WebAssign
2.7 - 3.1 Derivadas y Derivadas de Polinomios (Homework)

David Corzo
Diferencial, section B, Spring 2019
Instructor: Christiaan Ketelaar

Current Score : 60.4 / 49 **Due :** Wednesday, February 27, 2019 11:59 PM CST**Last Saved : n/a Saving... ()**

The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may *not* grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

Request Extension

1. 1.5/1.5 points | Previous AnswersSCalcET8 2.8.027.

Find the derivative of the function using the definition of derivative.

$$g(x) = \sqrt{1 - x}$$

$g'(x) =$
$$−12(1−x)12

$$\boxed{-\frac{1}{2\sqrt{1-x}}}$$

State the domain of the function. (Enter your answer using interval notation.)
$$(−∞,1]

$$\boxed{(-\infty, 1]}$$

State the domain of its derivative. (Enter your answer using interval notation.)
$$(−∞,1)

$$\boxed{(-\infty, 1)}$$

Solution or Explanation

$$\begin{aligned}
g'(x) &= \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h}\left[\frac{\sqrt{1-(x+h)} + \sqrt{1-x}}{\sqrt{1-(x+h)} + \sqrt{1-x}}\right] \\
&= \lim_{h \to 0} \frac{[1-(x+h)] - (1-x)}{h\left[\sqrt{1-(x+h)} + \sqrt{1-x}\right]} \\
&= \lim_{h \to 0} \frac{-h}{h\left[\sqrt{1-(x+h)} + \sqrt{1-x}\right]} \\
&= \lim_{h \to 0} \frac{-1}{\sqrt{1-(x+h)} + \sqrt{1-x}} \\
&= \frac{-1}{2\sqrt{1-x}}
\end{aligned}$$

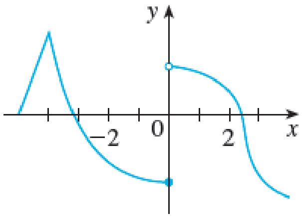
Domain of $g = (-\infty, 1]$, domain of $g' = (-\infty, 1)$.

2. 2/2 points | Previous AnswersSCalcET8 2.8.041.

The graph of *f* is given. State the numbers at which *f* is *not* differentiable.

x = -4 ✓ -4 (smaller value)

x = 0 ✓ 0 (larger value)



Solution or Explanation

Click to View Solution

3. 2/2 points | Previous Answers SCalcET8 2.8.055.

If $f(x) = 2x^2 - x^3$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

$$4x−3x2

$f'(x) =$ $4x - 3x^2$

$$4−6x

$f''(x) =$ $4 - 6x$

$$−6

$f'''(x) =$ -6

$$0

$f^{(4)}(x) =$ 0

Graph f, f', f'', and f''' on a common screen. Are the graphs consistent with the geometric interpretations of these derivatives?

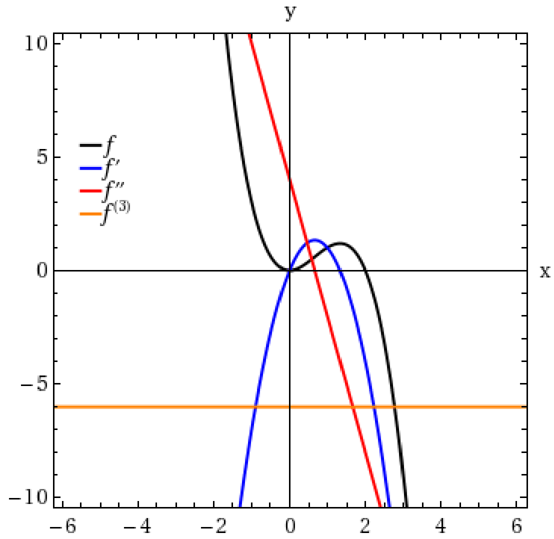
The graphs are are consistent with the geometric interpretations of the derivatives because f' crosses the x-axis crosses the x-axis where f has a slope of m = 0, f'' crosses the x-axis crosses the x-axis where f' has a slope of m = 0, and f''' is a straight line a straight line function equal to the slope of f''.

Solution or Explanation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{[2(x+h)^2 - (x+h)^3] - (2x^2 - x^3)}{h}$$
$$= \lim_{h \to 0} \frac{h(4x + 2h - 3x^2 - 3xh - h^2)}{h}$$
$$= \lim_{h \to 0} (4x + 2h - 3x^2 - 3xh - h^2)$$
$$= 4x - 3x^2$$
$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$
$$= \frac{[4(x+h) - 3(x+h)^2] - (4x - 3x^2)}{h}\}$$
$$= \lim_{h \to 0} \frac{h(4 - 6x - 3h)}{h}$$
$$= \lim_{h \to 0} (4 - 6x - 3h)$$
$$= 4 - 6x$$
$$f'''(x) = \lim_{h \to 0} \frac{f''(x+h) - f''(x)}{h} = \lim_{h \to 0} \frac{[4 - 6(x+h)] - (4 - 6x)}{h} = \lim_{h \to 0} \frac{-6h}{h} = \lim_{h \to 0} (-6) = -6$$
$$f^{(4)} = \lim_{h \to 0} \frac{f'''(x+h) - f'''(x)}{h} = \lim_{h \to 0} \frac{-6 - (-6)}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} (0) = 0$$

The graphs are consistent with the geometric interpretations of the derivatives because f' crosses the x-axis where f has a slope of m = 0, f

$''$ crosses the x-axis where f' has a slope of m = 0, and f''' is a straight line function equal to the slope of f''.



4. 2/2 points | Previous AnswersSCalcET8 2.8.058.

Consider the following function.

$g(x) = x^{2/3}$

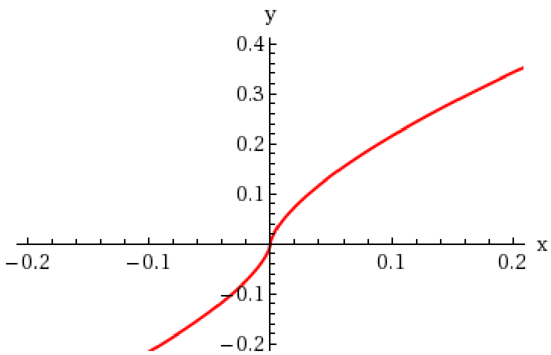
(a) If $a \neq 0$, find $g'(a)$.

