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Laboratorio # 12
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100

5 $x = e^{t}$; $y = t e^{-t}$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{e^{-t} - t e^{-t}}{e^{t}}$$

$$\frac{d^2y}{d^2x} = \frac{y''(t)}{x''(t)} = \boxed{\frac{e^{-t} \cdot t - 2e^{-t}}{e^{t}}}$$

1)a) ■ Eliminar el parámetro
■ Graficar e indicar la dirección

a) $x = 3 - 4\sin(t)$

Para $0 \leq t \leq 2\pi$

$y = 2 - 3\sin(t)$

$x = 3 - 4\sin(t)$

$x - 3 = -4\sin(t)$

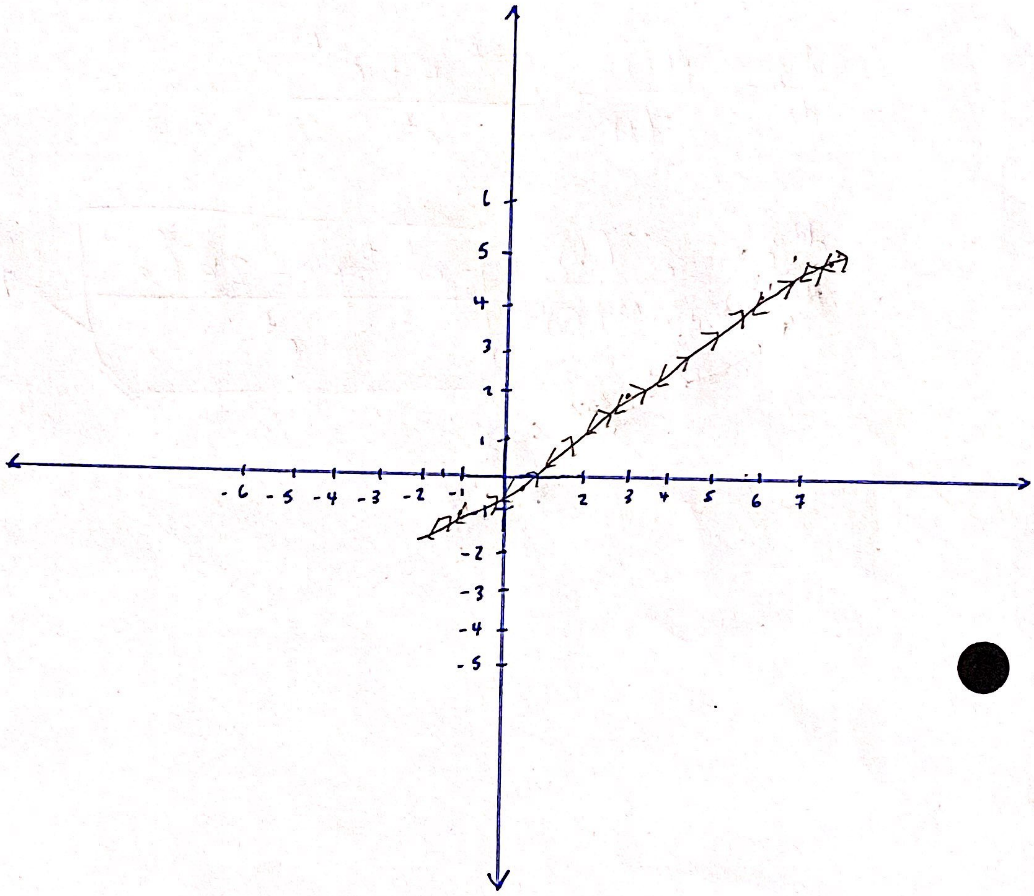
$\frac{x-3}{-4} = \sin(t)$

$y = 2 - 3\left(\frac{x-3}{-4}\right)$

$= 2 - \frac{3x+9}{4}$

$= 2 - \frac{3}{4}x + \frac{9}{4}$

$= \frac{13}{2} - \frac{3}{4}x$



t	x	y
0	3	2
$\frac{\pi}{4}$	≈ 0.17	≈ -0.12
$\frac{\pi}{2}$	-1	-1
$\frac{3\pi}{4}$	≈ 0.17	≈ -0.12
π	3	2
$\frac{5\pi}{4}$	≈ 5.82	≈ 4.12
$\frac{3\pi}{2}$	7	5

