Control Engineering (SC42095)

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Lecture outline

- Compensation for disturbances
- Servo problem (tracking)
- Actuator saturation
- Input–output design

More general disturbances

System:

$$\frac{dx}{dt} = Ax + Bu + v \qquad y = Cx$$

Describe disturbance as a dynamic system:

$$\frac{dw}{dt} = A_w w, \quad v = C_w w$$

Combine the two into an augmented system:

$$\frac{d}{dt}\begin{pmatrix} x \\ w \end{pmatrix} = \begin{pmatrix} A & C_w \\ 0 & A_w \end{pmatrix}\begin{pmatrix} x \\ w \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} C & 0 \end{pmatrix}\begin{pmatrix} x & w \end{pmatrix}^T$$

More general disturbances – cont'd

$$\begin{pmatrix} x(k+1) \\ w(k+1) \end{pmatrix} = \begin{pmatrix} \Phi & \Phi_{xw} \\ 0 & \Phi_w \end{pmatrix} \begin{pmatrix} x(k) \\ w(k) \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u(k)$$

State feedback:

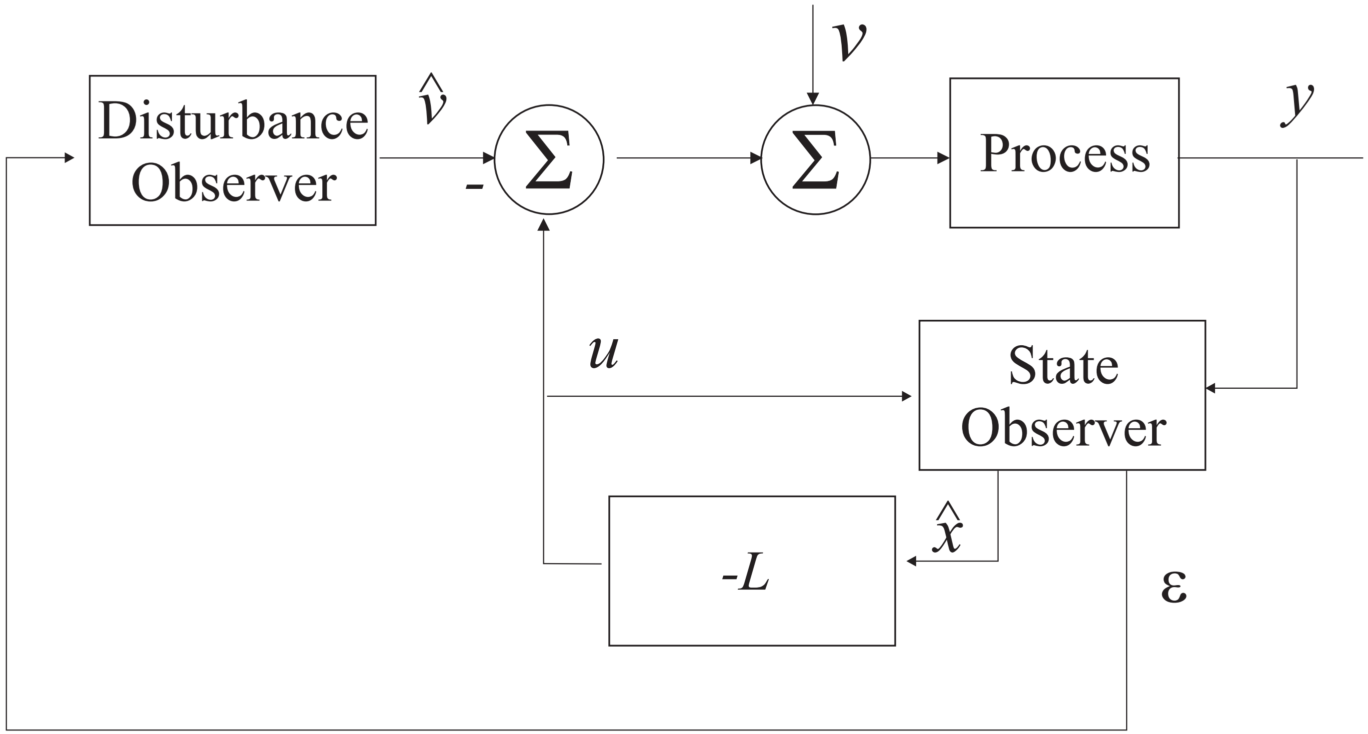
$$u(k) = -Lx(k) - L_w w(k)$$

Closed-loop system

$$\begin{aligned} x(k+1) &= (\Phi - \Gamma L)x(k) + \underbrace{(\Phi_{xw} - \Gamma L_w)}_{\approx 0?} w(k) \\ w(k+1) &= \Phi_w w(k) \end{aligned}$$

