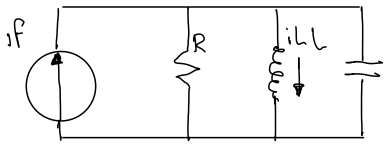
CELAYA GONZÁLEZ DAVID ALEJANDRO

PARCIAL 2

② EDO con $n=2$ Entrada i_f Salida i_L



$$C\begin{cases} V_C = \frac{1}{C}\int_0^{\tau} i_C(\tau)\,d\tau \;\dots (2.1) \\ i_C = C\frac{dV_C}{d\tau} \;\dots (2.2) \end{cases} \qquad R\begin{cases} V_R = R\,i_R \;\dots (1.1) \\ i_R = \frac{V_R}{R} \;\dots (1.2) \end{cases}$$

$$L\begin{cases} i_L = 1/L\int_0^{\tau} V_L(\tau)\,d\tau \;\dots (3.1) \\ V_L = L\frac{di_L}{dt} \;\dots (3.2) \end{cases}$$

Sustituyendo 1.2 y 2.2 en ⑤

$$\frac{V_R}{R} + C\frac{dV_C}{dt} + i_L = i_f \;\dots ⑥$$

$\Rightarrow V_R = V_L = V_C \;\dots ④$

$i_R + i_C + i_L = i_f \;\dots ⑤$

Ahora 4 en 6

$$\frac{V_L}{R} + C\frac{dC}{dL} + i_L = i_f \;\dots ⑦$$

$\Rightarrow$ 3.2 en ⑦

$$\frac{L}{R}\frac{di_L}{dt} + C\frac{dV_L}{dt} + i_L = i_f \;\dots ⑧$$

Ahora sust. ⑨ en ⑧

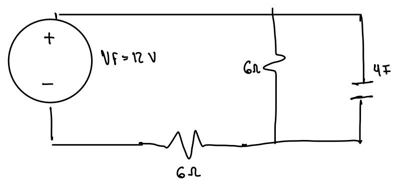
$$\frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} + i_L = i_f \;\dots ⑩$$

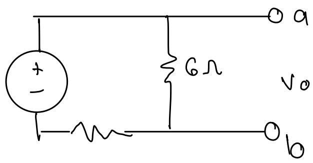
Derivando 3.2

$$\frac{dV_L}{dt} = L\frac{d^2 i_L}{dt^2} \;\dots ⑨$$

$$\therefore\; i_f = LC\frac{d^2 i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L$$

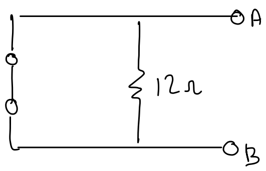
③

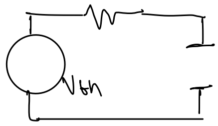




$$R_{TH} = \frac{16(6)}{6+6} = 3\,[\Omega]$$

$$V_o = \frac{V_f(6)}{12} = 6V \;\Rightarrow\; V_o = V_{TH}$$





$$\frac{1}{RC} = \frac{1}{3(4F)} = \frac{1}{12}$$

$\Rightarrow -V_1 + V_R + V_o = \emptyset$

$V_R + V_o = V_1$

$$\frac{1}{RC} = \frac{1}{3\Omega\,4F} = \frac{1}{12} \;\dots ②$$

$$RC\frac{dV_o}{dt} + V_o = V_1 \;\Rightarrow\; \frac{1}{RC}\left(RC\frac{dV_o}{dt} + V_o\right) = \frac{1}{RC}V_1 \;\Rightarrow\; \frac{dV_o}{dt} = \frac{1}{12}V_o = \frac{1}{12}\cdot 12 \;\dots ①$$

