WebAssign
2.8 Interpretaciones de la Derivada (Homework)

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Diferencial, section B, Spring 2019
Instructor: Christiaan Ketelaar

Current Score : 44.3 / 37 **Due :** Friday, March 8, 2019 11:59 PM CST**Last Saved : n/a Saving... ()**

The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may *not* grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

Request Extension

1. 1/1 points | Previous AnswersSCalcET8 2.7.001.

A curve has equation $y = f(x)$.

(a) Write an expression for the slope of the secant line through the points $P(5, f(5))$ and $Q(x, f(x))$.

- $\dfrac{f(x) - x}{f(5) - 5}$
- $\dfrac{x - 5}{f(x) - f(5)}$
- (selected) $\dfrac{f(x) - f(5)}{x - 5}$
- $\dfrac{f(5) - 5}{f(x) - x}$

(b) Write an expression for the slope of the tangent line at P.

- $\lim\limits_{x \to 5} \dfrac{x - 5}{f(x) - f(5)}$
- $\lim\limits_{x \to 0} \dfrac{f(x) - f(5)}{x - 5}$
- $\lim\limits_{x \to 0} \dfrac{f(x) - x}{f(5) - 5}$
- (selected) $\lim\limits_{x \to 5} \dfrac{f(x) - f(5)}{x - 5}$

Solution or Explanation

(a) This is just the slope of the line through two points: $m_{PQ} = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x) - f(5)}{x - 5}$.

(b) This is the limit of the slope of the secant line PQ as Q approaches P: $m = \lim\limits_{x \to 5} \dfrac{f(x) - f(5)}{x - 5}$.

2. 4/4 points | Previous Answers SCalcET8 2.7.009.

(a) Find the slope m of the tangent to the curve $y = 7 + 4x^2 - 2x^3$ at the point where $x = a$.

$m =$

$$8a−6a2

$\boxed{8a - 6a^2}$

(b) Find equations of the tangent lines at the points (1, 9) and (2, 7).

$$2(x−1)+9

$y(x) =$

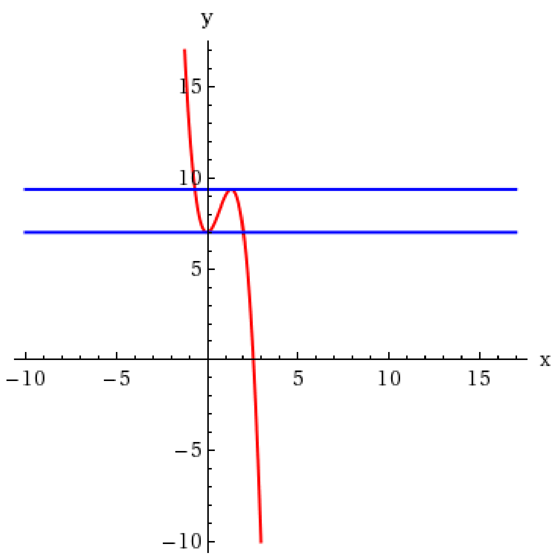
$\boxed{2x + 7}$ (at the point (1, 9))

