# Elements of linear system theory & Performance limitations

Jan-Willem van Wingerden

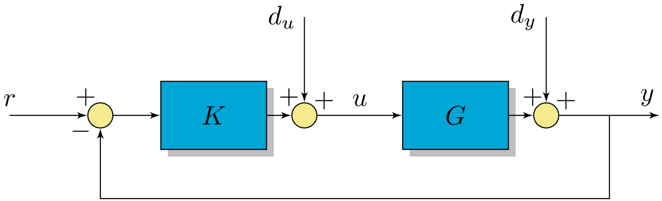
Delft Center for Systems and Control
Delft University of Technology
J.W.vanWingerden@TUDelft.nl

SC42145, 2024/25

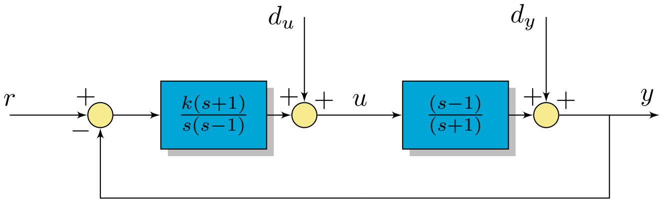
# Basic Elements (Section 4.1-4.6)

- Super Position $f(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 f(u_1) + \alpha_2 f(u_2)$
- System representations (Transfer Functions, State-Space, Impulse Response, FRF)
- Controllability and Observability
- Minimal Realization (smallest possible dimension of the state space realization)
- Stability, bounded input results in bounded output
- Stabilizable, if all unstable modes are controllable
- Detectable, if all unstable modes are observable
- Poles and zeros, note that they have associated direction for MIMO systems

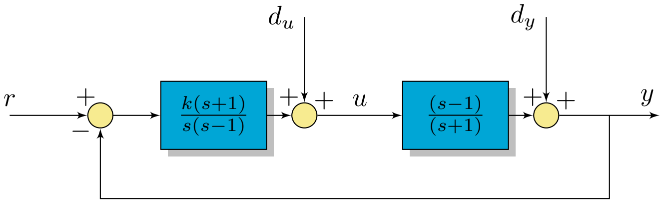
# Internal stability of feedback systems: Example



# Internal stability of feedback systems: Example

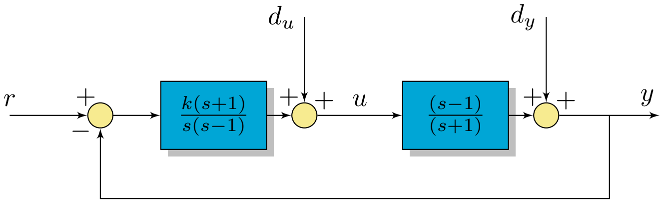


# Internal stability of feedback systems: Example




Q: What happens if we look at the loop transfer function $L$?

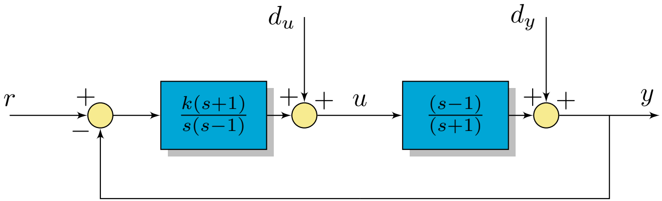
# Internal stability of feedback systems: Example




Q: What happens if we look at the loop transfer function $L$?

A: $L = \frac{k}{s}$ and $S = \frac{y}{d_y} = \frac{s}{s+k}$. So, stable for $k > 0$

# Internal stability of feedback systems: Example

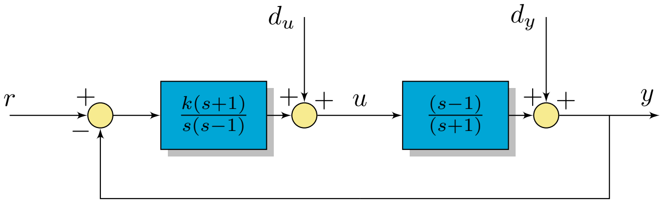



Q: What happens if we look at the loop transfer function $L$?

A: $L = \frac{k}{s}$ and $S = \frac{y}{d_y} = \frac{s}{s+k}$. So, stable for $k > 0$

Q: What happens if we look at $\frac{u}{d_y} = -K\left(I + GK\right)^{-1}$?

# Internal stability of feedback systems: Example



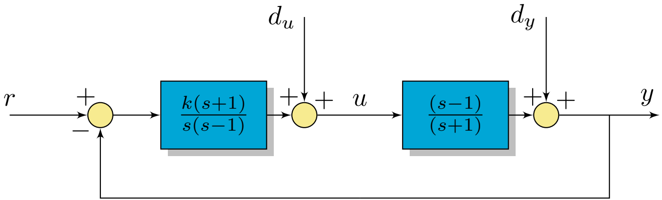

Q: What happens if we look at the loop transfer function $L$?

A: $L = \frac{k}{s}$ and $S = \frac{y}{d_y} = \frac{s}{s+k}$. So, stable for $k > 0$

Q: What happens if we look at $\frac{u}{d_y} = -K\left(I + GK\right)^{-1}$?

A: $\frac{u}{d_y} = \frac{-k(s+1)}{(s-1)(s+k)}$. Unstable $\forall k$

# Internal stability of feedback systems: Example




Q: What happens if we look at the loop transfer function $L$?

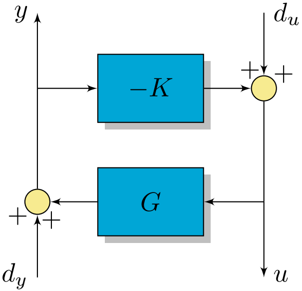
A: $L = \frac{k}{s}$ and $S = \frac{y}{d_y} = \frac{s}{s+k}$. So, stable for $k > 0$

Q: What happens if we look at $\frac{u}{d_y} = -K\left(I + GK\right)^{-1}$?

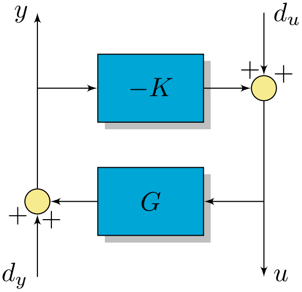
A: $\frac{u}{d_y} = \frac{-k(s+1)}{(s-1)(s+k)}$. Unstable $\forall k$

The system is internally unstable!!

# Internal stability of feedback systems

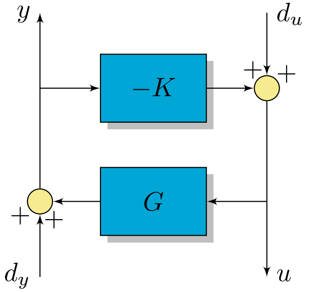


# Internal stability of feedback systems




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

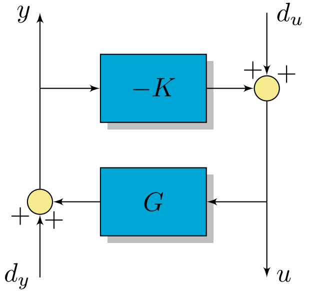
# Internal stability of feedback systems




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

**Internal stability I:** Assume $G$ and $K$ contain no unstable hidden modes. Internal stable iff all the above transfer functions are stable

# Internal stability of feedback systems




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

**Internal stability I:** Assume $G$ and $K$ contain no unstable hidden modes. Internal stable iff all the above transfer functions are stable

**Internal stability II:** All RHP-poles are contained in the min. realization of $GK$ and $KG$. The system is internally stable iff one of the above transfer functions is stable

# Internal stability of feedback systems: Implications

If we have internal stability. The following holds:

1. If $G(s)$ has a RHP-zero at $z$. The following TF's also have a RHP-zero at $z$: $L$, $L_I$, $T$, $SG$, $T_I$

# Internal stability of feedback systems: Implications

If we have internal stability. The following holds:

1. If $G(s)$ has a RHP-zero at $z$. The following TF's also have a RHP-zero at $z$: $L$, $L_I$, $T$, $SG$, $T_I$

   Sketch of proof: (1) there are no RHP pole-zero cancelations (2) $S$ is stable $\Rightarrow T = GKS$ there is not RHP-pole so no cancelation, etc.

# Internal stability of feedback systems: Implications

If we have internal stability. The following holds:

1. If $G(s)$ has a RHP-zero at $z$. The following TF's also have a RHP-zero at $z$: $L$, $L_I$, $T$, $SG$, $T_I$

   Sketch of proof: (1) there are no RHP pole-zero cancelations (2) $S$ is stable $\Rightarrow T = GKS$ there is not RHP-pole so no cancelation, etc.

