

Multiterminal Josephson Junctions (MTJJs): non-hermiticity, topology and reflectionless modes

Marco Coraiola
Tommaso Antonelli
Deividas Sabonis
Fabrizio Nichele

IBM Research | Zurich

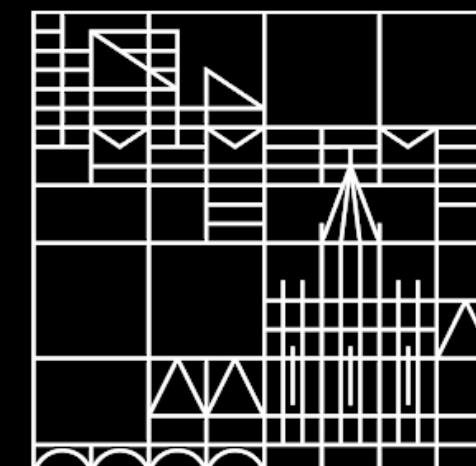
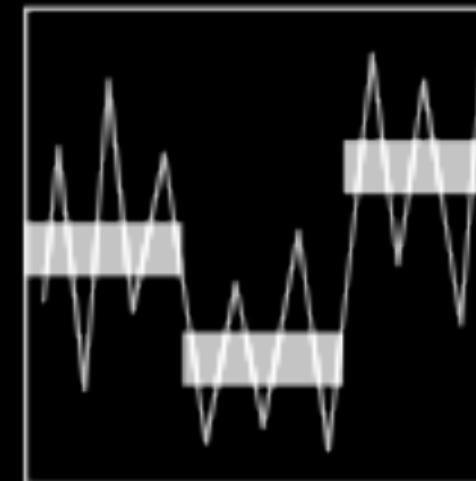
David Christian Ohnmacht

Wolfgang Belzig
Valentin Wilhelm
Hannes Weisbrich

Juan Carlos Cuevas Juan
José García-Estebaran

SFB 1432

Universität
Konstanz



UAM Universidad Autónoma
de Madrid

19. March 2025

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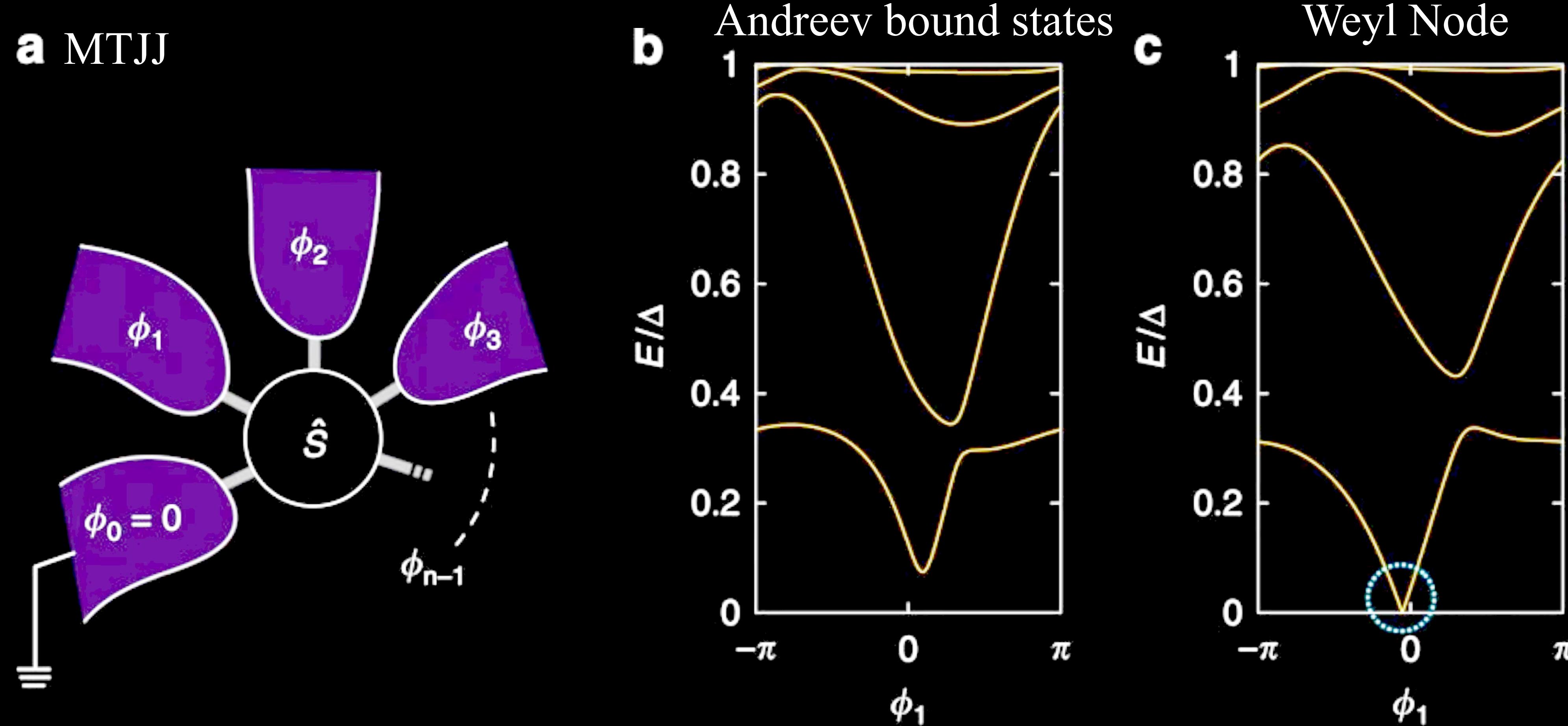
- 1) MTJJs are an excellent platform to study engineered non-hermitian topology
- 2) Refectionless scattering modes are a source of topology in MTJJs

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Topology in Multiterminal Josephson junctions (MTJJs)

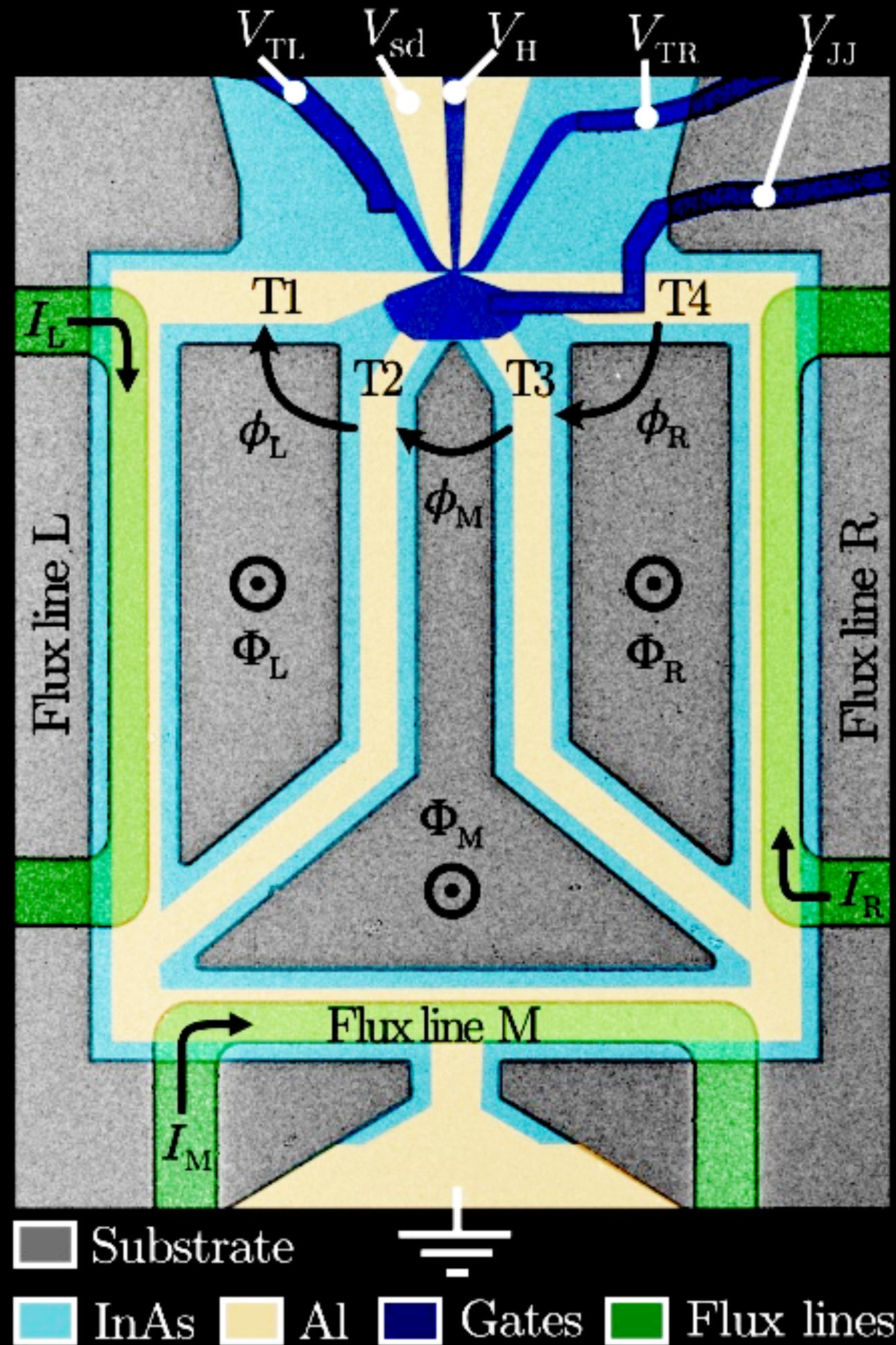


- R.-P. Riwar, et. al., Nature Commun. 7, 1 (2016)
E. Eriksson, et. al., Phys. Rev. B 95, 075417 (2017)
H.-Y. Xie, et. al., Phys. Rev. B 96, 161406(R) (2017)
R. L. Klees, et. al., Phys. Rev. Lett. 124, 197002 (2020)

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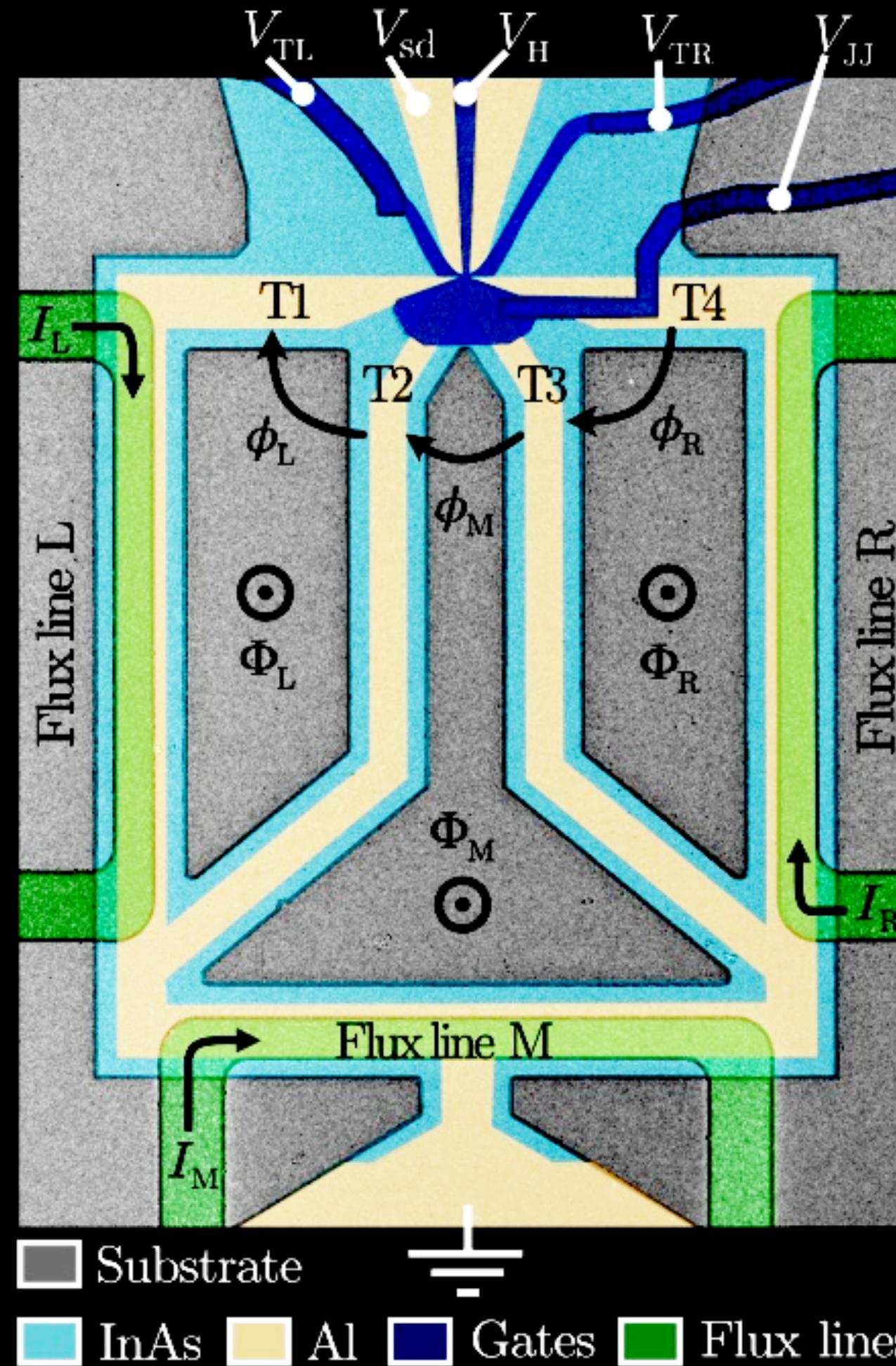
Experiments on MTJJs: Topology?

Topography of
4-terminal MTJJ

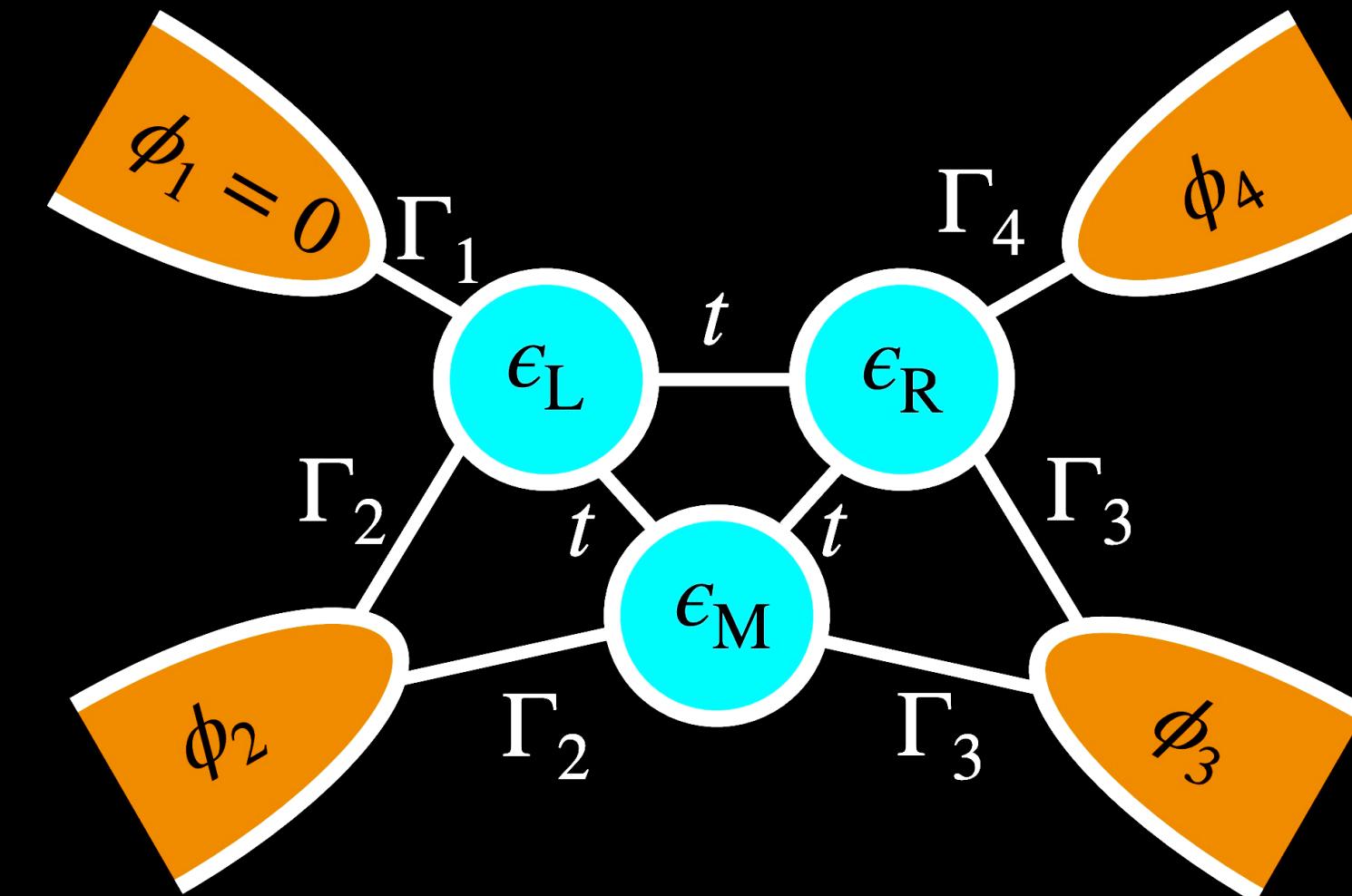


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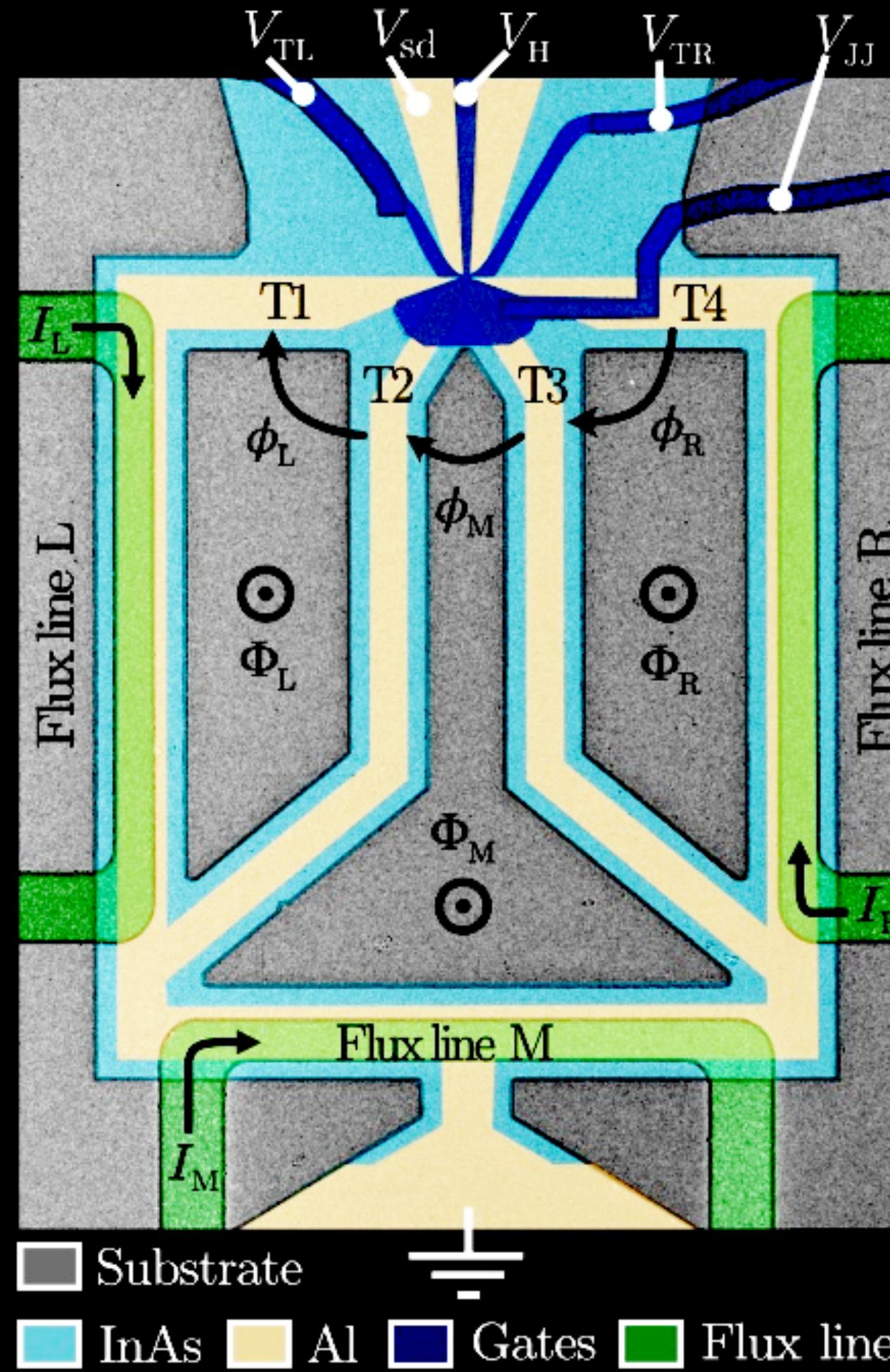


Three state Andreev molecule model

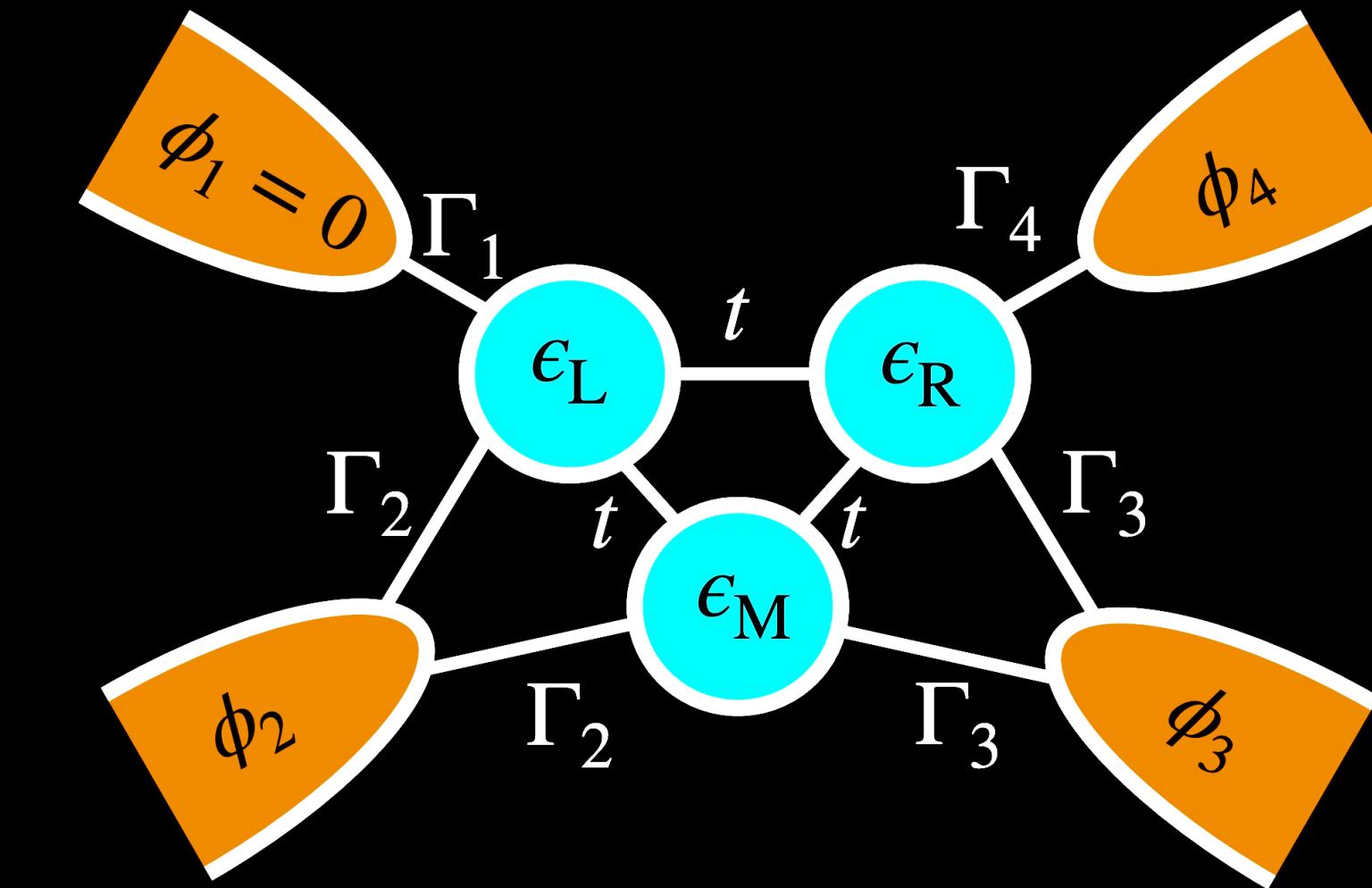


Experiments on MTJJs: Topology?

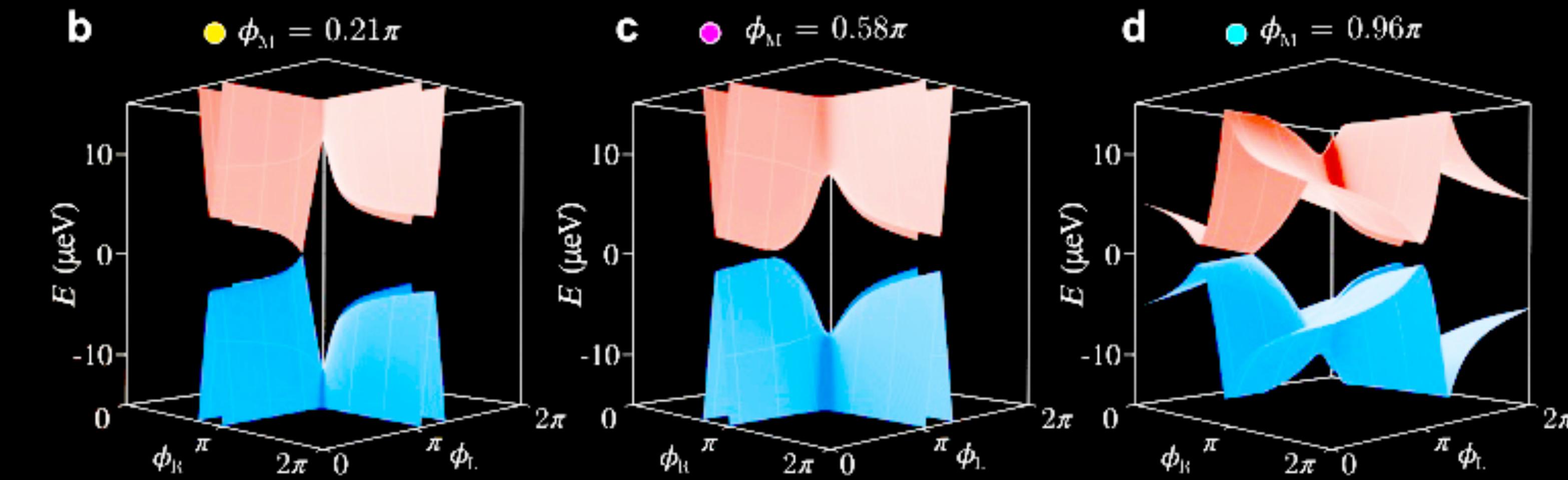
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Three state Andreev molecule model

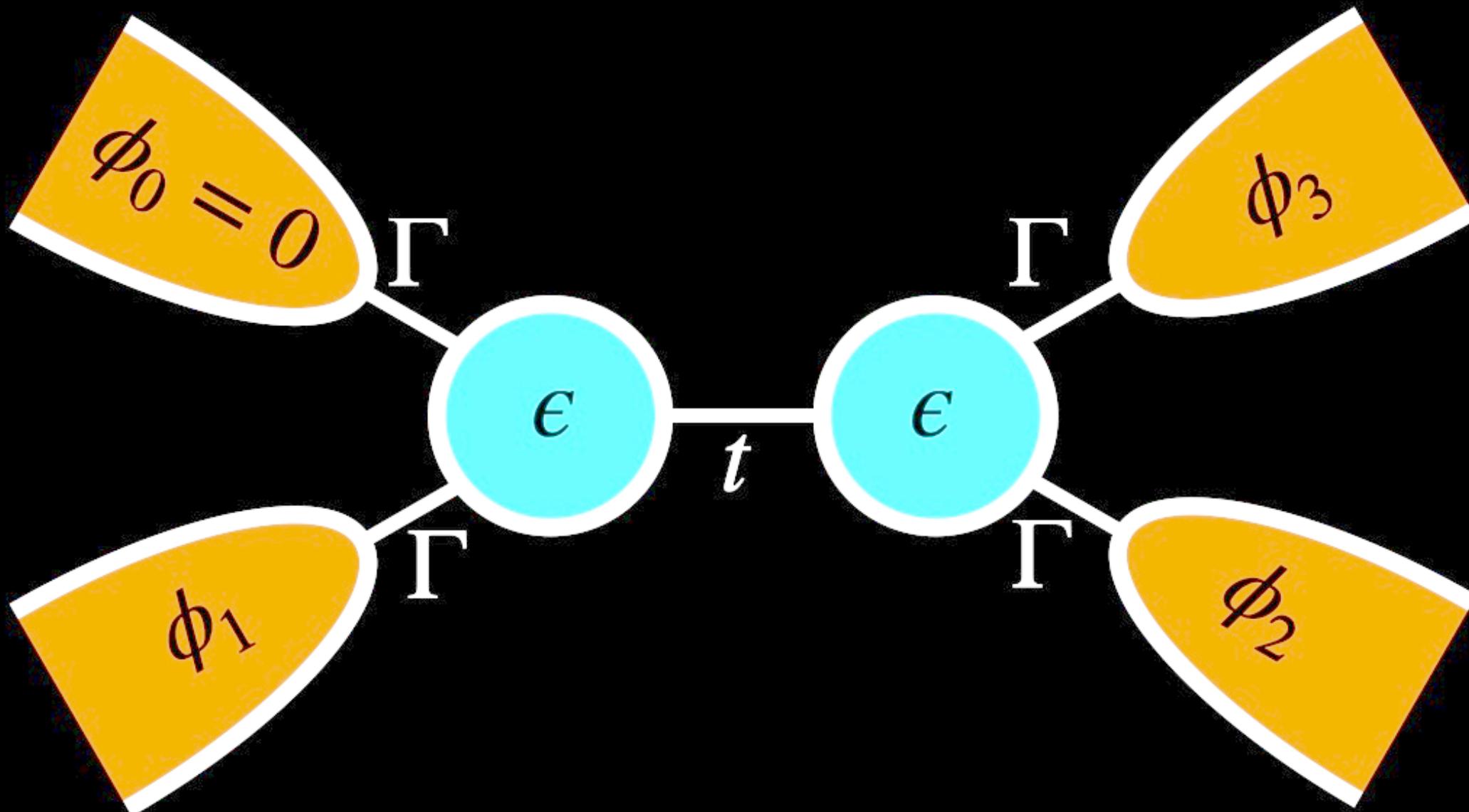


Model predicts non-trivial Topology



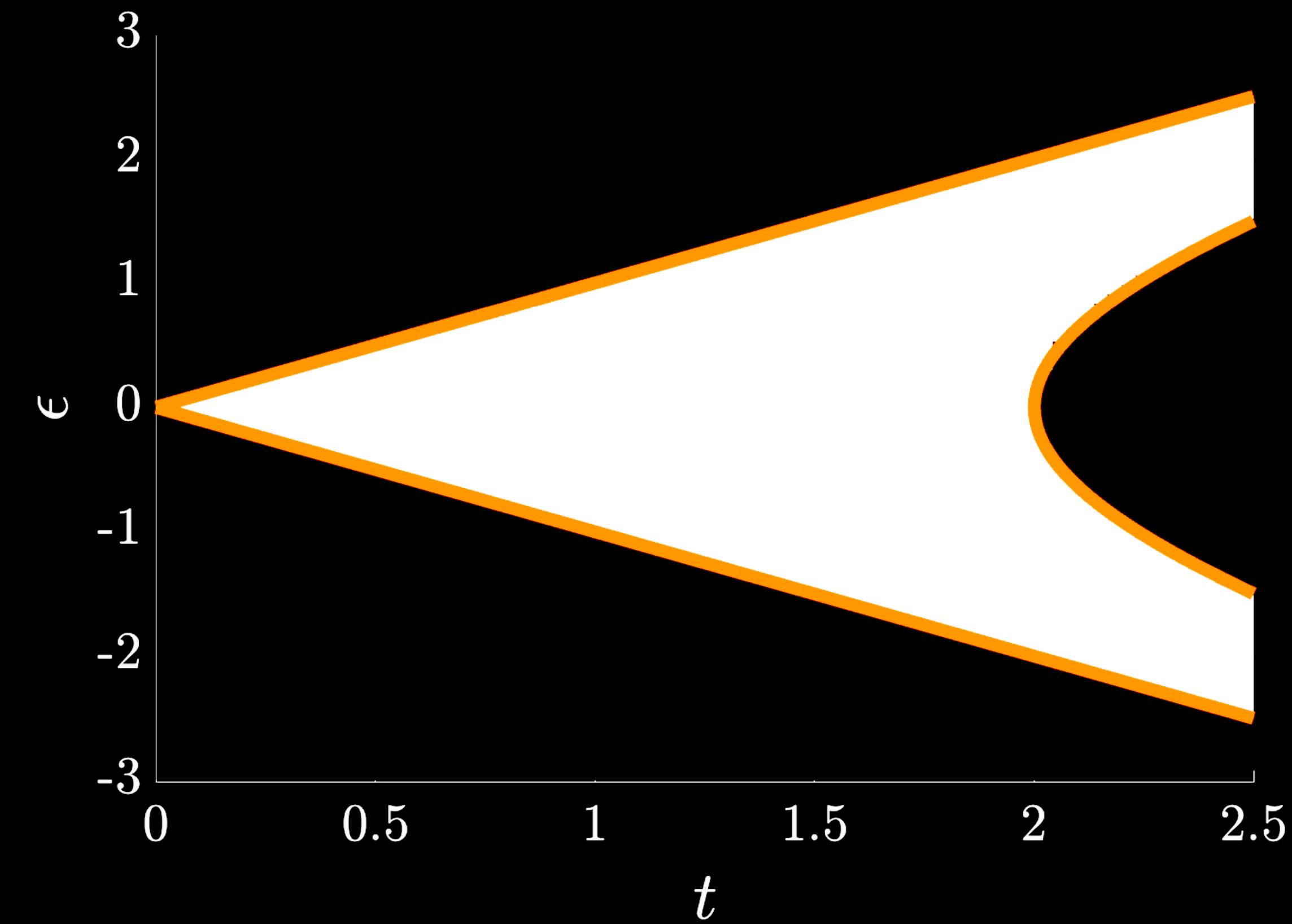
What's the source of topology in MTJJs?