w uncontrollable from u, not directly measurable $\longrightarrow$ use observer

Disturbance observer



$$\hat{x}(k+1) = \Phi\hat{x}(k) + \Phi_{xw}\hat{w}(k) + \Gamma u(k) + K\varepsilon(k)$$

$$\hat{w}(k+1) = \Phi_w\hat{w}(k) + K_w\varepsilon(k)$$

$$\hat{v}(k) = C_w\hat{w}(k), \quad \text{with} \quad \varepsilon(k) = y(k) - C\hat{x}(k)$$

Example – load disturbance at process input

$$w(k+1) = w(k) \qquad\qquad v(k) = w(k)$$

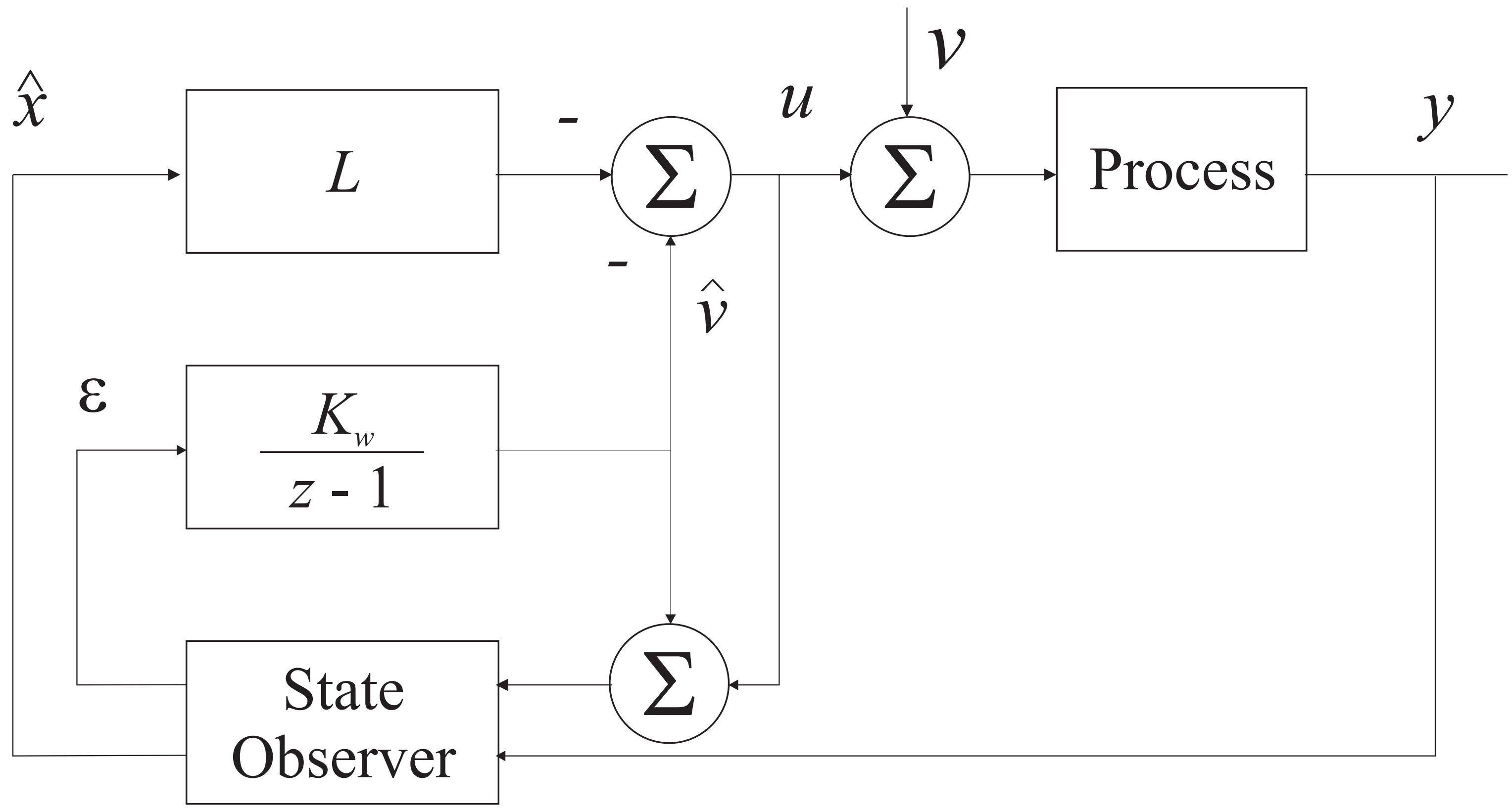
$$u(k) = -L\hat{x}(k) - \hat{w}(k) = -L\hat{x}(k) - \hat{v}(k)$$

