90pts

Corto #3 Cálculo Integral (20 min)

Nombre: David Corzo **Carnet:** 20190432

Resuelva las siguientes integrales:

1. (50 pts.) $\int \tan^5\theta \sec^4\theta \, d\theta$

$= \int \tan^5\theta \cdot \sec^2\theta \left(\tan^2\theta + 1\right) d\theta$

$= \int \tan^7\theta \sec^2\theta + \int \tan^5 \sec^2\theta$

$u = \tan\theta$ $\quad$ $du = \sec^2\theta$ $\qquad$ $u = \tan\theta$ $\quad$ $du = \sec^2\theta$

$\sin^2 x + \cos^2 x = 1$

$\int d\theta$ $\quad$ $\tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right)$

$\tan^2 x = \sec^2 x - 1$

$\therefore = \int u^7 \, du + \int u^5 \, du$

$= \frac{u^8}{8} + \frac{u^5}{5} + C$

$= \frac{\tan^8\theta}{8} + \frac{\tan^5\theta}{5} + C$

40pts

2. (50 pts.) $\int e^{x^8+1}(x^8+1)16x^7 \, dx$ $= 2\int e^u (u) \, du$

$u = x^8 + 1$

$du = 8x^7 \, dx$

$2(du) = 16x^7 \, dx$

$\upsilon = u$ $\quad$ $d\beta = e^u$

$d\upsilon = 1 du$ $\quad$ $\beta = e^u$

$\upsilon\beta - \int \beta \, d\upsilon$

$u \cdot e^u - \int e^u \, du$

$u \cdot e^u - e^u + C$

$\cancel{\frac{d}{dx}\left[2(x^8+1)e^{x^8+1}\right] - \left(2e^{x^8+1}\right)}$

$\cancel{2x^8 + 2 + 2e^{x^8+1} - 2e^{x^8+1}}$

$\cancel{2x^8 + 2}$

$2\left[(x^8+1)(e^{x^8+1}) - (e^{x^8+1}) + C\right]$

$2\left[(x^8+1)\left(e^{x^8+1}\right) - \left(e^{x^8+1}\right) + C\right]$

50pts

55pts

Corto #2 Cálculo Integral (15 min)

Nombre: David Gabriel Corzo Mcmath **Carnet:** 20190432

1. (50 pts.) ¿Cuál es la ecuación de la recta tangente a $h(x) = 2 + \int_1^{e^x} \cos^3\theta d\theta$ en $x = 0$?

$y = f(a) + f'(a)(x - a)$

$h(x) = 2 + \underbrace{\int_1^{e^x} \cos^3\theta \, d\theta}_{\text{derivo}}$ $x = 0$

$h'(x) = 0 + \cos^3(e^x) \cdot e^x$

$h'(0) = 0 + \cos^3(e^0) e^0$

$= 1$

$h(0) = 2 + \cos^3(0)$

$= 2 + 1 = 3$

$y = 3 + 1(x - 0)$

20pts

2. (50 pts.) Un resporte sin amortiguamiento y sujeto a una fuerza externa tiene aceleracion $a(t) = 2t - 4\cos t + 5\sin t$, velocidad y posición iniciales de 2 m/s y 0 m.

(a) (25 pts.) Encuentra la función de velocidad del resorte

$a''(t) = 2t - 4\cos t - 5\sin t$

$a'(t) = 2\int t - 4\int \cos t - 5\int \sin t$

$a'(t) = \frac{2t^2}{2} - 4\sin t - 5\cos t$

$a'(t) = t^2 - 4\sin t - 5\cos t + C_1$

$a'(0) = 0^2 - 4\sin 0 - 5\cos 0 + C_1$

$y = 0 - 0 - 5 + C_1$

$y = -5 + C_1$

$C_1 = -5$

15pts

(b) (25 pts.) Encuentra la función de desplazamiento del resorte.

$\sin' = \cos$

$\cos' = -\sin$

$a'(t) = t^2 - 4\sin t - 5\cos t$

$a(t) = \int t^2 - 4\int \sin t - 5\int \cos t$

$a(t) = \frac{t^3}{3} + 4\cos t - 5\sin t + C_1 + C_2$

20pts

$a(0) = \frac{0^3}{3} + 4\cos 0 - 5\sin 0 + (-5) + C_2$

$a(0) = 0 + 4 - 0 - 5 + C_2$

$a(0) = 4 - 5 + C_2$

$a(0) = -4 + C$

$C_2 = -4$

100pts

Corto #1 Cálculo Integral (20 min)

Nombre: David Gabriel Corzo Mcmath **Carnet:** 20190432

1. Evalúe las siguientes integrales

   (a) (30 pts.) $\int \left(x^2 - \frac{1}{2}x + \frac{1}{4x} - \frac{2}{x^2}\right)\,dx$ $= \int [x^2] dx - \frac{1}{2}\int [x] dx + \frac{1}{4}\int \left[\frac{1}{x}\right] dx - 2\int \left[\frac{1}{x^2}\right] dx$

   $-\frac{1}{x}$; $\frac{0 * x - 1x}{x^2} = -\frac{x}{x^2}$

   $= \frac{x^{2+1}}{2+1} - \left(\frac{1}{2}\right)\frac{x^{1+1}}{1+1} + \left(\frac{1}{4}\right)\ln(x) - (2)\frac{x^{-2+1}}{-2+1} + C$

   $+\left(\frac{2x^{-1}}{-1}\right)$

   $= \frac{x^3}{3} - \frac{x^2}{4} + \frac{\ln(x)}{4} + \frac{2}{x} + C$

   30pts

   (b) (30 pts.) $\int_0^{\pi/4} (2\sin x - 4\cos(x))dx$

   $\sin x = \cos x$

   $\cos x = -\sin x$

   $2\int \sin x\,dx - 4\int \cos x\,dx = 2(-\cos x) - 4(\sin x)\Big]_0^{\pi/4}$

   $\{-2\cos(\pi/4) - 4\sin(\pi/4)\} - \{-2\cos(0) - 4\sin(0)\}$

   $= \{-\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} - 4\frac{\sqrt{2}}{2}\} - \{-2\} = -\sqrt{2} - 2\sqrt{2} + 2$

   30pts

2. Considere la región entre $f(x) = |x-5|$, el eje-x, y las rectas verticales $x = 0$, $x = 10$.

   (a) (20 pts.) Trace la gráfica de la región.

   (b) (20 pts.) Encuentre el área de la región utilizando geometría.

