Control Engineering (SC42095)

Lecture 7, 2020

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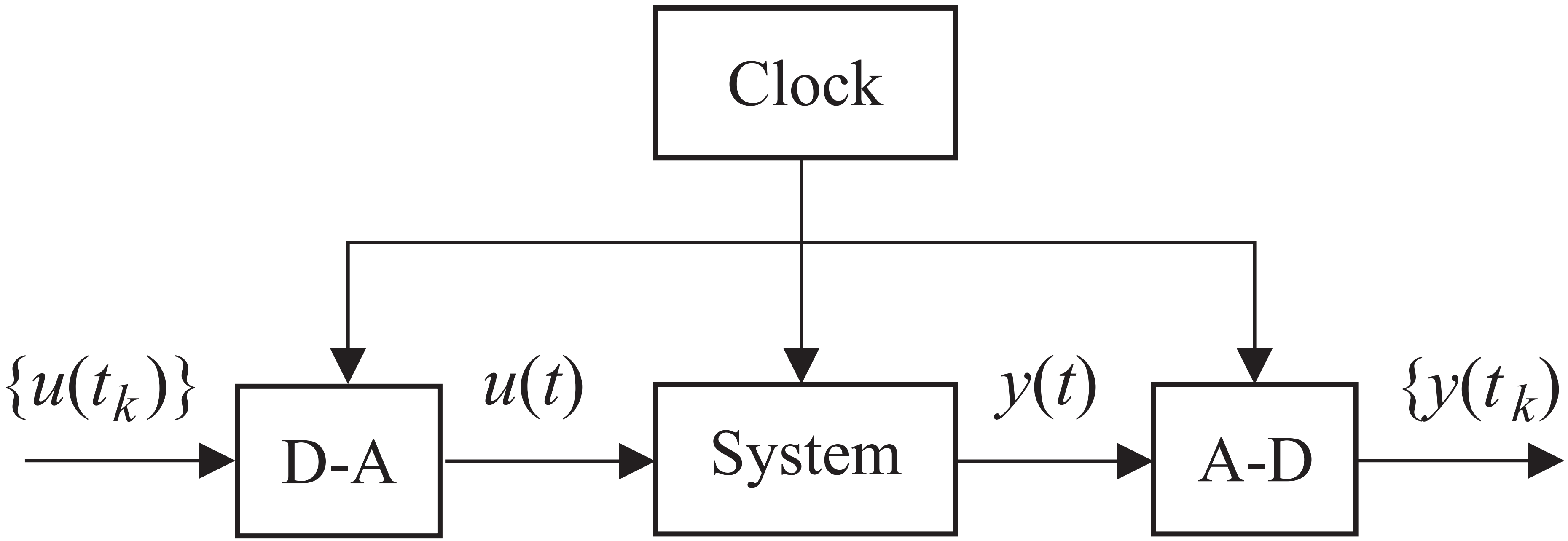
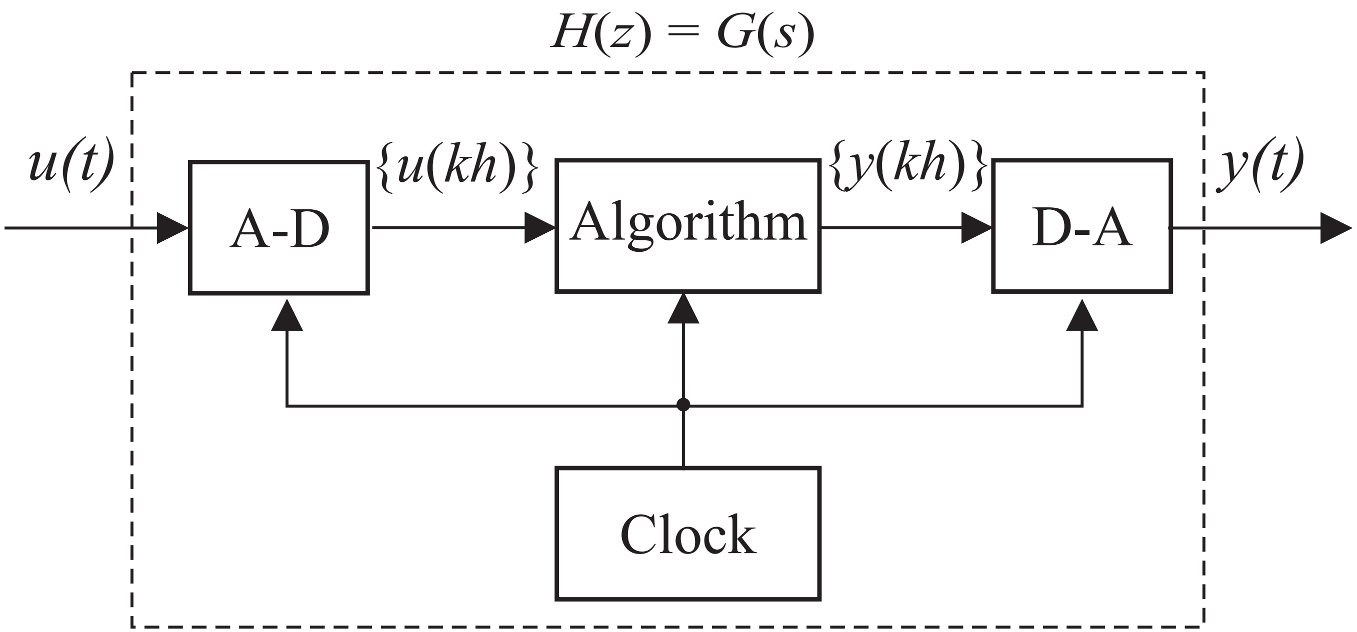
Lecture outline

- Redesign of continuous-time controllers
- First-order holds
- PID control
- Implementation issues

So far: system from computer's viewpoint



Computer implementation of analog controllers



$G(s)$ is designed by using continuous-time techniques

Want to get

$$\text{A/D} + \text{Algorithm} + \text{D/A} \approx G(s)$$

Approximation of derivatives

Forward difference (Euler method)

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t+h) - x(t)}{h} = \frac{q-1}{h}x(t)$$

Backward difference

$$px(t) = \frac{dx(t)}{dt} \approx \frac{x(t) - x(t-h)}{h} = \frac{q-1}{qh}x(t)$$

Trapezoidal method (Tustin, bilinear)

$$\frac{dx(t)}{dt} \approx \frac{\dot{x}(t+h) + \dot{x}(t)}{2} \quad \Rightarrow \quad px(t) = \frac{2}{h} \cdot \frac{q-1}{q+1}x(t)$$

Approximation of transfer functions

$$H(z) = G(s)$$

with the substitutions:

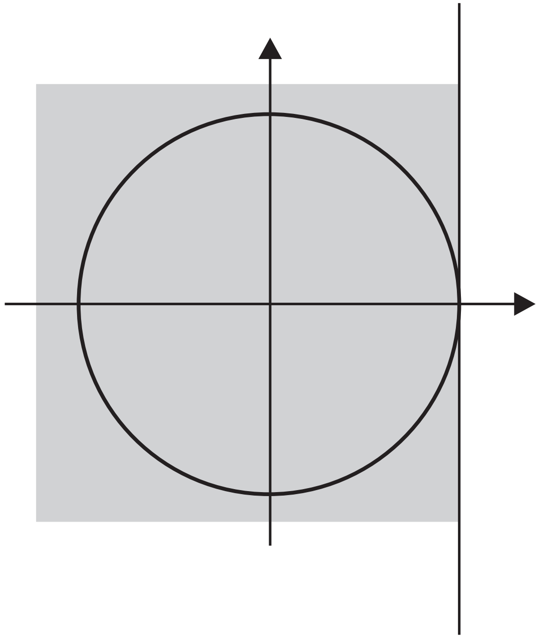
$s = \frac{z-1}{h}$ (forward difference or Euler method)

