

① a)  $r = 2 \sin \theta$

$\theta = \frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{\sin(2\theta) \cdot r}{\cos(2\theta) \cdot r}$$

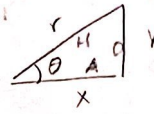
$$= \tan(2\theta) \quad ; \quad \theta = \frac{\pi}{6}$$

$$m = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$x\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$y\left(\frac{\pi}{6}\right) = 2 \sin^2\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$m = \sqrt{3}$$



$$\frac{SO}{H} \left( \frac{A}{H} \right) \frac{T}{A}$$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$r \cdot \sin \theta = y$$

$$r \cdot \cos \theta = x$$

$$2 \sin^2 \theta = y$$

$$2 \cdot \frac{1}{2} (1 - \cos(2\theta)) = y$$

$$1 - \cos(2\theta) = y$$

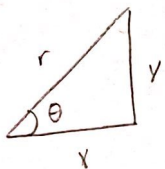
$$y'(\theta) = \sin(2\theta) \cdot 2$$

$$2 \sin \theta \cos \theta = x$$

$$\sin(2\theta) = x$$

$$x'(\theta) = \cos(2\theta) \cdot 2$$

b)  $r = \frac{1}{\theta} \quad \theta = \pi$



$$\frac{SO}{H} \left( \frac{A}{H} \right) \frac{T}{A}$$

$$r \cdot \sin \theta = y$$

$$r \cos \theta = x$$

$$\frac{1}{\theta} \sin \theta = y$$

$$\frac{1}{\theta} \cos \theta = x$$

$$y'(\theta) = -\frac{1}{\theta^2} \sin \theta + \frac{1}{\theta} \cos \theta$$

$$x'(\theta) = -\frac{1}{\theta^2} \cos \theta - \frac{1}{\theta} \sin \theta$$

$$\frac{dy}{dx} = \frac{-\frac{1}{\theta^2} \sin \theta + \frac{1}{\theta} \cos \theta}{-\frac{1}{\theta^2} \cos \theta - \frac{1}{\theta} \sin \theta} \quad \theta = \pi$$

$$= \frac{-\frac{1}{\pi^2} \sin \pi + \frac{1}{\pi} \cos \pi}{-\frac{1}{\pi^2} \cos \pi - \frac{1}{\pi} \sin \pi} = \frac{-\frac{1}{\pi}}{-\frac{1}{\pi^2}} = -\frac{\pi^2}{\pi} = -\pi$$

② a)  $r = e^{-\frac{\theta}{4}} \quad \frac{\pi}{2} \leq \theta \leq \pi$

$$A = \frac{1}{2} \int_a^b r^2 d\theta$$

$$= \frac{1}{2} \int \underbrace{\left( e^{-\frac{\theta}{4}} \right)^2}_{\underbrace{e^{-\frac{2\theta}{2}}}_{e^{-\frac{\theta}{2}}}} d\theta = -\frac{2}{2} \int e^u du$$

$$u = -\frac{\theta}{2}$$

$$du = -\frac{1}{2} d\theta$$

$$-2 du = d\theta$$

$$u\left(\frac{\pi}{2}\right) = -\left[\frac{\frac{\pi}{2}}{2}\right] = -\frac{\pi}{4}$$

$$u(\pi) = -\frac{\pi}{2}$$

$$= -1 \left[ e^u \right]_{u\left(\frac{\pi}{2}\right)}^{u(\pi)} = - \left[ \left( e^{-\frac{\pi}{4}} \right) - \left( e^{-\frac{\pi}{2}} \right) \right] = \boxed{-e^{-\frac{\pi}{4}} + e^{-\frac{\pi}{2}}}$$

b)  $r = \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{6}$

$$A = \frac{1}{2} \int \cos \theta d\theta = \frac{1}{2} \sin \theta \Big|_0^{\frac{\pi}{6}} = \frac{1}{2} \left[ \left( \sin \frac{\pi}{6} \right) - \left( \sin 0 \right) \right] = \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$$



