WebAssign
4.7 Optimización (Homework)

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Diferencial, section B, Spring 2019
Instructor: Christiaan Ketelaar

Current Score : 9.5 / 22 **Due :** Friday, May 17, 2019 11:59 PM CST**Last Saved : n/a Saving... ()**

The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may *not* grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

Request Extension

1. 2/2 points | Previous AnswersSCalcET8 4.7.003.

Find two positive numbers whose product is 64 and whose sum is a minimum. (If both values are the same number, enter it into both blanks.)

8 ✓ 8 (smaller number)
8 ✓ 8 (larger number)

Solution or Explanation
Click to View Solution

2. 2/2 points | Previous AnswersSCalcET8 4.7.007.MI.

Find the dimensions of a rectangle with perimeter 60 m whose area is as large as possible. (If both values are the same number, enter it into both blanks.)

15 ✓ 15 m (smaller value)
15 ✓ 15 m (larger value)

Solution or Explanation
Click to View Solution

3. 2/2 points | Previous AnswersSCalcET8 4.7.011.

Consider the following problem: A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

(a) Draw several diagrams illustrating the situation, some with shallow, wide pens and some with deep, narrow pens. Find the total areas of these configurations. Does it appear that there is a maximum area? If so, estimate it.

(b) Draw a diagram illustrating the general situation. Let x denote the length of each of two sides and three dividers. Let y denote the length of the other two sides.

(c) Write an expression for the total area A in terms of both x and y.

A =
$$x·y

xy

(d) Use the given information to write an equation that relates the variables.

$$750=5x+2y

$5x + 2y = 750$

(e) Use part (d) to write the total area as a function of one variable.

$A(x)$ =
$$x·750−5x2

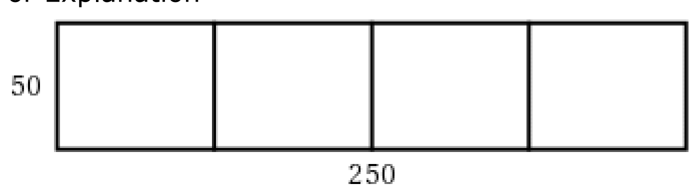
$375x - \dfrac{5x^2}{2}$

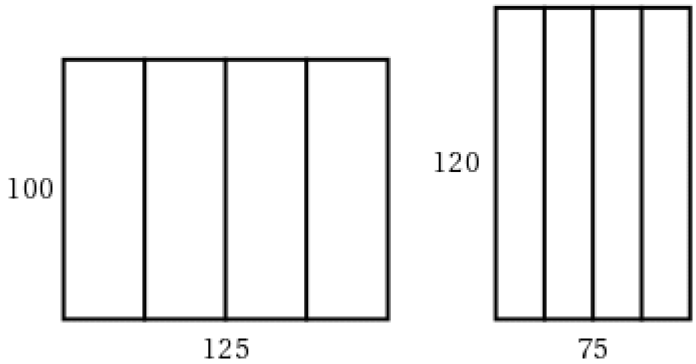
(f) Finish solving the problem by finding the largest area.

14062.5 14,062.5 ft^2

Solution or Explanation

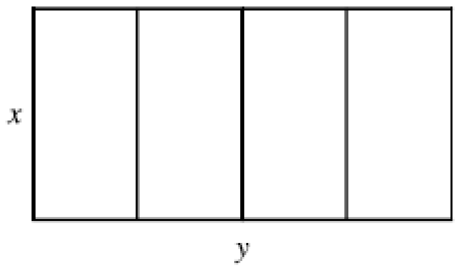
(a)





The areas of the three figures are 12,500, 12,500, and 9,000 ft^2. There appears to be a maximum area of at least 12,500 ft^2.

(b) Let x denote the length of each of two sides and three dividers. Let y denote the length of the other two sides.



