Thermodynamic uncertainty relation in superconducting junctions

David Christian Ohnmacht,
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3. September 2024

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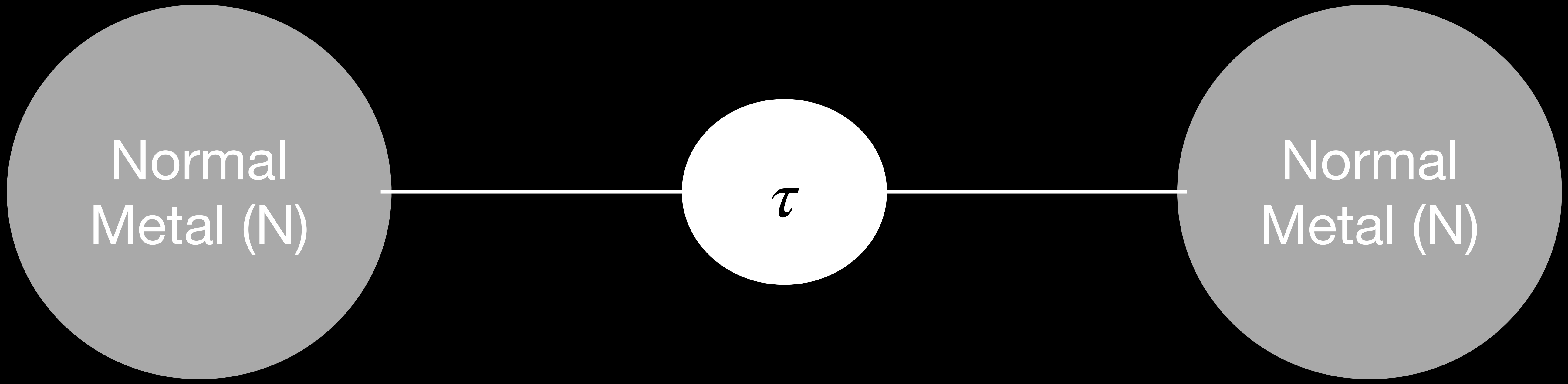
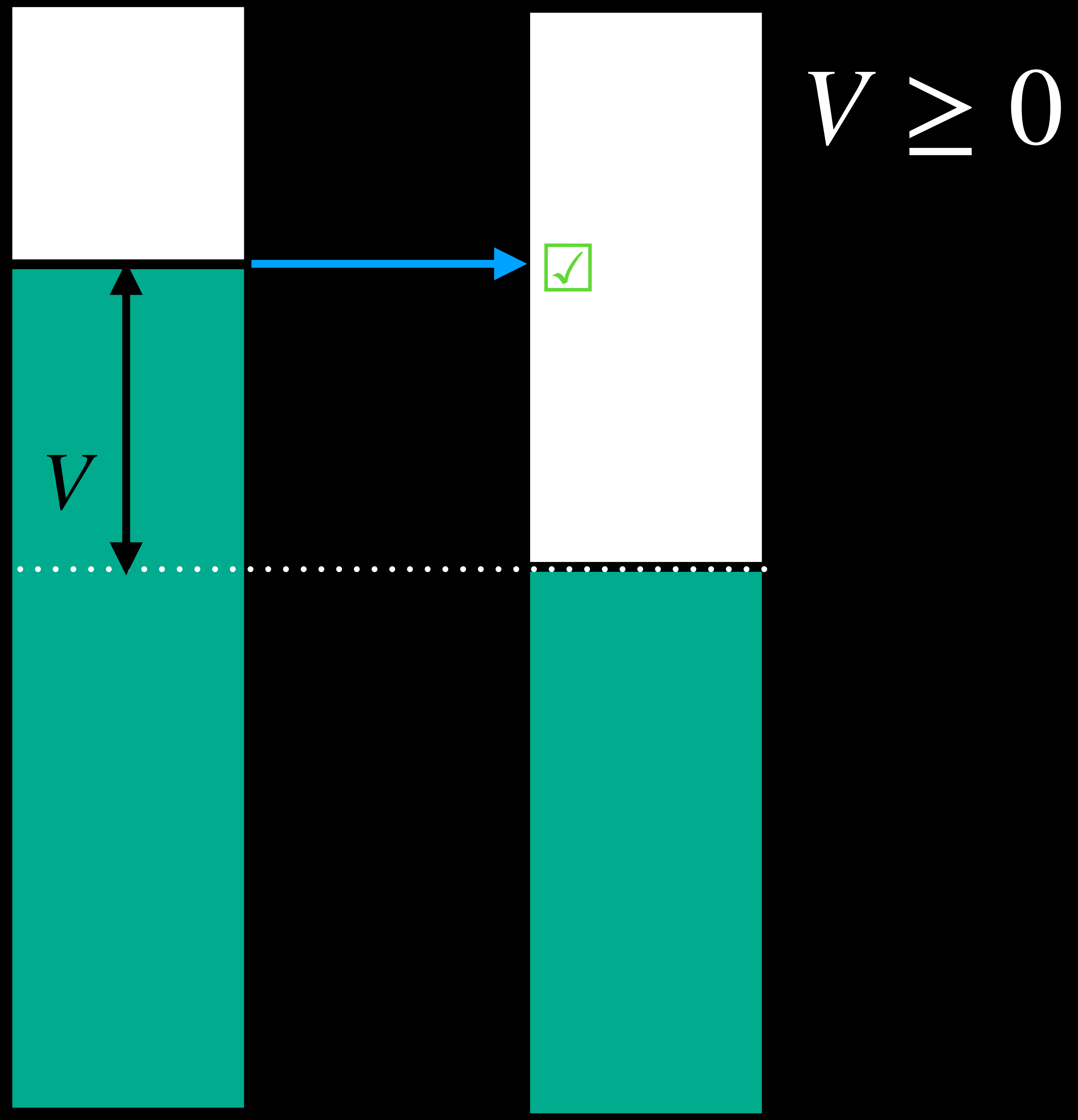
2) Breaking of TUR is rooted in competition of different transport processes

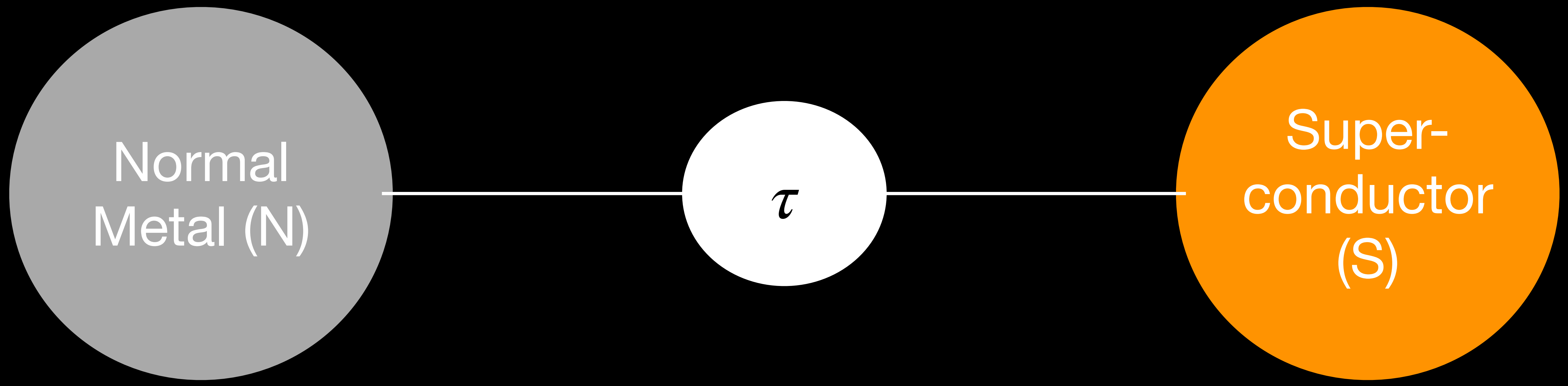
What’s special about superconducting junctions? (NN)

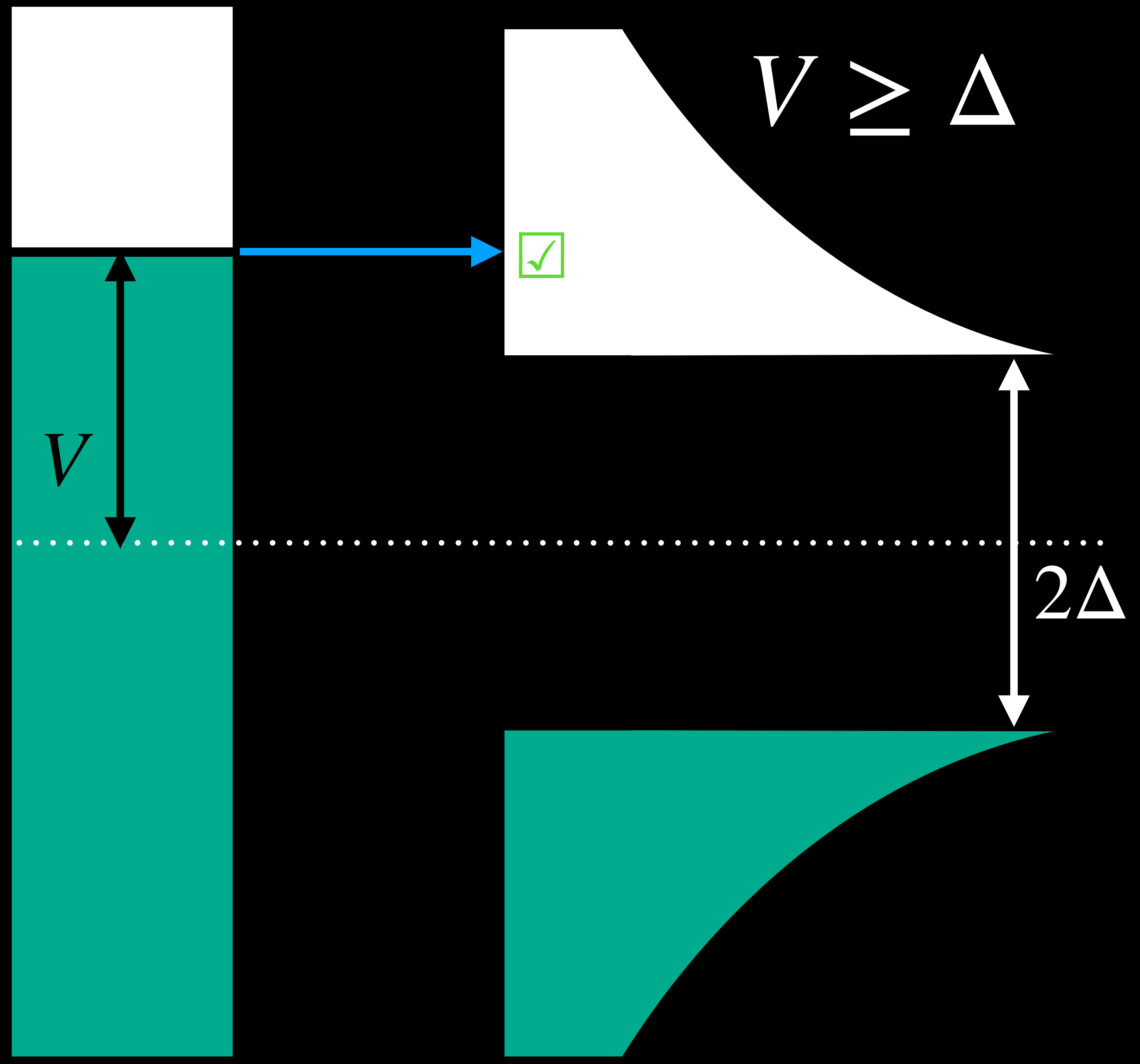




☑ Quasiparticle (QP) tunneling transferring 1 charge

What's special about superconducting junctions? (NS)





