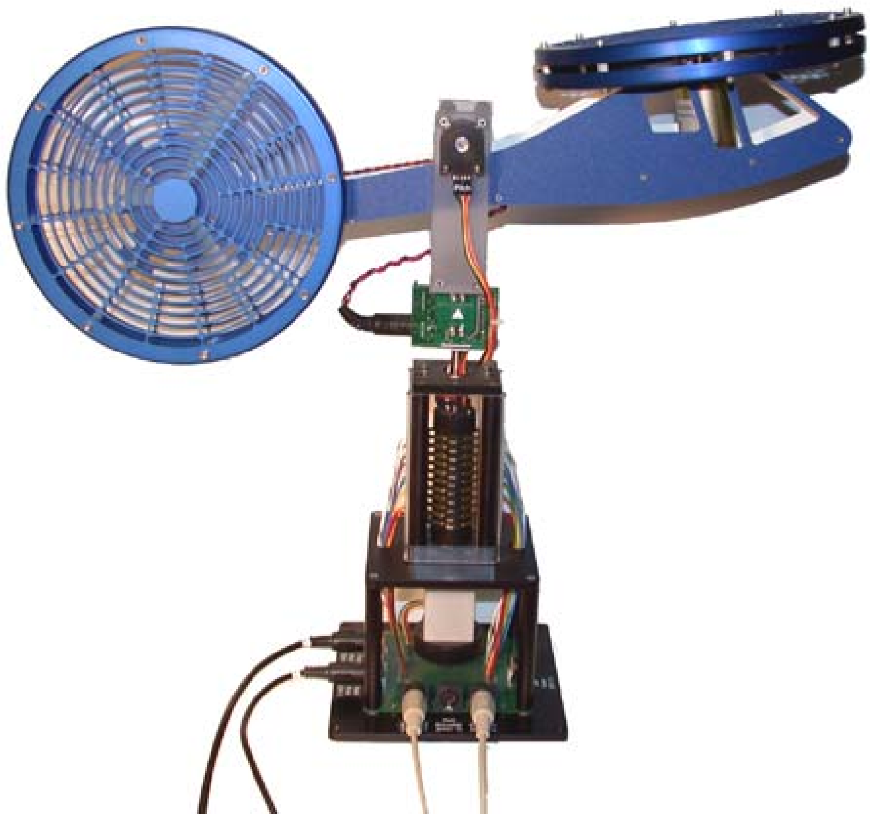
Specialty Experiments: 2 DOF Helicopter



Quanser 2 DOF Helicopter User and Control Manual

February 10, 2006

Contents

1 Introduction

The Quanser 2 DOF Helicopter experiment consists of a helicopter model mounted on a fixed base with two propellers that are driven by DC motors and is shown in Figure 1. The front propeller controls the elevation of the helicopter nose about the pitch axis and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders. The pitch encoder and motor signals are transmitted via a slipring. This eliminates the possibility of wires tangling on the yaw axis and allows the yaw angle to rotate freely about 360 degrees.

In Section 3 the components composing the 2 DOF Helicopter are described and the wiring procedure

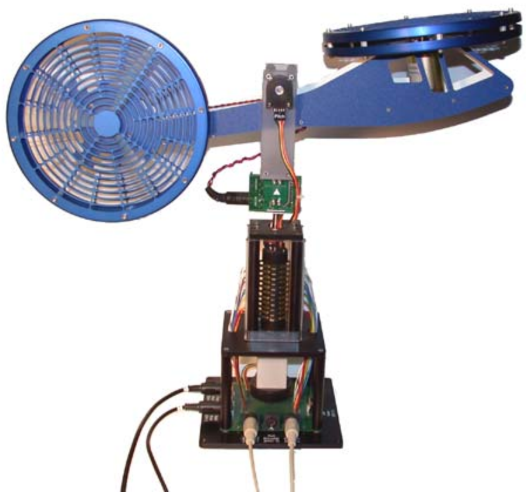


Figure 1: Quanser 2 DOF Helicopter.

is explained. The dynamics of the helicopter is developed and a position controller is designed using this model in Section 4. Several procedures are outlined in Section 5 that show the user how to simulate the position controller and then how to run this controller on the actual helicopter plant. Further, this section explains how to use the joystick to manually control the helicopter.

2 Prerequisites

The prerequisites to successfully carry out this laboratory are:

1. Become familiar with your Quanser 2 DOF Helicopter components (e.g. actuator, sensors), your data acquisition card (e.g. Q8), and your power amplifier (e.g. UPM), as described in Section 3 and References [2], [3], and [4].
2. Ensure you are comfortable in using WinCon to control and monitor the plant in real-time and in designing a controller through Simulink.
3. Complete the wiring and operating procedure as discussed in Section 3.5.
4. To be familiar with the LQR method enough to design a controller.

3 Experiment Setup

3.1 Main System Components

The following hardware and software are required to run the 2 DOF Helicopter experiment:

- **Power module**: Quanser UPM-1503 and UPM-2405, or equivalent.
- **Data acquisition board**: Quanser MultiQ, Q4, or Q8, National Instruments E-Series or M-Series, or equivalent.
- **Real-time control software**: Quanser WinCon and Ardence RTX configuration as detailed in [4], or the equivalent.
- **Helicopter Plant**: Quanser 2 DOF Helicopter System
- **Joystick**: Logitech Attack 3 USB joystick, or another windows-enabled joystick.

3.2 2 DOF Helicopter Components

The components of the 2 DOF Helicopter described in Table 1 are labeled in Figures 2, 3, 4, 5 and, 6.

ID	Description	ID	Description
1	Back propeller	13	Metal shaft rotating about yaw axis
2	Back propeller shield	14	Slip ring
3	Yaw/back motor	15	Yaw Encoder
4	Pitch encoder	16	Base platform
5	Yoke	17	Front motor connector
6	Helicopter body	18	Right motor connector (not used)
7	Front propeller	19	Back motor connector
8	Pitch/front motor	20	Yaw encoder connector
9	Front propeller shield	21	Roll encoder connector (not used)
10	Encoder/motor circuit	22	Pitch encoder connector
11	Encoder connector on circuit (not used)	23	Left motor connector (not used)
12	Motor connector on circuit		

Table 1: Quanser 2 DOF helicopter components.

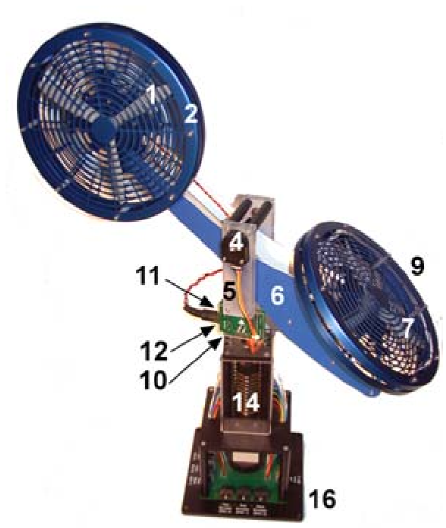


Figure 2: 2 DOF Helicopter components.

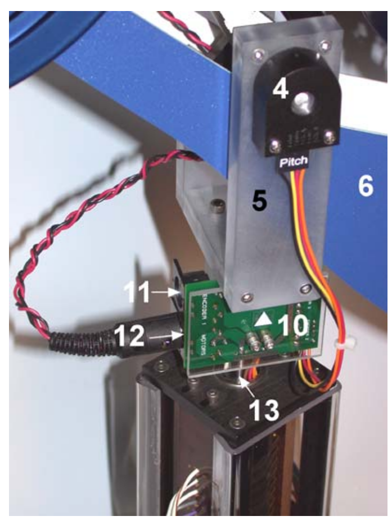


Figure 3: 2 DOF Helicopter yoke components.

3.3 Component Description

This section explain several of the components identified in Section 3.2.

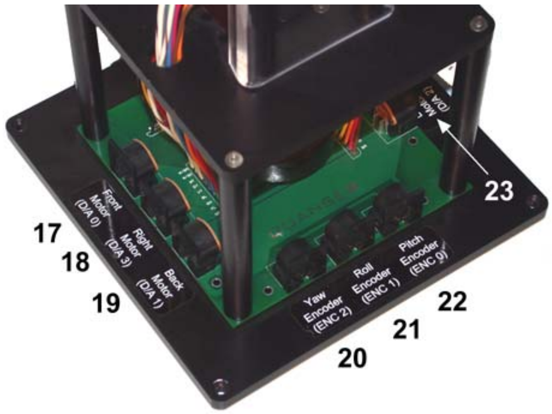


Figure 4: 2 DOF Helicopter base components.

Figure 5: 2 DOF Helicopter tail components.

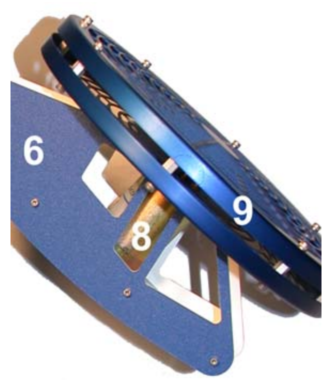


Figure 6: 2 DOF Helicopter front propeller assembly components.

3.3.1 DC Motors (Component #3 and #8)

The helicopter has two DC motors: the yaw motor, component #3, actuating the back propeller and the pitch motor, component #8, rotating the front propeller.

The yaw motor is a *Faulhaber Series 2842 Model 006C* motor. It has a terminal resistance of 1.6 Ω and a current-torque constant of 0.0109 $N \cdot m/A$. See [1] for the full specifications of this motor. The larger pitch motor is a *Pittman Model 9234* motor. It has an electrical resistance of 0.83 Ω and a current-torque constant of 0.0182 $N \cdot m/A$. See [5] for the full specifications of this motor.

3.3.2 Propellers

The propellers used are for both the pitch and yaw motors are *Graupner* 20/15 cm or 8/6". The pitch motor/propeller has an identified thrust force constant of 1.04 N/V and the yaw motor/propeller has a thrust force constant of 0.43 N/V.

3.3.3 Encoders (Component #4 and #15)

The helicopter has two encoders: the encoder measuring the pitch angle, component #4, and the encoder that measured the yaw angle, component #15. The pitch encoder outputs 4096 counts per revolution in quadrature mode. The resolution of the angular position about the pitch axis is therefore 0.791 degrees/count. The yaw encoder outputs 8192 counts per revolution. The position resolution about the yaw axis is 0.0439 degrees / count.

3.4 System Specifications and Model Parameters

Table 2 summarizes the main electrical properties of the motor and actuator parameters involving thrust. Table 3 lists various lengths, masses, and moment of inertias associated with the helicopter. Within these tables the only parameters used in the model are: K_{pp}, K_{yy}, K_{yp}, K_{py}, $J_{eq,p}$, $J_{eq,y}$, B_p, B_y, m_{heli}, l_{cm}, and g.

