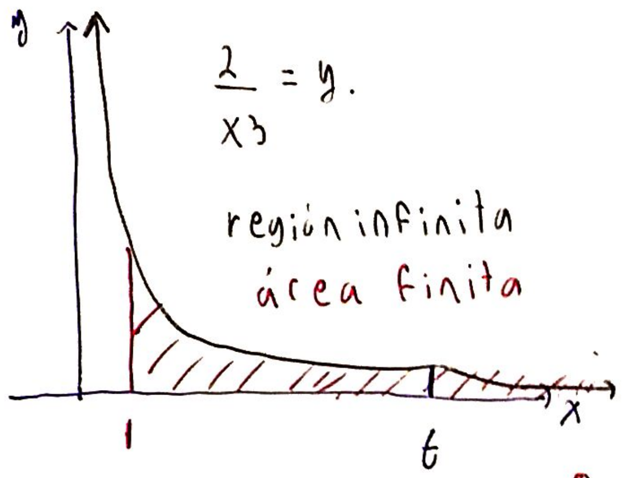
Fracciones Parciales Pág. 66 (Posteriormente). 1.

7.8 Integrales Impropias.

Considere la región bajo la curva $y = \frac{2}{x^3}$ encima del eje-x y a la derecha de la recta $x = 1$.



$$A = \int_1^t 2x^{-3}\,dx$$

$$A = \frac{2}{-2}\,x^{-2}\Big]_1^t$$

$$A = -1 \cdot t^{-2} + \underbrace{1 \cdot 1^{-2}}_{1} = 1 - \frac{1}{t^2}.$$

$$\lim_{t\to\infty} A = \lim_{t\to\infty} 1 - \cancel{\frac{1}{t^2}}^{\,0} = 1$$

$$\boxed{\int_1^\infty \frac{2}{x^3}\,dx = 1}$$

Limites Básicos

a. $\lim_{x\to\infty} \frac{1}{x^r} = 0$ $\frac{1}{\infty}.$ $\lim_{x\to\infty} x^r = +\infty.$

r positivo

b. $\lim_{x\to\infty} e^x = \infty$ e^∞ $\lim_{x\to-\infty} e^x = 0$ $\frac{1}{e^\infty} \to 0.$

c. $\lim_{x\to 0^+} \ln x = -\infty$ $\lim_{x\to\infty} \ln x = +\infty.$

$\log_{10} \underbrace{10^{-48}}_{0^+} = -48$ $\log_{10} 10^{-10{,}000} = -10{,}000$

Integrales Impropias:

Tipo 1: Intervalos infinitos $\pm\infty$.

Tipo 2: Funciones discontinuas. (AVs en $x=\pm a$).

Integrales Impropias Tipo 1: (Pág. 74)

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$$

Integre 1ro
Evalúe límite.

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty}\int_t^a f(x)\,dx.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

CONVERGENTE: (Se acerca a un número) el límite existe.

DIVERGENTE: (La integral a $\pm\infty$) el límite no existe.

Ejercicio 1: Evalúe. (p. 74) $\sqrt{\infty} \Rightarrow \infty$

a. $\displaystyle\int_1^{\infty} x^{-1/2}\,dx. = 2x^{1/2}\Big]_1^{\infty} = \lim_{x\to\infty} 2\sqrt{x} - 2. = \infty.$

no existe.

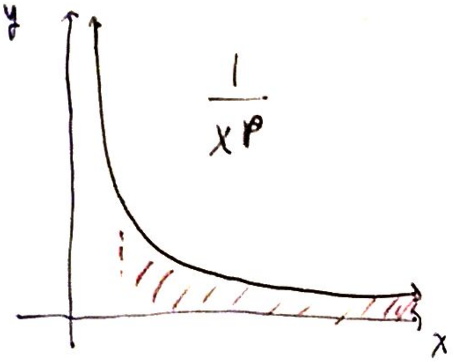
$p = \frac{1}{2}$ DIVERGENTE.

b. $\displaystyle\int_1^{\infty} \frac{1}{x}\,dx = \ln x\Big]_1^{\infty} = \lim_{x\to\infty} \ln x - 0 = \infty.$

no existe.

$p = 1$

DIVERGENTE



$\int_1^\infty \frac{1}{x^p}\,dx$ no necesariamente existe.

