WebAssign
4.3 Concavidad y Gráficas de Funciones (Homework)

David Corzo
Diferencial, section B, Spring 2019
Instructor: Christiaan Ketelaar

Current Score : 64.7 / 52 **Due :** Saturday, May 4, 2019 11:59 PM CST**Last Saved : n/a Saving... ()**

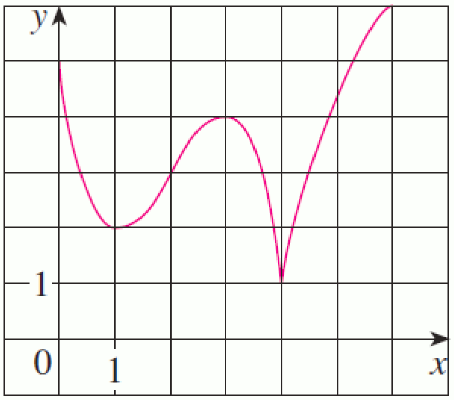
The due date for this assignment is past. Your work can be viewed below, but no changes can be made.

Important! Before you view the answer key, decide whether or not you plan to request an extension. Your Instructor may *not* grant you an extension if you have viewed the answer key. Automatic extensions are not granted if you have viewed the answer key.

Request Extension

1. 2.5/2.5 points | Previous AnswersSCalcET8 4.3.001.

Use the given graph of *f* over the interval (0, 6) to find the following.



(a) The open intervals on which *f* is increasing. (Enter your answer using interval notation.)

$$(1,3)∪(4,6)

$(1, 3), (4, 6)$

(b) The open intervals on which *f* is decreasing. (Enter your answer using interval notation.)

$$(0,1)∪(3,4)

$(0, 1), (3, 4)$

(c) The open intervals on which *f* is concave upward. (Enter your answer using interval notation.)

$$(0,2)

$(0, 2)$

(d) The open intervals on which *f* is concave downward. (Enter your answer using interval notation.)

$$(2,4)∪(4,6)

$(2, 4), (4, 6)$

(e) The coordinates of the point of inflection.

$(x, y) = ($

$$2,3

$2, 3)$

Solution or Explanation

Click to View Solution

2. 1/1 points | Previous AnswersSCalcET8 4.3.003.

Suppose you are given a formula for a function *f*.

(a) How do you determine where *f* is increasing or decreasing?

If $f'(x)$ > > 0 on an interval, then *f* is increasing on that interval.
If $f'(x)$ < < 0 on an interval, then *f* is decreasing on that interval.

(b) How do you determine where the graph of *f* is concave upward or concave downward?

If $f''(x)$ > > 0 for all x in I, then the graph of *f* is concave upward on I.
If $f''(x)$ < < 0 for all x in I, then the graph of *f* is concave downward on I.

(c) How do you locate inflection points?

- ○ At any value of x where the function changes from increasing to decreasing, we have an inflection point at $(x, f(x))$.
- ○ At any value of x where $f'(x) = 0$, we have an inflection point at $(x, f(x))$.
- ○ At any value of x where the function changes from decreasing to increasing, we have an inflection point at $(x, f(x))$.
- ◉ At any value of x where the concavity changes, we have an inflection point at $(x, f(x))$.
- ○ At any value of x where the concavity does not change, we have an inflection point at $(x, f(x))$.

Solution or Explanation
(a) Use the Increasing/Decreasing (I/D) Test.

(b) Use the Concavity Test.

(c) At any value of x where the concavity changes, we have an inflection point at $(x, f(x))$.

3. 3.5/3.5 points | Previous AnswersSCalcET8 4.3.009.

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 9x^2 - 21x + 7$

(a) Find the interval on which *f* is increasing. (Enter your answer using interval notation.)

$$(−∞,−1)∪(7,∞)

$(-\infty, -1), (7, \infty)$

Find the interval on which *f* is decreasing. (Enter your answer using interval notation.)

$$(−1,7)

$(-1, 7)$

(b) Find the local minimum and maximum values of *f*.

local minimum value $$−238

-238

local maximum value $$18

18

(c) Find the inflection point.

$(x, y) = ($ $$3,−110

$3, -110$ $)$

Find the interval on which *f* is concave up. (Enter your answer using interval notation.)

$$(3,∞)

$(3, \infty)$

Find the interval on which *f* is concave down. (Enter your answer using interval notation.)

$$(−∞,3)

$(-\infty, 3)$

Solution or Explanation

(a) $f(x) = x^3 - 9x^2 - 21x + 7 \Rightarrow f'(x) = 3x^2 - 18x - 21 = 3(x^2 - 6x - 7) = 3(x + 1)(x - 7)$.

Interval	$x + 1$	$x - 3$	$f'(x)$	f
$x < -1$	$-$	$-$	$+$	increasing on $(-\infty, -1)$
$-1 < x < 7$	$+$	$-$	$-$	decreasing on $(-1, 7)$
$x > 7$	$+$	$+$	$+$	increasing on $(7, \infty)$

(b) *f* changes from increasing to decreasing at $x = -1$ and from decreasing to increasing at $x = 7$. Thus, $f(-1) = 18$ is a local maximum value and $f(7) = -238$ is a local minimum value.

(c) $f''(x) = 6x - 18 = 6(x - 3)$. $f''(x) > 0 \iff x > 3$ and $f''(x) < 0 \iff x < 3$. Thus, f is concave upward on $(3, \infty)$ and concave downward on $(-\infty, 3)$. There is an inflection point at $(3, -110)$.

4. 3/0 points | Previous AnswersSCalcET8 4.3.013.MI.

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = 5\sin(x) + 5\cos(x), \quad 0 \le x \le 2\pi$

(a) Find the interval on which *f* is increasing. (Enter your answer using interval notation.)

$$(0,π4)∪(5π4,2π)

$\left(0, \frac{\pi}{4}\right), \left(\frac{5\pi}{4}, 2\pi\right)$

Find the interval on which *f* is decreasing. (Enter your answer using interval notation.)

$$(π4,5π4)

$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(b) Find the local minimum and maximum values of *f*.

local minimum value $$π4

$-5\sqrt{2}$

local maximum value $$5π4

$5\sqrt{2}$

(c) Find the inflection points.

$(x, y) =$ ($$3π4,0

$\frac{3\pi}{4}, 0$) (smaller *x*-value)

$(x, y) =$ ($$7π4,0

$\frac{7\pi}{4}, 0$) (larger *x*-value)

Find the interval on which *f* is concave up. (Enter your answer using interval notation.)

$$(3π4,7π4)

$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

Find the interval on which *f* is concave down. (Enter your answer using interval notation.)

$$(0,3π4)∪(7π4,2π)

$\left(0, \frac{3\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Solution or Explanation
Click to View Solution

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5. 3.5/3.5 points | Previous AnswersSCalcET8 4.3.015.

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = e^{3x} + e^{-x}$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)
$$(−ln(3)4,∞)

$\left(\frac{-\ln(3)}{4}, \infty\right)$

Find the interval on which f is decreasing. (Enter your answer using interval notation.)
$$(−∞,−ln(3)4)

$\left(-\infty, \frac{-\ln(3)}{4}\right)$

(b) Find the local minimum and maximum values of f.

local minimum value
$$e−(3ln(3)4)+e(ln(3)4)

$\frac{1}{3^{3/4}} + \sqrt[4]{3}$

local maximum value
$$DNE

DNE

(c) Find the inflection point.

$(x, y) =$ (
$$DNE

DNE)

Find the interval on which f is concave up. (Enter your answer using interval notation.)
$$(−∞,∞)

$(-\infty, \infty)$

Find the interval on which f is concave down. (Enter your answer using interval notation.)
$$DNE

DNE

Solution or Explanation
Click to View Solution

6. 2/2 points | Previous AnswersSCalcET8 4.3.019.

Find the local maximum and minimum values of f using both the First and Second Derivative Tests.

$f(x) = 3 + 12x^2 - 8x^3$

$$7

local maximum value

$\boxed{7}$

$$3

local minimum value

$\boxed{3}$

Solution or Explanation

$f(x) = 3 + 12x^2 - 8x^3 \Rightarrow f'(x) = 24x - 24x^2 = 24x(1 - x)$.

First Derivative Test: $f'(x) > 0 \Rightarrow 0 < x < 1$ and $f'(x) < 0 \Rightarrow x < 0$ or $x > 1$. Since f' changes from negative to positive at $x = 0$, $f(0) = 3$ is a local minimum value; and since f' changes from positive to negative at $x = 1$, $f(1) = 7$ is a local maximum value.

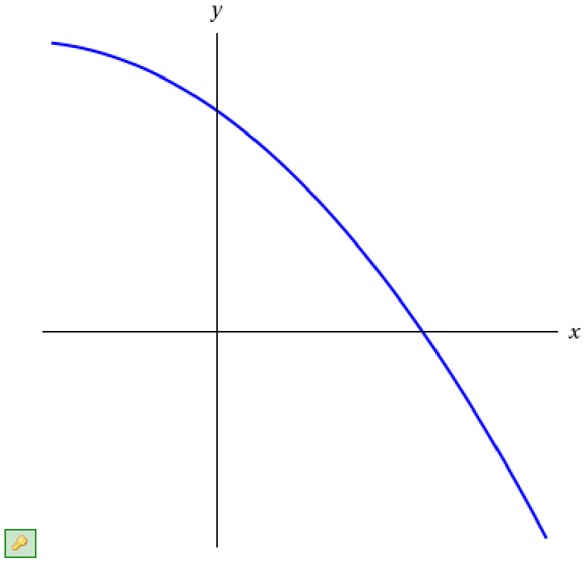
Second Derivative Test: $f''(x) = 24 - 48x$. $f'(x) = 0 \Leftrightarrow x = 0, 1$. $f''(0) = 24 > 0 \Rightarrow f(0) = 3$ is a local minimum value. $f''(1) = -24 < 0 \Rightarrow f(1) = 7$ is a local maximum value.

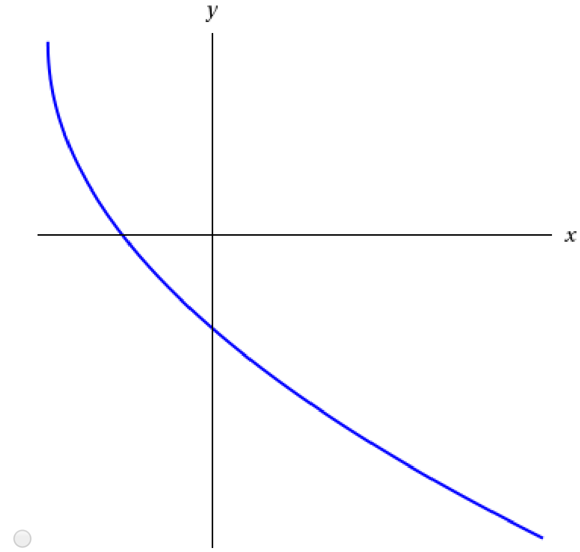
Preference: For this function, the two tests are equally easy.

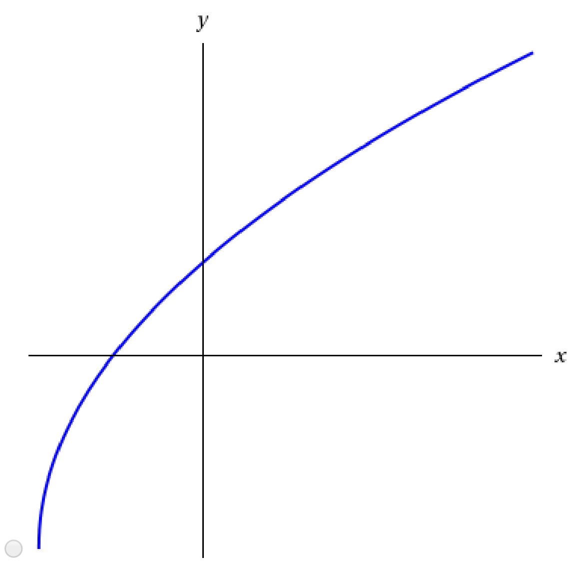
7. 2/2 points | Previous AnswersSCalcET8 4.3.024.

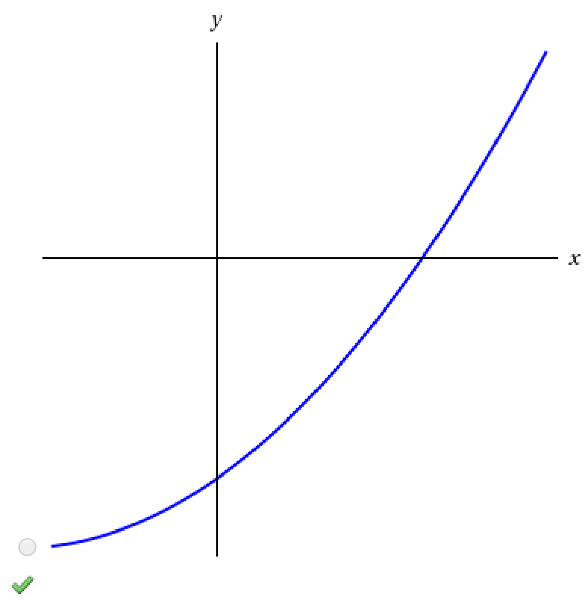
Sketch the graph of a function that satisfies all of the given conditions.

