SC42145: Preliminaries

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sc42145, 2024-25

SC42145: Robust Control

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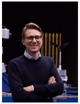


Daan van der Hoek

Anil Alan

Purpose of the course

- Formulate control objectives in a mixed-sensitivity design
- Define stability and performance for MIMO LTI systems
- Construct a generalized plant for complex system interconnections
- Design MIMO controllers on the basis of the mixed-sensitivity
- Describe parametric and dynamic uncertainties
- Translate concrete controller synthesis problem into the abstract framework of robust control
- Reproduce definition, properties and computation of the structured singular value
- Master application of structured singular value for robust stability and performance analysis
- Design robust controllers on the basis of the $\mathcal{H}_\infty$ control algorithm (D-K iterations)

Organization

- 7 Lectures (L), see BS
- Communication via BrightSpace (BS)
- We are available during the scheduled assisted lecture hours for questions
- Design exercise, see BS

Design exercise: Control of floating wind turbine

- Part 1a: Design a manually tuned SISO controller
- Part 1b: Design nominal $\mathcal{H}_\infty$ controller (mixed sensitivity)
- Part 2a: Add uncertainties to your model
- Part 2b: Test robustness
- Part 2c: Design robust (fixed structure) $\mathcal{H}_\infty$ controller

(Use: robust control toolbox)

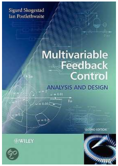
WindFloat (Principle Power USA)

Deliverables & Grade

1. Report deadlines:
   Part 1: 13th of Dec.,
   Part 2: 17th of Jan.,
2. 1 Report per team (team size max. 2)
3. Submit on BS
4. Grade: 50% Part 1 + 50% Part 2

Study material

- Chapter 1-9 and 12 of Skogestad & Postlethwaite (see BS for more details)
- Overhead slides of classes
- Assignments



Sigurd Skogestad, and Ian Postlethwaite
Multivariable Feedback Control
Analysis and Design (2005)

Short term requirements:

1. Enroll for the course on BS
2. Get book

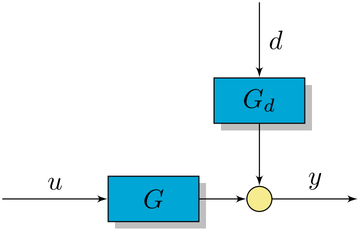
Lectures

1. Preliminaries (Chap. 1 & 2 (excluding 2.7))
2. Intro to MIMO control (Chap. 3)
3. Syst. Theory & Limit. (Chap. 4 & 5 (sect.5.2-5.4 & 5.6-5.9), Chap. 6 as ref. info)
4. Uncertainty & robustness SISO (Chap. 7)
5. Uncertainty & robustness MIMO (Chap. 8.1-8.7)
6. Structured Singular Value (Chap. 8.9-8.13+Chap. 12.1-12.2)
7. Overview +recap (Chap. 9 (sections 9.1-9.3))

The process of control system design

1. Study the system (process, plant)
2. Modeling, simplify, linearize
3. Scaling, and determine properties of the model
4. Select: Controllable inputs
5. Select: Sensors, which ones where to place?
6. Select: Control configuration
7. Select: Type of controller
8. Specify: Performance specs
9. Design a controller
10. Analyze the result
11. Simulate
12. Iterate (back to 6 or 8)
13. Choose hardware and software
14. Test and validate controller

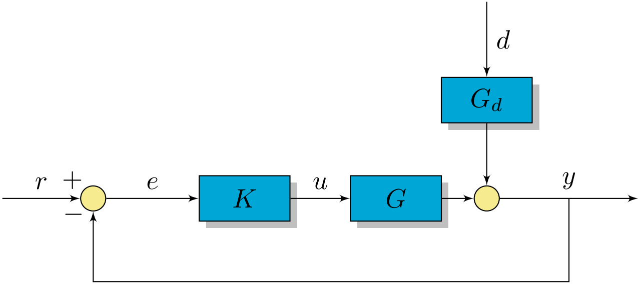
The control problem



- Manipulate u to counteract the effect of the disturbance d (Regulator problem)
- Manipulate u to track a reference r (Servo problem)

Minimize control error (e)!!

The control problem (cont'd)



Design K with a priori information of d and/or r, G, and G_d

What about uncertainties?? What about MIMO??

The control problem (cont'dd)

We define a class of models: $G_p = G + \Delta$ with $\Delta \leq 1$

- Nominal Stability (**NS**):
  G is stable
- Nominal Performance (**NP**):
  G satisfies certain performance bounds
- Robust Stability (**RS**):
  G_p is stable for all systems within the class
- Robust Performance (**RP**):
  G_p satisfies certain performance bounds for systems within the class

Transfer functions

We consider: $G(s) = \frac{\beta_{n_z} s^{n_z} + \ldots + \beta_1 s + \beta_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$

- Rational transfer function
- Linear Time Invariant (LTI)
- MIMO $\Rightarrow$ Matrix transfer function
  e.g. $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$
- If proper $\Rightarrow$ state space realization

Scaling

Why?

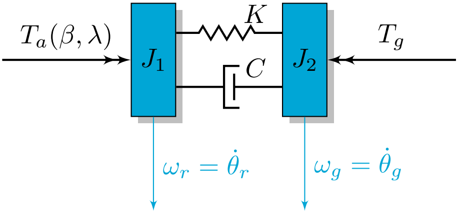
- For numerical stability
- For weighting purposes (MIMO)
- For norm bounded control

How?

