Control Engineering SC42095

Kalman filtering and LQG control

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Lecture outline

1. Review
   - LQ control
   - completing the squares
   - dynamic programming
2. Kalman filtering
3. Linear quadratic Gaussian control

Note: These slides are partly inspired by the slides for this course developed at the Department of Automatic Control, Lund Institute of Technology (see http://www.control.lth.se/~kursdr)

Lecture outline (continued)

- Linear Quadratic (LQ) control $\longrightarrow$ assumes *full state information*
- Estimating state from measurements of output = Kalman filtering
- Combination of LQ and state estimation = Linear Quadratic Gaussian (LQ) control based on *separation theorem*

1. Review

1.1 LQ control

- Minimize
$$J=\sum_{k=0}^{N-1}\Big(x^{\mathsf{T}}(k)Q_1x(k)+2x^{\mathsf{T}}(k)Q_{12}u(k)+u^{\mathsf{T}}(k)Q_2u(k)\Big)+x^{\mathsf{T}}(N)Q_0x(N)$$
subject to $x(k+1)=\Phi x(k)+\Gamma u(k)$ and $x(0)=x_0$

- Solution approach based on quadratic optimization problem and dynamic programming

- Results in state feedback controller $u=-L(k)x(k)$ with $L(k)$ determined by solution $S(k)$ of Riccati recursion

1.2 Completing the squares

- Find u that minimizes $x^\mathsf{T} Q_x x + 2x^\mathsf{T} Q_{xu} u + u^\mathsf{T} Q_u u$ with Q_u positive definite

- Let L be such that $Q_u L = Q_{xu}^\mathsf{T}$. Then

$$x^\mathsf{T} Q_x x + 2x^\mathsf{T} Q_{xu} u + u^\mathsf{T} Q_u u =$$

$$x^\mathsf{T} (Q_x - L^\mathsf{T} {\color{red} Q_u} L) x + (u + Lx)^\mathsf{T} Q_u (u + Lx)$$

  is minimized for $u = -Lx$

  and minimum value is $x^\mathsf{T} (Q_x - L^\mathsf{T} {\color{red} Q_u} L) x$

- If Q_u is positive definite, then $L = Q_u^{-1} Q_{xu}^\mathsf{T}$

1.3 Dynamic programming

- **Principle of optimality**: From any point on optimal trajectory, remaining trajectory is also optimal

$\rightarrow$ allows to determine best control law over period $[t_2, t_3]$ independent of how state at t_2 was reached

- For N-step problem:
  - start from end at time $k = N$
  - now we can determine best control law for last step independent of how state at time $N-1$ was reached
  - iterate backward in time to initial time $k = 0$

2. Kalman filtering

- LQ control requires full state information
- In practice: only output measured
  $\rightarrow$ how to estimate states from noisy measurements of output?
- Consider system

$$
\begin{aligned}
x(k+1) &= \Phi x(k) + \Gamma u(k) + v(k) \\
y(k) &= Cx(k) + e(k)
\end{aligned}
$$

with v, e Gaussian zero-mean white noise process with

$$
\mathsf{E}\left[v(k)v^{\mathsf{T}}(k)\right] = R_1, \quad \mathsf{E}\left[e(k)e^{\mathsf{T}}(k)\right] = R_2, \quad \mathsf{E}\left[v(k)e^{\mathsf{T}}(k)\right] = R_{12}
$$

and $x(0)$ Gaussian distributed with

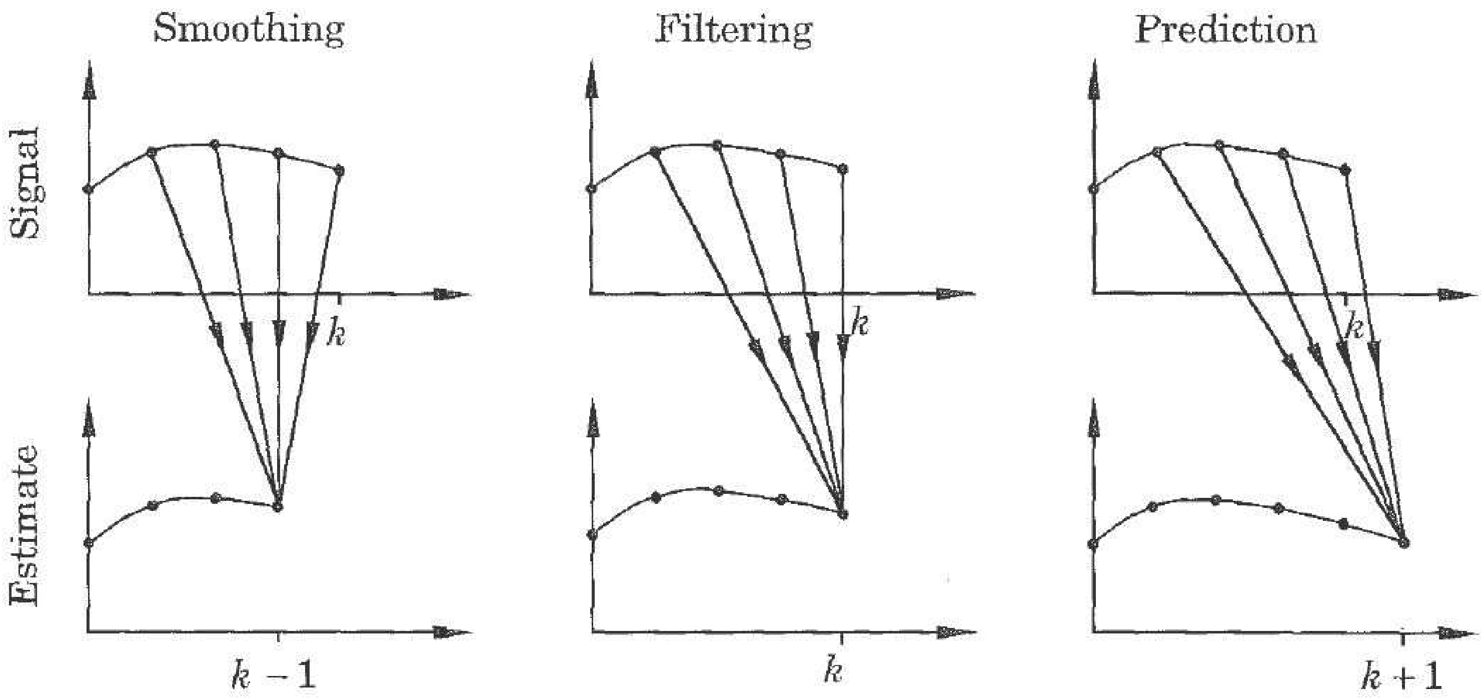
$$
\begin{aligned}
&\mathsf{E}\left[x(0)\right] = m_0 \\
&\operatorname{cov}\left(x(0)\right) := \mathsf{E}\left[\left(x(0) - \mathsf{E}\left[x(0)\right]\right)\left(x(0) - \mathsf{E}\left[x(0)\right]\right)^{\mathsf{T}}\right] = R_0
\end{aligned}
$$