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} p \leq 1 & \text{DIVERGE.} \\ p > 1 & \text{converge.} \end{cases}$$

$p = 0.99$ $\quad \int_1^\infty x^{-0.99}\,dx = \frac{x^{0.01}}{0.01}\Big]_1^\infty = \lim_{x\to\infty} x^{0.01} - \frac{1}{0.01} = +\infty.$

DIVERGE.

$p = 1.001$ $\quad \int_1^\infty x^{-1.001}\,dx = \frac{x^{-0.001}}{-0.001}\Big]_1^\infty = \lim_{x\to\infty} \frac{1000}{x^{0.001}} + \frac{1}{0.001}$

d.

$$= \frac{1000}{x^{0.001}}\Big]_\infty^1 = 1000 - \lim_{x\to\infty} \overset{0}{\frac{1000}{x^{0.001}}}$$

$\frac{1000}{\infty} \to 0$

$= 1000$ converge.

Ejercicio 3: Evalúe.

a. $\int_{-\infty}^0 e^{-x^2} \underbrace{x\,dx}_{du/-2} = \int_{-\infty}^0 e^u \frac{du}{-2} = -\frac{1}{2} e^u\Big]_{-\infty}^0 = -\frac{1}{2}e^0 + \overset{0}{\cancel{\frac{1}{2}e^{-\infty}}}$

$u = -x^2$ $\quad u(0) = -0^2$

$du = -2x\,dx$ $\quad u(-\infty) = -(-\infty)^2 = -\infty$

$= -\frac{1}{2} + 0$

$= -\frac{1}{2}$ converge.

Notación Abreviada:

$e^{-\infty} = \lim_{x\to -\infty} e^x = 0$

$f(\infty) = \lim_{x\to\infty} f(x)$

b. $\frac{1}{2}\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1}(x)\Big]_{-\infty}^{\infty} = \frac{\pi}{4}+\frac{\pi}{4} = \frac{\pi}{2}.$

$= \frac{1}{2}\tan^{-1}(\infty) - \frac{1}{2}\tan^{-1}(-\infty)$ CONVERGE

$\tan x$ $D: (-\frac{\pi}{2}, \frac{\pi}{2})$

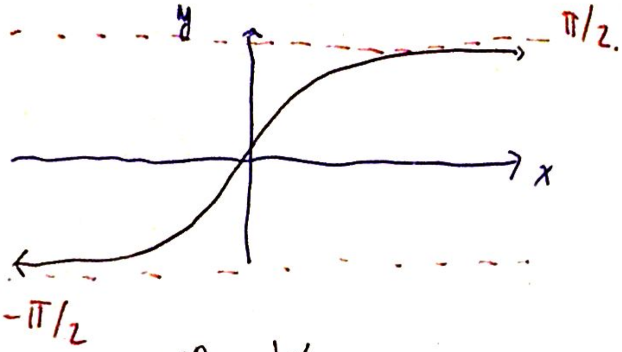
$R: (-\infty, \infty)$

A.V $x = -\pi/2, +\frac{\pi}{2}$

$\tan^{-1} x$ $D: (-\infty, \infty)$

$R: (-\frac{\pi}{2}, \frac{\pi}{2})$

A.H. $y = \pm \pi/2.$



$\tan^{-1}(\infty) = \pi/2$

$\tan^{-1}(-\infty) = -\pi/2.$

David Corzo.

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \underline{\underline{\pi.}} \approx \int_{-1000}^{1000}\frac{dx}{1+x^2} \approx \sum_{i=1}^{n}\frac{1}{1+x_i^2}\Delta x$$

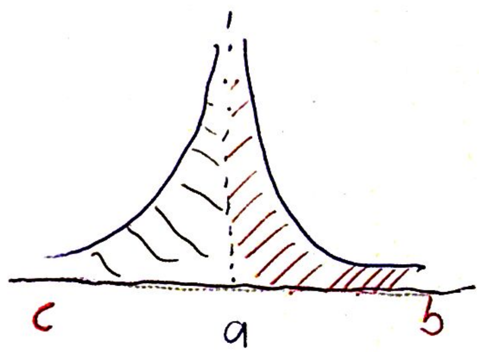
PYTHON.

Integrales Impropias Tipo 2.

