5.5 Regla de la Sustitución.

a $\int 3(x+2)^2\,dx = \int (3x^2+12x+12)\,dx$

$x^3+6x^2+12x+C.$

$= (x+2)^3 + C$

b. $\int 5(x+2)^4\,dx = (x+2)^5 + C.$

Reglas de Integración para funciones compuestas

$f(g(x)) \longrightarrow F(g(x)) + C.$

Regla de la Potencia para derivadas.

$$\frac{d}{dx}\left([g(x)]^{n+1}\right) = (n+1)[g(x)]^n g'(x).$$

$$\int [g(x)]^n g'(x)\,dx = \frac{1}{n+1}[g(x)]^{n+1} + C.$$

Nueva variable $u = g(x)$ $du = g'(x)\,dx$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \qquad n \neq -1$$

$\ln|u| + C$ si $n = -1.$

Ejercicio 1: Integre

a. $\int 5(\overbrace{x+2}^{u})^4\,dx = \int 5u^4\,du = u^5 + C$

$= \boxed{(x+2)^5 + C.}$

$u = x+2$ $du = dx$

2.

20. $\int (\overbrace{x^2+3}^{u})^5 \overbrace{2x\,dx}^{du} = \int u^5\,du = \frac{u^6}{6} + C.$

$u = x^2+3 \qquad du = 2x\,dx \qquad \frac{(x^2+3)^6}{6} + C.$

a. $\int (\underbrace{y^3+3y^2+11}_{u^{2/3}})^{2/3} \underbrace{(3y^2+6y)\,dy}_{du} = \int u^{2/3}\,du$

$u = y^3+3y^2+11 \qquad du = (3y^2+6y)\,dy.$

$= \frac{3}{5}u^{5/3} + C.$

$= \frac{3}{5}(y^3+3y^2+11)^{5/3} + C.$

b. $\int (10w^3-8)^{15} \underbrace{w^2\,dw}_{du/30} = \int \frac{u^{15}}{30}\,du$ sólo en términos de u.

$u = 10w^3-8$

$\frac{du}{30} = w^2\,dw \qquad dw = \frac{du}{30w^2}$

$\int (10w^3-8)^{15} w^2\,dw = \int u^{15}\,\frac{du}{30} = \frac{u^{16}}{30\cdot 16} + C.$

$= \frac{(10w^3-8)^{16}}{480} + C.$

c. $\int 8x^3\sqrt{8+x^4}\,dx = \int 8\cancel{x^3}\,u^{1/2}\frac{du}{4\cancel{x^3}} = \int 2u^{1/2}\,du.$

$u = 8+x^4 \qquad du = 4x^3\,dx \Rightarrow dx = \frac{du}{4x^3}$

$\int 2\sqrt{\underbrace{8+x^4}_{u}}\,\underbrace{4x^3\,dx}_{du} = \int 2u^{1/2}\,du = \frac{4}{3}u^{3/2} + C.$

$= \frac{4}{3}(8+x^4)^{3/2} + C.$

d. $\int (\underbrace{4x^3+x}_{u})^2\,dx = \int (16x^6+8x^4+x^2)\,dx$

$= \frac{16}{7}x^7 + \frac{8}{5}x^5 + \frac{1}{3}x^3 + C.$

Regla de la cadena. $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\, g'(x)$

Regla de la Sustitución: si $g(x)$ es derivable entonces.

$$\int f'(g(x))\, g'(x)\, dx = \int f'(u)\, du = f(u) + C.$$
$$f(g(x)) + C.$$

$u = g(x) \quad du = g'(x)\, dx$

Ejercicio 2: Integre.

a. $\displaystyle\int \frac{\overbrace{(2+4x+12x^2)}^{2\cdot du}}{x+x^2+2x^3}\, dx = \int \frac{2du}{u} = 2\ln|u| + C.$

$u = x + x^2 + 2x^3 \quad du = (1+2x+6x^2)\, dx$

$2du = (2+4x+12x^2)\, dx$

La integral es $2\ln|x+x^2+2x^3| + C$

a1. $\displaystyle\int e^{x^8} \overbrace{x^7 dx}^{du/8} = \int e^u \frac{du}{8} = \frac{e^u}{8} + C. = \frac{e^{x^8}}{8} + C$