2.1 Problem formulation

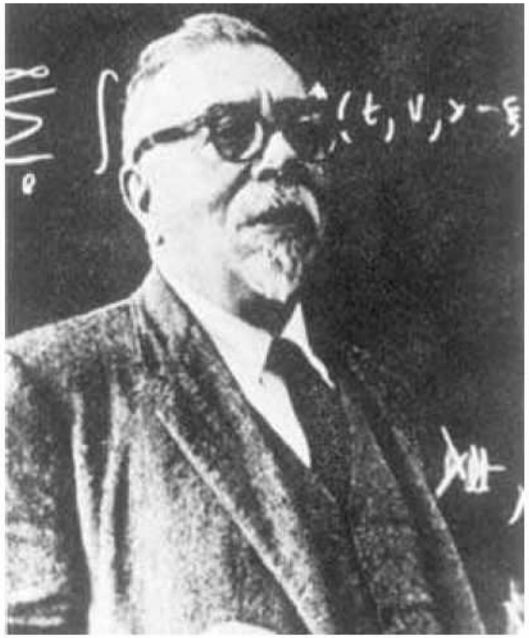
- Given the data

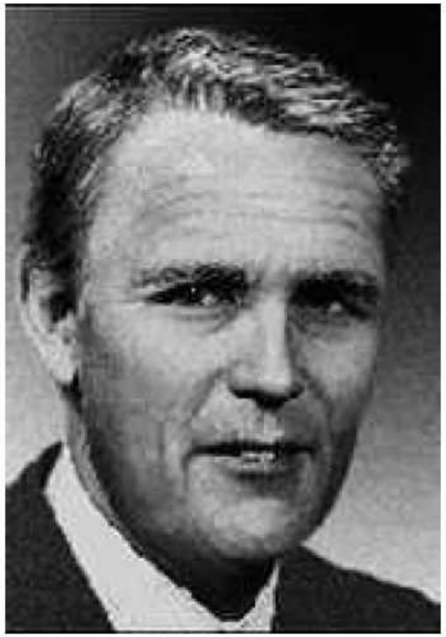
$$Y_k = \{y(i), u(i) \; : \; 0 \leq i \leq k\}$$

find the "best" (to be defined) estimate $\hat{x}(k+m)$ of $x(k+m)$.
($m = 0$ filtering, $m > 0$ prediction, $m < 0$ smoothing)



Some history





Norbert Wiener: Filtering, prediction, and smoothing using integral equations. Spectral factorizations.

Rudolf E. Kalman: Filtering, prediction, and smoothing using state-space formulas. Riccati equations.

2.2 Kalman filter structure

- Goal is to estimate $x(k+1)$ by linear combination of previous inputs and outputs
- Estimator (cf. lecture on observers):

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + \Gamma u(k) + K(k)\big(y(k) - C\hat{x}(k|k-1)\big)$$

with $\hat{x}(k+1|k)$ estimate of state x at sample step $k+1$ using information available at step k

- Error dynamics $\tilde{x} = x - \hat{x}$ governed by

$$\begin{aligned}
\tilde{x}(k+1) &= x(k+1) - \hat{x}(k+1|k) \\
&= \Phi x(k) + \Gamma u(k) + v(k) - \Phi\hat{x}(k|k-1) - \Gamma u(k) - K(k)\big(y(k) - C\hat{x}(k|k-1)\big) \\
&= \Phi x(k) + v(k) - \Phi\hat{x}(k|k-1) - K(k)\big(Cx(k) + e(k) - C\hat{x}(k|k-1)\big) \\
&= (\Phi - K(k)C)\tilde{x}(k) + v(k) - K(k)e(k)
\end{aligned}$$

2.3 Determination of Kalman gain

- Error dynamics: $\tilde{x}(k+1) = (\Phi - K(k)C)\tilde{x}(k) + v(k) - K(k)e(k)$
- If $\hat{x}(0) = m_0$, then $\mathsf{E}\left[\tilde{x}(0)\right] = \mathsf{E}\left[x(0) - \hat{x}(0)\right] = \mathsf{E}\left[x(0)\right] - m_0 = 0$ and thus $\mathsf{E}\left[\tilde{x}(k)\right] = 0$ for all k
- How to choose $K(k)$? $\rightarrow$ minimize covariance of $\tilde{x}(k)$
- We have

$$\begin{aligned} \text{cov}\left(\tilde{x}(k)\right) &= \mathsf{E}\left[\left(\tilde{x}(k) - \mathsf{E}\left[\tilde{x}(k)\right]\right)\left(\tilde{x}(k) - \mathsf{E}\left[\tilde{x}(k)\right]\right)^{\mathsf{T}}\right] \\ &= \mathsf{E}\left[\tilde{x}(k)\,\tilde{x}^{\mathsf{T}}(k)\right] \end{aligned}$$

