Non-hermitian topology in multiterminal superconducting junctions

David Christian Ohnmacht,
Valentin Wilhelm
Hannes Weisbrich
Wolfgang Belzig

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SFB 1432
Universität Konstanz

Non-hermitian topology in multiterminal superconducting junctions

1) Multiterminal Josephson Junctions (MTJJs) host stable exceptional points (EPs)

2) Topological classification provides useful insight for experiments

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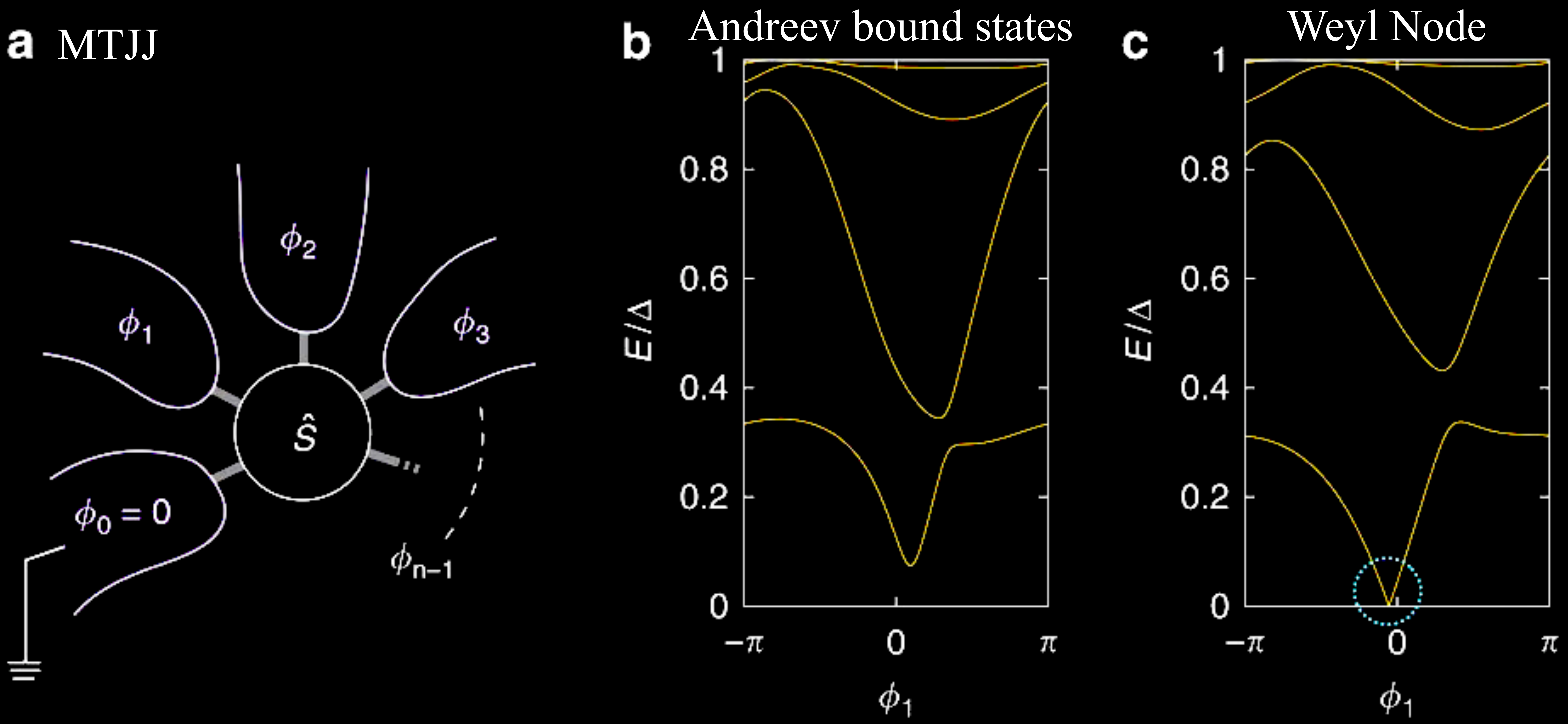
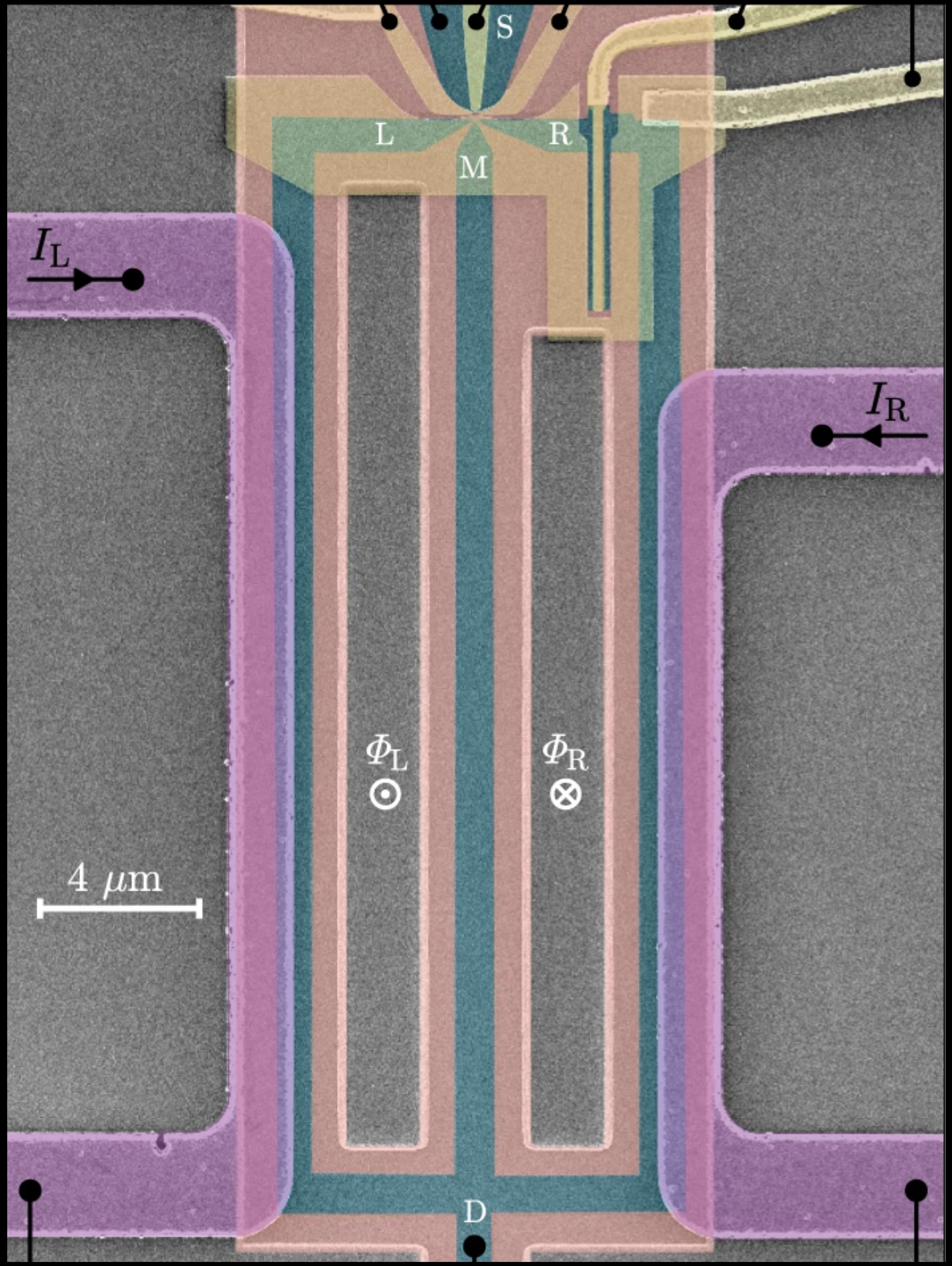
Multiterminal Josephson junctions (MTJJs): Theory



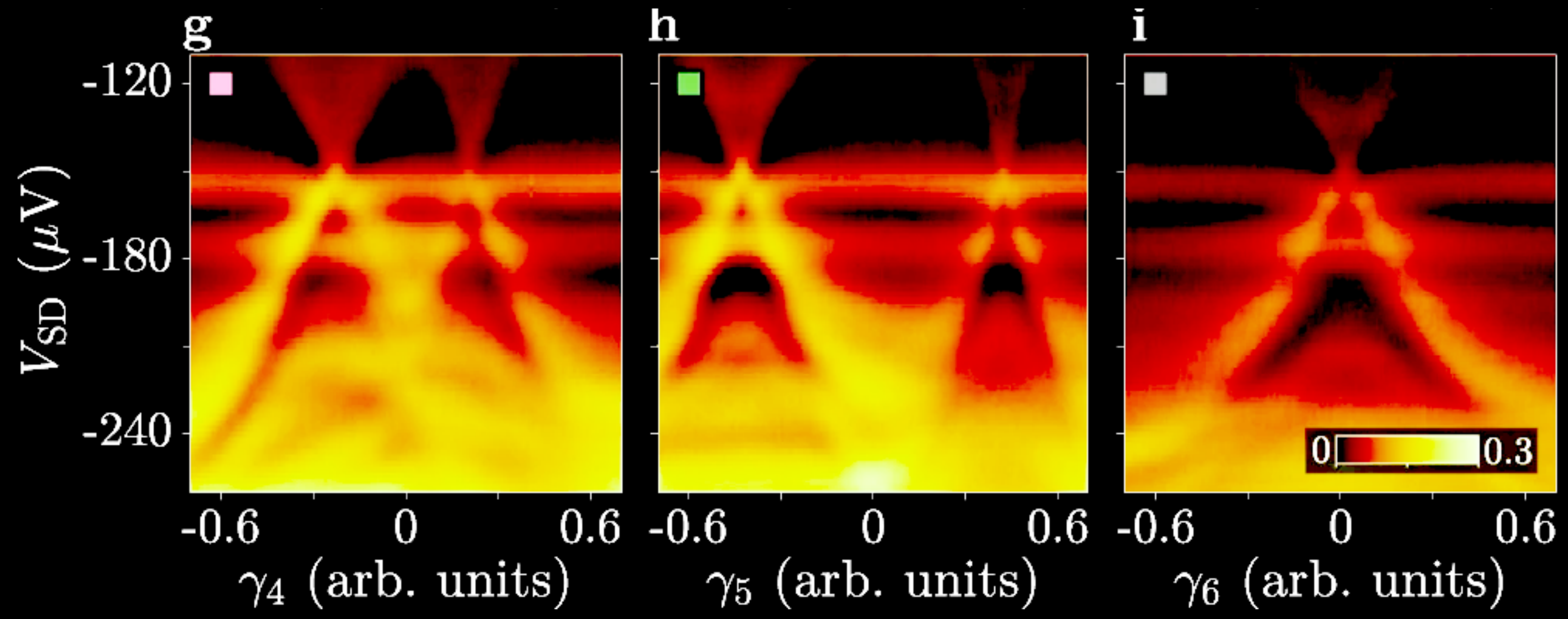
R.-P. Riwar, et. al., Nature Commun. 7, 1 (2016)
E. Eriksson, et. al., Phys. Rev. B 95, 075417 (2017)
H.-Y. Xie, et. al., Phys. Rev. B 96, 161406(R) (2017)
R. L. Klees, et. al., Phys. Rev. Lett. 124, 197002 (2020)
…

Multiterminal Josephson junctions (MTJJs): Experiment

Topography of MTJJ



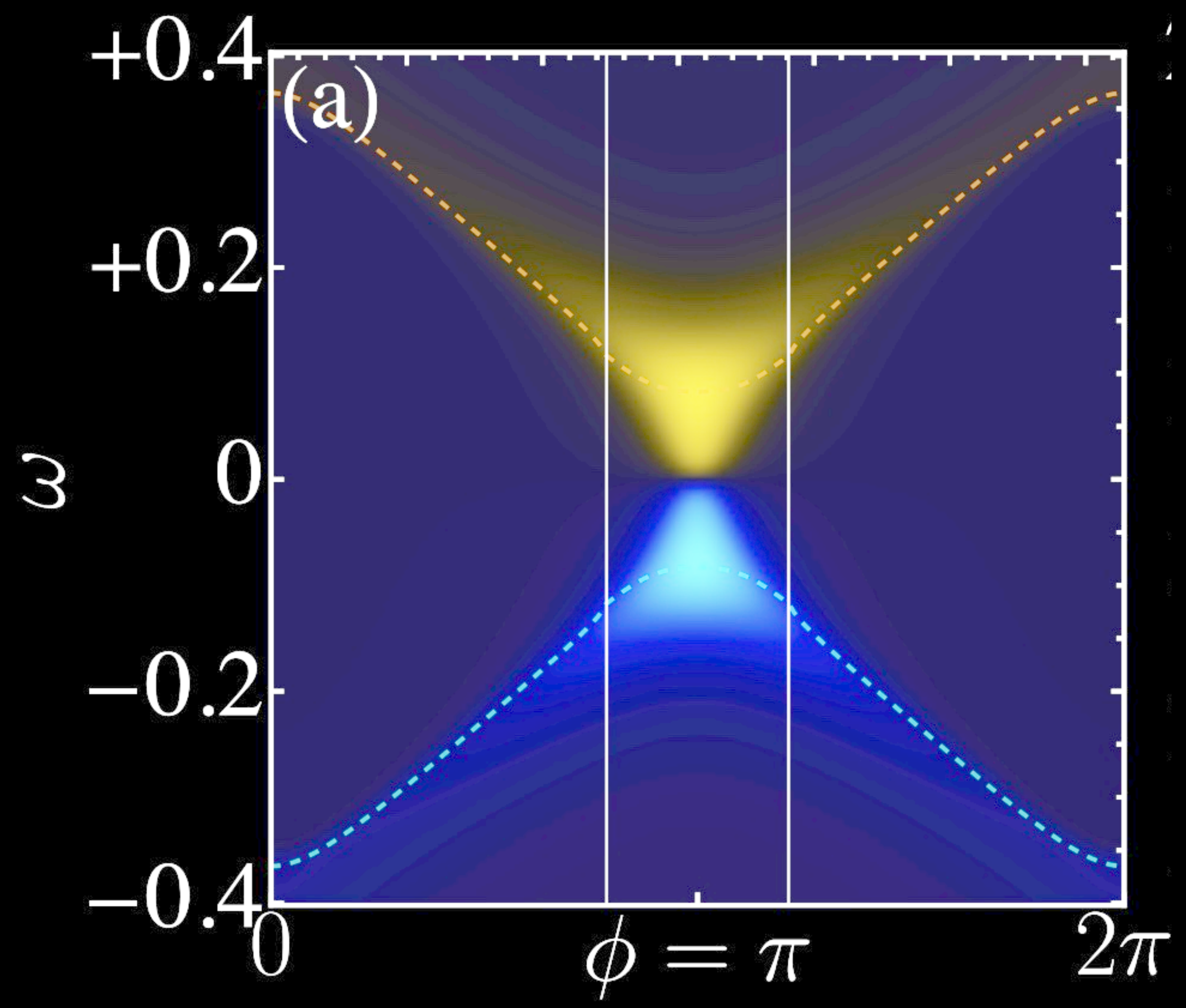
Conductance @ probe bias V_{SD} and phase γ



M. Coraiola, et. al., Nature Communications 14, 6784 (2023).
M. Coraiola, et. al., Phys. Rev. X 14, 031024 (2024)
…

Why care about non-Hermiticity and (MT)JJs?

Measurability in current susceptibility



Non-hermitian topology generalisation of hermitian topology

Superconductors offer tuneability via superconducting phases

Enhanced sensing, undirectional laser, bulk-fermi arcs, …

C. Dembowski, et. al., Phys. Rev. Lett. 86, 787 (2001)
J. Doppler, et. al., Nature 537, 76 (2016)
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H. Zhou, et. al., Science 359, 1009 (2018)
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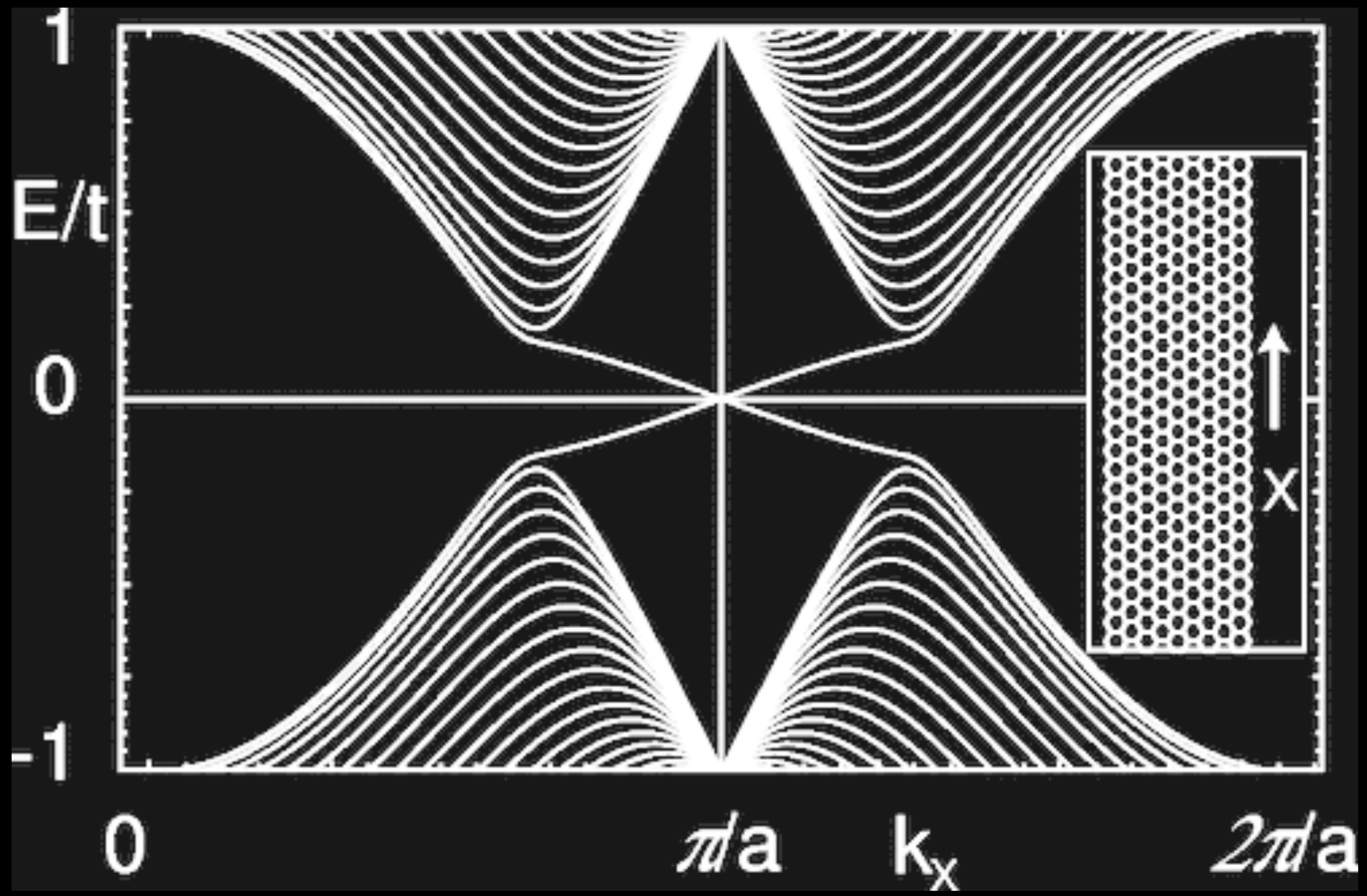
V. Kornich and B. Trauzettel Phys. Rev. Research 4, 033201 (2022)
C. A. Li, et. al. Phys. Rev. B 109, 214514 (2024)
J. Cayao and M. Sato, arXiv:2307.15472 (2024)
C.W.J. Beenakker, Appl. Phys. Lett. 125, 122601 (2024)
P.X. Shen, et. al. Phys. Rev. Lett. 133, 086301 (2024)
J. Cayao and M. Sato, arXiv:2408.17260 (2024)
…

Why care about topology?

