

```

In[237]:= SetDirectory[
  "/Volumes/MicroSD/2_PostDoc_SD/CaCO3/Common_Tangent-Mathematica/TEST/
  Files_Outputs"]
oneDat = Import["vol_Free_allTemps_ordered_I.dat"];
twoDat = Import["vol_Free_allTemps_ordered_II.dat"];
temps = Flatten@Import["temperatures.dat"]

Out[237]= /Volumes/MicroSD/2_PostDoc_SD/CaCO3/Common_Tangent-Mathematica/TEST/
  Files_Outputs

Out[240]= {10., 30.1, 50.2, 70.3, 90.4, 110.51, 130.61, 150.71, 170.81, 190.91, 211.01,
  231.11, 251.21, 271.31, 291.41, 311.52, 331.62, 351.72, 371.82, 391.92, 412.02,
  432.12, 452.22, 472.32, 492.42, 512.53, 532.63, 552.73, 572.83, 592.93}

(*30 temperatures. Each 11 for I*)

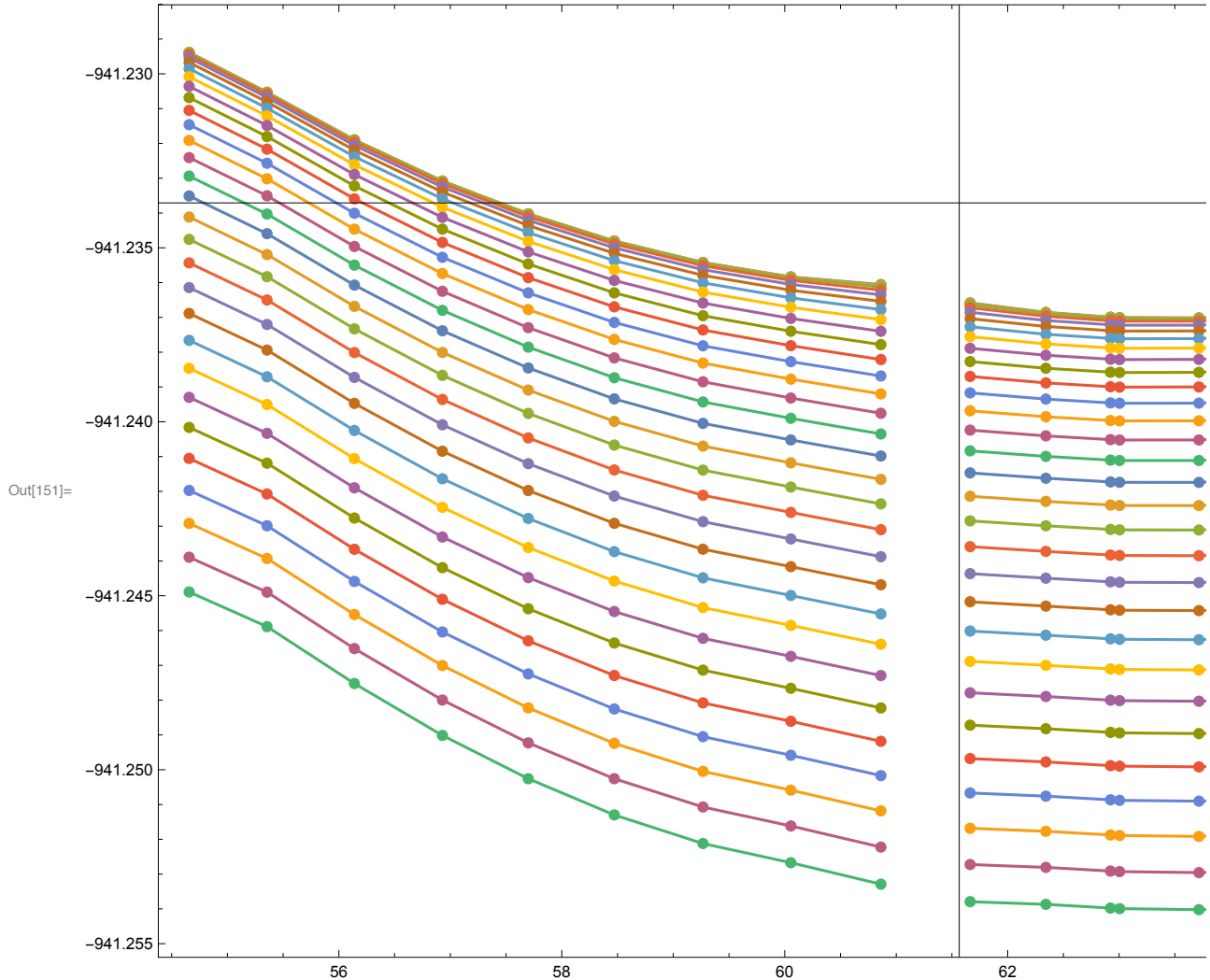
In[116]:= oneDataParts = Table[Partition[oneDat, 11][[i]], {i, 1, 30}];
twoDataParts = Table[Partition[twoDat, 9][[i]], {i, 1, 30}];

