

Full Counting Statistics of Yu-Shiba-Rusinov states

David Christian Ohnmacht,
Wolfgang Belzig,
Juan Carlos Cuevas

13. March 2024

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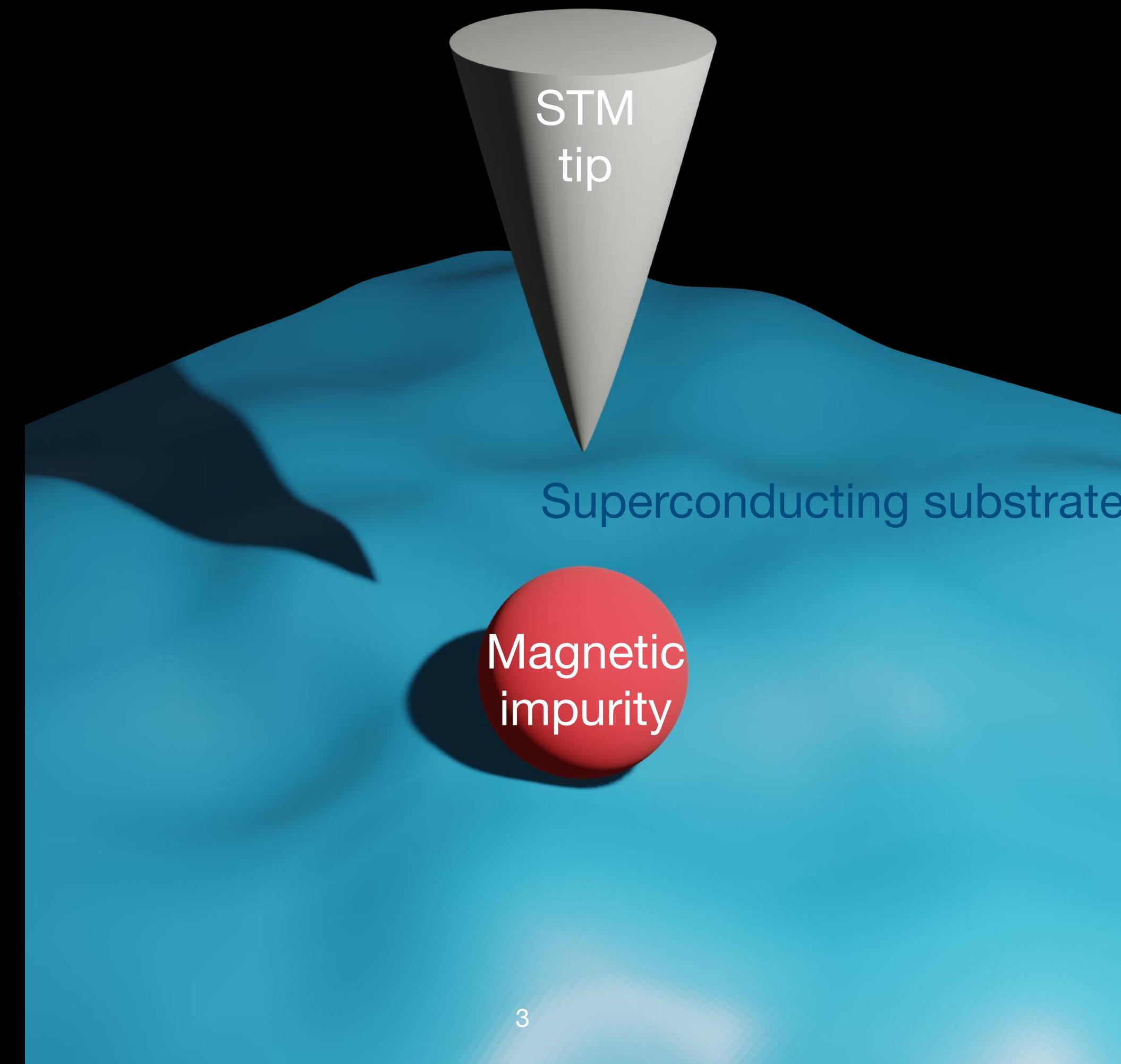
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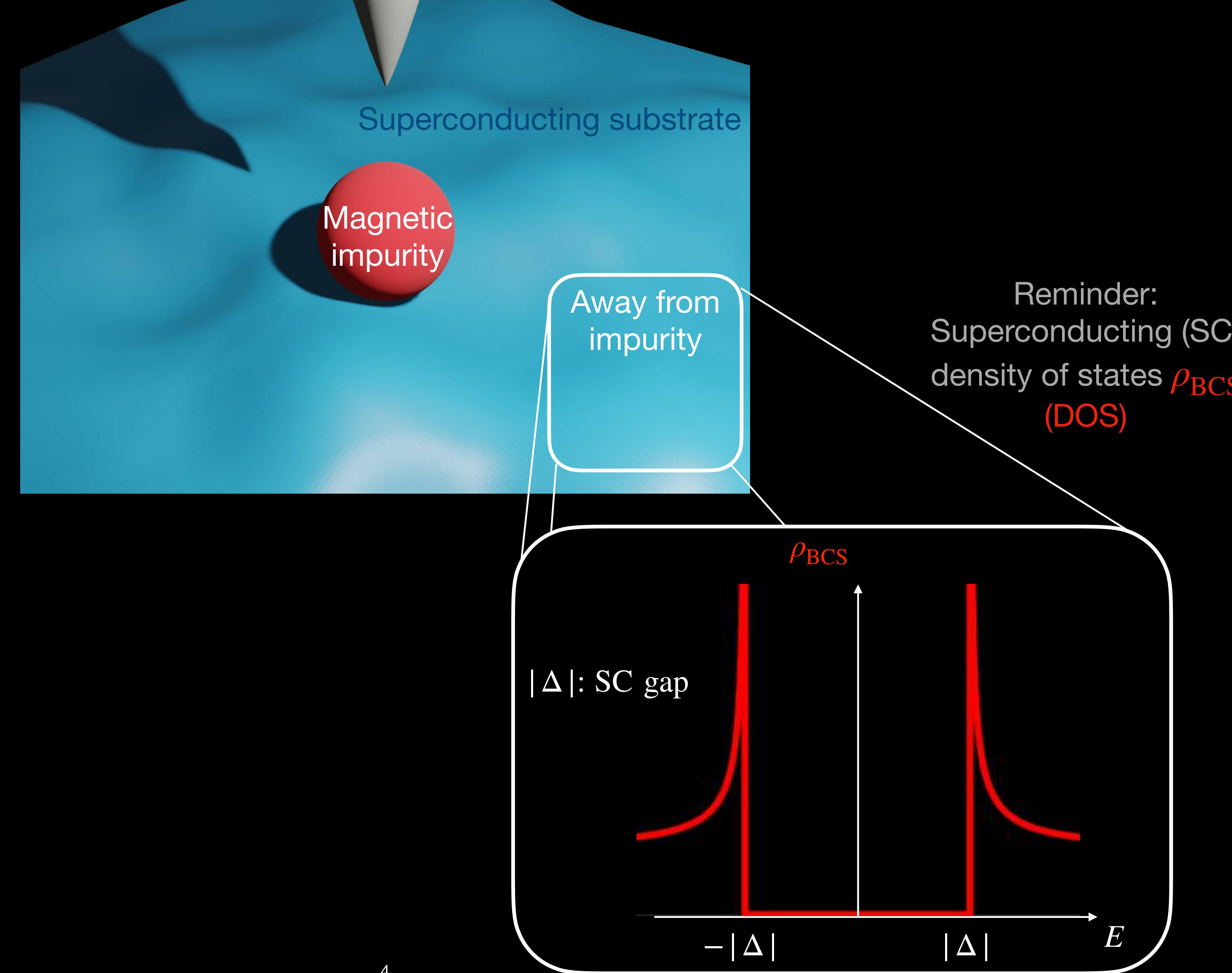
- 1) Full Counting Statistics (FCS) provides unprecedented insight into transport of junctions containing YSR states**

- 2) FCS calculations are needed to fully understand measurements**

What are Yu-Shiba-Rusinov (YSR) states?



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What are Yu-Shiba-Rusinov states? (YSR)

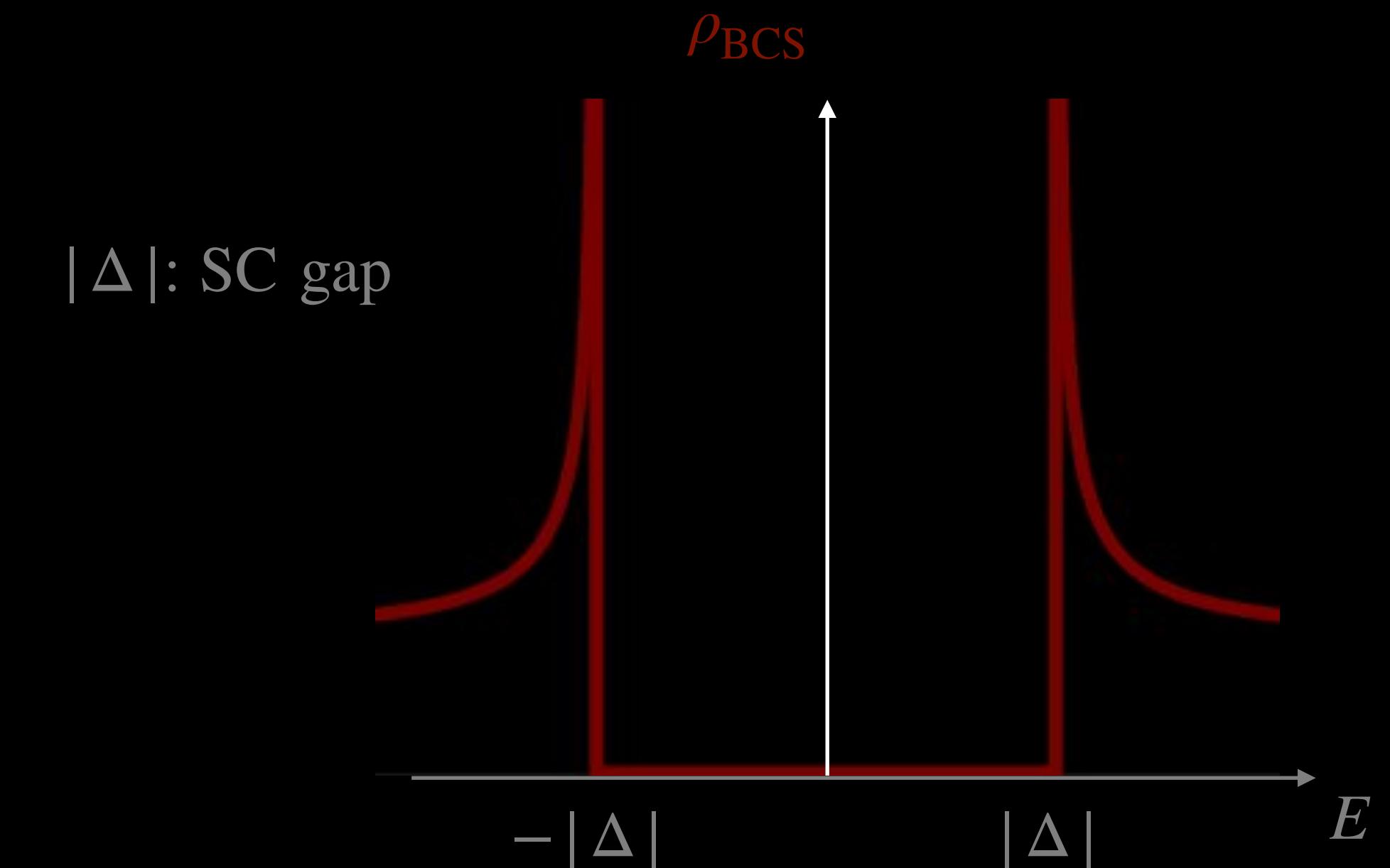
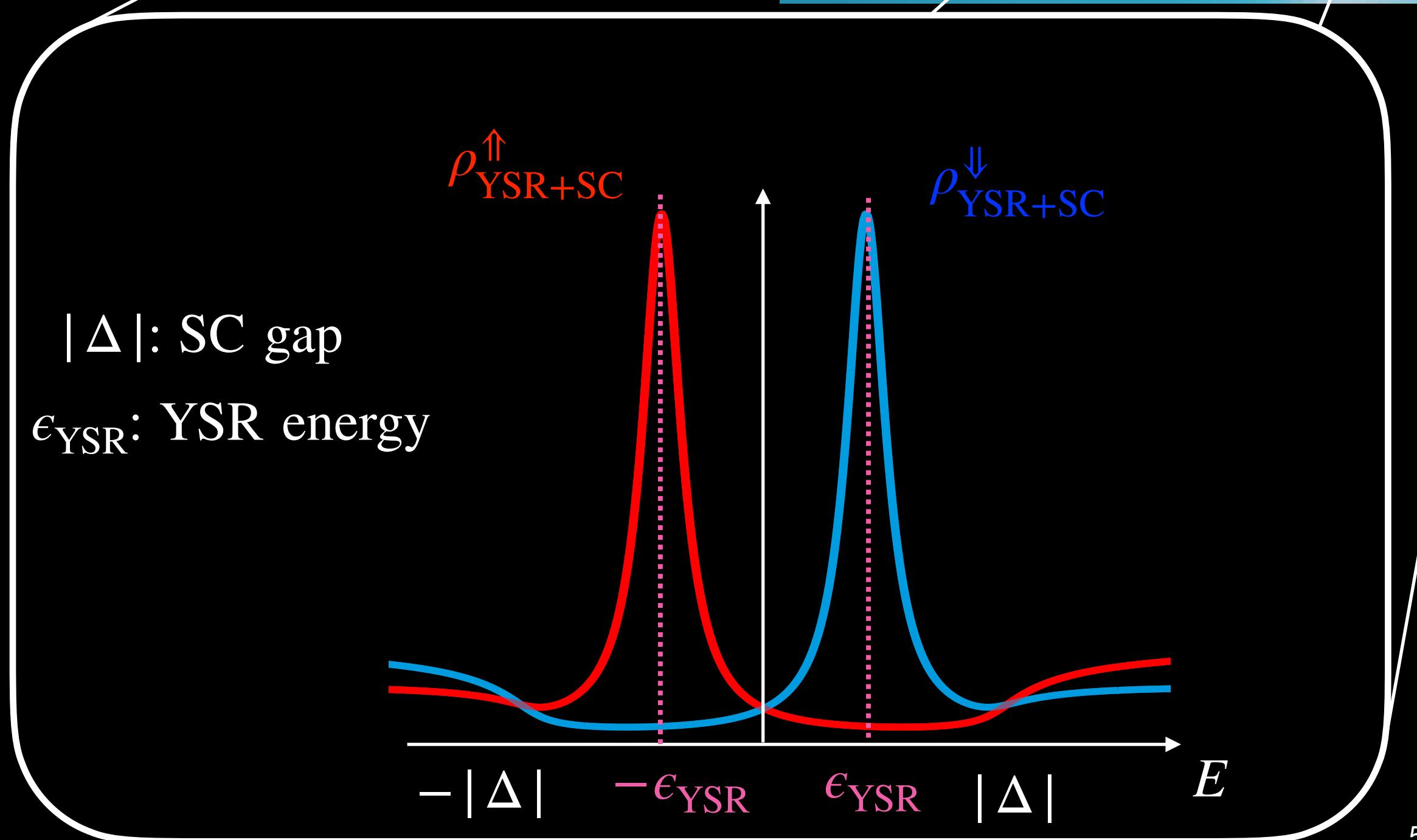
L. Yu, Acta Phys. Sin. **21**, 75 (1965).

H. Shiba, Prog. Theor. Phys. **40**, 435 (1968)

Superconducting substrate

Magnetic
impurity

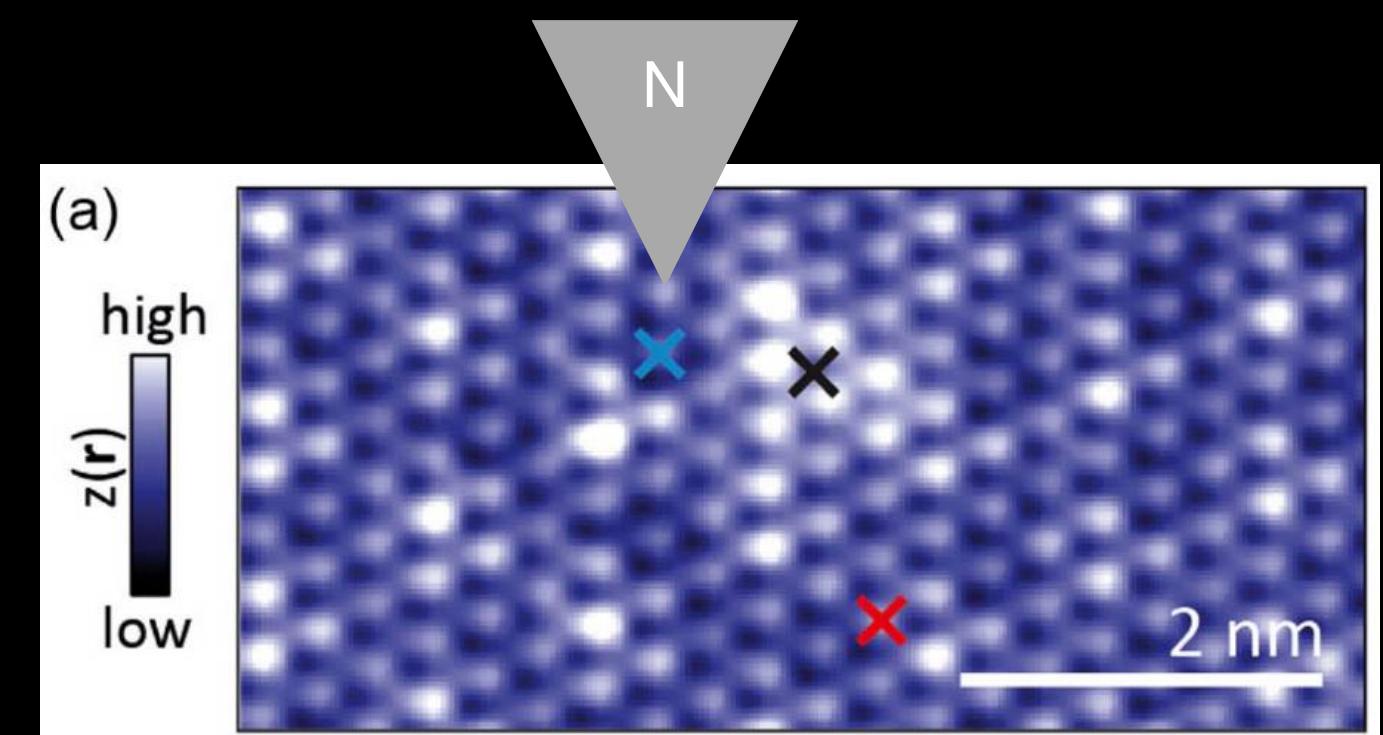
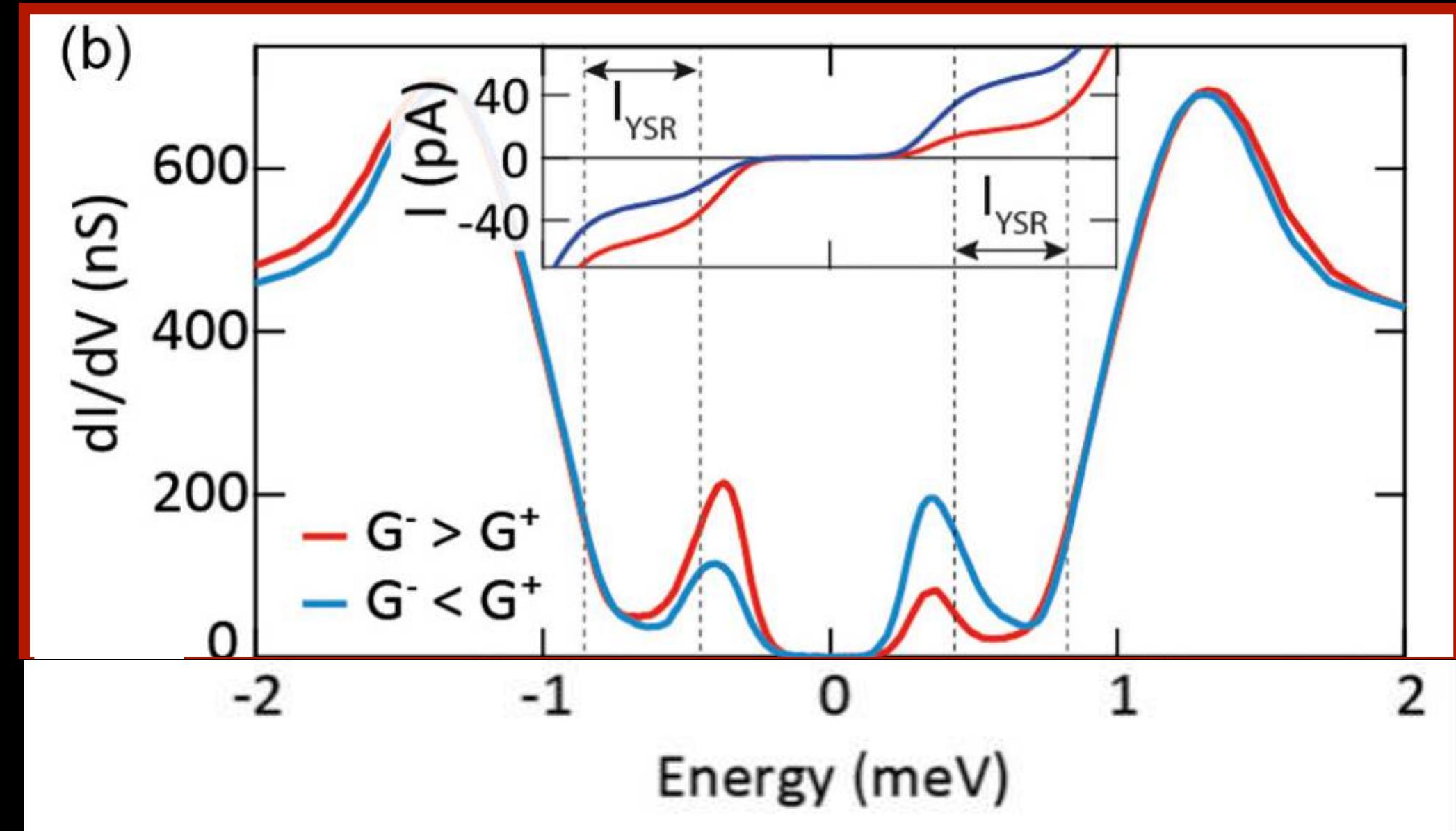
Reminder:
Superconducting (SC)
density of states ρ_{BCS}
(DOS)



Experiment on current and Shot Noise

U. Thupakula, et. al. Phys. Rev. Lett. **128**, 247001 (2022)

Conductance and Fano Factor measurement data



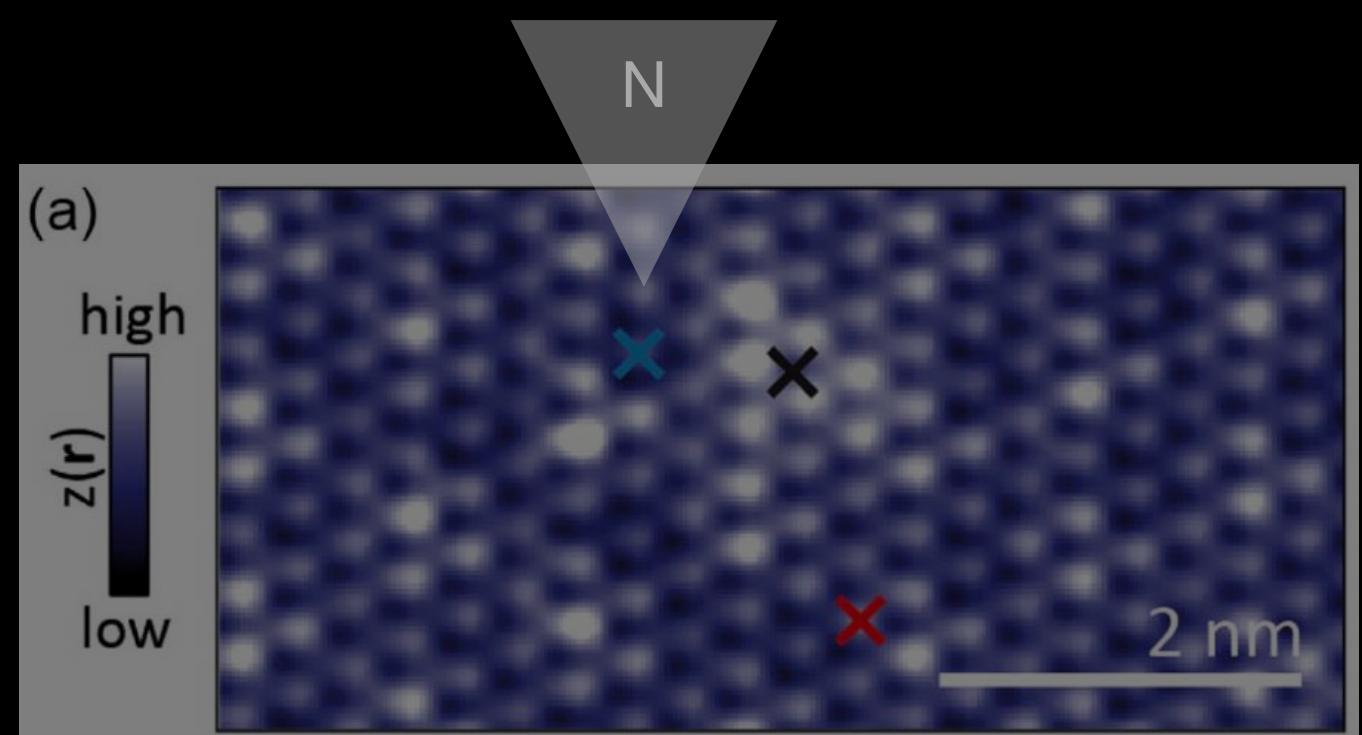
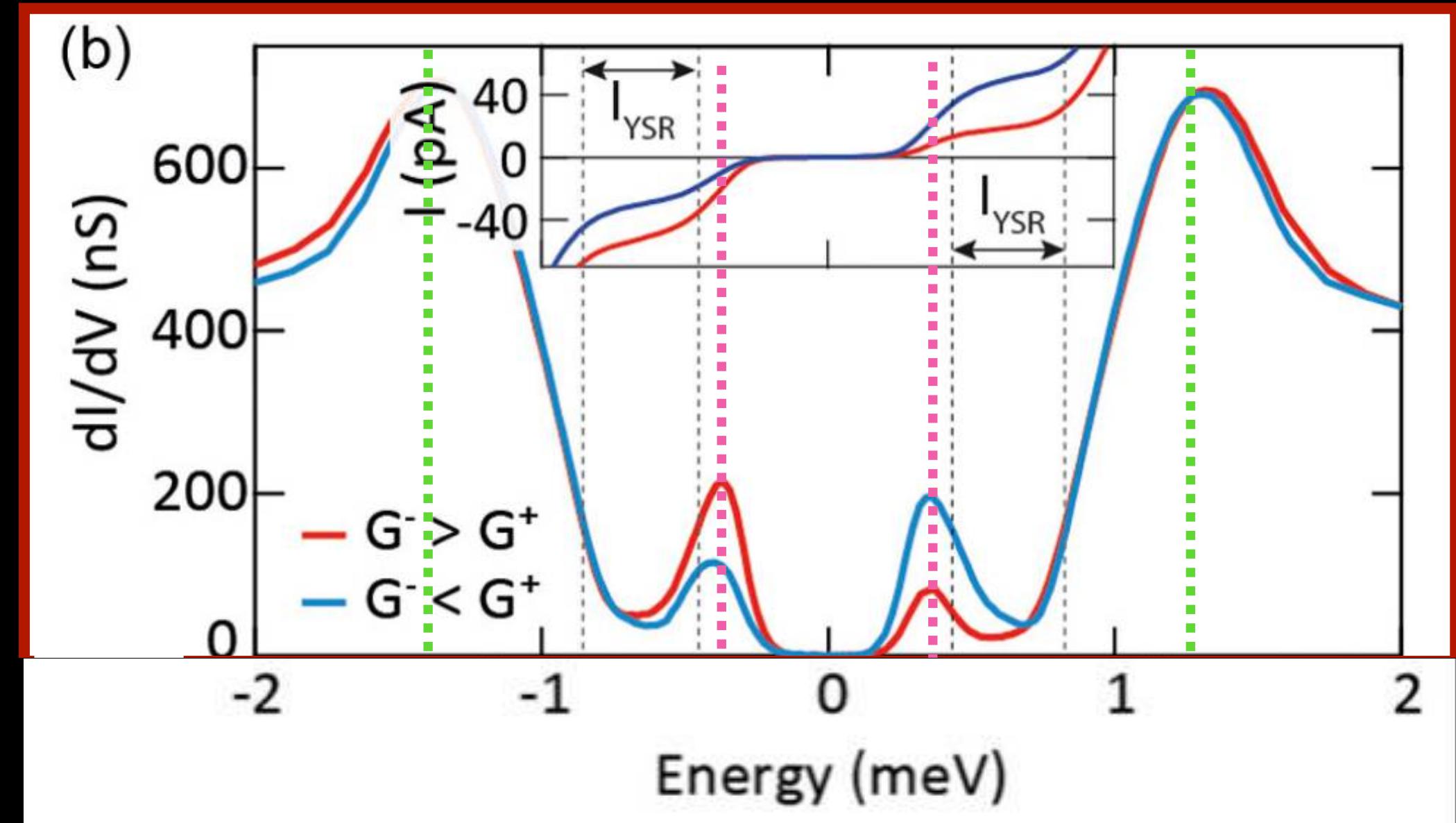
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Superconducting gap
 $|\Delta| \approx 1.30 \text{ meV}$

