Robust Control overview

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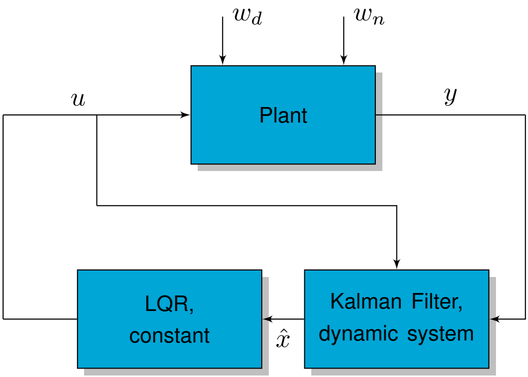
LQG control

Optimal control theory reached maturity in *the sixties* (aerospace, etc).

For everyday industrial problems: no accurate plant model available and white noise disturbances not always realistic.



LQG control (cont'd)

The problem: Given the covariances W (of w_d) and V (of w_n) and weighting matrices Q and R minimize

$$E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T\left[x^TQx+u^TRu\right]dt\right\}$$

LQG controller consists out of (separation principle):

1. Kalman Filter (solve one Riccati equation)
2. State feedback (solve one Riccati equation)

Trick required to include integral action

What about robustness margins?

Guaranteed Margins for LQG Regulators

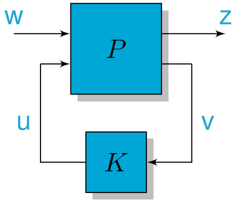
JOHN C. DOYLE

***Abstract*—There are none.**

$\mathcal{H}_2$ and $\mathcal{H}_\infty$ control

Due to robustness issues people looked at $\mathcal{H}_\infty$ optimization for robust control in *the eighties*.

$$P = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right]$$



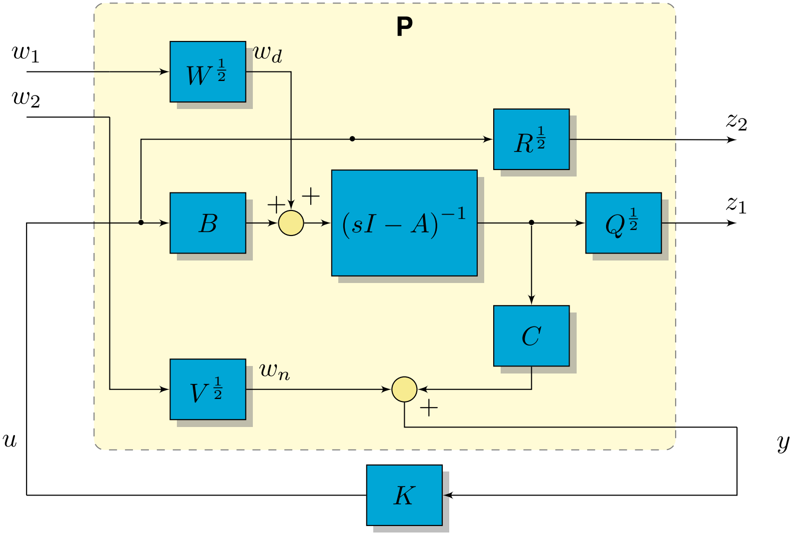
$$F_l(P,K) = P_{11} + P_{12}K\left(I - P_{22}K\right)^{-1}P_{21}$$

$\mathcal{H}_2$ control: $\sqrt{E\left\{\lim_{T\to\infty}\frac{1}{T}\int_0^T z^T z dt\right\}} = ||F_l(P,K)||_2$

$\mathcal{H}_\infty$ control: $\max_{w\neq 0}\frac{||z||_2}{||w||_2} = ||F_l(P,K)||_\infty$

+separation principle + we have to solve two riccati eqns

LQG: a special $\mathcal{H}_2$ controller ($||F_l(P,K)||_2$)



Trends

1. Formulate the $\mathcal{H}_\infty$ problem in the LMI framework

2. Research in the area of fixed-structure robust control

3. Research in the area of LPV control

$$\dot{x} = A(\mu)x + B(\mu)u + w_d$$
$$y = C(\mu)x + D(\mu)u + w_n$$

4. Identification for robust controller design

Purpose of the course

- Formulate control objectives in a mixed-sensitivity design
- Define stability and performance for MIMO LTI systems
- Construct a generalized plant for complex system interconnections
- Design MIMO controllers on the basis of the mixed-sensitivity
- Describe parametric and dynamic uncertainties
- Translate concrete controller synthesis problem into abstract framework of robust control
- Reproduce definition, properties and computation of the structured singular value
- Master application of structured singular value for robust stability and performance analysis
- Design robust controllers on the basis of the $\mathcal{H}_\infty$ control algorithm

The steps

1. Study the system (poles, zeros, dominant directions)
2. Explore decoupling possibilities
3. Define objectives of the controller and translate to open or closed loop properties
4. Design a nominal controller
5. Introduce uncertainty structures
6. Robustness analysis
7. If necessary design a robust controller