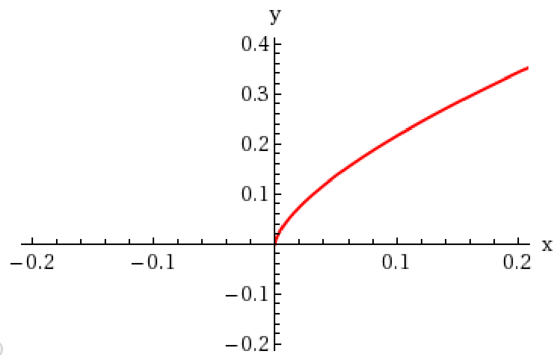
$g'(a) =$

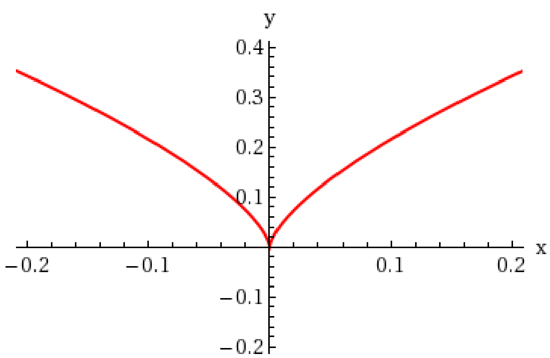
$$23a13

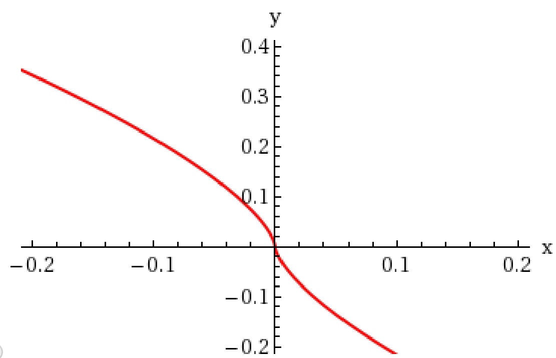
$$\boxed{\frac{2}{3a^{\frac{1}{3}}}}$$

(b) Illustrate that $y = x^{2/3}$ has a vertical tangent line at (0, 0) by graphing the equation.









Solution or Explanation

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5. 2/2 points | Previous AnswersSCalcET8 2.8.059.

Show that the function $f(x) = |x - 5|$ is not differentiable at 5.

We have

$$x-5

$\boxed{x - 5}$ if $x \geq 5$

$f(x) = |x - 5| =$

$$-(x-5)

$\boxed{5 - x}$ if $x < 5$.

The right-hand limit is

$$\lim_{x \to 5^+} \frac{f(x) - f(5)}{x - 5} =$$

$$1

$\boxed{1}$,

and the left-hand limit is

$$\lim_{x \to 5^-} \frac{f(x) - f(5)}{x - 5} =$$

$$-1

$\boxed{-1}$.

Since these limits are not equal, $f'(5) = \lim_{x \to 5} \frac{f(x) - f(5)}{x - 5}$ does not exist and f is not differentiable at 5.

Find a formula for f' and sketch its graph.

$$1

$\boxed{1}$ if $x > 5$

$f'(x) =$

$$-1

$\boxed{-1}$ if $x < 5$

Flash Player version 10 or higher is required for this question.

You can get Flash Player free from Adobe's website.

Submission Data

ray: ((5,-1),(-1,-1)); ray: ((5,1),(6,1))

Solution or Explanation

$$f(x) = |x - 5| = \begin{cases} x - 5 & \text{if } x - 5 \geq 0 \\ -(x - 5) & \text{if } x - 5 < 0 \end{cases} = \begin{cases} x - 5 & \text{if } x \geq 5 \\ 5 - x & \text{if } x < 5 \end{cases}$$

So the right-hand limit is

$$\lim_{x \to 5^+} \frac{f(x) - f(5)}{x - 5} = \lim_{x \to 5^+} \frac{|x - 5| - 0}{x - 5} = \lim_{x \to 5^+} \frac{x - 5}{x - 5} = \lim_{x \to 5^+} 1 = 1,$$

and the left-hand limit is

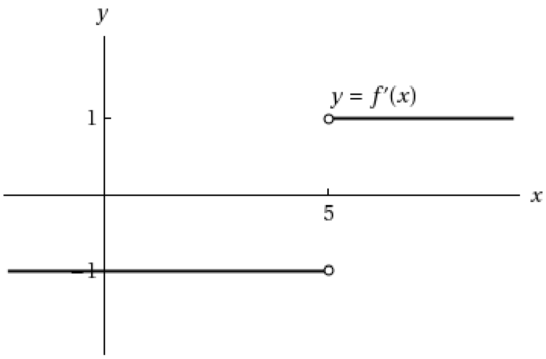
$$\lim_{x \to 5^-} \frac{f(x) - f(5)}{x - 5} = \lim_{x \to 5^-} \frac{|x - 5| - 0}{x - 5} = \lim_{x \to 5^-} \frac{5 - x}{x - 5} = \lim_{x \to 5^-} (-1) = -1.$$

Since these limits are not equal, $f'(5) = \lim_{x \to 5} \frac{f(x) - f(5)}{x - 5}$ does not exist and f is not differentiable at 5.

However, a formula for f' is

$$f'(x) = \begin{cases} 1 & \text{if } x > 5 \\ -1 & \text{if } x < 5. \end{cases}$$

Another way of writing the formula is $f'(x) = \frac{x - 5}{|x - 5|}$.



6. 1.5/1.5 points | Previous AnswersSCalcET8 2.8.029.