YSR energ
 $\epsilon_{YSR} \approx 0.3 \text{ meV}$

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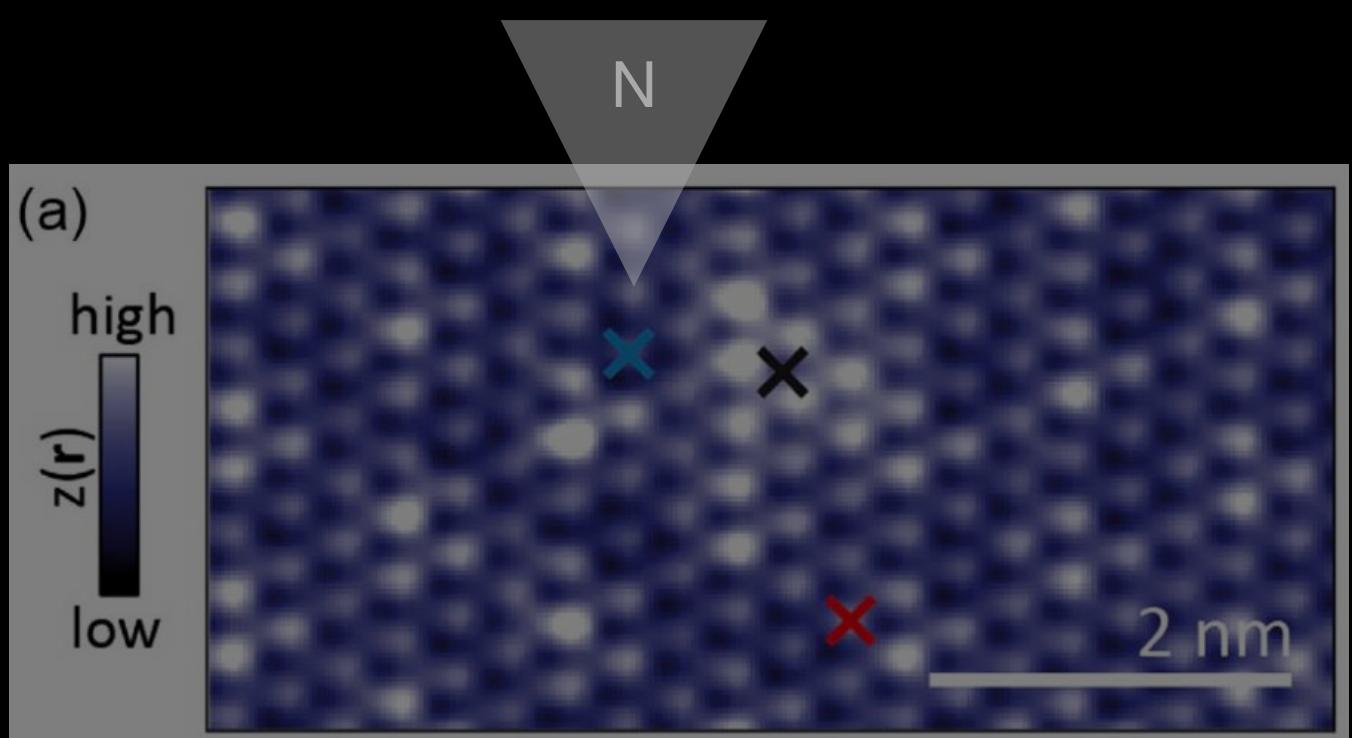
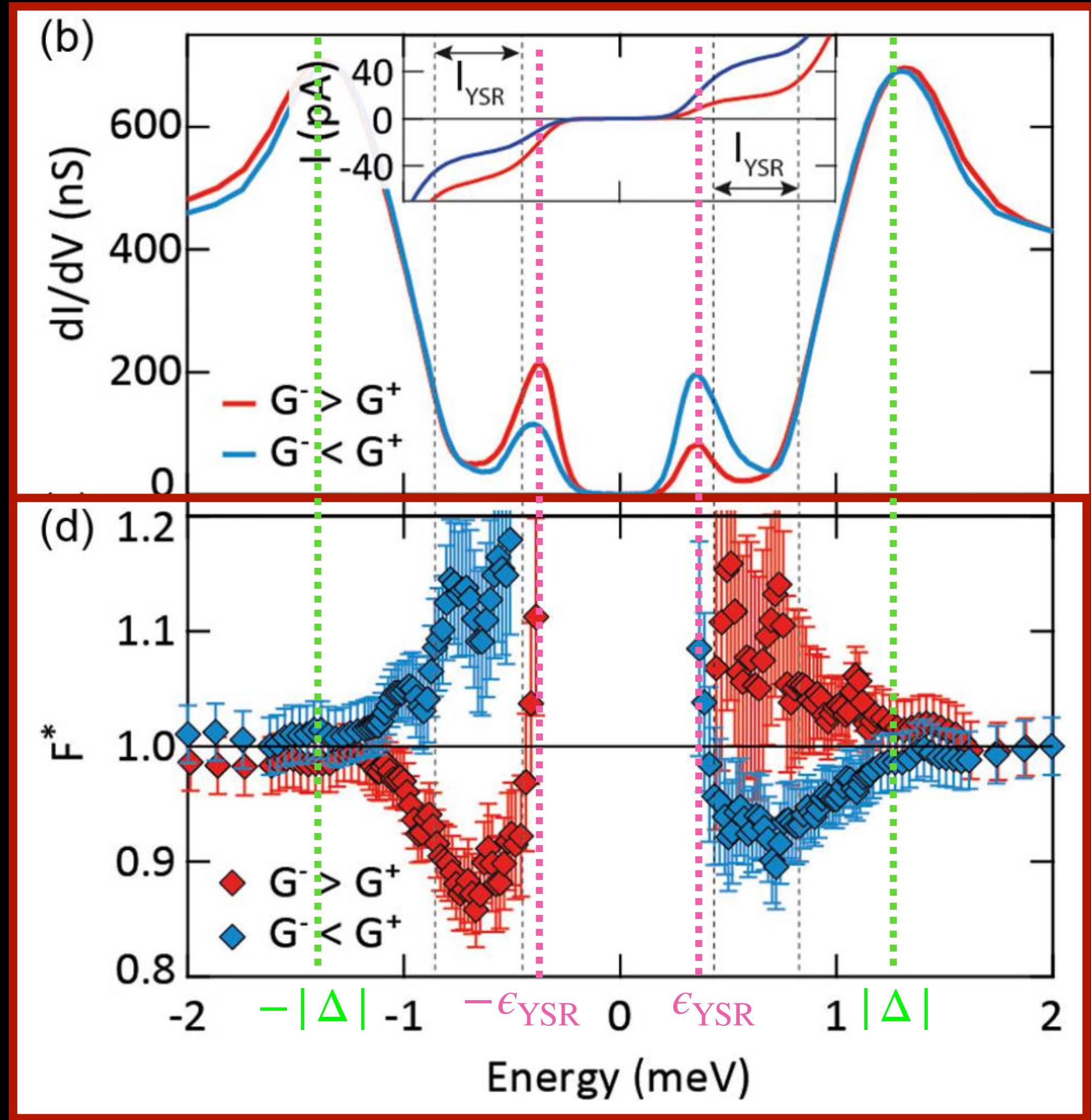
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Conductance and Fano Factor measurement data



Fano Factor:

$$F^* = \frac{S}{2e|I|} \approx q_{\text{eff}}$$

What is Full counting statistics (FCS)?

Theoretical framework computing (average) **current**, **shot noise** (variance of current) and **charge resolved currents**

L.S. Levitov and G.B. Lesovik, JETP Lett. **58**, 230 (1993)

Yu. V. Nazarov, Ann. Phys. (Berlin) **8**, SI-193 (1999)

W. Belzig and Yu.V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001)

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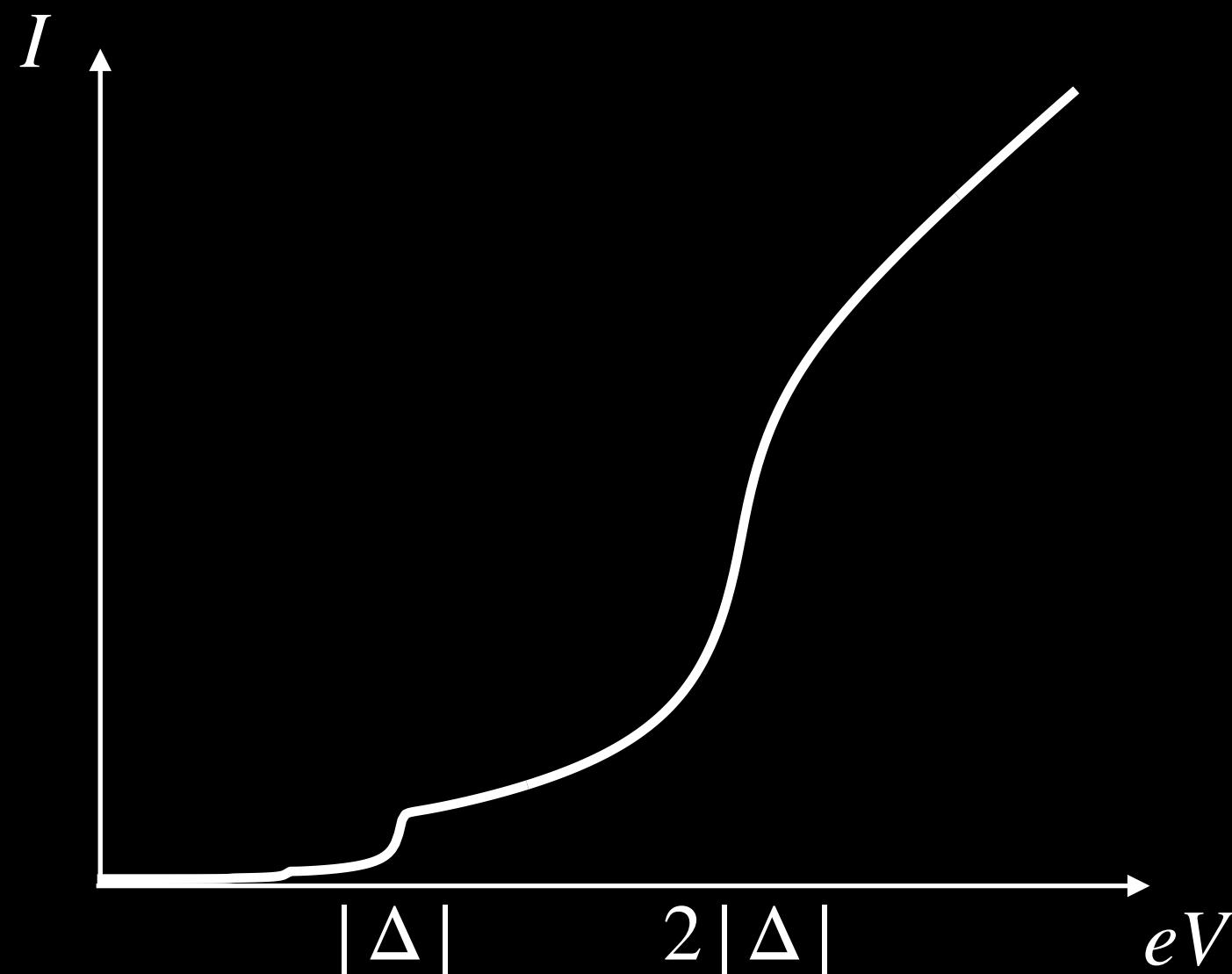
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Example: Superconductor-Superconductor (SS) contact

J. C. Cuevas and W. Belzig, Phys. Rev. Lett. **91**, 187001 (2003)



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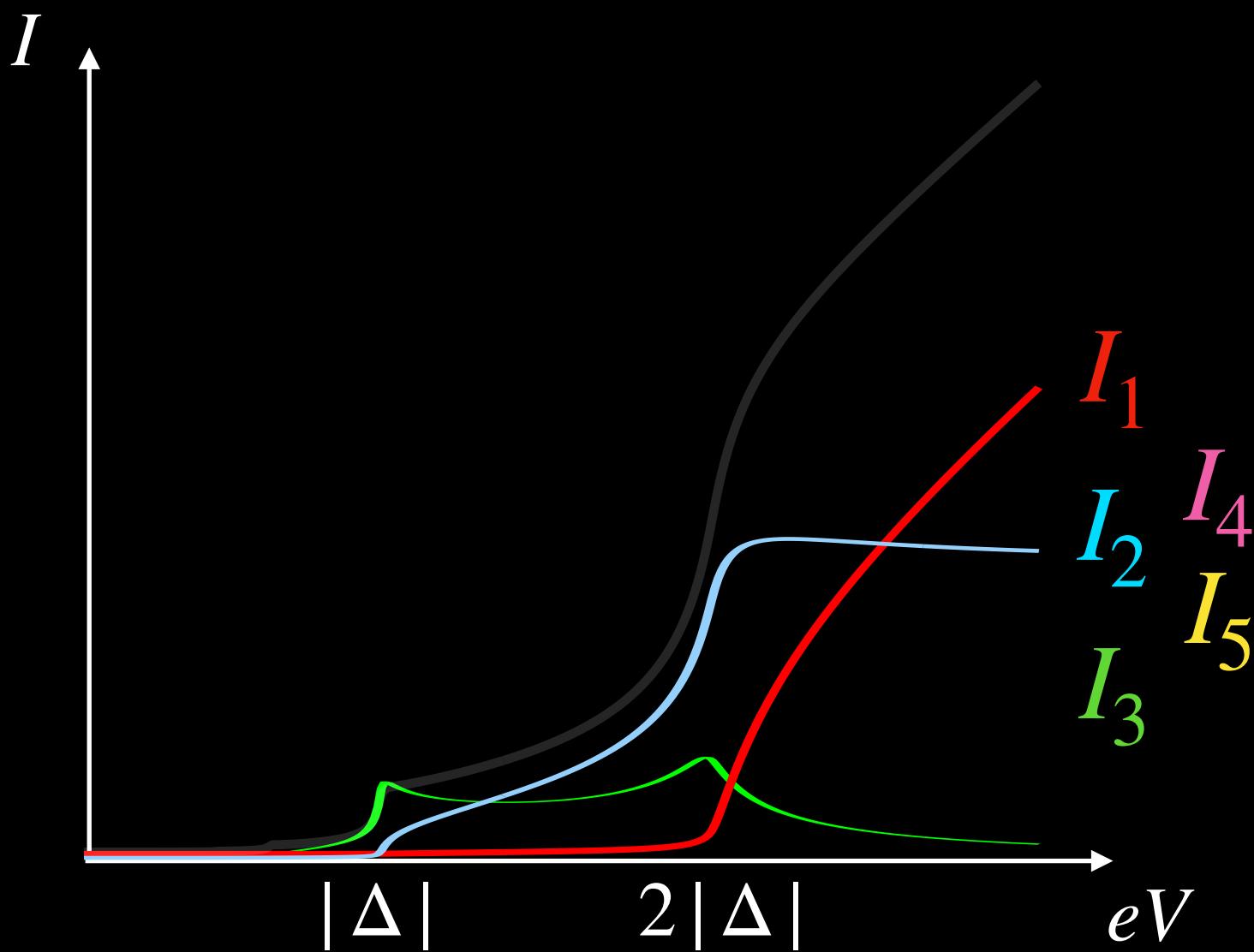
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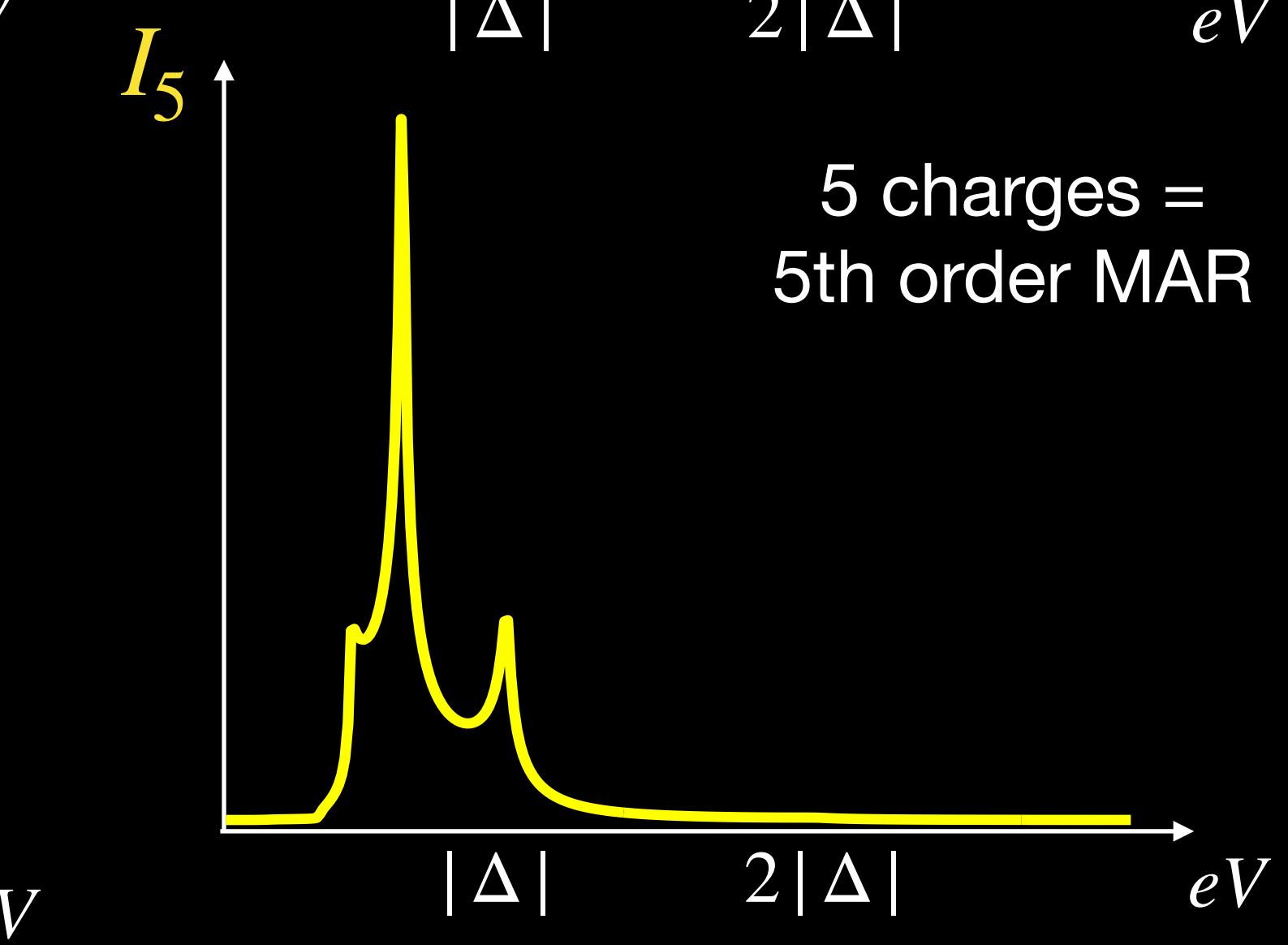
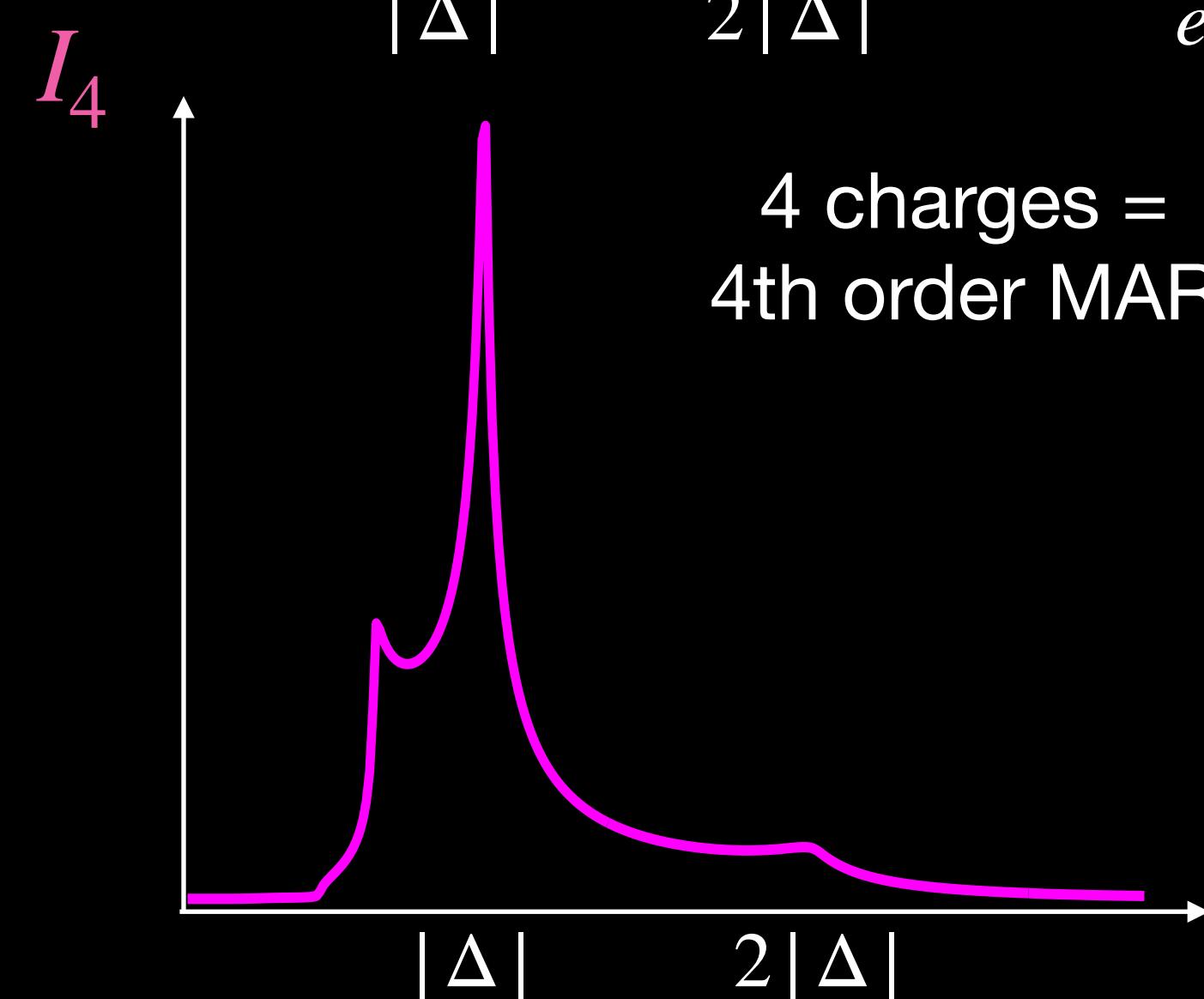
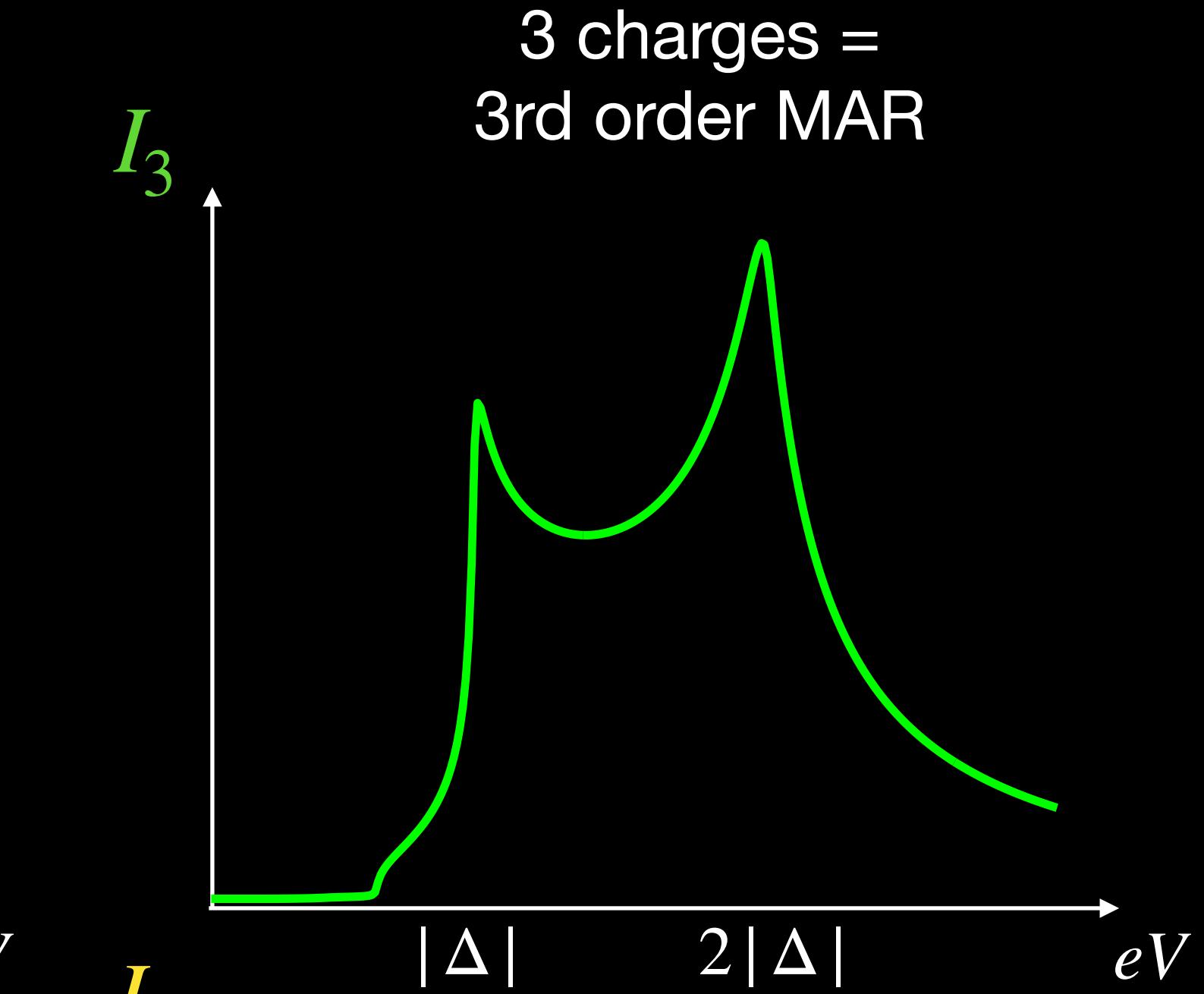
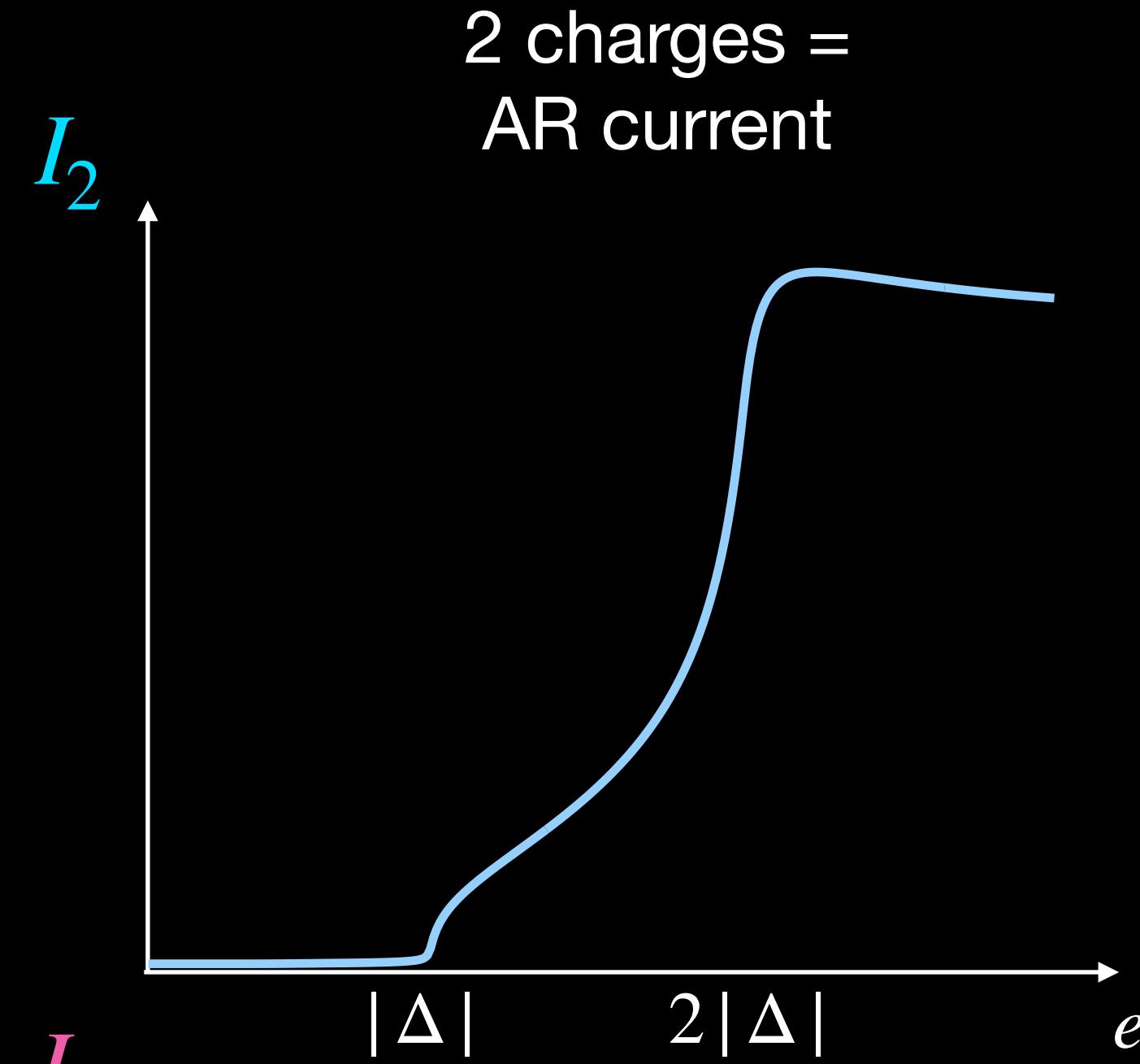
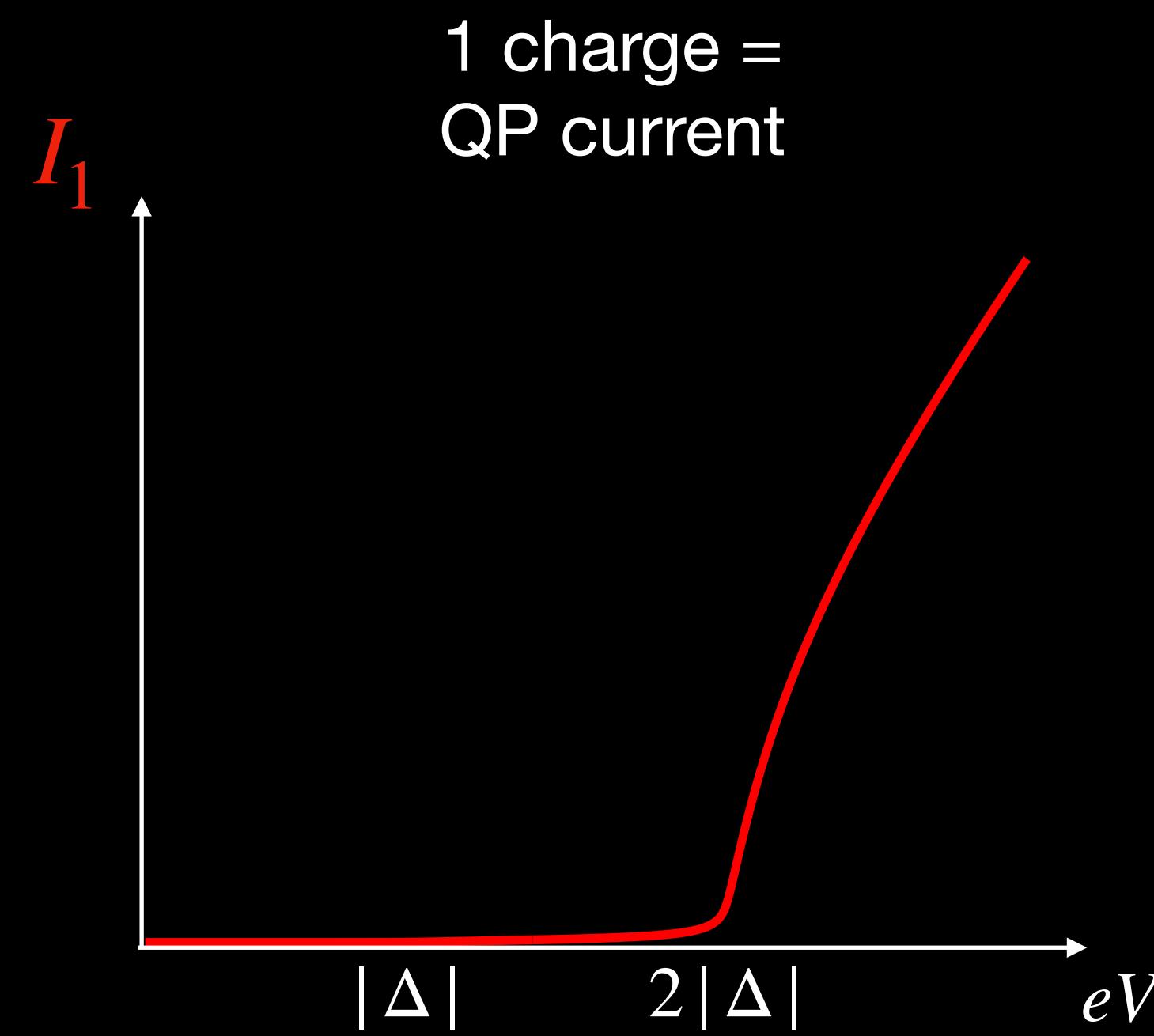
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Example: Superconductor-Superconductor (SS) contact