- Determine max. d, u change (scaling $\frac{d}{d_{max}}$ and $\frac{u}{u_{max}}$)
- Same for e, r, or y (note same units so pick one)

Modeling: WT example, drive-train

Vestas V164-8MW (D=164m)



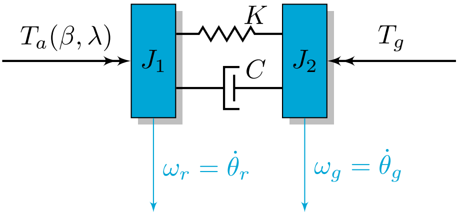
$$J_1 \ddot{\theta}_r = T_a(\beta, \lambda) + K(\theta_g - \theta_r) + C\left(\dot{\theta}_g - \dot{\theta}_r\right)$$

$$J_2 \ddot{\theta}_g = -T_g - K(\theta_g - \theta_r) - C\left(\dot{\theta}_g - \dot{\theta}_r\right)$$

Modeling: WT example, drive-train (cont'd)

The aerodynamic torque:
$T_a = \frac{1}{2}\rho R^3 C_Q(\beta,\lambda)V^2$

with:
$\lambda = \frac{\omega_r R}{V}$ - TSR
β -pitch angle
T_g -generator torque
C_Q - Torque coefficient



Taylor expansion:
$T_a(\beta,\lambda) \approx \frac{\partial T_a(\beta_o,\lambda_0)}{\partial \omega_r}\dot{\theta}_r$
Control ω_r or ω_g?

$$J_1\ddot{\theta}_r = T_a(\beta,\lambda) + K(\theta_g - \theta_r) + C\left(\dot{\theta}_g - \dot{\theta}_r\right)$$
$$J_2\ddot{\theta}_g = -T_g - K(\theta_g - \theta_r) - C\left(\dot{\theta}_g - \dot{\theta}_r\right)$$

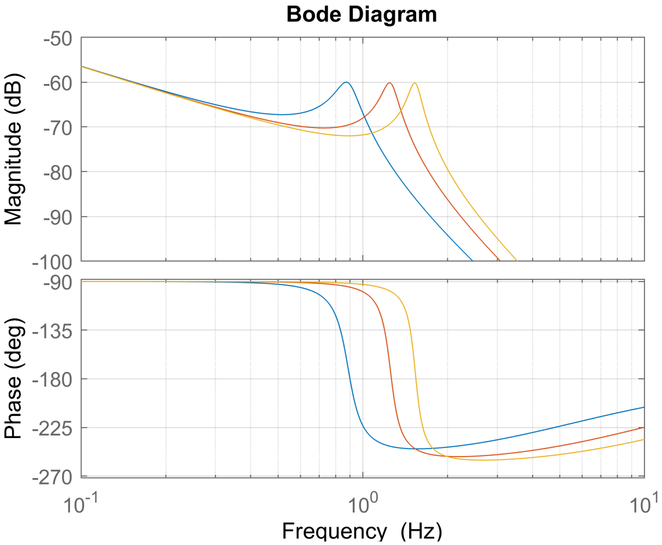
Modeling: WT example, drive-train (cont'dd)

$$\frac{\omega_r}{T_g}$$

Workpoint 1

Workpoint 2

Workpoint 3



Modeling: WT example, drive-train (cont'ddd)

$$\frac{\omega_g}{T_g}$$

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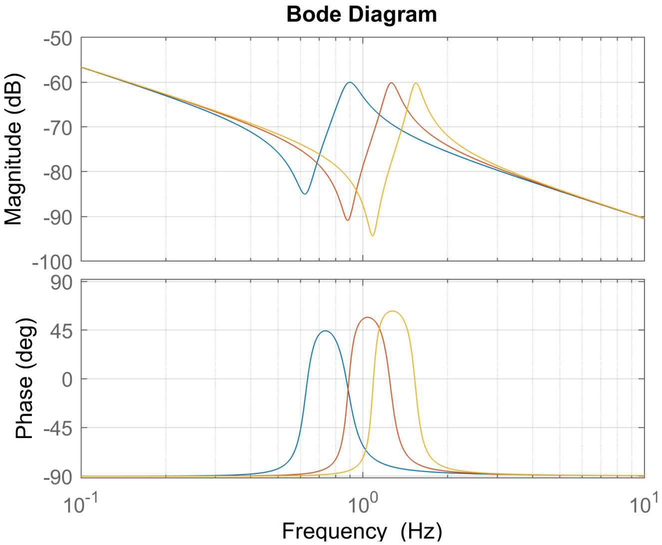


Modeling: WT example, drive-train (cont'ddd)

$$\frac{\omega_g}{T_g}$$

Workpoint 1

Workpoint 2

Workpoint 3

1. Actuator/Sensor placement plays a role
2. Linearization/Working point will play a role
3. Uncertainty in parameters will play a role

Frequency Response Function

FRF: $G(s) \Rightarrow G(j\omega)$

- A systems response to sinusoidal frequencies
- Frequency content of a deterministic signal (Fourier)
- Frequency distribution of a stochastic signal (PSD)

Frequency Response Function (cont'd)

- $u = u_0 \sin(\omega t + \alpha)$ (persistent)
- $y = y_0 \sin(\omega t + \beta)$
- $\|G(j\omega)\| = \frac{y_0}{u_0}$
- $\angle G(j\omega) = \beta - \alpha$

Example:
$G(s) = \frac{ke^{-\theta s}}{\tau s+1}, \qquad k = 5, \quad \theta = 2, \tau = 10;$

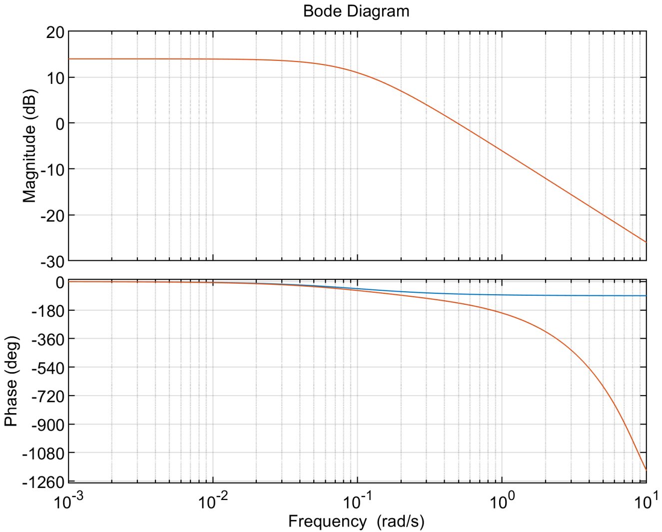
$$\|G(j\omega)\| = \frac{k}{\sqrt{(\tau\omega)^2+1}} = \frac{5}{\sqrt{(10\omega)^2+1}}$$

$$\angle G(j\omega) = -\omega\theta - \arctan(\tau\omega) = -\omega\theta - \arctan(10\omega)$$