Find the derivative of the function using the definition of derivative.

$$G(t) = \frac{1 - 2t}{5 + t}$$

$G'(t) =$
$$−11(5+t)2

$$-\frac{11}{(5+t)^2}$$

State the domain of the function. (Enter your answer using interval notation.)
$$(−∞,−5)∪(−5,∞)

$$(-\infty,-5)\cup(-5,\infty)$$

State the domain of its derivative. (Enter your answer using interval notation.)
$$(−∞,−5)∪(−5, ∞)

$$(-\infty,-5)\cup(-5,\infty)$$

Solution or Explanation

$$\begin{aligned}
G'(t) &= \lim_{h\to 0}\frac{G(t+h)-G(t)}{h}\\
&= \lim_{h\to 0}\frac{\frac{1-2(t+h)}{5+(t+h)}-\frac{1-2t}{5+t}}{h}\\
&= \lim_{h\to 0}\frac{\frac{[1-2(t+h)](5+t)-[5+(t+h)](1-2t)}{[5+(t+h)](5+t)}}{h}\\
&= \lim_{h\to 0}\frac{5+t-10t-2t^2-10h-2ht-(5-10t+t-2t^2+h-2ht)}{h[5+(t+h)](5+t)}\\
&= \lim_{h\to 0}\frac{-10h-h}{h(5+t+h)(5+t)}\\
&= \lim_{h\to 0}\frac{-11h}{h((5+t+h)(5+t)}\\
&= \lim_{h\to 0}\frac{-11}{(5+t+h)(5+t)}\\
&= \frac{-11}{(5+t)^2}
\end{aligned}$$

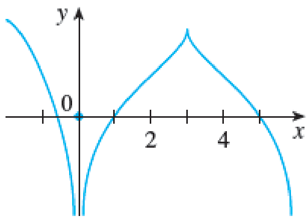
Domain of G = domain of G' = $(-\infty, -5) \cup (-5, \infty)$.

7. 2/2 points | Previous AnswersSCalcET8 2.8.504.XP.

The graph of f is given. State the numbers at which f is not differentiable.

$x =$ 0 ✓ 0 (smaller value)

$x =$ 3 ✓ 3 (larger value)



Solution or Explanation

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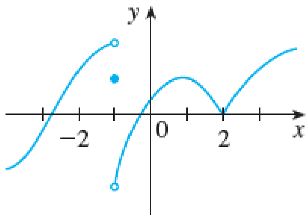
8. 2/2 points | Previous Answers SCalcET8 2.8.506.XP.

The graph of *f* is given. State the numbers at which *f* is not differentiable.

x = -1 ✓ -1 (smaller value)

x = 2 ✓ 2 (larger value)



Solution or Explanation

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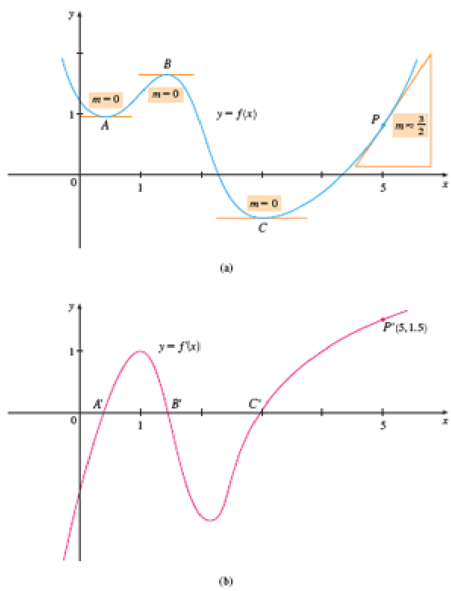
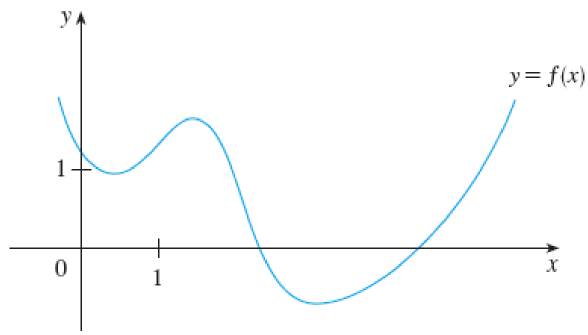
9. 1.8/0 points | Previous Answers SCalcET8 2.8.AE.001.



Video Example

EXAMPLE 1 The graph of a function *f* is given to the left. Use it to sketch the graph of the derivative *f* '.



SOLUTION We can estimate the value of the derivative at any value of *x* by drawing the tangent at the point $(x, f(x))$ and estimating its slope. For instance, for $x = 5$ we draw the tangent at *P* in the figure and estimate its slope to be about $\frac{3}{2}$, so $f'($ 5 ✓ 5 $)$ = 3/2 ✓ 1.5 . This allows us to plot the point $P'($ 5 ✓ 5 , 3/2 ✓ 1.5 $)$ on the graph of f' directly beneath *P*. Repeating this procedure at several points, we get the lower graph shown in the figure. Notice that the tangents at *A*, *B*, and *C* are horizontal, so the derivative is 0 ✓ 0 there and the graph of f' crosses the *x*-axis at the points *A*', *B*', and *C*', directly beneath *A*, *B*, and *C*. Between *A* and *B* the tangents have positive ✓ positive slope, so $f'(x)$ is positive ✓ positive there. But between *B* and *C* the tangents have negative ✓ negative slope, so $f'(x)$ is negative ✓ negative there.

10.2.5/2.5 points | Previous AnswersSCalcET8 2.7.003.

Consider the parabola $y = 7x - x^2$.

(a) Find the slope of the tangent line to the parabola at the point (1, 6).

5 5

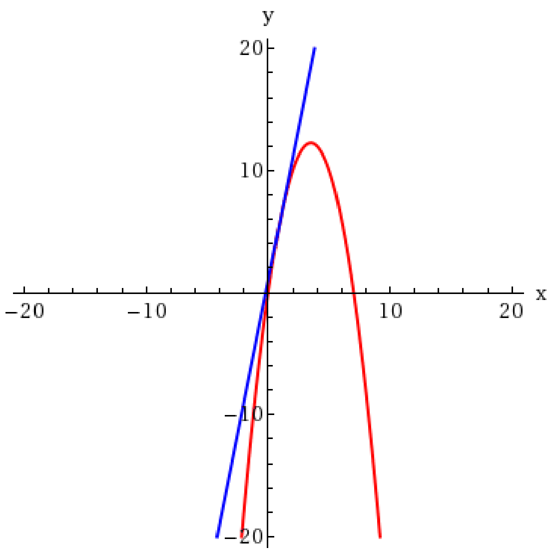
(b) Find an equation of the tangent line in part (a).