Topological Phases of Matter



C. L. Kane and E. J. Mele Phys. Rev. Lett. 95, 226801 (2005)
König, M. et al. Science 318, 766–770 (2007)

Fundamental Insights into Symmetry and Topology

Class	T	C	S	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$

Altland and M. R. Zirnbauer, Phys. Rev. B 55, 1142 (1997)
Andreas P. Schnyder, et. al., Phys. Rev. B 78, 195125 (2008)

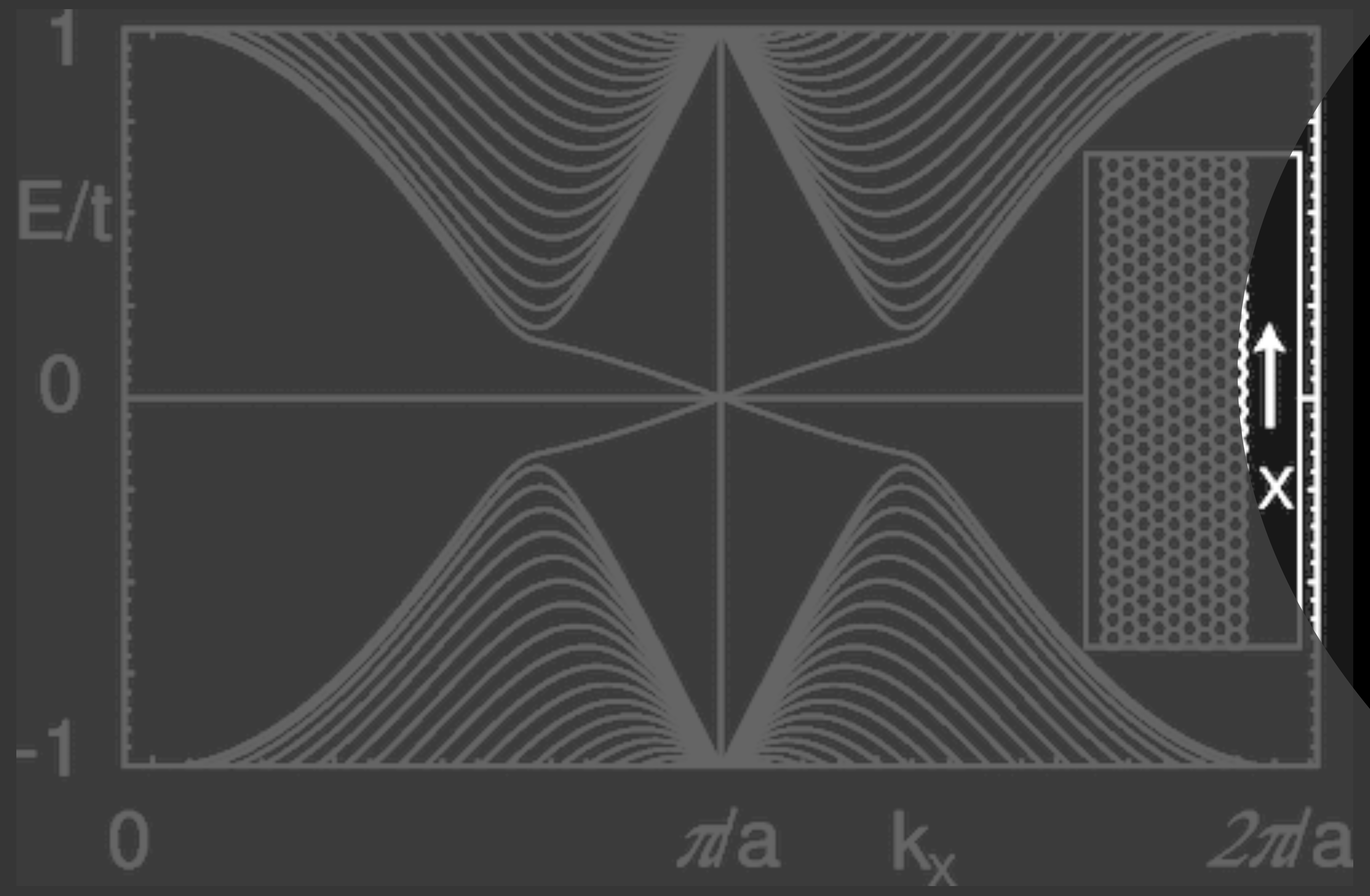
Stability Against Perturbations

Applications in Quantum Computing and Electronics

…

Why care about topological aspects of condensed matter physics?

Topological Phases of Matter



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Fundamental Insights into Symmetry and Topology

Class	T	C	S	1	2	3
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BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
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Stability Against Perturbations

Applications in Quantum Computing and Electronics

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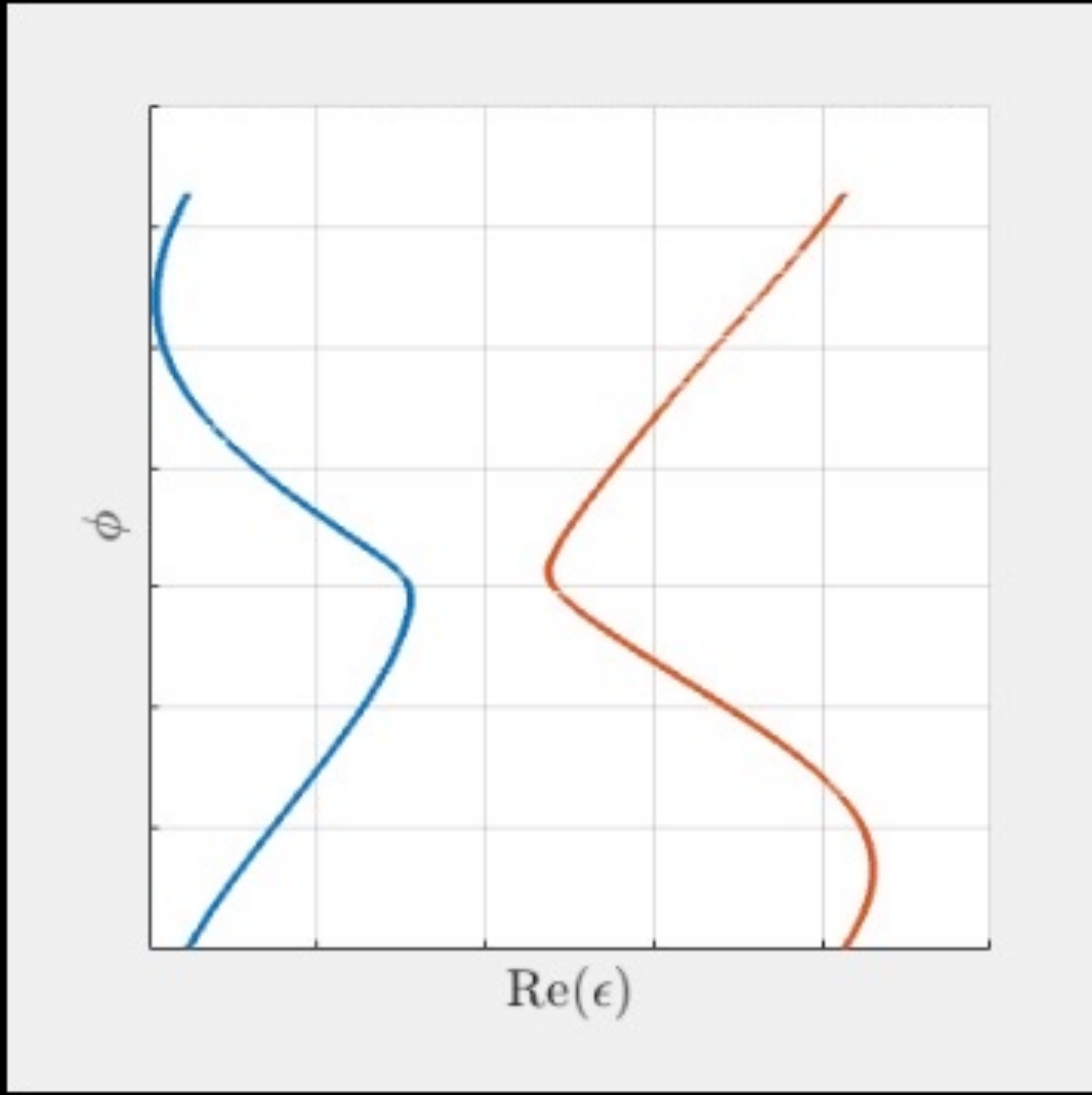
Non-hermitian physics

Hermitian

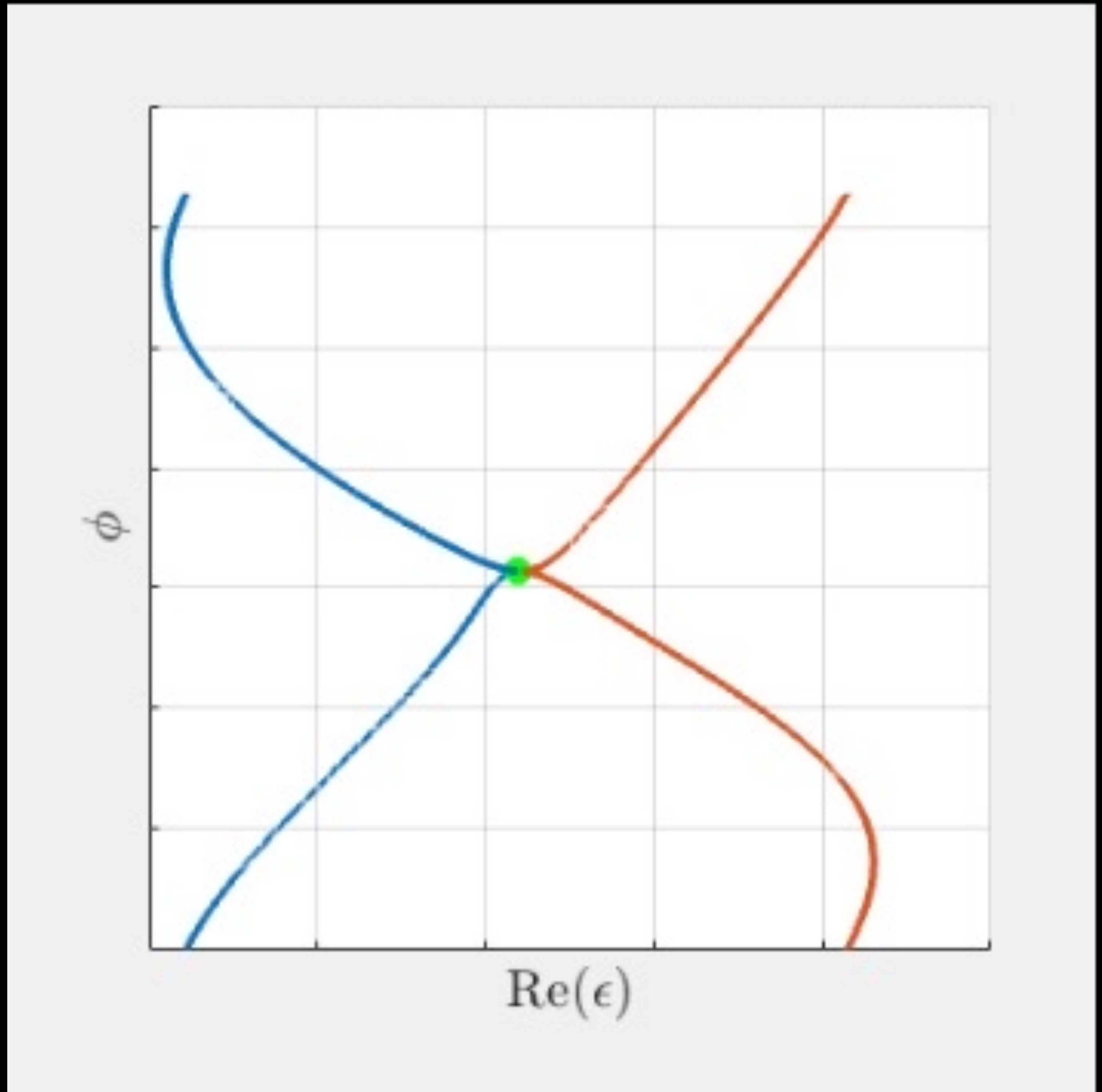
$$H = H^\dagger$$

$$\epsilon_\alpha \in \mathbb{R}$$

Gapped



Ungapped



Non-hermitian physics

Kohei Kawabata, et. al., Phys. Rev. X 9, 041015 (2019)

non-Hermitian

$$H \neq H^\dagger$$

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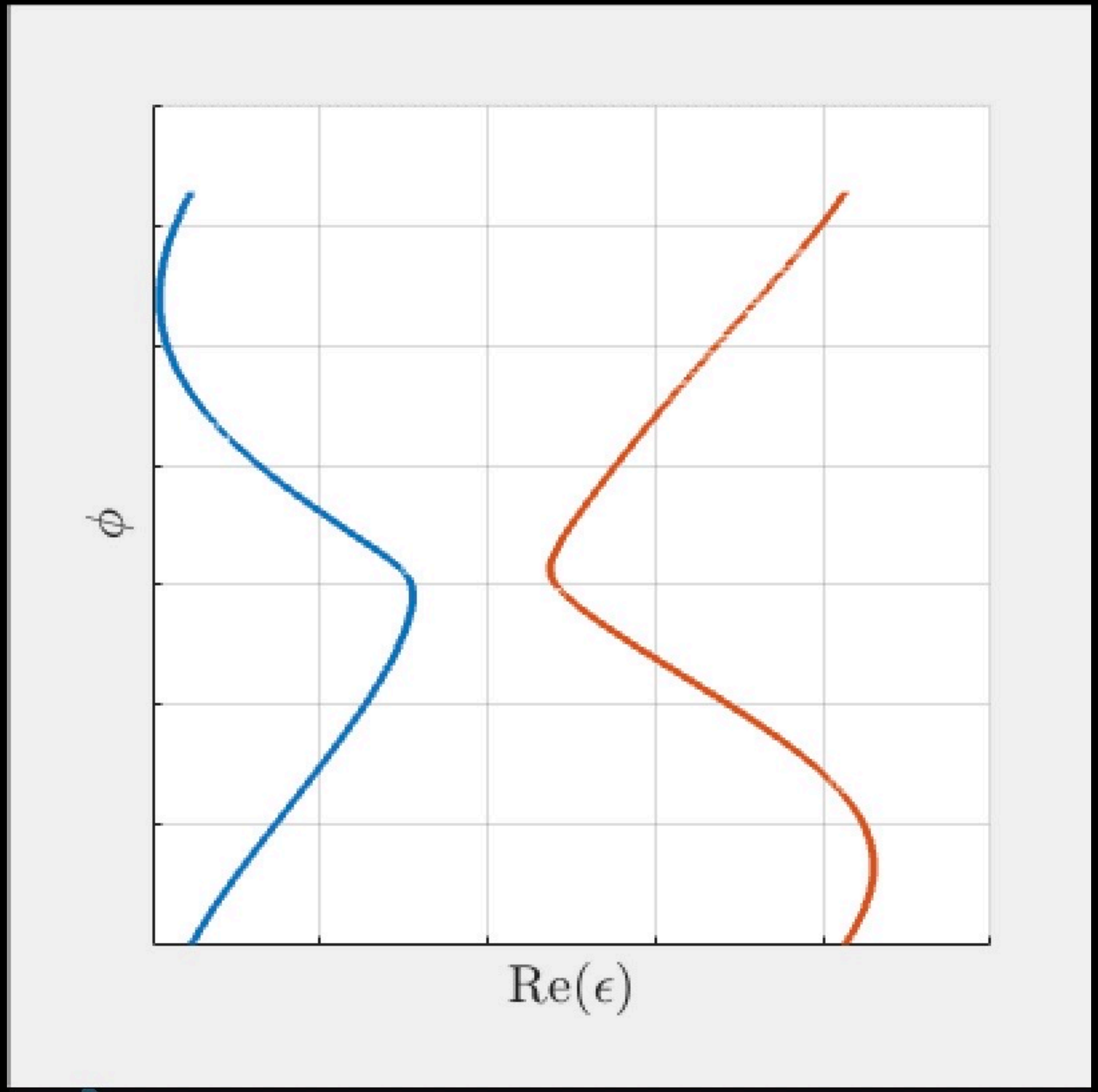

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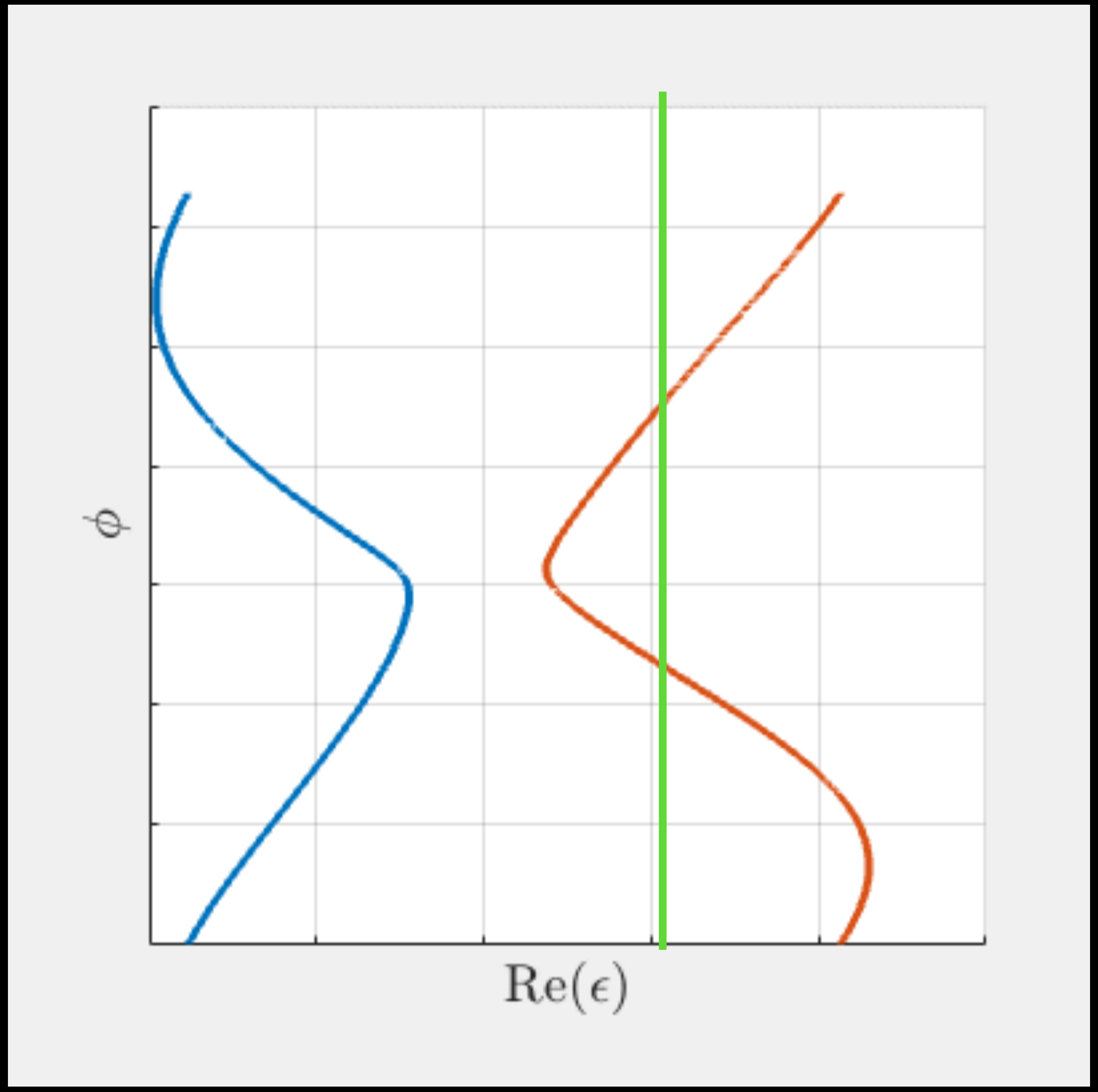
Line Gap



