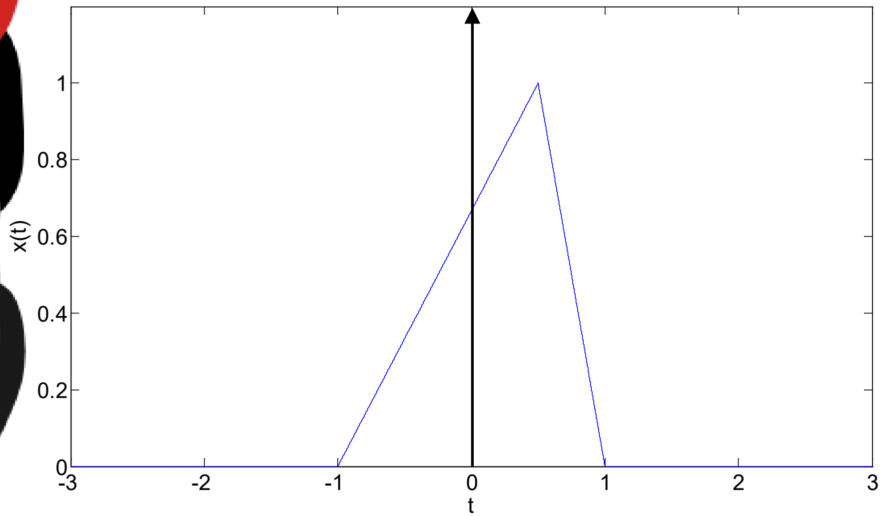


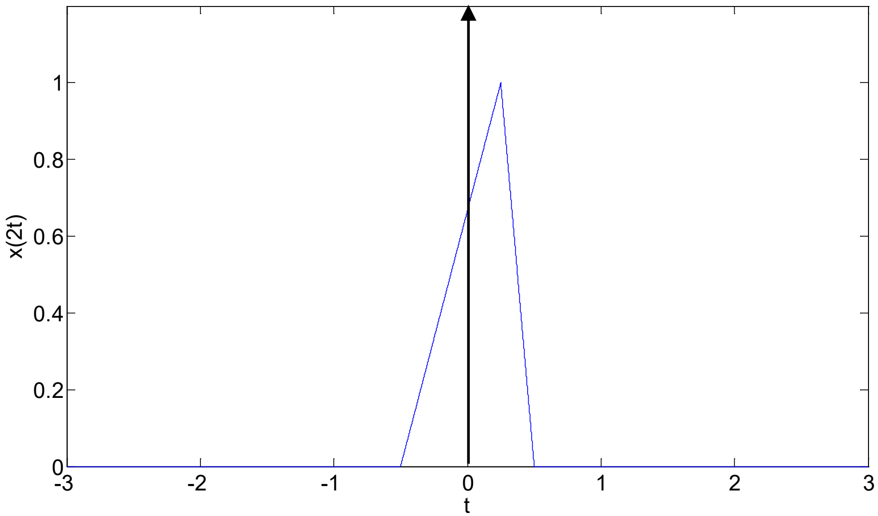
Basic operations with signals

- Time scaling

$$x(t) \longrightarrow x(at) \qquad x[n] \longrightarrow x[an]$$

a>1 compresses the signal (in discrete time samples must be removed)





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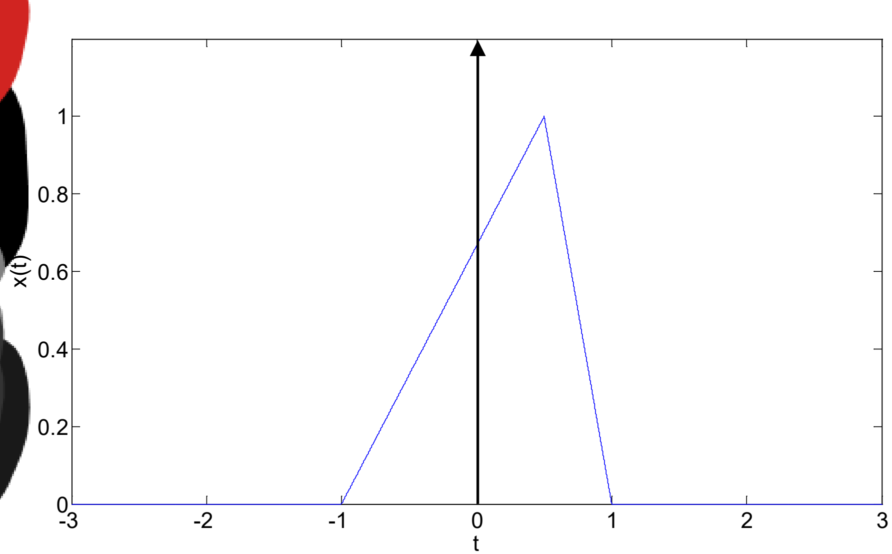
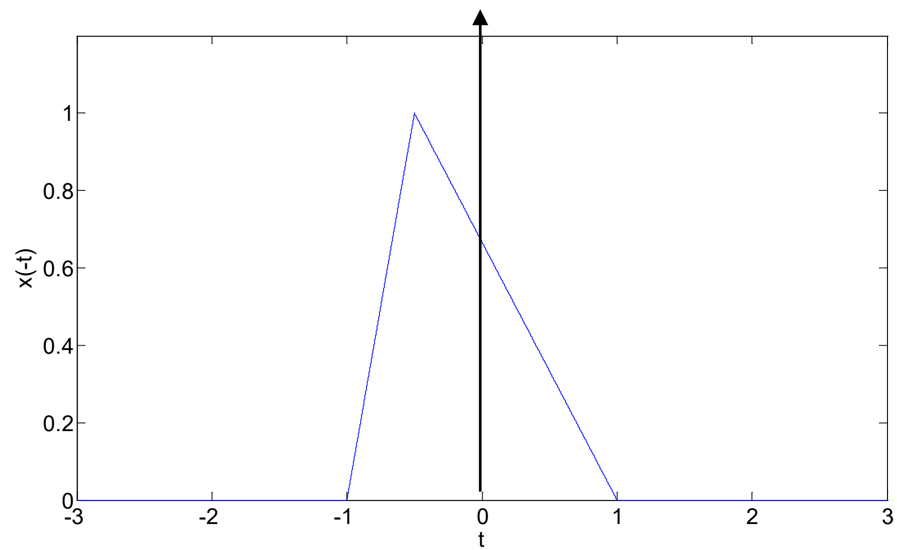
Basic operations with signals

