

WebAssign

3.11 Hiperbólicas y 4.4 Regla de L'Hospital (Homework)

David Corzo

Diferencial, section B, Spring 2019

Instructor: Christiaan Ketelaar

Current Score : 45 / 37

Due : Friday, April 19, 2019 11:59 PM CST Last Saved : n/a Saving... ()

The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

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1. 1.5/1.5 points | [Previous Answers](#) SCalcET8 3.11.008.

Prove the identity.

$$\cosh(-x) = \cosh(x)$$

(This shows that cosh is an even function.)

$$\begin{aligned} \cosh(-x) &= \frac{1}{2} \left[e^{-(-x)} + e^{-(-x)} \right] \\ &= \frac{1}{2} \left(e^{-x} + e^x \right) \\ &= \cosh(x) \end{aligned}$$

Solution or Explanation

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2. 1.5/1.5 points | [Previous Answers](#)SCalcET8 3.11.009.

Prove the identity.

$$\cosh(x) + \sinh(x) = e^x$$

$$\cosh(x) + \sinh(x) = \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x})$$

$$= \frac{1}{2}(2e^x)$$

$$= e^x$$

Solution or Explanation

$$\cosh(x) + \sinh(x) = \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}(2e^x) = e^x$$

3. 0/2 points | [Previous Answers](#)SCalcET8 3.11.024.

Prove the formulas given in [this table](#) for the derivatives of the functions cosh, tanh, csch, sech, and coth. Which of the following are proven correctly? (Select all that apply.)



$$\frac{d}{dx}(\coth(x)) = \frac{d}{dx}\left(\frac{\sinh(x)}{\cosh(x)}\right) = \frac{\cosh(x)\cosh(x) - \sinh(x)\sinh(x)}{\cosh^2(x)} = \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = -\frac{1}{\cosh^2(x)} = -\operatorname{csch}^2(x)$$



$$\frac{d}{dx}(\operatorname{sech}(x)) = \frac{d}{dx}\left(\frac{1}{\cosh(x)}\right) = -\frac{\sinh(x)}{\cosh^2(x)} = -\frac{1}{\cosh(x)} \cdot \frac{\sinh(x)}{\cosh(x)} = -\operatorname{sech}(x)\tanh(x)$$



$$\frac{d}{dx}(\operatorname{csch}(x)) = \frac{d}{dx}\left(\frac{1}{\sinh(x)}\right) = -\frac{\cosh^2(x)}{\sinh^2(x)} = -\frac{1}{\sinh(x)} \cdot \frac{\cosh^2(x)}{\sinh(x)} = -\operatorname{csch}(x)\coth(x)$$



$$\frac{d}{dx}(\operatorname{csch}(x)) = \frac{d}{dx}\left(\frac{1}{\sinh(x)}\right) = -\frac{\cosh(x)}{\sinh^2(x)} = -\frac{1}{\sinh(x)} \cdot \frac{\cosh(x)}{\sinh(x)} = -\operatorname{csch}(x)\coth(x)$$



$$\frac{d}{dx}(\cosh(x)) = \frac{d}{dx}\left[\frac{1}{2}(e^x - e^{-x})\right] = \frac{1}{2}(e^x + e^{-x}) = \sinh(x)$$



Solution or Explanation

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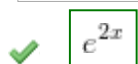
4. 1/1 points | [Previous Answers](#)SCalcET8 3.11.030.

Find the derivative.

$$f(x) = e^x \cosh(x)$$

$$f'(x) =$$

$$e^x \cosh(x) + e^x \sinh(x)$$



$$e^{2x}$$

Solution or Explanation

The notation $\overset{\text{PR}}{\Rightarrow}$ indicates the use of the Product Rule.

$$f(x) = e^x \cosh(x) \overset{\text{PR}}{\Rightarrow} f'(x) = e^x \sinh(x) + (\cosh(x))e^x = e^x(\sinh(x) + \cosh(x)), \text{ or } e^x(e^x) = e^{2x}.$$

5. 1/1 points | [Previous Answers](#)SCalcET8 3.11.031.