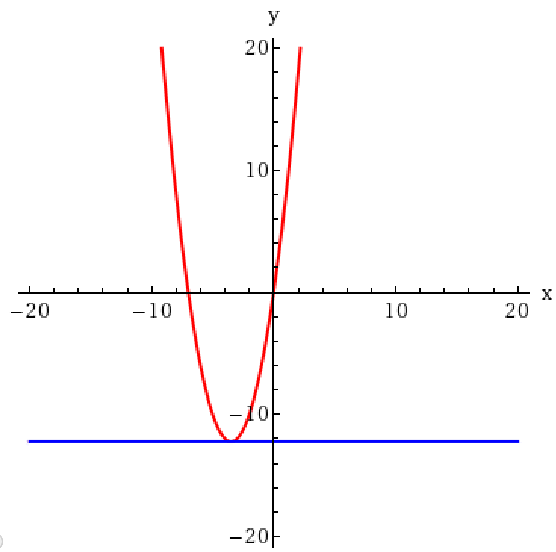
y =

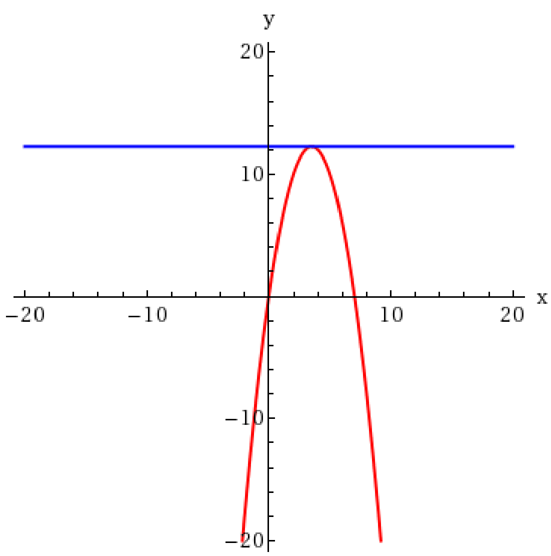
$$5(x−1)+6

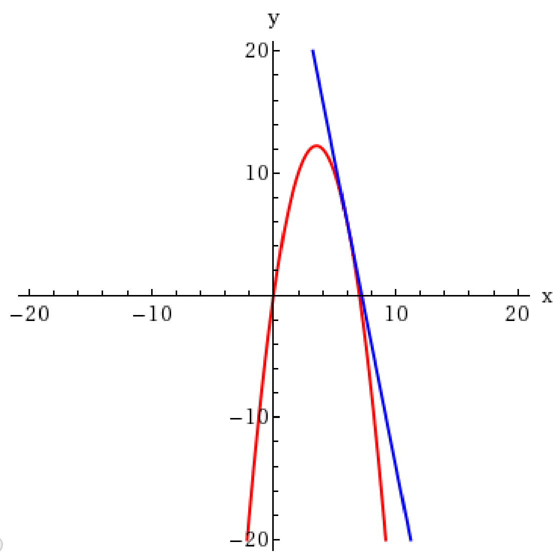
$5x + 1$

(c) Graph the parabola and the tangent line.









Solution or Explanation

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11.2/2 points | Previous AnswersSCalcET8 2.7.006.

Find an equation of the tangent line to the curve at the given point.

$$y = x^3 - 3x + 2, \quad (4, 54)$$

$y =$
$$45x−126

$\boxed{45x - 126}$

Solution or Explanation
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12.1/1 points | Previous AnswersSCalcET8 2.7.031.

Find $f'(a)$.

$$f(x) = 4x^2 - 4x + 3$$

$f'(a) =$
$$8a−4

$\boxed{8a - 4}$

Solution or Explanation
Using the definition with $f(x) = 4x^2 - 4x + 3$.

$$\begin{aligned} f'(a) &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \to 0} \frac{[4(a+h)^2 - 4(a+h) + 3] - (4a^2 - 4a + 3)}{h} \\ &= \lim_{h \to 0} \frac{4a^2 + 8ah + 4h^2 - 4a - 4h + 3 - 4a^2 + 4a - 3}{h} \\ &= \lim_{h \to 0} \frac{8ah + 4h^2 - 4h}{h} \\ &= \lim_{h \to 0} \frac{h(8a + 4h - 4)}{h} = \lim_{h \to 0} (8a + 4h - 4) = 8a - 4. \end{aligned}$$

13.1/1 points | Previous AnswersSCalcET8 2.7.041.

The limit represents the derivative of some function f at some number a. State such an f and a.

$$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$$

- ○ $f(x) = \cos(x), \ a = \pi/4$
- ◉ $f(x) = \cos(x), \ a = \pi$
- ○ $f(x) = \cos(x), \ a = 0$
- ○ $f(x) = \cos(x), \ a = \pi/3$
- ○ $f(x) = \cos(x), \ a = \pi/6$

Solution or Explanation
Click to View Solution

14.1/1 points | Previous AnswersSCalcET8 2.7.503.XP.

The limit represents the derivative of some function *f* at some number *a*. State such an *f* and *a*.

$$\lim_{h \to 0} \frac{\sqrt[4]{16 + h} - 2}{h}$$

- $f(x) = \sqrt[4]{x}$, $a = 2$
- $f(x) = x^4$, $a = 2$
- (selected) $f(x) = \sqrt[4]{x}$, $a = 16$
- $f(x) = \sqrt{x}$, $a = 4$
- $f(x) = \sqrt{x}$, $a = 16$

Solution or Explanation
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15.2/2 points | Previous AnswersSCalcET8 2.7.516.XP.

Find an equation of the tangent line to the curve at the given point.

$$y = \frac{7x}{(x + 1)^2}, \quad (0, 0)$$

$$y=7x

$y = 7x$

Solution or Explanation
Click to View Solution

16. 1.5/0 points | Previous Answers SCalcET8 2.7.AE.004.

Video Example

EXAMPLE 4 Find the derivative of the function
$f(x) = x^2 - 6x + 8$ at the number a.

SOLUTION From the definition we have

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$(a+h)2

$$= \lim_{h \to 0} \frac{\left[\boxed{(a+h)^2} - 6(a+h) + 8\right] - [a^2 - 6a + 8]}{h}$$

a^2 +
$$2ah+h2

$$= \lim_{h \to 0} \frac{\boxed{2ah+h^2} - 6a - 6h + 8 - a^2 + 6a - 8}{h}$$

$$2ah−6h+h2

$$= \lim_{h \to 0} \frac{\boxed{2ah+h^2-6h}}{h}$$

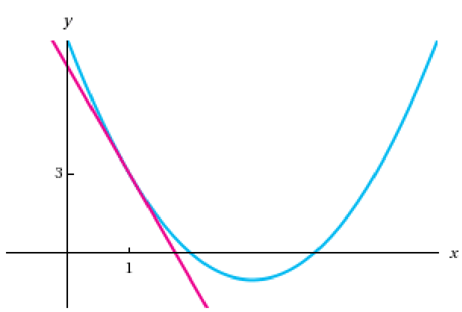
$$2a+h−6

$$= \lim_{h \to 0} \left(\boxed{2a+h-6}\right)$$

$$2a−6

$$= \boxed{2a-6}.$$

17. 1.5/0 points | Previous Answers SCalcET8 2.7.AE.005.