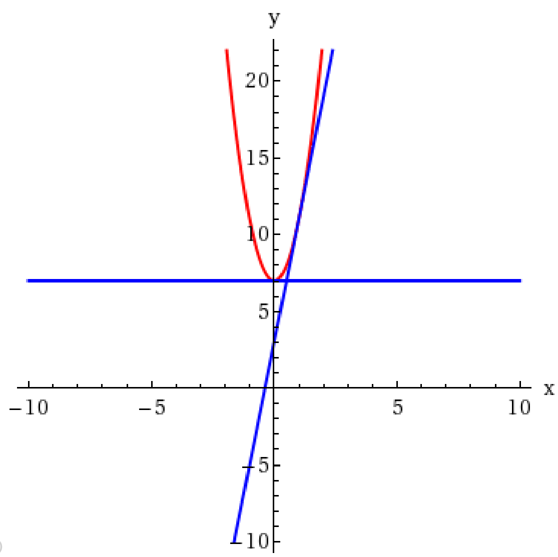
$$−8(x−2)+7

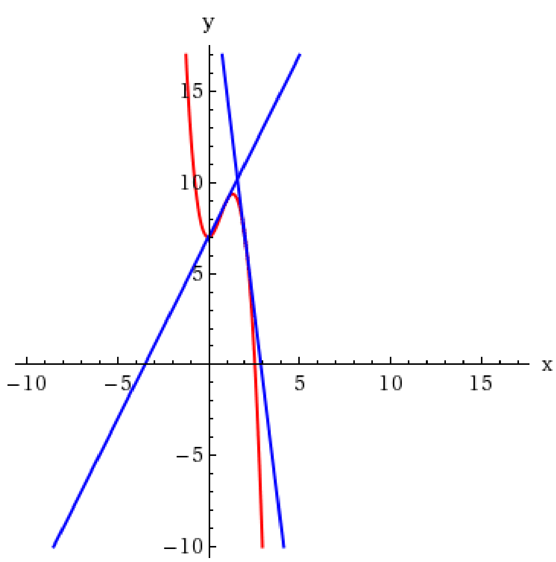
$y(x) =$

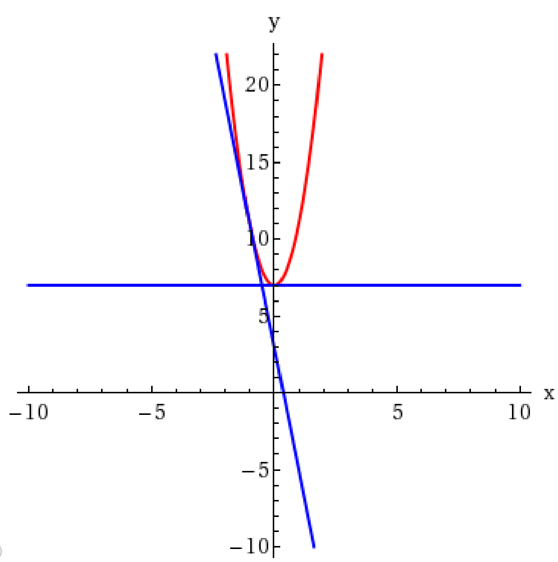
$\boxed{-8x + 23}$ (at the point (2, 7))

(c) Graph the curve and both tangents on a common screen.









Solution or Explanation
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3. 1/1 points | Previous AnswersSCalcET8 2.7.013.MI.

If a ball is thrown into the air with a velocity of 39 ft/s, its height (in feet) after t seconds is given by $y = 39t - 16t^2$. Find the velocity when $t = 1$.

7 ✓ 7 ft/s

Solution or Explanation
Click to View Solution

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4. 2/2 points | Previous AnswersSCalcET8 2.7.014.

If a rock is thrown upward on the planet Mars with a velocity of 13 m/s, its height (in meters) after t seconds is given by $H = 13t - 1.86t^2$.

(a) Find the velocity of the rock after one second.
9.28 ✓ 9.28 m/s

(b) Find the velocity of the rock when $t = a$.
$$13−2(1.86)a

✓ $13 - 3.72a$ m/s

(c) When will the rock hit the surface? (Round your answer to one decimal place.)
t = 7.0 ✓ 7.0 s

(d) With what velocity will the rock hit the surface?
-13.04 ✓ -13 m/s

Solution or Explanation
Click to View Solution

5. 2/2 points | Previous AnswersSCalcET8 2.7.015.

The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $s = 2/t^2$, where t is measured in seconds. Find the velocity of the particle at times $t = a$, $t = 1$, $t = 2$, and $t = 3$.

$$−4a−3

$t = a$ $v =$ $-\frac{4}{a^3}$ m/s

$t = 1$ $v =$ -4 ✔ -4 m/s

$t = 2$ $v =$ -1/2 ✔ -1/2 m/s

$t = 3$ $v =$ -4/27 ✔ -4/27 m/s

Solution or Explanation

$$v(a) = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h} = \lim_{h\to 0} \frac{\frac{2}{(a+h)^2} - \frac{2}{a^2}}{h} = \lim_{h\to 0} \frac{\frac{2a^2 - 2(a+h)^2}{a^2(a+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{2a^2 - 2\left(a^2 + 2ah + h^2\right)}{ha^2(a+h)^2} = \lim_{h\to 0} \frac{-2\left(2ah + h^2\right)}{ha^2(a+h)^2} = \lim_{h\to 0} \frac{-2h(2a+h)}{ha^2(a+h)^2}$$

$$= \lim_{h\to 0} \frac{-2(2a+h)}{a^2(a+h)^2} = \frac{-4a}{a^2 \cdot a^2} = \frac{-4}{a^3} \text{ m/s}$$

So $v(1) = \frac{-4}{1^3} = -4$ m/s, $v(2) = \frac{-4}{2^3} = -\frac{1}{2}$ m/s, $v(2) = \frac{-4}{3^3} = -\frac{4}{27}$ m/s.

6. 2/0 points | Previous AnswersSCalcET8 2.7.022.

If the tangent line to $y = f(x)$ at $(3, 2)$ passes through the point $(0, 1)$, find $f(3)$ and $f'(3)$.

$$2

$f(3) =$ 2

$$13

$f'(3) =$ $\frac{1}{3}$

Solution or Explanation

Since $(3, 2)$ is on $y = f(x)$, $f(3) = 2$. The slope of the tangent line between $(0, 1)$ and $(3, 2)$ is $\frac{1}{3}$, so $f'(3) = \frac{1}{3}$.

7. 2/2 points | Previous AnswersSCalcET8 2.7.044.