4-terminal MTJJ with two dots



L. Teshler, et. al., SciPost Phys. 15, 214 (2023)

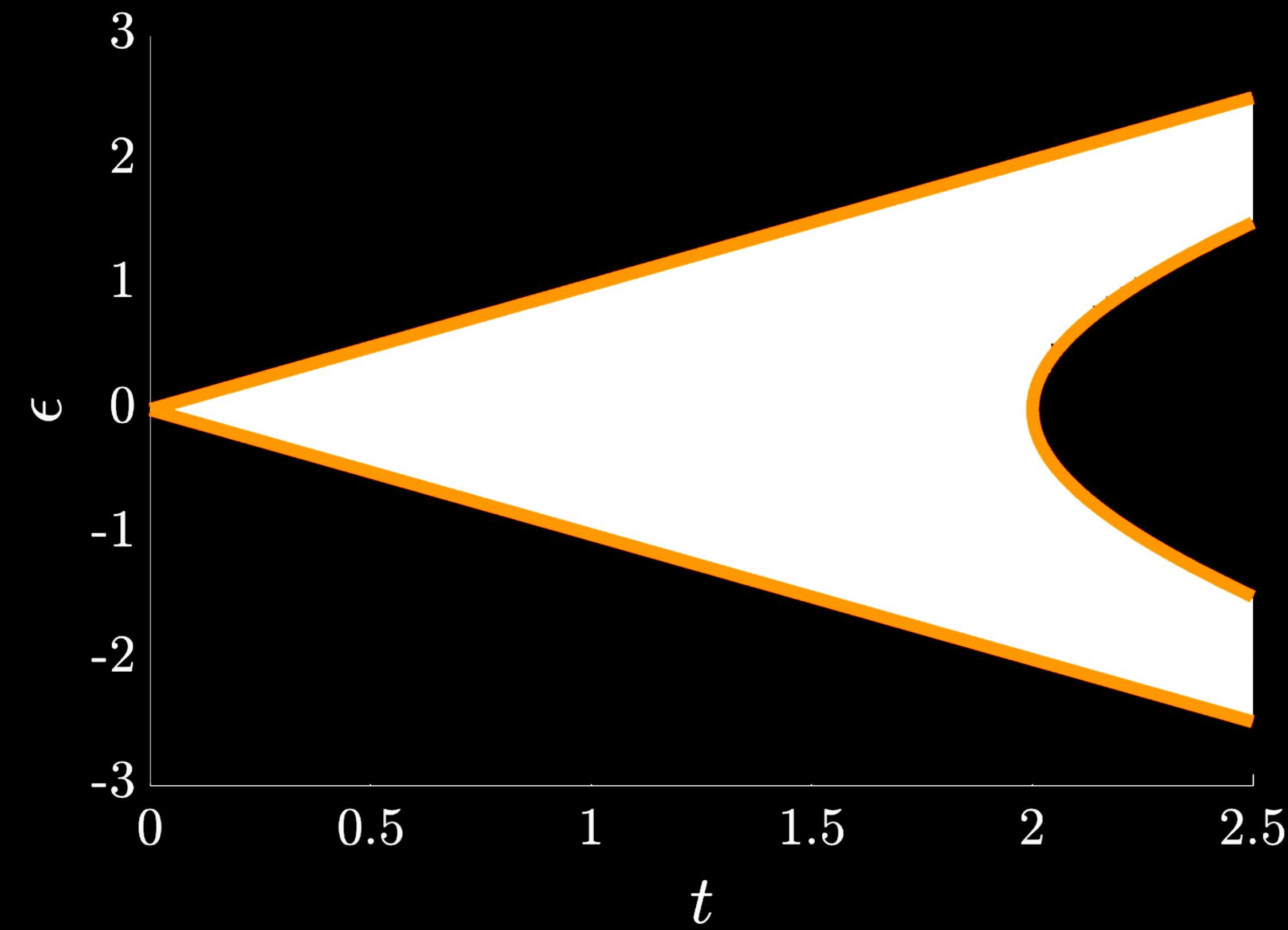
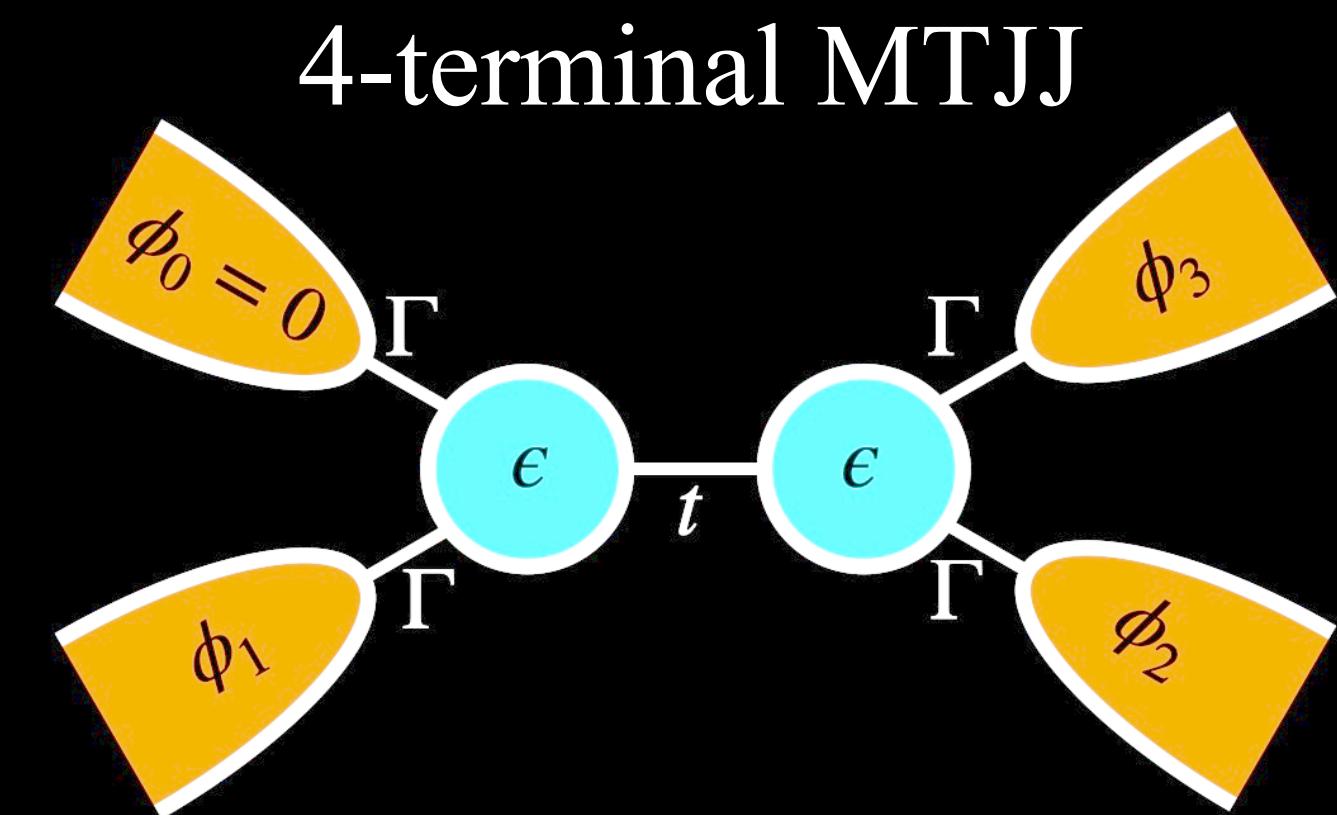
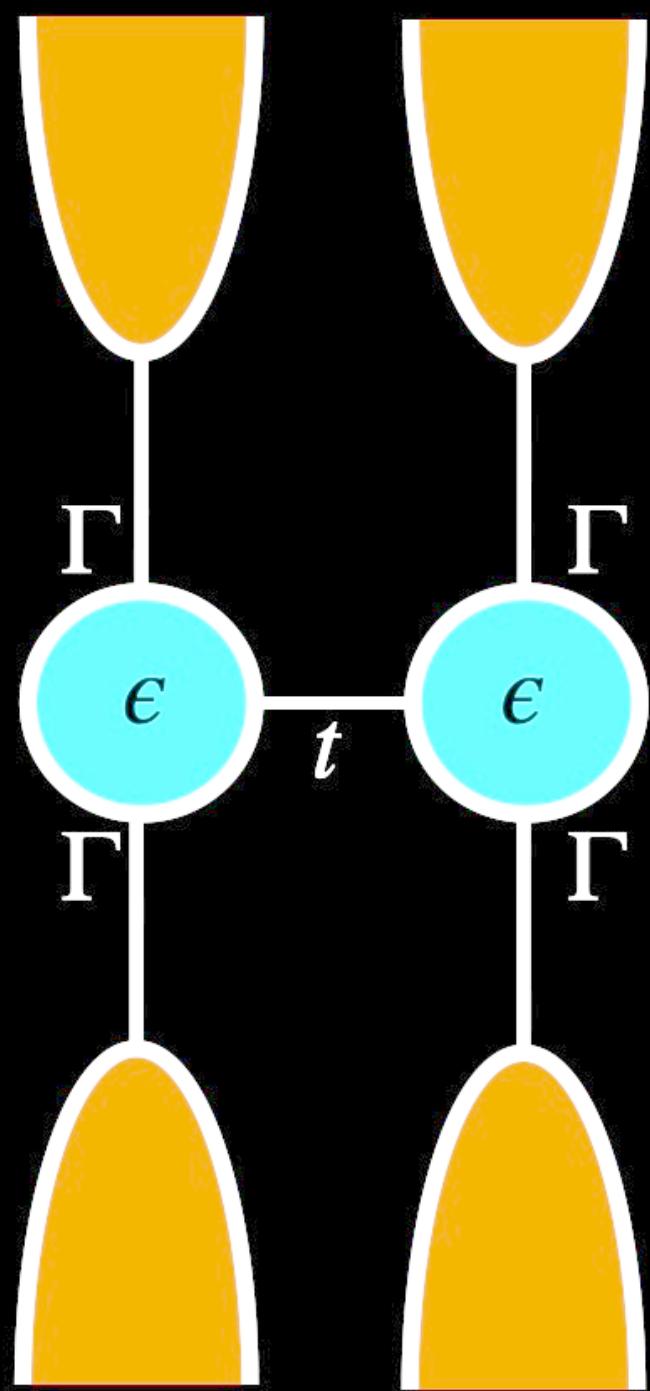
Topological phase diagram



■ non-trivial Chern number
in superconducting phase space $\{ \vec{\phi} \}$

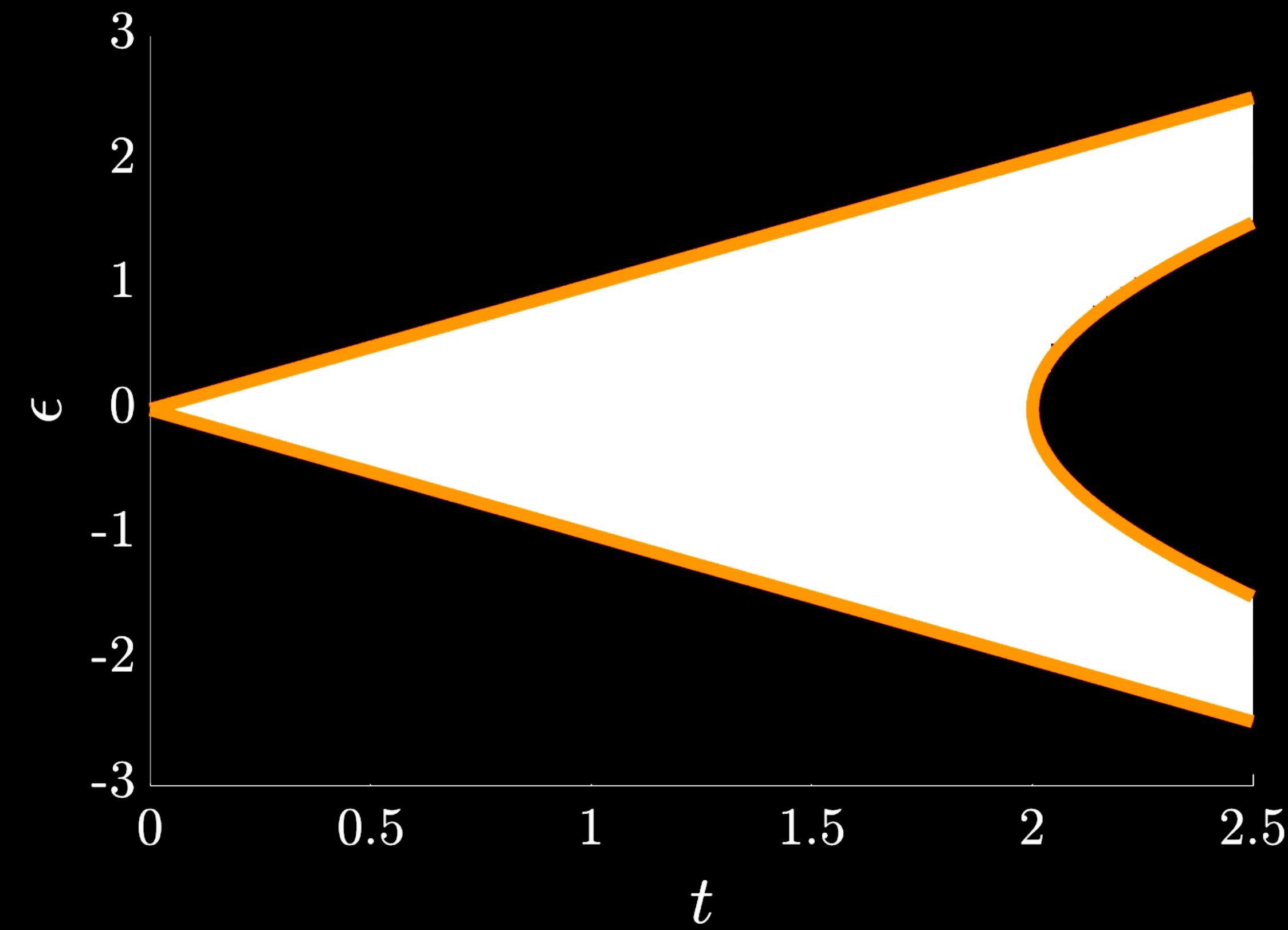
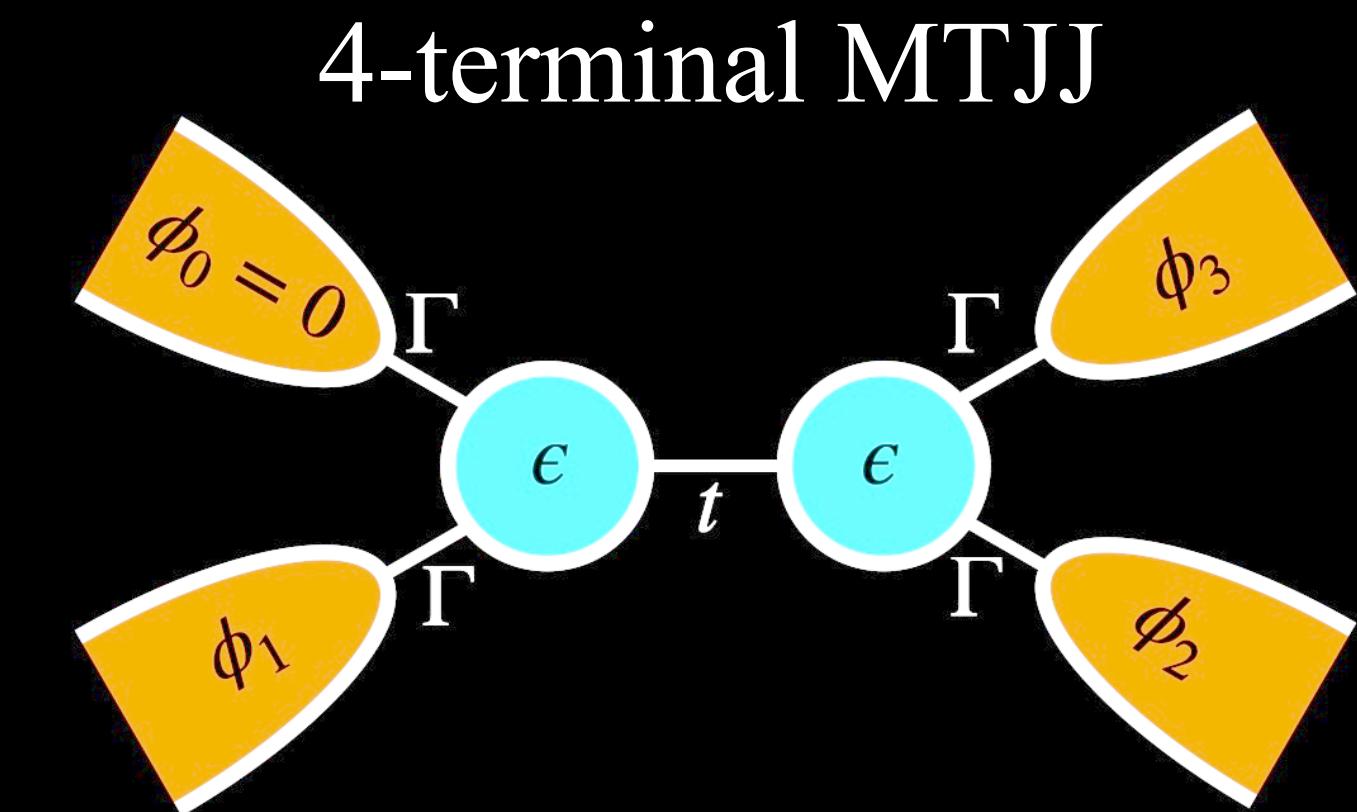
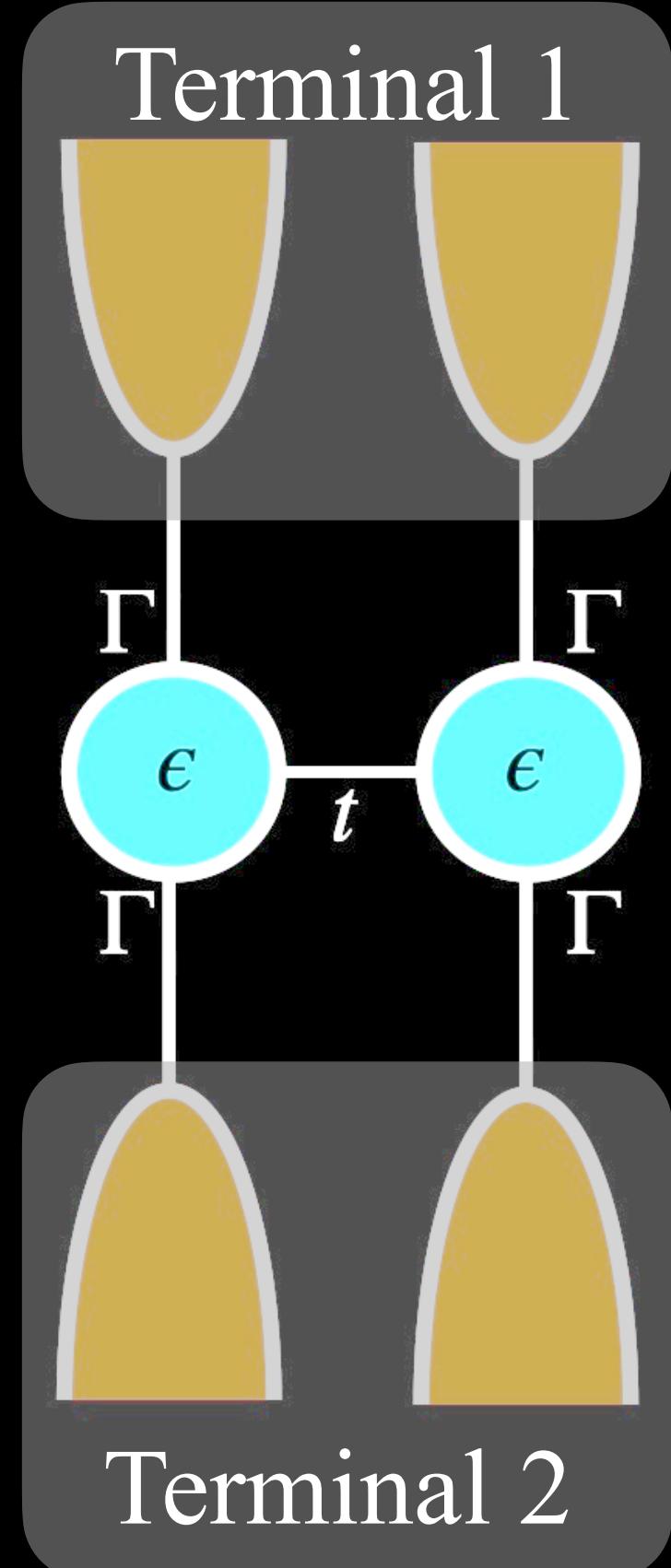
Reflectionless modes as source of Weyl Nodes

effective 2-terminal
MTJJ



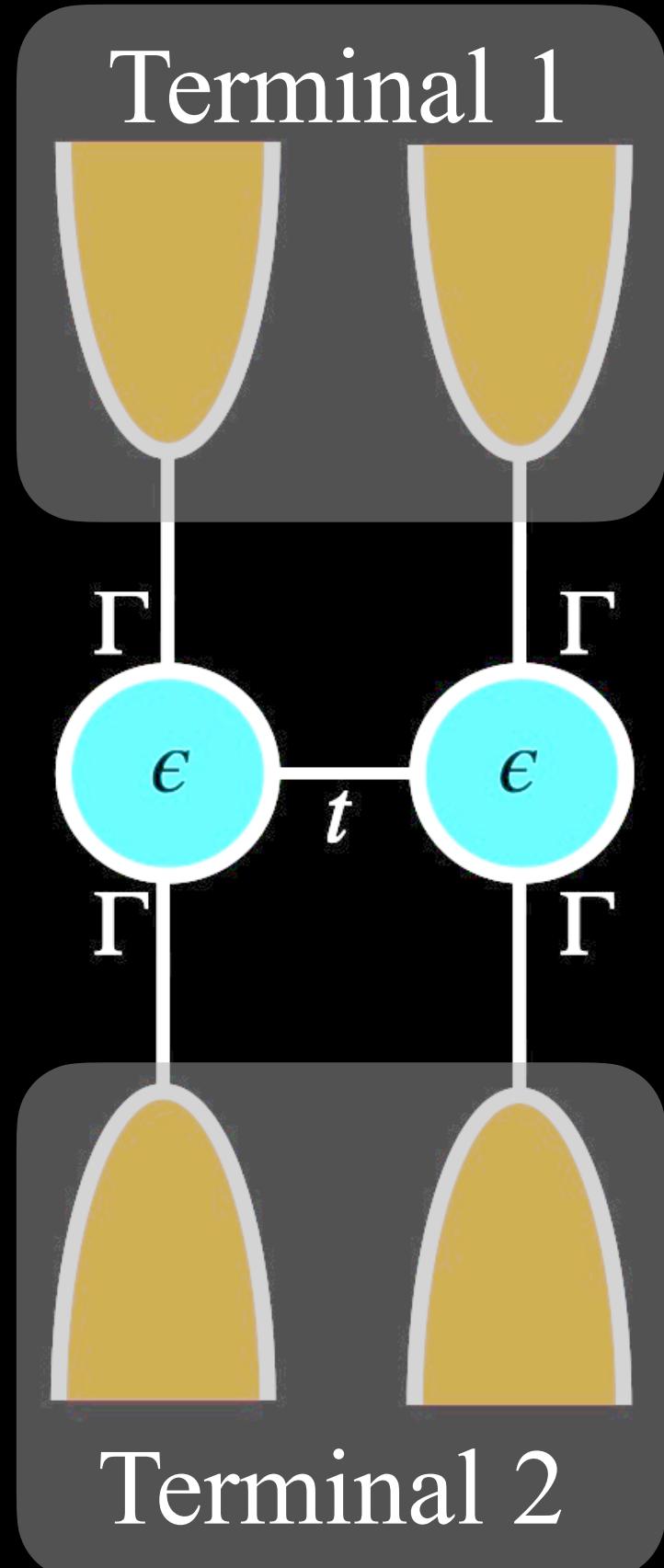
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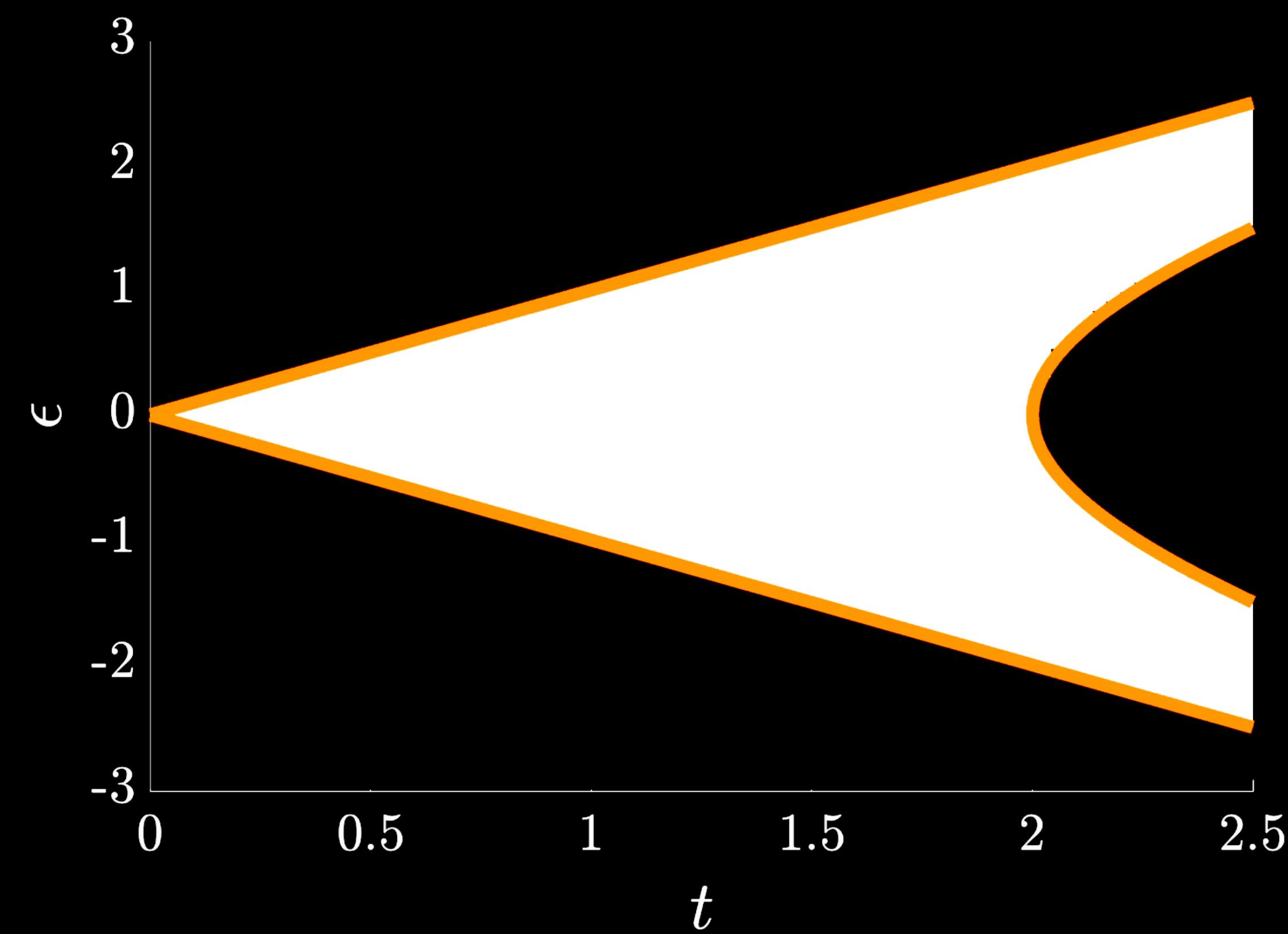
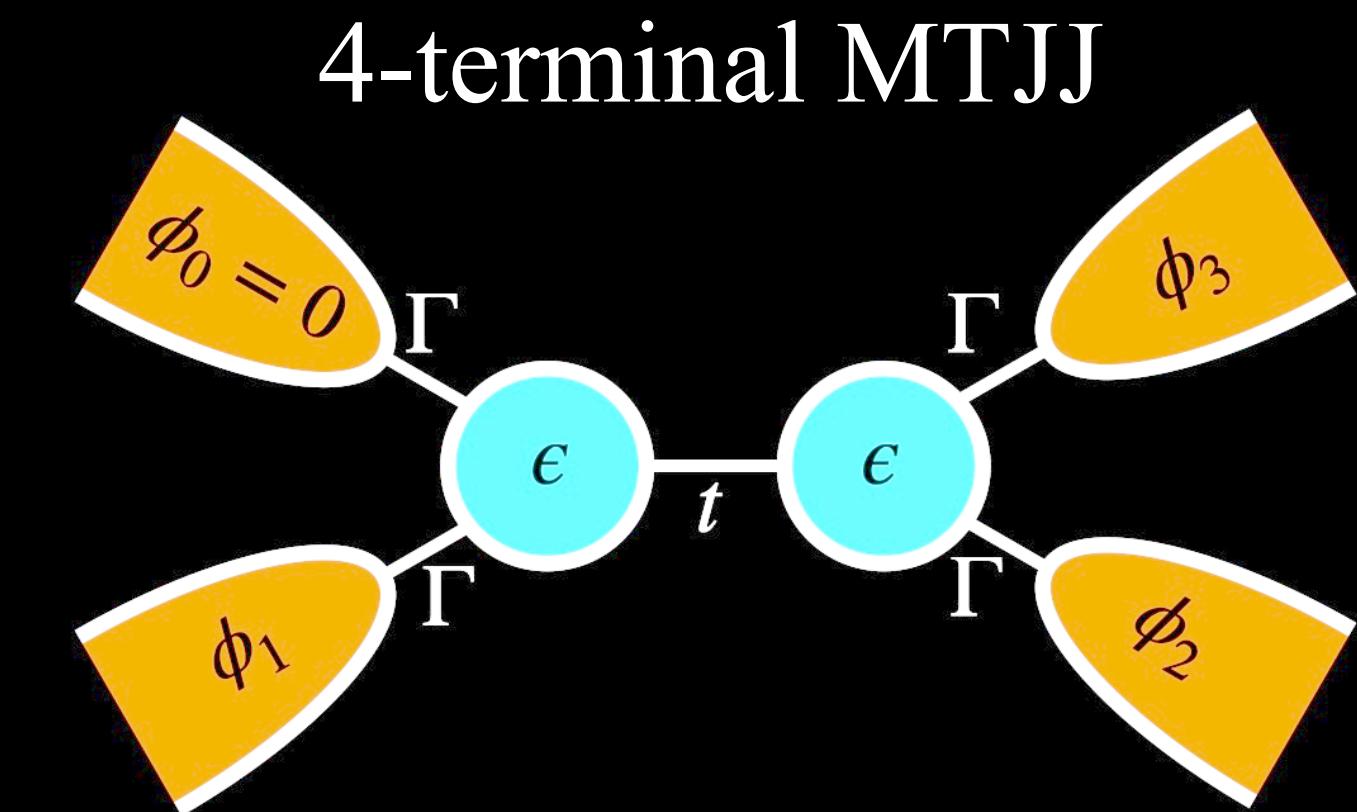
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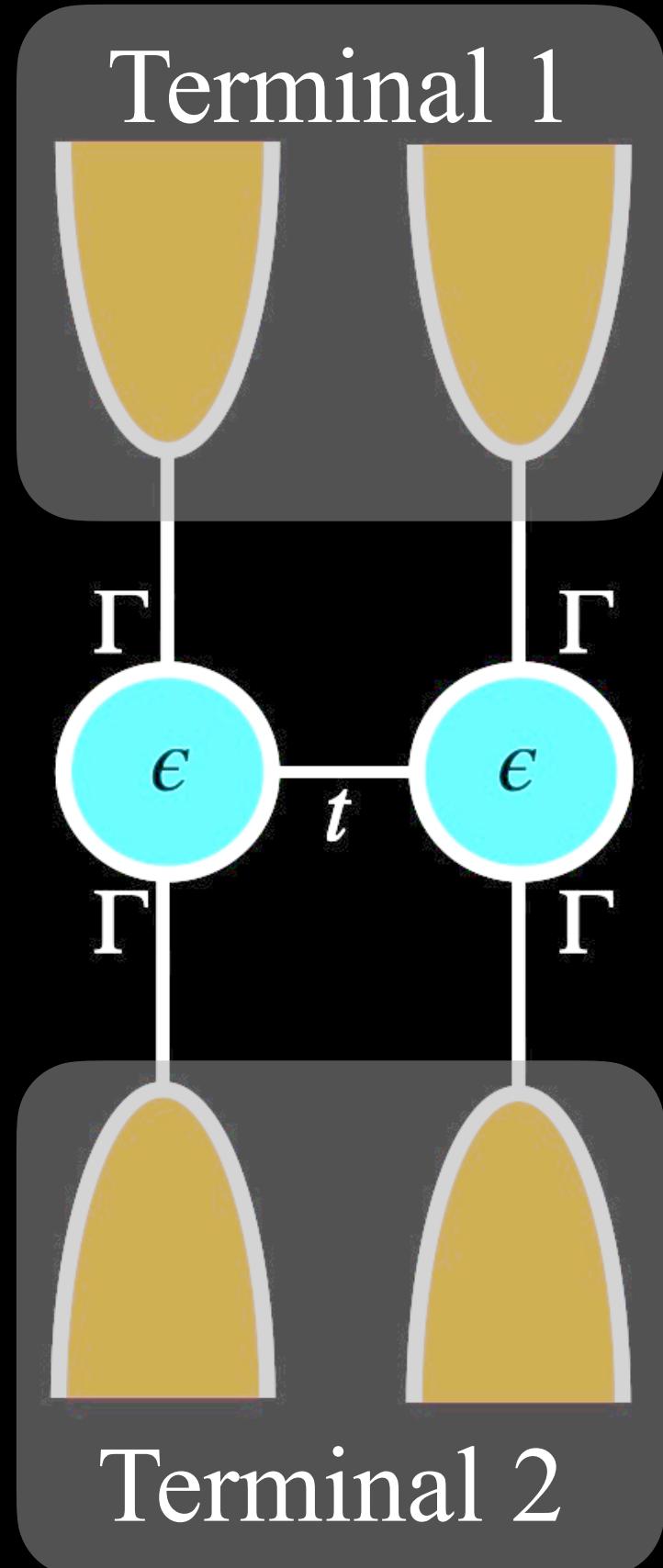
Normal state
Scattering matrix

$$S_N = \begin{pmatrix} r_{2 \times 2} & t_{2 \times 2} \\ t'_{2 \times 2} & r'_{2 \times 2} \end{pmatrix}$$