$$\begin{aligned}
\hat{x}(k+1) &= \Phi\hat{x}(k) + \Gamma(\hat{v}(k) + u(k)) + K(y(k) - C\hat{x}(k)) \\
\hat{v}(k+1) &= \hat{v}(k) + K_w(y(k) - C\hat{x}(k))
\end{aligned}$$

Here, the disturbance observer is an integrator.

Integrator in the controller

$$\hat{x}(k+1) = \Phi\hat{x}(k) + \Gamma(\hat{v}(k) + u(k)) + K(y(k) - C\hat{x}(k))$$
$$\hat{v}(k+1) = \hat{v}(k) + K_w(y(k) - C\hat{x}(k))$$



Integrator – another approach

- Include integrator in an outer loop
- Analogous to PID control
- Extended system:

$$\begin{pmatrix} x(k+1) \\ x_i(k+1) \end{pmatrix} = \begin{pmatrix} \Phi & 0 \\ C & I \end{pmatrix} \begin{pmatrix} x(k) \\ x_i(k) \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u(k)$$

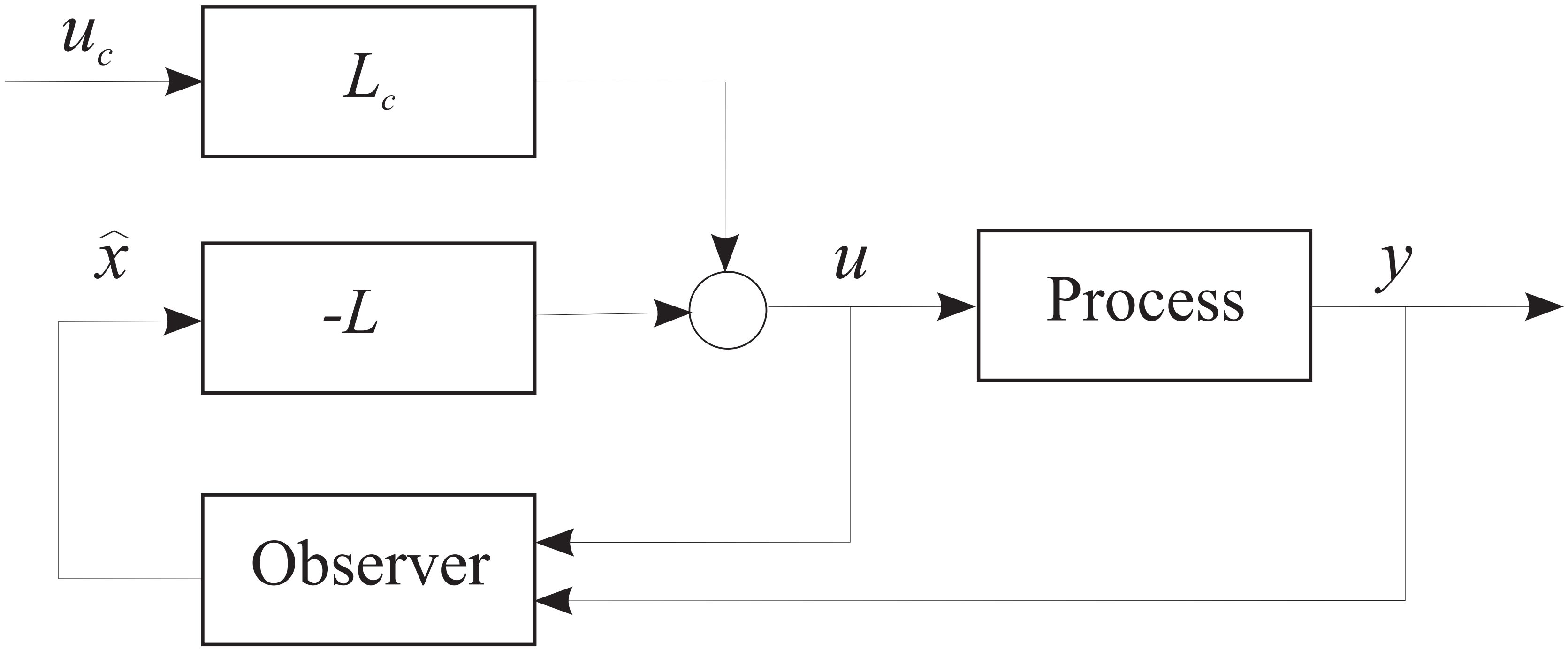
- State feedback:

$$u(k) = -\begin{pmatrix} L & L_i \end{pmatrix} \begin{pmatrix} x(k) \\ x_i(k) \end{pmatrix}$$

Servo case: simple approach

Goal: respond to a reference signal in a specified way.

Replace $u(k) = -L\hat{x}(k)$ by: $u(k) = -L\hat{x}(k) + L_C u_C(k)$



Servo case: simple approach (cont'd)

Closed-loop system:

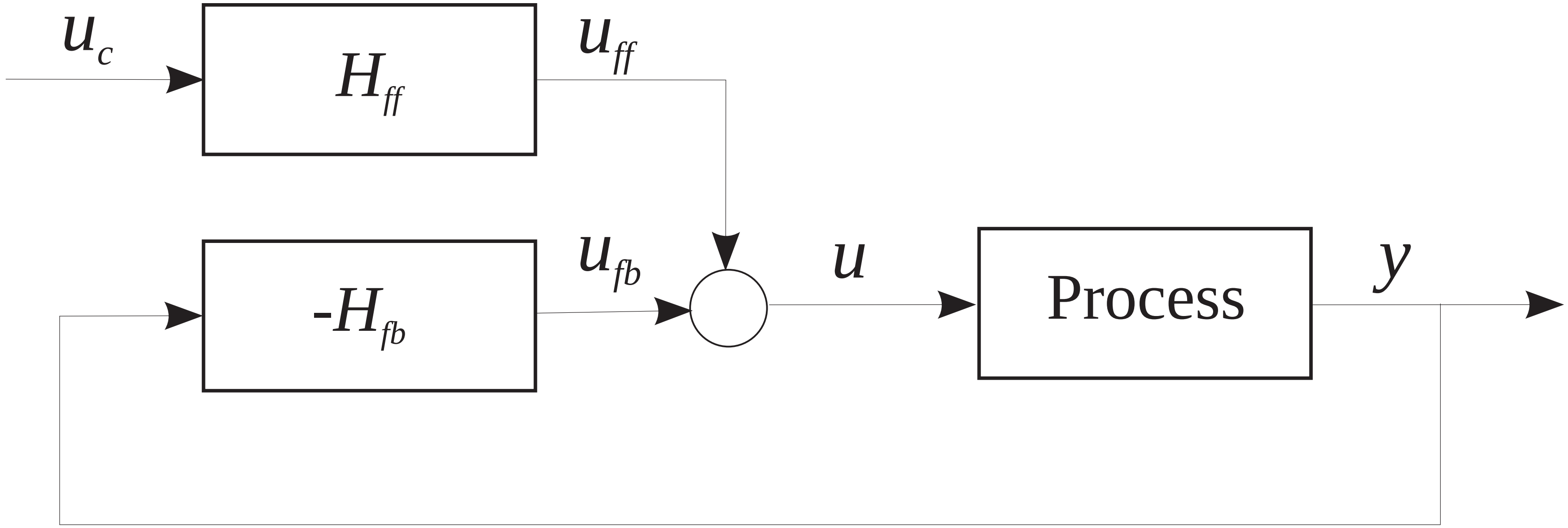
$$\begin{aligned} x(k+1) &= (\Phi - \Gamma L)x(k) + \Gamma L e(k) + \Gamma L_c u_c(k) \\ e(k+1) &= (\Phi - KC)e(k) \\ y(k) &= Cx(k) \end{aligned}$$