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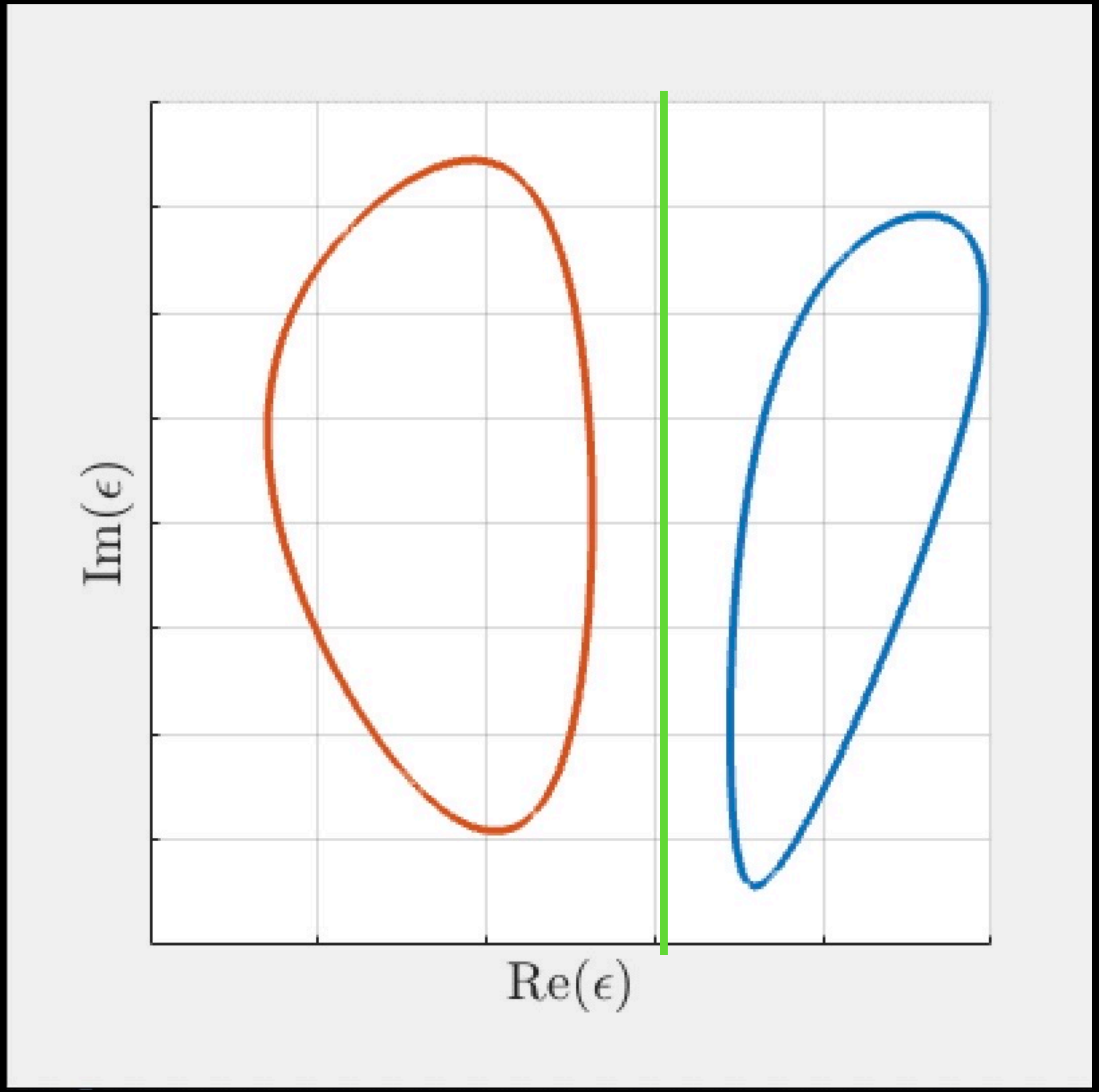

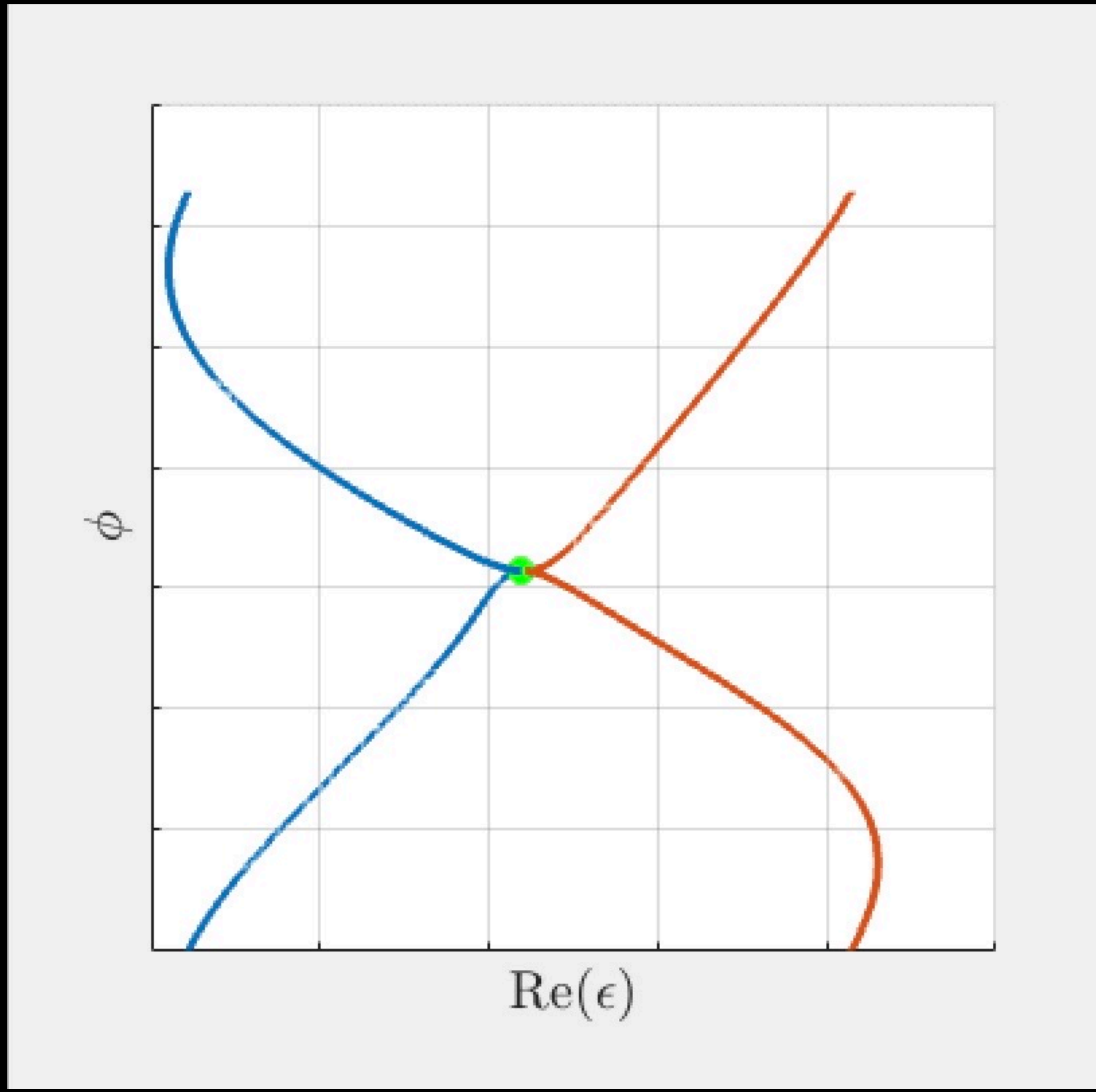

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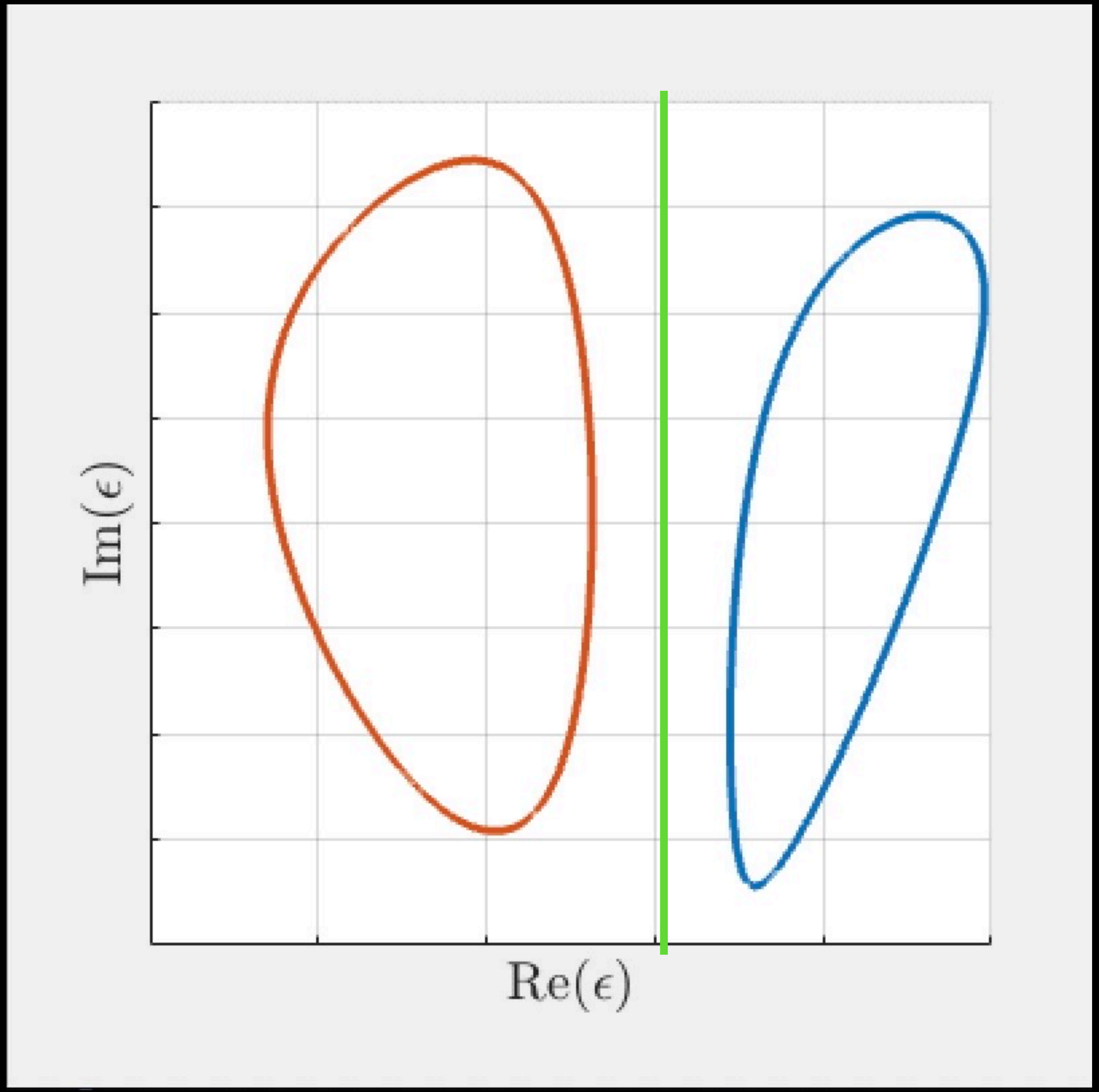
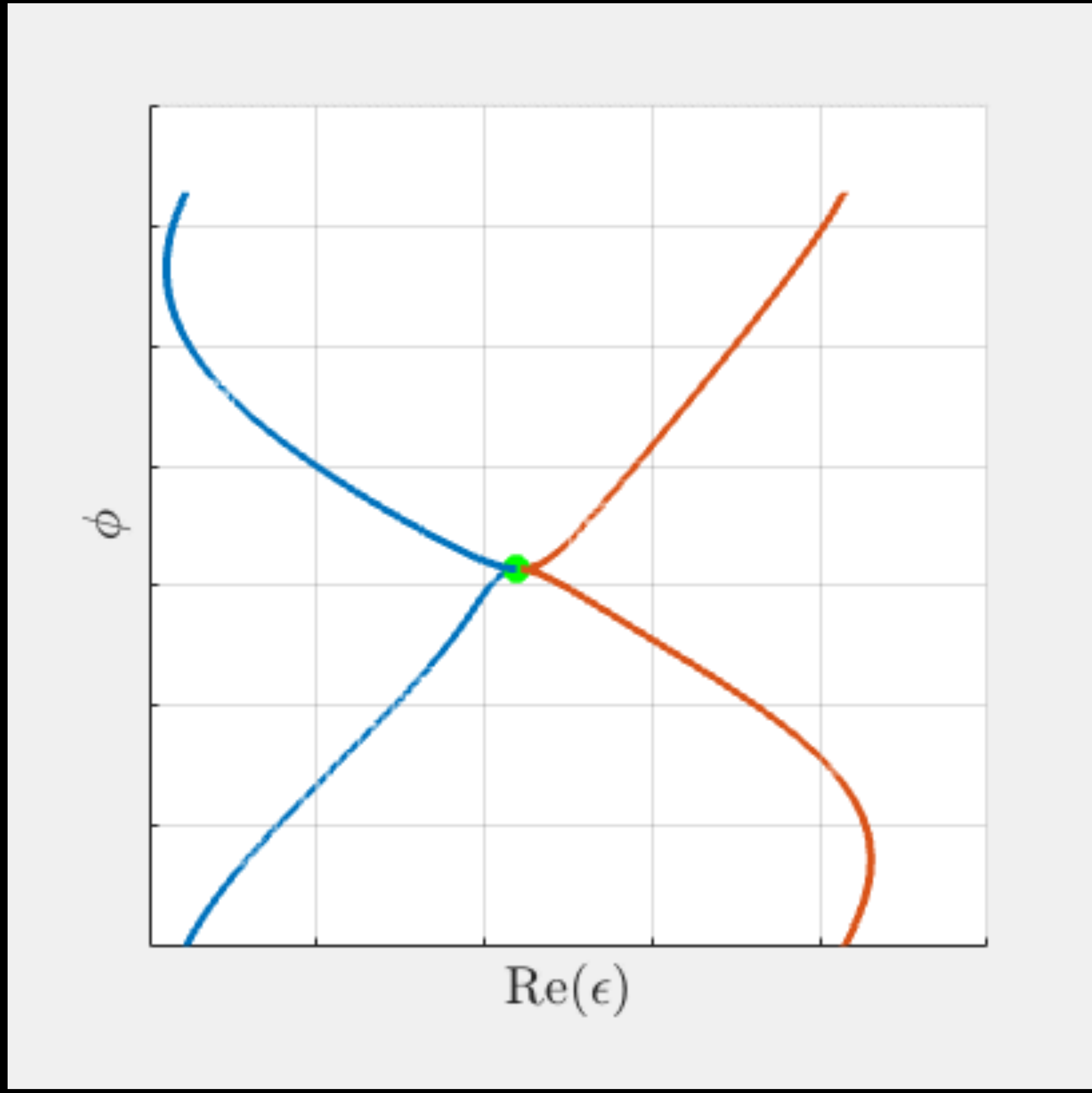


Exceptional Point (EPs)



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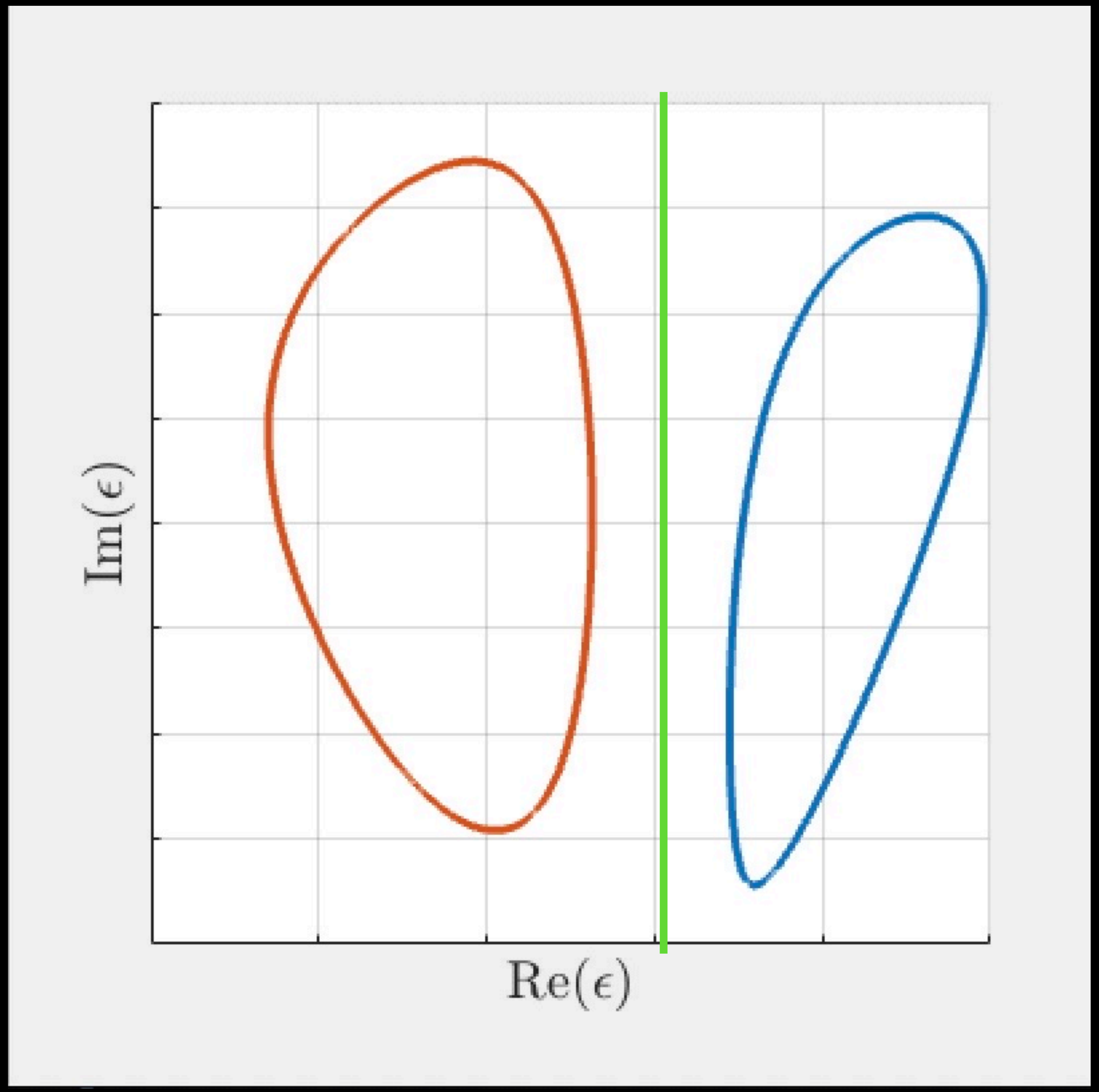

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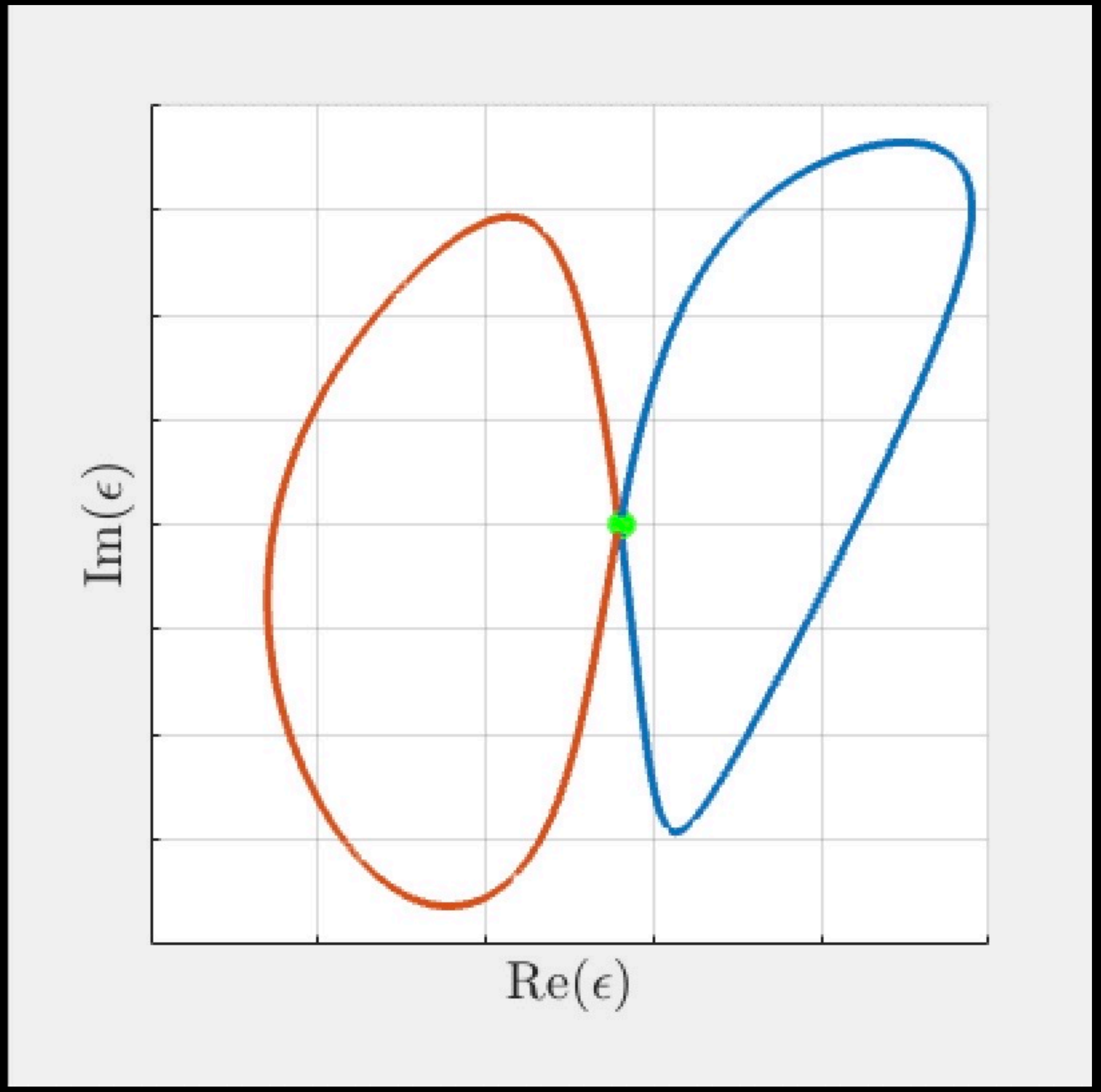