$f(x) = |x-5| = \begin{cases} x - 5 & x > 5 \\ -x + 5 & x < 5 \end{cases}$

$f(0) = 0$

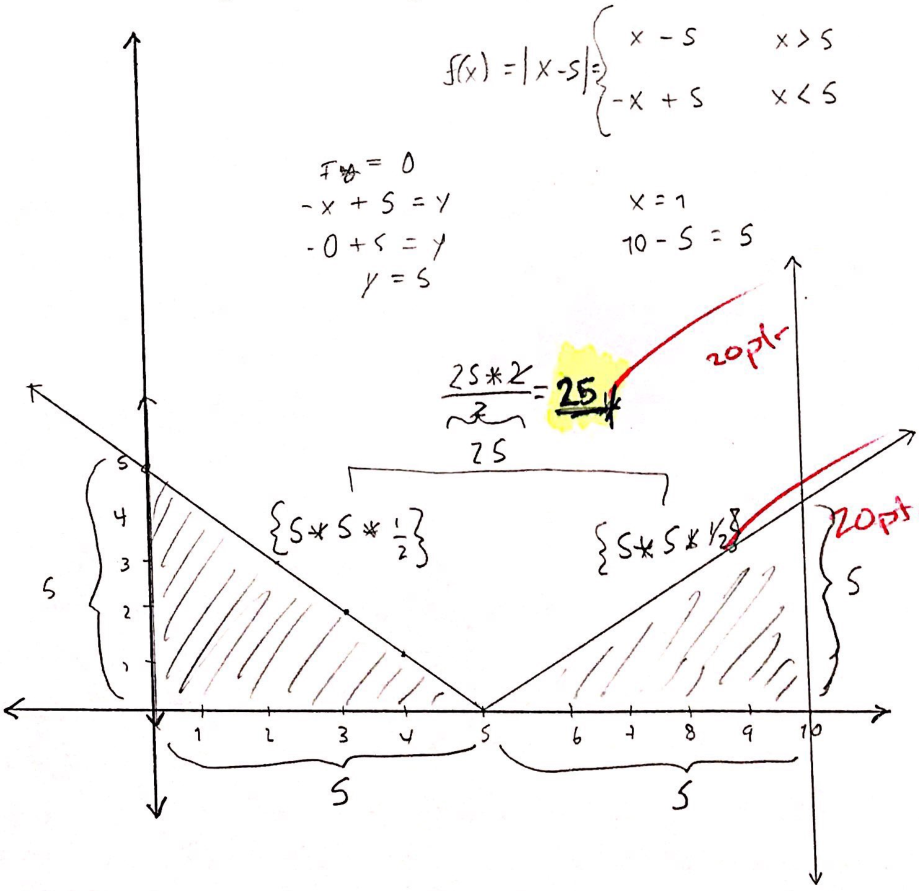
$-x + 5 = y$

$-0 + 5 = y$

$y = 5$

$x = 1$

$10 - 5 = 5$



$\frac{25 * \cancel{2}}{\cancel{2}} = 25$

25

20pts

20pts

18/08/2019

Webassign Desplazamiento & distancia.

① Calculate area $y = \sqrt{x}$ betwen 0 & 9.

$$\int_0^9 \sqrt{x}\, dx = \int_0^9 x^{1/2}\, dx = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3}\Bigg]_0^9 =$$

$$= \left\{\frac{2(9)^{3/2}}{3}\right\} - \left\{\frac{2(0)^{3/2}}{3}\right\} = \left(\frac{2(27)}{3}\right) = \frac{54}{3}$$

② $y = 4 - x^2$

$$2\int_0^2 4 - x^2\, dx = 2\left[(4x) - \frac{x^3}{3}\right] = 8x - \frac{2x^3}{3}\Bigg]_0^2$$

$$= \left\{8(2) - \frac{2(2)^3}{3}\right\} - \cancel{\left\{8(0) - \frac{2(0)^3}{3}\right\}}^{\,0}$$

$$\frac{3 \cdot 16}{3} - \frac{16}{3} = \frac{48-16}{3} = \frac{32}{3}$$

③ $y = \sec^2(x) \quad 0 \leq x \leq \pi/3$

$$\int \sec^2 x \, dx = \tan x \Big]_0^{\pi/3}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sin \pi/3}{\cos \pi/3}$$

$$= \{\tan(0)\} - \{\tan(\pi/3)\}$$

$$= \sqrt{3}$$

④ $$\int_{-1}^{2} x^3 \, dx = \frac{x^4}{4}\Big]_{-1}^{2} = \left\{\frac{2^4}{2^2}\right\} - \left\{\frac{(-1)^4}{4}\right\} = 4 - \frac{1}{4} = \frac{4 \cdot 4}{4} - \frac{1}{4}$$

$$= \frac{16 - 1}{4} = \frac{15}{4}$$

① Área B

$$A = \int_0^a e^x \, dx = e^x\Big]_0^a = e^a - 1$$

$$B = \int_0^b e^x \, dx = e^x\Big]_0^b = e^b - e^0 = e^b - 1$$

$B = 3A$

$(e^b - 1) = 3(e^a - 1)$

$e^b - 1 = 3e^a - 3$

$e^b = 3e^a - 3 + 1$

$e^b = 3e^a - 2$

$\ln(e^b) = \ln(3e^a - 2)$

$b = \ln(3e^a - 2)$

$\square$

(8) $w(t) = 3t - 8, \quad 0 \le t \le 3$

$$d(t) = \int_0^3 (3t-8)\,dt$$

$$d(t) = \frac{3t^2}{2} - 8t\Big]_0^3 = \left\{ \underbrace{\frac{3(3)^2}{2}}_{\frac{27}{2}} - \underbrace{8(3)}_{24} \right\} - \left\{ \cancel{\frac{3(0)^2}{2} - 8(0)} \right\} = \frac{27}{2} - \frac{24\cdot 2}{2} = \frac{27 - 24\cdot 2}{2} = \frac{27-48}{2}$$

$$= -\frac{21}{2}$$

$$\begin{array}{r} \cancel{2}\cancel{7} \\ -48 \\ \hline -29 \end{array}$$

Ix $y = 0$

$3t - 8 = 0$

$3t = 8$

$t = \frac{8}{3}$

Iy $= x = 0$

$-8 = y$

$v(2) = 6 - 8 = -2$

$v(1) = 3 - 8 = -5$

$v(3) = 9 - 8 = 1$

$v(8/3) = 3(\frac{8}{3}) - 8 = 0$

$$\left| \int_0^{8/3} (3t-8)\,dt \right| + \left| \int_{8/3}^3 (3t-8)\,dt \right|$$

$$\left\{ \frac{3\left(\frac{8}{3}\right)^2}{2} - 8\cdot\frac{8}{3} \right\} - 0 = -\frac{64}{6} = -\frac{32}{3}$$