$s = \frac{z-1}{zh}$ (backward difference)

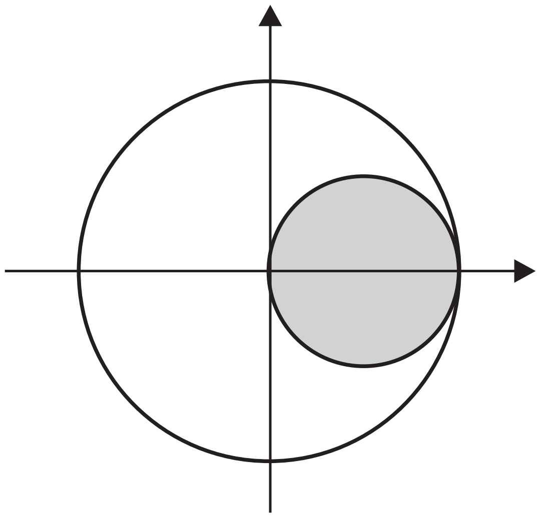
$s = \frac{2}{h}\frac{z-1}{z+1}$ (Tustin or bilinear approximation)

Note: The frequency scale gets distorted with these approximations (warping effect)!
See page 295 of CCS book about frequency prewarping.

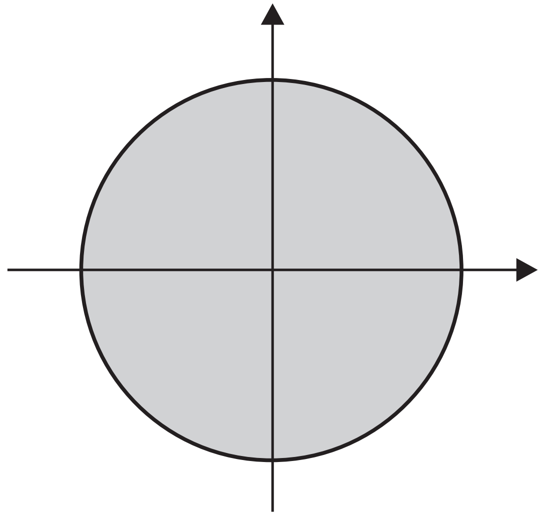
Stability of the approximations



Forward differences



Backward differences



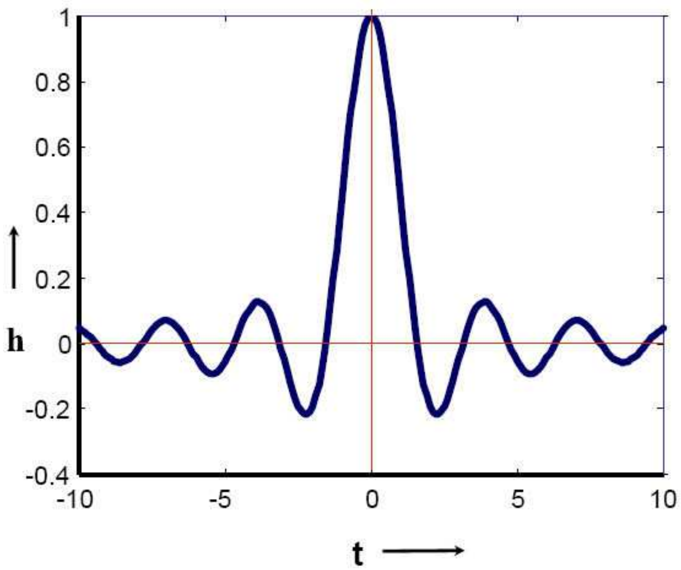
Tustin

$\forall$ unstable CT $\rightarrow$ unstable DT	$\forall$ stable CT $\rightarrow$ stable DT	$\forall$ unstable CT $\rightarrow$ unstable DT
$\exists$ stable CT $\rightarrow$ unstable DT	$\exists$ unstable CT $\rightarrow$ stable DT	$\forall$ stable CT $\rightarrow$ stable DT

Signal reconstruction

Shannon's sampling theorem:
If a signal $f(t)$ has frequency content $\omega < \omega_N$, where the ω_N Nyquist frequency is half of the sampling frequency ω_s, then it is uniquely determined by its sample points.

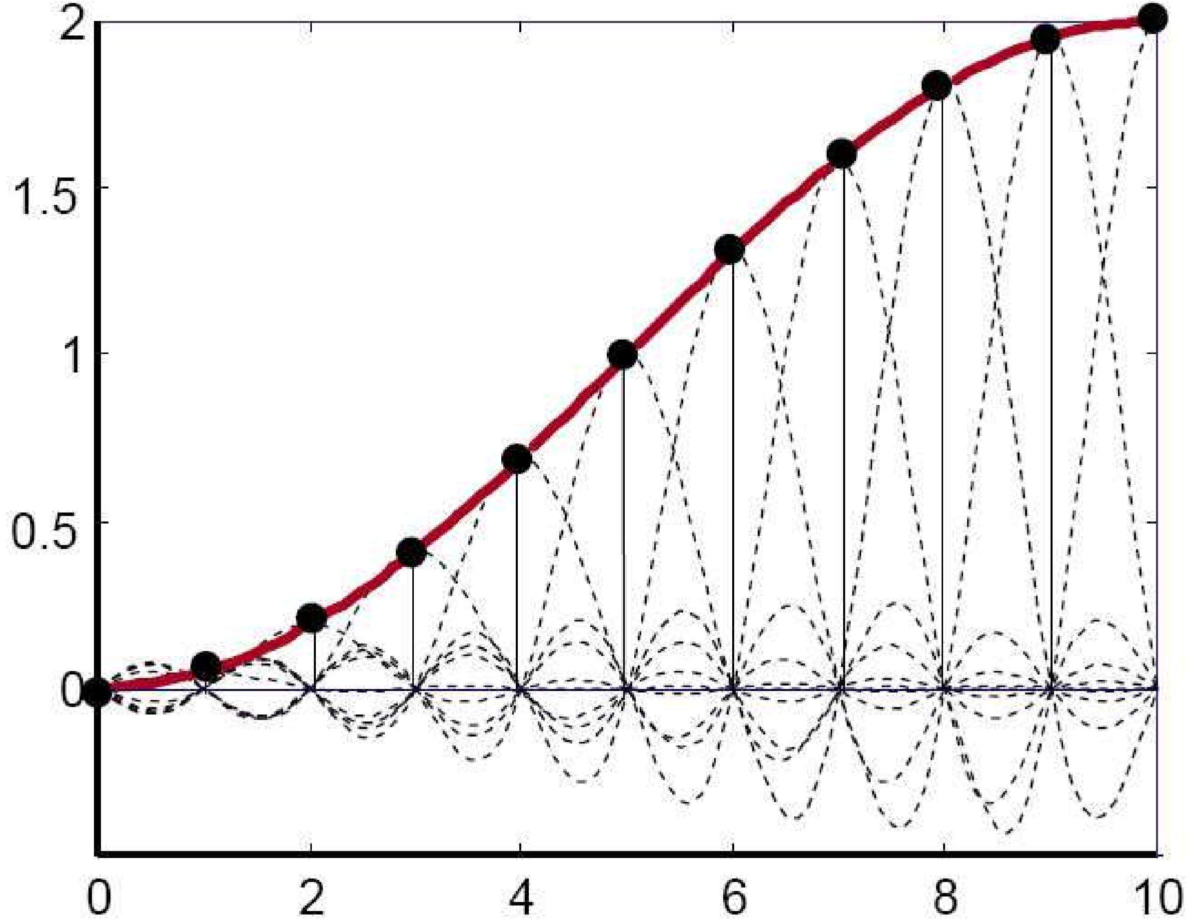
$$f(t) = \sum_{-\infty}^{\infty} f(k) \frac{\sin(\omega_s(t - kh)/2)}{\omega_s(t - kh)/2}$$



