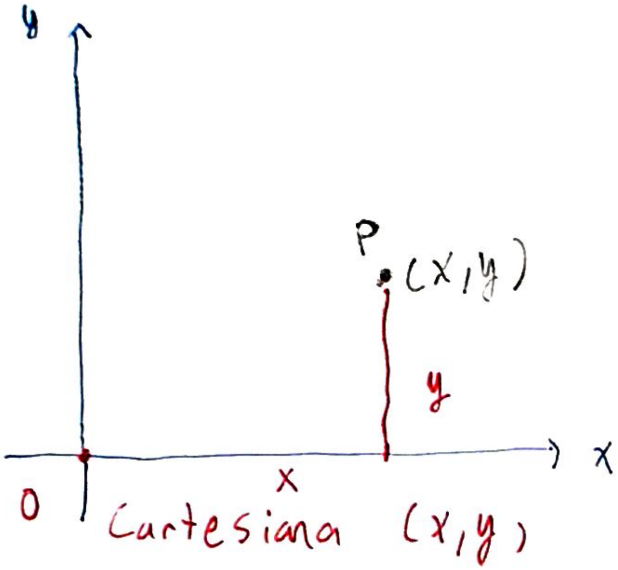
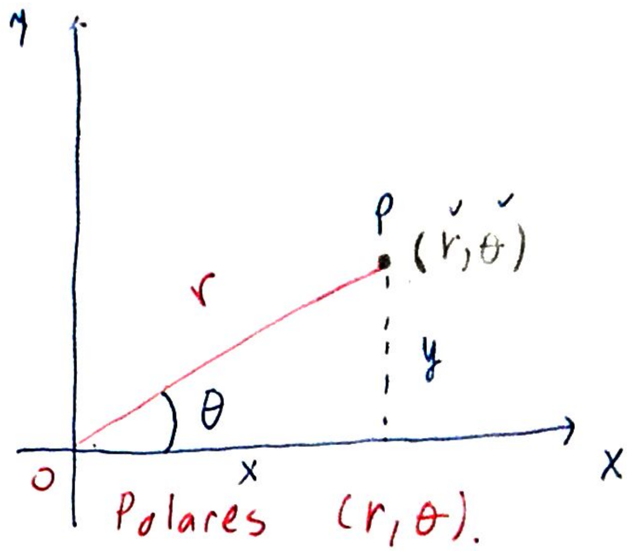
10.3 Coordenadas Polares p.147.





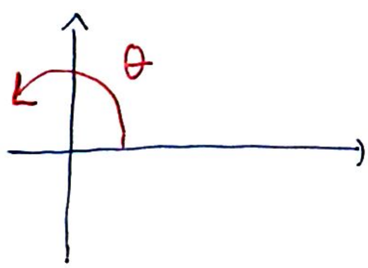
r : radio distancia del punto (x, y) al origen (0,0)

$$\boxed{r = \sqrt{x^2 + y^2}}$$

θ : ángulo entre la recta $\overline{OP}$ y el eje-x.

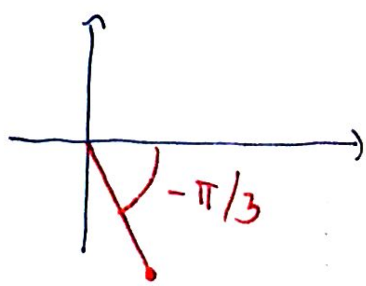
$$\frac{C.O}{C.A.} = \tan\theta = \frac{y}{x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

