The structure and function of complex network

Newman, M. E. (2003). The structure and function of complex networks. SIAM review, 45(2), 167-256.

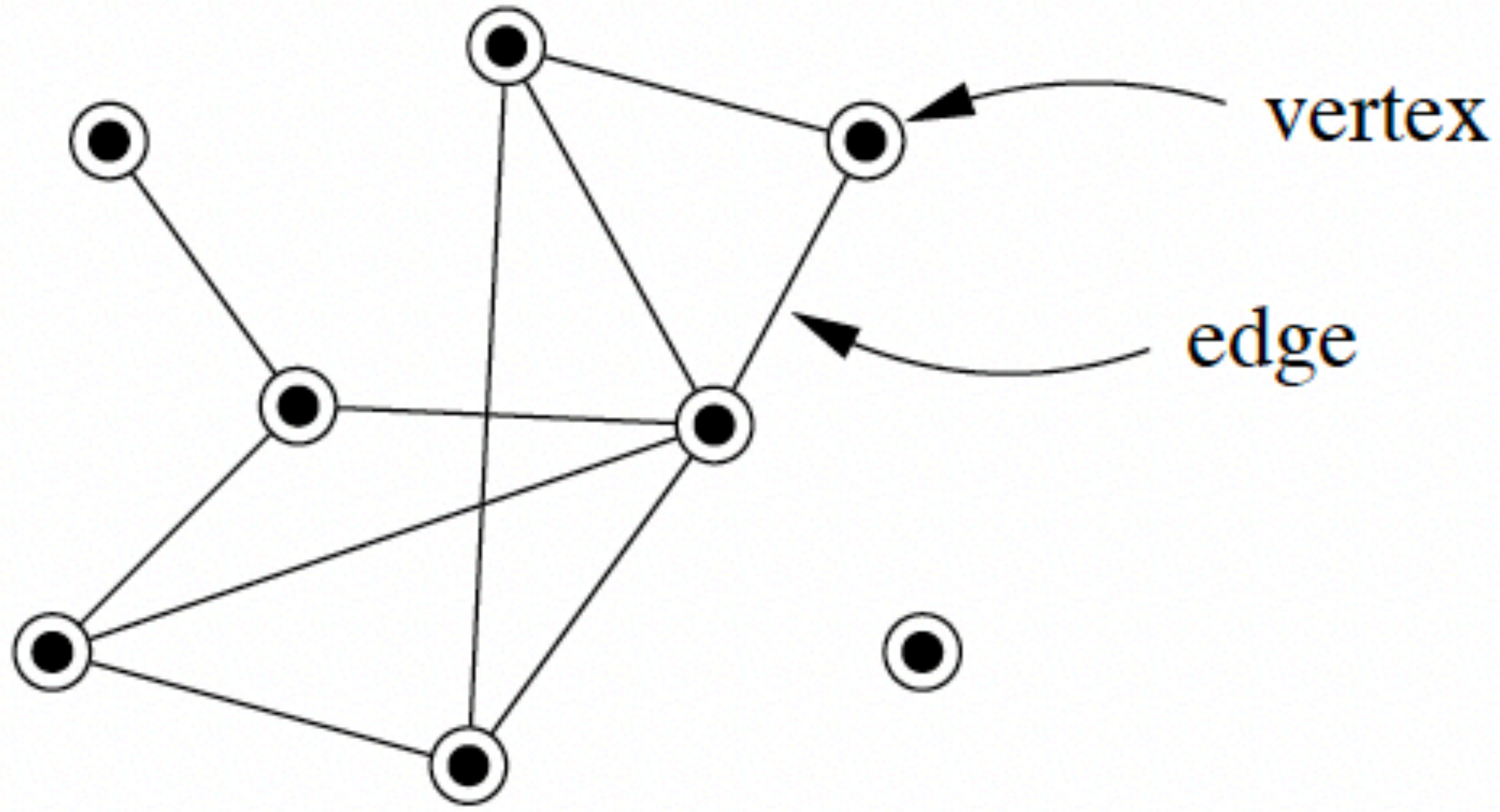
Yujin Kim 01142023

The body of theory

- This paper reviews work on the structure and function of networked systems.
- Primary focus is that it aims:
  - (1) to find appropriate ways to measure the properties that characterize the network through statistical approaches, e.g., path lengths and degree distributions,
  - (2) to create models of networks that can help us to understand the meaning of these properties; e.g., how they came to be as they are and how they interact with one another,
  - and (3) to predict what the behavior of networked systems will be on the basis of measured structures etc.

What is network?