Shannon reconstruction with

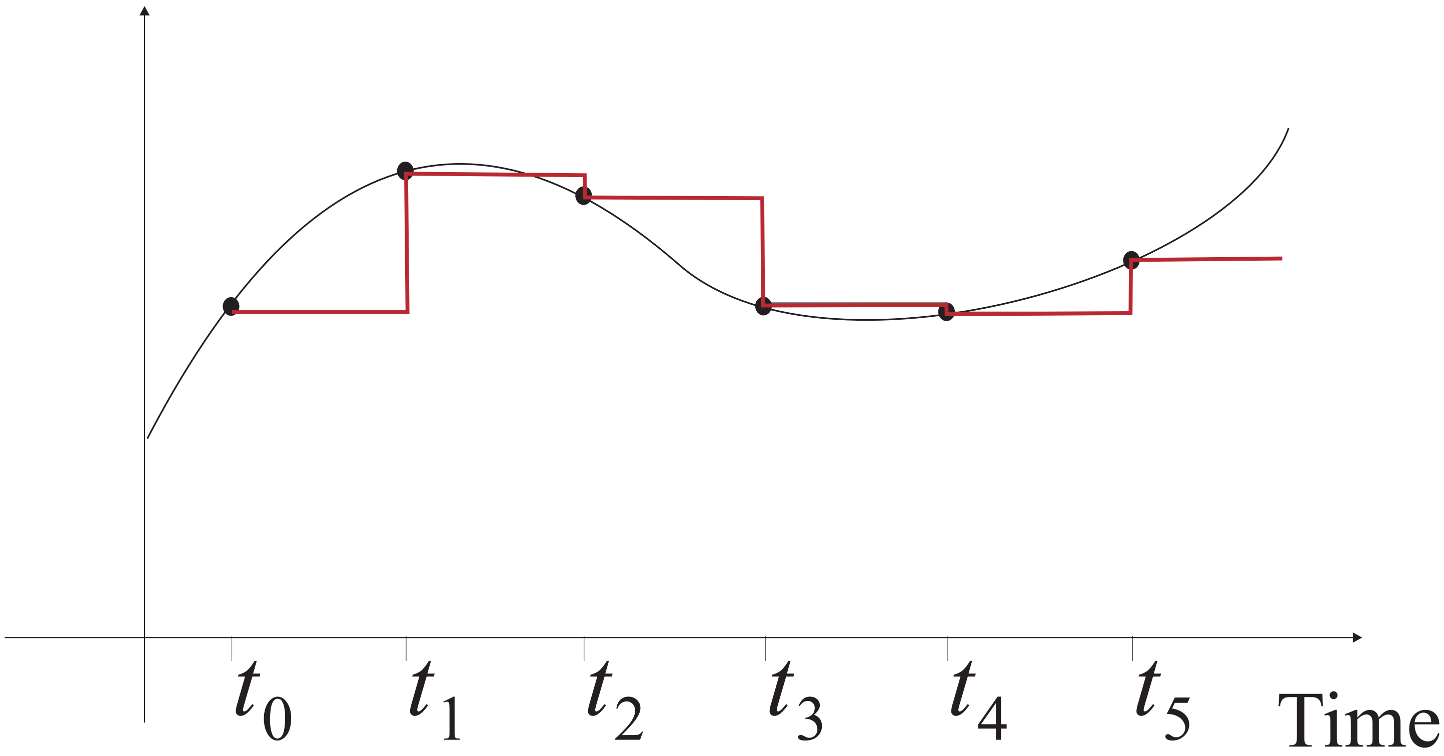
$h(t) = \dfrac{\sin(\omega_s t/2)}{\omega_s t/2}$ is not causal!

Shannon reconstruction (exact)