$$I_n(V) = \frac{e}{h} \int np_n(E, V)dE$$

Keldysh Action

All knowing **action** $\mathcal{A}(\chi)$ depending on **counting field** χ gives current I and shot noise

$$\mathcal{A}(\chi) = \text{Tr} \ln \left(\frac{1+s}{2} + \frac{1-s}{2} G(\chi) \right) - \mathcal{A}(0)$$

I. Snyman and Y. V. Nazarov Phys. Rev. B 77, 165118 (2008)

Reservoir Green's functions G and **normal state scattering matrix** s and as only input

Current

$$I = -\iota \frac{e}{h} \frac{\partial \mathcal{A}(\chi)}{\partial \chi} \Big|_{\chi=0} = \sum_{n=1}^{\infty} I_n$$

Shot noise

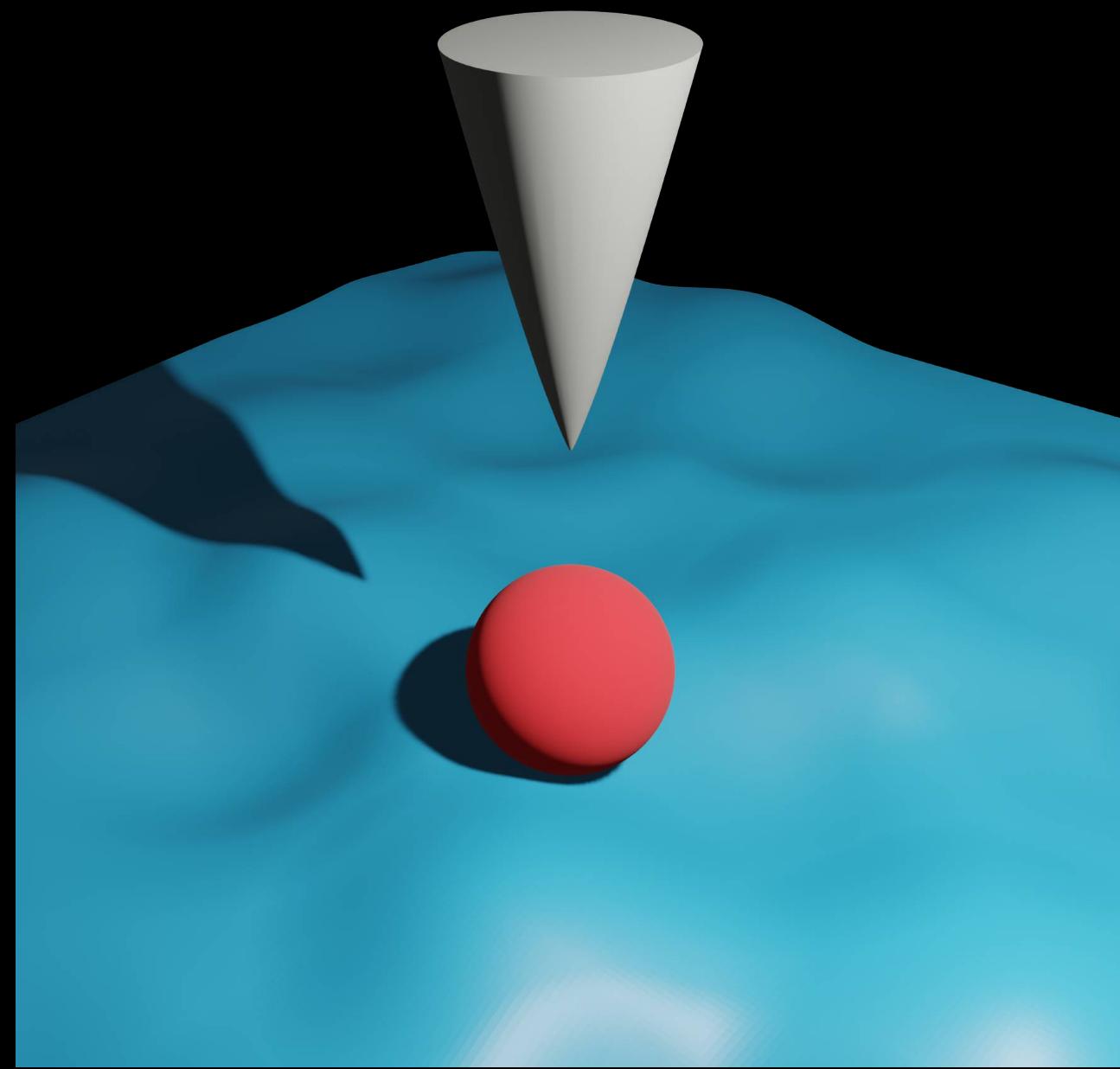
$$S = -\frac{2e^2}{h} \frac{\partial^2 \mathcal{A}(\chi)}{\partial \chi^2} \Big|_{\chi=0}$$

What systems are we describing?

Learn more:



Single-impurity



Normal tip

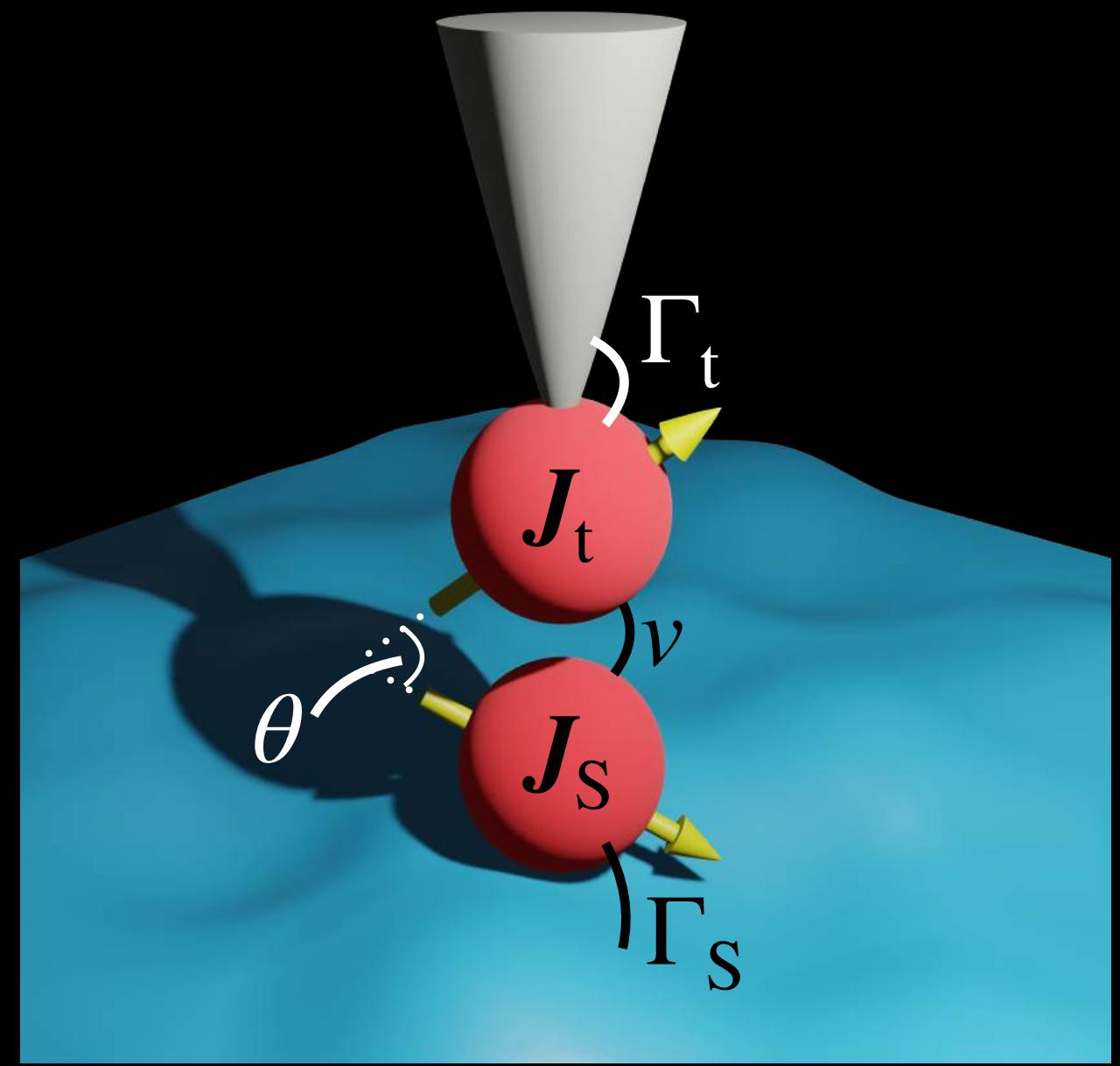
or

SC tip

QP vs AR

n -th order (M)AR vs
 $n+1$ -th order (M)AR

Double-impurity



$$F^* = 1/4$$
$$F_{\min}^* = 7/32$$

A. Yazdani, et. al., Science 275,
1767 (1997)

Shuai-Hua Ji, et. al. Phys. Rev.
Lett. 100, 226801 (2008)

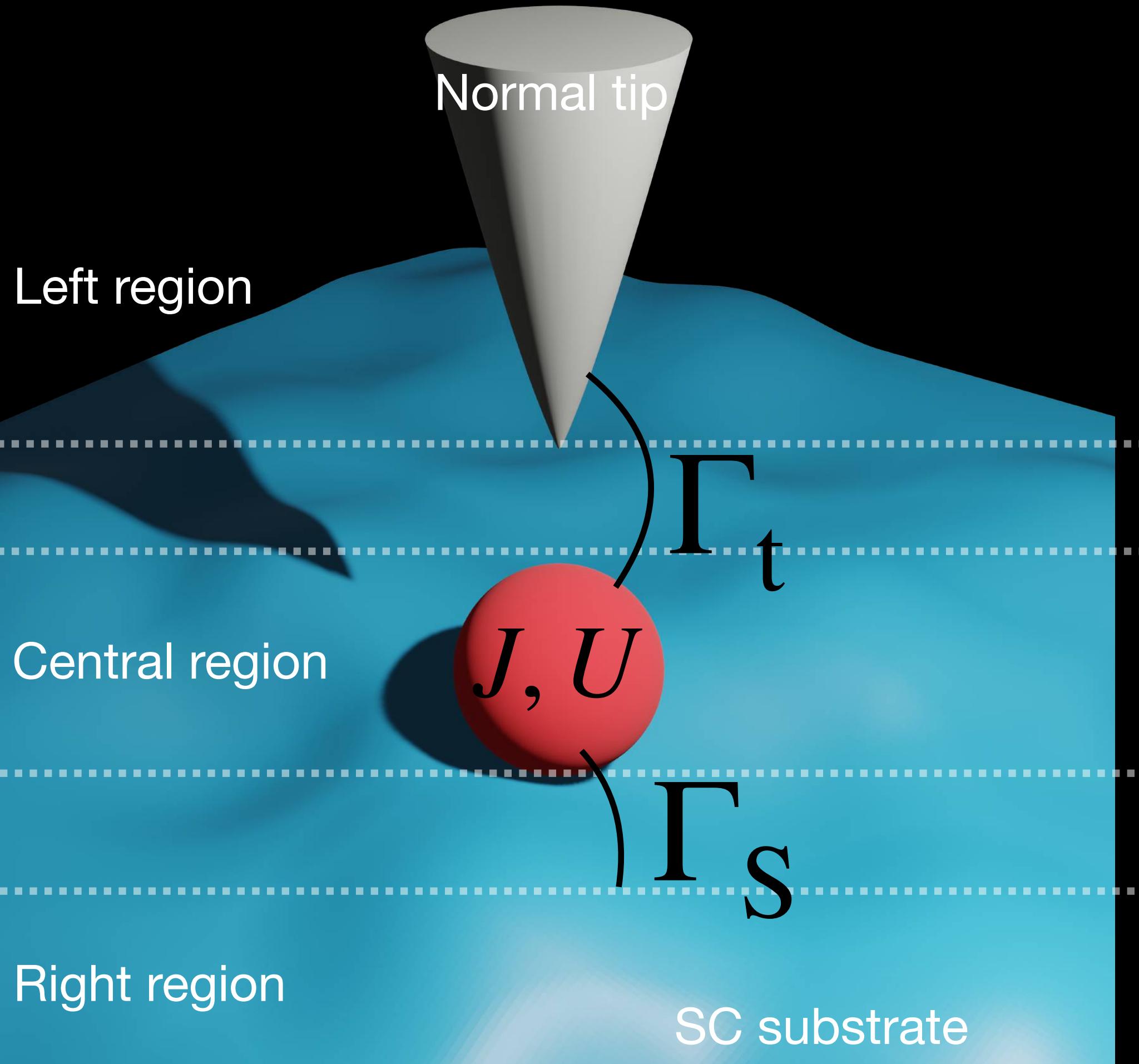
M. T. Randeria, et. al., Phys. Rev. B 93,
161115(R) (2016)

L. Farinacci, et. al., Phys. Rev. Lett. 121,
196803 (2018)

H. Huang, et. al., Nat. Phys. 16, 1227 (2020)

What systems are we describing? And how?

Single-impurity



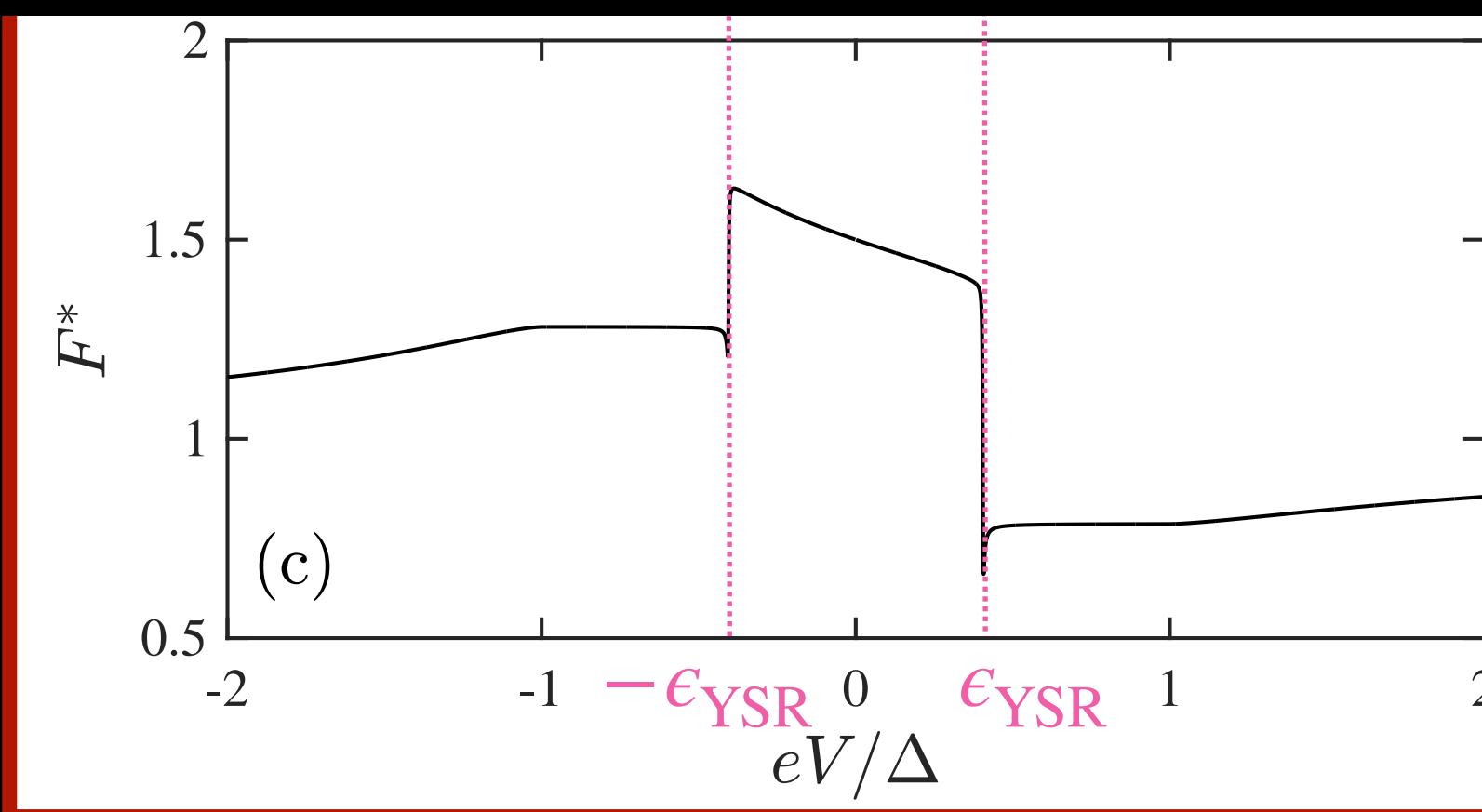
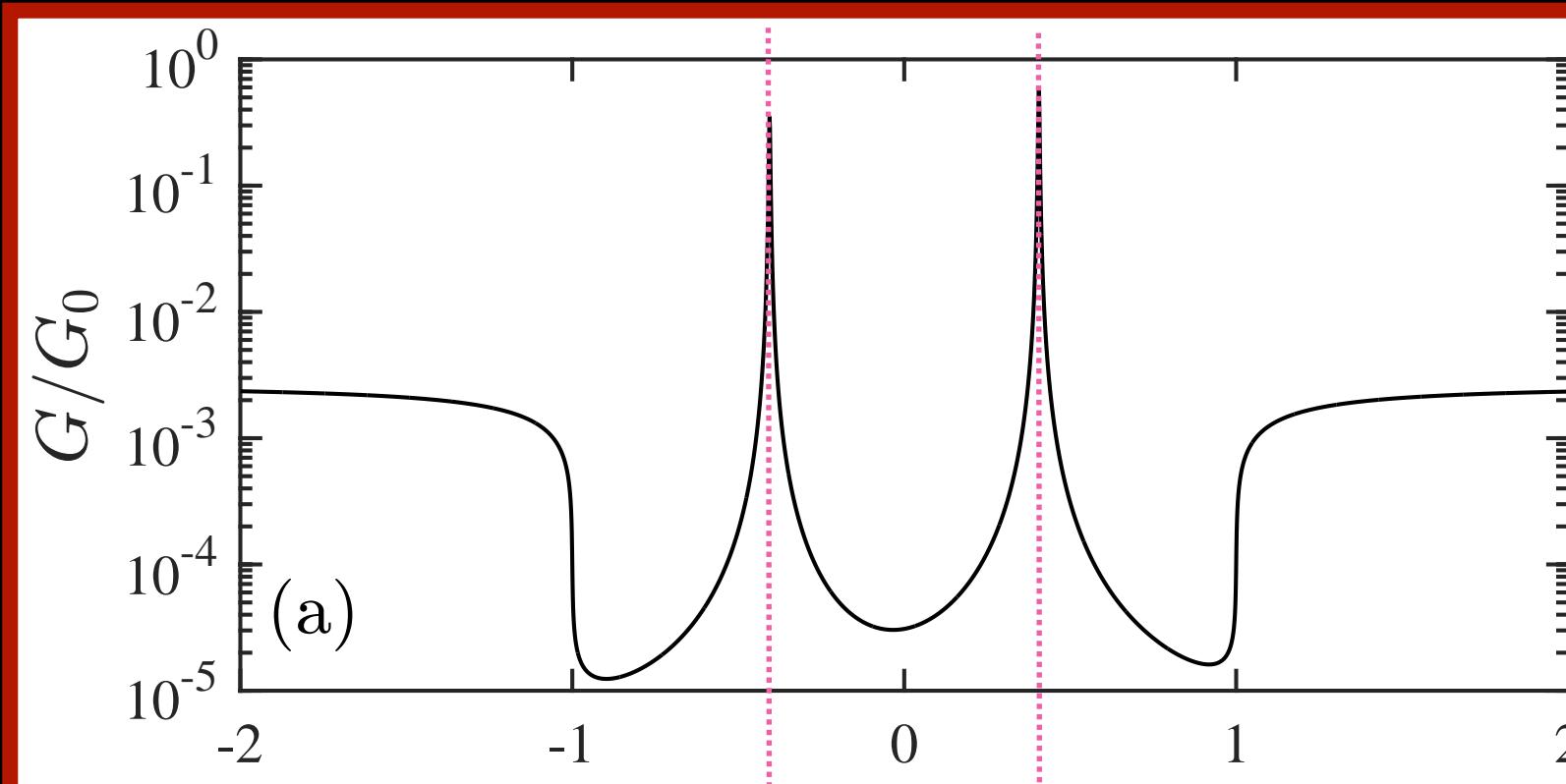
✗ Normal metal Hamilton and Green's function (GF)