$\frac{64}{6} - \frac{64\cdot 2}{3\cdot 2} = \frac{64 - 128}{6}$

$$\left| -\frac{32}{3} \right| + \frac{127}{6} =$$

$$\left\{ \frac{3(3)^2}{2} - 8(3) \right\} - \left\{ \frac{3\left(\frac{8}{3}\right)^2}{2} - 8\cdot\frac{8}{3} \right\}$$

$\frac{3(9)}{2} = \frac{27}{2} - \frac{24\cdot 2}{2}$

$\left\{ \frac{27 - 24\cdot 2}{2} \right\} - \frac{\frac{64}{3}}{\frac{2}{1}}$ $\frac{64}{6} - \frac{64\cdot 2}{6}$

$\frac{64 - 64\cdot 2}{6}$

$-\frac{21}{2} + \frac{64}{6}$ $-\frac{64}{6}$ $\frac{64(1-2)}{6}$

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(7) $y = 8x - x^2$

$x = 8y - y^2$

$$g(x) = \int_0^8 (8y - y^2)\,dy$$

$$\int_0^8 8y\,dy - \int_0^8 y^2\,dy$$

$$8\int_0^8 y\,dy - \int_0^8 y^2\,dy$$

$$8\frac{y^2}{2} - \frac{y^3}{3} + C\Big]_0^8$$

$$y^2\left[4 - \frac{y}{3}\right]$$

$$\left\{ 4(8)^2 - \frac{(8)^3}{3} \right\} - \left\{ \cancel{4(0)^2 - \frac{(0)^3}{3}} \right\}$$

$$= (8)^2\left[4 - \frac{8}{3}\right]$$

$$= 64\left[4 - \frac{8}{3}\right]$$

$$= \frac{3\cdot 256}{3} - \frac{512}{3}$$

$$= \frac{3256 - 512}{3}$$

$$= \frac{256}{3}$$

1 + 24

256

5

5

1

2

$$\left|\int_0^{8/3}(3t-8)\,dt\right| + \int_{8/3}^{3}(3t-8)\,dt$$

$$\left|\frac{3t^2}{2} - 8t\Big]_0^{8/3}\right| + \frac{3t^2}{2} - 8t\Big]_{8/3}^{3} + C$$

$$\left|\left\{\frac{\frac{3}{1}\cdot\left(\frac{8}{3}\right)^2}{\frac{2}{1}} - \frac{8}{1}\cdot\left(\frac{8}{3}\right)\right\} - \left\{0\right\}\right| + \left\{\frac{3(3)^2}{2} - 8(3)\right\} - \left\{\frac{\frac{3}{1}\left(\frac{8}{3}\right)^2}{\frac{2}{1}} - \frac{8}{1}\left(\frac{8}{3}\right)\right\} =$$

$$\left|\left\{\frac{\frac{\cancel{3}}{1}\cdot\frac{64}{\cancel{3}\cdot 3}}{\frac{2}{1}} - \frac{64}{3}\right\}\right| + \left\{\frac{27}{2} - \frac{24\cdot 2}{2}\right\} - \left\{\frac{\frac{\cancel{3}\cdot 64}{\cancel{3}\cdot 3}}{\frac{2}{1}} - \frac{8\cdot 8}{3}\right\} =$$

$$+ \left\{\frac{27-48}{2}\right\} - $$

$$\left|\left\{\left[\frac{\frac{64}{3}}{\frac{2}{1}}\right] - \frac{64}{3}\right\}\right| + \left\{\frac{-21}{2}\right\} - \left\{\frac{64}{6} - \frac{64\cdot 2}{3\cdot 2}\right\} =$$

$$\left|\left\{\frac{64}{6} - \frac{64\cdot 2}{3\cdot 2}\right\}\right| + \left\{\frac{-21}{2}\right\} - \left\{\frac{64-64\cdot 2}{6}\right\} =$$

$$-\frac{21\cdot 3}{2\cdot 3} + \frac{64}{6} = \frac{-21\cdot 3 + 64}{6} = \frac{1}{6}$$

$$\left|\left\{\frac{64-64\cdot 2}{6}\right\}\right| = -\frac{\cancel{64}}{\cancel{6}} = \left|-\frac{32}{3}\right| = \frac{32\cdot 2}{3\cdot 2} + \frac{1}{6} = \frac{32\cdot 2 + 1}{6} = \frac{64+1}{6} = \frac{65}{6} \quad \square$$

$$\frac{32}{3} + \frac{1}{6}$$

⑨ $a(t) = t + 8$ $\quad v(0) = 6 \quad 0 \le t \le 10$

$$v(t) = \frac{t^2}{2} + 8t + C$$

$$v(0) = \frac{(0)^2}{2} + 8(0) + 6$$

$$v(t) = \frac{t^2}{2} + 8t + 6$$

$$d(t) = \frac{t^3}{6} + 4t^2 + 6t\Big]_0^{10} = \left\{\frac{(10)^3}{6} + 4(10)^2 + 6(10)\right\} - \left\{\frac{(0)^3}{6} + 4(0)^2 + 6(0)\right\}$$

$$= \left\{\frac{1000}{6} + 400 + 60\right\}$$

$$= \left\{\frac{500}{3} + 400 + 60\right\}$$

$$= \frac{1880}{3}$$

⑩ $p(x) = 8 + 4\sqrt{x}$

$$\text{mass} = \int_0^4 8 + 4\sqrt{x}\,dx$$

$$= \int_0^4 8\,dx + 4\int_0^4 \sqrt{x}\,dx$$

- the linear density is $\frac{\text{mass}}{v}$
- the mass is the integration of the linear density

$$8x\Big]_0^4 + \frac{8x^{3/2}}{3}\Big]_0^4 = \left\{8(4) + \frac{8(4)^{\frac{3}{2}}}{3}\right\} - \left\{8(0) + \frac{8(0)^{3/2}}{3}\right\}$$

$$\left\{32 + \frac{64}{3}\right\} - \left\{0 + 0\right\}$$

$$\frac{160}{3}$$

(11) find the displacement

$$v(t) = t^2 - 2t - 15 \;, \quad 1 \leq t \leq 7$$

$$f(x) = \frac{t^3}{3} - \frac{\cancel{2}t^2}{\cancel{2}} - 15t + C \Big]_1^7$$

$$= \left\{ \frac{7^3}{3} - 7^2 - 15(7) \right\} - \left\{ \frac{1^3}{3} - 1^2 - 15(1) \right\}$$

$$= \frac{-119}{3} + \frac{47}{3}$$

$$= \underline{-24}$$

find the distance

$$v(t) = t^2 - 2t - 15 \quad \text{Ix}$$

$$(t - 5)(t + 3) = 0 \qquad \text{Ix} \quad y = -15$$

$$t = 5 \quad t = -3$$

nos interesa de 1 - 7
y pasa por 5 dividimos
la integral

$$= \left| \int_1^5 t^2 - 2t - 15 \, dt \right| + \left| \int_5^7 t^2 - 2t - 15 \, dt \right|$$

