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Topology identification of complex dynamical networks

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Recently, some researchers investigated the topology identification for complex networks via LaSalle's invariance principle. The principle cannot be directly applied to time-varying systems since the positive limit sets are generally not invariant. In this paper, we study the topology identification problem for a class of weighted complex networks with time-varying node systems. Adaptive identification laws are proposed to estimate the coupling parameters of the networks with and without communication delays. We prove that the asymptotic identification is ensured by a persistently exciting condition. Numerical simulations are given to demonstrate the effectiveness of the proposed approach. © 2010 American Institute of Physics. [doi:10.1063/1.3421947]

The topology identification, as an inverse problem, is a significant issue in the study of complex networks. For example, if a major malfunction occurs in a communication network, power network, or the Internet, it is very important to quickly detect the location of the faulty line. This paper proposes a novel adaptive identification approach for the topology identification of the weighted complex dynamical networks. We show that the concept of persistent excitation plays a key role in the process of topology identification. Our result overcomes the limitation of previous methods, which rely on the use of LaSalle's invariance principle, and is applicable to networks with time-varying node systems and diverse time-varying coupling delays.

I. INTRODUCTION

Today, complex networks have become an important part in our daily life. Examples of such networks include transportation and phone networks, Internet, wireless networks, and the World Wide Web, to name a few. A complex dynamical network can be described by a graph in mathematics. In such a graph, each node represents a basic element with certain dynamics, and edges represent interactive topology of the network. Analysis and control of the behaviors of complex networks consisting of a large number of dynamical nodes have attracted wide attention in different fields in the past decade.^{1–5} In particular, special attention has been focused on the control and synchronization of large-scale complex dynamical networks with certain types of topology.^{6–15} Another attractive topic on complex networks is to develop systematic schemes for the topology identifica-

tion. Applications can be found in various scientific and engineering fields. For instance, if a major malfunction occurs in a communication network, power network, or the Internet, it is very important to quickly localize the faulty spot or the failing edge. There are also other works that involve network identification such as the understanding of protein-DNA interactions in cell processes.¹⁶ Note that the information transmission delay (called communication delay in this paper) is ubiquitous in complex networks and should be regarded as a critical issue in both the control and identification problems.^{17–19}

Recently, many researchers have applied LaSalle's invariance principle in the topology identification of complex networks.^{20,21} However, for a general network with time-varying node systems, the principle is not applicable as the positive limit sets may not be invariant.^{22–24} On the other hand, it has been reported in Refs. 25–27 that the earlier results neglected a crucial condition, which requires that the inner function should be mutually linearly independent of the synchronization manifold. However, it is difficult to verify the linear independence condition, especially on the synchronization manifold.

In this paper, we address the topology identification for weighted time-varying dynamical networks with nonlinear inner coupling. An adaptive identification rule is first proposed for the networks without communication delay. Unlike previous results, the asymptotic identification of the topology is guaranteed under a persistency excitation condition, which is widely used in dealing with parameter identification and adaptive control problems.^{22–24,28–31} Based on similar idea, an identification rule is further presented for the networks with diverse and time-varying communication delays, which brings our approach closer to practical applications.

The rest of this paper is organized as follows. Section II introduces some mathematical preliminaries used in this work. The system description and main results on the topol-

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ogy identification are presented in Secs. III and IV, respectively. In Sec. V, illustrative simulations are provided by taking the chaotic system in Ref. 35 as the node systems in the network. Finally, some concluding remarks are given in Sec. VI.

II. PRELIMINARIES

In this section, we present some mathematical preliminaries, which will be used in this work. Throughout the paper, $\|A\|$ is used to denote the spectral norm of matrix A , and $\lambda_{\max}(A)$ [$\lambda_{\min}(A)$] represents the maximum (minimum) eigenvalue of A . L^2 denotes the square integrable space, and I_n is the identity matrix on the order of n .

Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a directed graph with the vertex set $\mathcal{V} = \{1, 2, \dots, N\}$ and the directed edge set $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$. The adjacency matrix $A = (a_{ij})_{N \times N}$ of a directed graph with weighted edge $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a non-negative matrix defined as $a_{ij} = w$ if and only if (i, j) is an edge with weight w . The out-degree of a vertex v is the sum of the weights of the edges emanating from v .³² See Ref. 33 for more basics of graph theory.

A matrix is reducible if it can be written as

$$P \begin{bmatrix} A & B \\ 0 & C \end{bmatrix} P^T,$$

where P is a permutation matrix. A matrix is irreducible if it is not reducible.³²

Definition 1: (Reference 28) A function $\varphi: R^+ \rightarrow R^{n \times m}$ is persistently exciting (PE) if there exist $T_0, \delta_1, \delta_2 > 0$ such that

$$\delta_1 I_n \leq \int_t^{t+T_0} \varphi(\tau) \varphi^T(\tau) d\tau \leq \delta_2 I_n \quad (1)$$

holds for all $t \geq 0$.