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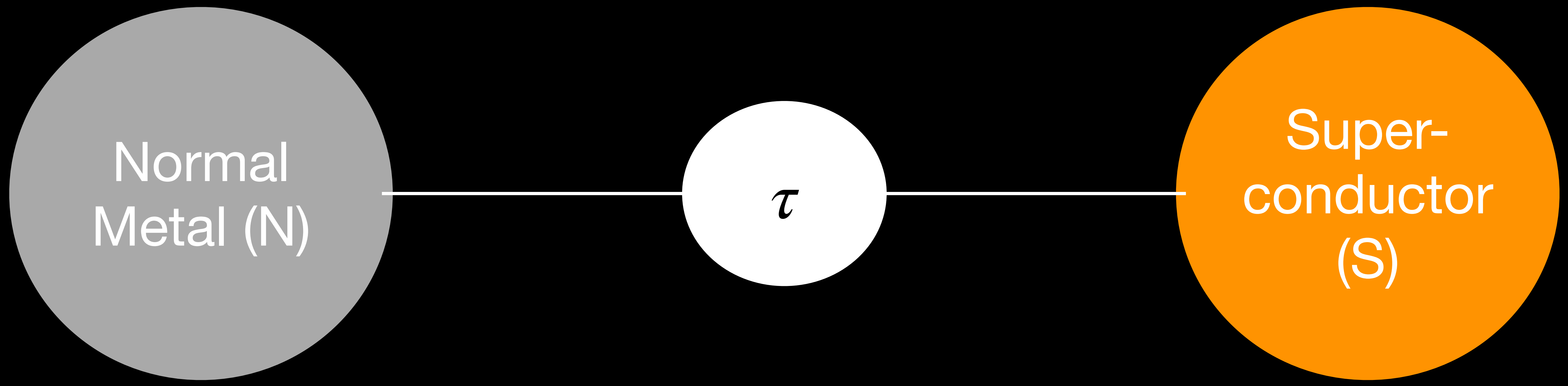
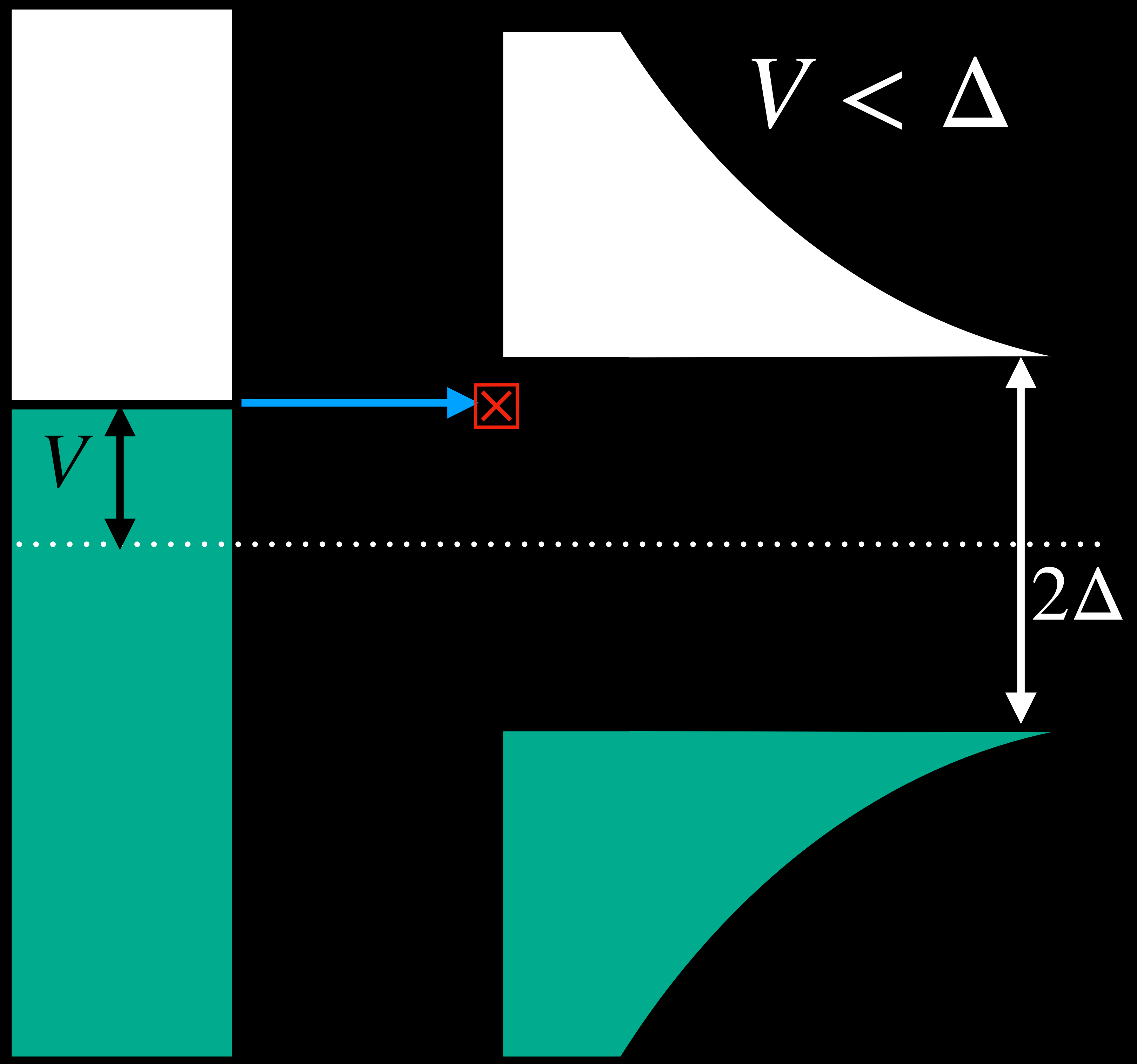
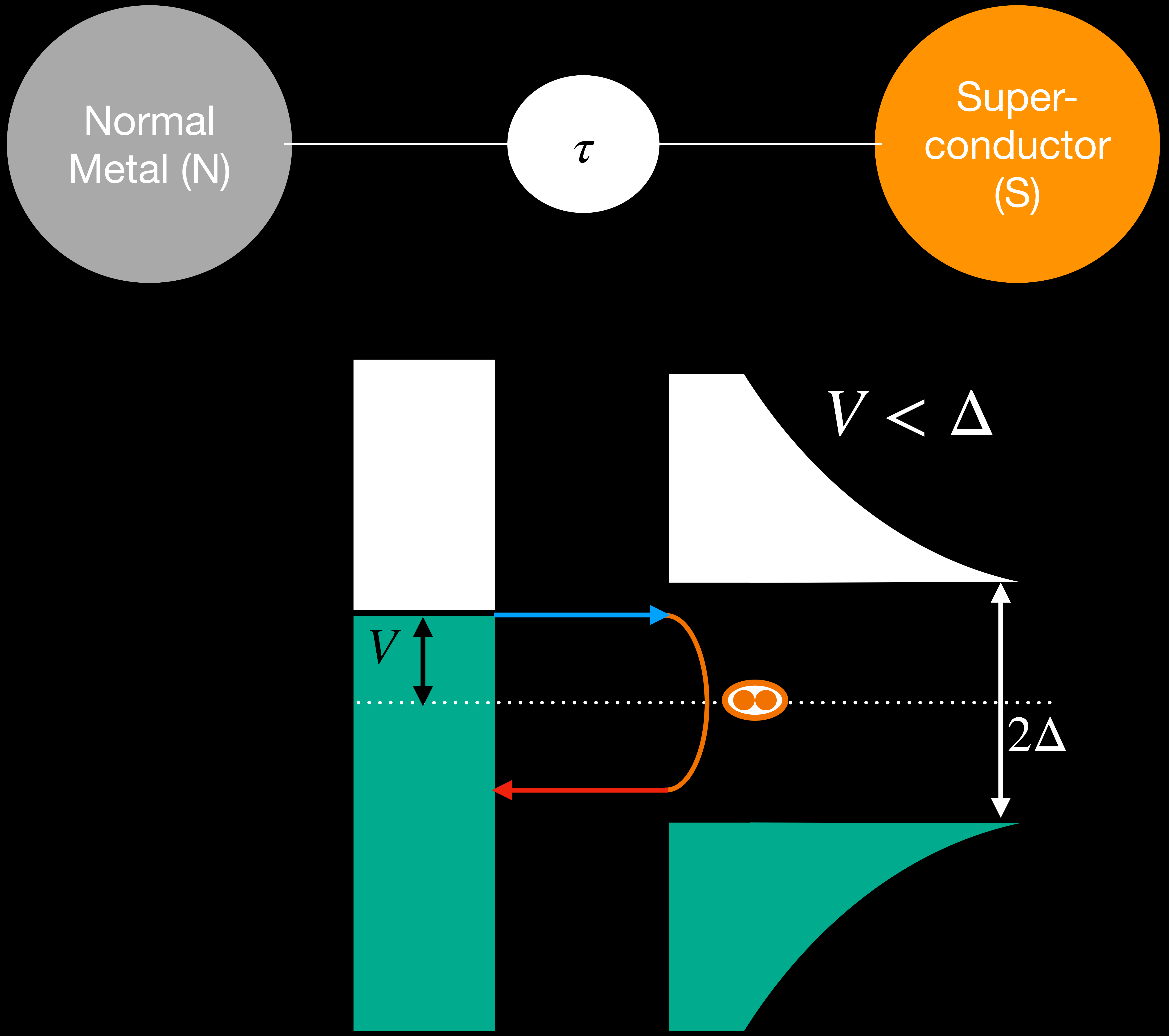




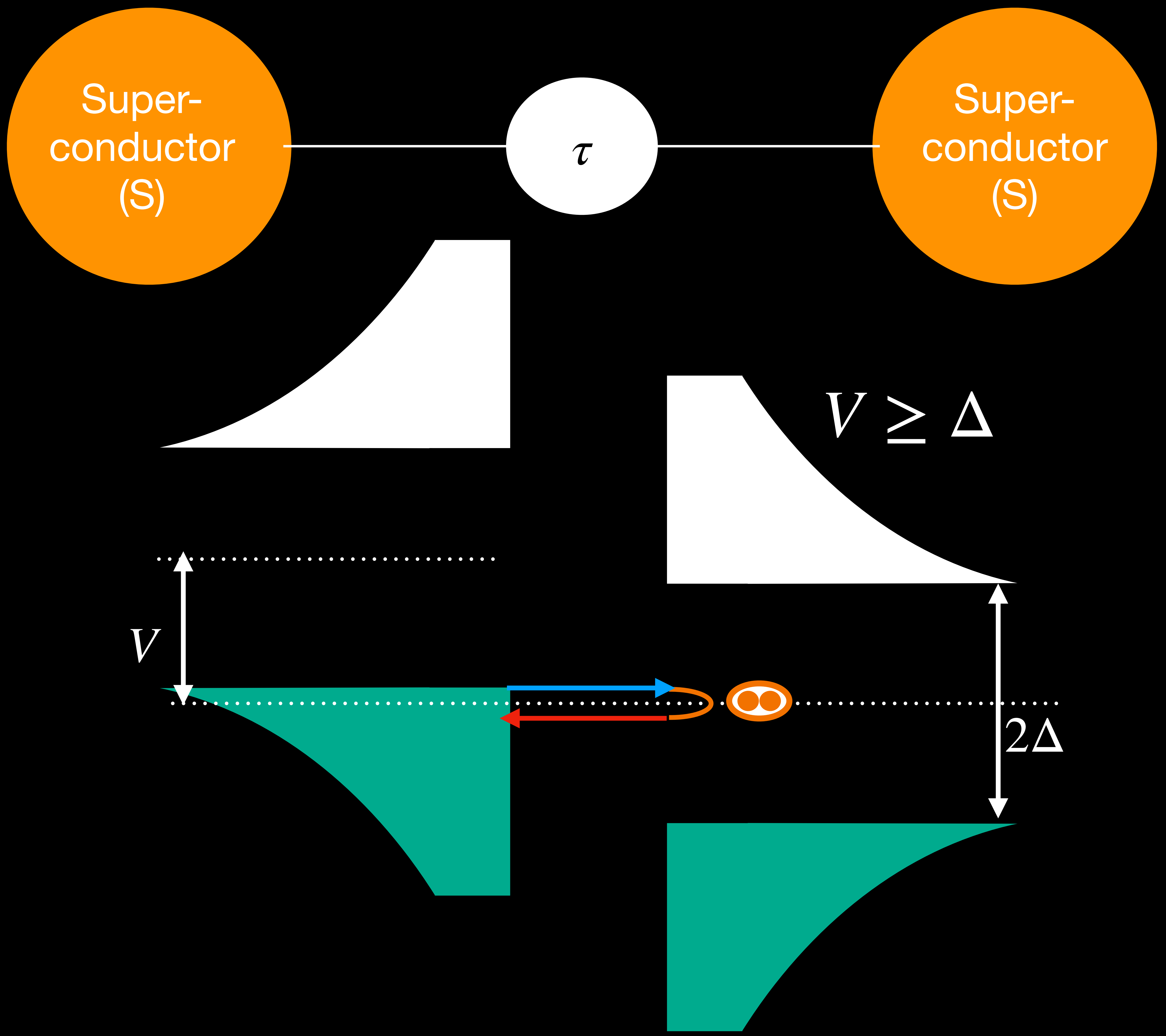
☒ No QP tunneling transferring 1 charge

What's special about superconducting junctions? (NS)



☑ Andreev reflection (AR) transferring 2 charges

What's special about superconducting junctions? (SS)



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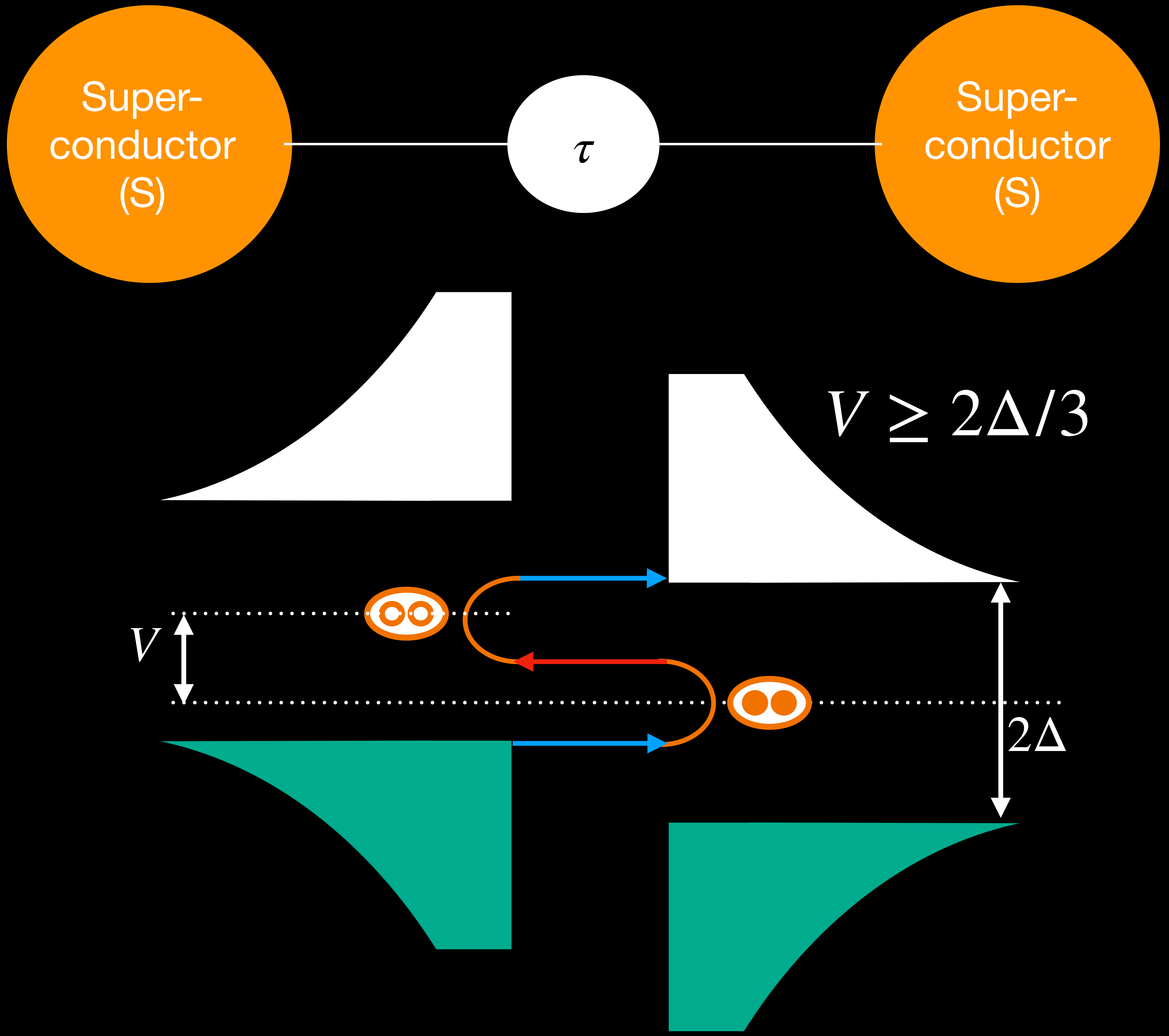

☑ Multiple Andreev reflection (MAR) transferring 3 charges

What is Full counting statistics (FCS)?