```

```

In[151]:= With[{p = 30}, Show[{ListLinePlot[Evaluate@Table[oneDataParts[[i]], {i, 1, p}],
    PlotLegends → temps], ListPlot[Evaluate@Table[oneDataParts[[i]], {i, 1, p}]],
    ListLinePlot[Evaluate@Table[twoDataParts[[i]], {i, 1, p}]],
    ListPlot[Evaluate@Table[twoDataParts[[i]], {i, 1, p}]]],
    PlotRange → All, Frame → True, ImageSize → 900]

```



```

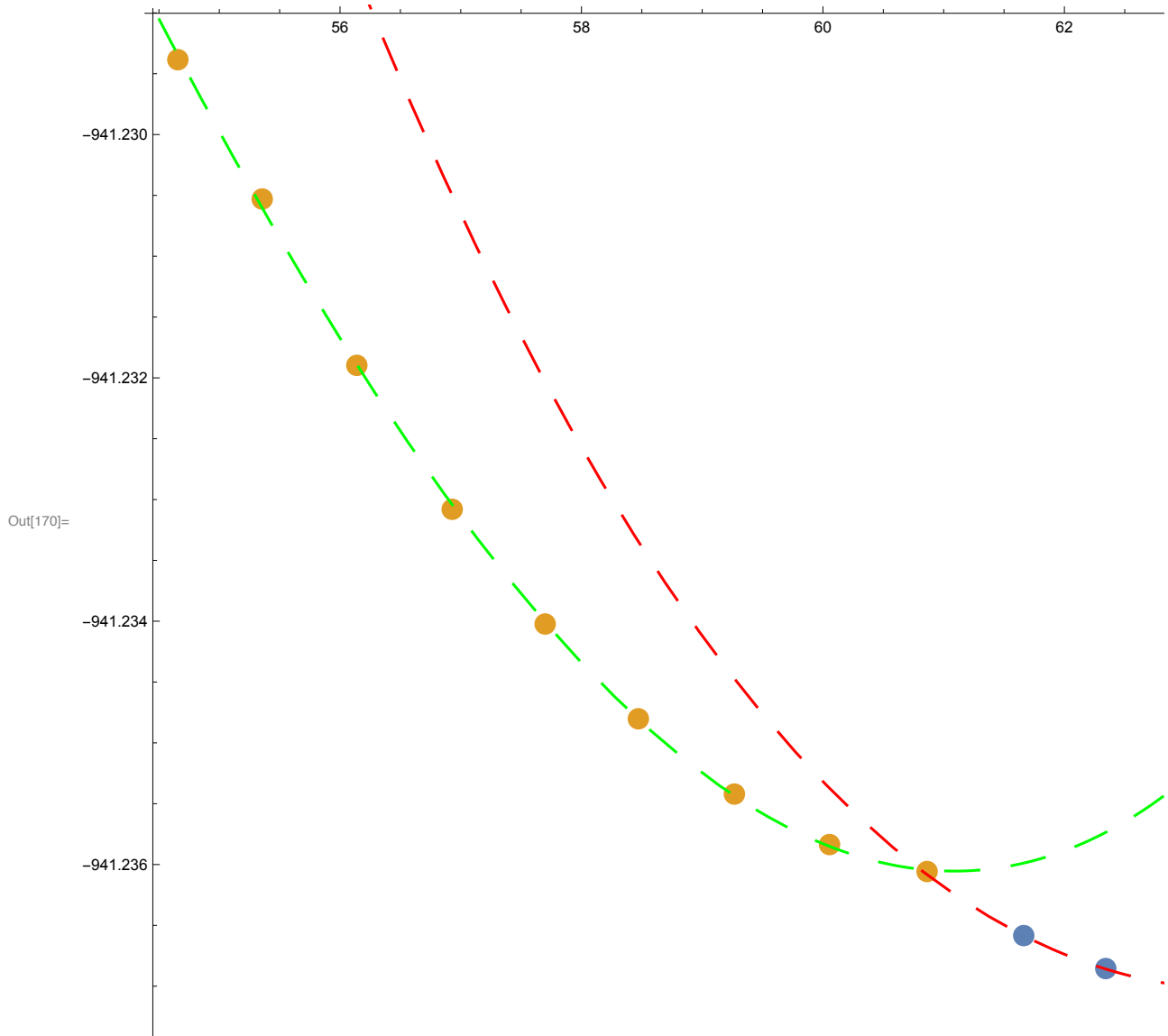
In[166]:= fitOneDatx = Table[Fit[oneDataParts[[i]], {1, x, x^2, x^3}, x], {i, 1, 30}];
fitTwoDatx = Table[Fit[twoDataParts[[i]], {1, x, x^2, x^3}, x], {i, 1, 30}];
fitOneDat = fitOneDatx /. x → y;
fitTwoDat = fitTwoDatx /. x → z;