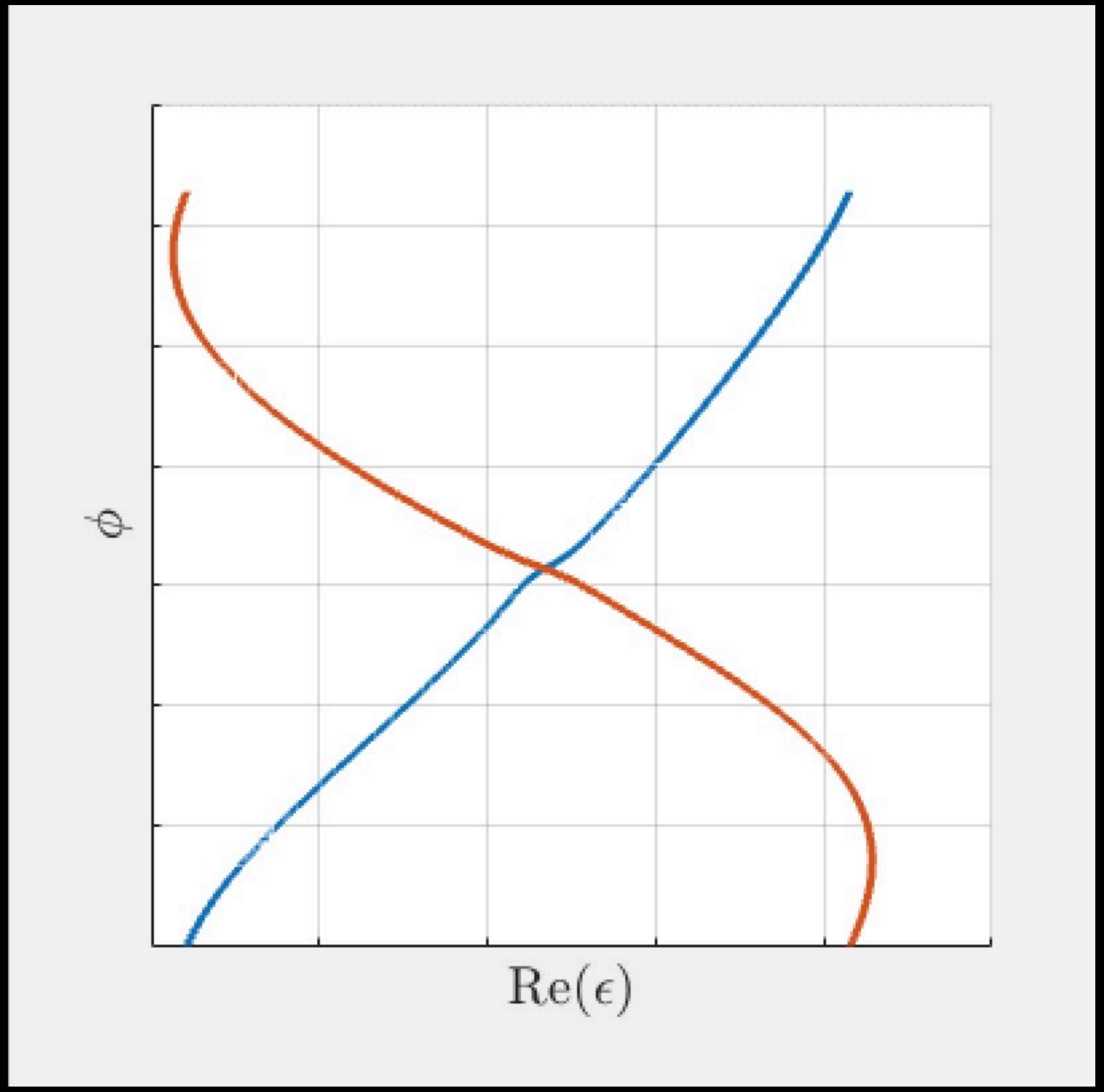

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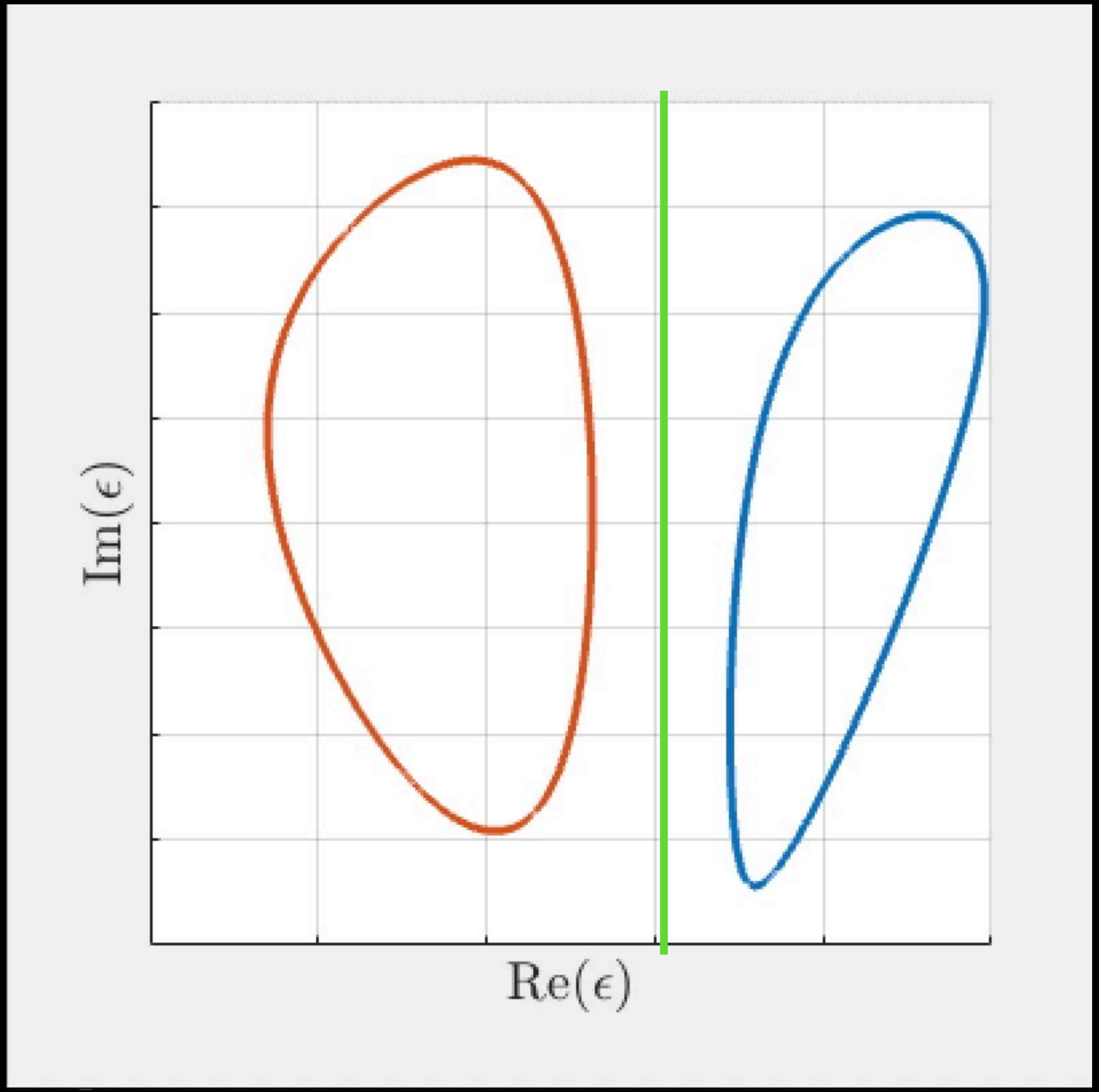
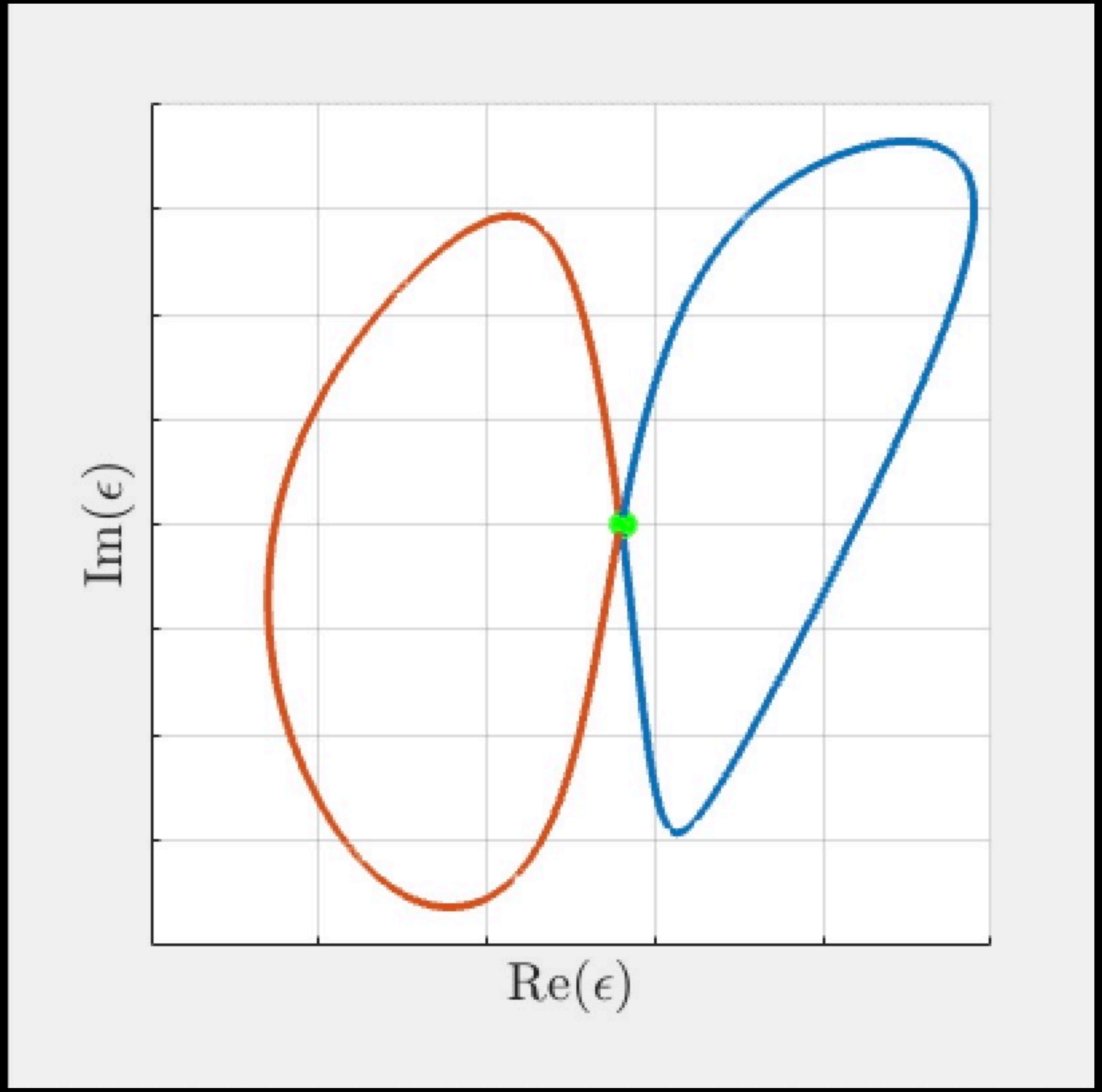
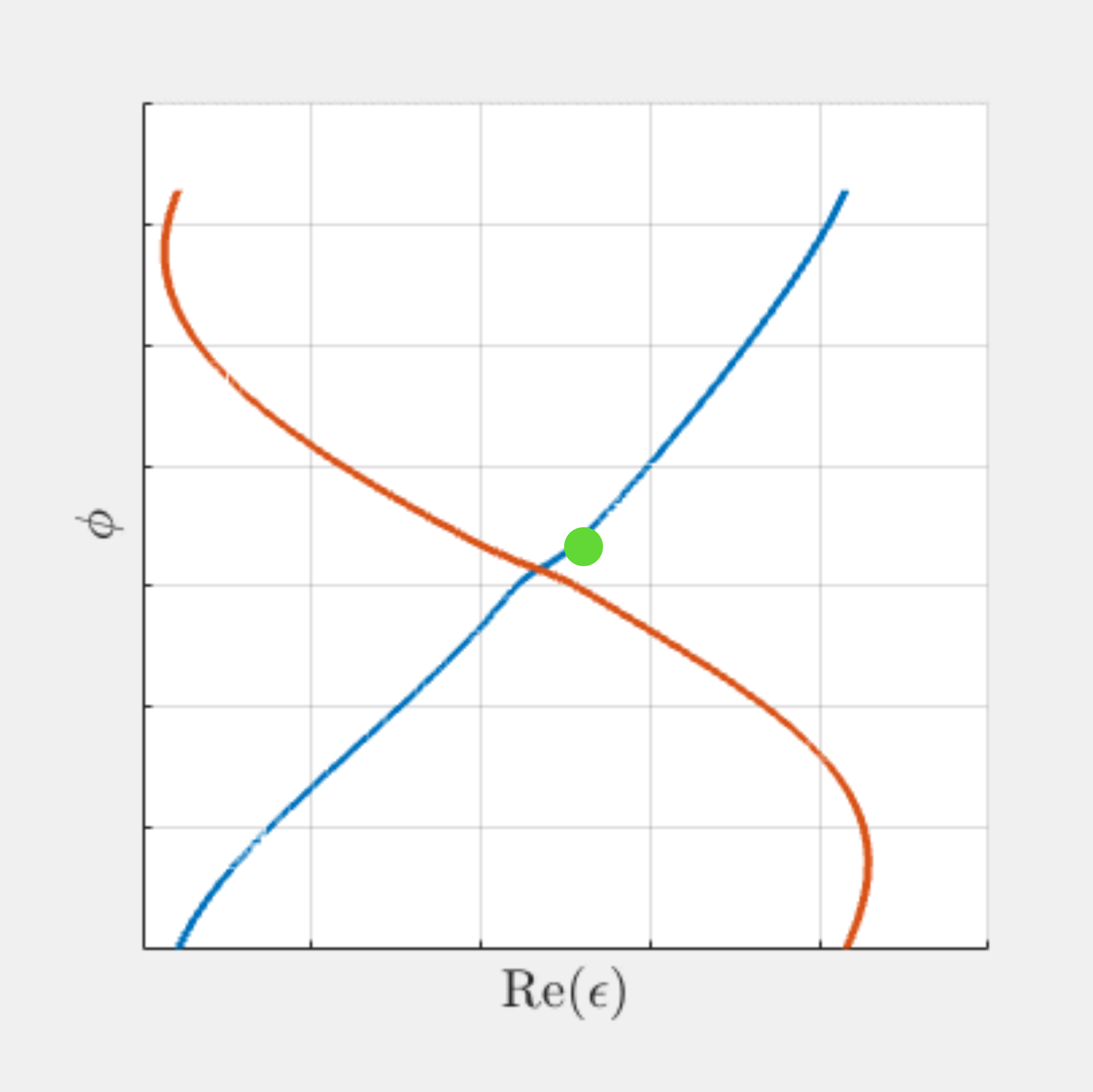


Exceptional Point (EPs)

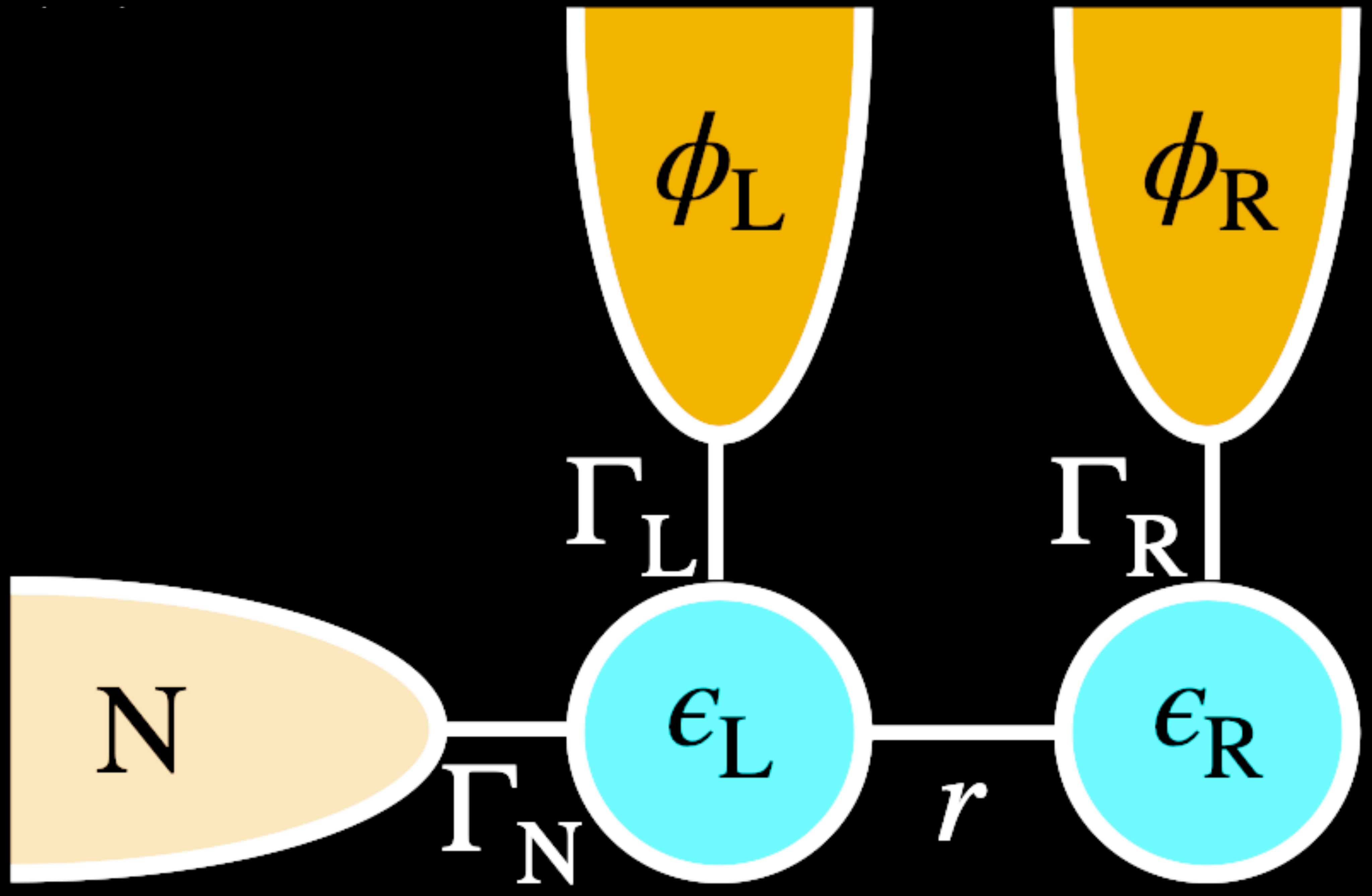


Point Gap



What's non-hermitian about (MT)JJs?

