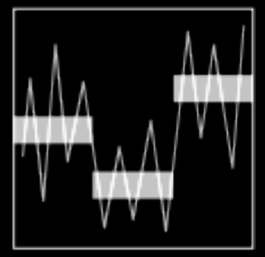
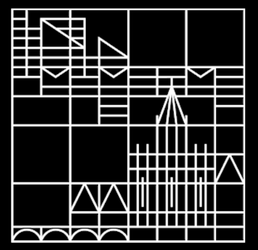
Universidad Autónoma de Madrid

SFB 1432



Universität Konstanz



Full Counting Statistics of Yu-Shiba-Rusinov states

13. March 2024

David Christian Ohnmacht, Wolfgang Belzig, Juan Carlos Cuevas

Full Counting Statistics of Yu-Shiba-Rusinov states

1) Full Counting Statistics (FCS) provides unprecedented insight into transport of junctions containing YSR states

2) FCS calculations are needed to fully understand measurements

13. March 2024

David Christian Ohnmacht, Wolfgang Belzig, Juan Carlos Cuevas

What is Full counting statistics (FCS)?

Theoretical framework computing (average) **current**, **shot noise** **(variance of current)** **and** **charge resolved currents**

L.S. Levitov and G.B. Lesovik, JETP Lett. **58**, 230 (1993)
Yu. V. Nazarov, Ann. Phys. (Berlin) **8,** SI-193 (1999)
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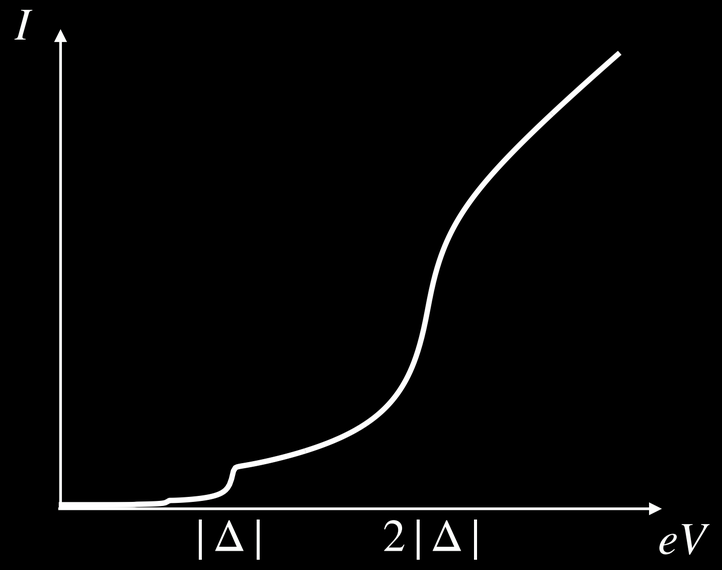
Example: Superconductor-Superconductor (SS) contact

J. C. Cuevas and W. Belzig, Phys. Rev. Lett. 91, 187001 (2003)



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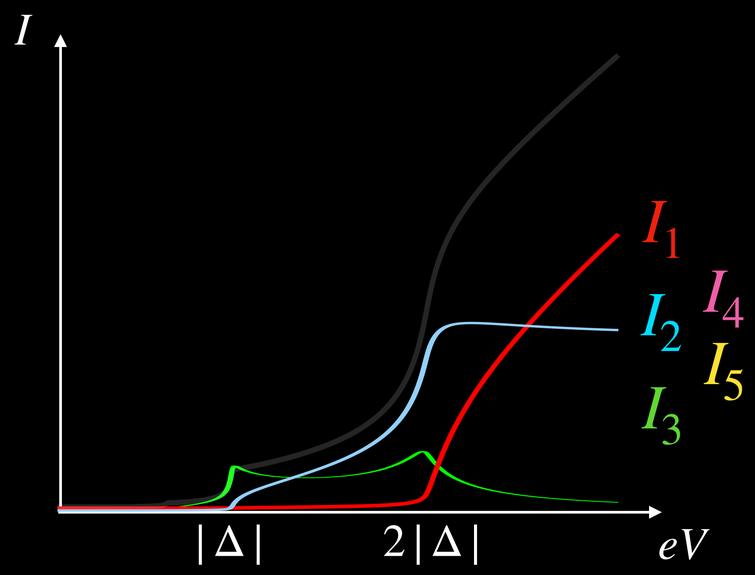
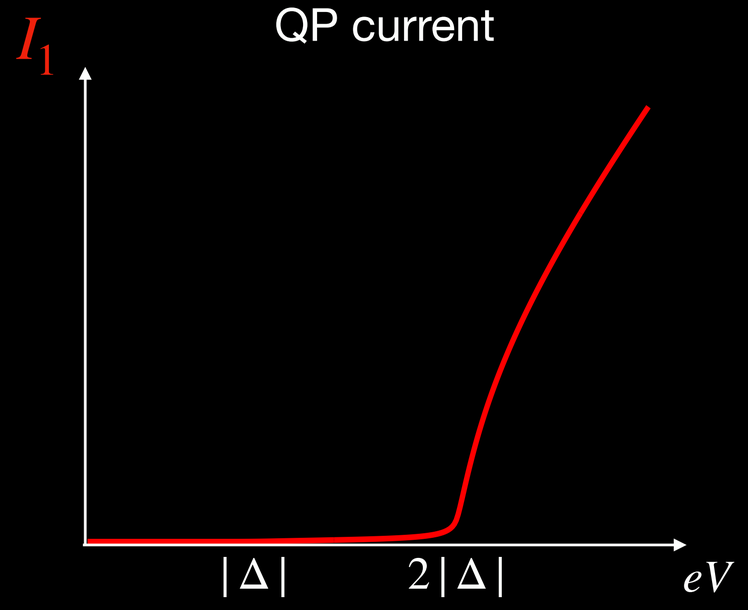
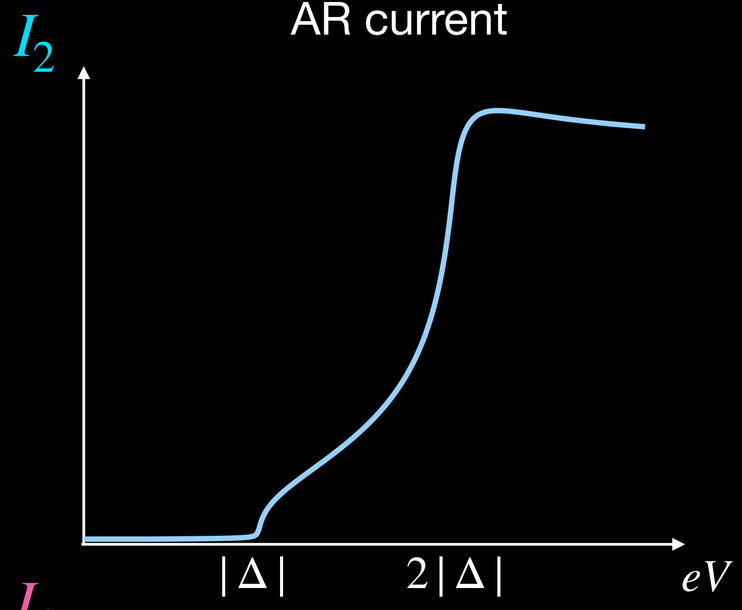


Example: Superconductor-Superconductor (SS) contact

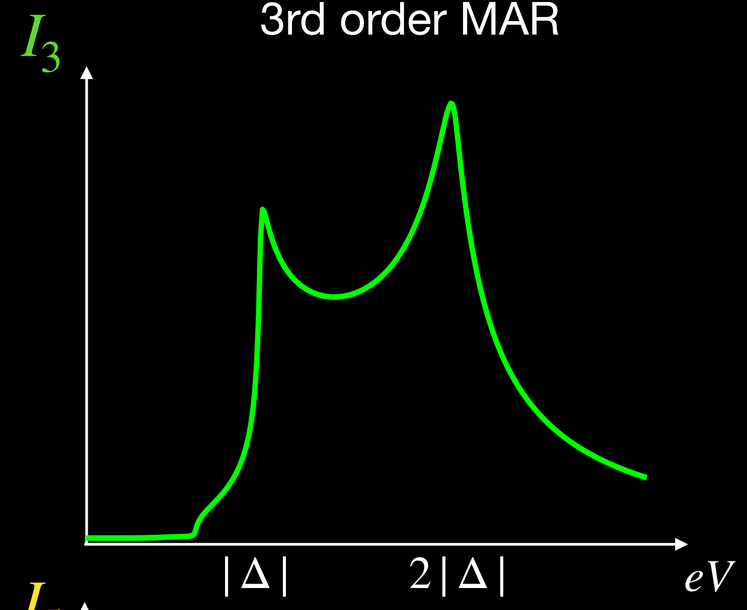
1 charge =
QP current



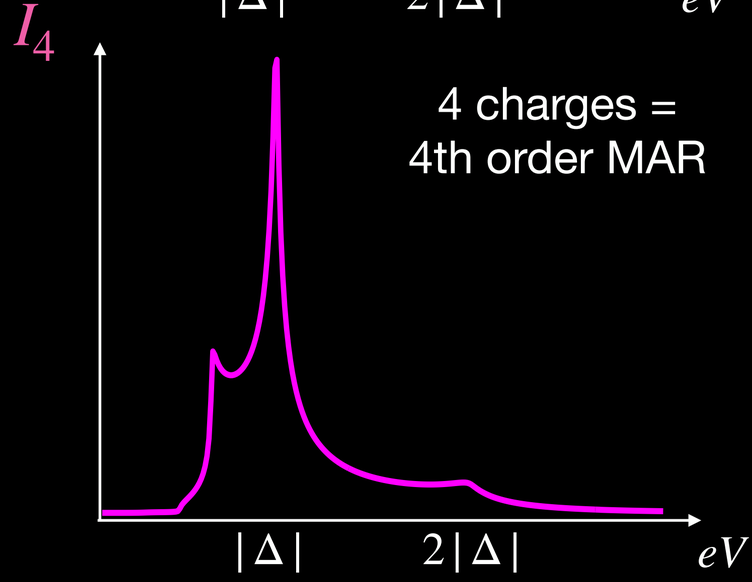
2 charges =
AR current



3 charges =
3rd order MAR



$$I_n(V) = \frac{e}{h}\int np_n(E,V)dE$$



4 charges =
4th order MAR

5 charges =
5th order MAR

J. C. Cuevas and W. Belzig, Phys. Rev. Lett. 91, 187001 (2003)

Keldysh Action

All knowing **action** $\mathcal{A}(\chi)$ depending on **counting field** χ gives current I and shot noise

$$\boxed{\mathcal{A}(\chi) = \mathrm{Tr}\ln\left(\frac{1+s}{2} + \frac{1-s}{2}G(\chi)\right) - \mathcal{A}(0)}$$

I. Snyman and Y. V. Nazarov Phys. Rev. B 77, 165118 (2008)

Current

$$I = -\imath\frac{e}{h}\frac{\partial\mathcal{A}(\chi)}{\partial\chi}\bigg|_{\chi=0}$$

Shot noise

$$S = -\frac{2e^2}{h}\frac{\partial^2\mathcal{A}(\chi)}{\partial\chi^2}\bigg|_{\chi=0}$$

<u>Resevoir Green's functions (GFs)</u> **and** <u>normal state scattering matrix</u> s and as only input

$$G = \begin{pmatrix} \check{g}_{\mathrm{N}}(\chi) & 0 \\ 0 & \check{g}_{\mathrm{S}} \end{pmatrix} \quad | \quad \check{g}_{\mathrm{N/S}}(\chi)\ \textbf{Keldysh GF}\ \text{of Normal/SC metal}$$

$$s = \begin{pmatrix} r & t \\ t & r' \end{pmatrix} \quad \text{(from Fisher-Lee-relation)}$$

Result for NS case (and temperature $T = 0$)

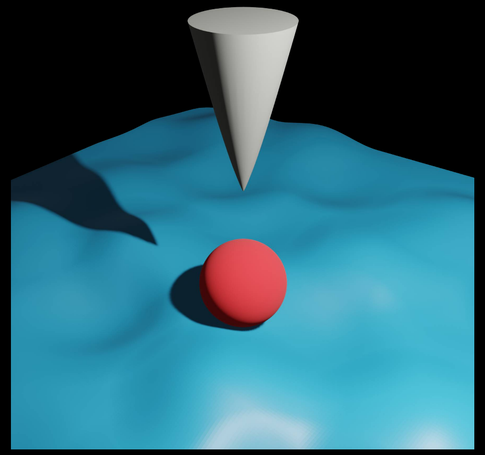
$$\mathcal{A}(\chi) = \int dE \ln\left[1 + p_1(E)(e^{1\imath\chi} - 1) + p_2(E)(e^{2\imath\chi} - 1)\right]$$

$$\Rightarrow I = I_1 + I_2 \Rightarrow G = G_1 + G_2$$

1 <-> QP
2 <-> AR

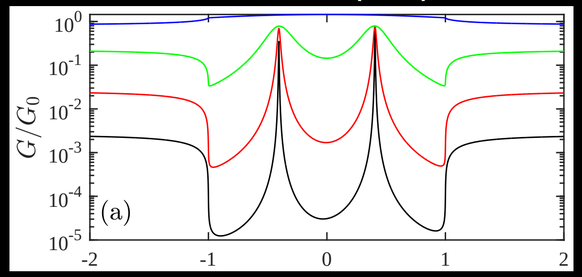
What systems are we describing?

