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# Smith Predictor Control and Internal Model Control - A Tutorial -

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**Abstract:** This paper is to provide a guide to the time-delay compensation scheme known as Smith predictor control. The internal model control (IMC) scheme is discussed together to understand the Smith predictor control in terms of newer theoretical concepts. Related topics such as disturbance rejection, $\mathcal{H}_2$ optimality, stability robustness and windup problem are also discussed.

**Keywords:** Time Delay Systems, Smith Predictor Control, Internal Model Control

## 1. Introduction

Time-delay has been a common phenomenon to overcome whenever we close a feedback loop for the purpose of controlling any system. Recent increase of control applications in the variety gives more importance to systematic methods to cope with time-delay. Motivated by this observation, we review one of the best known schemes for controlling systems with time-delay. The distinctive scheme was proposed by O.J.M.Smith approximately 50 years ago, and is still attracting much attention for its usefulness. From the present viewpoint, the internal model control (IMC) is closely related to the Smith predictor control as long as time-delay systems are concerned. After seeing the basic formulations of the two control schemes separately, we discuss equivalence between them. Discussions about related topics follow on the basis of the equivalence recognition. SISO systems are main object of this paper, and MIMO systems are also discussed for the case including an issue peculiar to MIMO systems.

## 2. Smith Predictor Control

Smith predictor control [1)] is a feedback control scheme that have a minor loop as shown in Fig.1. $G$ denotes a stable, strictly proper rational function characterizing the delay-free part of the plant. $L$ denotes a positive constant standing for the time-delay. $\tilde{G}$ and $\tilde{L}$ are nominal version of $G$ and $L$, respectively, obtained through modeling process. $C_S$ denotes a rational function characterizing the compensator called primary controller. The minor loop works to eliminate the actual delayed output as well as to feed the predicted output to the primary controller. This makes it possible to design the primary controller assuming no time-delay in the control loop. PID controllers can be successfully applied together with classical tuning techniques.

In the case where $G = \tilde{G}$ and $L = \tilde{L}$, the output $y$ depends on the reference input $r$ and the disturbance $d$

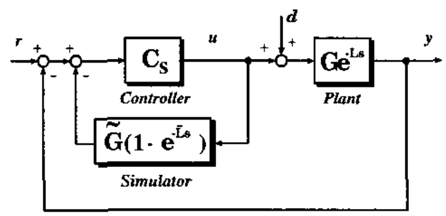


Figure 1: Smith predictor control

as follows.

$$y = \frac{GC_S}{1+GC_S}e^{-Ls}r + \frac{Ge^{-Ls}}{1+GC_S}d + \frac{GC_Se^{-Ls}}{1+GC_S}G(1-e^{-Ls})d \quad (1)$$

The closed-loop poles are at most finite in number since the denominator function $1 + GC_S$ is a rational function. It should be remarked that the response to the disturbance depends directly on the poles of $G$, which is out of control. This issue will be discussed later.

Fig.2 is another equivalent diagram of Smith predictor control scheme, which may possibly be more convenient for practical use.

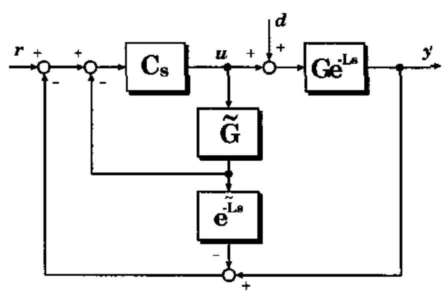


Figure 2: Practical installation of Smith scheme

# 3. Internal Model Control

Internal model control (IMC) [11] is a control scheme that incorporates plant model as shown in Fig.3. $G, L, \tilde{G}$ and $\tilde{L}$ are the same as those mentioned in Section 2. $C_{IMC}$ denotes a transfer function, which is suitably chosen from stable rational functions as design parameter. Suppose that the plant behaves perfectly the same behavior as its model, i.e., $\tilde{G}e^{-\tilde{L}s} = Ge^{-Ls}$. Then the output $y$ is simply related to the inputs $r, d$ as

$$y = GC_{IMC}e^{-Ls}r + (1 - GC_{IMC})Ge^{-Ls}d. \quad (2)$$

It can be seen that the internal stability is always assured as long as a stable parameter $C_{IMC}$ is used to control a stable plant.