Find the derivative.

$$f(x) = \tanh(\sqrt{x})$$

$$f'(x) =$$

$$\frac{\operatorname{sech}^2(\sqrt{x})}{2\sqrt{x}}$$



$$\frac{\operatorname{sech}^2(\sqrt{x})}{2\sqrt{x}}$$

Solution or Explanation

$$f(x) = \tanh(\sqrt{x}) \Rightarrow f'(x) = \operatorname{sech}^2(\sqrt{x}) \frac{d}{dx} \sqrt{x} = \operatorname{sech}^2(\sqrt{x}) \left(\frac{1}{2\sqrt{x}} \right) = \frac{\operatorname{sech}^2(\sqrt{x})}{2\sqrt{x}}$$

6. 1/1 points | [Previous Answers](#)SCalcET8 3.11.037.

Find the derivative.

$$y = e^{\cosh(7x)}$$

$$y'(x) =$$

$$7 \sinh(7x) e^{\cosh(7x)}$$



$$7 \sinh(7x) e^{\cosh(7x)}$$

Solution or Explanation

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7. -/0 pointsSCalcET8 3.11.054.

A model for the velocity of a falling object after time t is

$$v(t) = \sqrt{\frac{mg}{k}} \tanh\left(t\sqrt{\frac{gk}{m}}\right) +$$

where m is the mass of the object, $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity, k is a constant, t is measured in seconds, and v is in m/s.

(a) Calculate the terminal velocity of the object, that is, $\lim_{t \rightarrow \infty} v(t)$.

(No Response)

$$\sqrt{\frac{gm}{k}}$$


(b) If a person is falling from a building, the value of the constant k depends on his or her position. For a "belly-to-earth" position, $k = 0.515 \text{ kg/s}$, but for a "feet-first" position, $k = 0.067 \text{ kg/s}$. If a **70**-kg person falls in belly-to-earth position, what is the terminal velocity? (Round your answer to two decimal places.)

(No Response)

 36.50 m/s

What about feet-first? (Round your answer to two decimal places.)

(No Response)

 101.19 m/s

Solution or Explanation

$$(a) \quad \lim_{t \rightarrow \infty} v(t) = \lim_{t \rightarrow \infty} \sqrt{\frac{mg}{k}} \tanh\left(t\sqrt{\frac{gk}{m}}\right) = \sqrt{\frac{mg}{k}} \lim_{t \rightarrow \infty} \tanh\left(t\sqrt{\frac{gk}{m}}\right) = \sqrt{\frac{mg}{k}}$$

$$(b) \quad \text{Belly-to-earth:} \quad g = 9.8, k = 0.515, m = 70, \text{ so the terminal velocity is } \sqrt{\frac{70(9.8)}{0.515}} \approx 36.50 \text{ m/s.}$$

$$\text{Feet-first:} \quad g = 9.8, k = 0.067, m = 70, \text{ so the terminal velocity is } \sqrt{\frac{70(9.8)}{0.067}} \approx 101.19 \text{ m/s.}$$

8. 1/0 points | [Previous Answers](#)SCalcET8 3.11.057.

At what point of the curve $y = \cosh(x)$ does the tangent have slope 4?

$(x, y) = ($

$\ln(4 + \sqrt{17}), \sqrt{17}$

✓ $\ln(4 + \sqrt{17}), \sqrt{17}$

Solution or Explanation

The tangent to $y = \cosh(x)$ has slope 4 when $y' = \sinh(x) = 4 \Rightarrow x = \sinh^{-1}(4) = \ln(4 + \sqrt{17})$, since

$$\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1}), \quad x \in \mathbb{R}.$$

Since $\sinh(x) = 4$ and $y = \cosh(x) = \sqrt{1 + \sinh^2(x)}$, we have $\cosh(x) = \sqrt{17}$. The point is $(\ln(4 + \sqrt{17}), \sqrt{17})$.