✗ Mean-field Anderson impurity model
(U : on-site energy, J : exchange)

$$H_{\text{imp}} = U(n_\uparrow + n_\downarrow) + J(n_\uparrow - n_\downarrow)$$

✗ BCS Hamiltonian and GF

Theoretical curves - Changing Γ_t



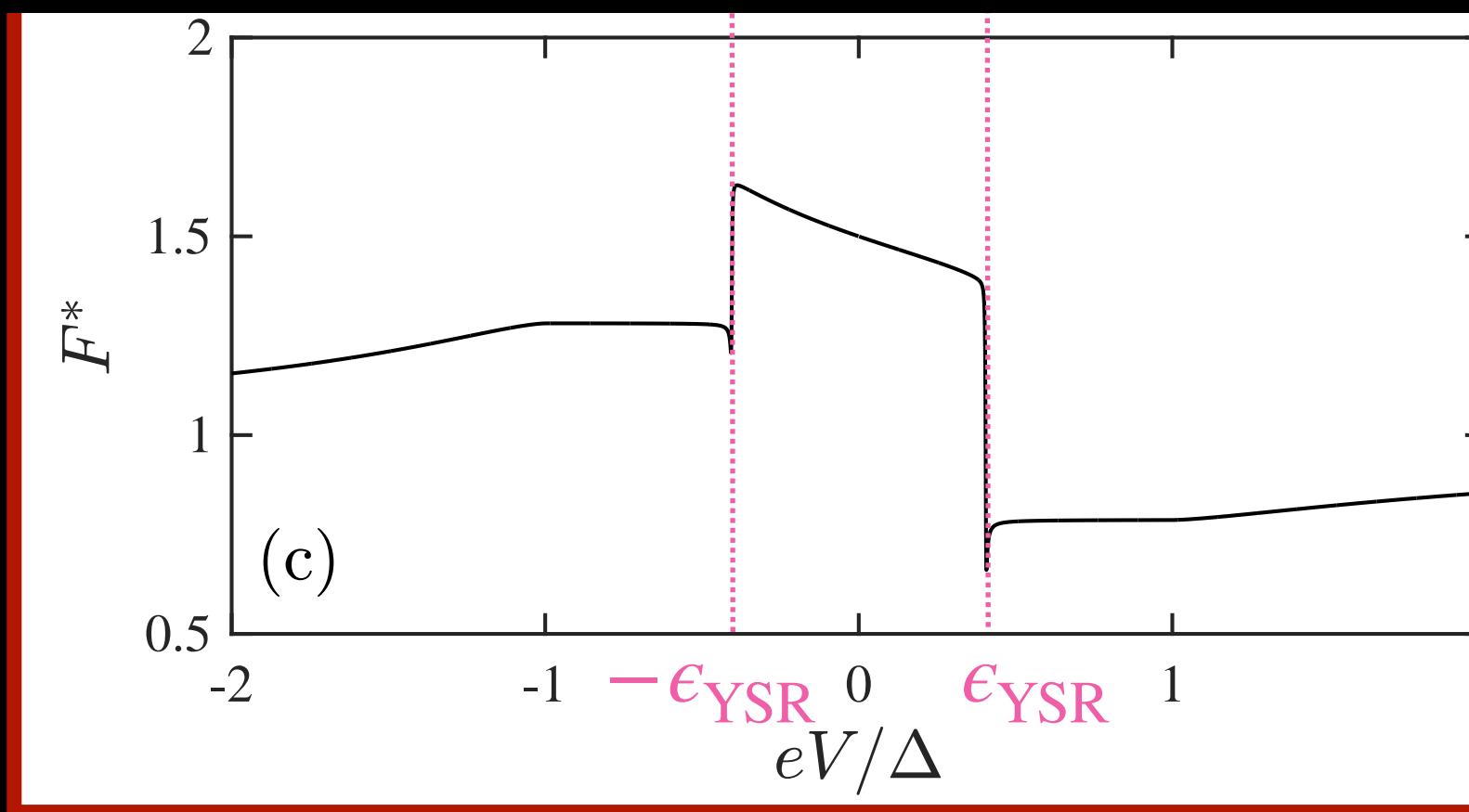
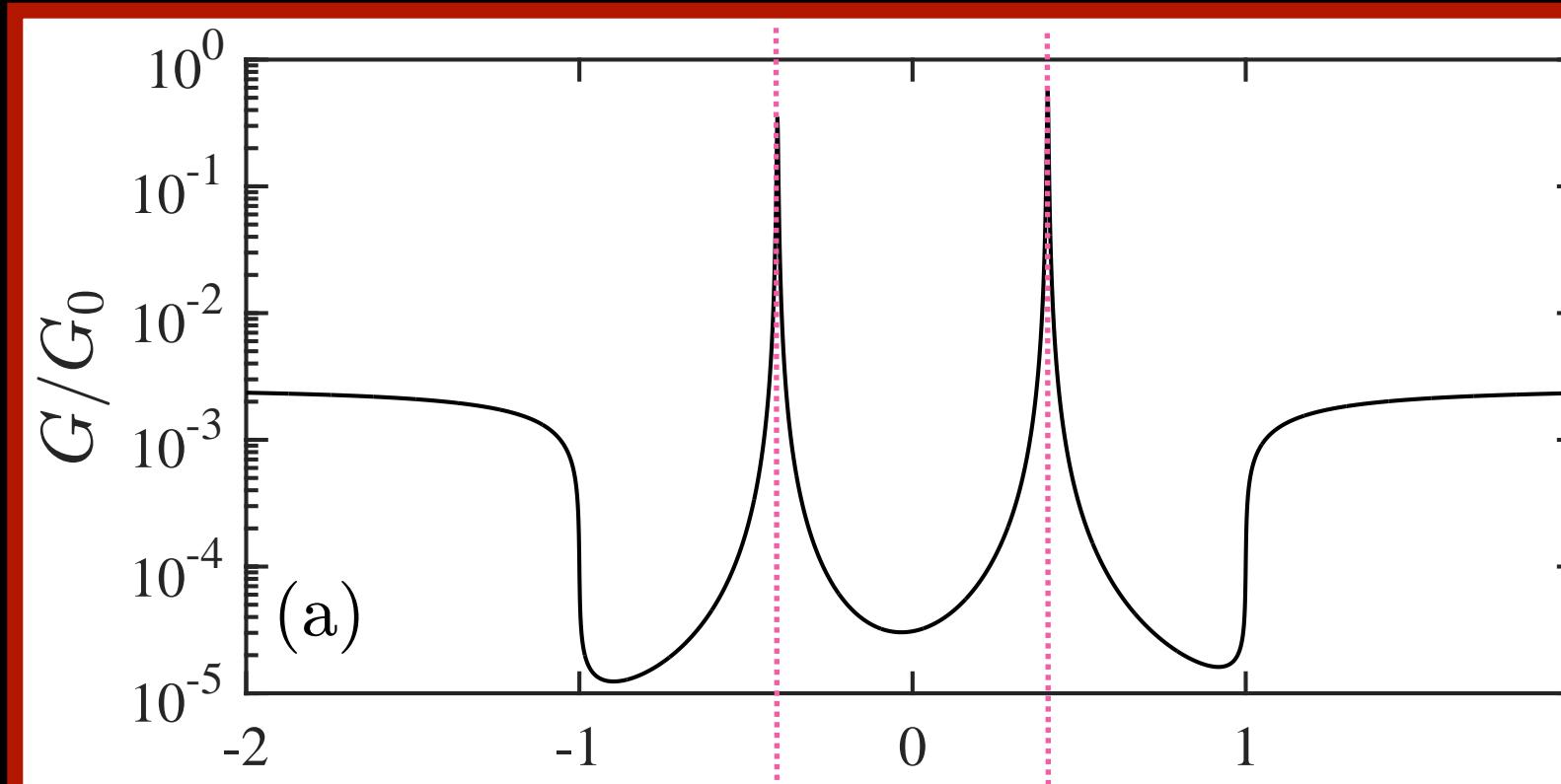
$$\Gamma_S = 100, J = 60, U = 80, \eta = 0.001$$

Charge resolved conductance

a) $\Gamma_t = 0.1\Delta$

— G_1 — G_2

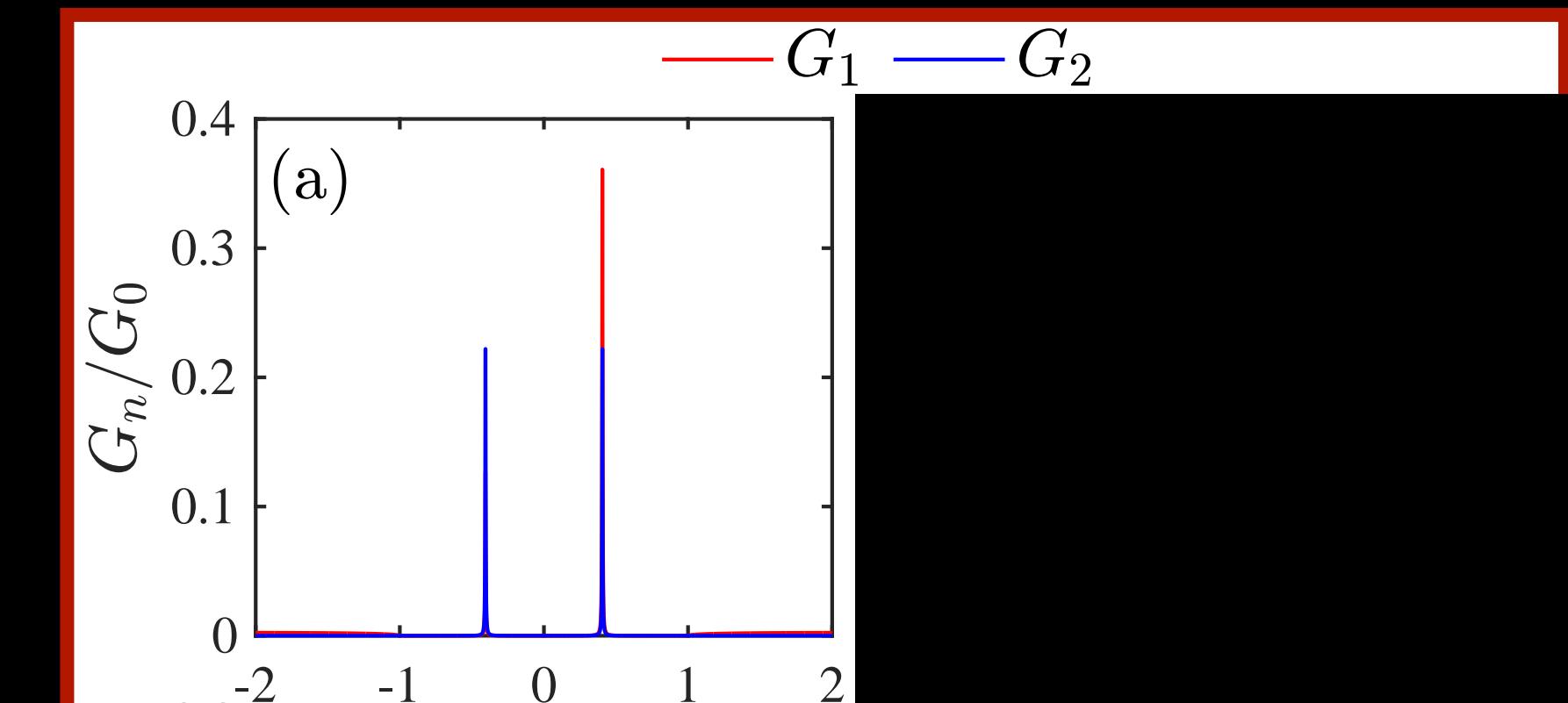
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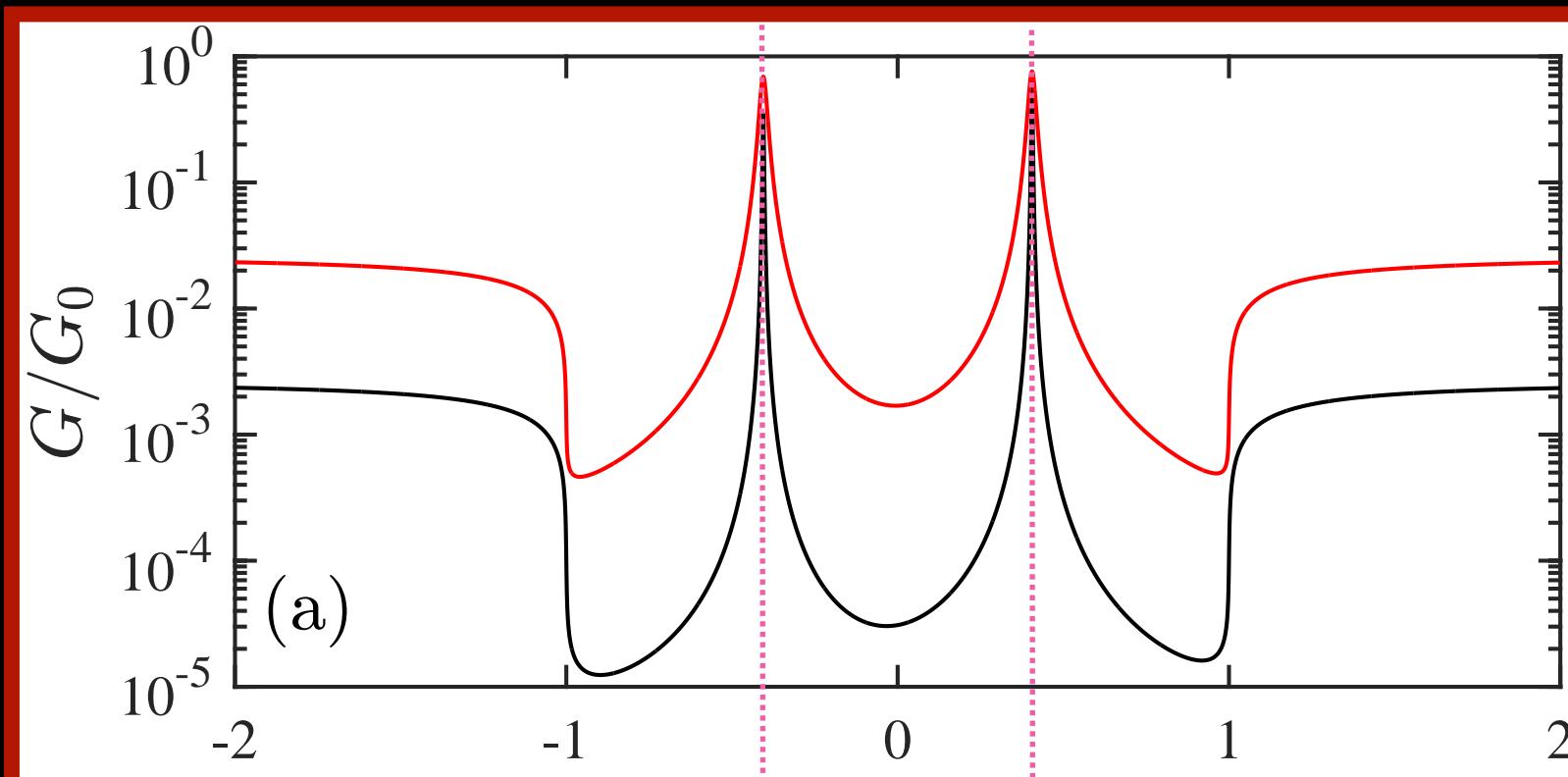
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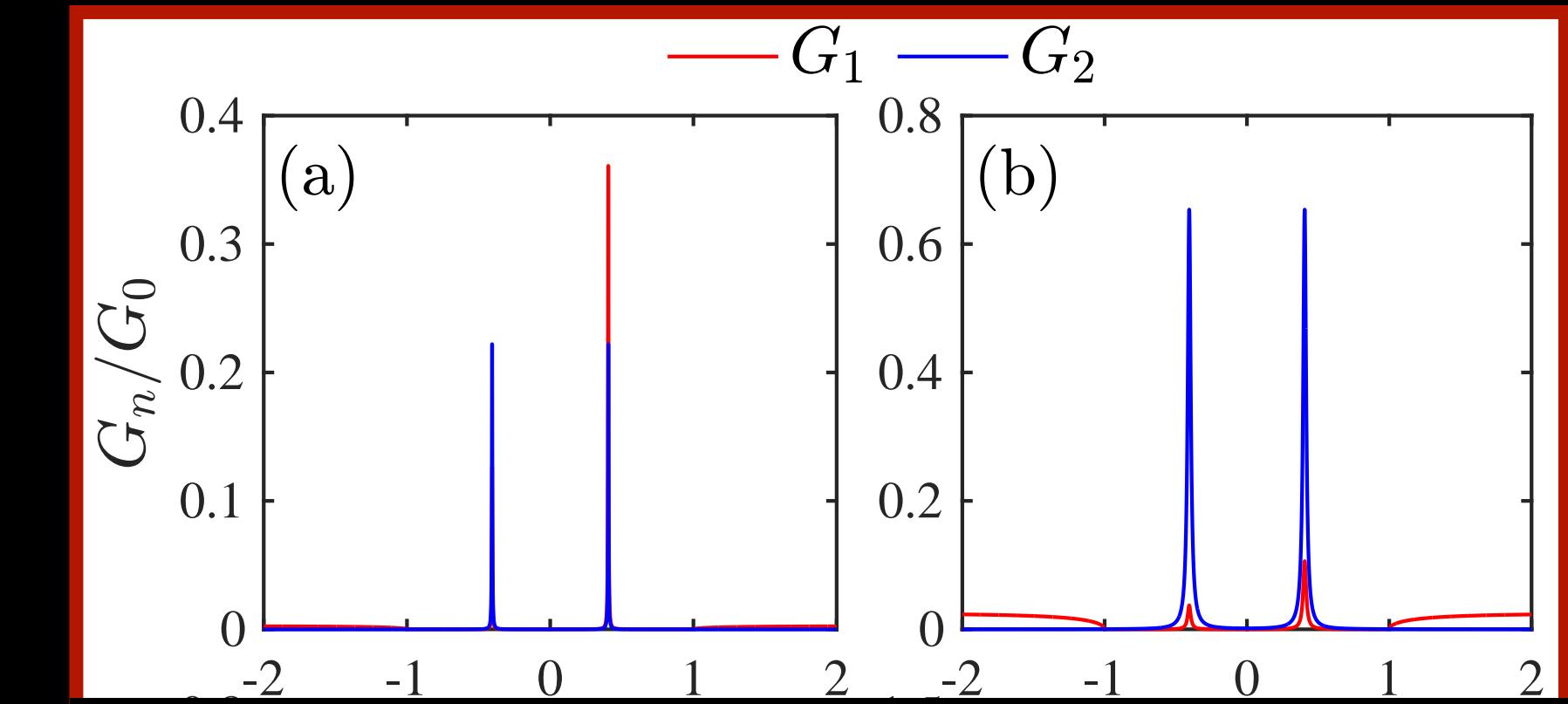


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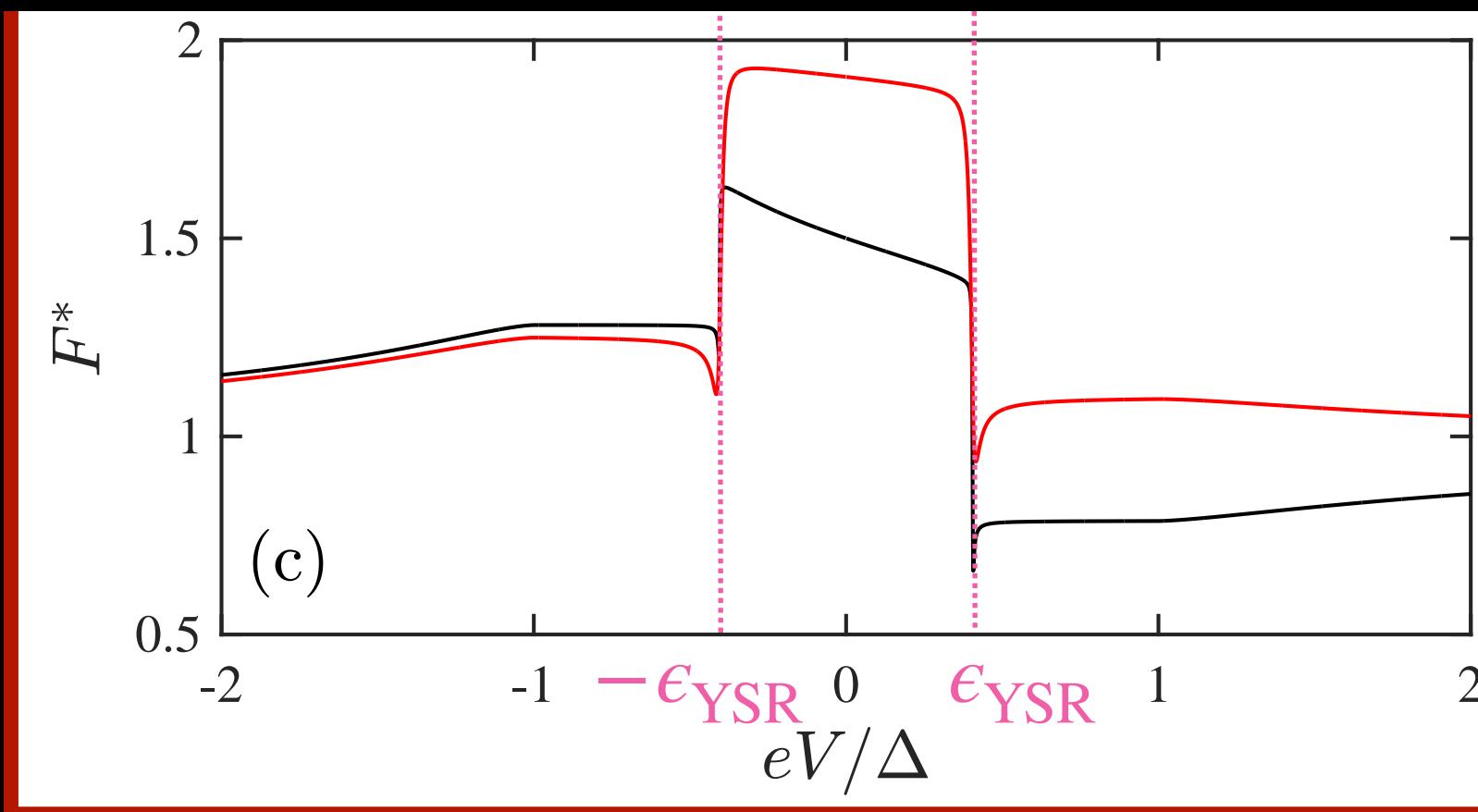


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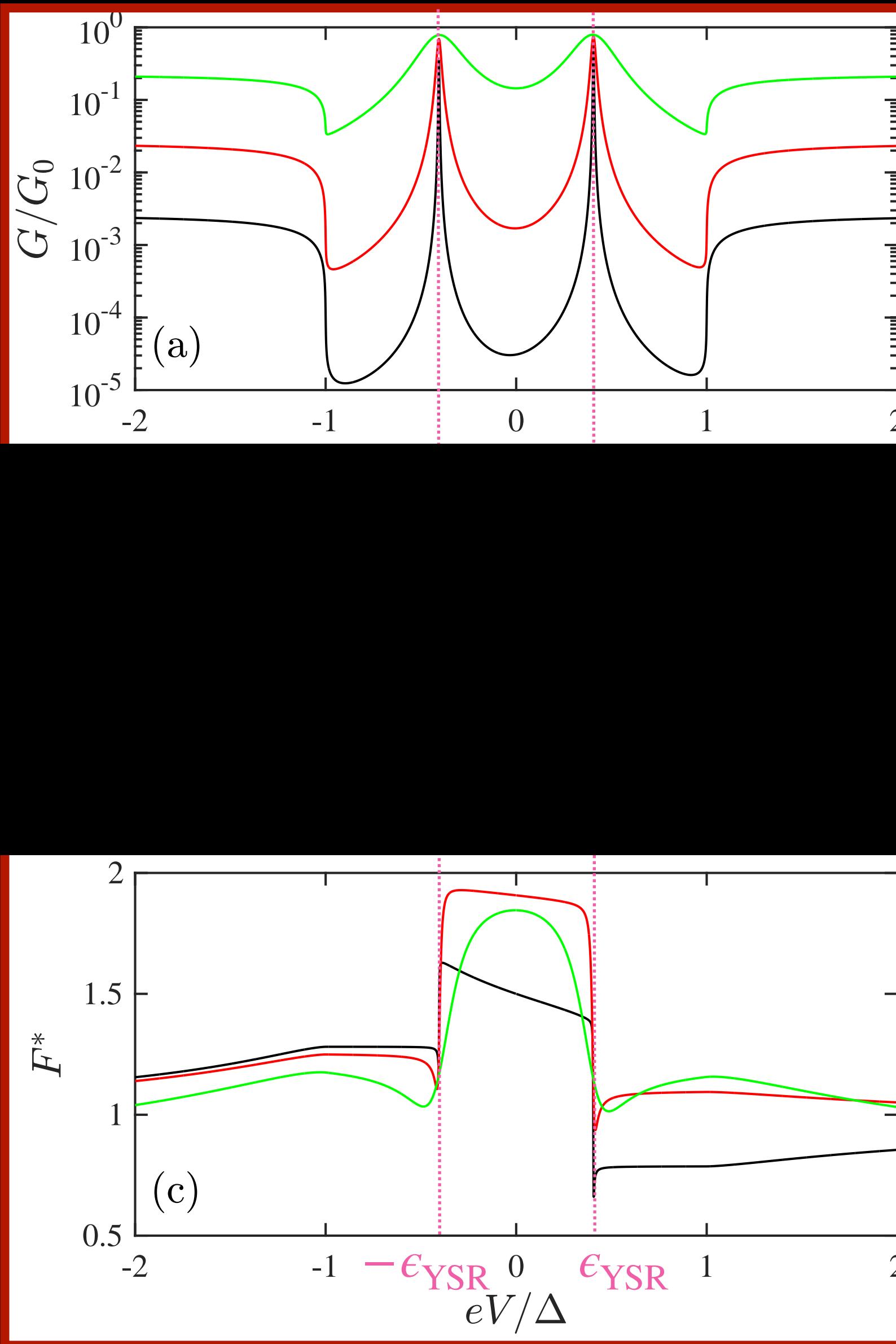
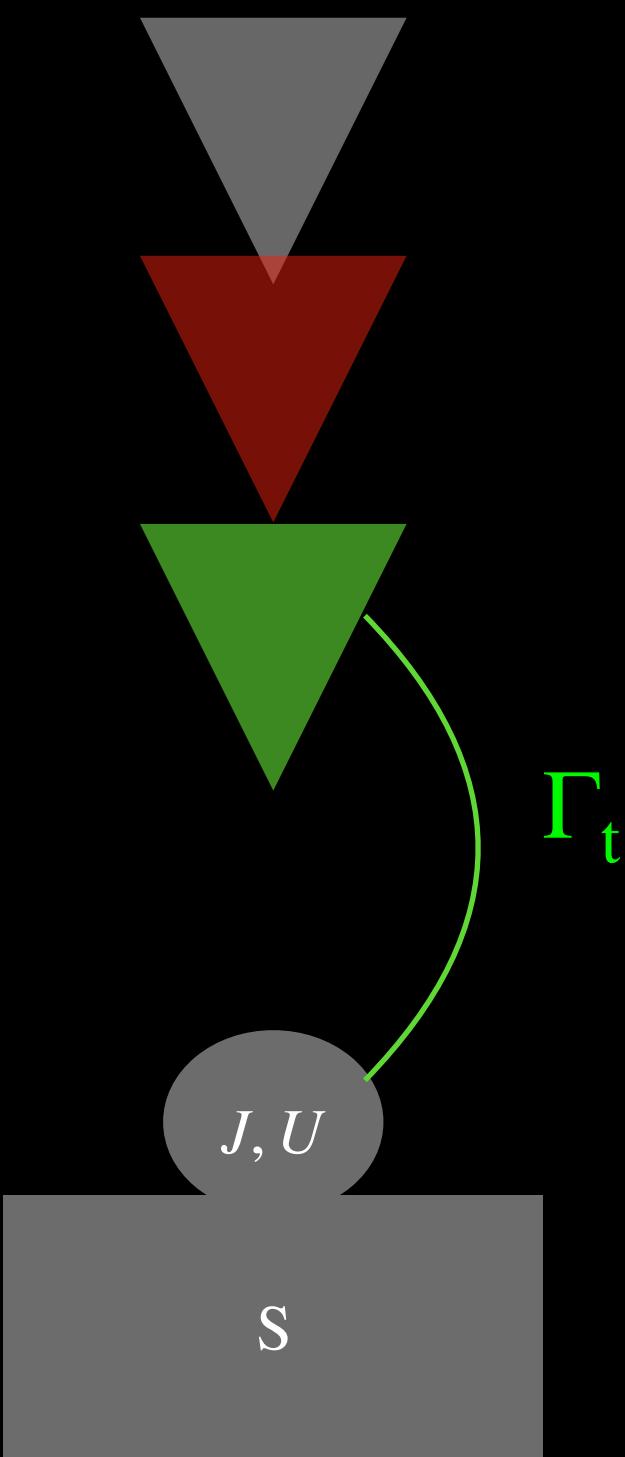


b) $\Gamma_t = 1\Delta$



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Theoretical curves - Changing Γ_t

