Tarea 2 Vide 1

=> Ejemplo 12,1 Control Systems

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \begin{cases} \text{Overshoot } OS\% \; 9,5\% \\ t_s = 0,74 \text{ seg} \end{cases}$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$x_1 = x_1$

$\dot{x}_3 \; .. \; \dddot{x}_1$

$x_2 = \dot{x}_1$

$$(s^3+5s^2+4s)X_1(s) = U(s)$$

$x_3 = \dot{x}_2 = \ddot{x}_1$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

$$\Rightarrow \boxed{\dot{x}_3 = -5x_3 - 4x_2 + u} \quad (1)$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0) X_1(s)$$

$$= (0s^2 + 20s + 100) X_1(s)$$

Escaneado con CamScanner

$Y(s) = (20s + 100)\, X_1(s)$

$= 20\dot{X}_1 + 100 X_1 \Rightarrow \boxed{Y = 20x_2 + 100x_1}$ ⑥

$\dot{X}_1 = x_2$, $X_1 = x_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} u \qquad Y = [100 \;\; 20 \;\; 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Overshut $= e^{-(\eta\pi/\sqrt{1-\eta^2})} \times 100$

$\ln(0{,}095) = \ln\left(e^{-(\eta\pi/\sqrt{1-\eta^2})}\right)$

$-2{,}3539 = \dfrac{-\eta\pi}{\sqrt{1-\eta^2}}$

$-(2{,}3539(\sqrt{1-\eta^2}))^2 = (-\eta\pi)^2$

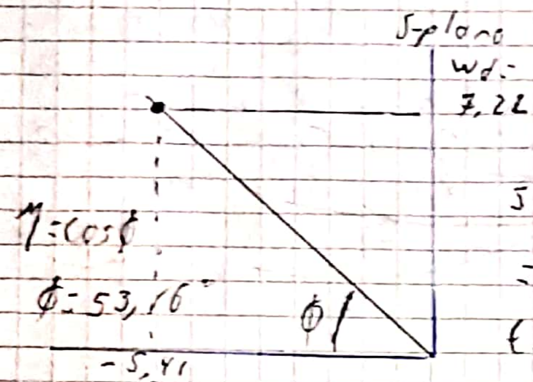
$5{,}5407(1-\eta^2) = \eta^2\pi^2$

$5{,}5407 - 5{,}5407\eta^2 = \eta^2\pi^2$

$5{,}5407 = \eta^2\pi^2 + 5{,}5407\eta$

$\eta^2 = \dfrac{5{,}5407}{\pi^2 + 5{,}5407}$

$\boxed{\eta = 0{,}5996}$



$s = \sigma \pm j\omega d$

$= \eta\omega_n$

$\phi = \cos^{-1}(0{,}5996) = 53{,}16°$

$t_s = 0{,}74$ $\quad t_s = \dfrac{4}{\sigma}$ $\quad 0{,}74 = \dfrac{4}{\sigma}$

$\boxed{\sigma = 5{,}41}$

$\sigma = \eta\omega_n$

$5{,}46 = 0{,}5996\,\omega_n$

$\boxed{\omega_n = 9{,}02 \text{ rad/s}}$

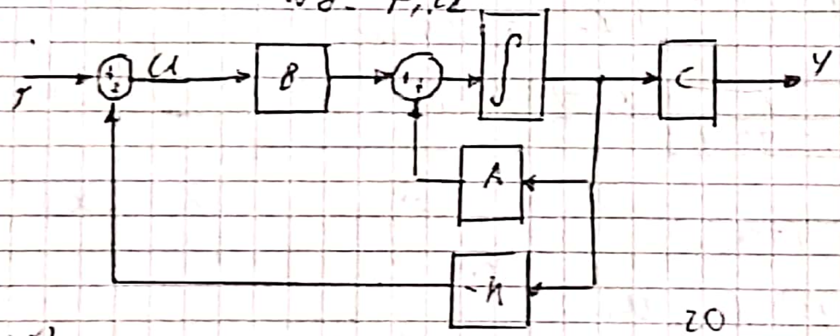
$\tan\phi = \dfrac{\omega d}{5{,}41}$

$\tan(53{,}16)5{,}41 = \omega d$

$\omega d = 7{,}22$

$\dot{X} = AX + BX$

$Y = C\alpha$

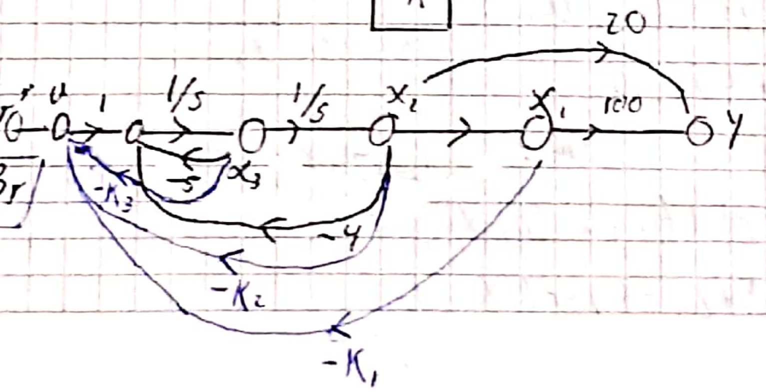


$\dot{\alpha} = Ax + Bu$

$= Ax + B(-kx + r)$

$= A\alpha - Bkx + Br$

$\boxed{\dot{X} = (A - Bk)x + Br}$



$\dot{x}_3 = -4x_2 - 5x_3 + u$

$\dot{x}_3 = -4x_2 - 5x_3 + (-k_3x_3 - k_2x_2 - k_1x_1) + r$

$\dot{x}_3 = -4x_2 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + r$

$\dot{x}_3 = -k_1x_1 - (4+k_2)x_2 - (5+k_3)x_3 + r$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\boxed{\det(sI - (A - BK)) = s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = 0}$$

$(s+5,4-j7,2)(s+5,4+j7,2)(s+5,1) = T$

$T = s^3 + 15,9s^2 + 136,22s + 413,83 = 0$ (5)

$s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = s^3 + 15,9s^2 + 136,22s + 413,83$

$(5+k_3)s^2 = 15,9s^2 \qquad (4+k_2)s = 136,22s \qquad \boxed{k_1 = 413,83}$

$5+k_3 = 15,9 \qquad\qquad 4+k_2 = 136,22$

$\boxed{k_3 = 10,9} \qquad\qquad \boxed{k_2 = 132,22}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,22 & -15,9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \qquad y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T(s) = \frac{20s + 100}{s^3 + 15,9s^2 + 136,22s + 413,8}$$ ✓