Remark 1: The PE condition requires that φ rotates sufficiently in space that the integral of the matrix $\varphi(\tau) \varphi^T(\tau)$ is uniformly positive definite over any interval of some length T_0 . The upper bound in Eq. (1) is satisfied whenever $\varphi(t)$ is bounded.

Lemma 1: (Reference 30) Given a system of the following form:

$$\begin{cases} \dot{e}_1 = g(t)e_2 + f_1(t), & e_1 \in R^p, \quad e_2 \in R^q \\ \dot{e}_2 = f_2(t) \end{cases} \quad (2)$$

such that

- (i) $\lim_{t \rightarrow \infty} \|e_1(t)\| = 0$, $\lim_{t \rightarrow \infty} \|f_1(t)\| = 0$, and $\lim_{t \rightarrow \infty} \|f_2(t)\| = 0$;
- (ii) $g(t)$ and $\dot{g}(t)$ are bounded and $g^T(t)$ is PE;

then $\lim_{t \rightarrow \infty} \|e_2(t)\| = 0$.

III. TOPOLOGY IDENTIFICATION OF NONDELAY DYNAMICAL NETWORKS

Consider a dynamical network consisting of N coupled oscillators, with each node being an m -dimensional dynamical system described by

$$\dot{x}_i = f_i(t, x_i) + \epsilon \sum_{j=1}^N c_{ij} H_{ij}(t, x_j), \quad i = 1, 2, \dots, N, \quad (3)$$

where $x_i \in R^m$ is the state variable of node i and $f_i: R \times R^m \rightarrow R^m$ is a continuous function, $\epsilon > 0$ is the known coupling strength, and H_{ij} is the nonlinear inner-coupling function. The outer-coupling matrix $C = (c_{ij})_{N \times N}$ is defined as $C = A - D$, where $A = (a_{ij})_{N \times N}$ is the adjacency matrix of the graph of the network, and $D = (d_{ij})$ is the diagonal matrix of vertex with $d_{ii} = \sum_{j=1}^N a_{ij}$ for all $i = 1, 2, \dots, N$. In this paper, we assume that the elements of the matrix C are unknown. Our purpose is to estimate these unknown parameters in an asymptotic manner.

Assumption 1: For all $i, j = 1, \dots, N$, the functions $f_i(t, x)$ and $H_{ij}(t, x)$ are continuous in t and satisfy that there exist constants $\alpha_1, \alpha_2 > 0$ such that

$$\|f_i(t, x) - f_i(t, y)\| \leq \alpha_1 \|x - y\| \quad (4)$$

and

$$\|H_{ij}(t, x) - H_{ij}(t, y)\| \leq \alpha_2 \|x - y\| \quad (5)$$

for all t and all $x, y \in R^m$

Design the response network as

$$\dot{y}_i = f_i(t, y_i) + \sum_{j=1}^N d_{ij}(t) H_{ij}(t, y_j) - k_i(t) e_i, \quad i = 1, 2, \dots, N, \quad (6)$$

with the adaptive feedback law

$$\begin{cases} \dot{d}_{ij} = -\delta_{ij} e_i^T H_{ij}(t, y_j) \\ \dot{k}_i = s_i e_i^T e_i, \end{cases} \quad (7)$$

where $e_i = y_i - x_i$ is called the synchronization error, k_i is adaptive feedback gain, d_{ij} is the adaptive parameters of the response system, and δ_{ij} and s_i are two positive constants.

In the following, we show how the unknown parameters c_{ij} can be asymptotically estimated from d_{ij} . The synchronization errors between network (3) and network (6) can be written as

$$\begin{aligned} \dot{e}_i(t) &= f_i(t, y_i) - f_i(t, x_i) + \sum_{j=1}^N d_{ij}(t) H_{ij}(t, y_j) \\ &\quad - \epsilon \sum_{j=1}^N c_{ij} H_{ij}(t, x_j) - k_i(t) e_i, \quad i = 1, 2, \dots, N. \end{aligned} \quad (8)$$

Theorem 1: Suppose that the functions f_i and H_{ij} satisfy Assumption 1, and H_{ij} is bounded. Then, by the updated law (7), $\lim_{t \rightarrow \infty} \|x_i - y_i\| = 0$ for all i . Furthermore, if for any i , $g_i^T(t) = [H_{i1}(t, y_1(t)), \dots, H_{iN}(t, y_N(t))]^T$ is PE, and $\dot{g}_i(t)$ is bounded, then $\lim_{t \rightarrow \infty} (d_{ij} - \epsilon c_{ij}) = 0$.