Reflectionless modes as source of Weyl Nodes

effective 2-terminal
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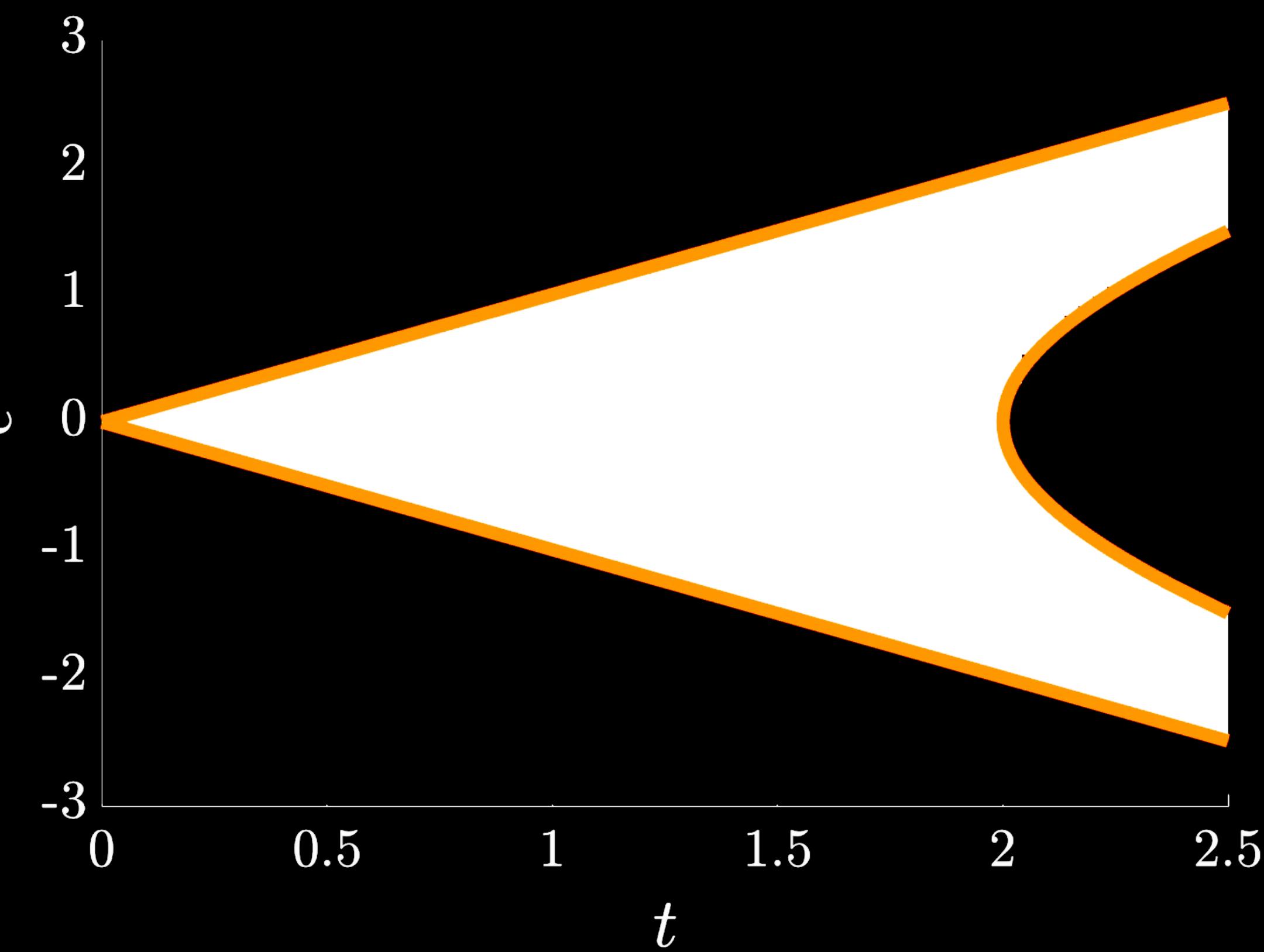
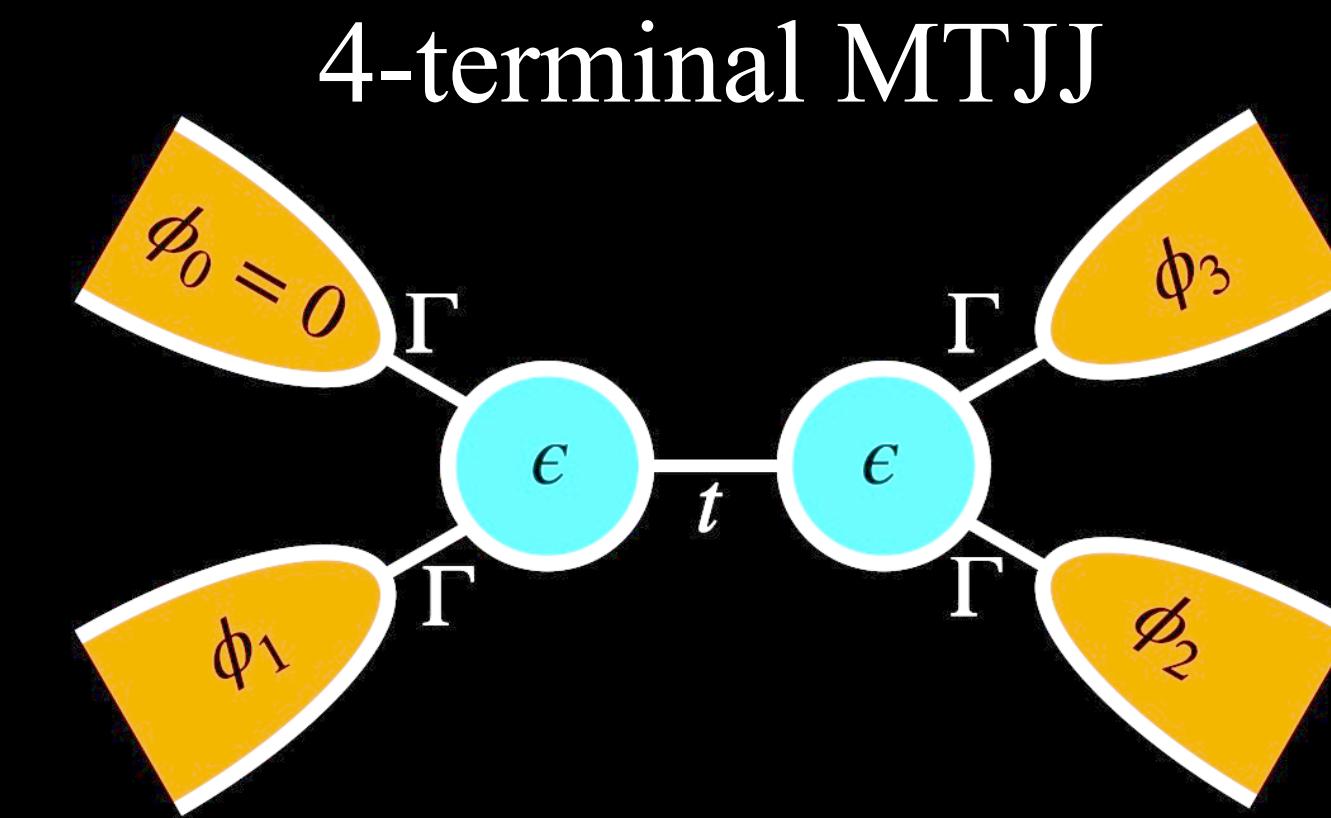


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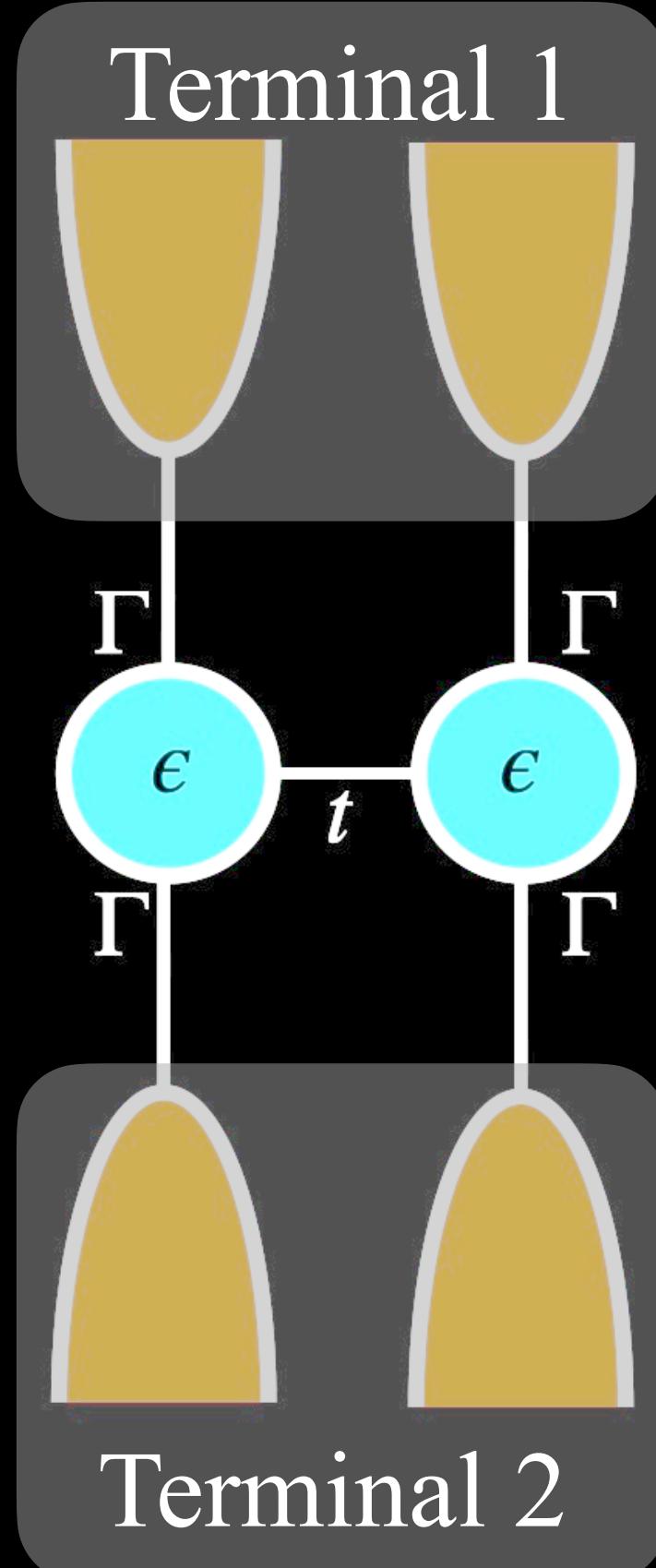
Diagonalized refl. matrix

$$D_{r_{2 \times 2}} = \begin{pmatrix} \frac{E - (\epsilon - t)}{E - (\epsilon - t - 2i\Gamma)} & 0 \\ 0 & \frac{E - (\epsilon + t)}{E - (\epsilon + t - 2i\Gamma)} \end{pmatrix}$$



Reflectionless modes as source of Weyl Nodes

effective 2-terminal
MTJJ

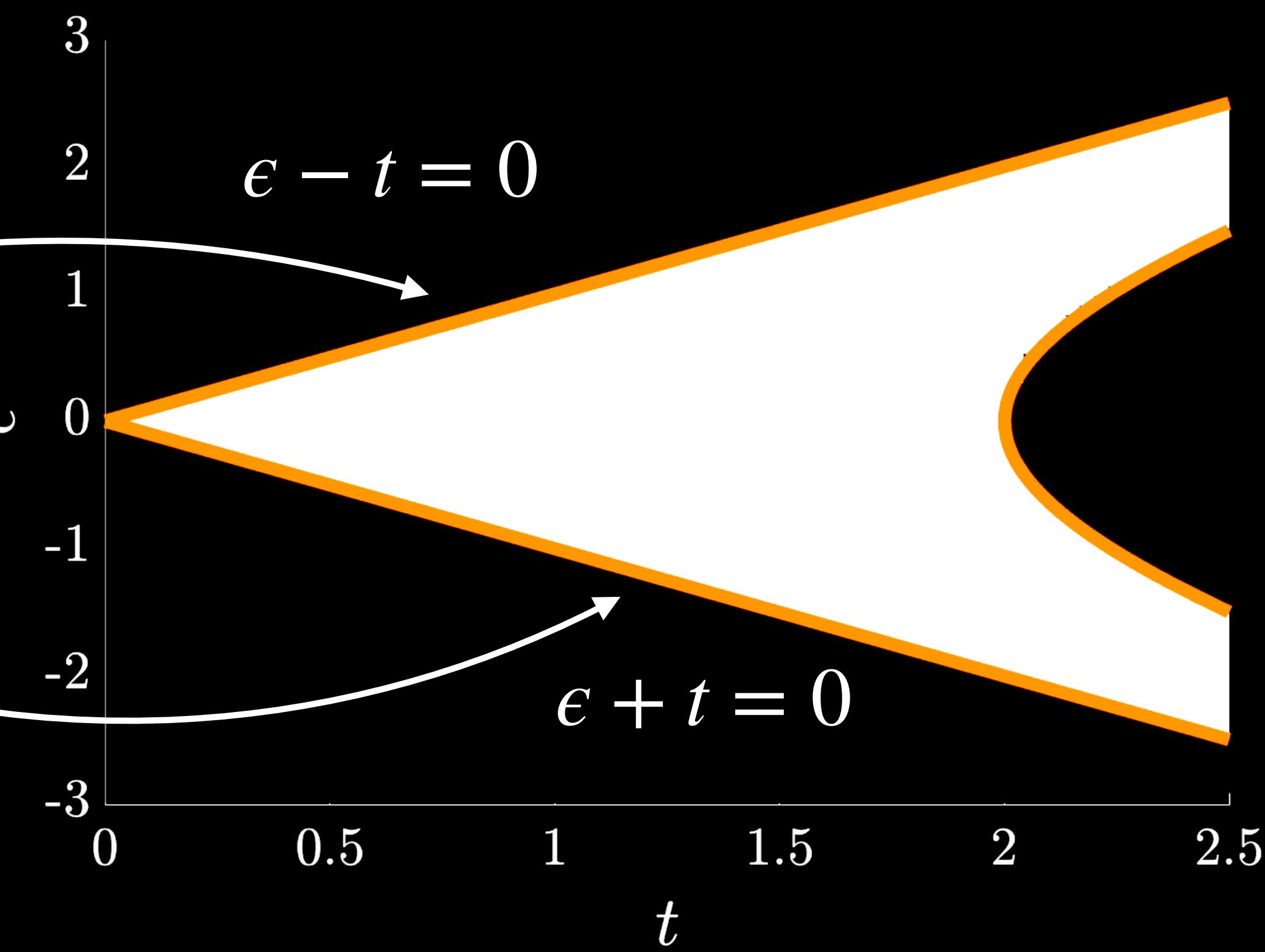
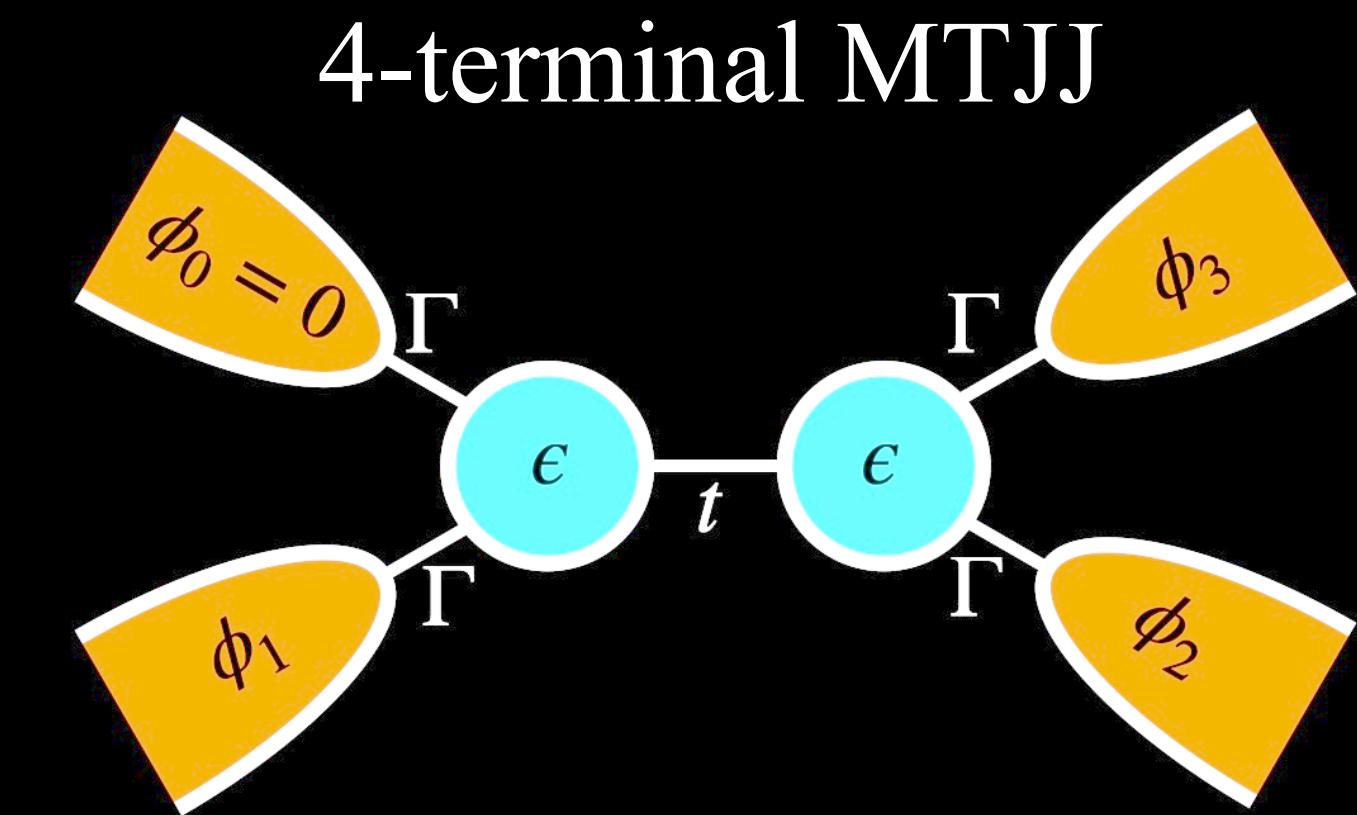


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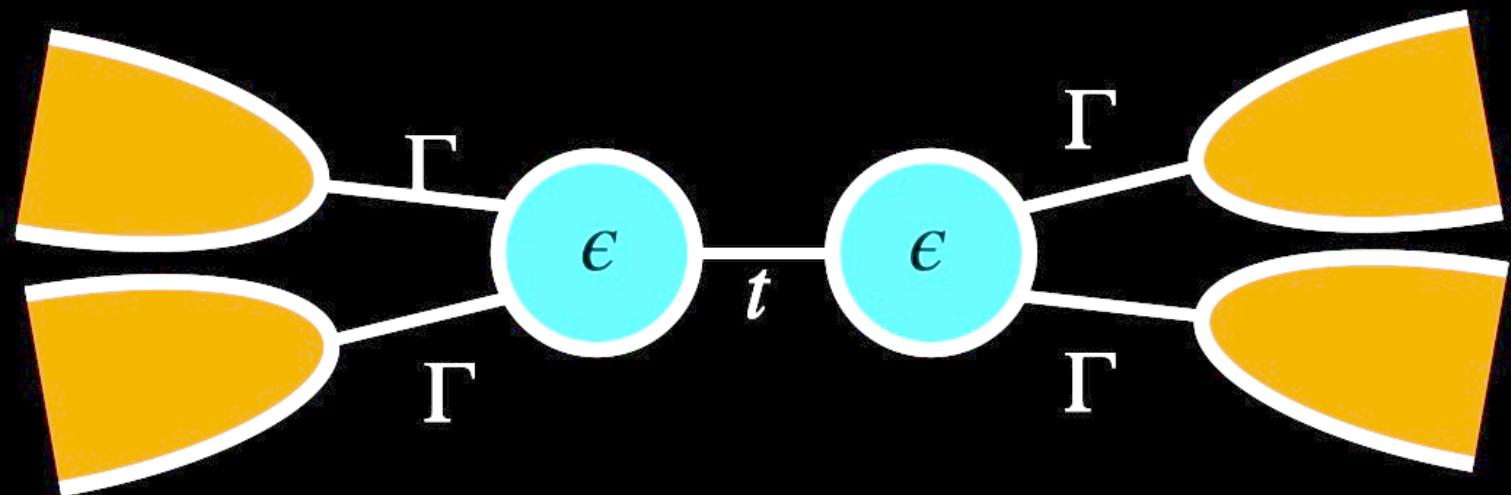
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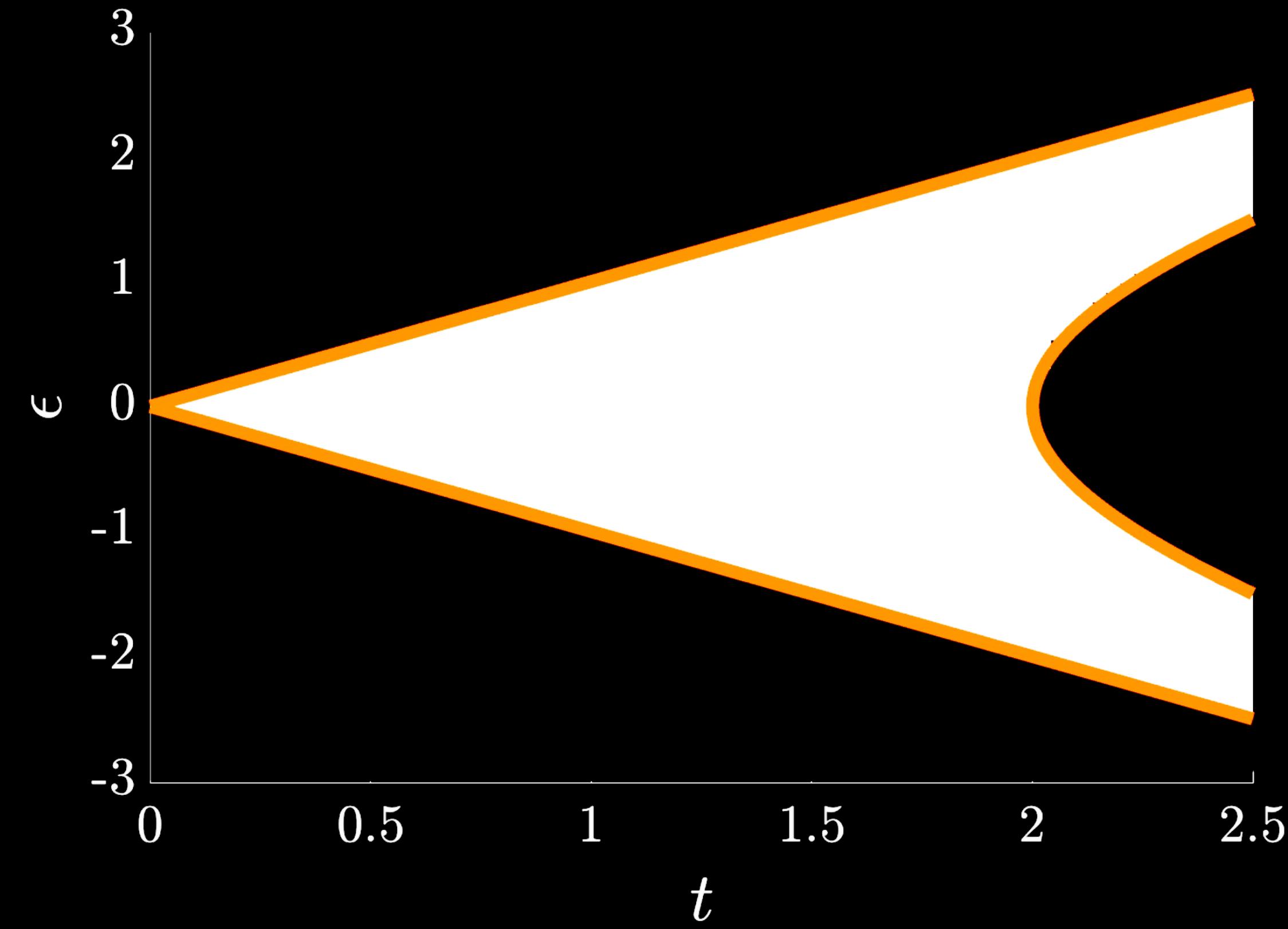
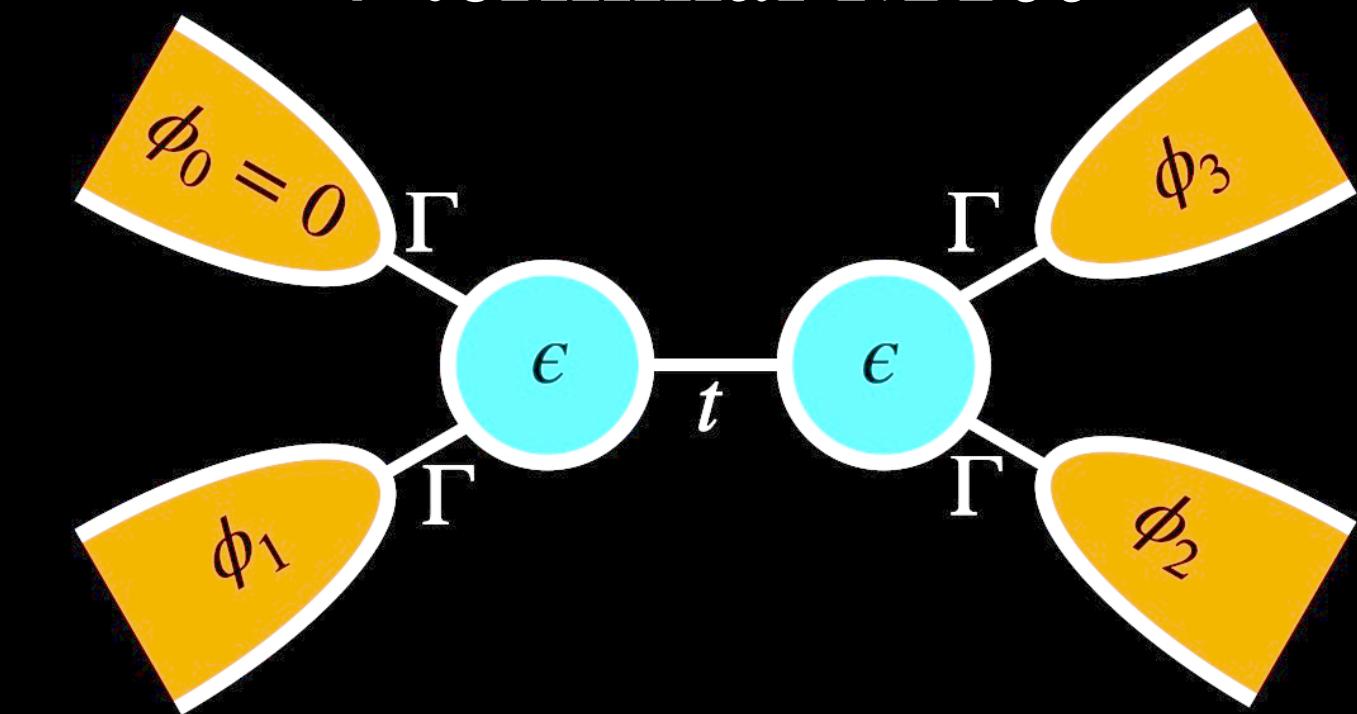


Reflectionless modes as source of Weyl Nodes

effective 2-terminal MTJJ

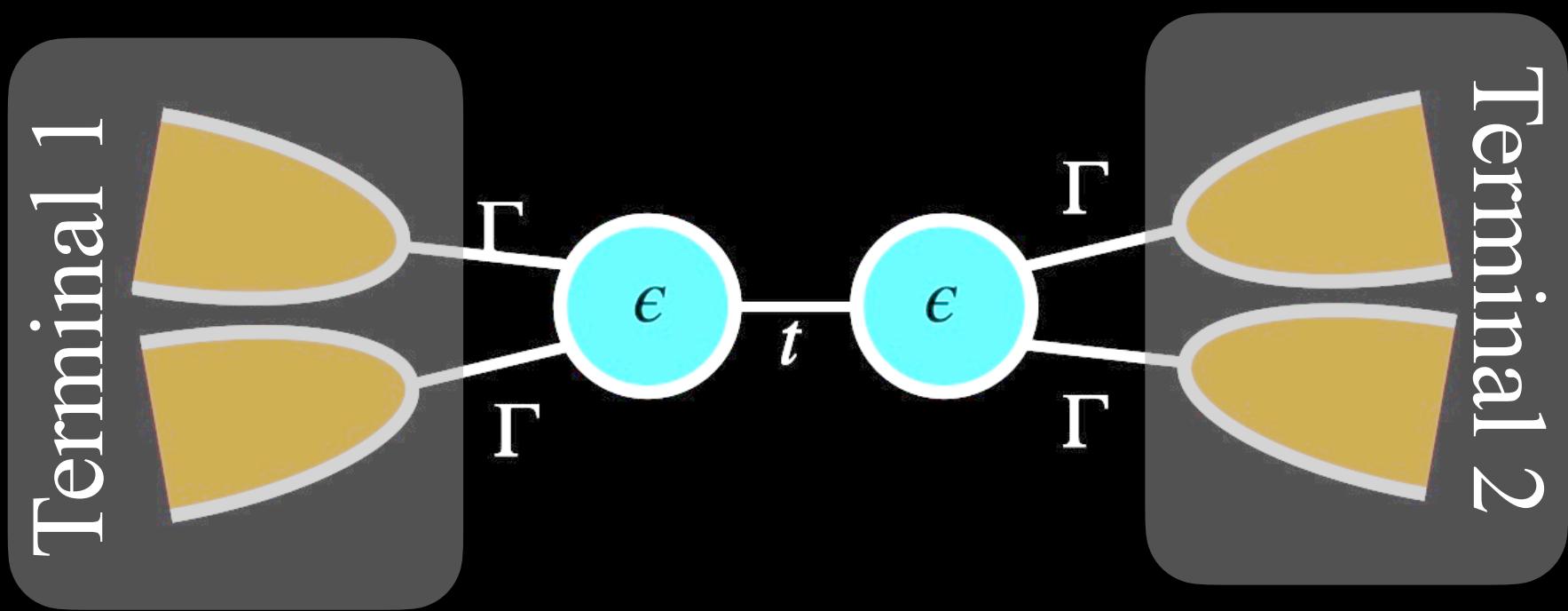


4-terminal MTJJ

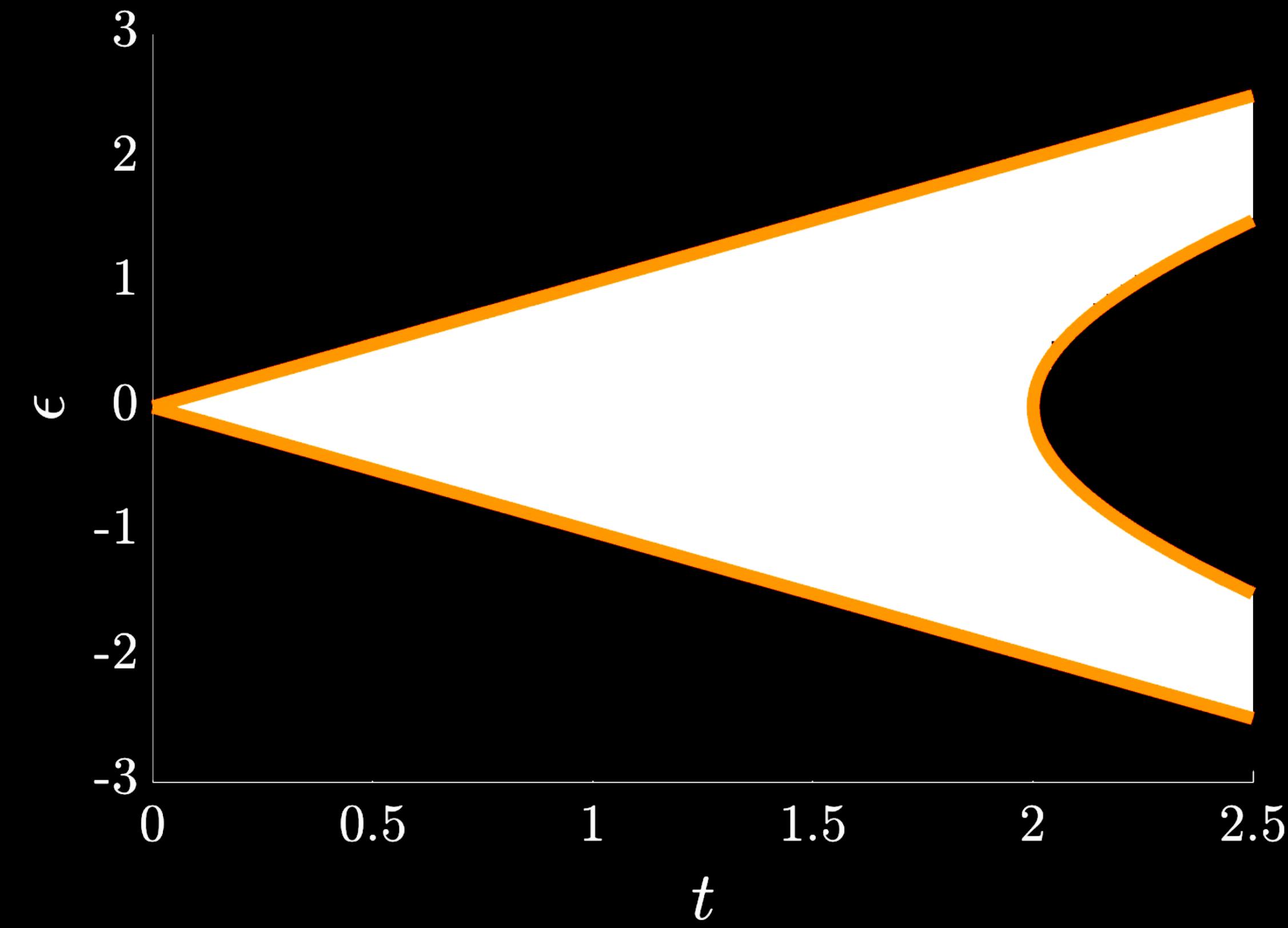
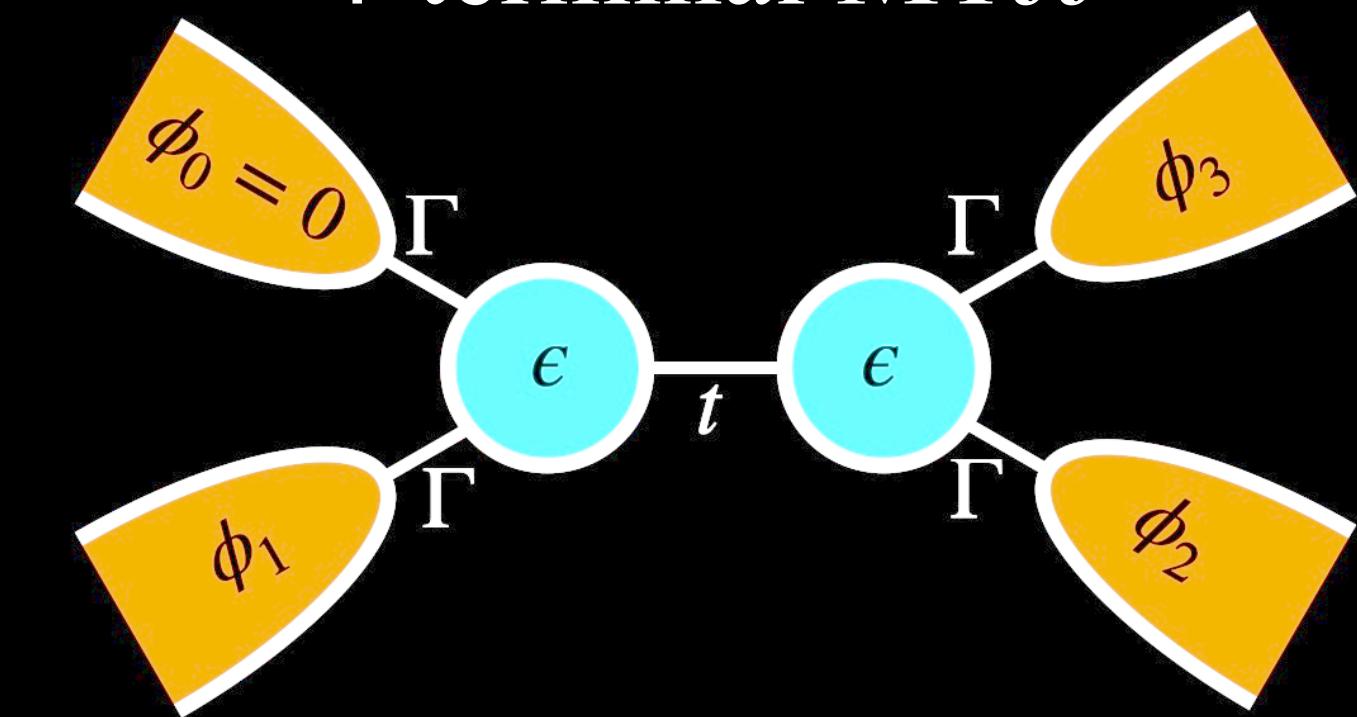