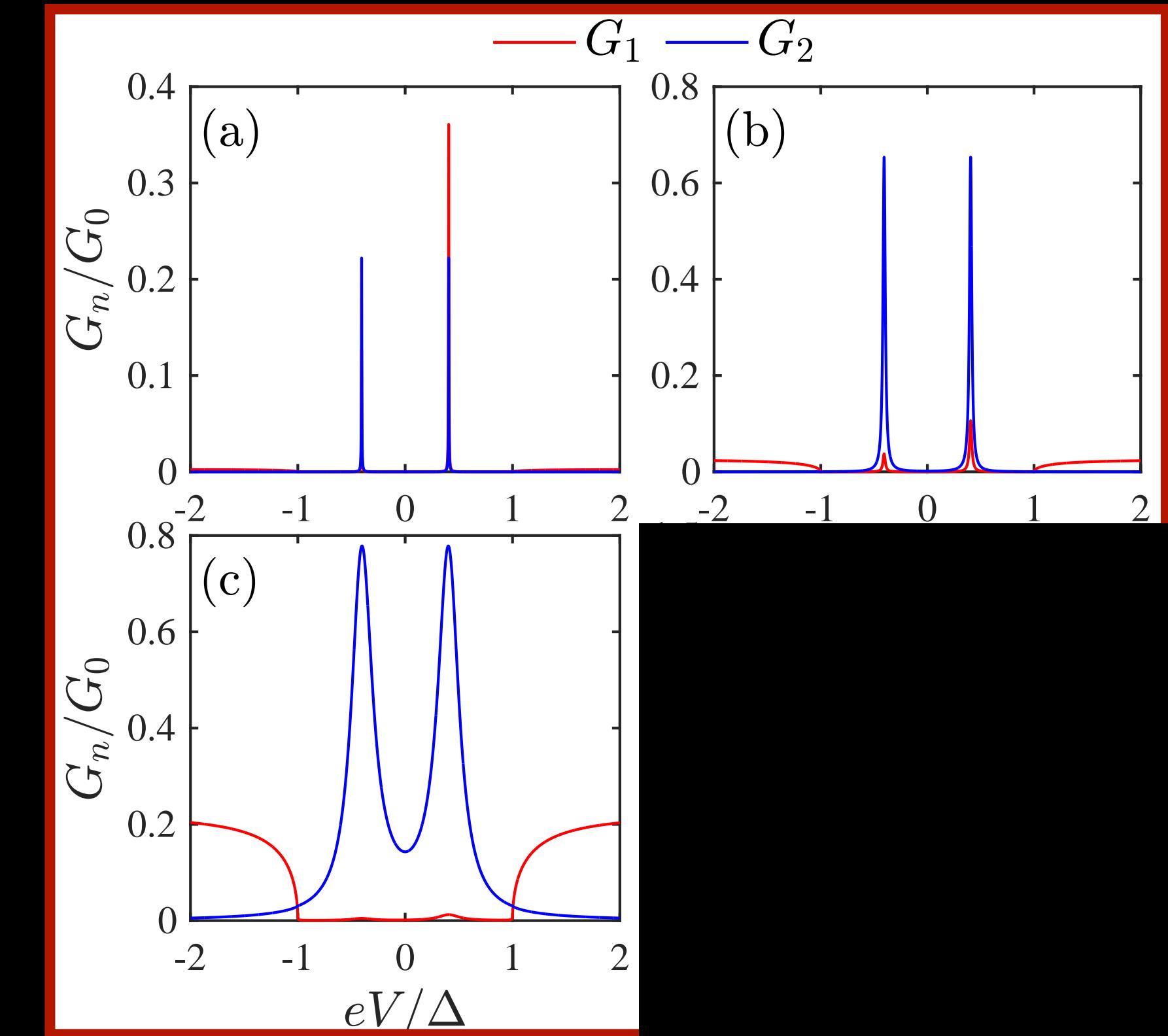


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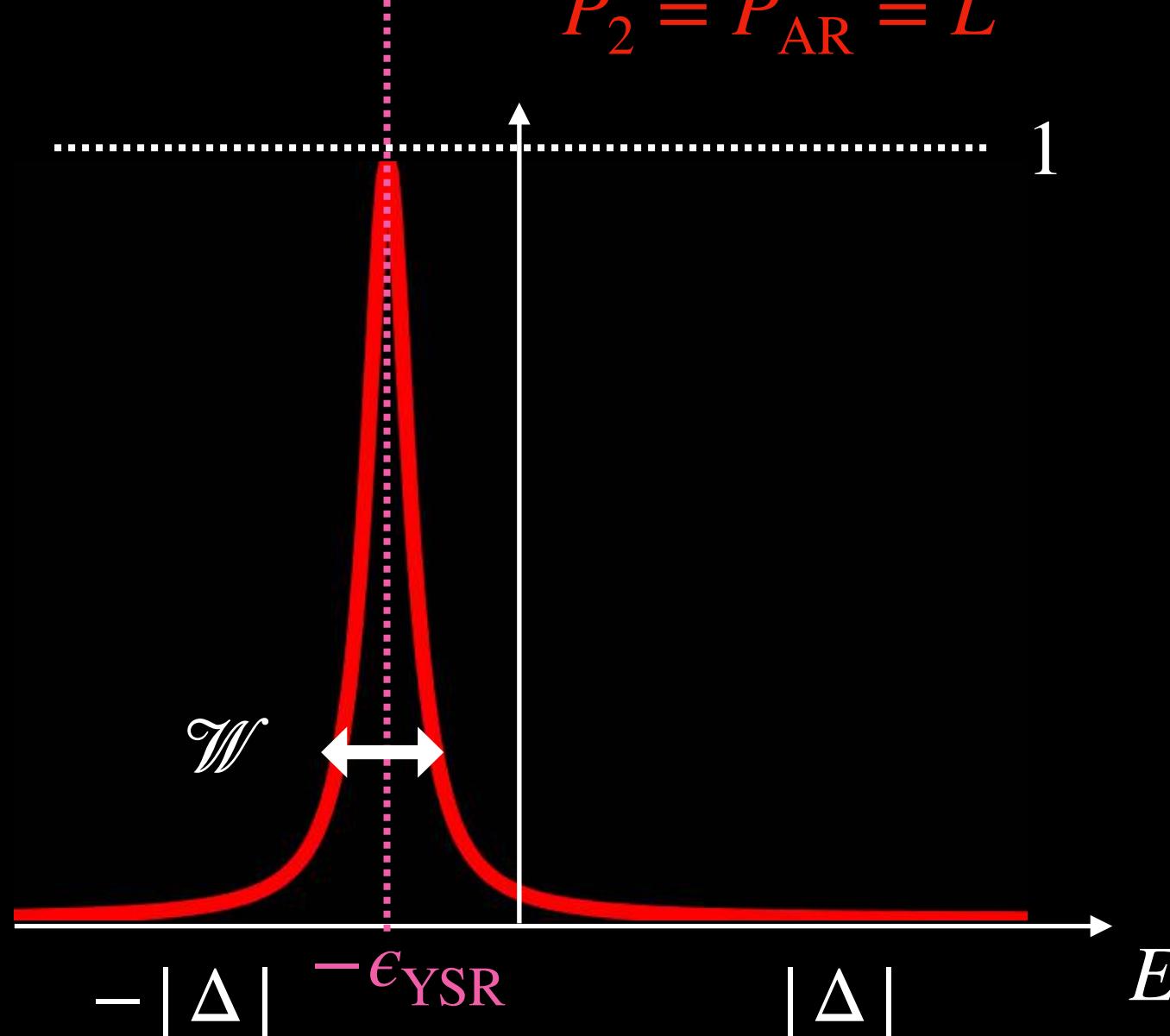
c) $\Gamma_t = 10\Delta$

Why does Fano Factor drop to 1 at YSR energy?

2 \leftrightarrow AR charge

Current:

$$P_2 = P_{\text{AR}} = L$$



$$\frac{I}{e/h} = \int_{-eV}^{eV} 2P_2(E) = 2\pi\mathcal{W}$$

Shot Noise:

$$\frac{S}{e^2/h} = 2 \int_{-eV}^{eV} 2^2 P_2(E) - 2^2 P_2(E)^2 = 2 \times \frac{1}{2} \times 2^2 \pi \mathcal{W}$$

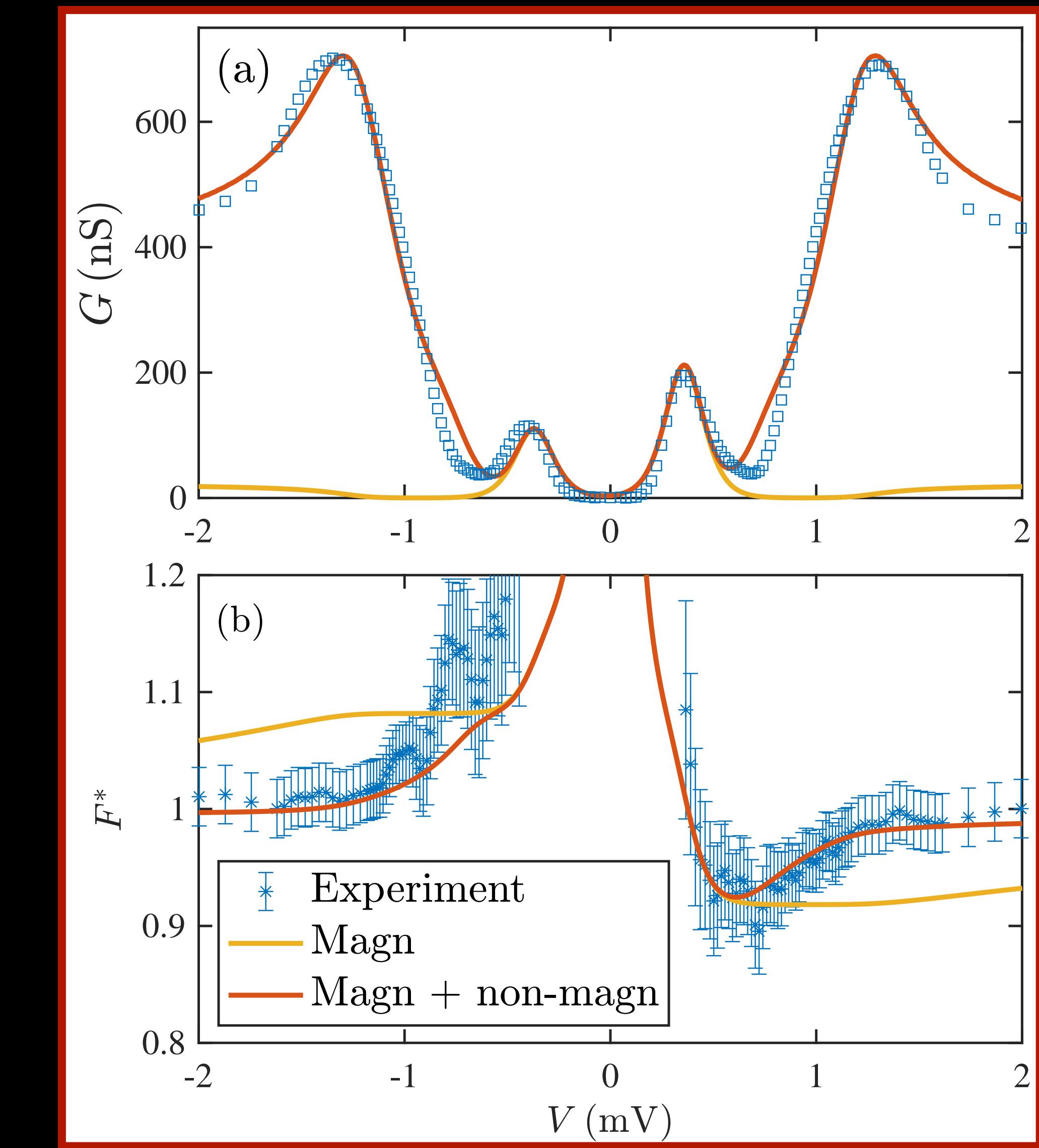
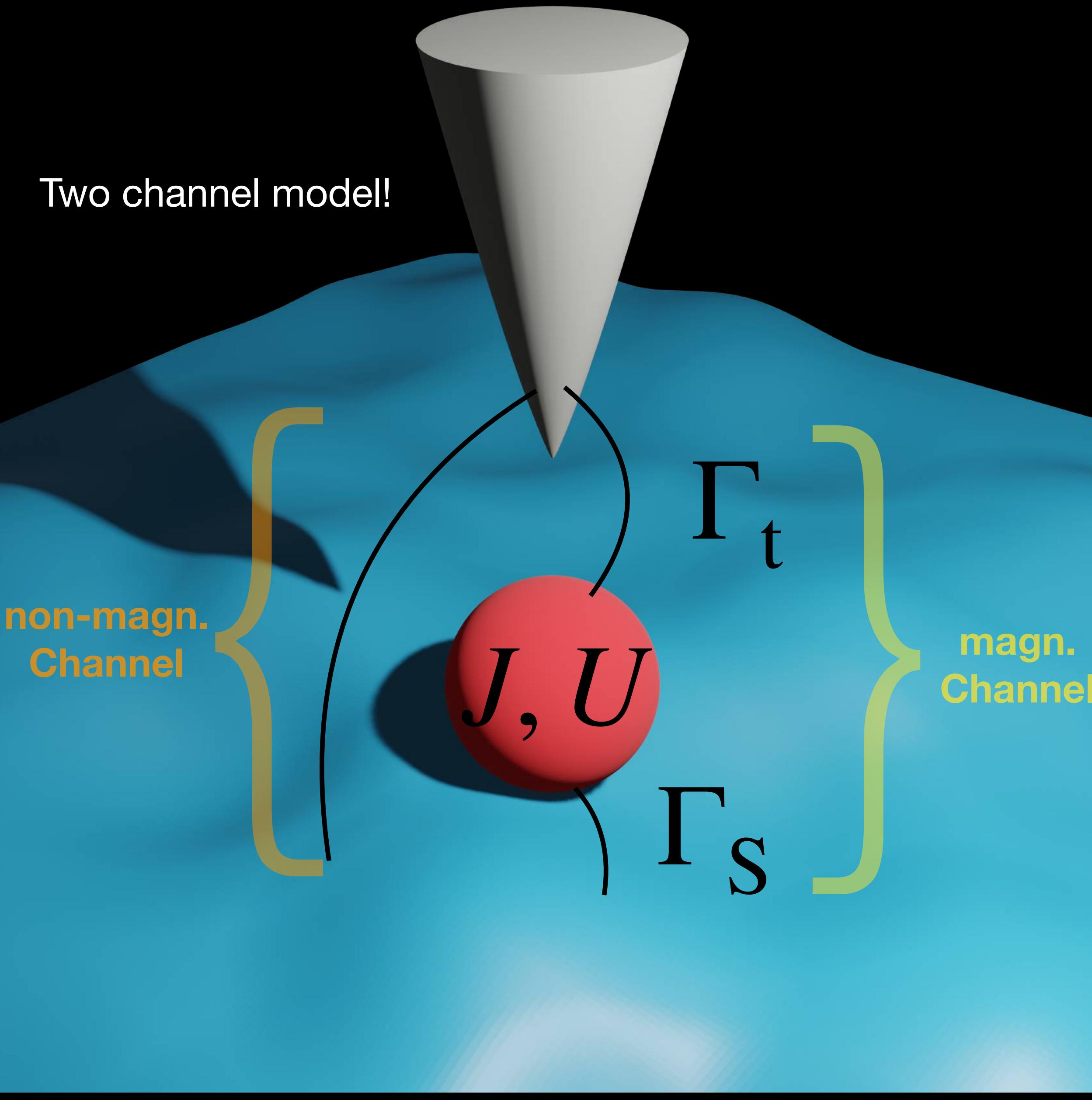
Fano Factor:

$$F = \frac{S}{2e|I|} = \frac{\frac{1}{2}2^2\pi\mathcal{W}}{2\pi\mathcal{W}} = \frac{1}{2} \cdot 2 = 1$$

resonant factor

AR charge

Comparison: Experiment and our Theory (Two-channel model)



D. C. O., et. al. Phys. Rev. Research 5, 033176 (2023)

Exp: U. Thupakula, et. al., Phys. Rev. Lett. 128, 247001 (2022)

Conclusion

1) Full Counting Statistics (FCS) provides unprecedented insight into transport of junctions containing YSR states

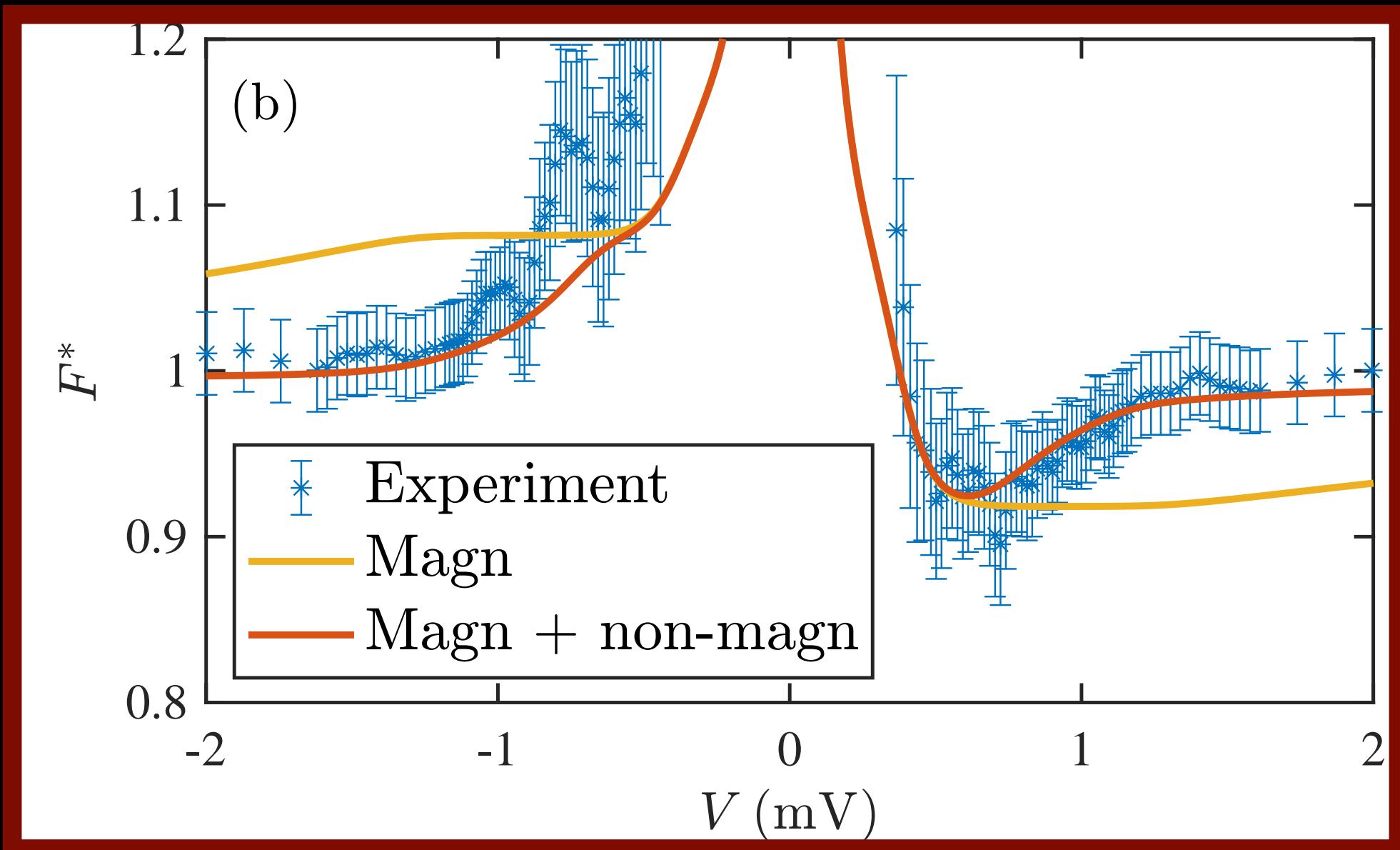
- Unveils competition between QP current and **resonant** AR in NS case

2) FCS calculations are needed to fully understand measurements

- Fano factor misleading as $1 = 2 \cdot 1/2 \neq 1$

D. C. O., et. al. Phys. Rev. Research 5, 033176 (2023)





$$1 = 2 \cdot \frac{1}{2}$$

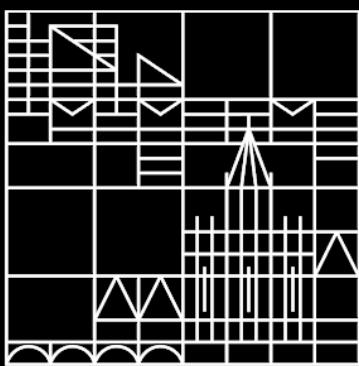


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david.ohnmacht@uni-konstanz.de

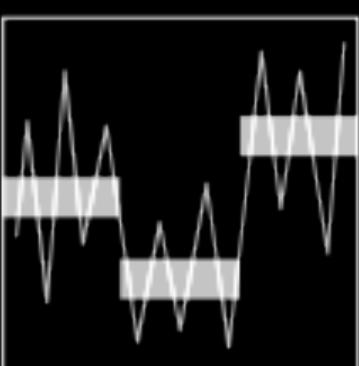
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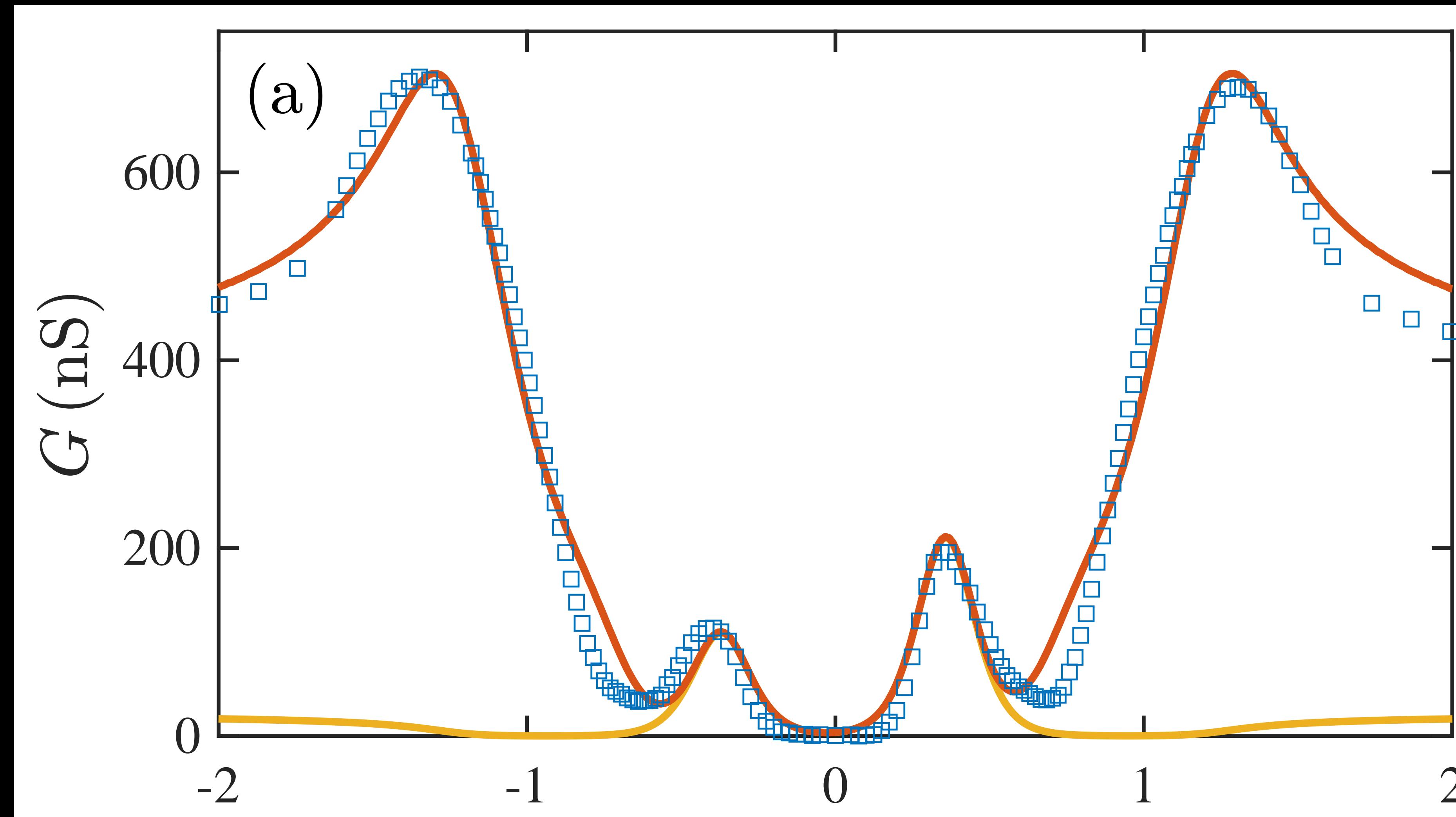


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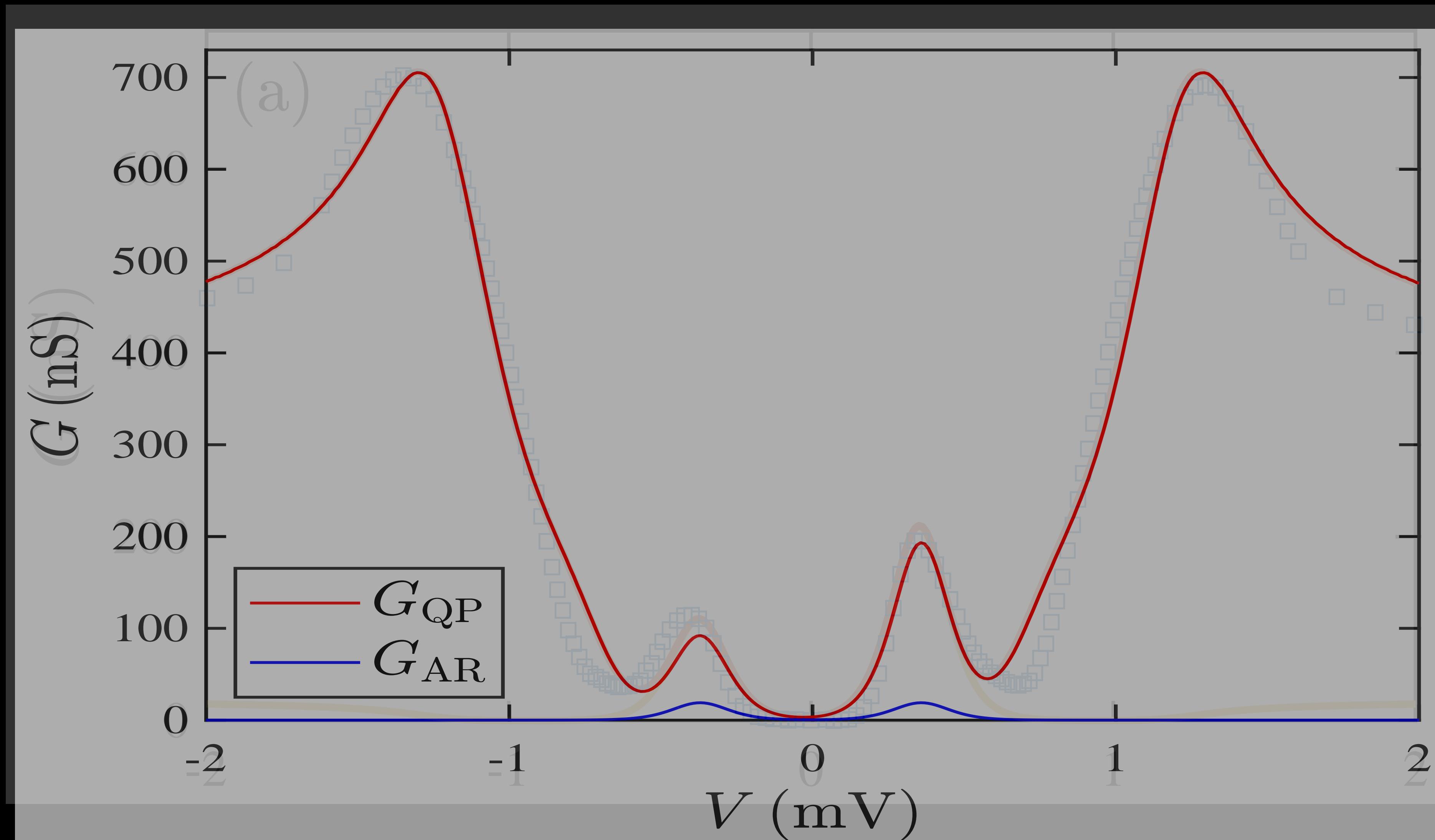