$\mathcal{L}\{1\}$

$\mathcal{L}^{-1}((4))$

$$sV_o(s) + \frac{1}{12}V_o(s) = 1 \;\dots ③$$

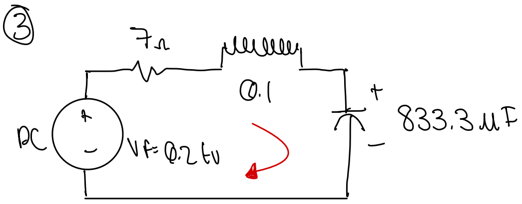
$$V_o(t) = 12$$

$$V_o(s) = 12/s \;\dots ④$$

$$\tau = RC = 12[s]$$

③

Entrada V_c Salida V_f



EDO con $n=3$

$$C\begin{cases} V_C = \frac{1}{C}\int_0^{\tau} i_C(\tau)\,d\tau \quad \dots (2.1) \\ i_C = C\frac{dv_C}{d\tau} \quad \dots (2.2) \end{cases}$$

$$R\begin{cases} V_R = R\,i_R \quad \dots (1.1) \\ i_R = \frac{V_R}{R} \quad \dots (1.2) \end{cases}$$

$$L\begin{cases} i_L = 1/L\int_0^{\tau} V_L(\tau)\,d\tau \quad \dots (3.1) \\ V_L = L\,\frac{di_L}{dt} \quad \dots (3.2) \end{cases}$$

$$V_R + V_L + V_C = V_F \quad \dots (4)$$
$$i_R + i_L + i_C = V_F \quad \dots (5)$$

(5) en ec (6)

Sustituyendo 1.1 y 3.2 en (4)

$$R i_C + L\frac{di_C}{dt} + V_C = V_F \quad \dots (7)$$

$$R i_R + L\frac{di_L}{dt} + V_C = V_F \quad \dots (6)$$

Derivada de (2.2)

$$\frac{di_C}{dt} = C\frac{dv^2_C}{dt^2} \quad \dots (8)$$

Ahora de 8 y 2.2

$$RC\frac{dV_C}{dt} + LC\frac{d^2V_C}{dt^2} + V_C = V_F \quad \dots (9)$$

$$\frac{d^2V_c}{dt^2} + \frac{L}{R}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{V_f}{LC} \quad \dots (10)$$

$$\Rightarrow V_0 = V_c$$

$$\frac{d^2V_0}{dt^2} + \frac{R}{L}\frac{dV_0}{dt} + \frac{V_0}{LC} = \frac{V_F}{LC} \quad \dots (11)$$

$\mathcal{L}(11)$

$$S^2V_0(S) + \frac{R}{L}S\,V_0(S) + \frac{1}{LC}V_0(S) = \frac{V_f(S)}{LC}$$

$$V_0(S)\left(S^2 + \frac{7}{0.1}S + \frac{1}{0.1(833.33\times10^{-6})}\right) = \frac{1\;V_f(S)}{0.1(833.33\times10^{-6})}$$

$$V_0(S)\left(S^2 + 70S + \frac{1}{12\times10^{-3}}\right) = \frac{2}{S^2\,12\times10^{-3}}$$

$$V_0(S) = \frac{2.4\times10^{-3}}{S^2(S^2+70S+12\times10^3)}$$

$$V_0(S) = \frac{2.4\times10^{-3}}{S^4+70S^3+12\times10^3S^2}$$

Raíces

$$S_1 = -35 + S\sqrt{43i} = -35 + 103.8i$$
$$S_2 = 0$$
$$S_3 = -35 - S\sqrt{43i} = -35 - 103.8i$$

$$V_0(S) = \frac{2.4 \times 10^{-3}}{S[(S+35)^2 + (103.8)^2]} = \frac{A}{S} + \frac{BS}{(S+32)^2 + 103.82}$$

$$\frac{A}{S} = \emptyset \quad \frac{2.4 \times 10^{3}}{12 \times 10^{3}} = 0.2$$