t

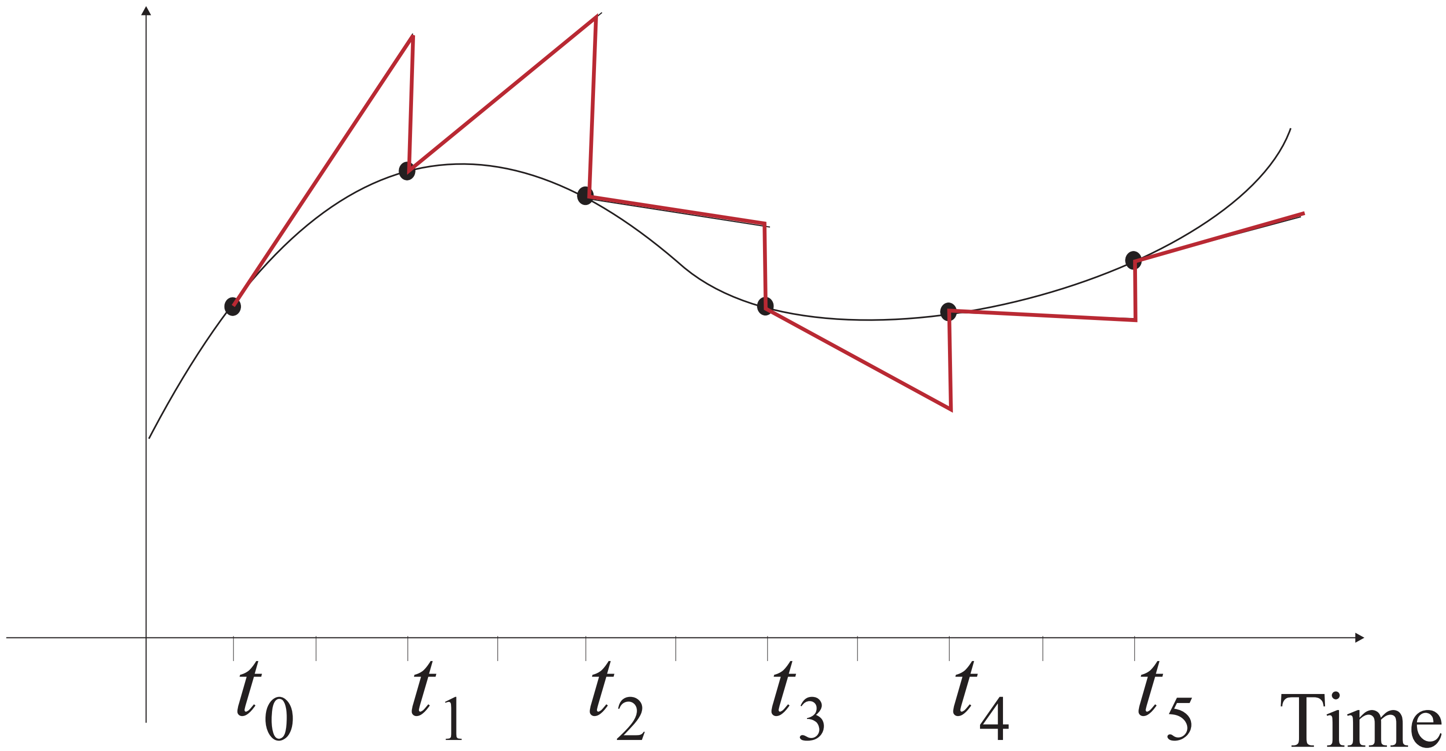
Zero-order hold



$$u(t) = u(t_k), \quad t_k \leq t < t_{k+1}$$

$$\frac{1 - e^{-hs}}{s}$$

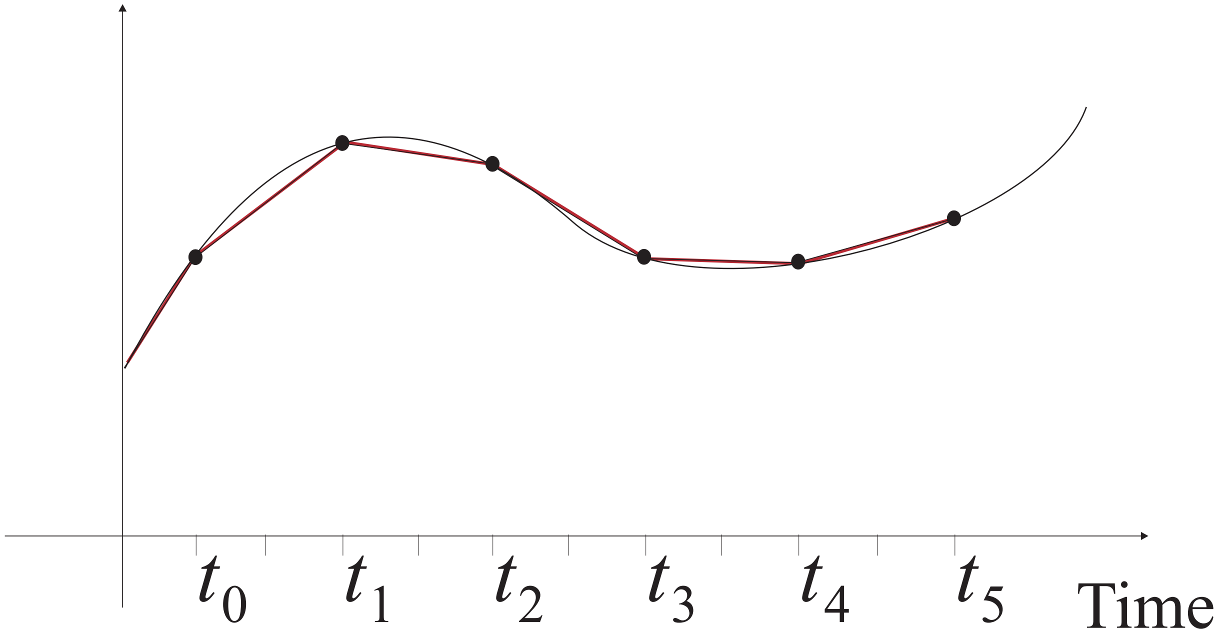
First-order hold



$$u(t) = u(t_k) + \frac{t - t_k}{t_k - t_{k-1}}\Big(u(t_k) - u(t_{k-1})\Big), \quad t_k \leq t < t_{k+1}$$

$$\frac{s+1}{s^2}(1 - 2e^{-hs} + e^{-2hs})$$

Predictive first order hold



$$u(t) = u(t_k) + \frac{t - t_k}{t_k - t_{k-1}}\Big(u(t_{k+1}) - u(t_k)\Big), \quad t_k \leq t < t_{k+1}$$

Input linear between samples, rather than constant (different sampling formulas).

PID control

The "textbook" version

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$

Common modifications:

- Filter on the derivative

$$sT_d \approx \frac{sT_d}{1 + sT_d/N}$$

- Derivative of $-y$ not of $e = u_c - y$
- Only fraction of u_c in the P-term (reduce overshoot)

More realistic PID controller

$$U(s) = K\left(bU_c(s) - Y(s) + \frac{1}{sT_i}(U_c(s) - Y(s)) - \frac{sT_d}{1 + sT_d/N}Y(s)\right)$$

Other modifications:

- high-frequency roll-off (extra filter)
- nonlinearities, e.g., $Ke|e|$
- anti-windup (later)

Discrete-time PID

P-term: $P(k) = K(bu_c(k) - y(k))$

I-term: $I(k+1) = I(k) + \frac{Kh}{T_i} e(k)$

D-term: $D(k) = \frac{T_d}{T_d + Nh} D(k-1) - \frac{KT_dN}{T_d + Nh} (y(k) - y(k-1))$

$$u(k) = P(k) + I(k) + D(k)$$

special case of RST: $R(q)u(k) = T(q)u_c(k) - S(q)y(k)$

table for $R(q)$, $S(q)$ and $T(q)$ (page 309)

Discrete-time PID (cont'd)

Simple position form:

$$u(k) = K\left(1 + \frac{h}{T_i}\frac{1}{q-1} + \frac{T_d}{h}\frac{q-1}{q}\right)e(k)$$

$$\Downarrow$$

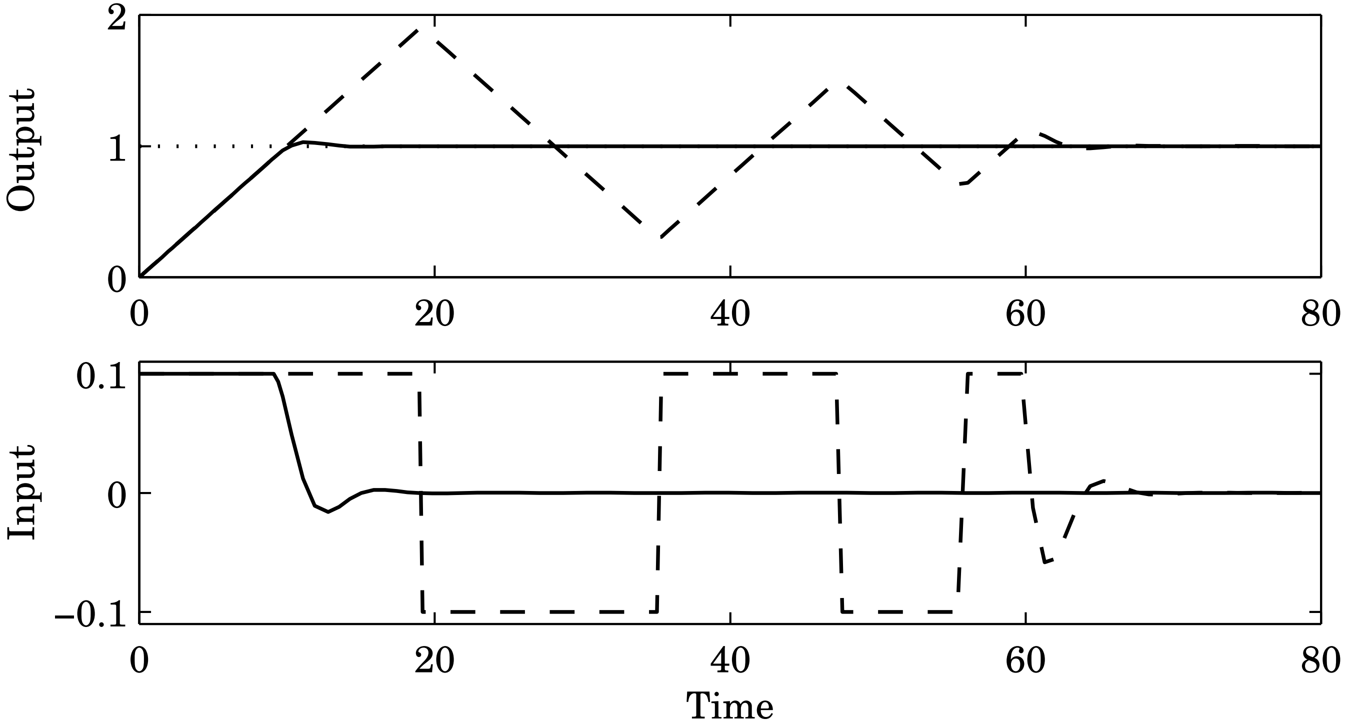
$$\Delta u(k) = u(k) - u(k-1)$$

$$\Downarrow$$

Simple velocity form:

$$\Delta u(k) = K\left(\frac{q-1}{q} + \frac{h}{T_i}\frac{1}{q-1} + \frac{T_d}{h}\frac{q-1}{q}\right)e(k)$$