EXAMPLE 5 Find an equation of the tangent line to the parabola
$y = x^2 - 6x + 8$ at the point $(1, 3)$.

SOLUTION From the previous example, we know the derivative of $f(x) = x^2 - 6x + 8$ at the number a is $f'(a) = 2a - 6$. Therefore the slope of the tangent line at $(1, 3)$ is $f'(1) = 2(1) - 6 = \boxed{-4}$ -4 . Thus an equation of the tangent line, shown in the figure, is

$$y - \left(\boxed{3}\ 3\right) = \boxed{-4}\ -4 \left(x - \boxed{1}\ 1\right)$$

or

$y =$
$$−4(x−1)+3

$$\boxed{7-4x}.$$

18.1/1 points | Previous AnswersSCalcET8 3.1.006.

Differentiate the function.

$$g(x) = \frac{5}{8}x^2 - 2x + 12$$

$g'(x) =$
$$54x−2

$\boxed{\frac{5x}{4} - 2}$

Solution or Explanation

$g(x) = \frac{5}{8}x^2 - 2x + 12 \quad \Rightarrow \quad g'(x)\ \frac{5}{8}(2x) - 2(1) + 0 = \frac{5}{4}x - 2$

19.2/2 points | Previous AnswersSCalcET8 3.1.013.

Differentiate the function.

$$F(r) = \frac{8}{r^3}$$

$F'(r) =$
$$−24r−4

$\boxed{-\frac{24}{r^4}}$

Solution or Explanation

$F(r) = \frac{8}{r^3} = 8r^{-3} \quad \Rightarrow \quad F'(r) = 8(-3r^{-4}) = -24r^{-4} = -\frac{24}{r^4}$

20.2/2 points | Previous AnswersSCalcET8 3.1.009.

Differentiate the function.

$$g(x) = x^2(1 - 7x)$$

$g'(x) =$

$$2x−14x2−7x2

$\boxed{2x - 21x^2}$

Solution or Explanation

$g(x) = x^2(1 - 7x) = x^2 - 7x^3 \Rightarrow g'(x) = 2x - 7(3x^2) = 2x - 21x^2$

21.2/2 points | Previous AnswersSCalcET8 3.1.015.

Differentiate the function.

$$R(a) = (5a + 1)^2$$

$R'(a) =$
$$50a+10

$\boxed{50a + 10}$

Solution or Explanation

$R(a) = (5a + 1)^2 = 25a^2 + 10a + 1 \Rightarrow R'(a) = 25(2a) + 10(1) + 0 = 50a + 10$

22.2/2 points | Previous AnswersSCalcET8 3.1.016.

Differentiate the function.

$$h(t) = \sqrt[4]{t} - 4e^t$$

$h'(t) =$
$$14t−34−4et

$$\boxed{\frac{1}{4t^{3/4}} - 4e^t}$$

Solution or Explanation

$h(t) = \sqrt[4]{t} - 4e^t = t^{1/4} - 4e^t \Rightarrow h'(t) = \frac{1}{4}t^{-3/4} - 4(e^t) = \frac{1}{4}t^{-3/4} - 4e^t$

23.2/2 points | Previous AnswersSCalcET8 3.1.019.

Differentiate the function.

$$y = 8e^x + \frac{8}{\sqrt[3]{x}}$$

$y' =$
$$8ex−83x−43

$$\boxed{8e^x - \frac{8}{3}x^{-\frac{4}{3}}}$$

Solution or Explanation

$y = 8e^x + \frac{8}{\sqrt[3]{x}} = 8e^x + 8x^{-1/3} \Rightarrow y' = 8(e^x) + 8\left(-\frac{1}{3}\right)x^{-8/3} = 8e^x - \frac{8}{3}x^{-4/3}$

24.2/2 points | Previous AnswersSCalcET8 3.1.023.MI.

Differentiate the function.

$$y = \frac{8x^2 + 8x + 4}{\sqrt{x}}$$

$y' =$
$$12x32+4x12−2x−12x

$$\boxed{12\sqrt{x} + \frac{4}{\sqrt{x}} - \frac{2}{x\sqrt{x}}}$$

Solution or Explanation
Click to View Solution

25.2/2 points | Previous AnswersSCalcET8 3.1.033.

Find an equation of the tangent line to the curve at the given point.

$$y = 2x^3 - x^2 + 7, \quad (2, 19)$$

$y =$
$$20(x−2)+19

$$\boxed{20x - 21}$$

Solution or Explanation

$y = 2x^3 - x^2 + 7 \Rightarrow y' = 6x^2 - 2x$. At $(2, 19)$, $y' = 6(2)^2 - 2(2) = 20$ and an equation of the tangent line is $y - 19 = 20(x - 2)$ or $y = 20x - 21$.

26. 2/2 points | Previous Answers SCalcET8 3.1.046.

Find the first and second derivative of the function.

$G(r) = \sqrt{r} + \sqrt[4]{r}$

$$12r−12+14r−34

$G'(r) = \boxed{\frac{1}{2}r^{-\frac{1}{2}} + \frac{1}{4}r^{-\frac{3}{4}}}$

$$−14r−32−316r−74

$G''(r) = \boxed{-\frac{1}{4}r^{-\frac{3}{2}} - \frac{3}{16}r^{-\frac{7}{4}}}$

Solution or Explanation

Click to View Solution

27. 2/2 points | Previous Answers SCalcET8 3.1.055.

Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 9$ where the tangent line is horizontal.

$$−2,29

$(x, y) = \left(\boxed{-2, 29}\right)$ (smaller x-value)

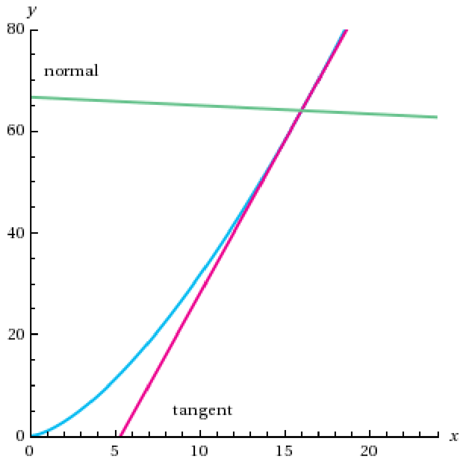
$$1,2

$(x, y) = \left(\boxed{1, 2}\right)$ (larger x-value)

Solution or Explanation

Click to View Solution

28. 1.6/0 points | Previous Answers SCalcET8 3.1.AE.003.



