AE4301 Automatic Flight Control System Design
Part I: Control Theory

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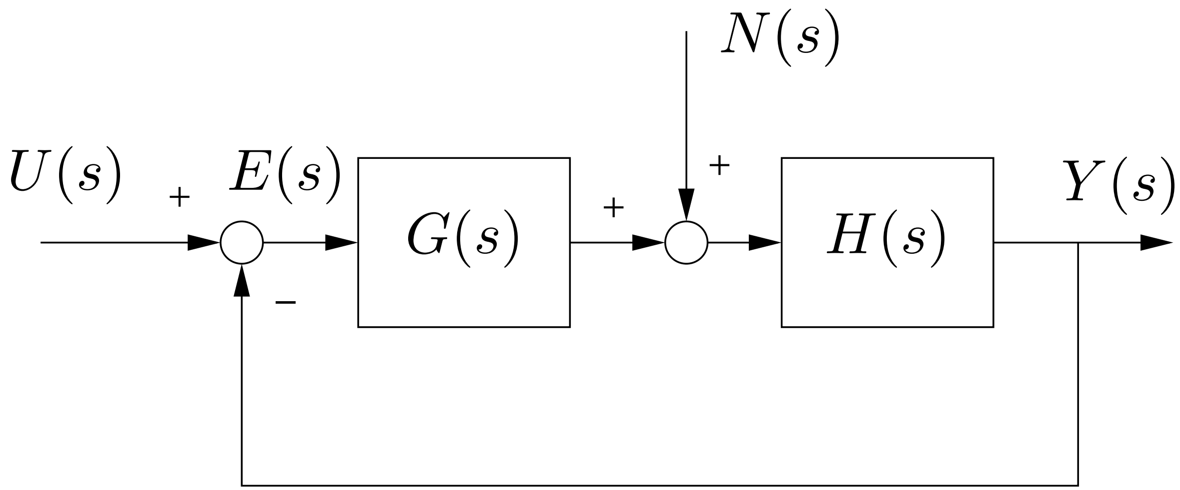
What we learned so far

- From differential equations to transfer function
- Characteristics of systems based on poles and zeros
- Root locus method
- State space representation, controllability, observability

Main objectives

- Being able to use initial & final value theorems
- Being able to determine steady-state error/response of various systems
- Being able to design standard controllers (e.g., P, PI, PD, PID) for given performance specifications

Design specifications



- Make steady state error ($e(t)$ for $t \to \infty$) small (e.g., 0)
- Reduce negative effects of noise and disturbances on output
- Make system stable
- Design desired transient behavior

Initial and final value theorems

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} \{sF(s)\}$$

Initial value theorem is calculated for 0^+ (right after time 0)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} \{sF(s)\}$$

Final value theorem is valid if $\lim_{t\to\infty} f(t)$ exists (not valid for unstable system)

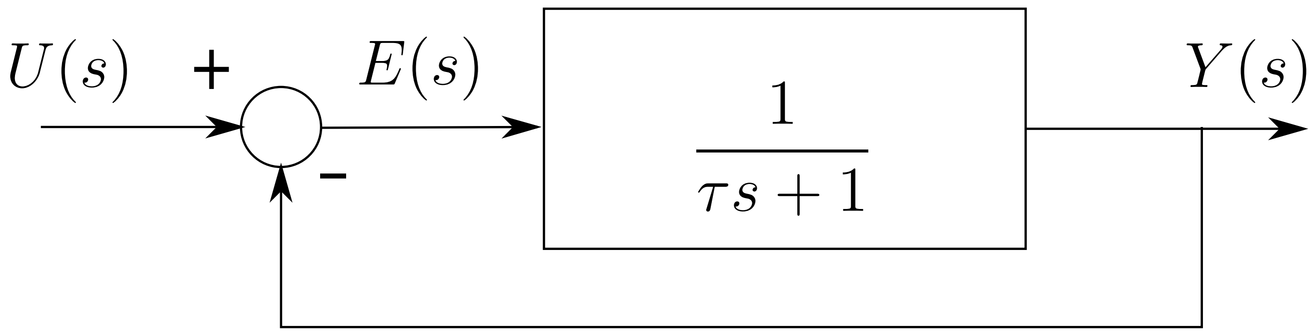
Control system $C(s)$

We will consider three types of control systems in this lecture:

- Proportional control
- Integral control
- Derivative control

We also consider combinations of these control systems

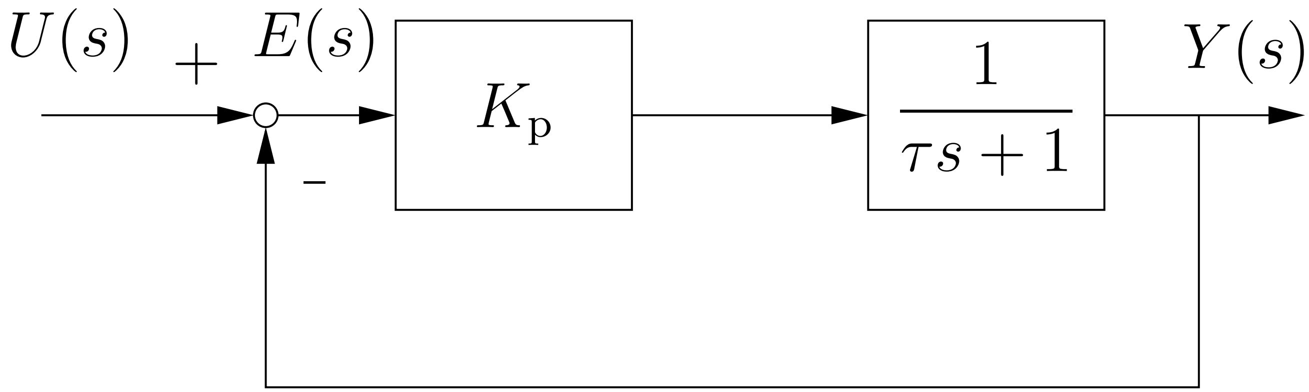
Uncontrolled system: Unit-step response



Determine the steady-state error of the closed-loop system to a unit-step input.

$$e_{\text{ss}} = \lim_{t \to \infty} e(t) = 0.5$$

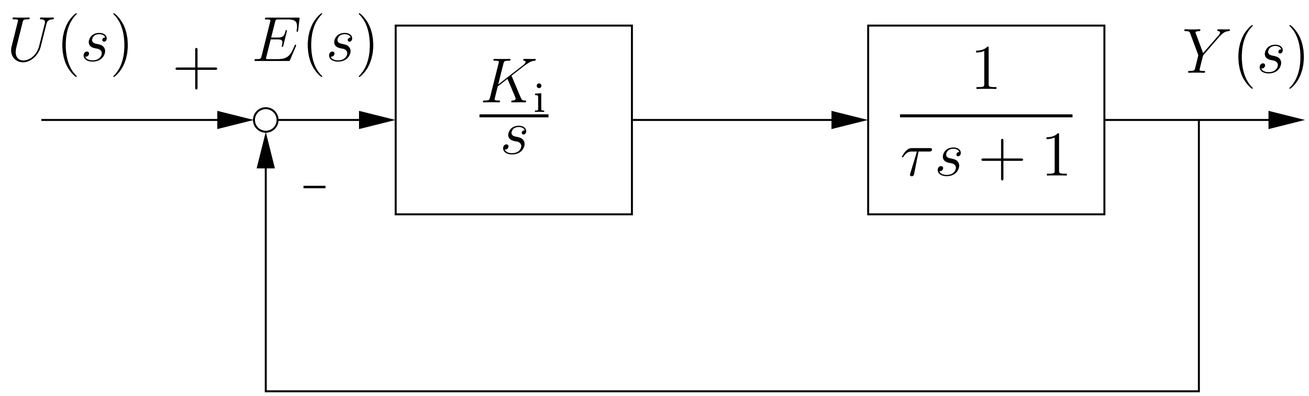
Proportional control: Unit-step response