Pulse-transfer function from u_c to y:

$$H_{cl}(z) = C(zI - \Phi + \Gamma L)^{-1}\Gamma L_c = L_c \frac{B(z)}{A_m(z)}$$

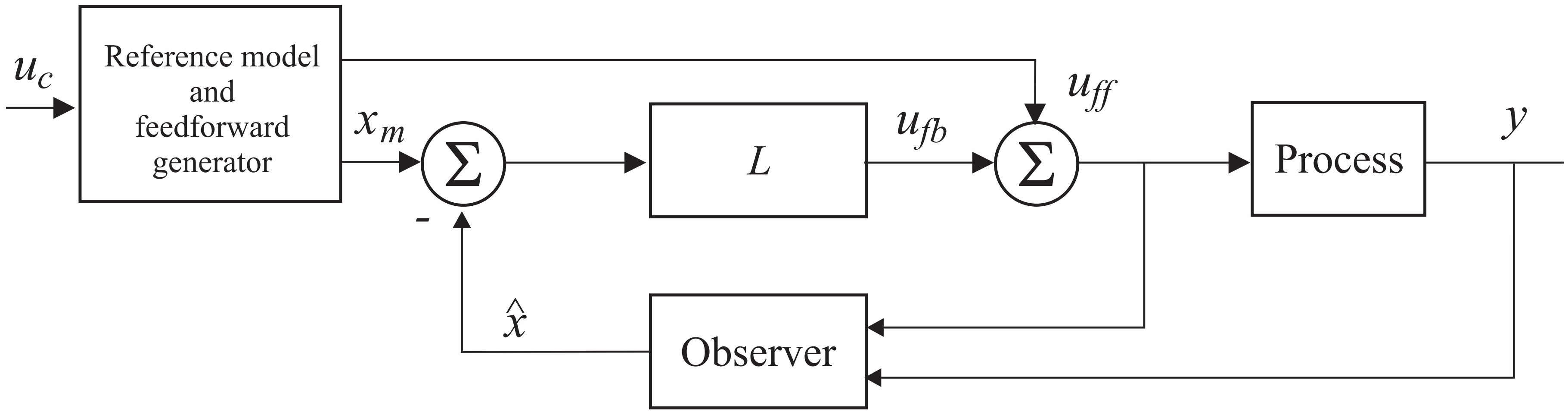
L_c for desired DC gain: $L_c = \dfrac{A_m(1)}{B(1)} \cdot H_{cl}(1)$

Servo case: two degrees of freedom



- H_{fb} is designed to obtain closed-loop system that is insensitive to disturbances, measurement noise, process uncertainties.
- H_{ff} is designed to obtain desired servo properties.

Servo case: model and feedforward



Controller

$$u(k) = \underbrace{L(x_m(k) - \hat{x}(k))}_{u_{fb}(k)} + u_{ff}(k)$$

Feedforward signal

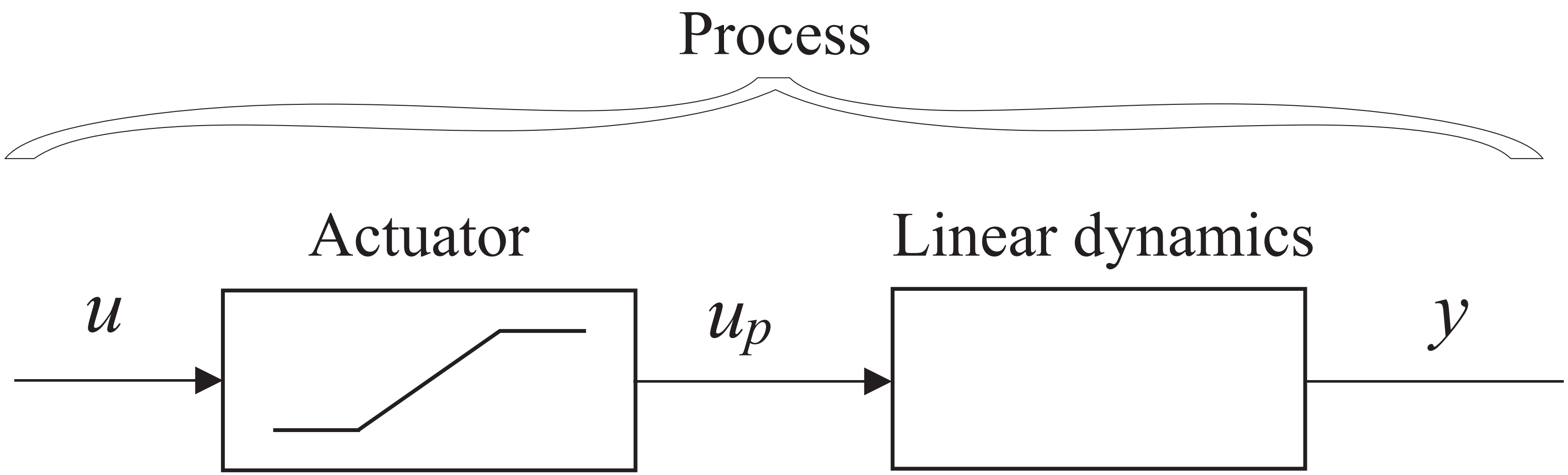
$$u_{ff}(k) = \frac{H_m(q)}{H(q)} u_c(k)$$