Single-impurity

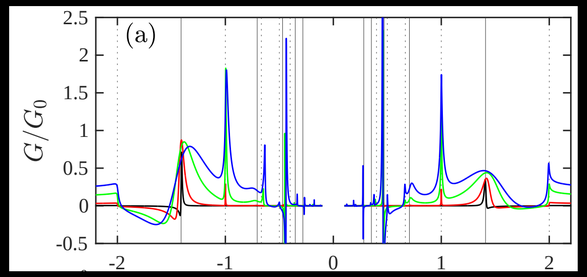


Normal tip or SC tip

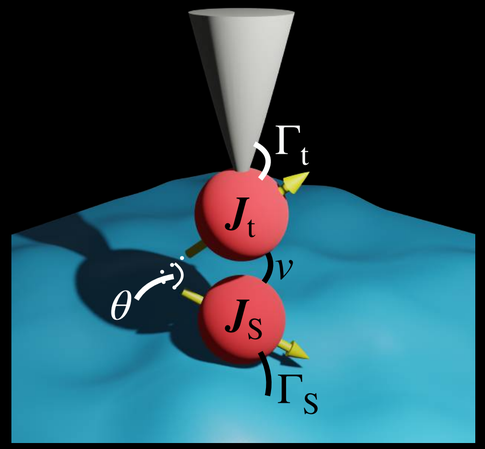
Competition between quasiparticle (QP) tunneling and Andreev reflection (AR)



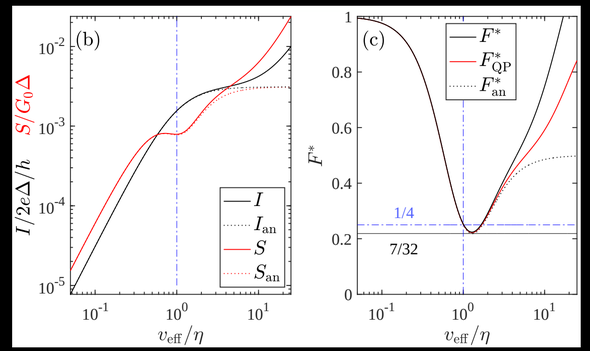
Competition between *n*-th order (M)AR and *n+1*-th order (M)AR



Double-impurity



Unveil fundamental Fano factor of 1/4 and 7/32

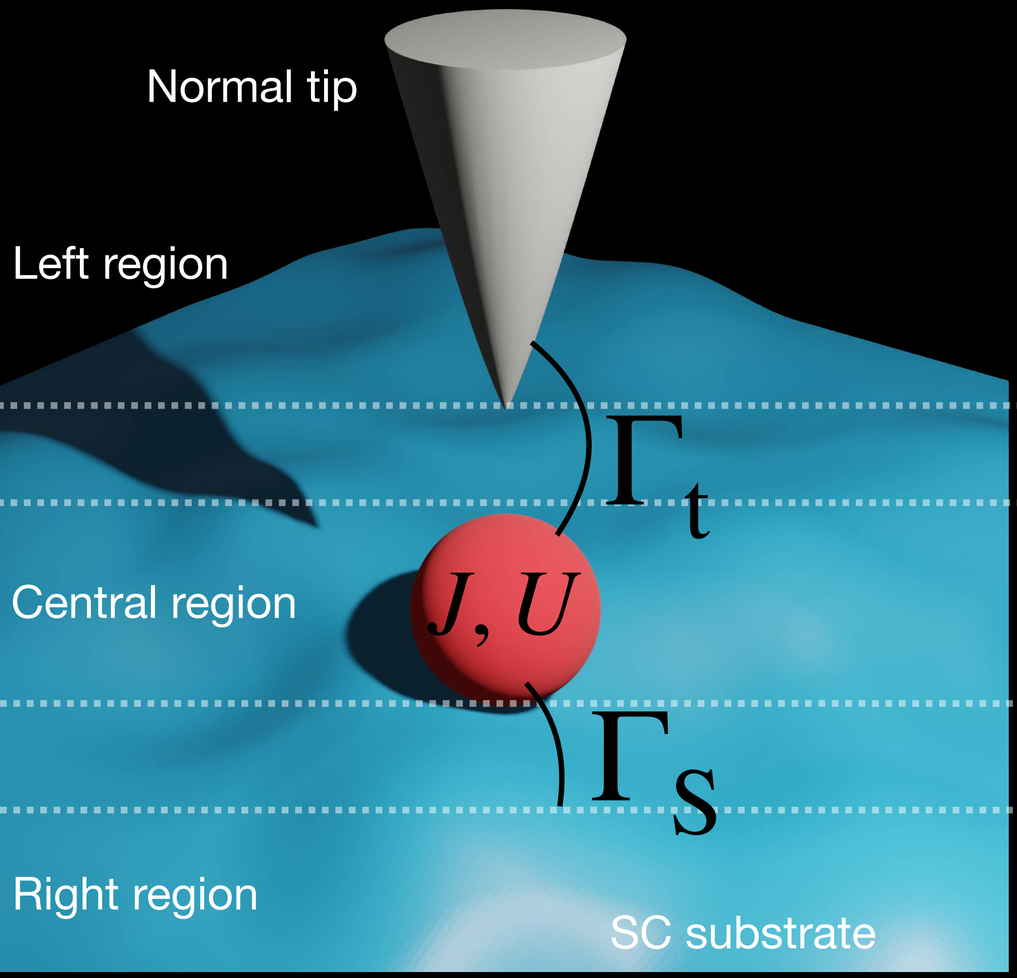


Learn more:

D. C. O., et. al. Phys. Rev. Research 5, 033176 (2023)

What systems are we describing?
And how?

Single-impurity



× Normal metal Hamilton and Green's function (GF)

× Mean-field Anderson impurity model
(U: on-site energy, J: exchange)

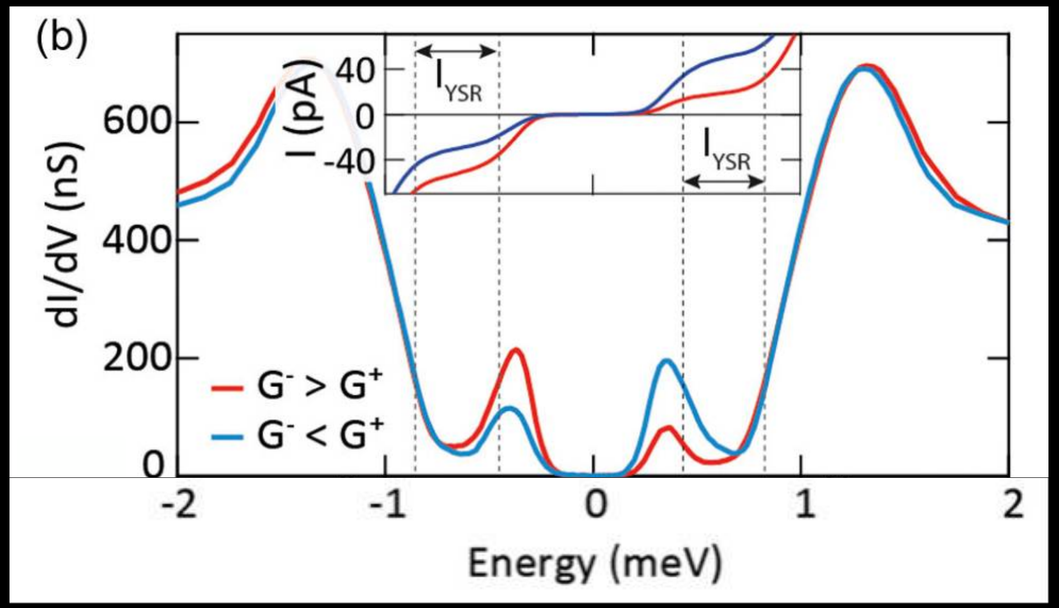
$$H_{\mathrm{imp}} = U(n_\uparrow + n_\downarrow) + J(n_\uparrow - n_\downarrow)$$

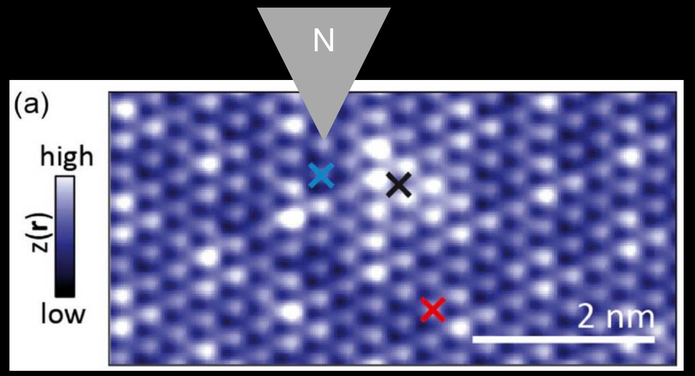
× BCS Hamiltonian and GF

Experiment on current and Shot Noise

U. Thupakula, V. Perrin, A. Palacio-Morales, L. Cario, M. Aprili, P. Simon, and F. Massee, Phys. Rev. Lett. **128**, 247001 (2022)

Conductance and Fano Factor measurement data





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System parameters:

Superconducting gap

$|\Delta| \approx 1.30\,\mathrm{meV}$

Conductance peak inside the SC gap

-> Yu-Shiba-Rusinov (YSR) bound states

-> Energy $\epsilon_{YSR} = 0.3\,\mathrm{meV}$

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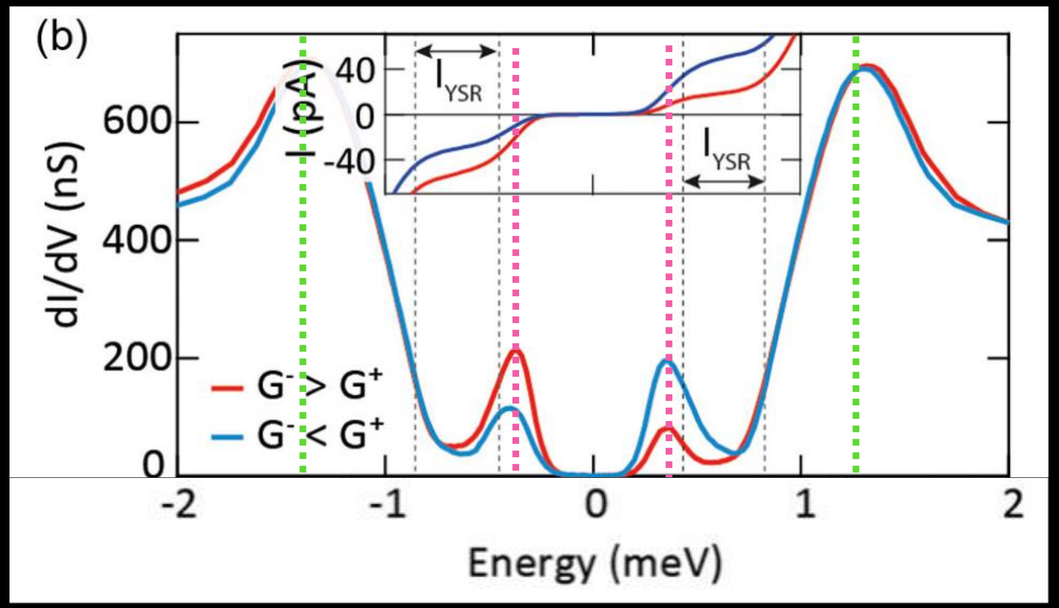

