Calculo Integral

25/07/2019

5.4 Área, Desplazamiento y Propiedades

Cómo se encontraba el área de una región

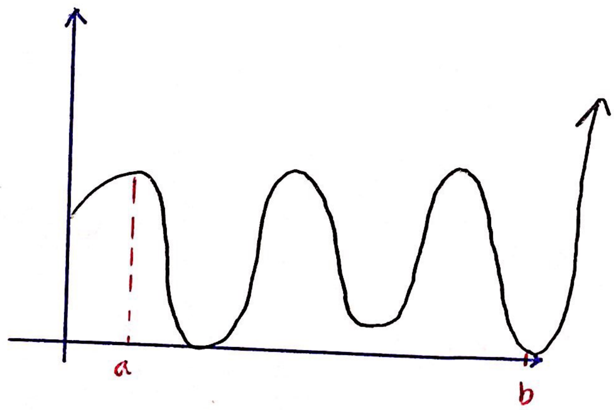
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, dx$$

La integral definida de f en $[a,b]$ f es continua

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, dx$$

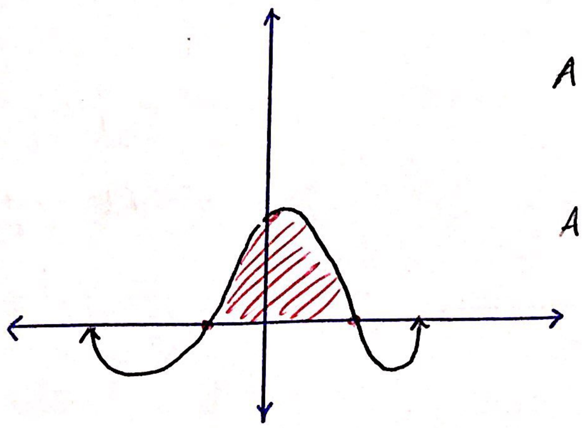
Interpretacion de integral definida

El area de la región bajo la curva $y = f(x)$, encima del eje $-x$ y entre las rectas verticales $x = a$ y $x = b$ en la integral definida de f en $[a,b]$ $f > 0$



$$A = \int_a^b f(x)\, dx$$

Considere el área bajo $y = \cos x$ en $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



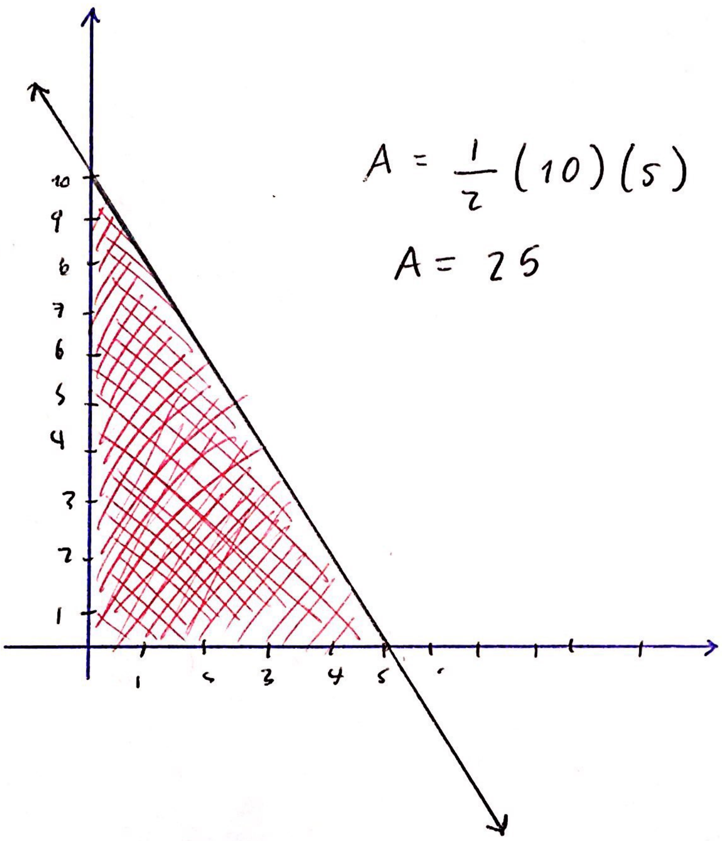
$$A = \int_{-\pi/2}^{\pi/2} \cos x \, dx = 2$$