- Time inversion

$x(t) \longrightarrow x(-t)$ $\qquad$ $x[n] \longrightarrow x[-n]$





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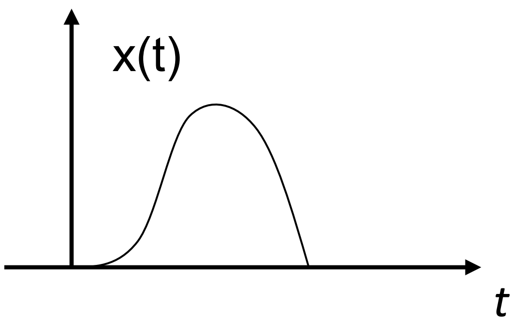
Basic operations with signals

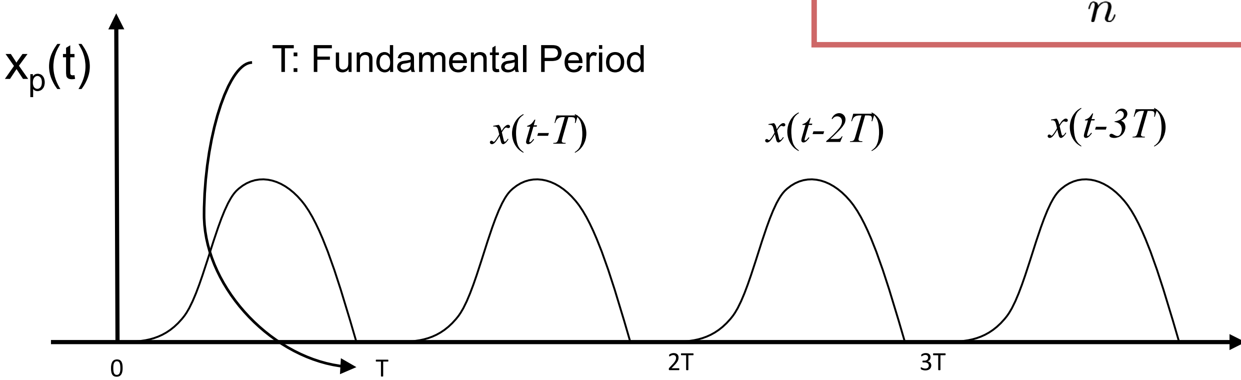
Periodic extension

We can build a periodic signal by repeating a non-periodic signal:



Periodic extension of x(t):

$$x_p(t) = \sum_n x(t - nT)$$



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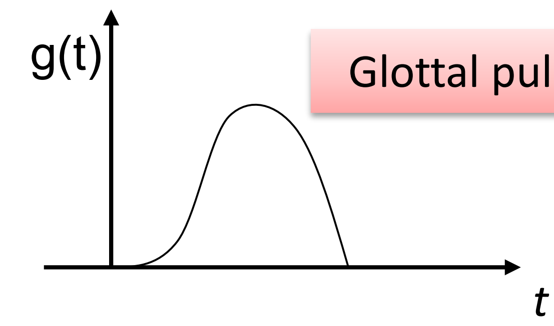
Basic operations with signals

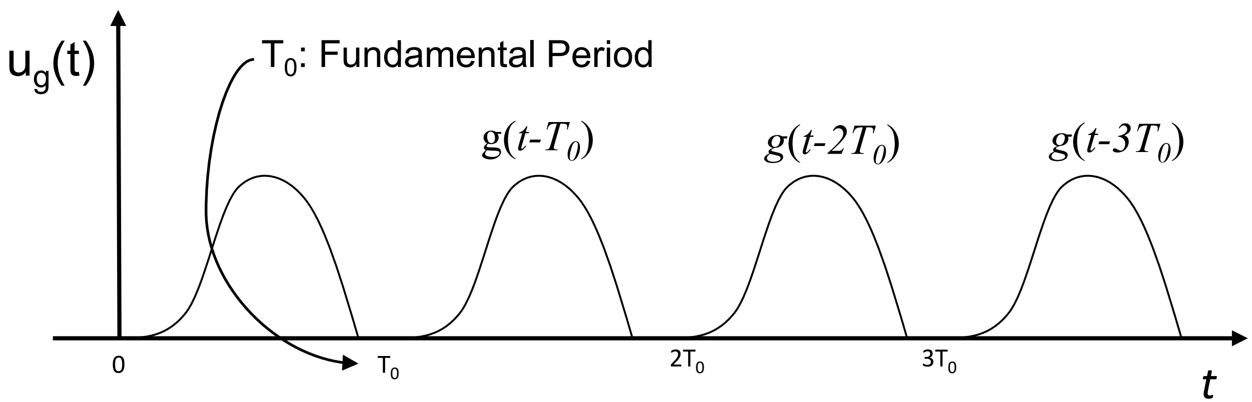
Periodic extension example: glottal signal

