2. Decaimiento Radioactivo $y(28) = 0.5 y_0$.

a. $y_0 = 64$ Ecuación $y(t)$.

Modelo Exponencial $y = 64 e^{Kt}$ Vida Media $y(28) = \frac{64}{2} = 32$.

$$64 e^{28K} = 32$$

$$e^{28K} = 0.5$$

$$28K = \ln(0.5)$$

$$K = \frac{1}{28} \ln(0.5)$$

$K = \frac{1}{\text{vidamedia}} \ln(0.5)$

$$y(t) = 64 e^{\frac{1}{28}\ln(0.5)\, t} = 64 e^{\frac{t}{28}\ln(0.5)} = 64 e^{\ln(0.5)^{t/28}}$$

$$= 64 \cdot (0.5)^{t/28}$$

Exponencial - Base 0.5

b. masa después de 84 días. $\left(\frac{1}{2}\right)^3$

$$y(84) = 64 \cdot (0.5)^{84/28} = 64 \cdot (0.5)^3 = 64 \cdot \frac{1}{8} = 8 \text{ mg}.$$

c. Tiempo para que $y(t) = 2$.

$$64 \cdot (0.5^{t/28}) = 2$$

$$0.5^{t/28} = \frac{1}{32} = 2^{-5}$$

Tome ln's $\ln(0.5)^{t/28} = \ln(2^{-5})$

$$\frac{t}{28} \ln(0.5) = -5 \ln 2$$

$$t = \frac{-5 \cdot 28 \ln 2}{\ln(2^{-1})} = \frac{-5 \cdot 28 \cancel{\ln 2}}{-\cancel{\ln(2)}} = 5 \cdot 28 \text{ días}$$

140 días

3. Asesinato. $T(t) = 17 + (\underbrace{37-17}_{20}) e^{Kt}$. $K < 0$.

a) 1:30 p.m. 33 $\cancel{T(1.5)}$ $T(t_0) = 33$

2:30 p.m. 29 $T(t_0+1) = 29$.

t_0 tiempo después del asesinato

K constante.

$T(t_0) = 17 + 20e^{Kt_0} = 33$ $\Rightarrow$ $20e^{Kt_0} = 16$ (1)

$T(t_0+1) = 17 + 20e^{K(t_0+1)} = 29$ $\quad$ $20e^{Kt_0+K} = 12$. (2)

Divida (2) por (1): $\dfrac{\cancel{20}\,\cancel{e^{Kt_0}}\,e^{K}}{\cancel{20}\,\cancel{e^{Kt_0}}} = \dfrac{12}{16}$

$e^K = \dfrac{3}{4}$ $\Rightarrow$ $K = \ln(3/4)$

b) ¿Cuándo ocurrió el asesinato?

$T(t) = 17 + 20e^{t\ln(3/4)}$ $\quad$ $T(t_0) = 33$

$T(t_0) = 17 + 20e^{t_0\ln(3/4)} = 33$

$20e^{t_0\ln(3/4)} = 16$

$e^{t_0\ln(3/4)} = 0.8$

Aplique ln's: $t_0\ln(3/4) = \ln(0.8)$

$t_0 = \dfrac{\ln(0.8)}{\ln(0.75)}$ = 0.77 horas antes de la 1 PM.

El asesinato ocurrió a las 1:30 - P.M - 0:42 = 12:43.8 PM.

5 a) Recta Tangente.