Convenciones y Observaciones Coordenadas Polares

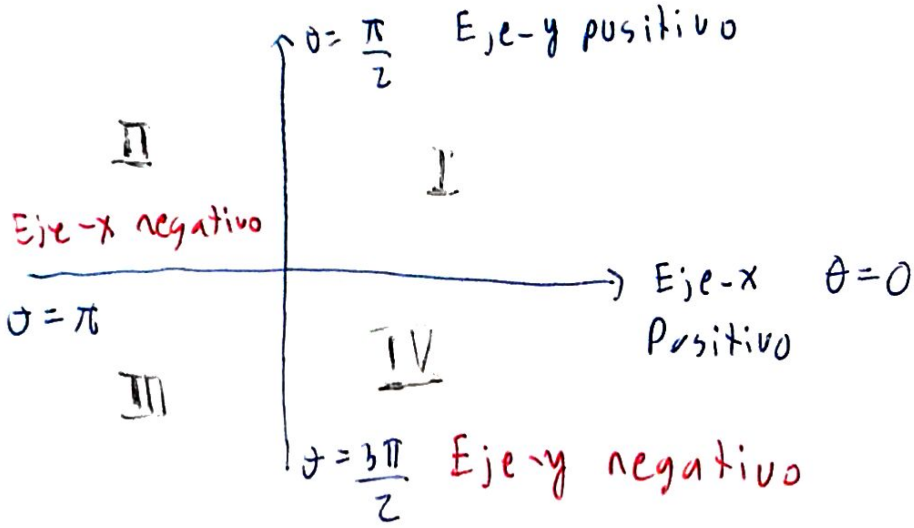


$\theta > 0$ en sentido antihorario.

$\theta < 0$ en sentido horario

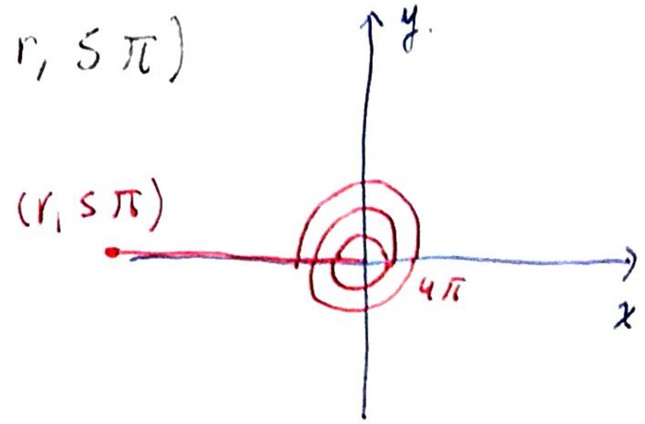


Usualmente $0 \leq \theta \leq 2\pi$.



Ángulos mayores a 2π se da un vuelta al plano.

$(r, 5\pi)$

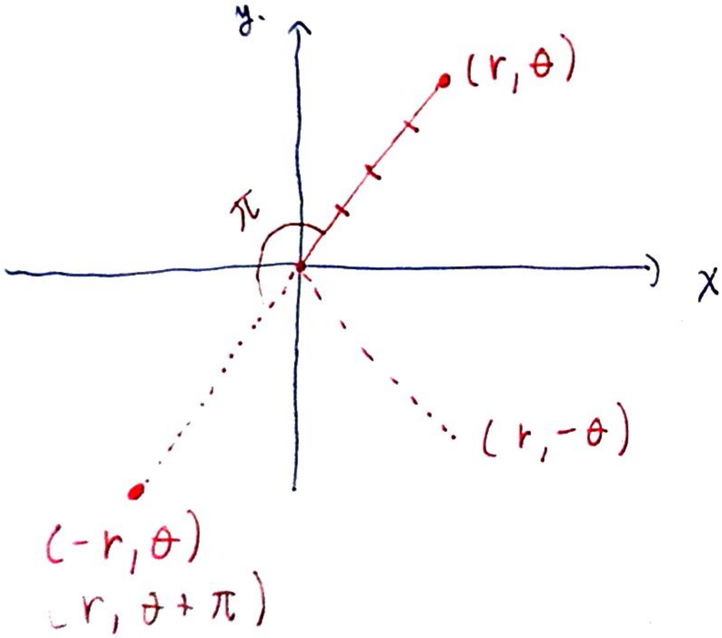


(r, π) $(r, -\pi)$ $(r, 3\pi)$ representan al mismo punto

Coordenadas cartesianas $(-r, 0)$

Usualmente $r > 0$

Reescribir radios negativos.



$(-r, \theta)$ es diamentralmente opuesto a (r, θ)

$(-r, \theta)$ se reescribe como $(r, \theta + \pi)$ ó $(r, \theta - \pi)$.

Origen : $(0,0)$ $r=0$

Cualquier punto de la forma $(0,\theta)$ representa al origen.