$$A = \sin x \Big]_{-\pi/2}^{\pi/2} = \sin \pi/2 - (-1) =$$

$$= 1 + 1 = 2$$

Ejercicio 2: Encuentre el área de las sigs. funciones bosqueje cada región

a) $f(x) = 10 - 2x$ $\quad f(x) \geq 0$ en $0 \leq x \leq 5$



$A = \frac{1}{2}(10)(5)$

$A = 25$

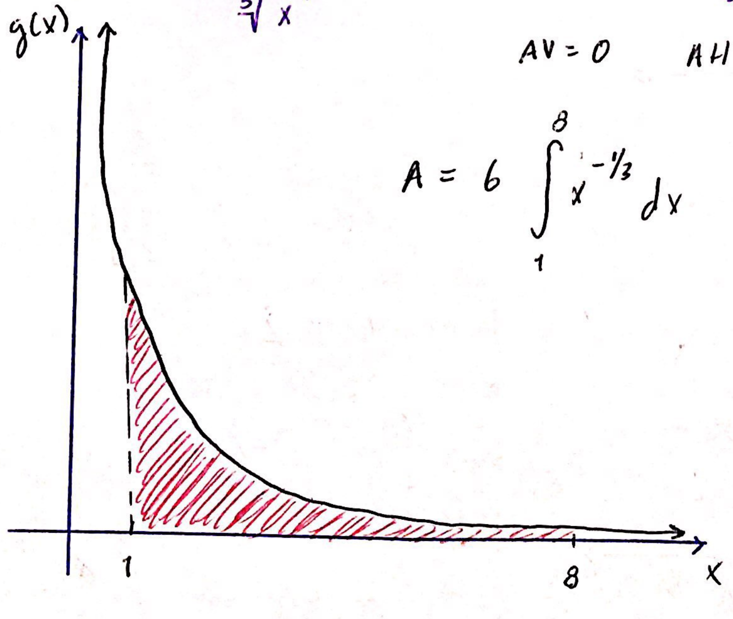
$A = \int_0^5 (10 - 2x)\,dx$

$A = 10x - x^2 \Big]_0^5$

$A = 10 \cdot 5^2 - (0 - 0)$

$A = 25$

b) $g(x) = \frac{6}{\sqrt[3]{x}}$ entre $1 \leq x \leq 8$



AV = 0 $\quad$ AH = 0

$A = 6 \int_1^8 x^{-1/3}\,dx = \frac{6 \cdot 3}{2} x^{2/3} \Big]_1^8$

$= 9\left(2^{3 \cdot 2/3} - 1^{2/3}\right)$

$= 9(4 - 1) = 27$

c) $h(x) = 2|x|$ entre $x = -2$ y $x = 3$

$$A = 2\int_{-2}^{3} |x|\, dx$$

$$A = \int_{-2}^{0} -2x\, dx + \int_{0}^{3} 2x$$

$$A = -2\int_{-2}^{0} x\, dx + 2\int_{0}^{3} x$$

