Integración por partes

2019-08/08

IPP: Generalmente se utiliza para intedrar productos

$$\int f(x)\,g(x) = ?$$

Regla del producto para derivadas

$$\frac{d}{dx}(fg) = f'g + fg'$$

$$\frac{d}{dx}(fg) - f'g = fg' \qquad \text{integre ésta expresión}$$

$$\underbrace{\int \frac{d}{dx}(fg)}_{\text{función original}} - \int f'g = \int fg'$$

$$\boxed{\int fg' = fg - \int f'g}$$

Integre esta expresión f deriva y g' integra

$\equiv$

f deriva
g integra

$$\int \underbrace{f(x)}_{u}\,\underbrace{g(x)\,dx}_{dv} = uv - \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u = f(x) \qquad dv = g(x)\,dx$

$du = f'(x)\,dx \qquad v = G(x)$

Ejercicio 1 pag 39 integre $\int xe^x\,dx$

Opción 1:

$$\boxed{\begin{array}{ll} u = x & dv = e^x\,dx \\ u' = dx & v = e^x \end{array}}$$

opción 2:

$$\cancel{\boxed{\begin{array}{ll} u = e^x & dv = x \\ du = e^x\,dx & v = \frac{x^2}{2} \end{array}}}$$

$$\int xe^x\,dx = \underbrace{xe^x}_{uv} - \int \underbrace{e^x\,dx}_{v\,du} = xe^x - e^x + C$$

derivar:

$$\cancel{e^x} + xe^x - \cancel{e^x} + 0'$$

$$xe^x$$

a) $\int 6x^2 \ln x \, dx$

$f = \ln(x)$ $\quad g' = 6x^2 dx$

$f' = \frac{1}{x} dx$ $\quad g = 2x^3$

Integre $\int fg' = fg - \int f'g$

$\therefore \int 6x^2 \ln x \, dx = \underbrace{(\ln x)}_{f} \underbrace{2x^3}_{g} - \int \underbrace{\frac{1}{x}}_{f'} \underbrace{2x^3}_{g} dx$

$= 2x^3 \ln x - \int 2x^2 \, dx$

$= 2x^3 \ln x - 2\int \frac{x^{2+1}}{2+1} dx$

$= 2x^3 \ln x - \frac{2x^3}{3} + C$

b) $\int \ln x \, dx$

$u = \ln x$ $\quad dv = 1\,dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$\ln x \cdot x - \int x \cdot \frac{1}{x} dx$

$\ln x \cdot x - x + C$

comprobemos

$(x\ln x - x)\frac{d}{dx} = \ln x + 1 - 1 = \ln x$

c) $\int \tan^{-1} x \, dx$ $\quad = \tan^{-1} x \cdot x - \int x \cdot \frac{1}{1+x^2} dx$

$u = \tan^{-1} x$ $\quad dv = 1\,dx$

$du = \frac{1}{1+x^2} dx$ $\quad v = x$

sustitución de

$\int \frac{x}{1+x^2} dx = \int \frac{1}{u} \frac{du}{2} = \frac{1}{2}\int \frac{1}{u}$

$u = 1 + x^2$

$du = (0 + 2x) dx$

$du = 2x dx$

$\frac{du}{2} = x dx$

$= \frac{\ln(u)}{2} + C$

$= \frac{\ln|1+x^2|}{2} + C$

$\therefore \int \tan^{-1} x \, dx = \tan^{-1}(x) \cdot x - \frac{\ln|1+x^2|}{2} - C$

d. $\int x^2 \cos x \, dx = x^2 \sin x - \int \underbrace{\sin x \, 2x \, dx}_{2 \text{ IPP}}$

IPP 1 $\begin{cases} u = x^2 & dv = \cos x \, dx. \\ du = 2x \, dx & v = \sin x \end{cases}$

$u = 2x \quad dv = \sin x$

$du = 2\,dx \quad v = -\cos x$

$\int \sin x \, 2x \, dx = -2x \cos x + \int \cos x \, 2dx$

$= x^2 \sin x - (-2x\cos x + 2\sin x) + C$

$= x^2 \sin x + 2x \cos x - 2\sin x + C$

Siempre dar prioridad de derivación a:

- **I**nversas trigonométricas $\boxed{\text{Mnemotecnia}}$
- **L**ogarítmicas
- **A**lgebraicas
- **T**rigonométricas
- **E**xponensiales

IPP: Integrales Definidas $\boxed{\int_a^b u \, dv = uv\Big]_a^b - \int_a^b v \, du}$

no cambian los límites de integración

Ejercicio 3: Evalúe pg. 41

b) $72 \int_1^2 \frac{\ln x}{x^4} dx = 72 \ln x \left(\frac{-1}{3x^3}\right)\Big]_1^2 - \int_1^2 \frac{x^{-3}}{-3} x^{-1} dx$

$u = \ln(x) \quad dv = x^{-4} dx$

$du = x^{-1} dx \quad v = \frac{x^{-3}}{-3}$

evaluación $\longrightarrow$

$$72\ln x\left(\frac{-1}{3x^3}\right)\Bigg]_1^2 - \int_1^2 \underbrace{\frac{x^{-3}}{-3}\, x^{-1}\,dx} \to -\int_1^2 \frac{1}{-3x^3\cdot x}\,dx \to -\int_1^2 \frac{1}{-3x^4} \to -\int_1^2 -\frac{x^{-4}}{3}$$

$$\to -1\cdot -\frac{1}{3}\int_1^2 x^{-4}$$

$$= -\frac{72\ln(x)}{3x^3} - \frac{72}{9x^3}\Bigg]_1^2$$

$$\to \frac{1}{3}\int_1^2 x^{-4}\,dx$$

$$= -\frac{24\ln(x)}{x^3} - \frac{8}{x^3}\Bigg]_1^2 = \left\{-\frac{24\ln(2)}{2^3} - \frac{8}{2^3}\right\} - \left\{-\frac{24\overset{0}{\cancel{\ln(1)}}}{\cancel{1^3}} - \frac{8}{1^3}\right\}$$

$$= \{-3\ln(2) - 1\} - \{-8\} = -3\ln(2) + 7$$

$$\to \frac{1}{3}\,\frac{x^{-3}}{3} = \frac{1}{9x^3}$$

$$\int e^x \cos x\,dx = e^x\cos x + \int \underbrace{e^x \sin x\,dx}_{\text{IPP 2}}$$

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x \qquad v = e^x$

$u = \sin x \qquad dv = e^x\,dx$

$du = -\cos x\,dx \qquad v = e^x$

$$\int e^x \sin x\,dx = \sin x\, e^x - \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx = -e^x\cos x + \sin x\, e^x - \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx + \int e^x \cos x\,dx = -e^x\cos x + \sin x\, e^x$$

$$2\int e^x \cos x\,dx = -e^x\cos x + \sin x\, e^x$$

$$\int e^x \cos x\,dx = \frac{-e^x\cos x + \sin x\, e^x}{2} + C$$