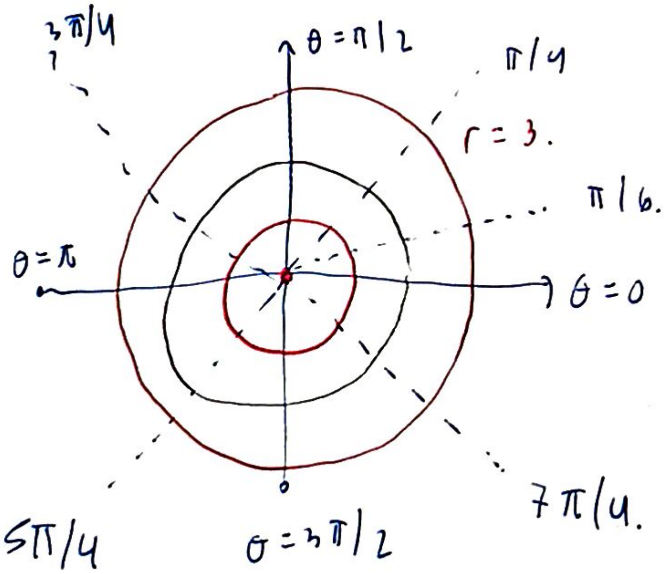
Infinitas representaciones de un punto en coordenadas polares: Como 2π es una vuelta

(r,θ) $(r,\theta+2\pi)$ $(r,\theta \pm 2n\pi)$ $n \in \mathbb{N}$.

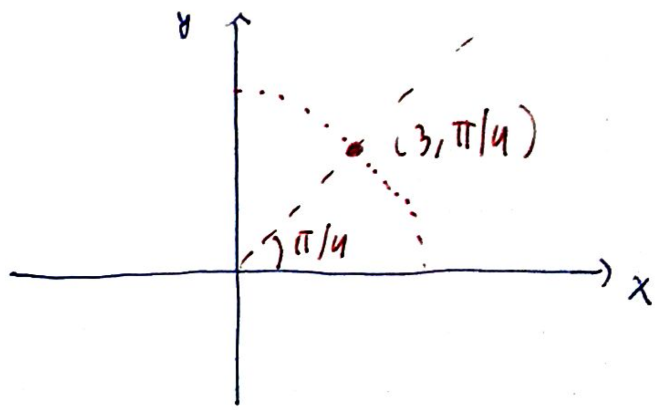
$(-r,\theta+\pi)$ $(-r,\theta+3\pi)$ $(-r,\theta \pm 2n\pi+\pi)$

representan al mismo punto en coordenadas polares.

Ejercicio 1: Grafique los puntos cuyas coordenadas están dadas



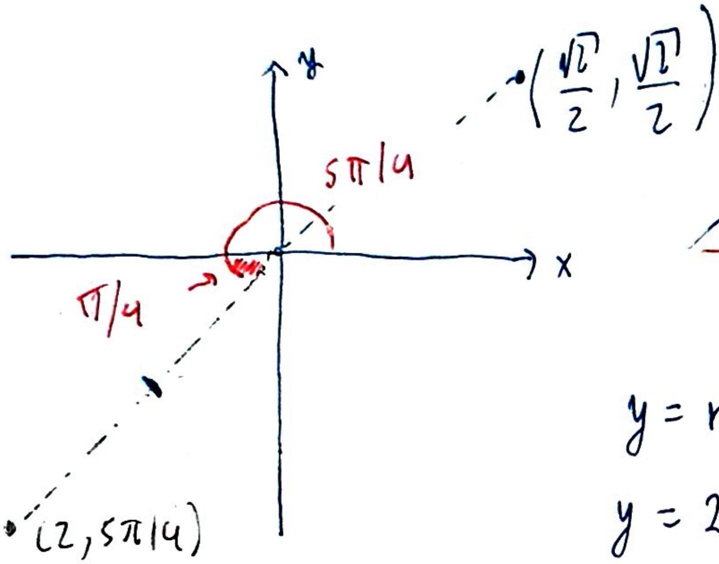
a. $(3, \pi/4)$

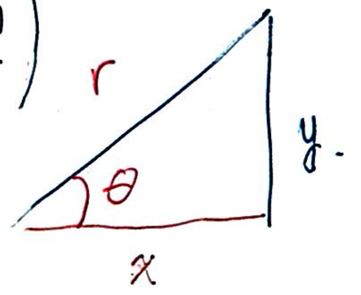


a. $(2, 5\pi/4)$ — $225°$ — 3er Cuadrante

$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

complementos
Suplementos de Ángulos.