$$= +\frac{128}{3} + \frac{56}{3}$$

$$= \underline{\frac{184}{3}}$$

Webassign 2019-08/14

$$g(x) = \int_0^x \sqrt{t^4 + t^6}\, dt = \sqrt{x^4 + x^6} \cdot 1$$

$$f(x) = \int_x^0 \sqrt{2 - \sec(8t)}\, dt = -\int_0^x \sqrt{2 - \sec(8t)}\, dt = -\sqrt{2 + \sec(8x)} \cdot 1$$

$$h(x) = \int_1^{e^x} \sin(t)\, dt = 5 \cdot \cancel{\ln}(e^x) = 5 \cdot x \cdot e^x$$

$$F'(x) = \int_x^{x^2} e^{t^4} dt = e^{x^8} \cdot 2x - e^{x^4} \cdot 1$$

$$y = \int_{\cos x}^{\sin x} \ln(1 + 7v)\, dv = \ln(1 + 7\sin x) \cdot \cos x + \underline{\ln(1 + 7\cos x)\sin x}$$

$\sin' = \cos$

$\cos = -\sin$

$$\int_4^6 (x^2 + 2x - 7)\, dx = \frac{x^3}{3} + \frac{2x^2}{2} - 7x$$

$$= \frac{x^3}{3} + x^2 - 7x \Big]_4^6$$

$$= \left\{ \frac{6^3}{3} + 6^2 - 7(6) \right\} - \left\{ \frac{4^3}{3} + \cancel{3}4^2 - 7(4) \right\}$$

$$= \{72 + 36 - 7(6)\} - \left\{ \frac{64}{3} + 16 - 28 \right\}$$

$$= \underline{\frac{170}{3}}$$

$$\int_4^9 \sqrt{x}\,dx = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \left[\frac{x^{3/2}}{\frac{1}{\frac{3}{2}}} = \frac{2x^{3/2}}{3}\right]_4^9 = \left\{\frac{2\sqrt[2]{9^3}}{3}\right\} - \left\{\frac{2\sqrt[2]{4^3}}{3}\right\}$$

$$= \frac{2(27)}{3} - \frac{2(8)}{3}$$

$$= \frac{54}{3} - \frac{16}{3} = \frac{38}{3}$$

$$\int_1^9 \frac{2+x^2}{\sqrt{x}}\,dx = \int_1^9 \frac{2}{\sqrt{x}} + \frac{x^2}{\sqrt{x}}\,dx$$

$$2\int_1^9 \frac{1}{x^{1/2}}\,dx + \int_1^9 x^2 x^{-1/2}\,dx$$

$$2\int_1^9 x^{-1/2}\,dx + \int_1^9 x^{3/2}\,dx$$

$$\left.\frac{2\sqrt{x}}{1/2} + \frac{x^{5/2}}{5/2}\right]_1^9 = \left\{\frac{2(3)}{1/2} + \frac{243}{5/2}\right\} - \left\{\frac{2}{1/2} + \frac{1}{5/2}\right\}$$

$$\{6 + 97.2\} - \{2 + 0.4\}$$

$$= \underline{104.8}$$

$$\int_1^2 \frac{v^3+4v^5}{v^2}\,dv = \int \frac{v^3}{v^2} + \frac{4v^5}{v^2}\,dv = \left.\frac{v^2}{2} + v^4\right]_1^2 = \left(\frac{4}{2}+16\right) - \left(\frac{1}{2}+\frac{2}{2}\right)$$

$$\int v + 4v^3$$

$$= (2+16) - \left(\frac{3}{2}\right)$$

$$= \frac{2 \cdot 18}{2} - \frac{3}{2}$$

$$= \frac{36}{2} - \frac{3}{2}$$

$$= \underline{\frac{33}{2}}$$

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{7}{1+x^2}\,dx = 7\int \frac{1}{1+x^2} \} \arctan(x) = 7\tan^{-1}(x)\Big]_{1/\sqrt{3}}^{\sqrt{3}} \; 7(\pi/3) - 7\pi/6$$

$$= \frac{2\cdot 7\pi}{2\cdot 3} - \frac{7\pi}{6}$$

$$\boxed{\frac{\pi}{3}} \quad \left[\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] = \frac{\cancel{2}}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \qquad = \frac{14\pi}{6} - \frac{7\pi}{6}$$

$$= \frac{7\pi}{6}$$

$$\frac{\sqrt{3}}{2}$$

$$\int_{-4}^{4} f(x)\,dx \qquad f(x) = \begin{cases} 3 & -4 \le x \le 0 \\ 6 - x^2 & 0 < x \le 4 \end{cases} \qquad = 12 + \frac{8}{3} = \frac{3\cdot 12}{3} + \frac{8}{3} = \frac{44}{3}$$

$$\int_{-4}^{0} 3\,dx = 3x\Big]_{-4}^{0} = \{3(0)\} - \{3(-4)\} = 0 - (-12) = 12$$

$$\int_{0}^{4} 6 - x^2\,dx = 6x - \frac{x^3}{3}\Big]_0^4 = \left\{6(4) - \frac{(4)^3}{3}\right\} - \{0 - 0\}$$

$$\frac{3\cdot 24}{3} - \frac{64}{3} = \frac{72-64}{3} = \frac{8}{3}$$

What is wrong?

$$\int_{-2}^{4} x^{-3}\,dx = \frac{x^{-2}}{2}\Bigg]_{-2}^{4} = \frac{3}{32}$$

$$\frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}\Bigg]_{2}^{4} = \left\{-\frac{1}{2(4)^2}\right\} - \left\{-\frac{1}{2(2)^2}\right\} = \left\{-\frac{1}{32}\right\} - \left\{-\frac{1}{8}\right\}$$

$$= -\frac{1}{32} + \frac{1\cdot 4}{8\cdot 4}$$

not continuos at 0

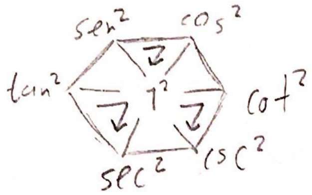
$$= -\frac{1}{32} + \frac{4}{32} = \frac{3}{32}$$

$$g(x) = \int_{4x}^{7x} \frac{u^2-3}{u^2+3}\,du = \left\{\frac{(7x)^2-3}{(7x)^2+3}\cdot 7\right\} - \left\{\frac{(4x)^2-3}{(4x)^2+3}\cdot 4\right\}$$

