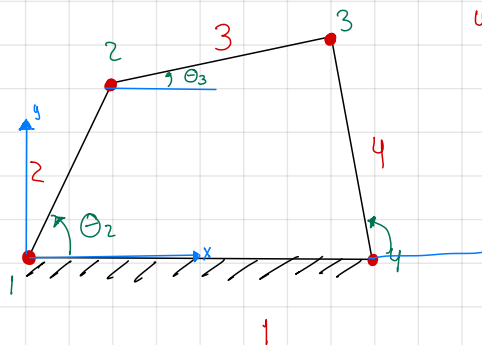


Celaya González David Alejandro 



4 JUNTAS COMPLETAS

4 ESCALABONES

$$GDL = 3(4-1) - 2(4) - 1(0)$$

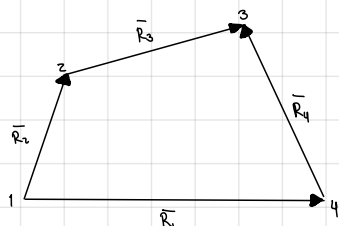
$$GDL = 1$$

VARIABLES = θ_3, θ_4

CONSTANTES = l_2, l_3, l_4

θ_2 : Es mi variable de control (yo la defino)

Definiendo lazos vectoriales



Ecuación de lazo

$$\vec{R}_2 + \vec{R}_3 - \vec{R}_4 - \vec{R}_1 = \vec{0}$$

Expresando vectores:

$$\vec{R}_1 = l_1 \hat{i}$$

$$\vec{R}_2 = l_2 \hat{u}_2 = l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$$

$$\vec{R}_3 = l_3 \hat{u}_3 = l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j})$$

$$\vec{R}_4 = l_4 \hat{u}_4 = l_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j})$$

Sustituyendo en ec. de lazo

$$l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j}) + l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) - l_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j}) - l_1 \hat{i} = \vec{0}$$

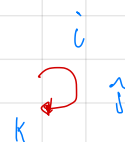
Separando por componentes:

$$\hat{i}: l_2 \cos \theta_2 + l_3 \cos \theta_3 - l_4 \cos \theta_4 - l_1 = 0 \quad \dots \textcircled{1}$$

$$\hat{j}: l_2 \sin \theta_2 + l_3 \sin \theta_3 - l_4 \sin \theta_4 = 0 \quad \dots \textcircled{2}$$

Derivando la ecuación de lazo con respecto al tiempo obtenemos:

$$\dot{\vec{v}}_2 + \dot{\vec{v}}_3 - \dot{\vec{v}}_4 - \dot{\vec{v}}_1 = \vec{0}$$



Caso 0 $\dot{\vec{v}}_1 = \vec{0}$

Caso 2 $\dot{\vec{v}}_2 = \vec{\omega}_2 \times \vec{R}_2 = \omega_2 \hat{k} \times l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j}) = -\omega_2 l_2 \sin \theta_2 \hat{i} + \omega_2 l_2 \cos \theta_2 \hat{j}$

Caso 2 $\dot{\vec{v}}_3 = \vec{\omega}_3 \times \vec{R}_3 = \omega_3 \hat{k} \times l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) = -\omega_3 l_3 \sin \theta_3 \hat{i} + \omega_3 l_3 \cos \theta_3 \hat{j}$

Caso 2 $\dot{\vec{v}}_4 = \vec{\omega}_4 \times \vec{R}_4 = \omega_4 \hat{k} \times l_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j}) = -\omega_4 l_4 \sin \theta_4 \hat{i} + \omega_4 l_4 \cos \theta_4 \hat{j}$

Separando por componentes

$$\hat{i}: -\omega_2 l_2 \sin \theta_2 - \omega_3 l_3 \sin \theta_3 + \omega_4 l_4 \sin \theta_4 = 0$$

$$\hat{j}: \omega_2 l_2 \cos \theta_2 + \omega_3 l_3 \cos \theta_3 - \omega_4 l_4 \cos \theta_4 = 0$$

Segunda derivada a ecuación de lazo con respecto al tiempo:

$$\bar{A}_2 + \bar{A}_3 - \bar{A}_4 - \bar{A}_1 = \bar{0}$$

Caso 0 $\bar{A}_1 = \bar{0}$

Caso 2 $\bar{A}_2 = \bar{\alpha}_2 \times \bar{R}_2 - \omega_2^2 \bar{R}_2 = \alpha_2 \hat{k} \times l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j}) - \omega_2^2 l_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$
 $\bar{A}_2 = (-\alpha_2 l_2 \sin \theta_2 - \omega_2^2 l_2 \cos \theta_2) \hat{i} + (\alpha_2 l_2 \cos \theta_2 - \omega_2^2 l_2 \sin \theta_2) \hat{j}$

Caso 2 $\bar{A}_3 = \bar{\alpha}_3 \times \bar{R}_3 - \omega_3^2 \bar{R}_3 = \alpha_3 \hat{k} \times l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) - \omega_3^2 l_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j})$
 $\bar{A}_3 = (-\alpha_3 l_3 \sin \theta_3 - \omega_3^2 l_3 \cos \theta_3) \hat{i} + (\alpha_3 l_3 \cos \theta_3 - \omega_3^2 l_3 \sin \theta_3) \hat{j}$

Caso 2 $\bar{A}_4 = \bar{\alpha}_4 \times \bar{R}_4 - \omega_4^2 \bar{R}_4 = \alpha_4 \hat{k} \times l_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j}) - \omega_4^2 l_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j})$
 $\bar{A}_4 = (-\alpha_4 l_4 \sin \theta_4 - \omega_4^2 l_4 \cos \theta_4) \hat{i} + (\alpha_4 l_4 \cos \theta_4 - \omega_4^2 l_4 \sin \theta_4) \hat{j}$

Separando por componentes:

$$\hat{i}: -\alpha_2 l_2 \sin \theta_2 - \omega_2^2 l_2 \cos \theta_2 - \alpha_3 l_3 \sin \theta_3 - \omega_3^2 l_3 \cos \theta_3 + \alpha_4 l_4 \sin \theta_4 + \omega_4^2 l_4 \cos \theta_4 = 0$$

$$\hat{j}: \alpha_2 l_2 \cos \theta_2 - \omega_2^2 l_2 \sin \theta_2 + \alpha_3 l_3 \cos \theta_3 - \omega_3^2 l_3 \sin \theta_3 - \alpha_4 l_4 \cos \theta_4 + \omega_4^2 l_4 \sin \theta_4 = 0$$