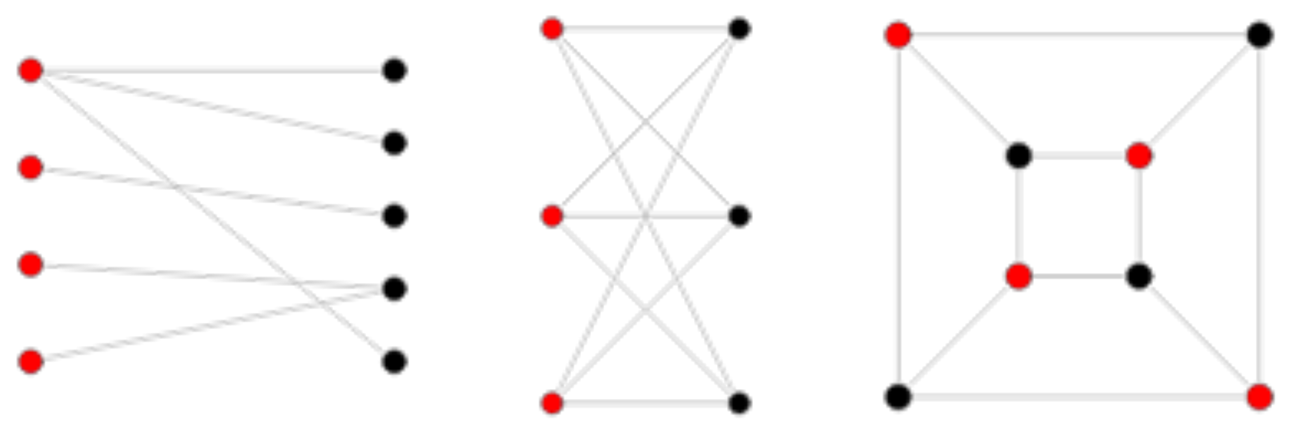
- Network = graph; a graph is a set of vertices (nodes) joined by a set of edges (lines or arrows)



- A small example network with eight vertices and ten edges

Types of networks

- digraphs (directed graphs); graphs composed of directed edges; e.g., email message networks,
- hyperedges: edges that join more than two vertices together
- bipartite graphs; graphs that contain vertices of two distinct types, with edges running only between unlike types



- Graphs may evolve over time, with vertices or edges appearing aor disappearing, or values defined on those vertices or edges changing.

Networks in the real world

A. Social networks

- A social network; a set of people or groups of people with some pattern of contacts or interactions between them.
- “small-world” experiments of Milgram;
    - probed the distribution of path lengths in an acquaintance network by asking participants to pass a letter to one of their first-name acquaintances to deliver it to an assigned target individual.
    - About 1/4 reached the target and passed on average through 6 people

Networks in the real world

B. Information networks (knowledge networks)

- e.g., the network of citations between academic papers (acyclic); The World Wide Web (cyclic; there is no natural ordering of sites and no constraints that prevent the appearance of closed loops)

- A preference network; a network with two kinds of vertices representing individuals and the objects of their preference such as books or movies, with an edge connecting each individual to the books or movies they like (e.g., bipartite information network)

    - *Collaborative filtering* algorithms and *recommender systems*; for predicting new likes or dislikes based on comparison of individuals' preferences.

Networks in the real world

C. Technological networks

- Man-made networks designed for distribution of commodity or resources such as electricity or information; e.g., electrical power grid
- The Internet (the network of physical connections between computers)

D. Biological networks

- e.g., metabolic pathways; a representation of metabolic substrates and products with directed edges joining them
- e.g., genetic regulatory network

Properties of Networks

A. The small-world effect

- Milgram in the 1960s; most pairs of vertices in most networks seem to be connected by a short path through the network

- Define l to be the mean geodesic (i.e., shortest) distance between vertex pairs in a network: where d_ij is the geodesic distance from vertex I to vertex j;

$$\ell = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij},$$

- e.g., information spread; aka Bacon numbers;

Properties of Networks

B. Transivity or clustering

- Transivity means the presence of a heightened number of triangles in the network — sets of three vertices each of which is connected to each of the others;

- A clustering coefficient C:

$$C = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}},$$

- The clustering coefficient measures the density of triangles in a network.

- Important in directed graphs where the two edges in question can join in opposite directions.

Properties of Networks

C. Degree distribution

- The degree of a vertex in a network is the number of edges connected to that vertex;
- Define p_k to be the fraction of vertices in the network that have degree k; a plot of p_k for any given network can be formed by making a histogram of the degrees of vertices;
- The histogram is the degree distribution for the network

Properties of Networks

D. Network resilience

- The property of resilience of networks to the removal of their vertices; networks vary in their level of resilience to such vertex removal

Properties of Networks

E. Mixing patterns

- Which vertices pair up with which others;
- In social networks, selective linking is called *assortative mixing* or *homophily*; e.g., social networks by race, age, or income

Rest of the review

- Random graphs
- Exponential random graphs and Markov graphs
- The small-world model
- Models of network growth
- Processes taking place on network