2. If $G(s)$ has a RHP-pole at $p$. The following TF's have a RHP-pole at $p$: $L$, $L_I$. While the following TF's have a RHP-zero at $p$. $S$, $KS$, $S_I$

# Internal stability of feedback systems: Implications

If we have internal stability. The following holds:

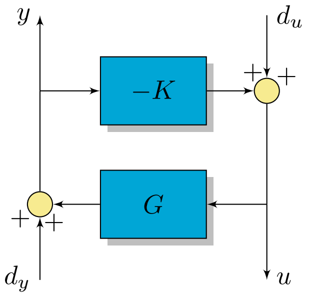
1. If $G(s)$ has a RHP-zero at $z$. The following TF's also have a RHP-zero at $z$: $L$, $L_I$, $T$, $SG$, $T_I$

   Sketch of proof: (1) there are no RHP pole-zero cancelations (2) $S$ is stable $\Rightarrow T = GKS$ there is not RHP-pole so no cancelation, etc.

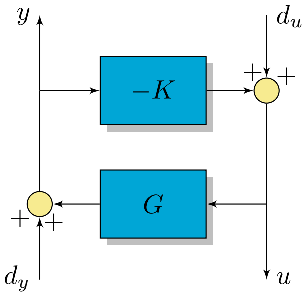
2. If $G(s)$ has a RHP-pole at $p$. The following TF's have a RHP-pole at $p$: $L$, $L_I$. While the following TF's have a RHP-zero at $p$. $S$, $KS$, $S_I$

   Sketch of proof: (1) part 1 is obvious (2) From $LS$ it follows that $S$ should contain RHP-zero at $p$ to cancel RHP-pole in $L$

# Internal Model Control (IMC)



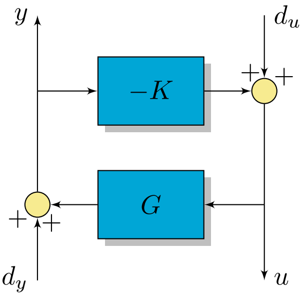
# Internal Model Control (IMC)




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$

$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

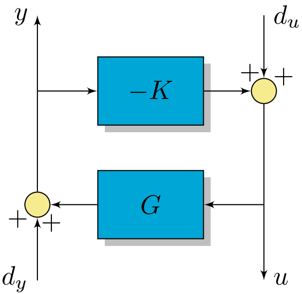
# Internal Model Control (IMC)




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

Given $Q = K\,(I + GK)^{-1}$.

# Internal Model Control (IMC)



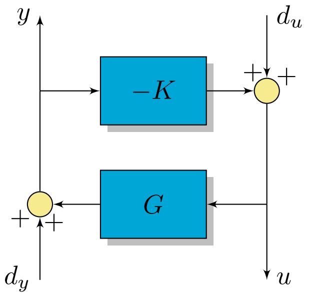

$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

$$u = (I - QG) d_u - Q d_y$$
$$y = G(I - QG) d_u + (I - GQ) d_y$$

(use $S + T = I$ and push through rule)

Given $Q = K\left(I + GK\right)^{-1}$.

# Internal Model Control (IMC)



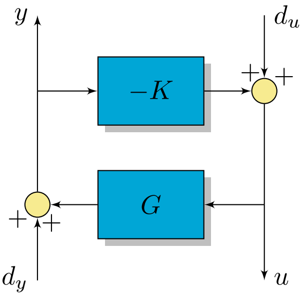

$$u = (I+KG)^{-1}d_u - K(I+GK)^{-1}d_y$$
$$y = G(I+KG)^{-1}d_u + (I+GK)^{-1}d_y$$

$$u = (I-QG)d_u - Qd_y$$
$$y = G(I-QG)d_u + (I-GQ)d_y$$

(use $S+T=I$ and push through rule)

Given $Q = K\left(I+GK\right)^{-1}$. If $G$ is **stable** the system above is internally stable iff $Q$ is stable

# Internal Model Control (IMC)




$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$
$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

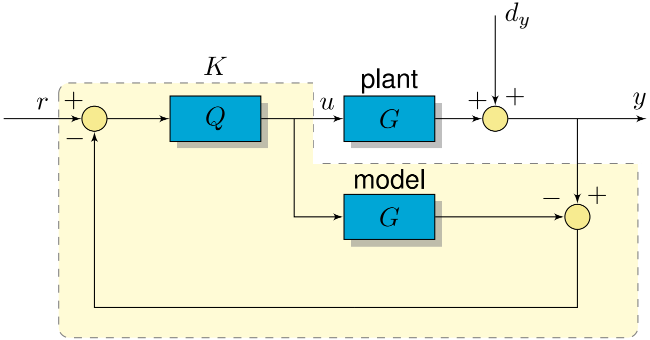
$$u = (I - QG) d_u - Q d_y$$
$$y = G(I - QG) d_u + (I - GQ) d_y$$

(use $S + T = I$ and push through rule)

Given $Q = K\left(I + GK\right)^{-1}$. If $G$ is **stable** the system above is internally stable iff $Q$ is stable

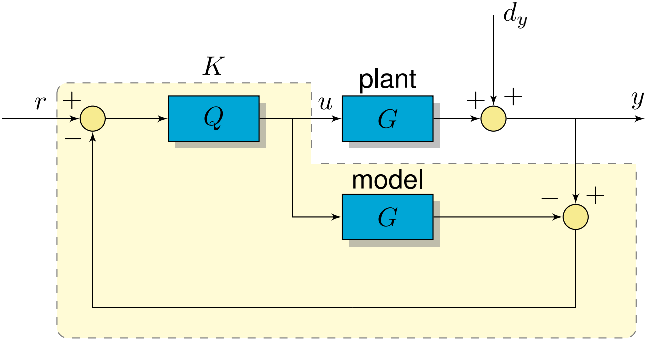
The set of all stabilizing controllers: $K = \left(I - QG\right)^{-1} Q = Q\left(I - GQ\right)^{-1}$ where $Q$ is *any stable TF.*

# Internal Model Control (IMC) /Youla parameterization




$K = (I - QG)^{-1} Q = Q(I - GQ)^{-1}$ where $Q$ is *any stable TF.*

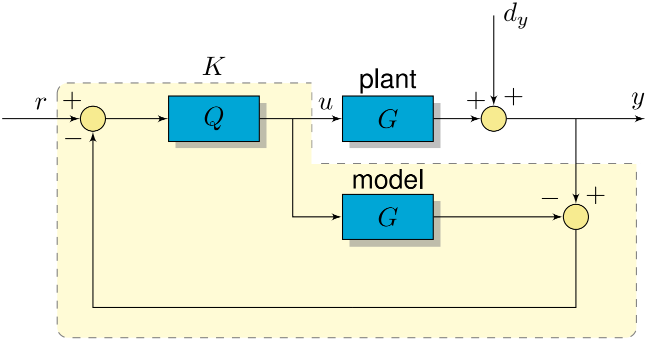
# Internal Model Control (IMC) /Youla parameterization




$K = (I - QG)^{-1} Q = Q(I - GQ)^{-1}$ where $Q$ is *any stable TF.*

What is the order of the controller?

# Internal Model Control (IMC) /Youla parameterization

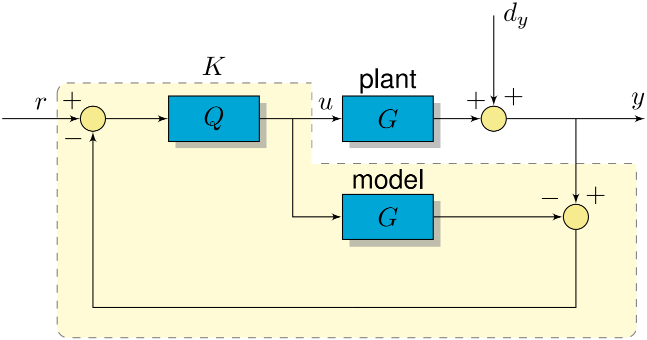



$K = (I - QG)^{-1} Q = Q(I - GQ)^{-1}$ where $Q$ is *any stable TF.*

What is the order of the controller?

Why useful?

# Internal Model Control (IMC) /Youla parameterization




$K = (I - QG)^{-1} Q = Q (I - GQ)^{-1}$ where $Q$ is *any stable TF.*

What is the order of the controller?

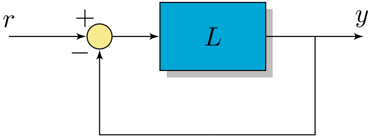
Why useful? For optimization, we only have to search over stable $Q$ and all the other TF's are affine in $Q$ (T or S in the form: $H_1 + H_2 Q H_3$)

# Stability analysis in the frequency domain

- Stability: NO RHP-poles of the closed loop

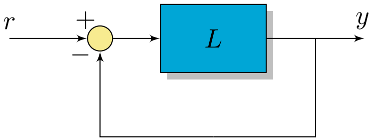
# Stability analysis in the frequency domain

- Stability: NO RHP-poles of the closed loop
- Bode stability criteria (only information $L(j\omega)$) enables design.
  Only SISO!