Theoretical framework computing (average) **current**, **shot noise** **(variance of current)** **and** **charge resolved currents**

L.S. Levitov and G.B. Lesovik, JETP Lett. **58**, 230 (1993)
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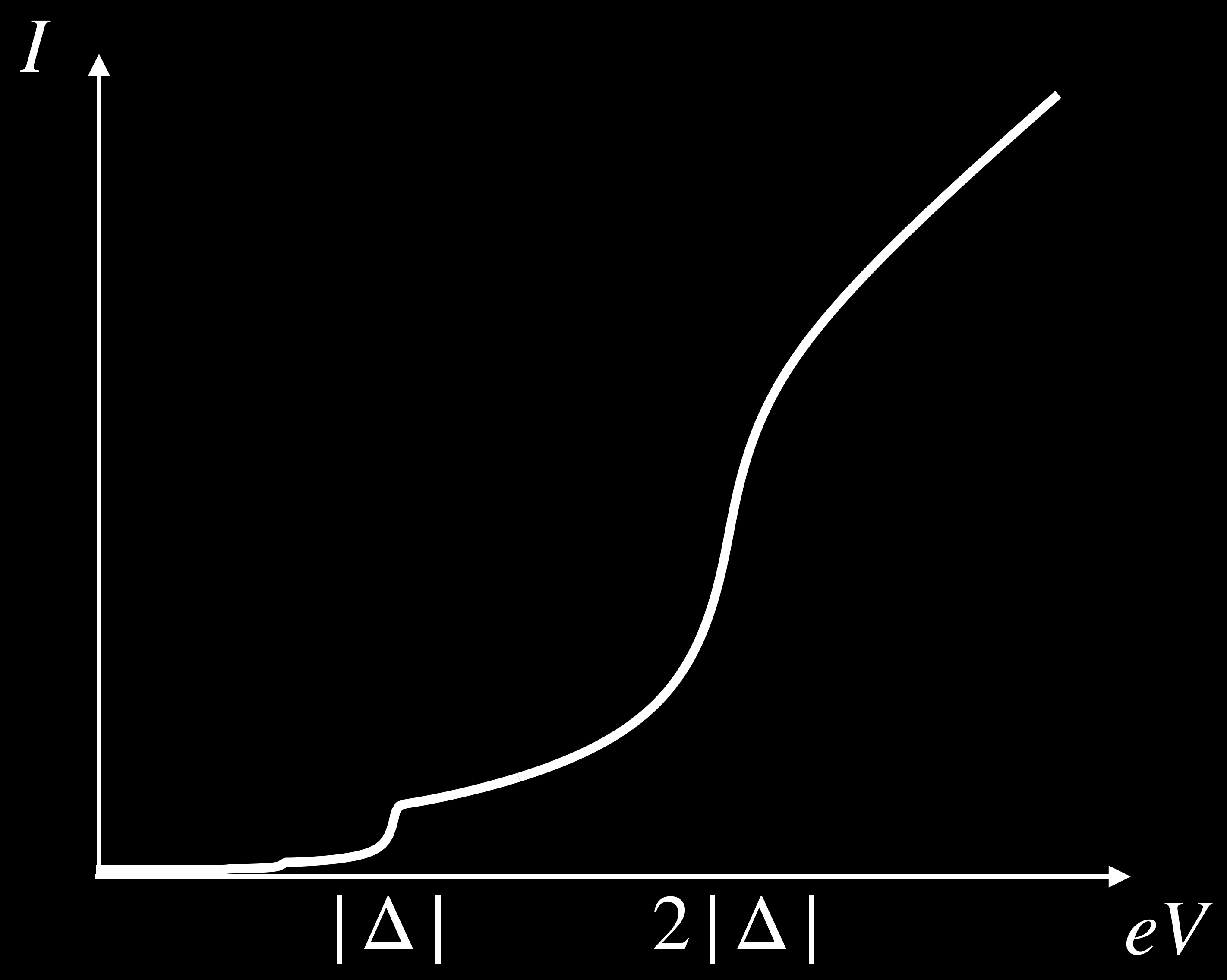
Example: Superconductor-Superconductor (SS) contact

J. C. Cuevas and W. Belzig, Phys. Rev. Lett. 91, 187001 (2003)



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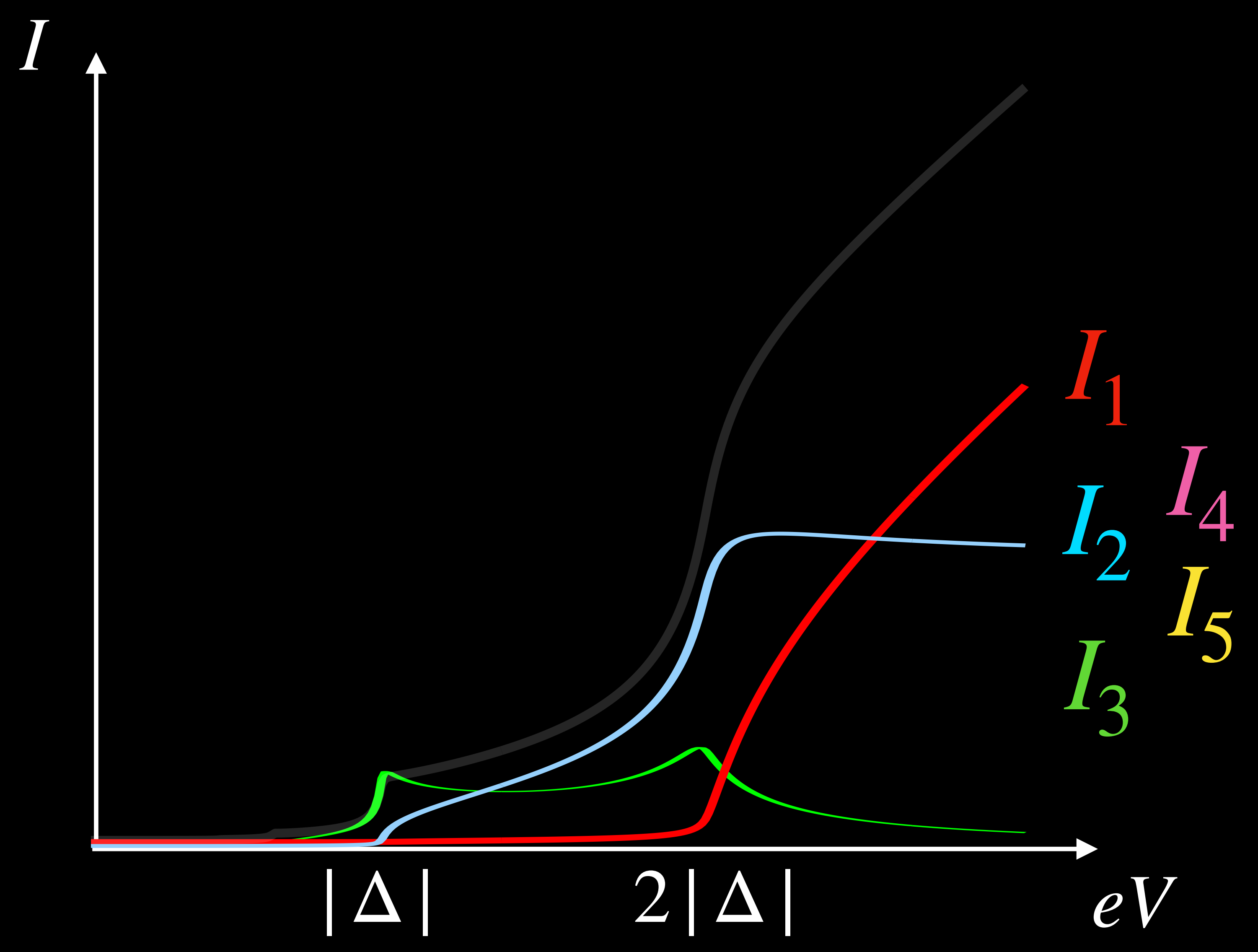

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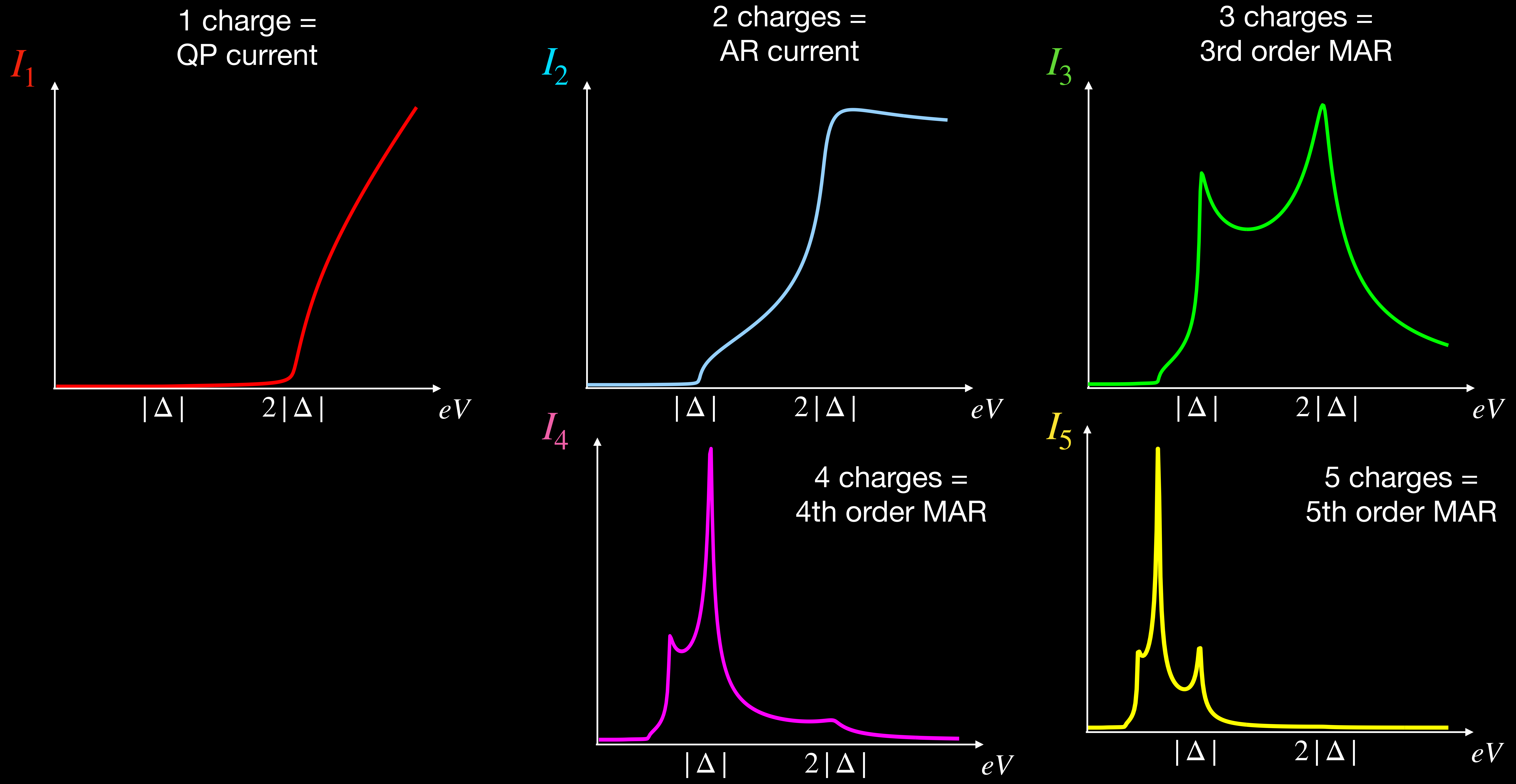

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Current and noise in FCS

Current

$$I = \int_{-\infty}^{\infty} \frac{dE}{h} \left(\sum_{n=-\infty}^{\infty} n p_n \right) = \sum_{n=-\infty}^{\infty} I_n$$

Noise

$$S = \int_{-\infty}^{\infty} \frac{dE}{h} \left[\sum_{n=-\infty}^{\infty} n^2 p_n - \left(\sum_{n=-\infty}^{\infty} n p_n \right)^2 \right]$$

For NN junction

$$I = \int_{-\infty}^{\infty} \frac{dE}{h} p_1 - p_{-1} = I_1 - I_{-1}$$

QP transferring 1 charge

For NS junction

$$I = \int_{-\infty}^{\infty} \frac{dE}{h} p_1 - p_{-1} + 2p_2 - 2p_{-2}$$

QP transferring 1 charge
+
AR transferring 2 charge

For SS junction

$$I = \int_{-\infty}^{\infty} \frac{dE}{h} p_1 - p_{-1} + 2p_2 - 2p_{-2} + 3p_3 - 3p_{-3} + \ldots$$

QP transferring 1 charge
+
AR transferring 2 charge
+
MAR transferring n charges

What is Thermodynamic uncertainty relation (TUR)?

Fundamental classical bound on entropy production concerning precision/uncertainty

A. C. Barato and U. Seifert, Phys. Rev. Lett. 114, 158101 (2015)
T. R. Gingrich, et. al., Phys. Rev. Lett. 116, 120601 (2016).
J. M. Horowitz and T. R. Gingrich, Nat. Phys. 16, 15 (2020)

Joule Heating

$$\dot{\Sigma} = IV/T$$

I: current
V: voltage
T: Temperature

TUR

$$\frac{S}{I^2}\dot{\Sigma} \geq 2k_\mathrm{B}$$

S: current fluctuations
k_B: Boltzmann constant

Rewritten TUR

$$\mathscr{F} \equiv \frac{S}{I} - \frac{2k_\mathrm{B}T}{V} \geq 0$$

$\mathscr{F}$: TUR-breaking coefficient

Units

$e = 1$

e : elementary charge

Thermodynamic uncertainty relation (NN)

What happens in the NN case?

B. K. Agarwalla and D. Segal, Phys. Rev. B 98, 155438 (2018)

$$\mathcal{F} = \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h}\tau(E)f(E)(1-f(E))\left[1-6\tau(E)f(E)(1-f(E))\right]+\mathcal{O}(V^2)$$

$f(E)$: Fermi function at equilibrium

$\tau(E)$: energy dependent transmission

1) $\tau = \text{const.}$

$\Rightarrow$ TUR can't be broken!

2) $\tau = \tau(E) \leftrightarrow \text{resonant}$

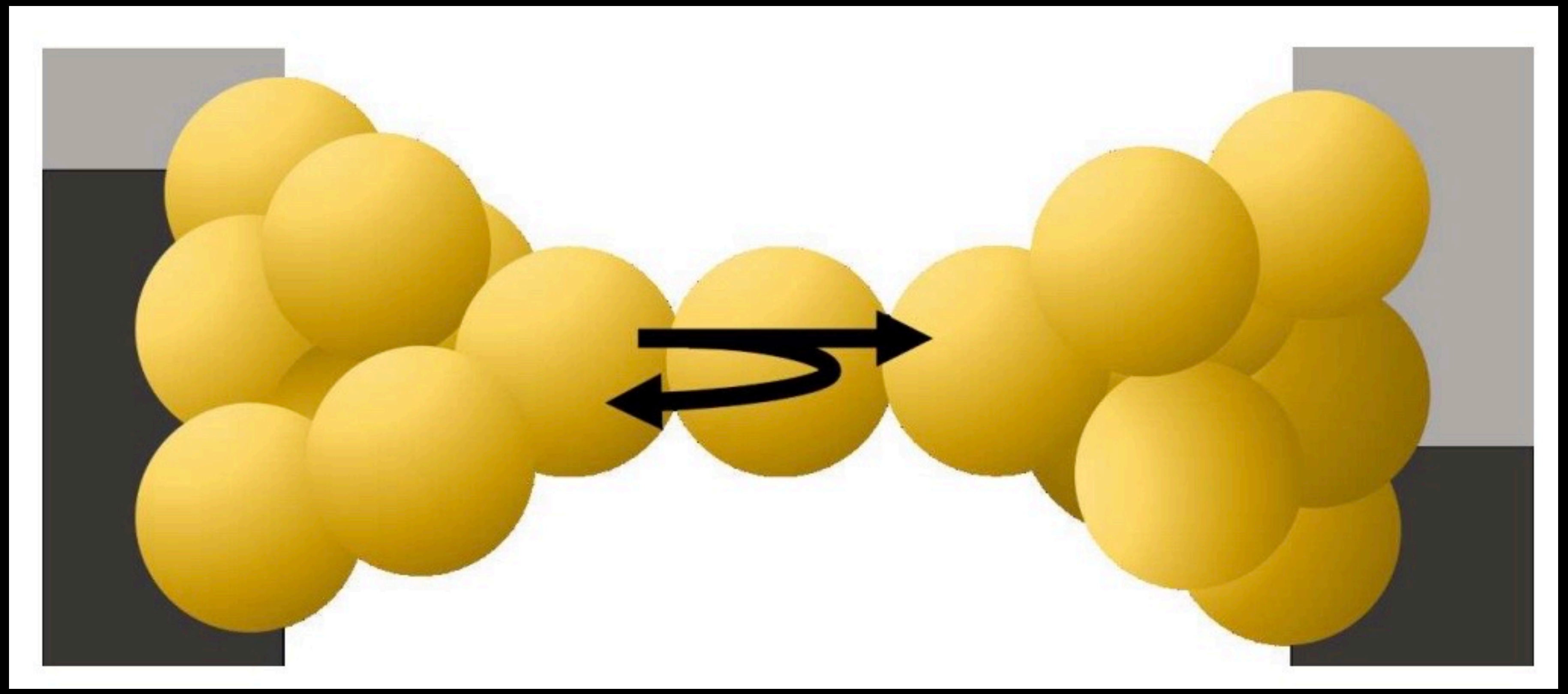
$$\frac{\tau_1}{\tau_2} > \frac{2}{3} \qquad \tau_n \equiv \int_{-\infty}^{\infty}\frac{dE}{h}\tau(E)^n$$