$$= \frac{49x^2-3}{49x^2+3}\cdot 7 - \frac{16x^2-3}{16x^2+3}\cdot 4$$

30/07/2019

Webassign 5.4 Integrales Indefinidas y definido



$$\int (x^{1.7} + 9x^{3.5})\,dx$$

$$\underline{\frac{x^{1.7+1}}{1.7+1} + \frac{9x^{4.5}}{4.5} + C}$$

$$\int \left(u^5 - 2u^4 - u^2 + \frac{4}{5}\right) du$$

$$\underline{\frac{u^6}{6} - \frac{2u^5}{5} - \frac{u^3}{3} + \frac{4}{5}x + C}$$

$$\int (v+6)(4u+7)\,dv$$

$$\int 4v^2 + 7u + 24u + 42\,du$$

$$\frac{4v^3}{3} + \frac{7v^2}{2} + \frac{24v^2}{2} + 42u$$

$$\frac{4u^3}{3} + \frac{7u^2}{2} + 12u^2 + \underline{42u + C}$$

▲ $$\int \frac{6 + \sqrt{x} + x}{x}\, dx$$

$$6\int \frac{1}{x}\,dx + \int \frac{\sqrt{x}}{x}\,dx + \int \frac{x}{x}\,dx$$

$$6 \cdot \ln x + \int (x)^{1/2}(x)^{-1}\,dx + x + C$$

$$6 \ln x + \int x^{-1/2}\,dx + x + C$$

$$6 \ln x + \frac{\sqrt{x}}{1/2} + x + C$$

$$6 \ln x + 2\sqrt{x} + x + C$$

▲ $$\int \sec(t)(9\sec(t) + 4\tan(t))\, dt$$

$$\int 9\sec^2 t + 4 \sec t \tan t \; dt$$

$$9\int \sec^2(t)\,dt + 4\int \sec(t)\tan(t)\,dt$$

$$9 \tan t + 4 \sec t + C$$

▲ $$\int 4 \frac{\sin(2x)}{\sin(x)}\, dx$$

ID => $\sin(2w) = 2\sin(w)\cos(w)$

entonces...

$$4\int \frac{2\cancel{\sin(x)}\cos(x)}{\cancel{\sin(x)}}\, dx$$

$$4 \cdot 2\int \cos(x)$$

$8\sin(x)$

$$\int_{-2}^{3} (x^2-3)\,dx$$

$$\int (x^2)\,dx - \int (3)\,dx$$

$$\frac{x^3}{3} - 3x + C$$

$$\left.\frac{x^3}{3}\right]_{-2}^{3} - \left.3x\right]_{-2}^{3} =$$

$$\frac{27}{3} - \frac{-8}{3} = \frac{27}{3} + \frac{8}{3} = \frac{35}{3}$$

$$3(3) - (3(-2))$$

$$3(3) + 6$$

$$9 + 6 = 15 - \frac{35}{6}$$

$$15 - \frac{35}{3} = \frac{10}{3}$$

$$\int_{0}^{2} (2x-3)(4x^2+4)\,dx$$

$$\int 8x^3 + 8x - 12x^2 - 12\,dx$$

$$8\int x^3\,dx + 8\int x\,dx - 12\int x^2\,dx - \int 12\,dx$$

$$\frac{8x^4}{4} + \frac{8x^2}{2} - \frac{12x^3}{3} - 12x$$

$$\left.2x^4 + 4x^2 - 4x^3 - 12x\right]_{0}^{2} = \left[2(2)^4 + 4(2)^2 - 4(2)^3 - 12(2)\right] -$$

$$[0]$$

▲ $$\int_0^{\pi} (5e^x + 6\sin(x))\, dx$$

$$5\int e^x\,dx + 6\int \sin(x)\,dx = 5e^x + 6(-\cos x)$$

$$5e^x - 6\cos x \Big]_0^{\pi}$$

$$\{5e^{\pi} - 6\cos(\pi)\} - \{5e^{\overset{1}{0}} - 6\,\overset{1}{\cancel{\cos(0)}}\}$$

$$5e^{\pi} - 6\cos[\pi] - 5 + 6$$

$$5e^{\pi} - 6(-1) - 5 + 6$$

$$5e^{\pi} + 6 - 5 + 6$$

$$5e^{\pi} + 12 - 5$$

$$5e^{\pi} + 7$$

▲ $$\int_1^{6} \left(\frac{x}{6} - \frac{2}{x}\right) dx$$

$$\frac{1}{6}\int x\,dx - 2\int \frac{1}{x}\,dx$$

$$\frac{1}{6} \cdot \frac{x^2}{2} - 2\ln x$$

$$\frac{x^2}{12} - 2\ln x \Big]_1^{6} = \left\{\frac{6^2}{12} - 2\ln(6)\right\} - \left\{\frac{1^2}{12} - 2\ln(1)\right\}$$

$$\left\{\frac{36}{12} - 2\ln(6)\right\} - \left\{\frac{1}{12} - 2\ln(1)\right\}$$

$$3 - 2\ln(6) - \frac{1}{12} - 2\cdot 0$$

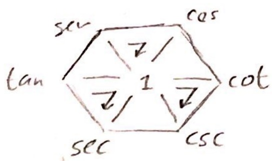
$$3 - 2\ln(6) - \frac{1}{12} = \frac{3 \cdot 12}{12} - \frac{1}{12} - 2\ln(6)$$

$$\frac{3 \cdot 12 - 1}{12} - 2\ln(6)$$

$$\frac{36 - 1}{12} - 2\ln(6)$$

$$\frac{35}{12} - 2\ln(6)$$

$$\int_0^{\pi/4} \frac{4 + 5\cos^2\theta}{\cos^2\theta}\, d\theta$$



$\tan^2 + 1 = \sec^2$

$\sec = \frac{1}{\cos}$ $\qquad \sin^2 + \cos^2 = 1$

$$\int \frac{4}{\cos^2\theta}\, d\theta + \int \frac{\cancel{5\cos^2\theta}}{\cancel{\cos^2\theta}}\, d\theta$$

$$4\int \frac{1}{\cos^2\theta}\, d\theta + 5\int 1\, d\theta$$

$$4\int \frac{1}{\cos\theta}\cdot\frac{1}{\cos\theta}\, d\theta + 5x$$

$$4\int \sec\theta \cdot \sec\theta\, d\theta + 5x$$

$$4\int \sec^2\theta\, d\theta + 5x$$

$$4\tan\theta + 5x\Big]_0^{\pi/4} = \Big\{4\tan(\pi/4) + 5(\pi/4)\Big\} - \Big\{\cancel{4\tan(0)} + \cancel{5(0)}\Big\} = \frac{16+5\pi}{4}$$

$$\int_0^{2\pi/3} \frac{5\sin(\theta) + 5\sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\, d\theta = \int \frac{5\sin\theta\,(1 + \tan^2\theta)}{\sec^2\theta}\, d\theta$$