Frequency Response Function (cont'dd)

$$G(s) = \frac{5}{10s+1}$$

$$G(s) = \frac{5e^{-2s}}{10s+1}$$



Minimum Phase System (MPS)

MPS: There exists an unique relation between the gain and phase of a frequency response function (for stable systems)

Problems: Right Half Plane (RHP) zeros, and time delays

Bode Gain-Phase relationship:

$$\angle G(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \underbrace{\frac{d\ln|G(j\omega)|}{d\ln\omega}}_{N(\omega)} \ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| \frac{d\omega}{\omega}$$

$$\approx \frac{\pi}{2} N(\omega_0)[rad/s] = 90^o N(\omega_0)$$

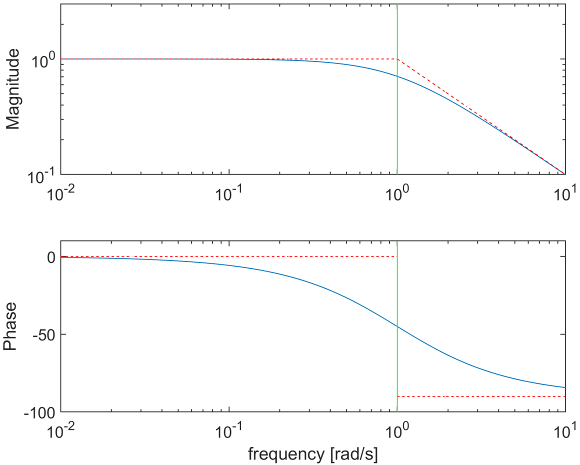
$$\text{Since :} \quad \left(\int_{-\infty}^{\infty} \ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| \frac{d\omega}{\omega} = \frac{\pi^2}{2}\right)$$

Asymptotes: single pole

$G(s) = \frac{1}{\tau s+1}$
(example: $\tau = 1$)

$\omega \ll \frac{1}{\tau}$
$N(\omega) = 0$
$\angle G(s) = 0$

$\omega \gg \frac{1}{\tau}$
$N(\omega) = -1$
$\angle G(s) = -90^o$

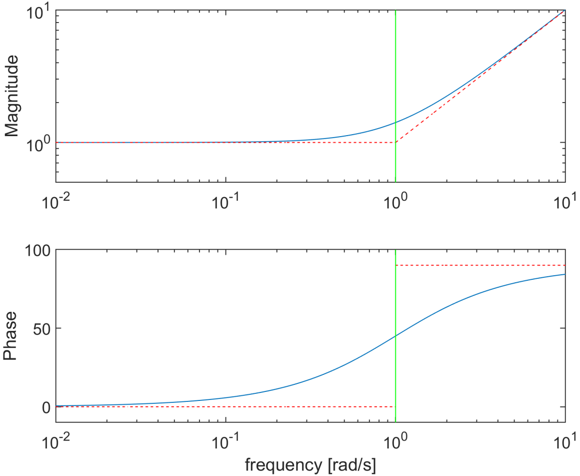


Asymptotes: single zero

$G(s) = \tau s + 1$
(example: $\tau = 1$)

$\omega \ll \frac{1}{\tau}$
$N(\omega) = 0$
$\angle G(s) = 0$

$\omega \gg \frac{1}{\tau}$
$N(\omega) = 1$
$\angle G(s) = 90^o$

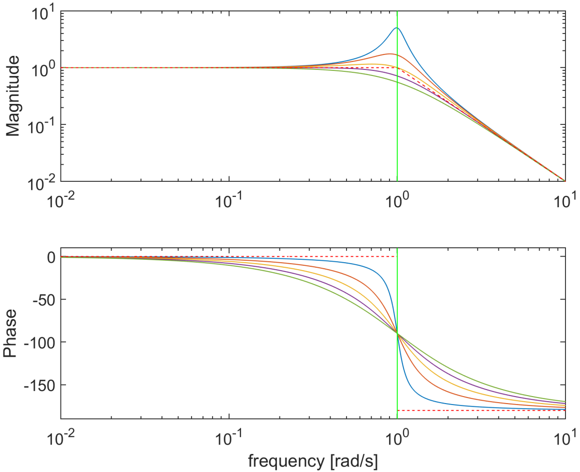


Asymptotes: double pole

$G(s) = \frac{\omega^2}{s^2+2\omega\beta s+\omega^2}$
(example: $\omega = 1$)

$\omega \ll \frac{1}{\tau}$
$N(\omega) = 0$
$\angle G(s) = 0$

$\omega \gg \frac{1}{\tau}$
$N(\omega) = -2$
$\angle G(s) = -180^o$

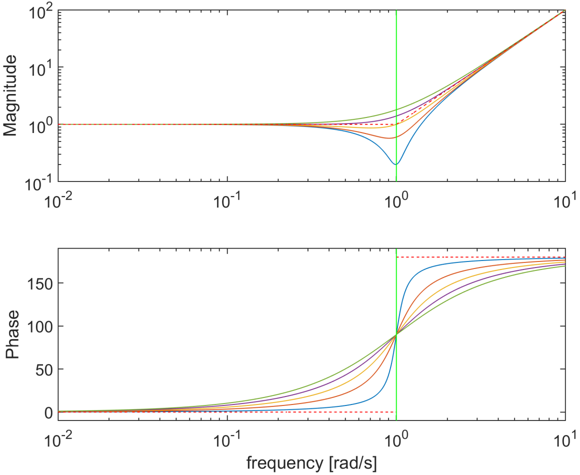


Asymptotes: double zero

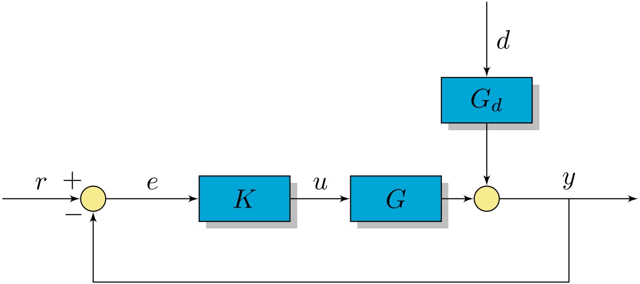
$G(s) =$
$s^2 + 2\omega\beta s + \omega^2$
(example: $\omega = 1$)

