AE4301 Automatic Flight Control System Design
Part I: Control Theory

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Lecture 6

What we learned so far

- Modeling and analysis of LTI systems in frequency and time domain
- System's response to external stimuli (transient component & steady-state component)
- We expect specific desired behaviors for both components of system's response $\rightarrow$ we design controllers
- Gain adjustment for pole placement: sometimes sufficient, sometimes not (increase of gain K can negatively affect system's stability and it may not completely eliminate steady-state error)
- Compensators: PID controllers
- Frequency-response analysis: Bode plots (magnitude and phase) for sinusoidal transfer function

What we will learn today

- Stability margin (gain and phase margins)
- Stability analysis based on Bode plots
- Compensator design in frequency domain
- We will also have a brief look at root locus method for control design

Gain margin

- Phase crossover frequency ω_{P}: Frequency corresponding to phase -180 degrees
- **Gain margin (GM):** reciprocal of magnitude $|G(j\omega_{\mathsf{P}})|$
- GM (dB) < 0 $\rightarrow$ unstable

Phase margin

- Gain crossover frequency ω_{G}: Frequency corresponding to magnitude 0 dB

- **Phase margin (PM):** $\angle G(j\omega_{\mathsf{G}}) - (-180)$ degrees

- PM > 0: system is stable
- PM = 0: system is neutrally stable
- PM < 0: system is unstable

Exercise

Consider a system with transfer function $G(s) = \dfrac{4\sqrt{3}}{s\left(s+1\right)^2}$.

Determine the phase margin (PM) and the gain margin (GM)

Compensator design

In control design if more than gain adjustment is needed compensators are designed.

- **Effect of adding zeros:** (e.g., derivative control):
  - pulling root locus to left: can make system more stable (steady-state effect)
  - speeds up settling time (transient response effect)
- **Effect of adding poles:** (e.g., integral control):
  - pulling root locus to right: may lower system's stability
  - slows down settling time

Options for fixing

We already saw proportional, derivative, and integral controllers. Additionally, the following categories of controllers have been found to be simple and effective:

- Lead filter – approximated PD function; speeds up response; lowers rise time; decreases overshoot; provides phase lead near cross-over frequency.
- Lag filter – approximates PI function; improves steady-state accuracy; provides additional gain at low frequencies.
- lag-lead – combination of the above two compensators

$$C(s) = K\frac{s+z}{s+p}$$

$z > p$: lead compensation
$z < p$: lag compensation

Lead compensator/filter

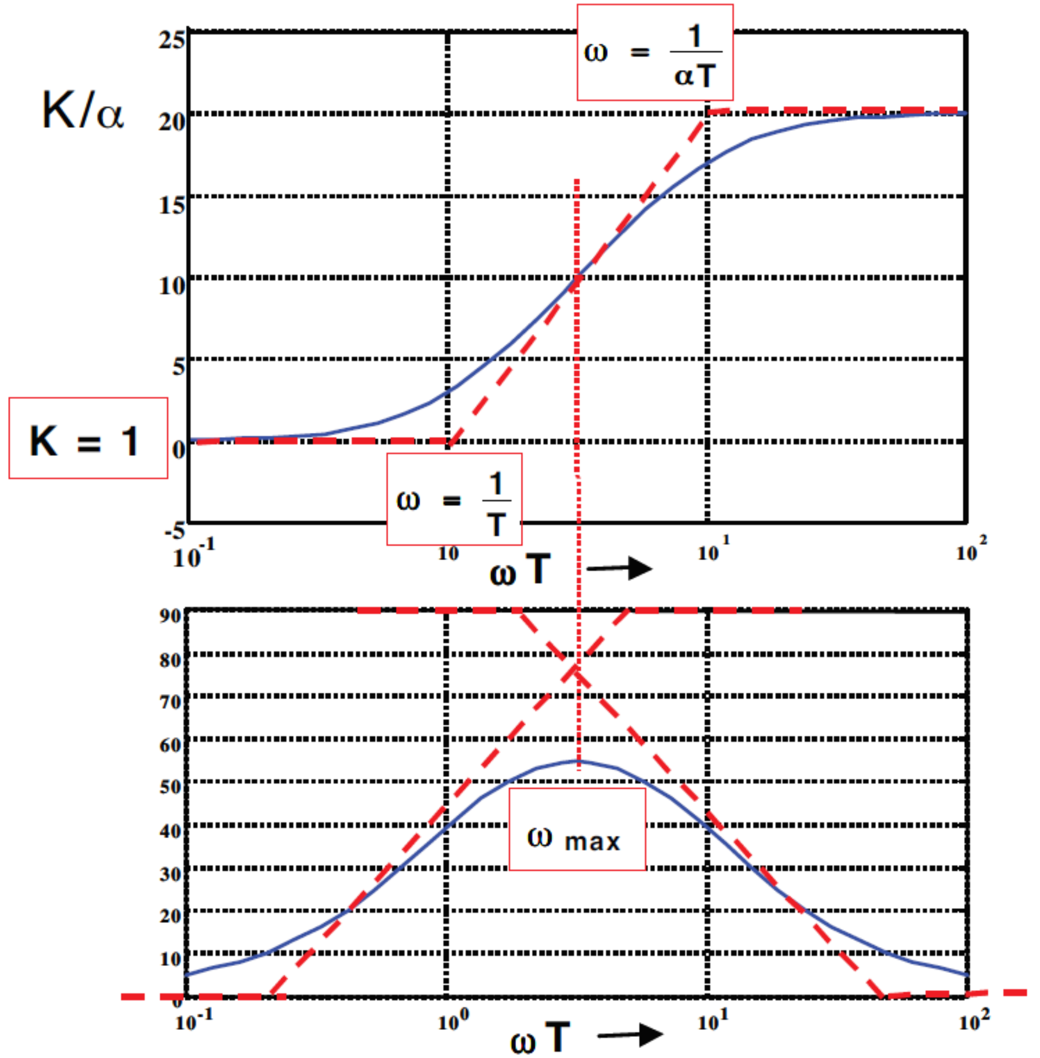
Consider a lead compensator with $z = -1/T$ and $p = -1/(\alpha T)$:

$$C(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1}, \qquad 0 < \alpha < 1$$

Since $0 < \alpha < 1$ zero is located to the right of pole in s-plane. When designing a lead compensator α may be a tuning parameter (assuming $\alpha > 0.05$):

$$\alpha = \frac{1 - \sin\left(\angle C(j\omega_{\text{max}})\right)}{1 + \sin\left(\angle C(j\omega_{\text{max}})\right)}$$

Bode plot & corner frequencies



Lead compensator

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$$\Rightarrow \boxed{\omega_{\max} = \frac{1}{\sqrt{\alpha} T}}$$

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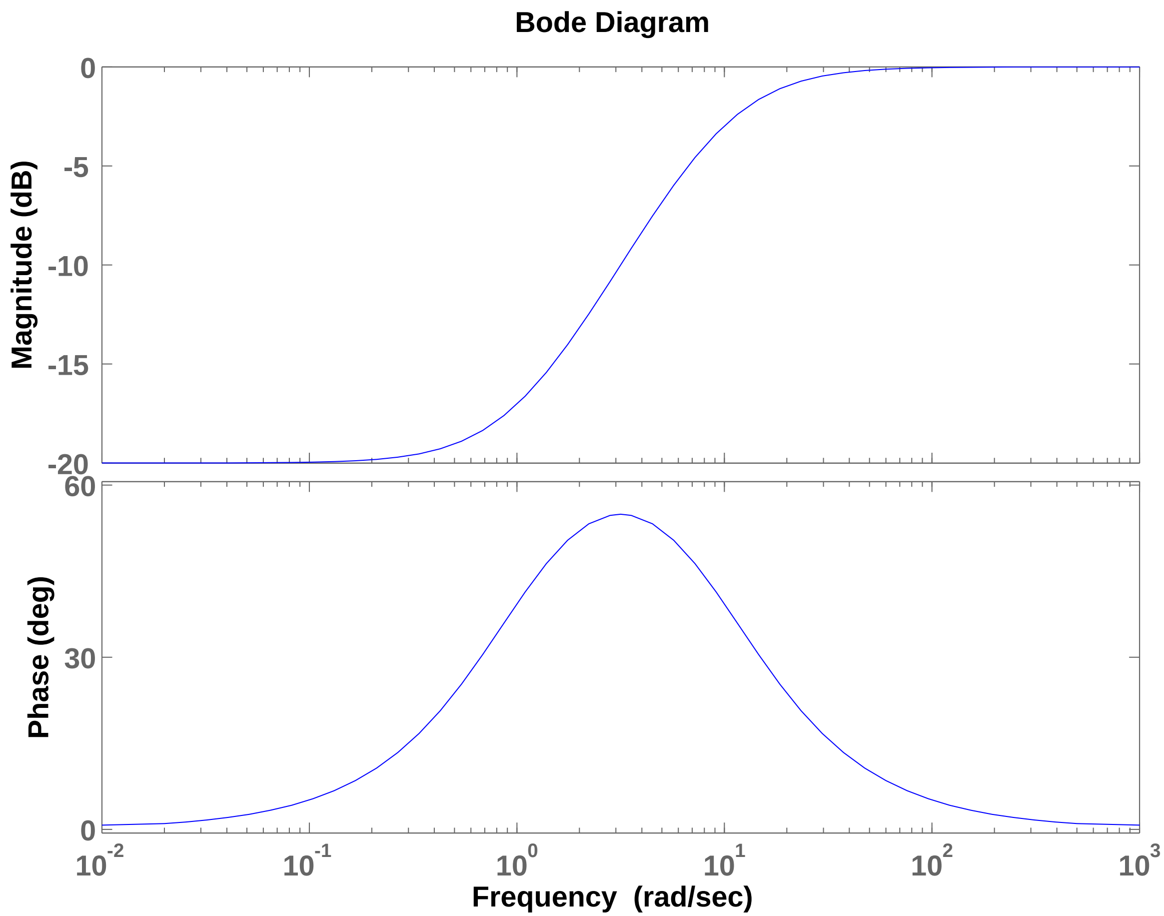
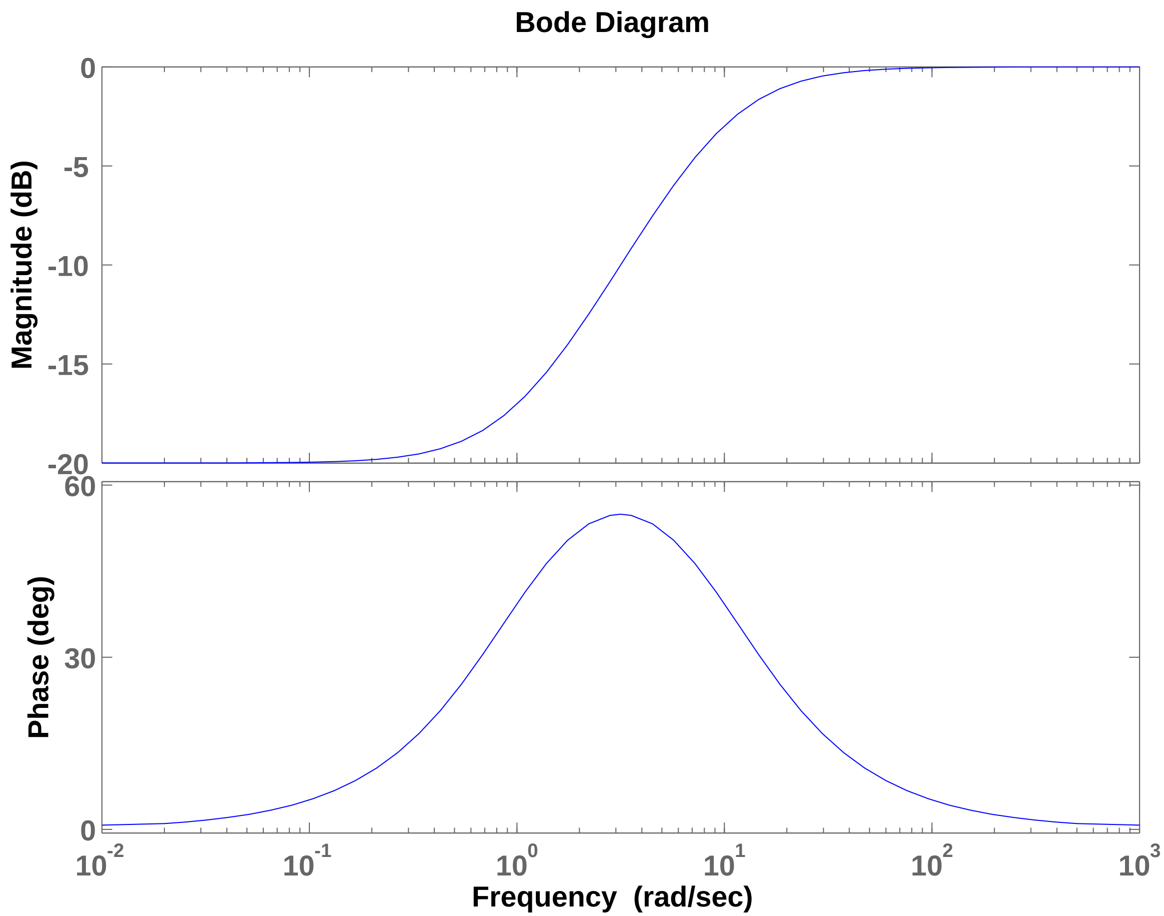
Why is it called a *lead* compensator?

