

minimum background test

1. Multivariate calculus

What is the partial derivative of y with respect to x ?
 $y = x \sin(z) e^{-x}$

2. Vectors and matrices

Consider the matrix \mathbf{X} and the vector \mathbf{y} below

$$\mathbf{X} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

What is the product \mathbf{Xy} ?

Is \mathbf{X} invertible? If so, give the inverse, if not explain why not.

What is the rank of \mathbf{X} ?

3. Probability and statistics

Consider a sample of data S obtained by flipping a coin x , where 0 denotes the coin turned up heads, and 1 denotes that it turned up tails.

$$S = \{1, 1, 0, 1, 0\}$$

What is the sample mean for this data ?

What is the sample variance ?

What is the probability of observing this data assuming that a coin with an equal probability of heads and tails was used (i.e., by the probability distribution $p(x=1)=0.5$, $p(x=0)=0.5$).

Note the probability of this data sample would be greater if the value of $p(x=1)$ was not 0.5, but some other value. What is the value that maximizes the probability of sample S ? [optional: can you prove your answer is correct?]

Given the following joint distribution between x and y , what is $P(x=T | y=b)$?

$P(x,y)$	y	a	b	c
x				
T		0.2	0.1	0.2
F		0.05	0.15	0.3

Modest Background Test

1 Probability and Random Variables

Probability

State true or false. Here A^c denotes complement of the event A .

- (a) $P(A \cup B) = P(A \cap (B \cap A^c))$
- (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- (c) $P(A) = P(A \cap B) + P(A^c \cap B)$
- (d) $P(A|B) = P(B|A)$
- (e) $P(A_1 \cap A_2 \cap A_3) = P(A_3|(A_2 \cap A_1))P(A_2|A_1)P(A_1)$

Discrete and Continuous Distributions

Match the distribution name to its formula.

Multivariate Gaussian	$p^x(1-p)^{1-x}$
Exponential	$\frac{1}{b-a}$ when $a \leq x \leq b$; 0 otherwise
Uniform	$\binom{n}{x}p^x(1-p)^{n-x}$
Bernoulli	$\lambda e^{-\lambda x}$ when $x \geq 0$; 0 otherwise
Binomial	$\frac{1}{\sqrt{(2\pi)^d \Sigma }} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^\top \Sigma^{-1}(\mathbf{x} - \mu)\right)$

Mean, Variance and Entropy

- (a) What is the mean, variance and entropy of a Bernoulli(p) random variable?
- (b) If the variance of a zero-mean random variable x is σ^2 , what is the variance of $2x$?
What about variance of $x + 2$?

Mutual and Conditional Independence

- (a) If X and Y are independent random variables, show that $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$.
- (b) Alice rolls a die and calls up Bob and Chad to tell them the outcome A . Due to disturbance in the phones, Bob and Chad think the roll was B and C , respectively. Is B independent of C ? Is B independent of C given A ?

Law of Large Numbers and Central Limit Theorem

Provide one line justifications.

- (a) If a die is rolled 6000 times, the number of times 3 shows up is close to 1000.