Symbol	Matlab Notation	Description	Value	Unit
$R_{m,p}$	R_m_p	Electrical resistance of pitch motor.	0.83	Ω
$R_{m,y}$	R_m_y	Electrical resistance of yaw motor.	1.60	Ω
$K_{t,p}$	K_t_p	Current-torque constant of pitch motor.	0.0182	$N \cdot m/A$
$K_{t,y}$	K_t_y	Current-torque constant of yaw motor.	0.0109	$N \cdot m/A$
$J_{m,p}$	J_m_p	Moment of inertia of pitch motor rotor.	1.91×10^{-6}	$kg \cdot m^2$
$J_{m,y}$	J_m_y	Moment of inertia of yaw motor rotor.	1.37×10^{-4}	$kg \cdot m^2$
$K_{f,p}$	K_f_p	Thrust force constant of pitch motor/propeller.	1.037	N/V
$K_{f,y}$	K_f_y	Thrust force constant of yaw motor/propeller.	0.428	N/V
K_{pp}	K_pp	Thrust torque constant acting on pitch axis from pitch motor/propeller.	0.204	$N \cdot m/V$
K_{yy}	K_yy	Thrust torque constant acting on yaw axis from yaw motor/propeller.	0.072	$N \cdot m/V$
K_{py}	K_py	Thrust torque constant acting on pitch axis from yaw motor/propeller.	0.0068	$N \cdot m/V$
K_{yp}	K_yp	Thrust torque constant acting on yaw axis from pitch motor/propeller.	0.0219	$N \cdot m/V$

Table 2: Actuator specifications and model parameters.

Symbol	**Matlab Notation**	**Description**	**Value**	**Unit**
$B_{eq,p}$	B_eq_p	Equivalent viscous damping about pitch axis.	0.800	N/V
$B_{eq,y}$	B_eq_p	Equivalent viscous damping about yaw axis.	0.318	N/V
m_{heli}	m_heli	Total moving mass of the helicopter.	1.3872	kg
$m_{m,p}$	m_motor_p	Mass of pitch motor.	0.292	kg
$m_{m,y}$	m_motor_y	Mass of yaw motor.	0.128	kg
m_{shield}	m_shield	Mass of propeller shield.	0.167	kg
m_{props}	m_props	Mass of pitch and yaw propellers, propeller shields and motors.	0.754	kg
$m_{body,p}$	m_body_p	Mass moving about the pitch axis.	0.633	kg
$m_{body,y}$	m_body_y	Mass moving about the yaw axis.	0.667	kg
m_{shaft}	m_shaft	Mass of metal shaft rotating about the yaw axis.	0.151	kg
L_{body}	L_body	Total length of helicopter body.	0.483	m
l_{cm}	l_cm	Center-of-mass length along helicopter body from pitch axis.	0.186	cm
L_{shaft}	L_shaft	Length of metal shaft rotating about the yaw axis.	0.280	m
$J_{body,p}$	J_m_y	Moment of inertia of helicopter body about pitch axis.	0.0123	$kg \cdot m^2$
$J_{body,y}$	J_m_y	Moment of inertia of helicopter body about yaw axis.	0.0129	$kg \cdot m^2$
J_{shaft}	J_shaft	Moment of inertia of metal shaft about yaw axis at end point.	0.0039	$kg \cdot m^2$
J_p	J_p	Moment of inertia of front motor/shield assembly about pitch pivot.	0.0178	$kg \cdot m^2$
J_y	J_y	Moment of inertia of back motor/shield assembly about yaw pivot.	0.0084	$kg \cdot m^2$
$J_{eq,p}$	J_eq_p	Total moment of inertia about pitch pivot.	0.0384	$kg \cdot m^2$
$J_{eq,y}$	J_eq_y	Total moment of inertia about yaw pivot.	0.0432	$kg \cdot m^2$

Table 3: Helicopter specifications and model parameters.

3.5 System Wiring

The pitch and yaw encoders are connected directly to the data-acquisition board. This provides the position feedback necessary to control the helicopter. The data-acquisition board outputs a control voltage that is amplified and drives the two motors. The pitch or front motor is connected to the UPM-2405 and the yaw motor or back motor is driven by the UPM-1503. The maximum voltage supplied to the pitch and yaw motors from the UPM-2405 and UPM-1503 power amplifiers is ± 24 V and ± 15 V.

The wiring procedure for the 2 DOF Helicopter system with the Q8 data-acquisition board, or DACB, is explained in the following steps:

1. Connect the 5-pin-DIN to RCA cable from *Analog Output Channel # 0* on the Q8 DAC board to the *From D/A* connector on the UPM-2405. This carries the attenuated pitch voltage control signal, $u_p/K_{a,p}$, where $K_{a,p}$ is the UPM-2405 amplifier gain.

2. Connect the 5-pin-DIN to RCA from *Analog Output Channel # 1* on the Q8 DAC board to the *From D/A* connector on the UPM-1503. This carries the attenuated yaw voltage control signal $u_y/K_{a,y}$, where $K_{a,y}$ is the UPM-1503 amplifier gain.

3. Connect the 4-pin-stereo-DIN to 6-pin-stereo-DIN that is labeled **Gain 5** from *To Load* on the UPM-2405 to the *Pitch Motor* connector, ID # 17 on Figure 4. This cable sets the gain of the amplifier to 5 and the connector on the UPM-side is gray in colour. The cable transmits the amplified voltage that is applied to the pitch motor, denoted $V_{m,p}$.

4. Connect the 4-pin-stereo-DIN to 6-pin-stereo-DIN is labeled **Gain 3** from *To Load* on the UPM-1503 to the *Yaw Motor* connector, ID # 19 on Figure 4. This cable sets the gain of the amplifier to 3 and the connector on the UPM-side is black in colour. The cable transmits the amplified voltage that is applied to the yaw motor, denoted $V_{m,y}$.

5. Connect the 5-pin-stereo-DIN to 5-pin-stereo-DIN cable from the *Encoder Input # 0* the *Pitch Encoder* connector on the 2 DOF Helicopter, ID # 22 in Figure 4, to the *Encoder Input # 0* on the Q8 DAC board. This carries the pitch angle measurement, θ.

6. Connect the 5-pin-stereo-DIN to 5-pin-stereo-DIN cable from the *Encoder Input # 1* the *Yaw Encoder* connector on the 2 DOF Helicopter, ID # 20 in Figure 4, to the *Encoder Input # 0* on the Q8 DAC board. This carries the yaw angle measurement, ψ.

The wiring is summarized in Table 4.

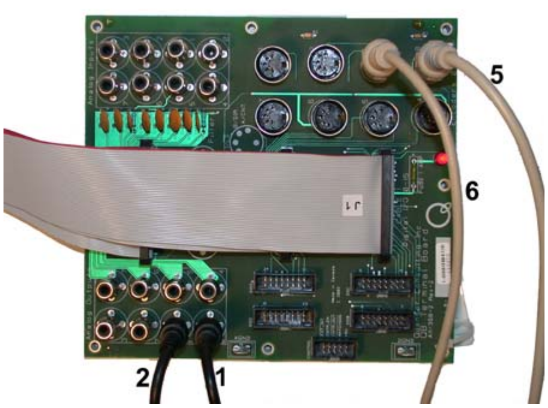


Figure 7: Q8 hardware-in-the-loop board wiring.

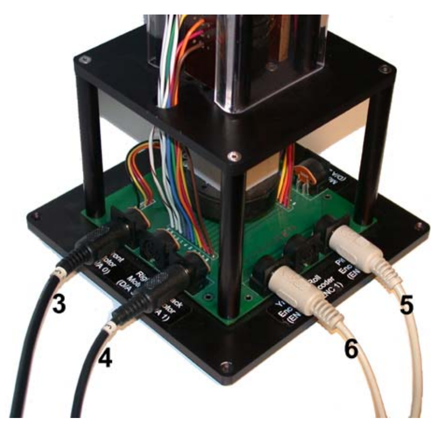


Figure 8: 2 DOF helicopter wiring.

Cable ID #	Cable Type	From	To	Function
1	5-pin-DIN to RCA	Analog Output 0 on the Terminal Board	"From D/A" connector on UPM-2405	Pitch control signal generated by controller running on PC divided by the amplifier gain, $u_p/K_{a,p}$.
2	5-pin-DIN to RCA	Analog Output 1 on the Terminal Board	"From D/A" connector on UPM-1503	Yaw control signal generated by controller running on PC divided by the amplifier gain, $u_y/K_{a,y}$.
3	Motor Cable with **Gain 5**: 4-pin-stereo-DIN to 6-pin-stereo-DIN	"To Load" connector on UPM-2405	Pitch motor connector on 2D HELI	Amplified control signal from PC applied to pitch motor, $V_{m,p}$.
4	Motor Cable with **Gain 3**: 4-pin-stereo-DIN to 6-pin-stereo-DIN	"To Load" connector on UPM-1503	Yaw motor connector on 2D HELI	Amplified control signal from PC applied to yaw motor, $V_{m,y}$.
5	Encoder Cable: 5-pin-stereo-DIN to 5-pin-stereo-DIN	2D HELI Pitch encoder connector	Encoder Channel 0 on the terminal board	Measures the pitch angle, θ.
6	Encoder Cable: 5-pin-stereo-DIN to 5-pin-stereo-DIN	2D HELI Yaw encoder connector	Encoder Channel 1 on the terminal board	Measures the yaw angle, ψ.

Table 4: 2 DOF helicopter system wiring.

Figure 9: UPM-2405 for pitch motor wiring.

Figure 10: UPM-1503 for yaw motor wiring.

3.6 Joystick Setup

The *Logitech Attack 3* USB joystick is supplied with the Quanser 2 DOF Helicopter. It used to control the voltage of the helicopter when running in open-loop and to change the desired position of the helicopter angles when running in closed-loop. The *Rate Command* knob shown in Figure 11 changes the rate at which a command is generated by the joystick. The rate is at its greatest when the knob is turned fully toward the joystick handle.

Figure 11: Logitech Attack 3 joystick.

The system requirements for the joystick are:

- PC with Pentium Processor or compatible
- 64 MB RAM
- USB port
- Windows 98, 2000, Me, or Xp

To setup the joystick, connect the USB cable from the joystick to a USB port on the PC while it is running. The system should detect the joystick and automatically install the driver (you will be prompted). See the *Logitech Installation Manual* for more information on the setup procedure.

CAUTION: Ensure the joystick is connected before opening any of the 2 DOF Helicopter Simulink models. Otherwise an error message will be prompted.