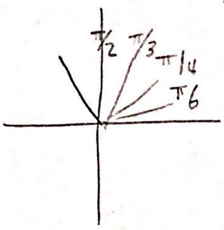
$$= \int \frac{5\sin\theta\,\cancel{(\sec^2\theta)}}{\cancel{\sec^2\theta}}\, d\theta$$

$$= \int 5\sin\theta\, d\theta = 5\int \sin\theta\, d\theta$$

$$-5\cos\theta\Big]_0^{2\pi/3}$$

$$\frac{2\cancel{\pi}}{3}\cdot\frac{180}{\cancel{\pi}} = \frac{2(180)}{3} = \frac{360}{3} = 120$$

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$



$$= \Big\{-5\cos\left(\tfrac{2\pi}{3}\right)\Big\} - \Big\{-5\cos(0)\Big\}$$

$$= -5\cos\left(\tfrac{2\pi}{3}\right) + 5\cancel{\cos(0)}^{\,1}$$

$$= -5\cos\left(\tfrac{\pi}{3} + \tfrac{\pi}{3}\right) + 5$$

$$= -5\left(\tfrac{2\sqrt{3}}{2}\right) + 5$$

$$= +\frac{5}{2} + \frac{5\cdot 2}{2} = \frac{15}{2}$$

$$\blacktriangle \int_0^{\sqrt{3}/2} \frac{dr}{\sqrt{1-r^2}} = \int \frac{1}{\sqrt{1-r^2}}\,dr = \sin^{-1}(r) = \frac{\pi}{3}$$

$$\blacktriangle \int_{-12}^{12} \frac{3e^x}{\sinh(x) + \cosh(x)}\,dx = \int \frac{3e^x}{\sinh x + \cosh x}\,dx = \ldots$$

$$= \int \frac{3e^x}{\underbrace{\left(\frac{e^x - e^{-x}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)}_{\frac{e^x - \cancel{\frac{1}{e^x}} + e^x + \cancel{\frac{1}{e^x}}}{2} = \frac{\cancel{2}e^x}{\cancel{2}}}}\,dx = \int \frac{3\cancel{e^x}}{\cancel{e^x}}\,dx = 3x\Big]_{-12}^{12} = \{3(12)\} - \{3(-12)\}$$

$$36 - \{-36\}$$

$$36 + 36$$

$$72$$

$$\blacktriangle \quad x = 5\sqrt[4]{y} \qquad \int_0^5 5\sqrt[4]{y}\,dy$$

$$x = 5$$

$$\frac{1}{4} + \frac{1 \cdot 4}{4} = \frac{5}{4}$$

$$5\int \frac{(y)^{1/4+1}}{1/4+1}\,dy = \frac{y^{5/4}}{5/4}\Big]_0^5 = \left\{\frac{\left(5^5\right)^{1/4}}{\frac{1}{\frac{5}{4}}}\right\} - \{0\}$$

$$y = 5\sqrt[4]{x}$$

$$\frac{y}{5} = \sqrt[4]{x}$$

$$\left(\frac{y}{5}\right)^4 = x$$

$$\int_0^5 \frac{y^4}{5^4}\,dy = \frac{1}{5^4} \cdot \frac{y^5}{5}\Big]_0^5 = \frac{1}{5^4} \cdot \frac{5^5}{5} = 1$$

$$\frac{4 \cdot \sqrt[4]{5^5}}{5} = 5.9813\ 95125$$

Laboratorio # 3

2019-08/15

① $\int \operatorname{sech}^2(\log_2 x)\,\frac{1}{x}\,dx =$ la derivada está presente. inclinación por sustitución

$$\log_2 x = \frac{\log x}{\log(2)}$$

$$\int \operatorname{sech}^2\left(\frac{\log x}{\log(2)}\right)\frac{1}{x} = \int \operatorname{sech}^2(u)\,\frac{du}{\log(2)}$$

$u = \frac{\log x}{\log(2)}$

$du = \frac{1}{x}\cdot\frac{1}{\log(2)}$

$$= \frac{1}{\log(2)}\int \operatorname{sech}^2(u)\,du$$

$$= \frac{1}{\log(2)}\tanh(u) + C$$

$$= \frac{1}{\log(2)}\cdot\tanh\left(\frac{\log x}{\log(2)}\right) + C$$

② $72\int_1^2 \frac{\ln(x)}{x^4}\,dx = 72\int_1^2 \ln(x)\cdot\frac{1}{x^4}\,dx$

$$= -\ln(x)\,\frac{1}{3x^3} - \underbrace{\int -\frac{1}{3x^3}\,\frac{1}{x}\,dx}$$

$u = \ln(x)$ $\quad dv = x^{-4}\,dx$

$du = \frac{1}{x}$ $\quad v = \frac{x^{-3}}{-3}$

$$-\int_1^2 -\frac{1}{3}\cdot x^{-3}\cdot x^{-1}\,dx$$

$$-1 \cdot -\frac{1}{3}\int_1^2 x^{-4}\,dx$$

$$+\frac{1}{3}\cdot\frac{x^{-3}}{-3}$$

$$-\frac{1}{3}\cdot\frac{1}{3}\cdot x^{-3}$$

$$-\frac{1}{9x^3}$$

$$72\left(-\frac{\ln(x)}{3x^3} - \frac{1}{9x^3}\right)\Bigg|_1^2$$

$$\left\{-\frac{72\ln(2)}{3(2)^3} - \frac{72}{9(2)^3}\right\} - \left\{-\frac{72\ln(1)}{3(1)^3} - \frac{72}{9(1)^3}\right\}$$

$$-\frac{24\ln(2)}{8} - \frac{72}{72} - \left(-\frac{24\overset{0}{\cancel{(0)}}}{\cancel{3(1)}} - 8\right)$$

$$-3\ln(2) - 1 + 0 + 8$$

$$-3\ln(2) + 7$$

③ $\int \sec^2\theta \tan^3\theta \, d\theta =$

$(\tan^3\theta)' = 3\tan^2\theta \cdot \sec^2\theta$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$= \int u^3 \, du = \frac{u^4}{4} + C$

$= \frac{\tan^4(\theta)}{4} + C$ //

④ $\int (x-1) \sin(\pi x) \, dx =$

$u = x - 1 \qquad dv = \sin(\pi x) \, dx$

$du = 1 \, dx \qquad v = -\frac{\cos(\pi x)}{\pi}$

$uv - \int v \, du$

$-\frac{(x-1)\cos(\pi x)}{\pi} - \underbrace{\int \frac{-\cos(\pi x)}{\pi} dx}_{\text{sustitución}}$