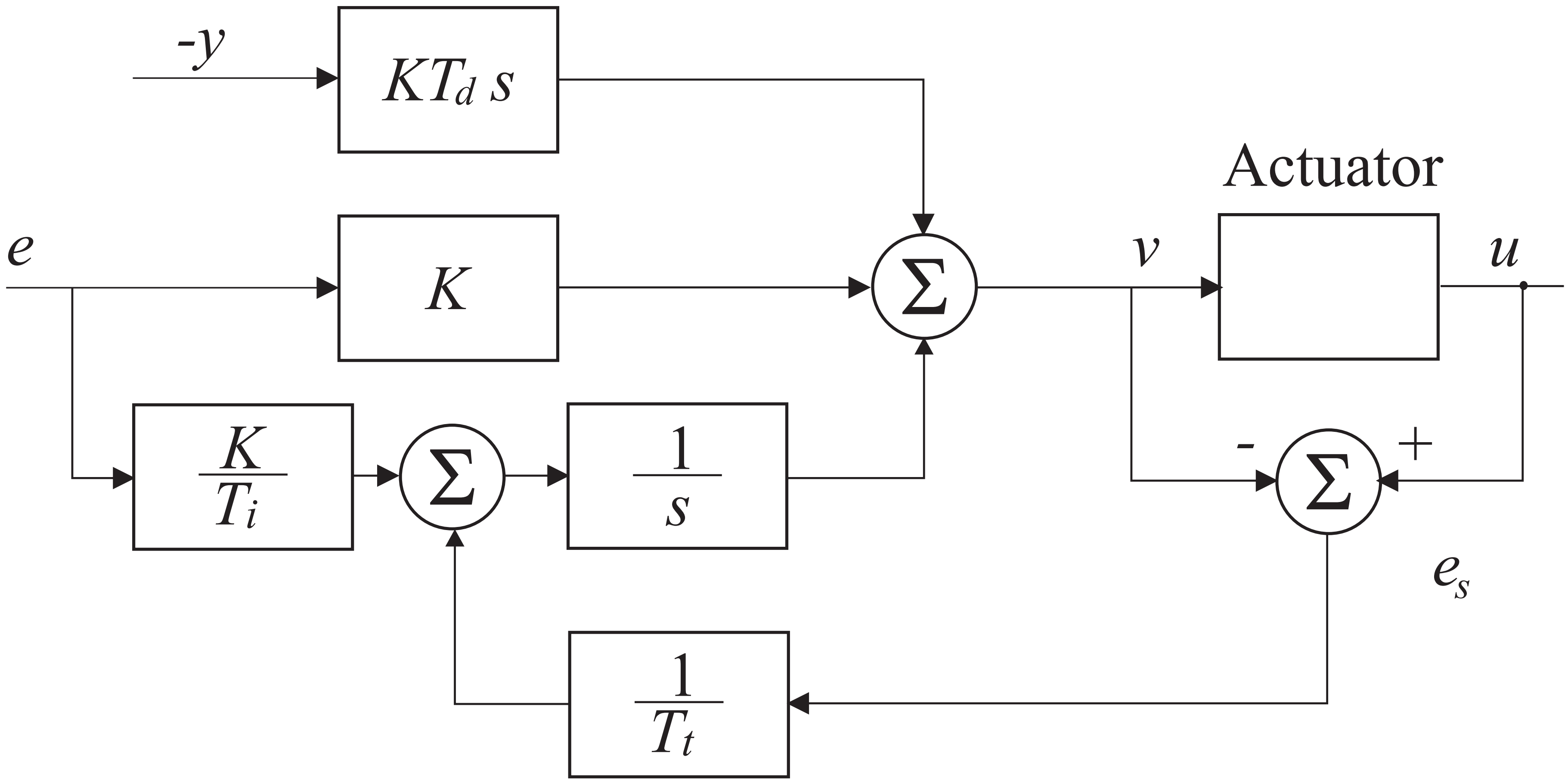
Integrator windup (due to actuator saturation)