Determine the steady-state error of the closed-loop system with a proportional controller in the feedforward path to a unit-step input.

$$e_{\mathrm{ss}} = \lim_{t \to \infty} e(t) = \frac{1}{1 + K_\mathrm{p}}$$

Integral control: Unit-step response



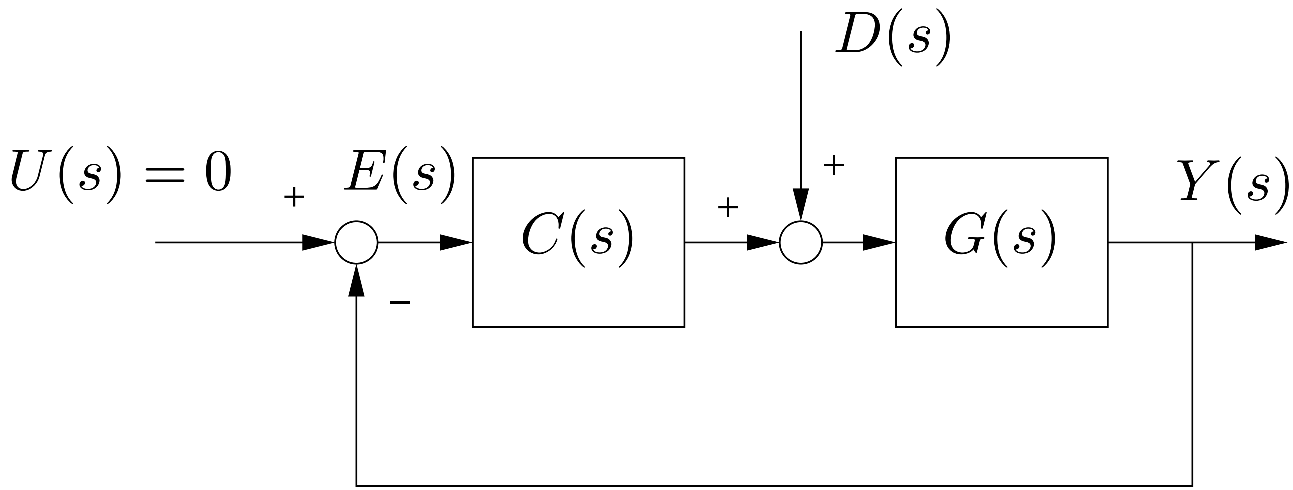
Determine the steady-state error of the closed-loop system with an integral controller in the feedforward path to a unit-step input.

$$e_{\mathrm{ss}} = \lim_{t\to\infty} e(t) = 0$$

Summary: Unit-step steady-state error for 1st-order system

- 1st-order system $G(s) = \dfrac{1}{\tau s + 1}$ has a steady-state error of 0.5.
- Adding only a proportional controller $C(s) = K_{\mathrm{p}}$ does not eliminate steady-state error $\dfrac{1}{K_{\mathrm{p}} + 1}$.
- Adding an integral controller $C(s) = \dfrac{K_{\mathrm{i}}}{s}$ eliminates steady-state error.