9. 1/1 points | [Previous Answers](#)SCalcET8 3.11.503.XP.

Find the derivative.

$$f(x) = \tanh(1 + e^{4x})$$

$f'(x) =$

$4e^{4x} \operatorname{sech}^2(1 + e^{4x})$

✓ $4e^{4x} \operatorname{sech}^2(1 + e^{4x})$

Solution or Explanation

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10. 1/1 points | [Previous Answers](#)SCalcET8 3.11.511.XP.

Find the derivative.

$$G(x) = \frac{9 - \cosh(x)}{9 + \cosh(x)}$$

$G'(x) =$

$\frac{-18 \sinh(x)}{(9 + \cosh(x))^2}$

✓ $\frac{-18 \sinh(x)}{(9 + \cosh(x))^2}$

Solution or Explanation

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11.2/2 points | [Previous Answers](#)SCalcET8 4.4.001.

Given that

$$\lim_{x \rightarrow a} f(x) = 0 \quad \lim_{x \rightarrow a} g(x) = 0 \quad \lim_{x \rightarrow a} h(x) = 1$$

$$\lim_{x \rightarrow a} p(x) = \infty \quad \lim_{x \rightarrow a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

(a) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

\$\$INDETERMINATE



INDETERMINATE

(b) $\lim_{x \rightarrow a} \frac{f(x)}{p(x)}$

\$\$0



0

(c) $\lim_{x \rightarrow a} \frac{h(x)}{p(x)}$

\$\$0



0

(d) $\lim_{x \rightarrow a} \frac{p(x)}{q(x)}$

\$\$INDETERMINATE



INDETERMINATE

Solution or Explanation

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12.3/3 points | [Previous Answers](#)SCalcET8 4.4.004.

Given that

$$\lim_{x \rightarrow a} f(x) = 0 \quad \lim_{x \rightarrow a} g(x) = 0 \quad \lim_{x \rightarrow a} h(x) = 1$$

$$\lim_{x \rightarrow a} p(x) = \infty \quad \lim_{x \rightarrow a} q(x) = \infty,$$

evaluate the limits below where possible. (If a limit is indeterminate, enter INDETERMINATE.)

(a) $\lim_{x \rightarrow a} [f(x)]^{g(x)}$

\$\$INDETERMINATE



INDETERMINATE

(b) $\lim_{x \rightarrow a} [f(x)]^{p(x)}$

\$\$0\infty



0

(c) $\lim_{x \rightarrow a} [h(x)]^{p(x)}$

\$\$INDETERMINATE



INDETERMINATE

(d) $\lim_{x \rightarrow a} [p(x)]^{f(x)}$

\$\$INDETERMINATE



INDETERMINATE

(e) $\lim_{x \rightarrow a} [p(x)]^{q(x)}$

\$\$\infty



INFINITY

(f) $\lim_{x \rightarrow a} \sqrt[q(x)]{p(x)}$

\$\$INDETERMINATE



INDETERMINATE

Solution or Explanation

[Click to View Solution](#)13.1/1 points | [Previous Answers](#)SCalcET8 4.4.008.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 9}$$

\$\$16

✓ $\frac{1}{6}$

Solution or Explanation

This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{x - 3}{(x + 3)(x - 3)} = \lim_{x \rightarrow 3} \frac{1}{x + 3} = \frac{1}{3 + 3} = \frac{1}{6}$

Note: Alternatively, we could apply l'Hospital's Rule.