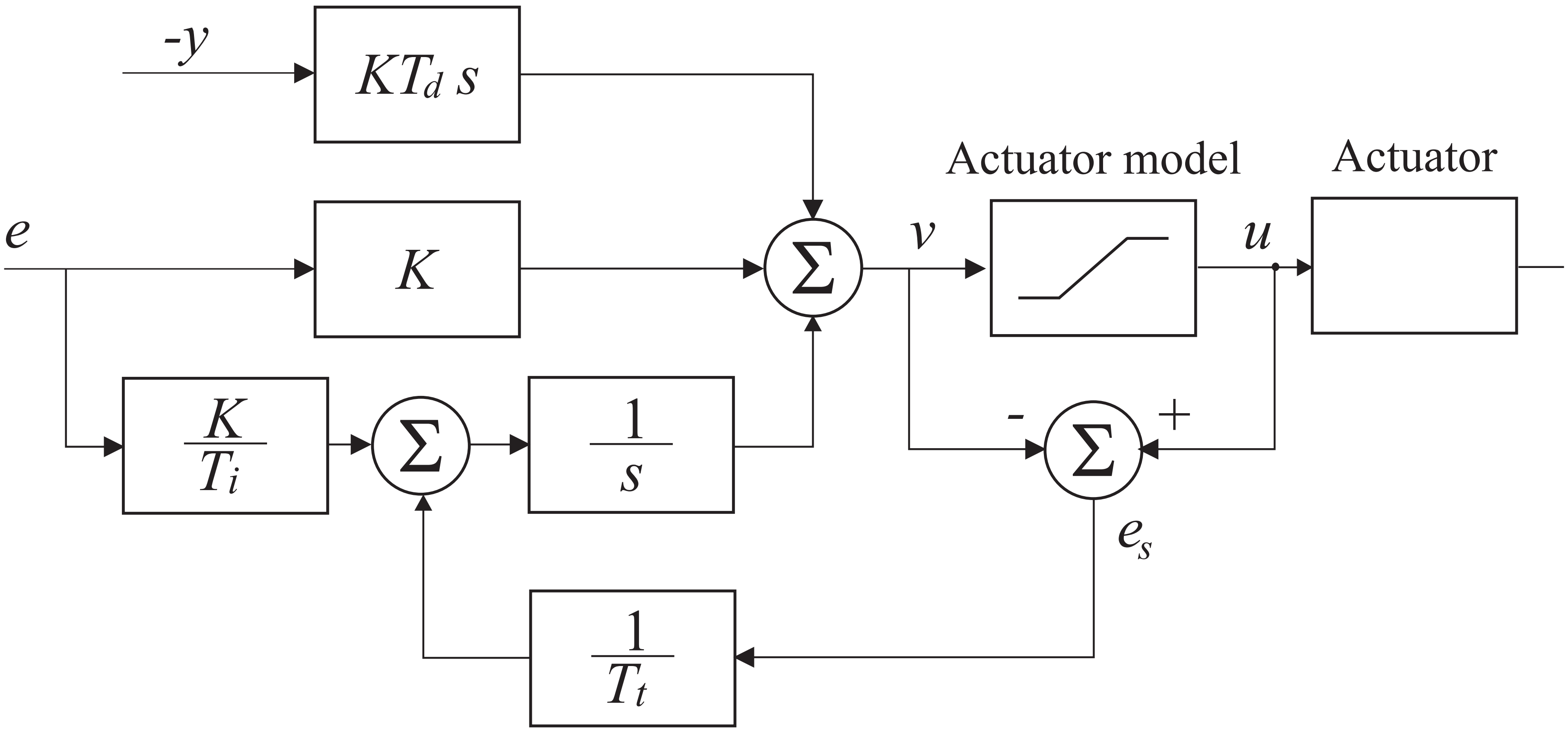
Solution: reset the integrator

- stop integrating (when actuator saturates)
- tracking schemes

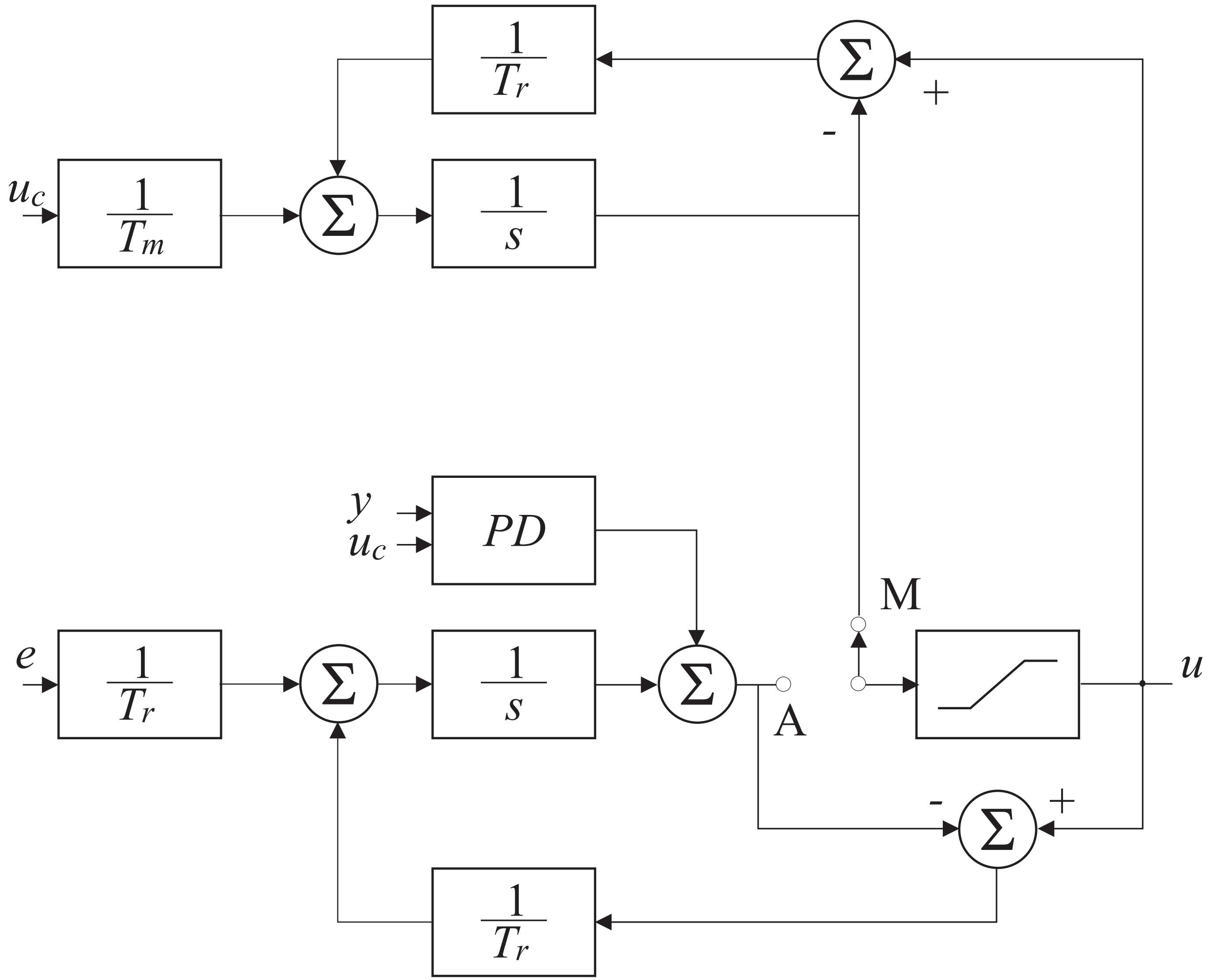
Tracking scheme for anti-windup



Tracking scheme for anti-windup



Bumpless transfer manual–automatic



Bumpless parameter change

Use:

$$x_I = \int^t \frac{K}{T_i} e(s) ds$$

instead of

$$x_I = \frac{K}{T_i} \int^t e(s) ds$$

Tuning of PID controllers

K, T_i, T_d and other parameters: N, T_t, u_{low}, u_{high}, h, b

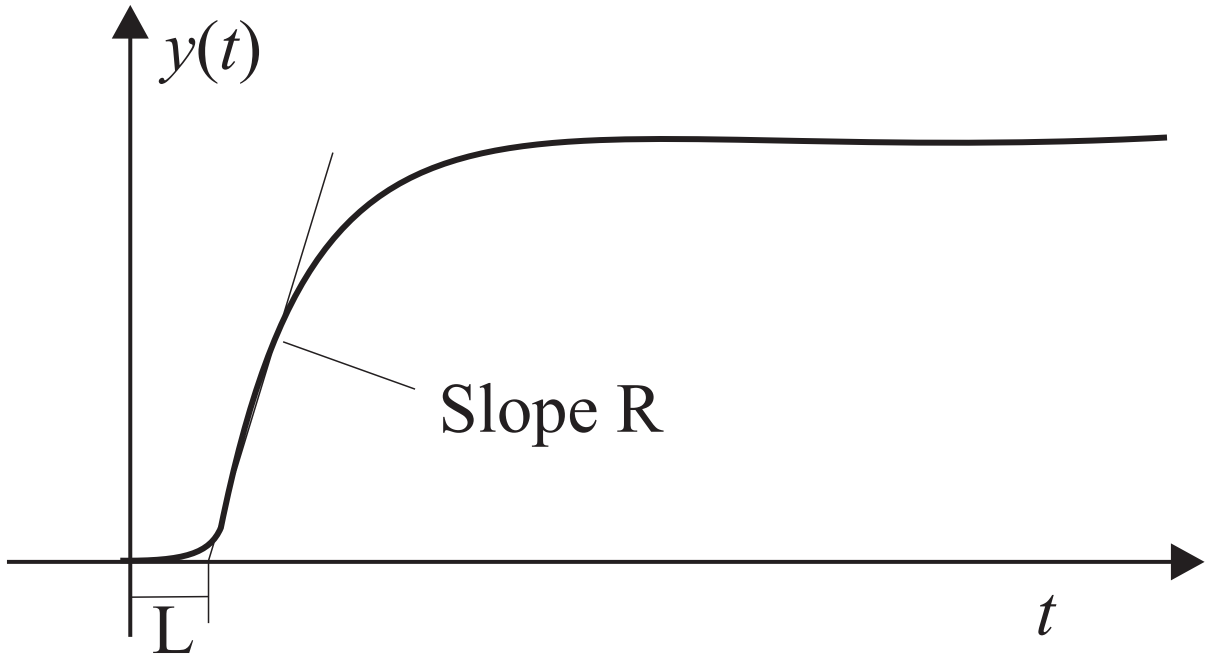
- $N = 10$ typically
- T_t equal to T_i or 0.1–0.5 times T_i
- u_{low} and u_{high} close to true saturation values
- Sampling period h

$$\text{For PI} : \frac{h}{T_i} \approx 0.1 - 0.3$$
$$\text{For PID} : \frac{hN}{T_d} \approx 0.2 - 0.6$$