Disturbance input



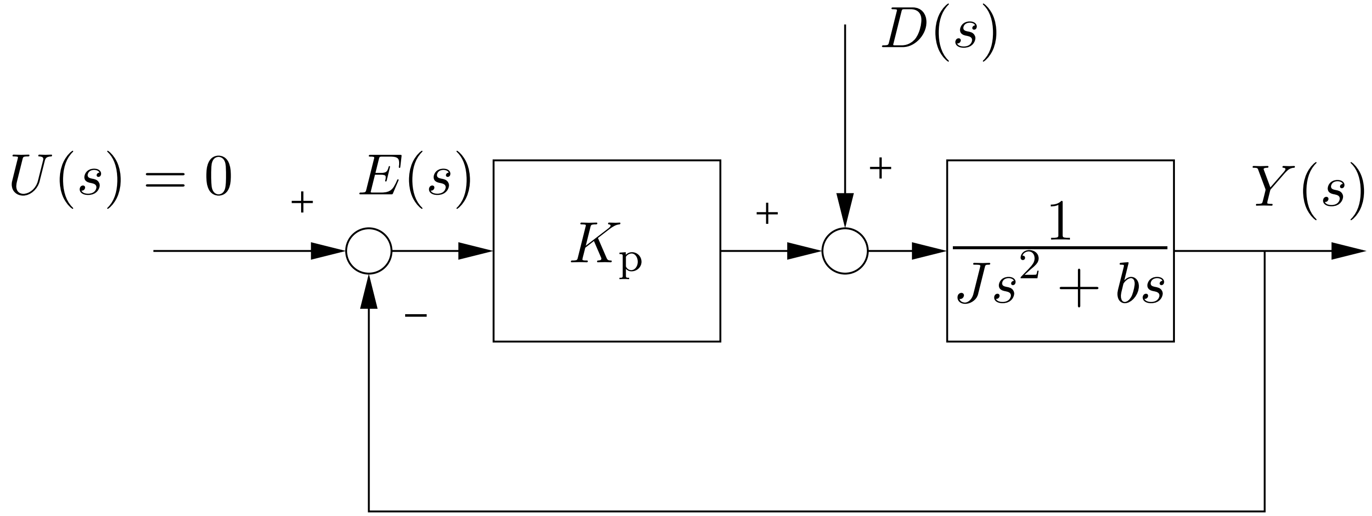
What is the transfer function between $Y(s)$ and $D(s)$?

$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + C(s)G(s)}$$

What is the transfer function between $E(s)$ and $D(s)$?

$$\frac{E(s)}{D(s)} = -\frac{G(s)}{1 + C(s)G(s)}$$

2nd-order system: Proportional control

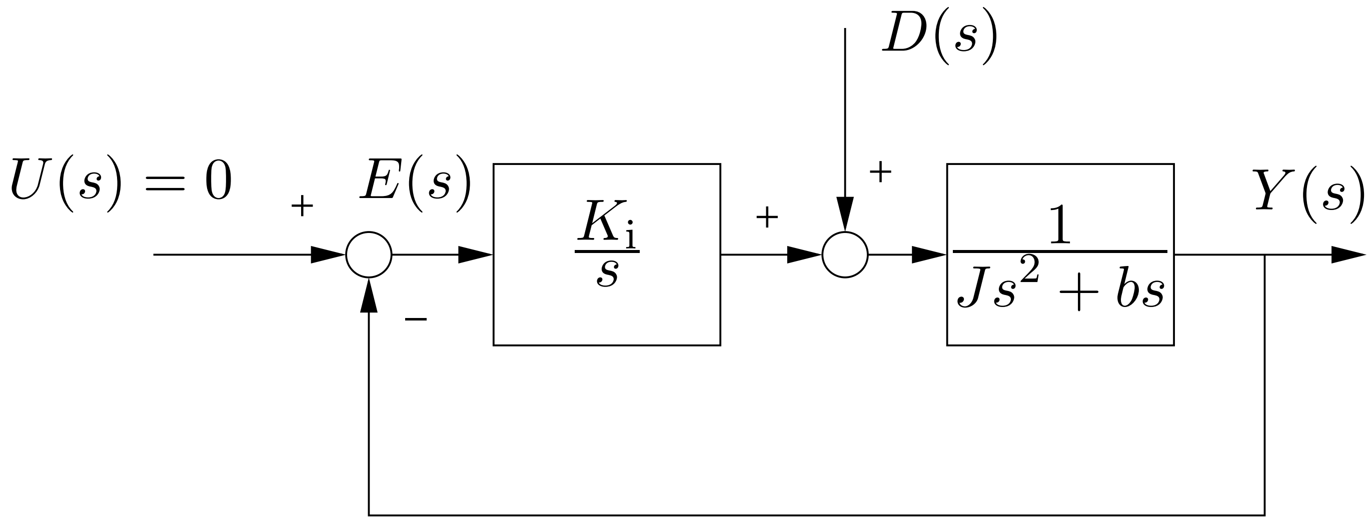


Determine the steady-state error due to a step disturbance of magnitude M_d.