```

```

In[170]:= Show[ListPlot[{oneDataParts[[1]], twoDataParts[[1]]},
  PlotLegends → {"OneDat", "TwoDat"}], Plot[fitOneDatx[[1]], {x, 55, 68},
  PlotStyle → Directive[Dashing[0.02], Red], PlotLegends → {"OneDatFit"}],
  Plot[fitTwoDatx[[1]], {x, 54, 68}, PlotStyle → Directive[Dashing[0.02], Green],
  PlotLegends → {"TwoDatFit"}], ImageSize → 1000]

```



```

In[171]:= fitOneDat
fitOneDatDash = D[fitOneDat, y]
fitTwoDat
fitTwoDatDash = D[fitTwoDat, z]

```

```

Out[171]= { -939.867 - 0.0563916 y + 0.000756843 y2 - 3.28316 × 10-6 y3,
            -939.913 - 0.0543148 y + 0.000725284 y2 - 3.12331 × 10-6 y3,
            -939.97 - 0.0517217 y + 0.000686071 y2 - 2.92559 × 10-6 y3,
            -940.028 - 0.0490838 y + 0.000646307 y2 - 2.72575 × 10-6 y3,
            -940.087 - 0.0464632 y + 0.000606878 y2 - 2.52805 × 10-6 y3,
            -940.144 - 0.0438599 y + 0.000567745 y2 - 2.33208 × 10-6 y3,
            -940.202 - 0.0412698 y + 0.000528817 y2 - 2.13727 × 10-6 y3,
            -940.259 - 0.03869 y + 0.000490041 y2 - 1.94327 × 10-6 y3,
            -940.317 - 0.0361186 y + 0.000451384 y2 - 1.74988 × 10-6 y3,
            -940.374 - 0.0335542 y + 0.000412822 y2 - 1.55696 × 10-6 y3,
            -940.431 - 0.0309954 y + 0.000374336 y2 - 1.36441 × 10-6 y3,
            -940.488 - 0.0284412 y + 0.00033591 y2 - 1.17215 × 10-6 y3,
            -940.545 - 0.0258906 y + 0.00029753 y2 - 9.80099 × 10-7 y3,
            -940.602 - 0.0233428 y + 0.000259187 y2 - 7.88215 × 10-7 y3,
            -940.658 - 0.0207973 y + 0.00022087 y2 - 5.96448 × 10-7 y3,
            -940.715 - 0.0182534 y + 0.000182572 y2 - 4.04765 × 10-7 y3,
            -940.772 - 0.0157108 y + 0.000144288 y2 - 2.13134 × 10-7 y3,
            -940.829 - 0.013169 y + 0.000106013 y2 - 2.15343 × 10-8 y3,
            -940.886 - 0.0106279 y + 0.0000677425 y2 + 1.70054 × 10-7 y3,
            -940.943 - 0.00808717 y + 0.000029474 y2 + 3.61643 × 10-7 y3,
            -940.999 - 0.00554665 y - 8.79497 × 10-6 y2 + 5.53246 × 10-7 y3,
            -941.056 - 0.00300622 y - 0.0000470661 y2 + 7.4487 × 10-7 y3,
            -941.113 - 0.000465769 y - 0.0000853407 y2 + 9.3652 × 10-7 y3,
            -941.17 + 0.00207479 y - 0.00012362 y2 + 1.1282 × 10-6 y3,
            -941.227 + 0.00461551 y - 0.000161904 y2 + 1.31992 × 10-6 y3,
            -941.284 + 0.00715644 y - 0.000200194 y2 + 1.51167 × 10-6 y3,
            -941.341 + 0.00969761 y - 0.00023849 y2 + 1.70346 × 10-6 y3,
            -941.398 + 0.012239 y - 0.000276792 y2 + 1.89529 × 10-6 y3,
            -941.455 + 0.0147808 y - 0.000315101 y2 + 2.08716 × 10-6 y3,
            -941.512 + 0.0173228 y - 0.000353416 y2 + 2.27907 × 10-6 y3 }