- So if we define $P(k) = \mathsf{E}\left[\tilde{x}(k)\tilde{x}^{\mathsf{T}}(k)\right]$, then we have to determine Kalman gain such that $P(k)$ is minimized

2.3 Determination of Kalman gain (continued)

- Error dynamics: $\tilde{x}(k+1) = (\Phi - K(k)C)\tilde{x}(k) + v(k) - K(k)e(k)$
- We have

$$P(k+1) = \mathsf{E}\left[\tilde{x}(k+1)\tilde{x}^{\mathsf{T}}(k+1)\right]$$
$$= \mathsf{E}\left[\Big((\Phi - K(k)C)\tilde{x}(k) + v(k) - K(k)e(k)\Big)\Big((\Phi - K(k)C)\tilde{x}(k) + v(k) - K(k)e(k)\Big)^{\mathsf{T}}\right]$$

- Since $\tilde{x}(k)$ is independent of $v(k)$ and $e(k)$, this results in

$$P(k+1) = \mathsf{E}\Big[(\Phi - K(k)C)\tilde{x}(k)\tilde{x}^{\mathsf{T}}(k)(\Phi - K(k)C)^{\mathsf{T}}$$
$$+ v(k)v^{\mathsf{T}}(k) + K(k)e(k)e^{\mathsf{T}}(k)K^{\mathsf{T}}(k) - v(k)e^{\mathsf{T}}(k)K^{\mathsf{T}}(k) - K(k)e(k)v^{\mathsf{T}}(k)\Big]$$
$$= (\Phi - K(k)C)\underbrace{\mathsf{E}\left[\tilde{x}(k)\tilde{x}^{\mathsf{T}}(k)\right]}_{P(k)}(\Phi - K(k)C)^{\mathsf{T}} + R_1 + K(k)R_2K^{\mathsf{T}}(k)$$
$$- R_{12}K^{\mathsf{T}}(k) - K(k)R_{12}^{\mathsf{T}}$$

2.3 Determination of Kalman gain (continued)

- So

$$P(k+1) = (\Phi - K(k)C)P(k)(\Phi - K(k)C)^{\mathsf{T}} + R_1 + K(k)R_2K^{\mathsf{T}}(k) - R_{12}K^{\mathsf{T}}(k) - K(k)R_{12}^{\mathsf{T}}$$

$$= K(k)\big(CP(k)C^{\mathsf{T}} + R_2\big)K^{\mathsf{T}}(k) - \big(\Phi P(k)C^{\mathsf{T}} + R_{12}\big)K^{\mathsf{T}}(k) - K(k)\big(CP^{\mathsf{T}}(k)\Phi^{\mathsf{T}} + R_{12}^{\mathsf{T}}\big) + \big(\Phi P(k)\Phi^{\mathsf{T}} + R_1\big)$$

- Minimize $P(k+1)$ with $K(k)$ as decision variable
- $P(k+1)$ is quadratic function of $K(k)$
- Use completing-of-squares solution with $u = K^{\mathsf{T}}(k)$ and $x = I$:

$$\underbrace{K(k)}_{u^{\mathsf{T}}}\underbrace{\big(CP(k)C^{\mathsf{T}} + R_2\big)}_{Q_u}\underbrace{K^{\mathsf{T}}(k)}_{u} + \underbrace{\big(-\Phi P(k)C^{\mathsf{T}} - R_{12}\big)}_{Q_{xu}}\underbrace{K^{\mathsf{T}}(k)}_{u}$$

$$+\underbrace{K(k)}_{u^{\mathsf{T}}}\underbrace{\big(-CP^{\mathsf{T}}(k)\Phi^{\mathsf{T}} - R_{12}^{\mathsf{T}}\big)}_{Q_{xu}^{\mathsf{T}}} + \underbrace{\big(\Phi P(k)\Phi^{\mathsf{T}} + R_1\big)}_{Q_x}$$

2.3 Determination of Kalman gain (continued)

- Results in:

$$K^{\mathsf{T}}(k) = -L(k)x = -L(k)$$
$$L(k) = Q_u^{-1} Q_{xu}^{\mathsf{T}} = \left(CP(k)C^{\mathsf{T}} + R_2\right)^{-1} \left(-\Phi P(k) C^{\mathsf{T}} - R_{12}\right)^{\mathsf{T}}$$

- So

$$K(k) = -L^{\mathsf{T}}(k) = \left(\Phi P(k) C^{\mathsf{T}} + R_{12}\right)\left(CP(k)C^{\mathsf{T}} + R_2\right)^{-1}$$

- Furthermore,

$$\begin{aligned} P(k+1) &= Q_x - L^{\mathsf{T}} Q_u L \\ &= \Phi P(k) \Phi^{\mathsf{T}} + R_1 - K(k)\left(CP(k)C^{\mathsf{T}} + R_2\right) K^{\mathsf{T}}(k) \\ &= \Phi P(k) \Phi^{\mathsf{T}} + R_1 - \left(\Phi P(k) C^{\mathsf{T}} + R_{12}\right)\left(CP(k)C^{\mathsf{T}} + R_2\right)^{-1}\left(CP(k)\Phi^{\mathsf{T}} + R_{12}^{\mathsf{T}}\right) \end{aligned}$$