2-terminal junction with added normal metal



Effective low-energy Hamiltonian from central Green's function

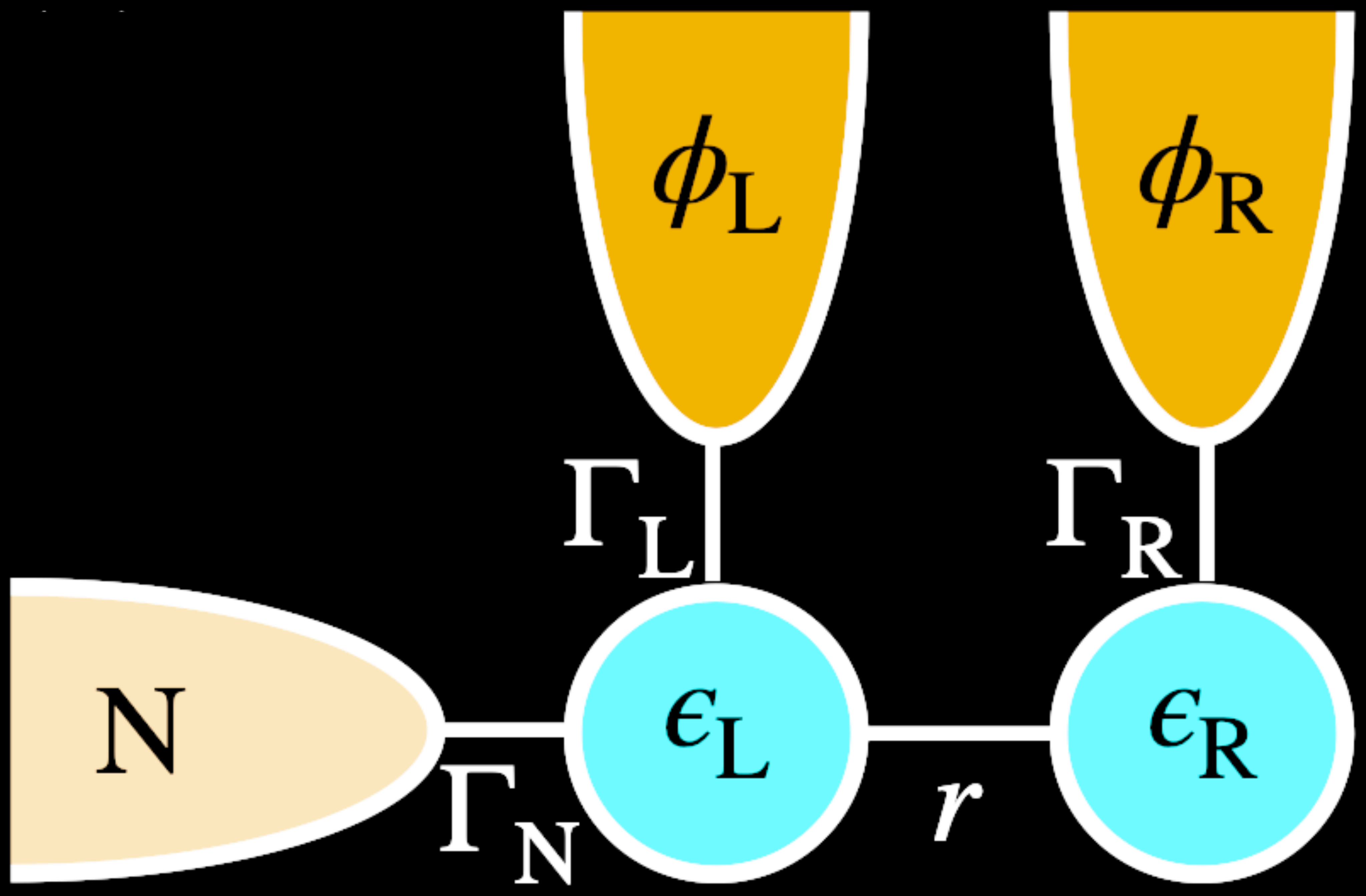
$$H_\mathrm{eff} = G_\mathrm{C}^{-1}(E=0)$$

$$= \begin{pmatrix} \epsilon_\mathrm{L}\tau_3 + \Gamma_\mathrm{L}\tau_1 - i\Gamma_\mathrm{N}\tau_0 & r\tau_3 \\ r\tau_3 & \epsilon_\mathrm{R}\tau_3 + \Gamma_\mathrm{R} e^{i\phi\tau_3}\tau_1 \end{pmatrix}$$

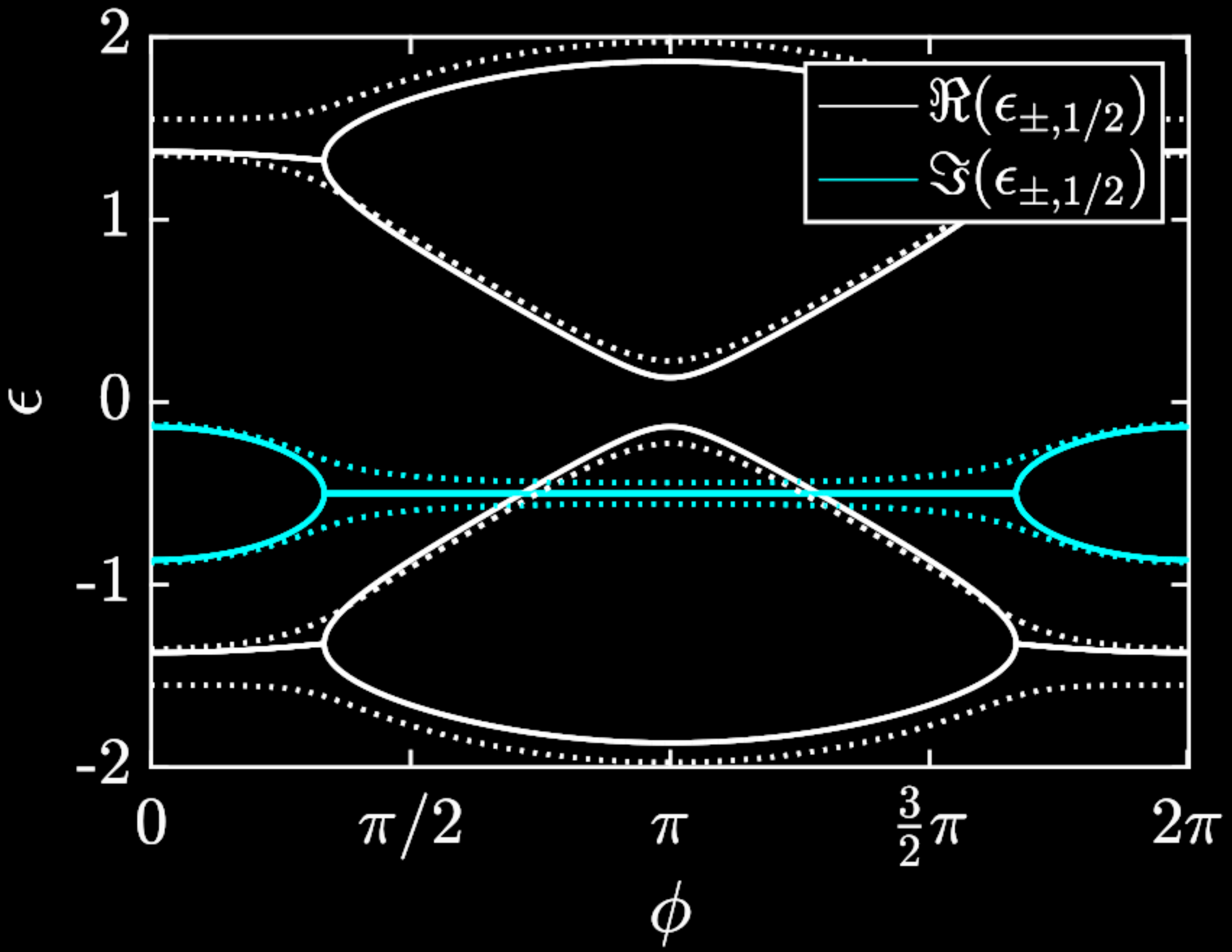
$$\neq H^\dagger$$

What's non-hermitian about (MT)JJs?

2-terminal junction with added normal metal



Real- and imaginary part of spectrum



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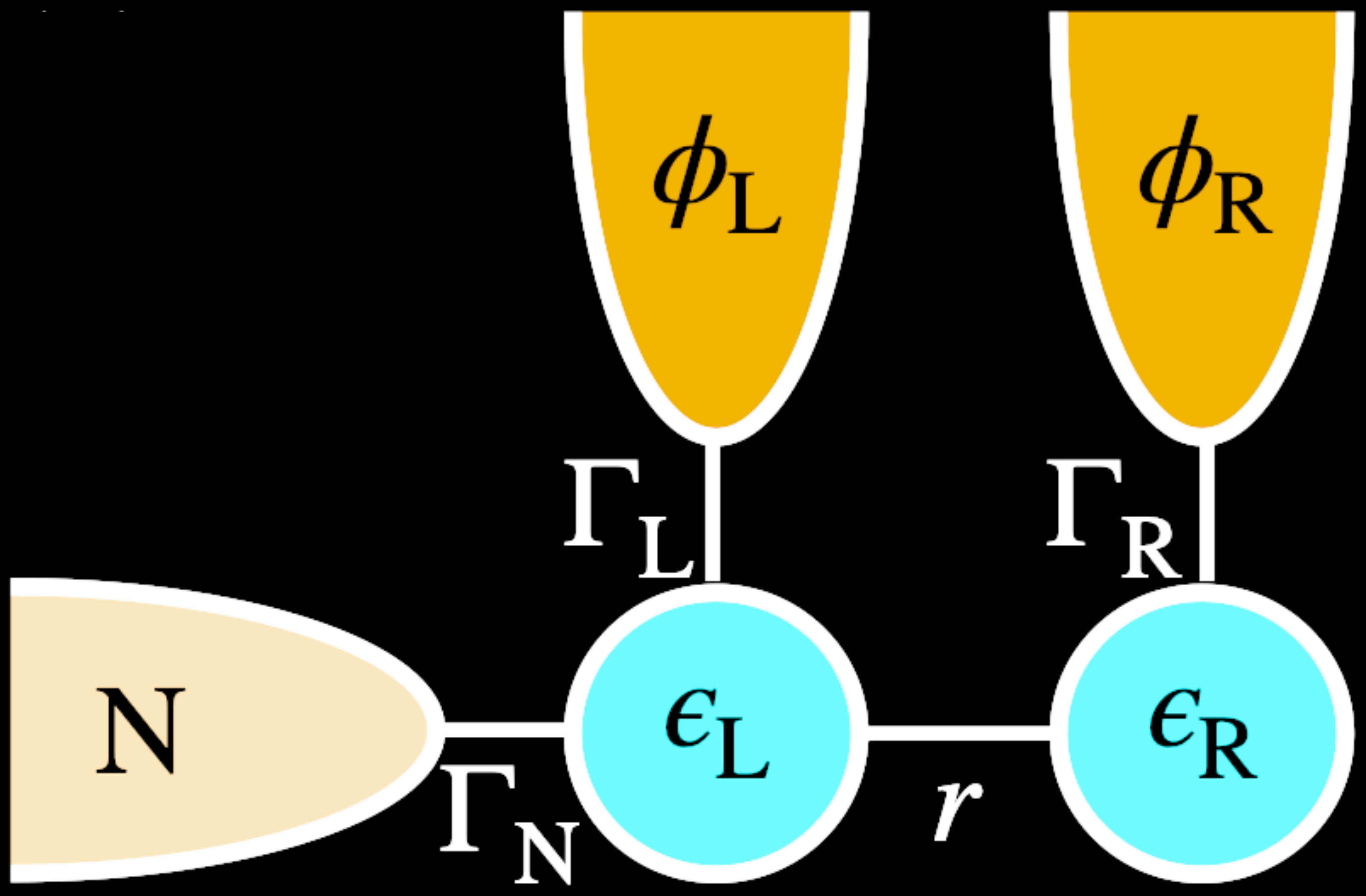

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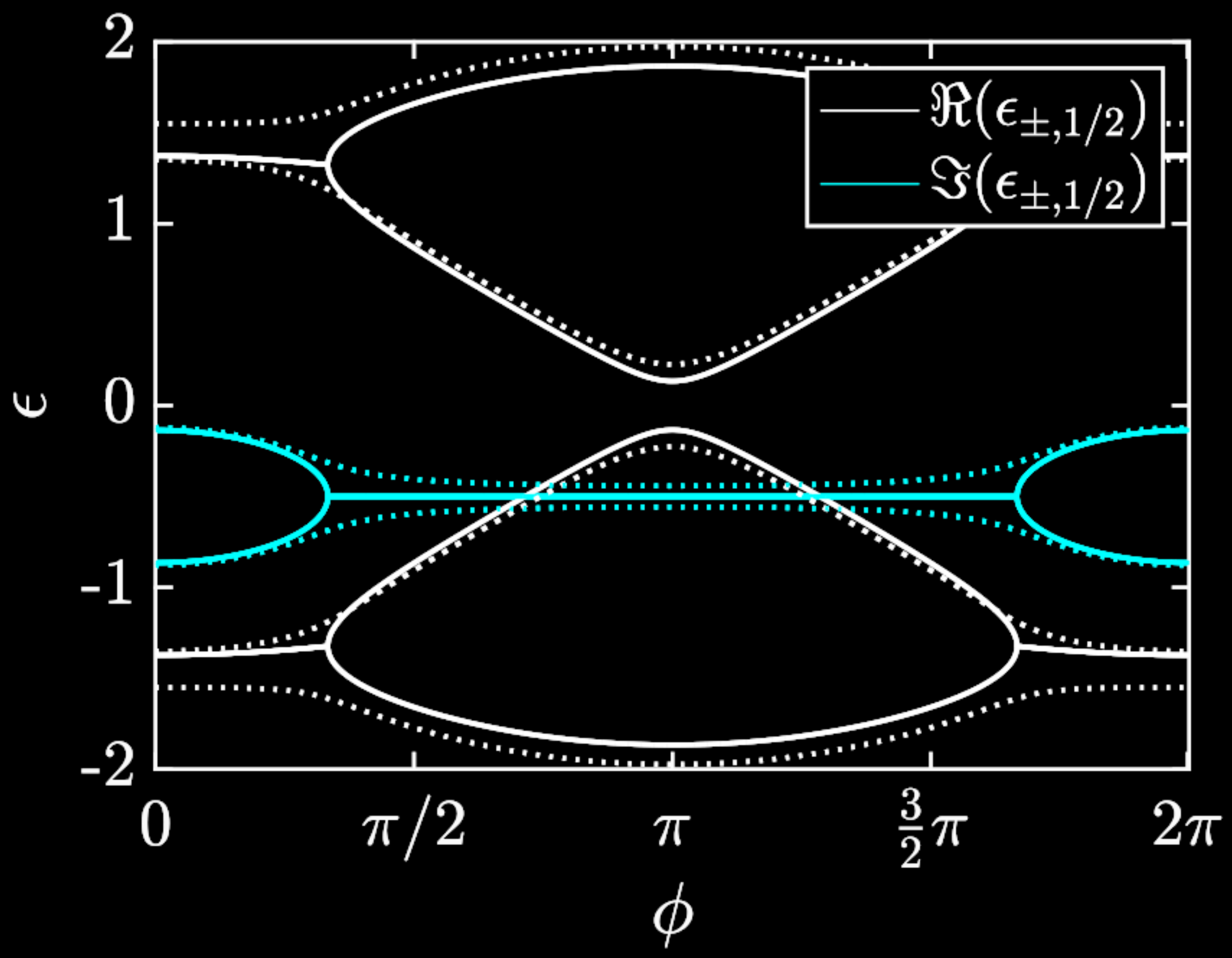
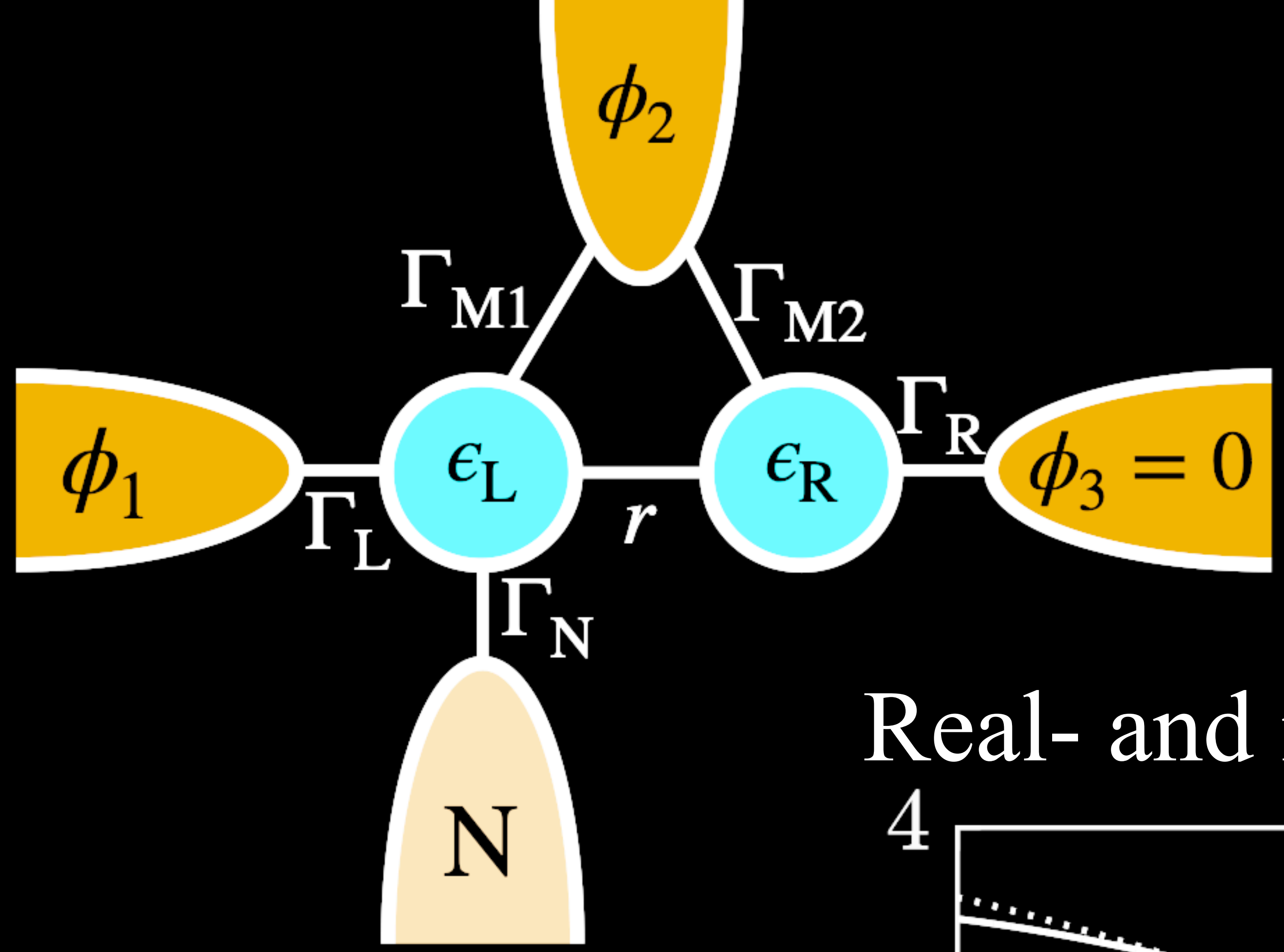
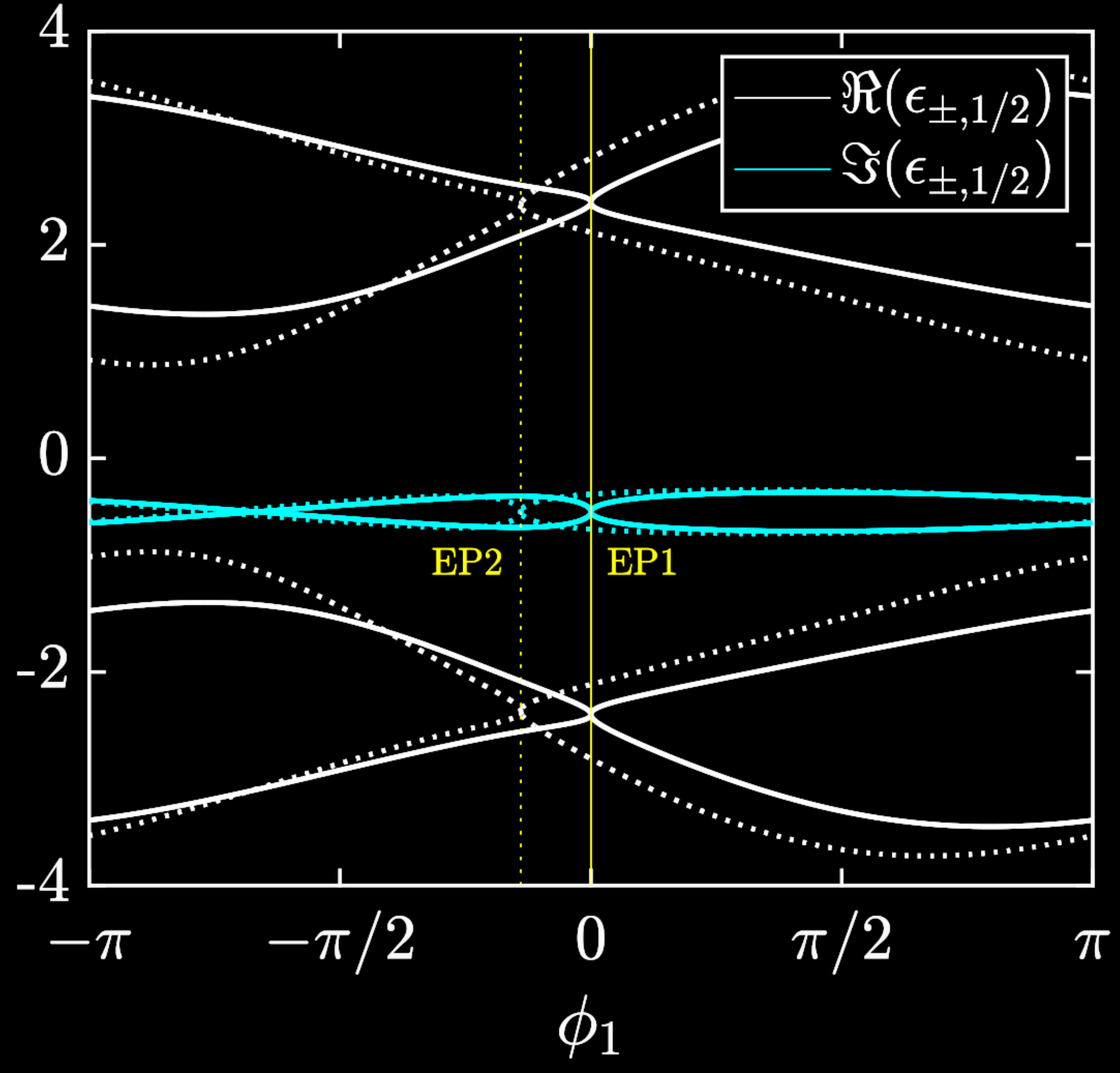