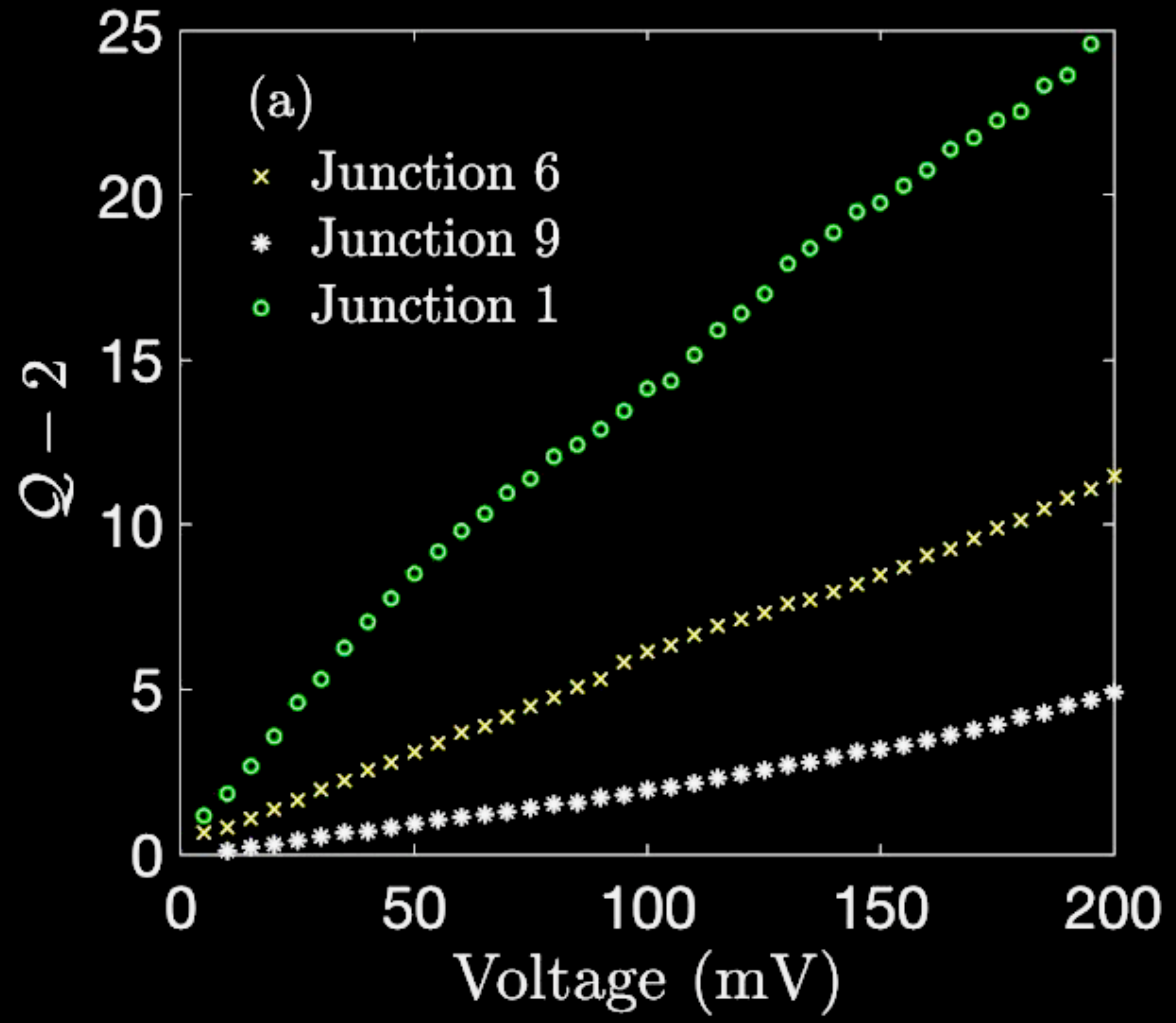
$\Rightarrow$ Transmission must be energy dependent!

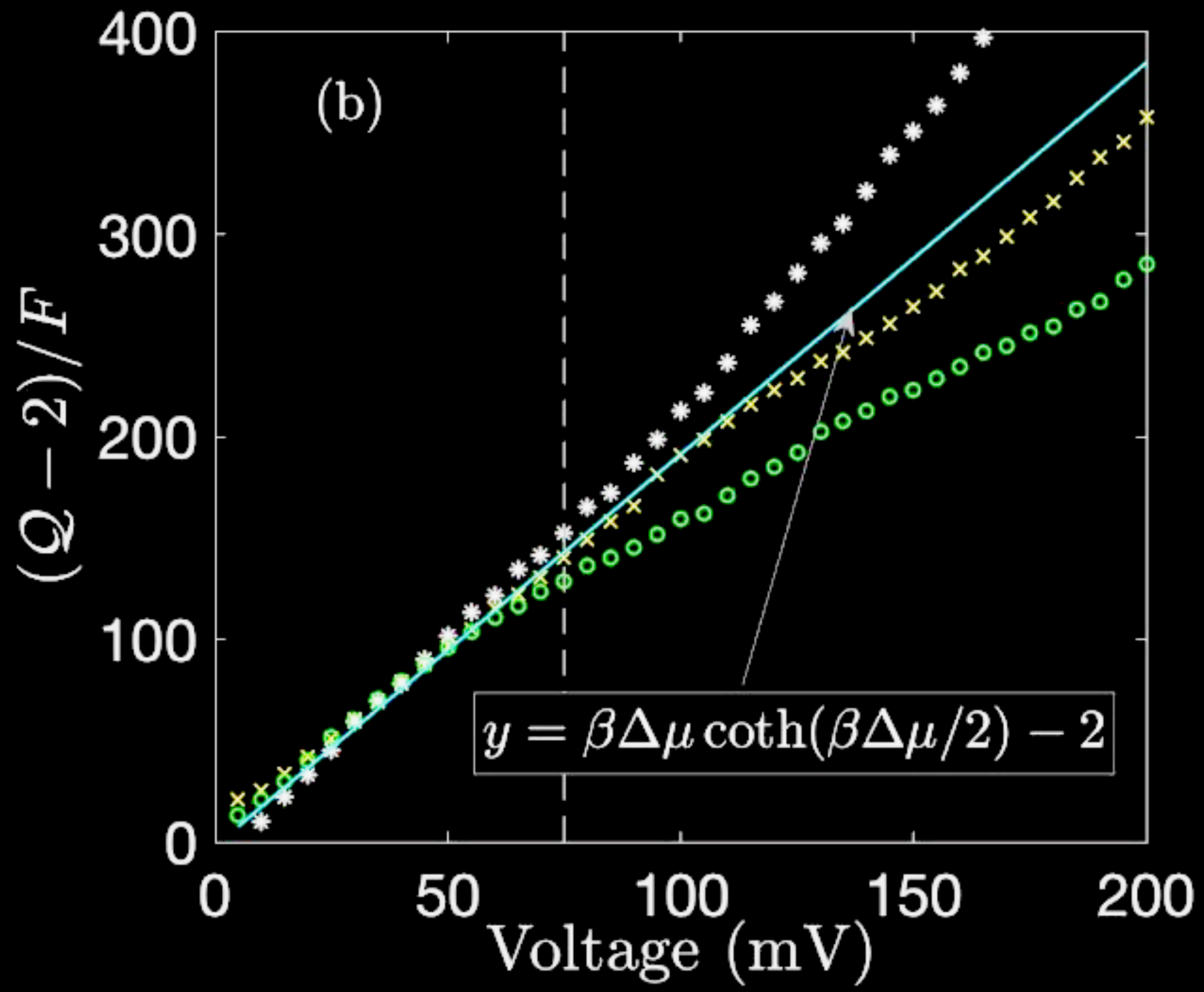
$\Rightarrow$ Transmission must be large!

Breaking TUR in experiment

H. M. Friedman, et. al. Thermodynamic uncertainty relation in atomic-scale quantum conductors, PRB 101, 195423 (2020)







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1) $\tau = \mathrm{const.}$

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2) $\tau = \tau(E) \leftrightarrow$ resonant

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$$= \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h} \quad p_1^{\mathrm{eq}}(E) \quad \left[1-6p_1^{\mathrm{eq}}(E)\right] \quad +\mathcal{O}(V^2)$$

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1) $\tau = \text{const}.$

⇒ TUR can't be broken!

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Meaning of probabilities

What happens in the NN case?

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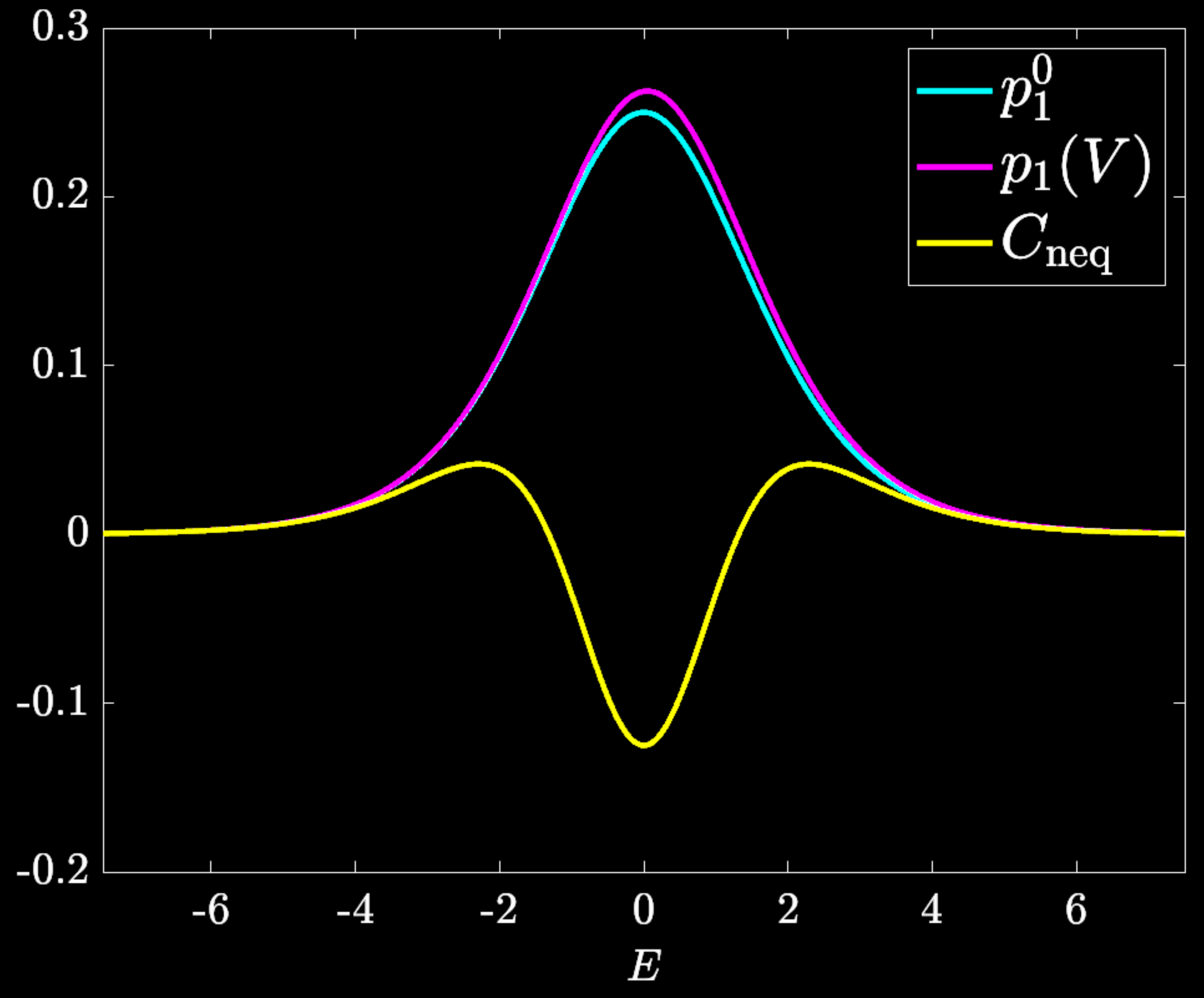
$$\mathscr{F} = \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h}\, p_1^{\mathrm{eq}}(E)\,\left[1 - 6p_1^{\mathrm{eq}}(E)\right] + \mathscr{O}(V^2) = \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h} C_{\mathrm{neq}} + \mathscr{O}(V^2)$$