Reflectionless modes as source of Weyl Nodes

effective 2-terminal MTJJ

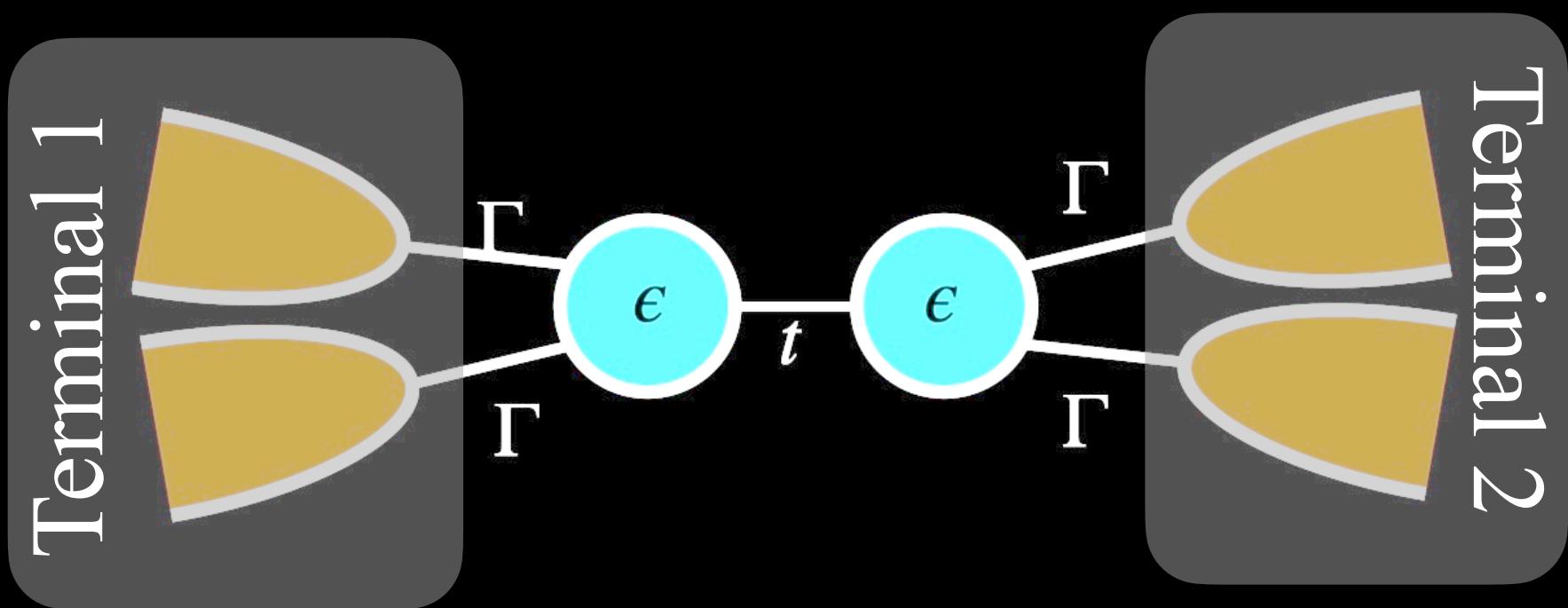


4-terminal MTJJ



Reflectionless modes as source of Weyl Nodes

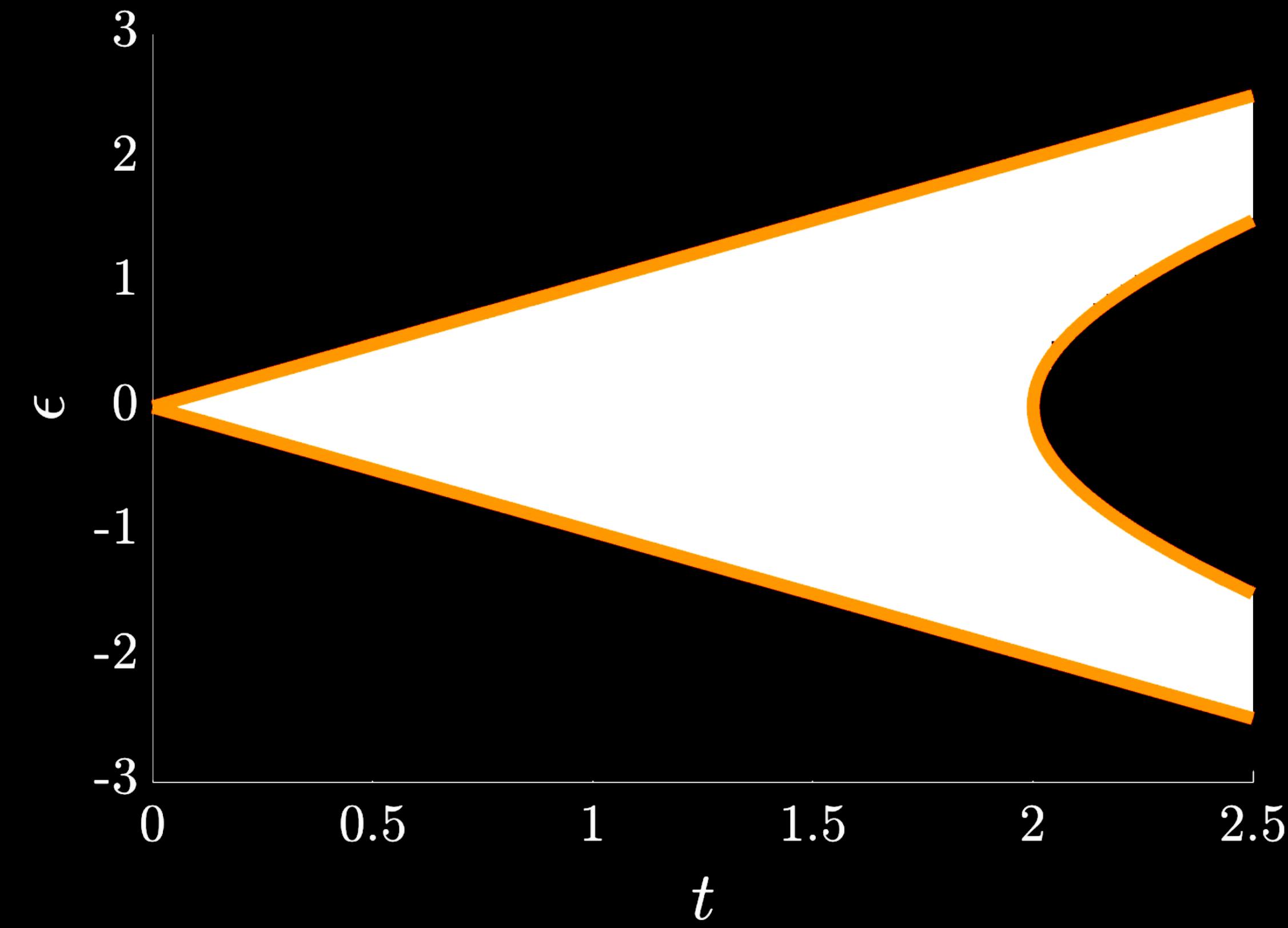
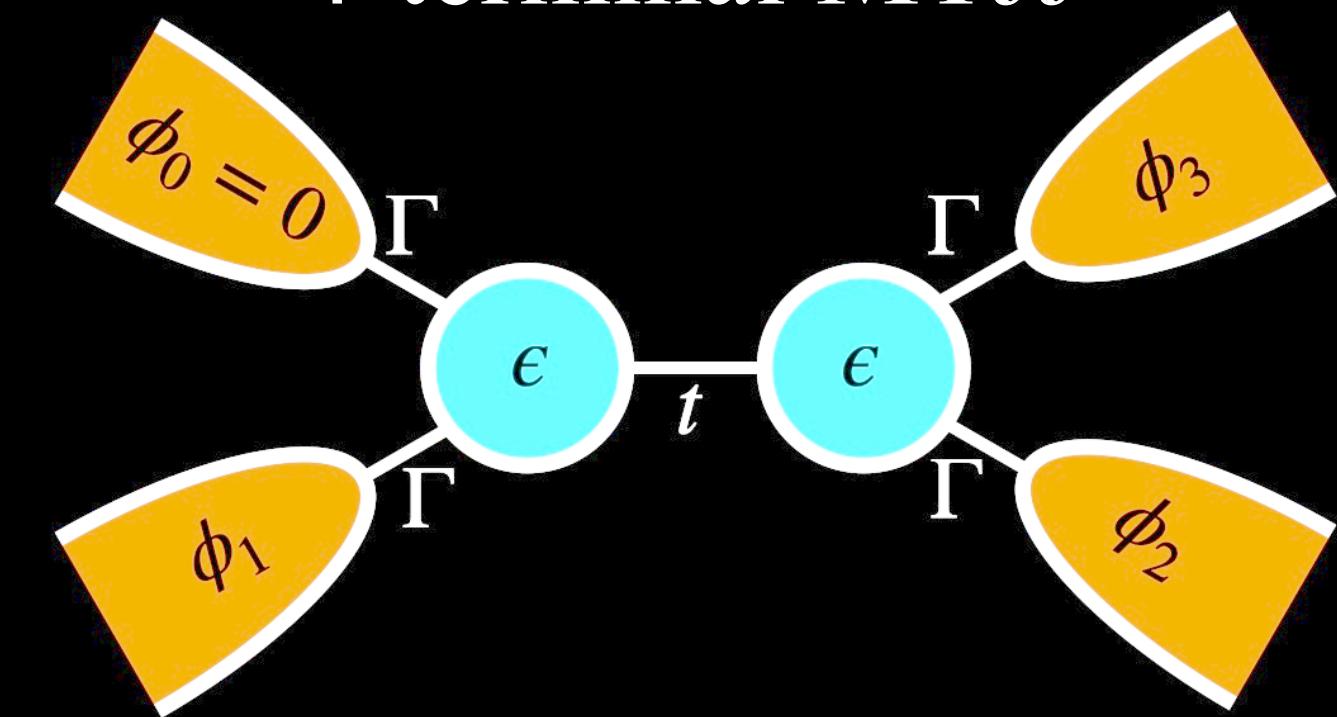
effective 2-terminal MTJJ



Scattering matrix

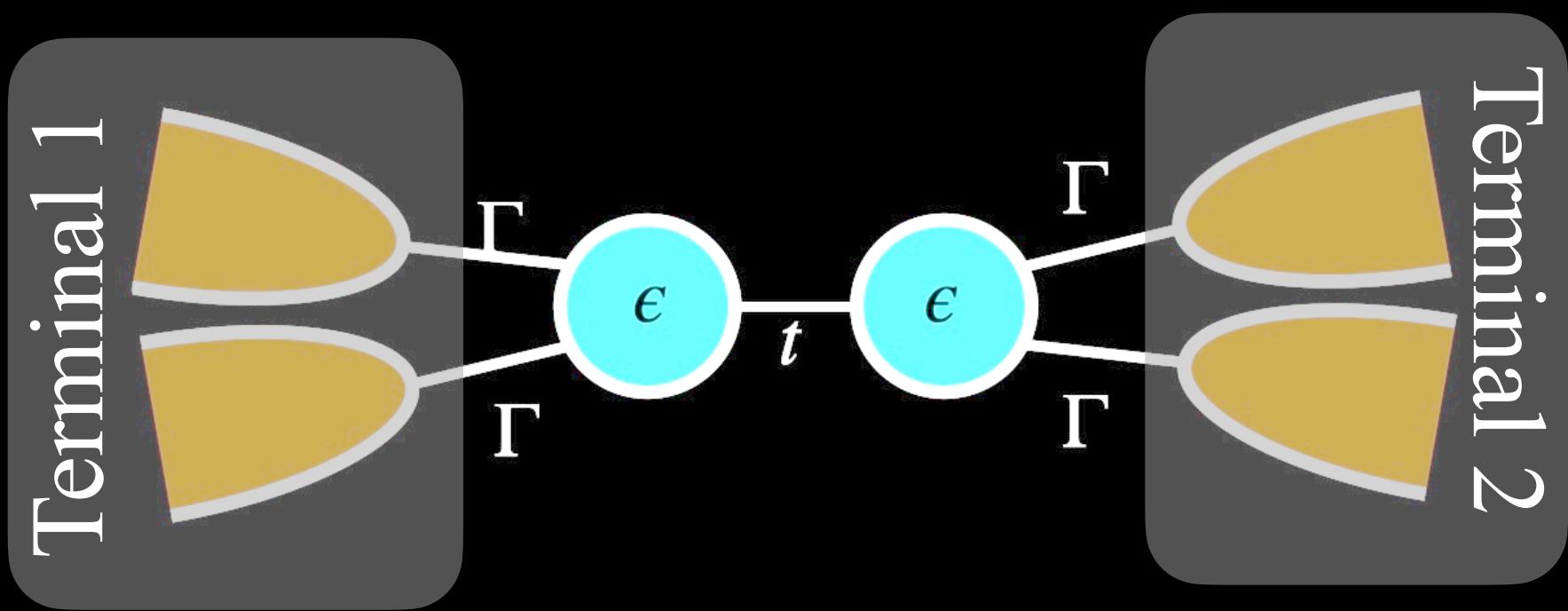
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4-terminal MTJJ



Reflectionless modes as source of Weyl Nodes

effective 2-terminal MTJJ



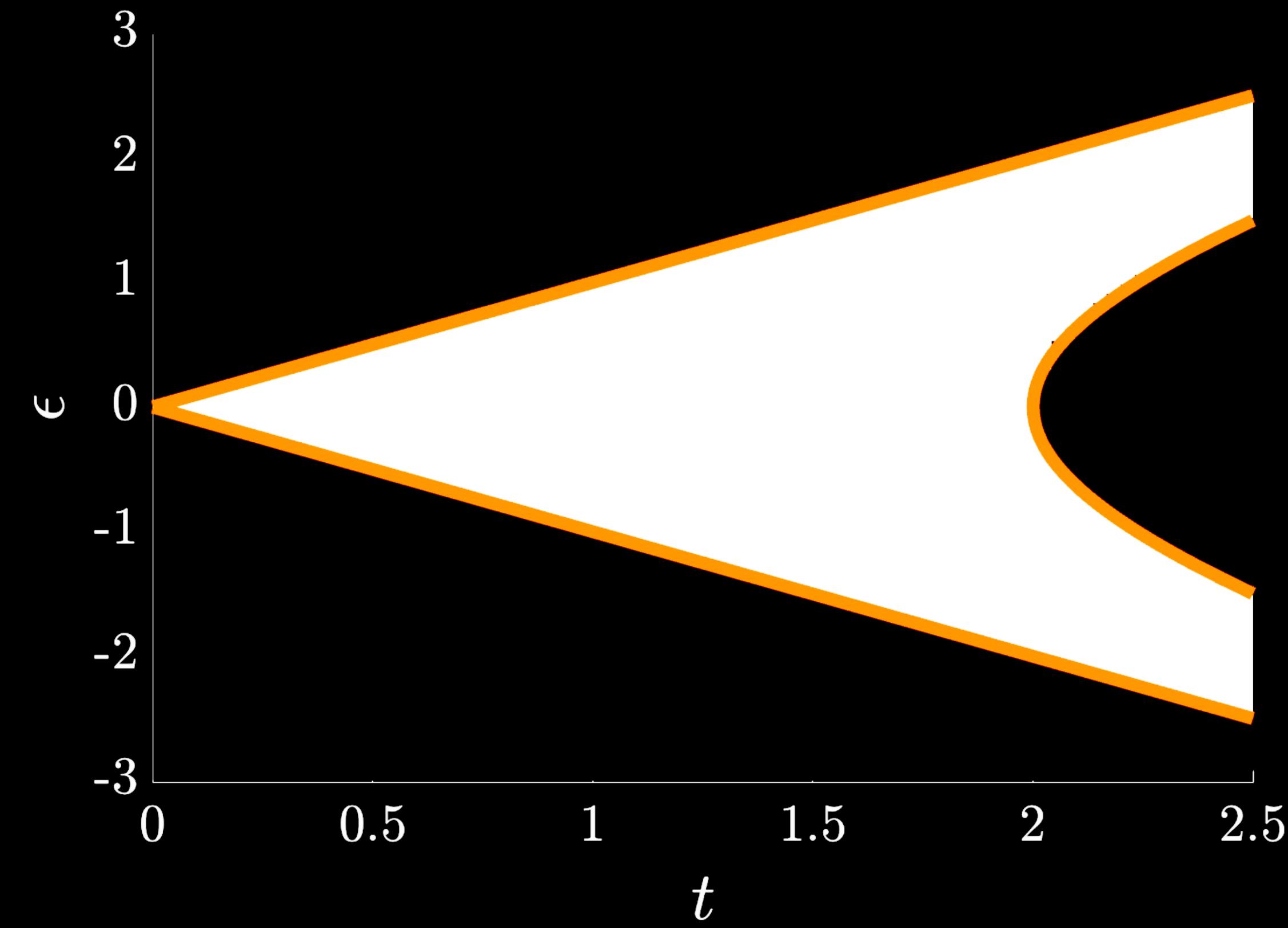
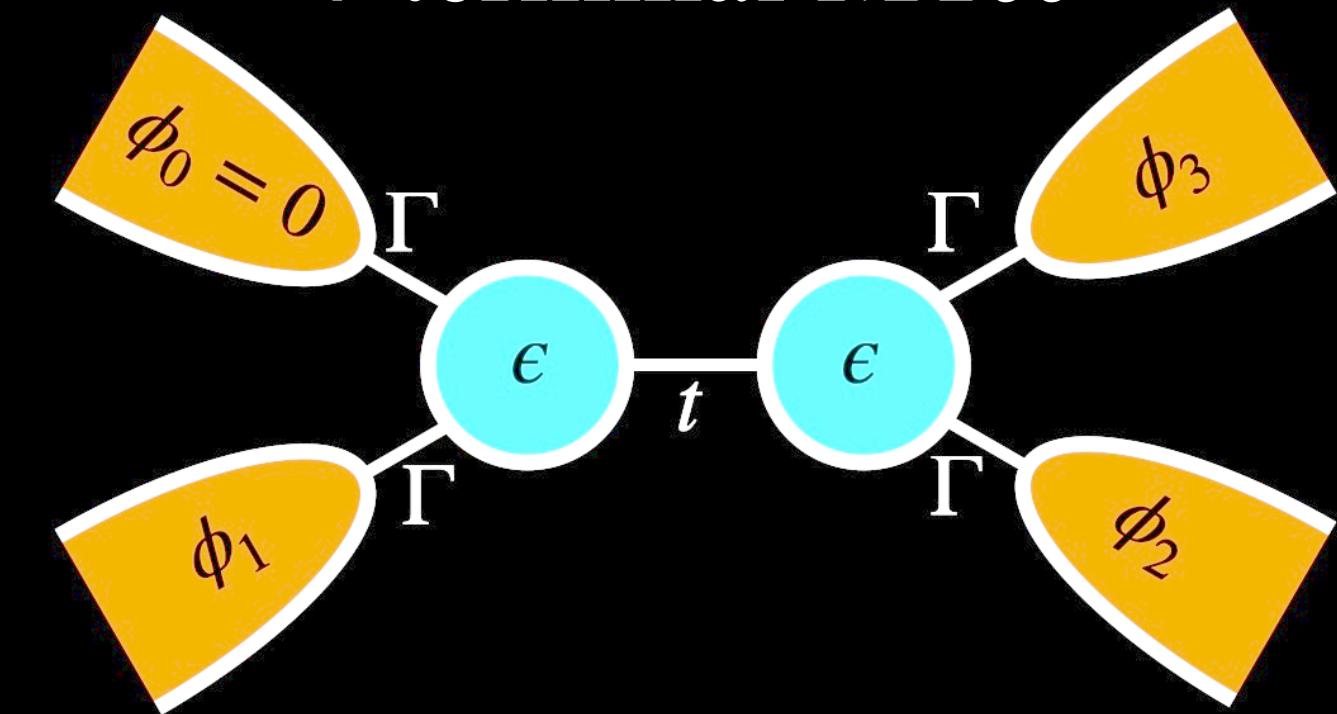
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Diagonalized refl. matrix

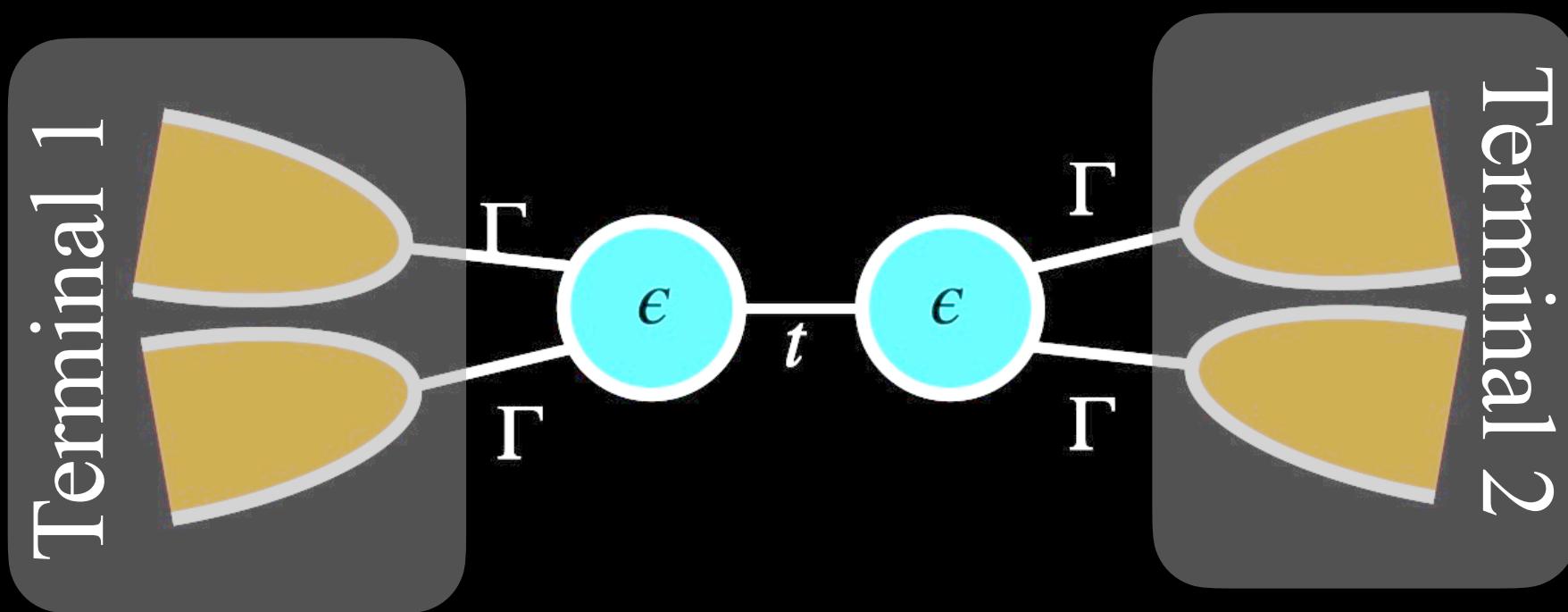
$$D_{r_{2 \times 2}} = \begin{pmatrix} \left[E(E - e) - (t^2 - \epsilon^2 - 4\Gamma^2) \right] / D(E) & 0 \\ 0 & 1 \end{pmatrix}$$

4-terminal MTJJ



Reflectionless modes as source of Weyl Nodes

effective 2-terminal MTJJ



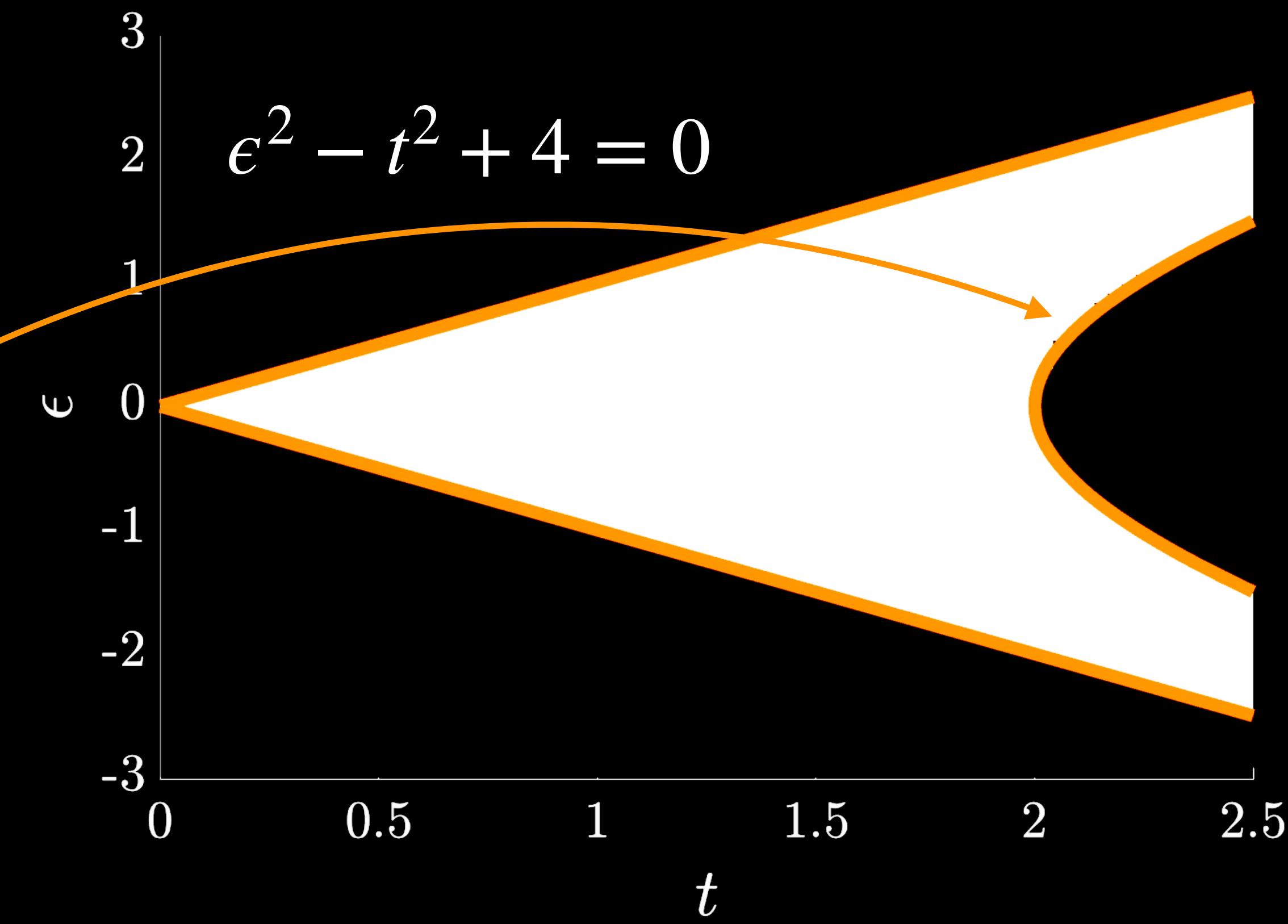
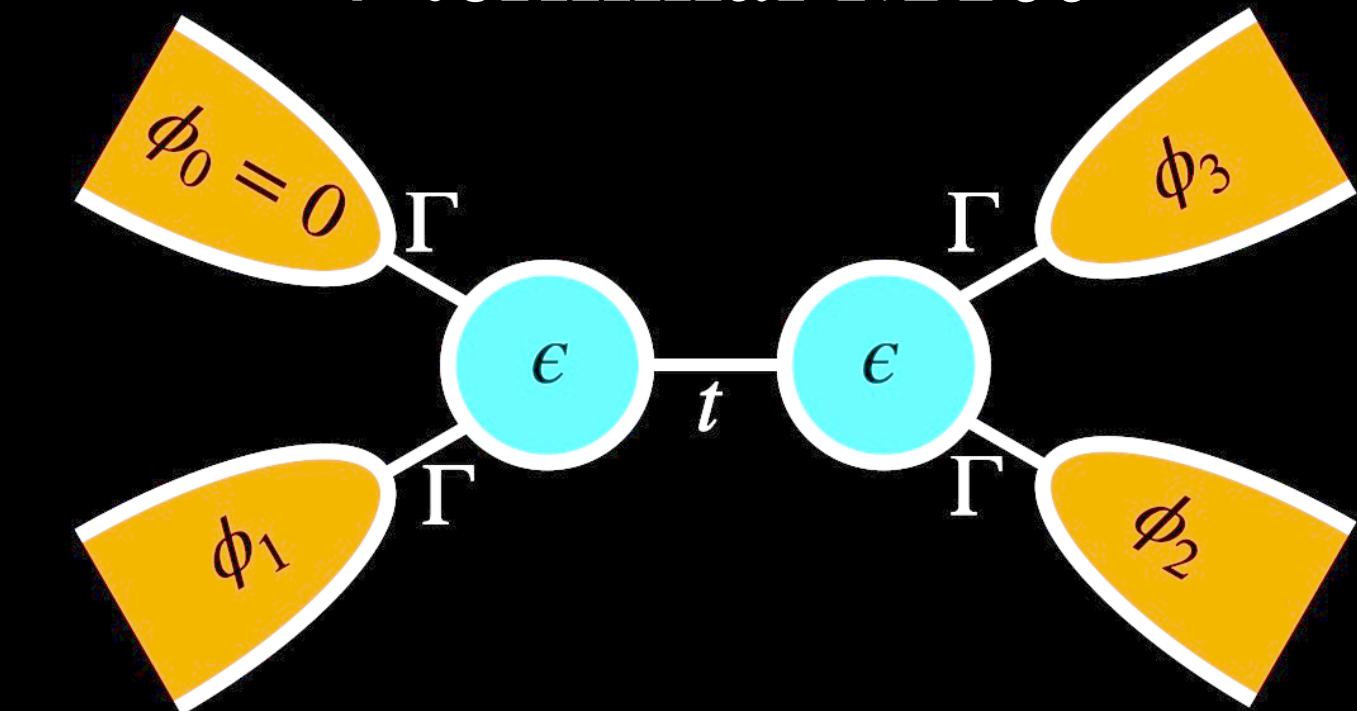
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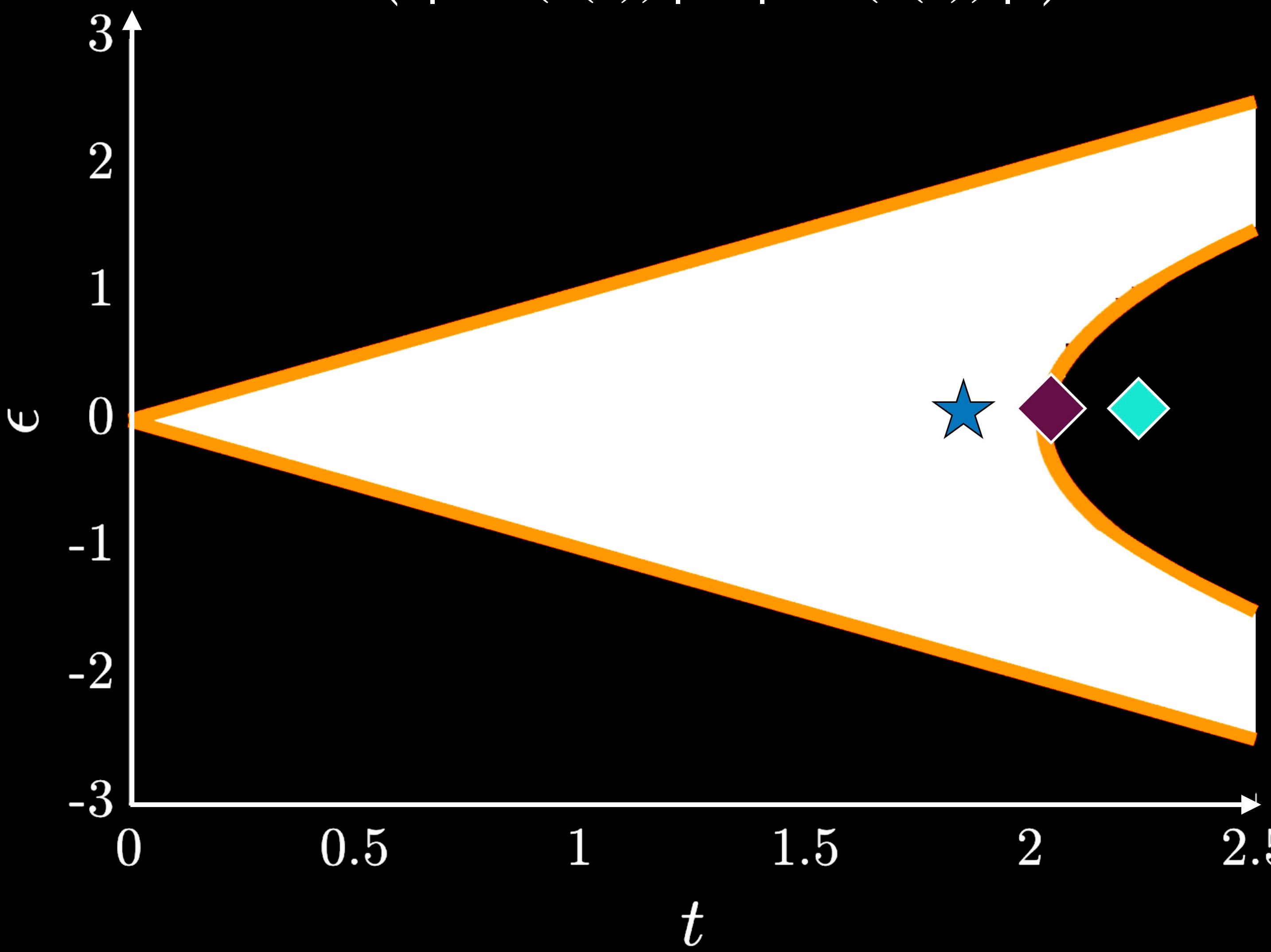
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4-terminal MTJJ



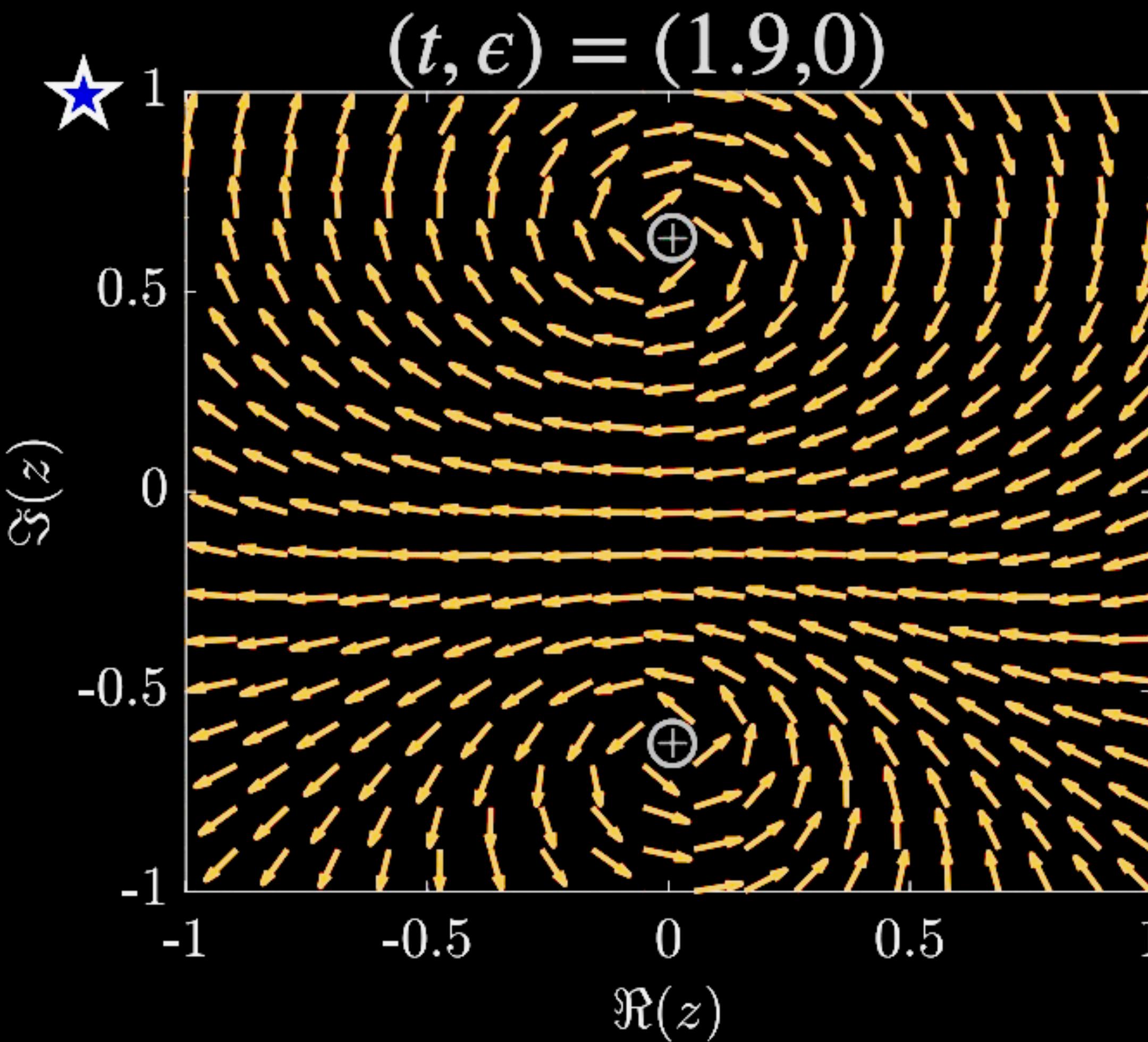
Topological charge of refl. less modes

$$\vec{V} = \left(\frac{\operatorname{Re} \det(r(z))}{|\det(r(z))|}, \frac{\operatorname{Im} \det(r(z))}{|\det(r(z))|} \right) \quad \begin{array}{l} \text{complex energy} \\ z \in \mathbb{C} \end{array}$$