4 Modeling and Control Design

The dynamics of the helicopter is developed in Section 4.1. This model is then used to design a position controller, as discussed in Section 4.2.

4.1 Model

The two degree of freedom helicopter pivots about the pitch axis by angle θ and about the yaw axis by angle ψ. As shown in Figure 12, the pitch is defined positive when the nose of the helicopter goes up and the yaw is defined positive for a clockwise rotation. Also in Figure 12, there is a thrust force F_p acting on the pitch axis that is normal to the plane of the front propeller and a thrust force F_y acting on the yaw axis that is normal to the rear propeller. Therefore a pitch torque is being applied at a distance r_p from the pitch axis and a yaw torque is applied at a distance r_y from the yaw axis. The gravitational force, F_g, generates a torque at the helicopter center of mass that pulls down on the helicopter nose. As shown in Figure 12, the center of mass is a distance of l_{cm} from the pitch axis along the helicopter body length.

The equations of motions are found using the Lagrangian method. First, in Section 4.1.1 the forward

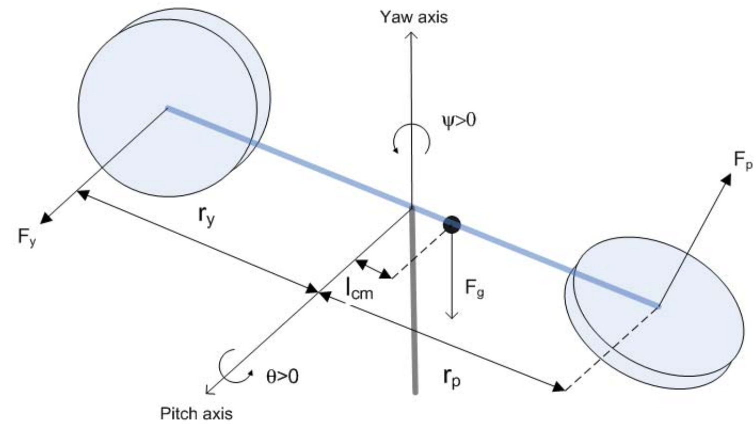


Figure 12: Dynamics of 2 DOF Helicopter.

kinematics of the helicopter center of mass is derived. The kinematics are then used to find the potential and kinetic energies involved in the helicopter system, detailed in Section 4.1.2. In Section 4.1.3, the Euler-Lagrange method is used to derive the nonlinear equations describing the motions of the helicopter. From its nonlinear equations of motions, the linear state-space model of the helicopter is found in Section 4.1.4.

See the Maple worksheet entitled *2DOF Helicopter Equations.mws* or its HTML equivalent, *2DOF Helicopter Equation.html* for the model derivation details.

4.1.1 Kinematics

The helicopter center of mass is to be described in xyz cartesian coordinates with respect the pitch, θ, and yaw, ψ, angles. As illustrated in Figure 13, the coordinate system $Ox_3y_3z_3$ is located at the center of mass of the helicopter and the base coordinate system $Ox_0y_0z_0$ is located at the pivot point of the helicopter. The pivot point of the helicopter is defined as the point where the pitch axis and yaw axis intersect, or the midpoint of the pitch axis. The base coordinate system $Ox_0y_0z_0$ is transformed to $Ox_3y_3z_3$ using the transformation matrix

$$\begin{aligned} T_0^3 &= Rot_{z_0,\psi} Rot_{y_1,\theta} Trans_{x_2,l_{cm}} \\ &= \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos\psi\cos\theta & \sin\psi & -\cos\psi\sin\theta & l_{cm}\cos\psi\cos\theta \\ -\sin\psi\cos\theta & \cos\psi & \sin\psi\sin\theta & -l_{cm}\sin\psi\cos\theta \\ \sin\theta & 0 & \cos\theta & l_{cm}\sin\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{1}$$

As described in Figure 13, the transformation includes a counter-clockwise yaw rotation about z_0 by $-\psi$, a clockwise pitch rotation about y_1 by $-\theta$, and a translation along the axis x_2 by a distance l_cm. The rotations are using negative angles because they were defined as shown in Figure 12. From the result in (1),

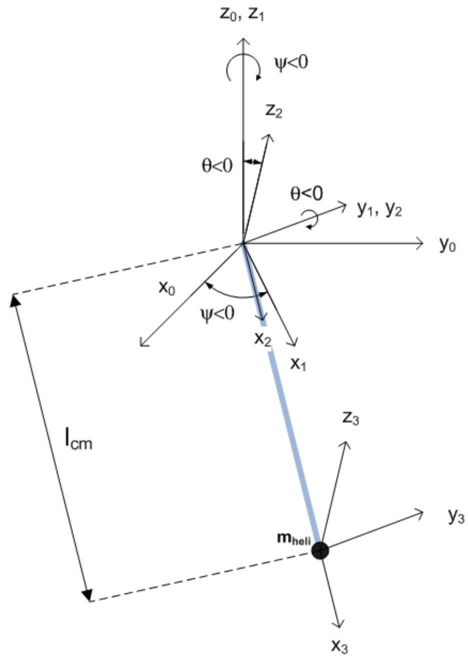


Figure 13: Kinematics of 2 DOF Helicopter.

the cartesian position of the helicopter center of mass is

$$\begin{aligned} x_{cm} &= l_{cm}\cos\psi\cos\theta \\ y_{cm} &= -l_{cm}\sin\psi\cos\theta \\ z_{cm} &= l_{cm}\sin\theta, \end{aligned} \tag{2}$$

where for the pitch motor mass $m_{m,p}$, the yaw motor mass $m_{m,y}$, and the propeller and shield mass assembly m_{shield} the center of mass is calculated with the expression

$$l_{cm} = \frac{(m_{m,p} + m_{shield})\, r_p + (m_{m,y} + m_{shield})\, r_y}{m_{m,p} + m_{m,y} + 2m_{shield}}. \tag{3}$$

As illustrated in Figure 12, r_p is the distance between the pivot and the pitch motor center and r_y is the distance between the pivot and the yaw motor center. The value of center of mass along with the various masses and lengths used to calculate it are defined in Table 3. These cartesian coordinates can now be used to model the energies of the helicopter system.

4.1.2 Kinetic and Potential Energy

The potential energy due to gravity is

$$\begin{aligned} V &= m_{heli} g z_{cm} \\ &= m_{heli} g l_{cm}\sin\theta. \end{aligned} \tag{4}$$

The total kinetic energy

$$T = T_{r,p} + T_{r,y} + T_t \tag{5}$$

is the sum of the rotational kinetic energies acting from the pitch, $T_{r,p}$, and from the yaw, $T_{r,y}$, along with the translational kinetic energy generated by the moving center of mass, T_t. The pitch rotational kinetic energy is

$$T_{r,p} = \frac{1}{2} J_{eq,p}\dot{\theta}^2 \tag{6}$$

and yaw rotational kinetic energy is

$$T_{r,y} = \frac{1}{2} J_{eq,y} \dot{\psi}^2, \tag{7}$$

where $J_{eq,p}$ and $J_{eq,y}$ are the equivalent moment of inertias of the pitch and yaw, respectively. The translational kinetic energy is

$$T_t = \frac{1}{2} m_{heli} \sqrt{\dot{x}_{cm}^2 + \dot{y}_{cm}^2 + \dot{z}_{cm}^2}, \tag{8}$$

where the three-dimensional velocity of the center of mass is found by taking the time-derivatives of the kinematic equations in (2),

$$\begin{aligned} \dot{x}_{cm} &= -l_{cm} \left(\dot{\psi} \sin\psi \cos\theta + \dot{\theta} \cos\psi \sin\theta \right) \\ \dot{y}_{cm} &= l_{cm} \left(-\dot{\psi} \cos\psi \cos\theta + \dot{\theta} \sin\psi \sin\theta \right) \\ \dot{z}_{cm} &= l_{cm} \dot{\theta} \cos\theta. \end{aligned} \tag{9}$$

In terms of the pitch and yaw angles the translational kinetic energy is

$$\begin{aligned} T_t =& \frac{1}{2} m_{heli} \left(-l_{cm} \dot{\psi} \sin\psi \cos\theta - l_{cm} \dot{\theta} \cos\psi \sin\theta \right)^2 + \left(-l_{cm} \dot{\psi} \cos\psi \cos\theta + l_{cm} \dot{\theta} \sin\psi \sin\theta \right)^2 \\ &+ l_{cm}^2 \dot{\theta}^2 \cos\theta^2. \end{aligned} \tag{10}$$

The potential and kinetic energy expressed here are used to derive the equations of motions in the next section.

4.1.3 Nonlinear Equation of Motion

For the 2 DOF Helicopter, the Euler-Lagrange equations are

$$\begin{aligned} \frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{q}_1} - \frac{\partial}{\partial q_1} L =& Q_1 \\ \frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{q}_2} - \frac{\partial}{\partial q_2} L =& Q_2, \end{aligned} \tag{11}$$

where L is the Lagrange variable and is the difference between the kinetic and potential energy of the system, $L = T - V$. The generalized coordinate are

$$\begin{aligned} \mathbf{q} &= \left[\begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \end{array} \right]^\top \\ &= \left[\begin{array}{cccc} \theta & \psi & \dot{\theta} & \dot{\psi} \end{array} \right]^\top \end{aligned} \tag{12}$$

and the generalized forces are

$$\begin{aligned} Q_1 =& \tau_p(V_{m,p}, V_{m,y}) - B_p \dot{\theta} \\ Q_2 =& \tau_y(V_{m,p}, V_{m,y}) - B_y \dot{\psi}. \end{aligned} \tag{13}$$

Equation (13) includes the viscous rotary friction acting about the pitch and yaw axes: B_p and B_y. The torques applied at the pitch and yaw axis from the motors are

$$\begin{aligned} \tau_p(V_{m,p}, V_{m,y}) =& K_{pp} V_{m,p} + K_{py} V_{m,y} \\ \tau_y(V_{m,p}, V_{m,y}) =& K_{yp} V_{m,p} + K_{yy} V_{m,y} \end{aligned} \tag{14}$$

where $V_{m,p}$ is the input pitch motor voltage and $V_{m,y}$ is the input yaw motor voltage. The torques acting on the pitch and yaw axes are *coupled*. The torque-constants used in Equation (14) are

$$\begin{aligned} K_{pp} =& K_{f,p} r_p \\ K_{yy} =& K_{f,y} r_y \\ K_{py} =& \frac{K_{t,y}}{R_{m,y}} \\ K_{yp} =& \frac{K_{t,p}}{R_{m,p}} \end{aligned} \tag{15}$$

where $K_{f,p}$ and $K_{f,y}$ are the thrust force constants of the pitch and yaw motor/propeller actuators found experimentally, $K_{t,p}$ and $K_{t,y}$ are the current-torque constants of the pitch and yaw motors, and $R_{m,p}$ and $R_{m,y}$ are the electrical resistances of the pitch and yaw motors. The values of these parameters are given in Table 2.