# Stability analysis in the frequency domain

- Stability: NO RHP-poles of the closed loop
- Bode stability criteria (only information $L(j\omega)$) enables design. Only SISO!
- Nyquist can be extended to MIMO (only information $L(j\omega) = GK(j\omega)$)



# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right] \text{ or in TF: } L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$$

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right]$ or in TF: $L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$

The OL Poles are given by: $\phi_{ol}(s) = \det\left(sI - A_{ol}\right)$

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right]$ or in TF: $L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$

The OL Poles are given by: $\phi_{ol}(s) = \det\left(sI - A_{ol}\right)$

Internal stability $\Rightarrow$ stability of $(I + L(s))^{-1}$. The state matrix of $S$:
$A_{cl} = A_{ol} - B_{ol}(I + D_{ol})^{-1}C_{ol}$

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right]$ or in TF: $L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$

The OL Poles are given by: $\phi_{ol}(s) = \det\left(sI - A_{ol}\right)$

Internal stability $\Rightarrow$ stability of $(I + L(s))^{-1}$. The state matrix of $S$:
$A_{cl} = A_{ol} - B_{ol}(I + D_{ol})^{-1}C_{ol}$

The CL Poles are given by:
$\phi_{cl}(s) = \det\left(sI - A_{ol} + B_{ol}(I + D_{ol})^{-1}C_{ol}\right)$

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right]$ or in TF: $L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$

The OL Poles are given by: $\phi_{ol}(s) = \det\left(sI - A_{ol}\right)$

Internal stability $\Rightarrow$ stability of $(I + L(s))^{-1}$. The state matrix of $S$:
$A_{cl} = A_{ol} - B_{ol}(I + D_{ol})^{-1}C_{ol}$

The CL Poles are given by:
$\phi_{cl}(s) = \det\left(sI - A_{ol} + B_{ol}(I + D_{ol})^{-1}C_{ol}\right)$

Determinant return difference operator:
$\det(I + L(s)) \; = \; \det(I + C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol})$

# Stability analysis in the frequency domain

Stability of the OL is determined by the poles of $L$.

$L(s) = \left[\begin{array}{c|c} A_{ol} & B_{ol} \\ \hline C_{ol} & D_{ol} \end{array}\right]$ or in TF: $L(s) = C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}$

The OL Poles are given by: $\phi_{ol}(s) = \det\left(sI - A_{ol}\right)$

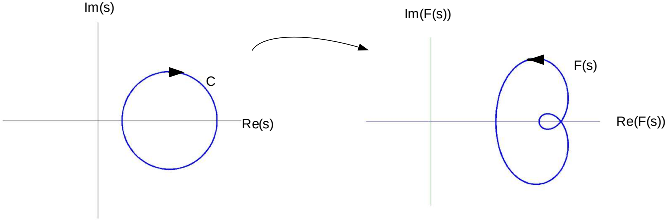
Internal stability $\Rightarrow$ stability of $(I + L(s))^{-1}$. The state matrix of $S$:
$A_{cl} = A_{ol} - B_{ol}(I + D_{ol})^{-1}C_{ol}$

The CL Poles are given by:
$\phi_{cl}(s) = \det\left(sI - A_{ol} + B_{ol}(I + D_{ol})^{-1}C_{ol}\right)$

Determinant return difference operator:
$\det(I + L(s)) = \det(I + C_{ol}(sI - A_{ol})^{-1}B_{ol} + D_{ol}) = c\frac{\phi_{cl}(s)}{\phi_{ol}(s)}$ (using Schur's formula)

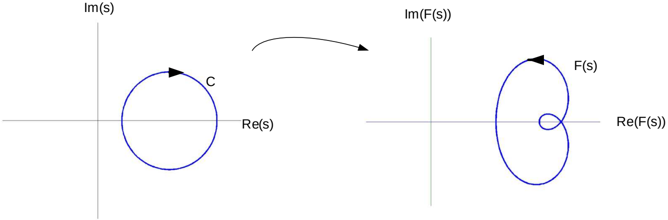
# Cauchy's argument principle

Evaluate a transfer function $F(s)$ along a clockwise contour $C$



# Cauchy's argument principle

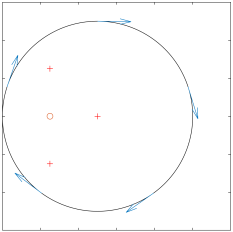
Evaluate a transfer function $F(s)$ along a clockwise contour $C$



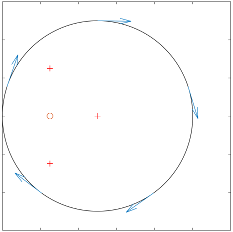

Contour Map encircles origin $N = Z - P$ times clock wise

- $Z$: number of zeros $F(s)$ inside $C$
- $P$: number of poles $F(s)$ inside $C$

# Example 1

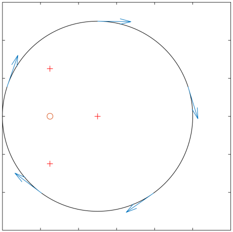


# Example 1



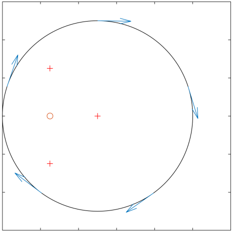
- $F(s)$ has 1 zero in $C \Rightarrow Z = 1$

# Example 1

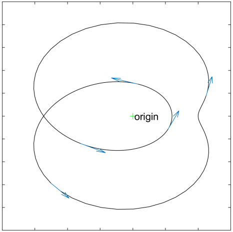


- $F(s)$ has 1 zero in $C \Rightarrow Z = 1$
- $F(s)$ has 3 poles in $C \Rightarrow P = 3$

# Example 1

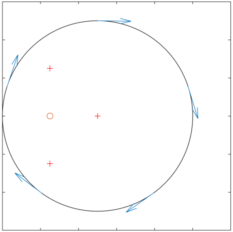


$\xrightarrow{F(s)}$

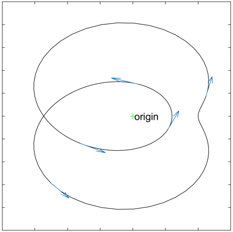



- $F(s)$ has 1 zero in $C \Rightarrow Z = 1$
- $F(s)$ has 3 poles in $C \Rightarrow P = 3$

# Example 1

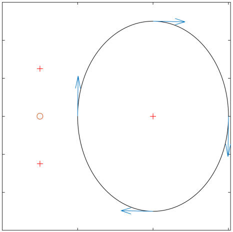


$\xrightarrow{F(s)}$

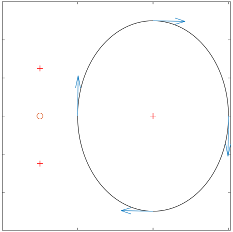



- $F(s)$ has 1 zero in $C \Rightarrow Z = 1$
- $F(s)$ has 3 poles in $C \Rightarrow P = 3$
- Hence: $N = Z - P = -2$

# Example 2

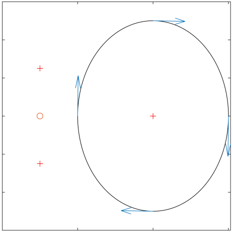


# Example 2



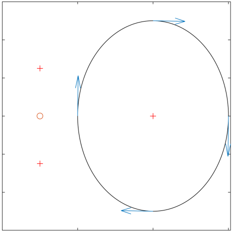
- $F(s)$ has 0 zeros in $C \Rightarrow Z = 0$

# Example 2

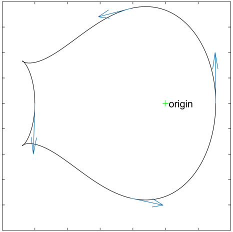


- $F(s)$ has 0 zeros in $C \Rightarrow Z = 0$
- $F(s)$ has 1 pole in $C \Rightarrow P = 1$

# Example 2

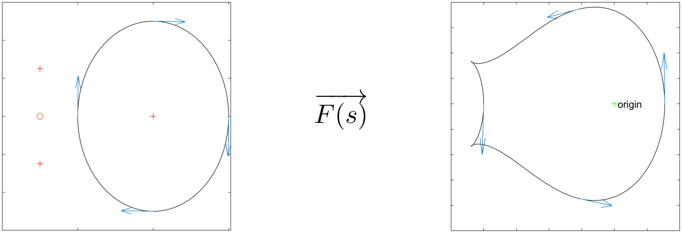


$\xrightarrow{F(s)}$




- $F(s)$ has 0 zeros in $C \Rightarrow Z = 0$
- $F(s)$ has 1 pole in $C \Rightarrow P = 1$

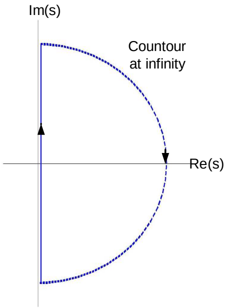
# Example 2



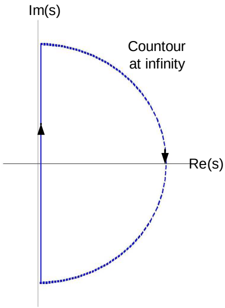

- $F(s)$ has 0 zeros in $C \Rightarrow Z = 0$
- $F(s)$ has 1 pole in $C \Rightarrow P = 1$
- Hence: $N = Z - P = -1$