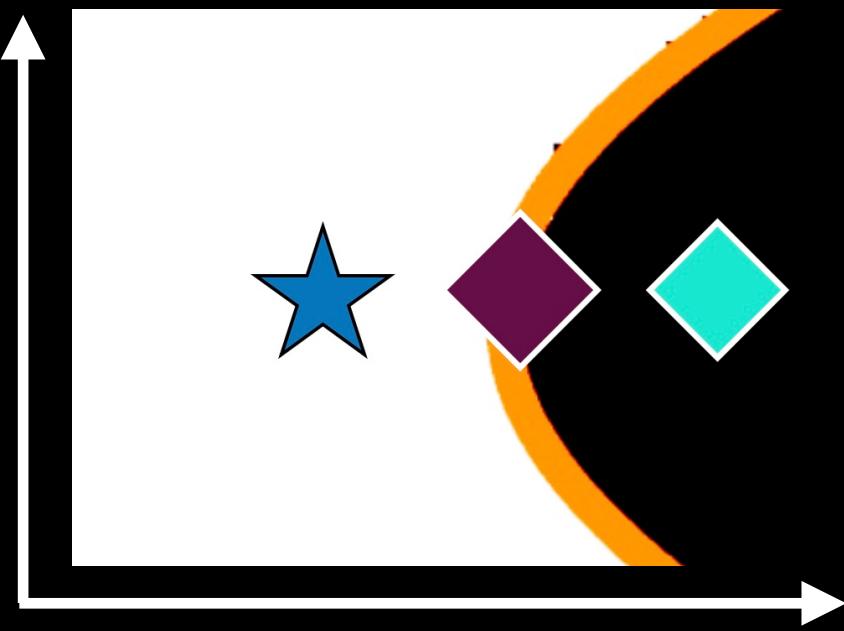


Topological charge of refl. less modes

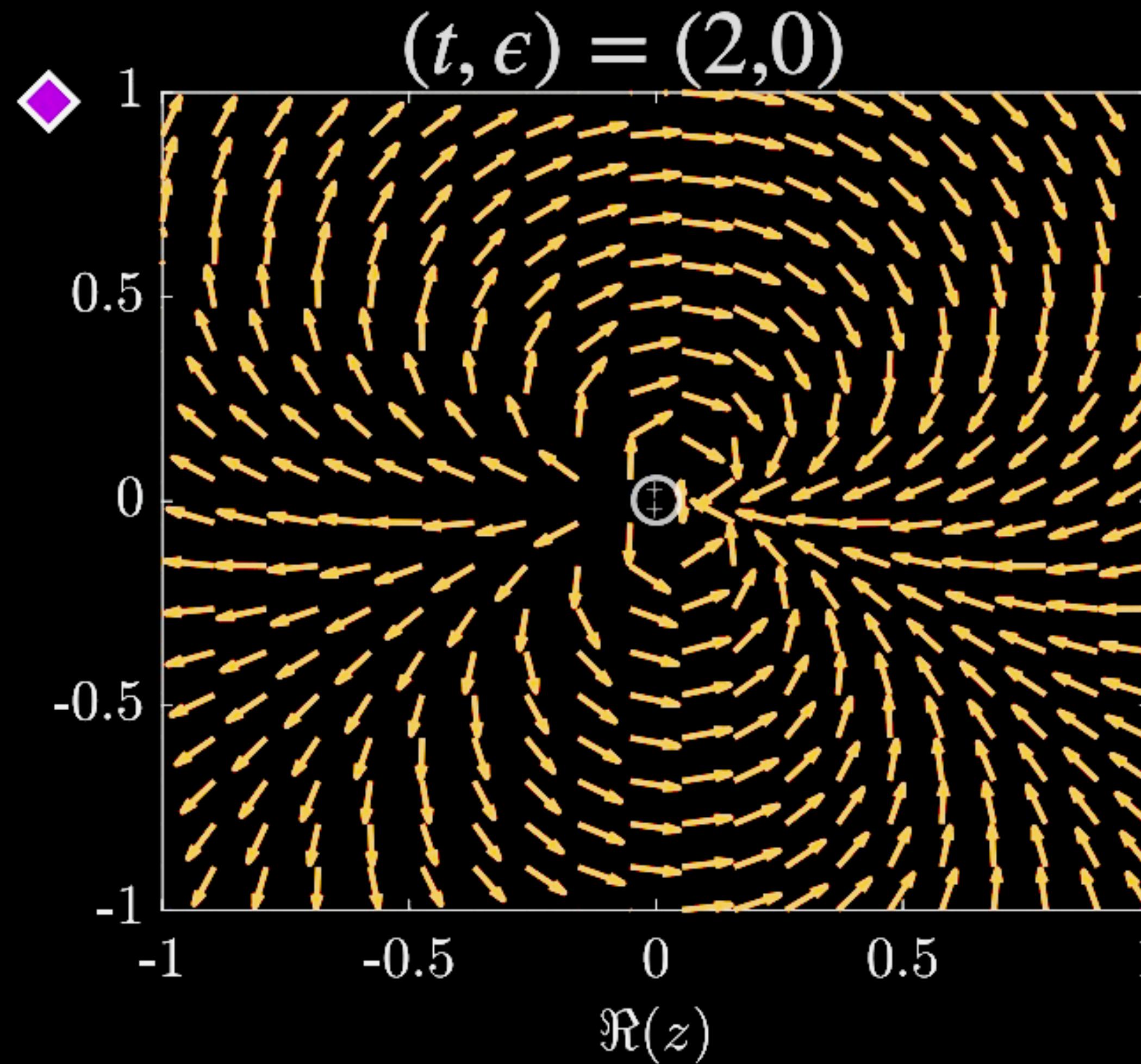
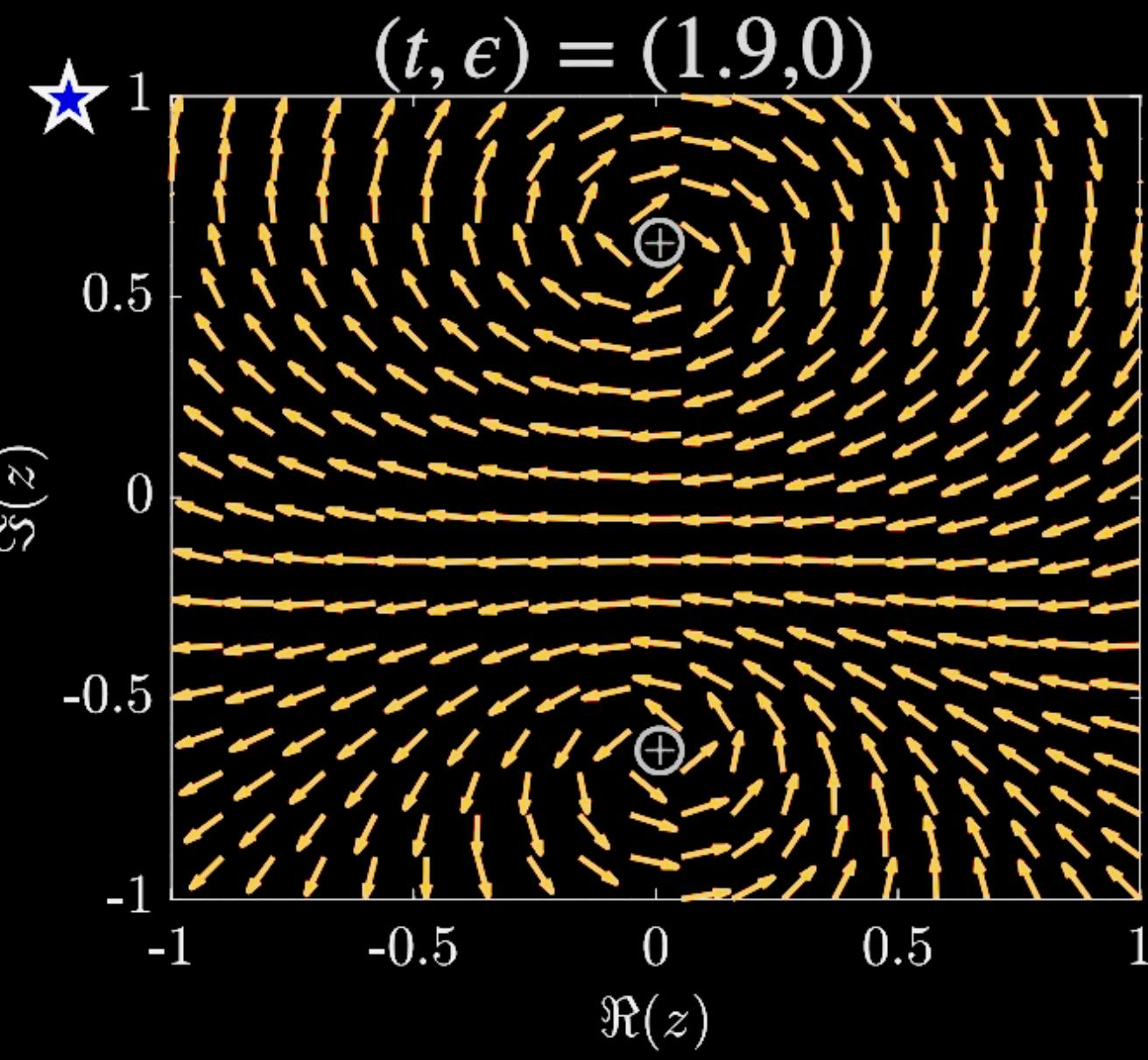
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Topological charge of refl. less modes



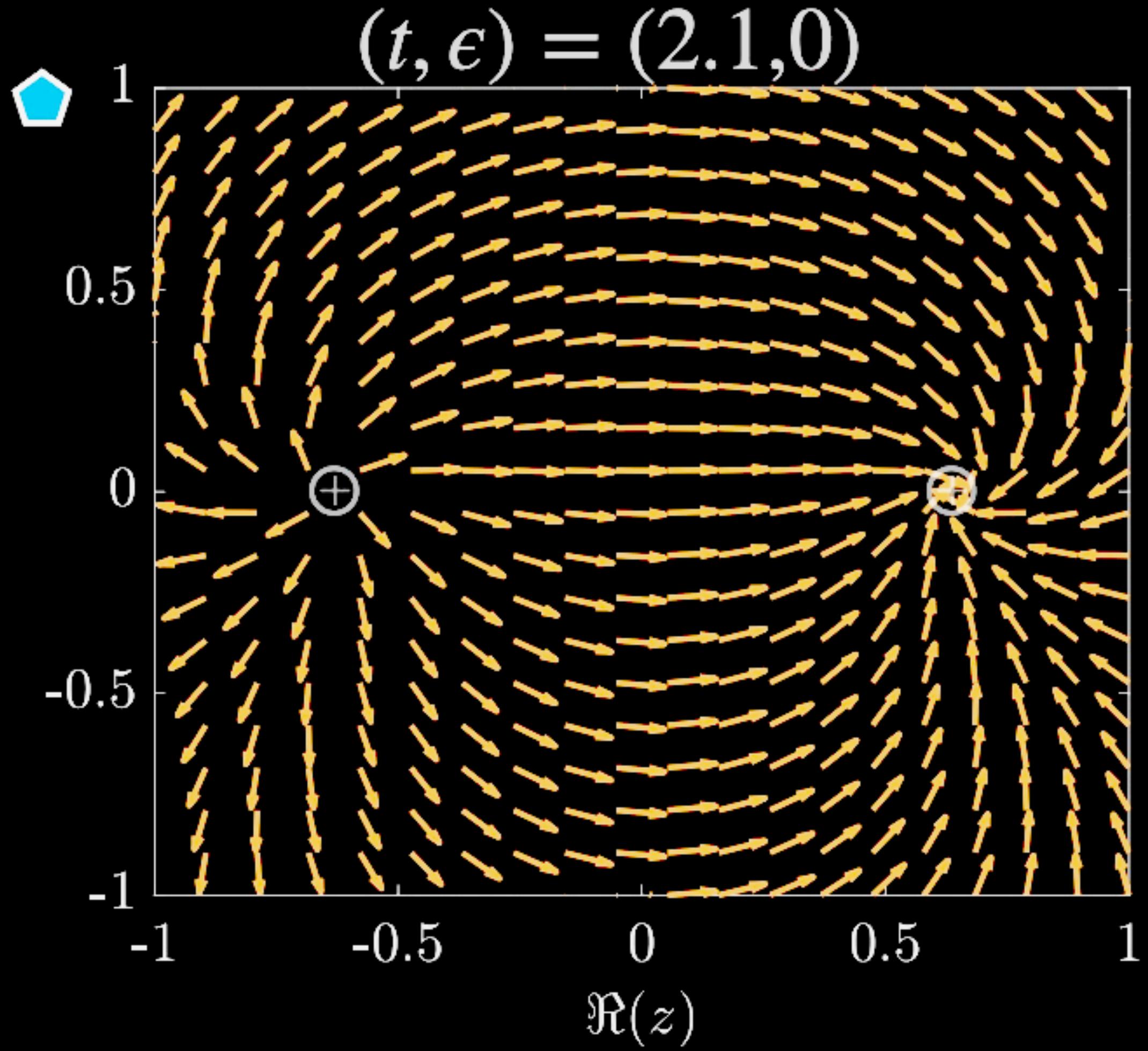
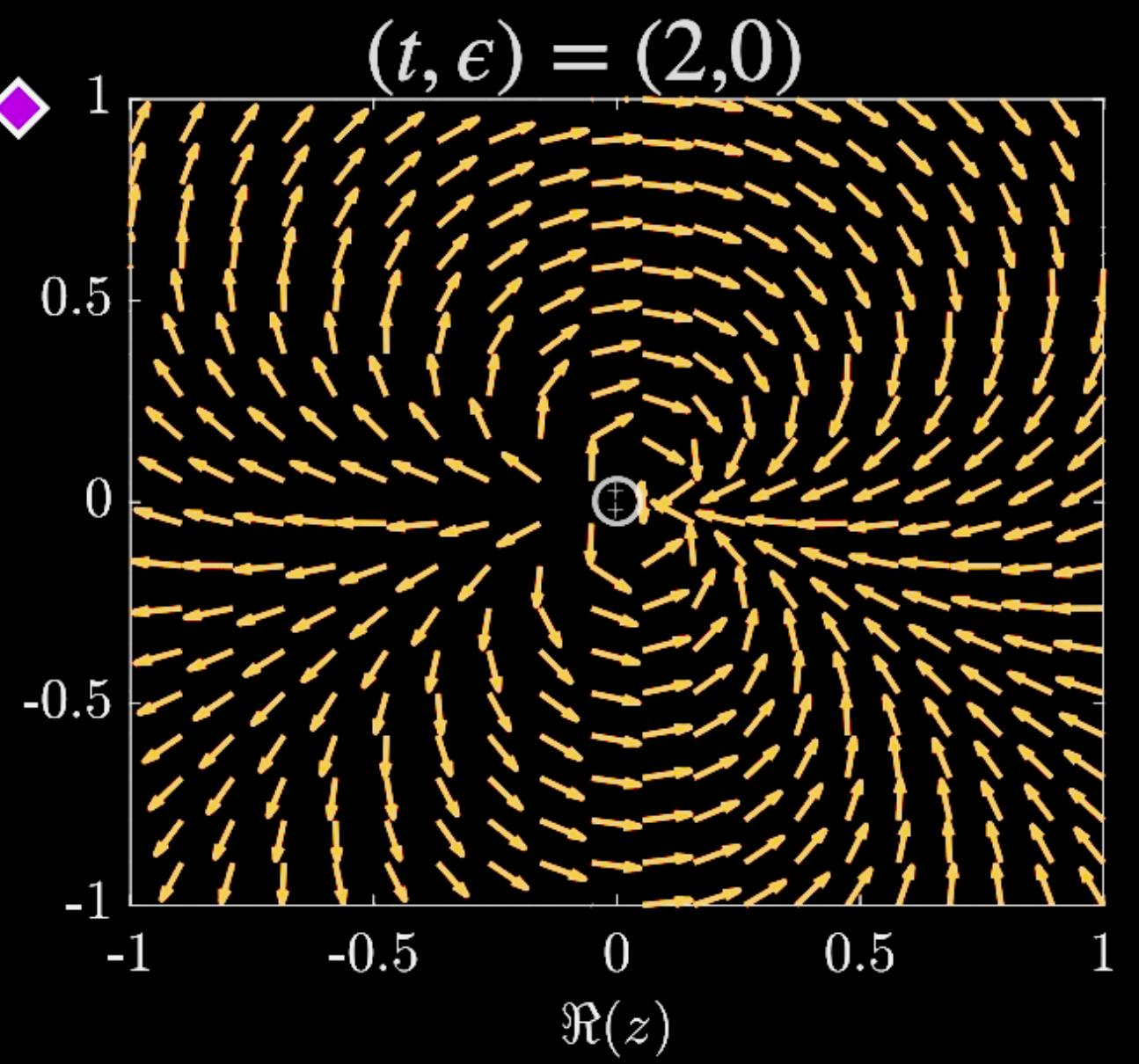
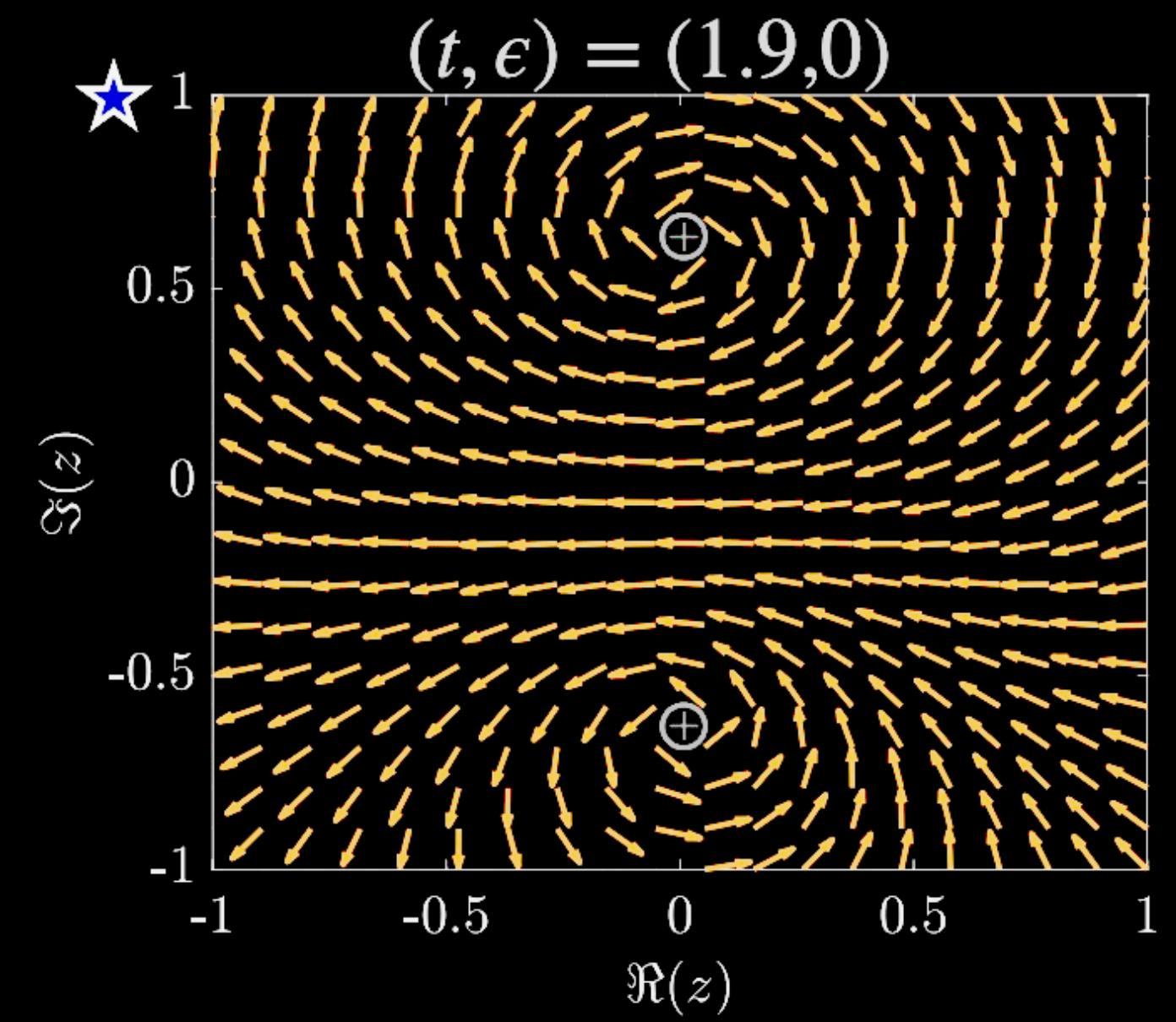
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Topological charge of refl. less modes



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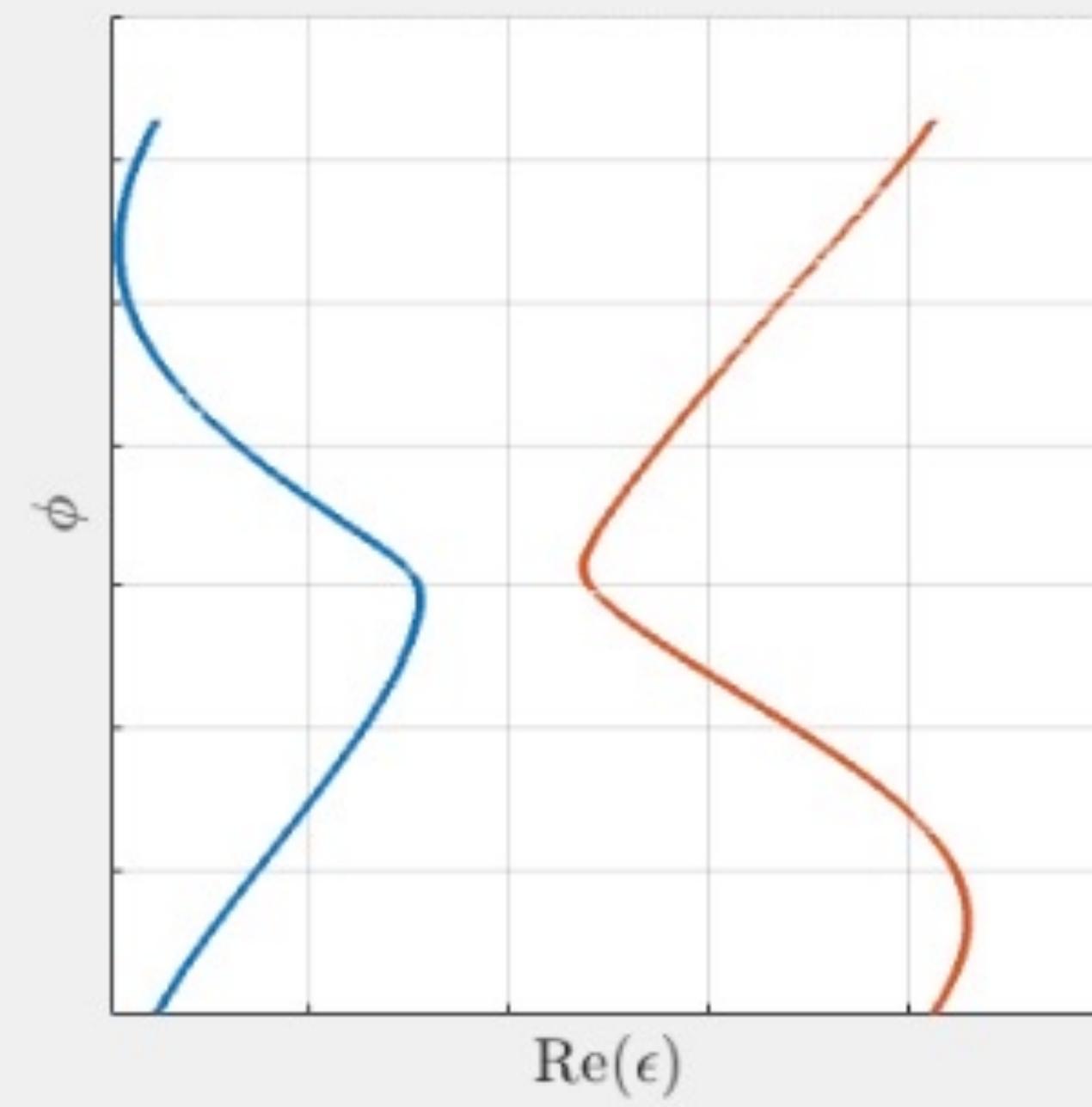
Hermitian physics

Hermitian

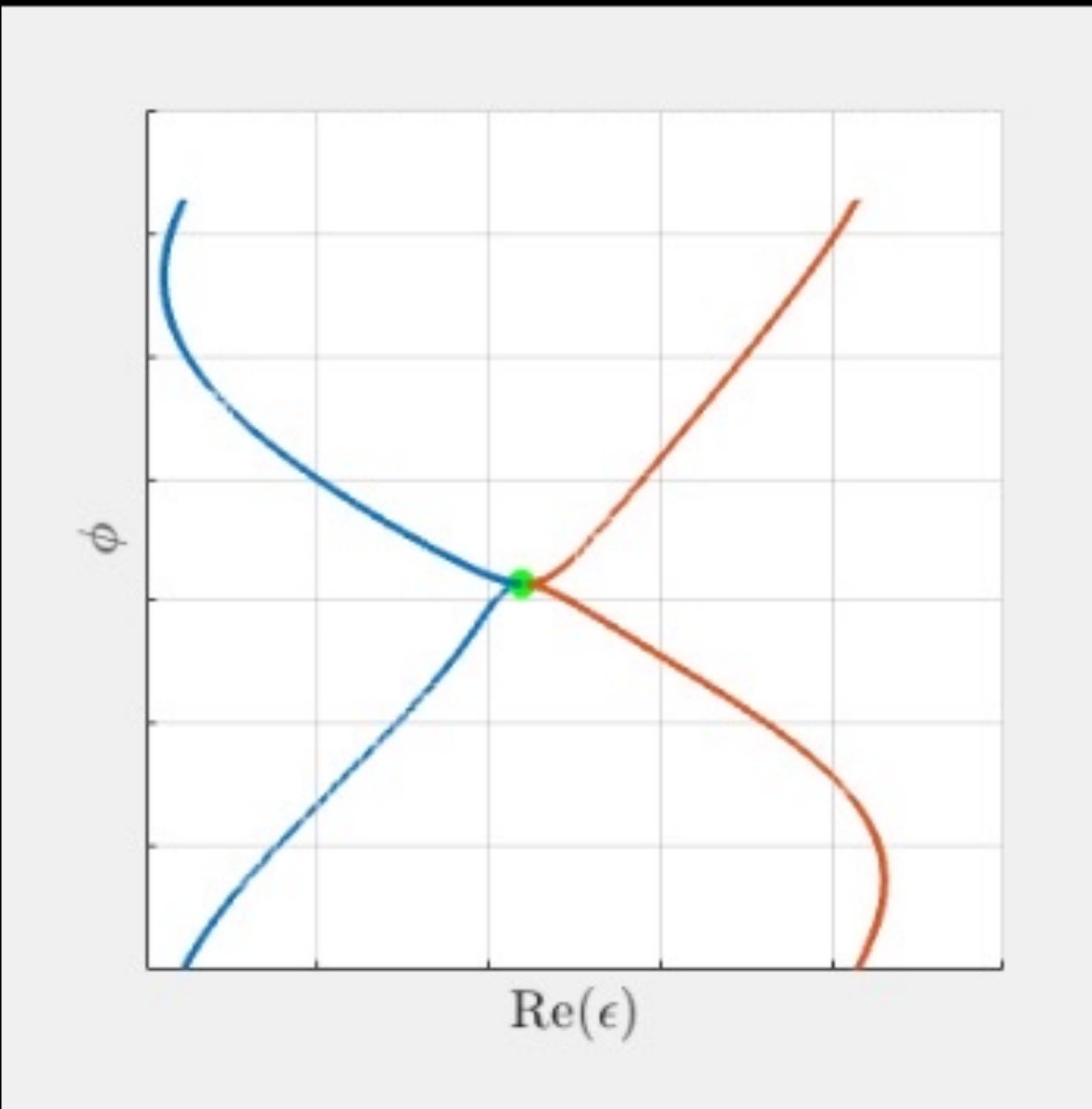
$$H = H^\dagger$$

$$\epsilon_\alpha \in \mathbb{R}$$

Gapped



Ungapped



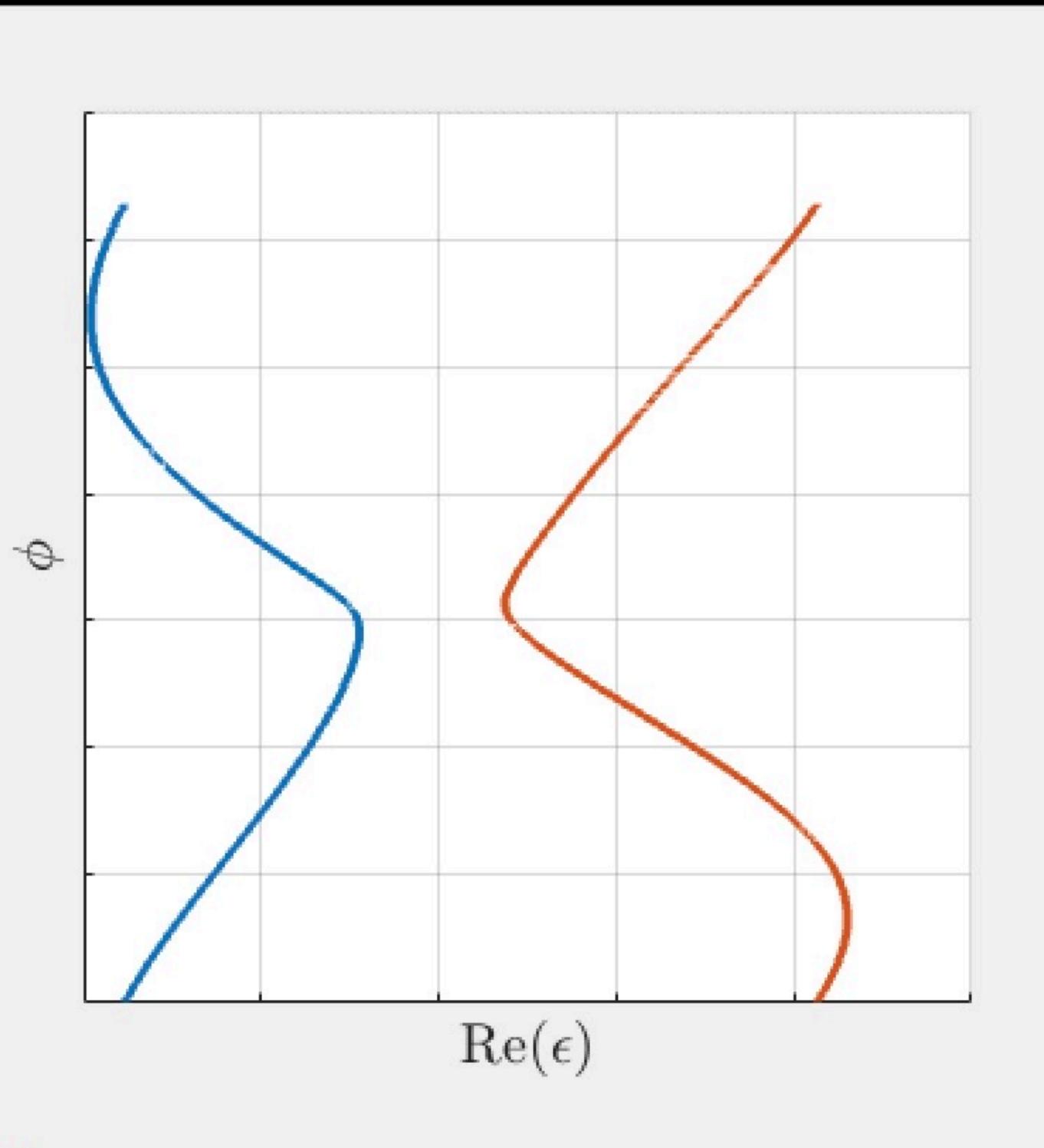
Non-hermitian physics

Kohei Kawabata, et. al., Phys. Rev. X 9, 041015 (2019)

non-Hermitian

$$H \neq H^\dagger$$

$$\epsilon_\alpha \in \mathbb{C}$$



Non-hermitian physics

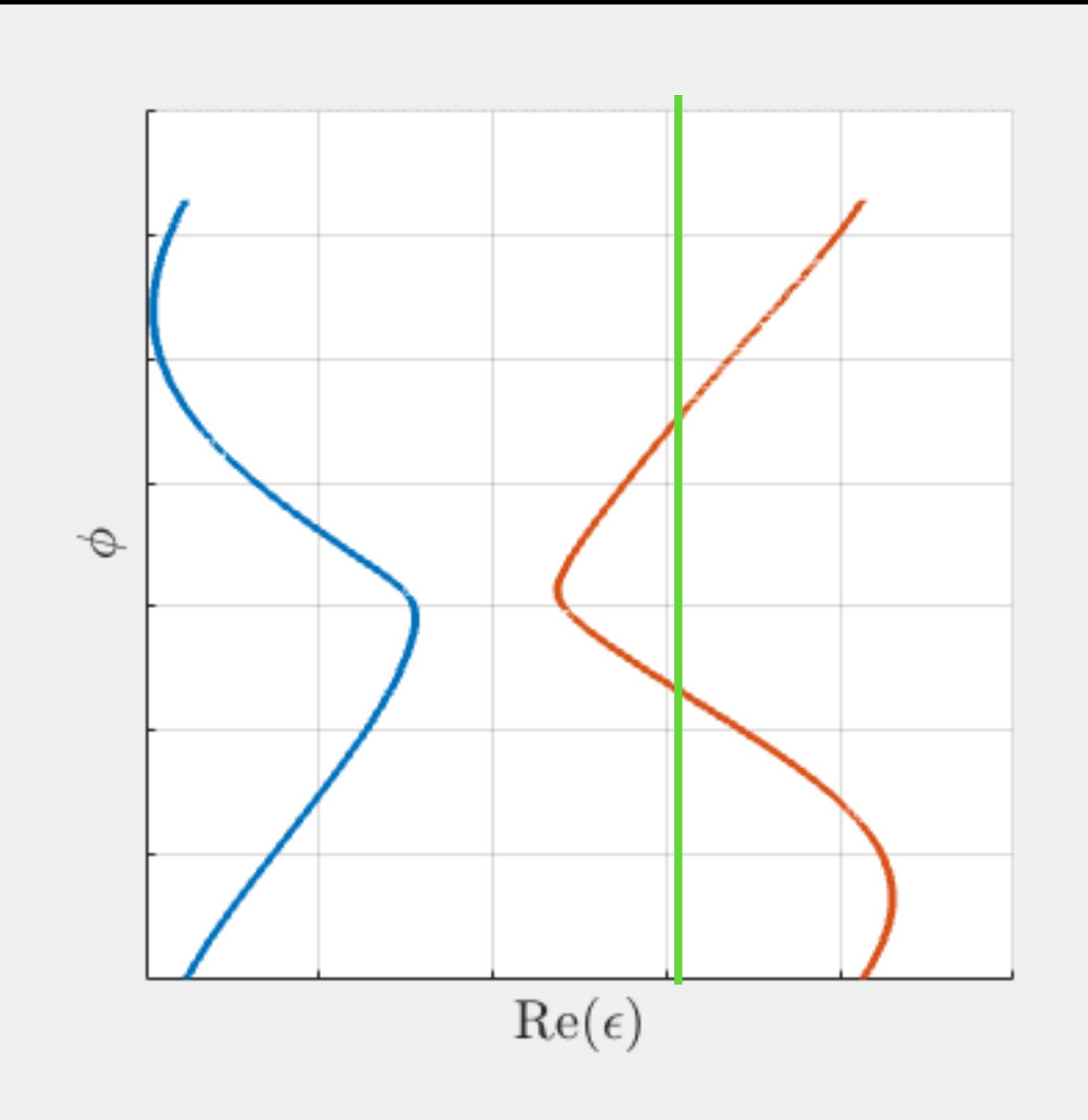
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Line Gap