$V = 0.1;\ k_B T = 1;\ \beta V = 0.1$



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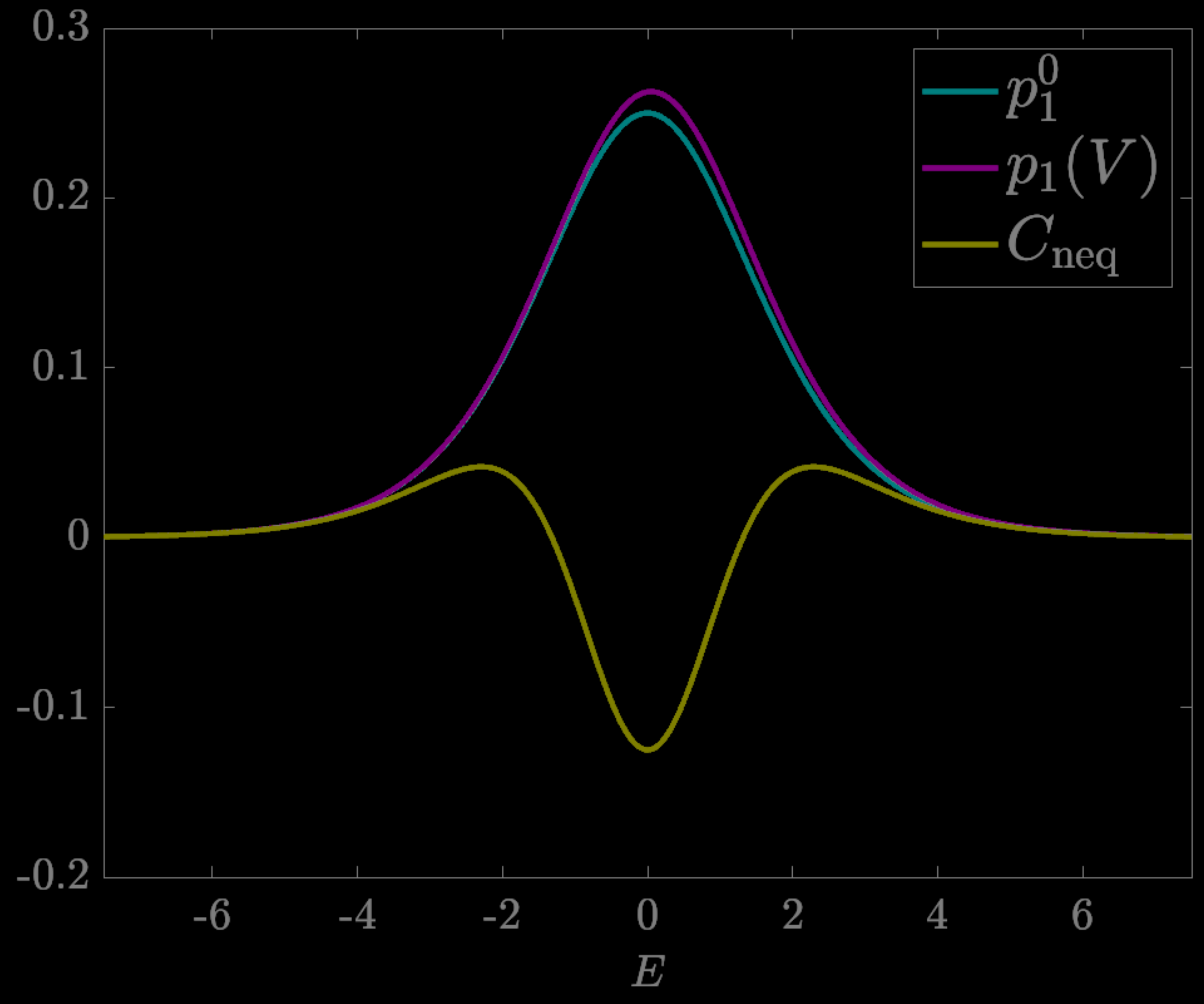
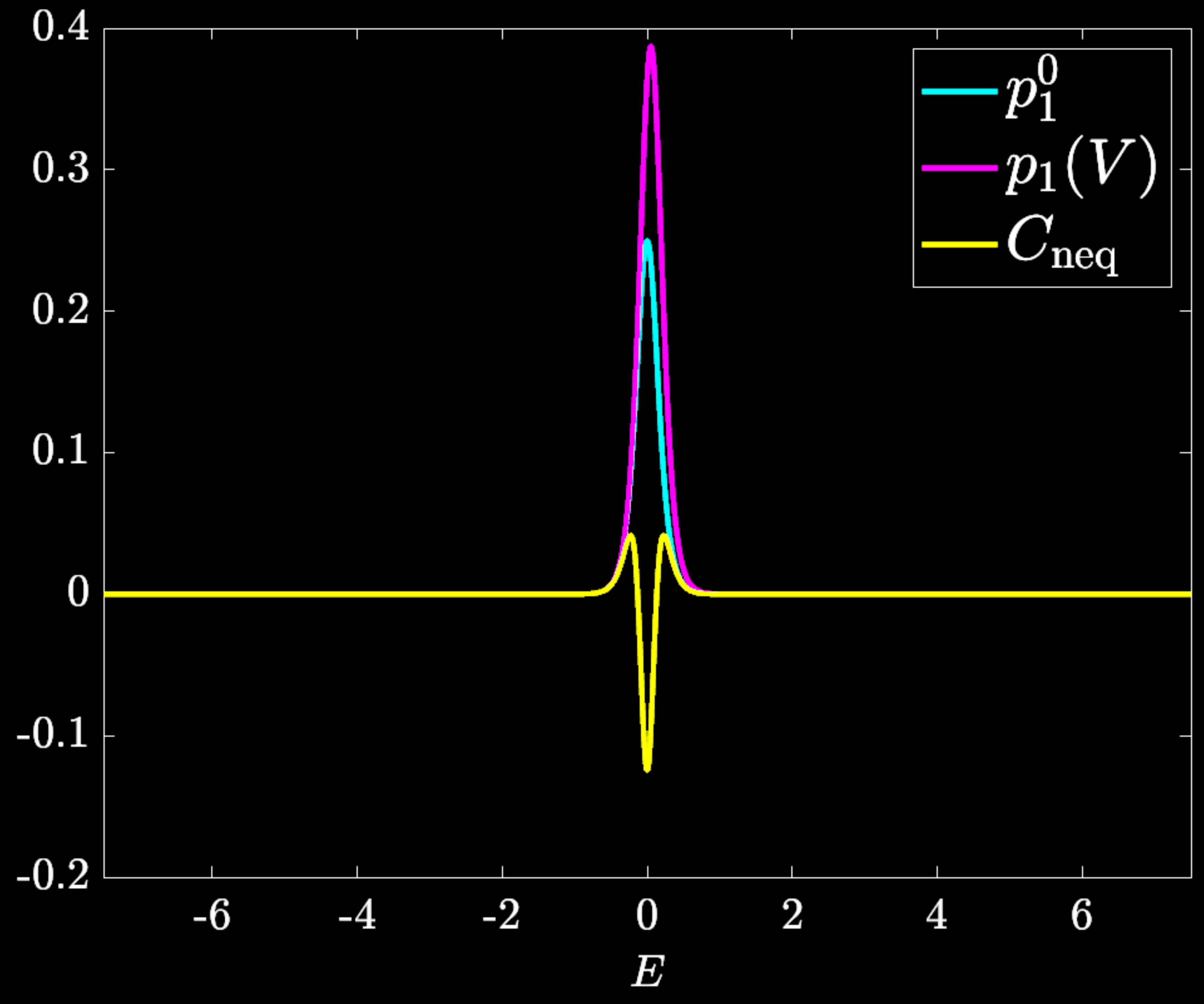


$V = 0.1;\ k_{\mathrm{B}}T = 0.1;\ \beta V = 1$



Thermodynamic uncertainty relation (General)

Full result for NS, SS, NN with ac bias, …

$$\mathscr{F} = \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h}\left\{\sum_{n=1}^{\infty} n^4 p_n^{\mathrm{eq}}(E)\left[1-6p_n^{\mathrm{eq}}(E)\right] - \sum_{m>n=1}^{\infty} 12n^2m^2p_n^{\mathrm{eq}}(E)p_m^{\mathrm{eq}}(E)\right\} + \mathscr{O}(V^2)$$

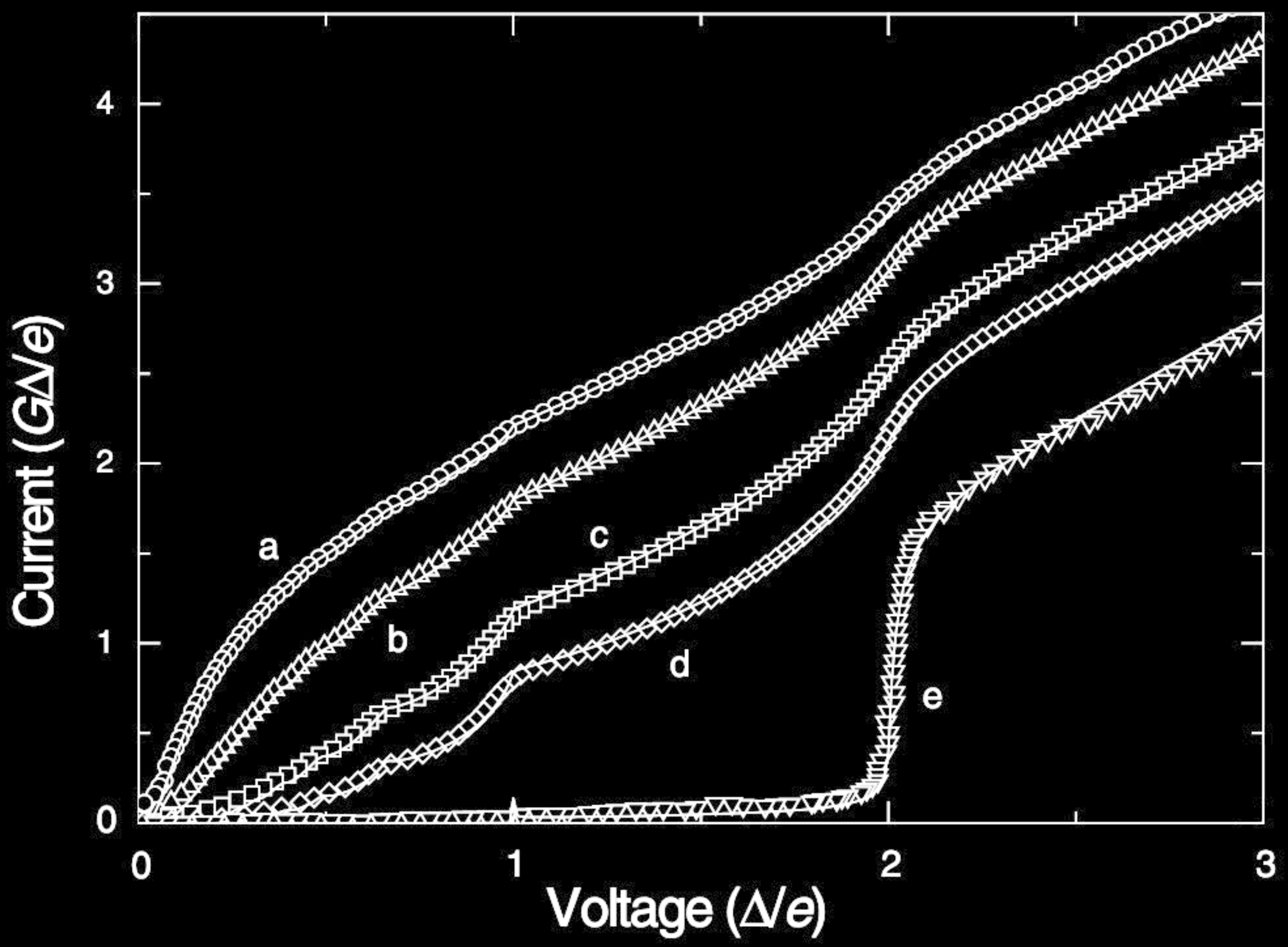
Same as for NN (for $n=1$)
but scales with factor n^4
$\Rightarrow$ stronger violations

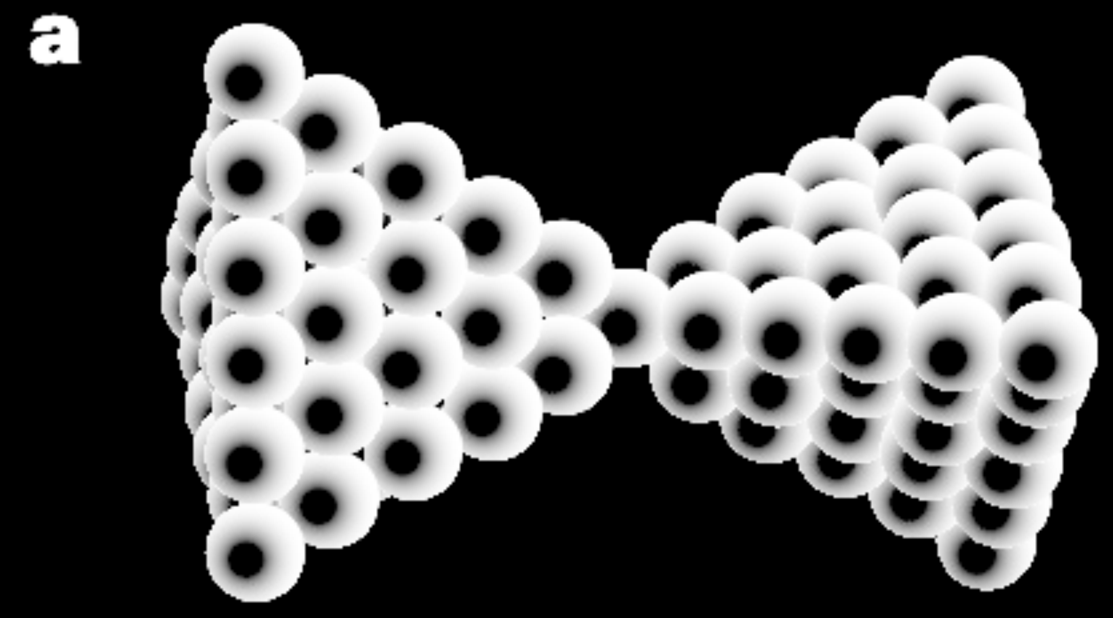
$$\mathscr{F} = \frac{\beta V}{6G_1}\int_{-\infty}^{\infty}\frac{dE}{h}\; p_1^{\mathrm{eq}}(E)\;\left[1-6p_1^{\mathrm{eq}}(E)\right] + \mathscr{O}(V^2)$$

Cross-coupling term always
increases chance to break TUR
$\Rightarrow$ multiple processes tend to violate TUR

Promising platform for breaking TUR in experiment

Examples of high transmission single channel contact in superconducting junctions





E. Scheer, et. al., Nature 394, 154 (1998)

C. Cuevas, et.al., Phys. Rev. Lett. 81, 2990 (1998)

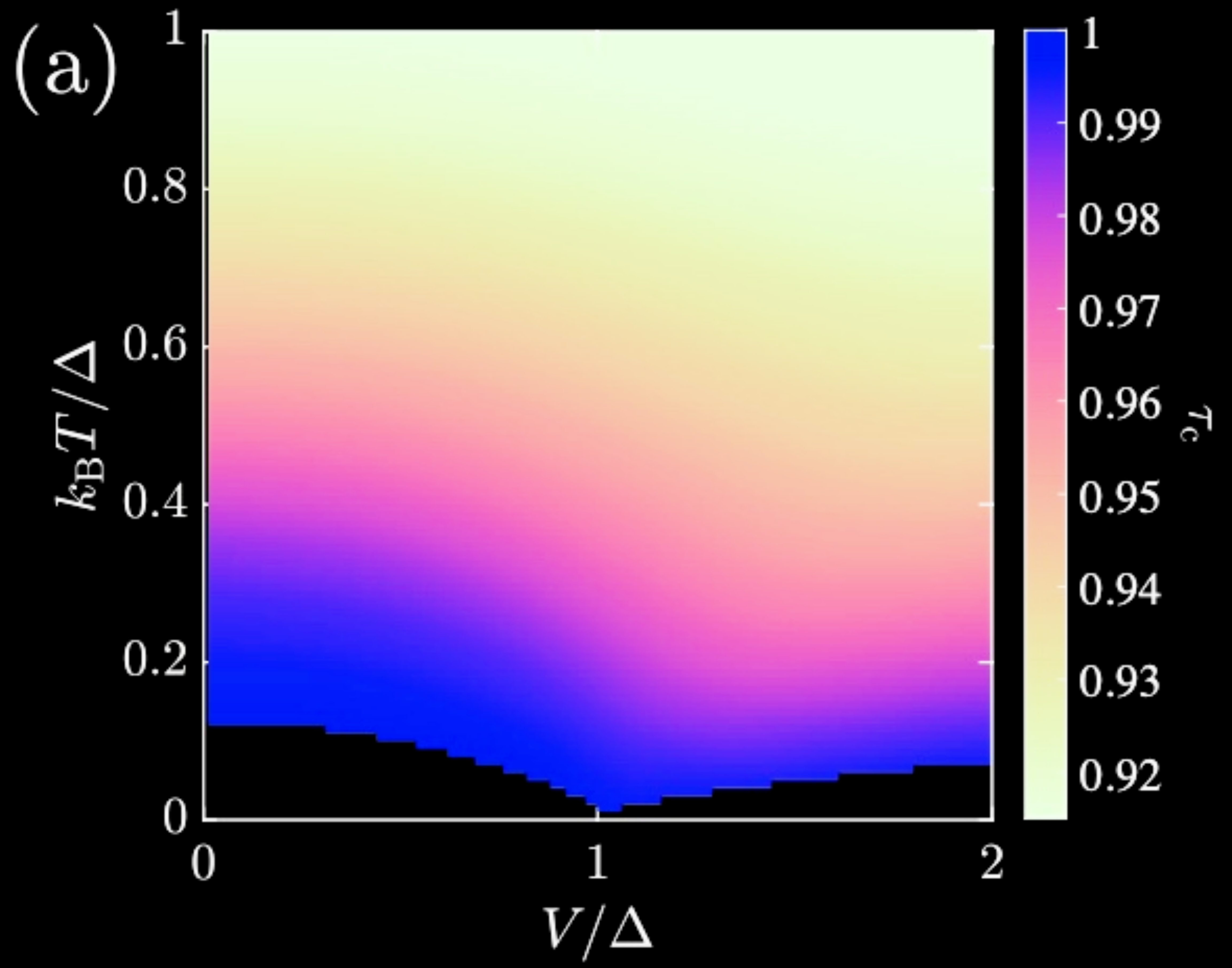
J. Senkpiel, et. al., Nat. Commun. 3, 1 (2020)