# Nyquist: Let $C$ contain RHP

# Nyquist: Let $C$ contain RHP

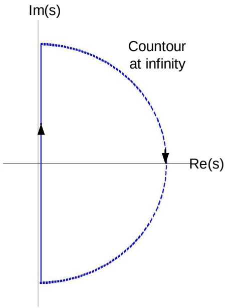


# Nyquist: Let $C$ contain RHP




Apply the argument principle on $\det(I + L(s))$

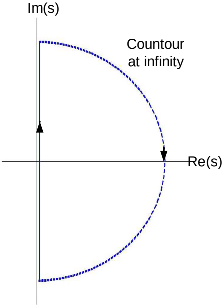
# Nyquist: Let $C$ contain RHP




Apply the argument principle on $\det(I + L(s))$

$$\det(I+L(s)) = c \cdot \frac{\text{Closed Lp characteristic polynomial}}{\text{Open Lp characteristic polynomial}}$$

# Nyquist: Let $C$ contain RHP

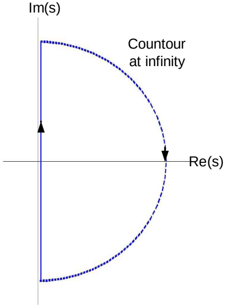



Apply the argument principle on $\det(I + L(s))$

$$\det(I+L(s)) = c \cdot \frac{\text{Closed Lp characteristic polynomial}}{\text{Open Lp characteristic polynomial}}$$

- $Z$ (zeros in $C =$ RHP) : unstable closed loop poles
- $P$ (poles in $C =$ RHP) : unstable open loop poles

# Nyquist: Let $C$ contain RHP




Apply the argument principle on $\det(I + L(s))$

$$\det(I+L(s)) = c \cdot \frac{\text{Closed Lp characteristic polynomial}}{\text{Open Lp characteristic polynomial}}$$

- $Z$ (zeros in $C$ = RHP) : unstable closed loop poles
- $P$ (poles in $C$ = RHP) : unstable open loop poles

**Generalized Nyquist:** $N$ is the number clock wise encirclements of origin by Contour Map $\det(I + L(s))$

# Small gain theorem

The small-gain theorem is a very general result.

# Small gain theorem

The small-gain theorem is a very general result.

**Small-gain theorem:** Consider a system with a stable $L(s)$. Then the CL is stable if

$$||L(j\omega)|| < 1 \forall \omega,$$

where $||\cdot||$ represents a matrix norm.

Says: if the system gain is less than 1 in all directions and for all frequencies, then all signal deviations will die out, system is stable

Conservative: consider SISO, $||L(j\omega)||_F = |L(j\omega)|$ we know from Bode stability condition we only require $|L(j\omega)| < 1$ for $\angle L(j\omega) = 180^o + n \cdot 360^o$

# Small gain theorem

The small-gain theorem is a very general result.

> **Small-gain theorem:** Consider a system with a stable $L(s)$. Then the CL is stable if
>
> $$||L(j\omega)|| < 1 \forall \omega,$$
>
> where $||\cdot||$ represents a matrix norm.

Says: if the system gain is less than 1 in all directions and for all frequencies, then all signal deviations will die out, system is stable

Conservative: consider SISO, $||L(j\omega)||_F = |L(j\omega)|$ we know from Bode stability condition we only require $|L(j\omega)| < 1$ for $\angle L(j\omega) = 180^o + n \cdot 360^o$

Theorem doesn't consider phase information.

Extremely useful for RS and RP synthesis (comes later)

# Norms

Norm gives a single number, of a: vector, matrix, signal or system.

# Norms

Norm gives a single number, of a: vector, matrix, signal or system.

**Norm:** A norm of $e$ is a real number, denoted by $||e||$, that satisfies:

1. Non-negative: $||e|| \geq 0$
2. Positive: $||e|| = 0 \Leftrightarrow e = 0$
3. Homogeneous: $||\alpha e|| = |\alpha| \cdot ||e|| \quad \forall \alpha \in \mathcal{C}$
4. Triangle inequality: $||e_1 + e_2|| \leq ||e_1|| + ||e_2||$
5. (for matrix norms) Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ where $A$ and $B$ are matrices.

# Norms

Norm gives a single number, of a: vector, matrix, signal or system.

**Norm:** A norm of $e$ is a real number, denoted by $||e||$, that satisfies:

1. Non-negative: $||e|| \geq 0$
2. Positive: $||e|| = 0 \Leftrightarrow e = 0$
3. Homogeneous: $||\alpha e|| = |\alpha| \cdot ||e|| \quad \forall \alpha \in \mathcal{C}$
4. Triangle inequality: $||e_1 + e_2|| \leq ||e_1|| + ||e_2||$
5. (for matrix norms) Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ where $A$ and $B$ are matrices.

Vector: $||a||_p = \left(\sum_i |a_i|^p\right)^{\frac{1}{p}}$

$||a||_1 = \sum_i |a_i|$, $||a||_2 = \sqrt{\left(\sum_i |a_i|^2\right)}$, $||a||_\infty = \max_i |a_i|$

# Norms

Norm gives a single number, of a: vector, matrix, signal or system.

**Norm:** A norm of $e$ is a real number, denoted by $||e||$, that satisfies:

1. Non-negative: $||e|| \geq 0$
2. Positive: $||e|| = 0 \Leftrightarrow e = 0$
3. Homogeneous: $||\alpha e|| = |\alpha| \cdot ||e|| \quad \forall \alpha \in \mathcal{C}$
4. Triangle inequality: $||e_1 + e_2|| \leq ||e_1|| + ||e_2||$
5. (for matrix norms) Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ where $A$ and $B$ are matrices.

Vector: $||a||_p = (\sum_i |a_i|^p)^{\frac{1}{p}}$

$||a||_1 = \sum_i |a_i|$, $||a||_2 = \sqrt{(\sum_i |a_i|^2)}$, $||a||_\infty = \max_i |a_i|$

Matrix:

$$||A||_F = \sqrt{\left(\sum_{i,j} |a_{ij}|^2\right)} = \sqrt{\mathrm{tr}(A^H A)}$$

# Norms

Norm gives a single number, of a: vector, matrix, signal or system.

**Norm:** A norm of $e$ is a real number, denoted by $||e||$, that satisfies:

1. Non-negative: $||e|| \geq 0$
2. Positive: $||e|| = 0 \Leftrightarrow e = 0$
3. Homogeneous: $||\alpha e|| = |\alpha| \cdot ||e|| \quad \forall \alpha \in \mathcal{C}$
4. Triangle inequality: $||e_1 + e_2|| \leq ||e_1|| + ||e_2||$
5. (for matrix norms) Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ where $A$ and $B$ are matrices.

Vector: $||a||_p = (\sum_i |a_i|^p)^{\frac{1}{p}}$

$||a||_1 = \sum_i |a_i|$, $||a||_2 = \sqrt{(\sum_i |a_i|^2)}$, $||a||_\infty = \max_i |a_i|$

Matrix:

$$||A||_F = \sqrt{\left(\sum_{i,j} |a_{ij}|^2\right)} = \sqrt{\mathrm{tr}(A^H A)}$$

For control (remember generalized plant): $\min_K ||N||$. Which norm?

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

For the output signal we use: $||z(t)||_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} |z_i(\tau)|^2 d\tau}$.

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

For the output signal we use: $||z(t)||_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} |z_i(\tau)|^2 d\tau}$.

Three types of inputs:

1. $w(t)$ is a series of unit impulses.
2. $w(t)$ any signal $||w(t)||_2 = 1$.
3. $w(t)$ any signal $||w(t)||_2 = 1$, but with $w(t) = 0$ for $t \geq 0$ measure $z$ for $t \geq 0$.

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

For the output signal we use: $||z(t)||_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} |z_i(\tau)|^2 d\tau}$.

Three types of inputs:

1. $w(t)$ is a series of unit impulses. $\mathcal{H}_2$-norm
2. $w(t)$ any signal $||w(t)||_2 = 1$.
3. $w(t)$ any signal $||w(t)||_2 = 1$, but with $w(t) = 0$ for $t \geq 0$ measure $z$ for $t \geq 0$.

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

For the output signal we use: $||z(t)||_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} |z_i(\tau)|^2 d\tau}$.

Three types of inputs:

1. $w(t)$ is a series of unit impulses. $\mathcal{H}_2$-norm
2. $w(t)$ any signal $||w(t)||_2 = 1$. $\mathcal{H}_\infty$-norm
3. $w(t)$ any signal $||w(t)||_2 = 1$, but with $w(t) = 0$ for $t \geq 0$ measure $z$ for $t \geq 0$.

# System norms


where: $w$ is an input signal, $z$ is an output signal and $G$ stable TF with impulse response $g(t)$.

Given $w$ how big can $z$ become?