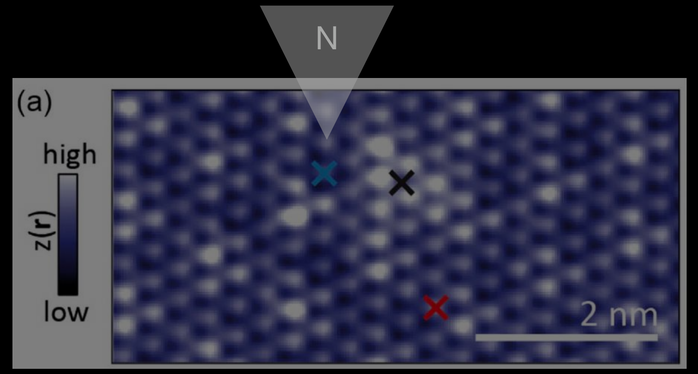

Temperature $T = 700\,\mathrm{mK}$

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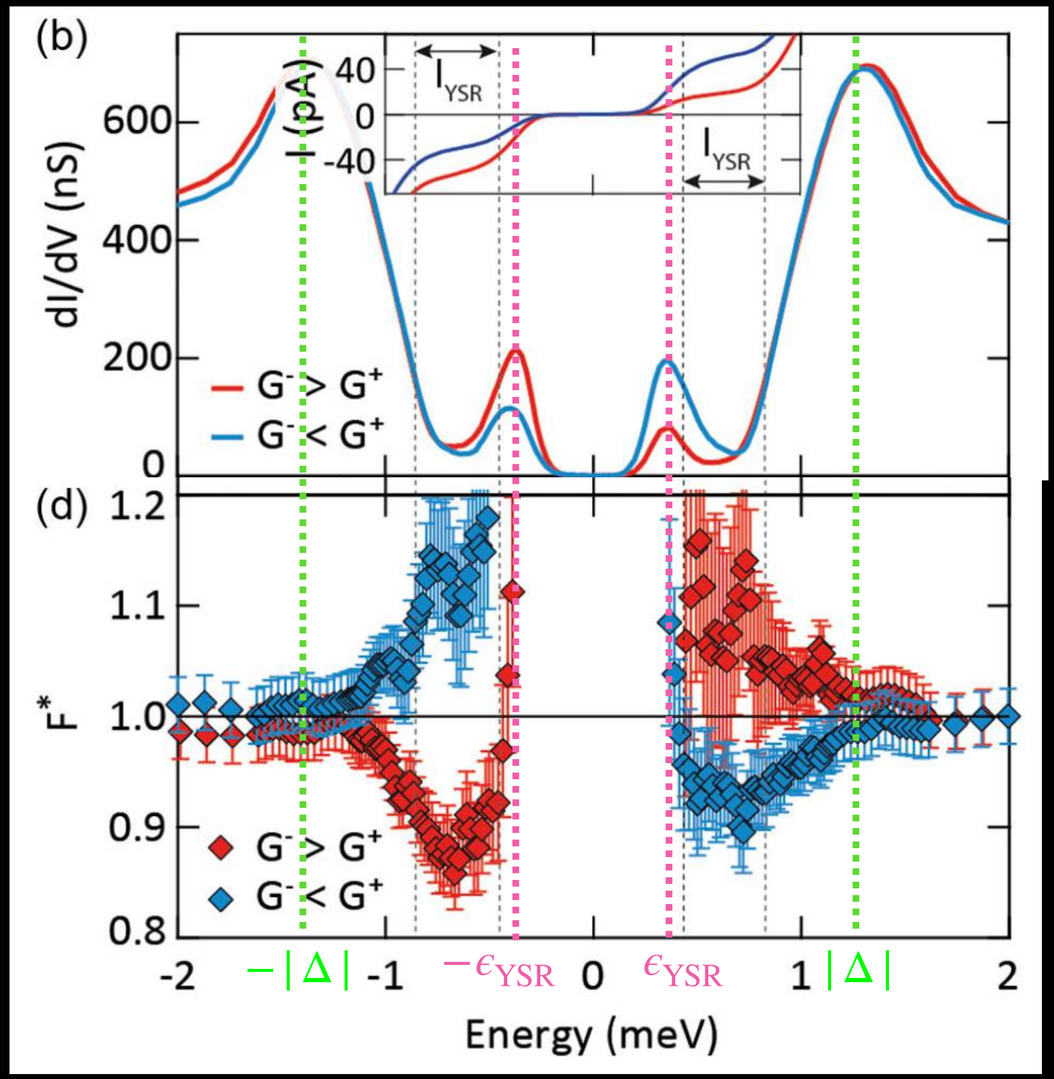
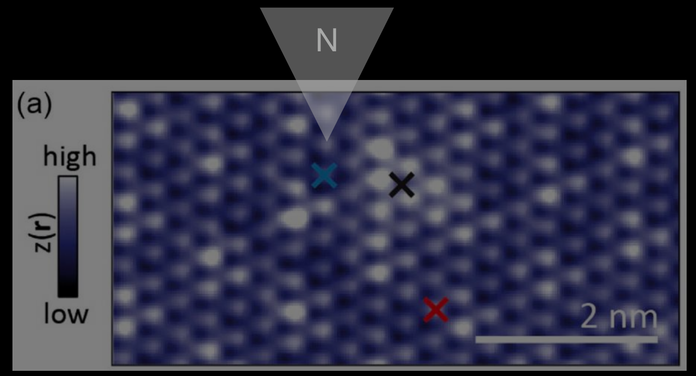




Fano Factor:

$$F^* = \frac{S}{2e\,|I|} \approx q_{\mathrm{eff}}$$

(for low transmission*)

Fano Factor asymmetry:

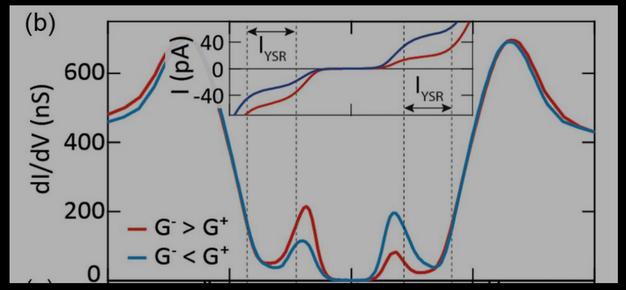
1) Drop from >1 to <1 at YSR bound state energy.

2) >1 for all voltages

Predicted Lifetime of YSR state via Shot noise $\Lambda = 1\,\mu\mathrm{eV}$

Theoretical curves - Changing Γ_t

Experiment: condutance



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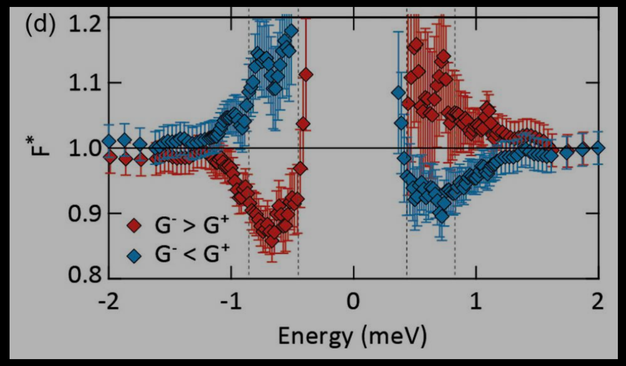
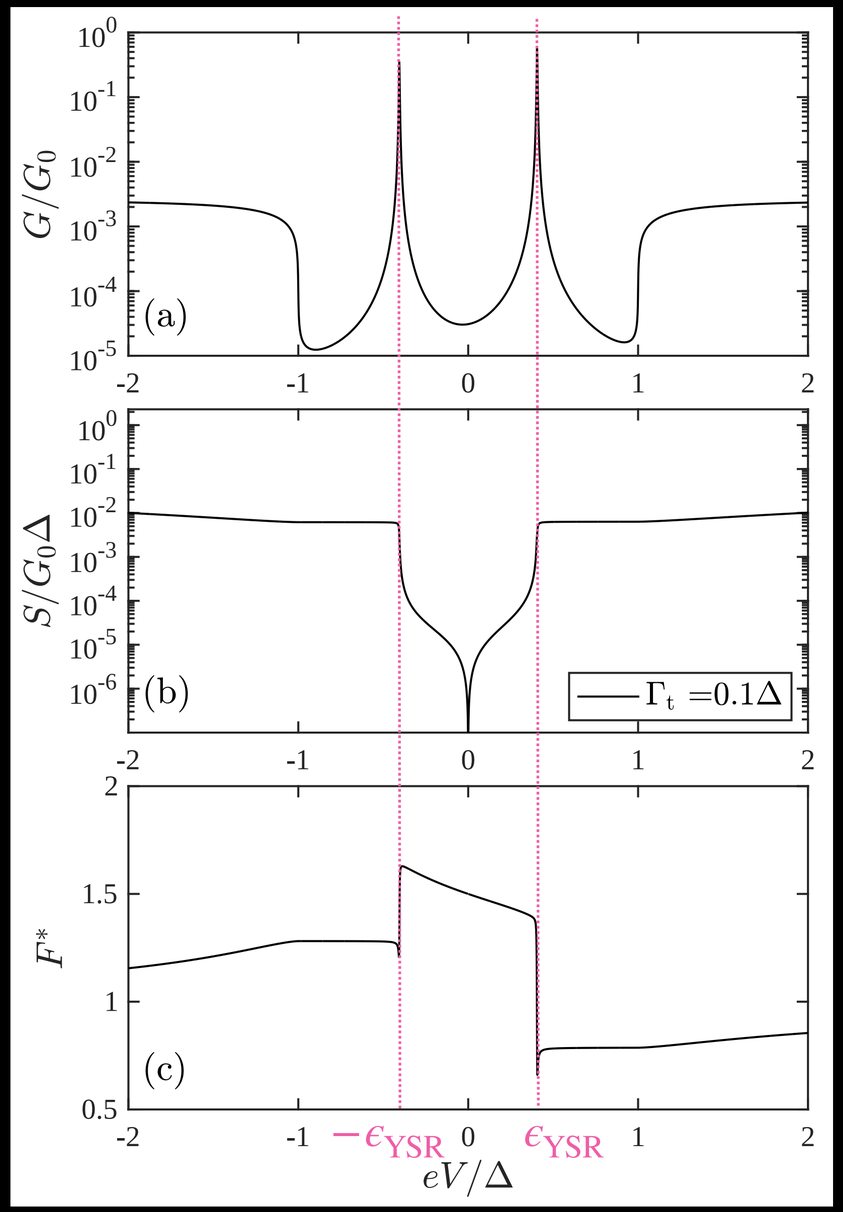
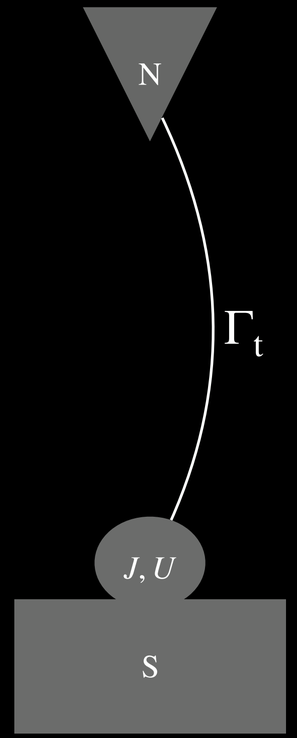




$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

$\epsilon_{YSR} = 0.48\Delta$

$\Gamma_t = 0.1\Delta \Rightarrow T_{E=0} = 0.002$



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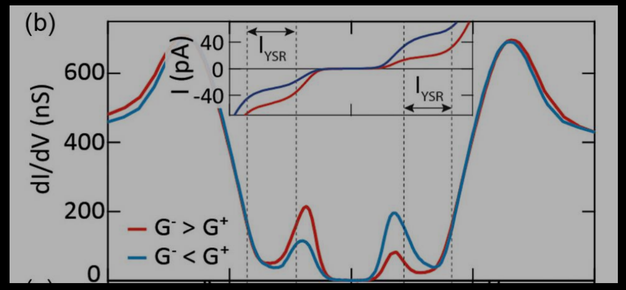

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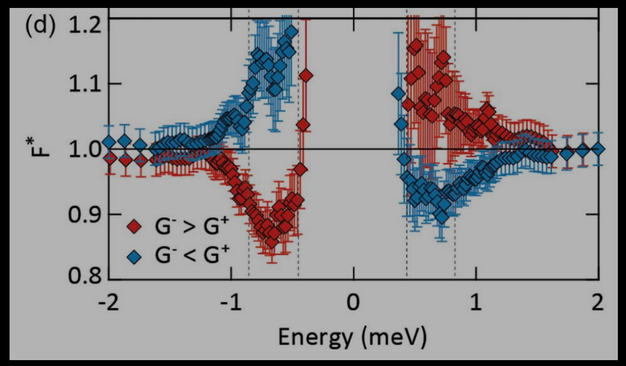