```

```

Out[172]= { -0.0563916 + 0.00151369 y - 9.84947 × 10-6 y2,
            -0.0543148 + 0.00145057 y - 9.36994 × 10-6 y2,
            -0.0517217 + 0.00137214 y - 8.77678 × 10-6 y2,
            -0.0490838 + 0.00129261 y - 8.17725 × 10-6 y2,
            -0.0464632 + 0.00121376 y - 7.58415 × 10-6 y2,
            -0.0438599 + 0.00113549 y - 6.99625 × 10-6 y2,
            -0.0412698 + 0.00105763 y - 6.41182 × 10-6 y2,
            -0.03869 + 0.000980082 y - 5.82982 × 10-6 y2,
            -0.0361186 + 0.000902768 y - 5.24965 × 10-6 y2,
            -0.0335542 + 0.000825644 y - 4.67089 × 10-6 y2,
            -0.0309954 + 0.000748672 y - 4.09324 × 10-6 y2,
            -0.0284412 + 0.00067182 y - 3.51644 × 10-6 y2,
            -0.0258906 + 0.000595061 y - 2.9403 × 10-6 y2,
            -0.0233428 + 0.000518374 y - 2.36464 × 10-6 y2,
            -0.0207973 + 0.00044174 y - 1.78935 × 10-6 y2,
            -0.0182534 + 0.000365145 y - 1.21429 × 10-6 y2,
            -0.0157108 + 0.000288577 y - 6.39403 × 10-7 y2,
            -0.013169 + 0.000212026 y - 6.46029 × 10-8 y2,
            -0.0106279 + 0.000135485 y + 5.10161 × 10-7 y2,
            -0.00808717 + 0.0000589479 y + 1.08493 × 10-6 y2,
            -0.00554665 - 0.0000175899 y + 1.65974 × 10-6 y2,
            -0.00300622 - 0.0000941321 y + 2.23461 × 10-6 y2,
            -0.000465769 - 0.000170681 y + 2.80956 × 10-6 y2,
            0.00207479 - 0.00024724 y + 3.38461 × 10-6 y2,
            0.00461551 - 0.000323808 y + 3.95976 × 10-6 y2,
            0.00715644 - 0.000400388 y + 4.53502 × 10-6 y2,
            0.00969761 - 0.00047698 y + 5.11039 × 10-6 y2,
            0.012239 - 0.000553585 y + 5.68588 × 10-6 y2,
            0.0147808 - 0.000630202 y + 6.26148 × 10-6 y2,
            0.0173228 - 0.000706831 y + 6.8372 × 10-6 y2}