Non-hermitian physics

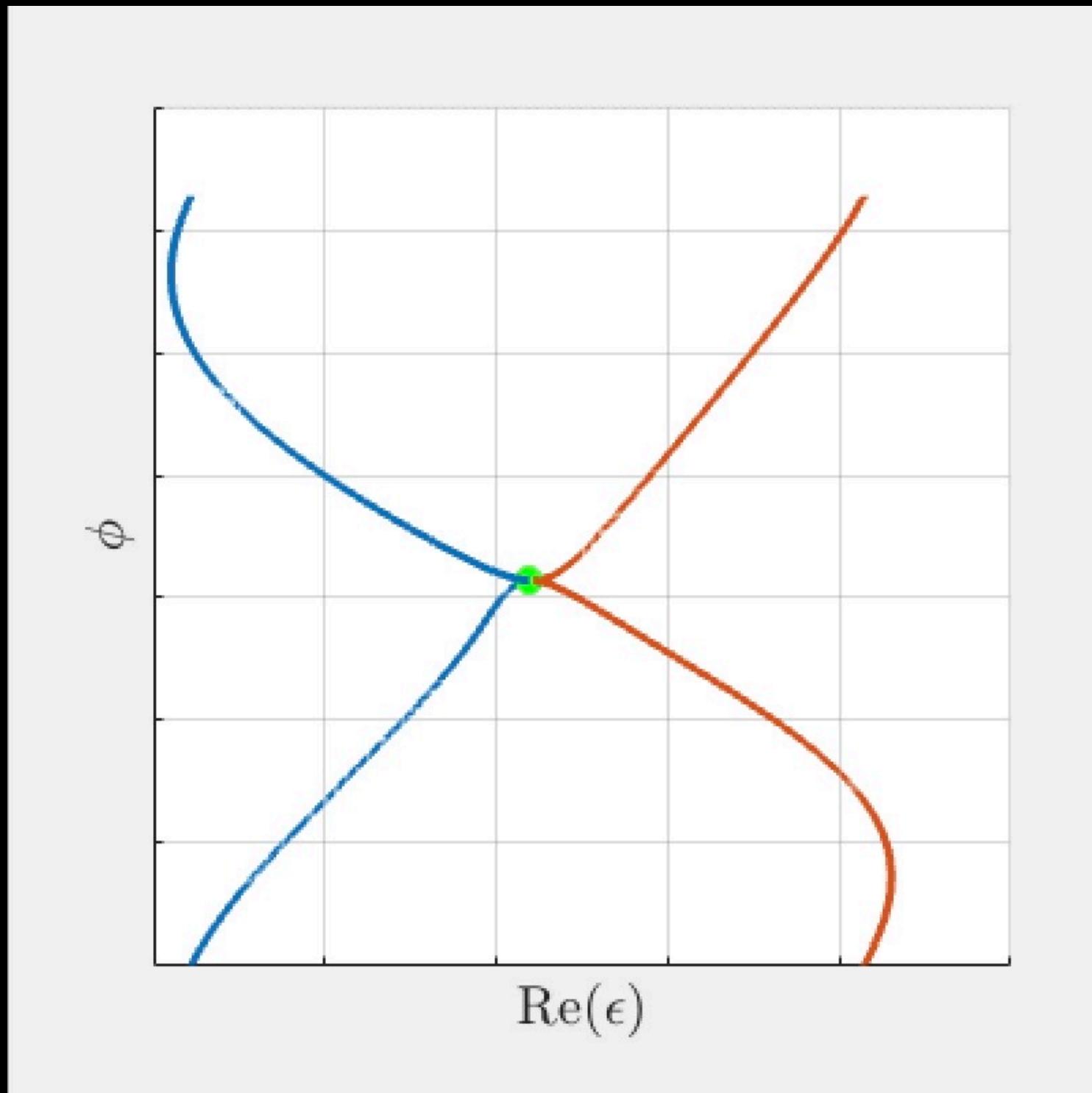
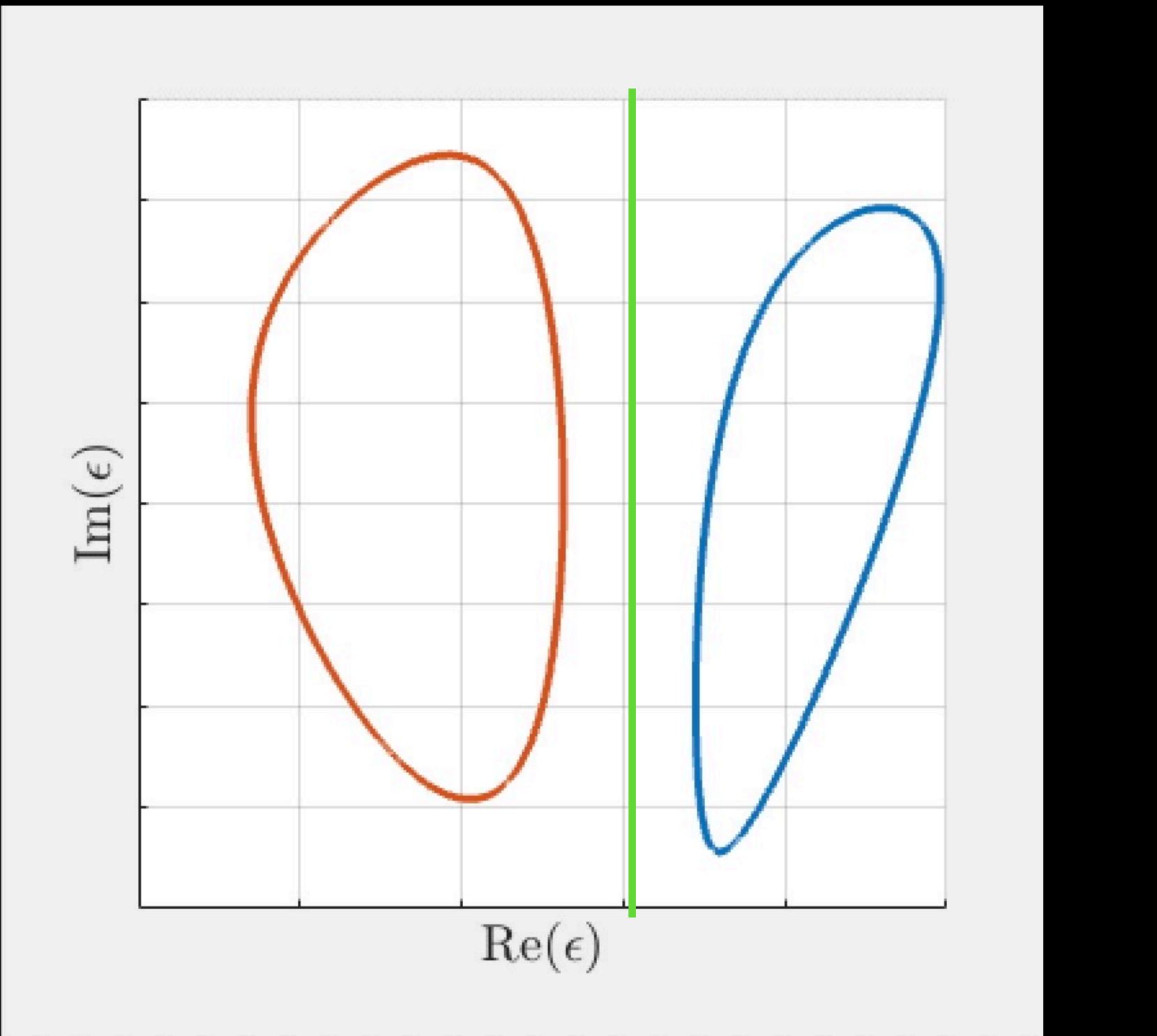
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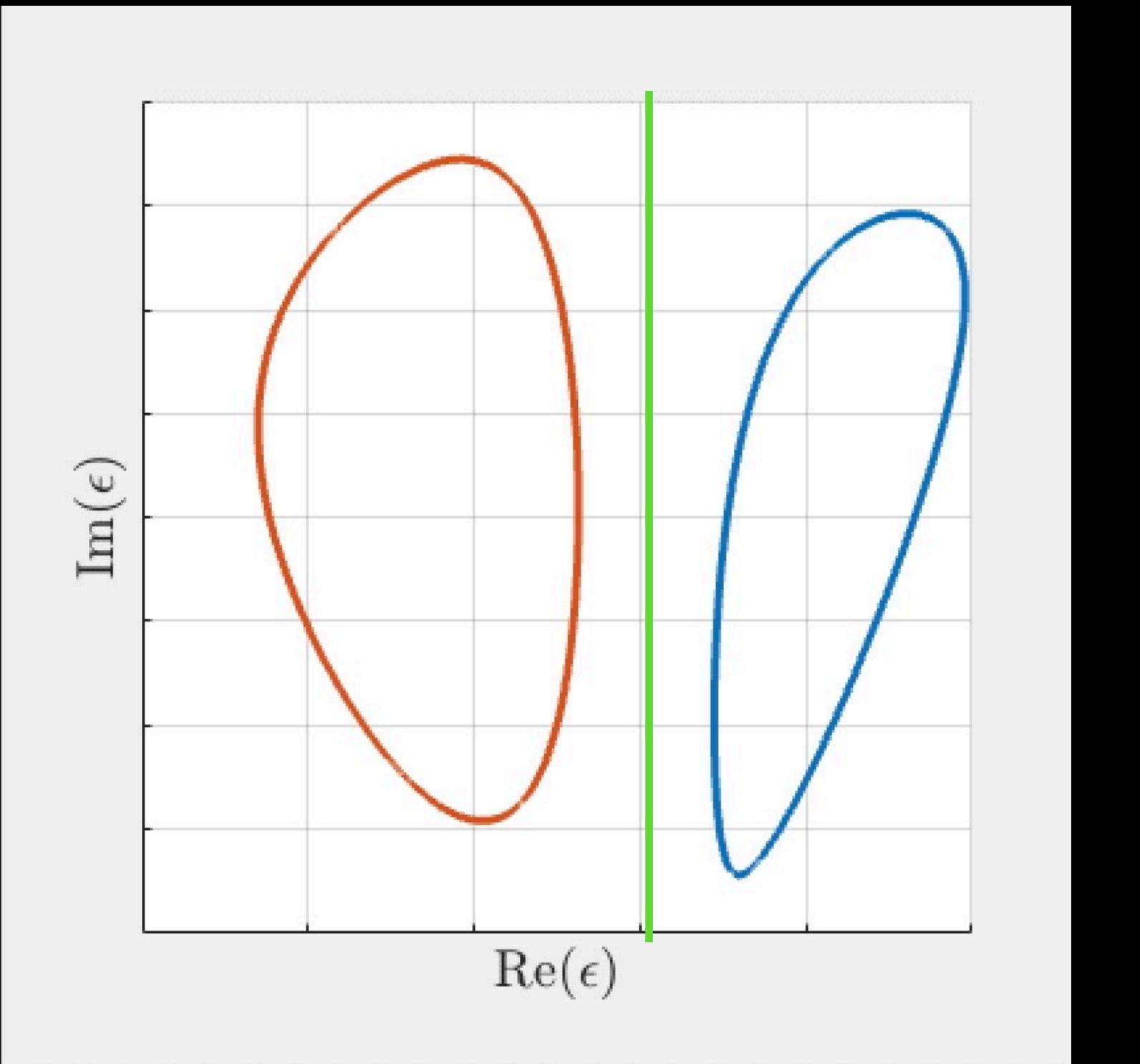
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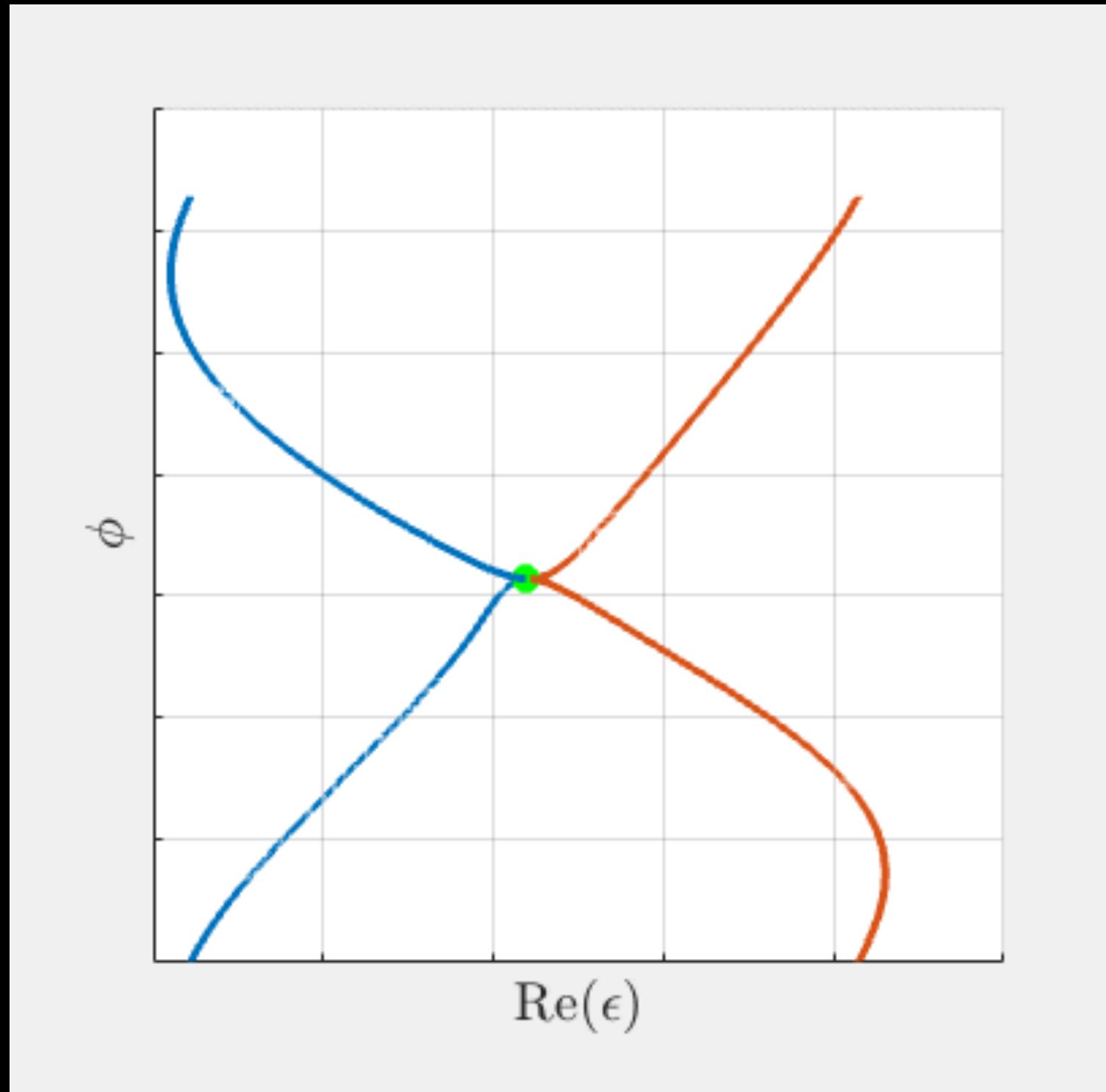
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Line Gap



Exceptional Point (EPs)



Non-hermitian physics

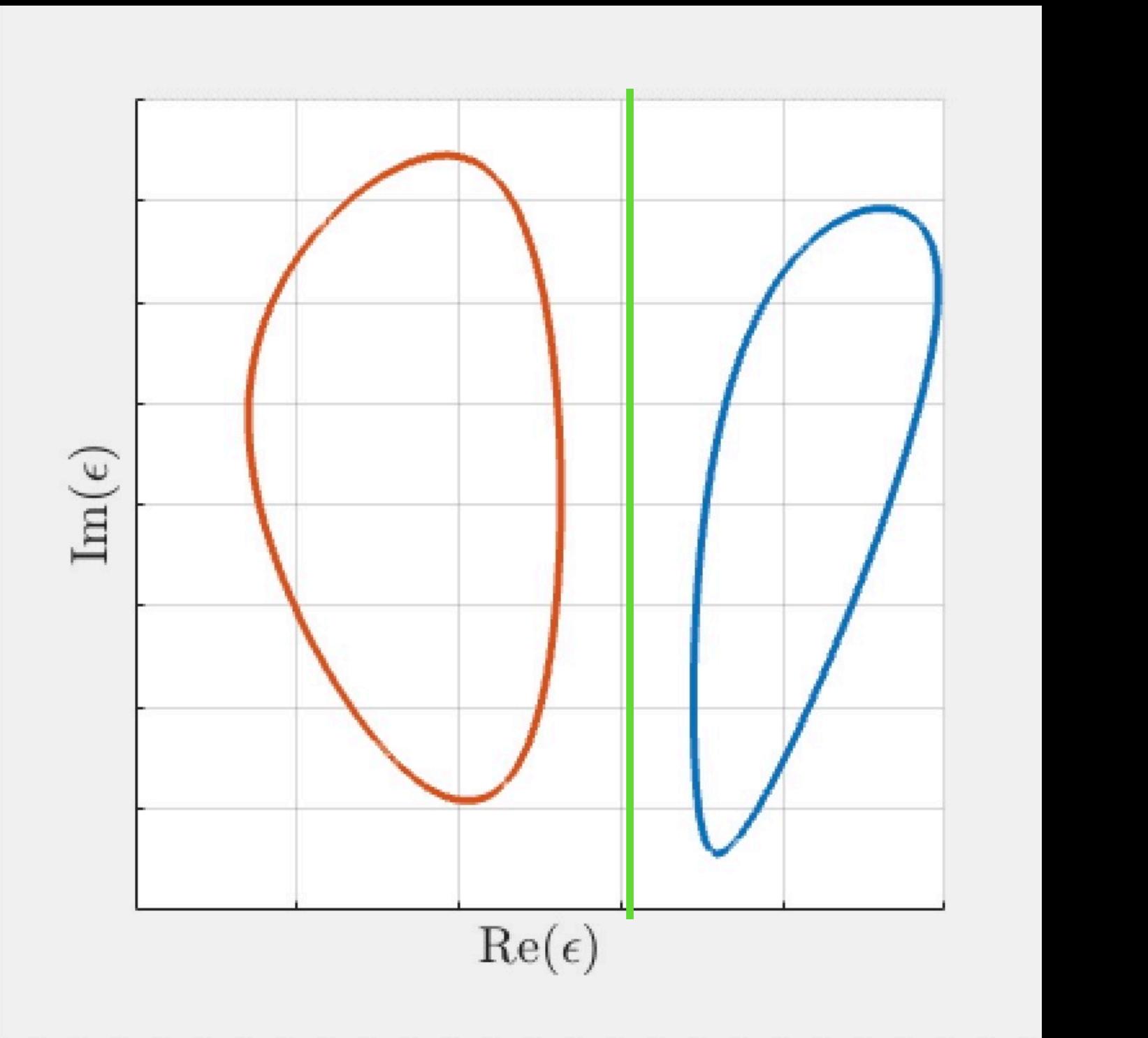
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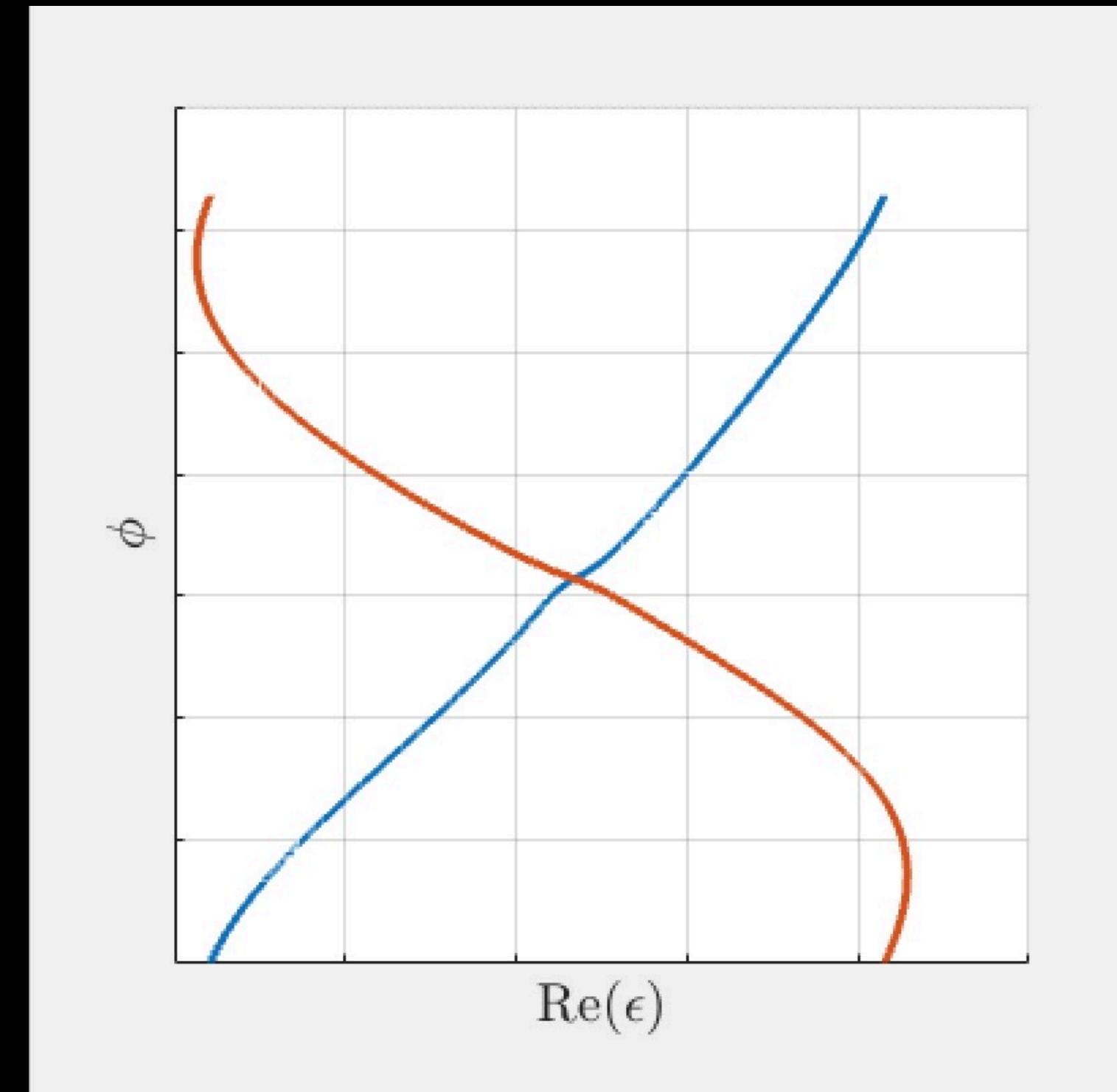
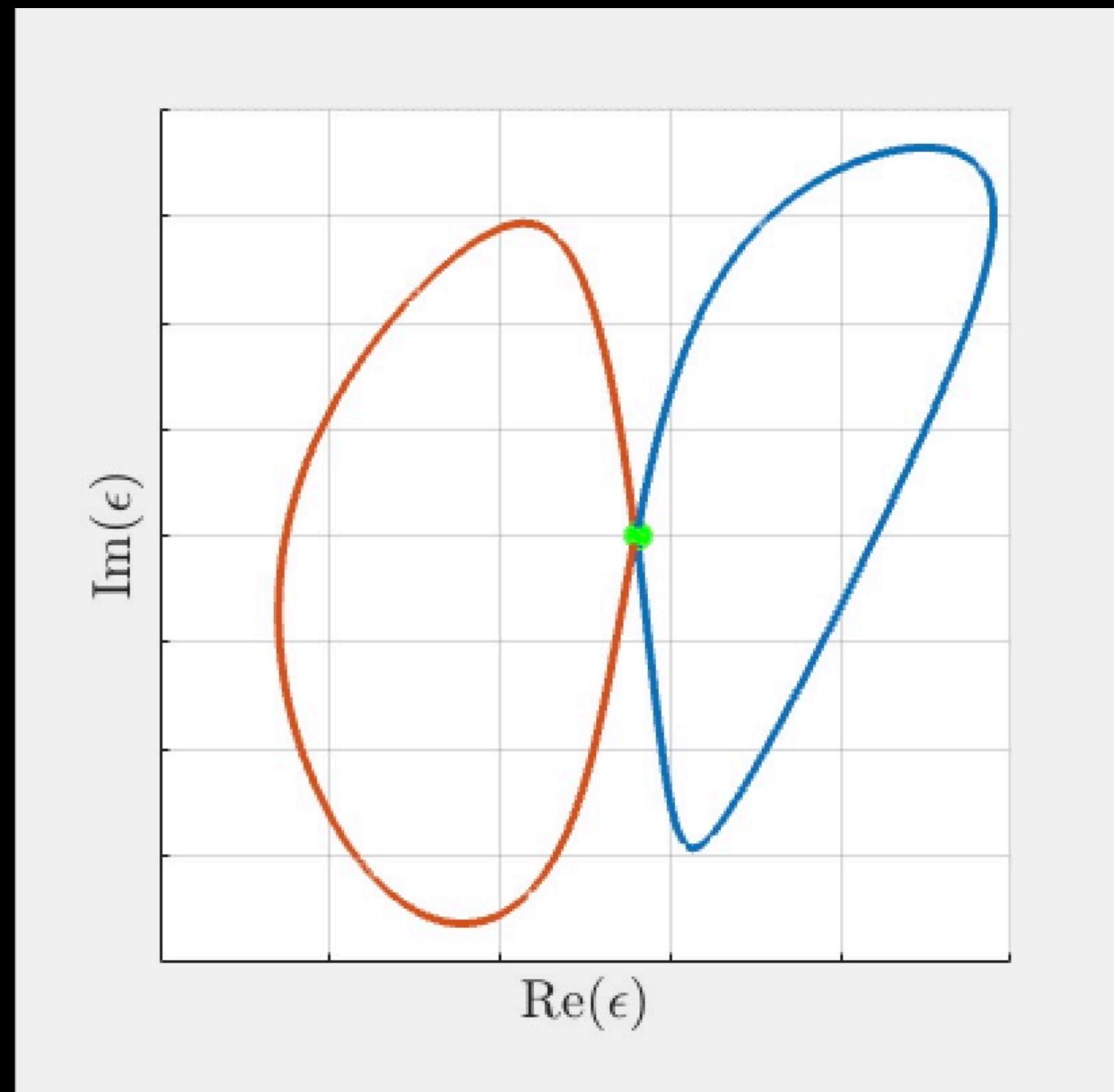
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Exceptional Point (EPs)



Non-hermitian physics

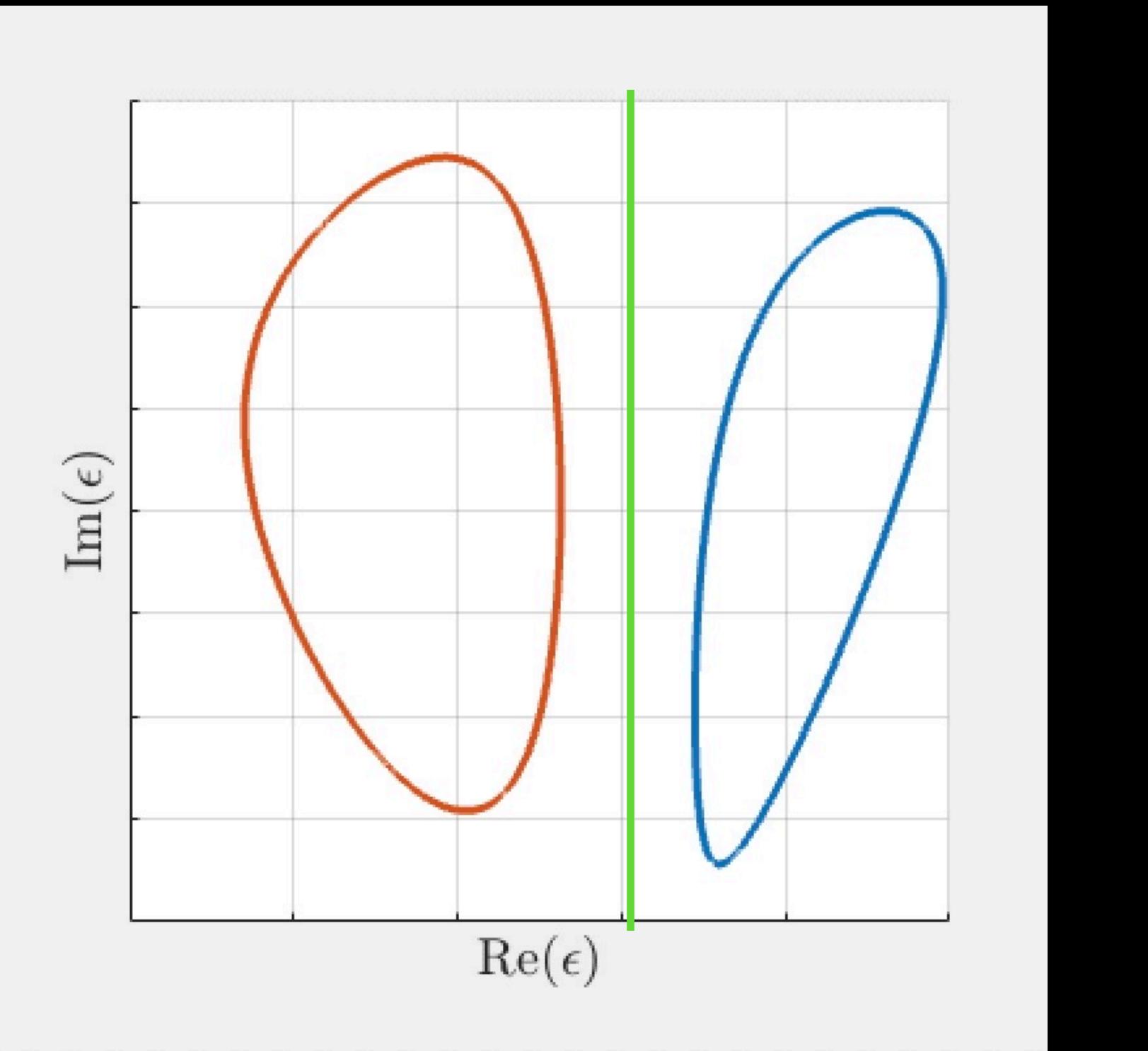
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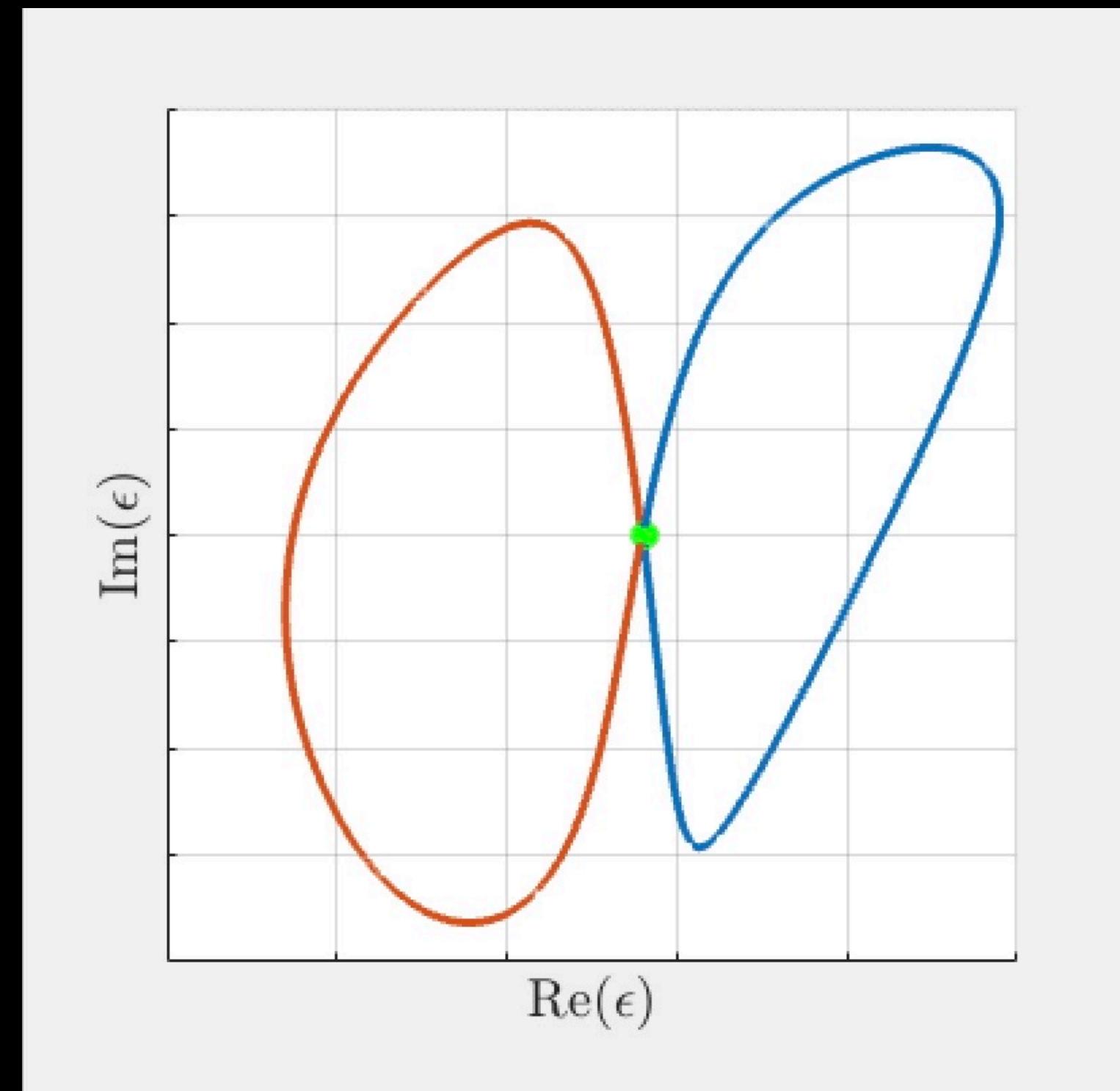
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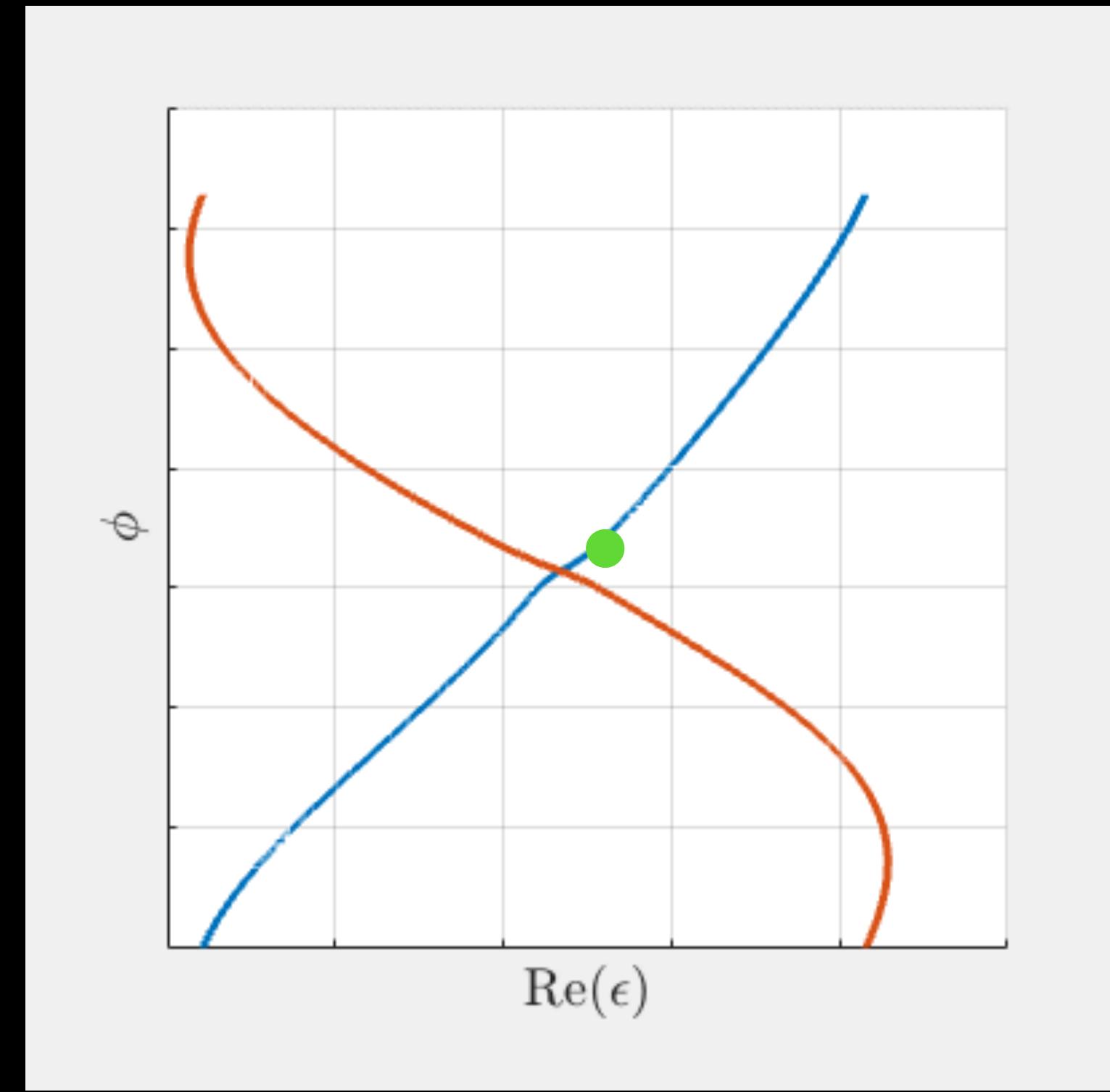
Line Gap