(a) $f'(x) < 0$ and $f''(x) < 0$ for all x

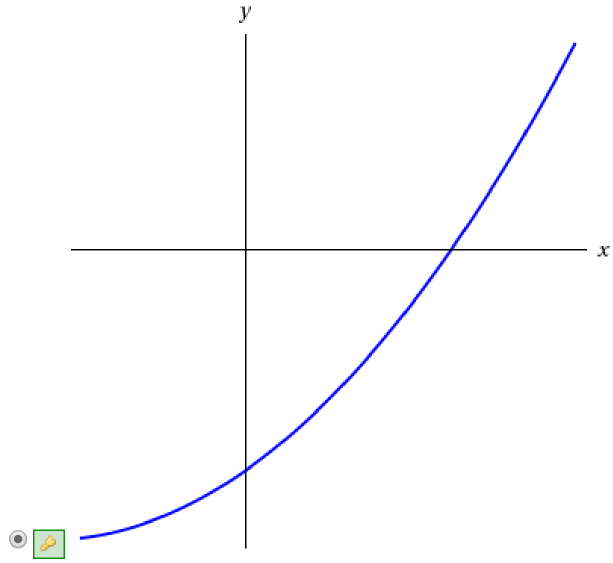


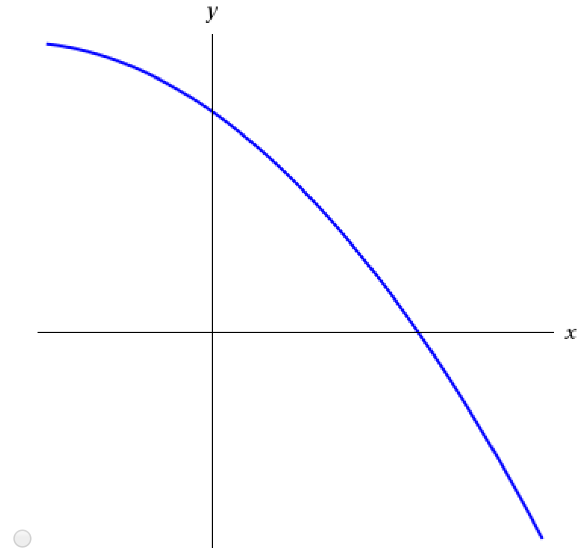


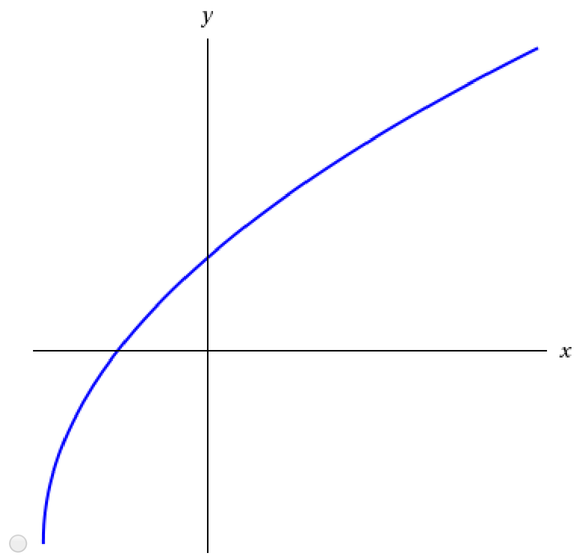


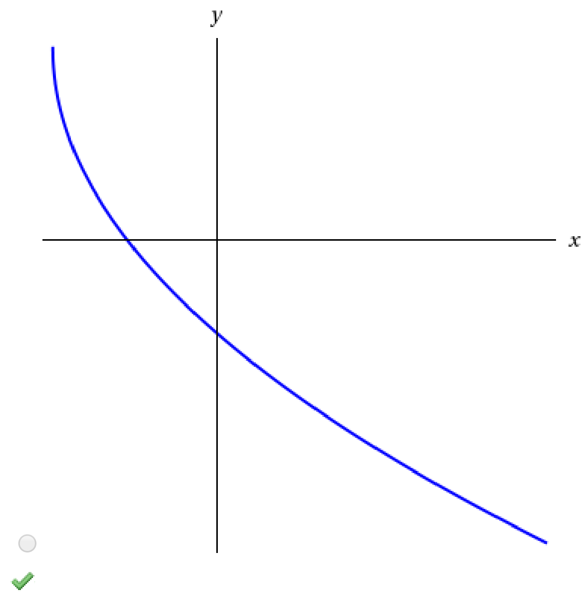


(b) $f'(x) > 0$ and $f''(x) > 0$ for all x





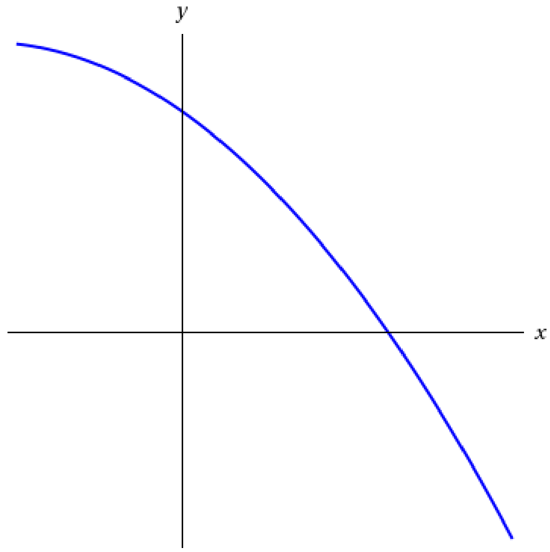




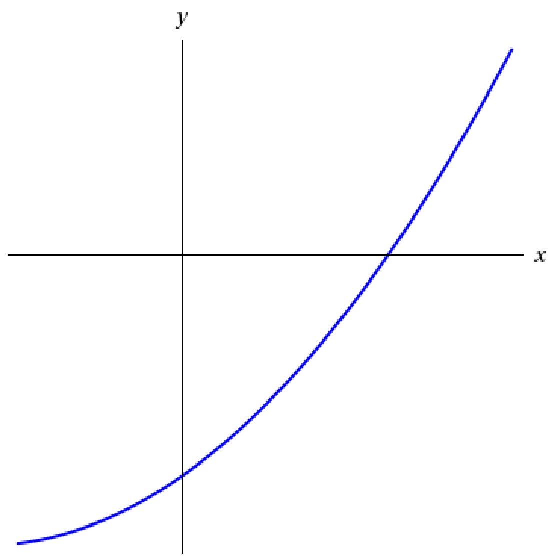
Solution or Explanation

(a) $f'(x) < 0$ and $f''(x) < 0$ for all x

The function must be always decreasing (since the first derivative is always negative) and concave downward (since the second derivative is always negative).



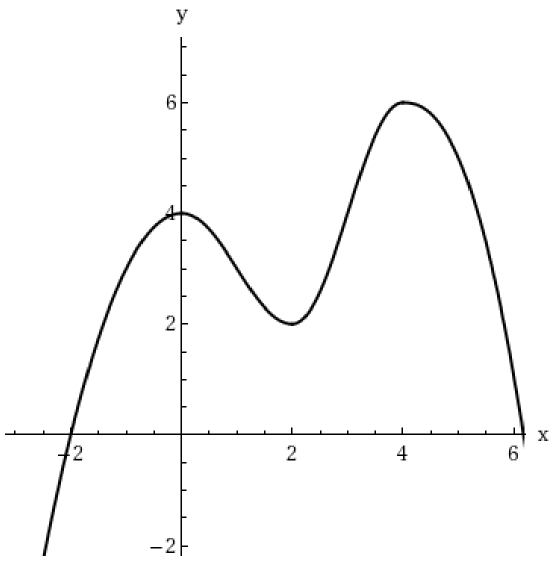
(b) $f'(x) > 0$ and $f''(x) > 0$ for all x

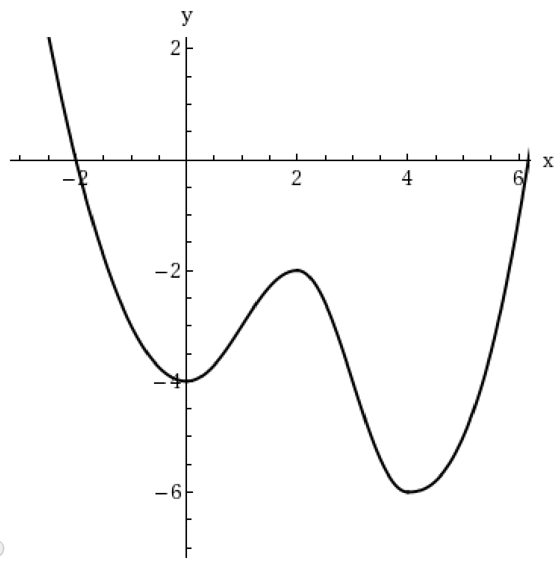


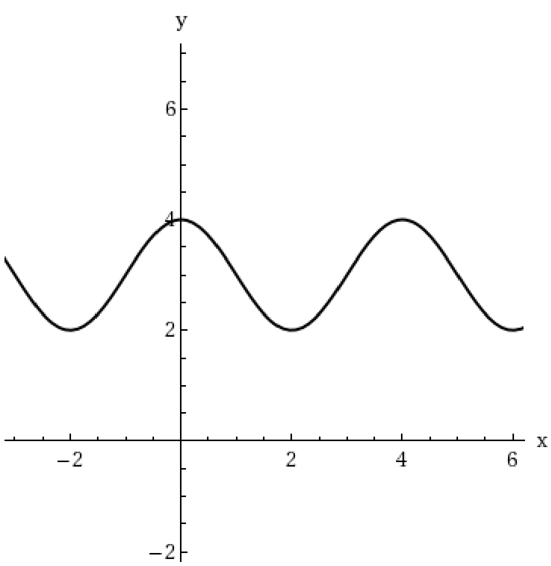
8. 1/1 points | Previous AnswersSCalcET8 4.3.027.

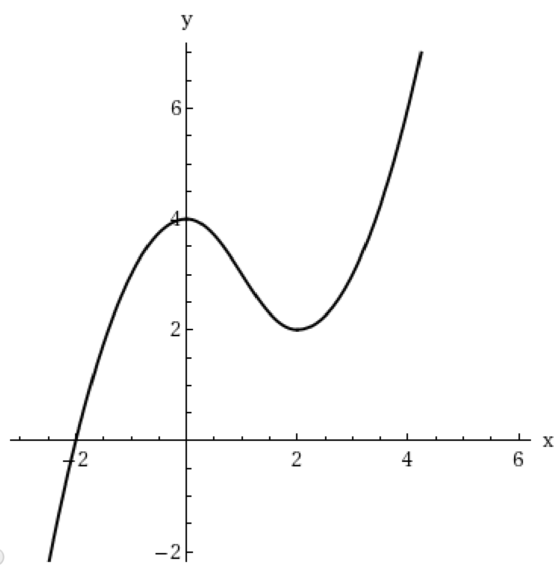
Sketch the graph of a function that satisfies all of the given conditions.

$f'(0) = f'(2) = f'(4) = 0,$
$f'(x) > 0$ if $x < 0$ or $2 < x < 4,$
$f'(x) < 0$ if $0 < x < 2$ or $x > 4,$
$f''(x) > 0$ if $1 < x < 3,$
$f''(x) < 0$ if $x < 1$ or $x > 3$









Solution or Explanation

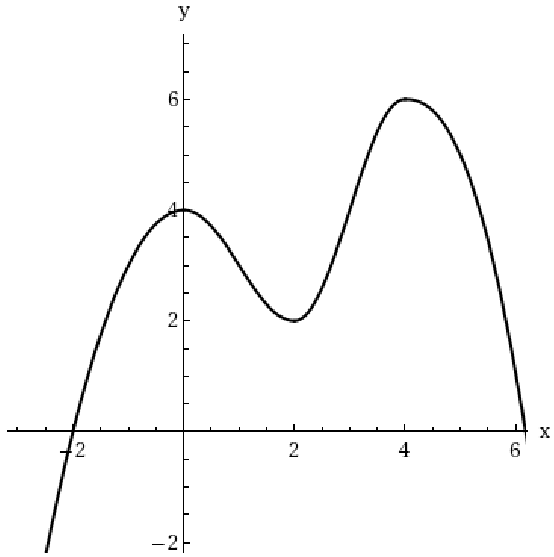
$f'(0) = f'(2) = f'(4) = 0 \Rightarrow$ horizontal tangents at $x = 0, 2, 4$.

$f'(x) > 0$ if $x < 0$ or $2 < x < 4 \Rightarrow f$ is increasing on $(-\infty, 0)$ and $(2, 4)$.

$f'(x) < 0$ if $0 < x < 2$ or $x > 4 \Rightarrow f$ is decreasing on $(0, 2)$ and $(4, \infty)$.

$f''(x) > 0$ if $1 < x < 3 \Rightarrow f$ is concave upward on $(1, 3)$.

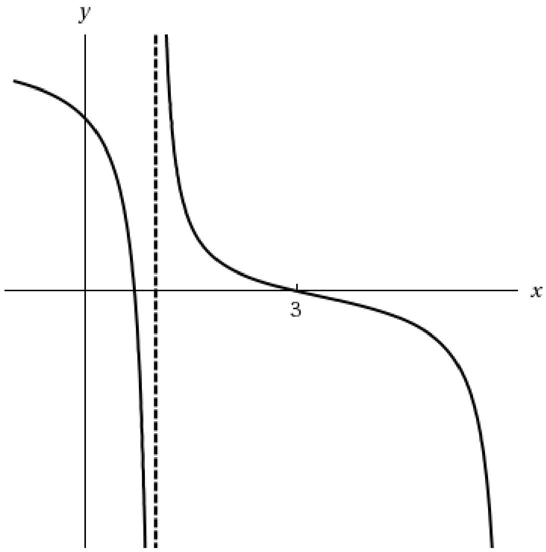
$f''(x) < 0$ if $x < 1$ or $x > 3 \Rightarrow f$ is concave downward on $(-\infty, 1)$ and $(3, \infty)$. there are inflection points where $x = 1$ and 3.

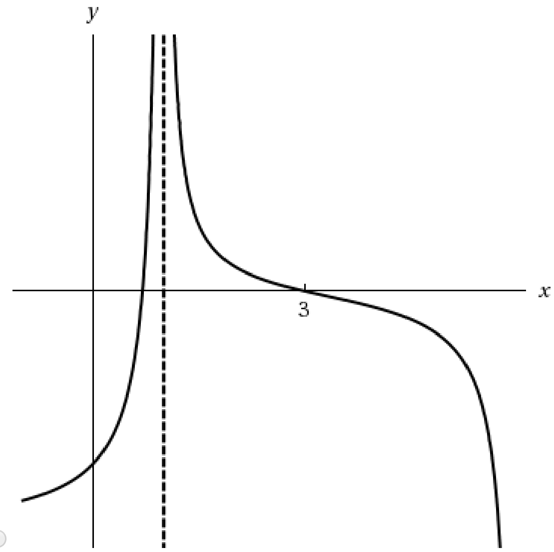

y
6
4
2
−2
−2
2
4
6
x

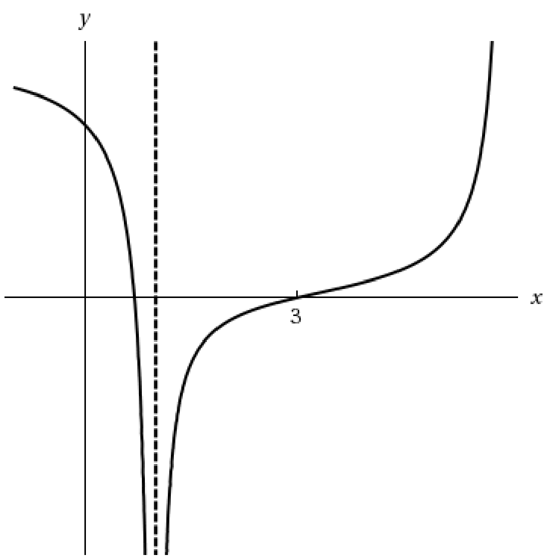
9. 1/1 points | Previous AnswersSCalcET8 4.3.028.

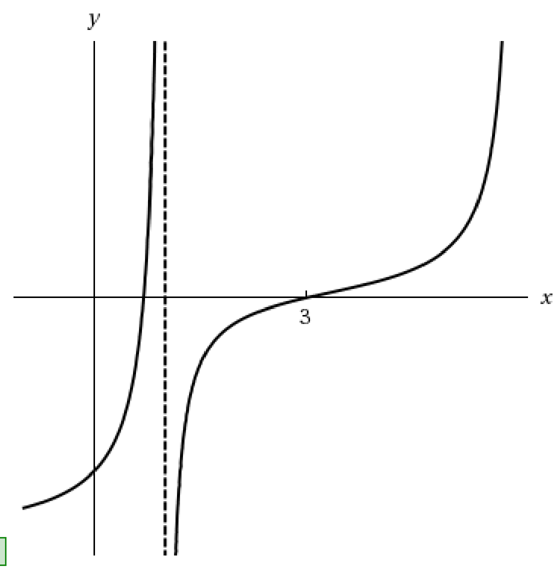
Sketch the graph of a function that satisfies all of the given conditions.

$f'(x) > 0$ for all $x \neq 1$,
vertical asymptote $x = 1$,
$f''(x) > 0$ if $x < 1$ or $x > 3$,
$f''(x) < 0$ if $1 < x < 3$









Solution or Explanation

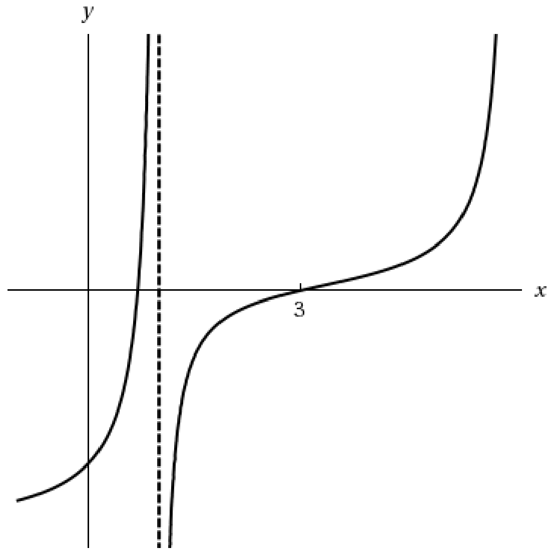
$f'(x) > 0$ for all $x \neq 1 \Rightarrow f$ is increasing on $(-\infty, 1)$ and $(1, \infty)$.