A particle moves along a straight line with equation of motion $s = f(t)$, where s is measured in meters and t in seconds. Find the velocity and the speed when $t = 5$.

$$f(t) = 16 + \frac{48}{t+1}$$

velocity -4/3 ✔ -4/3 m/s

speed 4/3 ✔ 4/3 m/s

Solution or Explanation

$$v(5) = f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h\to 0} \frac{\left(16 + \frac{48}{5+h+1}\right) - \left(16 + \frac{48}{5+1}\right)}{h} = \lim_{h\to 0} \frac{\frac{48}{6+h} - 8}{h}$$

$$= \lim_{h\to 0} \frac{48 - 8(6+h)}{h(6+h)} = \lim_{h\to 0} \frac{-8h}{h(6+h)} = \lim_{h\to 0} \frac{-8}{6+h} = -\frac{4}{3} \text{ m/s.}$$

The speed when $t = 5$ is $\left|-\frac{4}{3}\right| = \frac{4}{3}$ m/s.

8. 3/3 points | Previous AnswersSCalcET8 2.7.051.

The cost (in dollars) of producing x units of a certain commodity is $C(x) = 5000 + 11x + 0.1x^2$.

(a) Find the average rate of change of C with respect to x when the production level is changed from $x = 100$ to the given value. (Round your answers to the nearest cent.)

(i) $x = 105$
$ 31.5 ✓ 31.50 per unit

(ii) $x = 101$
$ 31.1 ✓ 31.10 per unit

(b) Find the instantaneous rate of change of C with respect to x when $x = 100$. (This is called the *marginal cost*.)
$ 31 ✓ 31.00 per unit

Solution or Explanation
Click to View Solution

9. 1.5/1.5 points | Previous AnswersSCalcET8 2.7.056.

The quantity (in pounds) of a gourmet ground coffee that is sold by a coffee company at a price of p dollars per pound is $Q = f(p)$.

(a) What is the meaning of the derivative $f\,'(6)$?

- ○ The rate of change of the price per pound with respect to the quantity of coffee sold.
- ◉ The rate of change of the quantity of coffee sold with respect to the price per pound when the price is $6 per pound.
- ○ The supply of coffee needed to be sold to charge $6 per pound.
- ○ The rate of change of the price per pound with respect to the quantity of coffee sold when the price is $6 per pound.
- ○ The price of the coffee as a function of the supply.

✓

What are the units of $f\,'(6)$?

- ○ pounds/dollar
- ◉ pounds/(dollars/pound)
- ○ dollars
- ○ pounds
- ○ dollars/(pound/pound)
- ○ dollars/pound

✓

(b) In general, will $f\,'(6)$ be positive or negative?

- ○ positive
- ◉ negative

✓

Solution or Explanation
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10.1.5/0 points | Previous AnswersSCalcET8 2.7.AE.007.

t	$D(t)$
1980	929.9
1985	1945.5
1990	3232.7
1995	4973.0
2000	5674.1
2005	7933.0

Video Example

EXAMPLE 7 Let $D(t)$ be the national debt at time t for a certain country. The table to the left gives approximate values of this function by providing end of year estimates, in billions of dollars, from 1980 to 2005. Interpret and estimate the value of $D'(1990)$.

SOLUTION The derivative $D'(1990)$ means the rate of change of D with respect to t when $t = 1990$, that is, the rate of increase of the national debt in 1990. According to an alternative form of the definition of derivative,

$$D'(1990) = \lim_{t \to 1990} \frac{D(t) - D(1990)}{t - 1990}.$$

So we compute and tabulate values of the difference quotient (the average rates of change) as follows.

t	$\frac{D(t) - D(1990)}{t - 1990}$
1980	230.28
1985	257.44 ✓ 257.44
1995	348.06
2000	244.14 ✓ 244.14
2005	313.35

From this table we see that $D'(1990)$ lies somewhere between 257.44 and 348.06 billion dollars per year. [Here we are making the reasonable assumption that the debt didn't fluctuate wildly between 1980 and 2000.] We estimate that the rate of increase of the national debt of the country in 1990 was the average of these two numbers, namely

$$D'(1990) \approx \boxed{302.75}\ ✓\ 302.75 \text{ billion dollars per year.}$$

Another method would be to plot the debt function and estimate the slope of the tangent line when $t = 1990$.

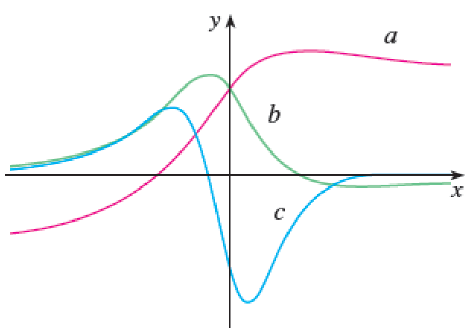
11.1.5/1.5 points | Previous AnswersSCalcET8 2.8.049.

The figure shows the graphs of f, f', and f''. Identify each curve.

f a ✓ a

f' b ✓ b

f'' c ✓ c



Solution or Explanation

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12. 4/4 points | Previous Answers SCalcET8 2.8.055.

If $f(x) = 2x^2 - x^3$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

$$4x−3x2

$f'(x) =$

$4x - 3x^2$

$$4−6x

$f''(x) =$

$4 - 6x$

$$−6

$f'''(x) =$

-6

$$0

$f^{(4)}(x) =$

0

Graph f, f', f'', and f''' on a common screen. Are the graphs consistent with the geometric interpretations of these derivatives?