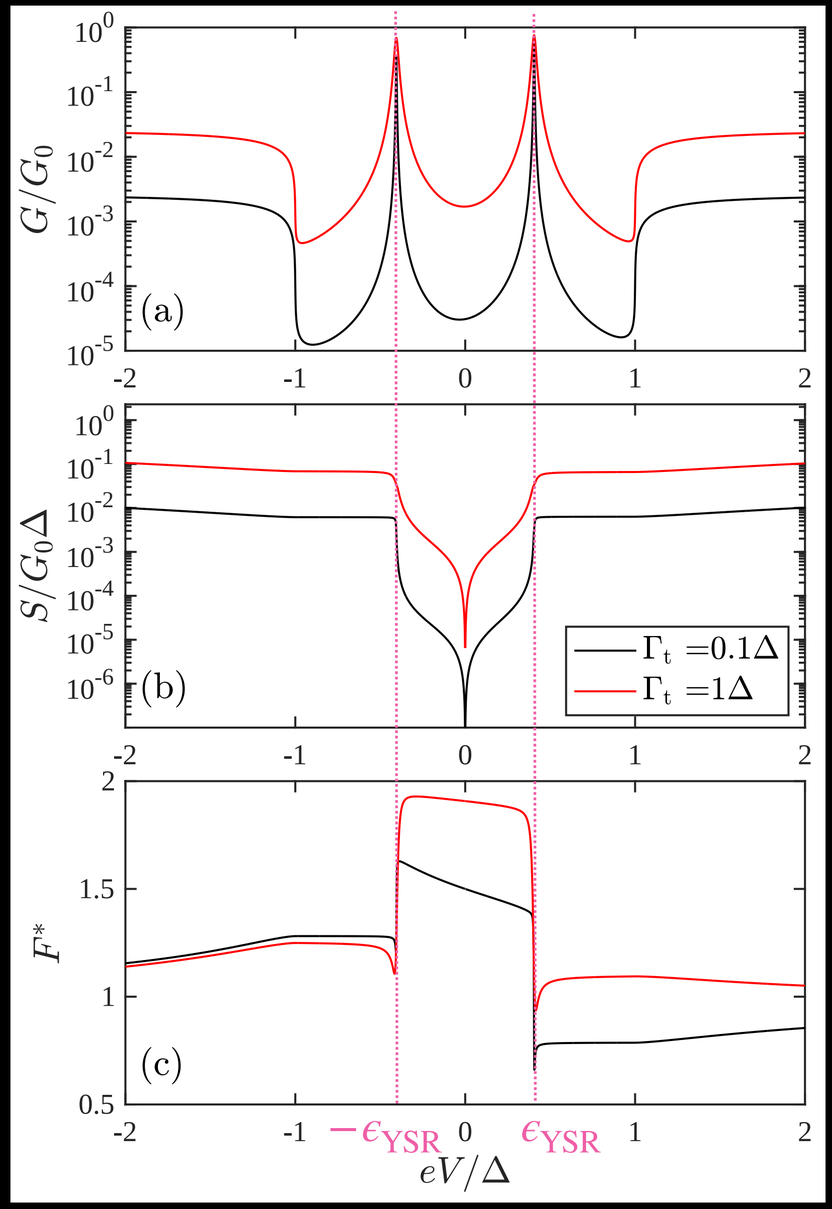

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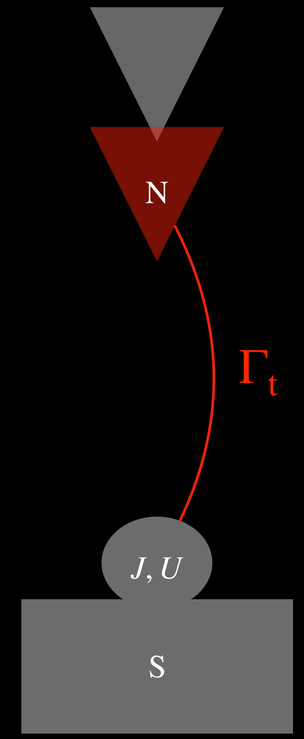
Width $\propto \Gamma_t$ -> Lifetime can't be probed by G

$\epsilon_{YSR} = 0.48\Delta$

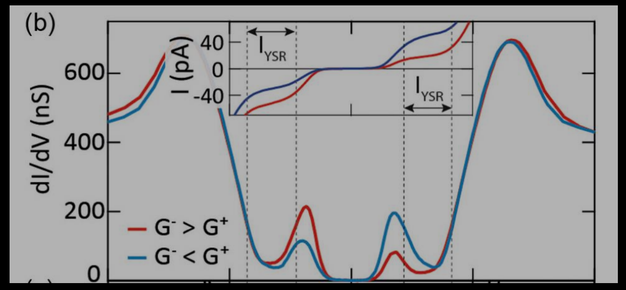
$\Gamma_t = 0.1\Delta \Rightarrow T_{E=0} = 0.002$

$\Gamma_t = 1\Delta \Rightarrow T_{E=0} = 0.02$

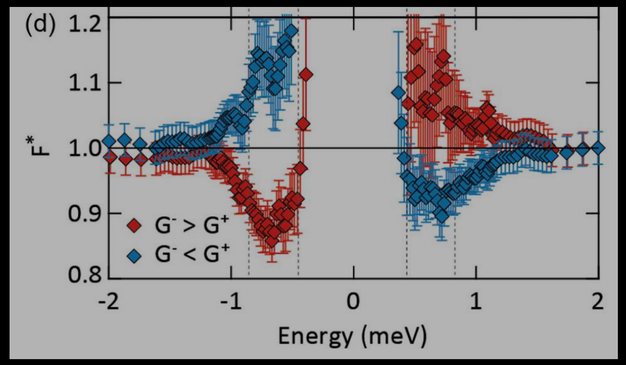


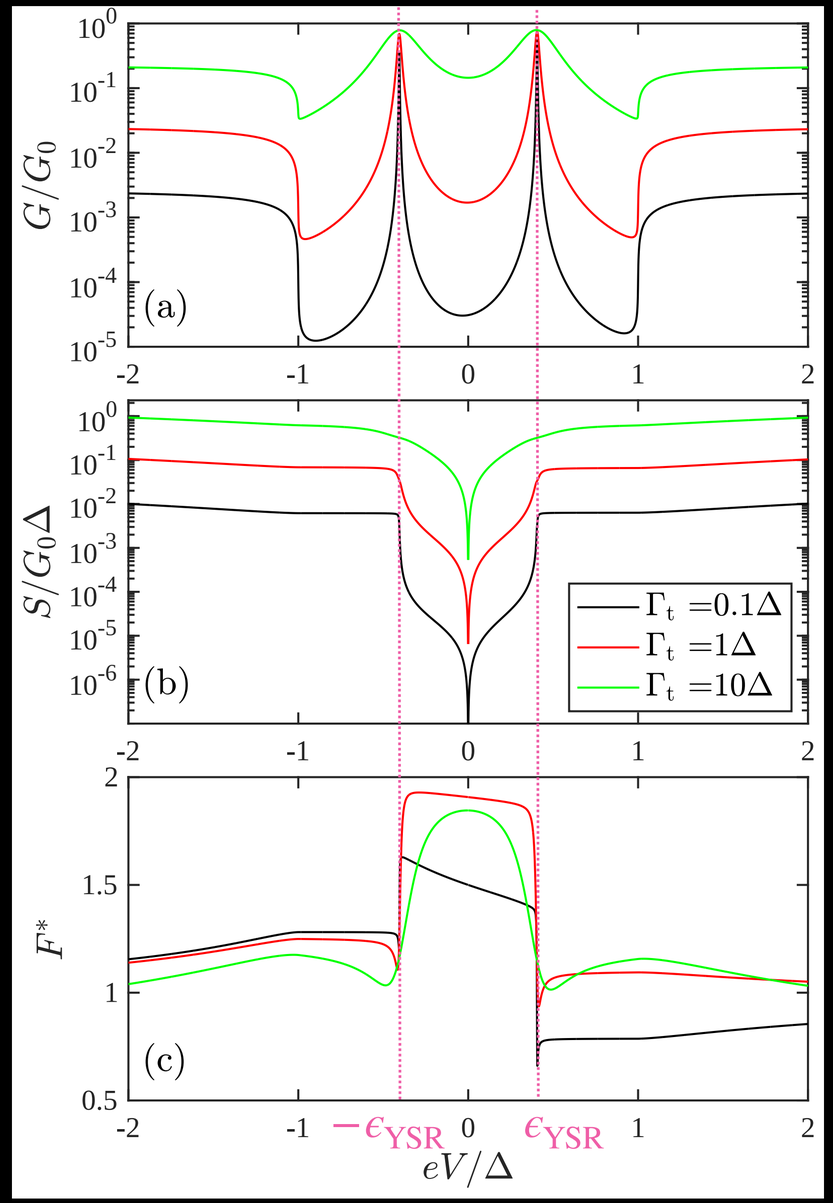
Theoretical curves - Changing Γ_t

Experiment: condutance



Experiment: Fano factor





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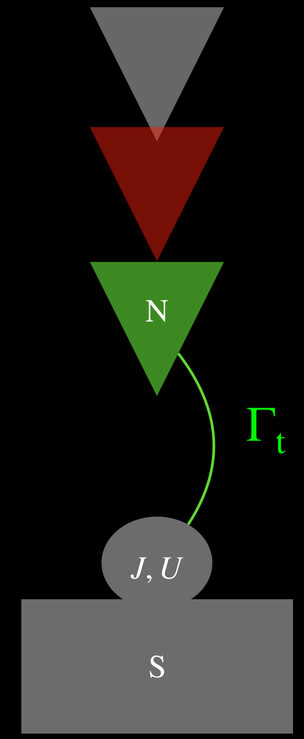
$\Gamma_t = 10\Delta \Rightarrow T_{E=0} = 0.2$



Theoretical curves - Changing Γ_t

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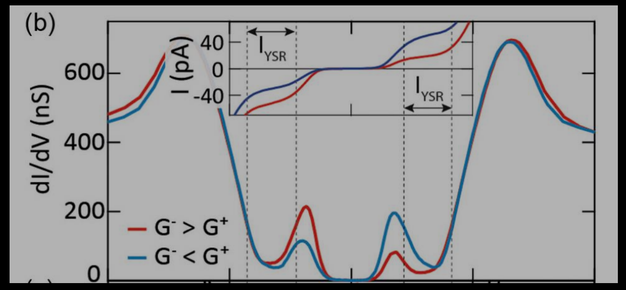

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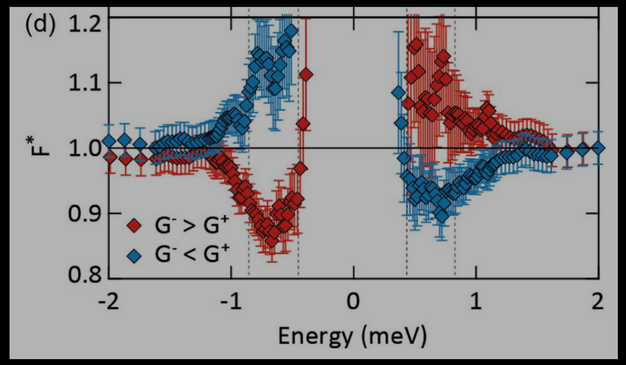