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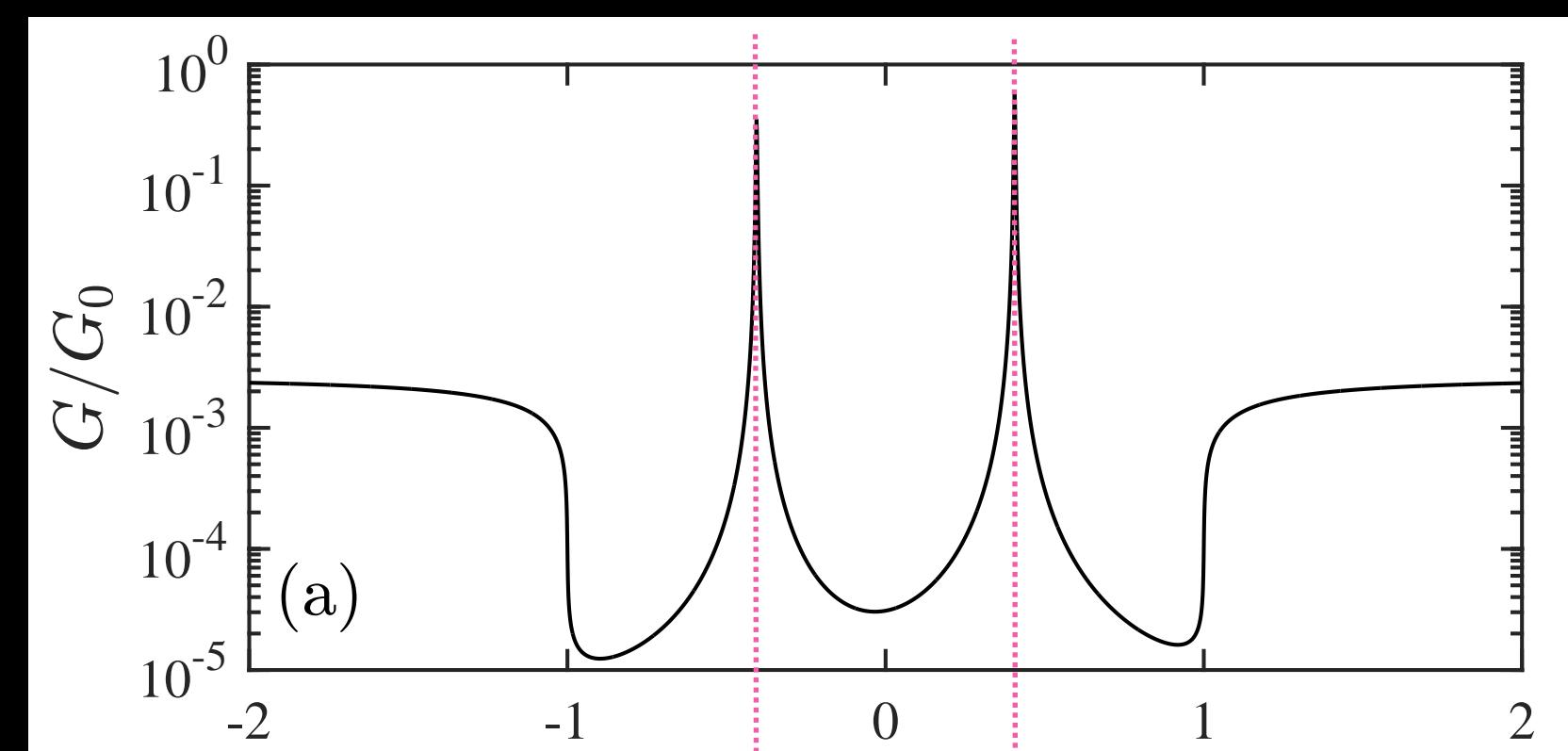
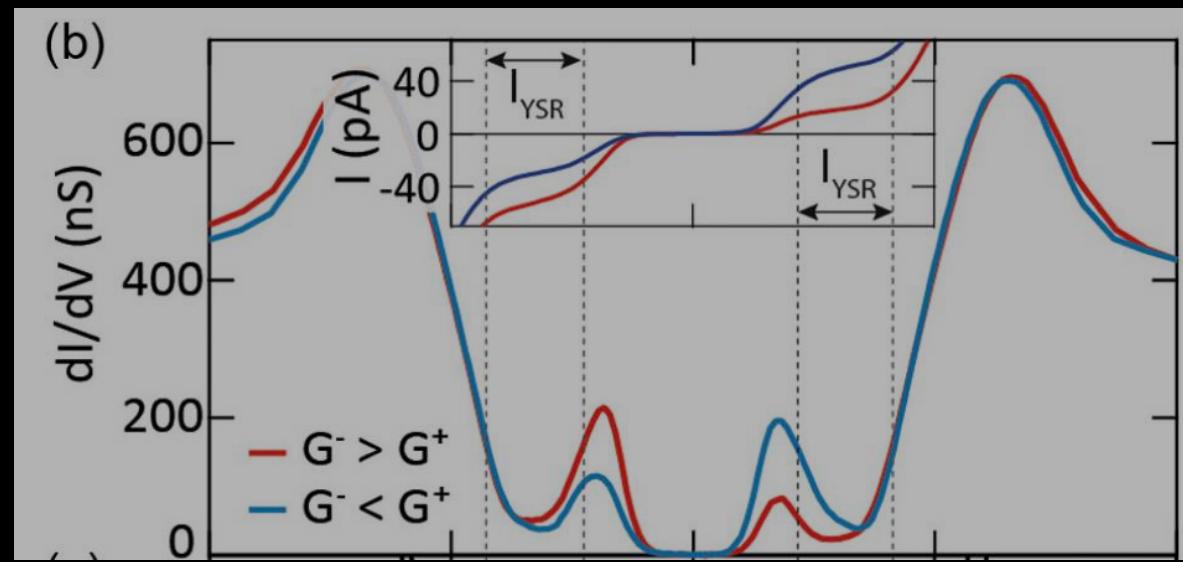
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Theoretical curves - Changing Γ_t

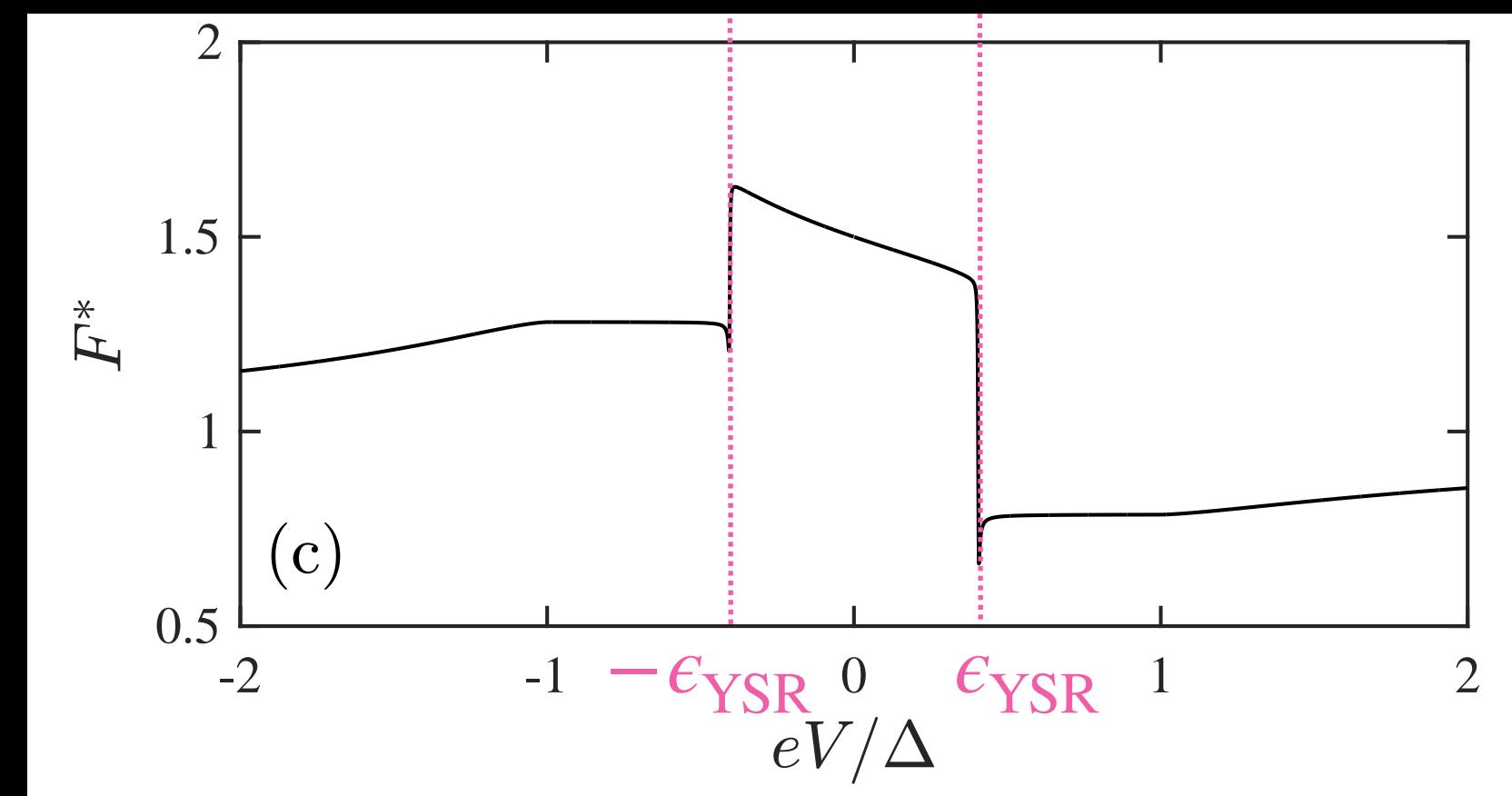
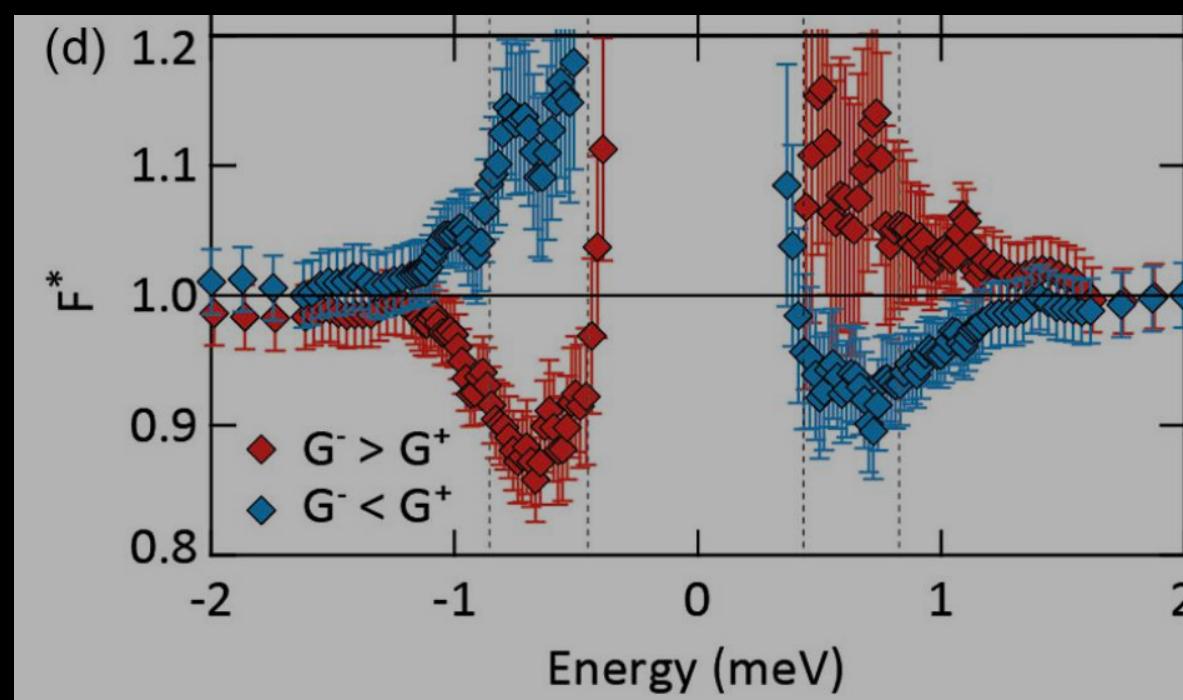
Experiment: conductance



$$\epsilon_{YSR} = 0.48\Delta$$

$$\Gamma_t = 0.1\Delta \Rightarrow T_{E=0} = 0.002$$

Experiment: Fano factor

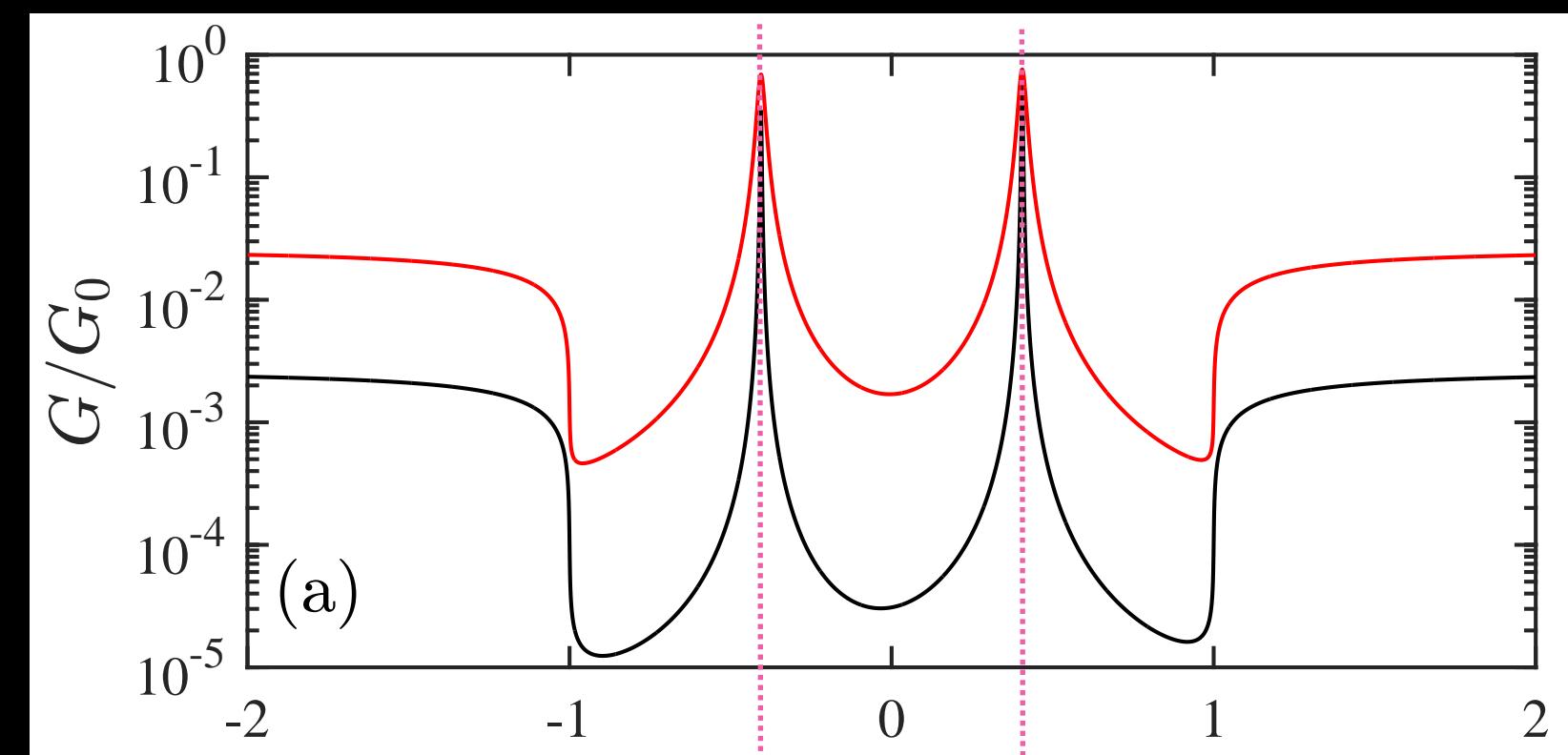
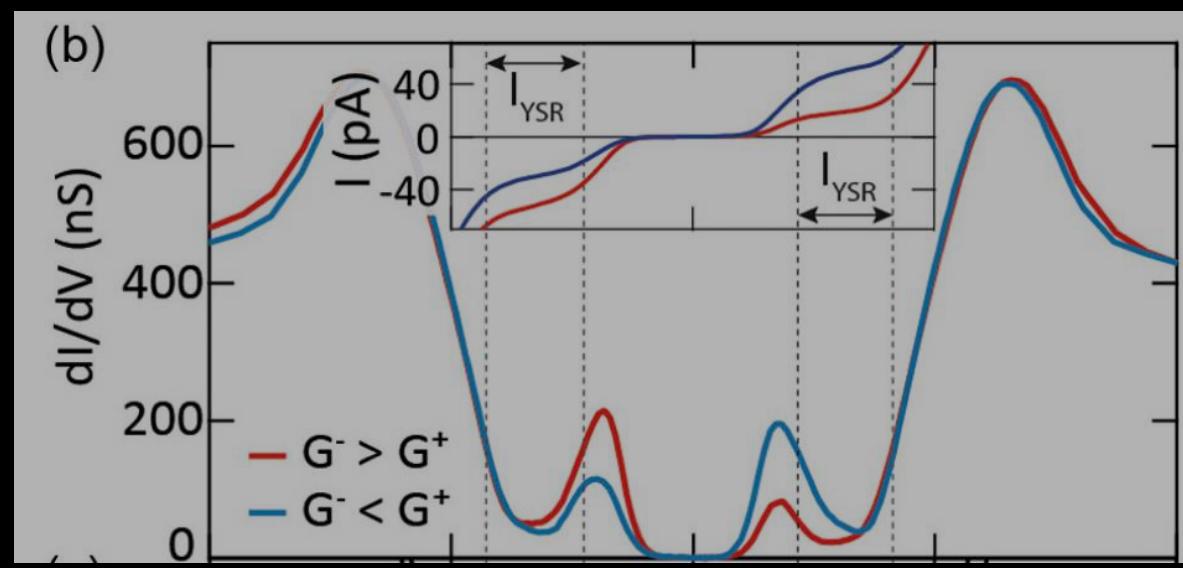


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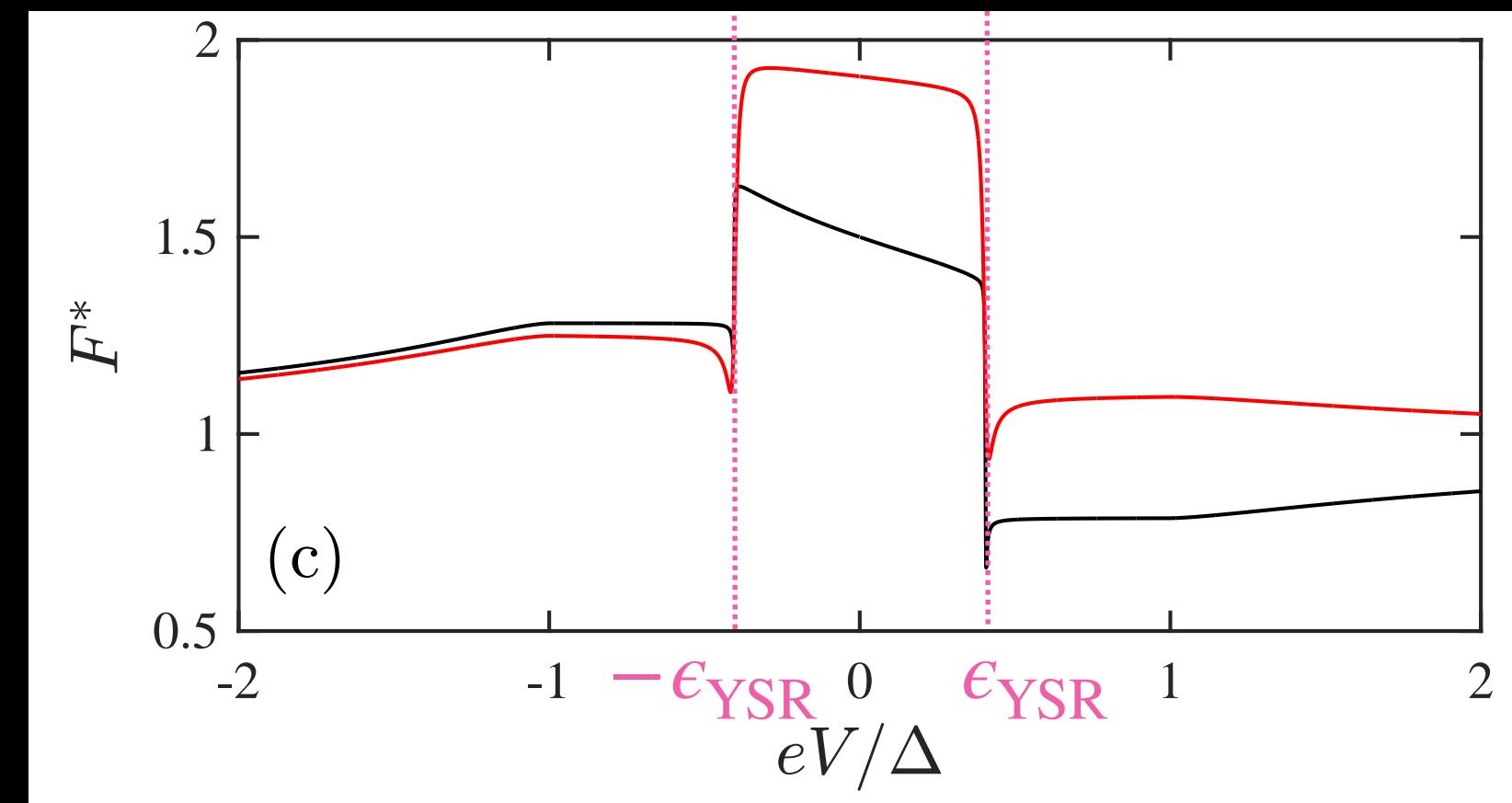
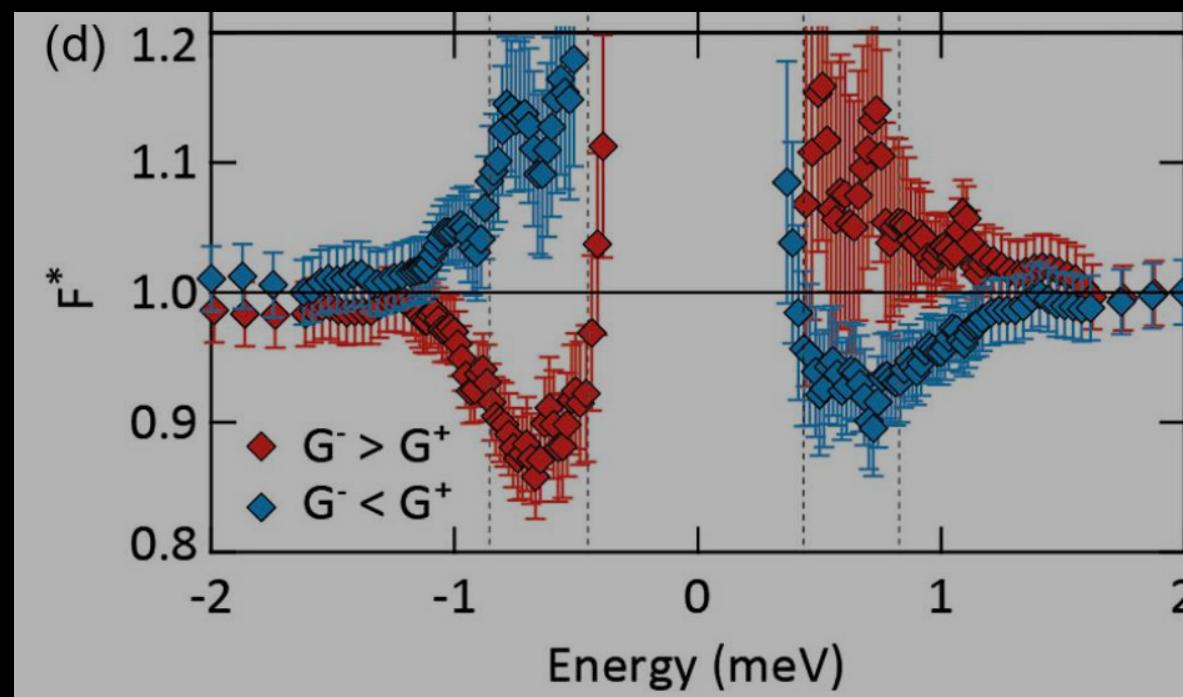
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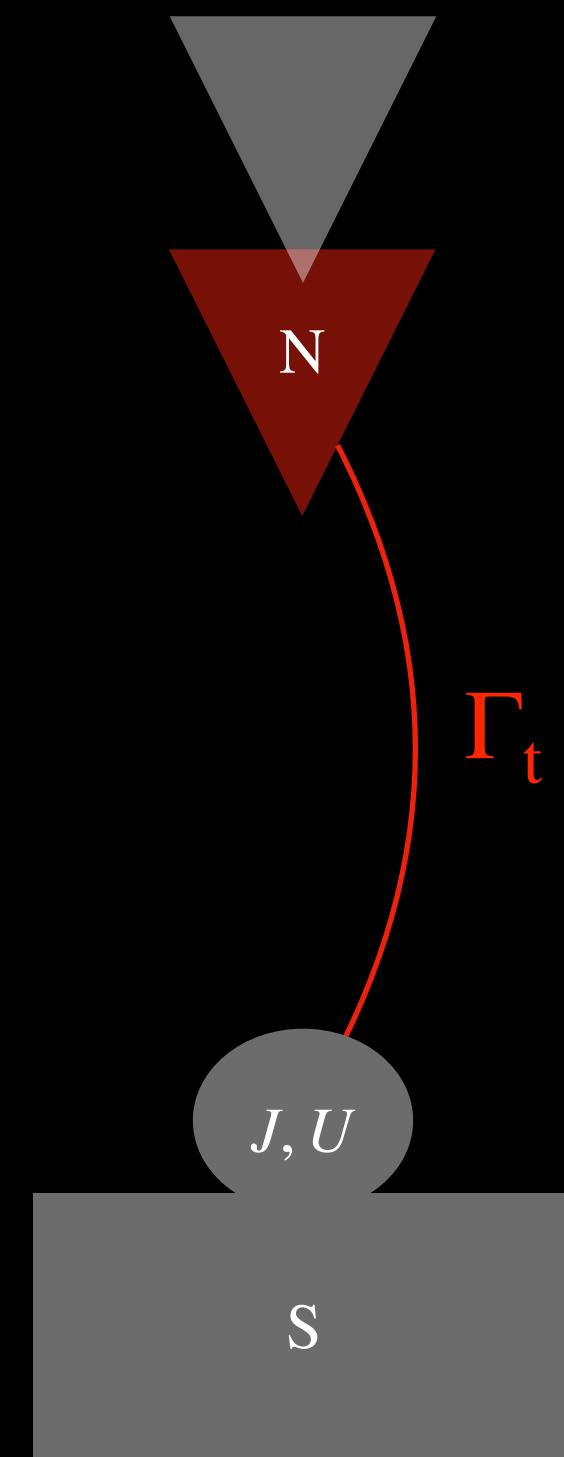
Width $\propto \Gamma_t \rightarrow$ Lifetime
can't be probed by G

