7.1 Integración por partes (Pág 39)

La regla de sustitución no se puede usar para integrar

$$\int \ln x \, dx \qquad \int x^n e^x \, dx \qquad \int x^n \sin x \, dx \qquad \int \sin^{-1} x \, dx$$

Regla del Producto para la Diferenciación.

$$(fg)' = f'g + fg'$$

$$(fg)' - f'g = fg'$$ Integre respecto a x.

$$\int fg' \, dx = \int (fg)' \, dx - \int f'g \, dx$$

$$\boxed{\int fg' \, dx = fg - \int f'g \, dx}$$ Regla del Producto para Integración

Objetivo: Pasar de una integral compleja $\int fg' \, dx$ a una más sencilla $\int f'g \, dx$

Integración por partes. IPP.

$$\int f(x) \overbrace{g'(x) \, dx}^{dv} = uv - \int v \, du \Rightarrow \boxed{\int u \, dv = uv - \int v \, du.}$$

Sea $u = f(x)$ $\quad dv = g'(x) dx$

$du = f'(x) dx$ $\quad v = g(x)$

ILATE.
$\xrightarrow{\text{derivar}}$

I = inversas $\sin^{-1} x$, $\tan^{-1} x$.

L = logaritmos. $\ln x$, $\log_a x$.

A = algebraicas, potencias x^2, x^n, x^r

T = trigonométricas $\sin x$, $\cos x$. E = exponenciales e^x

Ejercicio 1: Integre $\int x e^x dx$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$u = e^x \quad dv = x' dx$

$du = e^x dx \quad v = \frac{x^2}{2}$

$\int v du$

No es el adecuado

$$\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv} = \underbrace{xe^x}_{u \cdot v} - \underbrace{\int e^x dx}_{v \cdot du} = \boxed{xe^x - e^x + C.}$$

Compruebe: $(xe^x - e^x)' = e^x + xe^x - \cancel{e^x} = xe^x$

Ejercicio 2: Integre (Pág 40.)

Int. $2x^3$

Der. $12x$

$$a \quad \int \underbrace{6x^2}_{dv} \underbrace{\ln x}_{u} dx = (\ln x) 2x^3 - \int 2x^3 \frac{dx}{x}$$

$u = \ln x \quad dv = 6x^2 dx$

$du = \frac{dx}{x} \quad v = 2x^3$

$$= 2x^3 \ln x - \int 2x^2 dx$$

$$= 2x^3 \ln x - \frac{2}{3} x^3 + C.$$

En Resumen $\int u dv = uv - \int v du$ más fácil

$$b. \int \underbrace{\ln x}_{u} \underbrace{dx}_{dv.} = x \ln x - \int x \frac{dx}{x} = x \ln x - \int dx$$

$$= x \ln x - x + C.$$

$u = \ln x \quad dv = dx$

$du = \frac{dx}{x} \quad v = x$

c. $\int \underbrace{\sin^{-1}x}_{u} \; \underbrace{dx}_{dv}$

b2. $\int (1+x)\underbrace{(x+\frac{1}{2}x^2)^4}\,dx = \int u^4\,du = \frac{1}{5}u^5 + C.$

$u' = 1+x \qquad u = x+\frac{1}{2}x^2$

c. $\int \underbrace{\sin^{-1}x}_{u} \; \underbrace{dx}_{dv} = x\sin^{-1}x - \int \frac{\overbrace{x}^{-du/2}}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$\frac{1}{\sqrt{u}} = u^{-1/2}$

$du = \frac{1}{\sqrt{1-x^2}}, \; v = x$

$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$

$= x\sin^{-1}x + \frac{1}{2}\cdot 2\,u^{1/2} + C$

$= x\sin^{-1}x + \sqrt{1-x^2} + C.$

ILATE

d. $\int x^2\cos x\,dx = x^2\sin x - \underbrace{\int 2x\sin x\,dx}$

utilice IPP otra vez.

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$\int 2x\sin x\,dx = -2x\cos x + \int 2\cos x\,dx = -2x\cos x + 2\sin x + C_1$

$u = 2x \qquad dv = \sin x\,dx$

$du = 2 \qquad v = -\cos x$

$\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x - C$

IPP

Integrales Definidas

$$\int_a^b u\,dv = vu\Big]_a^b - \int_a^b v\,du$$

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Ejercicio 3: Evalúe.

$x\ln x - x$ derive. ILATE

a. $\displaystyle\int_1^e \sqrt{x}\,\ln x^9\,dx = \int_1^e 9x^{1/2}\cdot\ln x\,dx$

$\ln(x^9) = 9\ln x$

$u = \ln x \qquad dv = 9x^{1/2}\,dx$

$du = x^{-1}\,dx \qquad v = 6\,x^{3/2}$

$uv - \int v\,du \qquad = 6x^{3/2}\ln x\Big]_1^e - \int_1^e 6\,x^{3/2}\,x^{-1}\,dx$

$= 6e^{3/2}\ln e - 6\cdot 1^{3/2}\ln 1 - \int_1^e 6\,x^{1/2}\,dx$

$= 6e^{3/2} - \underbrace{6\cdot\frac{2}{3}}_{4}\,x^{3/2}\Big]_1^e = 6e^{3/2} - 4e^{3/2} + 4\cdot 1^{3/2}$

$= \boxed{2e^{3/2} + 4}$

Integre $\int e^x\cos x\,dx$ pág. 44

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x\,dx \qquad v = e^x$

$\int e^x\cos x\,dx = e^x\cos x + \int e^x\sin x\,dx.$

$\int e^x\sin x\,dx = e^x\sin x - \int e^x\cos x\,dx$

$u = \sin x \qquad dv = e^x\,dx$

$du = \cos x\,dx \qquad v = e^x$

$\int e^x\cos x\,dx = e^x\cos x + e^x\sin x - \int e^x\cos x\,dx$

$2\int e^x\cos x\,dx = e^x\cos x + e^x\sin x$

$\int e^x\cos x\,dx = \boxed{\frac{1}{2}(e^x\cos x + e^x\sin x) + C.}$