Vertical asymptote $x = 1$

$f''(x) > 0$ if $x < 1$ or $x > 3 \Rightarrow f$ is concave upward on $(-\infty, 1)$ and $(3, \infty)$.

$f''(x) < 0$ if $1 < x < 3 \Rightarrow f$ is concave downward on $(1, 3)$.

There is an inflection point at $x = 3$.


y
x
3

10. 4/4 points | Previous Answers SCalcET8 4.3.037.

Consider the function below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 12x + 3$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(−∞,−2)∪(2,∞)

$\boxed{(-\infty, -2), (2, \infty)}$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(−2,2)

$\boxed{(-2, 2)}$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$−13

$\boxed{-13}$

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$19

$\boxed{19}$

(c) Find the inflection point.

$(x, y) = \Big($

$$0,3

$\boxed{0, 3}\Big)$

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$$(0,∞)

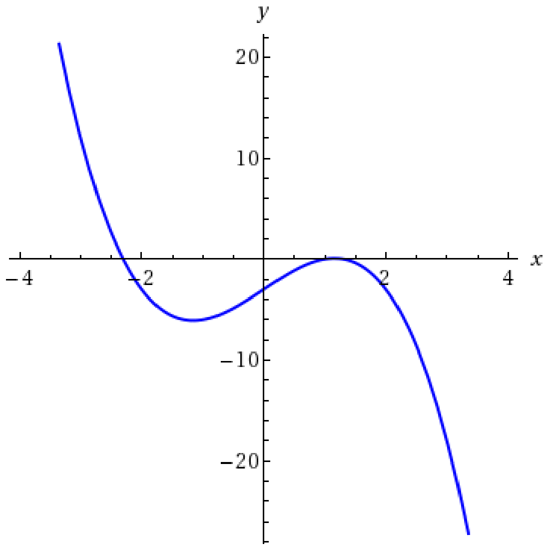
$\boxed{(0, \infty)}$

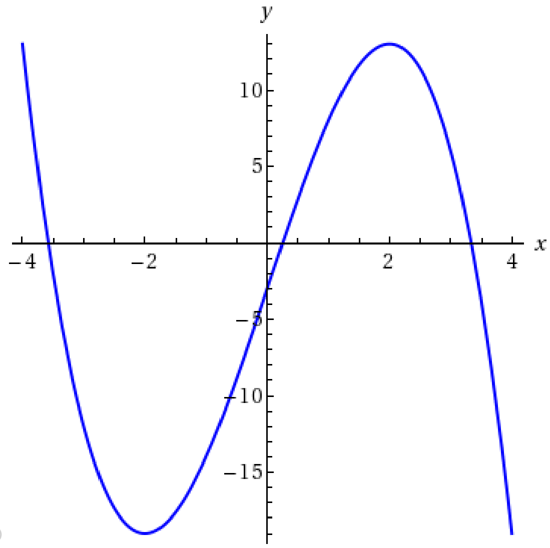
Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

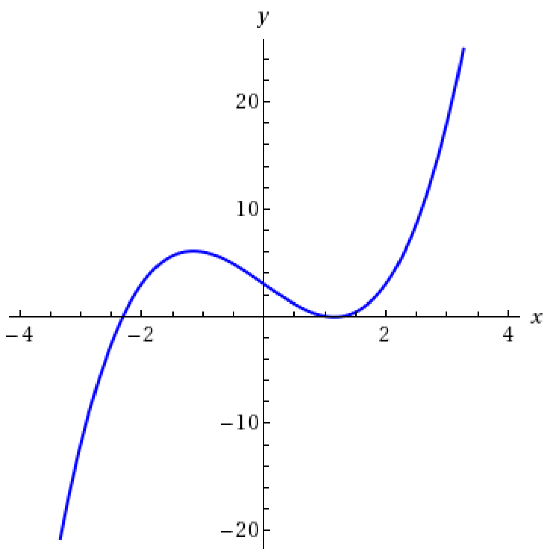
$$(−∞,0)

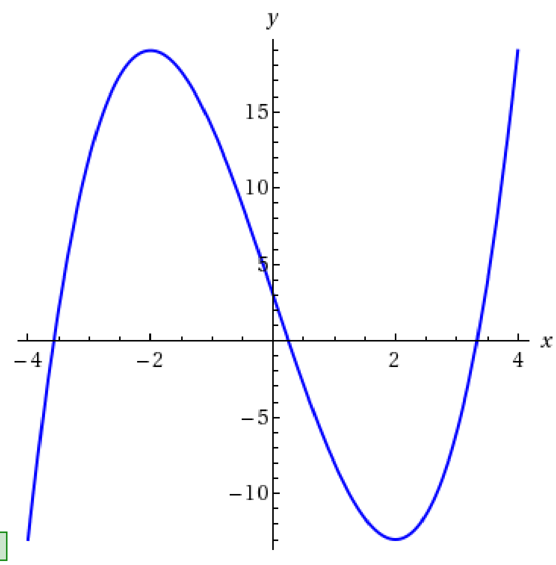
$\boxed{(-\infty, 0)}$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.









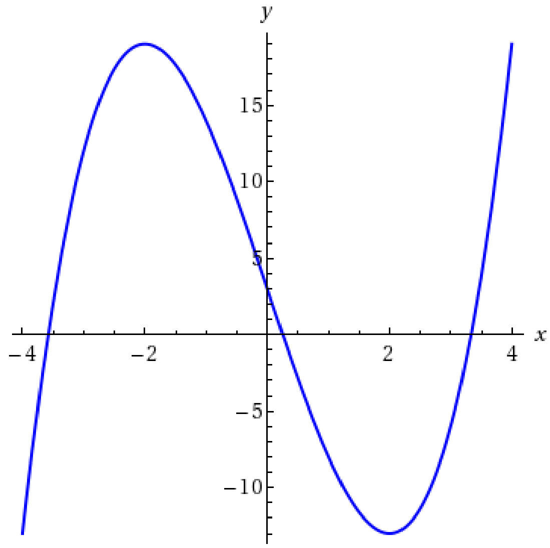
Solution or Explanation

(a) $f(x) = x^3 - 12x + 3 \;\Rightarrow\; f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x + 2)(x - 2)$. $f'(x) > 0 \;\Leftrightarrow\; x < -2$ or $x > 2$ and $f'(x) < 0 \;\Leftrightarrow\; -2 < x < 2$. So f is increasing on $(-\infty, -2)$ and $(2, \infty)$ and f is decreasing on $(-2, 2)$.

(b) f changes from increasing to decreasing at $x = -2$, so $f(-2) = 19$ is a local maximum value. f changes from decreasing to increasing at $x = 2$, so $f(2) = -13$ is a local minimum value.

(c) $f''(x) = 6x$. $f''(x) = 0 \;\Leftrightarrow\; x = 0$. $f''(x) > 0$ on $(0, \infty)$ and $f''(x) < 0$ on $(-\infty, 0)$. So f is concave upward on $(0, \infty)$ and f is concave downward on $(-\infty, 0)$. There is an inflection point at $(0, 3)$.

(d)


y
15
10
5
−5
−10
−4
−2
2
4
x

11.4.5/4.5 points | Previous AnswersSCalcET8 4.3.040.MI.

Consider the function below. (If an answer does not exist, enter DNE.)

$g(x) = 230 + 8x^3 + x^4$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(−6,∞)

$(-6, \infty)$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(−∞,−6)

$(-\infty, -6)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$−202

-202

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$*DNE*

DNE

(c) Find the inflection points.

$(x, y) = ($

$$−4,−26

$-4, -26$) (smaller x-value)

$(x, y) = ($

$$0,230

$0, 230$) (larger x-value)

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$$(−∞,−4)∪(0,∞)

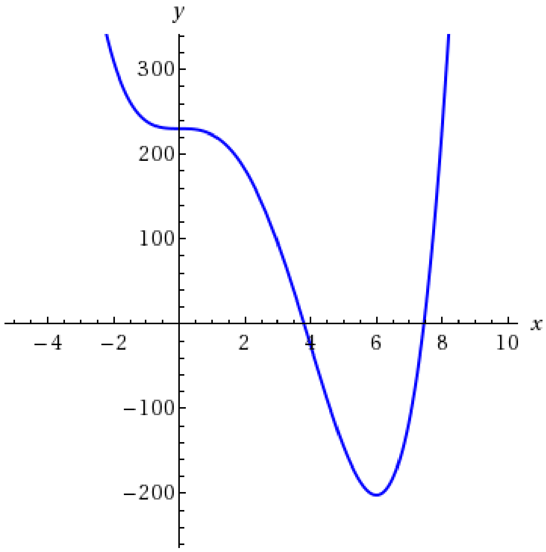
$(-\infty, -4), (0, \infty)$

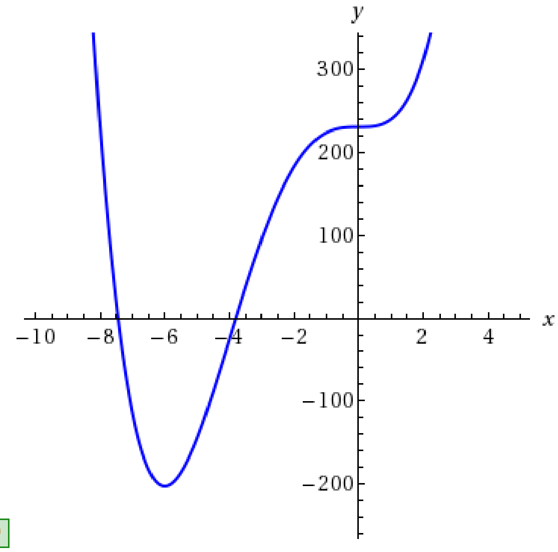
Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

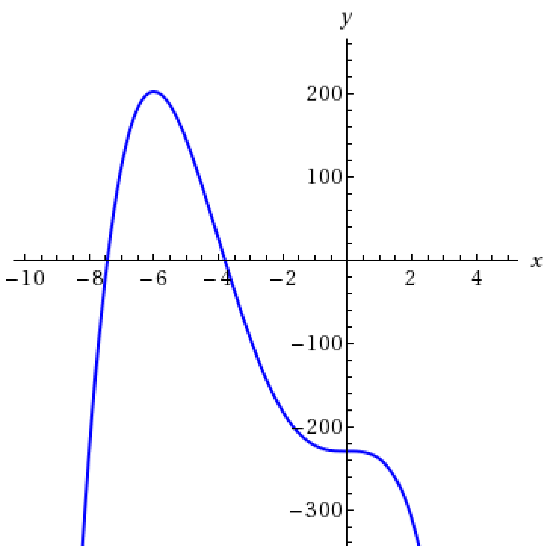
$$(−4,0)

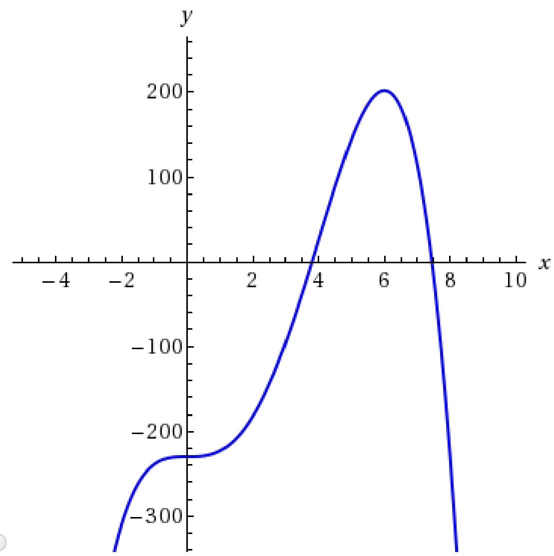
$(-4, 0)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.









Solution or Explanation

[Click to View Solution]

12. 4/4 points | Previous Answers SCalcET8 4.3.041.

Consider the function below. (If an answer does not exist, enter DNE.)

$h(x) = (x + 1)^9 - 9x - 3$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(−∞,−2)∪(0,∞)

$(-\infty, -2), (0, \infty)$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(−2,0)

$(-2, 0)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$−2

-2

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$14

14

(c) Find the inflection point.

$(x, y) = ($

$$−1,6

$-1, 6$)

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$$(−1,∞)

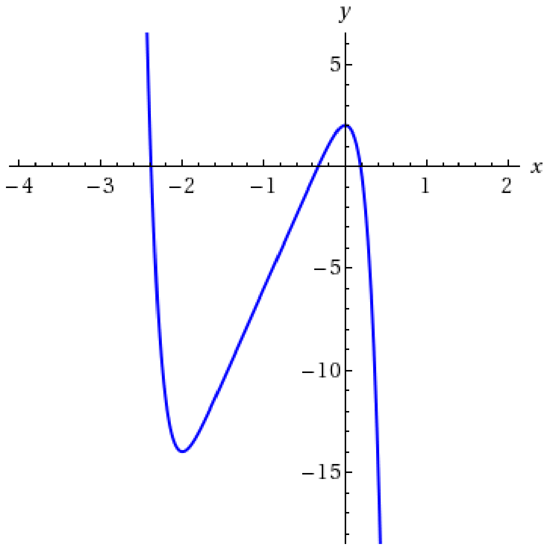
$(-1, \infty)$

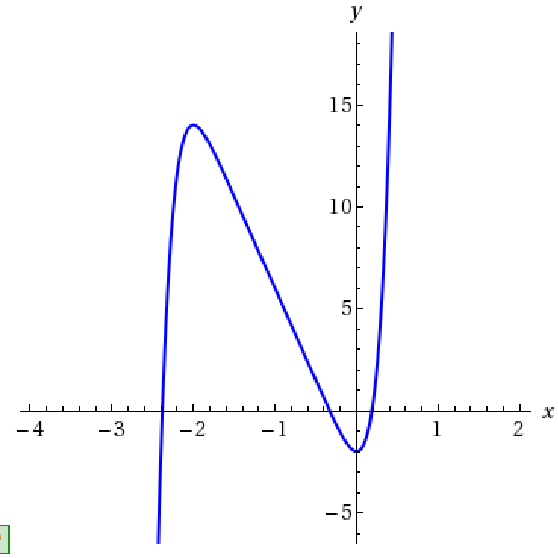
Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

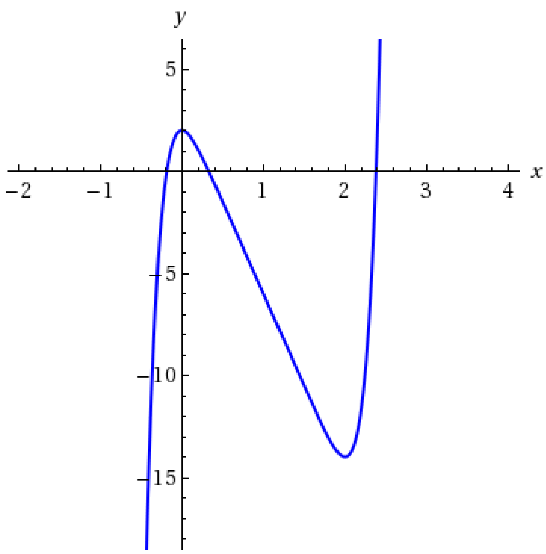
$$(−∞,−1)

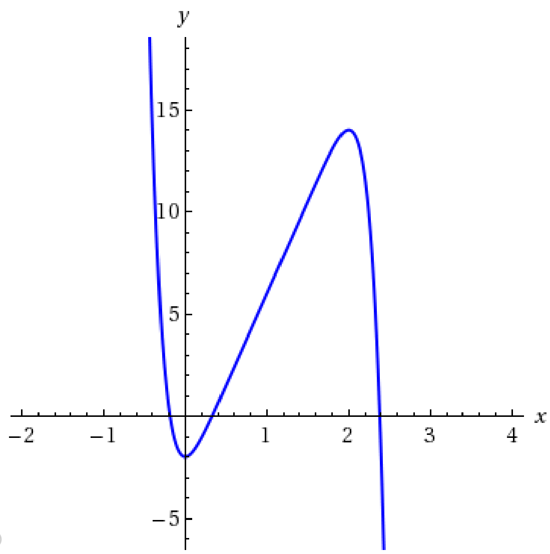
$(-\infty, -1)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.