Glottal pulse *model*



Gottal signal:

$$u_G(t) = \sum_n g(t - nT_0)$$

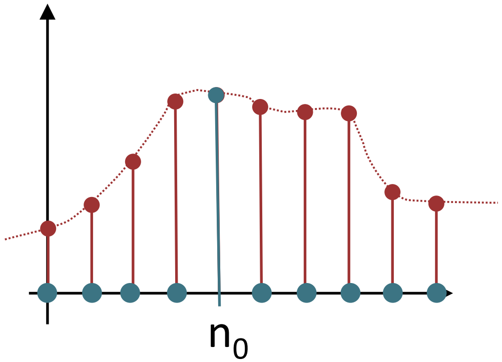


Basic operations with signals

- Properties
  - Sampling

$$x[n]\delta[n]=x[0]\delta[n]$$

$$x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$$



  - Expressing any signal as a combination of unit impulses

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\delta[n-k]$$

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Basic operations with signals

- Properties
  - Sampling

    $x(t)\delta(t)=x(0)\delta(t)$ $\quad$ $x[n]\delta[n]=x[0]\delta[n]$

    $x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$ $\quad$ $x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$

  - Expressing any signal as a combination of unit impulses

$$x(t)=\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \qquad x[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

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Fourier transform for continuous time signals

- Why treat signals and systems in frequency? Easier to be analyzed

$$X(f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

Fourier transform of x(t)
Analysis equation

$$x(t) \xrightarrow{F} X(f)$$

Time (s) Frequency (Hz)

$$x(t) = F^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

Inverse Fourier transform of X(f)
Synthesis equation

$$x(t) \xleftarrow{F^{-1}} X(f)$$

Time (s) Frequency (Hz)

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Fourier Transform for continuous time signals

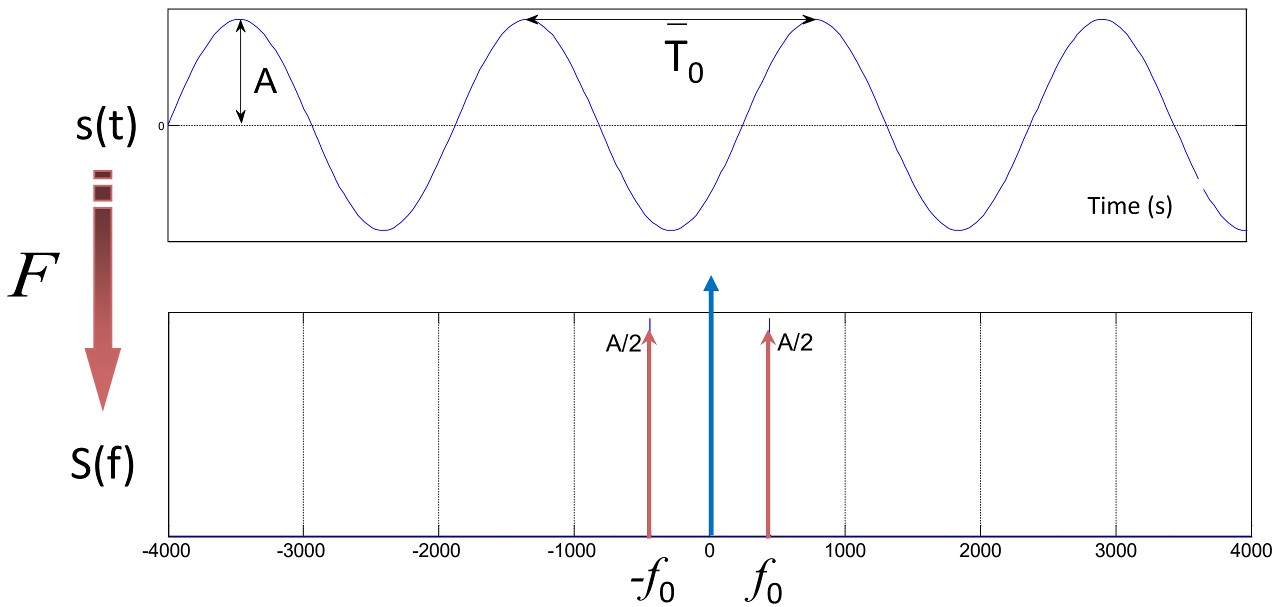
The Fourier Transform of a signal $x(t)$:

- Is a function of ***frequency*** (Hz): $X(f)$
- For each value of f, it measures the degree of similarity of $x(t)$ with a pure tone of frequency f
- It can be inverted to recover $x(t)$
- The module squared $|X(f)|^2$ is a measure of the amount of energy in the signal at each frequency f.
- The module $|X(f)|$ is called the **Spectrum** of $x(t)$.