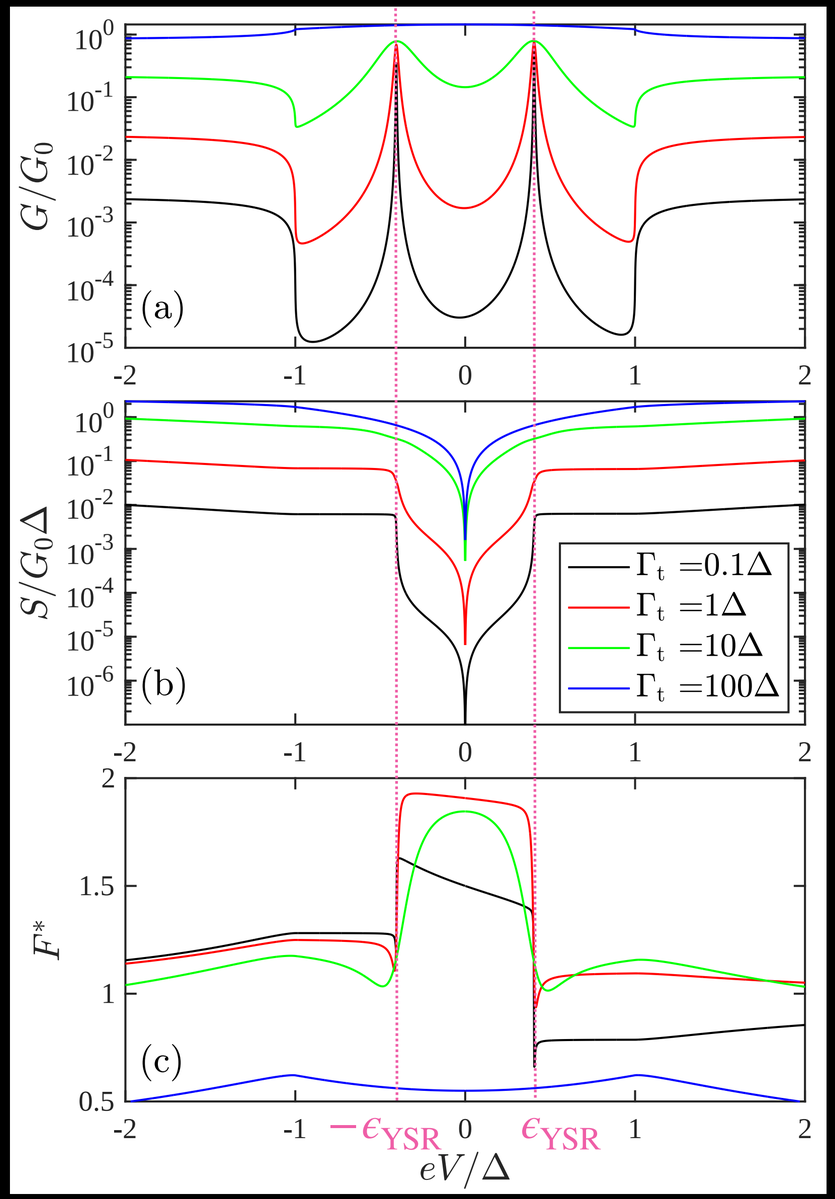

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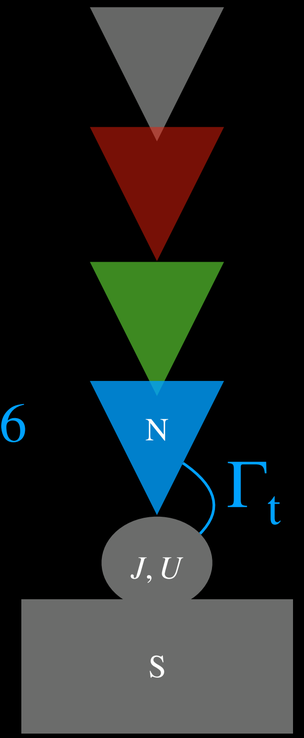
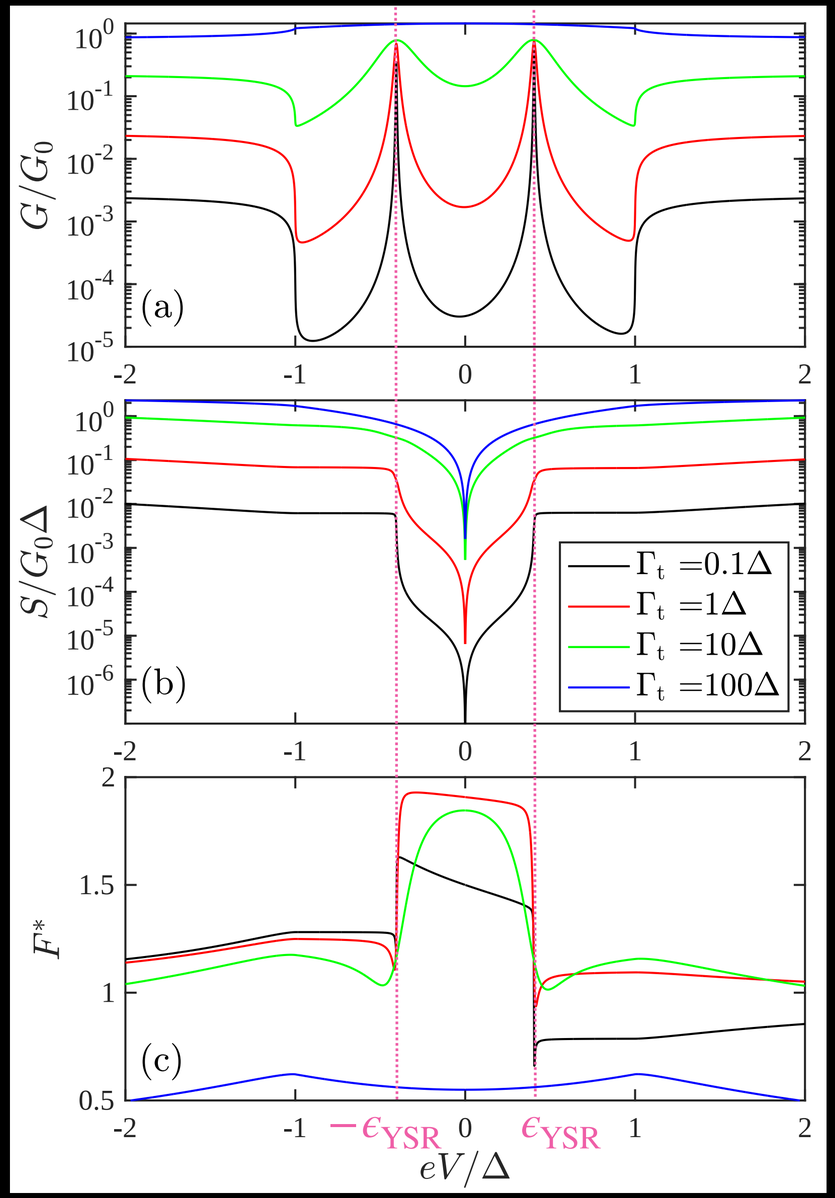
$\Gamma_t = 1\Delta \Rightarrow T_{E=0} = 0.02$

$\Gamma_t = 10\Delta \Rightarrow T_{E=0} = 0.2$

$\Gamma_t = 100\Delta \Rightarrow T_{E=0} = 0.86$



Charge resolved conductance



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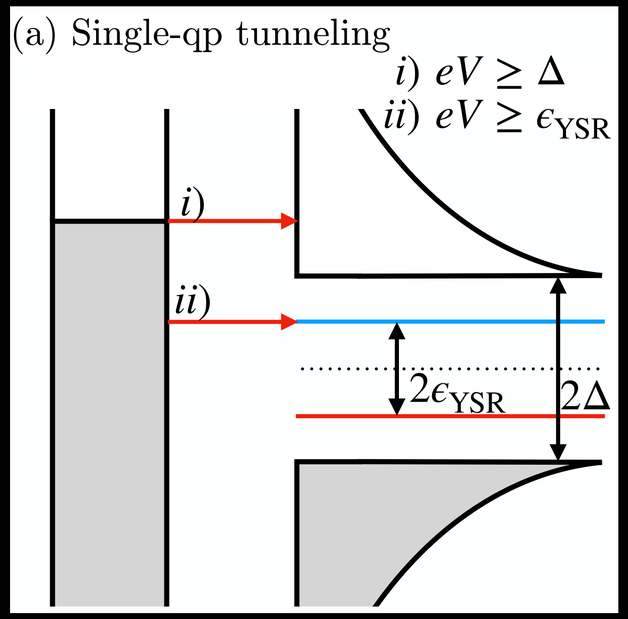
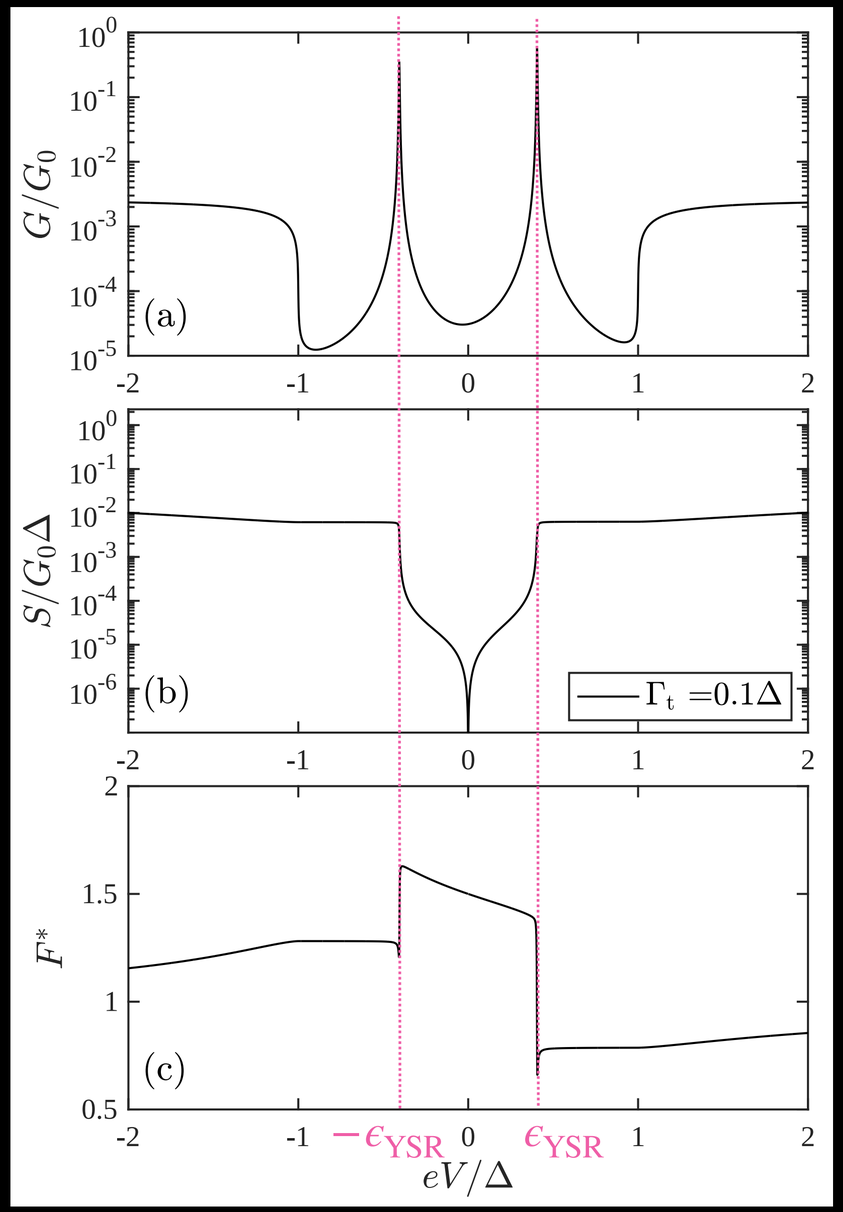
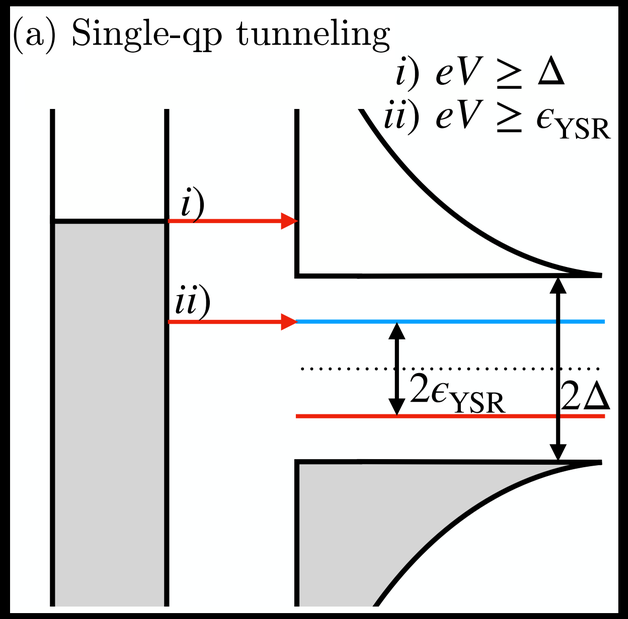