Video Example

EXAMPLE 3 Find the equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point (16, 64). Illustrate the curve and these lines.

SOLUTION The derivative of $f(x) = x\sqrt{x} = xx^{1/2} = x^{3/2}$ is

f '(x) = 3/2 ✓ 3/2 $x^{(3/2\ ✓\ 3/2\ -\ 1)}$ = 3/2 ✓

3/2 $x^{1/2}$ ✓ 1/2

$$32x12

= $3\frac{\sqrt{x}}{2}$ ✓.

So the slope of the tangent line at (16, 64) is $f'(16)$ = 6 ✓ 6 . Therefore an equation of the tangent line is

$y -$ 64 ✓ 64 = 6 ✓ 6 ($x -$ 16 ✓ 16)

or

$y =$

$$6(x−16)+64

$6x - 32$ ✓.

The normal line is perpendicular to the tangent line, so its slope is the negative reciprocal of 6 ✓ 6 , that is, -1/6 ✓ -1/6 . Thus the equation of the normal line is

$y -$ 64 ✓ 64 = -1/6 ✓ -1/6 ($x -$ 16 ✓ 16)

or

$y =$

$$−16(x−16)+64

$-\frac{1}{6}x + \frac{200}{3}$ ✓.

We graph the curve and its tangent line and normal line in the figure to the left.

29. 1/0 points | Previous Answers SCalcET8 3.1.061.

Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $6x + y = 1$.

$y =$

$$−6(x−9)+3

✓ $57 - 6x$

Solution or Explanation

The slope of $y = \sqrt{x}$ is given by $y = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$. The slope of $6x + y = 1$ (or $y = -6x + 1$) is -6, so the desired normal line must have slope -6, and hence, the tangent line to the curve must have slope $\frac{1}{6}$. This occurs if $\frac{1}{2\sqrt{x}} = \frac{1}{6}$ $\Rightarrow$ $\sqrt{x} = 3$ $\Rightarrow$ $x = 9$. When $x = 9$, $y = \sqrt{9} = 3$, and an equation of the normal line is $y - 3 = -6(x - 9)$ or $y = -6x + 57$.

30.2.5/2.5 points | Previous AnswersSCalcET8 3.1.073.

Consider the following function.

$f(x) = |x^2 - 25|$

(a) Find a formula for f'.

$$2x

$f'(x) = \begin{cases} 2x & \text{if } |x| > 5 \\ -2x & \text{if } |x| < 5 \end{cases}$

5

$$−(2x)

5

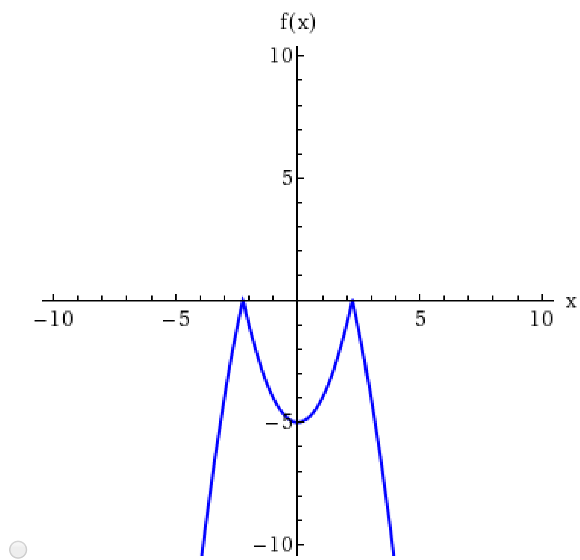
For what values of x is the function not differentiable? (Enter your answers as a comma-separated list.)

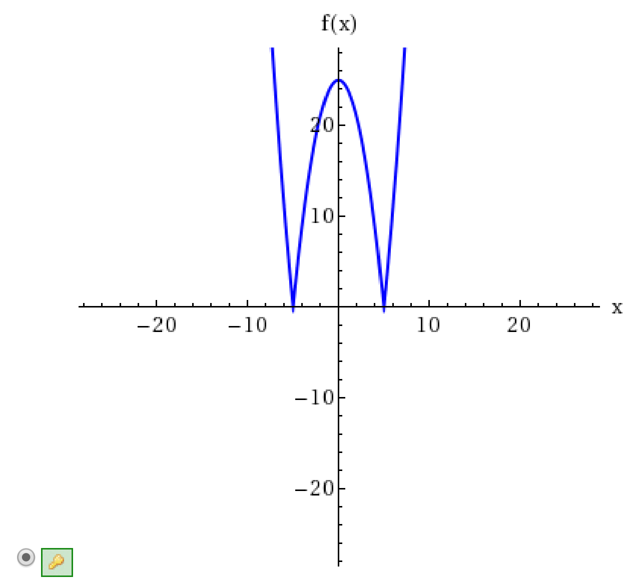
x =

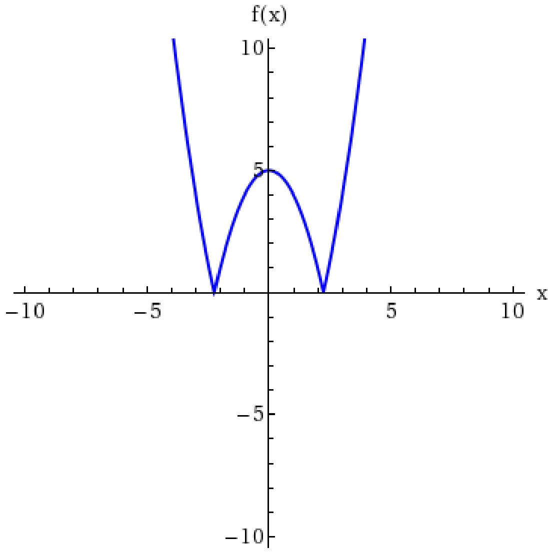
$$−5,5

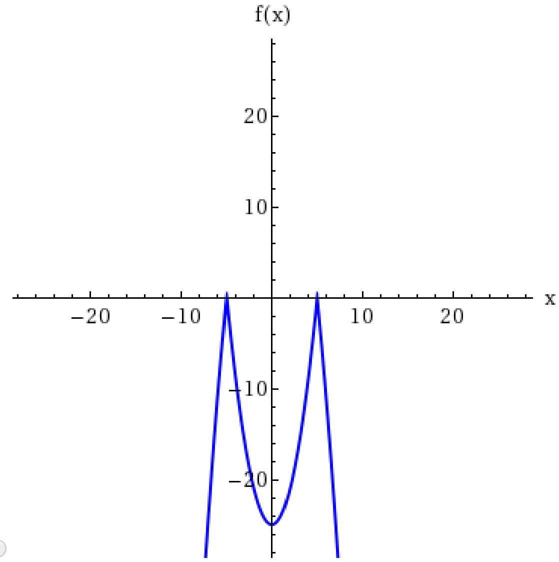
$-5, 5$

(b) Sketch the graph of f.