EPs are **not** stable

3-terminal model



Real- and imaginary part of spectrum



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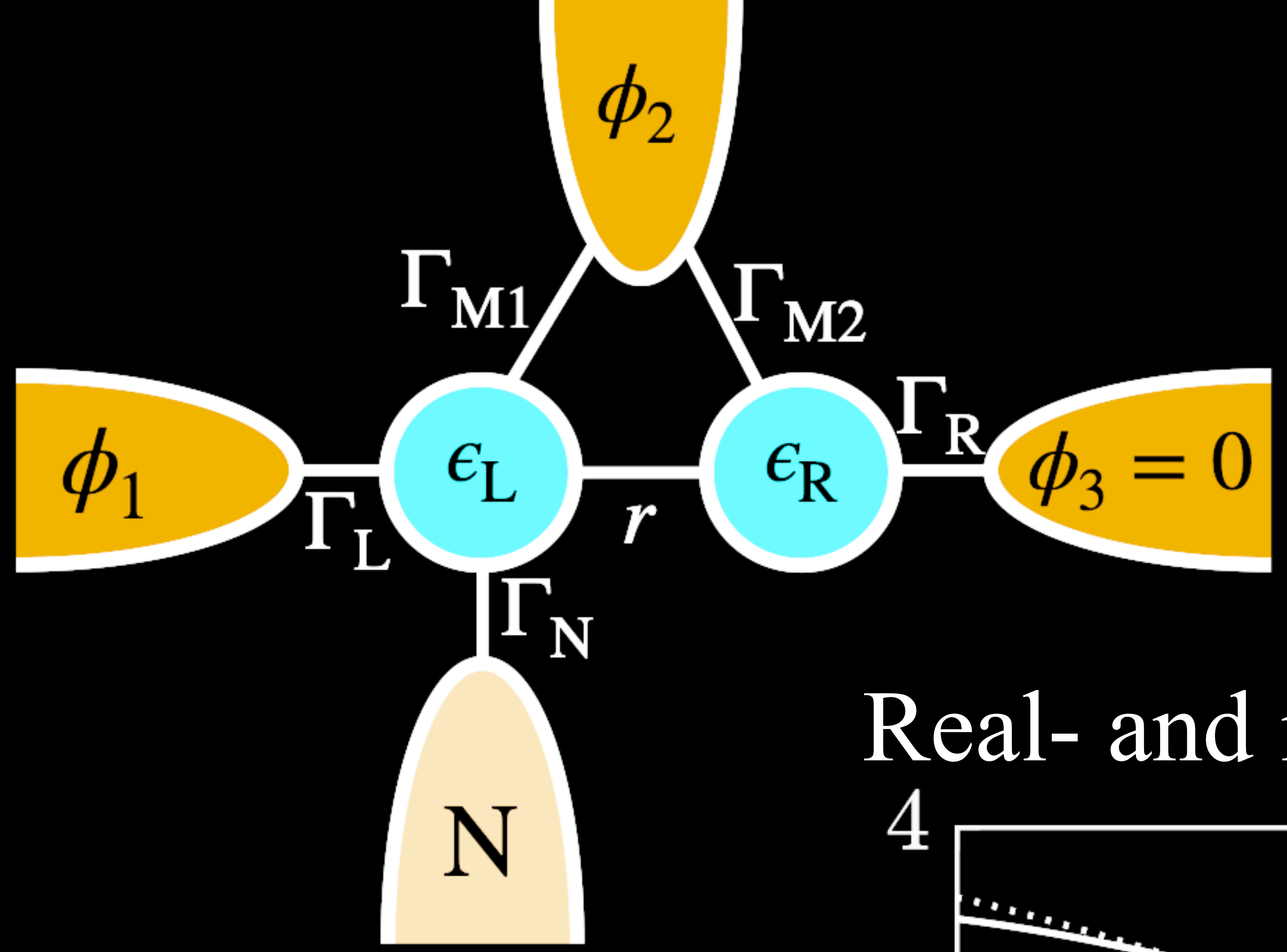
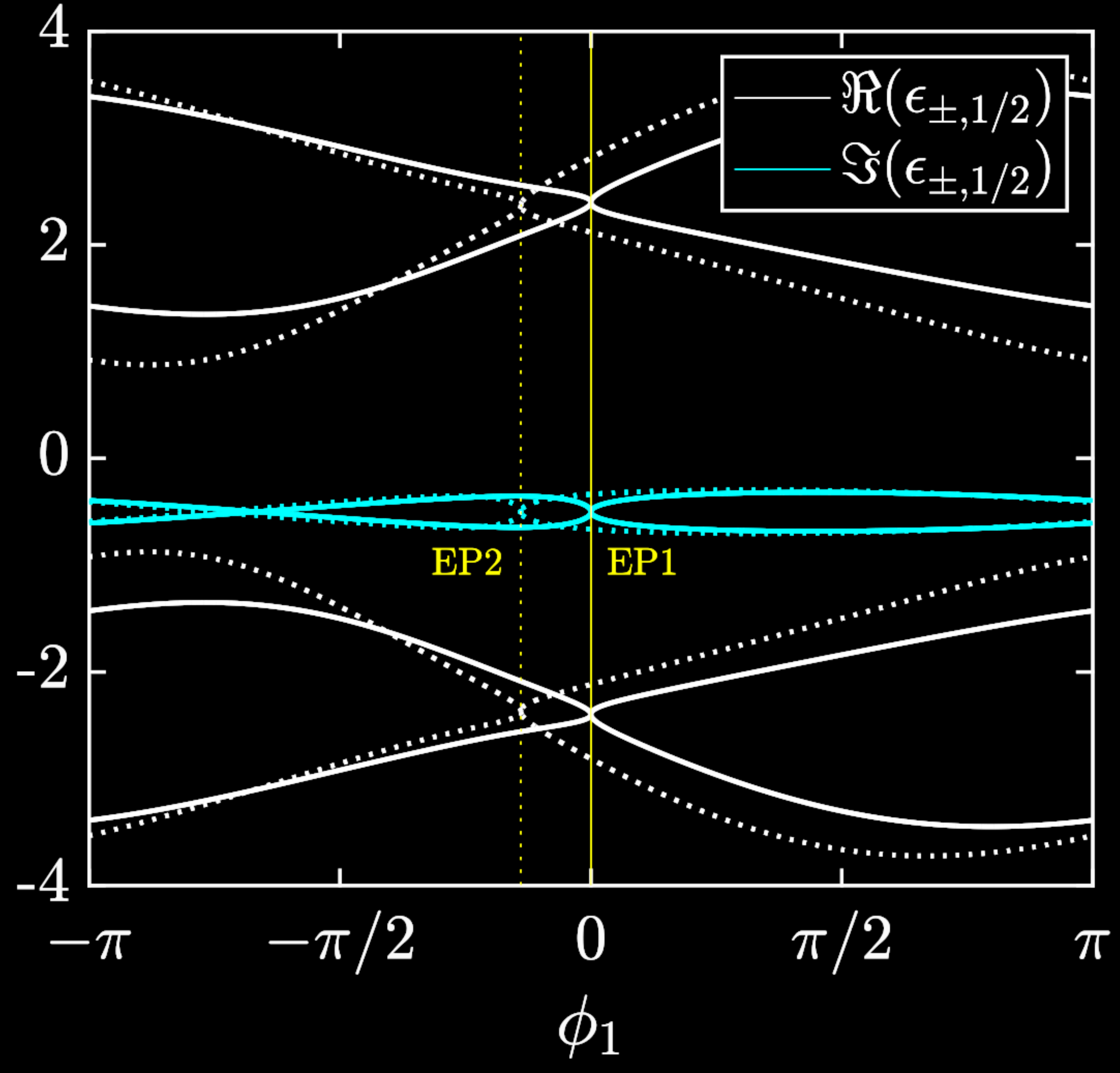
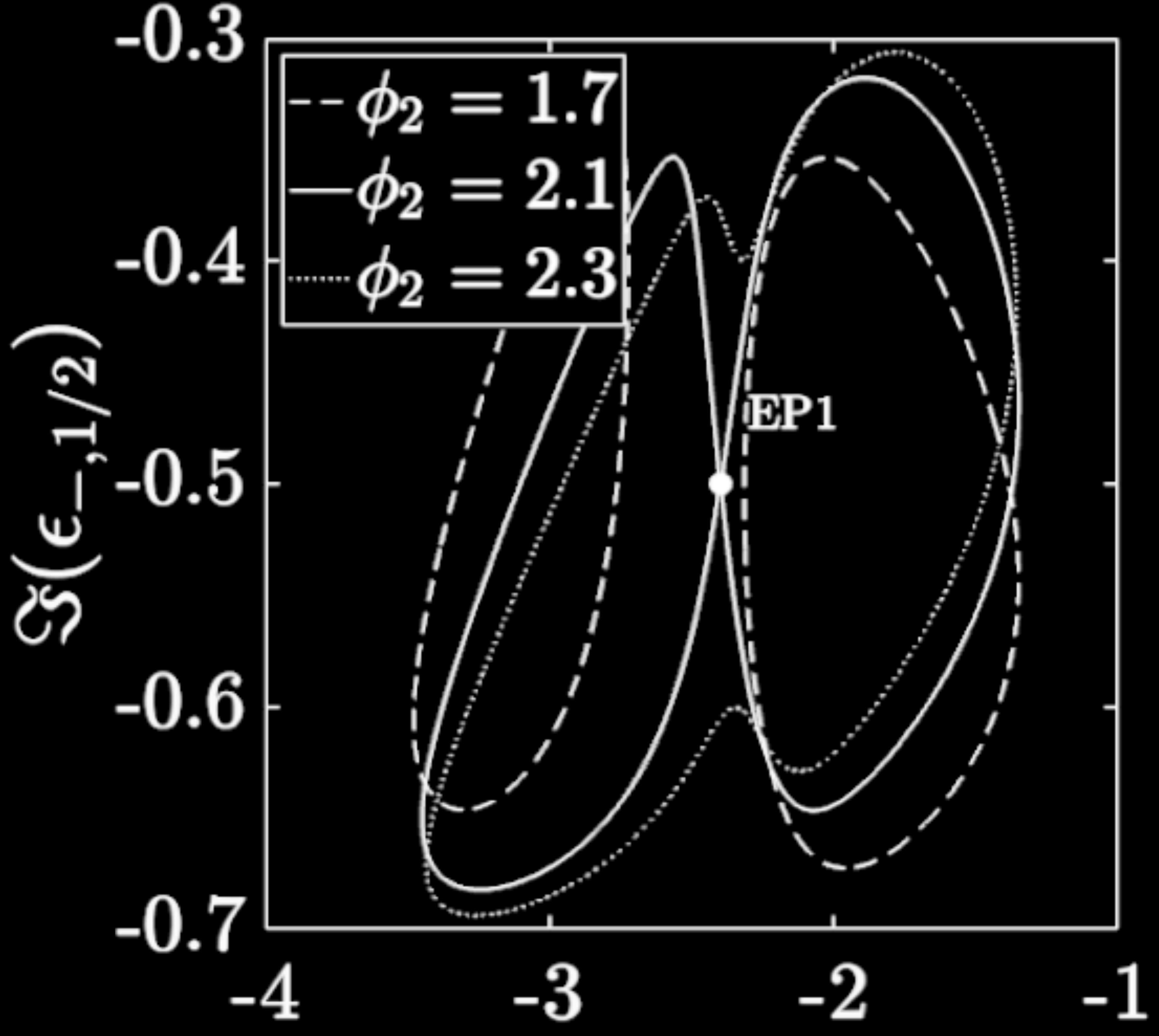


Real- and imaginary part of spectrum



Projected spectrum



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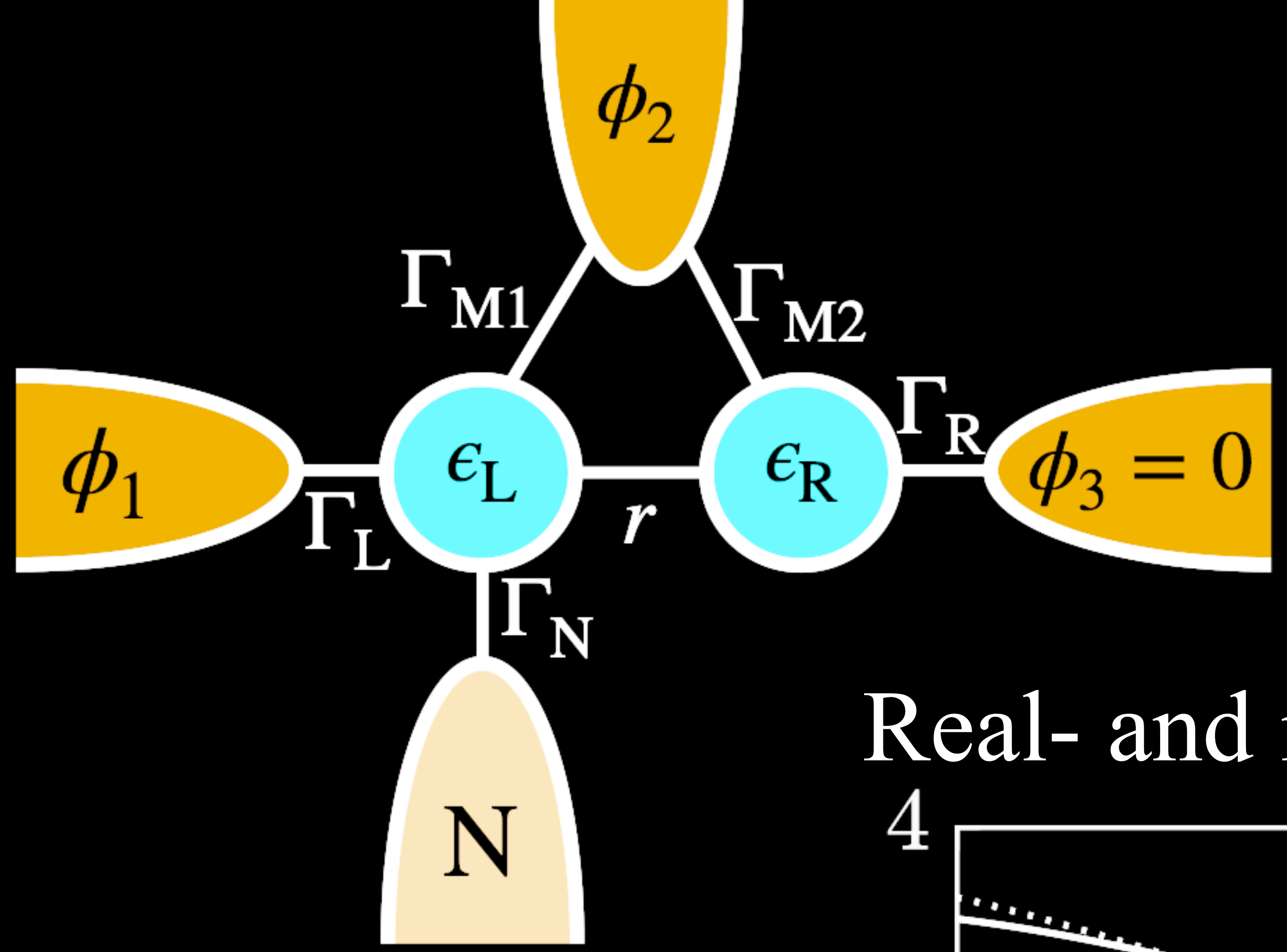
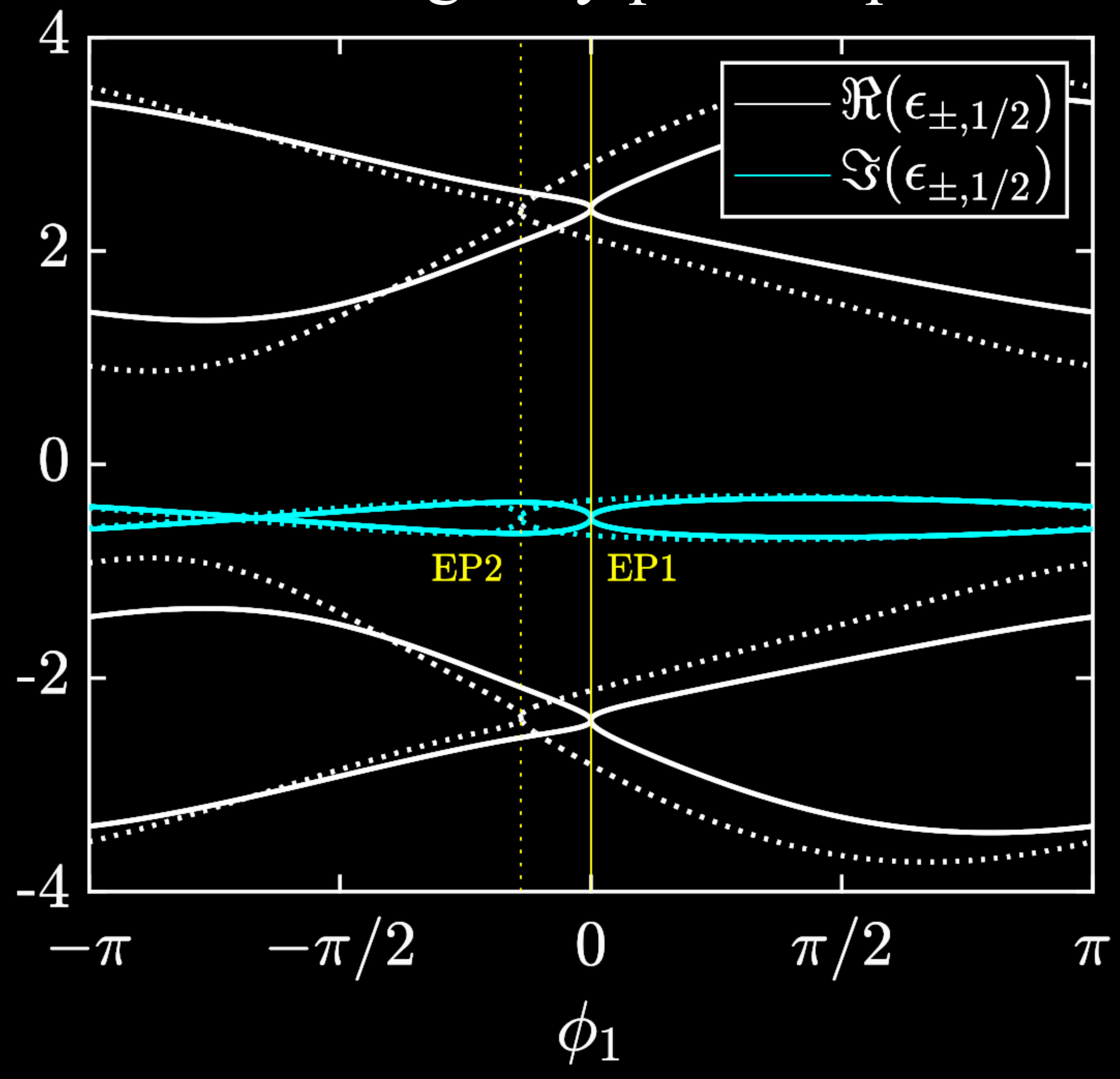