For the output signal we use: $||z(t)||_2 = \sqrt{\sum_i \int_{-\infty}^{\infty} |z_i(\tau)|^2 d\tau}$.

Three types of inputs:

1. $w(t)$ is a series of unit impulses. $\mathcal{H}_2$-norm
2. $w(t)$ any signal $||w(t)||_2 = 1$. $\mathcal{H}_\infty$-norm
3. $w(t)$ any signal $||w(t)||_2 = 1$, but with $w(t) = 0$ for $t \geq 0$ measure $z$ for $t \geq 0$. Hankel-norm

# System norms: $\mathcal{H}_2$-norm

(assume strictly proper, $D = 0$) For $\mathcal{H}_2$-norm: $w(t)$ is a series of unit impulses.

# System norms: $\mathcal{H}_2$-norm

(assume strictly proper, $D = 0$) For $\mathcal{H}_2$-norm: $w(t)$ is a series of unit impulses.


Frequency domain interpretation:

$$||G(s)||_2 \triangleq \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\mathrm{tr}\left(G(j\omega)^H G(j\omega)\right)}_{||G(j\omega)||_F^2=\sum_i \sigma_i^2(G(j\omega))} d\omega}$$

Time domain interpretation:

$$||G(s)||_2 = ||g(t)||_2 \triangleq \sqrt{\int_0^{\infty} \underbrace{\mathrm{tr}\left(g(\tau)^H g(\tau)\right)}_{||g(\tau)||_F^2} d\tau}$$

# System norms: $\mathcal{H}_\infty$-norm

(assume proper, $D \neq 0$) For $\mathcal{H}_\infty$-norm: $||w(t)||_2 = 1$ .

# System norms: $\mathcal{H}_\infty$-norm

(assume proper, $D \neq 0$) For $\mathcal{H}_\infty$-norm: $||w(t)||_2 = 1$ .


Frequency domain interpretation (peak of TF):

$$||G(s)||_\infty \triangleq \max_\omega \overline{\sigma}\left(G(j\omega)\right)$$

# System norms: $\mathcal{H}_\infty$-norm

(assume proper, $D \neq 0$) For $\mathcal{H}_\infty$-norm: $||w(t)||_2 = 1$ .


Frequency domain interpretation (peak of TF):

$$||G(s)||_\infty \triangleq \max_\omega \overline{\sigma}\left(G(j\omega)\right)$$

Time domain interpretation (worst case gain):

$$||G(s)||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{||w(t)||_2=1} ||z(t)||_2$$

# System norms: $\mathcal{H}_\infty$-norm

(assume proper, $D \neq 0$) For $\mathcal{H}_\infty$-norm: $||w(t)||_2 = 1$ .


Frequency domain interpretation (peak of TF):

$$||G(s)||_\infty \triangleq \max_\omega \overline{\sigma}\left(G(j\omega)\right)$$

Time domain interpretation (worst case gain):

$$||G(s)||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{||w(t)||_2=1} ||z(t)||_2$$

How to compute?

# System norms: $\mathcal{H}_\infty$-norm

(assume proper, $D \neq 0$) For $\mathcal{H}_\infty$-norm: $||w(t)||_2 = 1$ .


Frequency domain interpretation (peak of TF):

$$||G(s)||_\infty \triangleq \max_{\omega} \overline{\sigma}\left(G(j\omega)\right)$$

Time domain interpretation (worst case gain):

$$||G(s)||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{||w(t)||_2=1} ||z(t)||_2$$

How to compute? (assume $D = 0$) Smallest $\gamma$ for which $H = \begin{bmatrix} A & \frac{1}{\gamma^2}BB^T \\ -C^TC & -A^T \end{bmatrix}$ has no eigenvalues on imaginary axis.

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)
Push down the whole thing

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)
Push down the whole thing

Why do we like the $\mathcal{H}_\infty$ norm?

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)
Push down the whole thing

Why do we like the $\mathcal{H}_\infty$ norm?

Convenient to represent uncertainties, and it is an induced norm that satisfies the multiplicative property:$||AB|| \leq ||A|| \cdot ||B||$ (useful for cascade interconnections)

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)
Push down the whole thing

Why do we like the $\mathcal{H}_\infty$ norm?

Convenient to represent uncertainties, and it is an induced norm that satisfies the multiplicative property:$||AB|| \leq ||A|| \cdot ||B||$ (useful for cascade interconnections)

What is wrong with the $\mathcal{H}_2$ norm?

# Difference between $\mathcal{H}_2$ and $\mathcal{H}_\infty$

For $\mathcal{H}_\infty$-norm: Minimizing the peak, maximum singular value (worst case direction, frequency)
Push down the peak of largest singular value

For $\mathcal{H}_2$-norm: Minimizing all the singular values (all frequencies, directions)
Push down the whole thing

Why do we like the $\mathcal{H}_\infty$ norm?

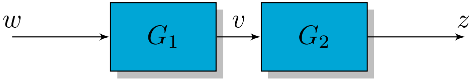
Convenient to represent uncertainties, and it is an induced norm that satisfies the multiplicative property:$||AB|| \leq ||A|| \cdot ||B||$ (useful for cascade interconnections)

What is wrong with the $\mathcal{H}_2$ norm?

Nothing, but no induced norm, no multiplicative property

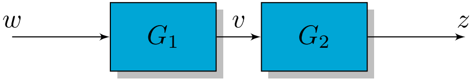
# Why do we like the multiplicative property

Consider two systems in serie:



# Why do we like the multiplicative property

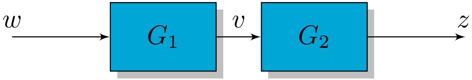
Consider two systems in serie:




Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ holds for $\mathcal{H}_\infty$ norm

# Why do we like the multiplicative property

Consider two systems in serie:

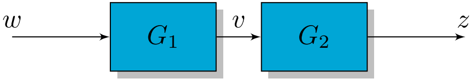



Multiplicative property: $||AB|| \leq ||A|| \cdot ||B||$ holds for $\mathcal{H}_\infty$ norm

We have $||G_2G_1||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{w\neq 0} \frac{||G_2G_1w||_2}{||w||_2}$.

# Why do we like the multiplicative property

Consider two systems in serie:




**Multiplicative property:** $||AB|| \leq ||A|| \cdot ||B||$ holds for $\mathcal{H}_\infty$ norm

We have $||G_2 G_1||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{w\neq 0} \frac{||G_2 G_1 w||_2}{||w||_2}$.

Multiply with $\frac{||v||_2}{||v||_2}$ we get:

$$\max_{w(t)\neq 0} \frac{||G_2 v||_2}{||v||_2} \cdot \frac{||G_1 w||_2}{||w||_2}$$

# Why do we like the multiplicative property

Consider two systems in serie:

$$w \rightarrow \boxed{G_1} \xrightarrow{v} \boxed{G_2} \rightarrow z$$

**Multiplicative property:** $||AB|| \leq ||A|| \cdot ||B||$ holds for $\mathcal{H}_\infty$ norm

We have $||G_2G_1||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{w\neq 0} \frac{||G_2G_1w||_2}{||w||_2}$.

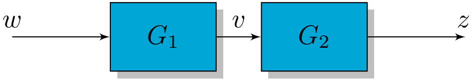
Multiply with $\frac{||v||_2}{||v||_2}$ we get:

$$\max_{w(t)\neq 0} \frac{||G_2v||_2}{||v||_2} \cdot \frac{||G_1w||_2}{||w||_2} \leq \max_{v(t)\neq 0} \frac{||G_2v||_2}{||v||_2} \cdot \max_{w(t)\neq 0} \frac{||G_1w||_2}{||w||_2}$$

Maximize of $v$ and $w$ independently

# Why do we like the multiplicative property

Consider two systems in serie:




**Multiplicative property:** $||AB|| \leq ||A|| \cdot ||B||$ holds for $\mathcal{H}_\infty$ norm

We have $||G_2G_1||_\infty = \max_{w(t)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} = \max_{w\neq 0} \frac{||G_2G_1w||_2}{||w||_2}$.

Multiply with $\frac{||v||_2}{||v||_2}$ we get:

$$\max_{w(t)\neq 0} \frac{||G_2v||_2}{||v||_2} \cdot \frac{||G_1w||_2}{||w||_2} \leq \max_{v(t)\neq 0} \frac{||G_2v||_2}{||v||_2} \cdot \max_{w(t)\neq 0} \frac{||G_1w||_2}{||w||_2}$$

Maximize of $v$ and $w$ independently

Example: $G = G_1 = G_2 = \frac{1}{s+a}$, we have $||G||_\infty = \frac{1}{a}$, $||G||_2 = \sqrt{\frac{1}{2a}}$,
$||GG||_\infty = \frac{1}{a^2}$, $||GG||_2 = \sqrt{\frac{1}{a}\frac{1}{2a}}$ so for $a < 1$ we have a problem for the $\mathcal{H}_2$-norm.