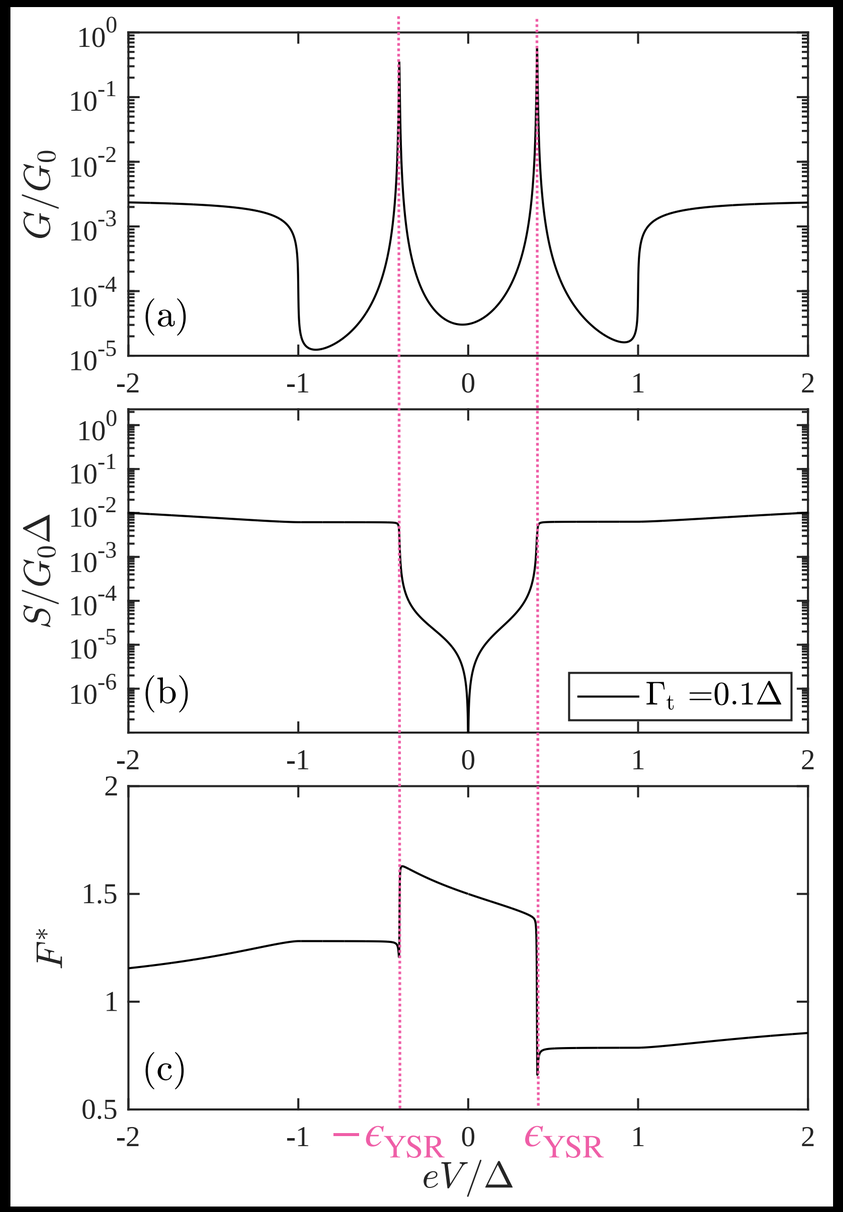


G_1 G_2

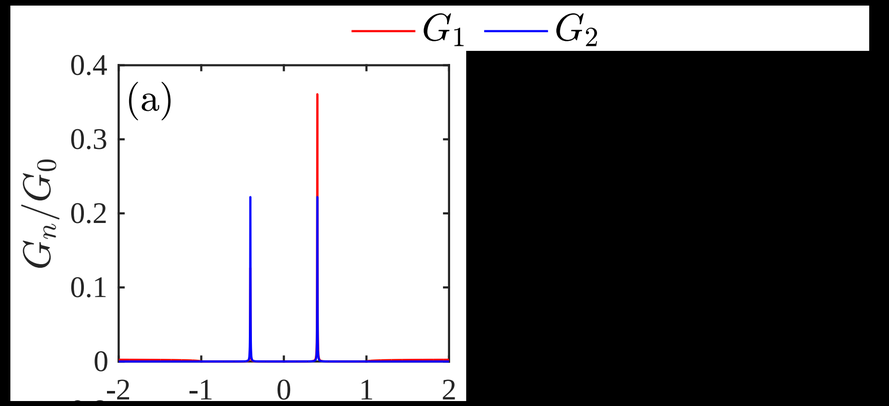
$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

Charge resolved conductance





a) $\Gamma_t = 0.1\Delta$



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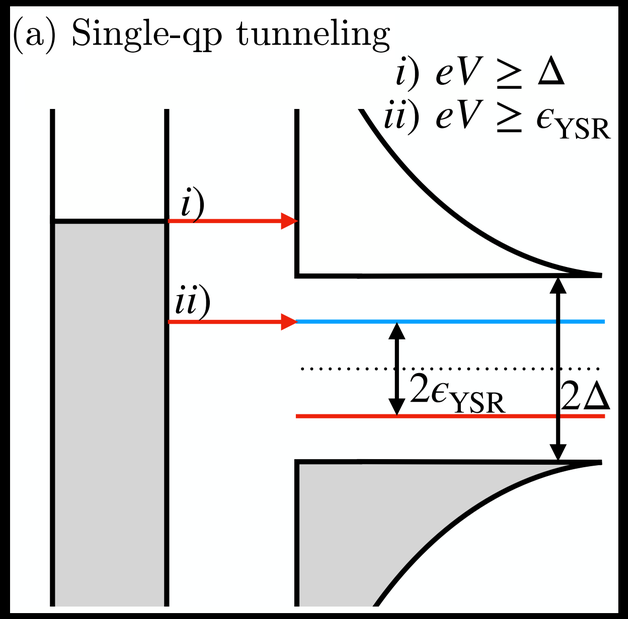
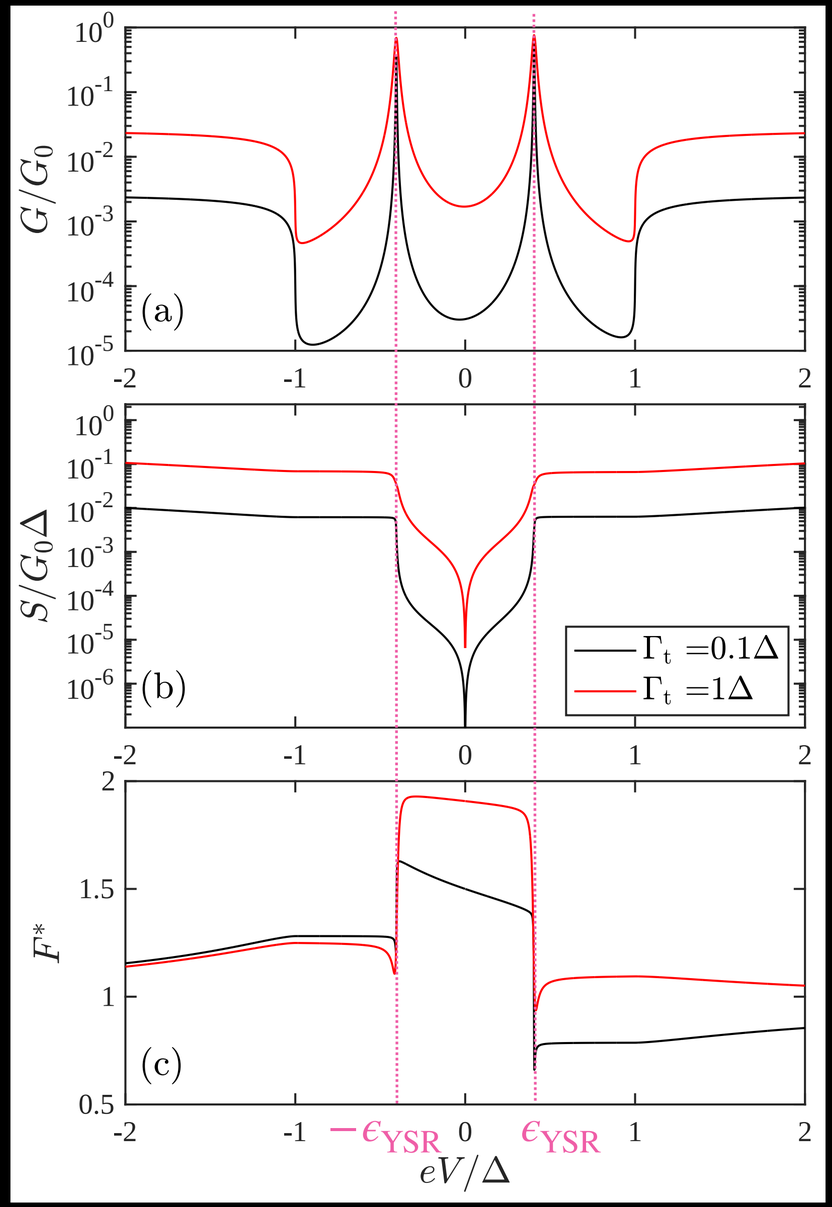
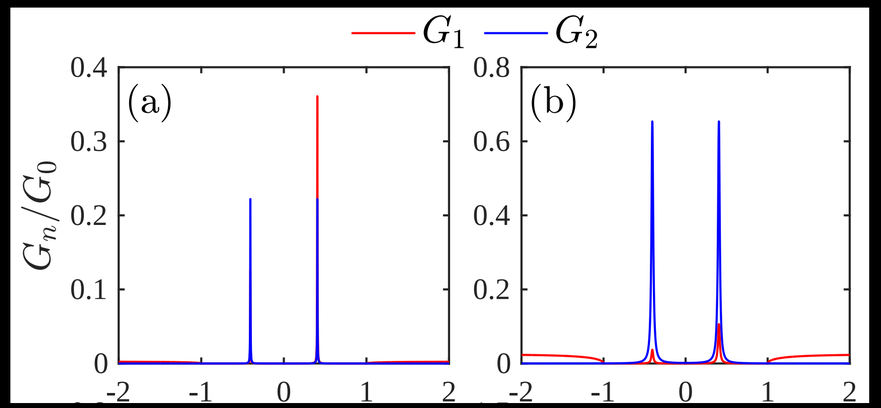
Charge resolved conductance



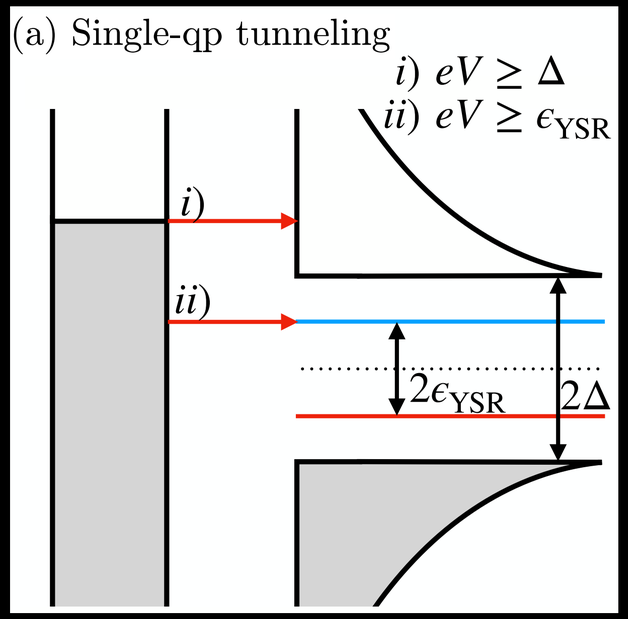


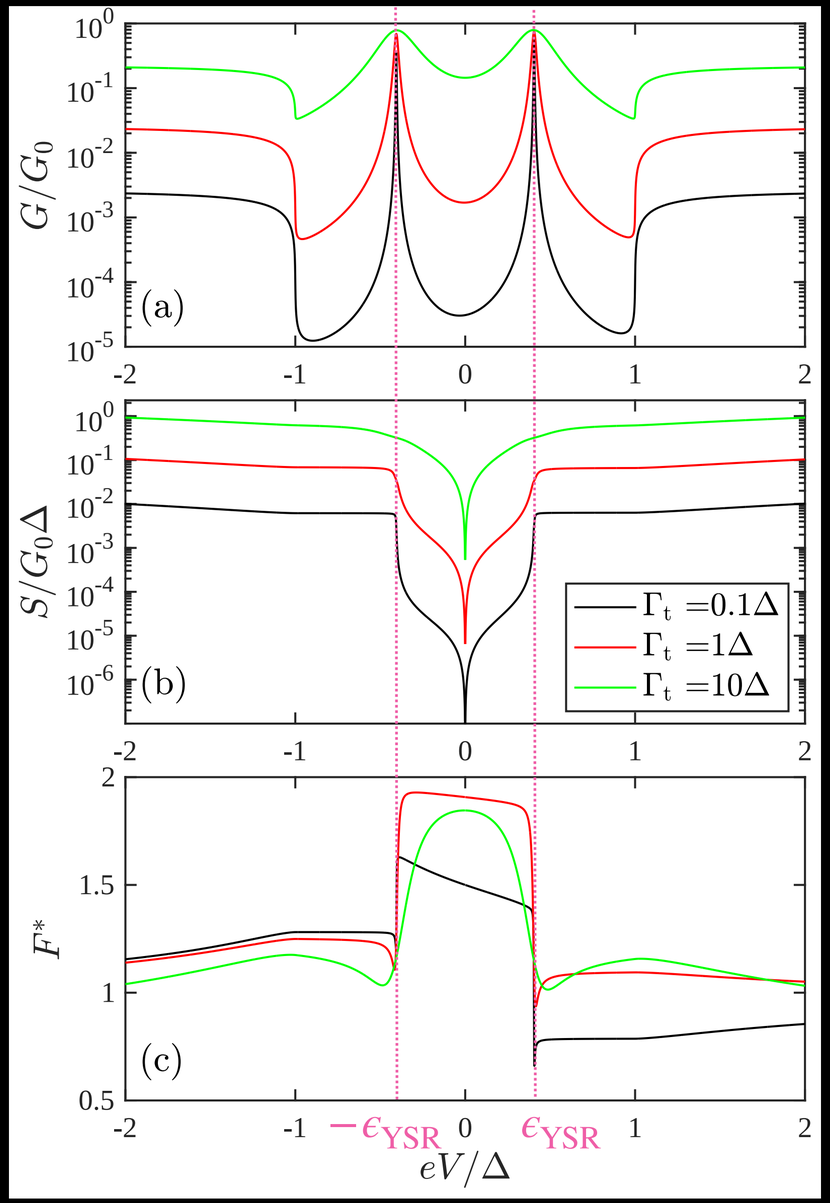
$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

