WebAssign

3.6 Derivadas Logaritmos (Homework)

Due: Friday, March 29, 2019 11:59 PM CSTLast Saved: n/a Saving... ()

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The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may *not* grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

Request Extension

Current Score : 51.5 / 46

1. 1.5/1.5 points | Previous Answers SPreCalc6 4.3.001.

 $\log x$ is the exponent to which the base 10 must be raised to get $x \neq 0$. So we can complete the following table for $\log x$.

x	10 ³	10 ²	10 ¹	10 ⁰	10 ⁻¹	10-2	10-3	10 ^{1/2}
log x	3 💉	2 💉	1 1	0 🗸	-1 /	-2 -2	-3 -3	1/2 🗸

2. 1/1 points | Previous Answers SPreCalc6 4.3.008.MI.

Express the equation in exponential form.

(a)
$$\log_8 0.125 = -1$$

\$\$8-1=0.125

$$8^{-1} = 0.125$$

(b)
$$\log_9 81 = 2$$

$$9^2 = 81$$

3. 1/1 points | Previous AnswersSPreCalc6 4.3.012.

Express the equation in exponential form.

(a)
$$ln(x + 5) = 7$$

$$e^7 = x + 5$$

(b)
$$ln(x-7) = 2$$

$$e^2 = x - 7$$

4. 1/1 points | Previous Answers SPreCalc6 4.3.017.

Express the equation in logarithmic form.

(a)
$$e^{x} = 8$$

\$\$x=In(8)
 $\ln 8 = x$
(b) $e^{6} = y$
\$\$6=In(y)

5. 1/1 points | Previous Answers SPreCalc6 4.3.018.

Express the equation in logarithmic form.

```
(a) e^{x} + \frac{1}{1} = 0.3

$$\frac{1}{3} = x + 1$

(b) e^{0.6x} = t

$$\frac{1}{3} = 0.6x
```

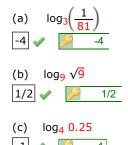
6. 1.5/1.5 points | Previous Answers SPreCalc6 4.3.020.MI.

Evaluate the expression.



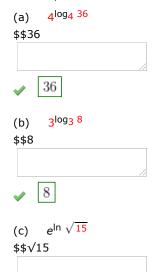
7. 1.5/1.5 points | Previous Answers SPreCalc6 4.3.023.MI.

Evaluate the expression.



8. 1.5/1.5 points | Previous Answers SPreCalc6 4.3.025.MI.

Evaluate the expression.



9. 1/1 points | Previous Answers SPreCalc6 4.3.030.MI.

Use the definition of the logarithmic function to find x.

(a)
$$\log_4 x = 3$$

 $x = 64$ \checkmark 64

10.1/1 points | Previous Answers SPreCalc6 4.3.032.MI.

Use the definition of the logarithmic function to find x.

(a)
$$\log_4 2 = x$$

 $x = 1/2$ \checkmark $1/2$

(b)
$$\log_4 x = 2$$

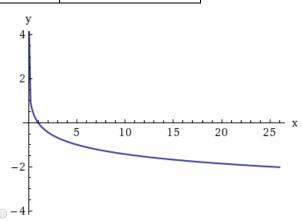
 $x = 16$ \searrow 16

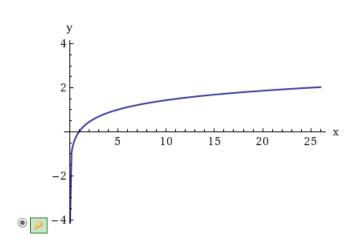
11.1/1 points | Previous Answers SPreCalc6 4.3.042.MI.

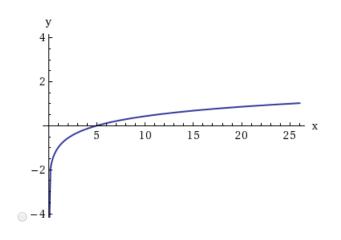
Sketch the graph of the function by plotting points.