The graphs are are consistent with the geometric interpretations of the derivatives because f' crosses the x-axis crosses the x-axis where f has a slope of m = 0, f'' crosses the x-axis crosses the x-axis where f' has a slope of m = 0, and f''' is a straight line a straight line function equal to the slope of f''.

Solution or Explanation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 - (x+h)^3] - (2x^2 - x^3)}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 3x^2 - 3xh - h^2)}{h}$$

$$= \lim_{h \to 0} (4x + 2h - 3x^2 - 3xh - h^2)$$

$$= 4x - 3x^2$$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \frac{[4(x+h) - 3(x+h)^2] - (4x - 3x^2)}{h}\}$$

$$= \lim_{h \to 0} \frac{h(4 - 6x - 3h)}{h}$$

$$= \lim_{h \to 0} (4 - 6x - 3h)$$

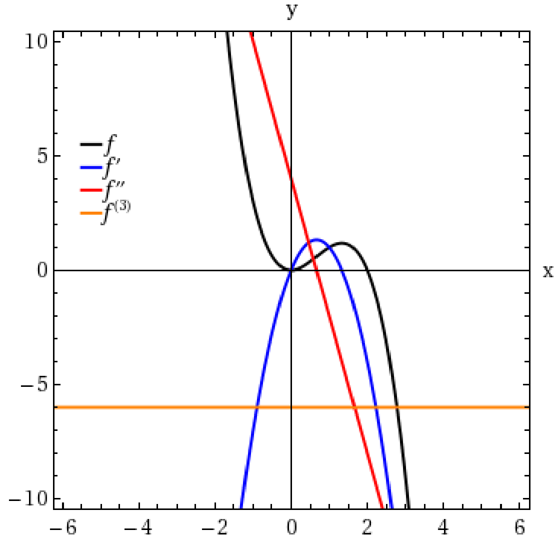
$$= 4 - 6x$$

$$f'''(x) = \lim_{h \to 0} \frac{f''(x+h) - f''(x)}{h} = \lim_{h \to 0} \frac{[4 - 6(x+h)] - (4 - 6x)}{h} = \lim_{h \to 0} \frac{-6h}{h} = \lim_{h \to 0} (-6) = -6$$

$$f^{(4)} = \lim_{h \to 0} \frac{f'''(x+h) - f'''(x)}{h} = \lim_{h \to 0} \frac{-6 - (-6)}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} (0) = 0$$

The graphs are consistent with the geometric interpretations of the derivatives because f' crosses the x-axis where f has a slope of m = 0, f

'' crosses the x-axis where f' has a slope of m = 0, and f''' is a straight line function equal to the slope of f''.



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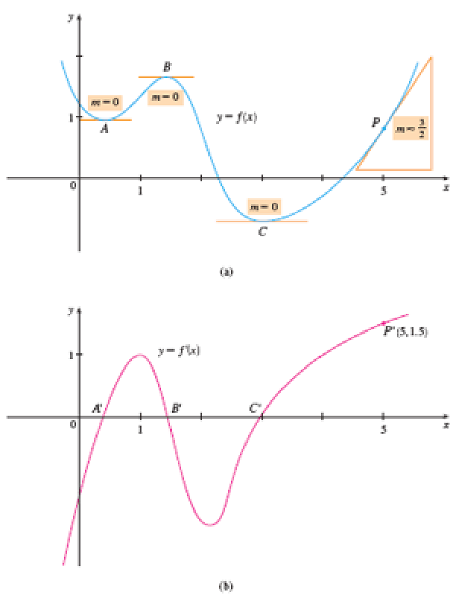
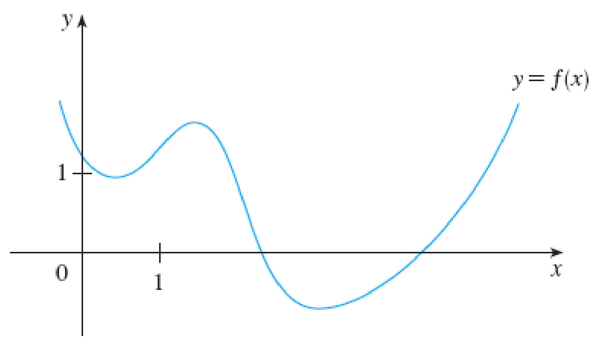
13. 1.8/0 points | Previous Answers SCalcET8 2.8.AE.001.



Video Example

EXAMPLE 1 The graph of a function f is given to the left. Use it to sketch the graph of the derivative f '.