$\omega \ll \frac{1}{\tau}$
$N(\omega) = 0$
$\angle G(s) = 0$

$\omega \gg \frac{1}{\tau}$
$N(\omega) = 2$
$\angle G(s) = 180^o$



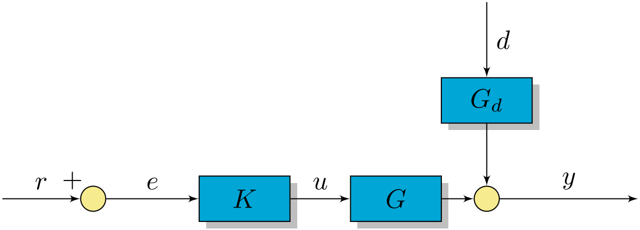
Feedback



- $y = Gu + G_d d$
- $y = GK(r - y) + G_d d$
- $y = \underbrace{(I + GK)^{-1}GK}_{T}\, r + \underbrace{(I + GK)^{-1}}_{S}\, G_d d$

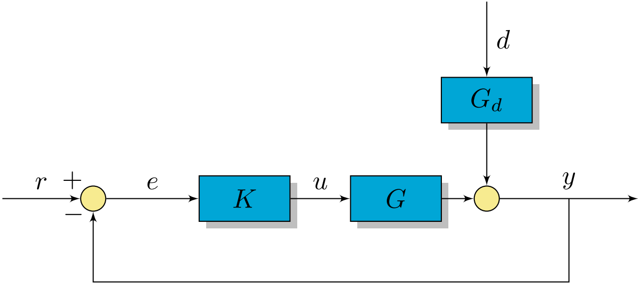
T=complementary sensitivity
S=sensitivity with $T + S = I$

Why Feedback



- Feedforward: $u = G^{-1}r - G^{-1}G_d d$
- G^{-1} can be hard to compute (RHP-zeros, delays)
- Signal uncertainty
- Model uncertainty Δ
- Unstable plant

High Gain Feedback



- $L = GK$ = loop transfer
- Consider $L \gg I$
- $S \approx 0$
- Since: $T + S = I$ we know $T \approx I$
- $u = KS(r - G_d d)$ and we know $KS = G^{-1}T$

High gain feedback: $u = G^{-1}r - G^{-1}G_d d$ (Similar as model inversion)

Stability

Problem with high gain feedback: Stability

How to check stability?

- Stable iff all the poles are in the LHP
- Nyquist: equating the number of encirclements and the number of RHP-poles
- For OL stable minimum phase systems: Bode's stability condition
  Stability $\Leftrightarrow |L(j\omega_{180})| < 1$
  ω_{180} is corresponding to $\angle L(j\omega_{180}) = -180^o$

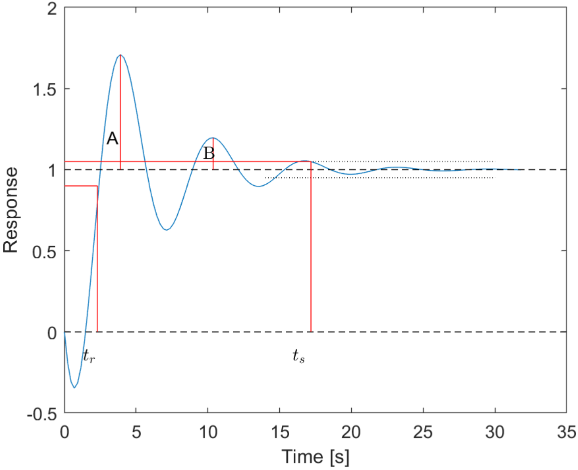
Performance: Time domain

Rise time (t_r)
90% steady state

Overshoot (A)
Max. peak

Damping ratio
(B/A)

Setting time
(within 5%)

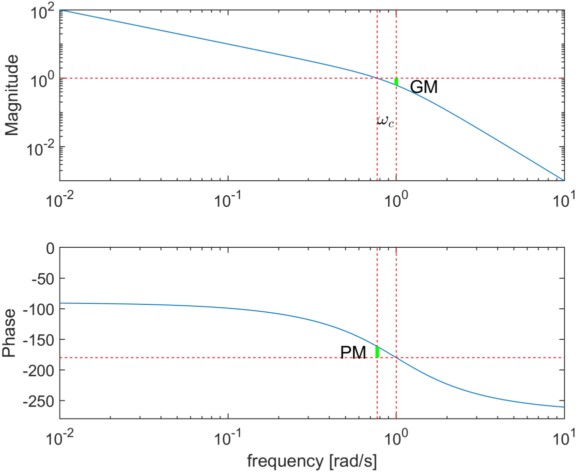


Performance: Frequency domain

Gain Margin (GM)

Phase Margin (PM)

Max. $\theta_{td} = \frac{\mathrm{PM}}{\omega_c}$

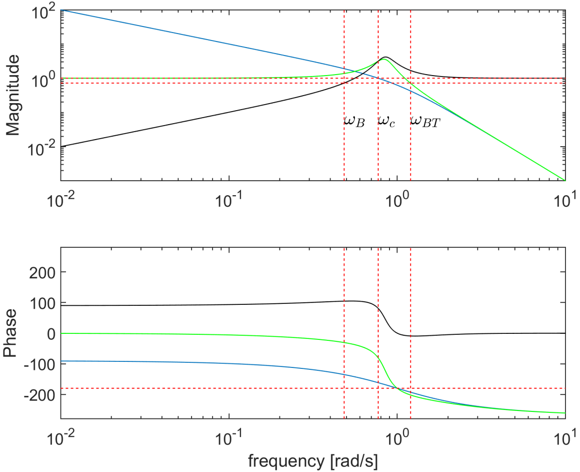


Performance: Frequency domain

L (ω_C 0dB cross-over)