Intuitively speaking, the thrust force when $V_{m,p} > 0$ causes the helicopter nose to elevate but also generates a torque about the yaw axis that makes the helicopter rotate clockwise, i.e. positive yaw angle. Similarly, a positive thrust by the yaw motor, $V_{m,y} > 0$, generates a torque about the yaw axis causing the helicopter to rotate clockwise but also generates a torque about the pitch axis that causes the nose of the helicopter to go upwards.

Thus the main torque generated by the pitch motor on the pitch axis is $\tau_{pp} = K_{pp}V_{m,p}$ and, similarly, the main torque acting on the yaw axis is $\tau_{yy} = K_{yy}V_{m,y}$. The torque generated by the yaw motor that acts on the the pitch axis is $\tau_{py} = K_{py}V_{m,y}$. Likewise, the pitch motor generates a rotary force about the yaw axis, $\tau_{yp} = K_{yp}V_{m,p}$.

Evaluating the Euler-Lagrange expressions in Equation (11) using the coordinates defined in (12) and the forces in (13) results in the nonlinear equations of motion

$$\begin{aligned} \left(J_{eq,p} + m_{heli}l_{cm}^2\right)\ddot{\theta} =& K_{pp}V_{m,p} + K_{py}V_{m,y} - m_{heli}gl_{cm}\cos\theta - B_p\dot{\theta} - m_{heli}l_{cm}^2\sin\theta\cos\theta\dot{\psi}^2 \\ \left(J_{eq,y} + m_{heli}l_{cm}^2\cos\theta^2\right)\ddot{\psi} =& K_{yy}V_{m,y} + K_{yp}V_{m,p} - B_y\dot{\psi} + 2m_{heli}l_{cm}^2\sin\theta\cos\theta\dot{\psi}\dot{\theta}. \end{aligned} \tag{16}$$

The equivalent moment of inertia about the center of mass in Equation (16) equals

$$\begin{aligned} J_{eq,p} =& J_{m,p} + J_{body,p} + J_p + J_y \\ J_{eq,y} =& J_{m,y} + J_{body,y} + J_p + J_y + J_{shaft} \end{aligned} \tag{17}$$

where $J_{m,p}$ and $J_{m,y}$ are the moment of inertias of the motor rotor given in the specifications and

$$\begin{aligned} J_{body,p} =& \frac{m_{body,p}L_{body}^2}{12} \\ J_{body,y} =& \frac{m_{body,y}L_{body}^2}{12} \\ J_{shaft} =& \frac{m_{shaft}L_{shaft}^2}{3} \\ J_p =& \left(m_{m,p} + m_{shield}\right)r_p^2 \\ J_y =& \left(m_{m,y} + m_{shield}\right)r_y^2. \end{aligned} \tag{18}$$

See Table 3 for the values of all these inertias.

Alternatively, the equations of motion can be packaged in the matrix form

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau \tag{19}$$

with the inertia, damping, gravitational, and applied torque matrices

$$\begin{aligned} \mathbf{D}(\mathbf{q}) &= \begin{bmatrix} J_{eq,p} + m_{heli}l_{cm}^2 & 0 \\ 0 & J_{eq,y} + m_{heli}l_{cm}^2\cos\theta^2 \end{bmatrix} \\ \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) &= \begin{bmatrix} B_p & m_{heli}l_{cm}^2\sin\theta\cos\theta\dot{\psi} \\ -2m_{heli}l_{cm}^2\sin\theta\cos\theta\dot{\psi} & B_y \end{bmatrix} \\ \mathbf{g}(\mathbf{q}) &= \begin{bmatrix} m_{heli}gl_{cm}\cos\theta \\ 0 \end{bmatrix} \\ \tau &= \begin{bmatrix} K_{pp}V_{m,p} + K_{py}V_{m,y} \\ K_{yp}V_{m,p} + K_{yy}V_{m,y} \end{bmatrix}. \end{aligned} \tag{20}$$

4.1.4 Linear State-Space Model

This section derives the linear state-space model of the helicopter that is used to design the position controller, later in Section 4.2. Linearizing the nonlinear equations of motion in (16) about the quiecent point ($\theta_0 = 0, \psi_0 = 0, \dot{\theta}_0 = 0, \dot{\psi}_0 = 0$) gives

$$\begin{aligned}\left(J_{eq,p} + m_{heli} l_{cm}^2\right)\ddot{\theta} &= K_{pp} V_{m,p} + K_{py} V_{m,y} - B_p \dot{\theta} - m_{heli} g l_{cm} \\ \left(J_{eq,y} + m_{heli} l_{cm}^2\right)\ddot{\psi} &= K_{yy} V_{m,y} + K_{yp} V_{m,p} - B_y \dot{\psi} + 2 m_{heli} l_{cm}^2 \theta \dot{\psi} \dot{\theta}.\end{aligned} \tag{21}$$

Substituting the state

$$\mathbf{x} = \left[\begin{array}{cccc} \theta & \psi & \dot{\theta} & \dot{\psi} \end{array}\right]^{\top} \tag{22}$$

in (21) and solving for the $\dot{x}$ results in the linear state-space model

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{B_p}{J_{eq,p}+m_{heli}l_{cm}^2} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{eq,y}+m_{heli}l_{cm}^2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{eq,p}+m_{heli}l_{cm}^2} & \frac{K_{py}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ \frac{K_{yp}}{J_{eq,y}+m_{heli}l_{cm}^2} & \frac{K_{yy}}{J_{eq,y}+m_{heli}l_{cm}^2} \end{bmatrix}\mathbf{u} \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\mathbf{x}\end{aligned} \tag{23}$$

where

$$\mathbf{u} = \left[\begin{array}{cc} V_{m,p} & V_{m,y} \end{array}\right]^{\top}. \tag{24}$$

The output variable in (23) is $y = [x_1, x_2, x_3, x_4]^{\top}$ and this implies that all the states are being measured. However in reality only the pitch and yaw positions are being measured by the encoder sensors and the velocity of these angles is calculated digitally using the Laplace transform

$$V_f(s) = \frac{\omega_c^2 s}{s^2 + 2\zeta\omega_c + \omega_c^2} X(s) \tag{25}$$

where $X(s)$ is the Laplace transform of the measured position, $V_f(s)$ is the Laplace transform of the output filtered derivative (i.e. the filtered velocity), ω_c is the cutoff frequency of the filter, and ζ is the damping ratio of the filter. In order to design a controller using the Linear-Quadratic Regulator technique, discussed in Section 4.2, it must be assumed that all the states are measured.

4.2 Control Design

Two controller are designed in this section: FF+LQR and FF+LQR+I. The FF+LQR is designed in Section 4.2.1 and it regulates the pitch axis of the helicopter using feed-forward (FF) and proportional-velocity (PV) compensators and the yaw axis using a PV control. As discussed in Section 4.2.2, the FF+LQR+I controller introduces an integrator in the feedback loop to improve the steady-state error. It uses a feed-forward and a proportional-integral-velocity (PIV) algorithms to regular the pitch and a PIV to control the yaw angle.

The input voltages of the pitch and yaw motors with respect to the pitch and yaw control output u_p and u_y, using either FF+LQR or FF+LQR+I, are

$$\begin{aligned}V_{m,p} &= \begin{cases} u_p & V_{m,p} > V_{p,min} \text{ and } V_{m,p} < V_{p,max} \\ V_{p,min} & V_{m,p} \leq V_{p,min} \\ V_{p,max} & V_{m,p} \geq V_{p,min} \end{cases} \\ V_{m,y} &= \begin{cases} u_y & V_{m,y} > V_{y,min} \text{ and } V_{m,y} < V_{y,max} \\ V_{y,min} & V_{m,y} \leq V_{y,min} \\ V_{y,max} & V_{m,y} \geq V_{y,min} \end{cases}\end{aligned} \tag{26}$$

These input voltages enter model (23). When using the UPM-2405, the pitch controller u_p is saturated by the maximum amplifier voltage $V_{p,max} = 24$ V and the minimum voltage $V_{p,min} = -24$ V. Similarly, the yaw control when using the UPM-1503 is limited to a maximum voltage of $V_{y,max} = 15$ V and a minimum voltage of $V_{y,min} = -15$ V. This is an important constraint in the control design.

In both the FF+LQR and the FF+LQR+I controllers, the pitch position is regulated using a nonlinear feed-forward loop that compensates for the gravitational torque $\tau_g = m_{heli} g l_{cm} \cos\theta$ shown in (16), the nonlinear equations of motion. The feed-forward control is

$$u_{ff} = K_{ff} \frac{m_{heli} g l_{cm} \cos\theta_d}{K_{pp}}, \tag{27}$$

where θ_d is the desired pitch angle and K_{ff} is the feed-forward control gain. This applies the bulk of the voltage needed to hover the helicopter at the commanded position. This feed-forward action works in conjunction with a PV or PIV compensator.

4.2.1 FF+LQR

In this section the feed-forward plus proportional-velocity control

$$\begin{aligned} u_p &= u_{ff} + u_{lqr,p} \\ u_y &= u_{lqr,y} \end{aligned} \tag{28}$$

is developed. The feed-forward control u_{ff} is given in (27). The proportional-velocity control $[u_{lqr,p}, u_{lqr,y}]^\top = -\mathbf{k} \cdot \mathbf{x}$ is designed using the Linear-Quadratic Regulator (LQR) technique. Using the linear state-space model (23) and the weighting matrices

$$\mathbf{Q} = \begin{bmatrix} 200 & 0 & 0 & 0 \\ 0 & 200 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{29}$$

the LQR algorithm generates the control gain

$$\mathbf{k} = \begin{bmatrix} 14.1 & 1.33 & 7.33 & 0.924 \\ -1.33 & 14.1 & -0.261 & 7.99 \end{bmatrix}. \tag{30}$$

In summary the FF+LQR control that converges $(\theta, \psi, \dot\theta, \dot\psi) \rightarrow (\theta_d, \psi_d, 0, 0)$, where θ_d is the desired pitch angle and ψ_d is the desired yaw angle, is

$$\begin{bmatrix} u_p \\ u_y \end{bmatrix} = \begin{bmatrix} K_{ff} \frac{m_{heli} g l_{cm} \cos\theta_d}{K_{pp}} \\ 0 \end{bmatrix} - \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \begin{bmatrix} \theta - \theta_d \\ \psi - \psi_d \\ \dot\theta \\ \dot\psi \end{bmatrix}. \tag{31}$$

This voltage calculated by this algorithm is, however, saturated by the amplifiers as described in (26).