14.1/1 points | [Previous Answers](#)SCalcET8 4.4.015.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{t \rightarrow 0} \frac{e^{9t} - 1}{\sin(t)}$$

\$\$9

✓ 9

Solution or Explanation

This limit has the form $\frac{0}{0}$. $\lim_{t \rightarrow 0} \frac{e^{9t} - 1}{\sin(t)} \stackrel{H}{=} \lim_{t \rightarrow 0} \frac{9e^{9t}}{\cos(t)} = \frac{9(1)}{1} = 9$

15.2/2 points | [Previous Answers](#)SCalcET8 4.4.017.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin(\theta)}{1 + \cos(2\theta)}$$

\$\$14

✓

Solution or Explanation

This limit has the form $\frac{0}{0}$. $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin(\theta)}{1 + \cos(2\theta)} \stackrel{H}{=} \lim_{\theta \rightarrow \pi/2} \frac{-\cos(\theta)}{-2 \sin(2\theta)} \stackrel{H}{=} \lim_{\theta \rightarrow \pi/2} \frac{\sin(\theta)}{-4 \cos(2\theta)} = \frac{1}{4}$

16.1/1 points | [Previous Answers](#)SCalcET8 4.4.019.MI.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$$

\$\$0

✓

Solution or Explanation

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17.1/1 points | [Previous Answers](#)SCalcET8 4.4.025.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+4x} - \sqrt{1-2x}}{x}$$

\$\$3



3

Solution or Explanation

This limit has the form $\frac{0}{0}$.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{1+4x} - \sqrt{1-2x}}{x} &\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+4x)^{-1/2} \cdot 4 - \frac{1}{2}(1-2x)^{-1/2}(-2)}{1} \\ &= \lim_{x \rightarrow 0} \left(\frac{2}{\sqrt{1+4x}} + \frac{1}{\sqrt{1-2x}} \right) = \frac{2}{\sqrt{1}} + \frac{1}{\sqrt{1}} = 3 \end{aligned}$$

18.2/2 points | [Previous Answers](#)SCalcET8 4.4.027.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{e^{9x} - 1 - 9x}{x^2}$$

\$\$812



$\frac{81}{2}$

Solution or Explanation

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19.1/0 points | [Previous Answers](#)SCalcET8 4.4.031.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{\sin^{-1}(x)}{9x}$$

\$\$19

✓ $\frac{1}{9}$

Solution or Explanation

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20.1/1 points | [Previous Answers](#)SCalcET8 4.4.033.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{x 6^x}{6^x - 1}$$

\$\$1/n(6)

✓ $\frac{1}{\ln(6)}$

Solution or Explanation

This limit has the form $\frac{0}{0}$.

$$\lim_{x \rightarrow 0} \frac{x 6^x}{6^x - 1} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{x 6^x \ln 6 + 6^x}{6^x \ln 6} = \lim_{x \rightarrow 0} \frac{6^x(x \ln 6 + 1)}{6^x \ln 6} = \lim_{x \rightarrow 0} \frac{x \ln 6 + 1}{\ln 6} = \frac{1}{\ln 6}$$

21.1/1 points | [Previous Answers](#)SCalcET8 4.4.040.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x} - 4x}{x - \sin(x)}$$

\$\$16

✓ 16

Solution or Explanation

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22.1/1 points | [Previous Answers](#)SCalcET8 4.4.045.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \sin(5x) \csc(3x)$$

\$\$53

✓ $\frac{5}{3}$

Solution or Explanation

This limit has the form $0 \cdot \infty$. We'll change it to the form $\frac{0}{0}$.

$$\lim_{x \rightarrow 0} \sin(5x) \csc(3x) = \lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(3x)} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{5 \cos(5x)}{3 \cos(3x)} = \frac{5 \cdot 1}{3 \cdot 1} = \frac{5}{3}$$

23.2/0 points | [Previous Answers](#)SCalcET8 4.4.051.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 1} \left(\frac{7x}{x-1} - \frac{7}{\ln(x)} \right)$$

\$\$72

✓ $\frac{7}{2}$

Solution or Explanation

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24.1/1 points | [Previous Answers](#)SCalcET8 4.4.056.