$$A = -2\left(\frac{x^2}{2}\right) + 2\left(\frac{x^2}{2}\right)$$

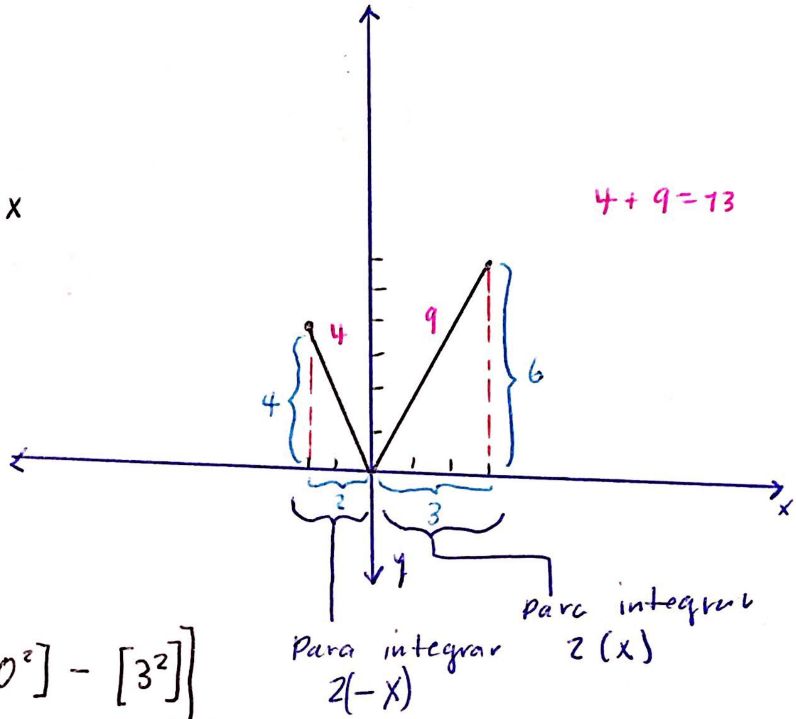
$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_{0}^{3}$$

$$A = \Big[[-(-2^2)] - [-0]\Big] + \Big[[0^2] - [3^2]\Big]$$

$$2^2 - 0 + 0 - 3^2$$

$$2^2 - 3^2 = 4 - 9 = \underline{13}$$

4 + 9 = 13



Regla de Integrales definidas

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

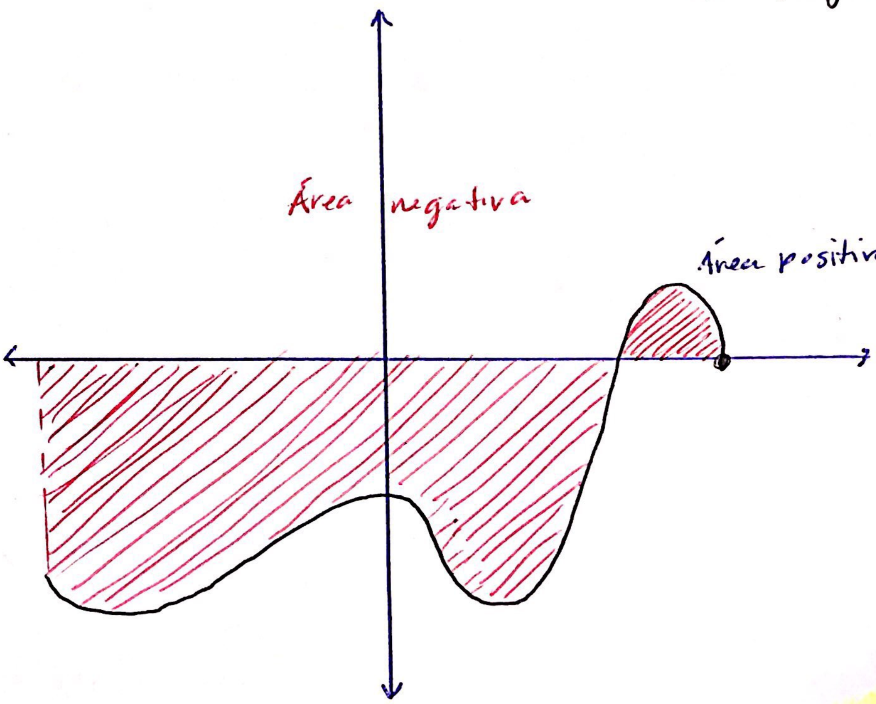
ej. $\int_{-2}^{0} -2x\, dx = \int_{0}^{-2} 2x\, dx = x^2\Big]_{0}^{-2} = 4 - 0 = 4$

---

b) $\int_0^{\pi} \sin x\, dx = -\int_{\pi}^{0} \sin x\, dx = \cos x\Big]_{\pi}^{0} = 1 - (-1) = \underline{2}$

ó $-\cos x\Big]_0^{\pi} = -\cos(\pi) - (-\cos 0) = 1 + 1 = \underline{2}$

¿Qué sucede cuando $f(x)$ es negativa?