Proof: Construct the function

$$V = \frac{1}{2} \sum_{i=1}^N e_i^T e_i + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{1}{\delta_{ij}} (d_{ij}(t) - \epsilon c_{ij})^2 + \frac{1}{2} \sum_{i=1}^N \frac{1}{s_i} (k_i(t) - k^*)^2,$$

where k^* is a positive constant to be determined. Meanwhile, the derivative of V along trajectories of the dynamical networks (7) and (8) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N e_i^T (f_i(t, y_i) - f_i(t, x_i)) + \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T H_{ij}(t, y_j) \\ &\quad - \sum_{i=1}^N \sum_{j=1}^N \epsilon c_{ij} e_i^T H_{ij}(t, x_j) - \sum_{i=1}^N \sum_{j=1}^N e_i^T ((d_{ij} - \epsilon c_{ij}) H_{ij}(t, y_j)) \\ &\quad - \sum_{i=1}^N k^* e_i^T e_i \\ &\leq \sum_{i=1}^N \alpha_i e_i^T e_i + \sum_{i=1}^N \sum_{j=1}^N \epsilon c_{ij} e_i^T (H_{ij}(t, y_j) - H_{ij}(t, x_j)) \\ &\quad - k^* \sum_{i=1}^N e_i^T e_i \\ &\leq \sum_{i=1}^N (\alpha_i - k^*) e_i^T e_i + \sum_{i=1}^N \sum_{j=1}^N \alpha_2 \epsilon |c_{ij}| \cdot \|e_i^T\| \cdot \|e_j\| \\ &= E^T Q E, \end{aligned}$$

where $E = (\|e_1\|, \dots, \|e_N\|)^T \in \mathbb{R}^N$, and

$$Q = \begin{cases} q_{ij} = \frac{1}{2} \alpha_2 \epsilon (|c_{ij}| + |c_{ji}|), & i \neq j \\ q_{ii} = \alpha_2 \epsilon \cdot |c_{ii}| + \alpha_1 - k^*. \end{cases}$$

It is obvious that there exists a sufficiently large positive constant k^* such that Q is negative definite. Namely,

$$\dot{V} \leq \lambda_{\max}(Q) E^T E \leq 0, \quad (9)$$

which means that $\|E\|$, $|d_{ij}(t) - \epsilon c_{ij}|$ are bounded. From Eq. (9), we have for any t ,

$$\begin{aligned} \int_0^t E^T(\tau) E(\tau) d\tau &\leq \frac{1}{\lambda_{\max}(Q)} \int_0^t \dot{V}(\tau) d\tau \\ &= \frac{-1}{\lambda_{\max}(Q)} (V(0) - V(t)) \\ &\leq \frac{-1}{\lambda_{\max}(Q)} V(0), \end{aligned}$$

then $E(t) \in L^2$, so $e_i(t) \in L^2$.

From Eq. (8) and conditions (4) and (5), where we have that \dot{e}_i ($i=1, \dots, N$) is bounded, it follows from Barbalat's lemma²² that $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ for $i=1, \dots, N$.

Note that

$$\begin{aligned} \dot{e}_i &= f_i(t, y_i) - f_i(t, x_i) + \sum_{j=1}^N \epsilon c_{ij} (H_{ij}(t, y_j) - H_{ij}(t, x_j)) - k_i e_i \\ &\quad + [H_{i1}(t, y_1), \dots, H_{iN}(t, y_N)] \xi_i, \end{aligned}$$

$$\dot{\xi}_i = -[\delta_{i1} e_i^T H_{i1}(t, y_1), \dots, \delta_{iN} e_i^T H_{iN}(t, y_N)]^T,$$

where $\xi_i = [d_{i1}(t) - \epsilon c_{i1}, \dots, d_{iN}(t) - \epsilon c_{iN}]^T$. Let $\eta_i(t) = f_i(t, y_i) - f_i(t, x_i) + \sum_{j=1}^N \epsilon c_{ij} (H_{ij}(t, y_j) - H_{ij}(t, x_j)) - k_i e_i$. It is easy to know that $\lim_{t \rightarrow \infty} \eta_i(t) = 0$ as $\lim_{t \rightarrow \infty} [y_i(t) - x_i(t)] = \lim_{t \rightarrow \infty} e_i(t) = 0$. Since $g_i^T(t) = [H_{i1}(t, y_1(t)), \dots, H_{iN}(t, y_N(t))]^T$ is PE, and $g_i(t)$ and $\dot{g}_i(t)$ are bounded, by Lemma 1, we can conclude that $\lim_{t \rightarrow \infty} (d_{ij}(t) - \epsilon c_{ij}) = 0$ for all $i, j=1, 2, \dots, N$. ■

Remark 2: Since the system described by Eqs. (7) and (8) is time varying (even if the functions f_i and H_{ij} do not explicitly depend on time t), LaSalle's invariance principle cannot be applied directly. Meanwhile, the feedback gains k_i ($i=1, \dots, N$) are convergent (see the proof in the Appendix).

Remark 3: In our method, the outer-coupling matrix C needs not to be symmetric or irreducible. So this theorem can be applied to a wide class of complex dynamical networks.

