

Laboratorio # 10  
David Corzo 20190432

100

(5)

media de 4 minutos

¿Qué sea atendida antes de 3 minutos?

Distribución exponencial:

$$f(x) = \int_{0}^{3} \frac{e^{-\frac{x}{4}}}{4} dx \quad \mu = 4$$

$$= \int_{0}^{3} \frac{1}{4} e^{-\frac{x}{4}} dx = - \int_{0}^{3} e^{\frac{x}{4}} \cdot d\ln u = - \int_{0}^{3} e^{\frac{x}{4}} \cdot du =$$

$$\begin{aligned} u &= -\frac{x}{4} \\ -du &= \frac{dx}{4} \end{aligned}$$

$$- \left[ \left( e^{-\frac{3}{4}} \right) - \left( e^0 \right) \right] = - \left[ e^{-\frac{3}{4}} - 1 \right]$$

$$= -e^{-\frac{3}{4}} + 1 \approx 0.52$$

④ Encuentre la media, mediana, varianza & desviación estandar

$$f(x) = \frac{1}{18}(6-x) \quad ; \quad 0 \leq x \leq 6$$

Media;

$$\begin{aligned} u &= \frac{1}{18} \int_0^6 x(6-x) dx \\ &= \frac{1}{18} \left[ (6x - x^2) \right]_0^6 = \frac{1}{18} \left( \frac{6}{2}x^2 - \frac{1}{3}x^3 \right)_0^6 \\ &= \frac{1}{18} \left[ \left( 3x^2 - \frac{1}{3}x^3 \right) \right]_0^6 = \frac{1}{18} \left[ \left( 3(6)^2 - \frac{1}{3}(6)^3 \right) - (0) \right] \\ &= \frac{1}{18} [108 - 72] = \frac{1}{18} [36] = 2 \end{aligned}$$

$$\text{mediana} = \frac{1}{18} \int_0^m (6 - x) dx = 0.5$$

$$\left[ \frac{1}{18} \left( 6x - \frac{1}{2}x^2 \right) \right]_0^m = 0.5$$

$$\frac{1}{18} \left[ \left( 6m - \frac{1}{2}m^2 \right) - (0) \right] = 0.5$$

$$\frac{1}{18} \left[ 6m - \frac{1}{2}m^2 \right] = 0.5$$

$$\frac{1}{18} \left[ m \left( 6 - \frac{1}{2}m \right) \right] = 0.5$$

$$\frac{m}{18} \left( 6 - \frac{1}{2}m \right) = 0.5$$

$$m \left( 6 - \frac{1}{2}m \right) = \frac{1}{2} \cdot 18$$

$$6m - \frac{1}{2}m^2 = 9$$

$$2 \left( 6m - \frac{1}{2}m^2 \right) = 18$$

$$-m^2 + 12m - 18 = 0$$

$$a = -1 \quad b = 12 \quad c = -18$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1, x_2 = \frac{-12 \pm \sqrt{12^2 - 4(-1)(-18)}}{2(-1)} = \frac{-12 \pm \sqrt{144 - 72}}{-2} = \frac{-12 \pm \sqrt{72}}{-2}$$

$$x_1 = \frac{6 - 3\sqrt{2}}{1}$$

$$x_1 \approx 1.75$$

$$x_2 = \frac{6 + 3\sqrt{2}}{1}$$

$$x_2 \approx 10.74$$

Dado el intervalo en  
cuestión  $0.66 \leq x_1 \leq x_2$   
es  $m = x'$

Varianza  $\Rightarrow$

$$\begin{aligned}\sigma^2 &= \frac{1}{18} \int_0^6 (x - \mu)^2 \cdot (6-x) dx \quad \mu = 2 \\ &= \frac{1}{18} \int_0^6 (x^2 - 2 \cdot x \cdot 2 + 2^2) (6-x) dx \\ &= \frac{1}{18} \int_0^6 (x^2 - 4x + 4)(6-x) dx \\ &= \frac{1}{18} \int_0^6 (6x^2 - 24x + 24) - x^3 + 4x^2 - 4x \cdot dx \\ &\quad -x^3 + 6x^2 + 4x^2 - 24x - 4x + 24 \\ &\quad -x^3 + 10x^2 - 28x + 24 \\ &= \frac{1}{18} \int_0^6 (-x^3 + 10x^2 - 28x + 24) dx \\ &= \frac{1}{18} \left[ -\frac{1}{4}x^4 + \frac{10}{3}x^3 - \frac{28}{2}x^2 + 24x \right] \\ &= \frac{1}{18} \left[ -\frac{1}{4}x^4 + \frac{19}{3}x^3 - \frac{14}{2}x^2 + 24x \right] \\ &= \frac{1}{18} \left[ \left( -\frac{1}{4}(6)^4 + \frac{10}{3}(6)^3 - 14(6)^2 + 24(6) \right) - (0) \right] = \\ &= \frac{1}{18} \left[ -324 + 720 - 504 + 144 \right] = \frac{1}{18} \cdot 36 = \frac{18 \cdot 2}{18} = 2 \text{ A}\end{aligned}$$