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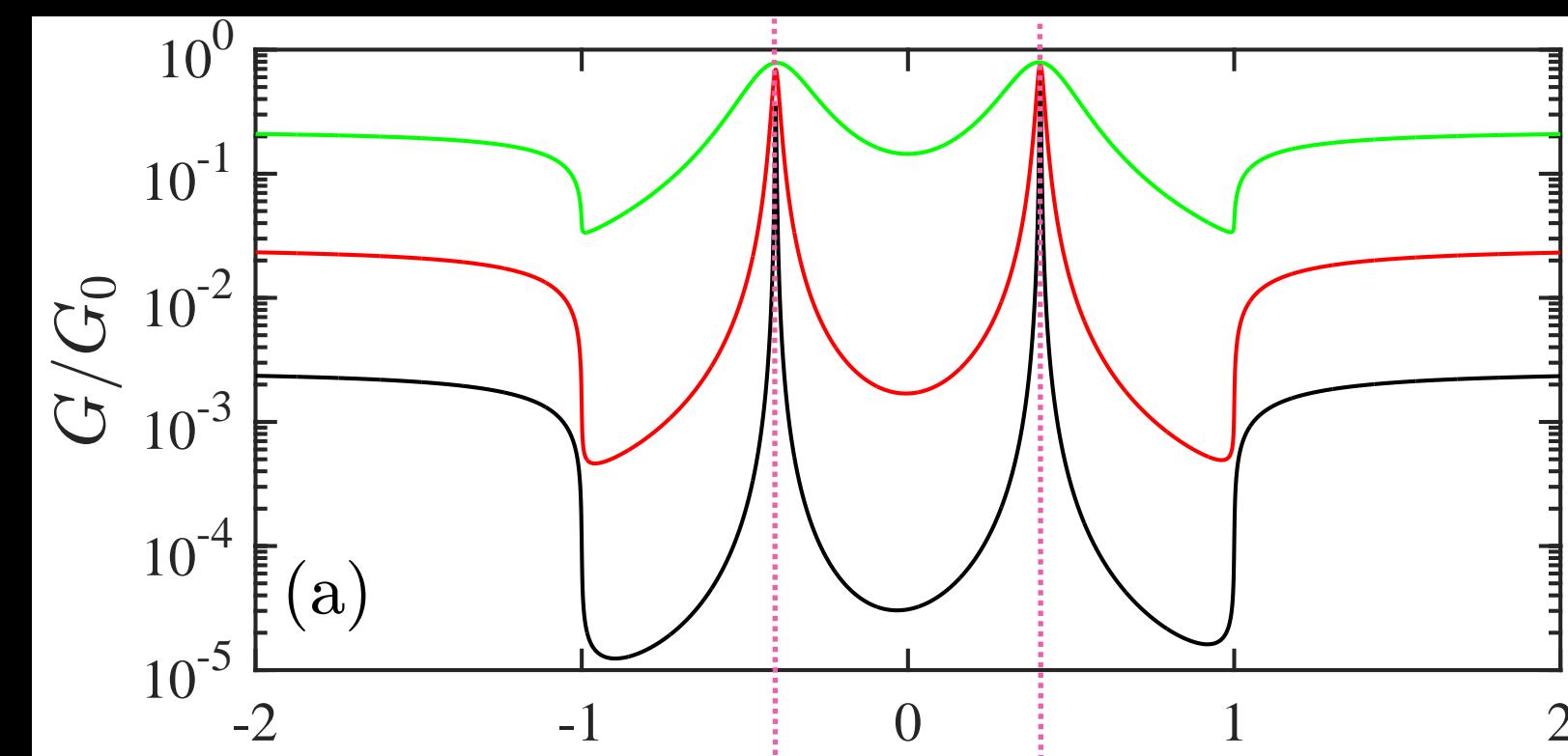
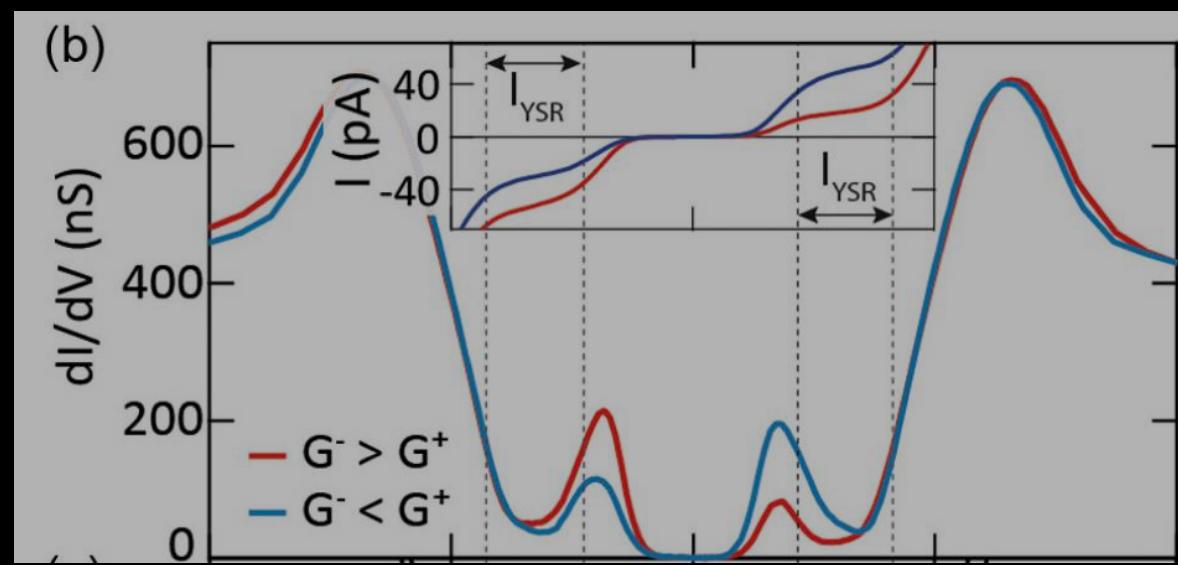


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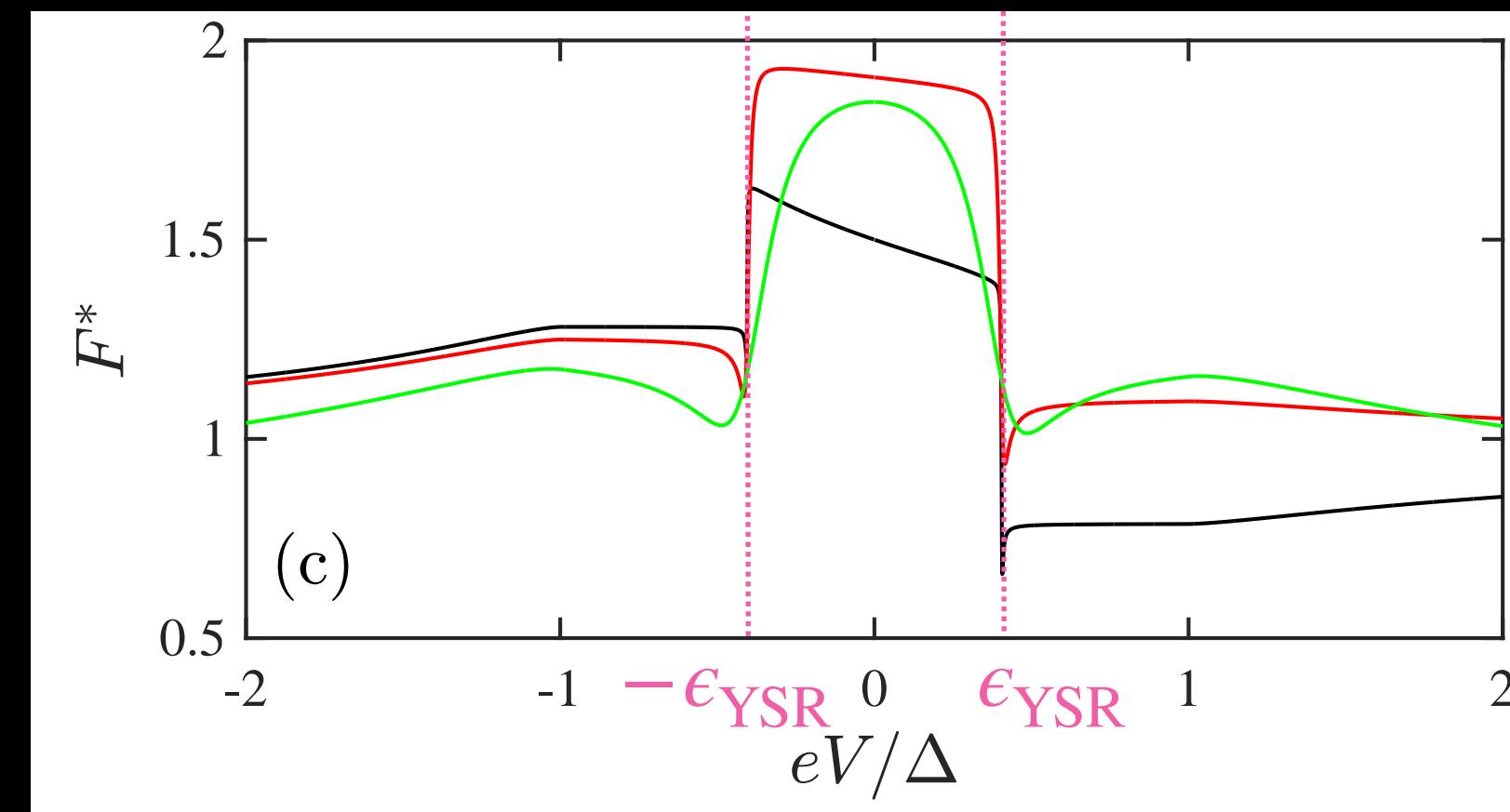
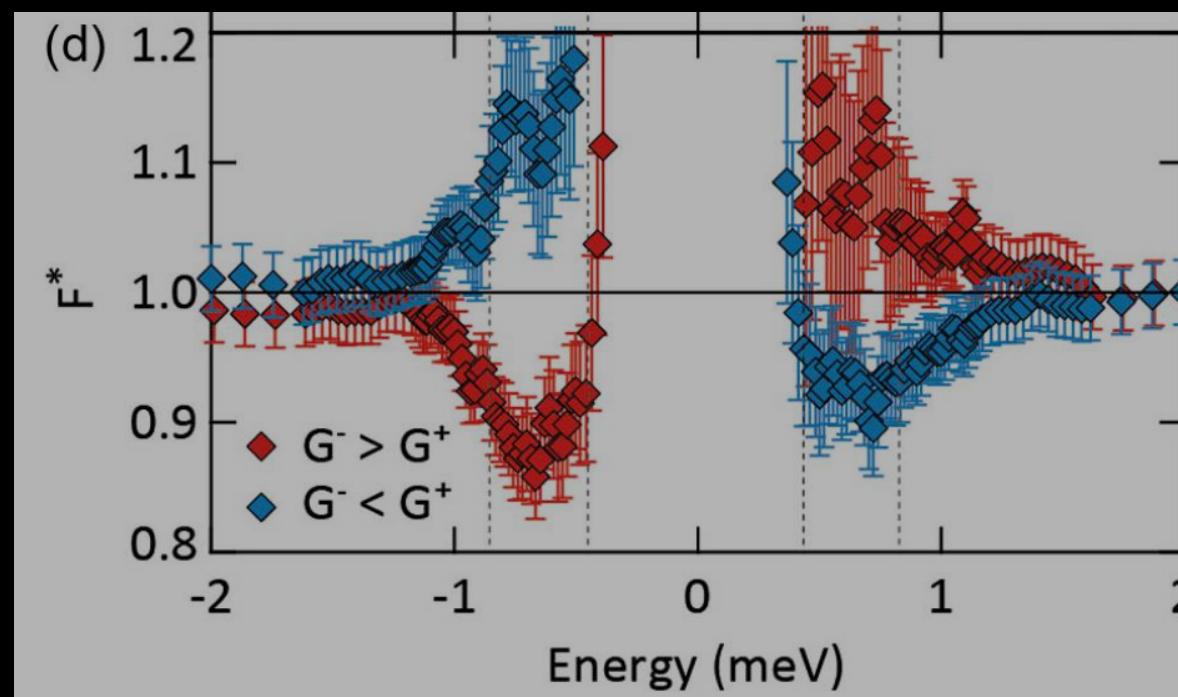
$$\epsilon_{YSR} = 0.48\Delta$$

$$\Gamma_t = 0.1\Delta \Rightarrow T_{E=0} = 0.002$$

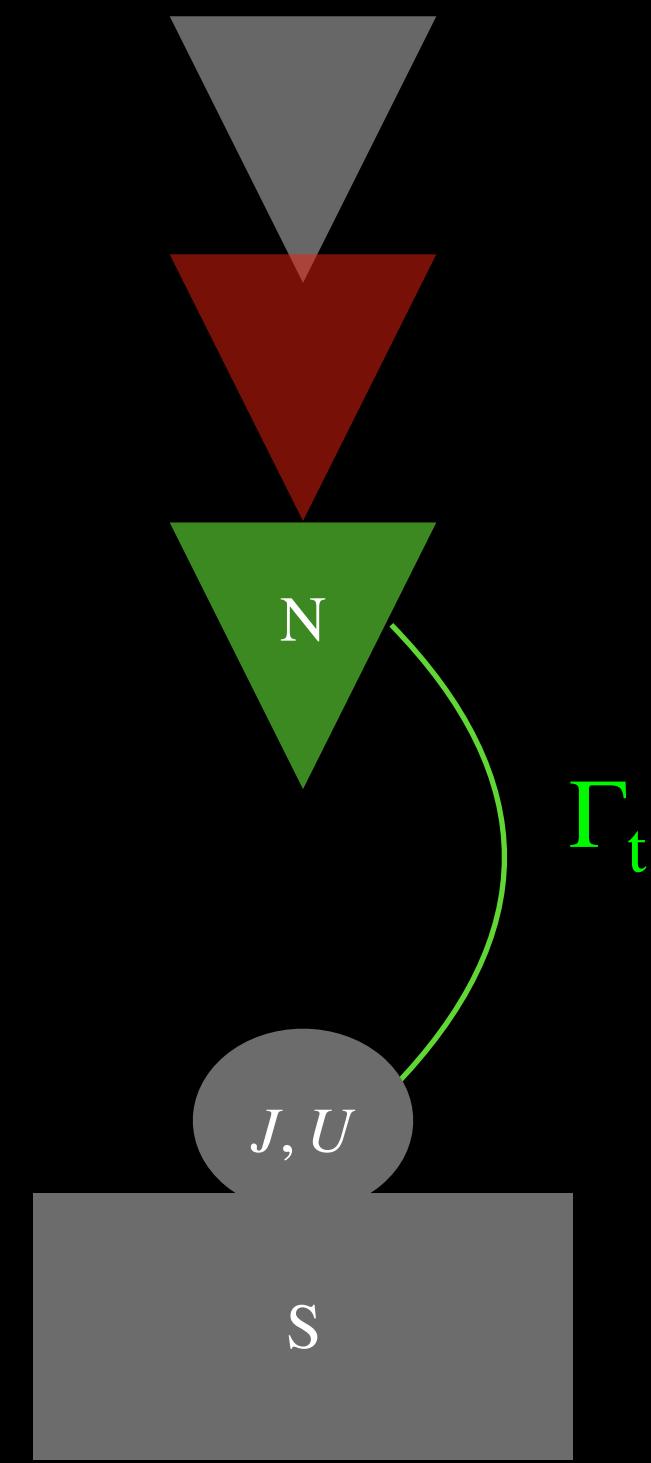
$$\Gamma_t = 1\Delta \Rightarrow T_{E=0} = 0.02$$

$$\Gamma_t = 10\Delta \Rightarrow T_{E=0} = 0.2$$

Experiment: Fano factor

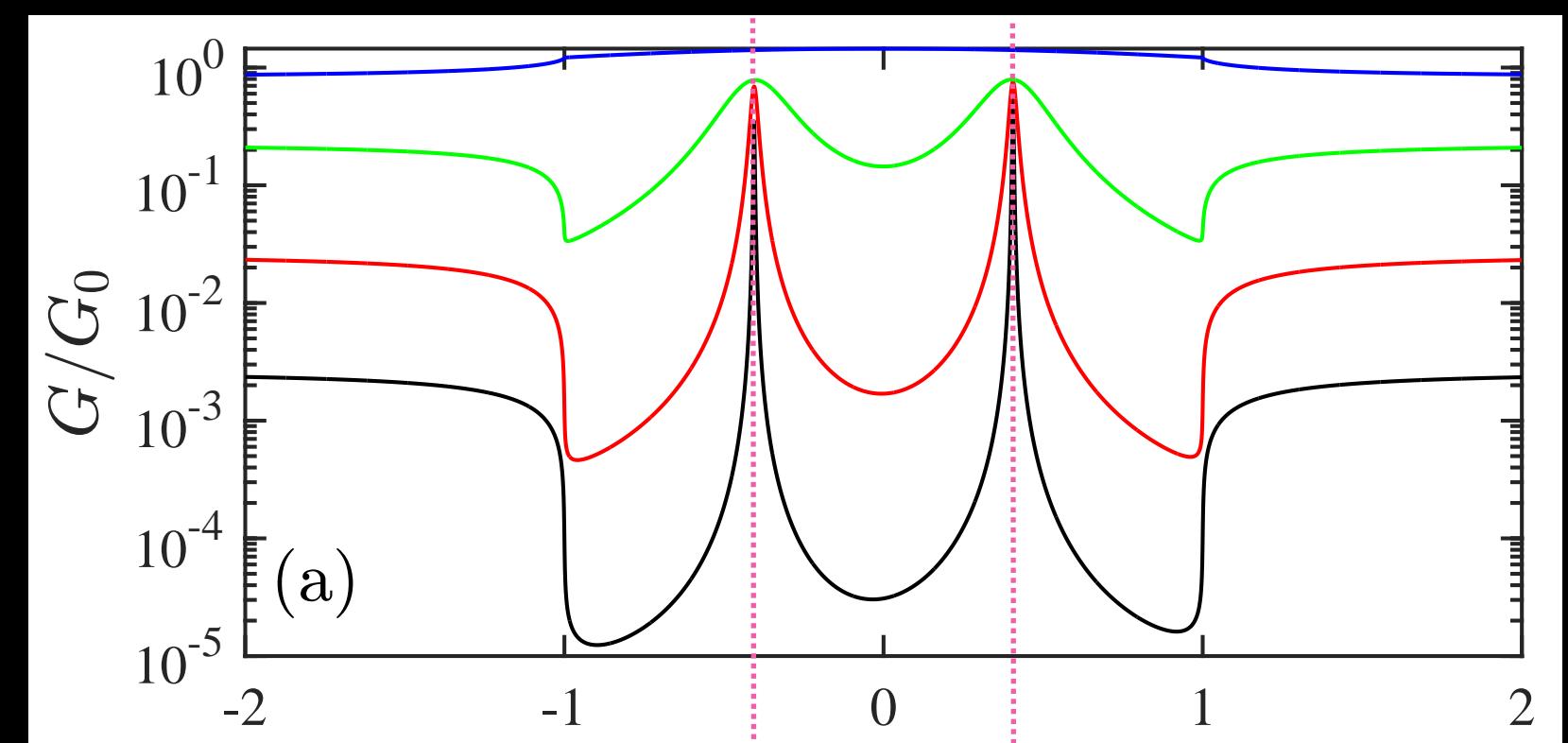
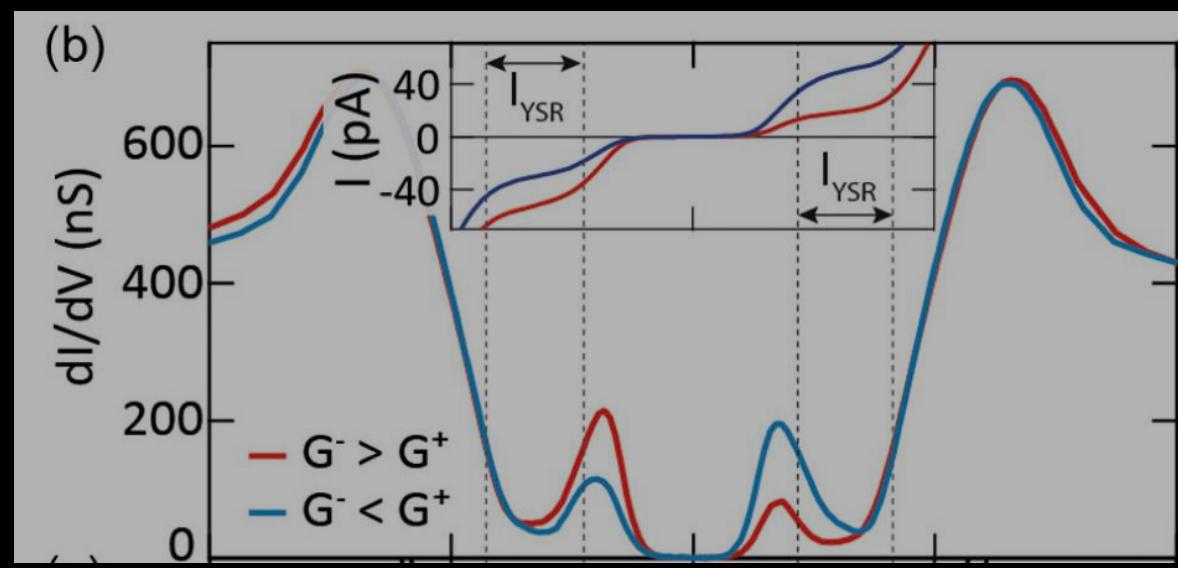


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Theoretical curves - Changing Γ_t

