Realization of Digital Controllers

Controller

$$y_k = H\left(q^{-1}\right) = \frac{b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}}{1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n\, q^{-n}} u_k$$

- Direct form
- Companion form
- Series form
- Parallel form
- δ-operator form

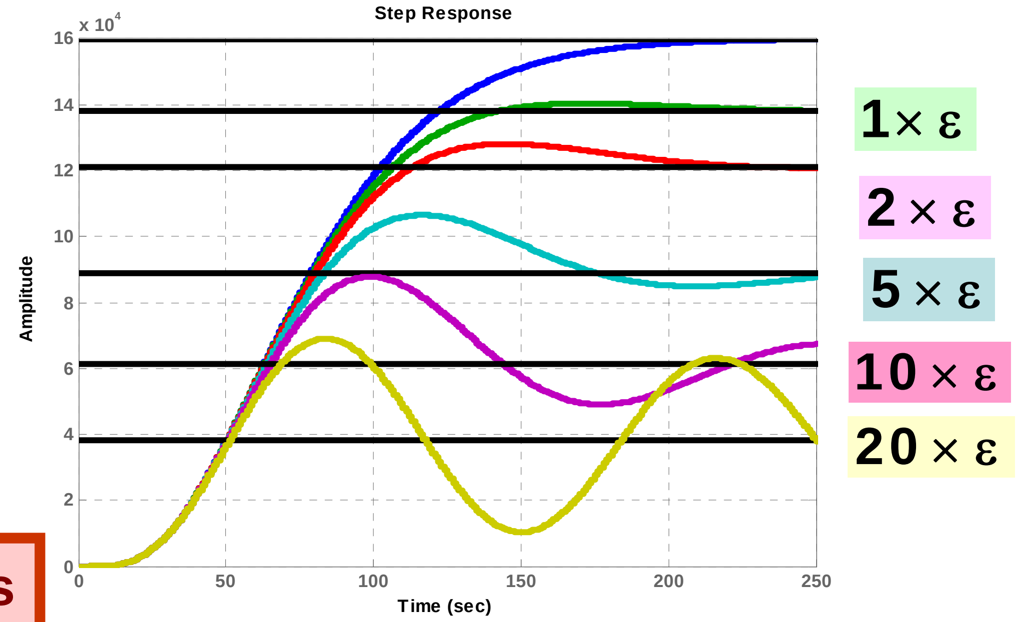
Direct form

$$y_k = \sum_{i=0}^{m} b_i u_{k-i} - \sum_{i=1}^{n} a_i y_{k-i}$$

$$y_k = u_{k-4} + 3.8y_{k-1} - 9.025y_{k-2} + 3.4561y_{k-3} - 0.8145062y_{k-4}$$

$\varepsilon = 10^{-6}$ deviation in a parameter

$$\frac{y_k}{u_k} = \frac{1}{(q-0.95)^4}$$



Sensitive for parameter errors

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Companion form

$$x_{k+1} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_n \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} x_k$$