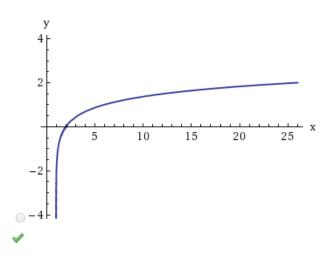
$$g(x) = \log_5 x$$

x	$g(x) = \log_5 x$
1 5 ³	-3 🗸 🛜 -3
1 5 ²	-2 🗸 🔁
<u>1</u> 5	-1 🛹 🔑 -1
1	0 🔷 👂 0
5	1 🕢 👂 1
5 ²	2 💉 👂 2







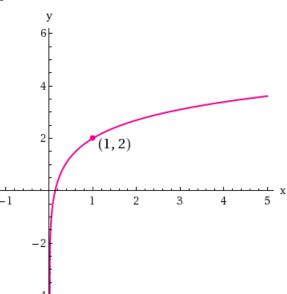


12.1/1 points | <u>Previous Answers</u>SPreCalc6 4.3.050.

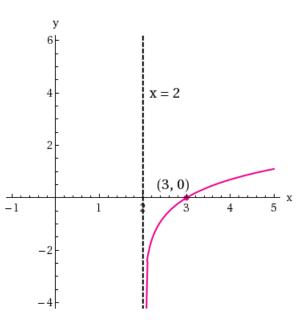
Match the logarithmic function with one of the graphs labeled I or II.

$$f(x) = \ln(x - 2)$$

I



II

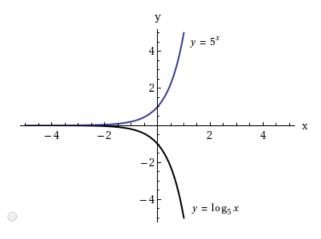


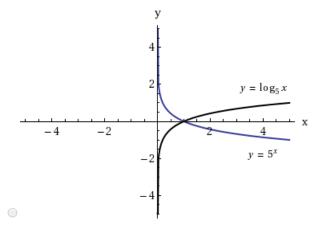
- The graph of $f(x) = \ln(x 2)$ is graph I.
- The graph of $f(x) = \ln(x 2)$ is graph II.

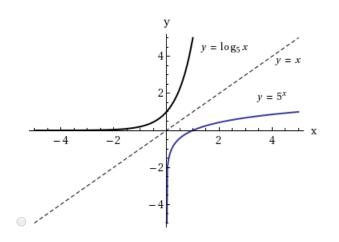
1

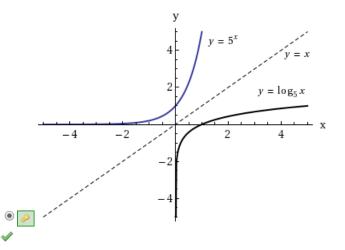
13.1/1 points | Previous Answers SPreCalc6 4.3.051.

Draw the graph of $y = 5^x$, then use it to draw the graph of $y = \log_5 x$.



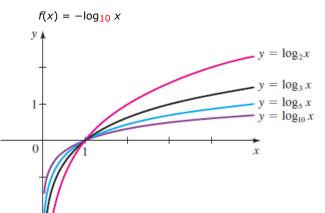


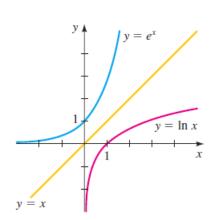


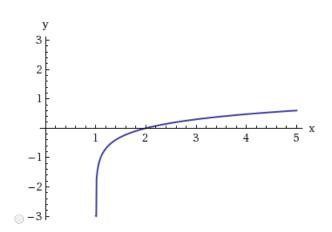


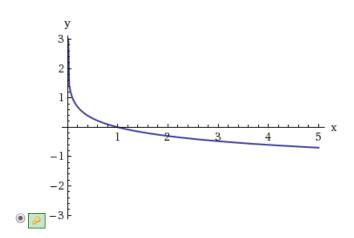
14.2/2 points | Previous Answers SPreCalc6 4.3.054.