Remark 4: Consider that the functions f_i and H_{ij} do not explicitly depend on time t and x_i , y_i are bounded for all i . Combine the i th node system of the drive network (3) and response network (6) into the system as follows:

$$\dot{z}_i = F_i + k_i(t) \sum_{j=1}^2 \tilde{a}_{ij} z_j, \quad i=1, 2,$$

where $z_1 = x_i, z_2 = y_i$, $F_1 = f_i(x_i) + \epsilon \sum_{j=1}^N c_{ij} H_{ij}(x_j)$, $F_2 = f_i(y_i) + \sum_{j=1}^N d_{ij}(t) H_{ij}(y_j)$, $k_i(t)$ is as in Eq. (6), and the coupling matrix $\tilde{A} = [\tilde{a}_{ij}] = [0, 0; 1, -1]$. Note that all the elements of z_1 and z_2 are coupled in z_2 system. From Theorem 1 and Remark 1 in Ref. 13, we can obtain that a sufficiently large coupling strength k_i will lead to synchronization. However, with only a partial coupling of z_1 and z_2 , for example, Rössler system with the first component coupling,⁹ the large coupling strength may lead to desynchronization. In this paper, all the states of the i th node system of the drive and response networks are coupled, thus avoid the case that the large coupling strength may lead to desynchronization. Therefore, our approach is consistent with the master stability function method.³

IV. TOPOLOGY IDENTIFICATION OF DYNAMICAL NETWORKS WITH COUPLING DELAY

In this section, we consider a dynamical network with coupling delay. Each node in the network represents an m -dimensional dynamical system described by

$$\dot{x}_i = f_i(t, x_i) + \epsilon \sum_{j=1}^N c_{ij} H_{ij}(t, x_j(t - \tau_j^i(t))), \quad i=1, 2, \dots, N, \quad (10)$$

where $x_i \in \mathbb{R}^m$ is the state variables of node i , $f_i: \mathbb{R} \times \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a continuous function, $\epsilon > 0$ is the known coupling strength, H_{ij} is the nonlinear inner-coupling function, $\tau_j^i(t) \geq 0$ represents the transmission delay of the data sent from agent j to agent i , and the outer-coupling matrix $C = (c_{ij})_{N \times N}$ is defined the same as in Sec. III.

Assumption 2: For any i, j , the delay function $\tau_j^i(t)$ is

differentiable with bounded derivative and satisfies $\dot{\tau}_j^i(t) < r < 1$.

Remark 5: Assumption 2 requires that the function $\phi_{ij}(t) = t - \tau_j^i(t)$ is increasing with respect to t . This coincides with the physical constraint that the data sent earlier from system j to system i will be received earlier by system i .

Design the response network as

$$\begin{aligned} \dot{y}_i &= f_i(t, y_i) + \sum_{j=1}^N d_{ij}(t) H_{ij}(t, y_j(t - \tau_j^i(t))) - k_i e_i, \\ i &= 1, 2, \dots, N, \end{aligned} \quad (11)$$

with the adaptive feedback law

$$\begin{cases} \dot{d}_{ij} = -\delta_{ij} e_i^T H_{ij}(t, y_j(t - \tau_j^i(t))) \\ \dot{k}_i = s_i e_i^T e_i, \end{cases} \quad (12)$$

where $e_i = y_i - x_i$ is the synchronization error, k_i and d_{ij} are the adaptive parameters of the response system, and δ_{ij} and s_i are two positive constants.

In the following, we show that c_{ij} can be identified using the response networks (11) and (12). The synchronization errors between network (10) and network (11) can be written as follows: for all $i = 1, 2, \dots, N$,

$$\begin{aligned} \dot{e}_i &= f_i(t, y_i) - f_i(t, x_i) + \sum_{j=1}^N d_{ij}(t) H_{ij}(t, y_j(t - \tau_j^i(t))) \\ &\quad - \epsilon \sum_{j=1}^N c_{ij} H_{ij}(t, x_j(t - \tau_j^i(t))) - k_i(t) e_i. \end{aligned} \quad (13)$$

Theorem 2: Suppose Assumptions 1 and 2 hold, and H_{ij} is bounded. Then, by updated law (12), $\lim_{t \rightarrow \infty} \|x_i - y_i\| = 0$ for all i . Furthermore, if for any i , $\tilde{g}_i^T(t) = [H_{i1}(t, y_1(t - \tau_1^i(t))), \dots, H_{iN}(t, y_N(t - \tau_N^i(t)))]^T$ is PE, and $\tilde{g}_i(t)$ is bounded, then $\lim_{t \rightarrow \infty} (d_{ij} - \epsilon c_{ij}) = 0$.