Reference trajectories for states

$$\begin{aligned} x_m(k+1) &= \Phi_m x_m(k) + \Gamma_m u_c(k) \\ y_m(k) &= C_m x_m(k) \end{aligned}$$

the same state coordinates as the process

Nonlinear actuators

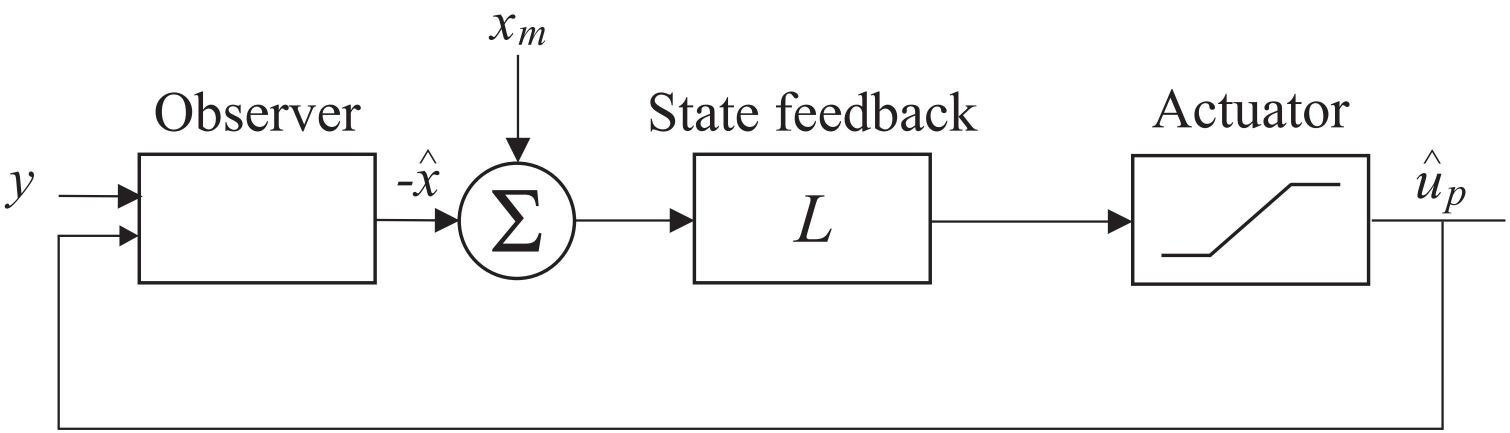


Feedback loop broken if control saturated

Is the controller stable?