4.2.2 FF+LQR+I

In this section an integrator is introduced to the FF+LQR control to minimize steady-state error. By introducing the states $\dot{x}_5 = \theta$ and $\dot{x}_6 = \psi$ the linear state-space model (23) is augmented to

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{B_p}{J_{eq,p}+m_{heli}l_{cm}^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{eq,y}+m_{heli}l_{cm}^2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{eq,p}+m_{heli}l_{cm}^2} & \frac{K_{py}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ \frac{K_{yp}}{J_{eq,y}+m_{heli}l_{cm}^2} & \frac{K_{yy}}{J_{eq,y}+m_{heli}l_{cm}^2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}. \tag{32}$$

Repeating the LQR process in Section 4.2.1 on model (32) above with the weighting matrices

$$\mathbf{Q} = \begin{bmatrix} 200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 200 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{33}$$

produces the control gain

$$\mathbf{k} = \begin{bmatrix} 18.9 & 1.98 & 7.48 & 1.53 & 7.03 & 0.770 \\ -2.22 & 19.4 & -0.450 & 11.9 & -0.770 & 7.03 \end{bmatrix}. \tag{34}$$

Thus the FF+LQR+I controller is

$$\begin{bmatrix} u_p \\ u_y \end{bmatrix} = \begin{bmatrix} K_{ff}\frac{m_{heli}gl_{cm}\cos\theta_d}{K_{pp}} \\ 0 \end{bmatrix} - \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \begin{bmatrix} \theta-\theta_d \\ \psi-\psi_d \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} \int k_{15}(\theta-\theta_d) + \int k_{16}(\psi-\psi_d) \\ \int k_{25}(\theta-\theta_d) + \int k_{26}(\psi-\psi_d) \end{bmatrix} \tag{35}$$

The helicopter system runs the risk of integrator windup. That is, given a large error in the between

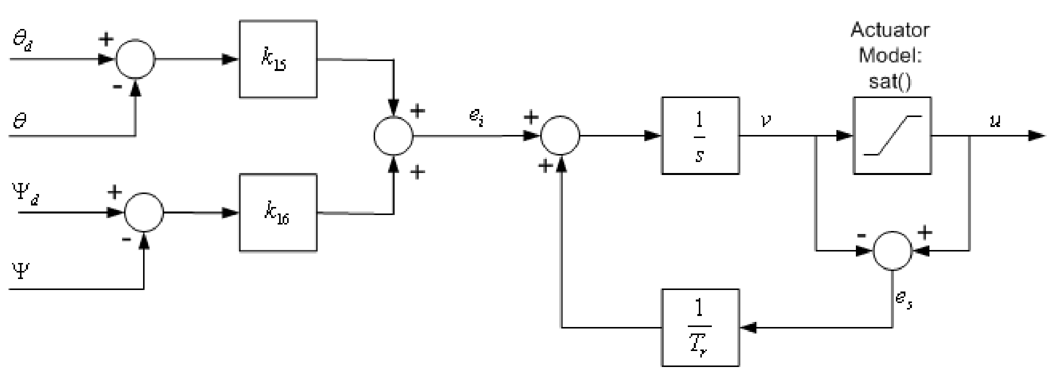


Figure 14: Anti-windup implementation.

the measured and desired pitch angle, $\theta - \theta_d$, or between the measured and desired yaw angle, $\psi - \psi_d$, the integrator outputs a large voltage that can saturate the amplifier. By the time the measured angle reaches the desired angle the integrator built-up so much energy that it remains saturated. This can cause large overshoots and oscillations in the response. To fix this, an integral windup protection algorithm is used. Figure 14 illustrates the anti-windup scheme implemented to control the pitch. When the integrator output voltage v is larger than the imposed integral saturation then the saturation error becomes negative, $e_s = u - v < 0$. The saturation error gets divided by the reset time, T_r, and its result is added to the integrator input. This effectively decreases the integrator input and winds-down the integrator. The integrator input when the saturation occurs is

$$e_i = k_{15}(\theta_d - \theta) + k_{16}(\psi_d - \psi) + \frac{u - v}{T_r}. \tag{36}$$

In the simulation and experimental results the saturation limit of the integrator is set to 5 Volts and the reset time to 1 second for maximum wind-down speed.

5 In-Lab Procedure

The various files supplied with the experiment such as Simulink models and Matlab scripts are summarized in Section 5.1. Section 5.2 overviews the various subsystems used to build the Simulink models supplied with the helicopter. The procedure to run a simulation of the closed-loop response using the position controller is explained in Section 5.3. Section 5.4 describes how to run the actual helicopter device in open-loop using the joystick. The closed-loop controller is then implemented on the helicopter in Section 5.5 allowing the user to command pitch and yaw angles. Finally, in Section 5.6, how to perform model validation and identify the viscous damping parameter on the yaw axis is described.

5.1 Experiment Design Files

The lists of files supplied with the Quanser 2 DOF Helicopter experiment is given in Table 5.

5.2 Description of 2 DOF Helicopter Library

The *heli_2d_lib.mdl* Simulink model shown in Figure 15 is the Quanser 2 DOF Helicopter Simulink library and it contains various subsystem that are used in the supplied Simulink models listed in Table 5. The *2DOF HELI: FF+LQR Controller* is the feed-forward and proportional-velocity control designed in Section 4.2.1 and *2DOF HELI: FF+LQR+I Controller* implements the feed-forward control and the LQR PIV position controller discussed in Section 4.2.2. The nonlinear feed-forward control, see Section 4.2, is constructed in the *Pitch feed-forward controller* block. The *2DOF HELI - Q8* Simulink subsystem interacts with the hardware of the 2 DOF Helicopter through the Q8 DAC board using the Quanser Toolbox blocks. The *2DOF HELI: Nonlinear Model* contains the nonlinear model developed in Section 4.1. Finally, the *Scopes* system has various Simulink scopes for displaying, for instance, the desired, measured, and simulated pitch and yaw angles of the 2 DOF Helicopter.

The interior of the *2DOF HELI: FF+LQR+I Controller* subsystem is displayed in Figure 16. As discussed in Section 4.2.2, the position and velocity states are multiplied by the corresponding elements of control gain **K**. The state includes the integral of the pitch and yaw angles and those are multiplied by the integral gains in **K**. Further the anti-windup scheme shown in Figure 14 is implemented in the *Pitch Integral Antiwindup* and *Yaw Integral Antiwindup*. Note that the green feed-forward block in Figure 16 is linked to the same block in the *heli_2d_lib.mdl* Simulink library shown in Figure 15. Similarly in the *2DOF HELI: FF+LQR Controller* subsystem, this block is referenced.

5.3 Closed-loop Simulation

5.3.1 Objectives

The objectives of simulating the 2 DOF Helicopter controller are:

File Name	Description
2d_heli.pdf	The user and control manual for the 2 DOF Helicopter device. It contains information to setup and configure the laboratory, explains how to derive the system model, outlines the control design, and gives procedures to simulate and implement the controllers.
2DOF Helicopter Equations.mws	Maple worksheet used to analytically derive the linear and nonlinear dynamics of the system as well as its linear state-space. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
2DOF Helicopter Equations.html	HTML presentation of the 2DOF Helicopter Equations.mws file. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
quanser_tools.mws	Executing this worksheet generates the quanser repository containing the Quanser_Tools package. The two package files are named: quanser.ind and quanser.lib. The Quanser_Tools module defines the generic procedures used in Lagrangian mechanics and resulting in the determination of a given system's equations of motion and state-space representation. It also contains data processing routines to save the obtained state-space matrices into a Matlab readable file.
quanser_tools.rtf	Rich Text Format presentation of the quanser_tools.mws file. It allows to view the content of the Maple worksheet without having Maple 9 installed. No modifications to the described Maple procedures can be performed when in this format.
setup_lab_heli_2d.m	The main Matlab script that calls setup_heli_2d_configuration.m, HELI_2D_ABCD_eqns.m, d_heli_2d_lqr.m, d_heli_2d_lqr_i.m to set the model and control parameters. It configures the encoder resolution, UPM voltage limits, filter cutoff frequency, joystick settings and so on. **Run this file only to setup the laboratory**.
setup_heli_2d_configuration.m	Sets the model attributes K_{pp}, K_{yy}, K_{yp}, K_{py}, $J_{eq,p}$, $J_{eq,y}$, B_p, B_y, m_{heli}, l_{cm}, and g.
HELI_2D_ABCD_eqns.m	Matlab script file generated using the Maple worksheet 2DOF Helicopter Equations.mws. It sets the A, B, C, and D matrices for the state-space representation of the 2 DOF Helicopter open-loop system used in d_heli_2d_lqr.mdl and d_heli_2d_lqr_i.mdl.
d_heli_2d_lqr.m	Matlab script that generates the position-velocity controller gain K using LQR. It is used in the Simulink models: s_heli_2d_ff_lqr_i.mdl and q_heli_2d_ff_lqr_i_zz.mdl.
d_heli_2d_lqr_i.m	Matlab function that the position-integral-velocity controller gain K using LQR. It is used in the Simulink models: s_heli_2d_ff_lqr_i.mdl and q_heli_2d_ff_lqr_i_zz.mdl.
heli_2d_lib.mdl	The supplied Simulink models are linked to the blocks in this Simulink library, as explained in Section 5.2.
s_heli_2d_ff_lqr_i.mdl	Simulink file that simulates the open-loop or closed-loop 2 DOF Helicopter system using a nonlinear model of the helicopter.
q_heli_2d_ff_lqr_i_zz.mdl	Simulink file that implements the real-time position controller for the 2 DOF Helicopter system with the board specified by the *zz*. For example if *zz* is *q8* then the file is setup for the Quanser Q8 DAC board.
q_heli_2d_open_loop_zz.mdl	Simulink file that implements the real-time open-loop algorithm for the 2 DOF Helicopter system with the board specified by the *zz*.