Real- and imaginary part of spectrum



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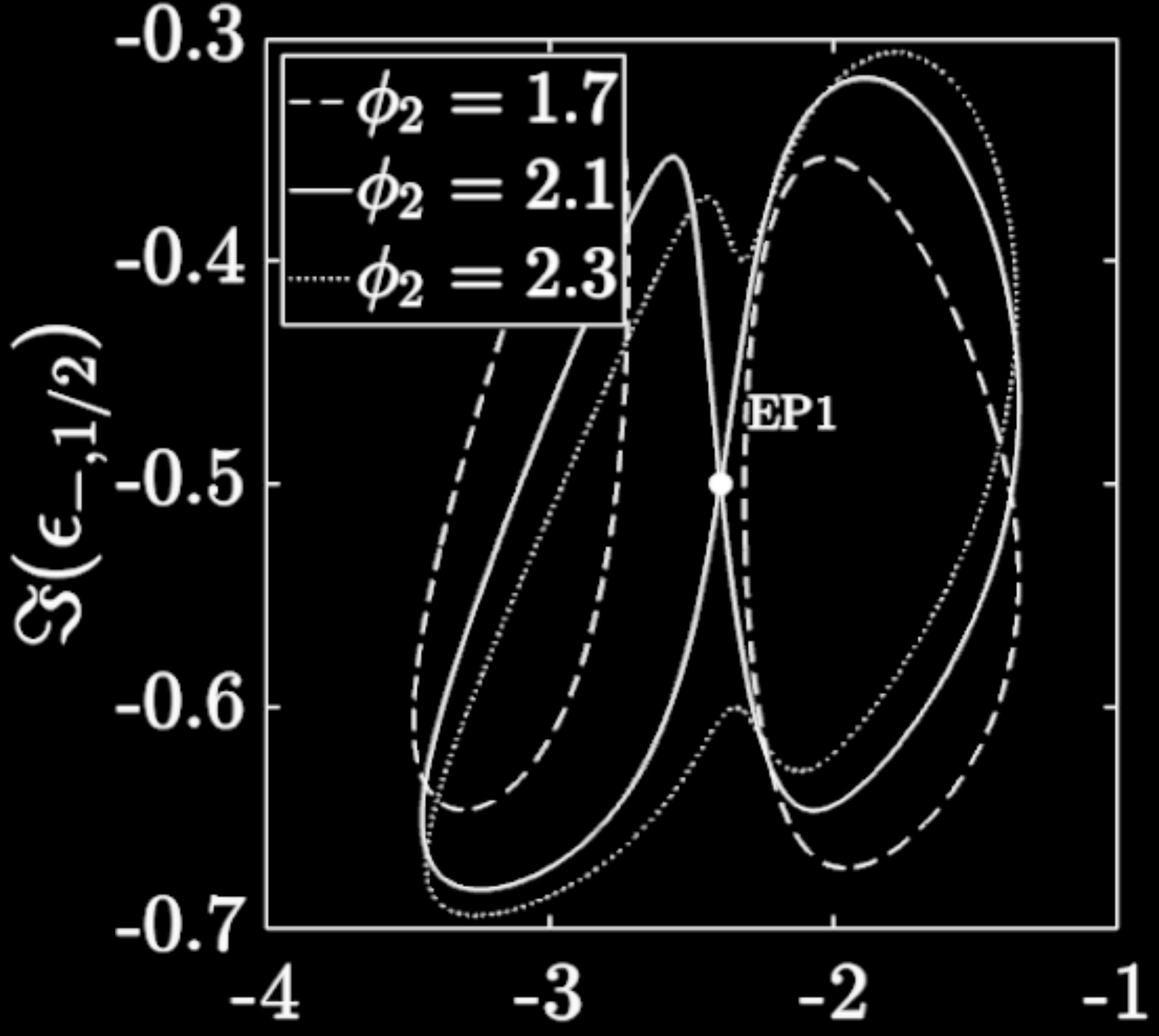
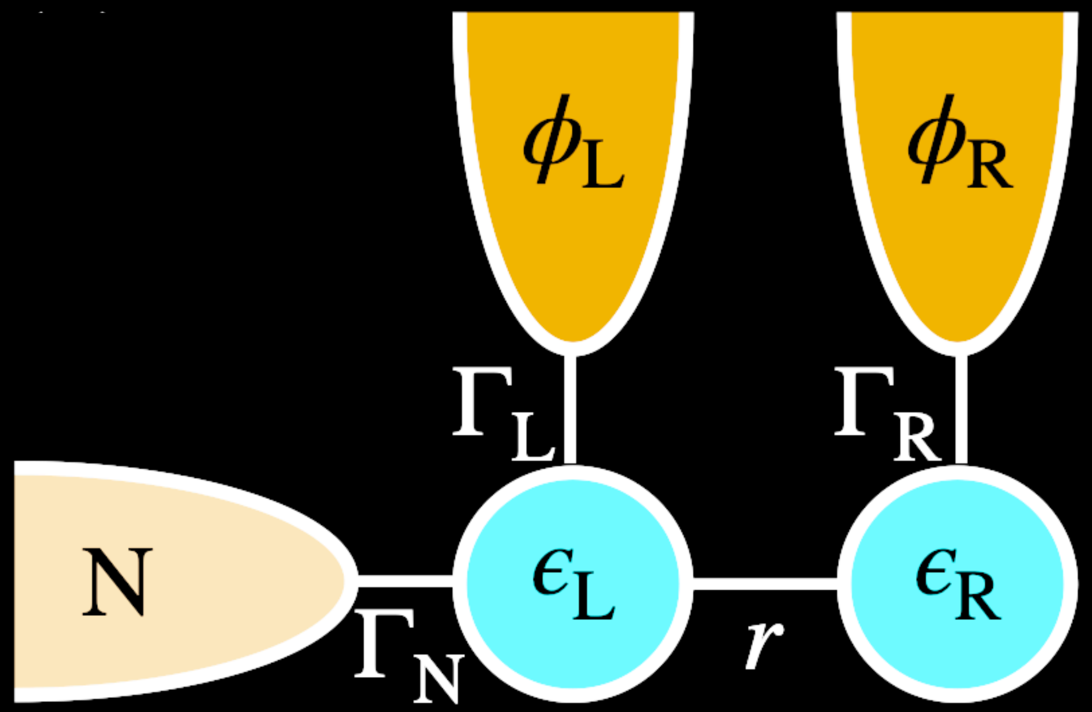
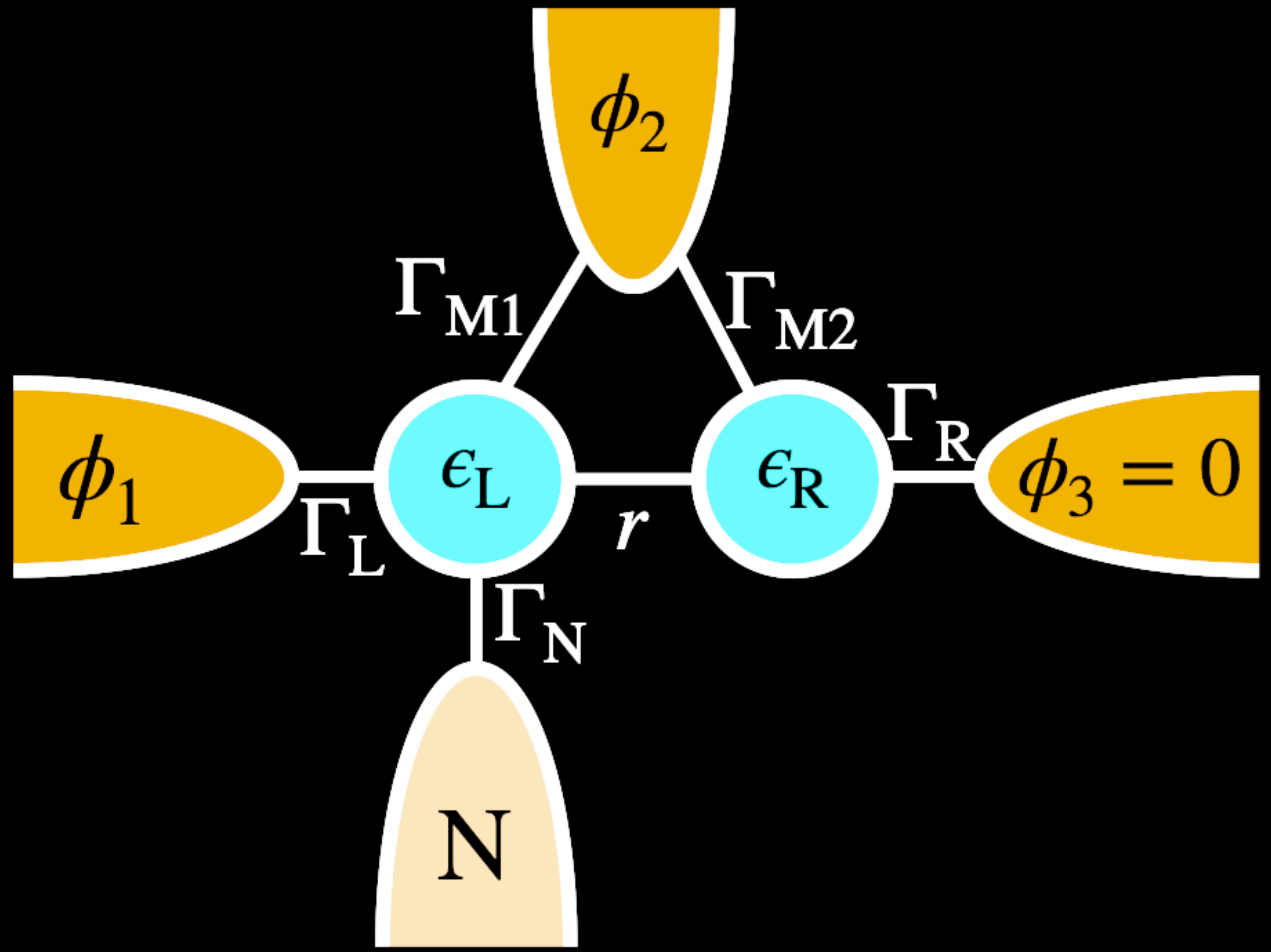


EPs are stable

Why the difference?



VS.



Effective Dimension of gapped phase: $d_\phi = 0$

Effective Dimension of gapped phase: $d_\phi = 1$

How does classification work?

AZ class	Gap	Classifying space	$d=0$	$d=1$
A	P	$\mathcal{C}_1$	0	$\mathbb{Z}$
	L	$\mathcal{C}_0$	$\mathbb{Z}$	0
AIII	P	$\mathcal{C}_0$	$\mathbb{Z}$	0
	$\mathrm{L_r}$	$\mathcal{C}_1$	0	$\mathbb{Z}$
	$\mathrm{L_i}$	$\mathcal{C}_0 \times \mathcal{C}_0$	$\mathbb{Z} \oplus \mathbb{Z}$	0

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What's the symmetry?

Particle-hole symmetry: Class $C^{\dagger}$

$$U_C H_i^*(\phi) U_C^{-1} = -H_i(\phi)$$

EP @ $E \neq 0$

$\Longrightarrow$

Broken spin-rotation symmetry

No symmetry: Class A

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	L	$\mathcal{C}_0$	$\mathbb{Z}$	0
AIII	P	$\mathcal{C}_0$	$\mathbb{Z}$	0
	L_r	$\mathcal{C}_1$	0	$\mathbb{Z}$
	L_i	$\mathcal{C}_0 \times \mathcal{C}_0$	$\mathbb{Z} \oplus \mathbb{Z}$	0

What's the dimension?

Number of superconducting terminals d_ϕ?

What's the symmetry?

Particle-hole symmetry: Class $C^\dagger$

$$U_C H_i^*(\phi) U_C^{-1} = -H_i(\phi)$$

EP @ $E \neq 0$

$\Longrightarrow$

Broken spin-rotation symmetry

No symmetry: Class A

Making sense of effective dimensions

RESEARCH ARTICLE | MAY 14 2009

Periodic table for topological insulators and superconductors

Alexei Kitaev

AIP Conf. Proc. 1134, 22–30 (2009)
https://doi.org/10.1063/1.3149495

The Hamiltonian of a translationally invariant systems can be written in the momentum representation:

$$\hat{H} = \frac{i}{4}\sum_{\mathbf{p}}\sum_{j,k} A_{jk}(\mathbf{p})\hat{c}_{-\mathbf{p},j}\hat{c}_{\mathbf{p},k}, \tag{19}$$

where j and k refer to particle flavors. The matrix $A(\mathbf{p})$ is skew-Hermitian but not real; it rather satisfies the condition $A_{jk}(\mathbf{p})^* = A_{jk}(-\mathbf{p})$. By abuse of terminology, such matrix-valued functions are called "functions from $\bar{\mathbb{R}}^d$ to real skew-symmetric matrices", where $\bar{\mathbb{R}}^d$ is the usual Euclidean space with the involution $\mathbf{p} \leftrightarrow -\mathbf{p}$ (cf. [29]).

By a *real vector bundle* over the *real space* X we mean a complex vector bundle E over X which is also a real space and such that

(i) the projection $E \to X$ is real (i.e. commutes with the involutions on E, X);

(ii) the map $E_x \to E_{\bar{x}}$ is anti-linear, i.e. the diagram

$$\begin{array}{ccc} \mathbf{C} \times E_x & \to & E_x \\ \downarrow & & \downarrow \\ \mathbf{C} \times E_{\bar{x}} & \to & E_{\bar{x}} \end{array}$$

commutes, where the vertical arrows denote the involution and $\mathbf{C}$ is given its standard real structure $(\tau(z) = \bar{z})$.

Normally, symmetries defined by

$$U_C H_i^*(k) U_C^{-1} = -H_i(-k)$$

But phase is **unusual** dimension:

$$U_C H_i^*(\phi) U_C^{-1} = -H_i(+\phi)$$

$\Rightarrow$ Phase is "just" a parameter

Solution: Effective Parameter

Effective classification parameter

$$d = d_k - d_\phi \bmod 8 = \begin{cases} 0 \ , \ d_k = d_\phi = 0 \\ 7 \ , \ d_k = 0 \ , \ d_\phi = 1 \end{cases}$$

Jeffrey C. Y. Teo and C. L. Kane Phys. Rev. B 82, 115120 (2010)
Fan Zhang and C. L. Kane Phys. Rev. B 90, 020501(R) (2014)

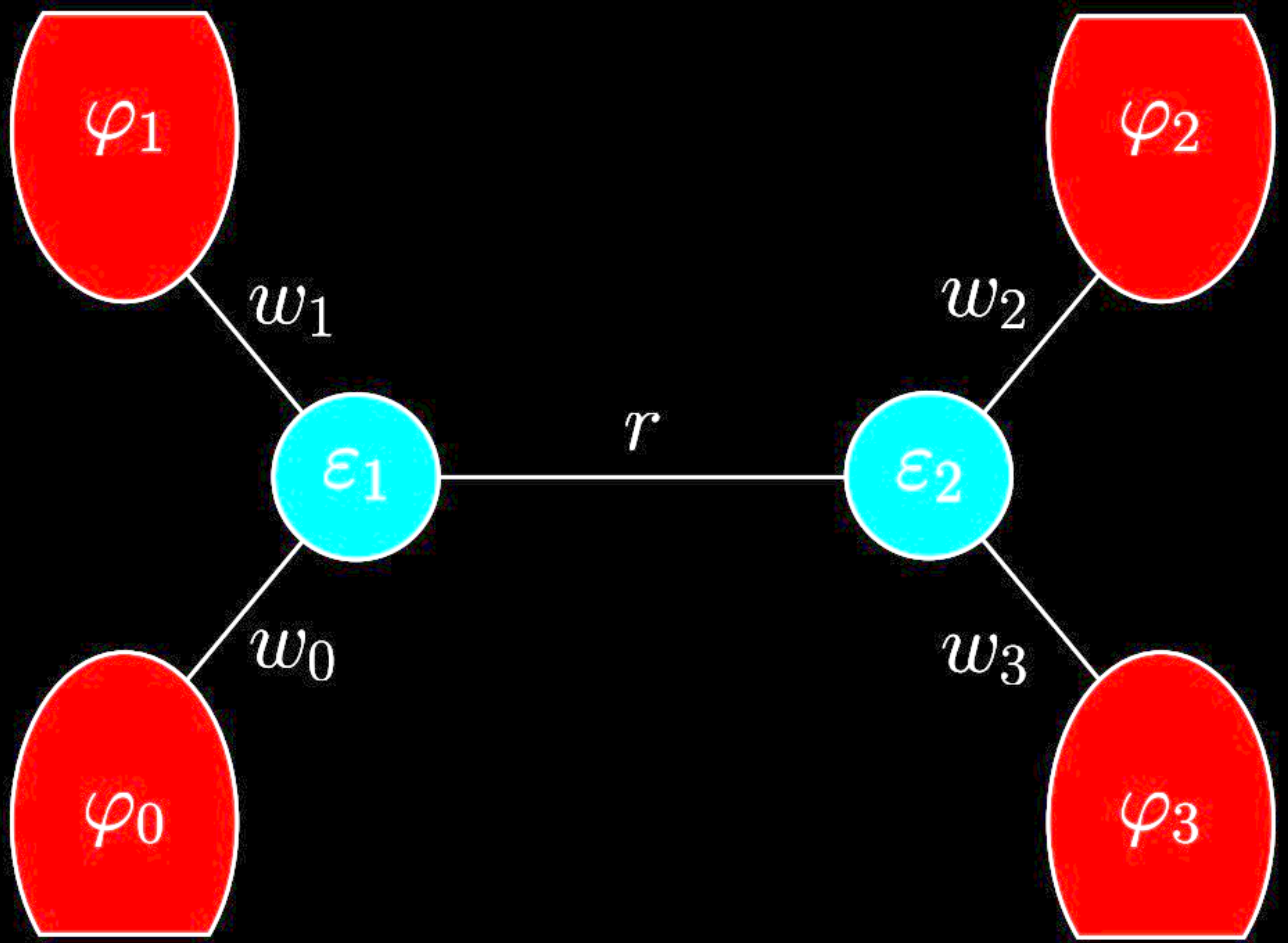
AZ class	Gap	Classifying space	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
A	P	$\mathcal{C}_1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
	L	$\mathcal{C}_0$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0

winding number
NOT allowed

winding number
allowed!