Equally sensitive for parameter errors

Series form

$$H(q)=\frac{num}{(q-\lambda)^4}$$

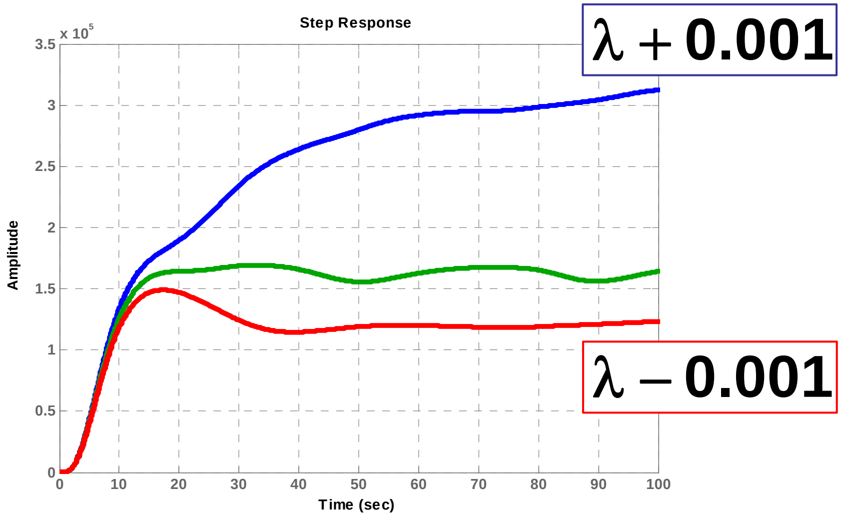
$$\Rightarrow$$

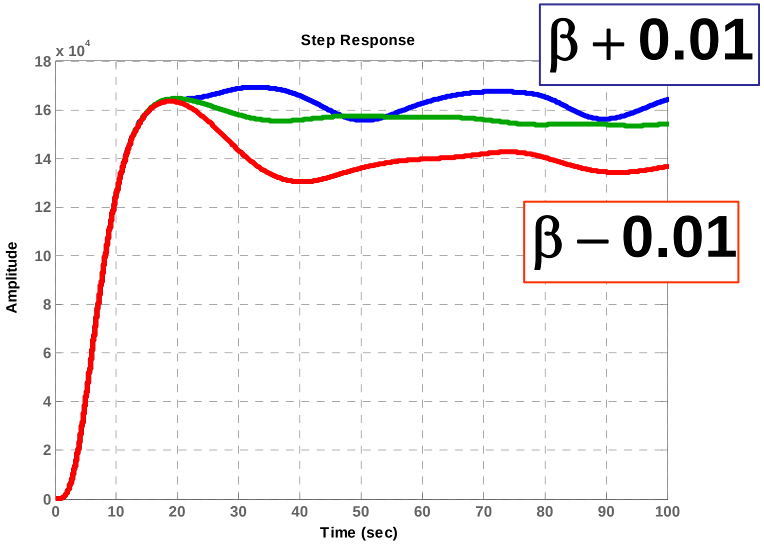
$$x_{k+1}=\begin{bmatrix}\lambda & 1 & 0 & 0\\ 0 & \lambda & 1 & 0\\ 0 & 0 & \lambda & 1\\ 0 & 0 & 0 & \lambda\end{bmatrix}x_k+\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix}u_k$$

$$y_k=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}x_k$$

JORDAN

$$\frac{y_k}{u_k}=\frac{\beta_1}{q-\lambda}+\frac{\beta_2}{(q-\lambda)^2}+\frac{\beta_3}{(q-\lambda)^3}+\frac{\beta_4}{(q-\lambda)^4}$$





$$\lambda = 0.95$$

$$\beta = 1$$

$$\frac{y_k}{u_k} = \frac{\beta_1}{q-\lambda} + \frac{\beta_2}{(q-\lambda)^2} + \frac{\beta_3}{(q-\lambda)^3} + \frac{\beta_4}{(q-\lambda)^4}$$

Sensitivity reasonable

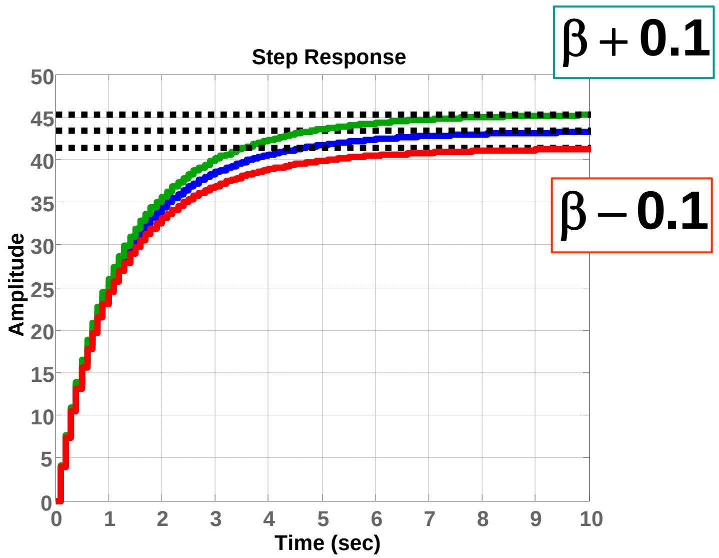
Parallel form

$$x_{k+1} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} x_k + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} u_k$$