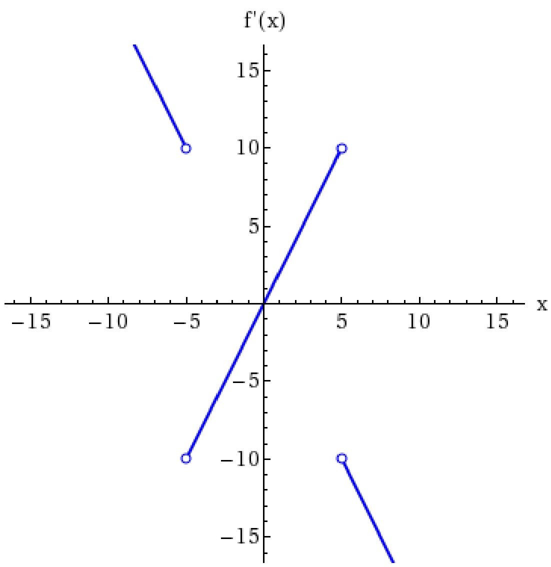


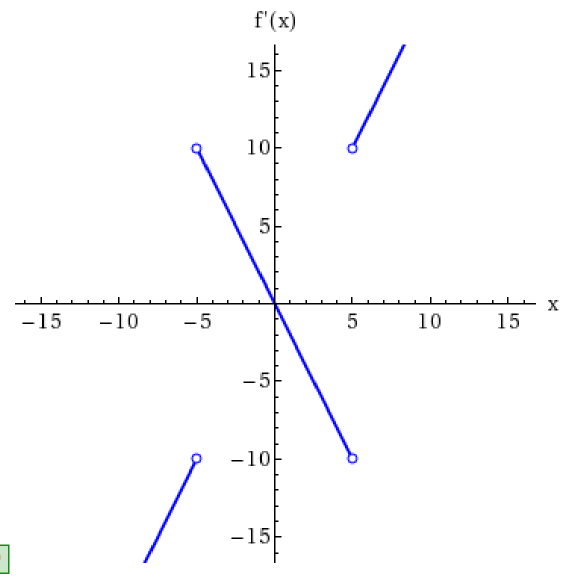


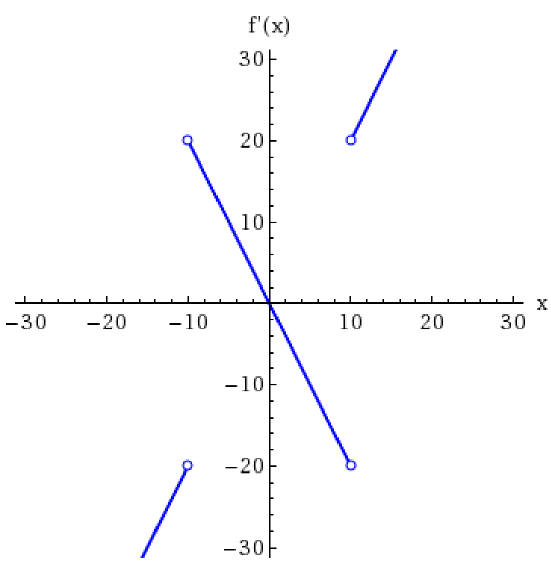


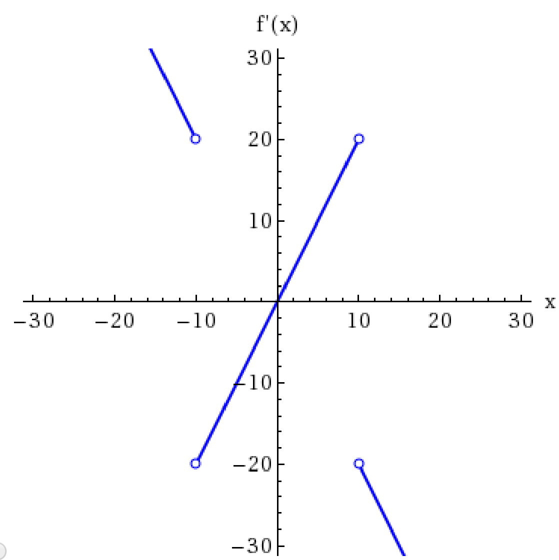


Sketch the graph of f'









Solution or Explanation

(a) Note that $x^2 - 25 < 0$ for $x^2 < 25 \Leftrightarrow |x| < 5 \Leftrightarrow -5 < x < 5$. So

$$f(x) = \begin{cases} x^2 - 25 & \text{if } x \le -5 \\ -x^2 + 25 & \text{if } -5 < x < 5 \\ x^2 - 25 & \text{if } x \ge 5 \end{cases} \Rightarrow f'(x) = \begin{cases} 2x & \text{if } x < -5 \\ -2x & \text{if } -5 < x < 5 \\ 2x & \text{if } x > 5 \end{cases} = \begin{cases} 2x & \text{if } |x| > 5 \\ -2x & \text{if } |x| < 5 \end{cases}$$