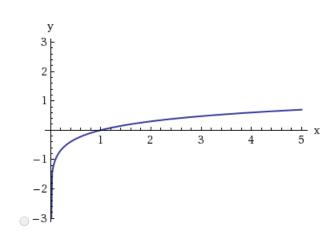
Graph the function, not by plotting points, but by starting from the graphs in the figures below.

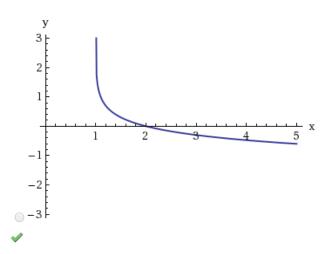












State the domain and the range. (Enter your answers using interval notation.)

 $(0,\infty)$ domain $(0,\infty)$ range $\$\$(-\infty,\infty)$



State the asymptote.

$$$\$x=0$$

$$x=0$$

15.1/1 points | Previous Answers SPreCalc6 4.3.064.

Find the domain of the function. (Enter your answer using interval notation.)

$$f(x) = \log_6(6 - 3x)$$

$$\$\$(-\infty,2)$$

$$\checkmark (-\infty,2)$$

16.1/1 points | Previous Answers SPreCalc6 4.3.065.

Find the domain of the function. (Enter your answer using interval notation.)

$$g(x) = \log_3(x^2 - 4)$$

$$\$\$(-\infty, -2) \cup (2, \infty)$$

$$\checkmark \qquad (-\infty, -2) \cup (2, \infty)$$

17.1/1 points | Previous Answers SPreCalc6 4.3.068.

Find the domain of the function. (Enter your answer using interval notation.)

$$h(x) = \sqrt{x - 3} - \log_3(9 - x)$$
\$\$[3,9)

18.1.5/1.5 points | Previous Answers SPreCalc6 4.5.002.

Let's solve the logarithmic equation $\log 3 + \log(x - 6) = \log x$.

- (c) Now we find x = 9

19.1/1 points | Previous Answers SPreCalc6 4.5.033.MI.

Solve the equation.

20.1/1 points | Previous Answers SPreCalc6 4.5.040.MI.

Solve the logarithmic equation for x.

$$\log(x - 6) = 2$$

$$x = \boxed{106} \checkmark \boxed{9} \qquad 106$$

21.1/1 points | Previous Answers SPreCalc6 4.5.043.

Solve the logarithmic equation for x.

$$4 - \log(5 - x) = 3$$

$$x = \begin{bmatrix} -5 \end{bmatrix} \checkmark \begin{bmatrix} 5 \end{bmatrix}$$

22.1/1 points | Previous Answers SCalcET8 3.6.003.MI.

Differentiate the function.

$$f(x) = \sin(3 \ln(x))$$

$$f'(x) =$$

$$\$ \cos(3\ln(x))(3x)$$

$$\sqrt[3]{\frac{3\cos(3\ln(x))}{x}}$$

Solution or Explanation

Click to View Solution

23.1/1 points | Previous Answers SCalcET8 3.6.004.

Differentiate the function.

$$f(x) = \ln(64 \sin^{2}(x))$$

$$f'(x) = $$[164sin2(x)][128(sin(x))][cos(x)]$$

$$2 \cot(x)$$

Solution or Explanation

Click to View Solution

24.2/0 points | Previous AnswersSCalcET8 3.6.013.

Differentiate the function.

$$G(y) = \ln\left(\frac{(4y+1)^5}{\sqrt{y^2+1}}\right)$$

$$G'(y) = \\ \$\$[(54y+1)(4)] - [(12)(1y2+1)(2y)]$$

$$\sqrt{\frac{16y^2 - y + 20}{(4y+1)(y^2+1)}}$$

Solution or Explanation

$$G(y) = \ln\left(\frac{(4y+1)^{5}}{\sqrt{y^{2}+1}}\right) = \ln(4y+1)^{5} - \ln(y^{2}+1)^{1/2} = 5\ln(4y+1) - \frac{1}{2}\ln(y^{2}+1) \Rightarrow$$

$$G'(y) = 5 \cdot \frac{1}{4y+1} \cdot 4 - \frac{1}{2} \cdot \frac{1}{y^{2}+1} \cdot 2y = \frac{20}{4y+1} - \frac{y}{y^{2}+1} \left[\text{or } \frac{16y^{2}-y+20}{(4y+1)(y^{2}+1)} \right]$$

25.1/1 points | Previous Answers SCalcET8 3.6.015.