(c) Area A = length × width = $y \cdot x$

(d) Length of fencing = 750 $\Rightarrow$ $5x + 2y = 750$

(e) $5x + 2y = 750 \Rightarrow y = 375 - \frac{5}{2}x \Rightarrow A(x) = \left(375 - \frac{5}{2}x\right)x = 375x - \frac{5}{2}x^2$

(f) $A'(x) = 375 - 5x = 0 \Rightarrow x = 75$. Since $A''(x) = -5 < 0$ there is an absolute maximum when $x = 75$. Then $y = \frac{375}{2} = 187.5$. The largest area is $75\left(\frac{375}{2}\right) = 14{,}062.5$ ft^2. These values of x and y are between the values in the first and second figures in part (a). Our original estimate was low.

4. 1.5/2 points | Previous AnswersSCalcET8 4.7.012.

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

(a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes.

(b) Draw a diagram illustrating the general situation. Let *x* denote the length of the side of the square being cut out. Let *y* denote the length of the base.

(c) Write an expression for the volume *V* in terms of both *x* and *y*.

V =

$$y2x

$\boxed{xy^2}$

(d) Use the given information to write an equation that relates the variables *x* and *y*.

$$y=3−2x

$\boxed{3 = 2x + y}$

(e) Use part (d) to write the volume as a function of only *x*.

V(*x*) =

$$(3−2x)2·x

$\boxed{(3 - 2x)^2 x}$

(f) Finish solving the problem by finding the largest volume that such a box can have.

V = $\boxed{6}$ ✗ $\boxed{2}$ ft^3

Solution or Explanation

Click to View Solution

5. 2/2 points | Previous AnswersSCalcET8 4.7.016.

A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs \$20 per square meter. Material for the sides costs \$12 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)

\$ 327.08 ✓ 327.08

Solution or Explanation
Click to View Solution

6. –/2 pointsSCalcET8 4.7.018.

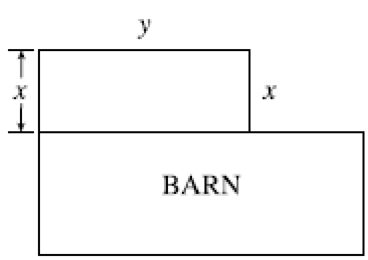
A farmer wants to fence in a rectangular plot of land adjacent to the north wall of his barn. No fencing is needed along the barn, and the fencing along the west side of the plot is shared with a neighbor who will split the cost of that portion of the fence. If the fencing costs \$24 per linear foot to install and the farmer is not willing to spend more than \$6000, find the dimensions for the plot that would enclose the most area.

(width, length) = (*(No Response)* $\frac{250}{3}, 125$)

Solution or Explanation
See the figure. The fencing cost \$24 per linear foot to install and the cost of the fencing on the west side will be split with the neighbor, so the farmer's cost C will be $C = \frac{1}{2}(24x) + 24y + 24x = 24y + 36x$. The area A will be maximized when $C = 6000$, so

$6000 = 24y + 36x \Leftrightarrow 24y = 6000 - 36x \Leftrightarrow y = 250 - \frac{3}{2}x$. Now

$A = xy = x\left(250 - \frac{3}{2}x\right) = 250x - \frac{3}{2}x^2 \Rightarrow A' = 250 - 3x$. $A' = 0 \Leftrightarrow x = \frac{250}{3}$ and since $A'' = -3 < 0$, we have a maximum for A when $x = \frac{250}{3}$ ft and $y = 250 - \frac{3}{2}\left(\frac{250}{3}\right) = 125$ ft. [The maximum area is $125\left(\frac{250}{3}\right) = 10{,}416.7$ ft^2.



7. –/0 pointsSCalcET8 4.7.027.

Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.

base (No Response) $\boxed{\frac{L}{2}}$

height (No Response) $\boxed{\frac{\sqrt{3}L}{4}}$

Solution or Explanation

The height h of the equilateral triangle with sides of length L is $\frac{\sqrt{3}}{2}L$, since

$$h^2 + \left(\frac{L}{2}\right)^2 = L^2 \Rightarrow h^2 = L^2 - \frac{1}{4}L^2 = \frac{3}{4}L^2 \Rightarrow h = \frac{\sqrt{3}}{2}L.$$