$-\frac{1}{\pi}\int -\cos(\pi x) = \left(-\frac{1}{\pi} \cdot -1\right) \int \cos(w) \, dw$

$w = \pi x$

$dw = \pi \, dx \qquad = \frac{1}{\pi} \frac{\sin(w)}{\pi} = \frac{1}{\pi^2} \sin(\pi x)$

$\frac{dw}{\pi} = dx$

$-\frac{(x-1)\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C$ //

⑤ $$\int e^{-\theta}\cos(2\theta)\,d\theta =$$

$\cos(2u) = 1 - \sin^2 u$

$u = \cos(2\theta)$ $\quad dv = e^{-\theta}\,d\theta$

$du = -2\sin(2\theta)\,d\theta$ $\quad v = -e^{-\theta}$

$$\underbrace{\ }_{u}\underbrace{\ }_{v} - \int \underbrace{\ }_{v}\underbrace{\ }_{du}$$

$$\underbrace{-\cos(2\theta)\,e^{-\theta} - \int -e^{-\theta}\cdot -2\sin(2\theta)}$$

$$-\frac{\cos(2\theta)}{e^{\theta}} - \underbrace{\int 2e^{-\theta}\sin(2\theta)\,d\theta}$$

$$\equiv 2\int e^{-\theta}\sin(2\theta)\,d\theta$$

$u = \sin(2\theta)$ $\quad dv = e^{-\theta}\,d\theta$

$du = 2\cos(2\theta)\,d\theta$ $\quad v = -e^{-\theta}$

$$\underbrace{\sin(2\theta)}_{u}\cdot\underbrace{-e^{-\theta}}_{v} - \int \underbrace{-e^{-\theta}}_{v}\cdot\underbrace{2\cos(2\theta)\,d\theta}_{du}$$

$$2\left\{-\frac{\sin(2\theta)}{e^{\theta}} - \int -2e^{-\theta}\cos(2\theta)\,d\theta\right\}$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} - 2\cdot -2\int e^{-\theta}\cos(2\theta)\,d\theta$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} + 4\int e^{-\theta}\cos(2\theta)\,d\theta$$

$$w = \int e^{-\theta}\cos(2\theta)\,d\theta$$

variable cíclica

$$w = -\frac{\cos(2\theta)}{e^{\theta}} - \left(-\frac{2\sin(2\theta)}{e^{\theta}} + 4w\right)$$

$$w = -\frac{\cos(2\theta)}{e^{\theta}} + \frac{2\sin(2\theta)}{e^{\theta}} - 4w$$

$$w + 4w = -\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}$$

$$w = \frac{-\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}}{5} + C$$

6 $$\int e^{\tan(x+\tan^{-1}x)} \cdot \sec^2(x+\tan^{-1}(x))\left(1+\frac{1}{1+x^2}\right)dx =$$

sustitución

$u = \tan(x+\tan^{-1}(x))$

$du = \sec^2(x+\tan^{-1}(x)) \cdot \left(1+\frac{1}{1+x^2}\right)$

$$= \int e^u \, du$$

$$= e^u + C$$

$$= e^{\tan(x+\tan^{-1}(x))} + C$$

8) $$\int 5^t \sin(5^t)\, dt = \frac{1}{\ln(5)} \cdot \int \sin(u)\, du = \frac{1}{\ln(5)} \Big[-\cos(u) \Big] = -\frac{\cos(5^t)}{\ln(5)} + C$$

$u = 5^t$

$du = 5^t \cdot \ln(5)\, dt$

$\frac{du}{\ln(5)} = 5^t\, dt$

9) $$\int_0^2 x \sin \pi x\, dx = -\frac{x \cdot \cos(\pi x)}{\pi} - \int -\frac{\cos(\pi x)}{\pi}\, dx = -\frac{x\cos(\pi x)}{\pi} - \frac{1}{\pi}\left[\int -\cos(\pi x)\, dx \right]$$

$u = x$ $\quad dv = \sin(\pi x)\, dx$

$du = 1\, dx$ $\quad v = -\frac{\cos(\pi x)}{\pi}$

$u = \pi x$

$du = \pi\, dx$

$\frac{du}{\pi} = dx$

$= \int -\cos(u) \frac{du}{\pi}$

$= -\sin(u)$

$= \frac{-\sin(\pi x)}{\pi}$

$$\frac{-x\cos(\pi x)}{\pi} - \frac{1}{\pi}\left(-\frac{\sin(\pi x)}{\pi} \right) + C$$

$$\left. \frac{-x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C \right]_0^2$$

$$\left\{ -\frac{2\cos(\pi 2)}{\pi} + \frac{\sin(\pi 2)}{\pi^2} \right\} - \left\{ \cancel{\frac{-0\cos(\pi(0))}{\pi}} + \cancel{\frac{\sin(\pi 0)}{\pi^2}} \right\}$$

0

$$\left\{ -\frac{2(1)}{\pi} \right\} = -\frac{2}{\pi}$$

(1)

$$\int_{-13}^{+13} \frac{4x}{\sqrt[3]{(1+2x^2)^2}}\,dx = \int_{-13}^{+13} \frac{du}{\sqrt[3]{(u)^2}} = \int_{-13}^{13} u^{-\frac{2}{3}}\,du$$

sustitución

$u = 1 + 2x^2$

$du = 0 + 4x\,dx$

$du = 4x\,dx$

$$\frac{u^{-\frac{2}{3}+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}} \qquad -\frac{2}{3}+\frac{3}{3} = \frac{-2+3}{3} = \frac{1}{3}$$

$$\frac{u^{\frac{1}{3}}}{1/3} = \left[\frac{\frac{u^{1/3}}{1}}{\frac{1}{3}}\right] = \frac{3u^{1/3}}{1} = 3u^{1/3}$$

$$f(-x) = \frac{4(-x)}{\sqrt[3]{(1+2(-x)^2}}$$

$$= -\frac{4x}{\sqrt[3]{(1+2x^2)}}$$

impar

$$3u^{1/3} = 3(1+2x^2)^{1/3} + C\Big]_{-13}^{13}$$

$$= \left\{3(1+2(13)^2)^{1/3}\right\} - \left\{3(1+2(-13)^2)^{1/3}\right\}$$

$$= 3\sqrt[3]{1+2(13)^2} - 3\sqrt{1+2(-13)^2}$$

$$= 3\sqrt[3]{339} - 3\sqrt{339}$$

$$= \emptyset$$

(10) $$\int \frac{e^{y/(y+1)}}{(y+1)^2}\,dy = \int \frac{e^{\frac{y}{(y+1)}}}{(y+1)^2}\,dy =$$