$\frac{y}{r} = \sin\theta.$

$\frac{x}{r} = \cos\theta.$

$y = r\sin\theta \qquad x = r\cos\theta.$

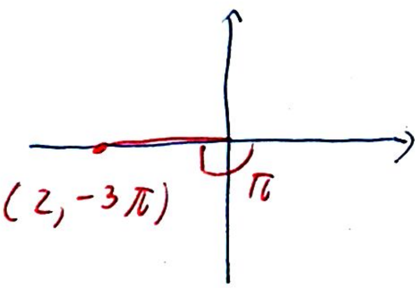
$y = 2\sin\frac{5\pi}{4} = -2\frac{\sqrt{2}}{2} = -\sqrt{2}$

$x = 2\cos\frac{5\pi}{4} = -2\frac{\sqrt{2}}{2} = -\sqrt{2}$

Cartesianas $(-\sqrt{2}, -\sqrt{2})$

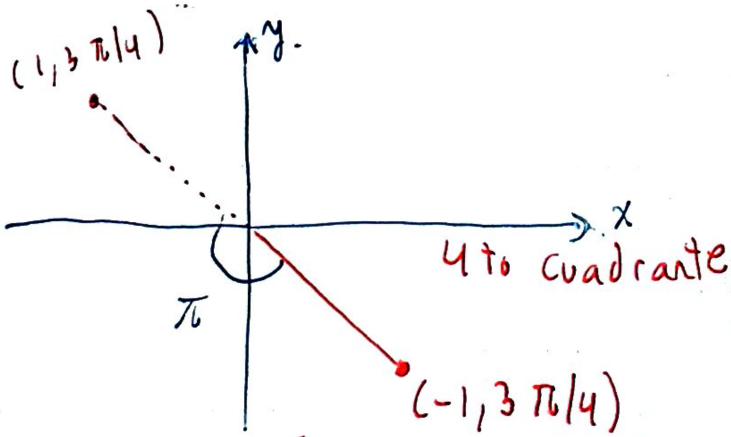
b. $(2, -3\pi)$ 1 vuelta en sentido horario, luego media vuelta.

$(2, -\pi)$ ó $(2, \pi)$



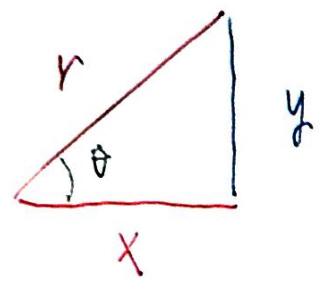
c. $(-1, 3\pi/4)$

está diametralmente opuesto a $(1, 3\pi/4)$



$180° \to \pi$ rad. Sume π $(1, 7\pi/4)$ ó $(1, 7\pi/4)$

Cambio de Coordenadas



r = hipotenusa.

Polares (r, θ) a Cartesianas (x, y)

Exprese x & y en términos de r, θ.

$$\boxed{\begin{aligned} x &= r\cos\theta. \\ y &= r\sin\theta. \end{aligned}}$$

Ecs. Paramétricas

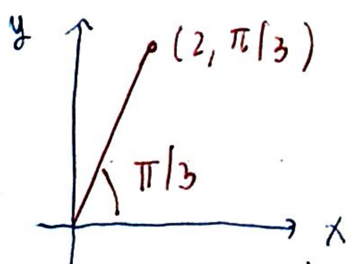
Cartesianas (x, y) a Polares (r, θ).

$$\boxed{r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

θ tiene que estar en el cuadrante correcto.

Ejercicio 2: Convierta los sigs. puntos de coordenadas polares a cartesianas

o. $\left(2, \frac{\pi}{3}\right)$



1er cuadrante.

$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1$

$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

Coordenadas Cartesianas $(1, \sqrt{3})$.