2.3 Determination of Kalman gain (continued)

- Initial value: $$\begin{aligned} P(0) &= \mathsf{E}\left[\tilde{x}(0)\tilde{x}^{\mathsf{T}}(0)\right] \\ &= \mathsf{E}\left[\left(x(0)-\hat{x}(0)\right)\left(x(0)-\hat{x}(0)\right)^{\mathsf{T}}\right] \\ &= \mathsf{E}\left[\left(x(0)-m_0\right)\left(x(0)-m_0\right)^{\mathsf{T}}\right] \\ &= \mathrm{cov}\left(x(0)\right) = R_0 \end{aligned}$$

- **Overall solution**:

$$\begin{aligned} \hat{x}(k+1|k) &= \Phi\hat{x}(k|k-1) + \Gamma u(k) + K(k)\left(y(k) - C\hat{x}(k|k-1)\right) \\ K(k) &= \left(\Phi P(k)C^{\mathsf{T}} + R_{12}\right)\left(CP(k)C^{\mathsf{T}} + R_2\right)^{-1} \\ P(k+1) &= \Phi P(k)\Phi^{\mathsf{T}} + R_1 - K(k)\left(CP(k)C^{\mathsf{T}} + R_2\right)K^{\mathsf{T}}(k) \\ P(0) &= R_0 \\ \hat{x}(0|-1) &= m_0 \end{aligned}$$

2.4 Common equivalent implementation

- **Step 0.** (Initialization)

$$P(0|-1) = P(0) = R_0$$
$$\hat{x}(0|-1) = m_0$$

Covariance and mean of initial state.

2.4 Common equivalent implementation (continued)

- **Step 1.** (Corrector - use the most recent measurement)

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\left(y(k) - C\hat{x}(k|k-1)\right)$$

$$K(k) = P(k|k-1)C^{\mathsf{T}}\left(CP(k|k-1)C^{\mathsf{T}} + R_2\right)^{-1}$$

$$P(k|k) = P(k|k-1) - K(k)CP(k|k-1)$$

  Update estimate with $y(k)$, compute Kalman gain, update error covariance.

- **Step 2.** (One-step predictor)

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k) + \Gamma u(k)$$

$$P(k+1|k) = \Phi P(k|k)\Phi^{\mathsf{T}} + R_1$$

  Project the state and error covariance ahead.

Iterate Step 1 and 2, increase k.

2.5 Comments on Kalman filter solution

- From the Kalman gain $K(k)$ update equation, we see that a large R_2 (much measurement noise) leads to low influence of error on estimate.

- The $P(k)$ estimation error covariance is a measure of the uncertainty of the estimate. It is updated as

$$P(k+1) = \Phi P(k)\Phi^{\mathsf{T}} + R_1 - K(k)\big(CP(k)C^{\mathsf{T}} + R_2)K^{\mathsf{T}}(k)$$

- The first two terms on the right represent the natural evolution of the uncertainty, the last term shows how much uncertainty the Kalman filter removes.

2.5 Comments on Kalman filter solution (continued)

- The Kalman filter gives an unbiased estimate, i.e.,

$$\mathsf{E}\left[\hat{x}(k|k)\right] = \mathsf{E}\left[\hat{x}(k|k-1)\right] = \mathsf{E}\left[x(k)\right]$$

- If the noise is uncorrelated with $x(0)$, then the Kalman filter is optimal, i.e., no other *linear* filter gives a smaller variance of the estimation error.
  (For non-Gaussian assumptions, nonlinear filters, particle filters or moving-horizon estimators do a much better job.)

2.6 Example

- Discrete-time system $x(k+1) = x(k)$

$$y(k) = x(k) + e(k)$$

  $\rightarrow$ constant state, to be reconstructed from noisy measurements

- Measurement noise e has standard deviation σ (so $R_2 = \sigma^2$)
  $x(0)$ has covariance $R_0 = 0.5$
  No process noise v, so $R_1 = 0$ and $R_{12} = 0$