Fourier transform for continuous time signals

FT of a sinusoidal signal (only module):

$$s(t) = Acos(2\pi f_0 t)$$

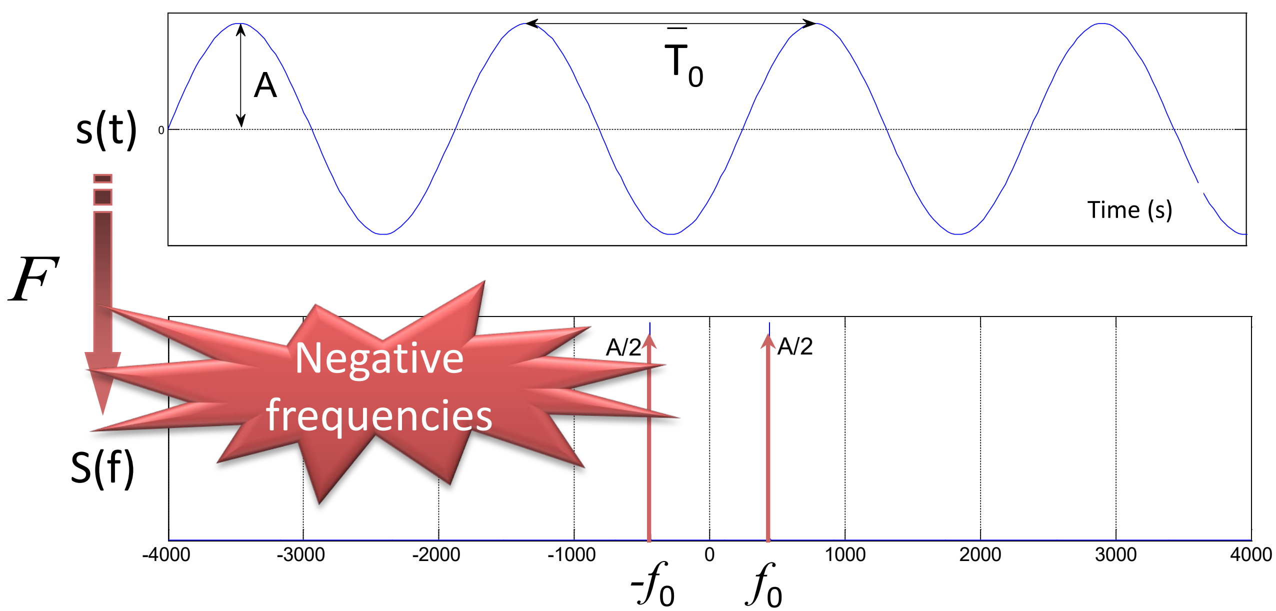


$$|X(f)| = \frac{A}{2}\{\delta(f - f_0) + \delta(f + f_0)\}$$

Fourier transform for continuous time signals

FT of a sinusoidal signal (only module):