Proof: Construct the function

$$\begin{aligned} V &= \frac{1}{2} \sum_{i=1}^N e_i^T e_i + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{1}{\delta_{ij}} (d_{ij} - \epsilon c_{ij})^2 \\ &\quad + \beta \sum_{i=1}^N \sum_{j=1}^N \int_{t-\tau_j^i(t)}^t e_j^T(\zeta) e_j(\zeta) d\zeta + \frac{1}{2} \sum_{i=1}^N \frac{1}{s_i} (k_i - k^*)^2, \end{aligned} \quad (14)$$

where k^* is positive constant. Meanwhile, the derivative of V along trajectories of the dynamical networks (12) and (13) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N e_i^T [f_i(y_i) - f_i(x_i)] + \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T H_{ij}(t, y_j(t - \tau_j^i(t))) \\ &\quad - \sum_{i=1}^N \sum_{j=1}^N \epsilon c_{ij} e_i^T H_{ij}(t, x_j(t - \tau_j^i(t))) \\ &\quad - \sum_{i=1}^N \sum_{j=1}^N e_i^T [(d_{ij} - \epsilon c_{ij}) H_{ij}(t, y_j(t - \tau_j^i(t)))] - \sum_{i=1}^N k^* e_i^T e_i \\ &\quad + \beta N \sum_{i=1}^N e_i^T e_i(t) - \beta \sum_{i=1}^N \sum_{j=1}^N e_i^T (t - \tau_j^i(t)) e_i(t - \tau_j^i(t)) \end{aligned}$$

$$\begin{aligned} &\leq \sum_{i=1}^N (\alpha_1 + \beta N - k^*) e_i^T(t) e_i(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \epsilon c_{ij} e_i^T (H_{ij}(t, y_j(t - \tau_j^i(t))) - H_{ij}(t, x_j(t - \tau_j^i(t)))) \\ &\quad - \beta \sum_{i=1}^N \sum_{j=1}^N (1 - \dot{\tau}_j^i(t)) e_j^T(t - \tau_j^i(t)) e_j(t - \tau_j^i(t)). \end{aligned}$$

As the element of the matrix C is constant, we can assume that the $c_M = \max_{i,j=1,\dots,N} \{c_{ij}\}$. By Assumption 2, it holds that

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^N (\alpha_1 + \beta N - k^*) \|e_i(t)\|^2 \\ &\quad + \gamma \sum_{i=1}^N \sum_{j=1}^N \|e_i\| \cdot \|e_j(t - \tau_j^i(t))\| \\ &\quad - \beta(1 - r) \sum_{i=1}^N \sum_{j=1}^N \|e_j(t - \tau_j^i(t))\|^2, \end{aligned} \quad (15)$$

$$\begin{aligned} &\leq \sum_{i=1}^N \left(\alpha + \beta N - k^* + \frac{\gamma N}{2} \right) \|e_i(t)\|^2 \\ &\quad - \left(\beta(1 - r) - \frac{\gamma}{2} \right) \sum_{i=1}^N \sum_{j=1}^N \|e_j(t - \tau_j^i(t))\|^2, \end{aligned}$$

where $\gamma = \epsilon c_M \alpha_2$ and r is in Assumption 2. Thus, letting

$$\beta = \frac{\gamma}{2(1 - r)}, \quad k^* = \alpha + \beta N + \frac{\gamma N}{2} + 1, \quad (16)$$

we have

$$\dot{V} \leq - \sum_{i=1}^N \|e_i(t)\|^2. \quad (17)$$

By similar reasoning as in the proof of Theorem 1, we can conclude that $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ for $i = 1, \dots, N$.

Now, from Eqs. (13) and (12), it follows that

$$\begin{aligned} \dot{e}_i(t) &= f_i(t, y_i) - f_i(t, x_i) \\ &\quad + \sum_{j=1}^N \epsilon c_{ij} (H_{ij}(t, y_j(t - \tau_j^i(t))) - H_{ij}(t, x_j(t - \tau_j^i(t)))) - k_i e_i \\ &\quad + [H_{i1}(t, y_1(t - \tau_1^i(t))), \dots, H_{iN}(t, y_N(t - \tau_N^i(t)))] \xi_i, \end{aligned} \quad (18)$$

$$\begin{aligned} \xi_i &= -[\delta_{i1} e_i^T H_{i1}(t, y_1(t - \tau_1^i(t))), \dots, \\ &\quad \delta_{iN} e_i^T H_{iN}(t, y_N(t - \tau_N^i(t)))]^T, \end{aligned} \quad (19)$$

where $\xi_i = [d_{i1} - \epsilon c_{i1}, \dots, d_{iN} - \epsilon c_{iN}]^T$. Since $\tilde{g}_i^T(t)$ is PE, and $\tilde{g}_i(t), \tilde{g}_i^T(t)$ are bounded, by Lemma 1, we can conclude that $\lim_{t \rightarrow \infty} (d_{ij}(t) - \epsilon c_{ij}) = 0$ for $i, j = 1, 2, \dots, N$.

Remark 6: By Lemma 2 in Ref. 34, we can conclude in the proof of Theorem 2 that $\lim_{t \rightarrow \infty} \tilde{g}_i(t) \xi_i(t) = 0$. It can also be proved that $\lim_{t \rightarrow \infty} \xi_i(t) = \xi_i^*$. Therefore, the result of Theo-

rem 2 can be justified if, instead of the PE assumption of $\tilde{g}_i^T(t)$, we have the following condition H1.

(H1) For any $N \times 1$ vector $\eta \neq 0$, $\tilde{g}_i^T(t)\eta \rightarrow 0$ as $t \rightarrow \infty$.