a. $\left(-3, \frac{\pi}{6}\right)$

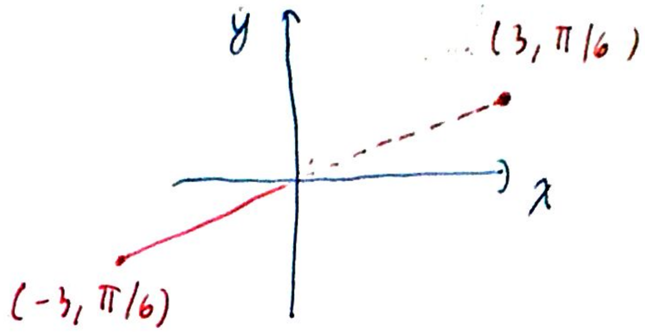
¿ $\left(3, \frac{7\pi}{6}\right)$

$x = r\cos\theta = -3\cos\frac{\pi}{6} = -3\frac{\sqrt{3}}{2}.$

$y = r\sin\theta. = -3\sin\frac{\pi}{6} = -\frac{3}{2}.$

Coordenadas Cartesianas

$$\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$$

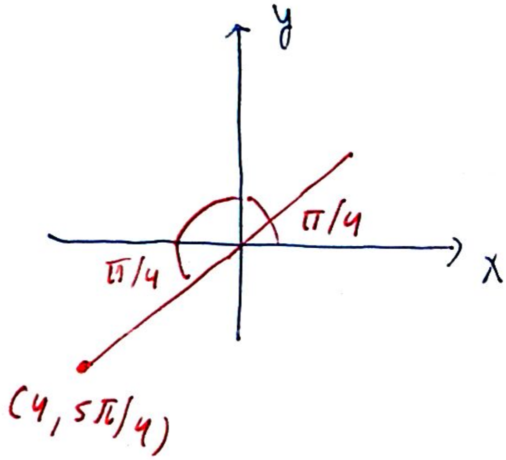


b. $\left(4, \frac{5\pi}{4}\right)$

3er cuadrante

$$\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}.$$



$$x = 4\cos\frac{5\pi}{4} = -2\sqrt{2}$$

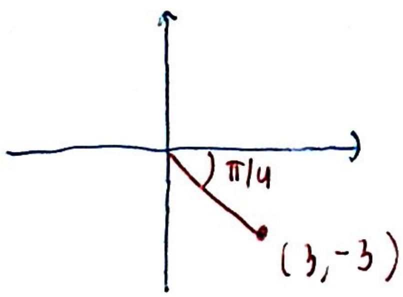
$$y = 4\sin\frac{5\pi}{4} = -2\sqrt{2}$$

Cartesianas $(-2\sqrt{2}, -2\sqrt{2})$

Ejercicio 3: Encuentre las coordenadas polares del punto (x, y).

a). $(3, -3)$

4to Cuadrante $\frac{3\pi}{2} \leq \theta \leq 2\pi$.



$$r = \sqrt{x^2 + y^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}.$$

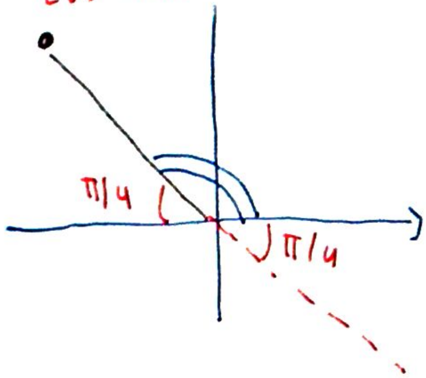
$(3\sqrt{2}, -\pi/4)$ ó $(3\sqrt{2}, 7\pi/4)$.

$$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

a 2. $(-3, 3)$ $r = 3\sqrt{2}$

$\theta = \tan^{-1}(-1) = -\pi/4.$

2do cuadrante



$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

Coordenadas Polares $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$

b. $(-1, -\sqrt{3})$

$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2.$

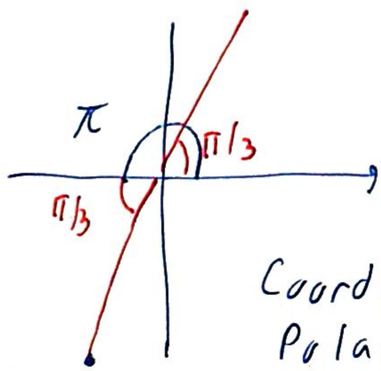
$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(\sqrt{3}) = \pi/3.$

$\sin\frac{\pi}{3} = \sqrt{3}/2$

$\cos\frac{\pi}{3} = 1/2.$

$\tan\frac{\pi}{3} = \sqrt{3}$

$(-1, -\sqrt{3})$ está en el 3er cuadrante.



$$\theta = \pi + \frac{\pi}{3} = \frac{7\pi}{3}.$$

Coordenadas Polares $(2, 7\pi/3)$