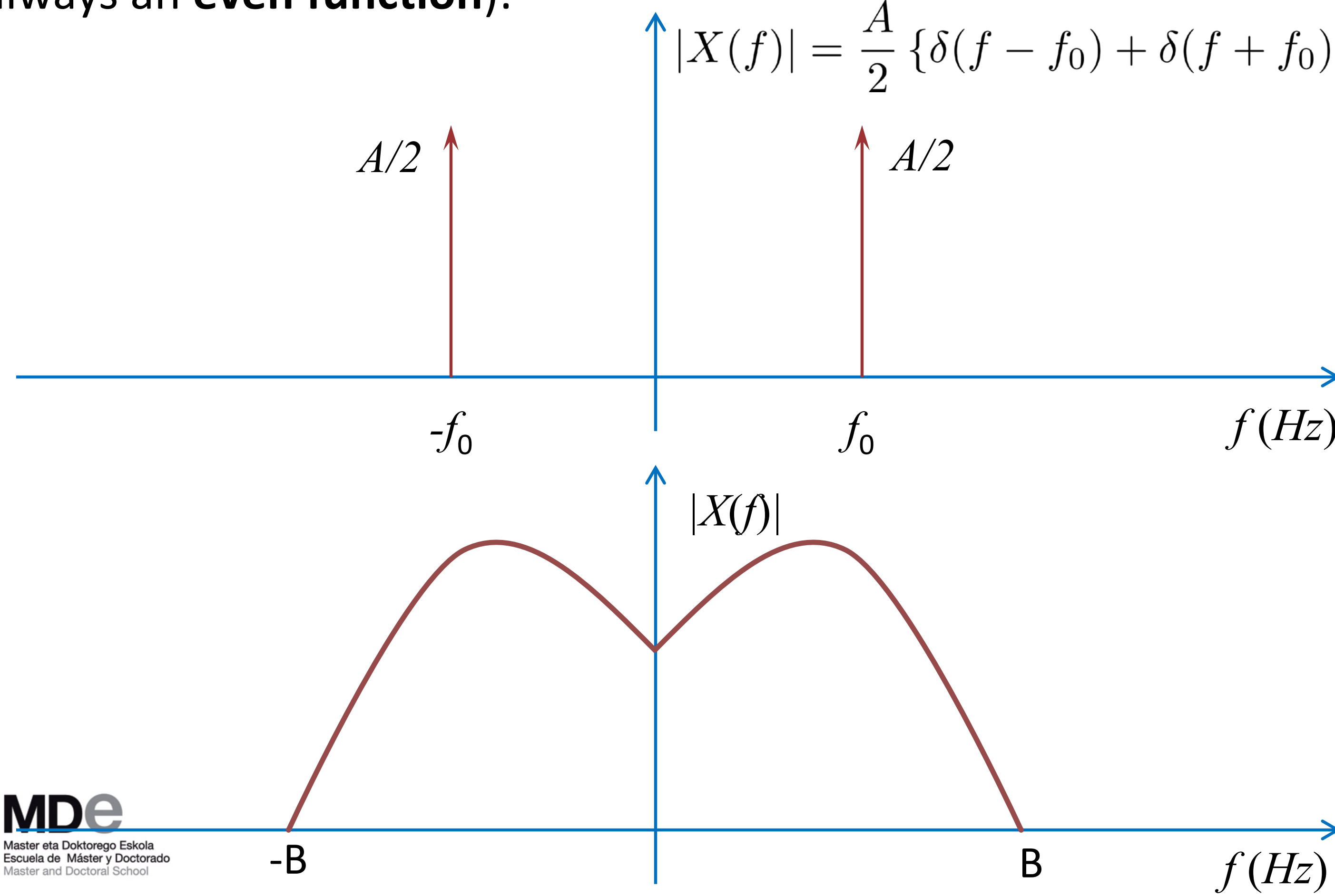
$$s(t) = A cos(2\pi f_0 t)$$



$$|S(f)| = \frac{A}{2}\{\delta(f - f_0) + \delta(f + f_0)\}$$

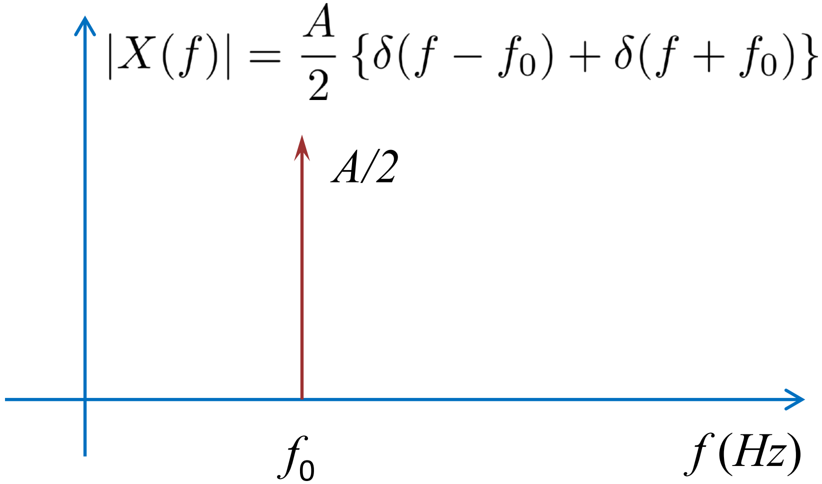
Fourier transform for continuous time signals

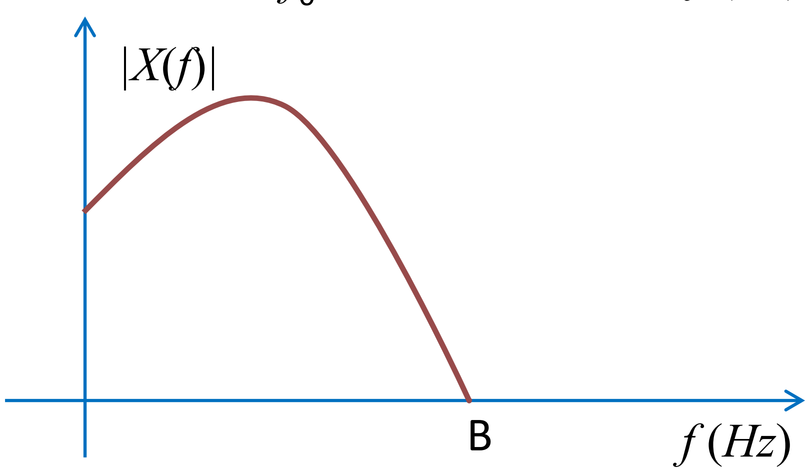
- In the Fourier Transform of a real signal there will always be components at negative and positive frequencies, with **reflected symmetry** (i.e., it is always an **even function**):



Fourier transform for continuous time signals

- And then we usually **only draw** the positive frequencies:





Fourier transform for continuous time signals: Linearity of the Fourier transform