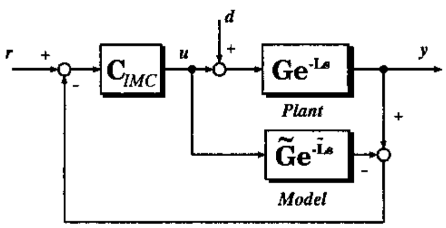


Figure 3: IMC Structure

**A design procedure of IMC** Assume that $G = \tilde{G}$, $L = \tilde{L}$ and $G$ is of minimum phase-shift. Then a simple choice of the design parameter is

$$C_{IMC} = \tilde{G}^{-1}(s)F(s), \quad (3)$$

where, $F$ is a rational function with $F(0) = 1$, called IMC filter, chosen so as to make $C_{IMC}$ proper. A simple choice of $F$ is as follows.

$$F(s) := \frac{1}{(\lambda s + 1)^n}, \quad (4)$$

where $\lambda$ is a positive real number, and $n$ is a positive integer, both suitably chosen. The resultant I-O property is

$$y = Fe^{-Ls}r + Ge^{-Ls}(1 - Fe^{-Ls})d. \quad (5)$$

Note that perfect tracking except pure time-delay is achieved by choosing $F \equiv 1$.

# 4. Equivalence

Comparing with the block diagrams of Fig.1 and Fig.3 with each other, we find a common feature in the Smith scheme and IMC scheme. Both incorporate the plant model to cancel the plant dynamics. Based on the equivalent block diagram of Fig.4, we can find a formula for converting one to the other as follows,

$$C_{IMC} = \frac{C_S}{1 + C_S\tilde{G}}, \quad (6)$$

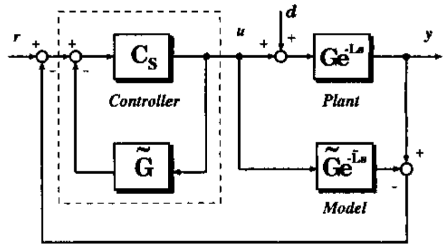


Figure 4: Equivalent of Smith scheme

$$C_S = \frac{C_{IMC}}{1 - C_{IMC}\tilde{G}} \quad (7)$$

Smith predictor control has an IMC structure in which $C_{IMC}$ is parametrized by $C_S$ as $C_{IMC} = C_S/(1 + C_S\tilde{G})$ and is designed as if the classical unity feedback structure is under consideration.

# 5. Relevant Topics

In this section we discuss about three relevant topics of Smith predictor control and IMC.

## 5.1 Disturbance rejection

In eq.(1) and eq.(2) the transfer function from $d$ to $y$ inherit the open-loop poles. When the open-loop system has slow modes, input channel disturbance influence harmfully for a long time. This can be solved in Smith predictor control by inserting a disturbance compensator in the feedback path [5]. This leads to Smith controller with two-degree of freedom.

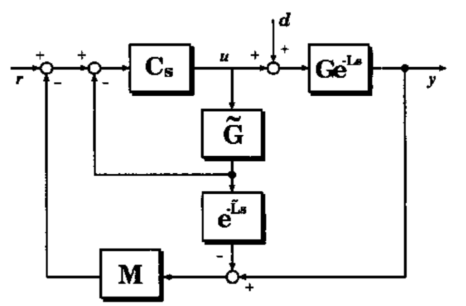


Figure 5: 2.D.O.F. Smith predictor control

The I-O relation (1) is modified into

$$\begin{aligned} y &= \frac{GC_Se^{-Ls}}{1 + GC_S}r \\ &+ \frac{Ge^{-Ls}}{1 + GC_S}d + \frac{GC_se^{-Ls}}{1 + GC_S}G(1 - M(s)e^{-Ls})d. \end{aligned} \quad (8)$$

The inserted compensator $M$ changes the response to disturbance. If the open-loop poles can be canceled by some zeros of $(1 - M(s)e^{-Ls})$, then the resoponse to disturbance is improved without changing the response to reference input.