Lead compensator example $\alpha = 0.1$



Lead compensator example $\alpha = 0.1$



Lead compensator is basically a high-pass filter, i.e., *high frequencies are passed but low frequencies are attenuated.*

Lag compensator

Consider a lag compensator with $z = -1/T$ and $p = -1/(\beta T)$:

$$C(s) = K_c \beta \frac{Ts+1}{\beta Ts+1}, \qquad \beta > 1$$

Since $\beta > 1$ pole is located to the right of zero in s-plane.
Corner frequencies are $1/T$ and $1/(\beta T)$.

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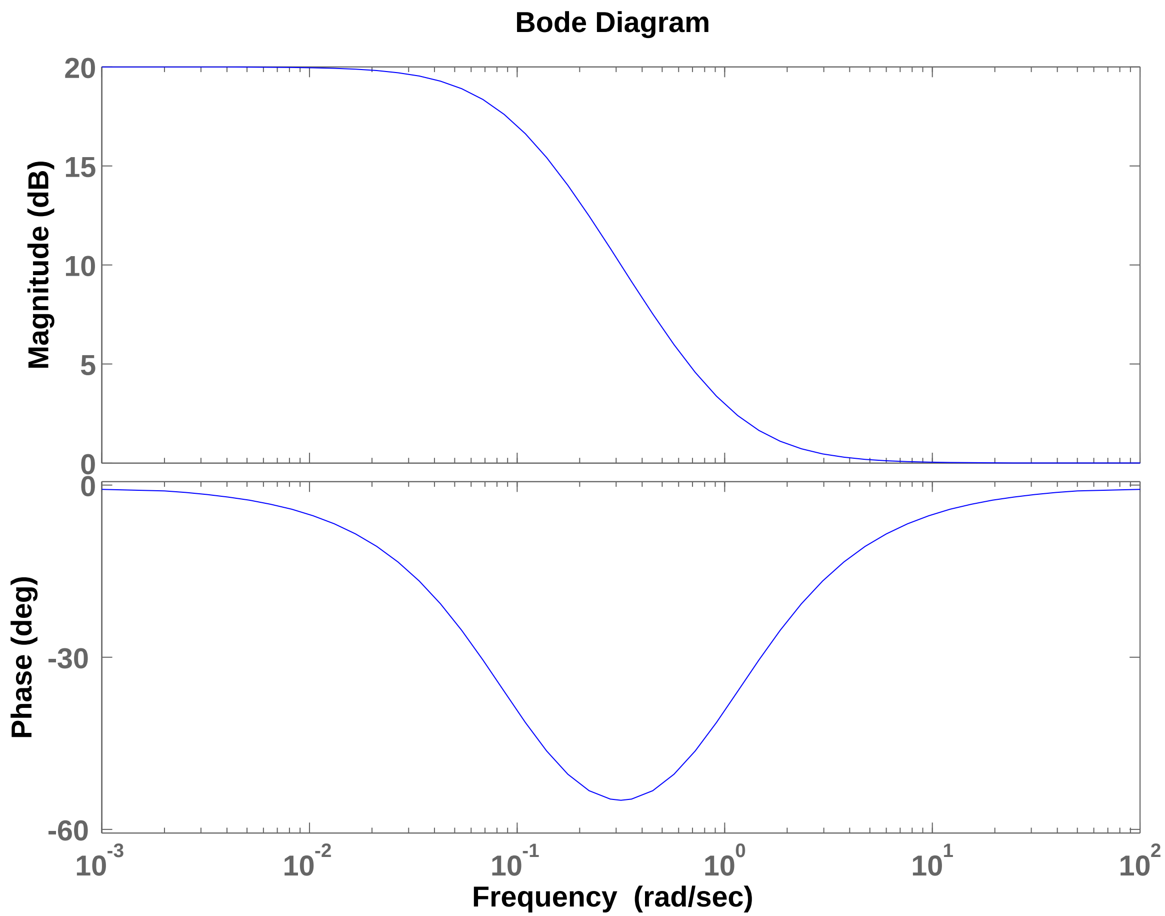
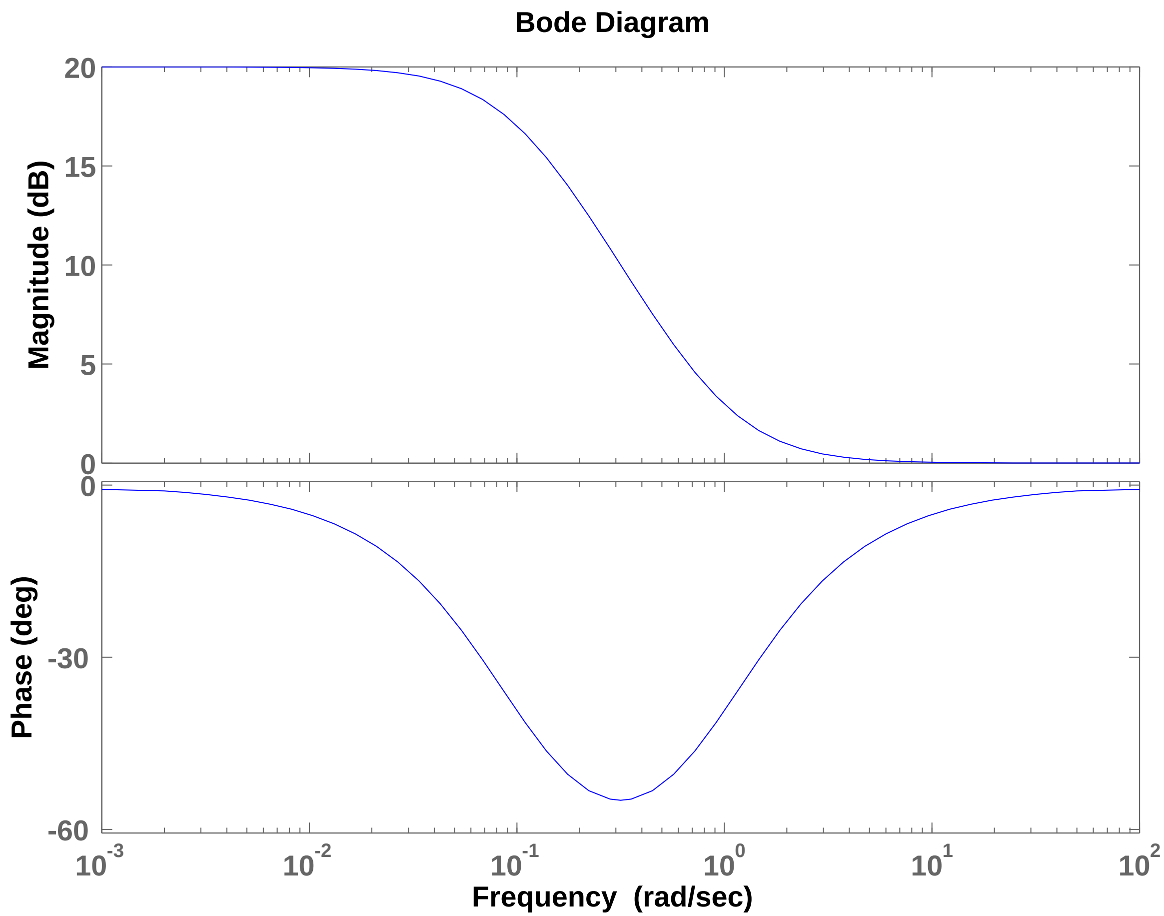
Lag compensator example $\beta = 10$



Lag compensator example $\beta = 10$



Lag compensator is basically a low-pass filter, i.e., *low frequencies are passed but high frequencies are attenuated.*

Lead compensator vs Lag compensator

Lead compensator	Lag compensator
Achieves desired results through phase lead contribution	Improves steady-state accuracy and achieves desired results through attenuation property at high frequencies
(Recall from previous lecture: higher cross-over frequency = larger bandwidth = faster response = reduced settling time) $\rightarrow$ Lead compensator improves transient response, but is sensitive to high-frequency noise signals	Low gain, low bandwidth, slower, while it can attenuate any high-frequency noise

Summary

- We learned about stability margins including gain margin and phase margin
- Desired gain and phase margin may be used to design controllers
- In particular, phase margin is directly linked with damping ratio, transient-response overshoot, and steady-state response resonant peak
- We learned about lead and lag compensators

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