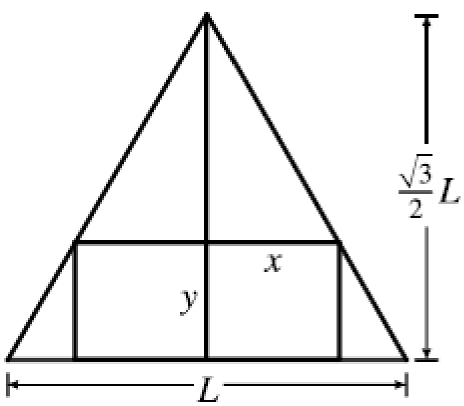
Using similar triangles,

$$\frac{\frac{\sqrt{3}}{2}L - y}{x} = \frac{\frac{\sqrt{3}}{2}L}{L/2} = \sqrt{3} \Rightarrow \sqrt{3}x = \frac{\sqrt{3}}{2}L - y \Rightarrow y = \frac{\sqrt{3}}{2}L - \sqrt{3}x \Rightarrow y = \frac{\sqrt{3}}{2}(L - 2x).$$

The area of the inscribed rectangle is $A(x) = (2x)y = \sqrt{3}x(L - 2x) = \sqrt{3}Lx - 2\sqrt{3}x^2$, where $0 \le x \le \frac{L}{2}$. Now

$$0 = A'(x) = \sqrt{3}L - 4\sqrt{3}x \Rightarrow x = \frac{\sqrt{3}L}{(4\sqrt{3})} = \frac{L}{4}.$$

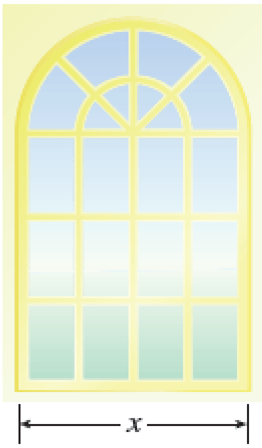
Since $A(0) = A\left(\frac{L}{2}\right) = 0$, the maximum occurs when $x = \frac{L}{4}$, and $y = \frac{\sqrt{3}}{2}L - \frac{\sqrt{3}}{4}L = \frac{\sqrt{3}}{4}L$, so the dimensions are $\frac{L}{2}$ and $\frac{\sqrt{3}}{4}L$.



8. –/0 pointsSCalcET8 4.7.034.MI.

A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See the figure below.) If the perimeter of the window is 16 ft, find the value of *x* so that the greatest possible amount of light is admitted.

x = (No Response) $\dfrac{32}{4+\pi}$ ft



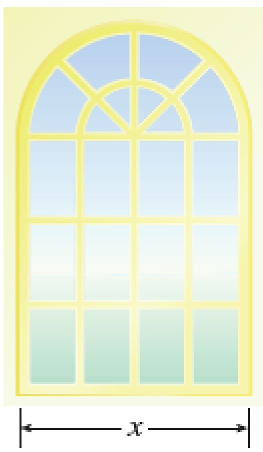
Solution or Explanation
Click to View Solution

9. –/3 pointsSCalcET8 4.7.034.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

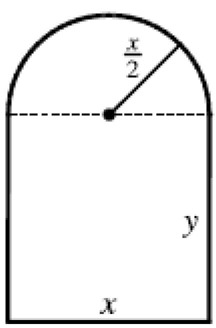
A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See the figure below.) If the perimeter of the window is 28 ft, find the value of x so that the greatest possible amount of light is admitted.



Step 1

Let x be the width and y be the height of the window. Thus, the semi-circle has radius $x/2$. We must maximize the area of the window, $A = xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$. The perimeter of the window is

$28 = 2y + x + \pi\left(\frac{x}{2}\right)$, and so $y =$ (No Response) 14 $- \frac{x}{2} - \frac{\pi x}{\text{(No Response) } 4}$.



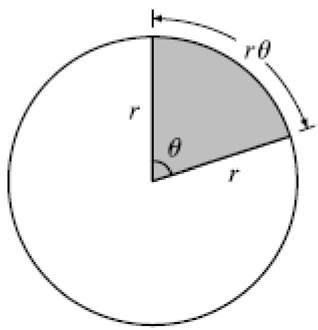
10.–/2 pointsSCalcET8 4.7.039.