T (ω_{BT} -3dB bandwidth T)

S (ω_B -3dB bandwidth S)



Performance: Frequency domain

Bandwidth: The closed-loop bandwidth, ω_B, is the frequency where $|S(j\omega)|$ first crosses $\frac{1}{\sqrt{2}} = 0.707(\approx -3dB)$ from below

Performance: Frequency domain

Typically: PM$> 30^o$ and GM> 2

Define: $M_S = \max_\omega |S(j\omega)|$ and $M_T = \max_\omega |T(j\omega)|$

Important for later: $M_S = ||S||_\infty$ and $M_T = ||T||_\infty$

Since: $T + S = I$ and $||S| - |T|| \leq |S + T| = 1$

So: M_S and M_T differ at most by one

Performance: Frequency domain $\Leftrightarrow$ Time domain

Given $M_S \Rightarrow \mathrm{GM} \geq \frac{\mathrm{M_S}}{\mathrm{M_S}-1}$ and $PM \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S}$

$M_S = 2$ we are guaranteed to have $\mathrm{GM} \geq 2$ and $\mathrm{PM} \geq 29^{\mathrm{o}}$

Given $M_T \Rightarrow \mathrm{GM} \geq 1 + \frac{1}{\mathrm{M_T}}$ and $PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T}$

$M_T = 2$ we are guaranteed to have $\mathrm{GM} \geq 1.5$ and $\mathrm{PM} \geq 29^{\mathrm{o}}$

Controller design

Approaches:

1. Shaping of transfer functions
   1. Shaping $L(j\omega)$ (Manual loop shaping B.Sc.)
   2. Shaping S, T, and KS
2. Signal based approach (LQG, $\mathcal{H}_2$, $\mathcal{H}_\infty$)
3. Direct numerical optimization (MPC)

Loop Shaping

Key Idea: Shape $L(s)$ using Nyquist/Bode for closed-loop performance and stability

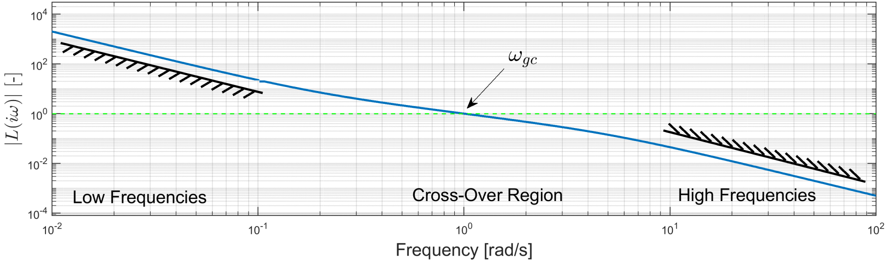
How (I): $C(s) = \frac{L_d(s)}{P(s)}$ where L_d is desired loop transfer function

How (II): place poles and zeros using Bode/Nyquist to get desired loop transfer function

Three key areas:

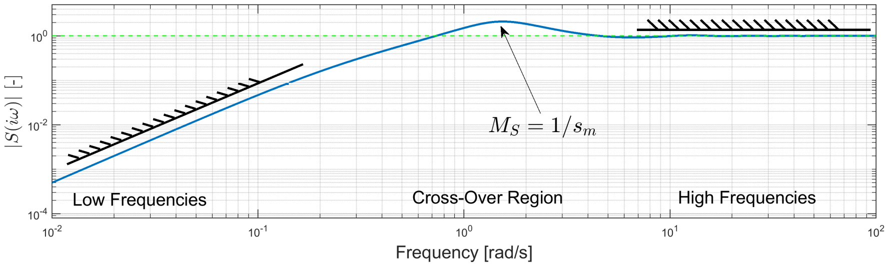
1. Low Frequencies, Load disturbance attenuation (High gain)
2. Cross-Over region, Robustness (Take care of margins)
3. High Frequencies, High frequency measurement noise (Low gain)

Loop Shaping: Loop Transfer Function



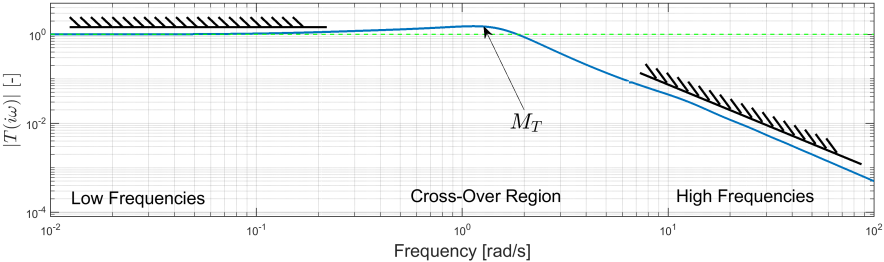
- Low Frequencies, High gain
- High Frequencies, Low gain
- Cross-Over region, Stability Margins, (PM=30^o, 45^o, 60^o equals Slope $-5/3$, $-3/2$, $-4/3$)

Loop Shaping: Sensitivity



- Low Frequencies, Low gain, Disturbance rejection
- High Frequencies, Gain=1, No difference between OL or CL
- Cross-Over region, Inevitable Peak M_S, OL will outperform CL at some freq.