Differentiate the function.

$$F(s) = \ln(\ln(6s))$$

$$F'(s) = $$[1ln(6s)][16s][6]$$

$$\frac{1}{s \ln(6s)}$$

Solution or Explanation

$$F(s) = \ln(\ln(6s)) \ \Rightarrow \ F'(s) = \frac{1}{\ln(6s)} \frac{d}{ds} (\ln(6s)) = \frac{1}{\ln(6s)} \cdot \frac{1}{s} = \frac{1}{s \ln(6s)}$$

26.0/1 points | Previous Answers SCalcET8 3.6.021.

Differentiate the function.

$$y = \tan(\ln(ax + b))$$

$$y' = $\$[\sec 2(\ln(ax+b))][1ax+b][a+1]$$

$$\sec^2(\ln(ax+b))\left(\frac{a}{ax+b}\right)$$

$$y = \tan(\ln(ax+b)) \Rightarrow y' = \sec^2(\ln(ax+b)) \cdot \frac{1}{ax+b} \cdot a = \sec^2(\ln(ax+b)) \cdot \frac{a}{ax+b}$$

27.1/2 points | Previous AnswersSCalcET8 3.6.023.

Find
$$y'$$
 and y'' .
$$y = \sqrt{x} \ln(x)$$

$$\$\$[12(x)-12][ln(x)]+[\sqrt{x}][1x]$$

$$y' = \frac{\ln(x)+2}{2\sqrt{x}}$$

$$y'' = \frac{\ln(x)}{(No\ Response)} - \frac{\ln(x)}{4x^{3/2}}$$

Solution or Explanation

$$y = \sqrt{x} \ln(x) \implies y' = \sqrt{x} \cdot \frac{1}{x} + (\ln(x)) \frac{1}{2\sqrt{x}} = \frac{2 + \ln(x)}{2\sqrt{x}}$$

$$\Rightarrow y'' = \frac{2\sqrt{x}(1/x) - (2 + \ln(x))(1/\sqrt{x})}{(2\sqrt{x})^2} = \frac{2/\sqrt{x} - (2 + \ln(x))(1/\sqrt{x})}{4x} = \frac{2 - (2 + \ln(x))}{\sqrt{x}(4x)} = -\frac{\ln(x)}{4x\sqrt{x}}$$

28.2/2 points | Previous Answers SCalcET8 3.6.027.

Differentiate f and find the domain of f. (Enter the domain in interval notation.)

Solution or Explanation

Click to View Solution

29.4/0 points | Previous AnswersSCalcET8 3.6.029.

Differentiate f and find the domain of f. (Enter the domain in interval notation.)

$$f(x) = \ln(x^2 - 18x) \\ \$ [1x2 - 18x][2x - 18]$$
 derivative
$$f'(x) = \frac{2(x - 9)}{x(x - 18)} \\ \$ (-\infty, 0) \cup (18, \infty)$$
 domain
$$(-\infty, 0) \cup (18, \infty)$$

Solution or Explanation

Click to View Solution

30.1/1 points | Previous Answers SCalcET8 3.6.033.

Find an equation of the tangent line to the curve at the given point.

$$y = \ln(x^2 - 4x + 1),$$
 (4, 0)
 $y =$
\$\$4(x-4)+0

Solution or Explanation

$$y = \ln(x^2 - 4x + 1) \Rightarrow y' = \frac{1}{x^2 - 4x + 1} \cdot (2x - 4) \Rightarrow y'(4) = \frac{1}{1} \cdot 4 = 4$$
, so an equation of a tangent line at (4, 0) is $y - 0 = 4(x - 4)$, or $y = 4x - 16$.

31.1/1 points | Previous Answers SCalcET8 3.6.037.