Table 5: Experiment Files

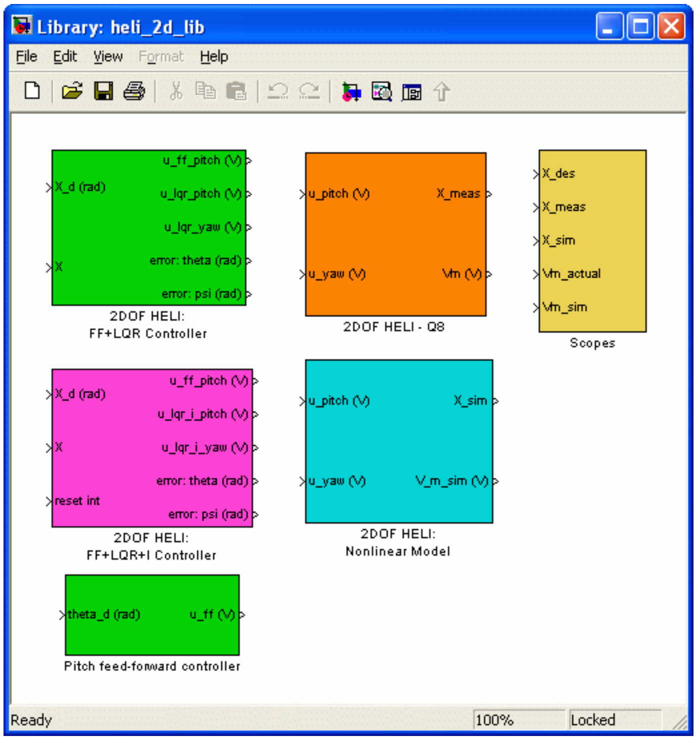


Figure 15: 2 DOF Helicopter Simulink Library.

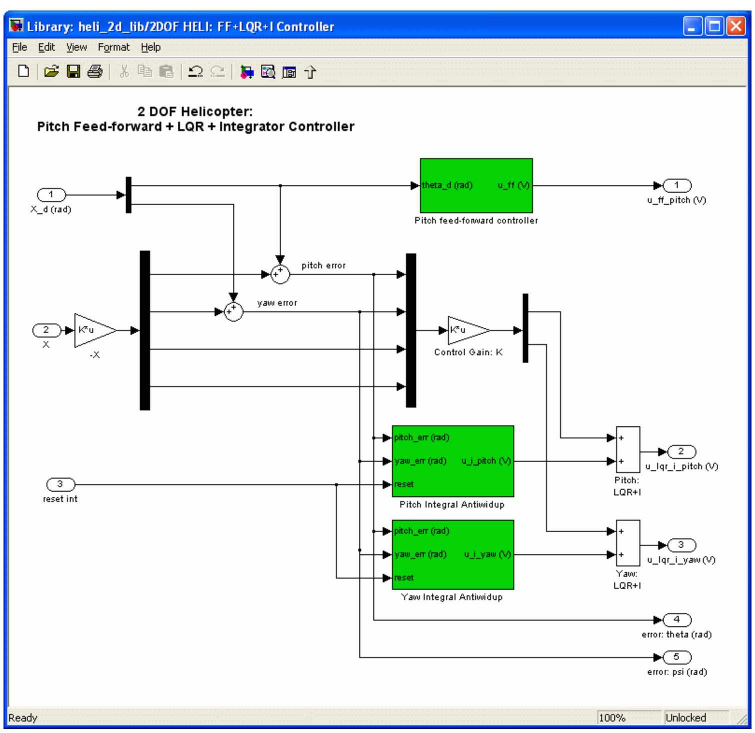


Figure 16: 2 DOF HELI: FF+LQR+I Controller subsystem.

- Become familiar with the supplied Simulink models and the some settings in the Matlab script *setup_lab_heli_2d.m*.
- Investigate the closed-loop performance of the FF+LQR and the LQR+I using the nonlinear model of the 2 DOF Helicopter system.
- Ensure the closed-loop controller does not saturate the actuator.

5.3.2 Procedure

Follow these steps to simulate the closed-loop response of the 2 DOF Helicopter:

1. Load the Matlab software.
2. Open the Simulink model entitled *s_heli_2d_ff_lqr_i.mdl* shown in Figure 17.

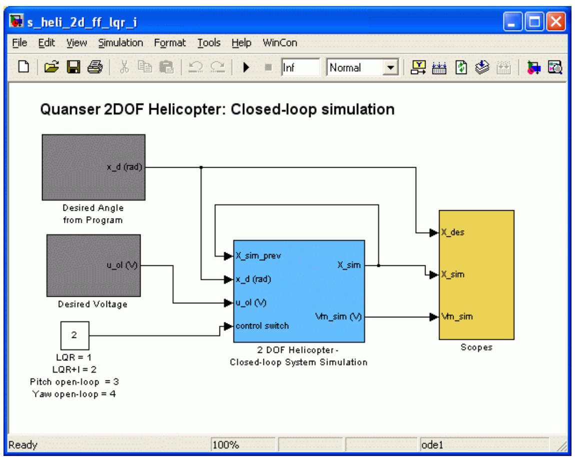


Figure 17: Simulink model used to simulate 2 DOF Helicopter response.

3. The subsystem labeled *Desired Angle from Program* is used to generate a desired pitch and yaw angle while the *Desired Voltage* block feeds open-loop voltages. The *Controller Switch* block implements the following switching logic:

   (a) switch = 1: FF+LQR closed-loop control.

   (b) switch = 2: FF+LQR+I closed-loop control.

   (c) switch = 3: Apply open-loop voltage to pitch motor.

   (d) switch = 4: Apply open-loop voltage to yaw motor.

   When the switch is 1 or 2 the system runs in closed-loop and when it is 3 or 4 the user can command voltages directly to the actuators. When the switch is made from the closed-loop mode to open-loop mode the controller voltage values are latched and the *Desired Voltage* block shown in Figure 17 is enabled. This is particularly useful in Section 5.6 when performing model validation and parameter tuning.

4. The interior of the *2DOF Helicopter - Closed-loop System Simulation* subsystem is shown in Figure 18. The LQR and LQR+I control blocks along with the nonlinear model are all linked to the 2 DOF Helicopter Library and are described in Section 5.2. The *Controller Switch* subsystem implements the logic to switch between the FF+LQR and FF+LQR+I controllers and between the pitch and yaw open-loop modes.

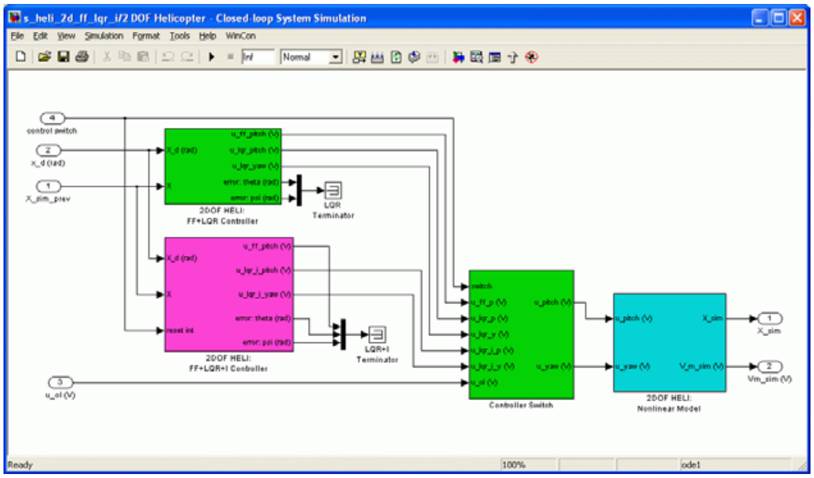


Figure 18: *2 DOF Helicopter - Closed-loop System Simulation* subsystem.

5. Open the Matlab script called *setup_lab_heli_2d.m*. This script sets the model parameters, control gains, amplifier limits, and so on that are used in the 2 DOF Helicopter Simulink models supplied, such as *s_heli_2d_ff_lqr_i.mdl*. By default, K_CABLE_P, K_CABLE_Y, VMAX_UPM_P, VMAX_UPM_Y, K_EC_P, and K_EC_Y is set to match the configuration used in Section 3.5. If, for example, a gain cable of 3 is used with the UPM-2405 instead of 5 then the script parameter K_CABLE_P must be changed to 3. The parameter *theta_0* initializes the integrator labeled *theta* in the *2DOF HELI: Nonlinear Model* that calculates the pitch position.

6. The saturation limit of the integrators that are used in the FF+LQR+I controller are set using the variables SAT_INT_ERR_PITCH and SAT_INT_ERR_YAW. The reset time of the anti-windup loop can be changed using *Tr_p* and *Tr_y*. For more information on the anti-windup algorithm see Section 4.2.2.

7. Ensure the CONTROLLER_TYPE is set to 'LQR_AUTO' to generate the controller automatically. Set the feed-forward gain $K_{ff} = 1$ V/V and the LQR and LQR+I **Q** and **R** weighting matrices as specified in sections 4.2.1 and 4.2.2.

8. Run the Matlab script to load the various parameter in the Matlab workspace. In the Matlab command window, the LQR gain shown in (30) and the LQR+I gain shown in (34) should be generated.

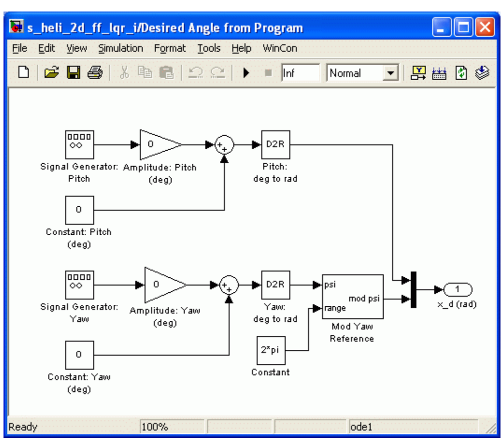


Figure 19: *Desired Angle from Program* subsystem.

9. In the Simulink model, open the subsystem labeled *Desired Angle from Program* as shown in Figure 19.
10. Set the *Amplitude: Pitch (deg)* to 10 and the *Amplitude: Yaw (deg)* block to 0.
11. Click on the *Start simulation* button, or on the *Start* item in the *Simulation* menu, to run the closed-loop system using LQR+I and the scopes should read as shown in figures 20 and 21.

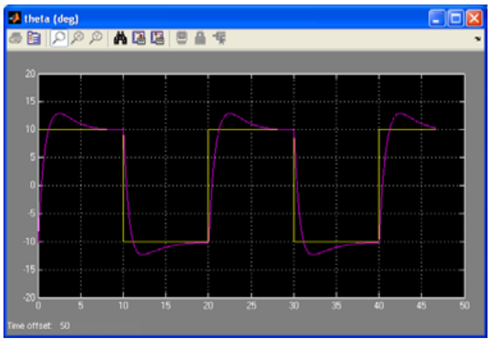


Figure 20: Simulated pitch angle response under pitch reference step using LQR+I.