$$e_\mathrm{ss} = \lim_{t \to \infty} e(t) = -\frac{M_\mathrm{d}}{K_\mathrm{p}}$$

Steady-state error can be reduced via increasing K_p.
Increasing K_p makes system's response more oscillatory.

2nd-order system: Integral control

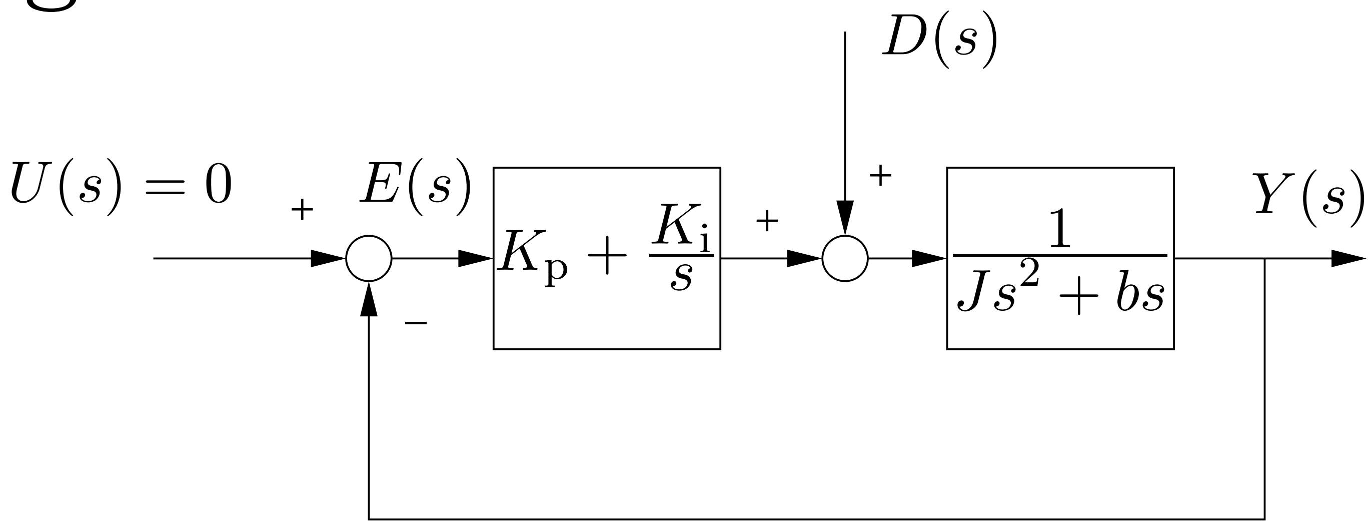


Determine the characteristic equation of $\dfrac{Y(s)}{D(s)}$.

$$\frac{Y(s)}{D(s)} = \frac{s}{Js^3 + bs^2 + K_i} \Rightarrow \boxed{Js^3 + bs^2 + K_i = 0}$$

What can be said about the stability of the system? There are roots with positive real parts. Therefore, the system becomes unstable.