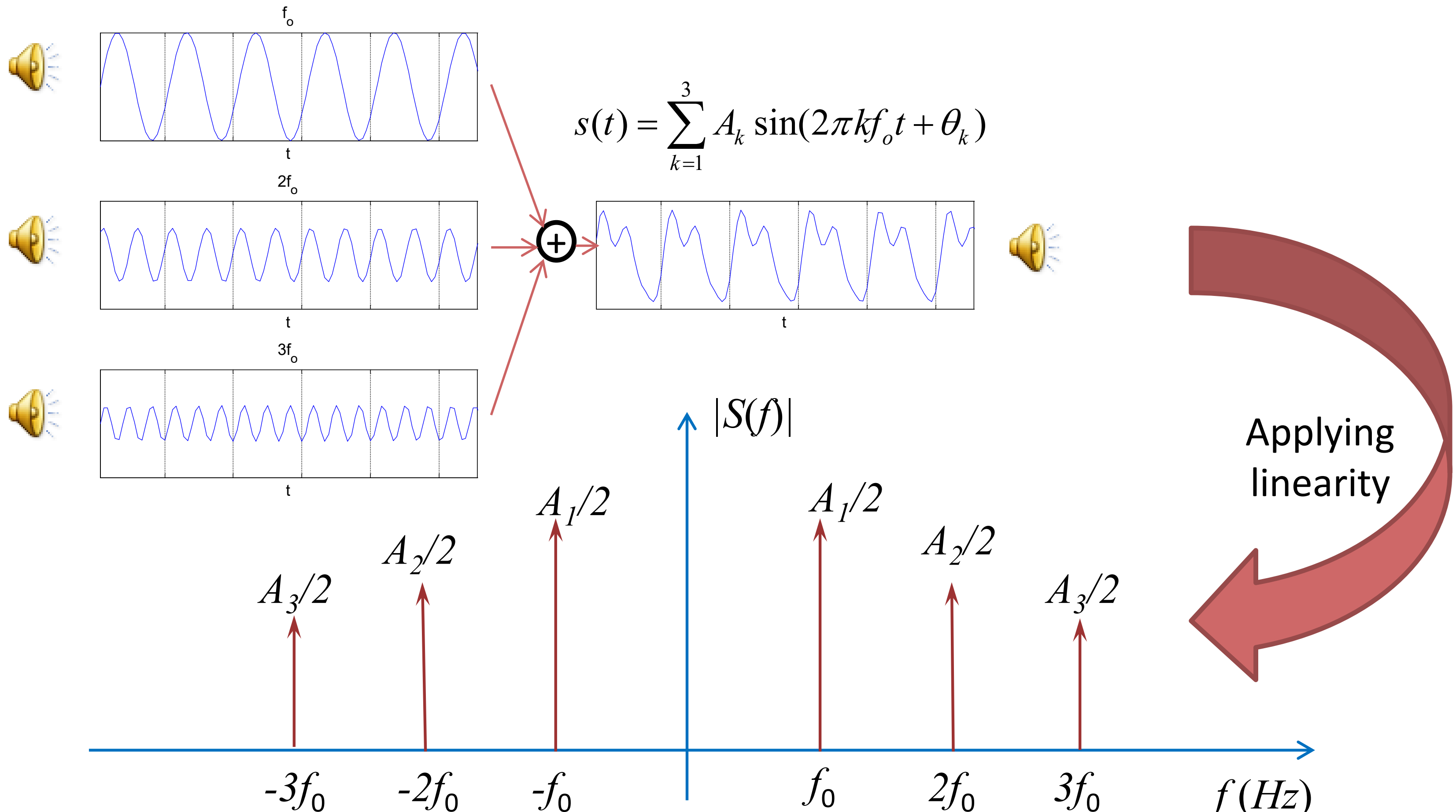
- Linearity

X(t)	X(f)
$k\ x(t)$	$k\ X(f)$
$x_1(t) + x_2(t)$	$X_1(f)+X_2(f)$
$k_1\ x_1(t) + k_2\ x_2(t)$	$k_1\ X_1(f)+ k_2\ X_2(f)$

- *The Fourier transform of an scaled version of a signal is the scaled version of the FT of the signal.*
- *The Fourier transform of a sum of two (scaled) signals is the sum of the (scaled) Fourier Transform of each signal.*

Fourier transform for continuous time signals: FT of multi-tone signals

$$s(t) = \sum_{k=1}^{3} A_k \sin(2\pi k f_o t + \theta_k)$$



$$|S(f)| = \frac{A_1}{2}\{\delta(f-f_0)+\delta(f+f_0)\} + \frac{A_2}{2}\{\delta(f-2f_0)+\delta(f+2f_0)\} + \frac{A_3}{2}\{\delta(f-3f_0)+\delta(f+3f_0)\}$$