- $b < 1$ to decrease overshoot after setpoint changes

Ziegler–Nichols tuning rules

1. Step-response method



$a = RL$, $a \longrightarrow K,\ T_i,\ T_d$ (page 315)

Ziegler–Nichols tuning rules – cont'd

2. Ultimate-sensitivity method

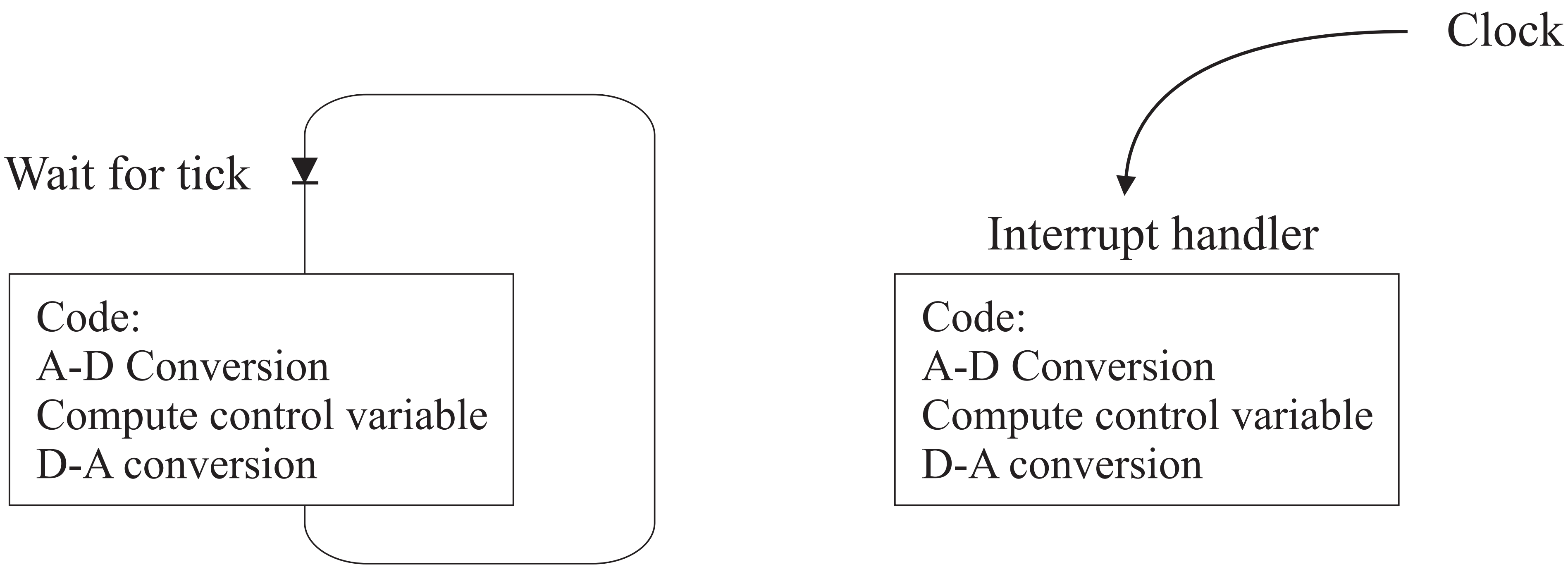
   Use a P controller and increase the gain $\Rightarrow K_u, T_u$

   $K_u,\ T_u \longrightarrow K,\ T_i,\ T_d$ (page 316)

Use with care (poor damping, $\zeta \approx 0.2$)

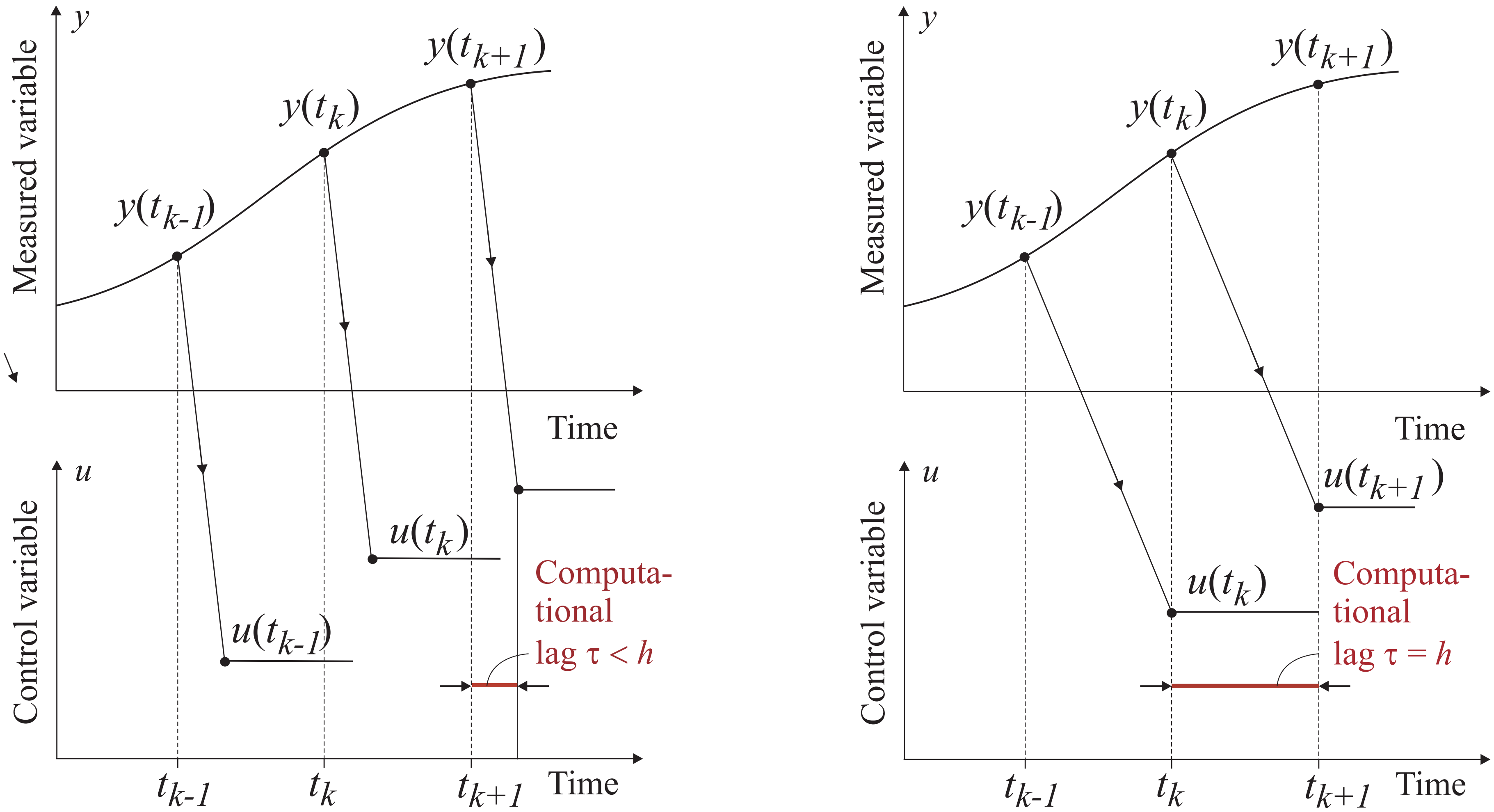
Many other tuning methods (including pole placement)

Implementation



Real-time operating systems, multi-tasking

Computational delay



Keep it constant, include in the process model.