```

```

Out[173]= { -941.696 + 0.0345721 z - 0.000761722 z2 + 5.22473 × 10-6 z3,
            -941.628 + 0.0309551 z - 0.000698302 z2 + 4.85419 × 10-6 z3,
            -941.556 + 0.0271721 z - 0.000631808 z2 + 4.46456 × 10-6 z3,
            -941.504 + 0.0244036 z - 0.000582947 z2 + 4.17682 × 10-6 z3,
            -941.475 + 0.0228477 z - 0.000555223 z2 + 4.01166 × 10-6 z3,
            -941.468 + 0.0224671 z - 0.000547997 z2 + 3.96549 × 10-6 z3,
            -941.481 + 0.0231085 z - 0.000558597 z2 + 4.02281 × 10-6 z3,
            -941.51 + 0.0245913 z - 0.000583878 z2 + 4.16538 × 10-6 z3,
            -941.552 + 0.0267519 z - 0.000620982 z2 + 4.37661 × 10-6 z3,
            -941.604 + 0.0294557 z - 0.000667562 z2 + 4.64287 × 10-6 z3,
            -941.665 + 0.0325962 z - 0.000721764 z2 + 4.9534 × 10-6 z3,
            -941.733 + 0.0360908 z - 0.000782144 z2 + 5.29981 × 10-6 z3,
            -941.806 + 0.039875 z - 0.00084758 z2 + 5.67561 × 10-6 z3,
            -941.884 + 0.0438988 z - 0.000917197 z2 + 6.07569 × 10-6 z3,
            -941.965 + 0.0481226 z - 0.000990305 z2 + 6.49606 × 10-6 z3,
            -942.05 + 0.0525151 z - 0.00106636 z2 + 6.93355 × 10-6 z3,
            -942.138 + 0.0570511 z - 0.00114492 z2 + 7.38562 × 10-6 z3,
            -942.228 + 0.0617103 z - 0.00122563 z2 + 7.8502 × 10-6 z3,
            -942.32 + 0.0664761 z - 0.00130821 z2 + 8.32561 × 10-6 z3,
            -942.414 + 0.0713347 z - 0.00139241 z2 + 8.81047 × 10-6 z3,
            -942.509 + 0.0762749 z - 0.00147803 z2 + 9.3036 × 10-6 z3,
            -942.606 + 0.0812869 z - 0.00156491 z2 + 9.80406 × 10-6 z3,
            -942.704 + 0.0863627 z - 0.00165291 z2 + 0.000010311 z3,
            -942.804 + 0.0914954 z - 0.0017419 z2 + 0.0000108237 z3,
            -942.904 + 0.096679 z - 0.00183178 z2 + 0.0000113416 z3,
            -943.005 + 0.101908 z - 0.00192246 z2 + 0.0000118642 z3,
            -943.107 + 0.107179 z - 0.00201387 z2 + 0.000012391 z3,
            -943.21 + 0.112487 z - 0.00210593 z2 + 0.0000129216 z3,
            -943.313 + 0.11783 z - 0.00219859 z2 + 0.0000134557 z3,
            -943.417 + 0.123203 z - 0.00229179 z2 + 0.000013993 z3 }

```

```
Out[174]= {0.0345721 - 0.00152344 z + 0.0000156742 z^2,
  0.0309551 - 0.0013966 z + 0.0000145626 z^2, 0.0271721 - 0.00126362 z + 0.0000133937 z^2,
  0.0244036 - 0.00116589 z + 0.0000125305 z^2,
  0.0228477 - 0.00111045 z + 0.000012035 z^2, 0.0224671 - 0.00109599 z + 0.0000118965 z^2,
  0.0231085 - 0.00111719 z + 0.0000120684 z^2,
  0.0245913 - 0.00116776 z + 0.0000124961 z^2,
  0.0267519 - 0.00124196 z + 0.0000131298 z^2,
  0.0294557 - 0.00133512 z + 0.0000139286 z^2,
  0.0325962 - 0.00144353 z + 0.0000148602 z^2,
  0.0360908 - 0.00156429 z + 0.0000158994 z^2,
  0.039875 - 0.00169516 z + 0.0000170268 z^2, 0.0438988 - 0.00183439 z + 0.0000182271 z^2,
  0.0481226 - 0.00198061 z + 0.0000194882 z^2,
  0.0525151 - 0.00213272 z + 0.0000208007 z^2,
  0.0570511 - 0.00228984 z + 0.0000221569 z^2,
  0.0617103 - 0.00245127 z + 0.0000235506 z^2,
  0.0664761 - 0.00261642 z + 0.0000249768 z^2,
  0.0713347 - 0.00278482 z + 0.0000264314 z^2,
  0.0762749 - 0.00295606 z + 0.0000279108 z^2,
  0.0812869 - 0.00312982 z + 0.0000294122 z^2,
  0.0863627 - 0.00330581 z + 0.000030933 z^2, 0.0914954 - 0.00348379 z + 0.0000324712 z^2,
  0.096679 - 0.00366356 z + 0.0000340249 z^2, 0.101908 - 0.00384492 z + 0.0000355927 z^2,
  0.107179 - 0.00402774 z + 0.0000371731 z^2, 0.112487 - 0.00421186 z + 0.0000387649 z^2,
  0.11783 - 0.00439718 z + 0.0000403672 z^2, 0.123203 - 0.00458358 z + 0.000041979 z^2}
```

```
sol = Solve[{(fitOneDat[[1]] - fitTwoDat[[1]]) / (y - z) == fitTwoDatDash[[1]] &&
  (fitOneDat[[1]] - fitTwoDat[[1]]) / (y - z) ==
  fitOneDatDash[[1]]}, {y, z}, Reals]
```

```
In[179]:= solConditions =
  Table[Solve[{(fitOneDat[[i]] - fitTwoDat[[i]]) / (y - z) == fitTwoDatDash[[i]] &&
    (fitOneDat[[i]] - fitTwoDat[[i]]) / (y - z) == fitOneDatDash[[i]] &&
    56 < z < 64 && 56 < y < 64}, {y, z}, Reals], {i, 1, 30}]
```