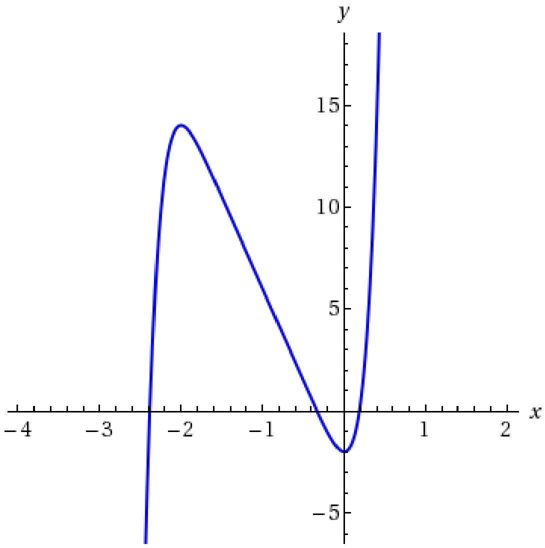
Solution or Explanation

(a) $h(x) = (x + 1)^{9} - 9x - 3 \;\Rightarrow\; h'(x) = 9(x + 1)^{8} - 9.$ $h'(x) = 0 \;\Leftrightarrow\; 9(x + 1)^{8} = 9 \;\Leftrightarrow\; (x + 1)^{8} = 1 \;\Rightarrow\; (x + 1)^{2} = 1 \;\Rightarrow$ $x + 1 = 1$ or $x + 1 = -1 \;\Rightarrow\; x = 0$ or $x = -2$. $h'(x) > 0 \;\Leftrightarrow\; x < -2$ or $x > 0$ and $h'(x) < 0 \;\Leftrightarrow\; -2 < x < 0$. So h is increasing on $(-\infty, -2)$ and $(0, \infty)$ and h is decreasing on $(-2, 0)$.

(b) $h(-2) = 14$ is a local maximum value and $h(0) = -2$ is a local minimum value.

(c) $h''(x) = 72(x + 1)^{7} = 0 \;\Leftrightarrow\; x = -1.$ $h''(x) > 0 \;\Leftrightarrow\; x > -1$ and $h''(x) < 0 \;\Leftrightarrow\; x < -1$, so h is CU on $(-1, \infty)$ and h is CD on $(-\infty, -1)$. There is a point of inflection at $(-1, h(-1)) = (-1, 6)$.

(d)


y
15
10
5
−5
x
−4
−3
−2
−1
1
2

13.5/5 points | Previous AnswersSCalcET8 4.3.042.

Consider the function below. (If an answer does not exist, enter DNE.)

$h(x) = 5x^3 - 3x^5$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(−1,0)∪(0,1)

$(-1, 1)$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(−∞,−1)∪(1,∞)

$(-\infty, -1), (1, \infty)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$−2

-2

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$2

2

(c) Find the inflection points.

(

$$−√12,−5(12)(32)+3(12)(52)

$(x, y) = \left(-\frac{1}{\sqrt{2}}, -\frac{7}{4\sqrt{2}}\right)$ (smallest x-value)

(

$$0,0

$(x, y) = (0, 0)$

(

$$√12,5(12)(32)−3(12)(52)

$(x, y) = \left(\frac{1}{\sqrt{2}}, \frac{7}{4\sqrt{2}}\right)$ (largest x-value)

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$$(−∞,−√0.5)∪(0,√0.5)

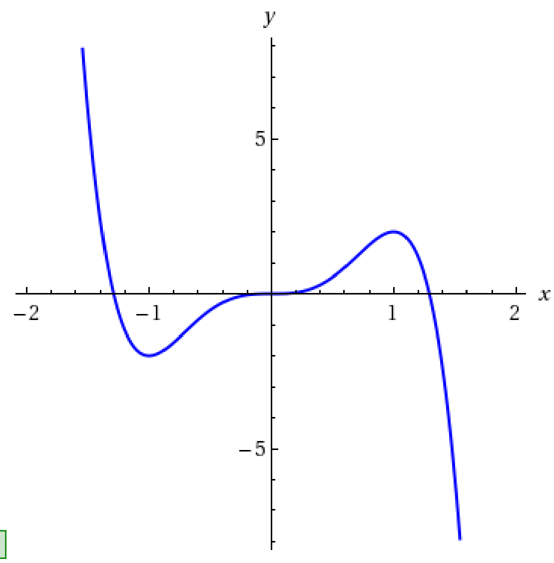
$\left(-\infty, -\frac{1}{\sqrt{2}}\right), \left(0, \frac{1}{\sqrt{2}}\right)$

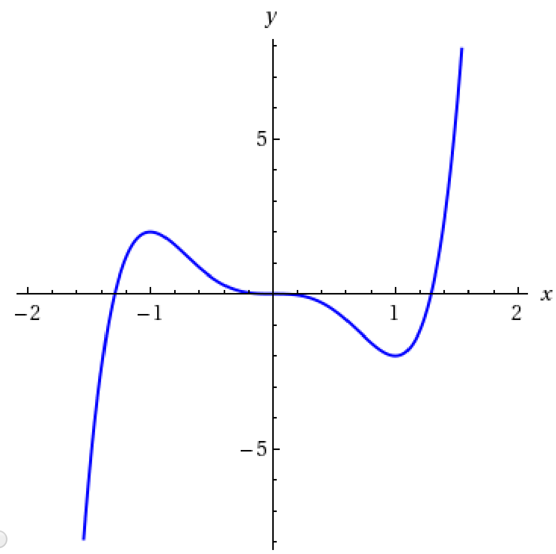
Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

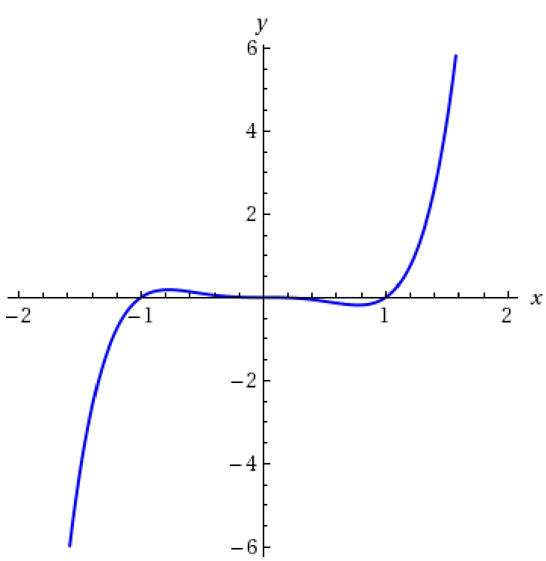
$$(−√0.5,0)∪(√0.5,∞)

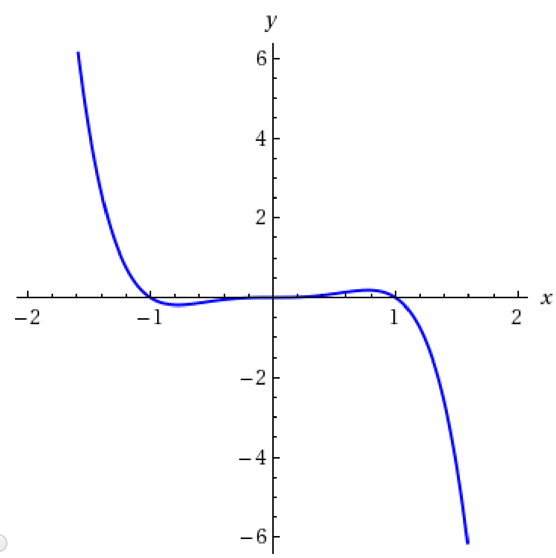
$\left(-\frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{2}}, \infty\right)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.









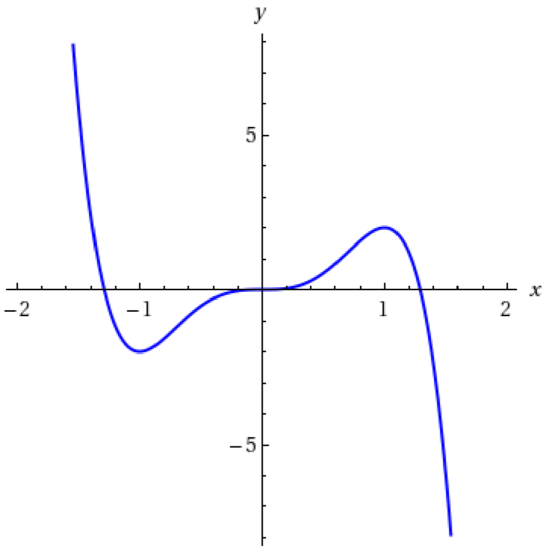
Solution or Explanation

(a) $h(x) = 5x^3 - 3x^5 \;\Rightarrow\; h'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2) = 15x^2(1 + x)(1 - x)$. $h'(x) > 0 \;\Leftrightarrow\; -1 < x < 0$ and $0 < x < 1$ [note that $h'(0) = 0$] and $h'(x) < 0 \;\Leftrightarrow\; x < -1$ or $x > 1$. So h is increasing on $(-1, 1)$ and h is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

(b) h changes from decreasing to increasing at $x = -1$, so $h(-1) = -2$ is a local minimum value. h changes from increasing to decreasing at $x = 1$, so $h(1) = 2$ is a local maximum value.

(c) $h''(x) = 30x - 60x^3 = 30x(1 - 2x^2)$. $h''(x) = 0 \;\Leftrightarrow\; x = 0$ or $1 - 2x^2 = 0 \;\Leftrightarrow\; x = 0$ or $x = \pm\frac{1}{\sqrt{2}}$. $h''(x) > 0$ on $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ and $\left(0, \frac{1}{\sqrt{2}}\right)$, and $h''(x) < 0$ on $\left(-\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$. So h is CU on $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ and $\left(0, \frac{1}{\sqrt{2}}\right)$, and h is CD on $\left(-\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$. There are inflection points at $\left(-\frac{1}{\sqrt{2}}, -\frac{7}{4\sqrt{2}}\right)$, $(0, 0)$, and $\left(\frac{1}{\sqrt{2}}, \frac{7}{4\sqrt{2}}\right)$.

(d)



14.3.2/0 points | Previous AnswersSCalcET8 4.3.043.

Consider the function below. (If an answer does not exist, enter DNE.)

$F(x) = x\sqrt{15 - x}$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(-\infty,10)

$(-\infty, 10)$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(10,15)

$(10, 15)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$*DNE*

DNE

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$10√5

$10\sqrt{5}$

(c) Find the inflection point.

$(x, y) = ($

$$*DNE*

DNE $)$

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$$*DNE*

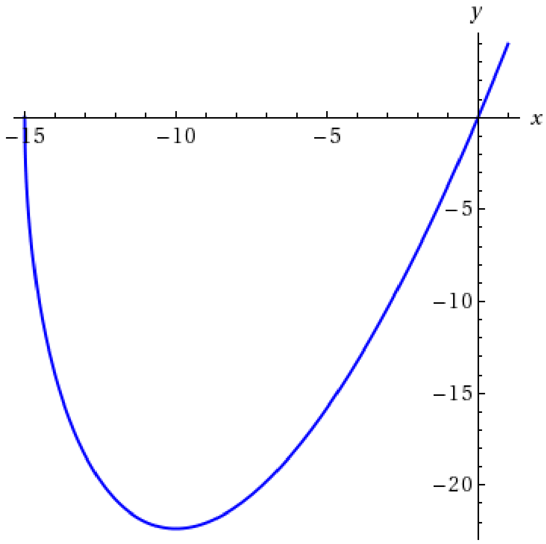
DNE

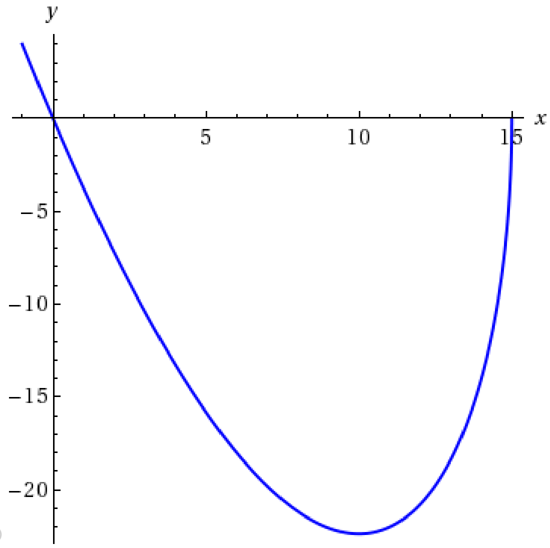
Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

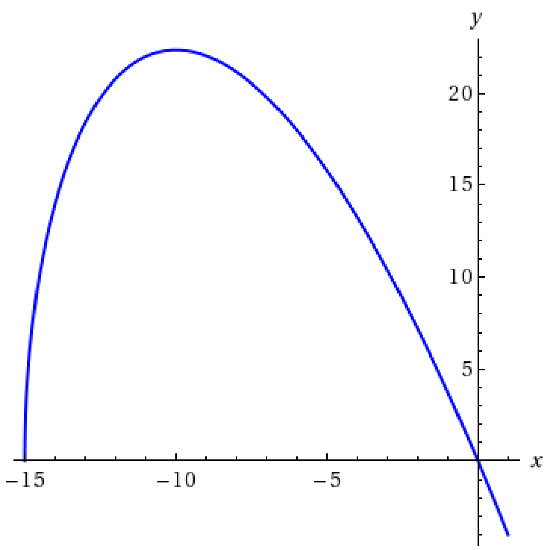
$$(-\infty,15)

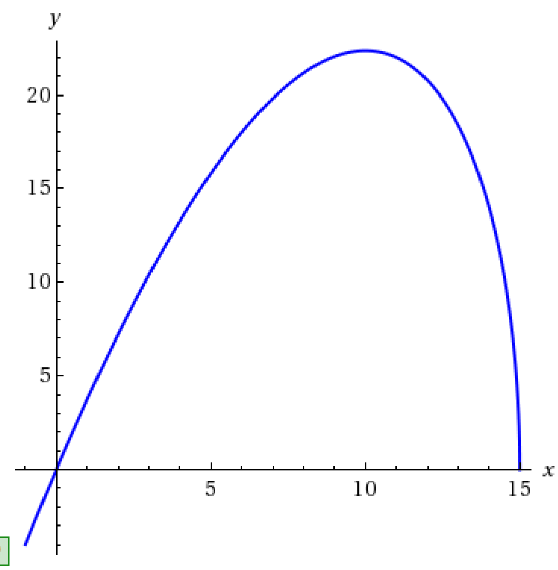
$(-\infty, 15)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.









Solution or Explanation

(a) $F(x) = x\sqrt{15 - x} \Rightarrow$

$$F'(x) = x \cdot \frac{1}{2}(15 - x)^{-1/2}(-1) + (15 - x)^{1/2}(1) = \frac{1}{2}(15 - x)^{-1/2}[-x + 2(15 - x)] = \frac{-3x + 30}{2\sqrt{15 - x}}.$$

$F'(x) > 0 \Leftrightarrow -3x + 30 > 0 \Leftrightarrow x < 10$ and $F'(x) < 0 \Leftrightarrow 10 < x < 15$. So F is increasing on $(-\infty, 10)$ and F is decreasing on $(10, 15)$.

(b) F changes from increasing to decreasing at $x = 10$, so $F(10) = 10\sqrt{5}$ is a local maximum value. There is no local minimum value.