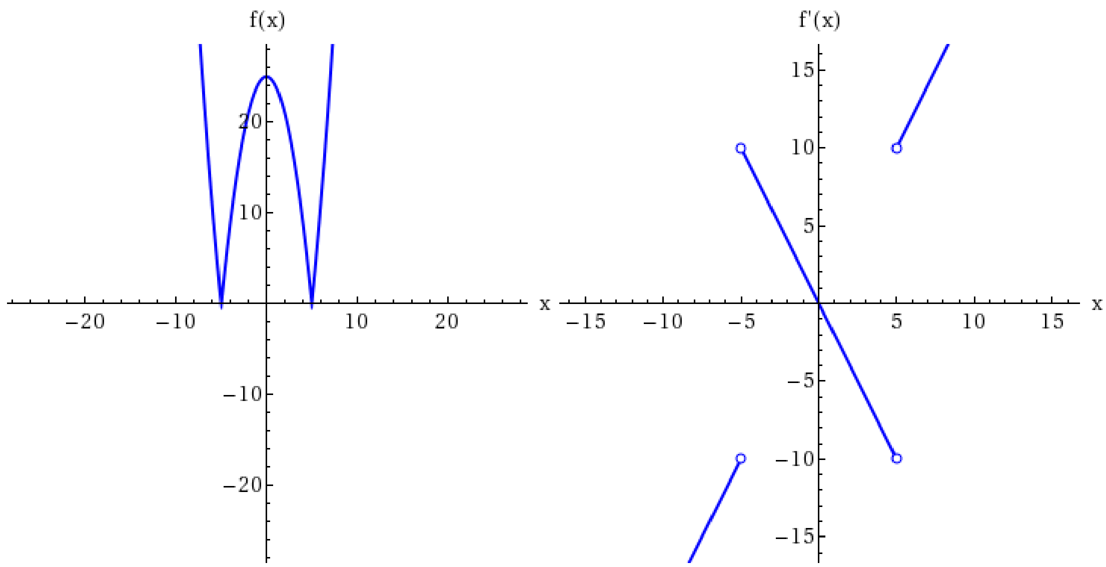
To show that $f'(5)$ does not exist we investigate $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$ by computing the left- and right-hand derivatives.

$$f'_{-}(5) = \lim_{h \to 0^-} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0^-} \frac{[-(5+h)^2 + 25] - 0}{h} = \lim_{h \to 0^-} (-10 - h) = -10 \text{ and}$$

$$f'_{+}(5) = \lim_{h \to 0^+} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0^+} \frac{[(5+h)^2 - 25] - 0}{h} = \lim_{h \to 0^+} \frac{10h + h^2}{h} = \lim_{h \to 0^+} (10 + h) = 10.$$

Since the left and right limits are different, $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$ does not exist, that is, $f'(5)$ does not exist. Similarly, $f'(-5)$ does not exist. Therefore, f is not differentiable at 5 or at -5.

(b)



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31. 2/0 points | Previous Answers SCalcET8 3.1.067.

Find a second-degree polynomial P such that $P(4) = 16$, $P'(4) = 11$, and $P''(4) = 4$.

$P(x) =$
$$2x2−5x+4

$2x^2 - 5x + 4$

Solution or Explanation

Let $P(x) = ax^2 + bx + c$. Then $P'(x) = 2ax + b$ and $P''(x) = 2a$. $P''(4) = 4 \Rightarrow 2a = 4 \Rightarrow a = 2$.

$P'(4) = 11 \Rightarrow 2(2)(4) + b = 11 \Rightarrow 16 + b = 11 \Rightarrow b = -5$.

$P(4) = 16 \Rightarrow 2(4)^2 + (-5)(4) + c = 16 \Rightarrow 12 + c = 16 \Rightarrow c = 4$. So $P(x) = 2x^2 - 5x + 4$.

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32. 2/0 points | Previous Answers SCalcET8 3.1.070.

Find a parabola with equation $y = ax^2 + bx + c$ that has slope 1 at $x = 1$, slope -11 at $x = -1$, and passes through the point $(2, 3)$.

$y =$
$$3x2−5x+1

$3x^2 - 5x + 1$

Solution or Explanation

Click to View Solution

33. 1/1 points | Previous Answers SCalcET8 3.1.075.

Find the parabola with equation $y = ax^2 + bx$ whose tangent line at $(2, 10)$ has equation $y = 11x - 12$.

$y =$

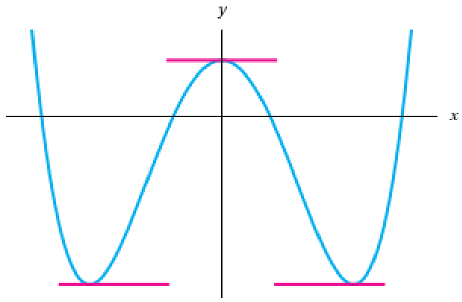
$$3x2−x

$3x^2 - x$

Solution or Explanation

Substituting $x = 2$ and $y = 10$ into $ax^2 + bx$ gives us $a(4) + b(2) = 10$ **(1)**. The slope of the tangent line $y = 11x - 12$ is 11 and the slope of the tangent to the parabola at (x, y) is $y' = 2ax + b$. At $x = 2$, $y' = 11 \Rightarrow 11 = 2a(2) + b$ **(2)**. Subtracting **(1)** from **(2)** gives us $3 = a$ and it follows that $b = -1$. The parabola has equation $y = 3x^2 - x$.

34.2/2 points | Previous AnswersSCalcET8 3.1.AE.006.



Video Example

EXAMPLE 6 Find the points on the curve $y = x^4 - 12x^2 + 9$ where the tangent line is horizontal.

SOLUTION Horizontal tangents occur where the derivative is zero. We have

$\frac{dy}{dx} = \frac{d}{dx}($ $$x4 $\boxed{x^4}$ $) - 12\frac{d}{dx}($ $$x2 $\boxed{x^2}$ $) + \frac{d}{dx}(9)$

$=$ $$4x3−24x $\boxed{4x^3 - 24x}$ $= 4x($ $$x2−6 $\boxed{x^2 - 6}$ $)$.

$$(x+√6)(x−√6)

Thus $dy/dx = 0$ if $x = 0$ or $\boxed{x^2 - 6} = 0$, that is,

$x = \pm$ $$√6 $\boxed{\sqrt{6}}$. So the given curve has horizontal tangents when

$x =$ $$−√6,0 ,√6 $\boxed{0, \sqrt{6}, -\sqrt{6}}$ (enter your answers as a comma-separated list). The corresponding points are then the following.

$(x, y) = ($ $$−√6,−27 $\boxed{-\sqrt{6}, -27}$ $)$ (smallest x-value)

$(x, y) = ($ $$0,9 $\boxed{0, 9}$ $)$

$(x, y) = ($ $$√6,−27 $\boxed{\sqrt{6}, -27}$ $)$ (largest x-value)