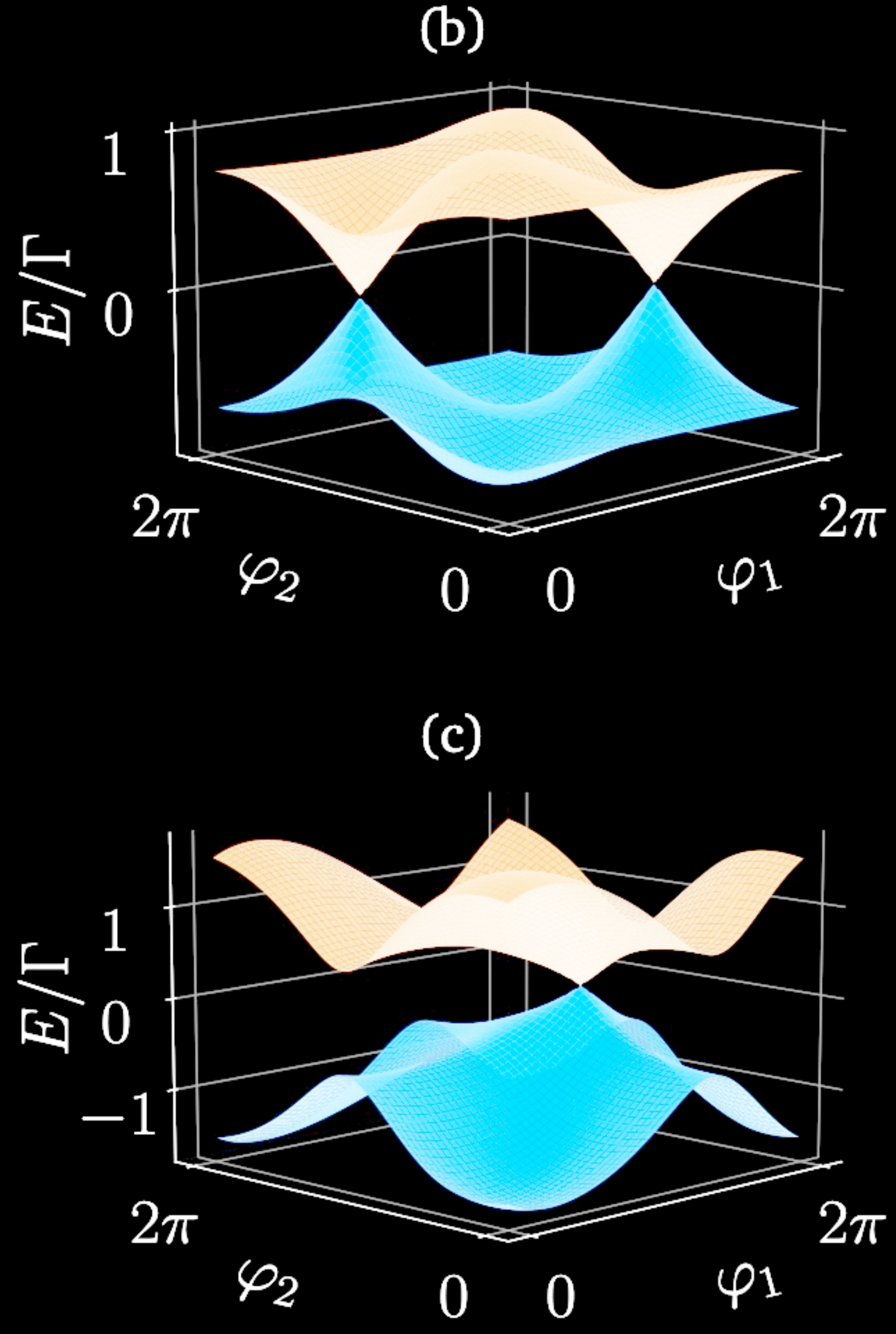
Further insight: Weyl node under non-hermiticity

2-dot 4-terminal MTJJ



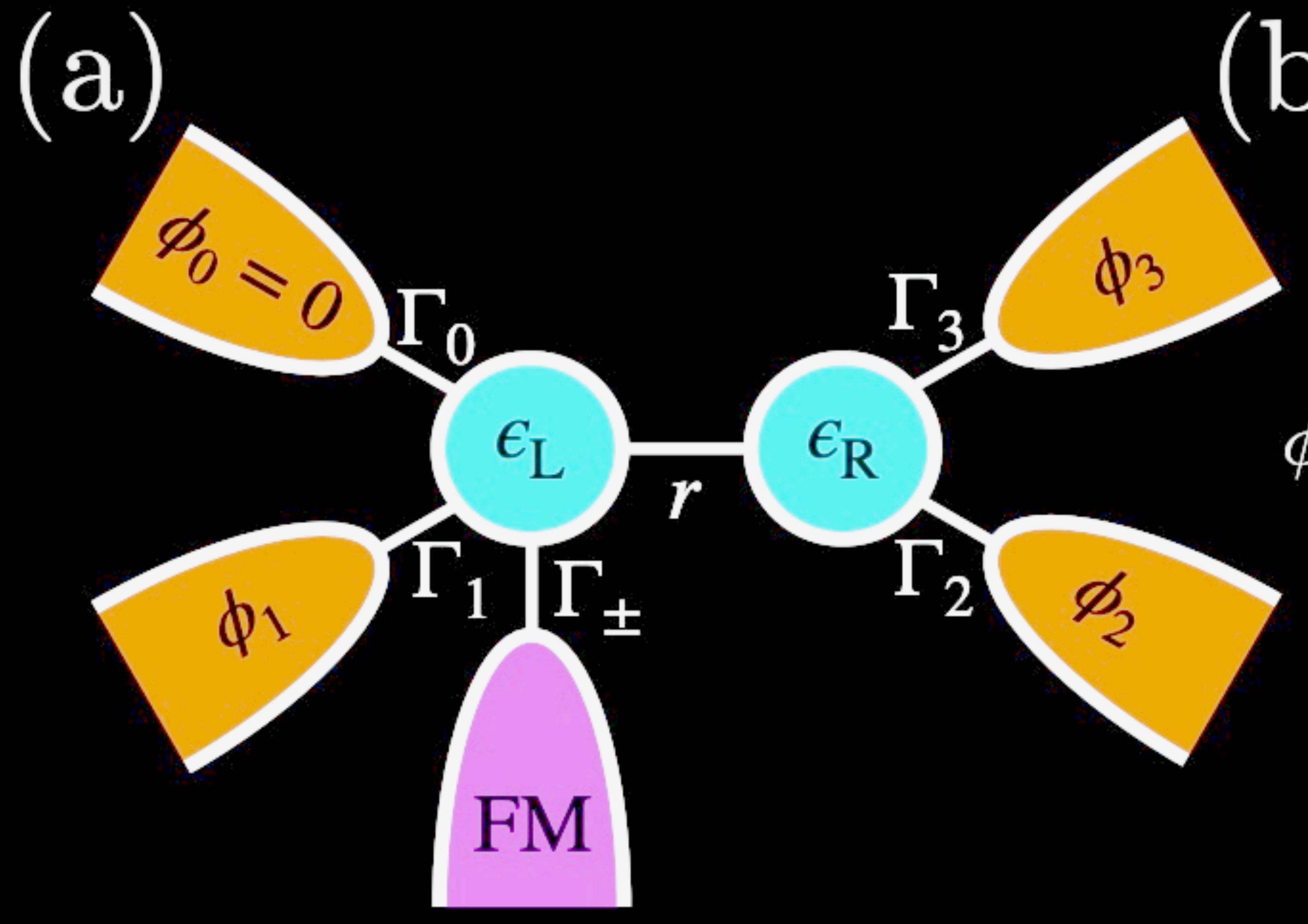
L. Teshler, et. al. SciPost Phys. 15, 214 (2023)

Weyl Nodes

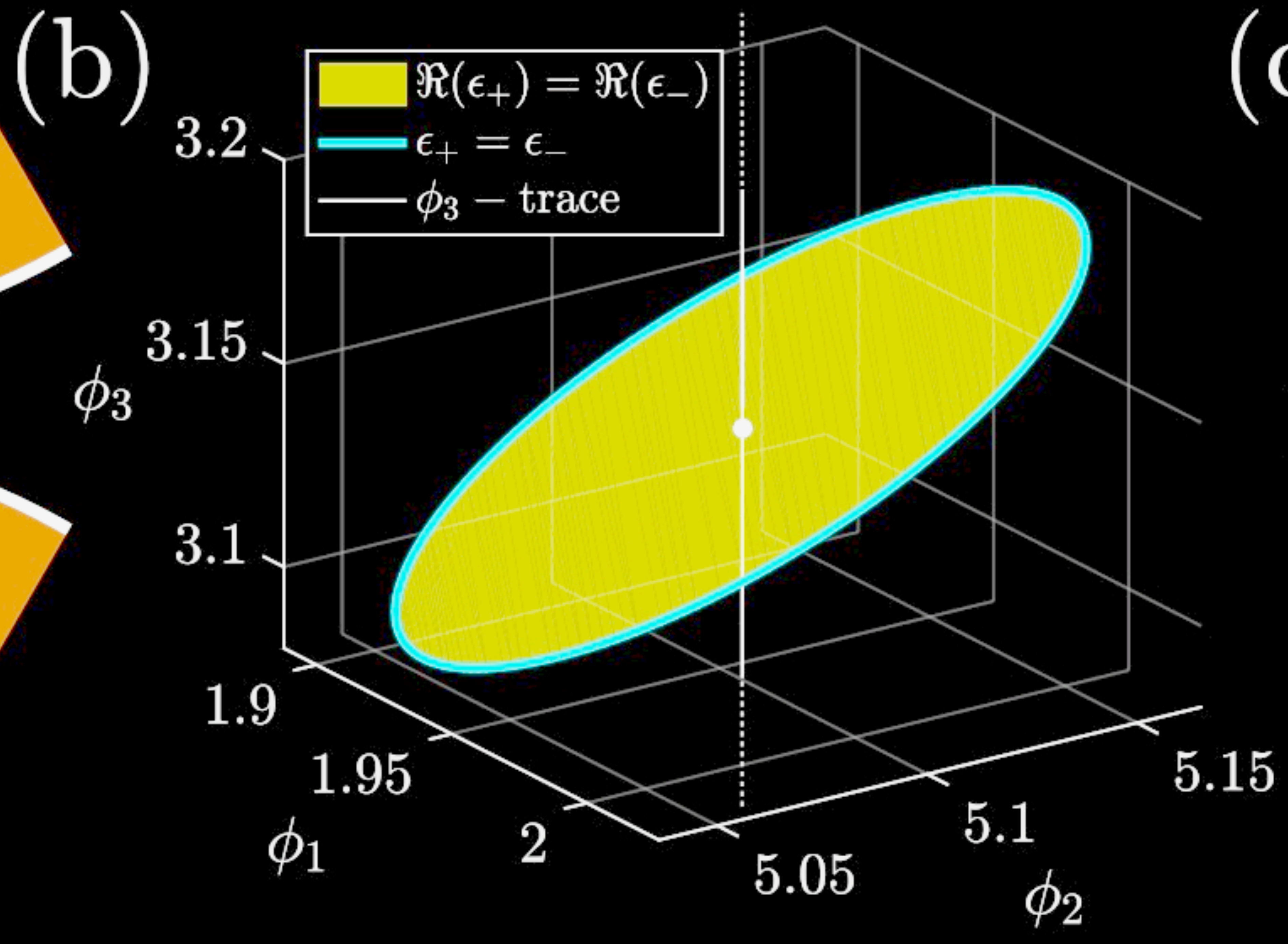


Weyl Disk via coupling to Ferromagnet

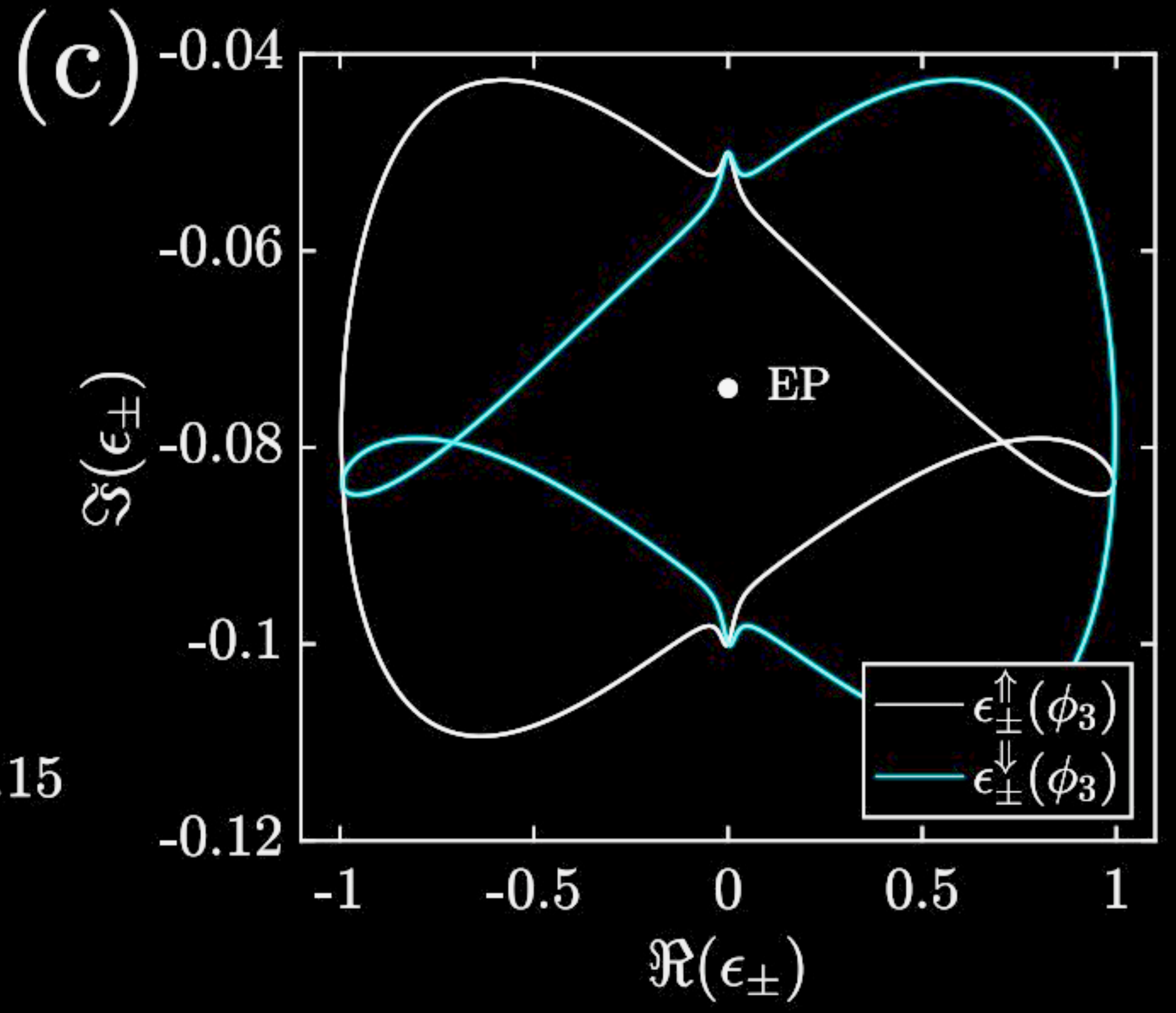
Coupling to FM



Weyl disk in phase space



Winding



Further insight: Weyl node under non-hermiticity

AZ class	Gap	Classifying space	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
$C^\dagger$	P	$\mathcal{R}_5$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	L_r	$\mathcal{R}_6$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	L_i	$\mathcal{R}_4$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0

winding number
NOT allowed

Break spin-rotation symmetry!

AZ class	Gap	Classifying space	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
A	P	$\mathcal{C}_1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
	L	$\mathcal{C}_0$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0

winding number
allowed!

Conclusion

david.ohnmacht@uni-konstanz.de

1) Multiterminal Josephson Junctions (MTJJs) host <u>stable</u> exceptional points (EPs)

- 3- and 4-terminal junctions host stable EPs whereas 2-terminal junction EPs are fragile

D. C. O., et. al. arXiv:2408.01289 (2024)

2) Topological classification provides useful insight for experiments

- Weyl disks are **<u>not</u>** observable without inducing spin-breaking
- EPs are **<u>not</u>** stable in 2-terminal JJs

Conclusion

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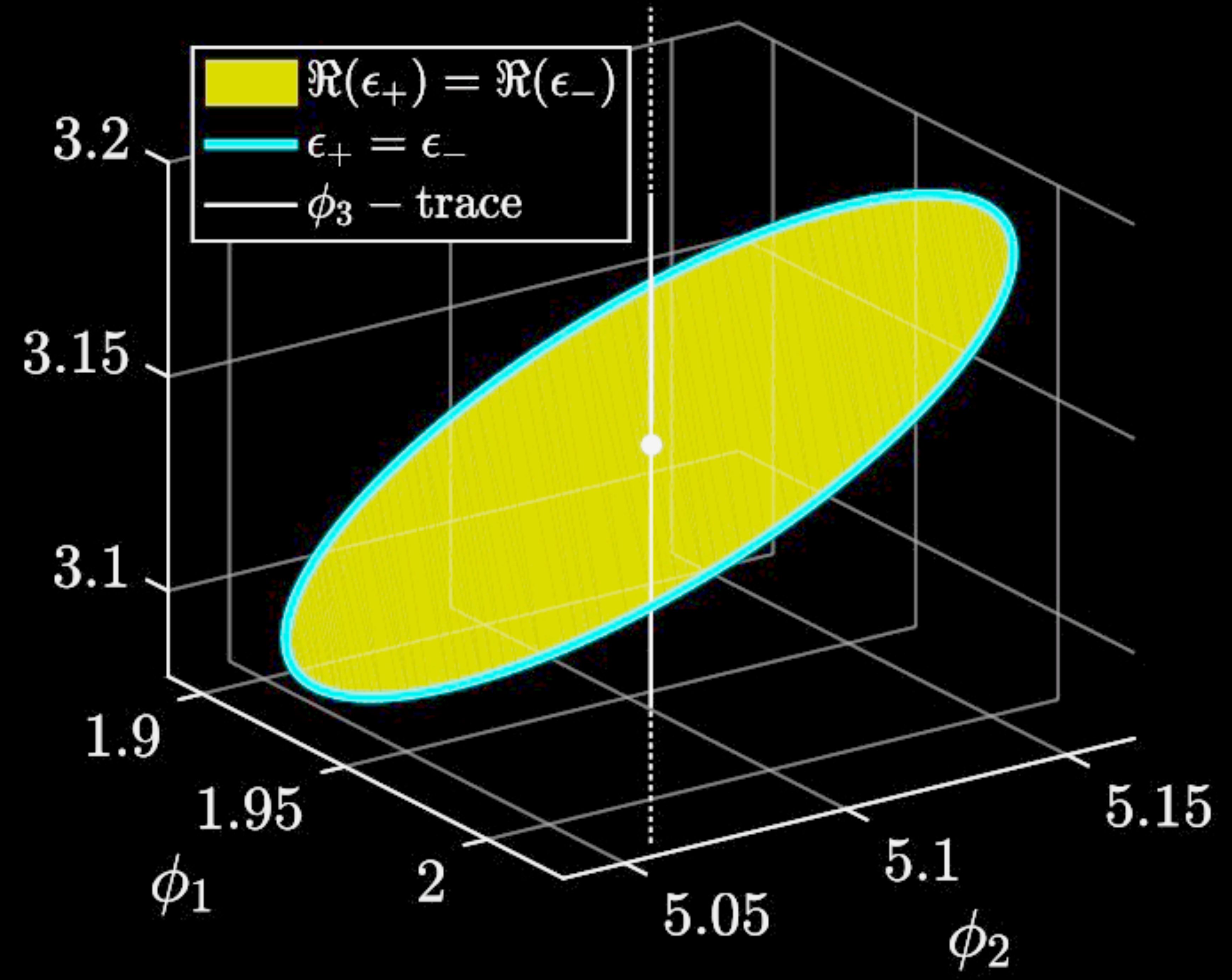

1) Multiterminal Josephson Junctions (MTJJs) host stable exceptional points (EPs)

2) Broken spin-rotation symmetry enables EPs at zero (real) energy

David Christian Ohnmacht,
Valentin Wilhelm
Hannes Weisbrich
Wolfgang Belzig