Let
$$f(x) = cx + \ln(\cos(x))$$
. For what value of c is $f'(\pi/4) = 4$? $c = \$$5$

$$f(x) = cx + \ln(\cos(x)) \Rightarrow f'(x) = c + \frac{1}{\cos(x)} \cdot (-\sin(x)) = c - \tan(x).$$

$$f'\left(\frac{\pi}{4}\right) = 4 \Rightarrow c - \tan\left(\frac{\pi}{4}\right) = 4 \Rightarrow c - 1 = 4 \Rightarrow c = 5.$$

32.1/1 points | Previous Answers SCalcET8 3.6.043.

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{5x}$$

$$y' = $$x5x[5ln(x)+5]$$

$$5x^{5x}(1 + ln(x))$$

Solution or Explanation

Click to View Solution

33.1/1 points | Previous Answers SCalcET8 3.6.046.

Use logarithmic differentiation to find the derivative of the function.

$$y = \left(\sqrt{x}\right)^{3x}$$

$$y' = \\ \$\$[32ln(x) + 32][\sqrt{x}]3x$$

$$\sqrt[3]{2} \left(\ln(x) + 1\right) \left(x^{\frac{3}{2}x}\right)$$

Solution or Explanation

Click to View Solution

34.2/0 points | Previous Answers SCalcET8 3.6.052.

Find
$$y'$$
 if $x^y = y^x$.
 $y' = $$In(y)-(yx)In(x)-xy$

$$\frac{\ln(y)-\frac{y}{x}}{\ln(x)-\frac{x}{y}}$$

$$x^{y} = y^{x} \Rightarrow y \ln(x) = x \ln(y)$$

$$\Rightarrow y \cdot \frac{1}{x} + (\ln(x)) \cdot y' = x \cdot \frac{1}{y} \cdot y' + \ln(y)$$

$$\Rightarrow y' \ln(x) - \frac{x}{y}y' = \ln(y) - \frac{y}{x}$$

$$\Rightarrow y' = \frac{\ln(y) - y/x}{\ln(x) - x/y}$$

35.1/1 points | Previous Answers SCalcET8 3.6.502.XP.

Differentiate the function.

$$f(x) = \log_3(xe^x)$$

$$f'(x) =$$

$$\$\$1xln(3) + [log3(e)]$$

$$\frac{1+x}{x \ln(3)}$$

Solution or Explanation Click to View Solution

36.1/0 points | Previous Answers SCalcET8 3.6.AE.006.

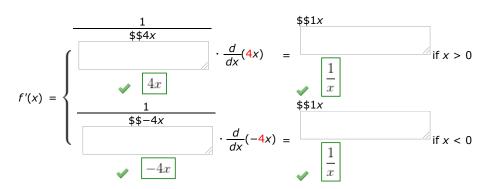
Video Example (1)

EXAMPLE 6 Find f'(x) if $f(x) = \ln(|4x|)$.

SOLUTION Since

$$f(x) = \begin{cases} \ln(4x) & \text{if } x > 0\\ \ln(-4x) & \text{if } x < 0 \end{cases}$$

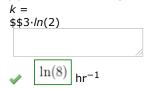
it follows that



37.2/2 points | Previous AnswersSCalcET8 3.8.002.

A common inhabitant of human intestines is the bacterium *Escherichia coli*, named after the German pediatrician Theodor Escherich, who identified it in 1885. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 40 cells.

(a) Find the relative growth rate.



(b) Find an expression for the number of cells after *t* hours.



(c) Find the number of cells after 3 hours.



(d) Find the rate of growth after 3 hours. (Round your answer to the nearest integer.)

(e) When will the population reach a million cells? (Round your answer to two decimal places.)

(a) By Theorem 2,
$$P(t) = P(0)e^{kt} = 40e^{kt}$$
. In 20 minutes $\left(\frac{1}{3} \text{ hour}\right)$, there are 80 cells, so $P\left(\frac{1}{3}\right) = 40e^{k/3} = 80 \implies e^{k/3} = 2 \implies k/3 = \ln(2) \implies k = 3\ln(2) = \ln(2^3) = \ln(8)$.