desviación estándar =  $\sqrt{2}$

③

$$\mu = 2 \text{ minutos}$$

a) más de 4 minutos

$$\square \mu = 2$$

$$f(x) = \int_4^{\infty} \frac{e^{-\frac{x}{\mu}}}{\mu} dx = \int_4^{\infty} e^{-\frac{x}{2}} \cdot \frac{1}{2} dx \quad u = -\frac{x}{2}$$

$$-du = \frac{dx}{2}$$

$$= \int_4^{\infty} e^{-u} du = \left[ -e^{-u} \right]_{u(4)}^{u(\infty)} = \left[ e^{-u} \right]_{u(4)}^{\infty} \quad u(\infty) = -\infty \\ u(4) = -2$$

$$= -e^{-\frac{\infty}{2}} = - \left[ \underbrace{\left( \lim_{t \rightarrow -\infty} (e^t) \right)}_{e^{-\infty} \rightarrow 0} - (e^{-2}) \right] = \frac{e^{-2}}{e^{\infty}}$$

(b) en menos de dos minutos.

$$f(x) = \int_0^2 \frac{1}{2} e^{-\frac{x}{2}} dx = - \int_0^2 e^u du = -e^u \Big|_0^{-1} =$$

$$\begin{aligned} u &= -\frac{x}{2} & u(2) &= -1 \\ -du &= \frac{dx}{2} & u(0) &= 0 \end{aligned}$$

$$= -1 \left[ (e^{-1}) - (e^0) \right] = -1 \left[ e^{-1} - 1 \right] =$$

$$= -e^{-1} + 1 \quad \text{Ans}$$

(c) Que solo 0.02 sirvan gratis.

$$\int_m^\infty f(x) dx = \frac{1}{50} \int_m^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = - \int_m^\infty e^u du = -e^u \Big|_{u(m)}^{-\infty} = -e^{\frac{m}{2}} =$$

$$\begin{aligned} u &= -\frac{x}{2} & u(\infty) &= -\infty \\ du &= -\frac{dx}{2} = -du & u(m) &= -\frac{m}{2} = -\frac{m}{2} \end{aligned}$$

$$- \left[ \underbrace{\left( \lim_{t \rightarrow \infty} (e^t) \right)}_{0} - \left( e^{-\frac{m}{2}} \right) \right] = - \left[ -e^{-\frac{m}{2}} \right] = +e^{-\frac{m}{2}}$$

$$\begin{aligned} e^{-\frac{m}{2}} &= \frac{1}{50} & -m &= 2 \ln(1/50) \\ -\frac{m}{2} &= \ln(1/50) & m &= -2 \ln(1/50) \\ & & m &\approx 7.824 \end{aligned}$$

(2)

$$f(x) = \begin{cases} K(8 - x^3) & 0 \leq x \leq 2 \\ 0 & x < 0 \text{ ó } x > 3 \end{cases}$$

a) Hallar el valor de  $K$ .

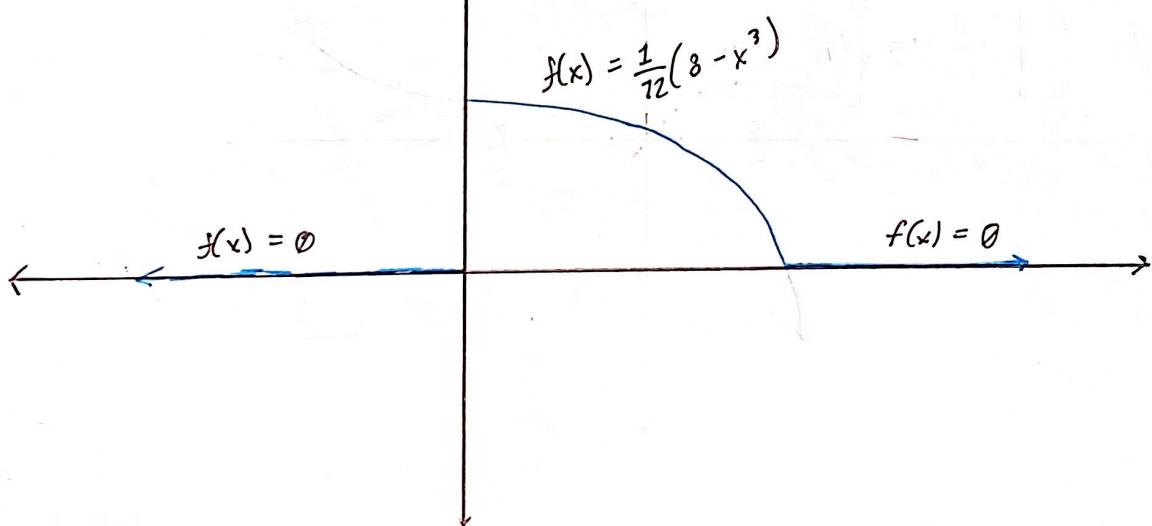
$$\int_0^2 K(8 - x^3) dx = K \int_0^2 (8 - x^3) dx = K \left[ \left( 8x - \frac{1}{4}x^4 \right) \right]_0^2 =$$