... **Solve:** Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

... **Solve:** Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

... **Solve:** Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

... **General:** Further output of Solve::ratnz will be suppressed during this calculation.

```
Out[179]= {{ {y → 60.8027, z → 60.8027}, {y → 61.8282, z → 59.8666}},
  {{y → 60.7951, z → 60.7951}, {y → 61.7938, z → 59.8806}},
  {{y → 60.7796, z → 60.7796}, {y → 61.7378, z → 59.8997}},
  {{y → 60.7591, z → 60.7591}, {y → 61.6727, z → 59.9202}},
  {{y → 60.7348, z → 60.7348}, {y → 61.6038, z → 59.9393}},
  {{y → 60.7072, z → 60.7072}, {y → 61.5332, z → 59.956}},
  {{y → 60.6764, z → 60.6764}, {y → 61.4609, z → 59.9694}},
  {{y → 60.6422, z → 60.6422}, {y → 61.3864, z → 59.9794}},
  {{y → 60.6043, z → 60.6043}, {y → 61.3083, z → 59.9856}},
  {{y → 60.562, z → 60.562}, {y → 61.2252, z → 59.9878}},
  {{y → 60.5142, z → 60.5142}, {y → 61.1352, z → 59.9854}},
  {{y → 60.4596, z → 60.4596}, {y → 61.0356, z → 59.9775}},
  {{y → 60.396, z → 60.396}, {y → 60.9229, z → 59.9627}},
  {{y → 60.3195, z → 60.3195}, {y → 60.7913, z → 59.9385}},
  {{y → 60.2235, z → 60.2235}, {y → 60.6309, z → 59.9004}},
  {{y → 56.6171, z → 56.9839}, {y → 56.8021, z → 56.8021},
    {y → 60.093, z → 60.093}, {y → 60.4207, z → 59.8377}},
  {{y → 58.0393, z → 58.3747}, {y → 58.2203, z → 58.2203}, {y → 59.8815, z → 59.8815},
    {y → 60.0966, z → 59.7163}}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {}, {} }
```



```
In[197]:= solConditions[[1 ;; 18]]
```

```
Out[197]= {{ {y → 60.8027, z → 60.8027}, {y → 61.8282, z → 59.8666}},
  {{y → 60.7951, z → 60.7951}, {y → 61.7938, z → 59.8806}},
  {{y → 60.7796, z → 60.7796}, {y → 61.7378, z → 59.8997}},
  {{y → 60.7591, z → 60.7591}, {y → 61.6727, z → 59.9202}},
  {{y → 60.7348, z → 60.7348}, {y → 61.6038, z → 59.9393}},
  {{y → 60.7072, z → 60.7072}, {y → 61.5332, z → 59.956}},
  {{y → 60.6764, z → 60.6764}, {y → 61.4609, z → 59.9694}},
  {{y → 60.6422, z → 60.6422}, {y → 61.3864, z → 59.9794}},
  {{y → 60.6043, z → 60.6043}, {y → 61.3083, z → 59.9856}},
  {{y → 60.562, z → 60.562}, {y → 61.2252, z → 59.9878}},
  {{y → 60.5142, z → 60.5142}, {y → 61.1352, z → 59.9854}},
  {{y → 60.4596, z → 60.4596}, {y → 61.0356, z → 59.9775}},
  {{y → 60.396, z → 60.396}, {y → 60.9229, z → 59.9627}},
  {{y → 60.3195, z → 60.3195}, {y → 60.7913, z → 59.9385}},
  {{y → 60.2235, z → 60.2235}, {y → 60.6309, z → 59.9004}},
  {{y → 56.6171, z → 56.9839}, {y → 56.8021, z → 56.8021},