Área de la región entre $f(x)$ a $y = 0$

$$\int_{-1}^{0} x^3\,dx = \frac{x^4}{4}\Big]_{-1}^{0} = 0 - 1/4$$
$$= -1/4$$

$$A \neq \int_a^c f(x)\,dx$$

$$A = -\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Ejercicio 3 : Pg 16 | considere

$f(x) = 4x^3 - 4$ en...

Definición más completa
$$A = \int_a^b |f(x)|\,dx$$

ⓐ Evalúe $\int_{-2}^{2} (4x^3 - 4)\,dx =$

$$= x^4 - 4x\Big]_{-2}^{2}$$
$$= (16 - 8) - (16 + 8)$$

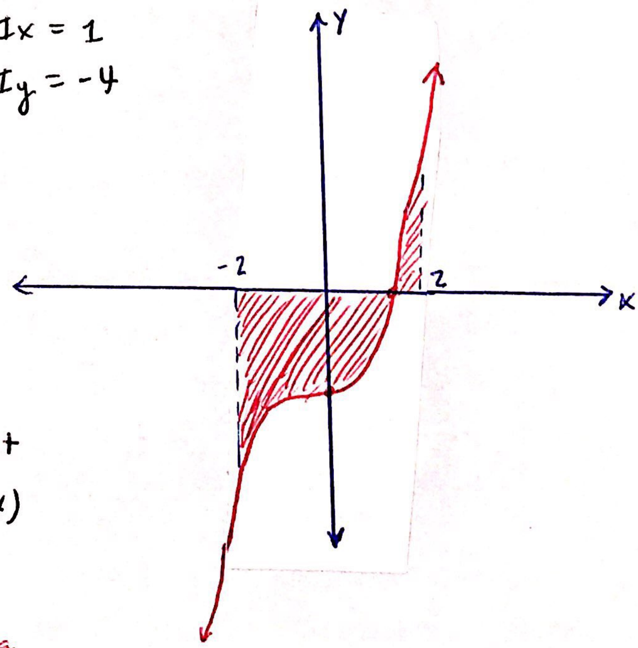
esto no está correcto.

$$= 8 - 24 = -16$$

$$A \neq \int_{-2}^{2} f(x)\,dx$$

ⓑ Bosqueje la región y explique si la integral definida es igual al area de la región

Ix = 1
Iy = -4



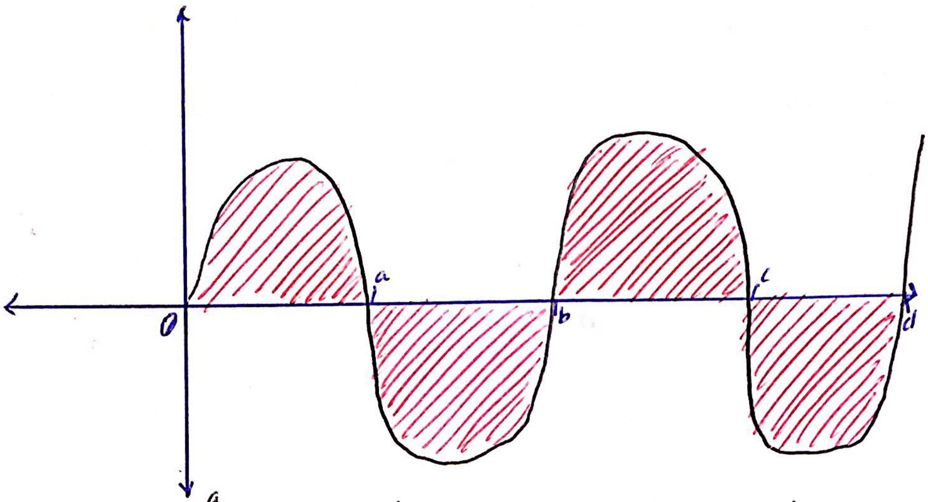
ⓒ encuentre el área de la región

$$A = \int_{2}^{1} (4 - 4x^3)\,dx + \int_{1}^{2} (4x^3 - 4)\,dx$$

$$A = 4x - x^4\Big]_{-2}^{1} + x^4 - 4x\Big]_{1}^{2} = (4-1) - (-8-16) + (16-8) - (1-4)$$