μ-The structured singular value

RS if $\det(I - M\Delta(j\omega)) \neq 0, \forall\omega, \forall\Delta, \overline{\sigma}(\Delta(j\omega)) \leq 1$

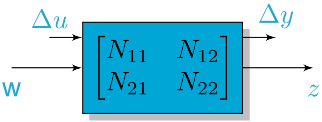
Note that this is a yes or no condition

Find the smallest k_m such that $\det(I - k_m M\Delta(j\omega)) = 0$

From the definition of μ we have $\mu = \frac{1}{k_m}$ and allowing only structured uncertainty

RS iff $\mu(M(j\omega)) < 1, \quad \forall\omega$

General conditions for analysis



NS: N internally stable

NP: $\overline{\sigma}(N_{22}) < 1 \quad \forall\omega$ (or $\mu_{\Delta_P}(N_{22}) < 1$) and **NS**

RS: $\mu_{\Delta}(N_{11}) < 1 \quad \forall\omega$ and **NS**

RP: $\mu_{\hat{\Delta}}(N) < 1 \quad \forall\omega,\ \hat{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix}$ and **NS**

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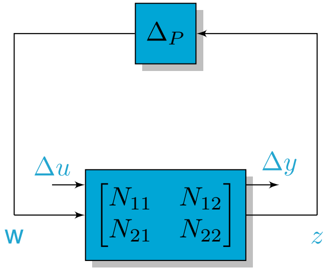

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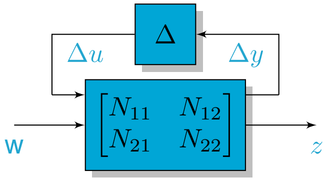

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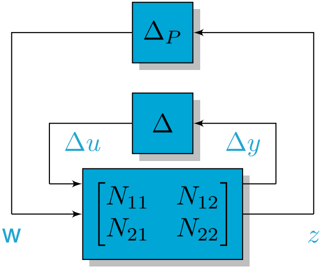

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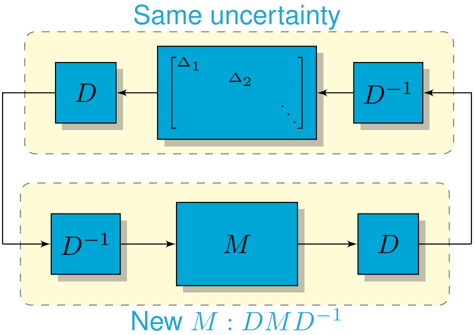
D-K iterations

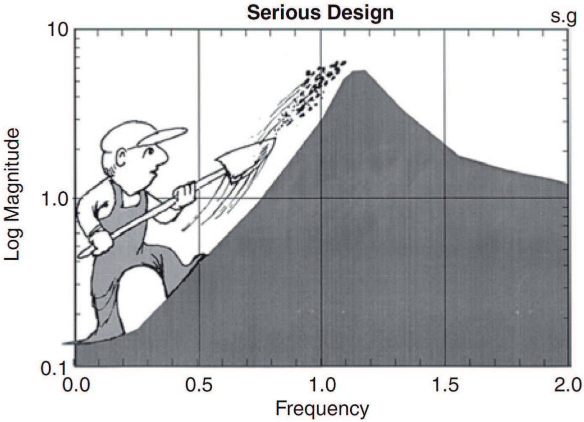


We have an upperbound: $\mu(N) \leq \min_{D\in\mathcal{D}} \overline{\sigma}(DND^{-1})$

Seek a controller that: $\min_K(\min_{D\in\mathcal{D}} ||DND^{-1}||_\infty)$

Alternate, between minimizing $||DND^{-1}||_\infty$ using D or K.

From manual loopshaping to modern tools



Pictures taken from: Respect the unstable, Gunter Stein

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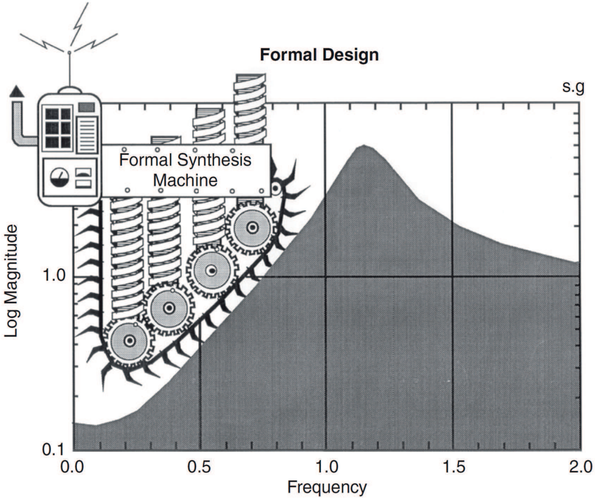

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