$f(4) = \sqrt{16-12} = \sqrt{4} = 2.$

$4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{2}.$

$f'(x) = \frac{1}{2}(x^2-3x)^{-1/2}(2x-3)$

$f'(4) = \frac{1}{2}(4)^{-1/2} \cdot (8-3) = \frac{1}{2} \cdot \frac{1}{2} 5 = \frac{5}{4}$

Aproximación Lineal $L(x) = f(4) + f'(4)(x-4)$

$L(x) = 2 + \frac{5}{4}(x-4)$

b. $g(x) = \log(x) \qquad a = 10.$

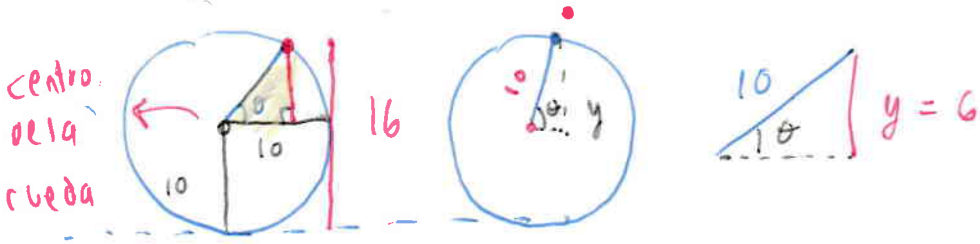
$g(10) = \log(10) = 1$

$g'(x) = \frac{1}{x \ln 10}$

$g'(10) = \frac{1}{10 \ln(10)}$

$L(x) = 1 + \frac{1}{10 \cdot \ln(10)}(x-10)$

8. Rueda de la Fortuna.



Relación entre y, θ & 10

$$\sin\theta = \frac{y}{10}$$

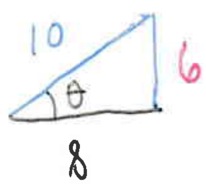
y - altura respecto al centro de la rueda.

Derive respecto a t: $\cos\theta \cdot \dfrac{d\theta}{dt} = \dfrac{1}{10}\dfrac{dy}{dt}$.

incógnita dy/dt

$\cos\theta$ y $\dfrac{d\theta}{dt}$.

$$\frac{d\theta}{dt} = \frac{2\pi \text{ rad}}{2 \text{ min}} = \pi \text{ rad/min.}$$



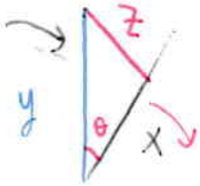
$\cos\theta = \dfrac{8}{10}$

$C.A. = \sqrt{100-36} = \sqrt{64} = 8$

Sustituya $\cos\theta = \dfrac{4}{5}$ $\quad \dfrac{d\theta}{dt} = \pi$

$$\frac{4}{5}\pi = \frac{1}{10}\frac{dy}{dt}.$$

$$\frac{dy}{dt} = \frac{4}{5}\cdot 10\pi = 8\pi \text{ m/min.}$$



4. Manecillas del Reloj

y = minutero = 8 $\qquad y' = 0$

x = horario = 4 $\qquad x' = 0$

z = distancia entre manecillas

No hay triángulo rectángulo

Ley de cosenos: $z^2 = x^2 + y^2 - 2xy\cos\theta.$

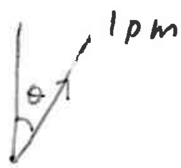
$$z^2 = 16 + 64 - 2\cdot 4\cdot 8\cos\theta. = 80 - 32\cos\theta.$$

Derive respecto a t: $2zz' = 32\,\text{sen}\,\theta\,\theta'$

Implícitamente dada θ, z, θ' ✓

$$\theta'_{minutero} = \frac{2\pi}{1\text{ hora}} = 2\pi \text{ rad/hora} \qquad \theta'_{horario} = \frac{2\pi}{12h} = \frac{\pi}{6}\text{ rad/hora}$$

$$\theta' = \theta'_{hora} - \theta'_{min} = \frac{\pi}{6} - 2\pi = \frac{-11\pi}{6}.$$