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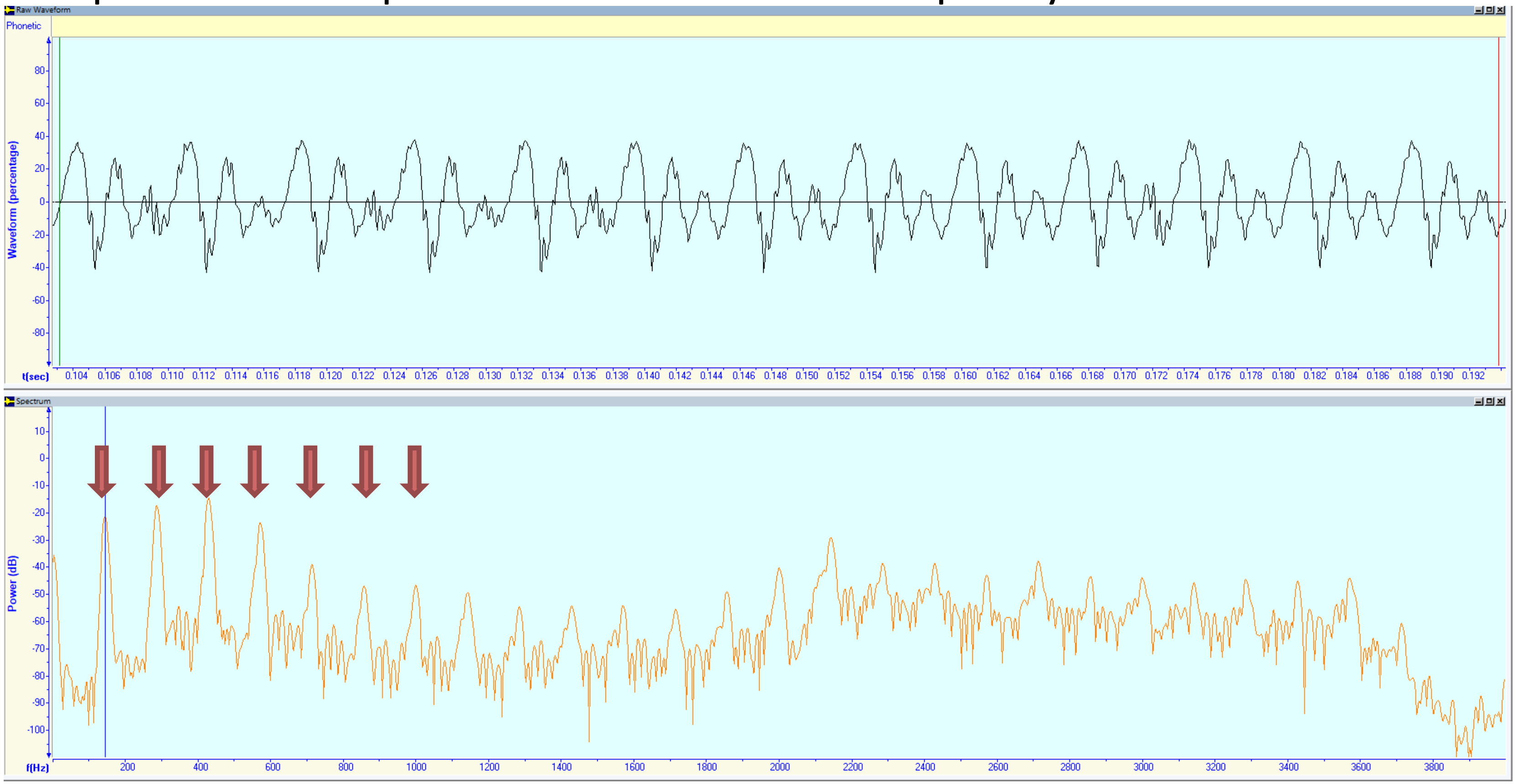
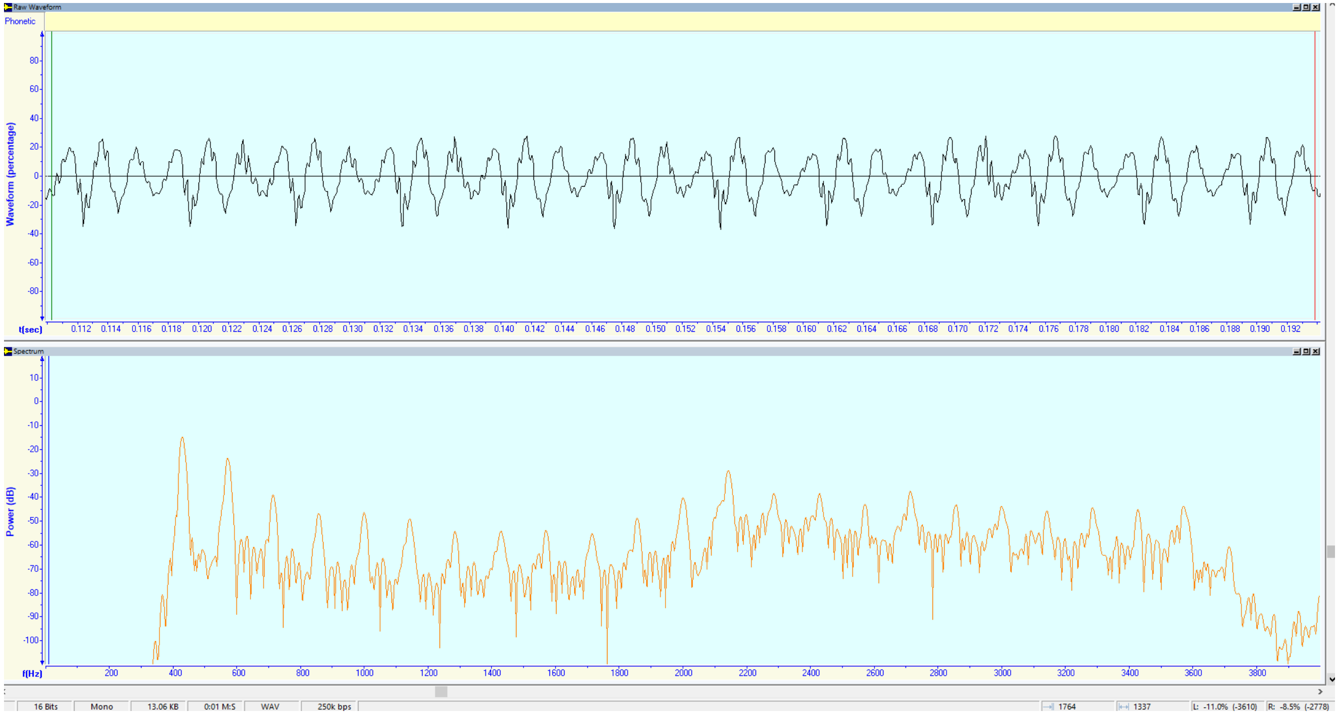
Fourier transform for continuous time signals: FT of periodic signals