Science **349**, 1199 (2015)

Nature 499, 312 (2013)

Results of TUR-breaking coefficient (NS)

Full numerical result with constant transmission $\tau = \text{const}\,.$



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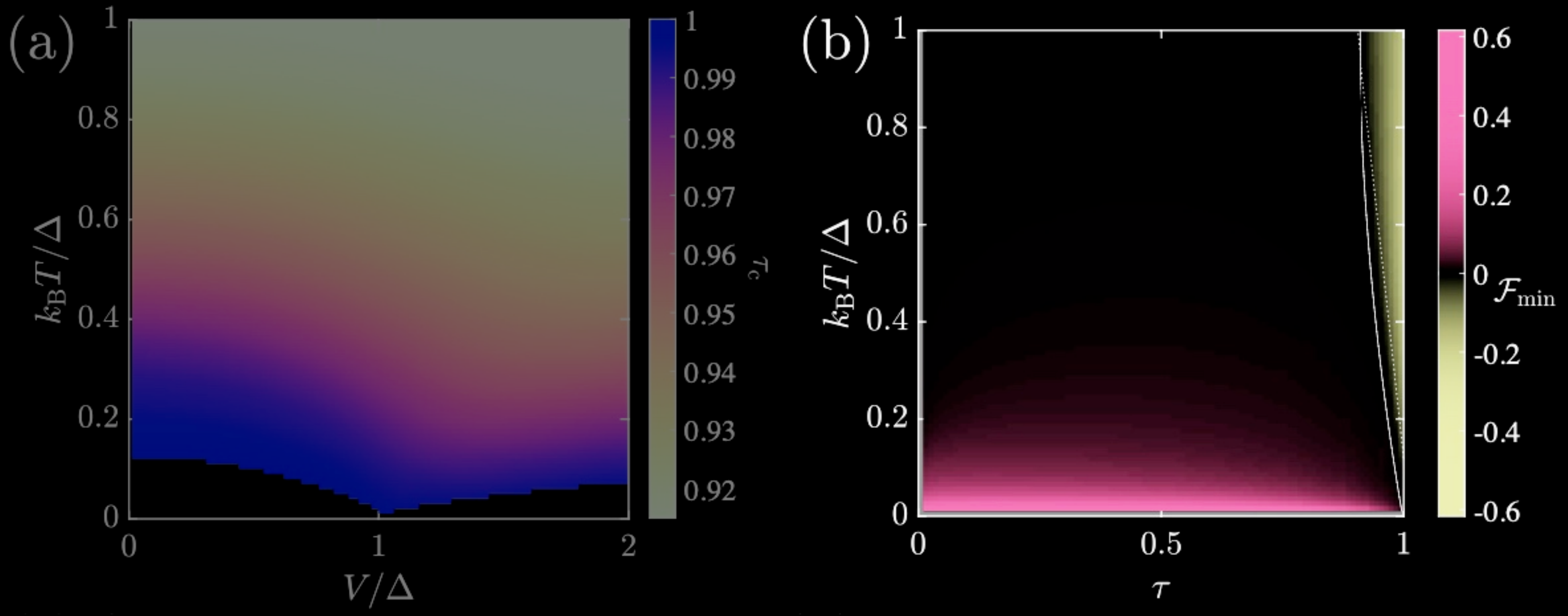
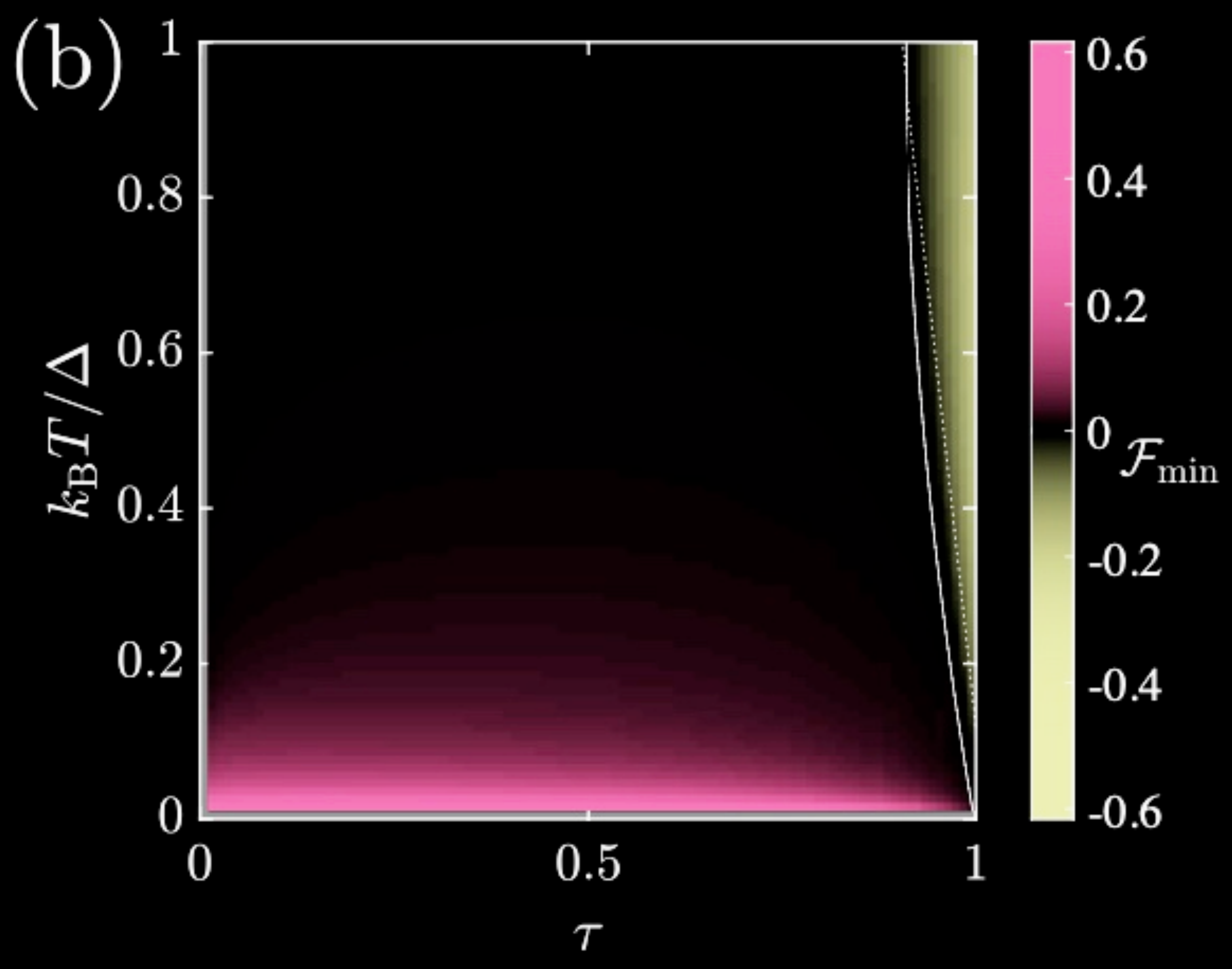
Full numerical result



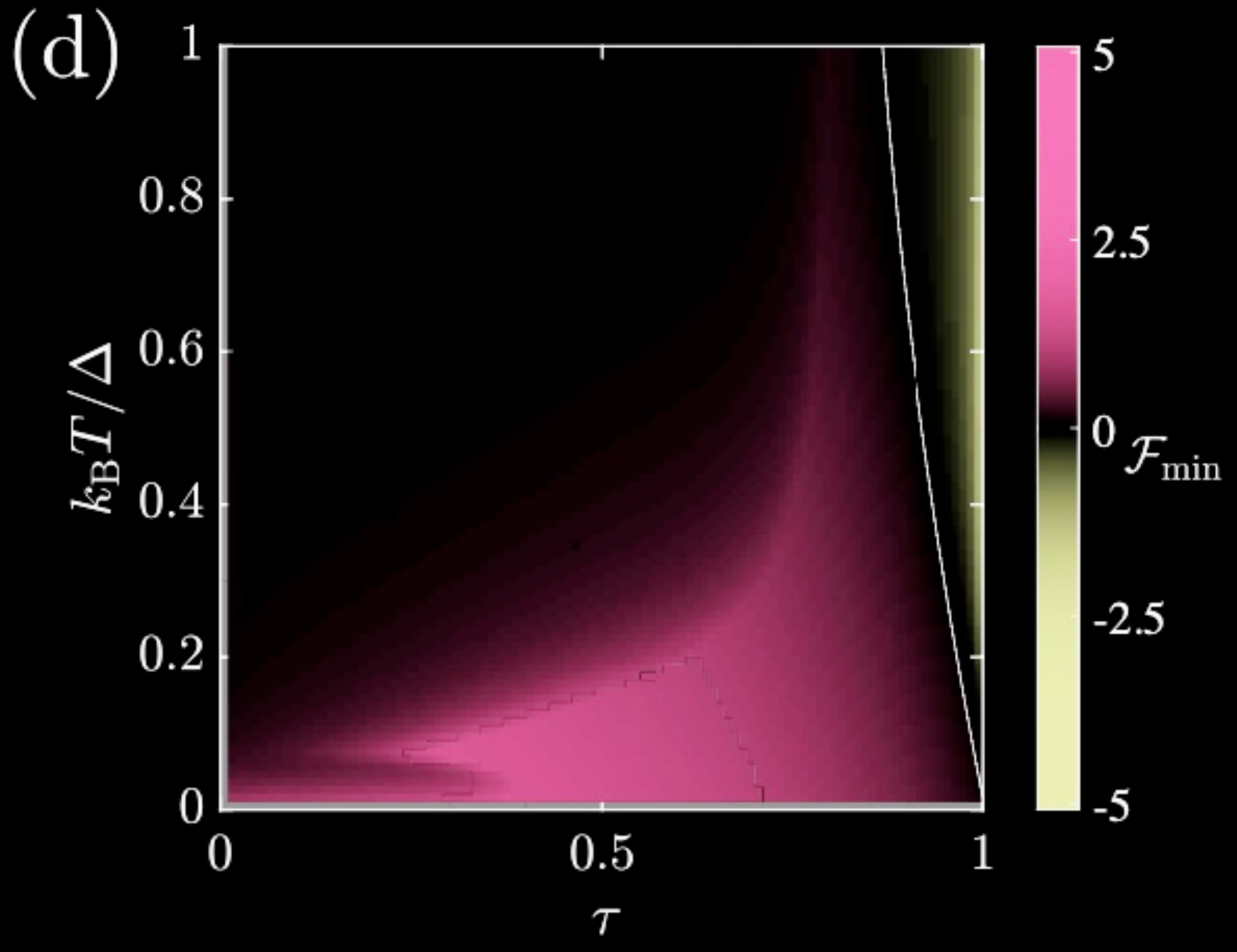
Results of TUR-breaking coefficient (NS+SS)

Full numerical result

NS junction

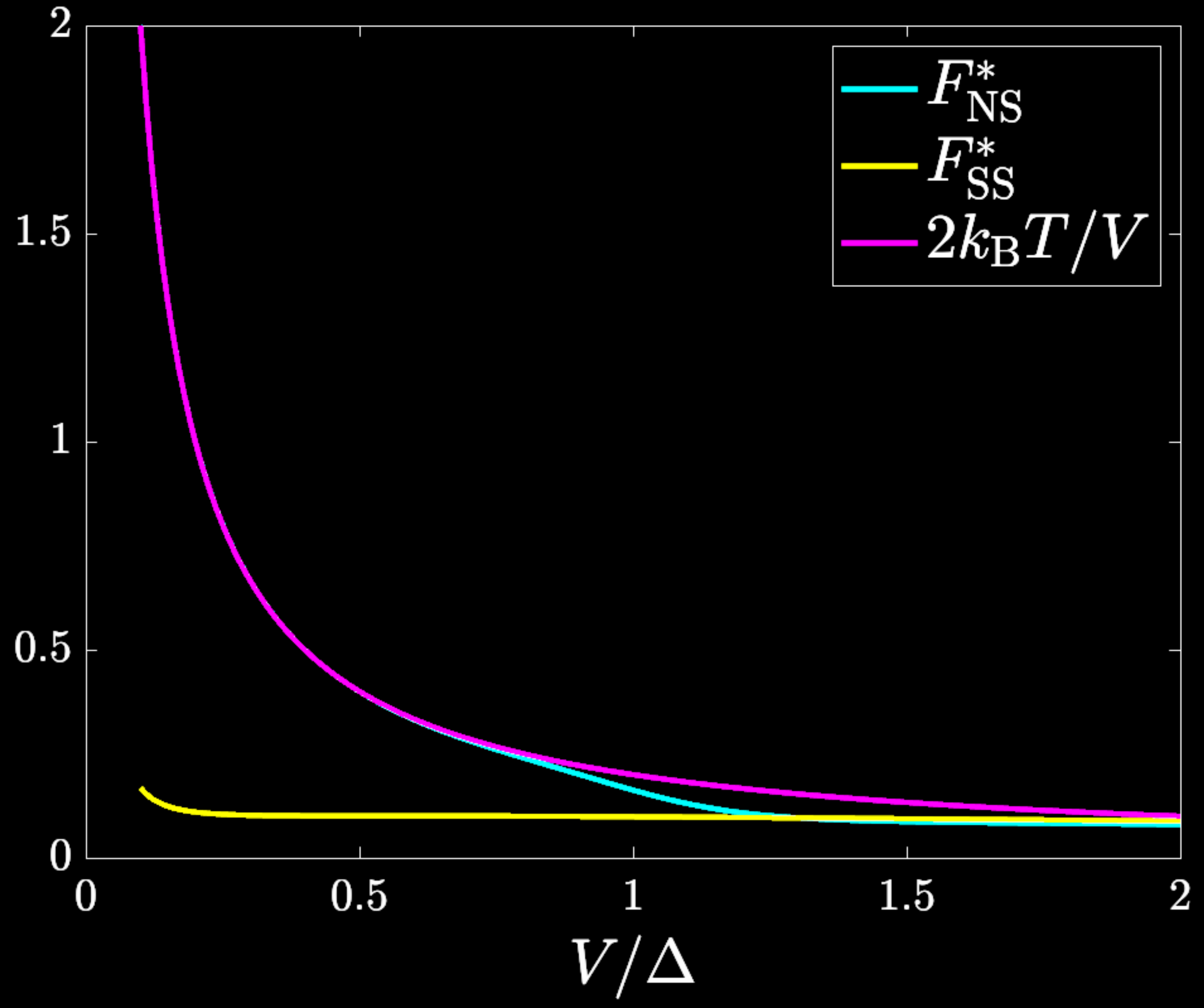


SS junction



Large breaking at low voltages and low temperatures

$$\tau = 1;\ k_B T = 0.1\Delta$$



Conclusion

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1) Superconducting junctions break the thermodynamic uncertainty relation in realistic setups

- **constant** transmission sufficient for **well-established** superconducting junctions (NS or SS)