## Laboratorio # 13

① a)  $r = 2 \sin \theta$  ;  $\theta = \frac{\pi}{6}$

$$r' = 2 \cos \theta$$

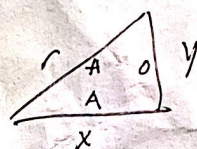
$$= 2 \cos\left(\frac{\pi}{6}\right)$$

$$= 2 \frac{\sqrt{3}}{2} = \sqrt{3} = m$$

$$r\left(\frac{\pi}{6}\right) = 2 \sin\left(\frac{\pi}{6}\right) = 1$$

$$\theta = \sqrt{3}\left(r - \frac{\pi}{6}\right) + 1$$

$$y = y(\pi/6) + \frac{dy}{dx} (x - x(\pi/6))$$



$$\cos \theta = \frac{x}{r}$$

$$r \cos \theta$$

$$2 \sin \theta \cos \theta = x$$

$$\sin(2\theta) = x$$

b)  $r = \frac{1}{\theta}$  ;  $\theta = \pi$

$$r'(\theta) = -1 \theta^{-1-1} = -1 \theta^{-2} = -\frac{1}{\theta^2}$$

$$r'(\pi) = -\frac{1}{\pi^2}$$

② a)  $r = e^{-\theta/4}$

$$\frac{\pi}{2} \leq \theta \leq \pi$$

$$A = \int_{\pi/2}^{\pi} \frac{1}{2} e^{-\frac{\theta}{2}} d\theta$$

$$u = \frac{\theta}{2} \quad du = \frac{1}{2} d\theta$$

$$= \int_{\pi/2}^{\pi} e^{-u} du = -e^{-u} \Big|_{\pi/2}^{\pi} = -e^{-\frac{\theta}{2}} \Big|_{\pi/2}^{\pi}$$

$$= - \left[ \left( e^{-\frac{\pi}{2}} \right) - \left( e^{-\frac{\pi}{4}} \right) \right] = -e^{-\frac{\pi}{2}} + e^{-\frac{\pi}{4}}$$



$$2b) \quad r = \cos \theta \quad ; \quad 0 \leq \theta \leq \pi/6$$

$$A = \int_a^b (\cos(\theta))^2 \cdot \frac{1}{2} d\theta$$

$$= \int_a^b \frac{1}{4} (1 + \cos(2\theta)) d\theta$$

$$= \left[ \frac{\theta}{4} + \frac{1}{8} \sin(2\theta) \right]_0^{\pi/6}$$

$$= \left[ \left( \frac{\pi}{24} + \frac{1}{8} \sin\left(\frac{\pi}{3}\right) \right) - (0 + 0) \right]$$

$$= \left[ \frac{\pi}{24} + \frac{\sqrt{3}}{16} \right]$$



④  $r = 2 \cos \theta$  ;  $r = 1$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\frac{\pi}{3}, \frac{5\pi}{3}$$

$$A = \frac{1}{2} \int_a^b (r_1^2 - r_2^2) d\theta$$

$$A = \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 4 \cos^2 \theta - 1 d\theta$$

$$A = \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \left( \frac{4}{2} (1 + \cos(2\theta)) - 1 \right) d\theta$$

$$= \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (2 + \cos(2\theta) - 1) d\theta$$

$$= \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (\cos(2\theta) + 1) d\theta = \left[ \frac{1}{4} \sin(2\theta) + \frac{\theta}{2} \right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$$

$$= \frac{1}{2} \left[ \left( \frac{1}{2} \sin\left(\frac{10\pi}{3}\right) + \frac{5\pi}{3} \right) - \left( \frac{1}{2} \sin\left(\frac{\pi}{3}\right) + \frac{\pi}{3} \right) \right] = \boxed{-\sqrt{3} + \frac{4}{3}\pi}$$

⑤  $r = 2 \cos \theta$  ;  $0 \leq \theta \leq \pi$

$$r'(\theta) = -2 \sin(\theta)$$

$$(r'(\theta))^2 = 4 \sin^2(\theta)$$

$$L = \frac{1}{2} \int_a^b \sqrt{(r'(\theta))^2 + r^2} d\theta$$

$$= \frac{1}{2} \int_0^\pi \sqrt{\underbrace{4 \sin^2 \theta + 4 \cos^2 \theta}_4} d\theta$$

$$= \frac{1}{2} \int_0^\pi \sqrt{4} d\theta = \frac{1}{2} \left[ 2\theta \right]_0^\pi = \left[ \theta \right]_0^\pi = [(\pi) - (0)] = \boxed{\pi}$$