Exceptional Point (EPs)



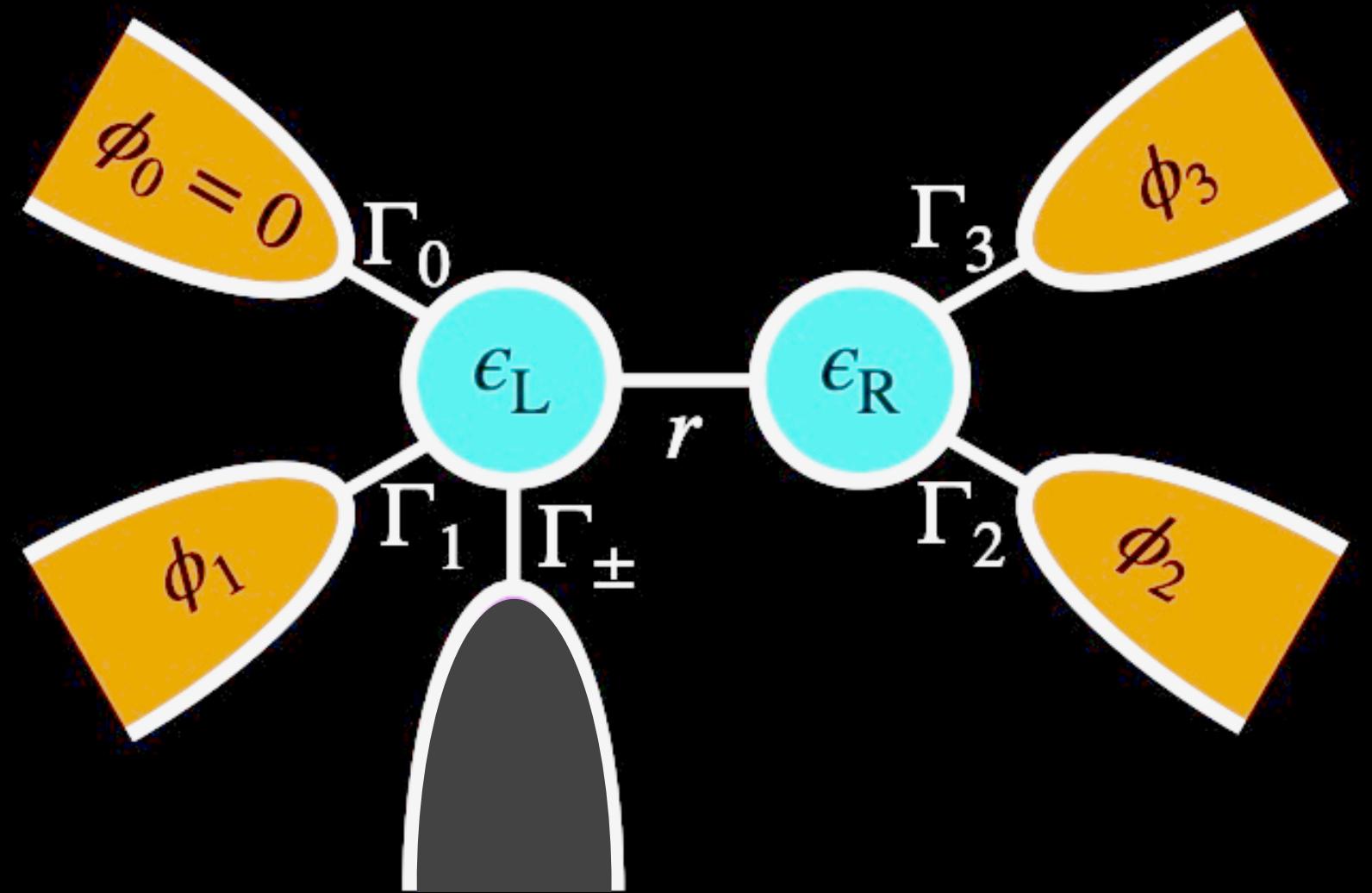
Point Gap



Non-hermitian topology in MTJJs

D. C. Ohnmacht, et. al., arXiv:2408.01289 (2024) (accepted in PRL)

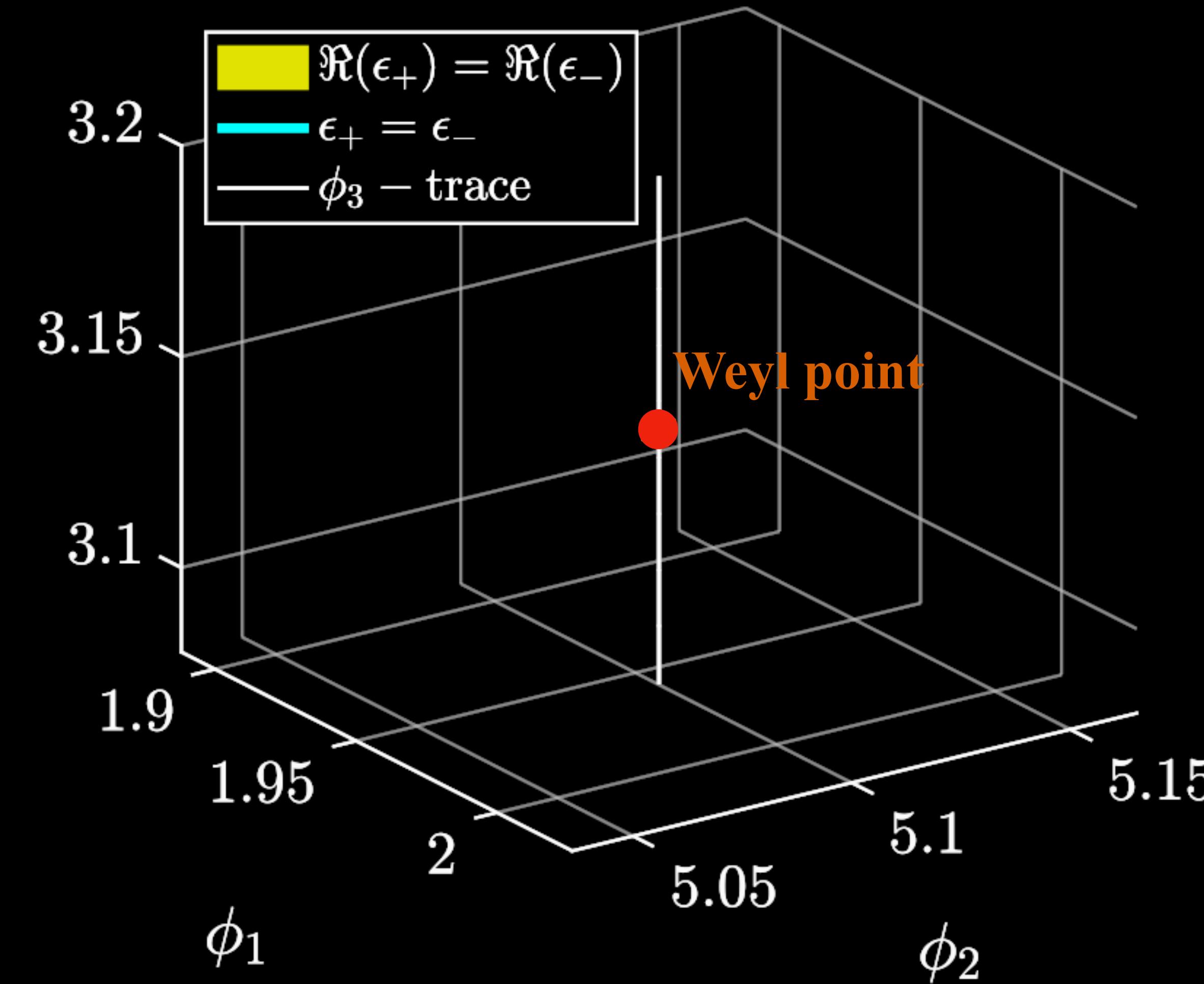
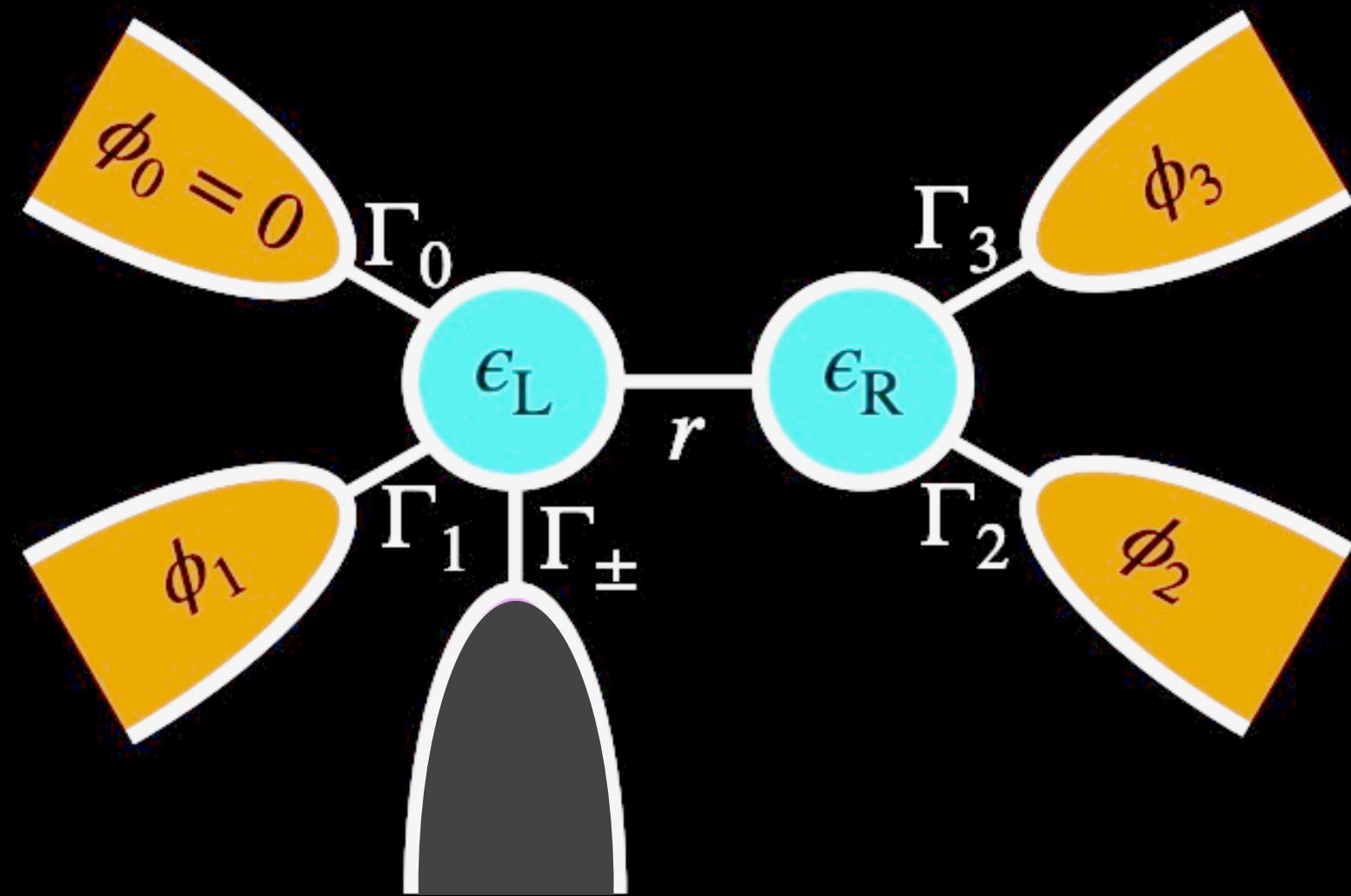
Breaking spin-rotation symm.



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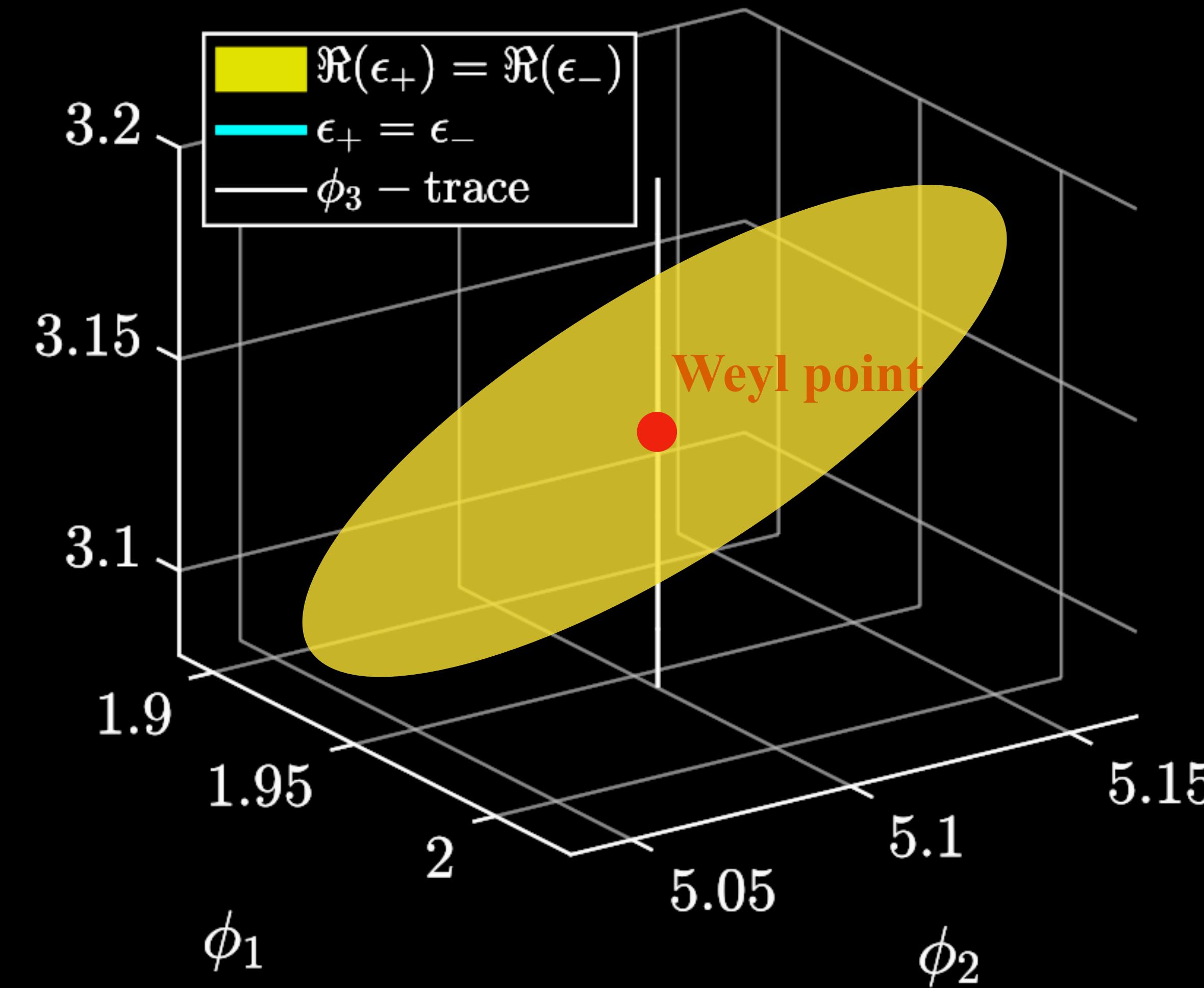
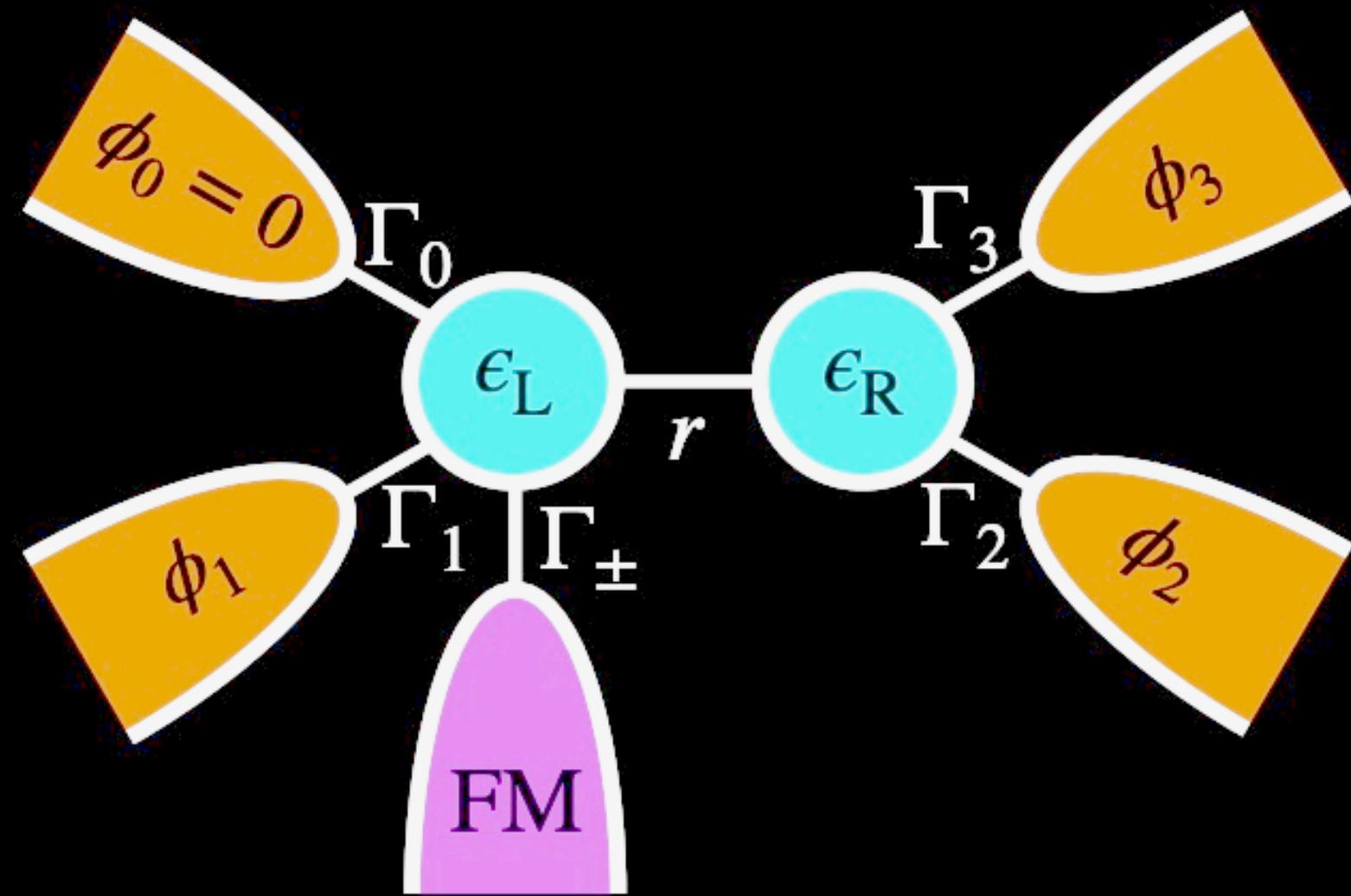
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Non-hermitian topology in MTJJs

D. C. Ohnmacht, et. al., arXiv:2408.01289 (2024) (accepted in PRL)

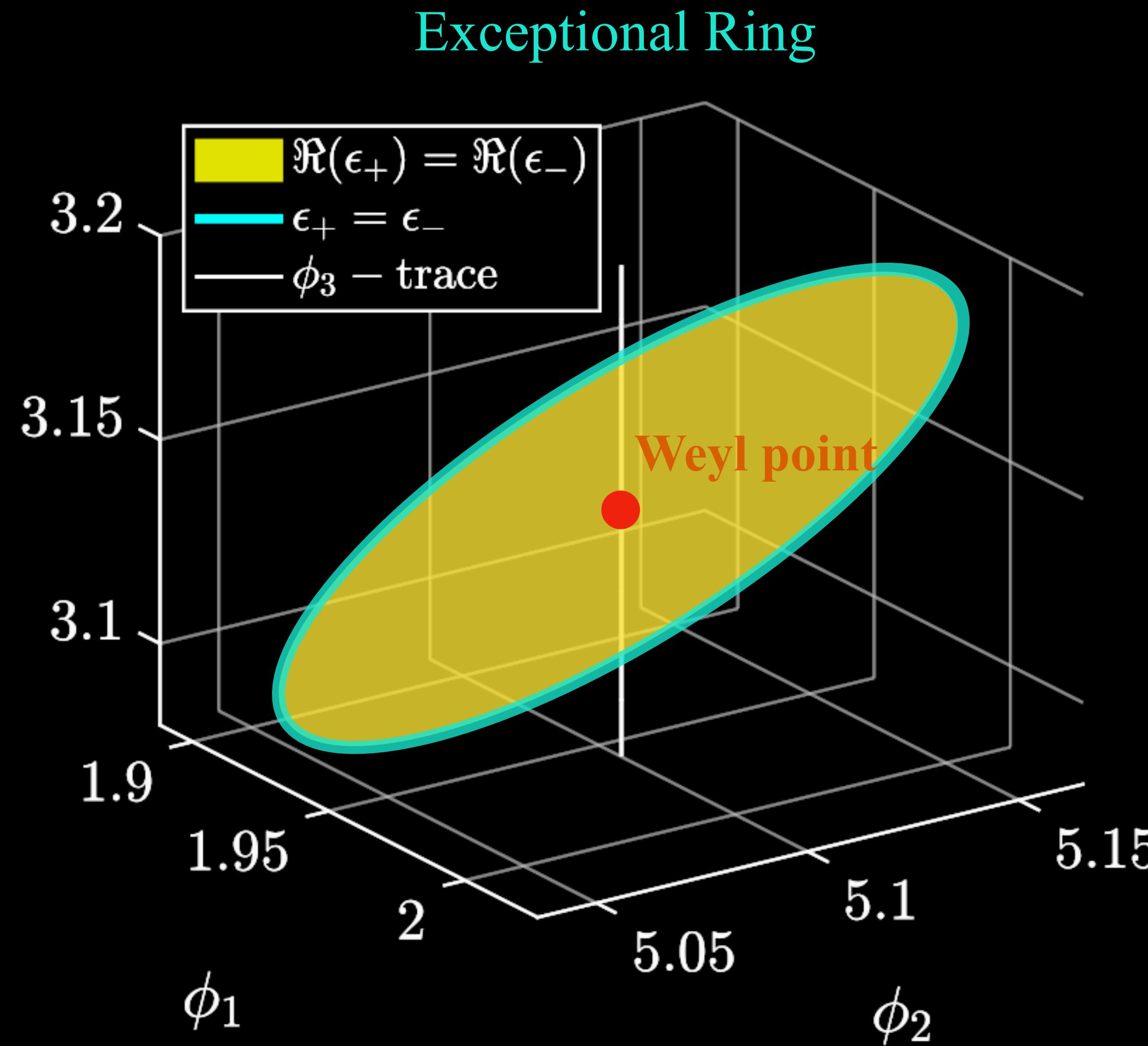
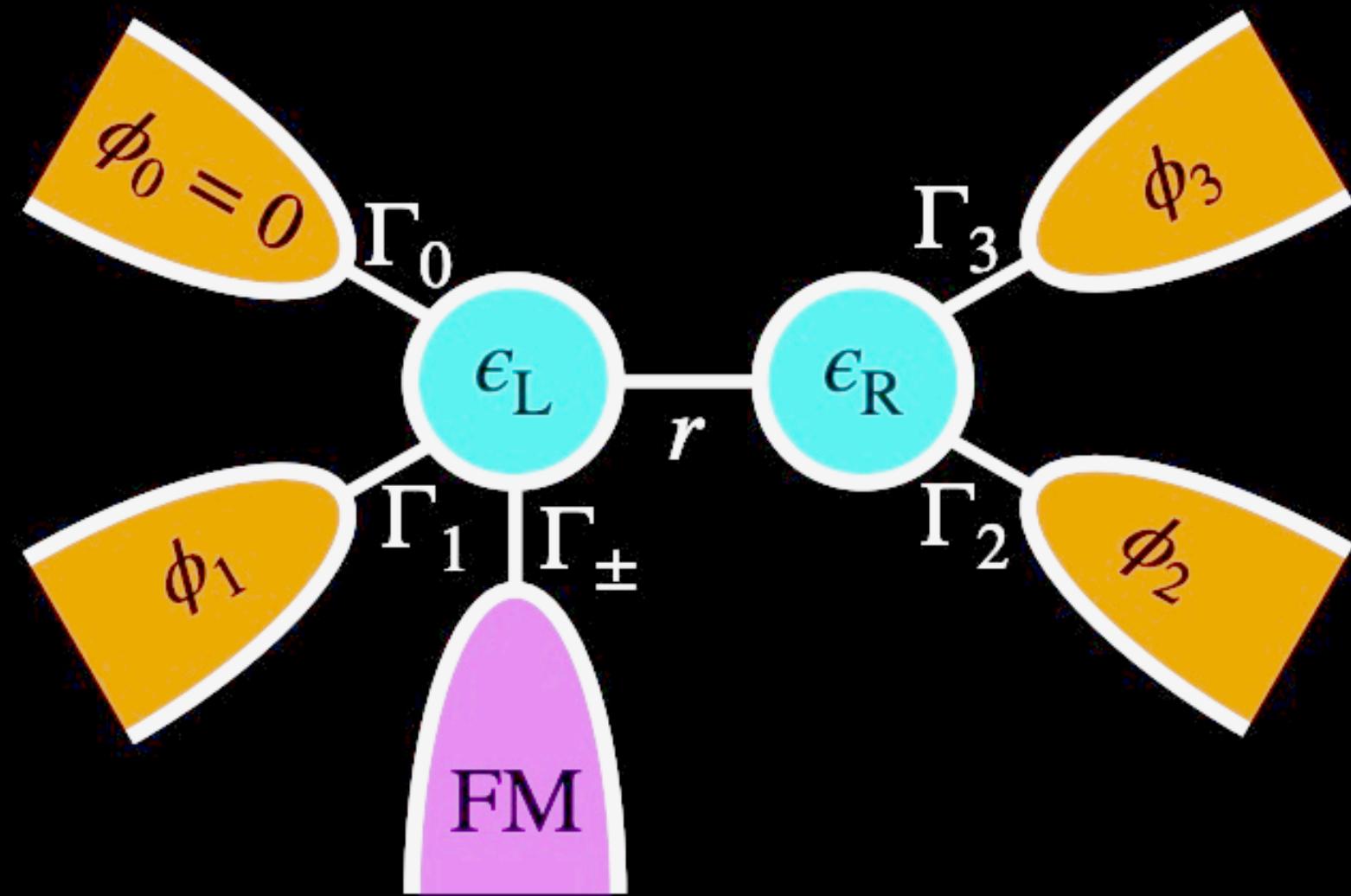
Breaking spin-rotation symm.



Non-hermitian topology in MTJJs

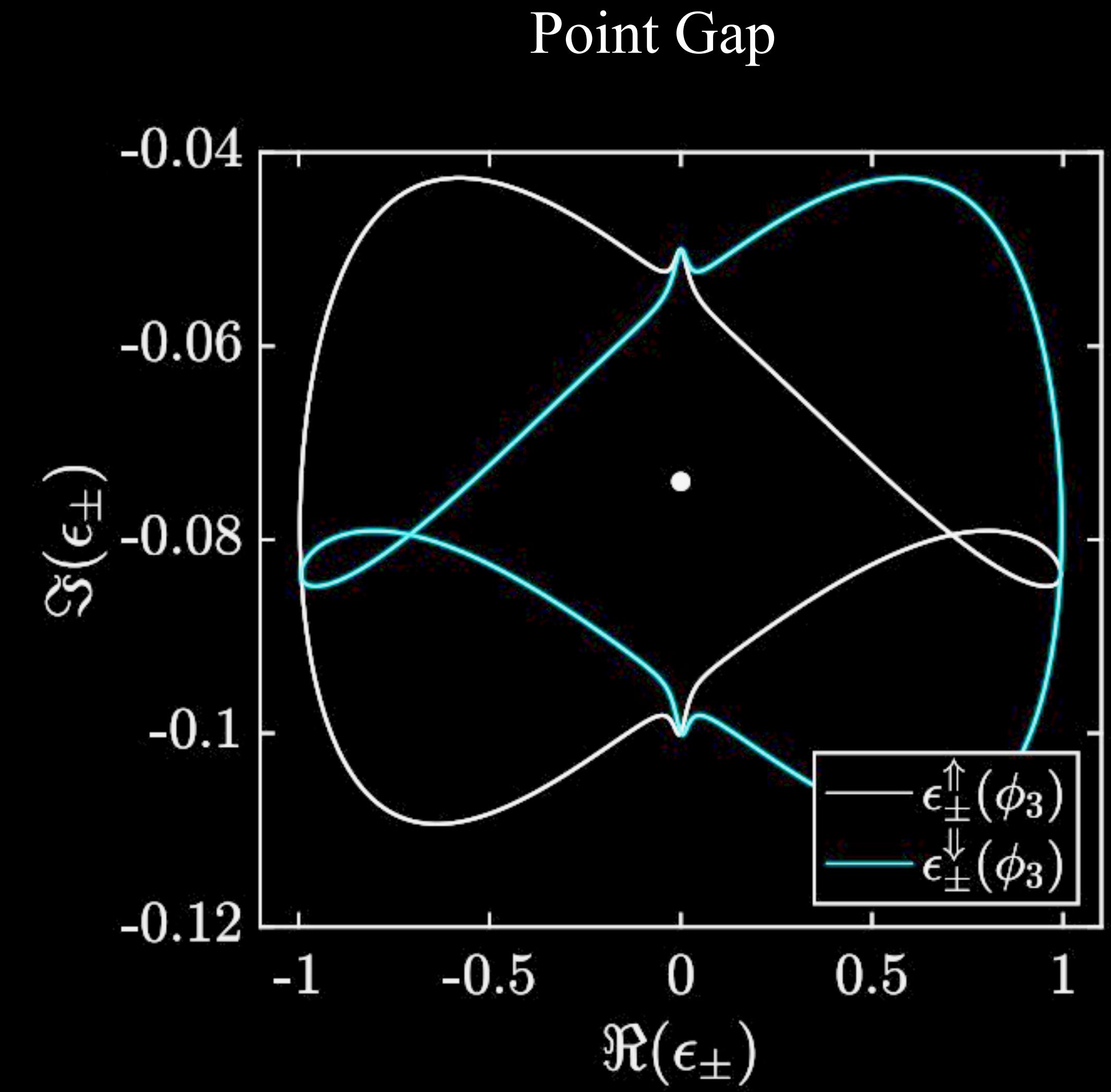
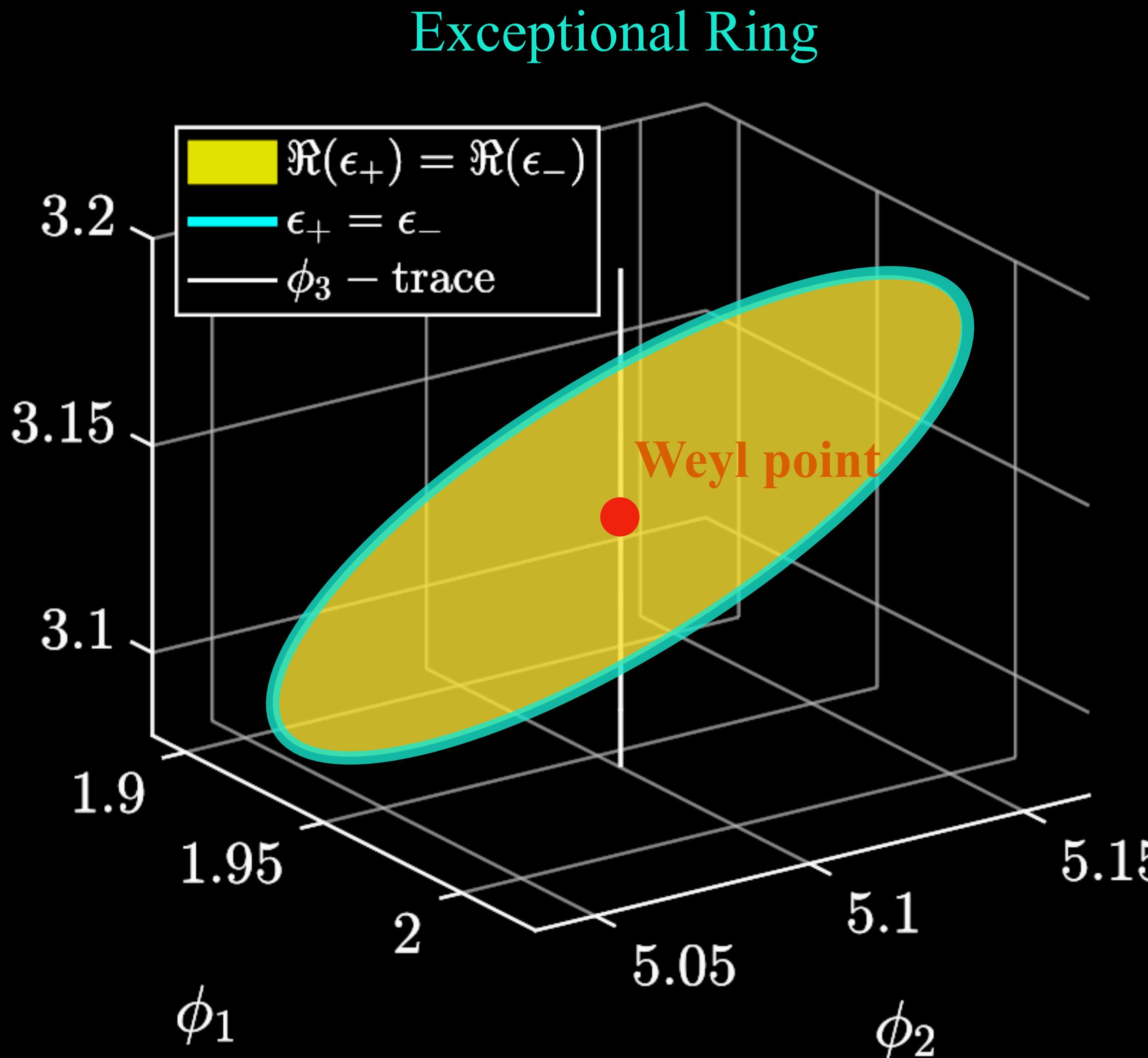
D. C. Ohnmacht, et. al., arXiv:2408.01289 (2024) (accepted in PRL)

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Effective classification parameter

Jeffrey C. Y. Teo and C. L. Kane Phys. Rev. B 82, 115120 (2010), Fan Zhang and C. L. Kane Phys. Rev. B 90, 020501(R) (2014)

Point gap in system with 1 superconducting phase $d_\phi = 1$ ($d_k = 0$)

$$d = d_k - d_\phi \bmod 8 = 7$$

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AZ class	Gap	Classifying space	$d = 0$	$d = 1$	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d = 7$
C^\dagger	P	\mathcal{R}_5	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
	L_r	\mathcal{R}_6	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
	L_i	\mathcal{R}_4	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0

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	L _r	\mathcal{R}_6	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
	L _i	\mathcal{R}_4	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0

winding number
NOT allowed

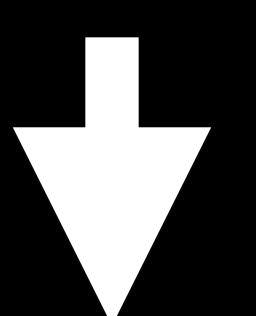
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	L _r	\mathcal{R}_6	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
	L _i	\mathcal{R}_4	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0

winding number
Break spin-rotation symmetry!  NOT allowed

AZ class	Gap	Classifying space	$d = 0$	$d = 1$	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d = 7$
A	P	\mathcal{C}_1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
	L	\mathcal{C}_0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0

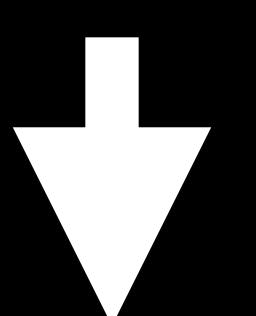
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C^\dagger	P	\mathcal{R}_5	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
	L _r	\mathcal{R}_6	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
	L _i	\mathcal{R}_4	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0

Break spin-rotation symmetry!  winding number
NOT allowed

AZ class	Gap	Classifying space	$d = 0$	$d = 1$	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d = 7$
A	P	\mathcal{C}_1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
	L	\mathcal{C}_0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0

winding number
allowed!