Experiment: conductance



Width $\propto \Gamma_t \rightarrow$ Lifetime
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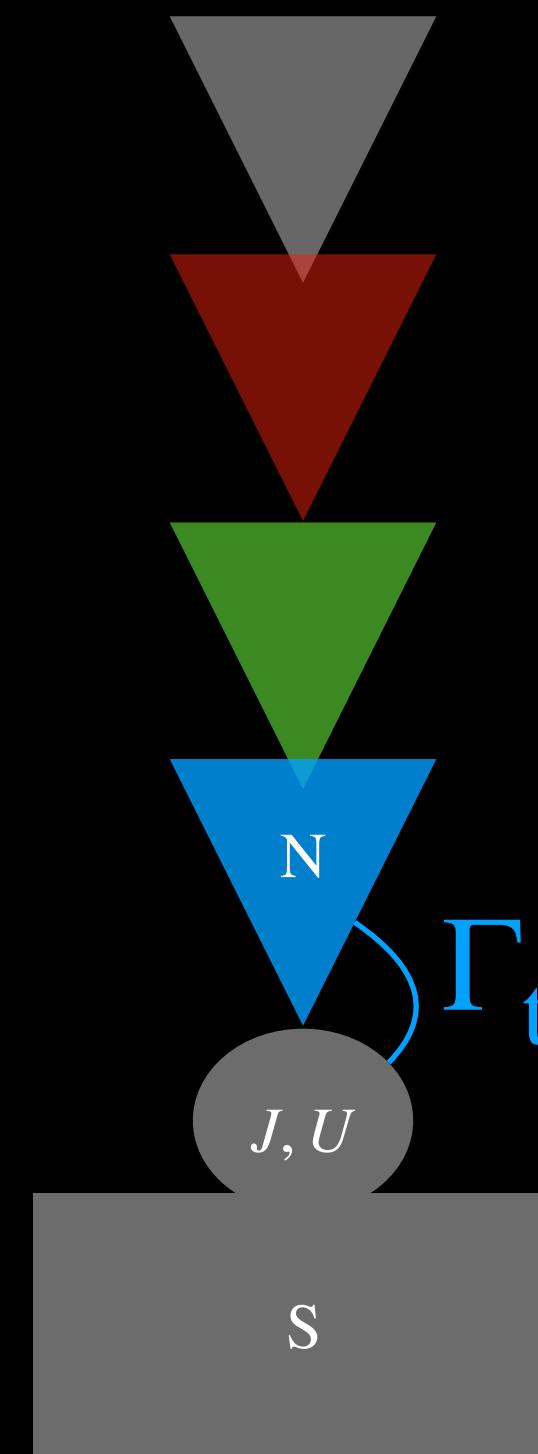
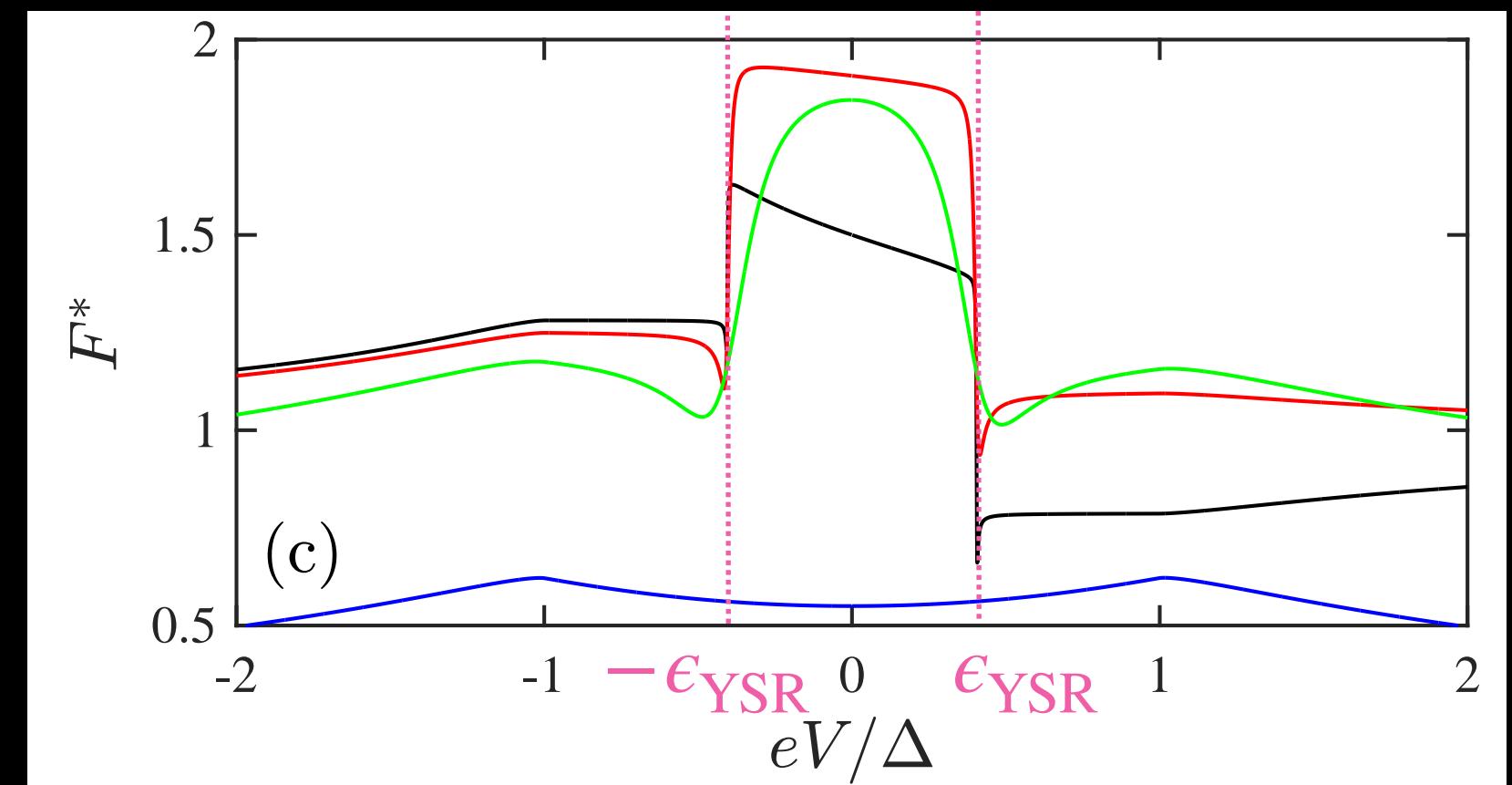
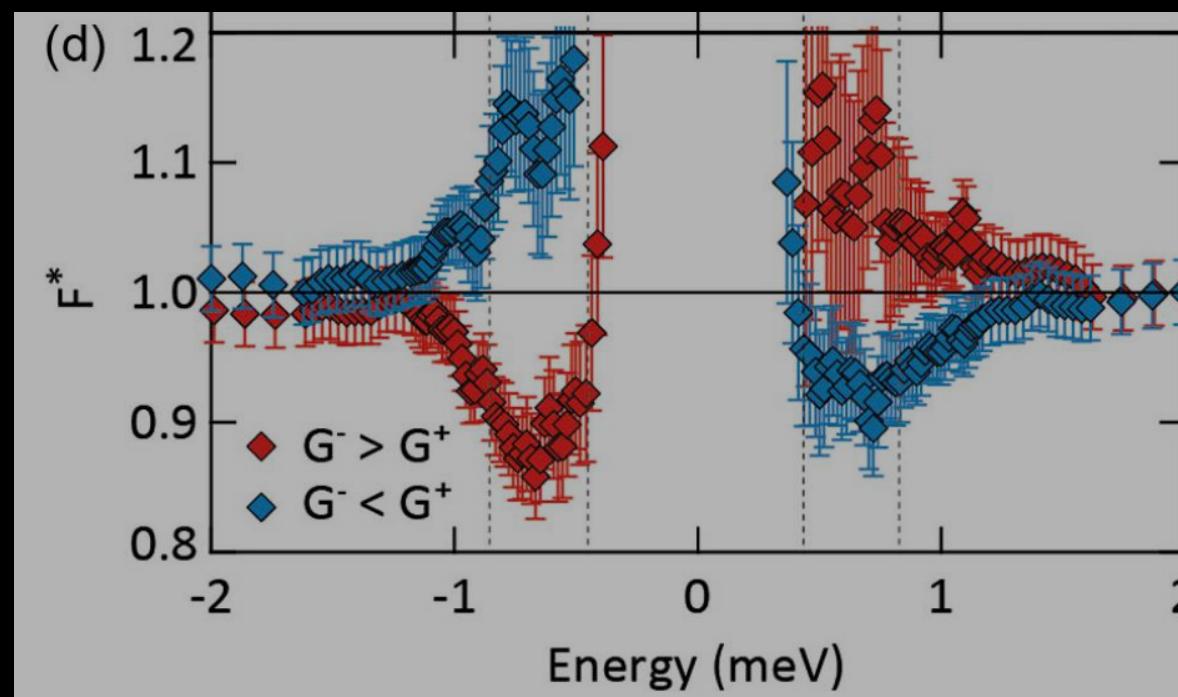
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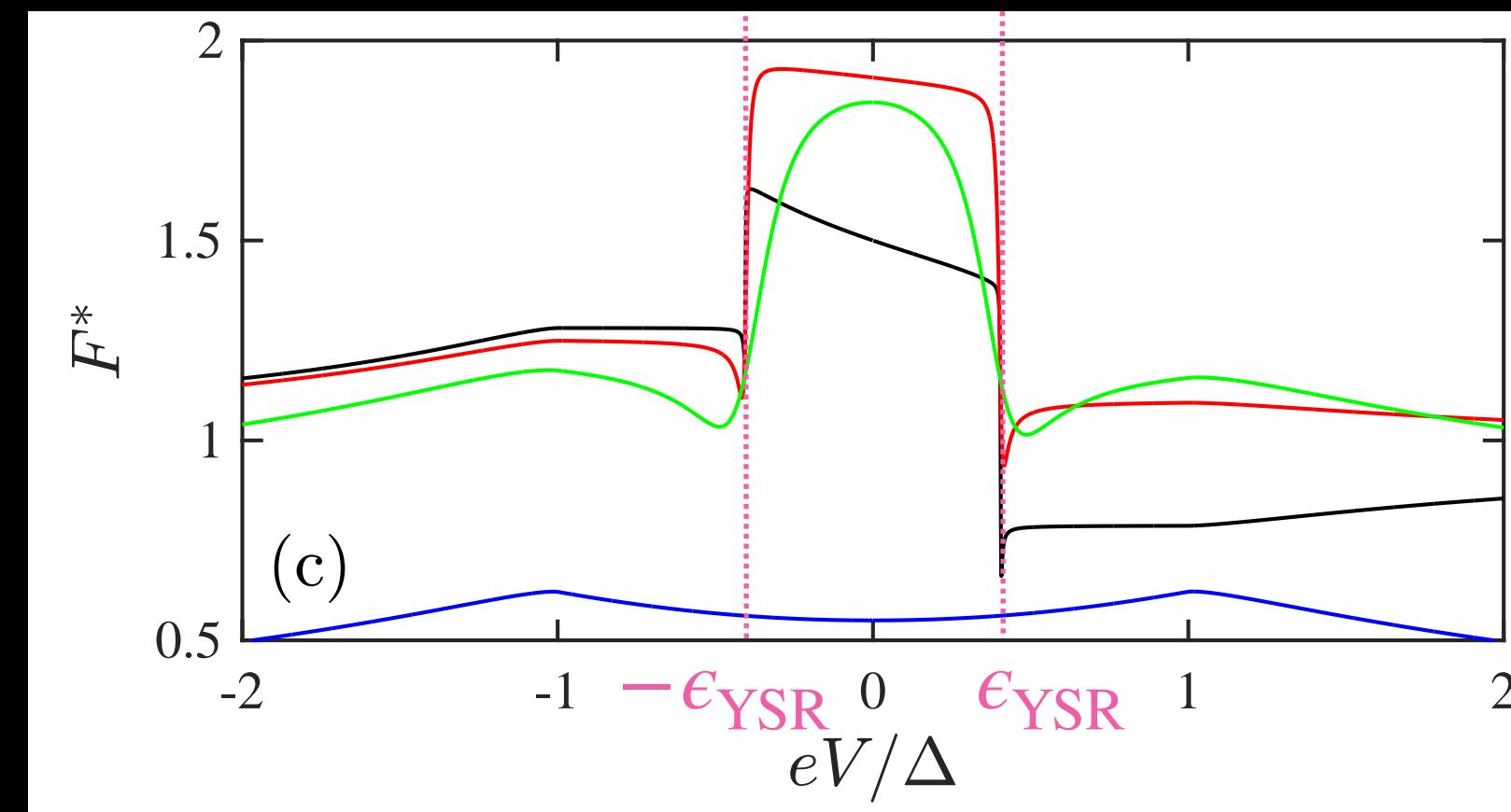
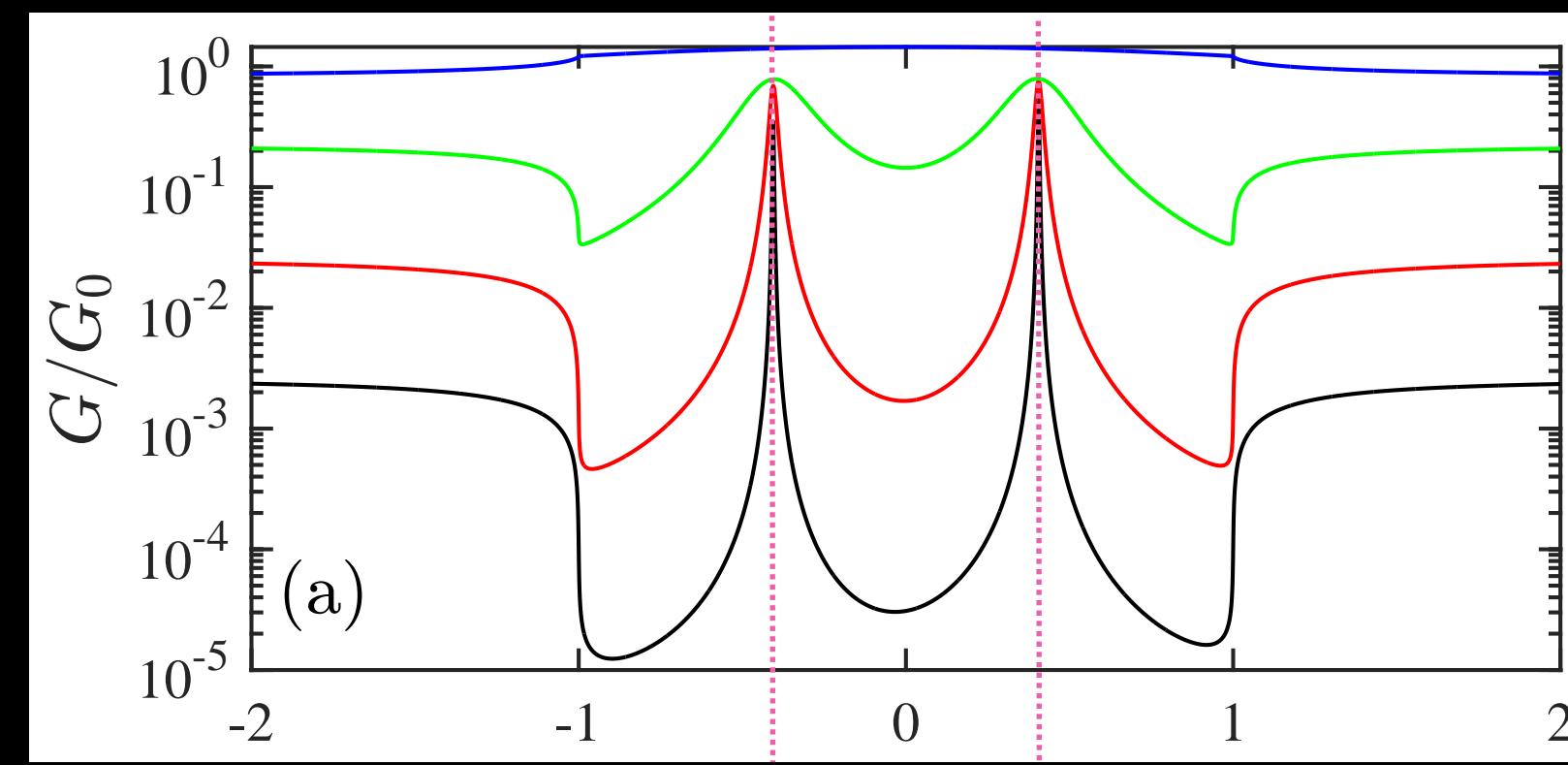
$$\Gamma_t = 100\Delta \Rightarrow T_{E=0} = 0.86$$

Experiment: Fano factor



$$\Gamma_S = 100, J = 60, U = 80, \eta = 0.001$$

Charge resolved conductance

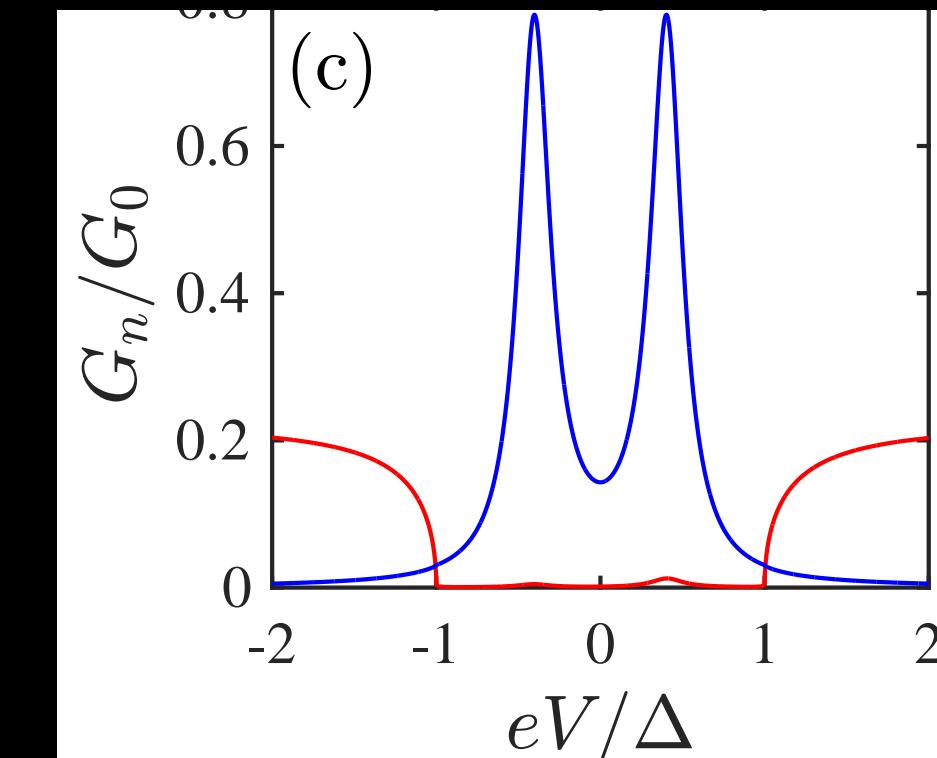
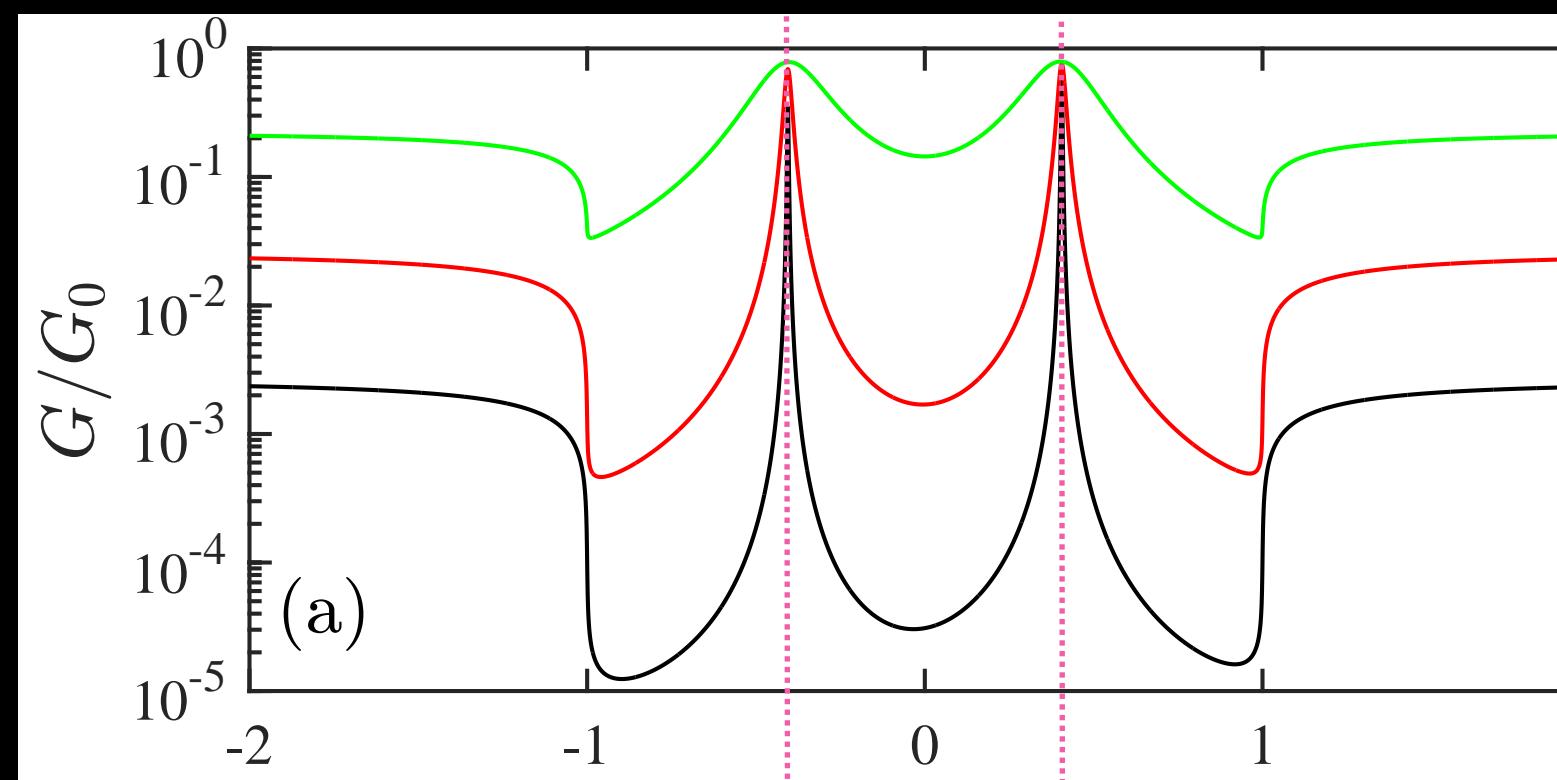
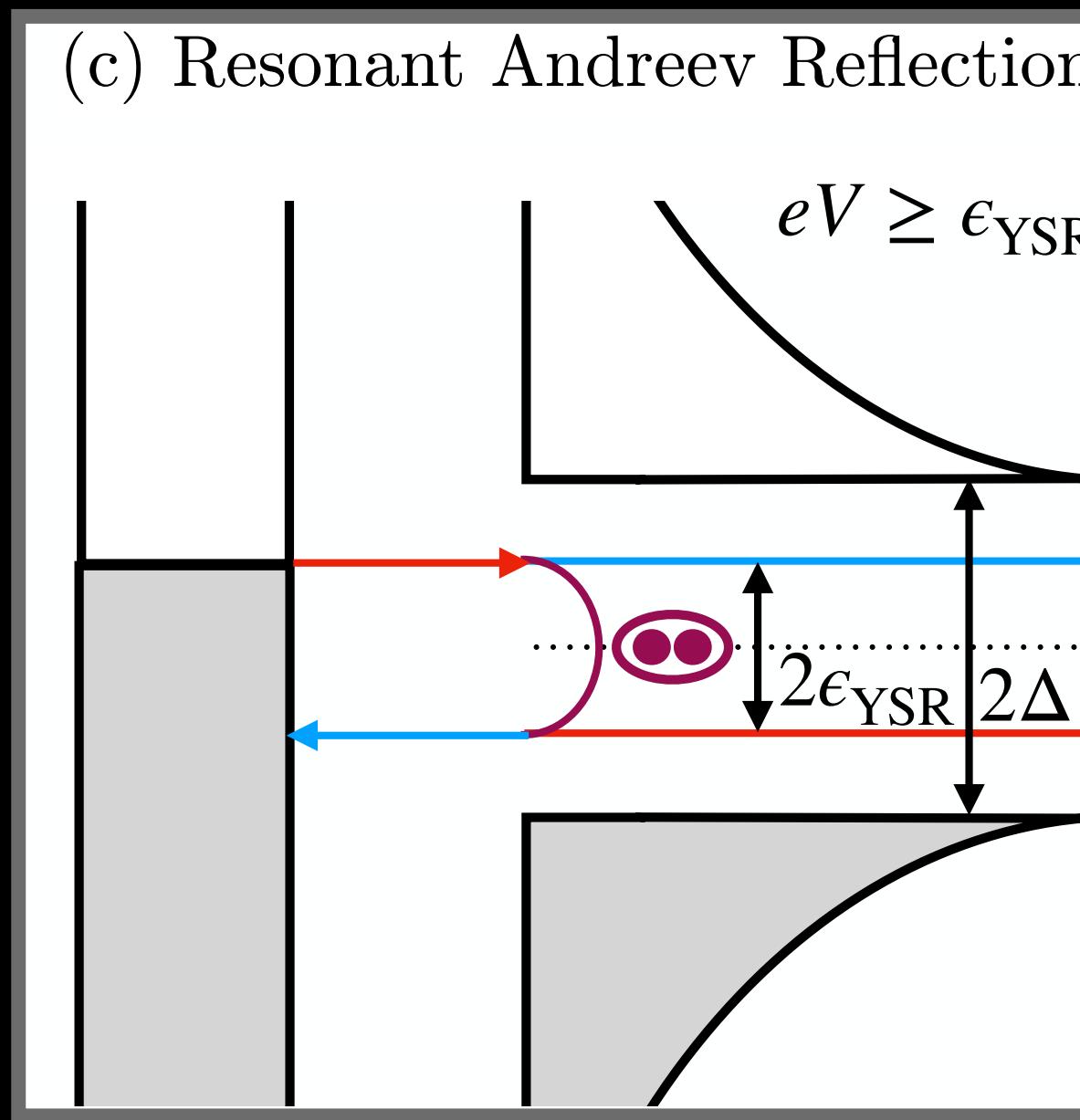


$\Gamma_S = 100, J = 60, U = 80, \eta = 0.001$

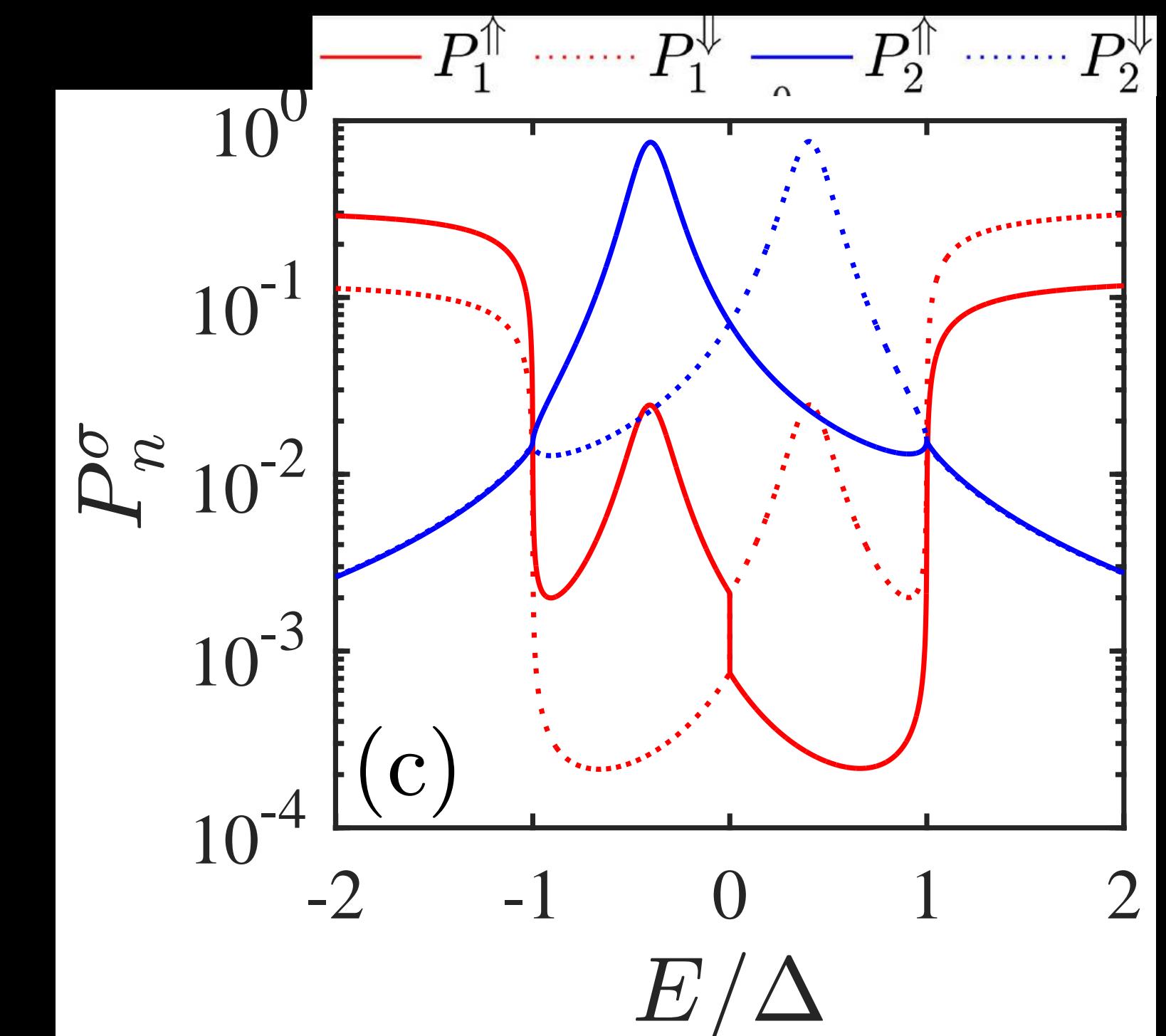
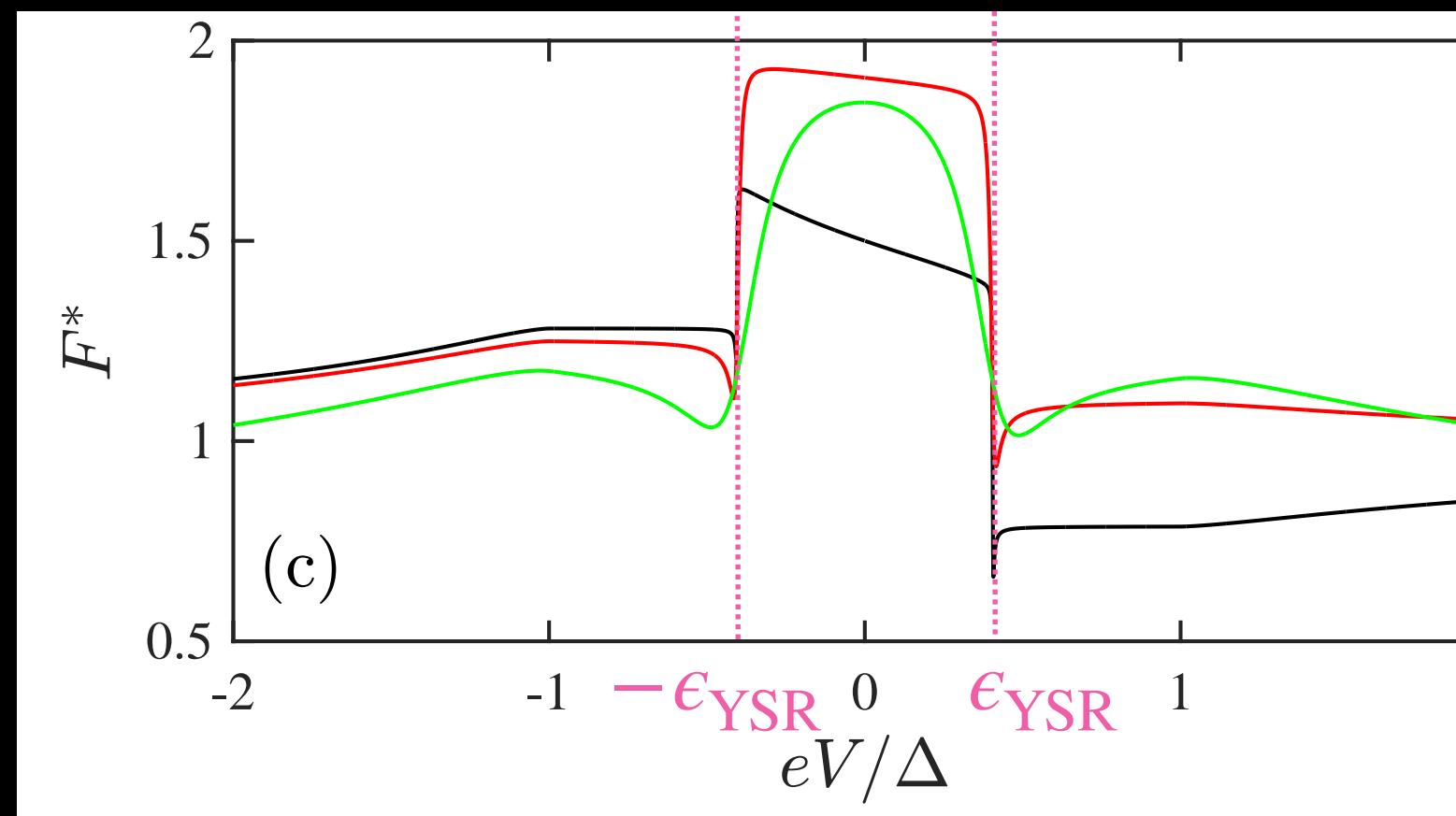
Charge resolved conductance

$$1 = 2 \cdot \frac{1}{2}$$

Charge
of AR Resonant
factor



$$c) \Gamma_t = 10\Delta$$



$$\Gamma_S = 100, J = 60, U = 80, \eta = 0.001$$