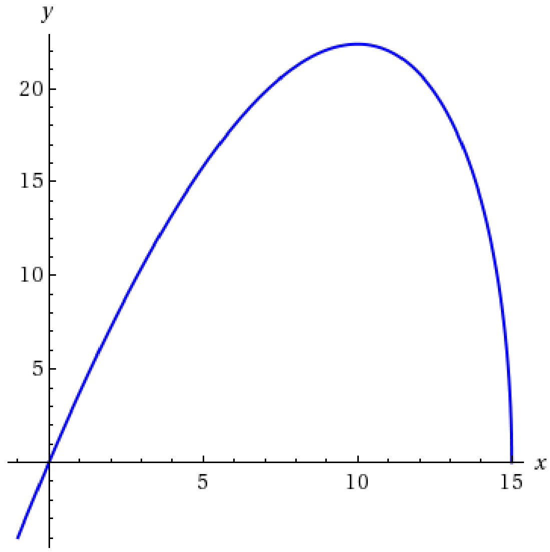
(c) $F'(x) = -\frac{3}{2}(x - 10)(15 - x)^{-1/2} \Rightarrow$

$$F''(x) = -\frac{3}{2}\left[(x - 10)\left(-\frac{1}{2}(15 - x)^{-3/2}(-1)\right) + (15 - x)^{-1/2}(1)\right]$$

$$= -\frac{3}{2} \cdot \frac{1}{2}(15 - x)^{-3/2}[(x - 10) + 2(15 - x)] = \frac{3(x - 20)}{4(15 - x)^{3/2}}$$

$F''(x) < 0$ on $(-\infty, 15)$, so F is CD on $(-\infty, 15)$. There is no inflection point.

(d)



y
20
15
10
5
5
10
15
x

15.4.5/0 points | Previous AnswersSCalcET8 4.3.047.

Consider the function below. (If an answer does not exist, enter DNE.)

$f(\theta) = 2\cos(\theta) + \cos^2(\theta), \quad 0 \le \theta \le 2\pi$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$$(π,2π)

$\boxed{(\pi, 2\pi)}$

Find the interval of decrease. (Enter your answer using interval notation.)

$$(0,π)

$\boxed{(0, \pi)}$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$$−1

$\boxed{-1}$

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$$*DNE*

$\boxed{\text{DNE}}$

(c) Find the inflection points.

(

$$π3,54

$(x, y) =$ $\boxed{\frac{\pi}{3}, \frac{5}{4}}$) (smaller *x*-value)

(

$$5π3,54

$(x, y) =$ $\boxed{\frac{5\pi}{3}, \frac{5}{4}}$) (larger *x*-value)

Find the intervals where the graph is concave upward. (Enter your answer using interval notation.)

$$(π3,5π3)

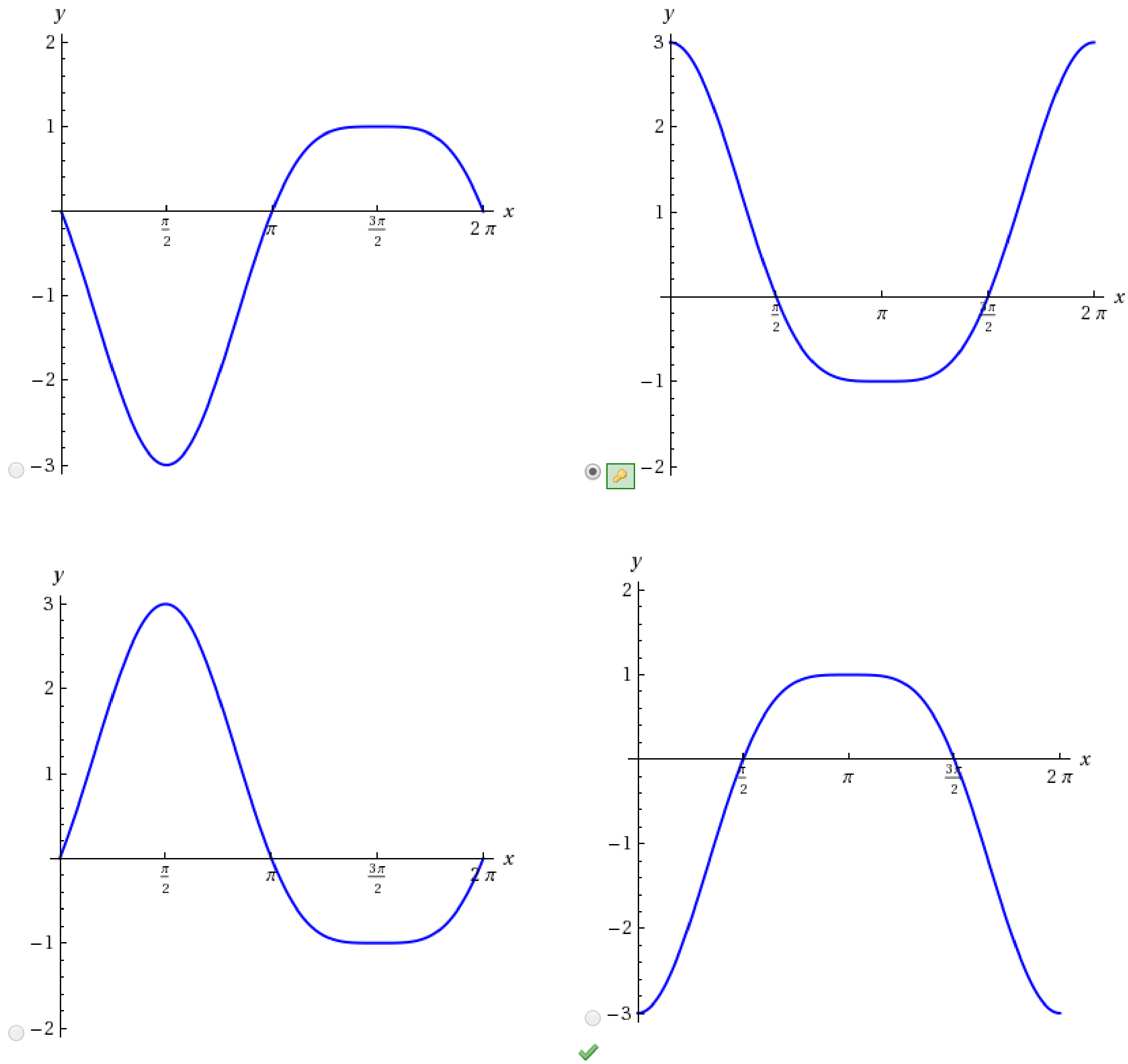
$\boxed{\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)}$

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$$(0,π3)∪(5π3,2π)

$\boxed{\left(0, \frac{\pi}{3}\right), \left(\frac{5\pi}{3}, 2\pi\right)}$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.



Solution or Explanation

(a) $f(\theta) = 2\cos(\theta) + \cos^2(\theta),\ 0 \le \theta \le 2\pi \ \Rightarrow\ f'(\theta) = -2\sin(\theta) + 2\cos(\theta)\,(-\sin(\theta)) = -2\sin(\theta)(1 + \cos(\theta))$. $f'(\theta) = 0 \ \Leftrightarrow\ \theta = 0, \pi,$ and 2π. $f'(\theta) > 0 \ \Leftrightarrow\ \pi < \theta < 2\pi$ and $f'(\theta) < 0 \ \Leftrightarrow\ 0 < \theta < \pi$. So f is increasing on $(\pi, 2\pi)$ and f is decreasing on $(0, \pi)$.

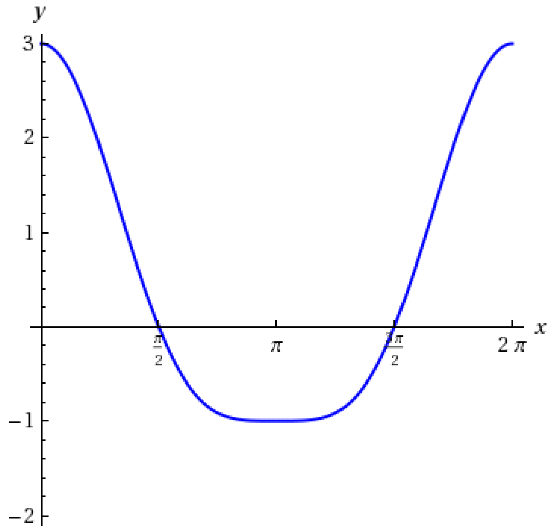
(b) $f(\pi) = -1$ is a local minimum value.

(c) $f'(\theta) = -2\sin(\theta)(1 + \cos(\theta)) \ \Rightarrow$

$$\begin{aligned} f''(\theta) &= -2\sin(\theta)(-\sin(\theta)) + (1 + \cos(\theta))(-2\cos(\theta)) \\ &= 2\sin^2(\theta) - 2\cos(\theta) - 2\cos^2(\theta) \\ &= 2(1 - \cos^2(\theta)) - 2\cos(\theta) - 2\cos^2(\theta) = -4\cos^2(\theta) - 2\cos(\theta) + 2 \\ &= -2(2\cos^2(\theta) + \cos(\theta) - 1) = -2(2\cos(\theta) - 1)(\cos(\theta) + 1) \end{aligned}$$

Since $-2(\cos(\theta) + 1) < 0$ [for $\theta \ne \pi$], $f''(\theta) > 0 \ \Rightarrow\ 2\cos(\theta) - 1 < 0 \ \Rightarrow\ \cos(\theta) < \frac{1}{2} \ \Rightarrow\ \frac{\pi}{3} < \theta < \frac{5\pi}{3}$ and $f''(\theta) < 0 \ \Rightarrow$ $\cos(\theta) > \frac{1}{2} \ \Rightarrow\ 0 < \theta < \frac{\pi}{3}$ or $\frac{5\pi}{3} < \theta < 2\pi$. So f is CU on $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$ and f is CD on $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$. There are points of inflection at $\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, \frac{5}{4}\right)$ and $\left(\frac{5\pi}{3}, f\left(\frac{5\pi}{3}\right)\right) = \left(\frac{5\pi}{3}, \frac{5}{4}\right)$.

(d)



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16. 2/2 points | Previous Answers SCalcET8 4.3.057.MI.

Suppose the derivative of a function f is $f'(x) = (x + 2)^4(x - 5)^7(x - 6)^6$. On what interval is f increasing? (Enter your answer in interval notation.)

$$(5,6)∪(6,∞)

$(5, \infty)$

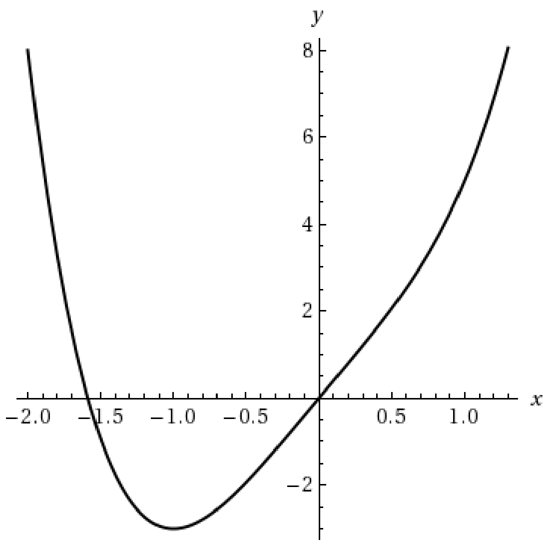
Solution or Explanation

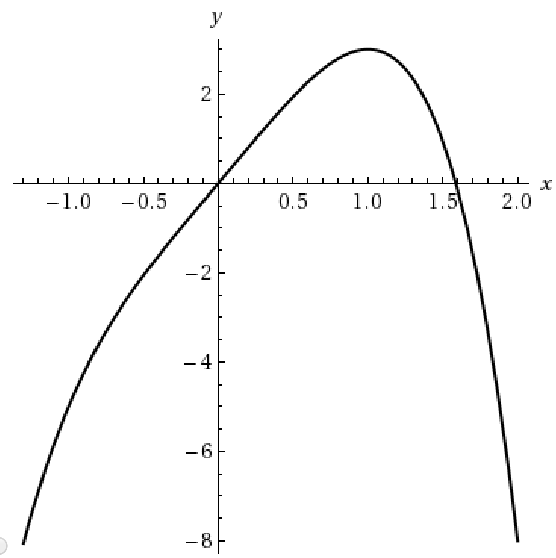
Click to View Solution

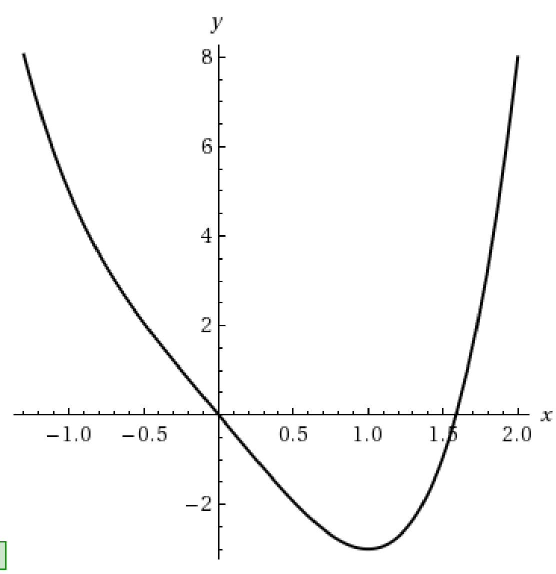
17. 2/2 points | Previous Answers SCalcET8 4.5.003.

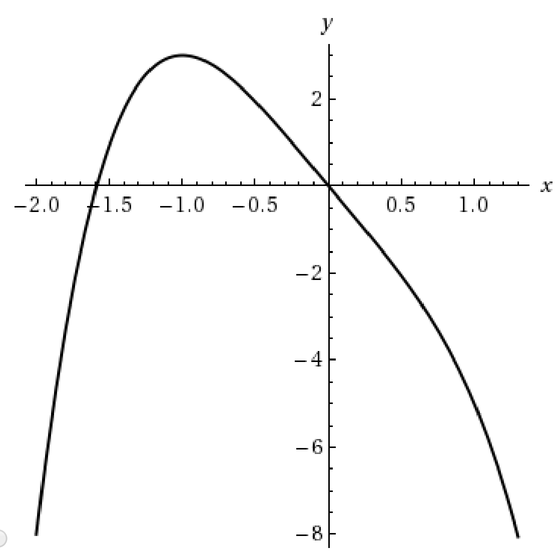
Use the guidelines of this section to sketch the curve.

$y = x^4 - 4x$









Solution or Explanation

$y = f(x) = x^4 - 4x = x(x^3 - 4)$

A. $D = \mathbb{R}$

B. x-intercepts are 0 and $\sqrt[3]{4}$, y-intercept $= f(0) = 0$

C. No symmetry

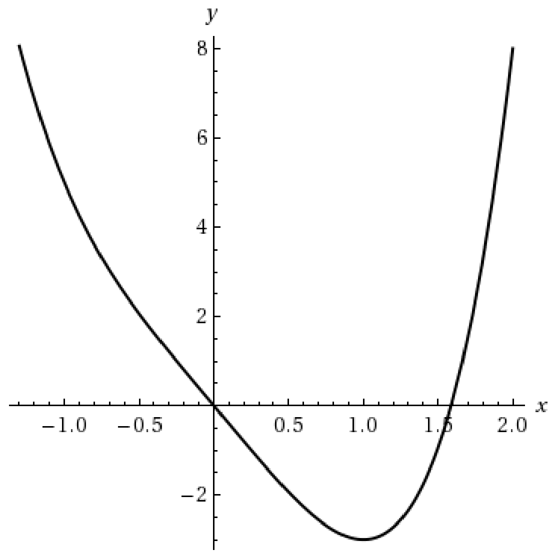
D. No asymptote

E. $f'(x) = 4x^3 - 4 = 4(x^3 - 1) = 4(x - 1)(x^2 + x + 1) > 0 \iff x > 1$, so f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.