Controller states may wind-up

Tracking scheme

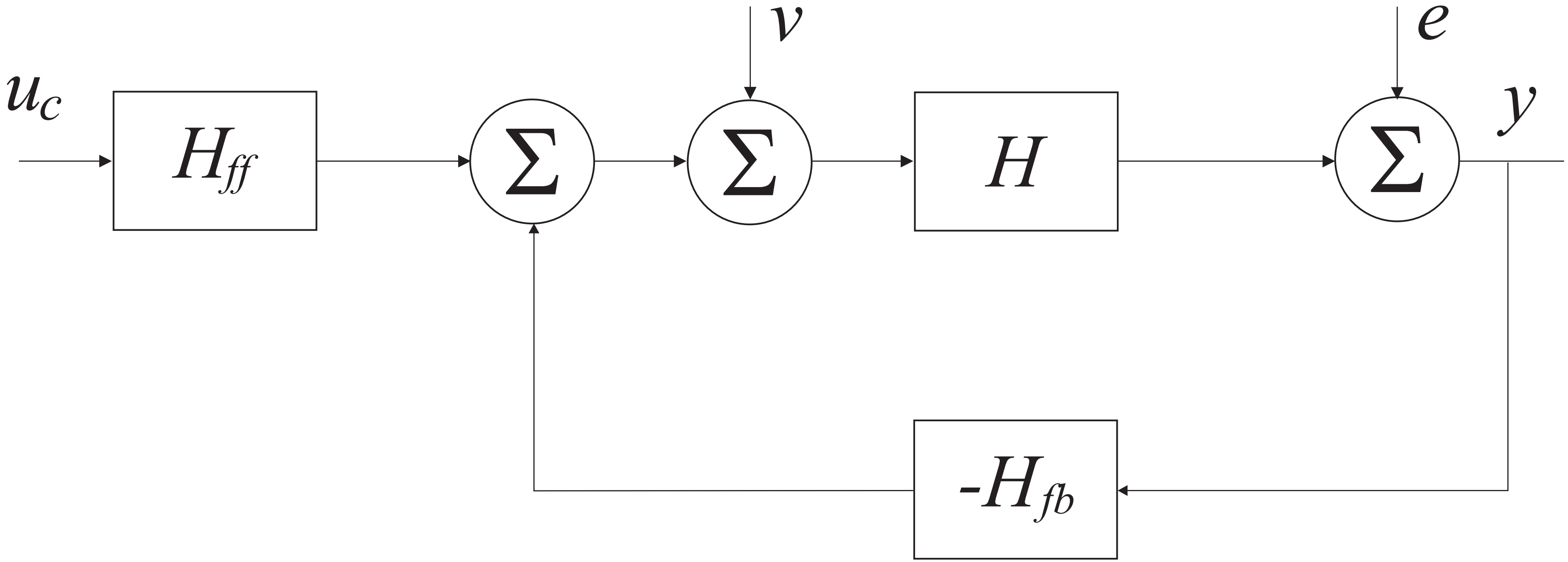


Measure or estimate the actual input u_p

$$\begin{aligned} \hat{x}(k+1) &= (\Phi - KC)\hat{x}(k) + Ky(k) + \Gamma\hat{u}_p(k) \\ \hat{u}_p(k) &= \mathsf{sat}(u(k)) \end{aligned}$$

A similar mechanism can be used for any type of controller (PID later)

Input/output design



Design H_{fb} and H_{ff} directly as rational transfer functions

Input/output design: formulation

- Process (SISO, including hold circuit, actuator, sensor, antialiasing filter):