SOLUTION We can estimate the value of the derivative at any value of x by drawing the tangent at the point $(x, f(x))$ and estimating its slope. For instance, for $x = 5$ we draw the tangent at P in the figure and estimate its slope to be about $\frac{3}{2}$, so $f'(\boxed{5}$ ✓ 5 $) = \boxed{3/2}$ ✓ 1.5 . This allows us to plot the point $P'(\boxed{5}$ ✓ 5 , $\boxed{3/2}$ ✓ 1.5 $)$ on the graph of f' directly beneath P. Repeating this procedure at several points, we get the lower graph shown in the figure. Notice that the tangents at A, B, and C are horizontal, so the derivative is $\boxed{0}$ ✓ 0 there and the graph of f' crosses the x-axis at the points A', B', and C', directly beneath A, B, and C. Between A and B the tangents have [positive] ✓ positive slope, so $f'(x)$ is [positive] ✓ positive there. But between B and C the tangents have [negative] ✓ negative slope, so $f'(x)$ is [negative] ✓ negative there.

14.1/1 points | Previous AnswersSCalcET8 3.1.019.

Differentiate the function.

$$y = 8e^x + \frac{2}{\sqrt[3]{x}}$$

$y' =$

$$8ex−23x−43

$8e^x - \frac{2}{3}x^{-\frac{4}{3}}$

Solution or Explanation

$$y = 8e^x + \frac{2}{\sqrt[3]{x}} = 8e^x + 2x^{-1/3} \Rightarrow y' = 8(e^x) + 2\left(-\frac{1}{3}\right)x^{-2/3} = 8e^x - \frac{2}{3}x^{-4/3}$$

15.2/2 points | Previous AnswersSCalcET8 3.1.037.

Find equations of the tangent line and normal line to the curve at the given point.

$$y = x^4 + 4e^x, \ (0, 4)$$

$$4(x−0)+4

tangent line $y =$

$4x + 4$

$$−14(x−0)+4

normal line $y =$

$-\frac{1}{4}x + 4$

Solution or Explanation

Click to View Solution

16. 2/2 points | Previous Answers SCalcET8 3.1.047.

Find the first and second derivative of the function. Check to see that your answers are reasonable by comparing the graphs of f, f', and f''.

$f(x) = 4x - 5x^{7/8}$

$$4−358x−18

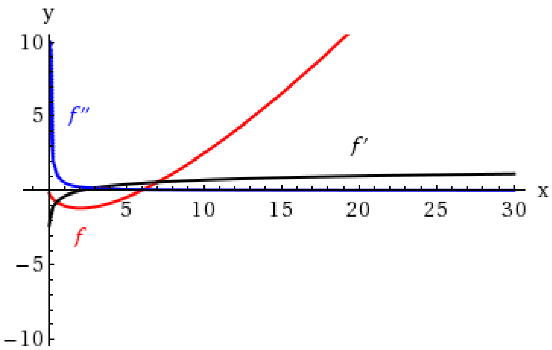
$f'(x)$ = $4 - \frac{35}{8}x^{-\frac{1}{8}}$

$$3564x−98

$f''(x)$ = $\frac{35}{64}x^{-\frac{9}{8}}$

Solution or Explanation

$f(x) = 4x - 5x^{7/8} \Rightarrow f'(x) = 4 - \frac{35}{8}x^{-1/8} \Rightarrow f''(x) = \frac{35}{64}x^{-9/8}$



Note that f' is negative when f is decreasing and positive when f is increasing. f'' is always positive since f' is always increasing.

17.2/2 points | Previous AnswersSCalcET8 3.1.048.

Find the first and second derivative of the function. Check to see that your answers are reasonable by comparing the graphs of *f*, *f* ', and *f* ''.

$f(x) = e^x - x^4$

$$ex−4x3

f '(*x*) =

$e^x - 4x^3$

$$ex−12x2

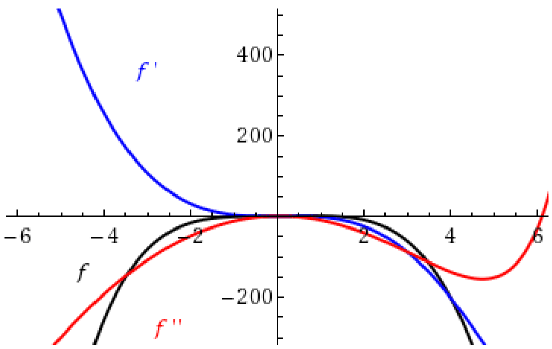
f ''(*x*) =

$e^x - 12x^2$

Solution or Explanation

$f(x) = e^x - x^4 \Rightarrow f'(x) = e^x - 4x^3 \Rightarrow f''(x) = e^x - 12x^2$

Note that $f'(x) = 0$ when *f* has a horizontal tangent and that $f''(x) = 0$ when *f* ' has a horizontal tangent.



18.2/2 points | Previous AnswersSCalcET8 3.1.049.

The equation of motion of a particle is $s = t^3 - 3t$, where *s* is in meters and *t* is in seconds. (Assume $t \geq 0$.)

(a) Find the velocity and acceleration as functions of *t*.

$v(t) =$

$$3t2−3

$3t^2 - 3$

$a(t) =$

$$6t

$6t$

(b) Find the acceleration after 5 s.

30 | 30 m/s^2

(c) Find the acceleration when the velocity is 0.

6 | 6 m/s^2

Solution or Explanation

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19. 2/2 points | Previous Answers SCalcET8 3.1.538.XP.