# Fundamental limitations on sensitivity

Three limitations:

1. We know: $S+T=I$ with $S=(I+L)^{-1}$ and $T=L\,(I+L)^{-1}$

# Fundamental limitations on sensitivity

Three limitations:

1. We know: $S + T = I$ with $S = (I + L)^{-1}$ and $T = L(I + L)^{-1}$

   Implies: For every $\omega$ either $|T| > 0.5$ or $|S| > 0.5$ and $||T| - |S|| < 1$

# Fundamental limitations on sensitivity

Three limitations:

1. We know: $S + T = I$ with $S = (I + L)^{-1}$ and $T = L(I + L)^{-1}$

   Implies: For every $\omega$ either $|T| > 0.5$ or $|S| > 0.5$ and $||T| - |S|| < 1$

2. Interpolation constraint I: If $p$ is a RHP-pole of $L$ then: $T(p) = 1$, $S(p) = 0$

   Interpolation constraint II: If $z$ is a RHP-zero of $L$ then: $T(z) = 0$, $S(z) = 1$

# Fundamental limitations on sensitivity

Three limitations:

1. We know: $S+T=I$ with $S=(I+L)^{-1}$ and $T=L(I+L)^{-1}$

   Implies: For every $\omega$ either $|T|>0.5$ or $|S|>0.5$ and $||T|-|S||<1$

2. Interpolation constraint I: If $p$ is a RHP-pole of $L$ then: $T(p)=1$, $S(p)=0$

   Interpolation constraint II: If $z$ is a RHP-zero of $L$ then: $T(z)=0$, $S(z)=1$

   Follows from internal stability requirement

# Fundamental limitations on sensitivity

Three limitations:

1. We know: $S + T = I$ with $S = (I + L)^{-1}$ and $T = L(I + L)^{-1}$

   Implies: For every $\omega$ either $|T| > 0.5$ or $|S| > 0.5$ and $||T| - |S|| < 1$

2. Interpolation constraint I: If $p$ is a RHP-pole of $L$ then: $T(p) = 1$, $S(p) = 0$

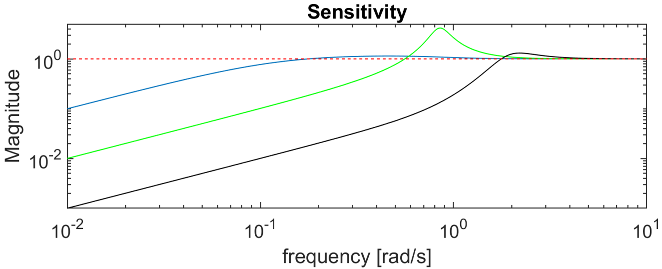
   Interpolation constraint II: If $z$ is a RHP-zero of $L$ then: $T(z) = 0$, $S(z) = 1$

   Follows from internal stability requirement

3. Waterbed effect $\rightarrow$

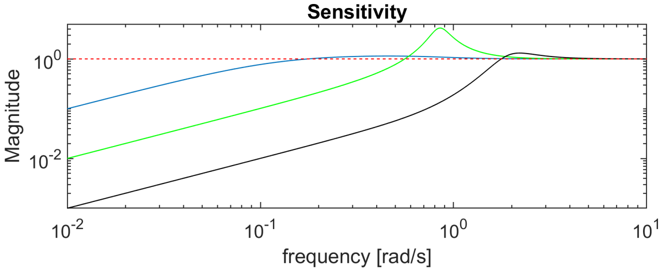
# The waterbed effects

Consider: $G(s) = \frac{1}{s^3+1.6s^2+s}$. Now $0.1 \cdot G(s)$, $1 \cdot G(s)$, $10 \cdot G(s)$



# The waterbed effects

Consider: $G(s) = \frac{1}{s^3+1.6s^2+s}$. Now $0.1 \cdot G(s)$, $1 \cdot G(s)$, $10 \cdot G(s)$

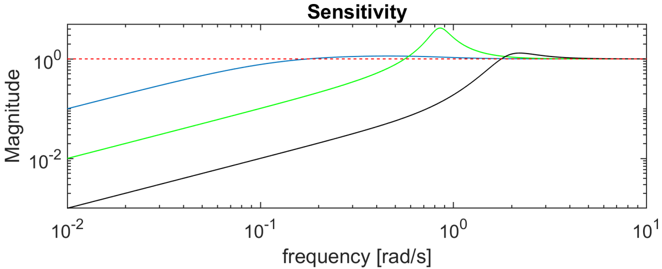



Assume $L$ has a relative degree $\geq 2$, and $N_p$ RHP-poles $p_i$. We have:

$$\int_0^\infty \ln |S(j\omega)| d\omega = \pi \cdot \sum_{i=1}^{N_p} Re(p_i)$$

# The waterbed effects

Consider: $G(s) = \frac{1}{s^3+1.6s^2+s}$. Now $0.1 \cdot G(s)$, $1 \cdot G(s)$, $10 \cdot G(s)$



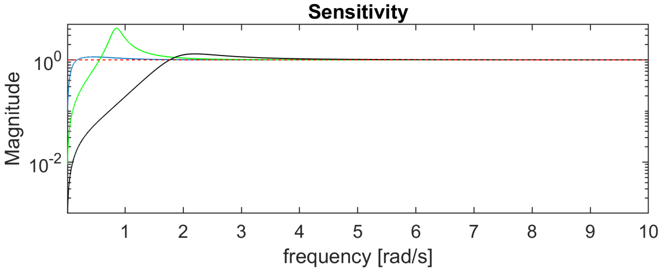

Assume $L$ has a relative degree $\geq 2$, and $N_p$ RHP-poles $p_i$. We have:

$$\int_0^\infty \ln|S(j\omega)|d\omega = \pi \cdot \sum_{i=1}^{N_p} Re(p_i)$$

So, if there are no RHP-poles surface below and below gain 1 should be equal.

# The waterbed effects

Consider: $G(s) = \frac{1}{s^3+1.6s^2+s}$. Now $0.1 \cdot G(s)$, $1 \cdot G(s)$, $10 \cdot G(s)$



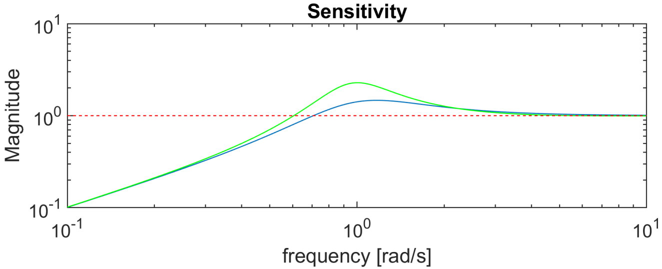

Assume $L$ has a relative degree $\geq 2$, and $N_p$ RHP-poles $p_i$. We have:

$$\int_0^\infty \ln|S(j\omega)|d\omega = \pi \cdot \sum_{i=1}^{N_p} Re(p_i)$$

So, if there are no RHP-poles surface below and below gain 1 should be equal.

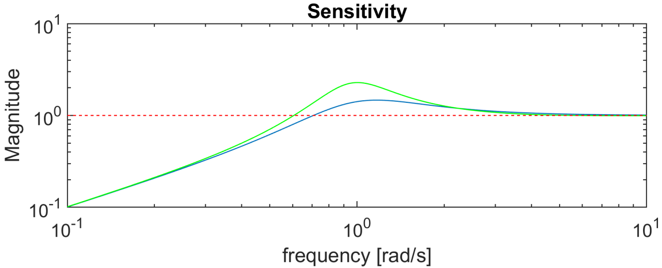
# The waterbed effects, effect of RHP-zero

Consider: $G(s) = \frac{2}{s^2+s}$. Now $G(s)$, $\frac{-s+5}{s+5}G(s)$



# The waterbed effects, effect of RHP-zero

Consider: $G(s) = \frac{2}{s^2+s}$. Now $G(s)$, $\frac{-s+5}{s+5}G(s)$



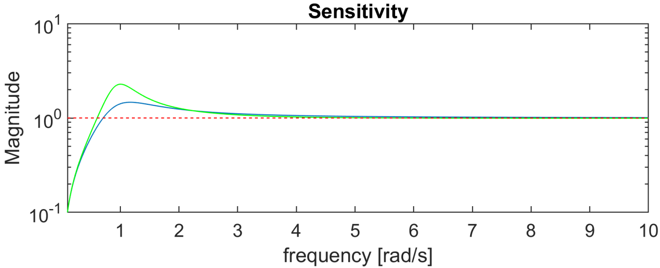

Assume $L$ has a single RHP-zero at $z$, and $N_p$ RHP-poles $p_i$ ($\overline{p}_i$ is complex conjugate). We have:

$$\int_0^\infty \ln|S(j\omega)|w(z,\omega)d\omega = \pi \cdot \ln \prod_{i=1}^{N_p} \left| \frac{p_i + z}{\overline{p}_i - z} \right|$$