In addition, the PE condition implies the condition H1. See the Appendix for the detailed proof of the claims above. According to Lemma 8 in Ref. 23, it can be shown that H1 also implies the PE condition.

V. NUMERICAL SIMULATIONS

In this section, we show three illustrative examples that validate our results in Secs. III and IV. In all the three examples, each node in the network represents a three-dimensional neural system, which is described by

$$\frac{dx_i}{dt} = -I_3 x_i + TM(x_i), \quad (20)$$

where $x_i = [x_{i1}, x_{i2}, x_{i3}]^T \in \mathbb{R}^3$,

$$T = \begin{bmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{bmatrix}, \quad (21)$$

I_3 is the 3×3 unity matrix, and $M(x_i) = [m(x_{i1}), m(x_{i2}), m(x_{i3})]^T$ with $m(u) = (|u+1| - |u-1|)/2$. As indicated in Ref. 35, system (20) has a double-scrolling chaotic attractor.

Example 1: First, let us consider a weighted complex dynamical network being composed of $N=6$ nodes and without coupling delay. The outer-coupling matrix is chosen as

$$C = \begin{bmatrix} -4 & 0 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 0 & 2 & 0 \\ 0 & 1 & -4 & 1 & 2 & 0 \\ 1 & 0 & 4 & -6 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 4 & 3 & -8 \end{bmatrix}.$$

For simplicity, let $H_{ij}(t, x_j) = x_j$ and $\epsilon = 0.01$ in Eq. (3). In the simulations, the parameters in the adaptive feedback law (7) are set as $\delta_{ij} = 1$ and $s_i = 1$. The initial conditions of the drive network $x_{ij}(0)$, $i=1,2,\dots,N$, $j=1,2,3$ are randomly selected in $[-10,10]$; and the initial condition of the response network $y_{ij}(0)$, $i=1,2,\dots,N$, $j=1,2,3$ are randomly selected in $[0,1]$. In addition, we set $k_i(0)=0, d_{ij}(0)=0$. Figures 1(a) and 1(b) show that the subsystems of response network do not achieve synchronization and the response network synchronizes the drive network asymptotically, respectively. Meanwhile, the feedback gains k_i , $i=1,\dots,N$, are convergent, as shown in Fig. 1(c). Define $W_i(t) = \int_t^{t+T_0} g_i(\tau) g_i^T(\tau) d\tau$. Figure 2(a) shows that $\lambda_{\min}(W_i(t)) > 0$ for all i, t , and $T_0 = 15$, which indicates that $g_i = [y_1, \dots, y_N]^T$ satisfies the PE condition for all i . The parameter identification errors vanish as time increases, as can be seen in Fig. 2(b), which means that $d_{ij}(t)$ approaches ϵc_{ij} asymptotically.

Example 2: Now we choose $\epsilon = 0.5$ in Eq. (3) and other conditions the same as in Example 1. In this case, the coupling strength of the drive network is sufficiently large to result in complete inner synchronization, i.e., $\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0$ for all i, j . By the first part of Theorem

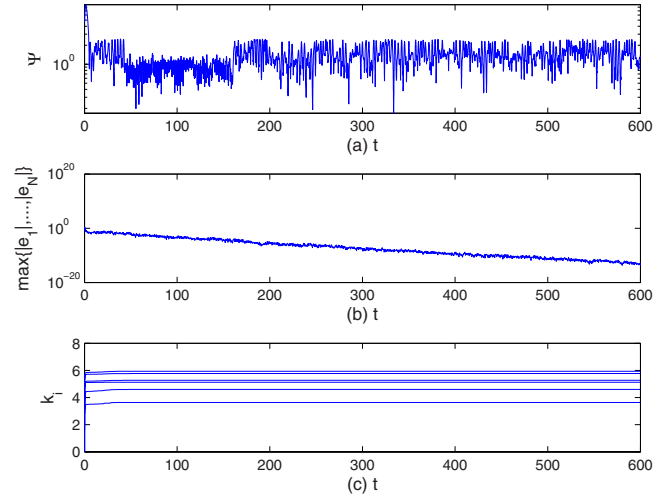


FIG. 1. (Color online) (a) The maximum inner synchronization error Ψ of the response network, where the coupling strength $\epsilon = 0.01$ in Eq. (3). (b) The synchronization error between the drive network and the response network. (c) Time evolution of adaptive feedback gains.

1, the response network still synchronizes the drive network and, as a consequence, also asymptotically achieves complete inner synchronization. However, it can be easily verified that the PE condition is not satisfied, and it is seen from Figs. 3 and 4 that the identification procedure fails.