To summarize, the additional compensator $M$ is required to satisfy the following conditions. [5].

1. The poles of $M$ are suitably located for desirable response.
2. The function $1-M(s)e^{-Ls}$ has such zeros that cancel all the "slow" poles of $G$.
3. $M(0) = 1$

The third condition is necessary to remove the steady-state output error from a constant reference. For the details of design procedure, see Reference [5].

The same problem arises in IMC scheme, and is solved by the 2 degree-of-freedom IMC scheme of Fig.6.

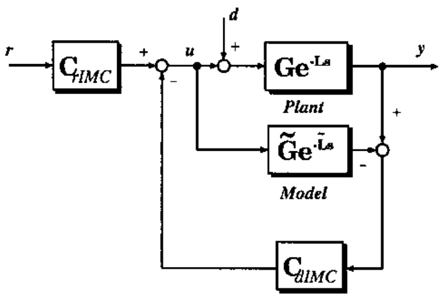


Figure 6: 2 D.O.F. IMC

The I-O relation (2) is modified into

$$y = GC_{rIMC}e^{-Ls}r + (1 - GC_{dIMC})Ge^{-Ls}d. \quad (9)$$

The compensator $C_{dIMC}$ can be used to improve the response to disturbance without changing the response to reference input [12].

## 5.2 $\mathcal{H}_2$ optimization

It has been recognized [2, 4, 7] that a structure of the Smith predictor control results from a certain class of $\mathcal{H}_2$ optimization problems. To state in terms of IMC, the optimal IMC compensator $C_{IMC}$ can be found within those characterized by rational transfer functions [13]. We discuss this issue below with the assumption $\tilde{G} = G$ and $\tilde{L} = L$.

We saw in the previous section that a simple design procedure applies to an IMC scheme, provided the delay-free part of the plant is of minimum phase-shift. The key was in canceling all the dynamics except the time-delay by using the compensator $C_{IMC} = G^{-1}$, which is multiplied by a suitable low-pass filter. On the other hand, the optimal compensator takes somewhat different form in terms of the $\mathcal{H}_2$ performance criterion. Let $\langle f, g\rangle$ denote the inner product of meromorphic functions $f$ and $g$, defined as

$$\langle f, g\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(j\omega)g(j\omega)^* d\omega, \quad (10)$$

as long as the integral exists. Here $*$ denotes complex conjugate. Let us consider a performance criterion of the form

$$J(C_{IMC}) = \langle (1 - DGC_{IMC})R, (1 - DGC_{IMC})R\rangle, \quad (11)$$

where $D(s) := exp(-Ls)$, and $R$ is a strictly proper rational function having no poles nor zeros in the closed right half-plane. This is the 2-norm of the sensitivity function weighted by the function $R$. If we consider $R(j\omega)R(j\omega)^*$ to be the power spectral density of reference input that is a stationary stochastic process, then the optimization in terms of $J$ amounts to the minimum-variance tracking.

Due to the all-pass characteristic of the time-delay, we can rewrite the right-hand side of (11) as

$$\langle \hat{D}R - GC_{IMC}, \hat{D}R - GC_{IMC}\rangle,$$
$$\hat{D}(s) := D(s)^{-1} = e^{Ls}. \quad (12)$$

Thus all we have to do is to find the $\mathcal{H}_2$-function $GC_{IMC}$ closest to the function $\hat{D}R$ lying in $\mathcal{L}_2$-space [2, 8]. This can be done as follows. First, the orthogonal projection of $\hat{D}R$ on to $\mathcal{H}_2$-space is $ce^{AL}(sI - A)^{-1}b$, which can be obtained as the "causal part" of $\hat{D}R$ [13]. Here, $(c, A, b)$ is the state-space realization of $R$:

$$R(s) = c(sI - A)^{-1}b. \quad (13)$$

Second, the optimal IMC compensator is derived by equating $GC_{IMC}$ and the orthogonal projection. The result is $C_{IMC} = G^{-1}ce^{AL}(sI - A)^{-1}b$.