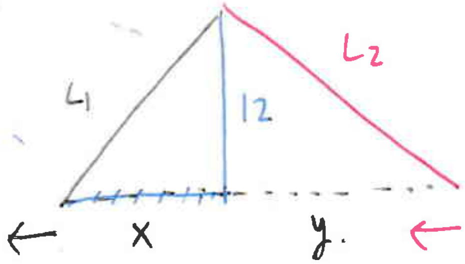


$$\theta = 360°\frac{1}{12.} = 30° \text{ ó } \frac{\pi}{6.}$$

$$z^2 = 80 - 32\cos\pi/6 = 80 - 32\frac{\sqrt{3}}{2} = 80 - 16\sqrt{3}$$

$$z' = \frac{16}{z}\,\text{sen}\,\theta\,\theta' = \frac{16}{\sqrt{80-16\sqrt{3}}}\,\frac{1}{2}\left(\frac{-11\pi}{6}\right) \text{ cm/hora}$$

10.



$\leftarrow y' =$

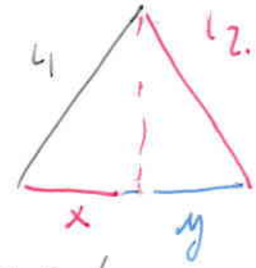
$x' = 2.$, $x = 5$

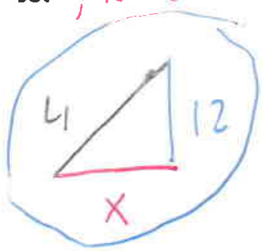
$$L_1 + L_2 = 39.$$

dada $x = 5,\ x' = 2,$

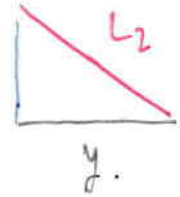
incógnita $y' = ?$

x, y cambian respecto a t.





$L_1 = \sqrt{144 + x^2}$



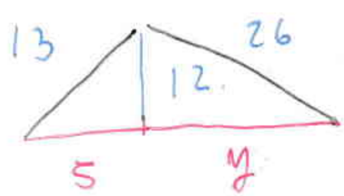
$L_2 = \sqrt{144 + y^2}$

Ec. Suga $\quad \sqrt{144 + x^2} + \sqrt{144 + y^2} = 39.$

Derive: $\quad \frac{1}{2}(144 + x^2)^{-1/2} 2x\, x' + \frac{1}{2}(144 + y^2)^{-1/2} 2y\, y' = 0$

$x = 5,\ x' = 2,$ $\quad \dfrac{x\, x'}{\sqrt{144 + x^2}} = -\dfrac{y\, y'}{\sqrt{144 + y^2}}$ $\quad \Rightarrow \quad y' = \dfrac{\sqrt{144 + y^2}\; x\, x'}{\sqrt{144 + x^2}}$

$$\frac{5 \cdot 2}{\underbrace{\sqrt{144 + 25}}_{13}} = \frac{-y}{\sqrt{144 + y^2}}\, y'$$



$y = \sqrt{26^2 - 12^2}$

$y \approx 23.06$

$y' = \dfrac{-10}{13} \dfrac{\sqrt{144 + y^2}}{y}$

$y' = -\dfrac{10}{13} \dfrac{\sqrt{144 + (23.06)^2}}{23.06}$

pies/s.

11. Evalúe los sigs. límites.

a) $\lim_{x\to 0} \frac{x-\sin(x)}{x-\tan(x)} \underset{LH}{=} \lim_{x\to 0} \frac{1-\cos x}{1-\sec^2 x} \underset{LH}{=} \lim_{x\to 0} \frac{\sin x}{-2\sec x \sec x \tan x}$

$\frac{0-\sin(0)}{0-\tan(0)} = \frac{0}{0}$ $\qquad \frac{1-\cos 0}{1-\sec^2 0} = \frac{1-1}{1-1}$

$\lim_{x\to 0} \frac{\sin x}{-2\sec^2 x \tan x} = \lim_{x\to 0} \frac{\cos^3 x}{-2} = \frac{(\cos 0)^3}{-2} = \frac{1}{-2} = -\frac{1}{2}$