The equation of motion of a particle is $s = 2t^3 - 9t^2 + 3t + 3$, where s is in meters and t is in seconds. (Assume $t \geq 0$.)

(a) Find the velocity and acceleration as functions of t.

$$6t2−18t+3

$v(t)$ = $6t^2 - 18t + 3$

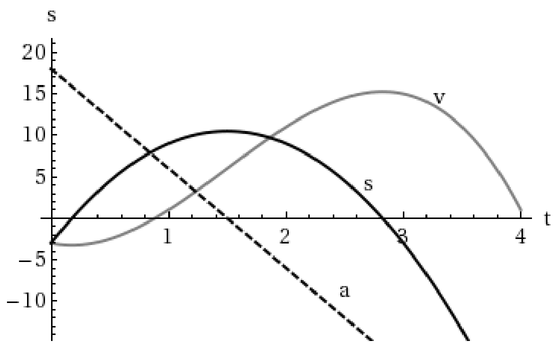
$$12t−18

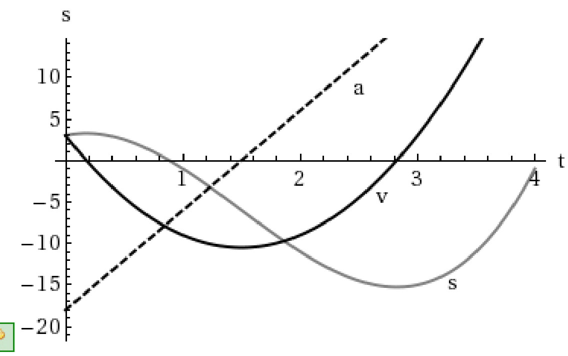
$a(t)$ = $12t - 18$

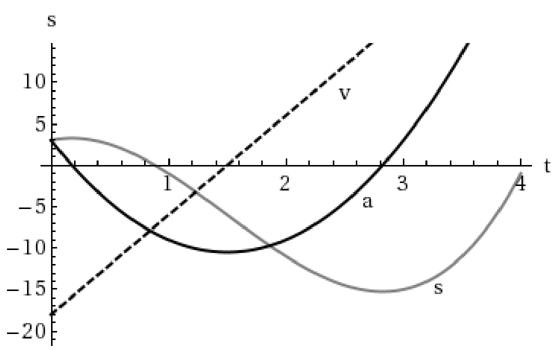
(b) Find the acceleration after 1 s.

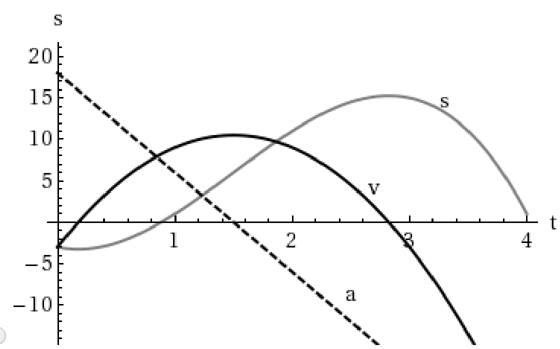
$a(1)$ = -6 -6 m/s^2

(c) Graph the position, velocity, and acceleration functions on the same screen.









Solution or Explanation

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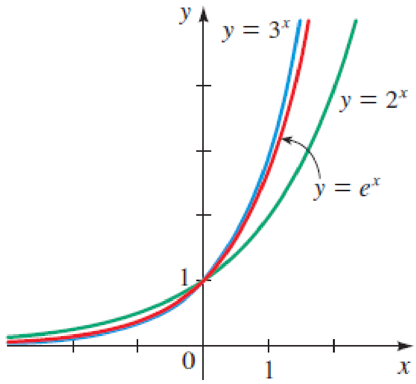
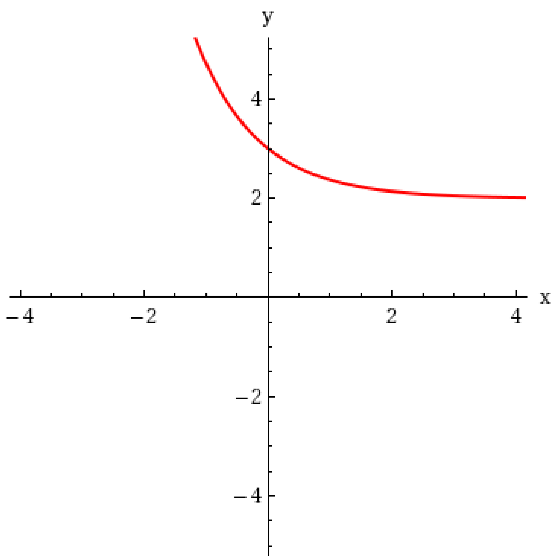
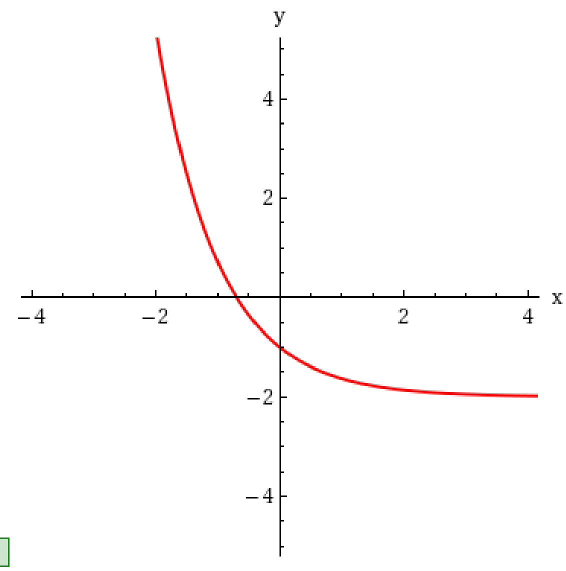
20.2/2 points | Previous AnswersSCalcET8 3.1.JIT.004.