Example 3: We consider another network with coupling delays, whose outer-coupling matrix is chosen as

$$C = \begin{bmatrix} -4 & 0 & 1 & 1 & 1 & 1 \\ 1 & -3 & 1 & 0 & 0 & 1 \\ 0 & 1 & -5 & 1 & 2 & 1 \\ 1 & 0 & 4 & -6 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 4 & 2 & -7 \end{bmatrix}.$$

We assume that $H_{ij}(t, x_j(t - \tau_{ij}^j)) = x_j(t-1)$ for all $i, j = 1, 2, \dots, N$ and $\epsilon = 0.01$ in Eq. (10). The initial conditions are set as follows: $k_i(0)=0$, $d_{ij}(0)=0$, $x_{ij}(0)$, $j=1,2,3$ are randomly selected in $[-5,5]$, and $y_{ij}(0)$, $j=1,2,3$ are randomly selected in $[0,1]$. Additionally, δ_{ij} and s_i are selected

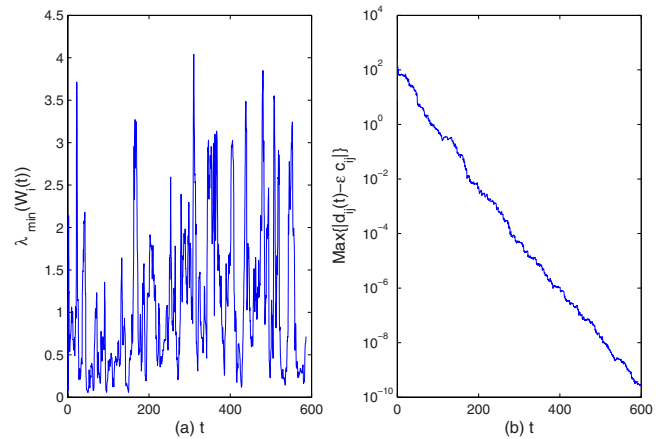


FIG. 2. (Color online) (a) The minimum eigenvalue of the matrix $W_i(t)$. (b) Parameter identification errors.

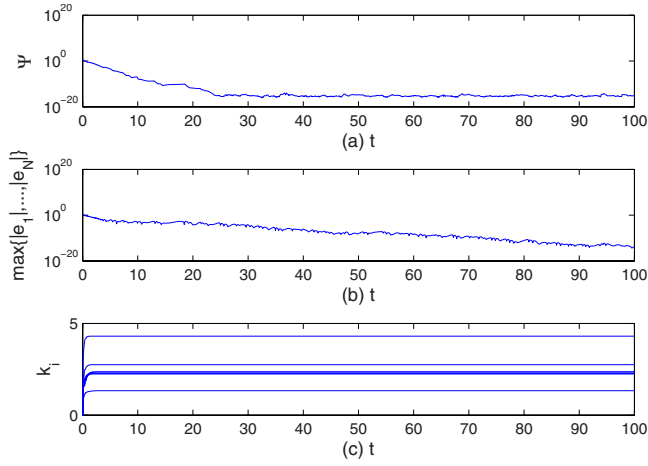


FIG. 3. (Color online) (a) The maximum inner synchronization error Ψ of the response network, where the coupling strength $\epsilon=0.5$ in Eq. (3). (b) The synchronization error between the drive network and the response network. (c) Time evolution of adaptive feedback gains.

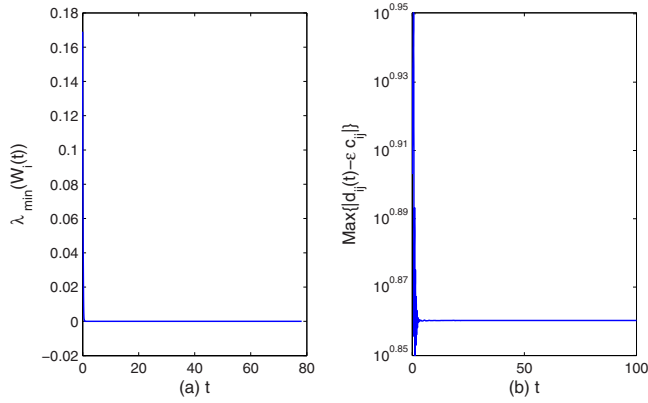


FIG. 4. (Color online) (a) The minimum eigenvalue of the matrix $W_i(t)$. (b) Parameter identification error.

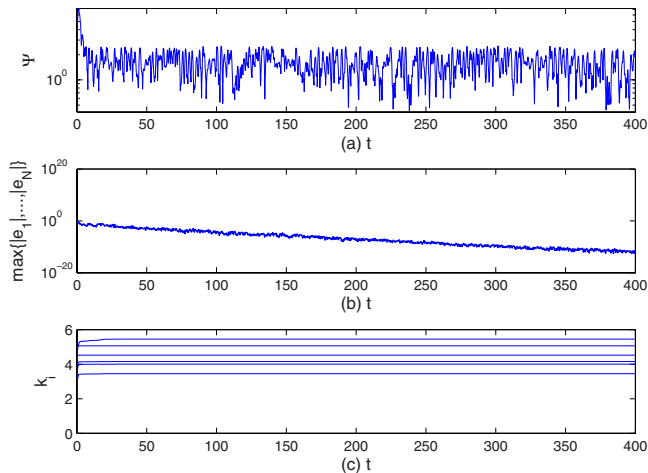


FIG. 5. (Color online) (a) The maximum inner synchronization error Ψ of the response network, where the coupling strength $\epsilon=0.01$ in Eq. (10). (b) The synchronization error between the drive network and the response network. (c) Time evolution of adaptive feedback gains.