D. C. O., et. al. arXiv:2408.01281 (2024)

2) Breaking of TUR is rooted in competition of different transport processes

- Breaking of TUR originates at small temperature at onset of QP tunnelling

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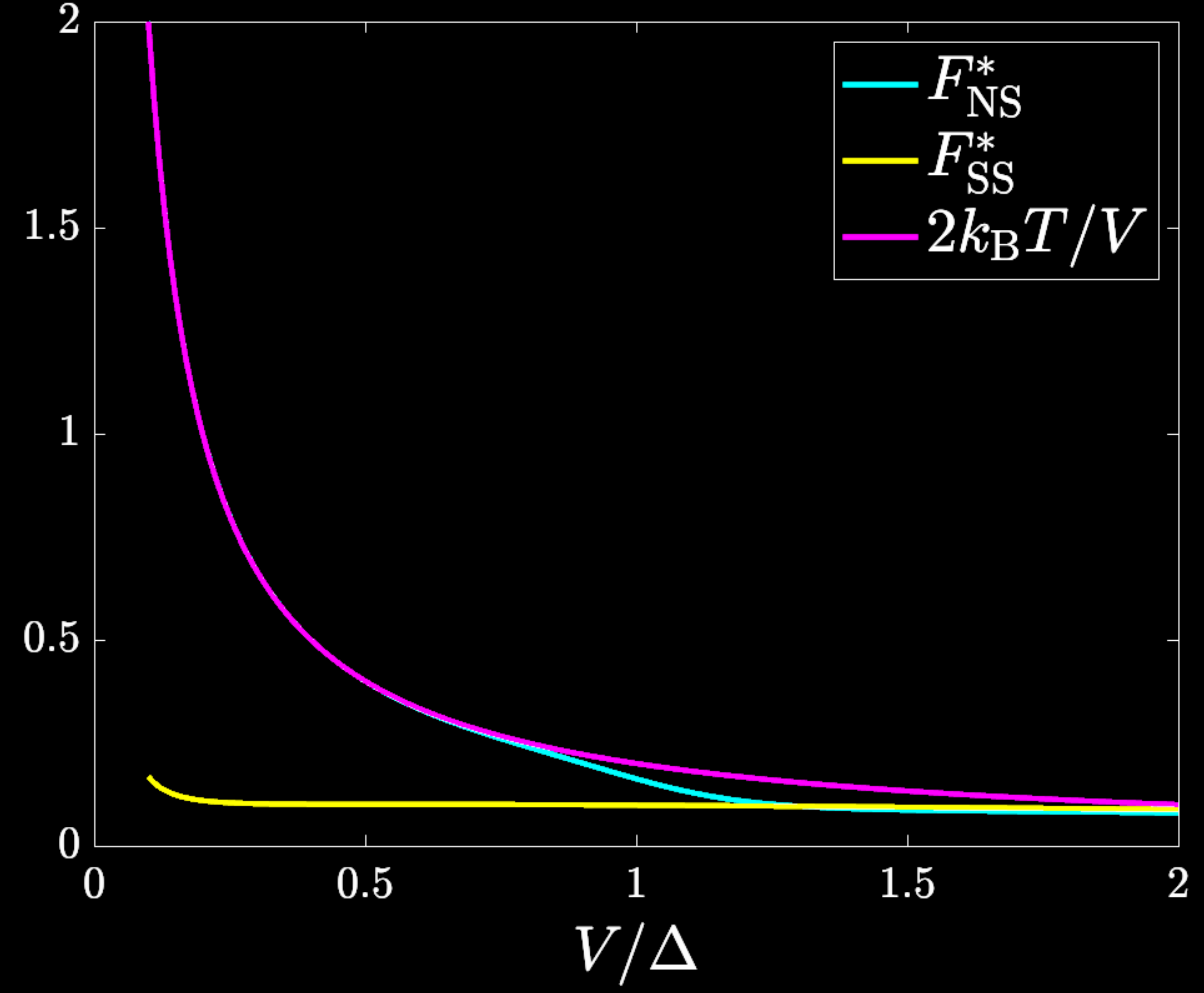

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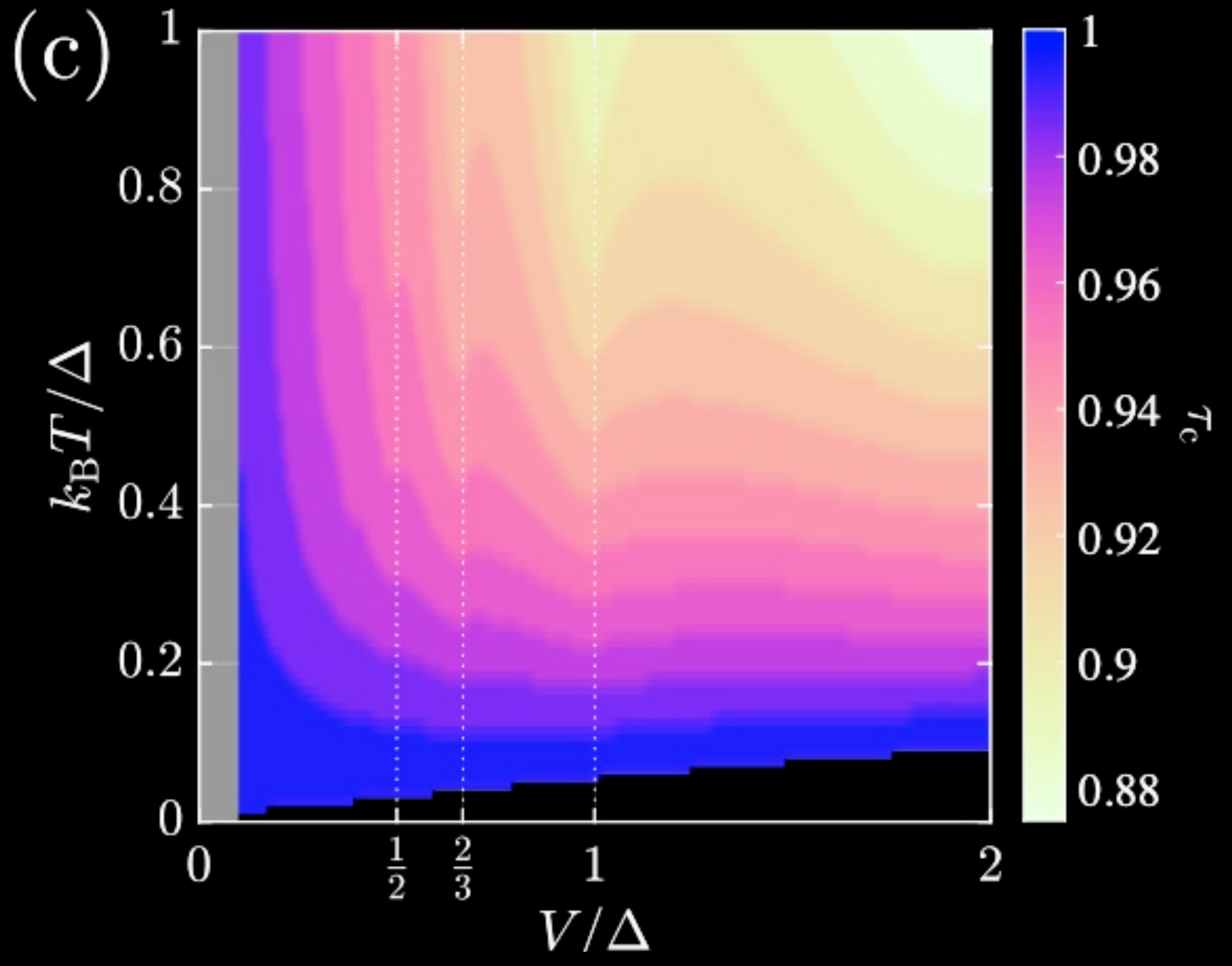

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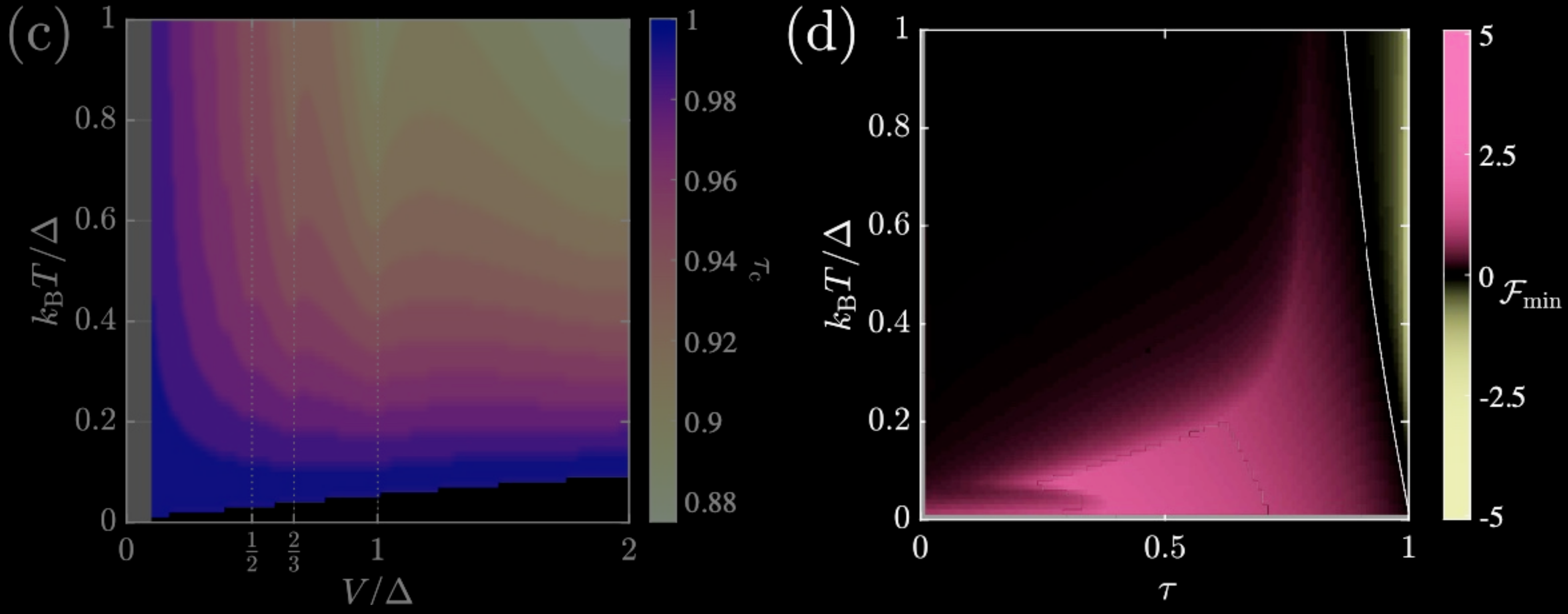