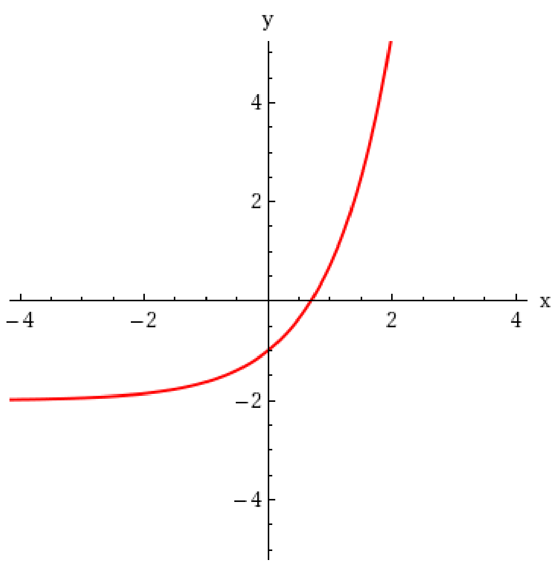
Graph the function, not by plotting points, but by starting from the graph of $y = e^x$ in the figure below.

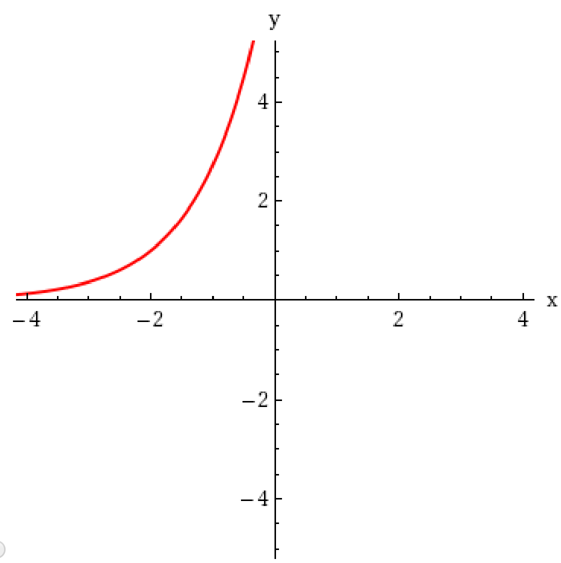
$y = e^{-x} - 2$











State the domain and range. (Enter your answers using interval notation.)

domain $$(−∞,∞)

$\boxed{(-\infty, \infty)}$

$$(−2,∞)

range

$\boxed{(-2, \infty)}$

State the asymptote.