(b)
$$P(t) = 40e^{(\ln(8))t} = 40 \cdot 8^t$$

(c)
$$P(3) = 40 \cdot 8^3 = 40 \cdot 2^9 = 20,480$$
 cells

(d)
$$\frac{dP}{dt} = kP \Rightarrow P'(3) = kP(3) = (\ln(8))P(3) \approx 42,587 \text{ cells/h}$$

(e)
$$P(t) = 10^6 \Leftrightarrow 40 \cdot 8^t = 1,000,000 \Leftrightarrow 8^t = 25,000 \Leftrightarrow t \ln(8) = \ln(25,000) \Leftrightarrow t = \frac{\ln(25,000)}{\ln(8)} \text{ h} \approx 4.87$$

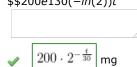
38.1/1.5 points | Previous Answers SCalcET8 3.8.009.MI.

The half-life of cesium-137 is 30 years. Suppose we have a 200-mg sample.

(a) Find the mass that remains after t years.

$$y(t) =$$

\$\$200e130(-In(2))t



(b) How much of the sample remains after 130 years? (Round your answer to two decimal places.)

(c) After how long will only 1 mg remain? (Round your answer to one decimal place.)

$$t = 199.3 \times 229.3 \text{ yr}$$

Solution or Explanation

If y(t) is the mass (in mg) remaining after t years then (a)

$$y(t) = y(0)e^{kt}$$
$$= 200e^{kt}.$$

$$y(30) = \frac{200e^{30k}}{2}$$
$$= \frac{1}{2}(200)$$

$$e^{30k} = \frac{1}{2}$$

$$k = -\frac{\ln(2)}{30}$$

$$y(t) = 200e^{-(\ln(2))t/30}$$

= 200 · 2^{-t/30} mg

(b)
$$y(130) = 200 \cdot 2^{-130/30}$$

 $\approx 9.92 \text{ mg}$

(c)
$$200e^{-(\ln(2))t/30} = 1$$

 $-\frac{(\ln(2))t}{30} = \ln(\frac{1}{200})$
 $t = -30 \frac{\ln(\frac{1}{200})}{\ln(2)}$
 $\approx 229.3 \text{ years}$

39.-/1 pointsSCalcET8 3.8.011.

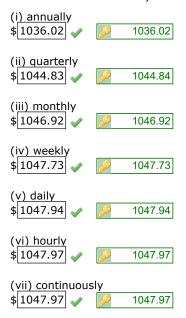
Scientist can determine the age of ancient objects by a method called *radiocarbon dating*. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, ¹⁴C, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates ¹⁴C through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of ¹⁴C begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially.

A parchment fragment was discovered that had about 74% as much 14 C radioactivity as does plant material on Earth today. Estimate the age of the parchment. (Round your answer to the nearest hundred years.)

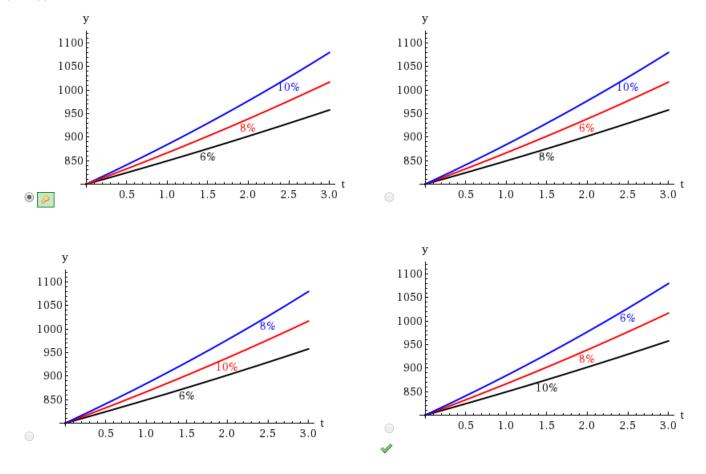
[(No Response)] 2500 yr

Solution or Explanation Click to View Solution 40.2/2 points | Previous Answers SCalcET8 3.8.020.