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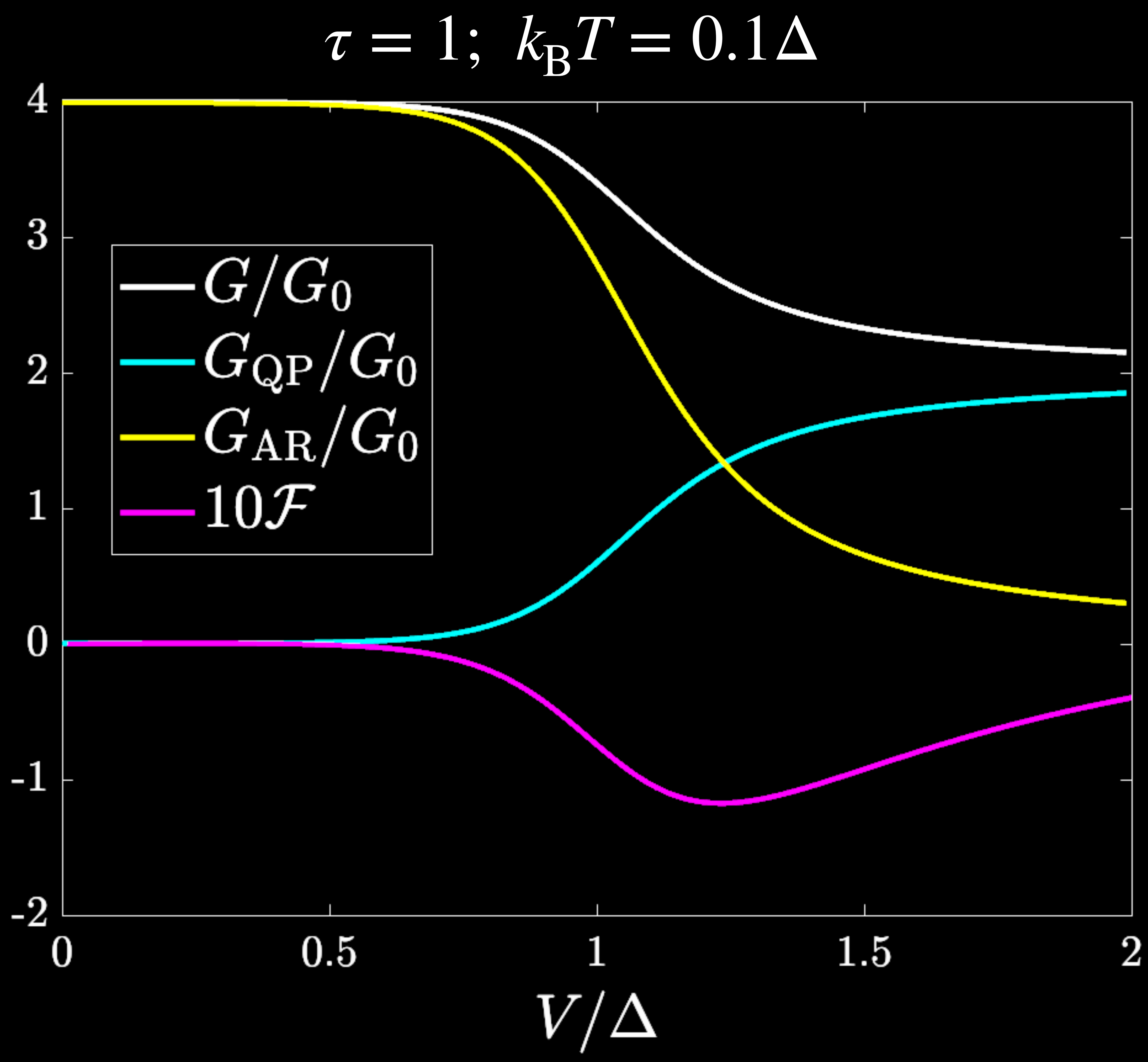
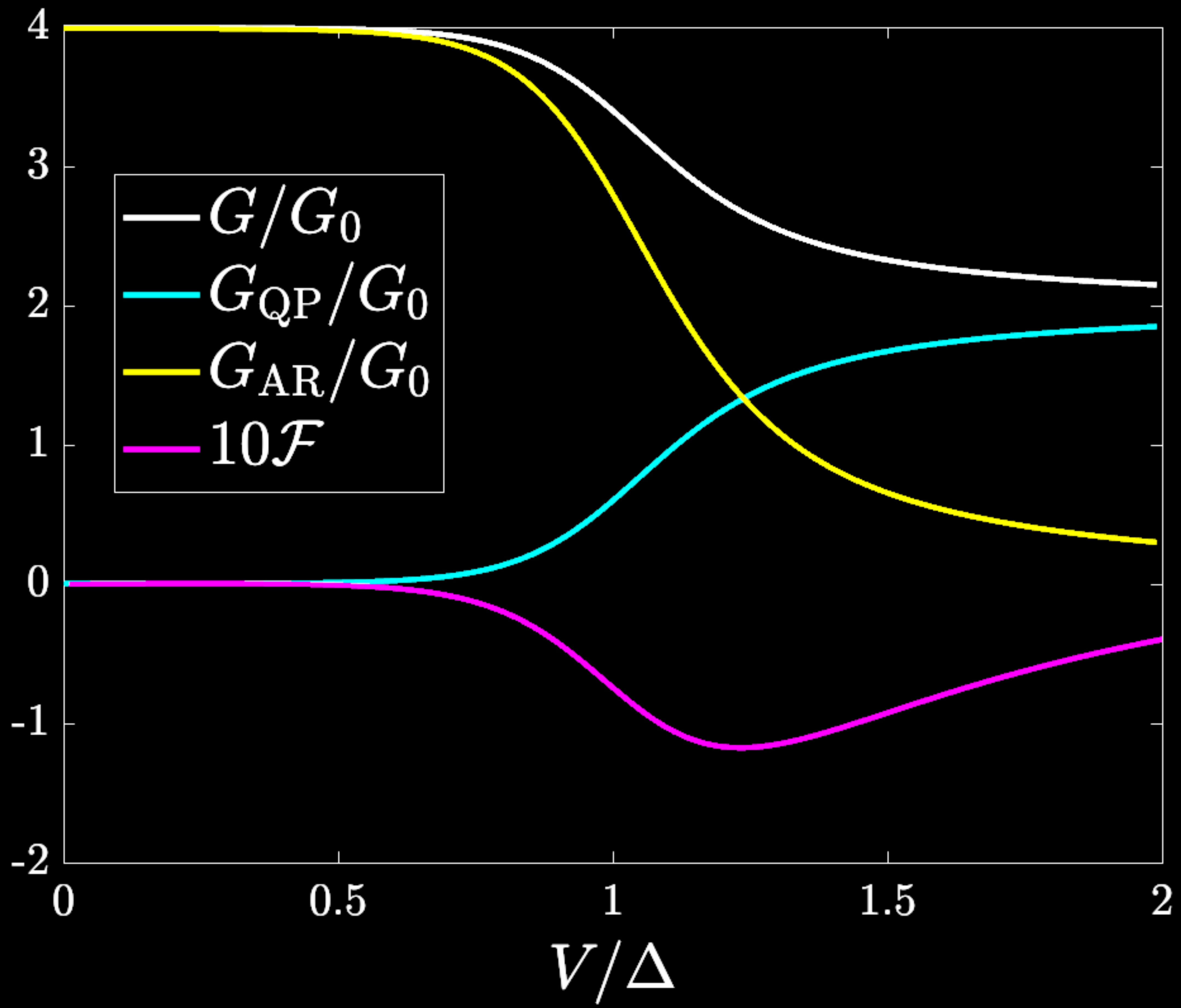
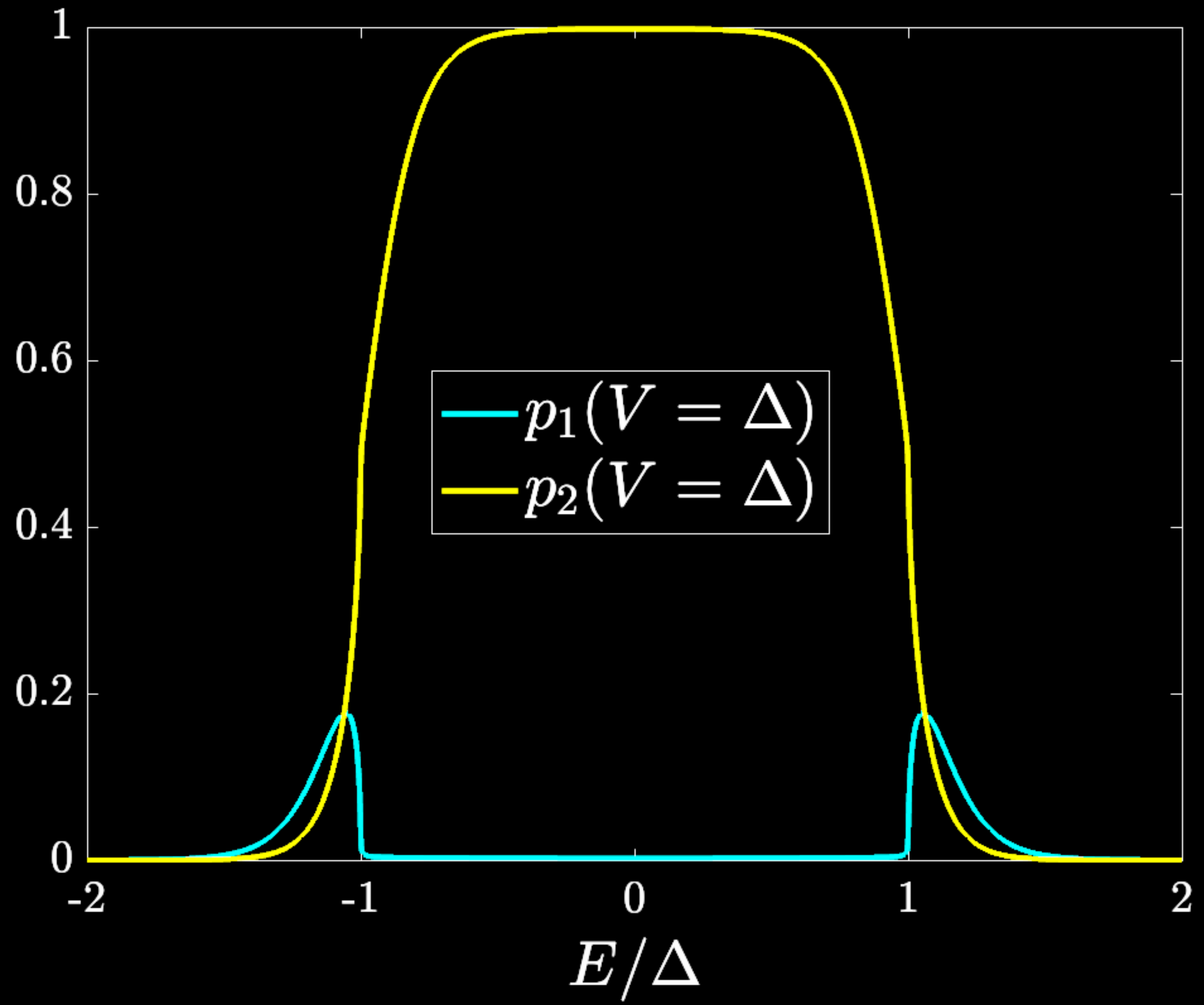


Thermodynamic uncertainty relation (NS)

$$\tau = 1; \;\; k_\mathrm{B}T = 0.1\Delta$$





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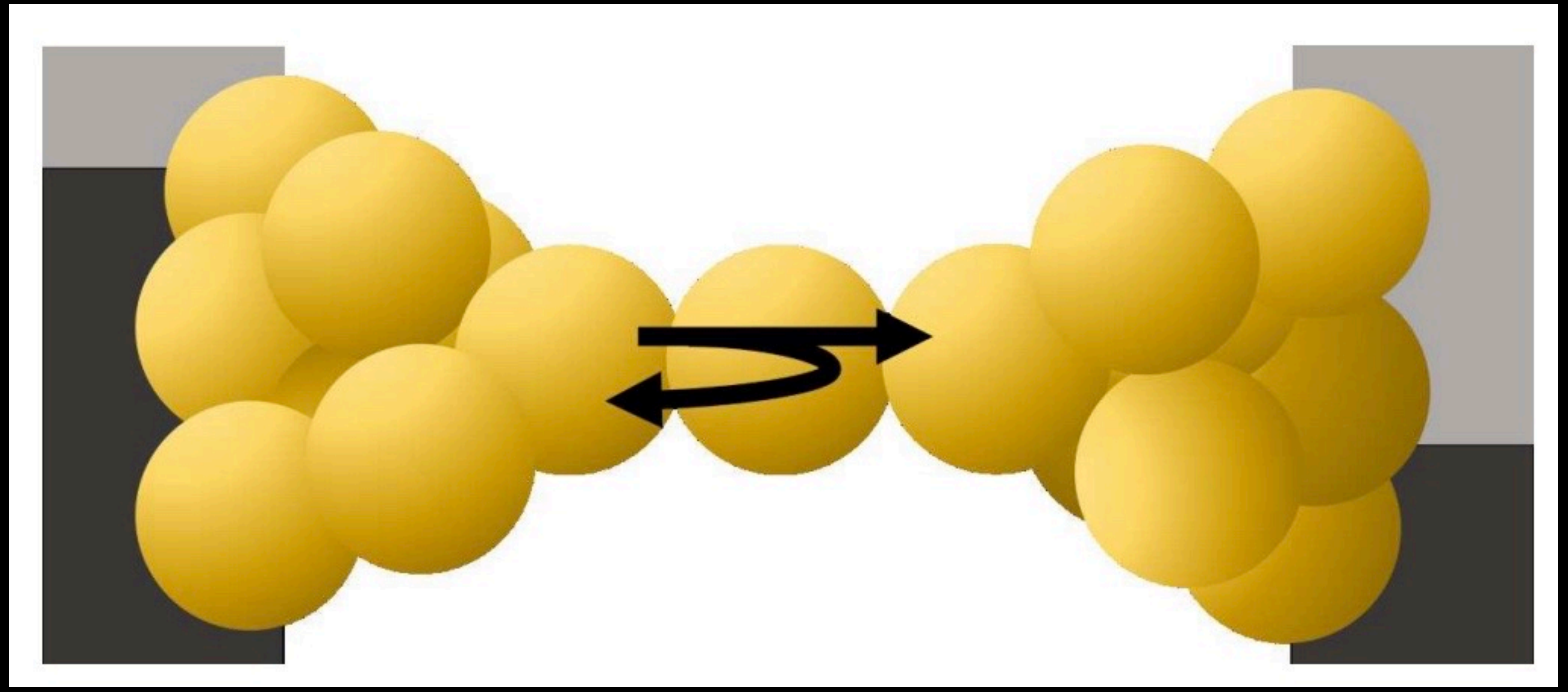

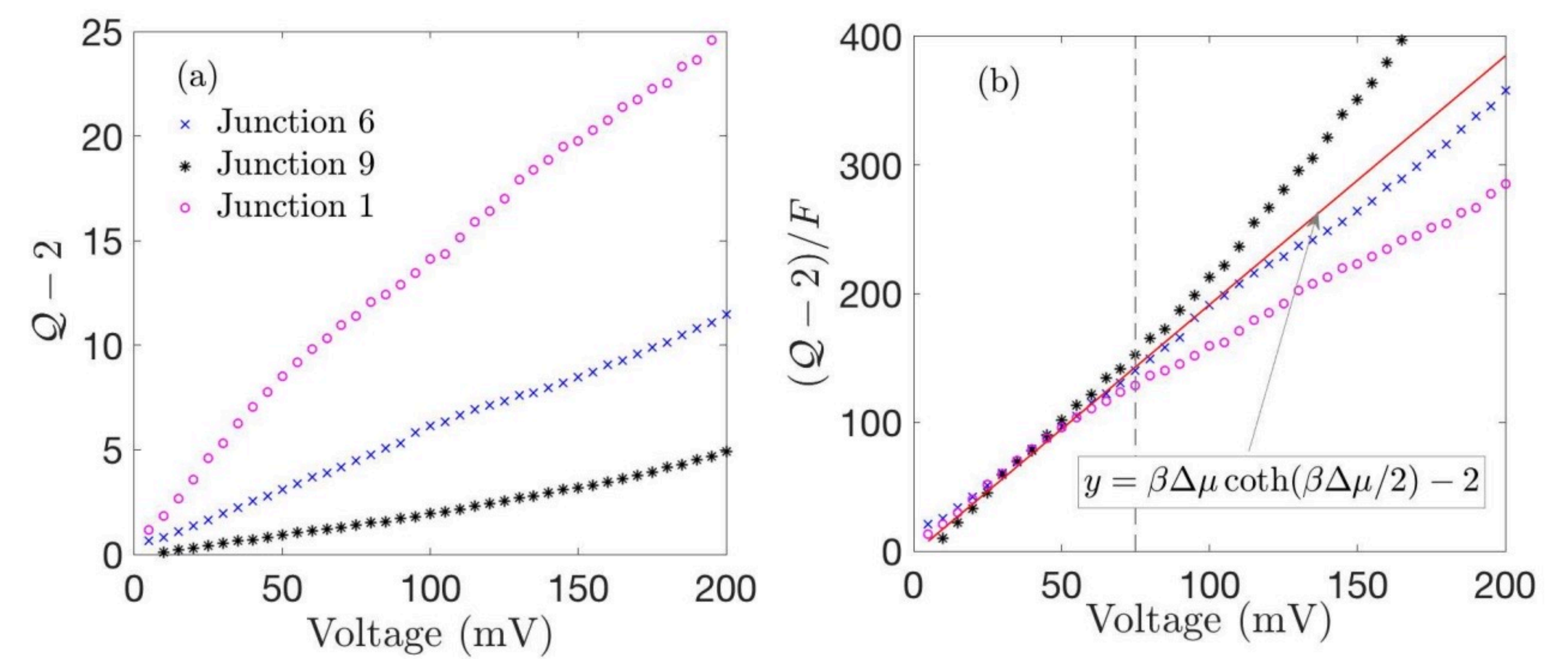

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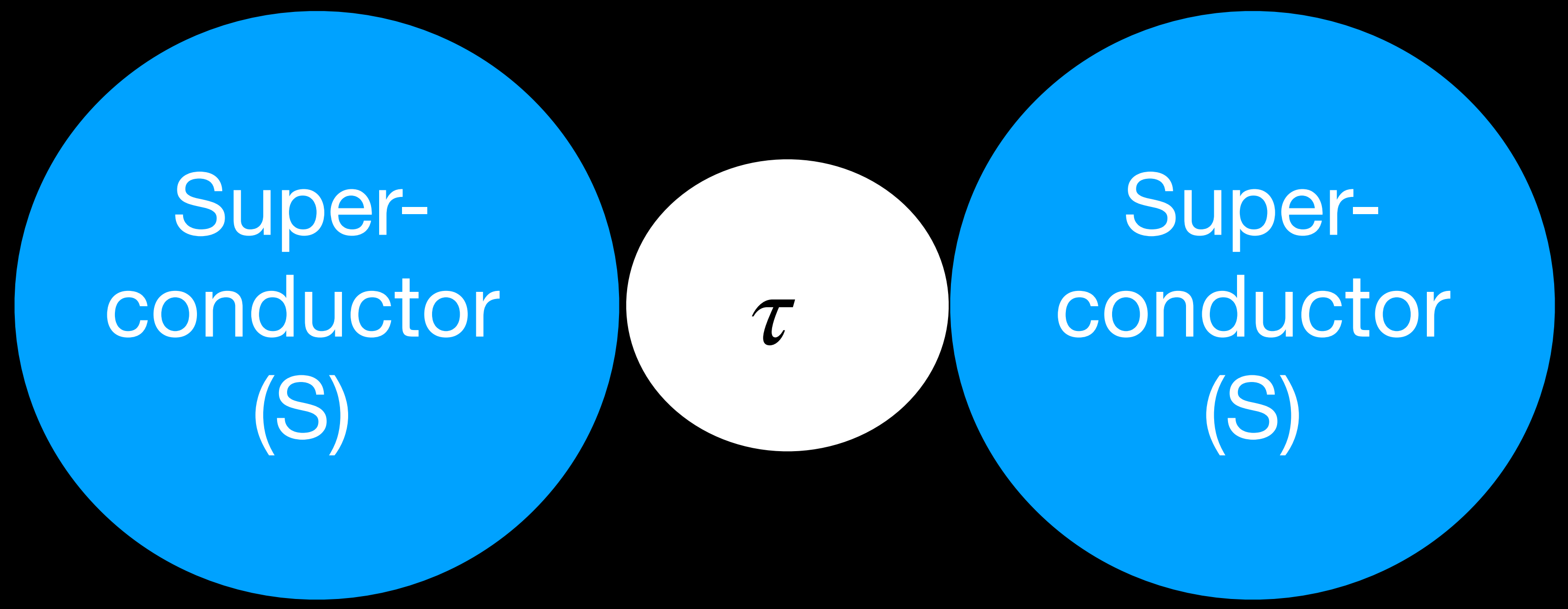
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