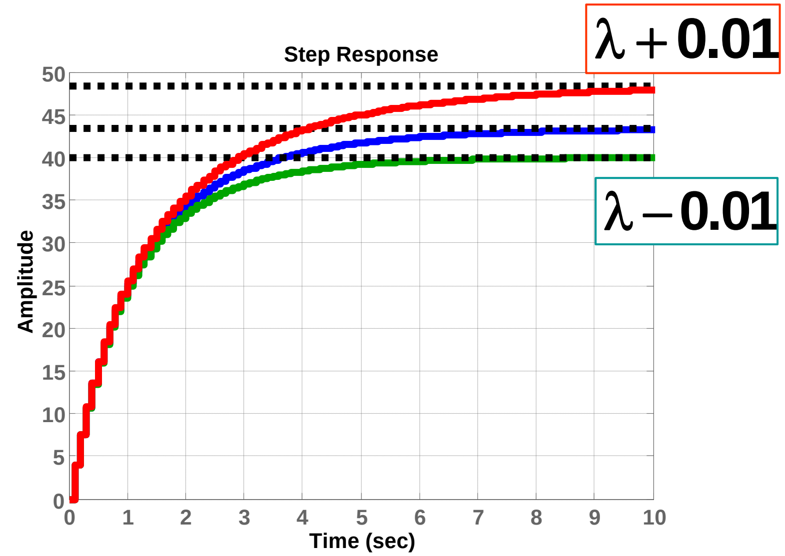
DIAGONAL

$$y_k = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} x_k$$

$$\frac{y_k}{u_k} = \frac{\beta_1}{q-\lambda_1} + \frac{\beta_2}{q-\lambda_2} + \frac{\beta_3}{q-\lambda_3} + \frac{\beta_4}{q-\lambda_4}$$



$\beta = 1$



$\lambda = 0.95$

$$\frac{y_k}{u_k} = \frac{\beta_1}{q-\lambda_1} + \frac{\beta_2}{q-\lambda_2} + \frac{\beta_3}{q-\lambda_3} + \frac{\beta_4}{q-\lambda_4}$$

Sensitivity good

δ-operator

$$\delta = \frac{q-1}{h} \quad \Rightarrow \quad \delta\, f_k = \frac{f_{k+1} - f_k}{h}$$

$$x_{k+1} = Fx_k + Gu_k$$

$$\Downarrow$$

$$x_{k+1} - x_k = (F - I)x_k + Gu_k$$

$$\Downarrow$$

$$\frac{x_{k+1} - x_k}{h} = \frac{F - I}{h} x_k + \frac{G}{h} u_k$$

$$\Downarrow$$

$$\delta x_k = \overline{F}\, x_k + \overline{G}\, u_k$$

Sensitivity

Characteristic equation:

$$(q - 0.95)^4 = q^4 - 3.8\,q^3 + 5.415\,q^2 - 3.4295\,q + 0.8145 = 0$$

Poles: 0.95 **Stable**

$$q^4 - 3.8\,q^3 + 5.415\,q^2 - 3.4295\,q + 0.814\boxed{6} = 0$$

Poles: $1.02 \pm j\,0.07$, $0.88 \pm j\,0.07$ **UNSTABLE**

$$q^4 - 3.8\,q^3 + 5.415\,q^2 - 3.4295\,q + 0.814\boxed{4} = 0$$

Poles: 1.05, $0.95 \pm j\,0.1$, 0.85 **UNSTABLE**

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