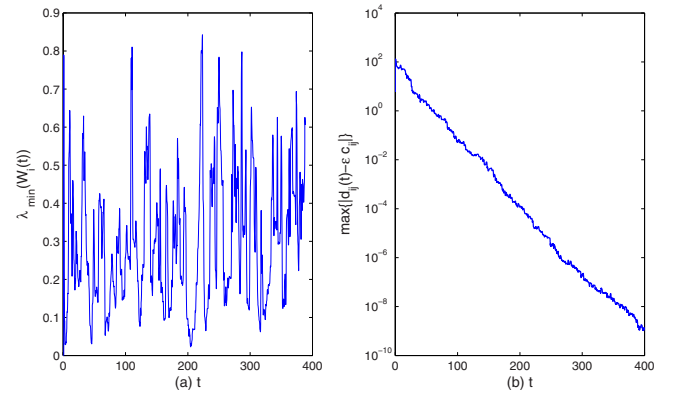


FIG. 6. (Color online) (a) The minimum eigenvalue of the matrix $W_i(t)$. (b) Parameter identification error.

the same as in Example 1. Figures 5(a) and 5(b) show that the subsystems of response network do not achieve synchronization and the response network synchronizes the drive network, respectively. Meanwhile, the feedback gains are convergent, as shown in Fig. 5(c). Similarly as in Example 1, the PE condition is verified with $T_0=10.5$, as shown in Fig. 6(a). The parameter identification error is depicted in Fig. 6(b).

VI. CONCLUSIONS

In this paper, we have proposed adaptive feedback laws to identify the exact topology of the weighted complex dynamical networks. PE conditions prove to guarantee the efficiency of our method for networks with and without coupling delays. The results are applicable to a large class of networks since the coupling matrix need not to be symmetrical or irreducible, and the inner coupling can be nonlinear and time varying.

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APPENDIX: PROOFS OF THE CLAIMS IN REMARK 6

- (1) $\lim_{t \rightarrow \infty} \tilde{g}_i(t) \xi_i(t) = 0$. From Eq. (18), we know that

$$\dot{e}_i(t) = \tilde{f}_i(t) + \tilde{g}_i(t) \xi_i(t),$$

where $\tilde{f}_i(t) = f_i(t, y_i) - f_i(t, x_i) + \sum_{j=1}^N \epsilon c_{ij} (H_{ij}(t, y_j(t - \tau_j^i(t))) - H_{ij}(t, x_j(t - \tau_j^i(t)))) - k_i e_i$. By the boundedness of $\tilde{g}_i(t)$, $\dot{\tilde{g}}_i(t)$, $\xi_i(t)$, and $\dot{\xi}_i(t)$, we have that $\tilde{g}_i(t) \xi_i(t)$ is uniformly continuous. In addition, it follows from $\lim_{t \rightarrow \infty} e_i(t) = 0$ that $\int_{t_0}^{\infty} (\tilde{f}_i(t) + \tilde{g}_i(t) \xi_i(t))$ exists and $\lim_{t \rightarrow \infty} \tilde{f}_i(t) = 0$. Hence, by Lemma 2 in Ref. 34, we conclude that $\lim_{t \rightarrow \infty} \tilde{g}_i(t) \xi_i(t) = 0$.

- (2) $\lim_{t \rightarrow \infty} \xi_i(t)$ exists. From Eq. (12), we see that k_i is increasing as t increases. However, since V is bounded, we know that $\lim_{t \rightarrow \infty} k_i(t)$ exists. On the other hand, from Eq. (17), we have $\int_{t_0}^{\infty} \|e_i(t)\|^2 dt < \infty$. This, together with the fact that $\lim_{t \rightarrow \infty} (t - \tau_j^i(t)) = \infty$, gives that the third term on the right-hand side of Eq. (14) converges to zero as t goes to infinity. Then, by Eq. (14), the convergence of ξ_i follows from that of the Lyapunov function V .
- (3) $\tilde{g}_i^T(t)$ is PE \Rightarrow H1. Since $\tilde{g}_i(t)$ is bounded, let $\|\tilde{g}_i(t)\| < b$, where b is some positive number. According to Definition 1, we have that if $\tilde{g}_i^T(t)$ is PE, then there exist $T_0, \delta_1 > 0$ such that for any $\|\eta\| = 1$,

$$\int_t^{t+T_0} \eta^T \tilde{g}_i^T(\tau) \tilde{g}_i(\tau) \eta d\tau \geq \delta_1 \quad \forall t \geq 0,$$

which leads to

$$\int_t^{t+T_0} \|\tilde{g}_i(\tau) \eta\| d\tau \geq \frac{\delta_1}{b} \quad \forall t \geq 0.$$

Therefore, $\tilde{g}_i(t) \eta \rightarrow 0$.

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