- The Fourier Transform (***spectrum***) of a periodic signal will always show impulses at multiples of the fundamental frequency:



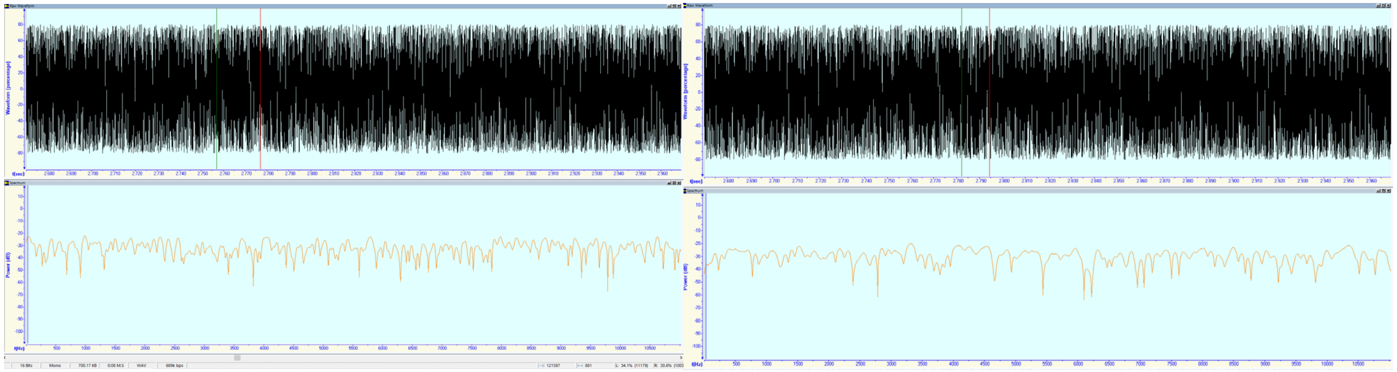
Fourier transform for continuous time signals: FT of periodic signals

- The fundamental F0 is not always present in the periodic speech signal.
- Equally, not all the harmonics are always present in the periodic speech signal.
- The fundamental frequency can be measured in the speech signal as the distance between two consecutive harmonics.



Fourier transform for continuous time signals: FT of noise

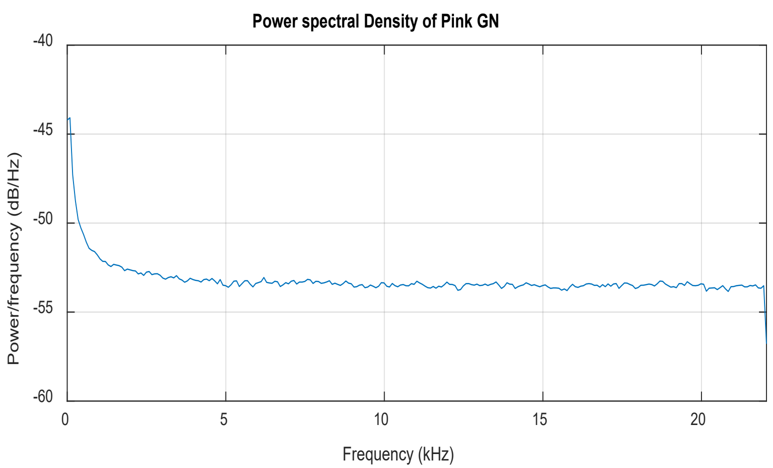
- Being noise of random nature, its FT will strongly depend on the selected segment
- It will also look very random

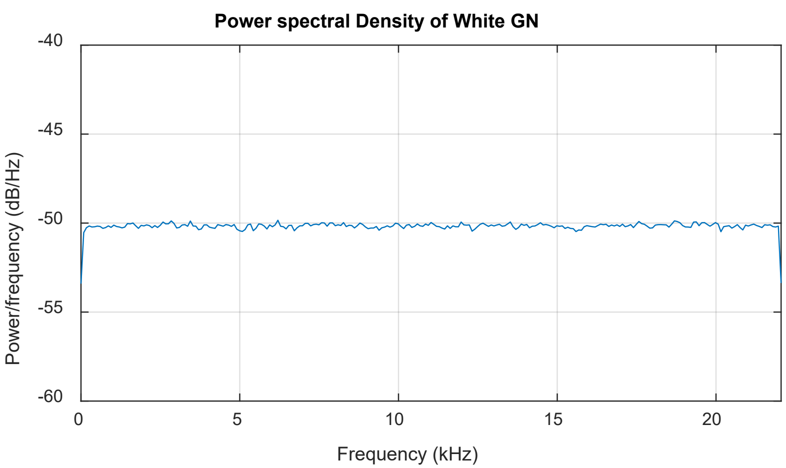


FT of two different frames of the same wgn signal

Fourier transform for continuous time signals: FT of noise & PSD

- **Power Specral Density**: the spectrums calculated from consecutive frames are averaged to obtain a better estimation of the spectral content of a random signal.





Recommended lectures

- Signals and systems: a Matlab integrated approach

Oktay Alkin, 2014 CRC Press, ISBN: 978-1-4665-9853-9

http://www.signalsandsystems.org/