with $w(z,\omega) = \frac{2z}{z^2+\omega^2}$

# The waterbed effects, effect of RHP-zero

Consider: $G(s) = \frac{2}{s^2+s}$. Now $G(s)$, $\frac{-s+5}{s+5}G(s)$



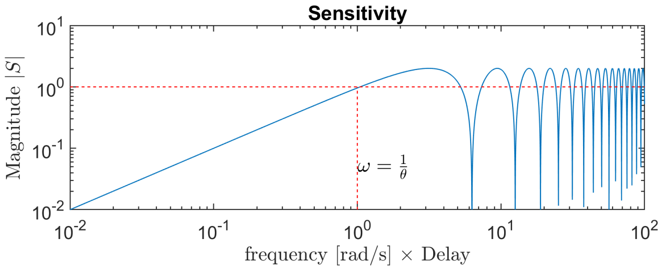

Assume $L$ has a single RHP-zero at $z$, and $N_p$ RHP-poles $p_i$ ($\overline{p}_i$ is complex conjugate). We have:

$$\int_0^{\infty} \ln|S(j\omega)| w(z,\omega) d\omega = \pi \cdot \ln \prod_{i=1}^{N_p} \left| \frac{p_i + z}{\overline{p}_i - z} \right|$$

with $w(z,\omega) = \frac{2z}{z^2+\omega^2}$

# Limitations imposed by time delays

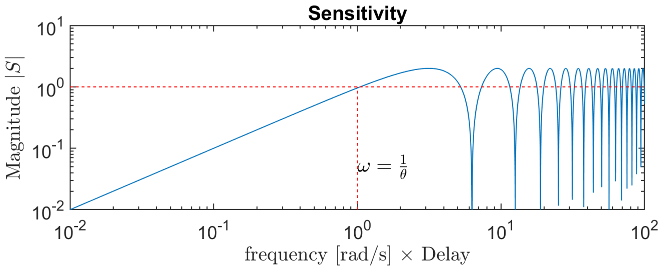
It should be clear that time delays $(e^{-\theta s})$ can form a significant limitation.



# Limitations imposed by time delays

It should be clear that time delays $(e^{-\theta s})$ can form a significant limitation.

The bandwidth is limited to be less than $\frac{1}{\theta}$.

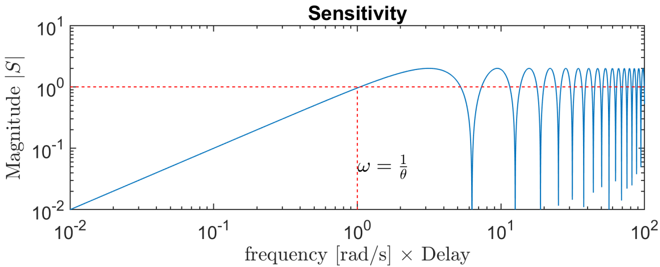


# Limitations imposed by time delays

It should be clear that time delays $(e^{-\theta s})$ can form a significant limitation.

The bandwidth is limited to be less than $\frac{1}{\theta}$.

Consider the ideal $T$ which is given by $T(s) = e^{-\theta s}$ then $S = 1 - e^{-\theta s}$

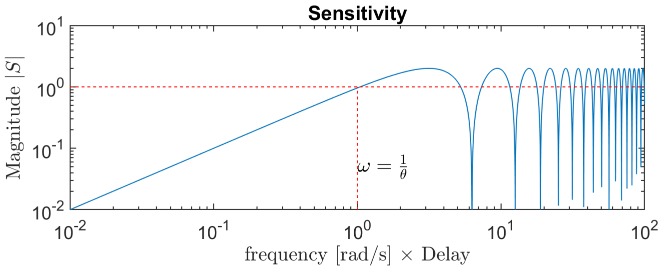


# Limitations imposed by time delays

It should be clear that time delays $(e^{-\theta s})$ can form a significant limitation.

The bandwidth is limited to be less than $\frac{1}{\theta}$.

Consider the ideal $T$ which is given by $T(s) = e^{-\theta s}$ then $S = 1 - e^{-\theta s}$

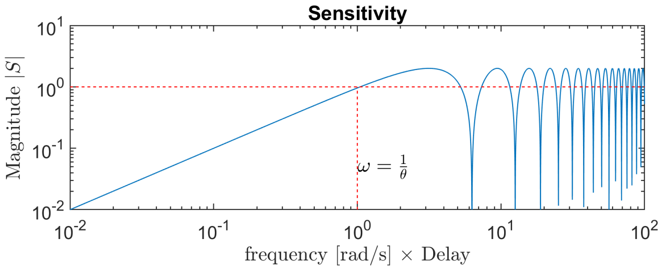


# Limitations imposed by time delays

It should be clear that time delays $(e^{-\theta s})$ can form a significant limitation.

The bandwidth is limited to be less than $\frac{1}{\theta}$.

Consider the ideal $T$ which is given by $T(s) = e^{-\theta s}$ then $S = 1 - e^{-\theta s}$




Pade approximation: $e^{-\theta s} \approx \frac{\left(1-\frac{\theta}{2n}s\right)^n}{\left(1+\frac{\theta}{2n}s\right)^n}$, where $n$ is the order.

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

From high gain feedback we know that poles will go to zeros

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

From high gain feedback we know that poles will go to zeros

From interpolation constraint we know $S(z) = 1$. When designing $w_P$ we have to realize this.

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

From high gain feedback we know that poles will go to zeros

From interpolation constraint we know $S(z) = 1$. When designing $w_P$ we have to realize this.

Example: We consider $w_P(s) = \frac{s/M+\omega_B}{s+\omega_B A}$.

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

From high gain feedback we know that poles will go to zeros

From interpolation constraint we know $S(z) = 1$. When designing $w_P$ we have to realize this.

Example: We consider $w_P(s) = \frac{s/M+\omega_B}{s+\omega_B A}$.

Example: $|w_P(z)| \leq ||W_p S||_\infty < 1$ and therefore $\left|\frac{z/M+\omega_B}{z+\omega_B A}\right| < 1$

# Limitations imposed by RHP-zero

Remember, system zeros are invariant under feedback!

From high gain feedback we know that poles will go to zeros

From interpolation constraint we know $S(z) = 1$. When designing $w_P$ we have to realize this.

Example: We consider $w_P(s) = \frac{s/M+\omega_B}{s+\omega_B A}$.

Example: $|w_P(z)| \leq ||W_p S||_\infty < 1$ and therefore $\left| \frac{z/M+\omega_B}{z+\omega_B A} \right| < 1$

Example: If $z$ is real: $\omega_B < z\frac{1-1/M}{1-A}$

# MIMO: Fundamental limitations on sensitivity

Again we have to think about directions!!

# MIMO: Fundamental limitations on sensitivity

Again we have to think about directions!!

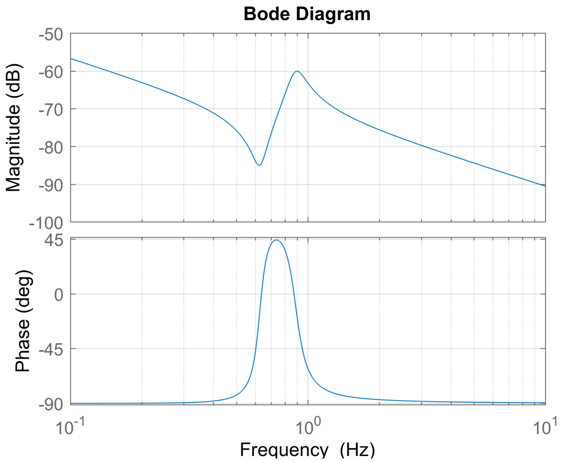
Same effects are present but expressions are far more complicated (See book).

# Drive train dynamics

Let's consider again our drive-train example:

$$\frac{\omega_g}{T_g}$$

Workpoint 1



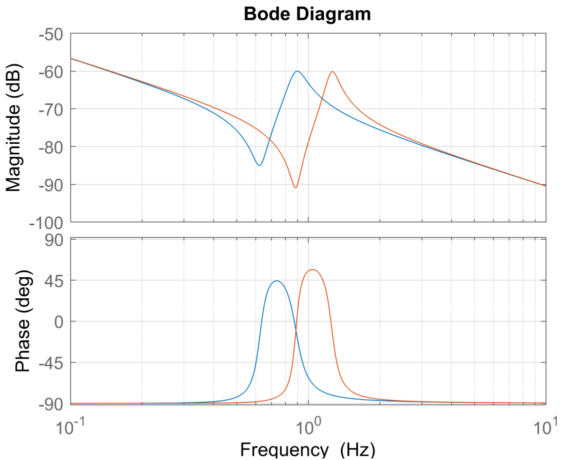
# Drive train dynamics

Let's consider again our drive-train example:

$$\frac{\omega_g}{T_g}$$

Workpoint 1

Workpoint 2



# Drive train dynamics

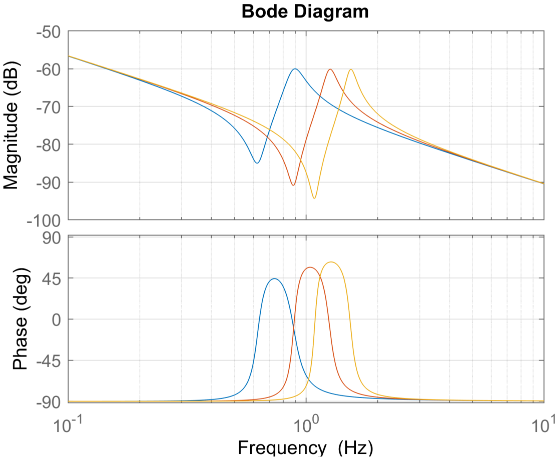
Let's consider again our drive-train example:

$$\frac{\omega_g}{T_g}$$

Workpoint 1

Workpoint 2

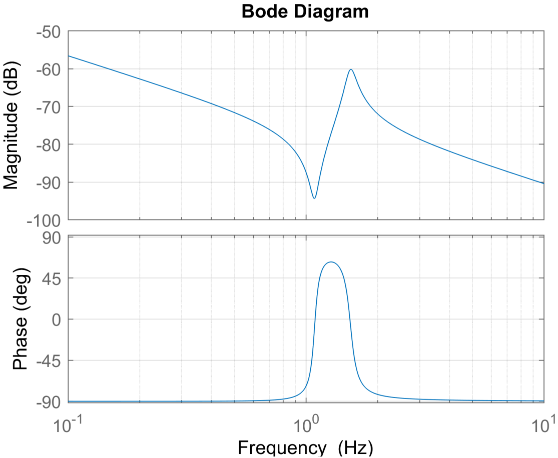
Workpoint 3



# Drive train dynamics

Let's consider again our drive-train example:

$$\frac{\omega_g}{T_g}$$




Workpoint 3

Let's design a controller for this WP and TF

# Mixed-sensitivity code

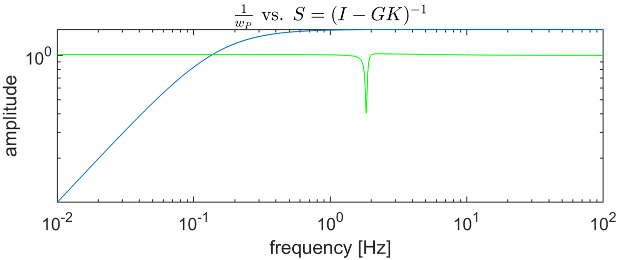
G is given

```
>>wB1=0.1*2*pi; % desired closed-loop bandwidth of 0.1Hz
>>A=1/100; % desired disturbance attenuation inside bandwidth
>>M=1.5 ; % desired bound on hinfnorm(S)
>>Wp=[(s/M+wB1)/(s+wB1*A)]; % Sensitivity weight
>>Wu=1; % Control weight
>>systemnames ='G Wp Wu '; % Define systems
>>inputvar ='[w(1); u(1)]'; % Input generalized plant
>>input_to_G= '[u]';
>>input_to_Wu= '[u]';
>>input_to_Wp= '[w+G]';
>>outputvar= '[Wp; Wu; G+w]'; % Output generalized plant
>>sysoutname='P';
>>sysic;
>>[K,CL,GAM,INFO] = hinfsyn(P,1,1); % Hinf design
```

# Design results

Results: $\dfrac{\omega_g}{T_g}$

Sensitivity

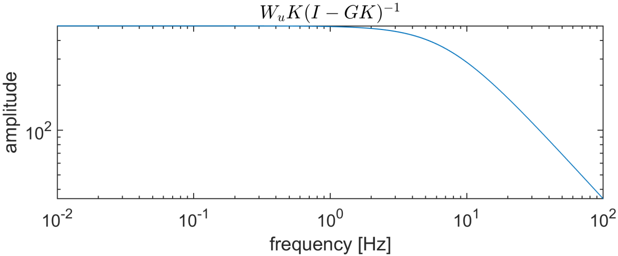



$||N||_\infty = 498!!$

# Design results

Results:
$$\frac{\omega_g}{T_g}$$

Weight on input

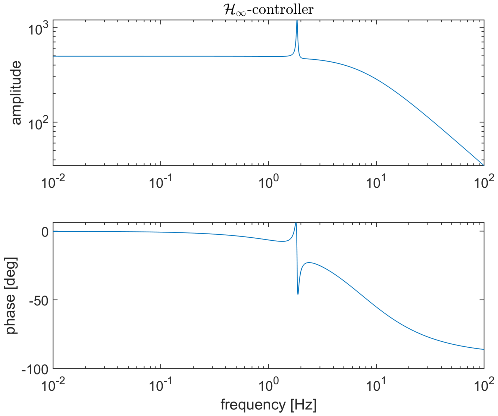



$$||N||_\infty = 498!!$$

# Design results

Results:

$$\frac{\omega_g}{T_g}$$



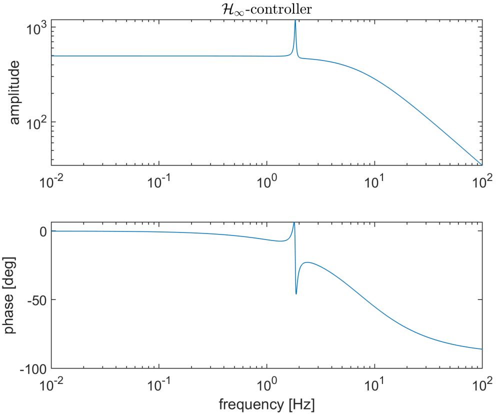

Controller

$$||N||_\infty = 498!!$$

# Design results

Results:

$$\frac{\omega_g}{T_g}$$




Controller

$$||N||_\infty = 498!!$$

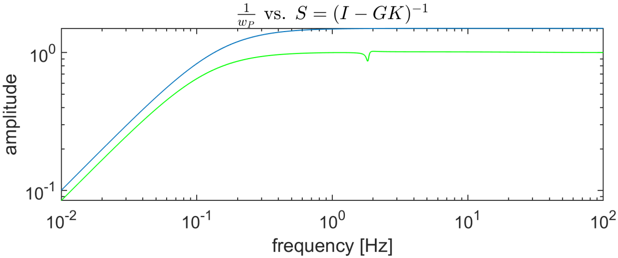
We didn't apply scaling!!!

# Design results (cont'd)

Results with scaling of $w_U$:

$$\frac{\omega_g}{T_g}$$

Sensitivity




$$||N||_\infty = 0.85$$

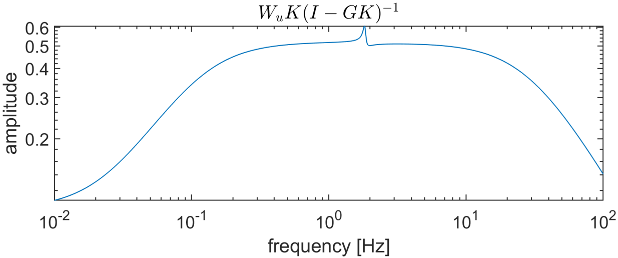
# Design results (cont'd)

Results with scaling of $w_U$:

$$\frac{\omega_g}{T_g}$$

Weight on input

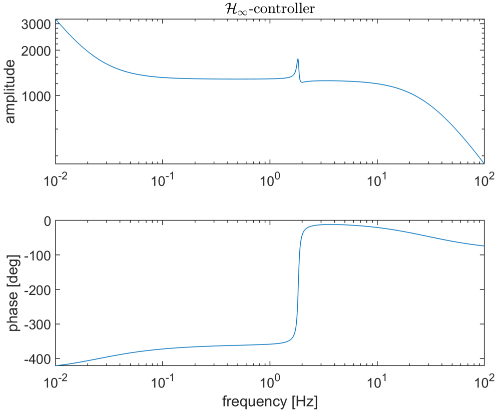
$||N||_\infty = 0.85$



# Design results (cont'd)

Results with scaling of $w_U$:

$$\frac{\omega_g}{T_g}$$




Controller

$$||N||_\infty = 0.85$$

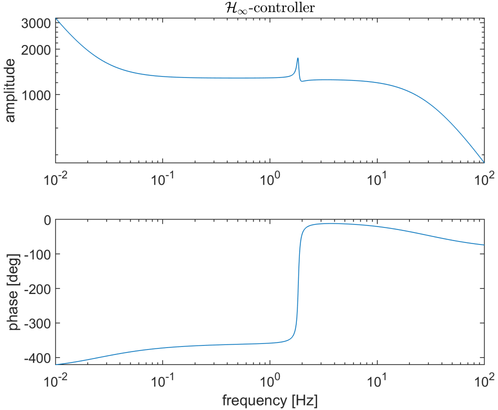
# Design results (cont'd)

Results with scaling of $w_U$:

$$\frac{\omega_g}{T_g}$$

Controller

$$||N||_\infty = 0.85$$




Controller unstable but $S$ stable with RHP-zeros that cancel RHP-poles of K!!!