2nd-order system: Proportional + Integral control

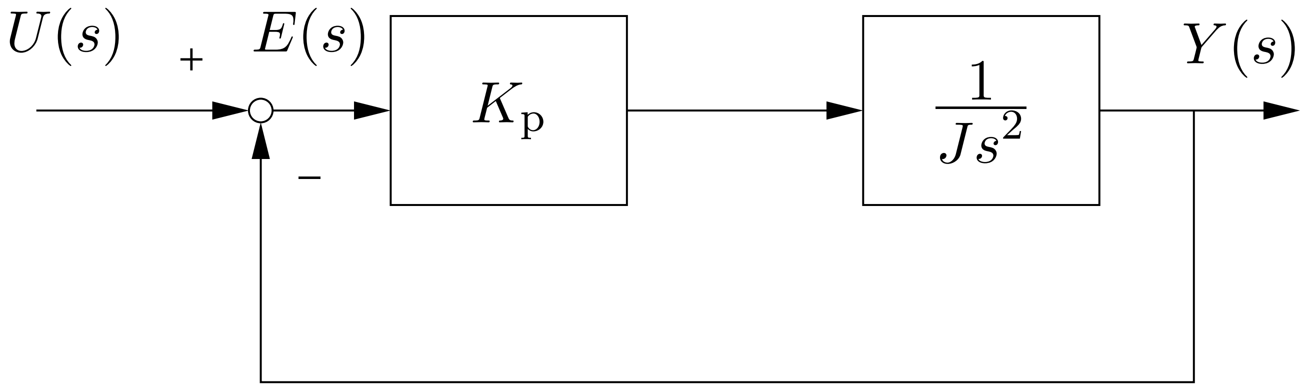


Determine the steady-state error due to a step disturbance of magnitude M_d.

$$e_{\mathrm{ss}} = \lim_{t \to \infty} e(t) = 0$$

Steady-state error to step disturbance can be eliminated with proportional-plus-integral controller.

Oscillatory response



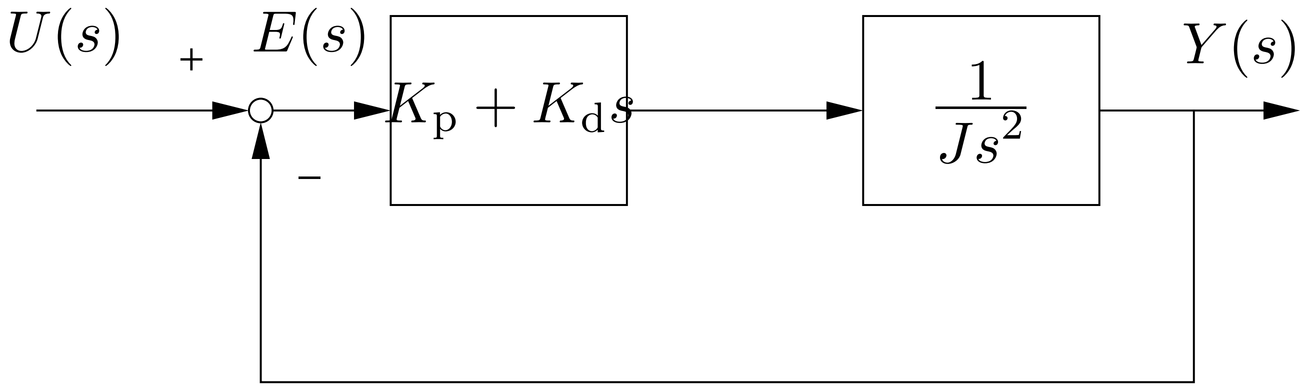
Determine the closed-loop transfer function.

$$\frac{Y(s)}{U(s)} = \frac{K_\mathrm{p}}{Js^2 + K_\mathrm{p}}$$

What can you deduce about the system's response to unit-step input from closed-loop poles?

Closed-loop poles are imaginary $\rightarrow$ response to unit-step input will oscillate infinitely.

Stabilization: Proportional + Derivative control



Determine the closed-loop transfer function.

$$\frac{Y(s)}{U(s)} = \frac{K_p + K_d s}{Js^2 + K_d s + K_p}$$

What can you deduce about the system's response to unit-step input from closed-loop poles?

Closed-loop poles have negative real parts $\rightarrow$ Derivative control introduces a damping effect.