$u = x^8 \quad du = 8x^7\, dx.$

a2 $\displaystyle\int e^{x^8} \underbrace{x^6}_{\neq x^7} dx$ No se puede integrar

b. $\displaystyle\int (x^4+3)^2 \sin(x^4+3)^3 x^3 dx = \int u^2 \sin u^3 \frac{du}{4}.$

$u = x^4 + 3 \quad du = 4x^3\, dx$

$$\int u^2 \sin u^3 \frac{du}{4} = \int \sin w \frac{dw}{12} = \frac{-\cos(w)}{12} + C$$

$w = u^3 \quad dw = 3u^2 du$

$u^2 du = \frac{dw}{3}$

$$= -\frac{1}{12} \cos(u^3) + C$$

$$= -\frac{1}{12} \cos(x^4+3)^3 + C.$$

c. $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx = \int \frac{1}{u} du = \ln|u| + C.$

$= \ln|\sin x| + C.$

$u = \sin x, \; du = \cos x \, dx$

d. $\int \sec^2(\ln x) \frac{1}{x} dx = \int \sec^2 u \, du = \tan u + C$

$= \tan(\ln x) + C.$

$u = \ln x, \quad du = \frac{dx}{x}$

e $\int (1000x + 2{,}000)^{1000} dx = \frac{1}{1000} \frac{(1000x + 2000)^{1001}}{1001} + C.$

$\int e^{x^2} 2x \, dx = e^{x^2} + C.$

Sustitución incompleta.

$x = u - 3.$

f. $\int 28x(x+3)^{1/3} dx. = \int 28x \, u^{1/3} du.$

$u = x+3 \quad du = dx$

$$= \int 28(u-3) u^{1/3} du.$$

$$= \int 28u^{4/3} - 28 \cdot 3 u^{1/3} du.$$

$$= \frac{28 \cdot 3}{7} u^{7/3} - 28 \cdot \frac{3 \cdot 3}{4} u^{4/3} + C.$$

$$= 12u^{7/3} - 63u^{4/3} + C.$$

$$= 12(x+3)^{7/3} - 63(x+3)^{4/3} + C.$$

Regla de la Sustitución para Integrales Definidas

por ejemplo. Integre respecto a u.

$$\int_0^{1/2} \cos(\pi x)\, \underbrace{dx}_{du/\pi} = \int_0^{\pi/2} \cos u \,\frac{du}{\pi} = \frac{1}{\pi} \sin u \Big]_0^{\pi/2}$$

$u = \pi x$ $\quad u(1/2) = \pi/2$ $\quad = \frac{1}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{1}{\pi}.$

$du = \pi\, dx$ $\quad u(0) = 0$

Regla de la Sustitución

$$\boxed{\int_a^b f(\underbrace{g(x)}_{u})\, \underbrace{g'(x)dx}_{du} = \int_{u(a)}^{u(b)} f(u)\,du.}$$

Ejercicio 3: Evalúe

a. $\displaystyle\int_{-4}^{0} \frac{1}{3x-2}\,dx = \int_{-14}^{-2} \frac{1}{3}\frac{du}{u} = \frac{1}{3}\ln|u|\Big]_{-14}^{-2} = \frac{1}{3}\ln 2 - \frac{1}{3}\ln 14.$

$u = 3x-2$, $du = 3dx$ $\quad u(0) = -2 \quad u(-4) = -14$

b. $\displaystyle\int_0^1 \frac{8\sin^{-1}t}{\sqrt{1-t^2}}\,dt = \int_0^{\pi/2} 8u\,du. = 4u^2\Big]_0^{\pi/2} = 4\frac{\pi^2}{4} = \pi^2.$

$u = \sin^{-1}t$ $\quad u(1) = \sin^{-1}(1) = \pi/2$

$du = \dfrac{1}{\sqrt{1-t^2}}\,dt.$ $\quad u(0) = \sin^{-1}(0) = 0$