F. Local minimum value $f(1) = -3$, no local maximum

G. $f''(x) = 12x^2 > 0$ for all x, so f is CU on $(-\infty, \infty)$. No IP.

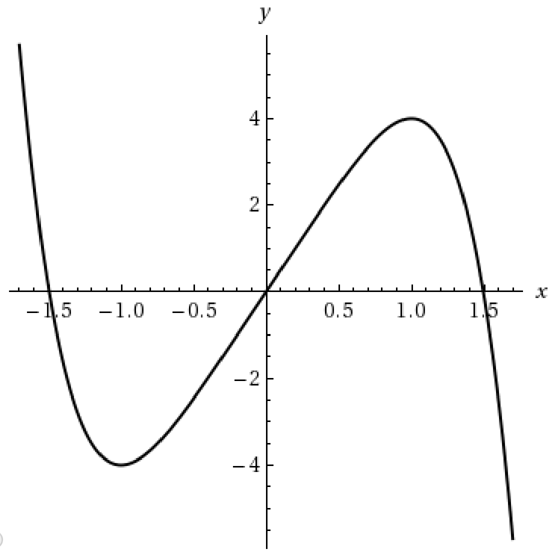
H.

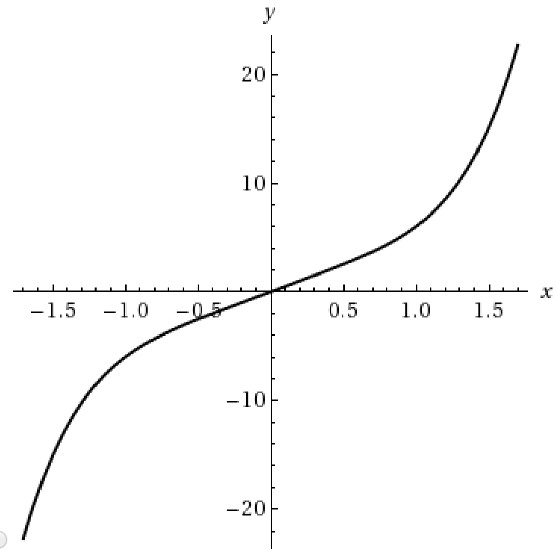


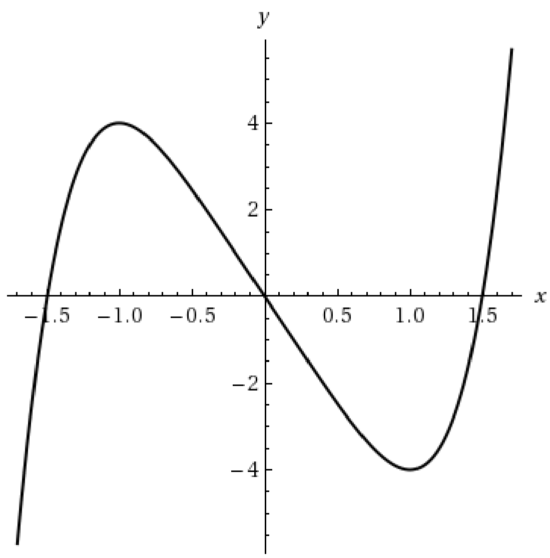
18. 2/2 points | Previous Answers SCalcET8 4.5.006.

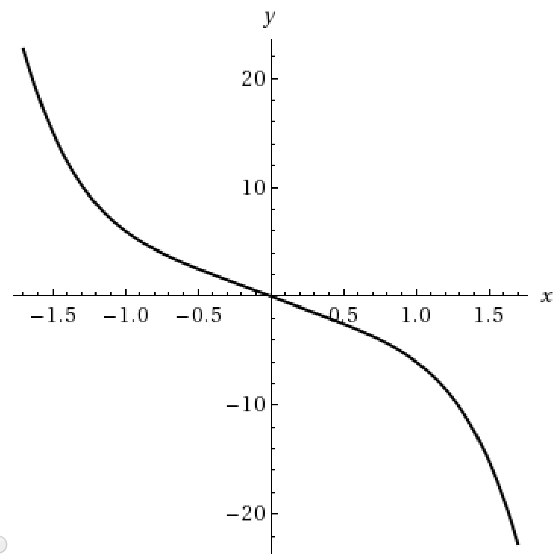
Use the guidelines of this section to sketch the curve.

$y = x^5 - 5x$









Solution or Explanation

$y = f(x) = x^5 - 5x = x(x^4 - 5)$

A. $D = \mathbb{R}$

B. x-intercepts $\pm\sqrt[4]{5}$ and 0, y-intercept $= f(0) = 0$

C. $f(-x) = -f(x)$, so f is odd; the curve is symmetric about the origin.

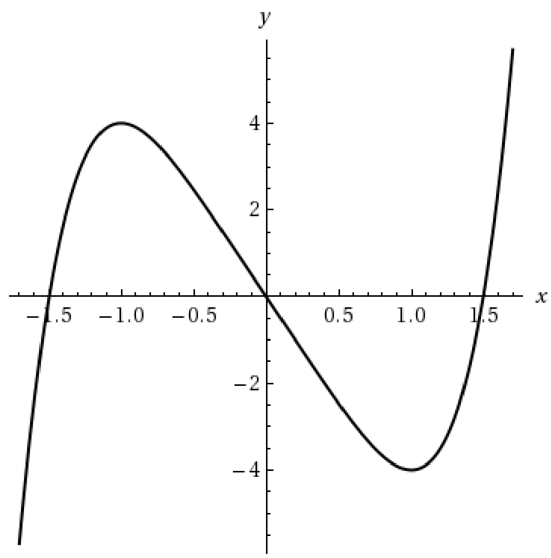
D. No asymptote

E. $f'(x) = 5x^4 - 5 = 5(x^4 - 1) = 5(x^2 - 1)(x^2 + 1) = 5(x + 1)(x - 1)(x^2 + 1) > 0 \iff x < -1$ or $x > 1$, so f is increasing on $(-\infty, -1)$ and $(1, \infty)$, and f is decreasing on $(-1, 1)$.

F. Local maximum value $f(-1) = 4$, local minimum value $f(1) = -4$.

G. $f''(x) = 20x^3 > 0 \iff x > 0$, so f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. IP at $(0, 0)$.

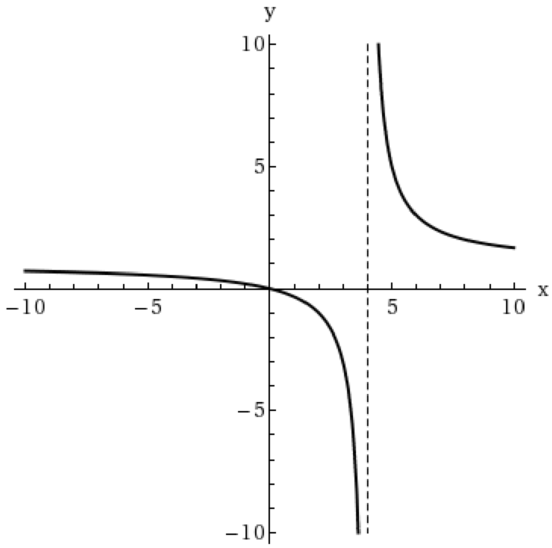
H.

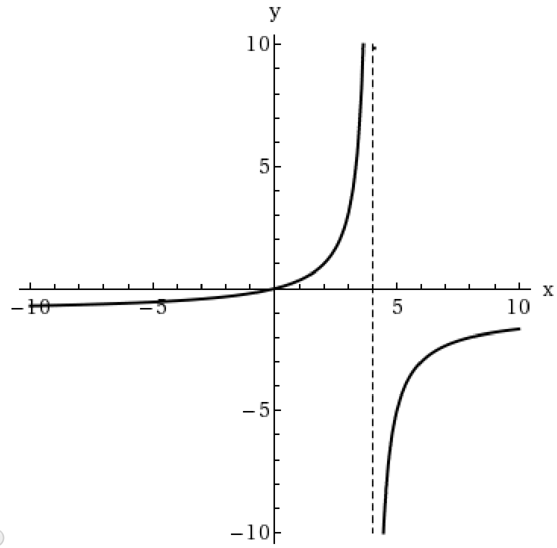


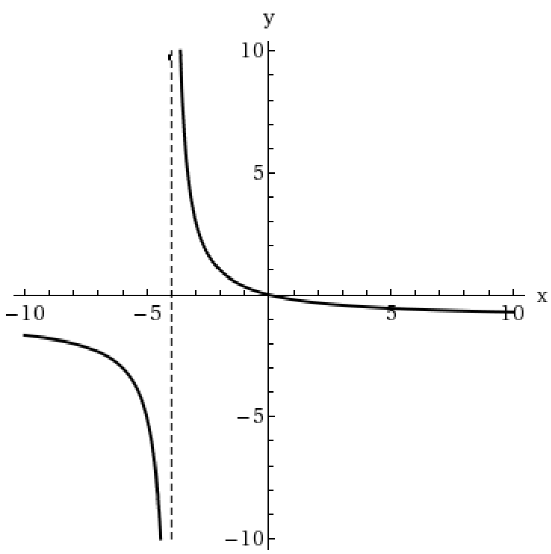
19.2/2 points | Previous AnswersSCalcET8 4.5.009.

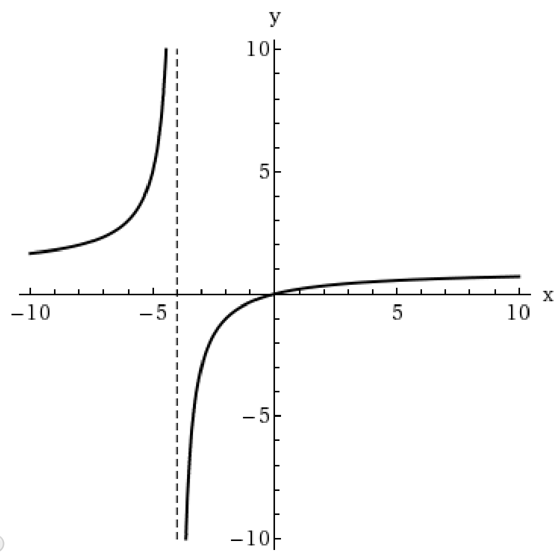
Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{x - 4}$$









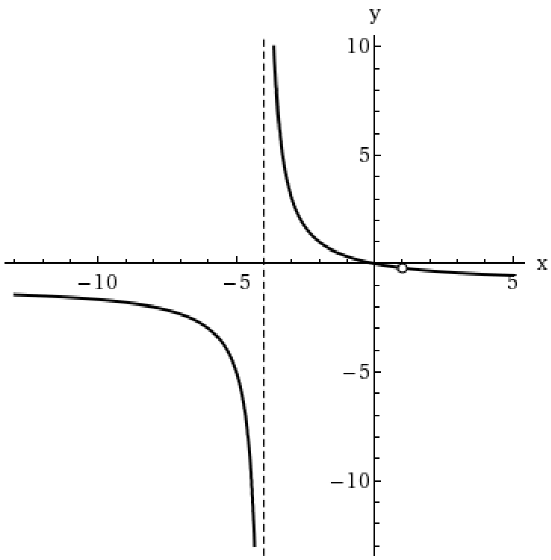
Solution or Explanation

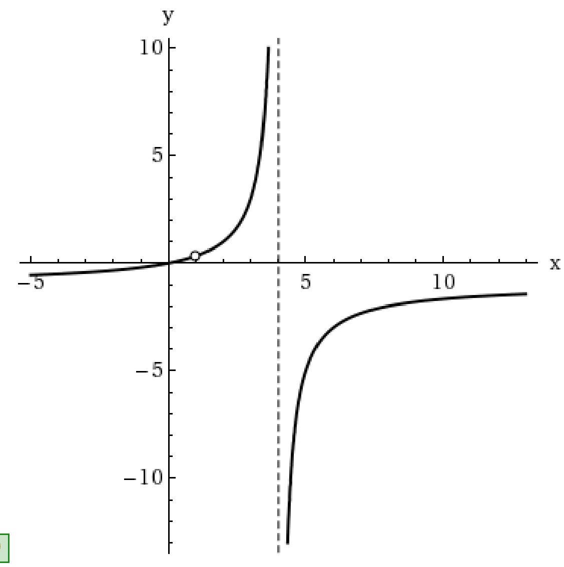
Click to View Solution

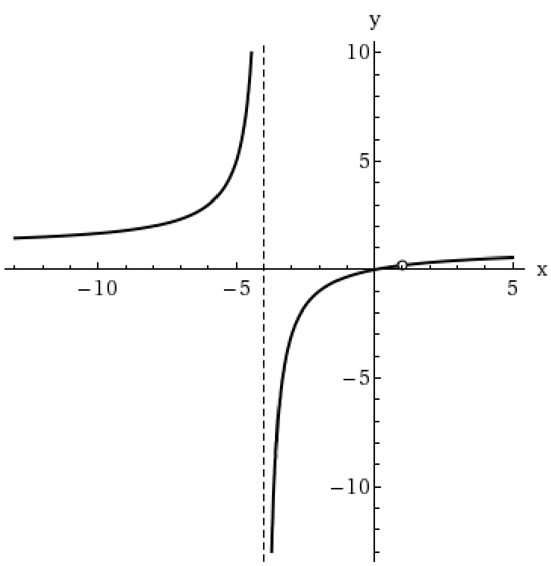
20. 2/2 points | Previous Answers SCalcET8 4.5.011.

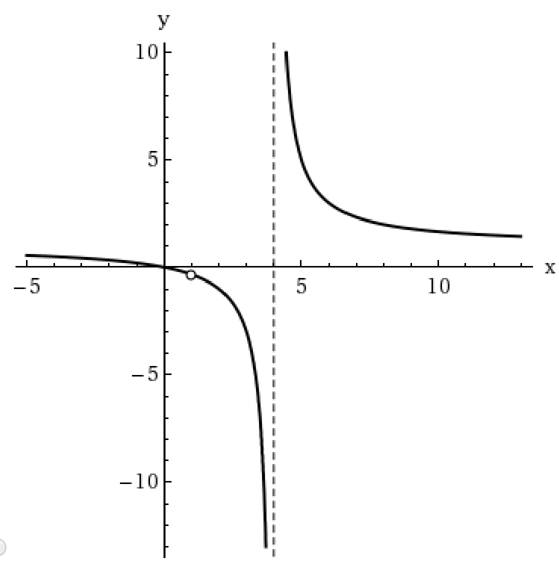
Use the guidelines of this section to sketch the curve.

$$y = \frac{x - x^2}{4 - 5x + x^2}$$









Solution or Explanation

$y = f(x) = \dfrac{x - x^2}{4 - 5x + x^2} = \dfrac{x(1 - x)}{(1 - x)(4 - x)} = \dfrac{x}{4 - x}$ for $x \neq 1$. There is a hole in the graph at $\left(1, \dfrac{1}{3}\right)$.

A. $D = \{x \mid x \neq 1, 4\} = (-\infty, 1) \cup (1, 4) \cup (4, \infty)$

B. x-intercept $= 0$, y-intercept $= f(0) = 0$

C. No symmetry

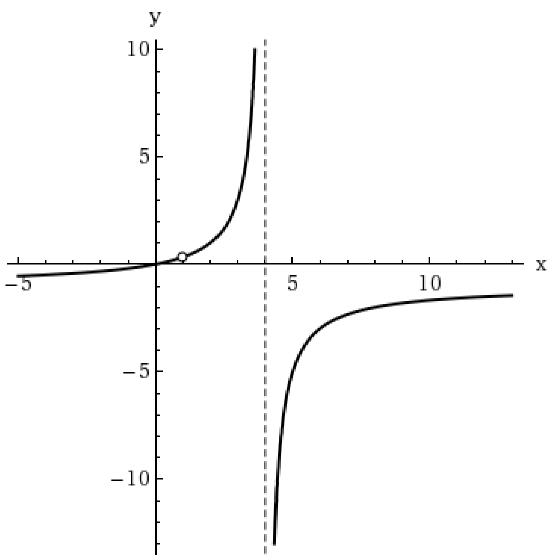
D. $\displaystyle\lim_{x \to \pm\infty} \frac{x}{4 - x} = -1$, so $y = -1$ is a HA. $\displaystyle\lim_{x \to 4^-} \frac{x}{4 - x} = \infty$, $\displaystyle\lim_{x \to 4^+} \frac{x}{4 - x} = -\infty$, so $x = 4$ is a VA.