Propiedades Func. Trigonométricas $\frac{\sin x}{\sec^2 x \tan x} = \frac{\frac{\sin x}{1}}{\frac{1}{\cos^2 x}\frac{\sin x}{\cos x}} = \frac{\cancel{\sin x}\cdot\cos^3 x}{\cancel{\sin x}}$

b) $\lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{\overbrace{2x+1}^{\infty}} = y.$ $\qquad \lim_{x\to\infty} \frac{2x-3}{2x+5} = 1$ Forma 1^{∞}

$\ln y = \lim_{x\to\infty} (2x+1)\ln\left(\frac{2x-3}{2x+5}\right)$ Forma $\infty \cdot 0$

$\ln y = \lim_{x\to\infty} \frac{\ln\left(\frac{2x-3}{2x+5}\right)}{(2x+1)^{-1}} = \lim_{x\to\infty} \frac{\ln(2x-3)-\ln(2x+5)}{(2x+1)^{-1}}$ $\infty \cdot \infty.$

$\ln y \underset{LH}{=} \lim_{x\to\infty} \frac{\frac{2}{2x-3} - \frac{2}{2x+5}}{-1(2x+1)^{-2}\cdot 2} = \lim_{x\to\infty} \frac{\frac{\cancel{4x}+10-\cancel{4x}+6}{(2x-3)(2x+5)}}{\frac{-2}{(2x+1)^2}}$

$\lim_{x\to\infty} \frac{\frac{16}{(2x-3)(2x+5)}}{\frac{-2}{(2x+1)^2}} = \lim_{x\to\infty} \frac{16(2x+1)^2}{-2(2x-3)(2x+5)} = \frac{16}{-2}$

$\ln y = \lim_{x\to\infty} \frac{16(4x^2+4x+1)}{-2(4x^2+4x-15)} = -8$

$y = e^{-8}$

12. Regla de L'Hospital.

a) $\lim_{x \to 1} (2-x)^{\tan(\pi x/2)}$ $L \to 1$ $1^{\tan \pi/2} = 1^{\infty}$

Tome ln's: $\ln y = \lim_{x \to 1} \tan(\pi x/2) \ln(2-x)$ $\infty \cdot 0$

$$\ln y = \lim_{x \to 1} \frac{\ln(2-x)}{\cot(\pi x/2)} \quad \frac{0}{0} \quad \cot\frac{\pi}{2} = \frac{\cos \pi/2}{\sin \pi/2} = 0$$

L'Hospital $$\ln y = \lim_{x \to 1} \frac{\frac{-1}{2-x}}{-\csc^2\left(\frac{\pi x}{2}\right) \cdot \frac{\pi}{2}} \quad \csc\frac{\pi}{2} = \frac{1}{\sin \pi/2} = 1$$

$$\ln y = \frac{\frac{-1}{2-1}}{-(1)^2 \frac{\pi}{2}} = \frac{-1}{-\frac{\pi}{2}} = \frac{2}{\pi}$$

$$y = \lim_{x \to 1} (2-x)^{\tan(\pi x/2)} = e^{2/\pi}$$

b. $$\lim_{x \to \infty} \frac{\ln(\sqrt{x})}{x^2} = \lim_{x \to \infty} \frac{1}{2} \frac{\ln x}{x^2} \underset{LH}{=} \lim_{x \to \infty} \frac{\frac{1}{2} \cdot \frac{1}{x}}{2x}$$

∞/∞.

$$= \lim_{x \to \infty} \frac{1}{4x^2} = 0 \quad \frac{1}{\infty} \to 0$$

$$\lim_{x \to \infty} \frac{\ln(\sqrt{x})}{x^2} = 0$$

c) $$\lim_{t \to 0} \frac{8^t - 5^t}{t} \underset{LH}{=} \lim_{t \to 0} \frac{8^t \ln 8 - 5^t \ln 5}{1} = \ln 8 - \ln 5$$