$$K = \left[ \left( 8(2) - \frac{1}{4}(2)^4 \right) - (0) \right] = K [16 - 4] = 12K =$$

$$\therefore K = \frac{1}{12}$$

b) Graficar la función de densidad:

$$f(x) = \begin{cases} \frac{1}{12}(8 - x^3) ; & 0 \leq x \leq 2 \\ 0 ; & x < 0 \text{ ó } x > 3 \end{cases}$$



c) Para que valor de  $K$   $P(x \leq 1)$

$$\frac{1}{12} \int_0^1 (8 - x^3) dx = \frac{1}{12} \left[ \left( 8x - \frac{1}{4}x^4 \right) \right]_0^1 = \frac{1}{12} \left[ \left( 8 - \frac{1}{4} \right) - (0) \right] = \dots$$

$$\dots = \frac{1}{12} \left( \frac{31}{4} \right) = \frac{31}{48}$$

d) La media  $\mu$ .

$$\mu = \int_0^2 x f(x) dx = \frac{1}{12} \int_0^2 x (8 - x^3) dx = \frac{1}{12} \int_0^2 (8x - x^4) dx = \dots$$

$$\dots = \frac{1}{12} \left[ 4x^2 - \frac{1}{5}x^5 \right]_0^2 = \frac{1}{12} \left[ \left( 4(2)^2 - \frac{1}{5}(2)^5 \right) - (0) \right] = \dots$$

$$= \frac{1}{12} \left[ 16 - \frac{32}{5} \right] = \frac{48}{12 \cdot 5} = \frac{48}{60} = \frac{24}{30} = \frac{12}{15} = \frac{4}{5} \approx 0.8$$

① función de densidad standard de Cauchy:

a) Para que valor de  $c$ ,  $f$  es una función de densidad de densidad?

$$\int_{-\infty}^{\infty} \frac{c}{1+x^2} dx = c \left[ \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx \right] = c \arctan(x) \Big|_{-\infty}^{\infty} =$$
$$= c \left[ \underbrace{\arctan(\infty)}_{\frac{\pi}{2}} - \underbrace{\arctan(-\infty)}_{-\left(\frac{\pi}{2}\right)} \right] = c \left[ \frac{\pi}{2} + \frac{\pi}{2} \right] =$$
$$= c \left[ \pi \right] = c = \frac{1}{\pi}$$

b) Hallar  $P(-1 \leq x \leq 1)$  en  $f(x) = \frac{1}{\pi(1+x^2)}$

$$f(x) = \frac{1}{\pi} \int_{-1}^1 \frac{1}{1+x^2} dx = \left[ \frac{\arctan(x)}{\pi} \right]_{-1}^1 =$$
$$= \frac{1}{\pi} \left[ \underbrace{\arctan(1)}_{\frac{\pi}{4}} - \underbrace{\arctan(-1)}_{-\left(\frac{\pi}{4}\right)} \right] = \frac{1}{\pi} \left[ \frac{\pi}{4} + \frac{\pi}{4} \right] = \frac{1}{\pi} \left[ \frac{\pi}{2} \right]$$
$$= \frac{1}{2}$$

La probabilidad es de  $\frac{1}{2}$  ó 50%.

c) Encontrar la media de la función.

$$\int_{-\infty}^{\infty} x f(x) dx = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x}{1+x^2} dx = \frac{1}{\pi} \cdot \frac{1}{2} \int_{-\infty}^{\infty} \frac{du}{u} =$$

$$u = x^2 + 1$$

$$du = 2x dx \quad u(\infty) = (\infty)^2 + 1 = \infty$$

$$\frac{du}{2} = x dx \quad u(-\infty) = (-\infty)^2 + 1 = \infty$$

$$= \frac{1}{2\pi} \left[ \ln|u| \right]_{u(-\infty)}^{u(\infty)} = \frac{1}{2\pi} \left[ \left( \lim_{t_1 \rightarrow \infty} (\ln|t_1|) \right) - \left( \lim_{t_2 \rightarrow -\infty} (\ln|t_2|) \right) \right] =$$

# Reescribir los logaritmos naturales

$$= \frac{1}{2\pi} \left[ \underbrace{\lim_{t \rightarrow \infty} (\ln(t) - \ln t)}_{\Downarrow} \right] = \frac{1}{2\pi} (\emptyset) = \emptyset$$

$$\ln\left(\frac{t}{t}\right) \equiv \ln(1) = \emptyset$$

Por la imparidad de la función se cancelan las magnitudes.