E. $f'(x) = \dfrac{(4 - x)(1) - x(-1)}{(4 - x)^2} = \dfrac{4}{(4 - x)^2} > 0$ $[x \neq 1, 4]$, so f is increasing on $(-\infty, 1)$, $(1, 4)$, and $(4, \infty)$.

F. No extrema

G. $f'(x) = 4(4 - x)^{-2} \Rightarrow f''(x) = -8(4 - x)^{-3}(-1) = \dfrac{8}{(4 - x)^3} > 0 \iff x < 4$, so f is CU on $(-\infty, 1)$ and $(1, 4)$, and f is CD on $(4, \infty)$. No IP

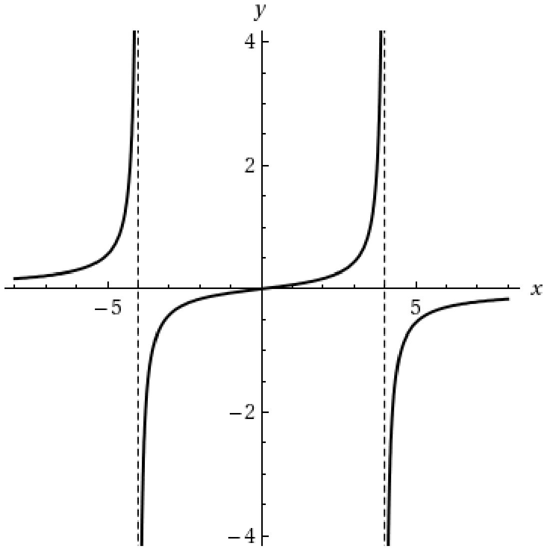
H.

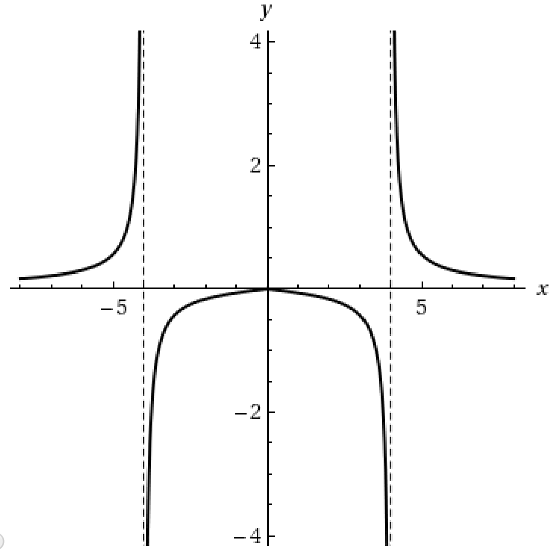


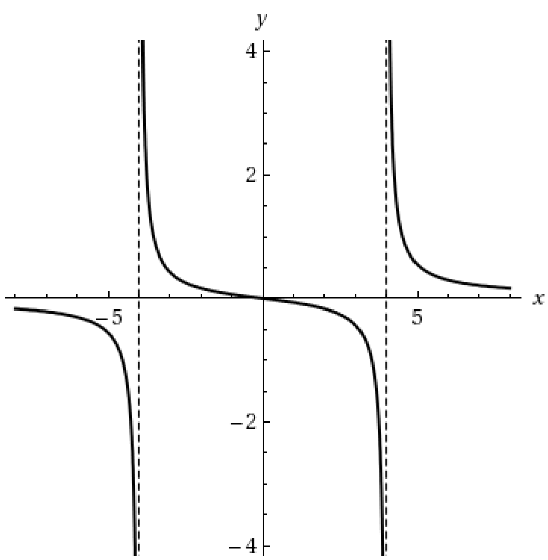
21. 2/2 points | Previous Answers SCalcET8 4.5.013.

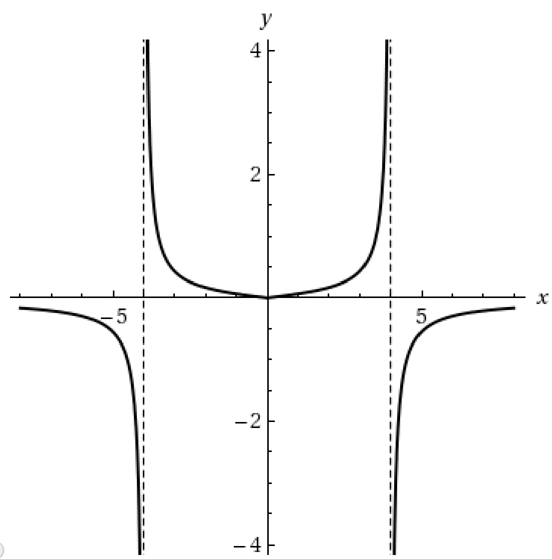
Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{x^2 - 16}$$









Solution or Explanation

$$y = f(x) = \frac{x}{x^2 - 16} = \frac{x}{(x + 4)(x - 4)}$$

A. $D = (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

B. x-intercept $= 0$, y-intercept $= f(0) = 0$

C. $f(-x) = -f(x)$, so f is odd; the graph is symmetric about the origin.

D. $\lim\limits_{x \to 4^+} \dfrac{x}{x^2 - 16} = \infty$, $\lim\limits_{x \to 4^-} f(x) = -\infty$, $\lim\limits_{x \to -4^+} f(x) = \infty$, $\lim\limits_{x \to -4^-} f(x) = -\infty$, so $x = \pm 4$ are VAs. $\lim\limits_{x \to \pm\infty} \dfrac{x}{x^2 - 16} = 0$, so $y = 0$ is a HA.

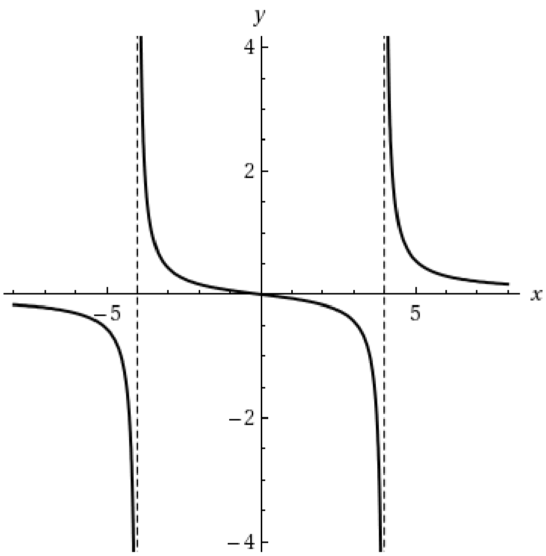
E. $f'(x) = \dfrac{(x^2 - 16)(1) - x(2x)}{(x^2 - 16)^2} = -\dfrac{x^2 + 16}{(x^2 - 16)^2} < 0$ for all x in D, so f is decreasing on $(-\infty, -4)$, $(-4, 4)$ and $(4, \infty)$.

F. No local extrema

G.
$$\begin{aligned} f''(x) &= -\frac{(x^2-16)^2(2x)-(x^2+16)2(x^2-16)(2x)}{[(x^2-16)^2]^2} \\ &= -\frac{2x(x^2-16)[(x^2-16)-2(x^2+16)]}{(x^2-16)^4} \\ &= -\frac{2x(-x^2-48)}{(x^2-16)^3} \\ &= \frac{2x(x^2+48)}{(x+4)^3(x-4)^3}. \end{aligned}$$

$f''(x) < 0$ if $x < -4$ or $0 < x < 4$, so f is CD on $(-\infty, -4)$ and $(0, 4)$, and CU on $(-4, 0)$ and $(4, \infty)$. IP at $(0, 0)$

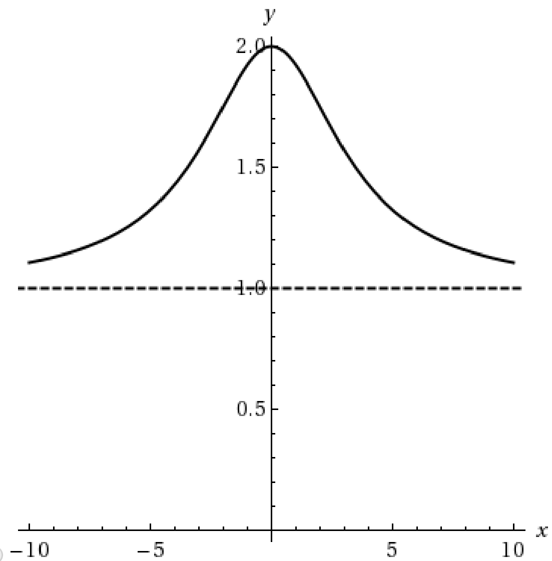
H.

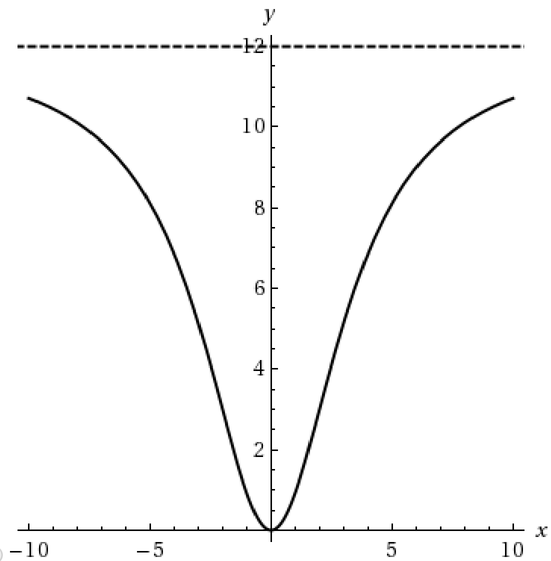


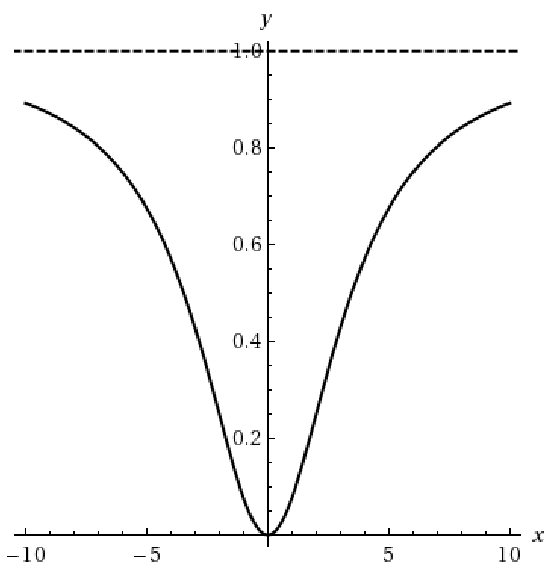
22. 2/2 points | Previous Answers SCalcET8 4.5.015.

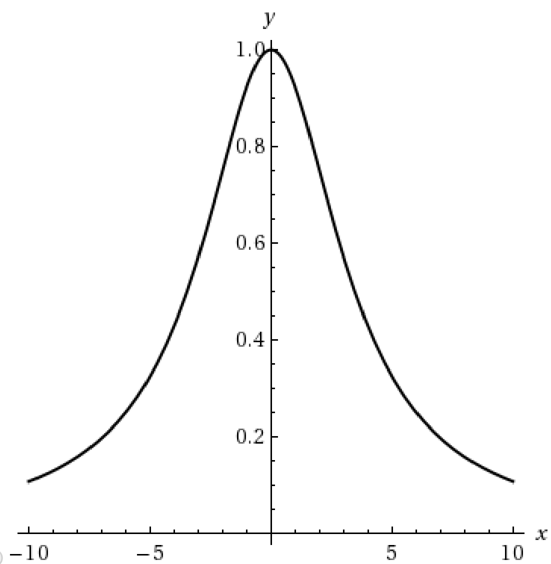
Use the guidelines of this section to sketch the curve.

$$y = \frac{x^2}{x^2 + 12}$$









Solution or Explanation

$$y = f(x) = \frac{x^2}{x^2 + 12} = \frac{(x^2 + 12) - 12}{x^2 + 12} = 1 - \frac{12}{x^2 + 12}$$

A. $D = \mathbb{R}$

B. y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \iff x = 0$

C. $f(-x) = f(x)$, so f is even; the graph is symmetric about the y-axis.

D. $\lim\limits_{x \to \pm\infty} \dfrac{x^2}{x^2 + 12} = 1$, so $y = 1$ is a HA. No VA.

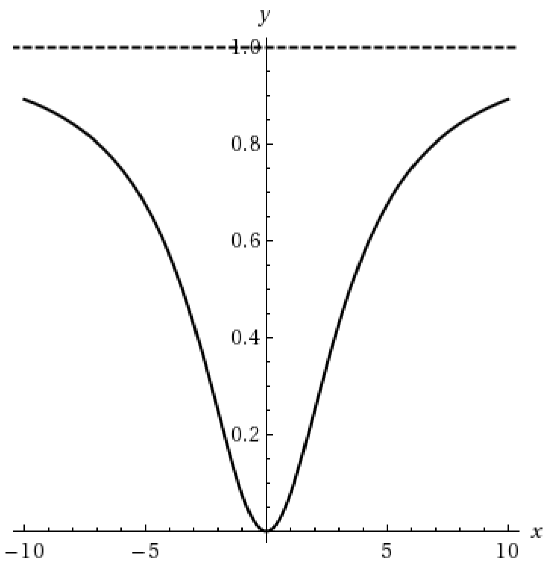
E. Using the Reciprocal Rule, $f'(x) = -12 \cdot \dfrac{-2x}{(x^2 + 12)^2} = \dfrac{24x}{(x^2 + 12)^2}$. $f'(x) > 0 \iff x > 0$ and $f'(x) < 0 \iff x < 0$, so f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

F. Local minimum value $f(0) = 0$, no local maximum.

G. $f''(x) = \dfrac{(x^2 + 12)^2 \cdot 24 - 24x \cdot 2(x^2 + 12) \cdot 2x}{[(x^2 + 12)^2]^2} = \dfrac{24(x^2 + 12)[(x^2 + 12) - 4x^2]}{(x^2 + 12)^4} = \dfrac{24(12 - 3x^2)}{(x^2 + 12)^3} = \dfrac{-72(x + 2)(x - 2)}{(x^2 + 12)^3}$

$f''(x)$ is negative on $(-\infty, -2)$ and $(2, \infty)$ and positive on $(-2, 2)$, so f is CD on $(-\infty, -2)$ and $(2, \infty)$ and CU on $(-2, 2)$. IP at $\left(\pm 2, \frac{1}{4}\right)$.

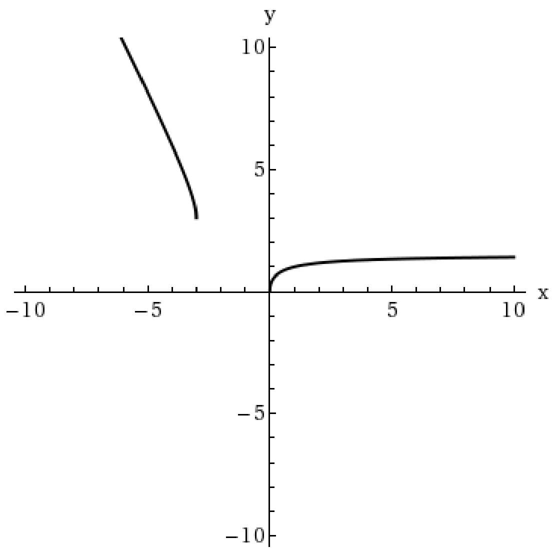
H.