t	x	y
$\frac{7}{4}\pi$	≈ 5.82	≈ 4.12
2π	3	2

1) b) $x = \sqrt{t-1}$

$y = 2 - t$

Para $1 \leq t$

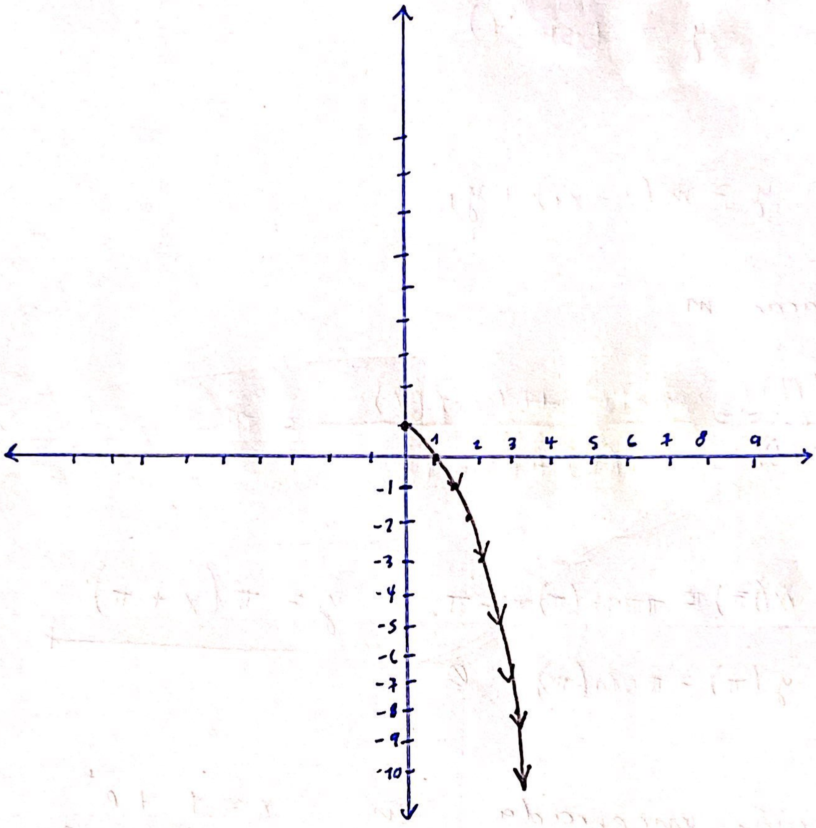
$x = \sqrt{t-1}$

$x^2 = t - 1$

$x^2 + 1 = t$

$y = 2 - x^2 - 1$

$\boxed{y = 1 - x^2}$



t	x	y
1	0	1
2	1	0
3	$\sqrt{2}$	-1
4	$\sqrt{3}$	-2
5	2	-3
6	$\sqrt{5}$	-4
7	$\sqrt{6}$	-5
8	$\sqrt{7}$	-6
9	$\sqrt{8}$	-7
10	3	-8

2) Ecuación de la recta tangente

$$x = t\cos(t)$$
$$y = t\sin(t)$$

$$y = m(x - x_1) + y_1$$

Sacar m

$$\frac{y'(t)}{x'(t)} = \frac{\sin(t) + t\cos(t)}{\cos(t) - t\sin(t)} = \pi$$

$$x(\pi) = \pi\cos(\pi) = -\pi \qquad \underline{y = \pi(x + \pi)}$$
$$y(\pi) = \pi\sin(\pi) = 0$$

3) Región encerrada en $x = 1 + e^t$, $y = t - t^2$

interalos

$$0 = t - t^2$$
$$= t(1 - t)$$
$$t = 0 \quad \& \quad t = 1$$

$$A = \int_{t_1}^{t_2} x(t)\, y'(t)\, dt$$

$$= \int_0^1 (1 + e^t)(1 - 2t)\, dt = \int_0^1 \underbrace{1 - 2t + e^t}_{1} \underbrace{- 2te^t}_{2}\, dt$$

Continuación 3

$$= \int_0^1 (1+e^t)(1-2t)\,dt$$

$$= \int_0^1 1 - 2t + e^t - 2te^t \, dt$$

$$= \int_0^1 1\,dt - \int_0^1 2t\,dt + \int_0^1 e^t\,dt - \underbrace{\int_0^1 2te^t\,dt}$$

$$= t - \frac{2}{2}t^2 + e^t - 2te^t + 2e^t$$

$$\underbrace{= t - t^2 + e^t - 2te^t + 2e^t \Big]_0^1}$$

$u = 2t \quad dv = e^t\,dt$

$du = 2\,dt \quad v = e^t$

$2te^t - 2\int e^t\,dt$

$2te^t - 2e^t$

$$= \left\{\left[1 - 1^2 + e^1 - 2(1)e^1 + 2e^1\right] - \left[0 - 0^2 + e^0 - 2(0)e^0 + 2e^0\right]\right\}$$

$$= \left\{\left[\cancel{1} - \cancel{1} + e - \cancel{2e} + \cancel{2e}\right] - \left[-1 + 2\right]\right\} = \left\{e - 3\right\}$$

$$\boxed{e - 3}$$

4) Longitud $x = 2 + 4t^2$

$y = 3 - \frac{8}{3}t^3$

$$\mathcal{L} = \int_a^b \sqrt{(f'(x))^2 + (f'(y))^2}$$

$x'(t) = 8t \Rightarrow 64t^2$

$y'(t) = -\underbrace{\frac{24}{3}}_{-8t^2}t^2 \Rightarrow 64t^4$

$$\mathcal{L} = \int_a^b \sqrt{64t^2 + 64t^4}\,dt = \int_a^b \sqrt{64t^2(1+t^2)}\,dt$$

$$= \int_a^b \sqrt{64t^2}\sqrt{(1+t^2)}\,dt = \int_a^b 8t\sqrt{1+t^2}\,dt$$

$u = 1 + t^2$

$du = 2t$

$4du = 8t$

$$= 4\int_a^b \sqrt{u}\,du =$$

$$= 4\,\frac{1}{\frac{3}{2}}\,u^{\frac{3}{2}} = 4\cdot\frac{2}{3}\,u^{3/2} = \frac{8}{3}(1+t^2)^{\frac{3}{2}}\Big]_0^1$$

$$= \frac{8}{3}\left[\left\{\sqrt{(1+1^2)^3}\right\} - \left\{\sqrt{(1+0^2)^3}\right\}\right]$$

$$= \frac{8}{3}\left[\sqrt{8} - \sqrt{1}\right] = \frac{8}{3}\cdot\sqrt{8} - \frac{8}{3}$$