Loop Shaping: Compl. Sensitivity



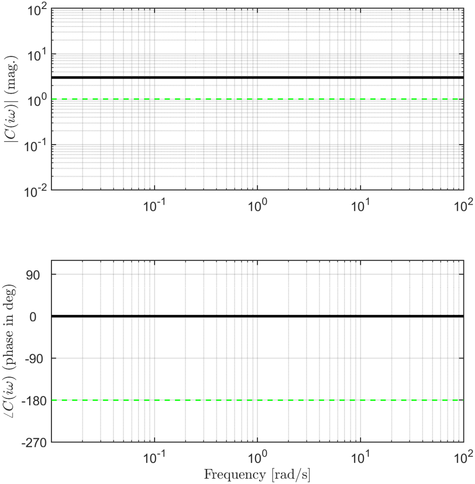
- Low Frequencies, Gain=1, Tracking of the reference
- High Frequencies, Low gain, No tracking of the reference
- Cross-Over region, Inevitable Peak M_T, Remember the equality $S + T = I$, Peak in S results in peak in T

Control elements

We have e.g.:

- P-action
- I-action
- D-action
- Low pass filter
- Notch
- PI
- PID
- Lead-lag

$C(s) = K_p$



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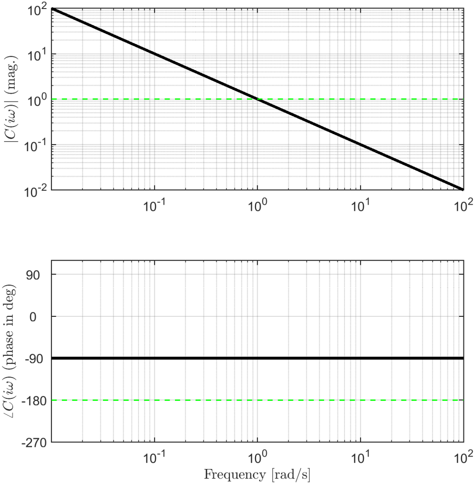


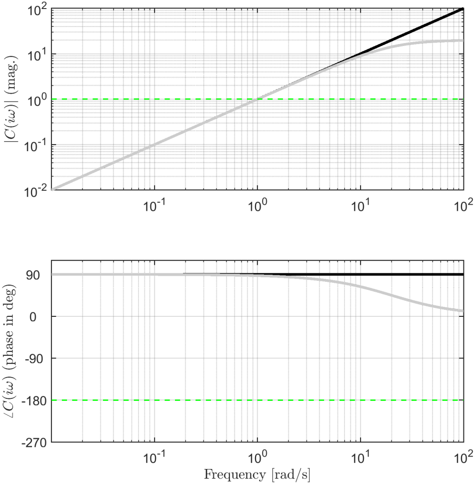
$$C(s) = \frac{K_i}{s} \quad \text{or} \quad \left(\frac{T_i}{s}\right)$$

Control elements

We have e.g.:

- P-action
- I-action
- D-action
- Low pass filter
- Notch
- PI
- PID
- Lead-lag

$C(s) = K_d s$ or $\left(\frac{K_d s}{T_f s + 1}\right)$



Control elements

We have e.g.:

- P-action
- I-action
- D-action
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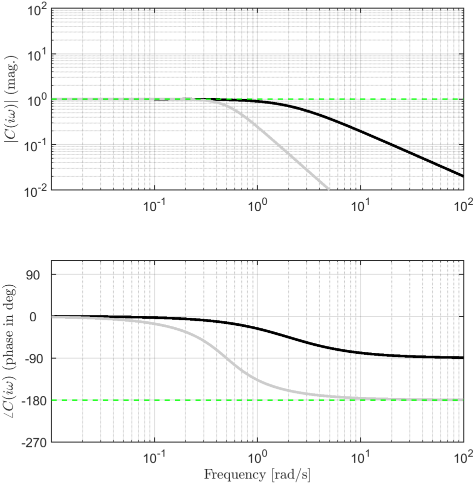
- PI
- PID
- Lead-lag

$C(s) = \quad \frac{1}{\tau s+1}$ or $\frac{\omega^2}{s^2+2\omega\beta s+\omega^2}$

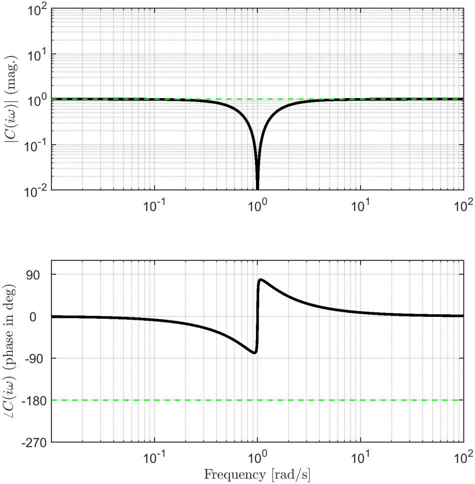


Control elements

We have e.g.:

- P-action
- I-action
- D-action
- Low pass filter
- Notch
- PI
- PID
- Lead-lag

$$C(s) = \frac{s^2+2\omega\beta_1 s+\omega^2}{s^2+2\omega\beta_2 s+\omega^2}$$



Control elements

We have e.g.:

- P-action
- I-action
- D-action
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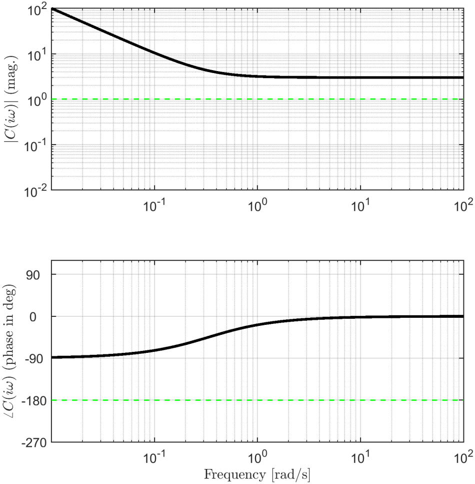
$$C(s) = \quad K_p\left(1 + \frac{T_I}{s}\right)$$



Control elements

We have e.g.:

- P-action
- I-action
- D-action
- Low pass filter
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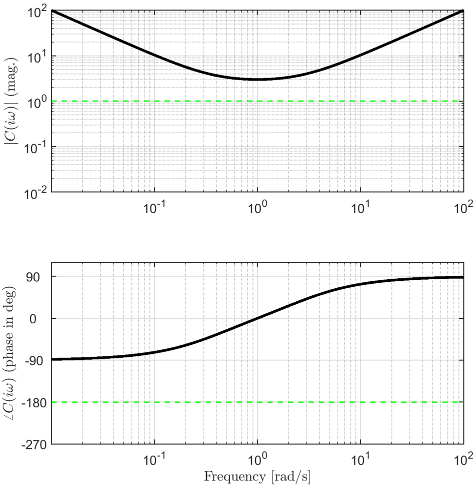
- Lead-lag

$$C(s) = \quad K_p + \frac{K_i}{s} + K_d s$$



Control elements

We have e.g.:

- P-action
- I-action
- D-action
- Low pass filter
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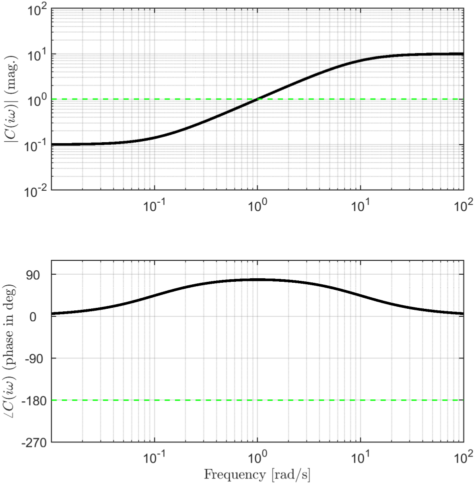
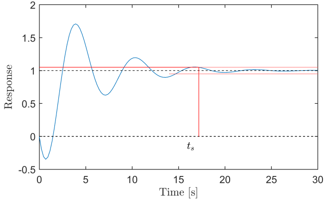
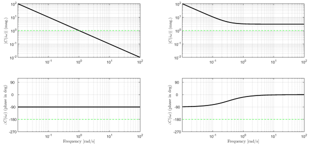
$$C(s) = \frac{\tau_1 s+1}{\tau_2 s+1}$$

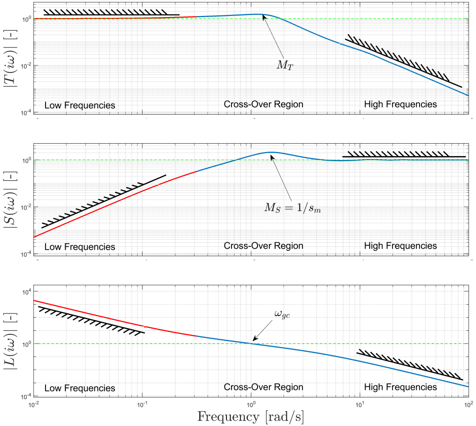


Design process (Tracking): Low Frequencies



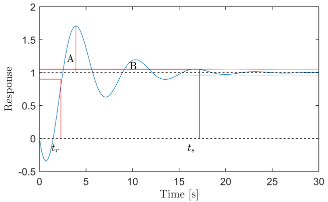
$e.g.$ I or PI



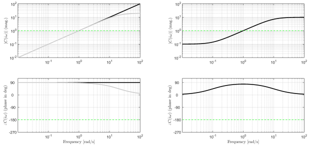


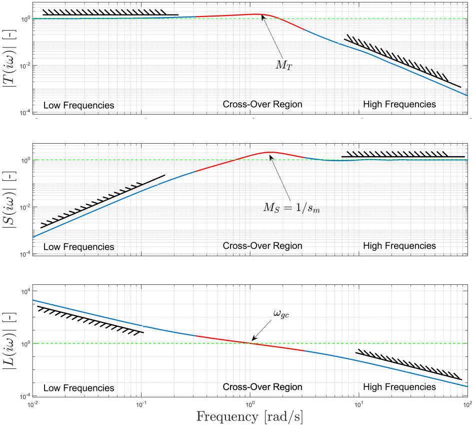
Reduce steady state errors by increasing the loop gain

Design process (Tracking): Cross-Over



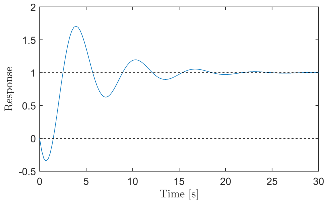
$e.g.$ D or Lead-Lag



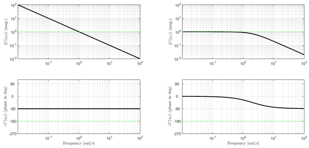


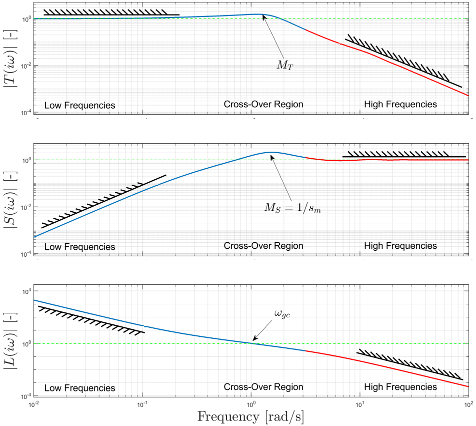
Increase the phase to satisfy stability margins

Design process (Tracking): High frequencies



$e.g.$ I or Low-Pass



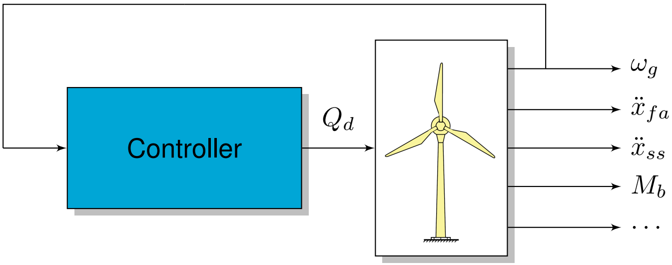


Reduce the gain to mitigate the effect of measurement noise

Design process (Tracking)

- Determine gain cross-over frequency ($|L(i\omega_{gc})|$ crosses 1)
- Add low pass filter to suppress the effect of high frequent dynamics (integrator, low-pass, notch)
- Add phase (D-action, lead-lag)
- Set cross-over frequency (P-action)
- Reference tracking (I-action)