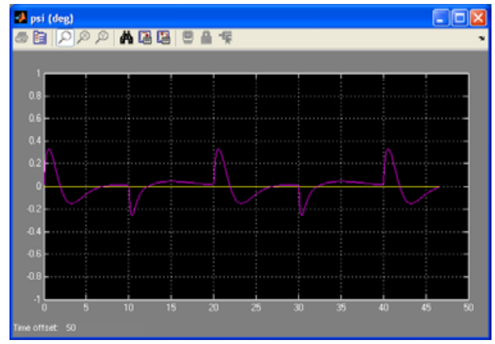


Figure 21: Simulated yaw angle response under pitch reference step using LQR+I.

12. To view the response under a yaw step reference of 100 degrees, set the *Amplitude: Pitch (deg)* to 0 and the *Amplitude: Yaw (deg)* block to 50 in the *Desired Angle from Program* subsystem. The scopes should read as shown in figures 22 and 23.

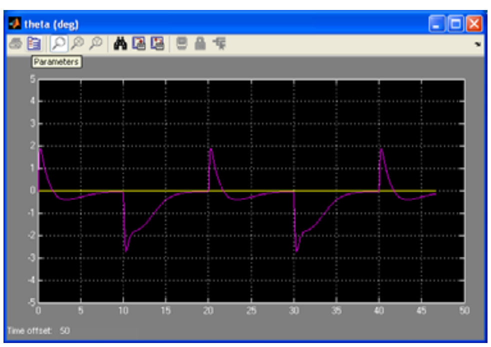


Figure 22: Simulated FF+LQR+I pitch angle response under yaw reference step using.

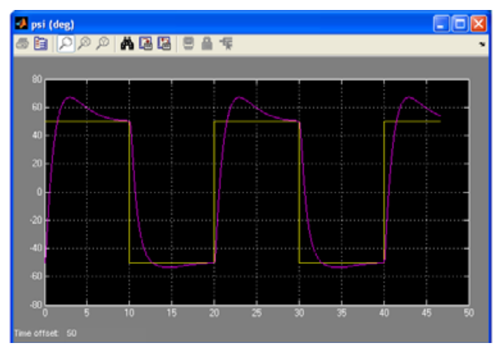


Figure 23: Simulated FF+LQR+I yaw angle response under yaw reference step.

13. The *Vm_sim(V)* scope shown in Figure 24 displays the simulated voltages used to control the helicopter model. The yaw voltage saturates the amplifier at ±15 V.
14. Select the *Stop simulation* button, or click on *Stop* in the *Simulation* menu, to end the session.

5.4 Open-loop Implementation

5.4.1 Objectives

The objectives of running the 2 DOF Helicopter in open-loop are to:

- Gain an intuition of the dynamics, in particular the coupling effect that exists between the pitch and yaw actuators.

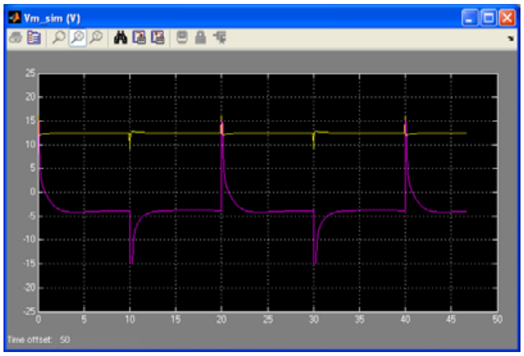


Figure 24: Simulated FF+LQR+I input voltage under yaw reference step.

- Obtain an idea on how difficult it is to control the apparatus in order to compare human operator performance with computer control.

5.4.2 Procedure

1. Before starting, confirm that the experiment has been connected properly as instructed in Section 3.5 and that the joystick is setup as discussed in Section 3.6.
2. Set the *Rate Command* knob on the joystick approximately at the midpoint, as shown in Section 3.6.
3. Load the Matlab software.
4. Open the Simulink model entitled *q_heli_2d_open_loop* shown in Figure 25.

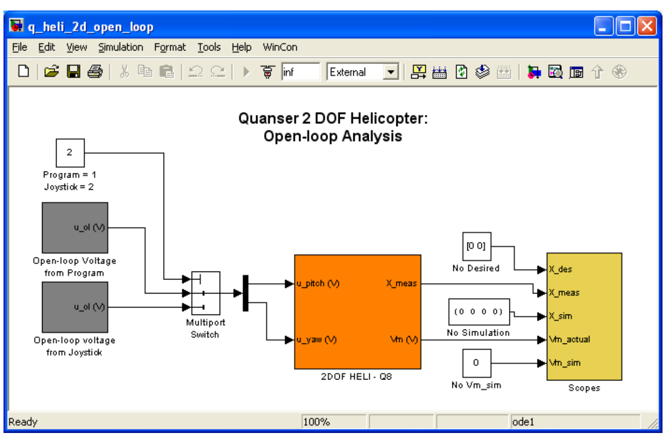


Figure 25: Simulink/WinCon model used to run the helicopter in open-loop.

5. Set the *Program/Joystick* source block to 2 so the open-loop voltage is generated using the joystick.
6. Run the Matlab script *setup_lab_heli_2d.m* to set the model parameters, control gains, power amplifier limits, and so on. The K_JOYSTICK_V_X and K_JOYSTICK_V_Y parameters control the rate that a voltage command is generated. The JOYSTICK_X_DZ and JOYSTICK_Y_DZ specify the deadzone of the joystick. The deadzone is used to remove negligible joystick outputs due to noise or from small motions in the joystick handle.
7. Select the *Build* item in the *WinCon* menu of the Simulink diagram to compile the real-time code.

8. Turn the UPM-1503 and UPM-2405 amplifiers ON. The red LED in the top-left corner of each UPM should be lit.

9. When complete, the WinCon Controller is downloaded to the client and the WinCon Server windows loads. Click on the green START button to run the controller in real-time.

10. Try to bring the helicopter body to a horizontal by pulling the joystick handle toward you. This supplies a positive voltage to the pitch motor and causes the pitch angle to increase.

11. As depicted in Figure 26, you will notice that the yaw angle moves clockwise in the positive direction as the voltage in the pitch motor increases. Compensate for this coupling effect by moving the joystick arm to the left and apply a negative voltage to the yaw motor. As illustrated in Figure 26 the pitch is eventually stabilized at about 0 degrees when the pitch motor voltage is approximately 12.5 V and the yaw begins to stabilize when feeding a voltage of about -6.0 V to the yaw motor. The voltage does not surpass the limit of the power amplifiers: ± 24 V for UPM-2405 and ± 15 V for UPM-1503.

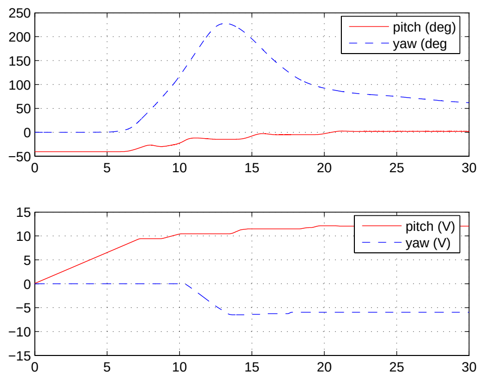


Figure 26: Effect of pitch on yaw.

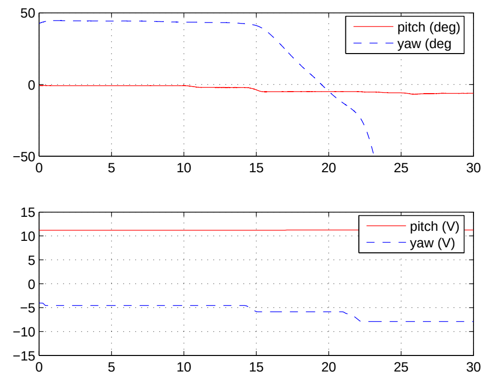


Figure 27: Effect of yaw on pitch.

12. From this point, now try decreasing the yaw voltage and observe its effect on the pitch angle.

13. As depicted in Figure 27, applying a negative voltage to the yaw motor causes the pitch angle decrease, i.e. the helicopter nose goes down. Similarly to the effect the pitch motor has on the yaw motion, the voltage applied to the yaw motor generates a torque on the pitch axis.

14. To stop running the WinCon controller, click on the red STOP button in the WinCon Server window. Make sure both the UPM-1503 and UPM-2405 are powered off if the session is complete.

5.5 Closed-loop Implementation

5.5.1 Objectives

The objectives of running the 2 DOF Helicopter in closed-loop are to:

- Investigate the closed-loop performance between the FF+LQR and the FF+LQR+I controllers running on the actual 2 DOF Helicopter plant.

- Compare the measured closed-loop response with the simulated response.

5.5.2 Procedure

Follow this procedure to run the FF+LQR and the FF+LQR+I controllers on the actual helicopter plant:

1. Before starting, confirm that the experiment has been connected properly as instructed in Section 3.5 and that the joystick is setup as discussed in Section 3.6.
2. Set the *Rate Command* knob on the joystick to the midpoint.
3. Set the Program/Joystick source block to 1 so the desired position is generated using Simulink blocks and not through the joystick.
4. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to -30.
5. To run the LQR controller set the control switch source block to 1.
6. Run the Matlab script setup_lab_heli_2d.m to set the model parameters, control gains, power amplifier limits, and so on. Ensure the LQR and LQR+I weighting matrices, **Q** and **R**, in the script are set as specified in Section 4.2. Thus the LQR and LQR+I control gain generated and displayed in the Matlab command window should be the same as in (30) and (34).
7. Select the *Build* item in the *WinCon* menu of the Simulink diagram to generate the real-time code.
8. When complete, the WinCon Controller is downloaded to the client and the WinCon Server windows loads. Click on the green START button to run the controller in real-time. You should hear the helicopter motors go on and the nose of the device's body should be at -30 degrees from the horizontal.
9. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 10 and the *Pitch: Constant (deg)* to 0. The helicopter should be tracking this reference position about its horizontal.
10. In the WinCon Server window, click on *Open Plot* button and select the *theta (deg)*, *psi (deg)*, and *Vm_Actual (V)* items under the *Scopes* folder. In the *theta (deg)* scope, the desired pitch position is the *green* line, the measured pitch position is the *red* plot, and the simulated pitch position is *light blue* trace. Similarly for the yaw *psi (deg)* scope. The *Vm_actual (V)* scope plots the voltage being applied to the pitch motor in *green* and yaw motor in *red*. Here are some helpful tips when using WinCon scopes:
    - To increase the time scale in each scope, select *Update* in the *Buffer* menu and change the value from 5 seconds to, for instance, 30 seconds.
    - To re-scale the y-axis in a scope immediately select the *Axis* menu and click on *Auto-Scale*. Note that the re-scaling is also done automatically after the plotted data runs passed the buffer set in the scope.