- Kalman filter is given by

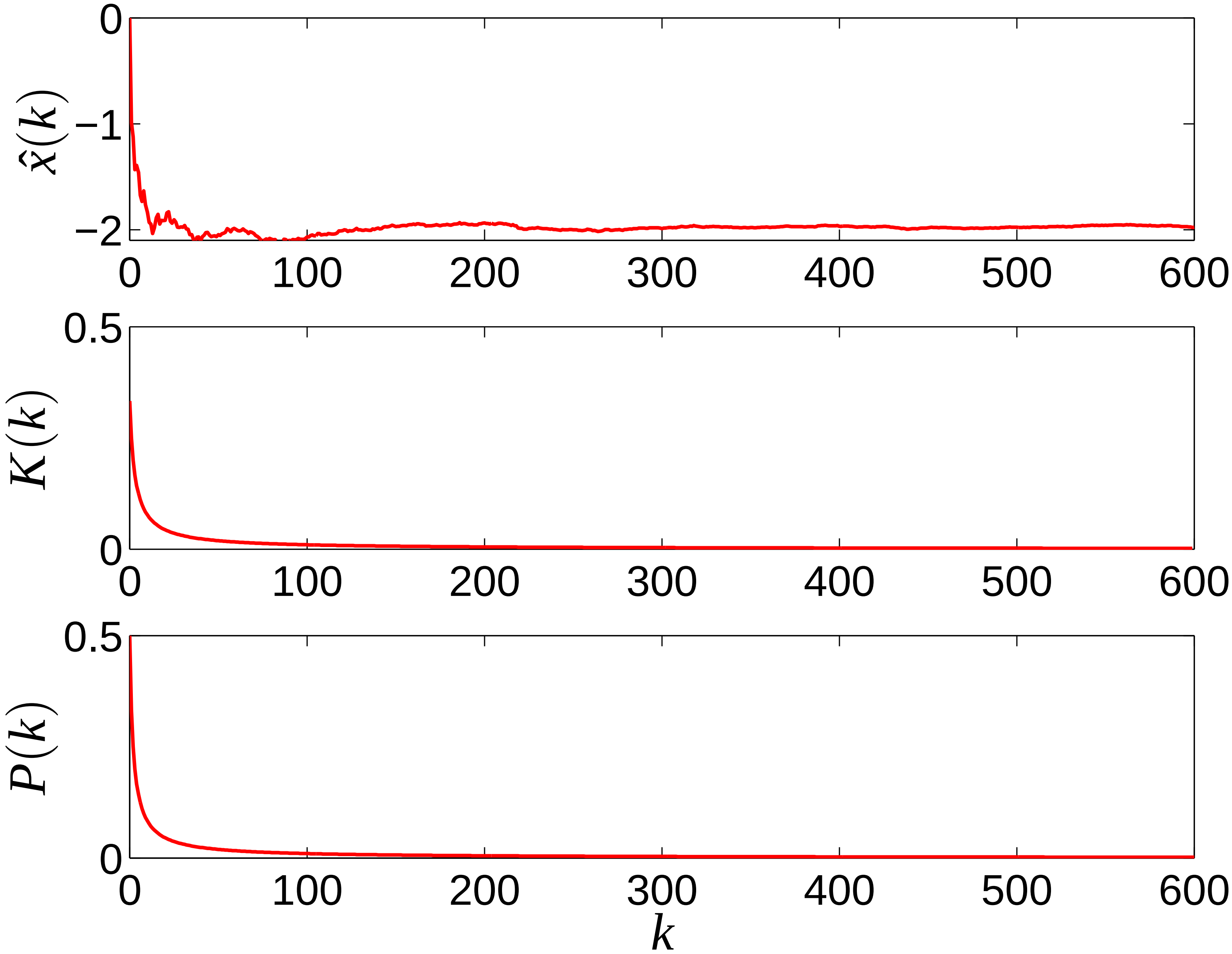
$$\hat{x}(k+1|k) = \hat{x}(k|k-1) + K(k)\big(y(k) - \hat{x}(k|k-1)\big)$$

$$K(k) = \frac{P(k)}{P(k) + \sigma^2}$$

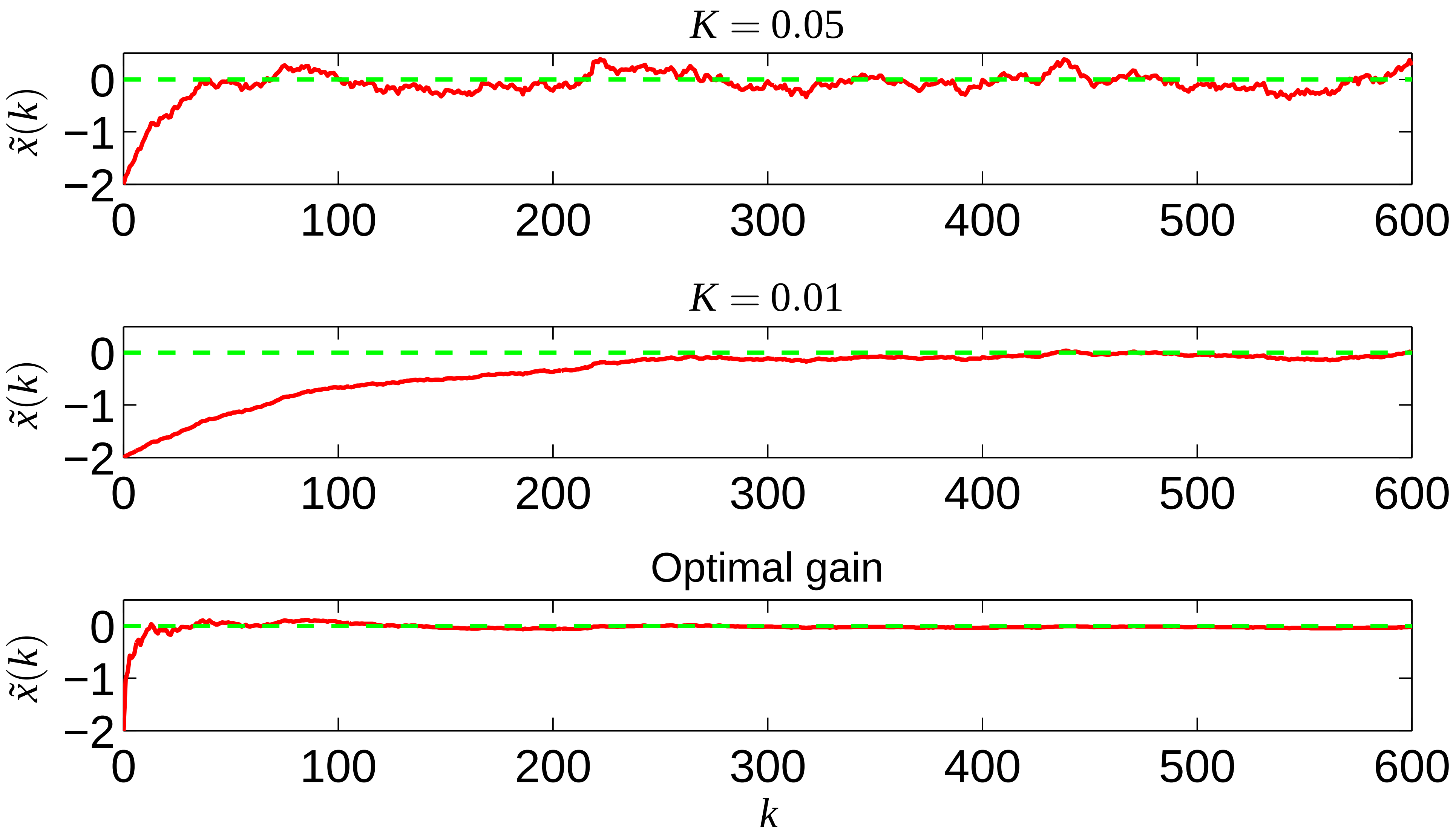
$$P(k+1) = P(k) - K(k)(P(k) + \sigma^2)K^{\mathsf{T}}(k) = \frac{\sigma^2 P(k)}{P(k) + \sigma^2}$$