a) $\Gamma_t = 0.1\Delta$ b) $\Gamma_t = 1\Delta$



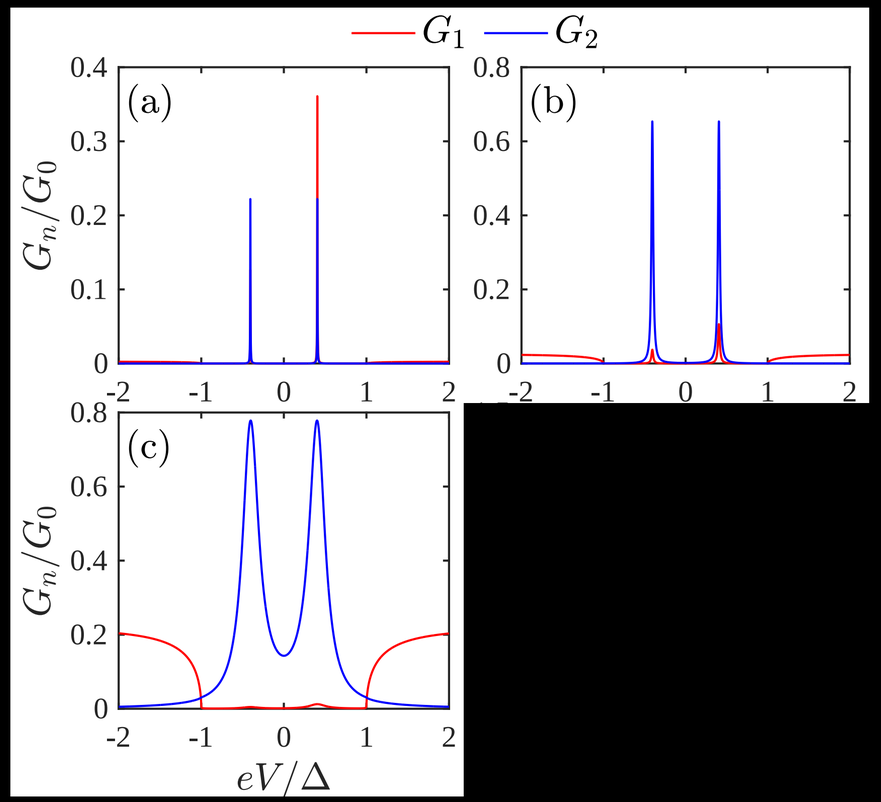
Charge resolved conductance





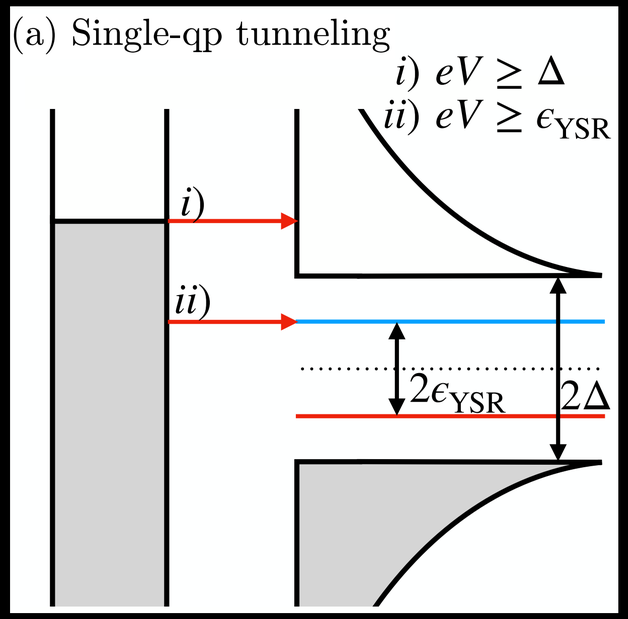
$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

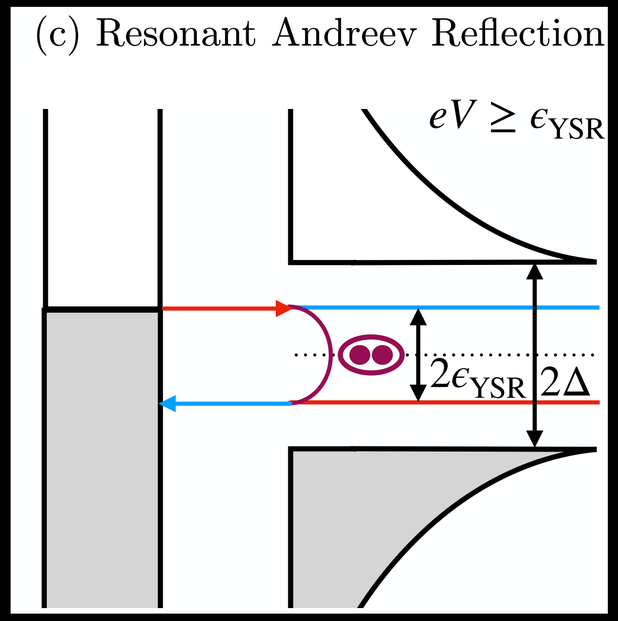
a) $\Gamma_t = 0.1\Delta$ b) $\Gamma_t = 1\Delta$