Numerical aspects

- Word-length, computer, A/D and D/A converters

- Fixed or floating point computations (IEEE standard)

$$\text{example: } (100 \quad 1 \quad 100)(100 \quad 1 \quad -100)^T$$

- Use higher precision for internal calculations

- Influence of noise and quantization

- Choice of realization (different sensitivity, see on extra slides)

Effects of roundoff and quantization

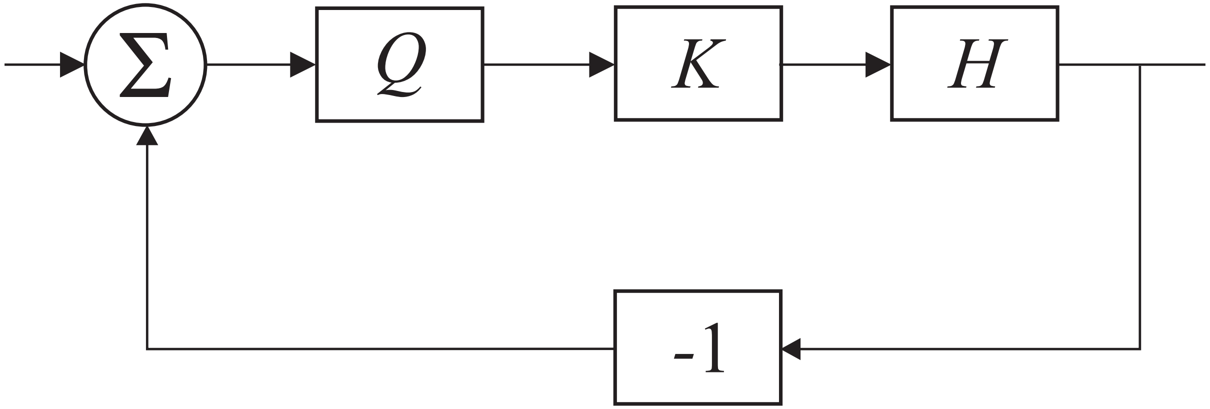
- Nonlinear phenomena

$$Q(a+b) \neq Q(a) + Q(b)$$

- Limit cycles and/or bias

- Analysis tools
  - Nonlinear analysis
  - Describing function approximation
  - Model quantization as stochastic processes

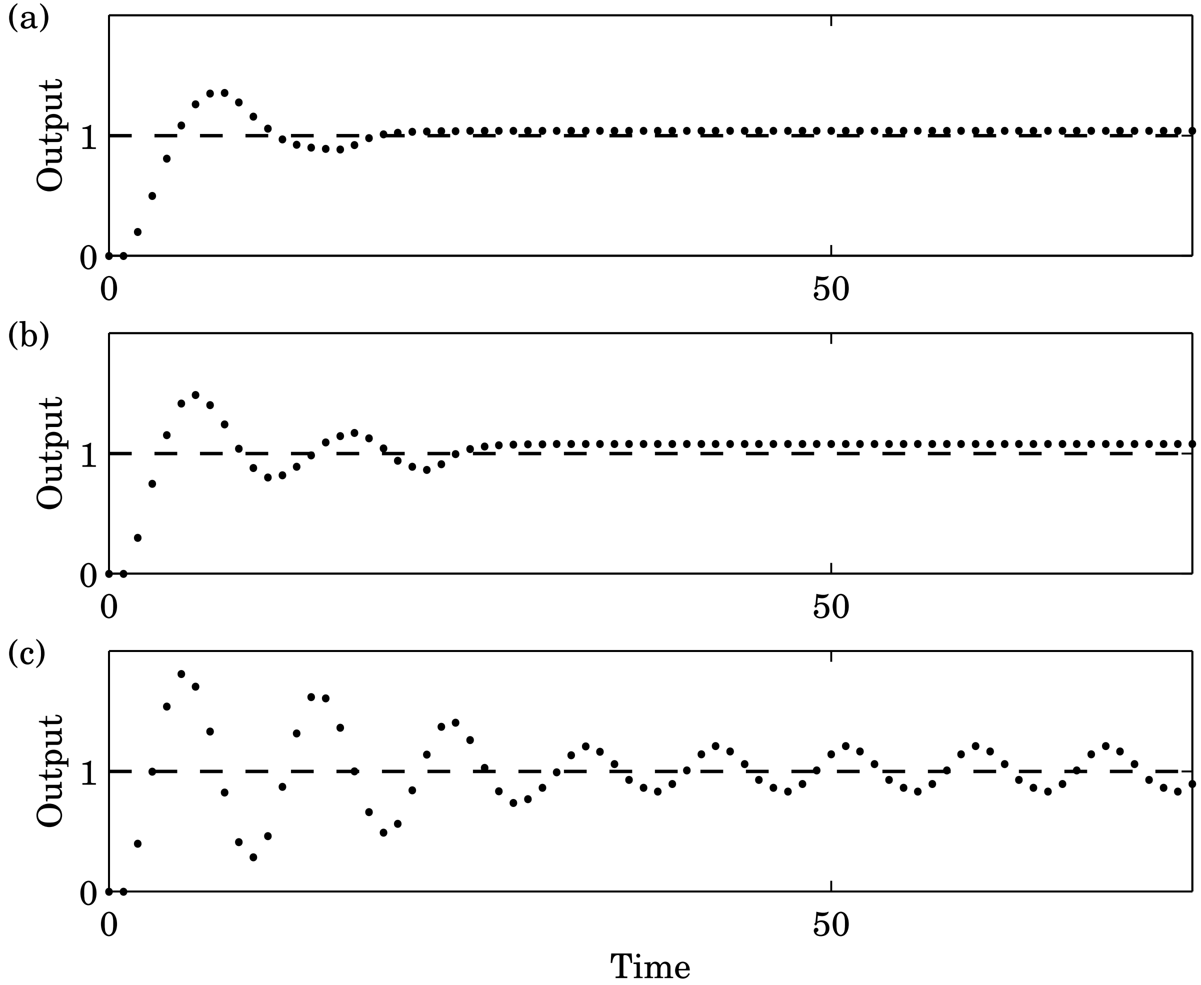
Effect of roundoff



$$H(z) = \frac{0.25}{(z-1)(z-0.5)}$$

Without quantization: asymptotically stable for $K < 2$

With quantization



a) $K = 0.8$, b) $K = 1.2$, c) $K = 1.6$

Summary

- Redesign of continuous-time controllers
- First-order holds
- PID control
- Implementation issues