  {y → 60.093, z → 60.093}, {y → 60.4207, z → 59.8377}},
  {{y → 58.0393, z → 58.3747}, {y → 58.2203, z → 58.2203},
  {y → 59.8815, z → 59.8815}, {y → 60.0966, z → 59.7163}}, {} }
```

```
In[181]:= solConditions[[1]][[2]]
```

```
Out[181]= {y → 61.8282, z → 59.8666}
```

```
In[198]:= a = Table[fitTwoDat[[i]] /. solConditions[[i]][[2]], {i, 1, 17}]
b = Table[-fitOneDatDash[[i]] * z /. solConditions[[i]][[2]], {i, 1, 17}]
c = Table[(fitOneDatDash[[i]] /. solConditions[[i]][[2]]) * x, {i, 1, 17}]
commonTangent = Table[a[[i]] + b[[i]] + c[[i]], {i, 1, 17}]
```

```
Out[198]= {-941.236, -941.236, -941.236, -941.236, -941.236,
  -941.236, -941.236, -941.237, -941.237, -941.237, -941.238,
  -941.238, -941.239, -941.239, -941.24, -941.237, -941.24}
```

```
Out[199]= {0.0272354, 0.0273987, 0.0276701, 0.0279973, 0.0283648,
  0.0287662, 0.0292013, 0.0296743, 0.0301929, 0.0307683, 0.0314167,
  0.0321614, 0.0330376, 0.0341031, 0.0354627, 0.081245, 0.0627073}
```

```
Out[200]= {-0.000454935 x, -0.000457556 x, -0.000461941 x, -0.000467243 x,
  -0.000473225 x, -0.000479789 x, -0.000486937 x, -0.000494742 x,
  -0.000503335 x, -0.000512909 x, -0.000523739 x, -0.000536224 x,
  -0.00055097 x, -0.000568968 x, -0.000592028 x, -0.00143032 x, -0.00107707 x}
```

```
Out[201]= {-941.209 - 0.000454935 x, -941.208 - 0.000457556 x,
  -941.208 - 0.000461941 x, -941.208 - 0.000467243 x, -941.208 - 0.000473225 x,
  -941.207 - 0.000479789 x, -941.207 - 0.000486937 x, -941.207 - 0.000494742 x,
  -941.207 - 0.000503335 x, -941.207 - 0.000512909 x, -941.206 - 0.000523739 x,
  -941.206 - 0.000536224 x, -941.206 - 0.00055097 x, -941.205 - 0.000568968 x,
  -941.204 - 0.000592028 x, -941.156 - 0.00143032 x, -941.177 - 0.00107707 x}
```

```
In[229]:= phaseBoundaryData = Transpose[{temps[[1 ;; 17]], (c /. x → -1) * 4.3597482 * 10^3}]
Out[229]= {{10., 1.9834}, {30.1, 1.99483}, {50.2, 2.01395}, {70.3, 2.03706}, {90.4, 2.06314},
{110.51, 2.09176}, {130.61, 2.12292}, {150.71, 2.15695}, {170.81, 2.19441},
{190.91, 2.23616}, {211.01, 2.28337}, {231.11, 2.3378}, {251.21, 2.40209},
{271.31, 2.48056}, {291.41, 2.58109}, {311.52, 6.23582}, {331.62, 4.69575}}
```

```
In[236]:= phaseBoundaryData // TableForm
Out[236]//TableForm=
```

10.	1.9834
30.1	1.99483
50.2	2.01395
70.3	2.03706
90.4	2.06314
110.51	2.09176
130.61	2.12292
150.71	2.15695
170.81	2.19441
190.91	2.23616
211.01	2.28337
231.11	2.3378
251.21	2.40209
271.31	2.48056
291.41	2.58109
311.52	6.23582
331.62	4.69575