Find the limit. Use [l'Hospital's Rule](#) where appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)]$$

\$\$\$ln(75)



$$\ln\left(\frac{7}{5}\right)$$

Solution or Explanation

This limit has the form $\infty - \infty$.

$$\lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)] = \lim_{x \rightarrow 1^+} \ln\left(\frac{x^7 - 1}{x^5 - 1}\right) = \ln\left(\lim_{x \rightarrow 1^+} \frac{x^7 - 1}{x^5 - 1}\right) \stackrel{H}{=} \ln\left(\lim_{x \rightarrow 1^+} \frac{7x^6}{5x^4}\right) = \ln\left(\frac{7}{5}\right)$$

25.2/2 points | [Previous Answers](#)SCalcET8 4.4.058.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0^+} (\tan(5x))^x$$

\$\$\$1



$$1$$

Solution or Explanation

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26.2/2 points | [Previous Answers](#)SCalcET8 4.4.059.MI.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} (1 - 6x)^{1/x}$$

\$\$\$1e6



$$\frac{1}{e^6}$$

Solution or Explanation

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27.2/0 points | [Previous Answers](#)SCalcET8 4.4.060.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$$

\$\$eab



e^{ab}

Solution or Explanation

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28.1/1 points | [Previous Answers](#)SCalcET8 4.4.062.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow \infty} x^{(\ln(9))/(1 + \ln(x))}$$

\$\$9



9

Solution or Explanation

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29.1/1 points | [Previous Answers](#)SCalcET8 4.4.063.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow \infty} x^{9/x}$$

\$\$1



1

Solution or Explanation

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30.2/2 points | [Previous Answers](#)SCalcET8 4.4.065.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0^+} (3x + 1)^{\cot(x)}$$

\$\$e^3



e^3

Solution or Explanation

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31.1/0 points | [Previous Answers](#)SCalcET8 4.4.078.

If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is


$$v = \frac{mg}{c}(1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant describing air resistance.

(a) Calculate $\lim_{t \rightarrow \infty} v$.

(No Response) $\frac{mg}{c}$

What is the meaning of this limit?


- ☐ It is the time it takes for the object to stop.
- ☐ It is the time it takes the object to reach its maximum speed.
- ☒  It is the speed the object approaches as time goes on.
- ☐ It is the speed the object reaches before it starts to slow down.



(b) For fixed t , use [l'Hospital's Rule](#) to calculate $\lim_{c \rightarrow 0^+} v$.

(No Response) gt

What can you conclude about the velocity of a falling object in a vacuum?

- ☐ An object falling in a vacuum will accelerate at a slower rate than an object not in a vacuum.
- ☐ The velocity of a falling object is proportional to its mass in a vacuum.
- ☐ The heavier the object is the faster it will fall in a vacuum.
- ☒  The velocity of a falling object in a vacuum is directly proportional to the amount of time it falls.



Solution or Explanation

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32.1/1 points | [Previous Answers](#)SCalcET8 4.4.070.

Use [l'Hospital's Rule](#) to find the exact value of the limit.

$$\lim_{x \rightarrow 0} \frac{7^x - 6^x}{5^x - 4^x}$$

\$\$\$ln(76)ln(54)\$

$$\frac{\ln\left(\frac{7}{6}\right)}{\ln\left(\frac{5}{4}\right)}$$



Solution or Explanation

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33.2/0 points | [Previous Answers](#)SCalcET8 4.4.068.

Find the limit. Use [l'Hospital's Rule](#) if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow \infty} \left(\frac{5x - 4}{5x + 2} \right)^{5x + 1}$$

\$\$\$e-6\$

$$\frac{1}{e^6}$$



Solution or Explanation

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34.1/0 points | [Previous Answers](#)SCalcET8 4.4.087.

If f' is continuous, $f(1) = 0$, and $f'(1) = 9$, evaluate

$$\lim_{x \rightarrow 0} \frac{f(1 + 3x) + f(1 + 4x)}{x}$$

63



63

Solution or Explanation

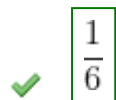
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35.1/1 points | [Previous Answers](#)SCalcET8 4.4.505.XP.

Find the limit. Use l'Hospital's Rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(6x)}$$

\$\$16



Solution or Explanation

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