It should be noted that a rational function $C_{IMC}$ is obtained as optimal compensator function. This means that the optimal control scheme can also be constructed in the form of Smith predictor control with rational $C_S$. As for MIMO systems, it depends on the plant dynamics whether a rational transfer matrix result from $\mathcal{H}_2$ optimization or not. So far we have the following result on this issue [13]. Assume that the plant transfer matrix is given in the form

$$e^{-Ls}G(s), \quad (14)$$

where $L$ is a diagonal matrix with $m$ positive diagonal entries, and $G$ is such a $m \times m$ matrix that both $G$ and $G^{-1}$ are composed of stable rational functions. Then, the plant is optimally controlled by an IMC scheme with rational stable $C_{IMC}$, or a Smith scheme with rational stabilizing $C_S$. This consequence can not necessarily valid without the above assumption.

## 5.3 Practical stablity

Since the plant model incorporated in the Smith scheme is not the plant itself, it is difficult to assume the actual behavior of the plant perfectly the same as the model. The possible difference of the actual plant from the model gives rise to mismatch between the plant and the controller. A certain Smith predictor control scheme is stable only when there is no mismatch, i.e. an arbitrarily small difference in the time-delays makes the system unstable. This was pointed out by Z.Palmor about fairly general class of systems, and such a system was said to be practically unstable [3]. On the other hand, a Smith predictor control scheme satisfying some conditions remains stable in the presence of small mismatch, and is called practically stable.

Suppose that the mismatch is caused only by identification error in the time-delay, i.e., $\tilde{G} = G$. Then, a Smith scheme or an IMC scheme is said to be practically stable, iff there exists a $\delta > 0$ such that the closed loop is stable whenever $\left|L - \tilde{L}\right| < \delta$ . Here, we use the word "stable" to mean that we can find some $\alpha > 0$ such that the closed-loop sensitivity function has no poles to the right of the line $\mathbf{Re}s = -\alpha$.

The inequality

$$\lim_{\omega\to\infty}\left|\frac{G(j\omega)C_S(j\omega)}{1+G(j\omega)C_S(j\omega)}\right| < \frac{1}{2} \tag{15}$$

was first shown to be necessary for practical stability of Smith schemes [3], before proving to be also sufficient [6, 9]. Equivalently, this necessary and sufficient condition can be rewritten in terms of IMC [12], as

$$|G(s)C_{IMC}(s)|_{s=\infty} < \frac{1}{2}. \tag{16}$$

The condition (15) [or (16)] can not be violated in usual situation since $G$ is strictly proper. From the practical viewpoint, however, we should be careful in such a case that use of differential action in $C_S$ prevents the complementary sensitivity from suitably decaying in high frequency range.

To state the result on practical stability for more general cases, we consider the following inequality that specifies the plant uncertainty [9].

$$\left|\frac{G(j\omega)}{\tilde{G}(j\omega)} - 1\right| < K\omega^\lambda, \omega > \gamma \tag{17}$$

Parameters $K > 0$, $\lambda \in \{0, 1, 2, \ldots\}$, $\gamma > 0$ are assumed to be given together with $\tilde{G}$ to define the class of all the perturbed plant dynamics $G$. It is taken into consideration here that uncertainty generally increases with the frequency. Now, let us define a Smith scheme to be practically stable, iff we can find $\delta > 0$ and $\beta > 0$ such that the closed loop remains stable against all the mismatch satisfying

$$\begin{aligned} \left|L - \tilde{L}\right| &< \delta \\ &\text{and} \\ \left|\frac{G(j\omega)}{\tilde{G}(j\omega)} - 1\right| &< \delta, \quad 0 \le \omega < \beta. \end{aligned} \tag{18}$$