Para $BS+C$ elegimos una raiz

$\therefore S = 35 + 103.8i$

$$C + B(-35 + 103.8i) = \frac{2.4 \times 10^{-3}}{-35 + 103.8i} \; \frac{(35 - 103.8i)}{(35 - 103.8i)} = \frac{84000 - 249120i}{-1225 + 10779.4} = \frac{8400 - 249120i}{9549.4}$$

$$C - 35B + 103.8Bi = 8.796 - 260.86i$$

$\Rightarrow \mathbb{R} = \mathbb{R}$ y $i = i$

- $C - 35B = 8.796$
- $103.8Bi = -260.86i$

$$B = \frac{-260.86i}{103.8i} = 2.51$$

$$C = 8.796 + 35(2.51)$$

$$C = 96$$

$$V_0(S) = 0.2\frac{1}{S^2} - \underbrace{\frac{2.51S + 96.646}{(S+35)^2 + (103.8)^2}}_{A}$$

$$V_0(t) = 0.2t - \mathcal{L}^{-1}|A|$$

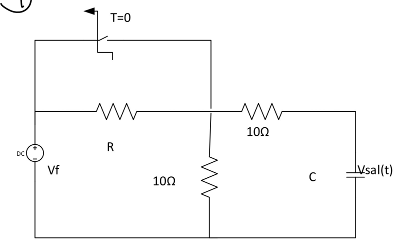
Si $V_0(0) = 0$ y $V_0(\infty) = 0.2$

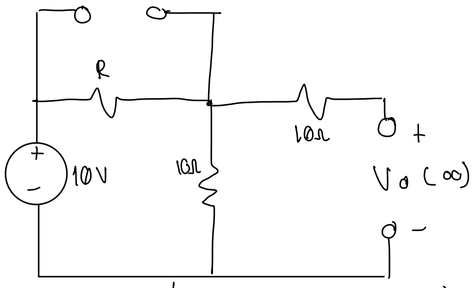
$$S^2 V_0(S) + 70S\, V_0(S) + 12 \times 10^{-3} V_0(S) = 12 \times 10^{-3} V_f(S)$$

$$V_0(S)(S^2 + 70S + 12 \times 10^{-3}) = 12 \times 10^{-3} V_f(S)$$

$$\frac{V_0(S)}{V_f(S)} = \frac{12 \times 10^{-3}}{S^2 + 70S + 12 \times 10^{-3}}$$

①





=> $V_o(t) = 2 + 8e^{-0.8t}$ para $t > 0$

=> $t = 0$

$V_o(t) = 2 + 8e^{-0.8(0)} = 2 + 8 = 10\,V$

=> Usando divisor de voltaje

$$V_o(\infty) = \frac{10}{R+10}(10)$$

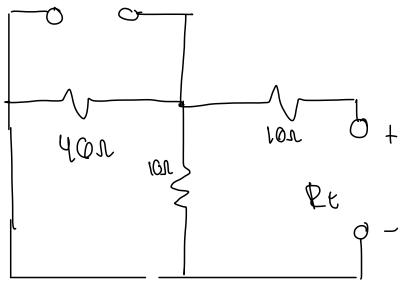
=> $t = \infty$ en $V_o(t)$

$$V_o(\infty) = 2 + 8e^{-\infty} = 2\,V$$

Segudo de:

$$2 = \frac{10}{R+10}(10) \Rightarrow 2R + 20 = 100 \Rightarrow R = 40\,\Omega$$

Una parte de $V_o(t)$ puede ser $e^{-t/\tau}$ recordando que $\tau = R_T C$ y R_T es la de Thevenin.



$$\Rightarrow R_T = 10 + \frac{(40)(10)}{40+10} = 18\,\Omega$$

$$\Rightarrow \tau = 18\,C$$

$$-0.5t = -t/\tau$$

$$\Rightarrow \tau = 2s$$

$$\Rightarrow 2 = 18C$$

$$C = 0.11$$

$$C = 111{,}000\,\mu F$$

El estado transitorio y el tiempo en que inicia el estado permanente estará dado por 5τ

=>

$5(2s) = 10[s]$