$$A = 3 + 24 + 8 + 3 = 27 + 11 = 38$$

esta es la respuesta



$$A = \int_0^a f(x)\,dx - \int_a^b f(x)\,dx - \int_b^c f(x)\,dx - \int_c^d f(x)\,dx$$

Propiedades Integrales definidas

①,② $$\int_a^b k_1 f(x) \pm k_2\, g(x)\, dx = k_1 \int_a^b f(x)\,dx \pm k_2 \int_a^b g(x)\,dx$$

---

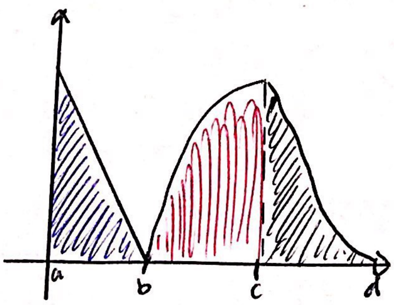
③ $$\int_a^a f(x)\,dx = 0$$

$$\int_{\sqrt{2}}^{\sqrt{2}} e^{y^2 + \ln x + \sinh x}\, dx = 0$$

④ $$\int_a^b h\, dx = h x\Big]_a^b = h(b-a)$$

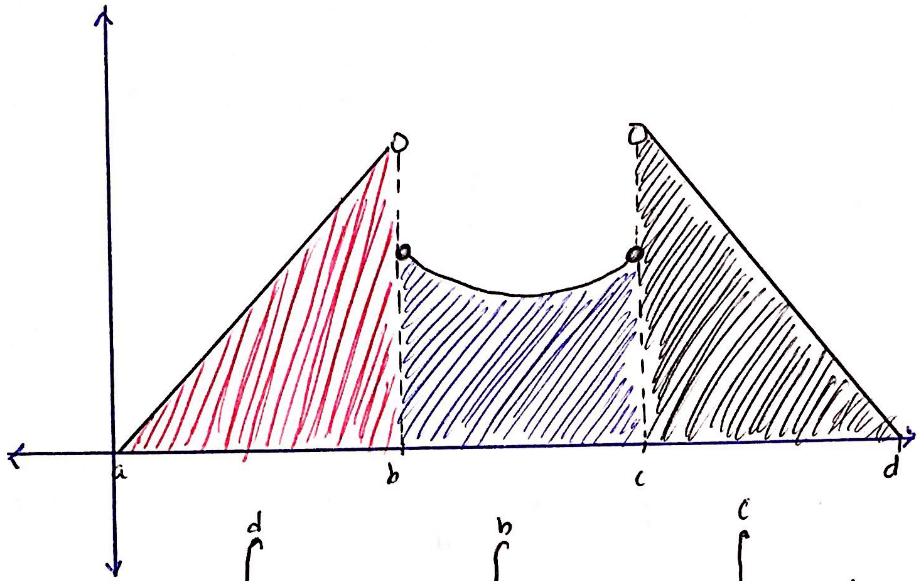
$$\int_e^{\sqrt{10}} \ln(10)\, dx = \ln(10)\left[\sqrt{10} - e\right]$$

⑤



$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$$

⑥ Continuidad por tramos, piecewise continuos



$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$$

Ejercicio 5: Evalué las sig. integrales definida.

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2 & \text{si } 0 \le x \le 1 \\ 4-2x & \text{si } 1 \le x \le 2 \\ 6x-12 & \text{si } 2 \le x \le 3 \end{cases}$$

$$\int_0^3 f(x)dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx$$

$$= 2 + (4x - x^2)\Big]_1^2 + (3x^4 - 12x)\Big]_2^3$$

$$= 2 + (4-3) - (-9 - (-12))$$

$$= 2 + 1 + 3 = 6$$