A necessary and sufficient condition for a Smith scheme to be practically stable is represented by the inequality

$$\lim_{\omega\to\infty}\left(K\omega^\lambda + 2\right)|W(j\omega)| < 1, \tag{19}$$

where $W$ denotes the delay-free part of the complementary sensitivity function of the closed loop with no mismatch, i.e.,

$$W(s) := \frac{G(s)C_S(s)}{1 + G(s)C_S(s)}. \tag{20}$$

This result involves the preceding result, since arbitrarily small $K$ suits to the case where $\tilde{G} = G$. The result also applies to IMC schemes due to the equivalence between Smith predictor control and IMC. The equivalent expression $W = GC_{IMC}$ replases (20) for IMC.

## 6. Windup problem

It is known that the input saturation with an integral action cases an overshoot response, which is called "Windup". In the classical PID control scheme, bumples transfer and anti-windup are actual important problemes. Windup problem is not exceptional in both Smith and IMC schemes with input saturation [14]. When there are some nonlinearityes, such as the saturation, both schemes are generally inconvertible. In this section we show some solutions for both schemes.

Duaring the time-delay interval, optput of the plant has no effect from input, then the integral error increase such that time delay period amplifis the windup phenomena.

**Smith scheme with PI controller** One of the typical and simple anti-windup solution of PID control is a limited integral action, however, since it uses discontinuous switches, it will cause the other problem.

Self conditioning anti-windup PI controller was proposed [10], which includes saturation model in PI controller. The structure is shown in Fig.7. The amount of

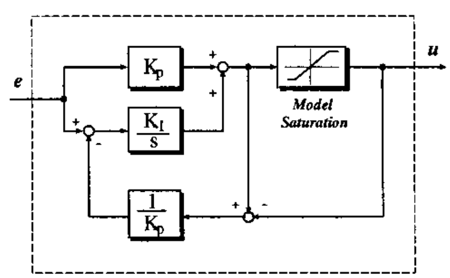


Figure 7: Self Conditioning Anti-Windup PI Controller

the saturated input eliminates the integral output, then the excessive overshoot response is restrained. This controller can be easily used for Smith predictor primary controller $C_S$ and it has good anti-windup property [14]. If the controller saturation is larger than the plant saturation, then the windup effect remains, therefore, the controller saturation is chosen smaller than the plant saturation.

**IMC scheme** If the model has no saturation, then the difference between the plant output and the model output causes the windup. It is necessary to coincide the plant with the model, therefore, inserting the saturation at the model such as Fig.8, then anti-windup is achieved [11].

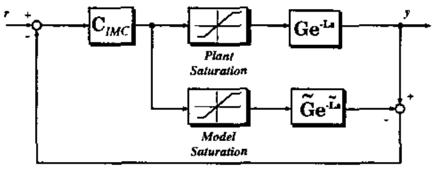


Figure 8: Anti-Windup IMC

Considering saturation mismatch between the plant and the model, the following scheme, in which the model saturation is smaller than the plant saturation, is effective [14].

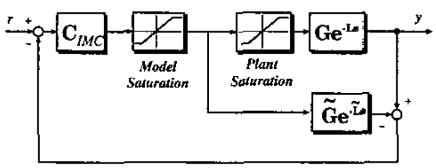


Figure 9: Anti-Windup IMC

In Fig.9 and Fig.8 case, the controller can not be converted to the Smith primary controller like as section 4, since there are nonlinear blocks in the control loop.

## 7. Conclusion

In this paper, some topics about Smith Predictor Control and Internal Model Control are shown. The structure of Smith predictor control was devised to remove the delay effect from the closed-loop design, and is equivalent to IMC in the sense that the delayed behavior of the plant is cancelled by the plant model. That is, these methodologies lead substantially to a common structure for control systems with time-delay.

Both control schemes owe their performance to the plant model, therefore model identification has a particular importance in the procedure of closed-loop synthesis [15].

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