General case: PID control

- Type N system has a pole of multiplicity N at origin:

$$G(s) = \frac{1}{s^N}\left(\frac{b_0 + b_1 s + b_2 s^2 + \dots}{a_0 + a_1 s + a_2 s^2 + \dots}\right)$$

- Adding an integral controller can remove steady-state error
- Adding a differential controller can remove oscillations
- A PID controller combines proportional, integral, and derivative controllers (three parameters K_p, K_i, K_d to be tuned
- Process of selecting parameters to meet given performance specifications is tuning

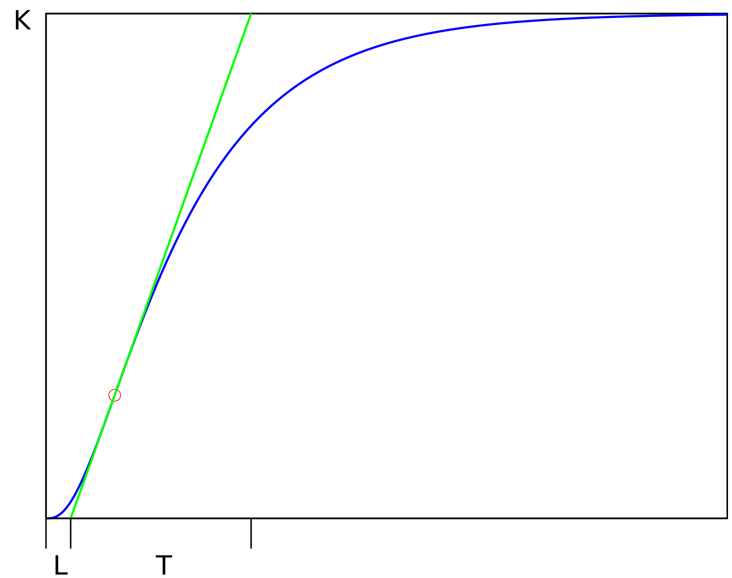
PID tuning: Ziegler-Nichols method

- Transfer function corresponding to PID controller:

$$G_{\mathrm{PID}}(s) = K_{\mathrm{p}} + \frac{K_{\mathrm{i}}}{s} + K_{\mathrm{d}}s = K_{\mathrm{p}}\left(1 + \frac{1}{T_i s} + T_d s\right)$$

- Ziegler and Nichols suggested rules to tune K_{p}, T_i and T_d based on transient response characteristics of system
- Ziegler-Nichols tuning includes 2 methods:
  - **First method:** Based on experimental unit-step response
  - **Second method:** Critical gain method

First method



- Obtain experimentally response of system to unit-step input.
- If response is S-shaped:
  - Graphically determine delay time L and time constant T.
  - Tangent line is drawn at inflection point to intersect with $y(t) = 0$ and $y(t) = k$.

First method

Parameters are tuned for various controllers using the following table:

controller	K_p	T_i	T_d
P	T/L	NA	NA
PI	$0.9T/L$	$L/0.3$	NA
PID	$1.2T/L$	$2L$	$0.5L$

Second method

- First consider only a proportional controller.
- Increase K_p from 0 until the output exhibit sustained oscillations.
- The corresponding K_p is called critical gain K_cr.
- Corresponding period of oscillations is called critical period P_cr.

Second method

Parameters are tuned for various controllers using the following table:

controller	K_p	T_i	T_d
P	$0.5K_\mathrm{cr}$	NA	NA
PI	$0.45K_\mathrm{cr}$	$P_\mathrm{cr}/1.2$	NA
PID	$0.6K_\mathrm{cr}$	$0.5P_\mathrm{cr}$	$0.125P_\mathrm{cr}$

Summary

- System type determines whether the system will suffer from steady-state error in response to various inputs
- Adding an integral controller can eliminate the steady-state error
- Adding a derivative controller can introduce influence of damping
- PID controllers combine proportional, integral, and derivative controllers to get desired behavior

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