$$P(0) = R_0 = 0.5 \qquad \hat{x}(0|-1) = m_0 = 0$$

2.6 Example (continued)



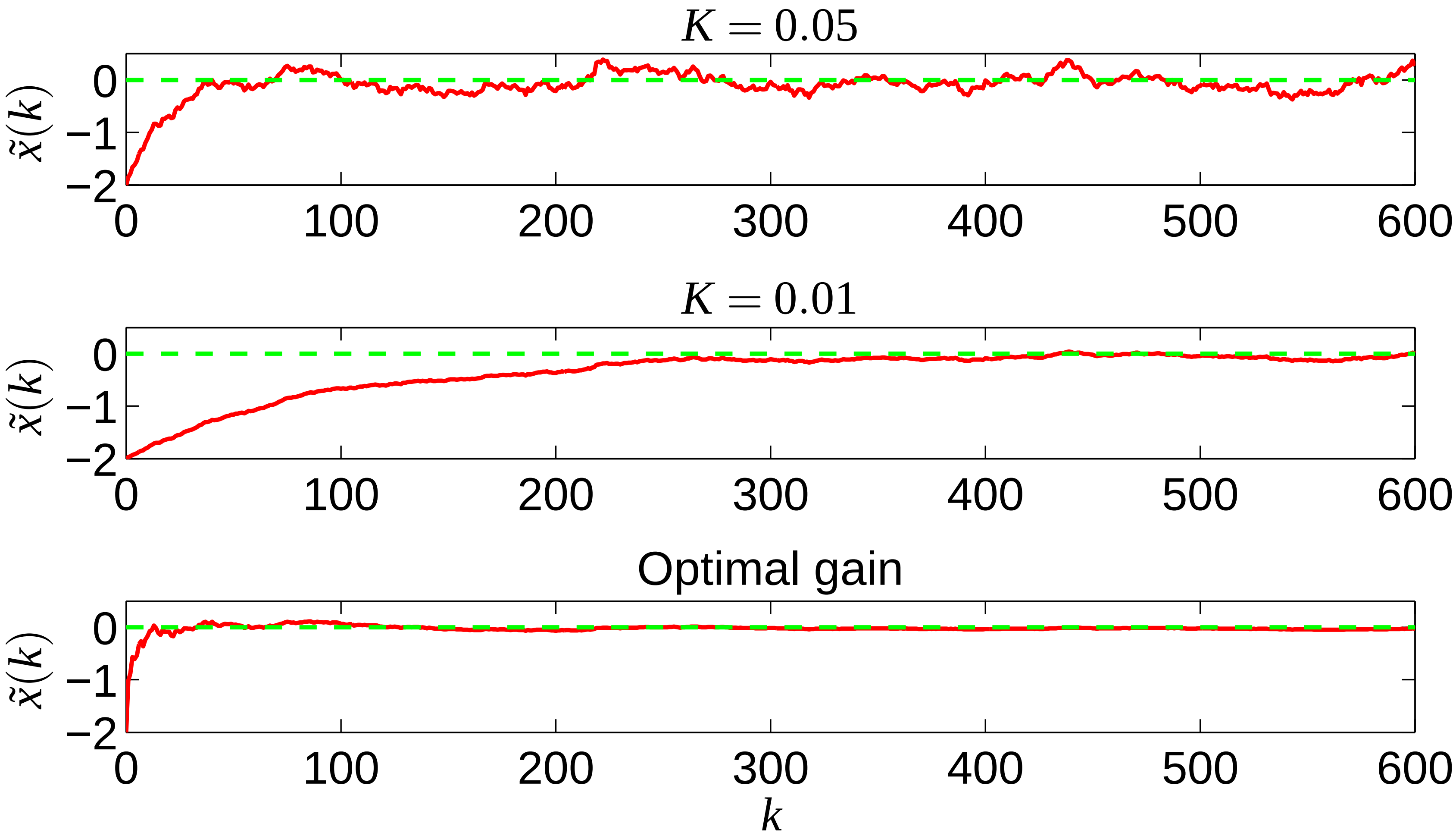
2.6 Example (continued)



Example – To think about

1. What is the problem with a large (small) K in the observer?
2. What does the Kalman filter do?
3. How would you change $P(0)$ in the Kalman filter to get a smoother (but slower) transient in $\tilde{x}(k)$?
4. In practice, R_1, R_2, and R_0 are often tuning parameters. What are their influence on the estimate?

Example – To think about (continued)



- Large fixed $K \rightarrow$ rapid initial convergence, but large steady-state variance
  Small fixed $K \rightarrow$ slower convergence, but better performance in steady state

Steady-state Kalman gain

- Recall:

$$P(k+1) = \Phi P(k)\Phi^{\mathsf{T}} + R_1 - \left(\Phi P(k)C^{\mathsf{T}} + R_{12}\right)\left(CP(k)C^{\mathsf{T}} + R_2\right)^{-1}\left(CP(k)\Phi^{\mathsf{T}} + R_{12}^{\mathsf{T}}\right)$$

- Steady-state solution given by

$$\bar{P} = \Phi\bar{P}\Phi^{\mathsf{T}} + R_1 - \left(\Phi\bar{P}C^{\mathsf{T}} + R_{12}\right)\left(C\bar{P}C^{\mathsf{T}} + R_2\right)^{-1}\left(C\bar{P}\Phi^{\mathsf{T}} + R_{12}^{\mathsf{T}}\right)$$

$\rightarrow$ is also Riccati equation!