If you are offered one slice from a round pizza (in other words, a sector of a circle) and the slice must have a perimeter of 16 inches, what diameter pizza will reward you with the largest slice?

(No Response) 8 in

Solution or Explanation

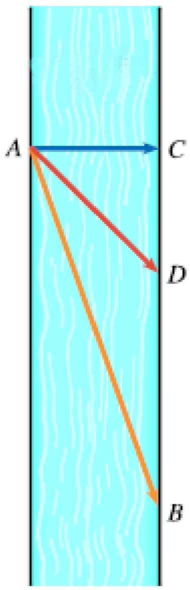
From the figure, the perimeter of the slice is $2r + r\theta = 16$, so $\theta = \dfrac{16 - 2r}{r}$. The area A of the slice is $A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{16 - 2r}{r}\right) = r(8 - r) = 8r - r^2$ for $0 \le r \le 8$. $A'(r) = 8 - 2r$, so $A' = 0$ when $r = 4$. Since $A(0) = 0$, $A(8) = 0$, and $A(4) = 16$ in^2, the largest piece comes from a pizza with radius 4 in and diameter 8 in. Note that $\theta = 2$ radians $\approx 114.6°$, which is about 32% of the whole pizza.



11.–/0 pointsSCalcET8 4.7.049.

A man launches his boat from point *A* on a bank of a straight river, 1 km wide, and wants to reach point *B*, 1 km downstream on the opposite bank, as quickly as possible (see the figure below). He could row his boat directly across the river to point *C* and then run to *B*, or he could row directly to *B*, or he could row to some point *D* between *C* and *B* and then run to *B*. If he can row 6 km/h and run 8 km/h, where should he land to reach *B* as soon as possible? (We assume that the speed of the water is negligible compared to the speed at which the man rows.) (Hint: This question is based on EXAMPLE 4 in Section 4.7 of the textbook. However, for this question, the textbook has added a challenge which may require an unexpected solution. Look for it!)

(No Response) 1 km from *C*



Solution or Explanation

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12.–/2 pointsSCalcET8 4.7.060.

If $C(x) = 19000 + 600x - 4.8x^2 + 0.004x^3$ is the cost function and $p(x) = 4200 - 9x$ is the demand function, find the production level that will maximize profit. (Hint: If the profit is maximized, then the marginal revenue equals the marginal cost.)

(No Response) 300 units

Solution or Explanation

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13.–/3 pointsSCalcET8 4.7.063.

A retailer has been selling 1200 tablet computers a week at $250 each. The marketing department estimates that an additional 80 tablets will sell each week for every $10 that the price is lowered.

(a) Find the demand function.

$p(x)$ = (No Response) $400 - \frac{x}{8}$

(b) What should the price be set at in order to maximize revenue?

$ (No Response) 200

(c) If the retailer's weekly cost function is

$C(x) = 35{,}000 + 150x$

what price should it choose in order to maximize its profit?

$ (No Response) 275

Solution or Explanation

(a) As in Example 6, we see that the demand function p is linear. We are given that $p(1200) = 250$ and deduce that $p(1280) = 240$, since a $10 reduction in price increases sales by 80 per week. The slope for p is $\frac{240 - 250}{1280 - 1200} = -\frac{1}{8}$, so an equation is $p - 250 = -\frac{1}{8}(x - 1200)$ or $p(x) = -\frac{1}{8}x + 400$.

(b) $R(x) = xp(x) = -\frac{1}{8}x^2 + 400x$. $R'(x) = -\frac{1}{4}x + 400 = 0$ when $x = 4(400) = 1600$. $p(1600) = 200$, so the price should be set at $200 to maximize revenue.

(c)
$C(x) = 35{,}000 + 150x \Rightarrow P(x) = R(x) - C(x) = -\frac{1}{8}x^2 + 400x - 35{,}000 - 150x = -\frac{1}{8}x^2 + 250x - 35{,}000$.
$P'(x) = -\frac{1}{4}x + 250 = 0$ when $x = 4(250) = 1000$. $p(1000) = 275$, so the price should be set at $275 to maximize profit.