Hay una asintota vertical en $x = a$.

$$\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$$

$$\int_c^a f(x)dx = \lim_{t\to a^-}\int_c^t f(x)dx$$



AV $x=a$ $\int_c^b f(x)dx = \int_c^a f(x)dx + \int_a^b f(x)dx$

Ejercicio 4: Evalúe. Indique donde es discontinua.

a. $\int_{1}^{9} \frac{1}{\sqrt[3]{x-1}}\,dx = \int_{0}^{8} u^{-1/3}\,du = \frac{3}{2}u^{2/3}\Big]_{0^+}^{8} =$

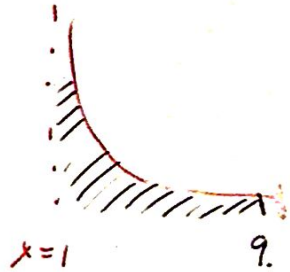
Discontinua: denominador igual a cero $1/0$.

en $x=1$ logaritmo de cero $\ln(0) \to -\infty$.

raiz cuadrada de un número negativo.

$u = x-1 \qquad u(9) = 8$

$du = dx \qquad u(1) = 0$



$\frac{3}{2}u^{2/3}\Big]_{0^+}^{8} = \frac{3}{2}(8^2)^{1/3} - \frac{3}{2}\lim_{u\to 0^+} u^{2/3}$

$\frac{3}{2}\sqrt[3]{64} - 0 = \frac{3}{2}\cdot 4 = 6$ CONVERGE

b. $\int_{-2}^{3} \frac{3}{x^4}\,dx = \underset{(1)}{\int_{-2}^{0} 3x^{-4}\,dx} + \underset{(2)}{\int_{0}^{3} 3x^{-4}\,dx} = \infty$ diverge.

Discontinua en $x=0$

(2) $\int_{0}^{3} 3x^{-4}\,dx = -x^{-3}\Big]_{0^+}^{3} = -\frac{1}{3^3} + \overbrace{\lim_{x\to 0^+}\frac{1}{x^3}}^{+\infty} = +\infty.$

(1) $\int_{-2}^{0} 3x^{-4}\,dx = -x^{-3}\Big]_{-2}^{0^-} = \lim_{x\to 0^-} \underbrace{-\frac{1}{x^3}}_{+\infty} + \frac{1}{(-2)^3} = +\infty.$

i. $\int_0^1 \ln x \, dx$

$\int \ln x \, dx = x\ln x - \int dx = x\ln x - x + C.$

$u = \ln x \qquad dv = dx$

$du = \dfrac{dx}{x} \qquad v = x$

$0 \cdot \infty$

$0 \cdot \ln 0$

discontinua en $x = 0$

$$\int_0^1 \ln x \, dx = x\ln x - x\Big]_{0^+}^{1} = 1\cdot\ln(1) - 1 - \lim_{x\to 0} x\ln x$$

A.V.

$$= -1 - \boxed{\lim_{x\to 0^+} x\ln x}$$

Regla de L'Hospital $\displaystyle \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$

$0/0$ ó ∞/∞.

También aplica para $0\cdot\infty$, 1^∞, ∞^0, 0^0.

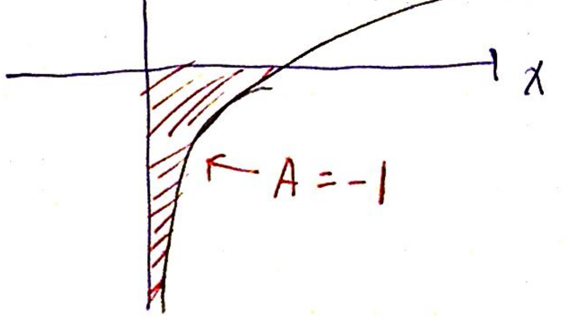
$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1}} \overset{LH}{=} \lim_{x\to 0^+} \frac{1}{x(-x^{-2})}$$

∞/∞

$$\lim_{x\to 0^+} \frac{1}{-x^{-1}} = \lim_{x\to 0^+} -x = 0$$

$(\ln x)^{-1} \xrightarrow{\ '\ } -(\ln x)^{-2}\dfrac{1}{x}.$

$$\int_0^1 \ln x \, dx = -1 - \lim_{x\to 0} x\ln x = -1 + 0 \quad \text{CONVERGE.}$$



$\dfrac{\ln x}{x} \to \dfrac{x^{-1}}{-x^{-2}} = -x$

$\lim_{x\to 0^+} -x = 0.$