Conclusion

**1) MTJJs are an excellent platform to study engineered
(non-hermiticity) topology**

Non-hermitian topology in MTJJs

Accepted in PRL: arXiv:2408.01289 (2024)

Topology in three state Andreev Molecule

arXiv:2501.07982 (2025)

**2) Refectionless scattering modes are a source of
topology in MTJJs**

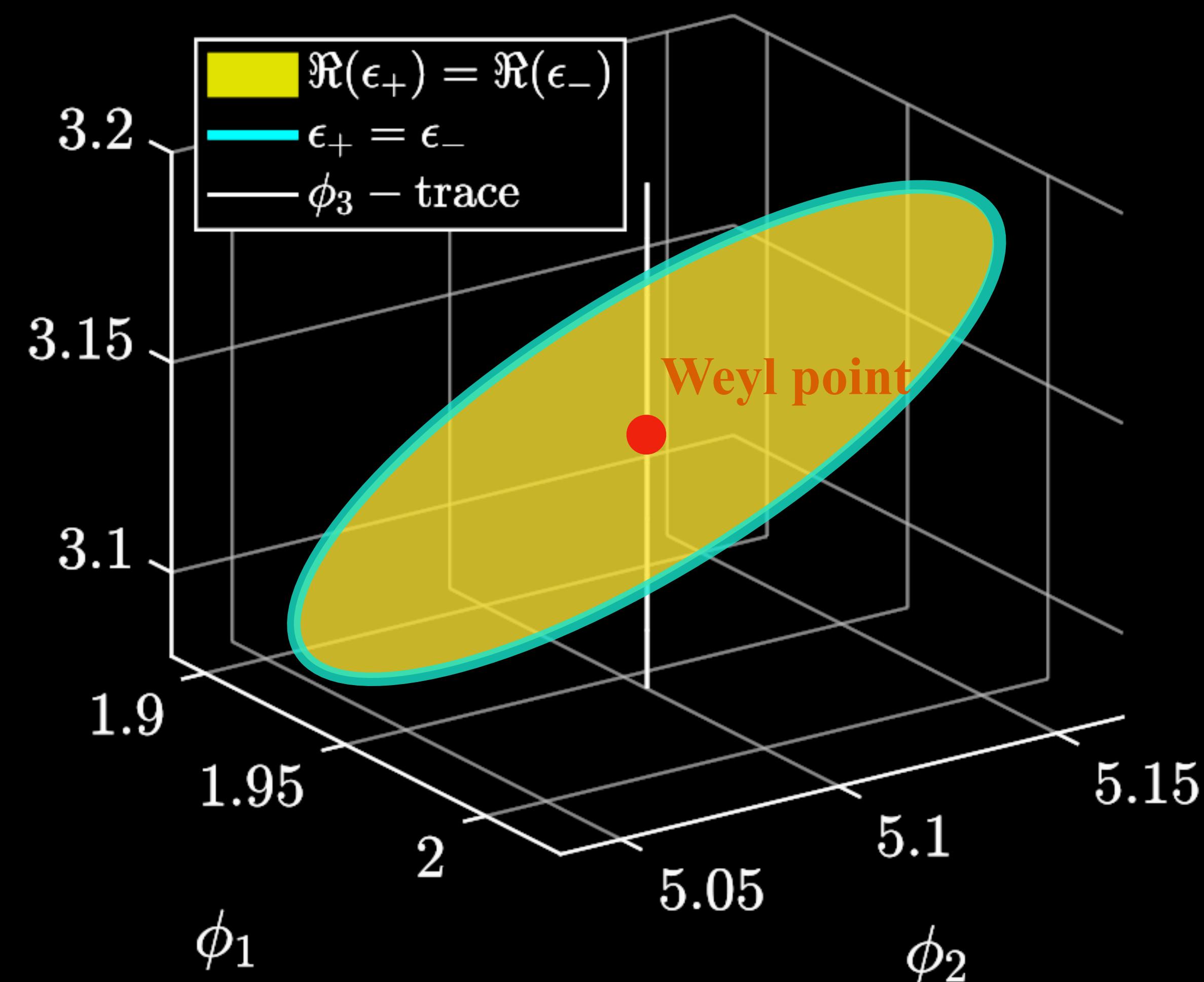
Reflectionless Modes lead to Weyl nodes in MTJJs

arXiv:2503.10874 (2025)

Thank you!

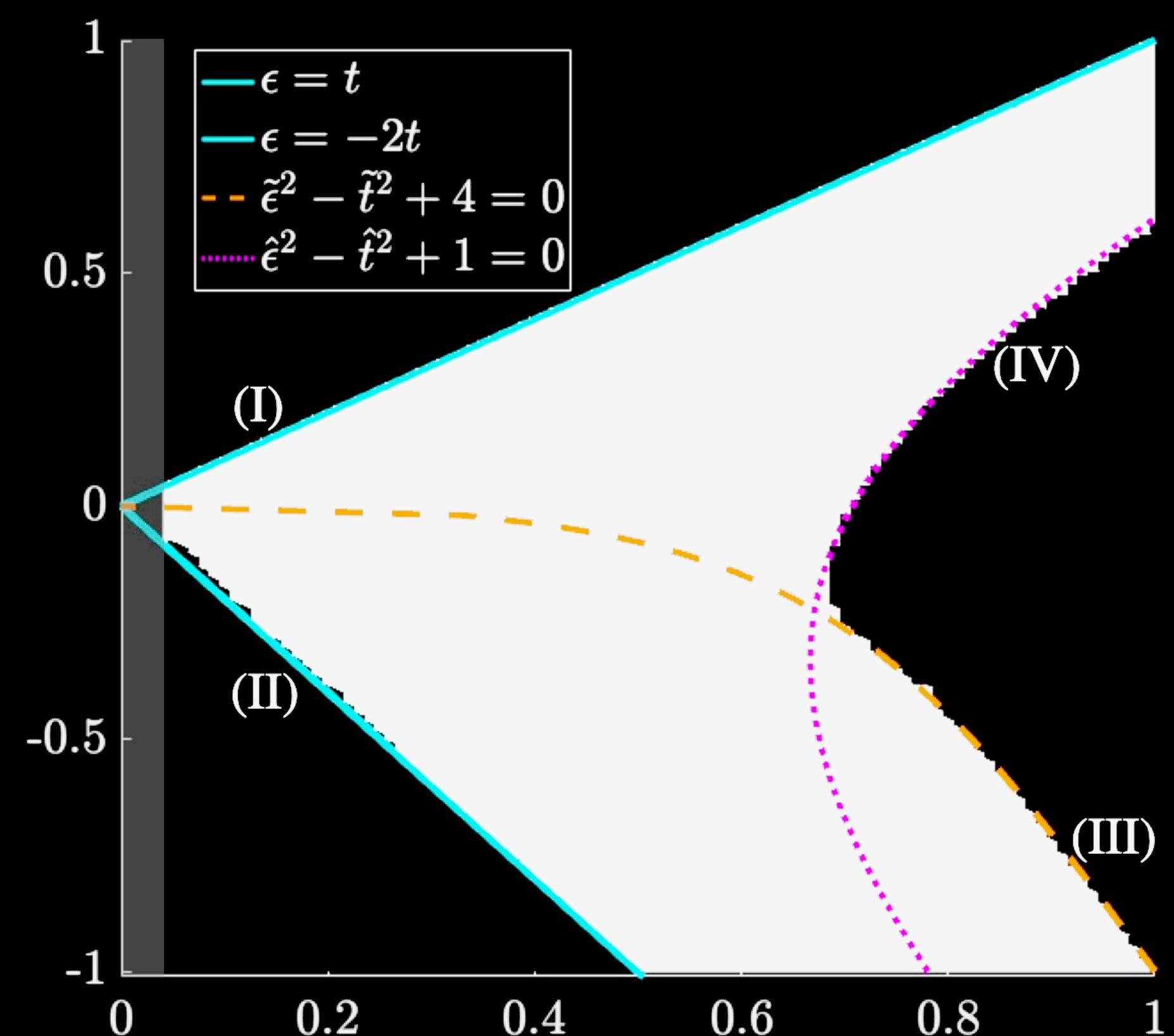
Non-hermitian topology in MTJJs

D. C. Ohnmacht, et. al., arXiv:2408.01289
(2024) (accepted in PRL)



Reflectionless modes as a source of Weyl nodes in multiterminal Josephson junctions

D. C. Ohnmacht, et. al.,
arXiv:2503.10874 (2025)



Why care about non-Hermiticity and (MT)JJs?

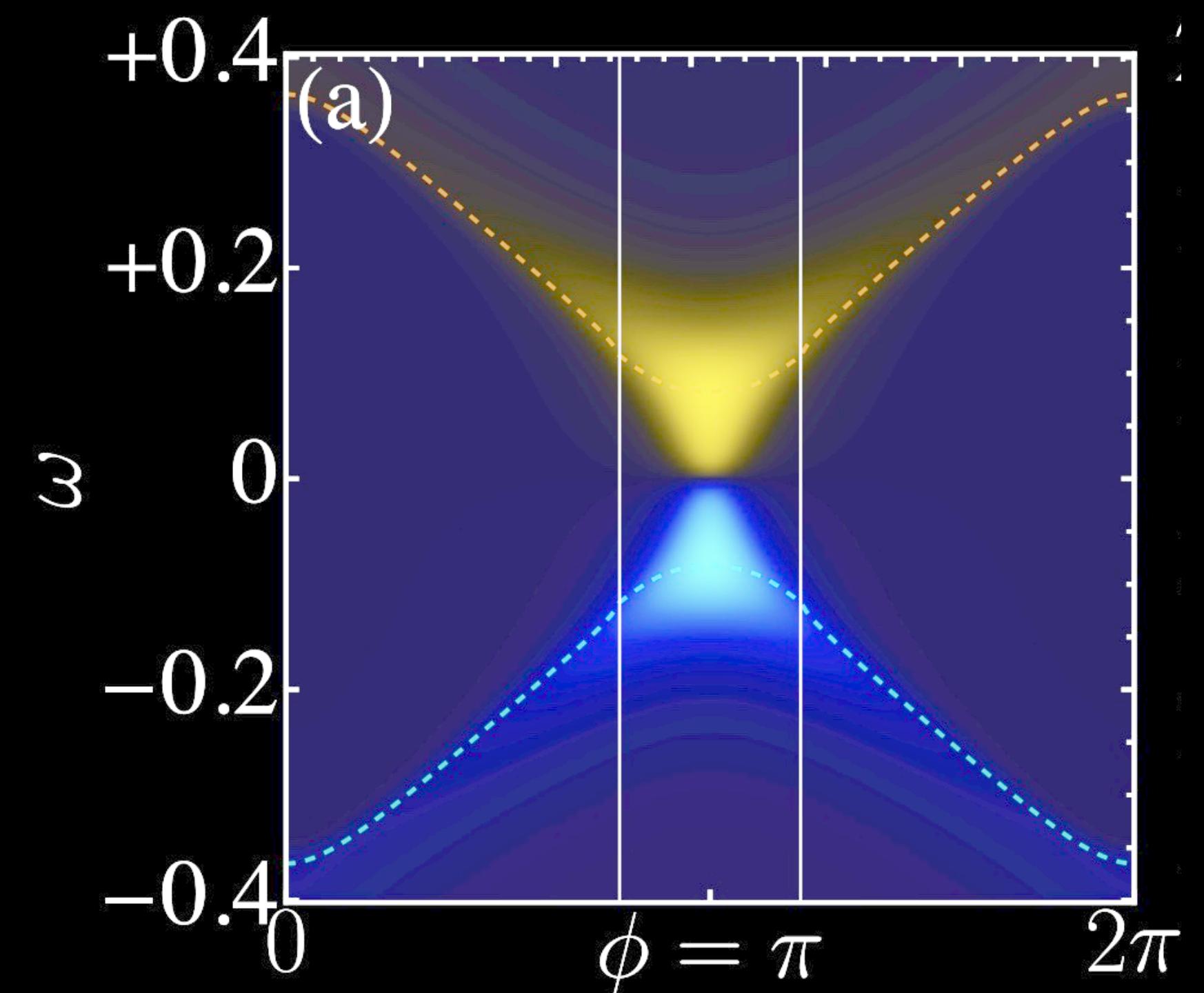
Non-hermitian topology
generalisation of hermitian
topology

Superconductors offer
tunability via
superconducting phases

Enhanced sensing,
undirectional laser, bulk-
fermi arcs, ...

C. Dembowski, et. al., Phys. Rev. Lett. 86, 787 (2001)
J. Doppler, et. al., Nature 537, 76 (2016)
W. Chen, et. al., Nature 548, 192 (2017)
B. Peng, et. al., PNAS 113, 6845 (2016)
H. Zhou, et. al., Science 359, 1009 (2018)
...

Measurability in current susceptibility



V. Kornich and B. Trauzettel Phys. Rev. Research 4, 033201 (2022)
C. A. Li, et. al. Phys. Rev. B 109, 214514 (2024)
J. Cayao and M. Sato, arXiv:2307.15472 (2024)
C.W.J. Beenakker, Appl. Phys. Lett. 125, 122601 (2024)
P.X. Shen, et. al. Phys. Rev. Lett. 133, 086301 (2024)
J. Cayao and M. Sato, arXiv:2408.17260 (2024)
...

Making sense of effective dimensions

RESEARCH ARTICLE | MAY 14 2009

Periodic table for topological insulators and superconductors

Alexei Kitaev

AIP Conf. Proc. 1134, 22–30 (2009)

<https://doi.org/10.1063/1.3149495>

The Hamiltonian of a translationally invariant systems can be written in the momentum representation:

$$\hat{H} = \frac{i}{4} \sum_{\mathbf{p}} \sum_{j,k} A_{jk}(\mathbf{p}) \hat{c}_{-\mathbf{p},j} \hat{c}_{\mathbf{p},k}, \quad (19)$$

where j and k refer to particle flavors. The matrix $A(\mathbf{p})$ is skew-Hermitian but not real; it rather satisfies the condition $A_{jk}(\mathbf{p})^* = A_{jk}(-\mathbf{p})$. By abuse of terminology, such matrix-valued functions are called “functions from $\bar{\mathbb{R}}^d$ to real skew-symmetric matrices”, where $\bar{\mathbb{R}}^d$ is the usual Euclidean space with the involution $\mathbf{p} \leftrightarrow -\mathbf{p}$ (cf. [29]).

By a *real vector bundle* over the *real space* X we mean a complex vector bundle E over X which is also a real space and such that

- (i) the projection $E \rightarrow X$ is real (i.e. commutes with the involutions on E, X);
- (ii) the map $E_x \rightarrow E_x$ is anti-linear, i.e. the diagram

$$\begin{array}{ccc} \mathbf{C} \times E_x & \xrightarrow{\quad} & E_x \\ \downarrow & & \downarrow \\ \mathbf{C} \times E_x & \xrightarrow{\quad} & E_x \end{array}$$

commutes, where the vertical arrows denote the involution and \mathbf{C} is given its standard real structure ($\tau(z) = \bar{z}$).

Normally, symmetries defined by

$$U_C H_i^*(k) U_C^{-1} = -H_i(-k)$$

But phase is unusual dimension:

$$U_C H_i^*(\phi) U_C^{-1} = -H_i(+\phi)$$

⇒ Phase is “just” a parameter

Solution: Effective Parameter

Effective classification parameter

$$d = d_k - d_\phi \bmod 8 = \begin{cases} 0 , d_k = d_\phi = 0 \\ 7 , d_k = 0 , d_\phi = 1 \end{cases}$$

Jeffrey C. Y. Teo and C. L. Kane Phys. Rev. B 82, 115120 (2010)
Fan Zhang and C. L. Kane Phys. Rev. B 90, 020501(R) (2014)

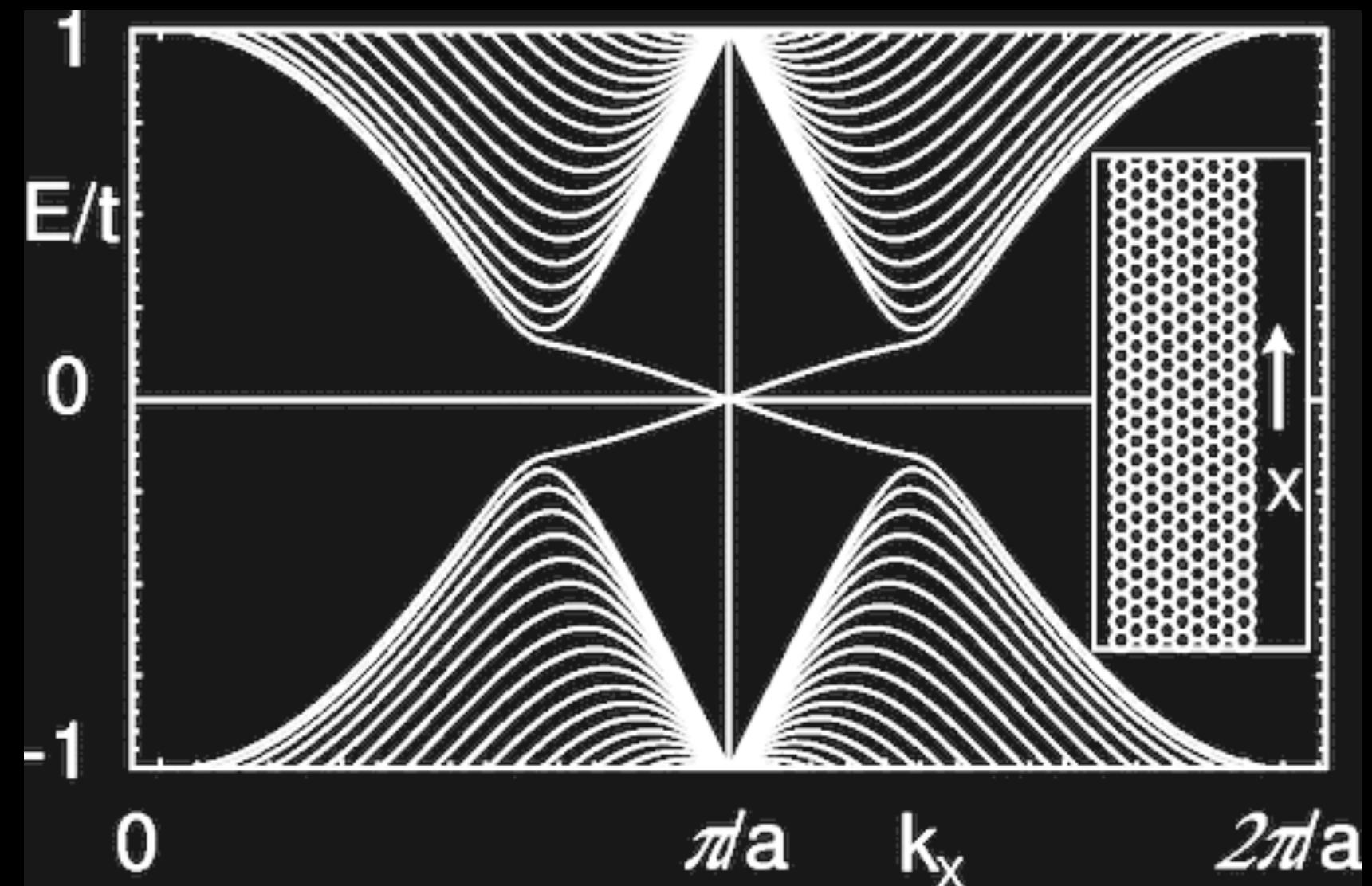
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	L	\mathcal{C}_0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0

winding number
NOT allowed

winding number
allowed!

Why care about topology?

Topological Phases of Matter



C. L. Kane and E. J. Mele Phys. Rev. Lett. 95, 226801 (2005)
König, M. et al. Science 318, 766–770 (2007)

Fundamental Insights into Symmetry and Topology

Class	T	C	S	1	2	3
A	0	0	0	0	\mathbb{Z}	0
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}
AI	1	0	0	0	0	0
BDI	1	1	1	\mathbb{Z}	0	0
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}
All	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2

Altland and M. R. Zirnbauer, Phys. Rev. B 55, 1142 (1997)
Andreas P. Schnyder, et. al., Phys. Rev. B 78, 195125 (2008)

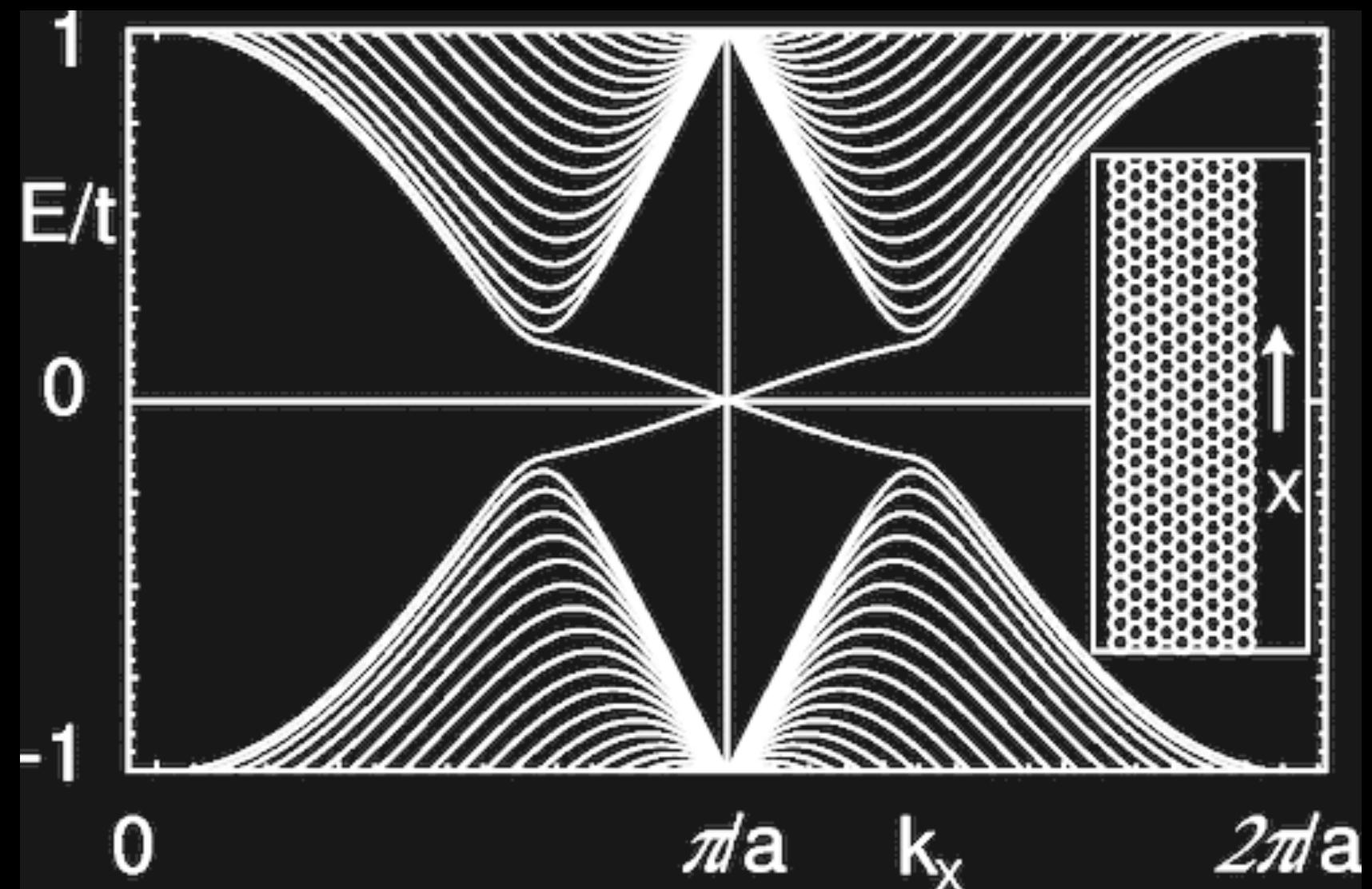
Stability Against Perturbations

Applications in Quantum Computing and Electronics

...

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DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}
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Altland and M. R. Zirnbauer, Phys. Rev. B 55, 1142 (1997)
Andreas P. Schnyder, et. al., Phys. Rev. B 78, 195125 (2008)

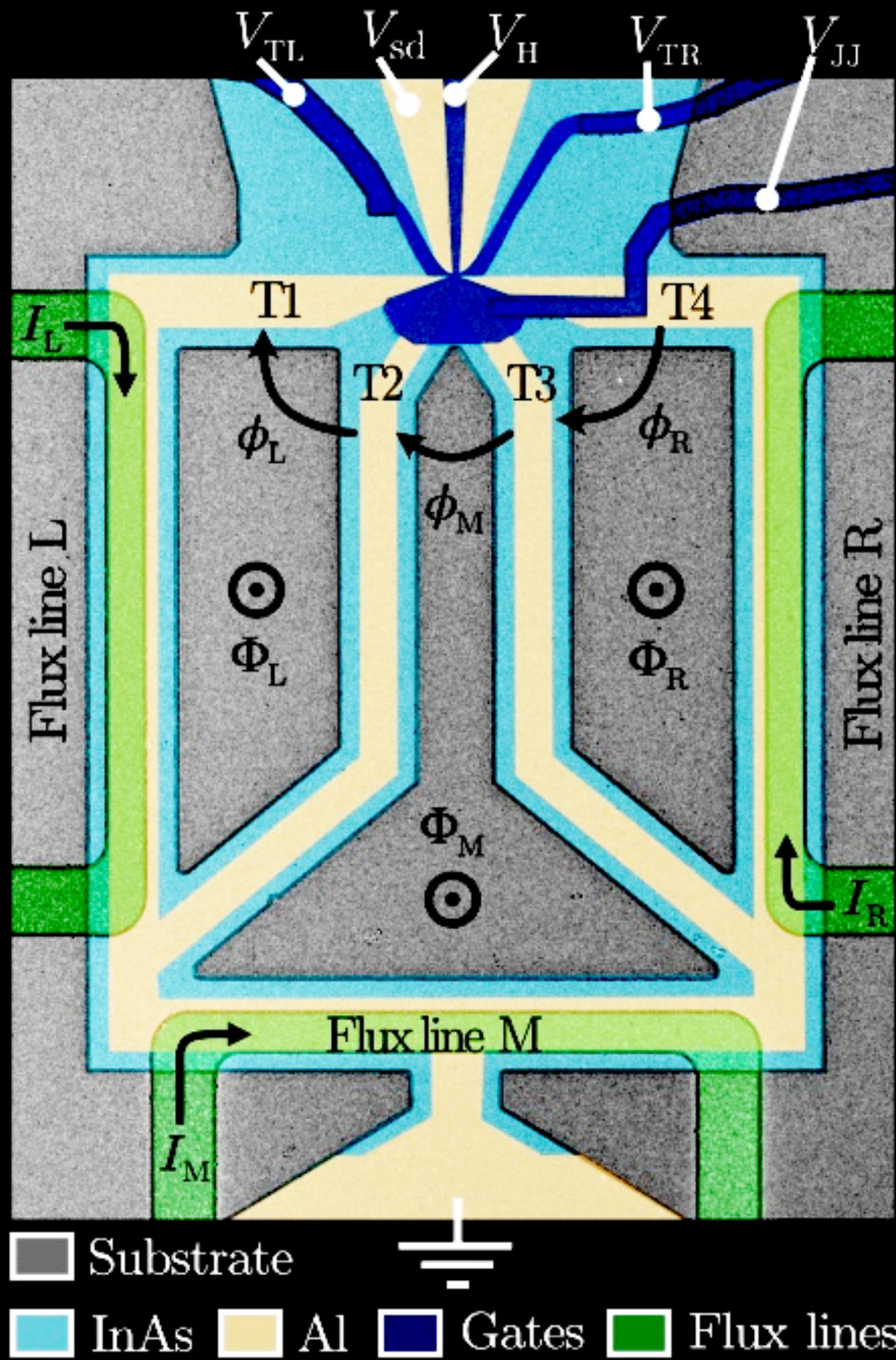
Stability Against Perturbations

Applications in Quantum Computing and Electronics

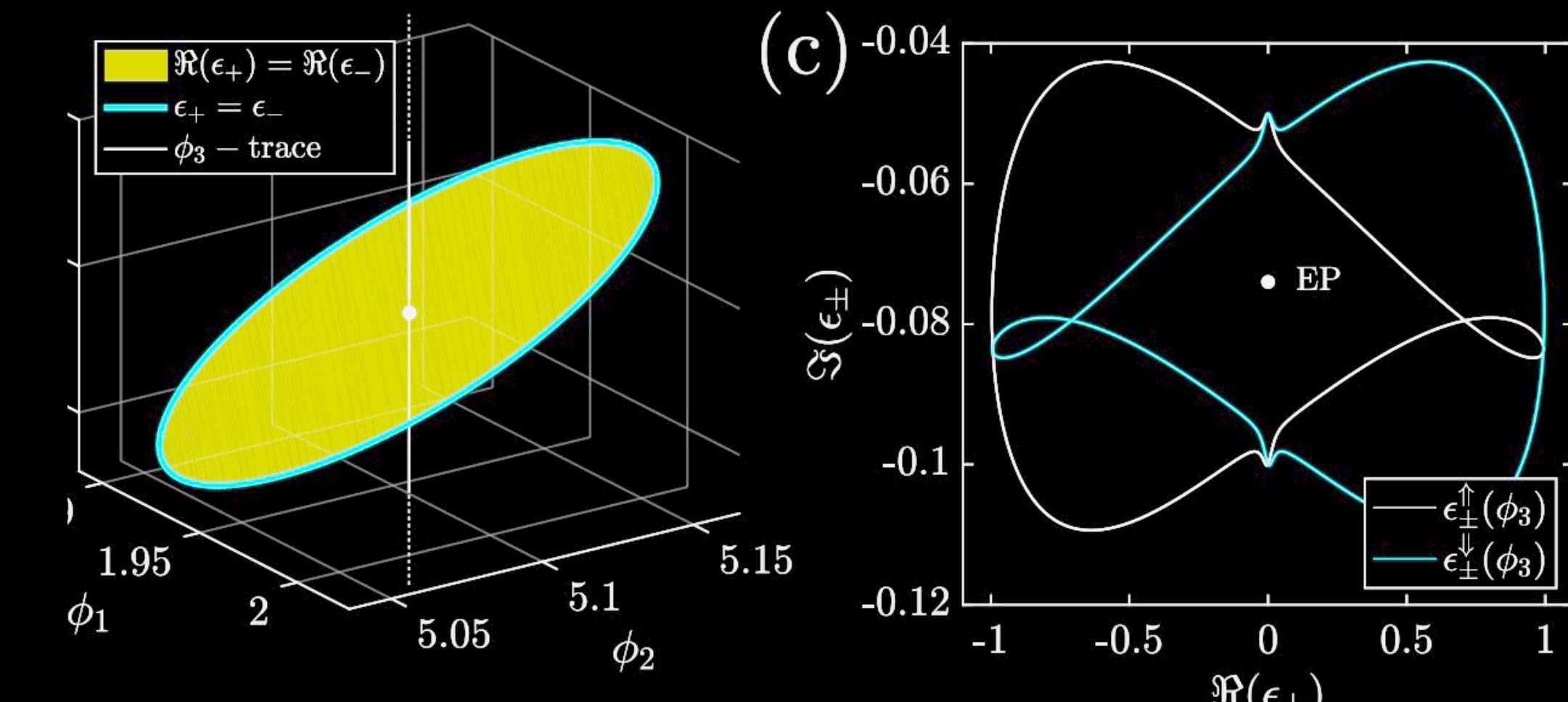
...

Thank you!

4-terminal MTJJ



MTJJ as non-herm. top. matter



arXiv:2408.01289 (2024)

Reflectionless Modes