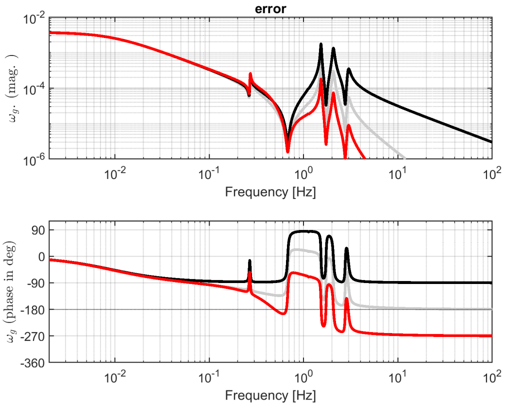
Torque control



Torque control: PI+

Reference tracking:

- $L(s)$
- Add LP $(1^{st}, 2^{nd})$



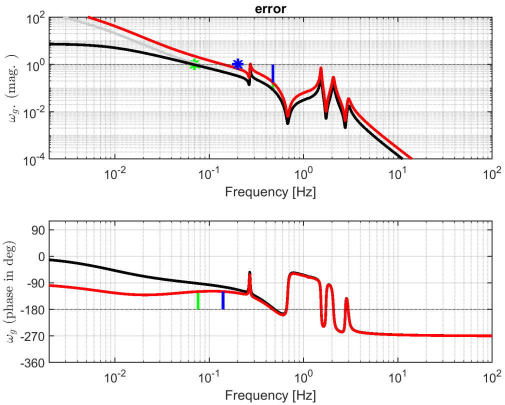
L(s):

- $P(s)LP(s)$

Torque control: PI+

Reference tracking:

- $L(s)$
- Add LP $(1^{st}, 2^{nd})$
- Tune Gain
- Add I-action
- Add more gain



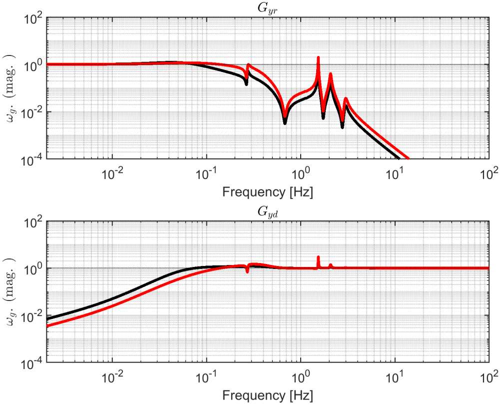
L(s):

$$-2e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$
$$-5e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$

Torque control: PI+

Reference tracking:

- $L(s)$
- Add LP $(1^{st}, 2^{nd})$
- Tune Gain
- Add I-action
- Add more gain
- $\frac{L(s)}{1+L(s)}$



L(s):

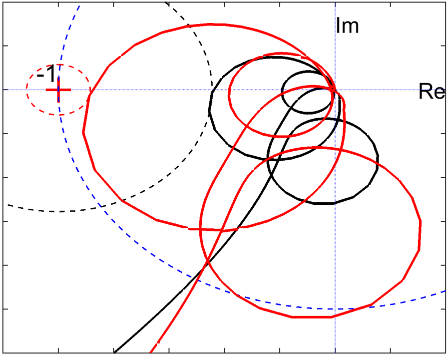
$-2e3\left(1 + \frac{0.25}{s}\right)P(s)LP(s)$

$-5e3\left(1 + \frac{0.25}{s}\right)P(s)LP(s)$

Torque control: PI+

Reference tracking:

- $L(s)$
- Add LP $(1^{st}, 2^{nd})$
- Tune Gain
- Add I-action
- Add more gain
- Nyquist



L(s):

$$\text{-}2e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$

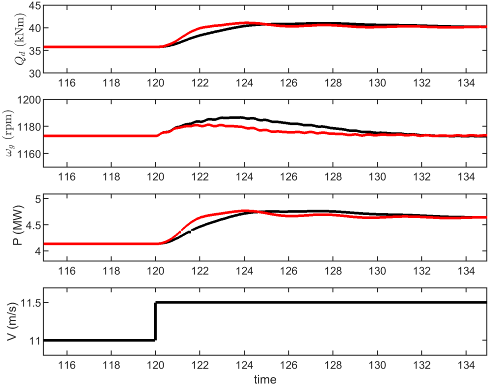
$$\text{-}5e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$

Torque control: PI+

Reference tracking:

- $L(s)$
- Add LP $({1^{st}}, {2^{nd}})$
- Tune Gain
- Add I-action
- Add more gain

- Time Domain



L(s):

$$\text{-}2e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$

$$\text{-}5e3\left(1+\frac{0.25}{s}\right)P(s)LP(s)$$

Shaping closed-loop transfer functions

We have seen a relation between GM/PM and M_S/M_T

$\mathcal{H}_\infty$ norm of a SCALAR and STABLE transfer function f(s) is simply the peak value of $|f(j\omega)|$ as a function of frequency, i.e.

$$||f(s)||_\infty \triangleq \max_\omega |f(j\omega)|$$

Other norms:

$$||f(s)||_p \triangleq \max_{p\to\infty} \left(\int_{-\infty}^{\infty} |f(j\omega)|^p d\omega\right)^{\frac{1}{p}}$$

Note: $\mathcal{H}$ represents a "Hardy Space" which is a space that contains

all the stable and proper transfer functions.

Sensitivity

Advantages:

1. Good performance indicator
2. Need only amplitude information

We can specify:

1. Minimum Bandwidth (ω_B)
2. Maximum tracking error at selected frequencies
3. Shape of S
4. Max peak of S

Weighted Sens. design: $|S(j\omega)| < 1/|w_P(j\omega)|, \forall\omega$ or $||w_P S||_\infty < 1$

Sensitivity

w_P sets a lower bound on bandwidth, doesn't allow us to specify roll-off.

Solution: bound other closed-loop transfer functions (Mixed sensitivity)

$$||N||_\infty = \max_\omega \overline{\sigma}(N(j\omega)) < 1; \qquad N = \begin{bmatrix} w_P S \\ w_T T \\ w_u K S \end{bmatrix}$$

$\mathcal{H}_\infty$ optimal controller is obtained by solving the problem

$$\min_K ||N(K)||_\infty$$

How to set-up synthesis? How to deal with MIMO?