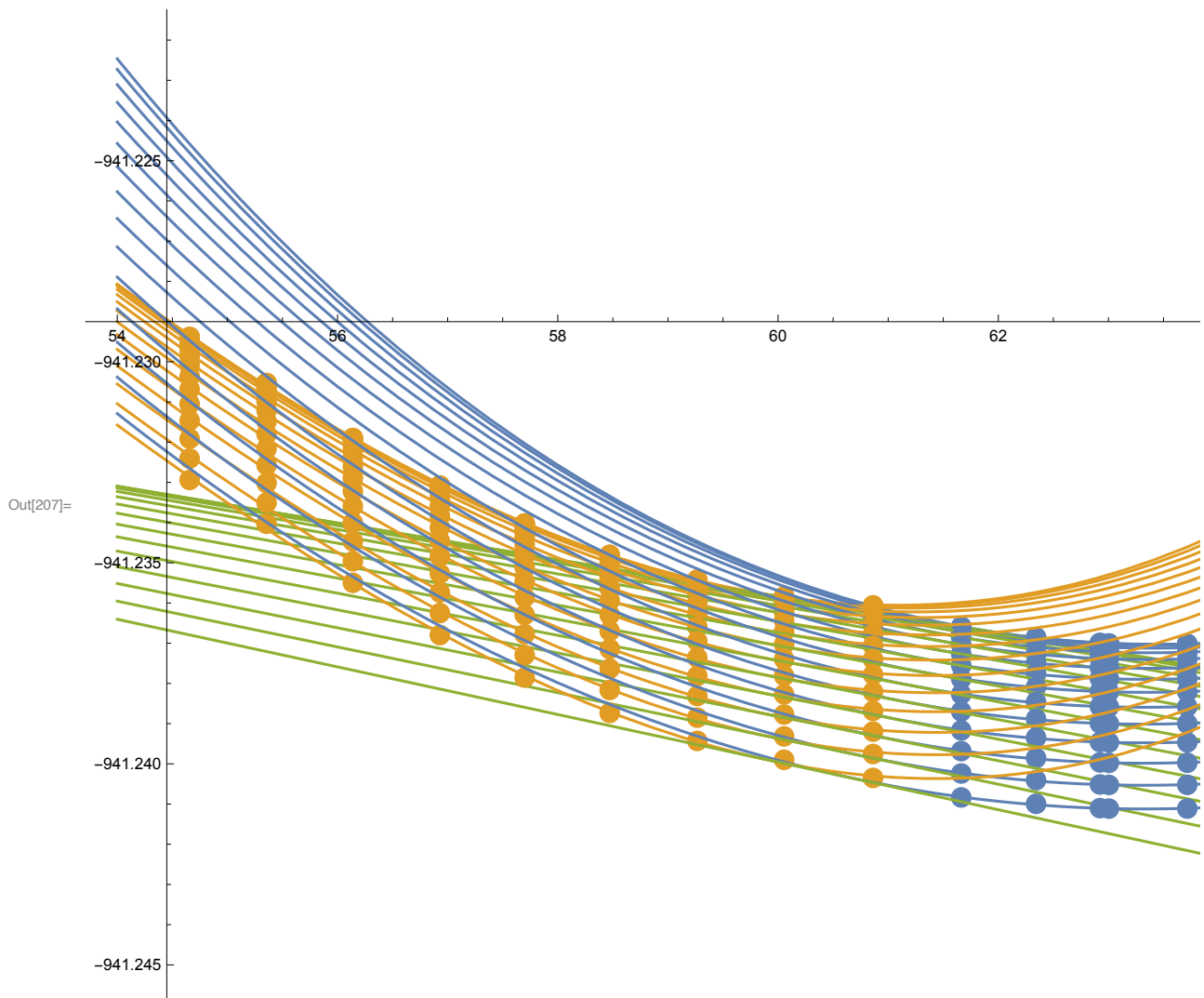
```
In[230]:= Export["absolutePressuresMathematica.dat", phaseBoundaryData // TableForm]
```

```
Out[230]= absolutePressuresMathematica.dat
```

```

In[207]:= Show[Table[Show[ListPlot[{oneDataParts[[i]], twoDataParts[[i]]}],
  Plot[{fitOneDatx[[i]], fitTwoDatx[[i]], commonTangent[[i]]}, {x, 54, 68}],
  ImageSize -> 900], {i, 1, 15}], PlotRange -> All]

```



```

In[231]:= ListPlot@phaseBoundaryData

```