(a) If \$800 is borrowed at 9% interest, find the amounts due at the end of 3 years if the interest is compounded as follows. (Round your answers to the nearest cent.)



(b) Suppose \$800 is borrowed and the interest is compounded continuously. If A(t) is the amount due after t years, where $0 \le t \le 3$, graph A(t) for each of the interest rates 6%, 8%, and 10% on a common screen.



(a) Using $A = A_0 \left(1 + \frac{r}{n}\right)^{nt}$ with $A_0 = 800$, r = 0.09, and t = 3 we have:

(i) Annually:
$$n = 1$$
; $A = 800 \left(1 + \frac{0.09}{1}\right)^{1 \cdot 3} = 1036.02

(ii) Quarterly:
$$n = 4$$
; $A = 800 \left(1 + \frac{0.09}{4}\right)^{4 \cdot 3} = 1044.84

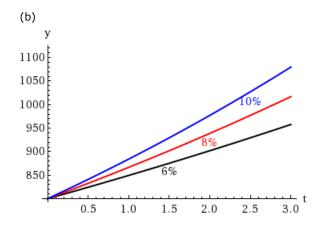
(iii) Monthly:
$$n = 12$$
; $A = 800 \left(1 + \frac{0.09}{12}\right)^{12 \cdot 3} = 1046.92

(iv) Weekly:
$$n = 52$$
; $A = 800 \left(1 + \frac{0.09}{52}\right)^{52 \cdot 3} = 1047.73

(v) Daily:
$$n = 365$$
; $A = 800 \left(1 + \frac{0.09}{365}\right)^{365 \cdot 3} = 1047.94

(vi) Hourly:
$$n = 365 \cdot 24$$
; $A = 800 \left(1 + \frac{0.09}{365 \cdot 24}\right)^{365 \cdot 24 \cdot 3} = 1047.97

(vii) Continuously:
$$A = 800e^{(0.09)3} = $1047.97$$



 $A_{0.10}(3) = $1079.89,$ $A_{0.08}(3) = $1017.00,$ and

 $A_{0.06}(3) = $957.77.$

41.-/0 pointsSCalcET8 3.8.012.

Dinosaur fossils are too old to be reliably dated using carbon-14, which has a half-life of about 5730 years. Suppose we had a 69 million year old dinosaur fossil. How much of the living dinosaur's ¹⁴C would be remaining today? (Round your answer to five decimal places.)

(No Response) D %

Suppose the minimum detectable amount is 0.6%. What is the maximum age of a fossil that we could date using 14 C? (Round your answer to the nearest integer.)

(No Response) 22,292 yr

Solution or Explanation

Let y(t) be the amount of ¹⁴C at time t. Thus, $y(t) = y(0)e^{-kt}$ and k is determined by using the half-life:

$$y(5730) = \frac{1}{2}y(0) \Rightarrow y(0)e^{-k(5730)} = \frac{1}{2}y(0) \Rightarrow e^{-5730k} = \frac{1}{2} \Rightarrow -5730k = \ln \frac{1}{2} \Rightarrow k = -\frac{\ln \frac{1}{2}}{5730} = \frac{\ln 2}{5730}.$$

The fraction of ¹⁴C remaining in a 69 million year old dinosaur is $\frac{y(69 \cdot 10^6)}{y(0)} = e^{-(69 \cdot 10^6) \ln 2/5730} \approx 0$.

With a 0.6% threshold, detecting ¹⁴C requires that

$$\frac{y(t)}{y(0)} \ge 0.006 \ \Rightarrow \ e^{-(\ln 2/5730)t} \ge 0.006 \ \Rightarrow \ -\frac{\ln 2}{5730}t \ge \ln 0.006 \ \Rightarrow \ t \le -\frac{5730 \cdot \ln 0.006}{\ln 2} \approx 42,292 \ \text{years}.$$

42.2/2 points | Previous Answers SCalcET8 3.8.022.

(a) How long will it take an investment to double in value if the interest rate is 11% compounded continuously? (Round your answer to two decimal places.)

(b) What is the equivalent annual interest rate? (Round your answer to two decimal places.)

Solution or Explanation

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