- Note: compare with steady-state LQ controller:

$$\bar{S} = \Phi^{\mathsf{T}}\bar{S}\Phi + Q_1 - \left(\Phi^{\mathsf{T}}\bar{S}\Gamma + Q_{12}\right)^{\mathsf{T}}\left(\Gamma^{\mathsf{T}}\bar{S}\Gamma + Q_2\right)^{-1}\left(\Gamma^{\mathsf{T}}\bar{S}\Phi + Q_{12}^{\mathsf{T}}\right)$$

Duality

- Equivalence between LQ control problem and Kalman filter state estimation problem

LQ	Kalman
k	$N-k$
Φ	Φ^{T}
Γ	C^{T}
Q_0	R_0
Q_1	R_1
Q_{12}	R_{12}
S	P
L	K^{T}

How to find Kalman filter using matlab?

- Command `[KEST,L,P] = kalman(SYS,QN,RN,NN)`
- Calculates (full) Kalman estimator `KEST` for system `SYS`

```
x[n+1] = Ax[n] + Bu[n] + Gw[n]          {State equation}
y[n]   = Cx[n] + Du[n] + Hw[n] + v[n] {Measurements}
```

  with known inputs `u`, process noise `w`, measurement noise `v`, and noise covariances

```
E{ww'} = QN,      E{vv'} = RN,      E{wv'} = NN,
```

  Note: Construct `SYS` as `SYS=ss(A,[B G],C,[D H],-1)`

- Also returns steady-state estimator gain `L` and steady-state error covariance

```
P = E{(x - x[n|n-1])(x - x[n|n-1])'}  (Riccati solution)
```

3. Linear quadratic Gaussian control

- Given discrete-time LTI system

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) + v(k) \\ y(k) &= Cx(k) + e(k) \end{aligned}$$

with

$$\mathsf{E}\left[x(0)\right] = m_0, \quad \mathsf{cov}\left(x(0)\right) = R_0, \quad \mathsf{E}\left[\begin{bmatrix} v(k) \\ e(k) \end{bmatrix} \begin{bmatrix} v(k) \\ e(k) \end{bmatrix}^{\mathsf{T}}\right] = \begin{bmatrix} R_1 & R_{12} \\ R_{12}^{\mathsf{T}} & R_2 \end{bmatrix}$$

find linear control law $y(0), y(1), \ldots, y(k-1) \mapsto u(k)$ that minimizes

$$\mathsf{E}\left[\sum_{k=0}^{N-1}\left(x^{\mathsf{T}} Q_1 x + 2x^{\mathsf{T}} Q_{12} u + u^{\mathsf{T}} Q_2 u\right) + x^{\mathsf{T}}(N) Q_0 x(N)\right]$$

- Solution: Separation principle

3.1 Separation principle

- Makes it possible to use control law

$$u(k) = -L\hat{x}(k|k-1)$$

(so $u(k) = -Lx(k) + L\tilde{x}(k)$) with closed-loop dynamics

$$x(k+1) = \Phi x(k) - \Gamma L x(k) + \Gamma L \tilde{x}(k) + v(k)$$

and to view term $\Gamma L \tilde{x}(k)$ as part of noise

$\rightarrow$ solve LQ problem and estimation problem separately

Proof of separation principle

- Solution of optimal observer design problem does not depend on input u
  So using state feedback does not influence optimality
  $\rightarrow$ Kalman filter still optimal

- Using $u(k) = -L(k)\hat{x}(k|k-1)$ results in closed-loop system

$$\hat{x}(k+1|k) = \big(\Phi - \Gamma L(k)\big)\hat{x}(k|k-1) + K(k)\underbrace{\big(y - C\hat{x}(k|k-1)\big)}_{w(k)}$$

- It can be shown that for optimal $K(k)$, $w(k)$ is white noise
- So dynamics become

$$\hat{x}(k+1|k) = (\Phi - \Gamma L(k))\hat{x}(k|k-1) + K(k)w(k)$$

Proof of separation principle (continued)

- For simplicity we assume $Q_0 = 0$
- For $u = -L\hat{x}$, control design problem in terms of $\hat{x}$ and $\tilde{x}$ becomes

$$\begin{aligned}
&\min_L \mathsf{E}\left[\sum_{k=0}^{N-1} x^\mathsf{T} Q_1 x + 2x^\mathsf{T} Q_{12} u + u^\mathsf{T} Q_2 u\right] \\
&= \min_L \mathsf{E}\left[\sum_{k=0}^{N-1} (\hat{x}+\tilde{x})^\mathsf{T} Q_1 (\hat{x}+\tilde{x}) + 2(\hat{x}+\tilde{x})^\mathsf{T} Q_{12} L\hat{x} + \hat{x}^\mathsf{T} L^\mathsf{T} Q_2 L\hat{x}\right]
\end{aligned}$$

- Since it can be shown that $\mathsf{E}\left[\tilde{x}^\mathsf{T} Q \hat{x}\right] = 0$, we get

$$\min_L \mathsf{E}\left[\sum_{k=0}^{N-1} \hat{x}^\mathsf{T} Q_1 \hat{x} + \tilde{x}^\mathsf{T} Q_1 \tilde{x} + 2\hat{x}^\mathsf{T} Q_{12} L\hat{x} + \hat{x}^\mathsf{T} L^\mathsf{T} Q_2 L\hat{x}\right]$$

- $\mathsf{E}\left[\tilde{x}^\mathsf{T} Q \tilde{x}\right]$ does not depend on L and is in fact minimal for Kalman gain K

Proof of separation principle (continued)