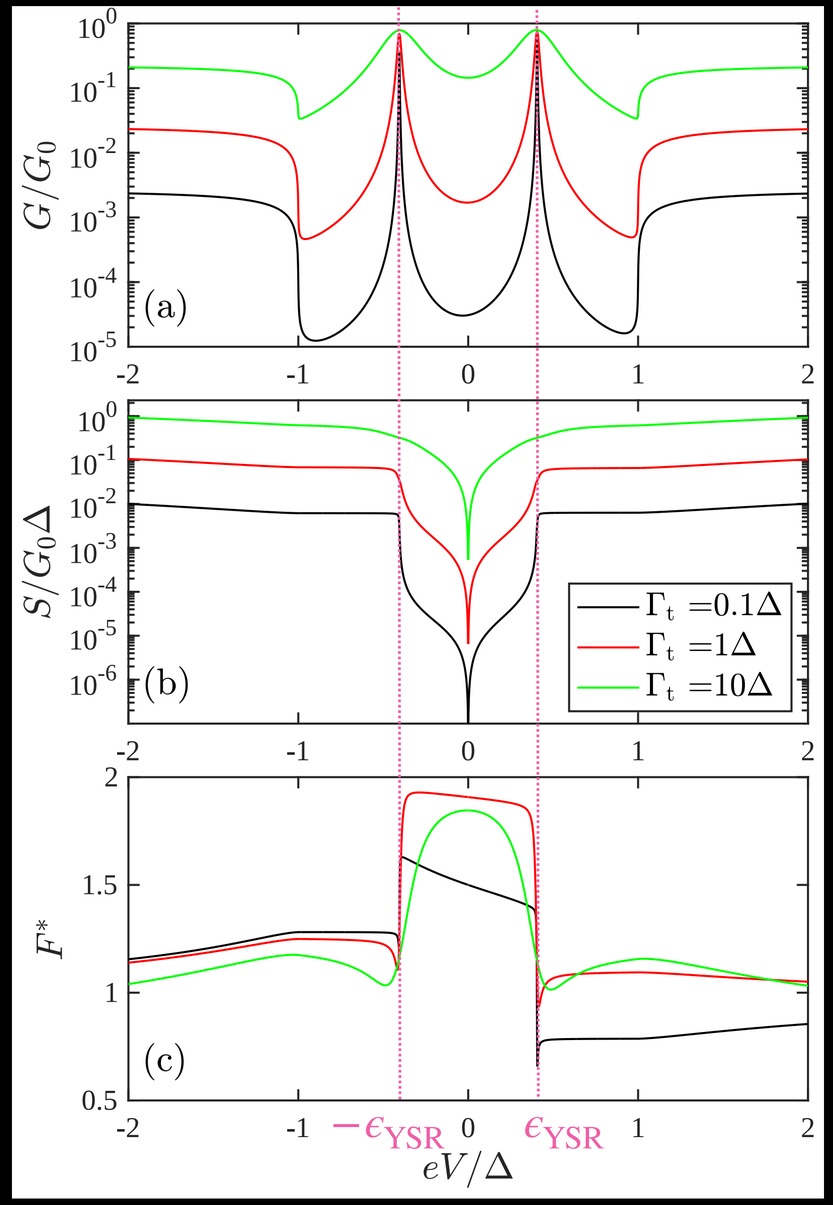


c) $\Gamma_t = 10\Delta$

Charge resolved conductance

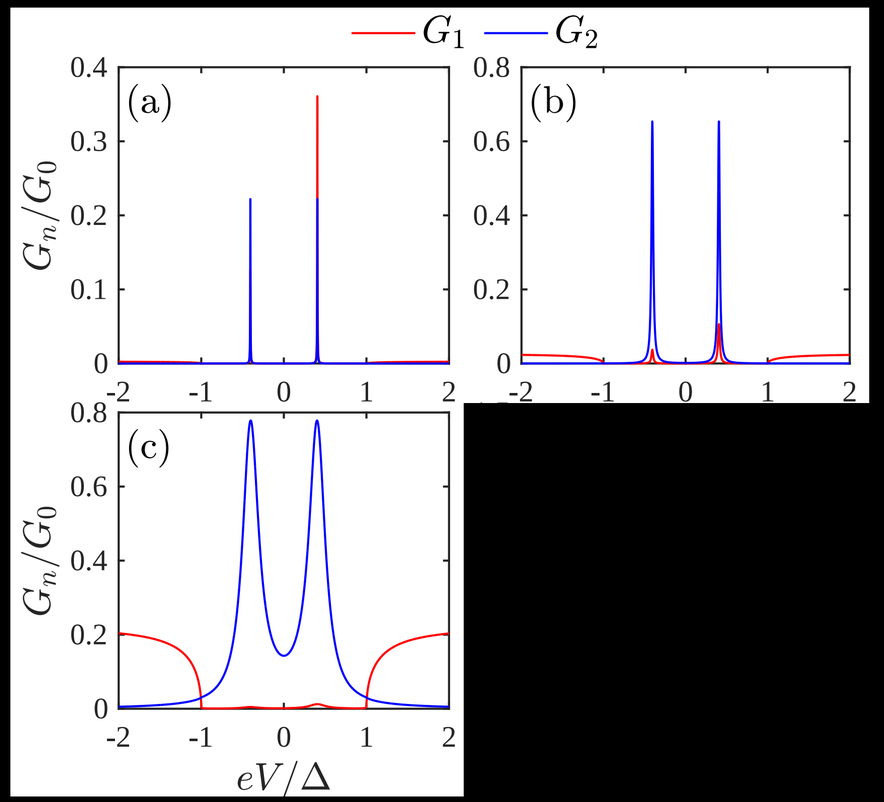






$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

a) $\Gamma_t = 0.1\Delta$ b) $\Gamma_t = 1\Delta$



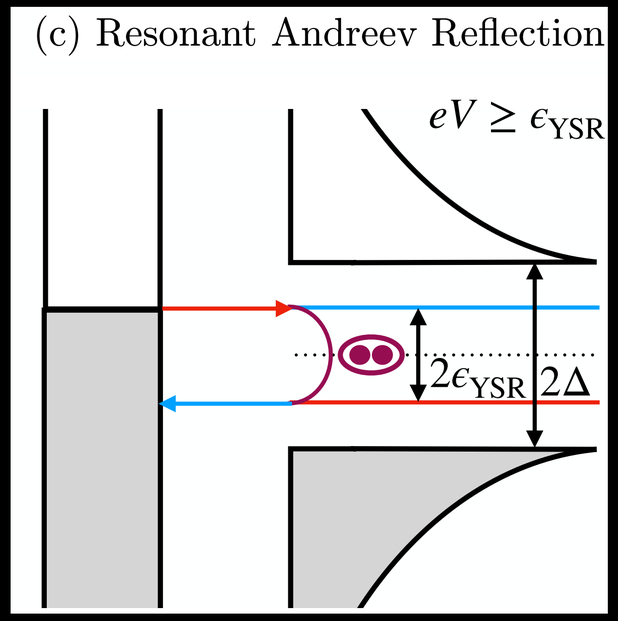
c) $\Gamma_t = 10\Delta$

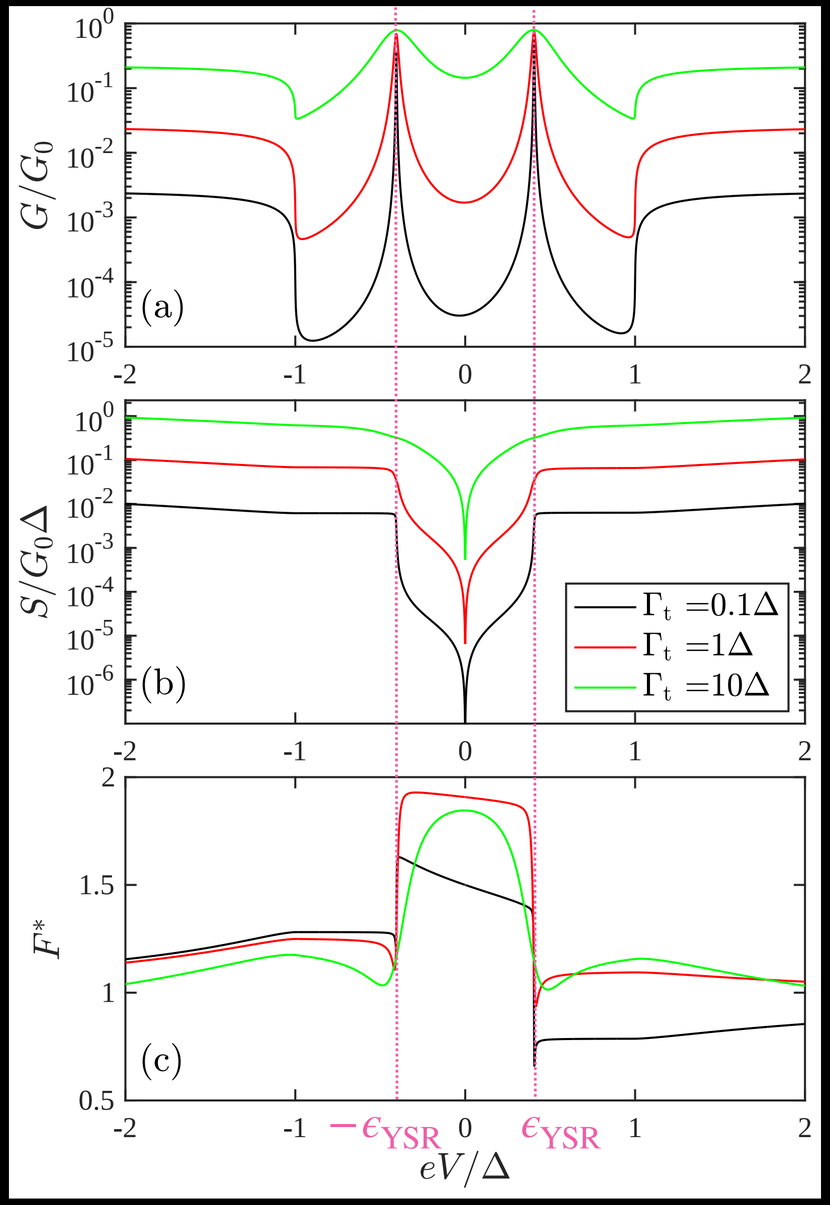
Charge resolved conductance

$$1 = 2 \cdot \frac{1}{2}$$

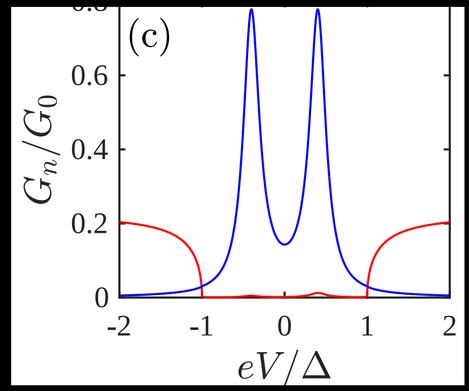
Charge of AR

Resonant factor

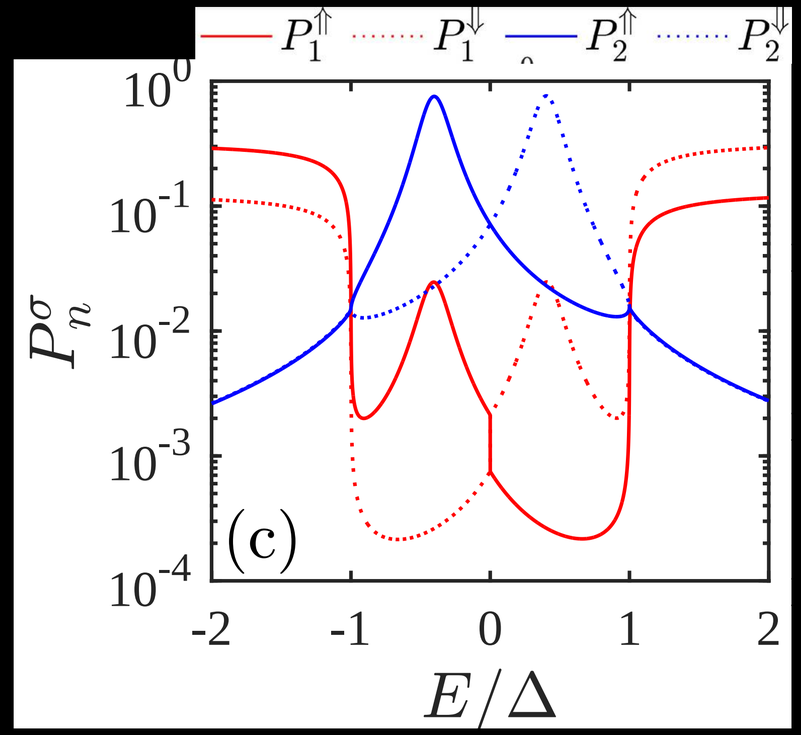




$\Gamma_S = 100,\ J = 60,\ U = 80,\ \eta = 0.001$

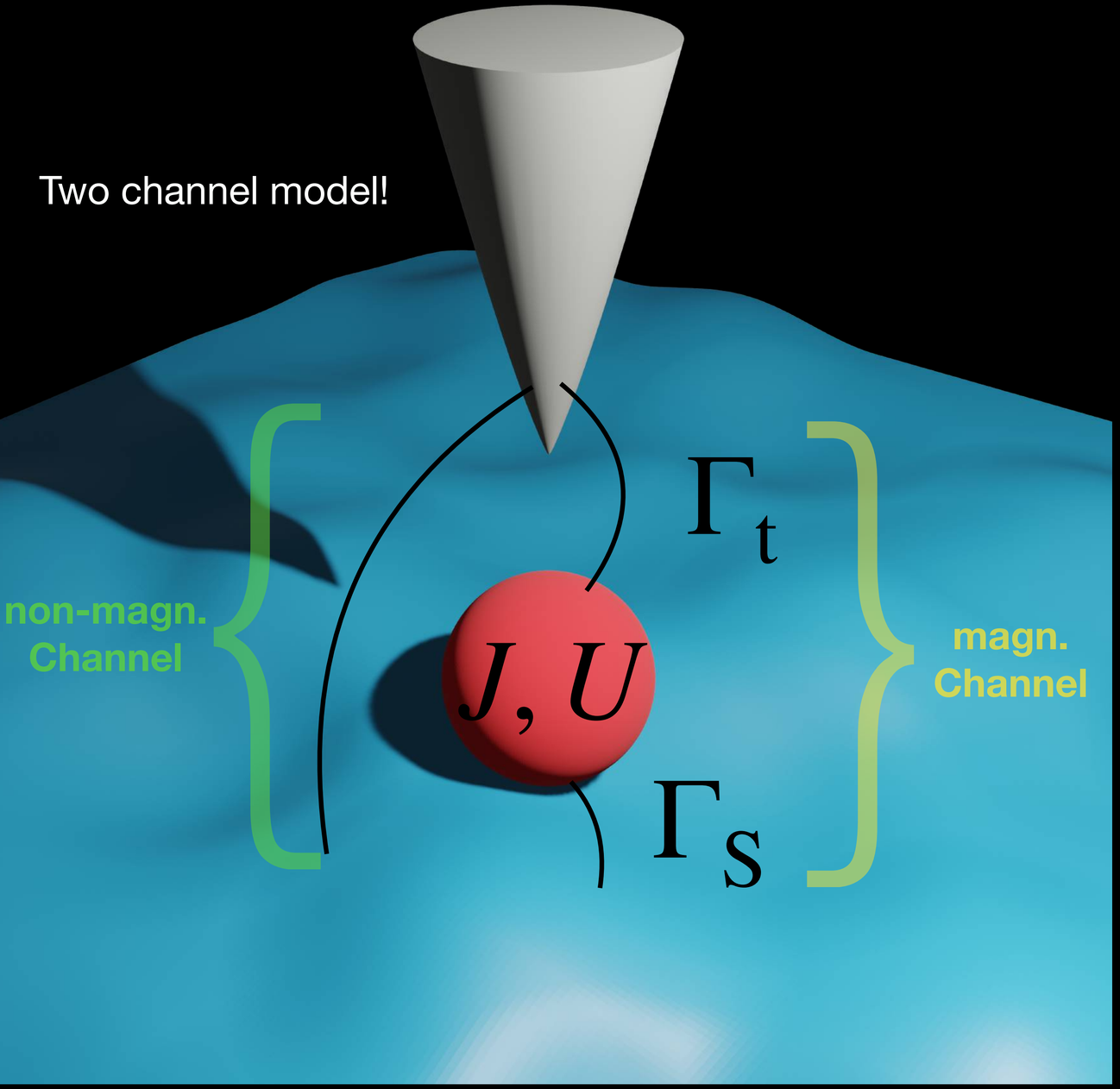


c) $\Gamma_t = 10\Delta$

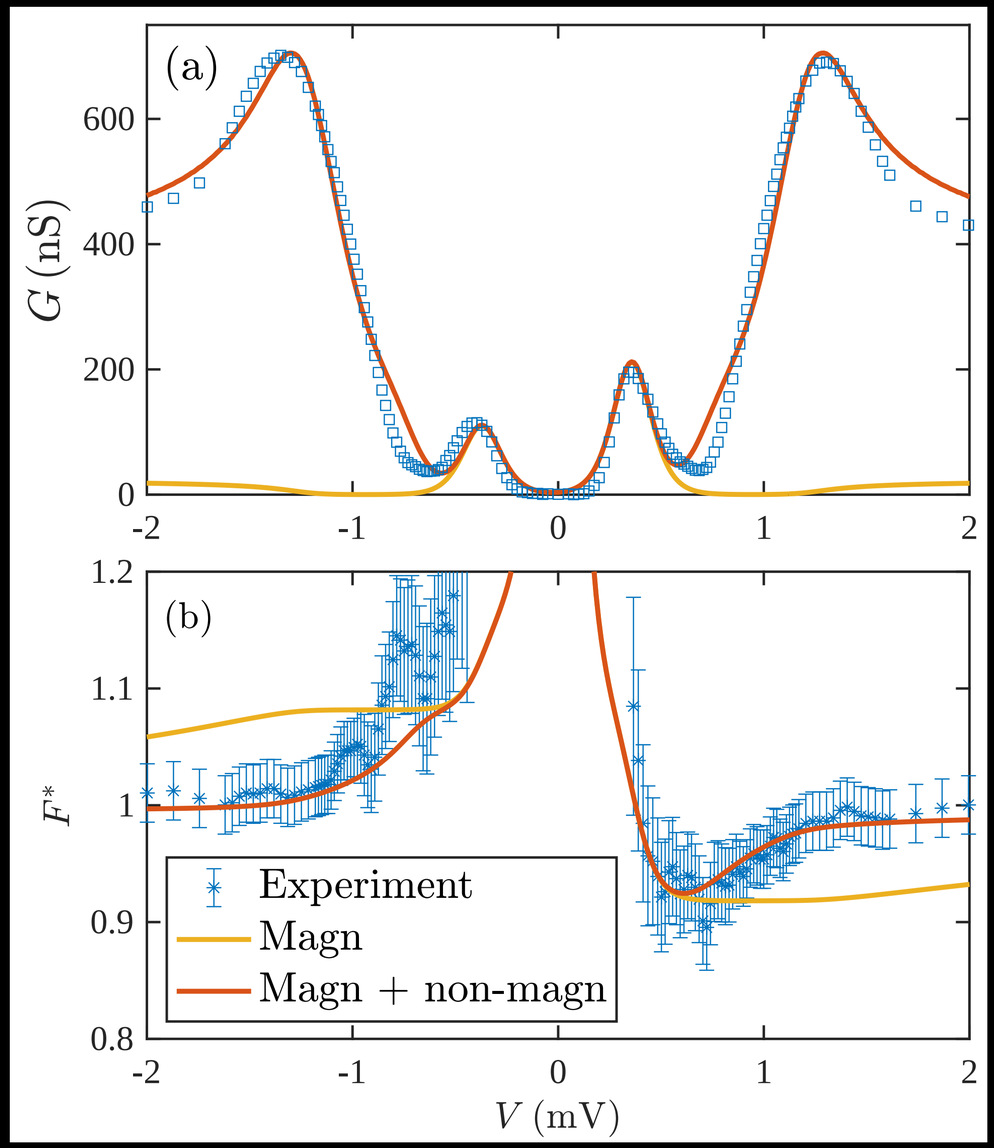


Comparison: Experiment and our Theory (Two-channel model)

Two channel model!



$\Gamma_t = 0.01\Delta,\ T_{\text{nonmagn}} = 0.0044$



D. C. Ohnmacht, W. Belzig, J. C. Cuevas, accepted in PRR, (2023)

Exp: U. Thupakula, et. al., Phys. Rev. Lett. 128, 247001 (2022)

Summary

1) Full Counting Statistics (FCS) provides unprecedented insight into transport of junctions containing YSR states

- Unveils competition between QP current and **resonant** AR in NS case
- Unveils dominance of resonant (M)AR over YSR-mediated (M)AR in SS case
- Unveils that current in double-impurity case is dominated by QP tunneling and characterized by Fano factors of 7/32 and 1/4
- Unveils all transport processes in double-impurity case

D. C. O., et. al. Phys. Rev. Research 5, 033176 (2023)

2) FCS calculations are needed to fully understand measurements

- Fano factor misleading as $1 = 2 \cdot 1/2 \neq 1$
- SS case example: resonant MAR instead of YSR-mediated AR