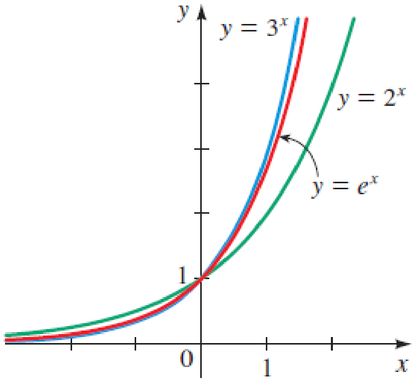
$$y=−2

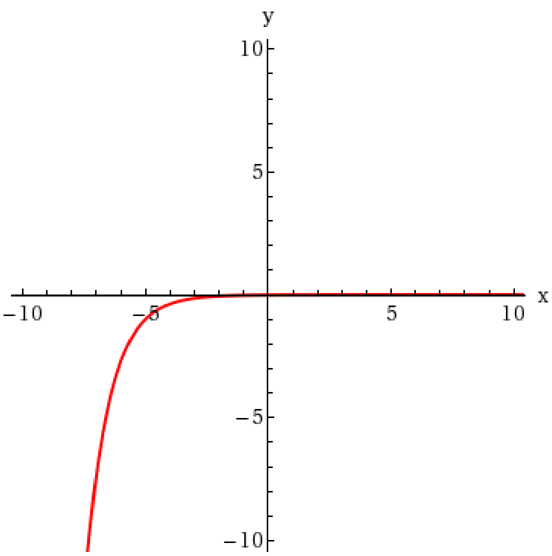
$\boxed{y = -2}$

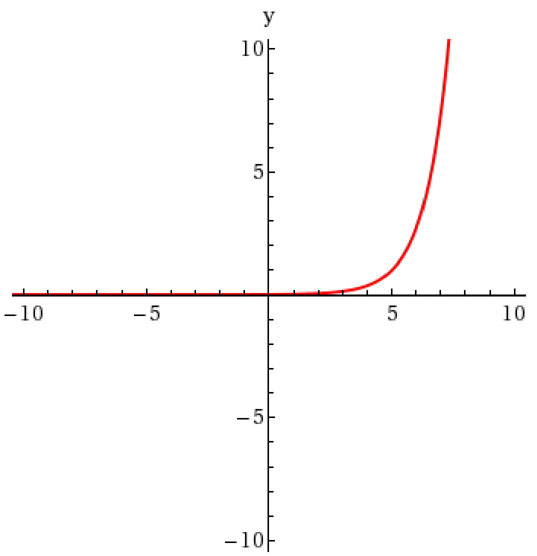
21. 2/2 points | Previous Answers SCalcET8 3.1.JIT.005.MI.

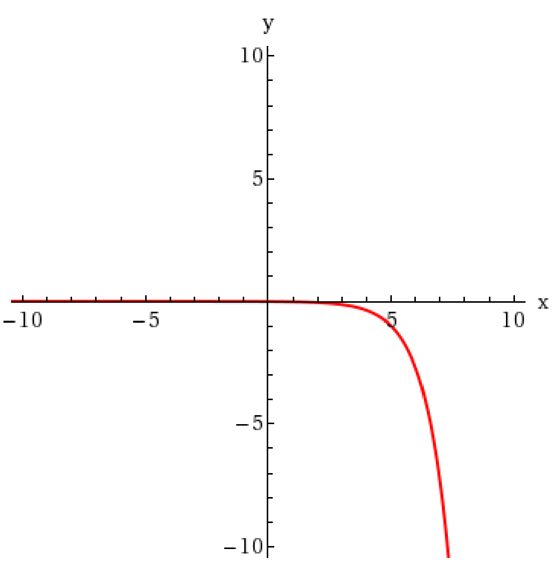
Graph the function, not by plotting points, but by starting from the graph of $y = e^x$ in the figure below.

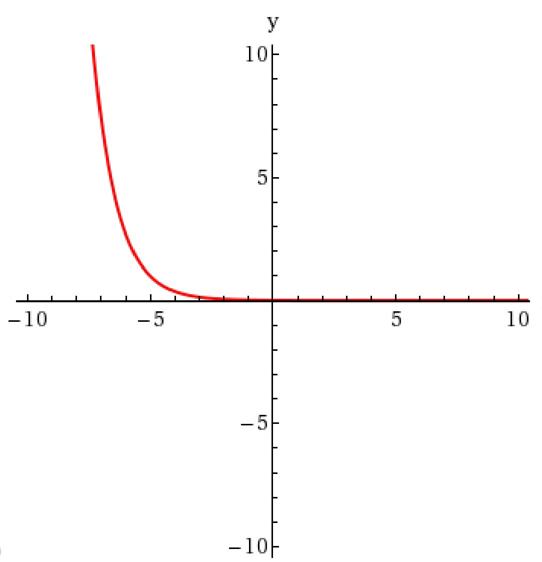
$f(x) = e^x - 5$











State the domain and range. (Enter your answers using interval notation.)

domain $(-\infty,\infty)$

$(-\infty, \infty)$

$$(0,∞)

range

$(0, \infty)$

State the asymptote.

$$y=0

$y = 0$

22.2/0 points | Previous AnswersSCalcET8 3.1.051.

Biologists have proposed a cubic polynomial to model the length L of rock bass at age A:

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

where L is measured in inches and A in years. Calculate

$\left.\frac{dL}{dA}\right|_{A\,=\,19}$. (Round your answer to three decimal places.)

$\left.\frac{dL}{dA}\right|_{A\,=\,19}$ = 6.601 ✓ 6.601 in/yr

Interpret your answer.

- (●) A 19-year old rock fish grows at a rate of 6.601 in/yr.
- ( ) A 19-year old rock fish shrinks at a rate of 6.601 in/yr.
- ( ) A 19-year old rock fish grows at a rate of 69.487 in/yr.
- ( ) A 19-year old rock fish grows at a rate of 62.886 in/yr.
- ( ) A 19-year old rock fish shrinks at a rate of 69.487 in/yr.

Solution or Explanation

$$\begin{aligned} L(A) &= 0.0155A^3 - 0.372A^2 + 3.95A + 1.21 \Rightarrow \\ \frac{dL}{dA} &= 0.0155(3A^2) - 0.372(2A^1) + 3.95(A^0) + 0 = 0.0465A^2 - 0.744A + 3.95 \Rightarrow \\ \left.\frac{dL}{dA}\right|_{A\,=\,19} &= 0.0465(19)^2 - 0.744(19) + 3.95 = 6.601 \end{aligned}$$

Therefore, a 19-year old rock fish grows at a rate of 6.601 inches/year.