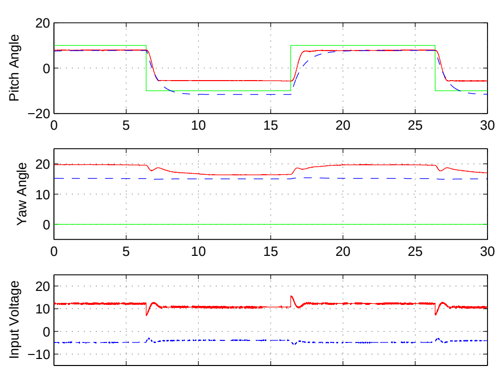


Figure 28: LQR closed-loop response under pitch step.

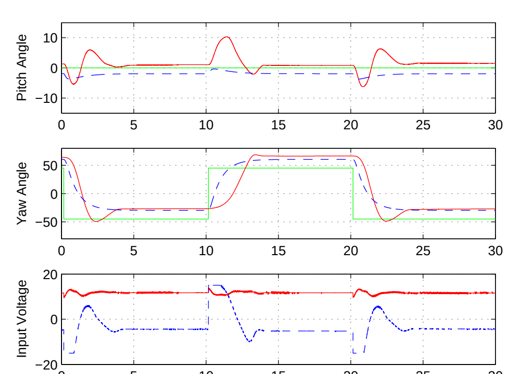


Figure 29: LQR closed-loop response under yaw step.

11. Figure 28 depicts the typical measured and simulated pitch and yaw response under a desired step pitch angle. The measured response is the solid red line (-), the simulation is the dashed blue line (–), and the reference is the solid green line (-). Notice in both the measured and simulated responses the yaw angle has a steady-state error.

12. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to 50. The helicopter should tracking the commanded yaw angle.

13. Figure 29 depicts the typical measured and simulated pitch and yaw response given a desired step yaw angle. Again, there is a steady-state error in the yaw angle.

14. The steady-state error can be removed using integral action. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to 0.

15. Switch to the FF+LQR+I control by setting the control switch source block to 2. The pitch and yaw angle should both, eventually, converge to 0 degrees.

16. Set the *Amplitude: Pitch (deg)* to 10 and the *Pitch: Constant (deg)* to 0 to observe the LQR+I response under a step pitch reference. The result should be similar to the response shown in Figure 30. Note that the steady-state error is removed.

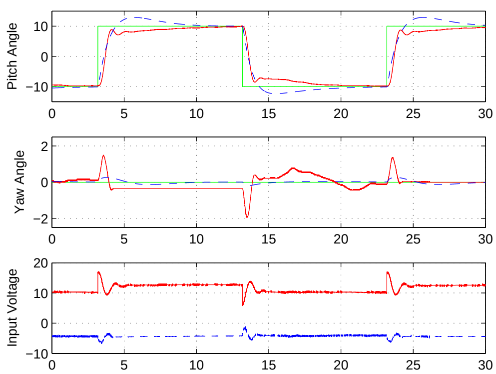


Figure 30: LQR+I closed-loop response under pitch step.

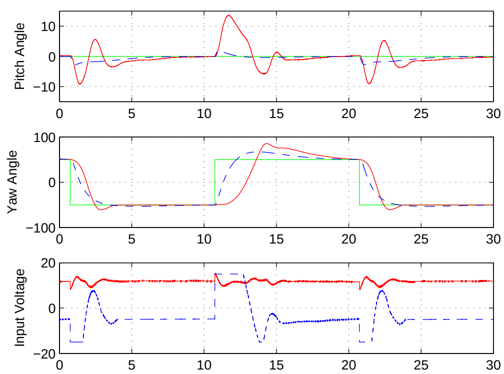


Figure 31: LQR+I closed-loop response under yaw step.

17. Set the *Amplitude: Pitch (deg)* to 0 and *Amplitude: Yaw (deg)* to 50 observe the LQR+I response under a desired yaw step. The obtained response should be similar to Figure 31. There is no steady-state error but the controller saturates the yaw amplifier due to the large step reference.

18. The desired angles can also be generated using the joystick, similarly in Section 3.6 when it was used to command voltage to the motors in open-loop. To use the joystick, set the *Program/Joystick* source block to 2. Notice the difference between the user controlling the helicopter manually as in Section 5.4 versus the computer controlling the helicopter position.

19. To stop running the WinCon controller, click on the red STOP button in the WinCon Server window. Make sure both the UPM-1503 and UPM-2405 are powered off if the session is complete.

5.6 Model Validation Implementation

5.6.1 Objectives

The objectives of running the model validation controller on the 2 DOF Helicopter are to:

1. Verify that the nonlinear model produced in Section 4.1.3 represents the actual device with reasonable accuracy.

2. Roughly identify the rotary viscous friction parameter about pitch and yaw axis.

5.6.2 Procedure

Follow this procedure to identify the viscous rotary friction on the yaw axis and do model validation of the pitch:

1. Before starting, confirm that the experiment has been connected properly as instructed in Section 3.5 and that the joystick is setup as discussed in Section 3.6.

2. Set the *Program/Joystick* source block to 1 so the desired position is generated using Simulink blocks and not through the joystick.

3. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to -30.

4. To run the LQR+I controller set the control switch source block to 2.

5. Run the Matlab script setup_lab_heli_2d.m. Ensure the LQR and LQR+I weighting matrices, **Q** and **R**, in the script are set as specified in Section 4.2. Thus the LQR and LQR+I control gain generated and displayed in the Matlab command window should be the same as in (30) and (34).

6. Select the *Build* item in the *WinCon* menu of the Simulink diagram to generate the real-time code. If the WinCon controller has already been built choose the *Download* instead.

7. When complete, the WinCon Controller is downloaded to the client and the WinCon Server windows loads. Click on the green START button to run the controller in real-time. You should hear the helicopter motors go on and the nose of the device's body should be at -30 degrees from the horizontal.

8. Inside the *Desired Angle from Program* set the *Amplitude: Pitch (deg)* to 0 and the *Pitch: Constant (deg)* to 0. The helicopter should be stabilized about $\theta = 0$ and $\psi = 0$.

9. In the WinCon Server window, click on *Open Plot* button and select the *theta (deg)*, *psi (deg)*, and *Vm_Actual (V)* items under the *Scopes* folder. In the *theta (deg)* scope, the desired pitch position is the *green* line, the measured pitch position is the *red* plot, and the simulated pitch position is *light blue* trace. Similarly for the yaw *psi (deg)* scope. The *Vm_actual (V)* scope plots the voltage being applied to the pitch motor in *green* and yaw motor in *red*.

10. To identify the viscous rotary friction parameter in the yaw axis a voltage step command must be applied to the yaw motor. In the *Desired Voltage* subsystem, set the *Signal Generator: Yaw (V)* block to 2.5.

11. Increase the buffer time of the scopes to 30 seconds in order to view the entire response from the voltage step, as explained in Section 5.5.

12. Change the control switch to 4. The yaw input motor voltage is now the control voltage used to stabilize the yaw angle, $u^*_{lqr,y}$, added to the commanded open-loop voltage, i.e. $V_{m,p} = u^*_{lqr,y} + 2.5$. The plot should resemble Figure 32.

13. For calculating the viscous damping parameter B_y, take the linear equation describing the yaw motion in (21). Given that the helicopter is rotating more or less at a constant speed, $\ddot{\psi} = 0$. Solving for the viscous damping term gives

$$B_y = \frac{K_{yy} V_{m,y} + K_{yp} V_{m,p}}{\dot{\psi}_{avg}} = 0.3180 \text{ N} \cdot \text{m/(rad/s)} \tag{37}$$

where from Figure 32 $\dot{\psi}_{avg} = -0.9481$ rad/s, $V_{m,y}$ = -7.65 V, and $V_{m,p}$ = 11.36 V, and K_{yy} and Kyp are defined in Table 2.

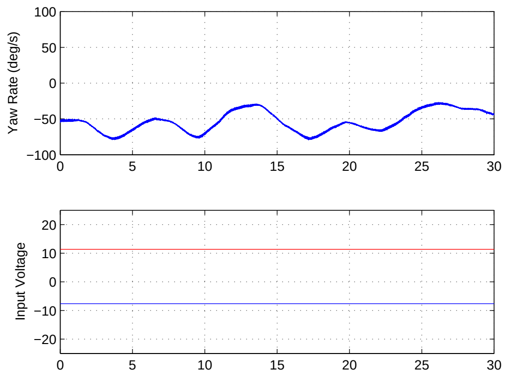


Figure 32: Open-loop voltage on yaw.

14. Bring the helicopter back to ($\theta = 0$, $\psi = 0$) by setting the control switch block to 2 to run the LQR+I controller.
15. To apply a voltage directly to the pitch motor set the control switch source block to 3.
16. In the *Desired Voltage* subsystem, set the frequency of the *Signal Generator: Pitch (V)* block to 0.4 Hertz, the *Amplitude: Pitch (V)* gain block to 0.2, and the *Constant Pitch (V)* to 0.

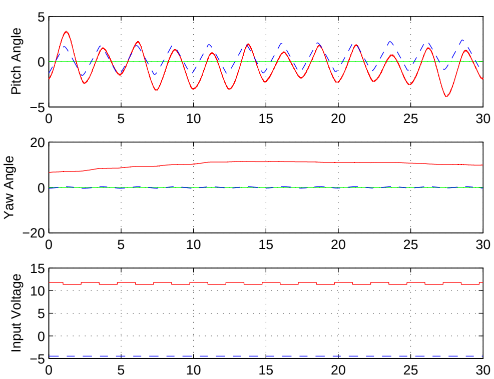


Figure 33: Open-loop pitch step.

17. As shown in Figure 33, the measured and simulated pitch angles are quite close. The pitch viscous damping term B_p was estimated by tuning its value online as the controller is running. To do this enter a value for B_p in the Matlab command window, for example $B_p = 0.8$. Because the parameter change is made in Matlab (and not directly in the Simulink model), the controller that is running must be updated for the changes of this parameter to take effect. The change can be applied by clicking on the *Edit* menu in the Simulink model and selecting the *Update Diagram* item. Alternatively, controller parameters can be updated by use the keystroke *CTRL-D* whenever the Simulink model is active.
18. To stop running the WinCon controller, click on the red STOP button in the WinCon Server window. Make sure both the UPM-1503 and UPM-2405 are powered off if the session is complete.

References

[1] Faulhaber. *DC-Micromotors Graphite Commutation Series 2842C.*

[2] Quanser Inc. *MultiQ, Q4, or Q8 User Manual.*

[3] Quanser Inc. *Universal Power Module User Manual.*

[4] Quanser Inc. *WinCon User Manual.*

[5] Pittman. *LO_COG DC Servo Motors Series 8000,9000, 14000.*