- Hence, we get

$$\min_{L} \mathsf{E}\left[\sum_{k=0}^{N-1} \hat{x}^{\mathsf{T}} Q_1 \hat{x} + 2\hat{x}^{\mathsf{T}} Q_{12} L \hat{x} + \hat{x}^{\mathsf{T}} L^{\mathsf{T}} Q_2 L \hat{x}\right]$$

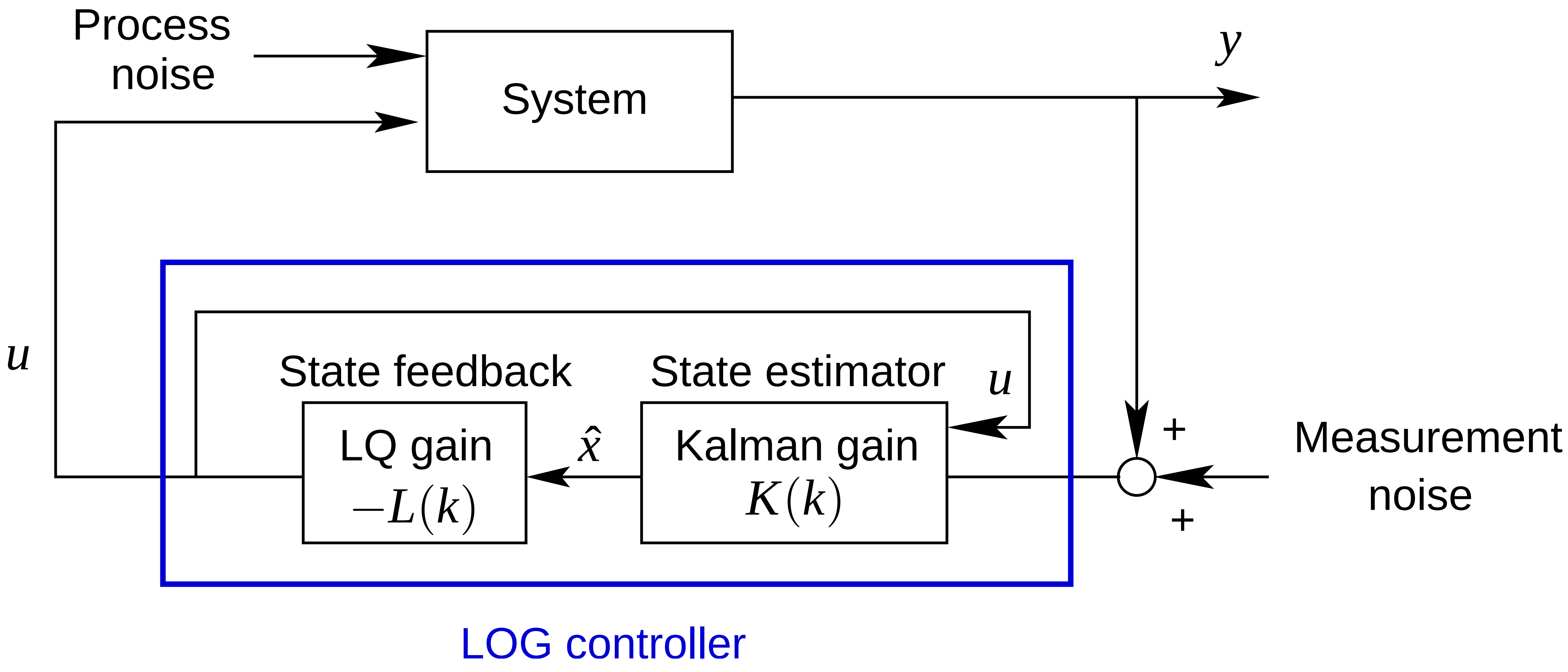
subject to

$$\hat{x}(k+1|k) = (\Phi - \Gamma L(k))\hat{x}(k|k-1) + K(k)w(k)$$

= stochastic LQ problem (but with $\hat{x}$ instead of x and with $u = -L\hat{x}$ already filled in)

$\rightarrow L(k)$ as computed before still optimal

3.2 LQG problem: Solution



Stationary LQG control

- Solution:

$$u(k) = -L\hat{x}(k|k-1)$$

with

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + \Gamma u(k) + K\big(y(k) - C\hat{x}(k|k-1)\big)$$

- Closed-loop dynamics (with error state $\tilde{x}(k)$, see slide ce_kf_lqg.8):

$$x(k+1) = (\Phi - \Gamma L)x(k) + \Gamma L\tilde{x}(k) + v(k)$$
$$\tilde{x}(k+1) = (\Phi - KC)\tilde{x}(k) + v(k) - Ke(k)$$

or

$$\begin{bmatrix} x(k+1) \\ \tilde{x}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi - \Gamma L & \Gamma L \\ 0 & \Phi - KC \end{bmatrix} \begin{bmatrix} x(k) \\ \tilde{x}(k) \end{bmatrix} + \begin{bmatrix} I \\ I \end{bmatrix} v(k) + \begin{bmatrix} 0 \\ -K \end{bmatrix} e(k)$$

$\rightarrow$ dynamics of closed-loop system determined by dynamics of LQ controller and of optimal filter

How to construct LQG controller using matlab?

- Command `RLQG = lqgreg(KEST,K)` produces LQG controller by connecting Kalman estimator `KEST` designed with `kalman` and state-feedback gain `K` designed with `dlqr`

Pros and cons of LQG

+ stabilizing
+ good robustness for SISO
+ works for MIMO

- robustness can be very bad for MIMO
- high-order controller
- how to choose weights?

Summary

- Kalman filtering
  - state estimator such that error covariance is minimized
- LQG control
  - separation principle $\rightarrow$ LQ + Kalman