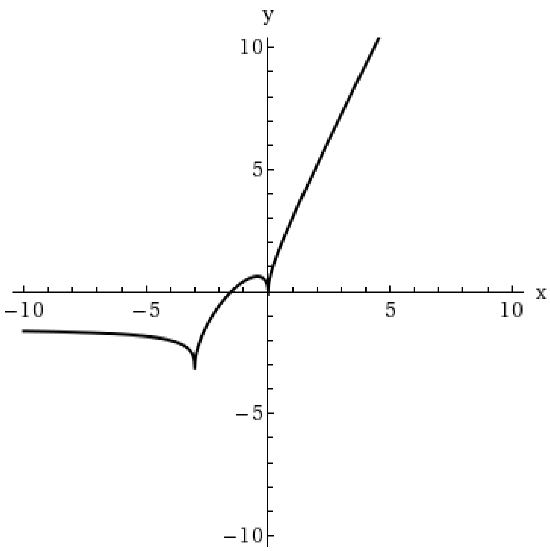


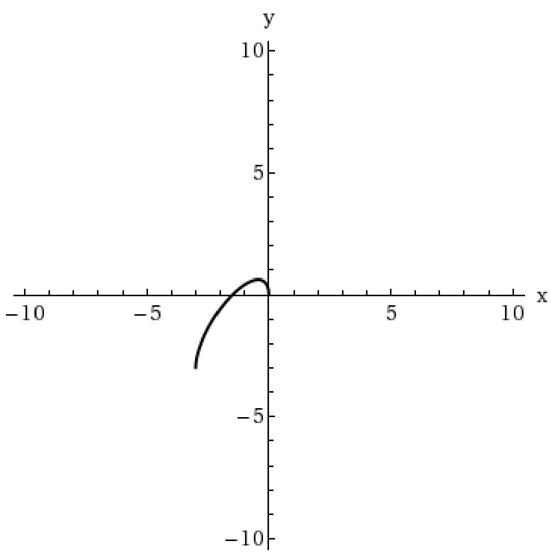
23. 1/0 points | Previous Answers SCalcET8 4.5.024.

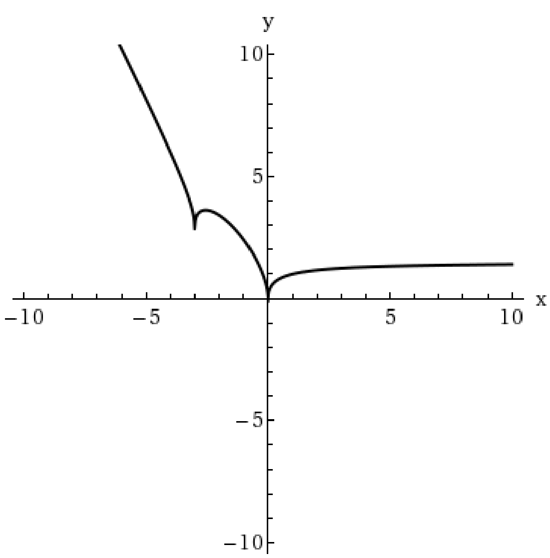
Use the guidelines of this section to sketch the curve.

$y = \sqrt{x^2 + 3x} - x$









Solution or Explanation

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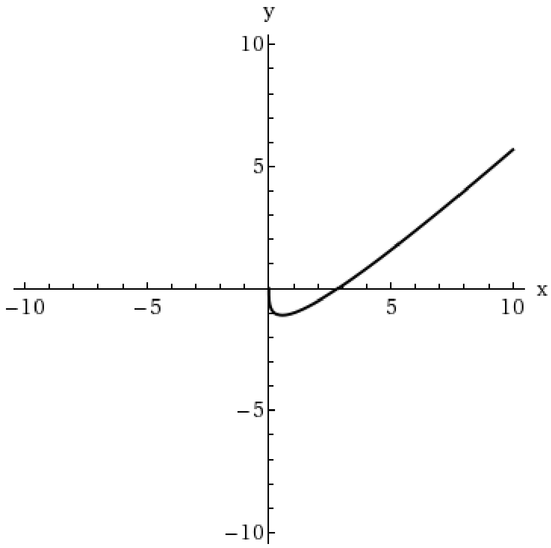
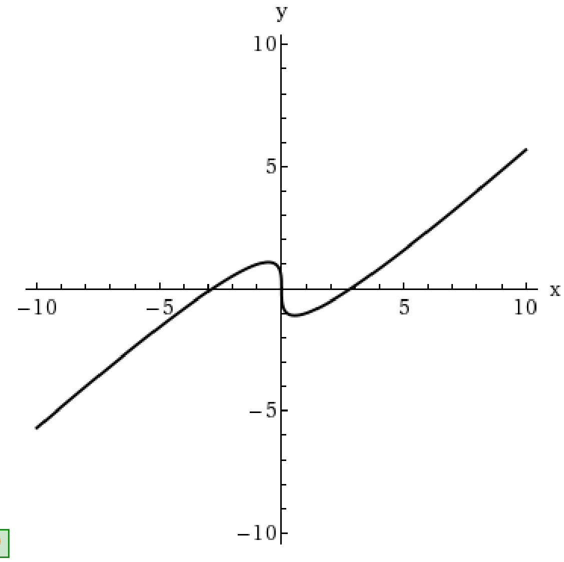
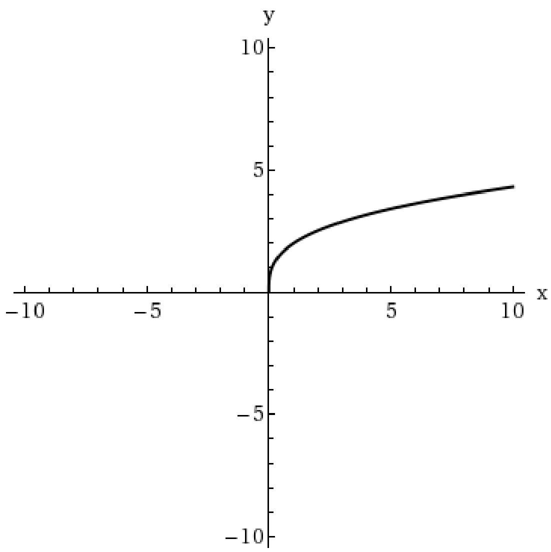
24.2/2 points | Previous AnswersSCalcET8 4.5.029.

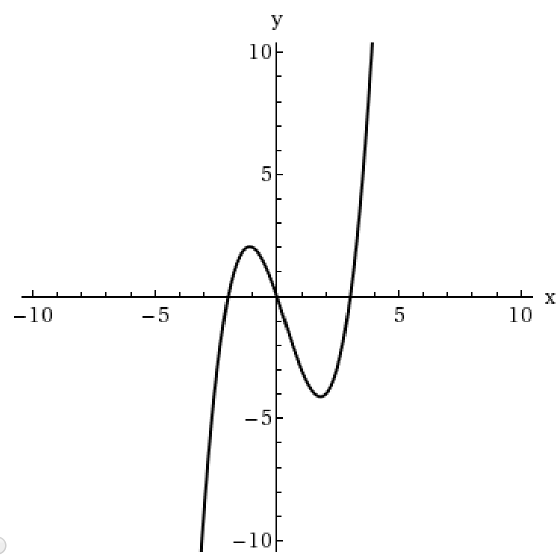
Use the guidelines of this section to sketch the curve.

$y = x - 2x^{1/3}$









Solution or Explanation

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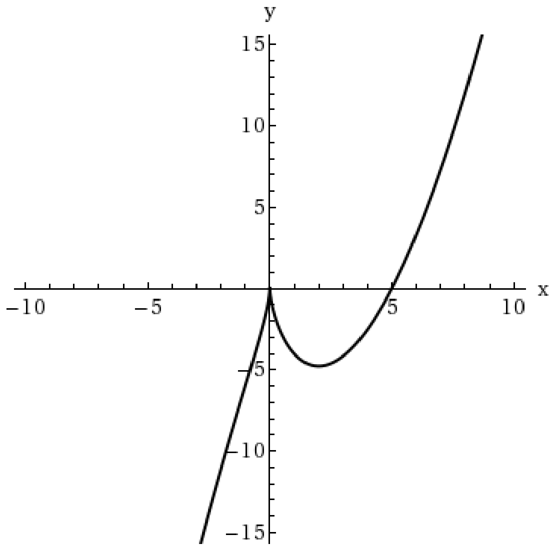
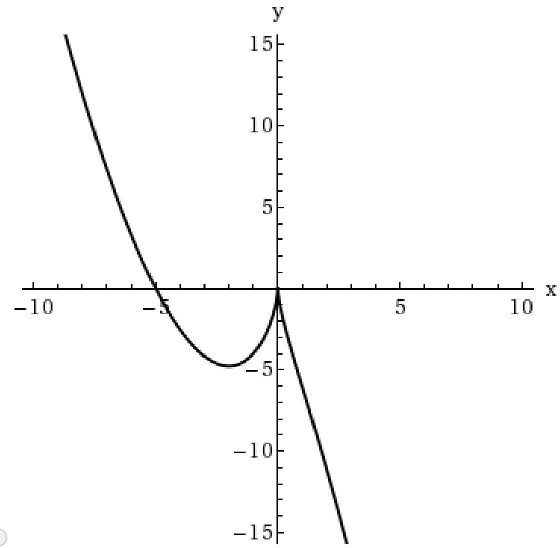
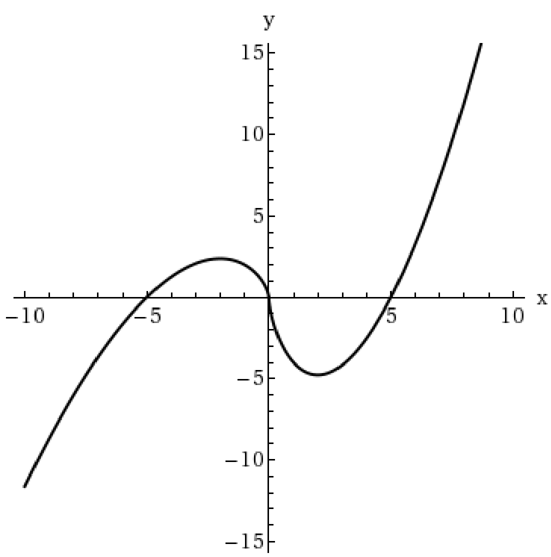
25. 1/0 points | Previous Answers SCalcET8 4.5.030.

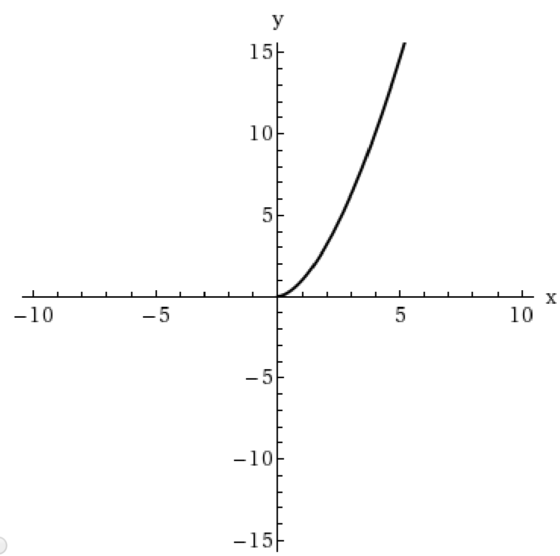
Use the guidelines of this section to sketch the curve.

$y = x^{5/3} - 5x^{2/3}$









Solution or Explanation

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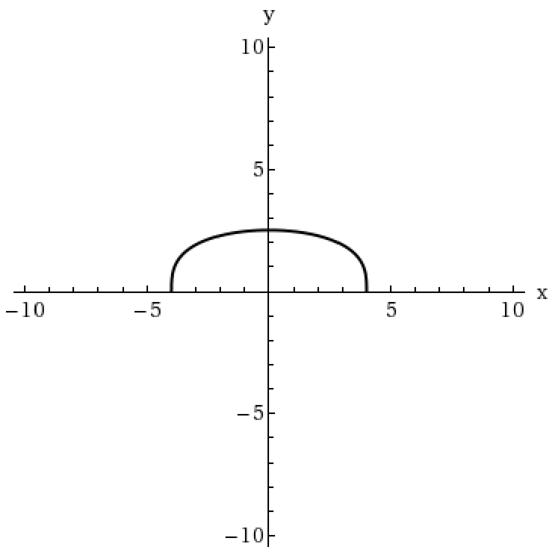
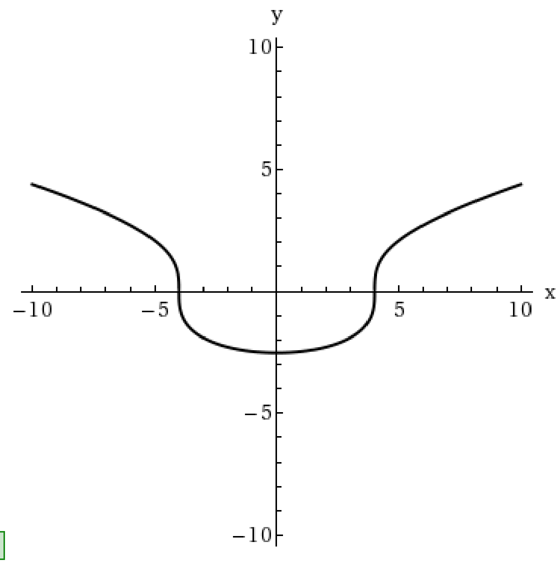
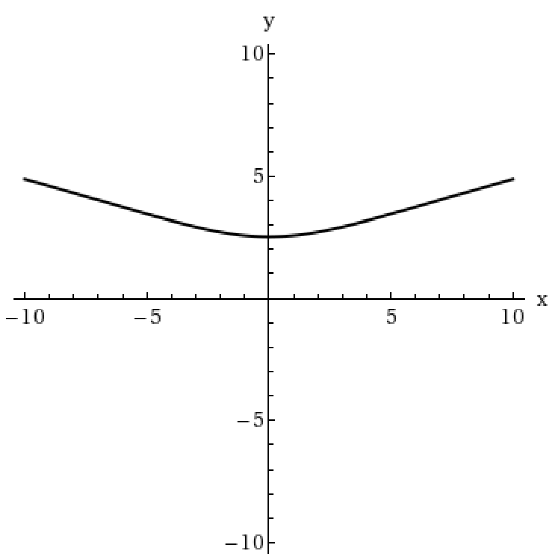
26.2/2 points | Previous AnswersSCalcET8 4.5.031.

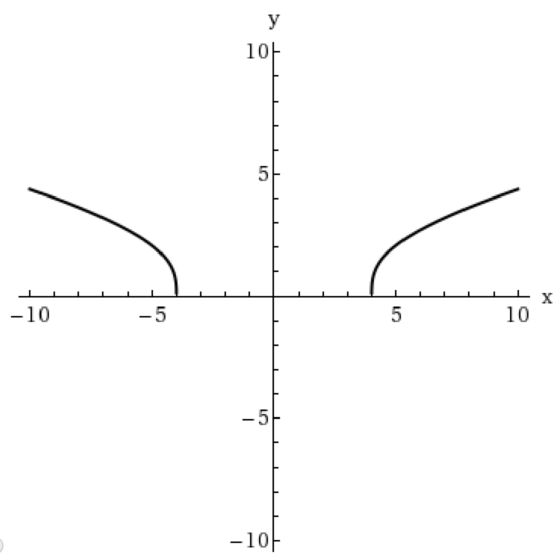
Use the guidelines of this section to sketch the curve.

$$y = \sqrt[3]{x^2 - 16}$$









Solution or Explanation
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