$$H(z) = \frac{B(z)}{A(z)}$$

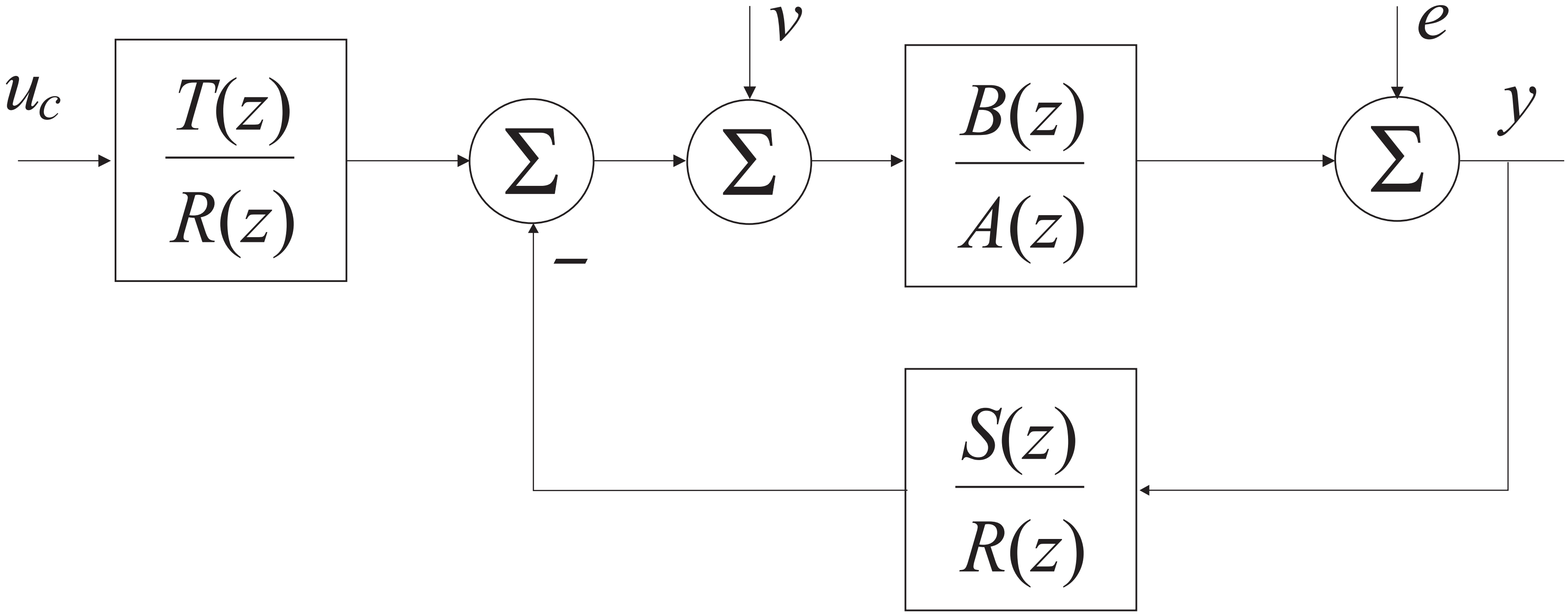
- RST Controller:

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

Causality implies: $\deg R \geq \deg T, \quad \deg R \geq \deg S$

$$H_{fb}(z) = S(z)/R(z), \quad H_{ff}(z) = T(z)/R(z)$$

RST controller



Closed loop

$$
\begin{aligned}
A(q)y(k) &= B(q)u(k) \\
R(q)u(k) &= T(q)u_c(k) - S(q)y(k)
\end{aligned}
$$

Closed loop system:

$$y = \frac{BT}{AR + BS} u_c$$

Desired input-output relation:

$$\frac{BT}{AR + BS} = \frac{BT}{A_{cl}} = \frac{BT}{A_c A_o} = \frac{t_o B}{A_c}$$

I/O pole-placement design

1. Find $R(z)$ and $S(z)$ $\left(\deg S(z) \leq \deg R(z)\right)$ satisfying

$$A(z)R(z) + B(z)S(z) = A_{cl}(z)$$

2. Factor $A_{cl}(z)$ as $A_{cl}(z) = A_c(z)A_o(z)$ with $\deg A_o(z) \leq \deg R(z)$ and choose

$$T(z) = t_o A_o(z)$$

where $t_o = A_c(1)/B(1)$ is chosen for desired static gain.

Diophantine equation

$$A(z)X(z) + B(z)Y(z) = C(z)$$

(Diophantus $\approx$ A.D. 300, also called Bezout identity)

- One equation, two unknowns.
- When does the Diophantine equation have a (unique) solution?
- Analogy: algebraic example.

Simple algebraic example

Assume x, y integers and

$$3x + 2y = 5$$

Some solutions are

$$\begin{array}{lrrrrrrr} x: & -5 & -3 & -1 & 1 & 3 & 5 & 7 \\ y: & 10 & 7 & 4 & 1 & -2 & -5 & -8 \end{array}$$

General solution:

$$\begin{aligned} x &= x_0 + 2n \\ y &= y_0 - 3n \end{aligned} \qquad n \text{ integer}$$

Unique solution if $0 \leq x < 2$ or $0 \leq y < 3$

Solution of the Diophantine equation

$$A(z)X(z) + B(z)Y(z) = C(z)$$

- Solution exists if and only if the greatest common factor of A and B is also a factor in C.
- Many solutions. If X_0 and Y_0 is a solution then for arbitrary Q

$$\begin{aligned} X &= X_0 + QB \\ Y &= Y_0 - QA \end{aligned}$$

  is also a solution
- Uniqueness if $\deg X < \deg B$ or $\deg Y < \deg A$

Euclid's algorithm, Sylvester matrix

Summary

- Compensate disturbances – integrator (see also PID)
- Servo control (reference following)
- Actuator saturation
- Input–output design (transfer functions)