$$\frac{d}{dx}\left(\frac{y}{y+1}\right) = \frac{1\cdot(y+1) - y(1)}{(y+1)^2} = \frac{\cancel{y}+1-\cancel{y}}{(y+1)^2} = \frac{1}{(y+1)^2}$$

$$\frac{f'g - fg'}{(g)^2}$$

$$u = \frac{y}{y+1}$$

$$du = \frac{1}{(y+1)^2}\,dy$$

$$= \int e^{u}\,du = e^{u} = \underline{e^{y/(y+1)} + C}$$

11 $\displaystyle\int \frac{xe^{2x}}{(1+2x)^2}\,dx = \int xe^{2x}(1+2x)^{-2}$

$u = xe^{2x}$ $\qquad dv = (1+2x)^{-2}$

$du = e^{2x} + xe^{2x}\cdot 2\,dx \quad v = -\dfrac{1}{2+4x}$

$\displaystyle\int \frac{1}{(1+2x)^2}\,dx$

$w = 1+2x$

$dw = 2\,dx$

$\dfrac{dw}{2} = dx$

$\displaystyle = \int w^{-2}\frac{dw}{2}$

$= \dfrac{w^{-1}}{-1}\cdot\dfrac{1}{2}$

$= \dfrac{1}{-2(1+2x)}$

$= \dfrac{1}{-2-4x}$

$\underbrace{xe^{2x}}_{u}\cdot\underbrace{-\frac{1}{2+4x}}_{v} - \underbrace{\boxed{\int -\frac{1}{2+4x}\cdot e^{2x} + xe^{2x}\cdot 2\,dx}}_{-\frac{e^{2x}}{4}}$

$\displaystyle -\int \frac{e^{2x} + xe^{2x}\cdot 2}{2+4x}\,dx$

$\displaystyle \frac{e^{2x}(\cancel{1+2x})}{2(\cancel{1+2x})}\,dx$

$\displaystyle \frac{e^{2x}}{2}\,dx \longrightarrow \frac{e^{2x}}{4}$

$-\dfrac{xe^{2x}}{2+4x} + \dfrac{e^{2x}}{4} + C$

CORTO #6 Cálculo Integral (20 min)

Nombre:________________________________ **Carnet:** ____________

1. Considera la región limitada por las curvas $y_1 = x^3$ y $y_2 = 4x$.

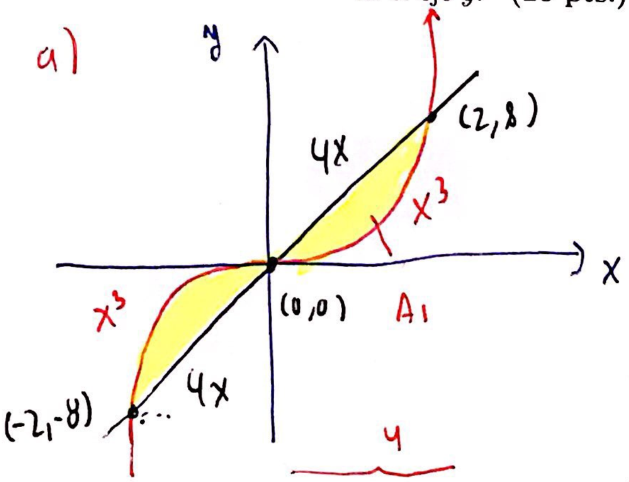
   a) Dibuja las regiones acotadas por cada una de las curvas dadas. **(40 pts.)**

   b) Plantea las integrales para encontrar el área de la región. **(40 pts.)**

   c) Evalúa la integral y encuentra el área de la región. **(20 pts.)**

   d) **BONO:** Plantea las integrales para encontrar el área de la región integrando en el eje-y. **(20 pts.)**

a)



$y_1 = y_2$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0, \pm 2.$

$(0,0) \quad (2,8) \quad (-2,8)$

b) $$A = \underbrace{\int_{-2}^{0} (x^3 - 4x)\,dx}_{4} + \underbrace{\int_{0}^{2} (4x - x^3)\,dx}_{4} = 2\int_0^2 (4x - x^3)\,dx$$

c) $$A = 2\left(2x^2 - \frac{1}{4}x^4 \Big]_0^2 \right) = 2\left(8 - \frac{16}{4}\right) = 2(4) = 8.$$

d) 1ra curva $y = x^3 \Rightarrow \cancel{x} = y^{1/3}$ $\quad -8 \le y \le 8.$

2da curva $y = 4x \Rightarrow x = y/4$

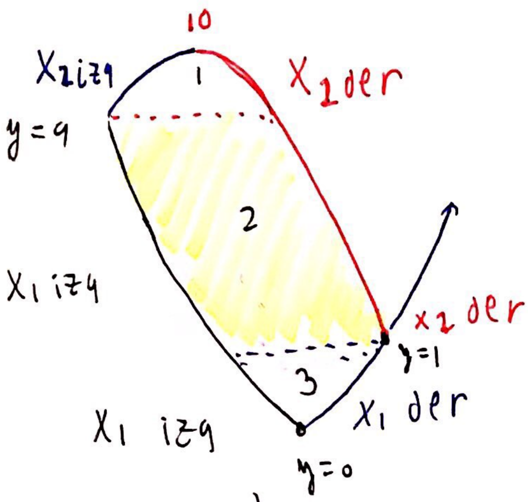
$8^{4/3} = (2^3)^{4/3}$

$2^4 = 16.$

$$A = \int_{-8}^{0} \left(\frac{y}{4} - y^{1/3}\right) dy + \int_0^8 \left(y^{1/3} - \frac{1}{4}y\right) dy = 2\int_0^8 \left(y^{1/3} - \frac{y}{4}\right) dy$$

$$A = 2\left(\frac{3}{4}y^{4/3} - \frac{y^2}{8}\Big]_0^8\right) = 2\left(\frac{3\cdot 16}{4} - \frac{64}{8}\right) = 2(12 - 8) = 8$$

d) (30 pts.).



10 pts.

$$A = \int_0^1 x_1 - x_2 \, dy + \int_1^9 x_2 - x_1 \, dy + \int_9^{10} x_1 - x_2 \, dy.$$

$$A = \int_0^1 2+\sqrt{y} - (2-\sqrt{y}) \, dy + \int_1^9 \sqrt{10-y} - (2-\sqrt{y}) \, dy + \int_9^{10} \sqrt{10-y} - (-\sqrt{10-y}) \, dy.$$

+10 pts. $$A = \int_0^1 2\sqrt{y} \, dy + \int_1^9 \sqrt{10-y} - 2 + \sqrt{y} \, dy + \int_9^{10} 2\sqrt{10-y} \, dy.$$

2019-09-17

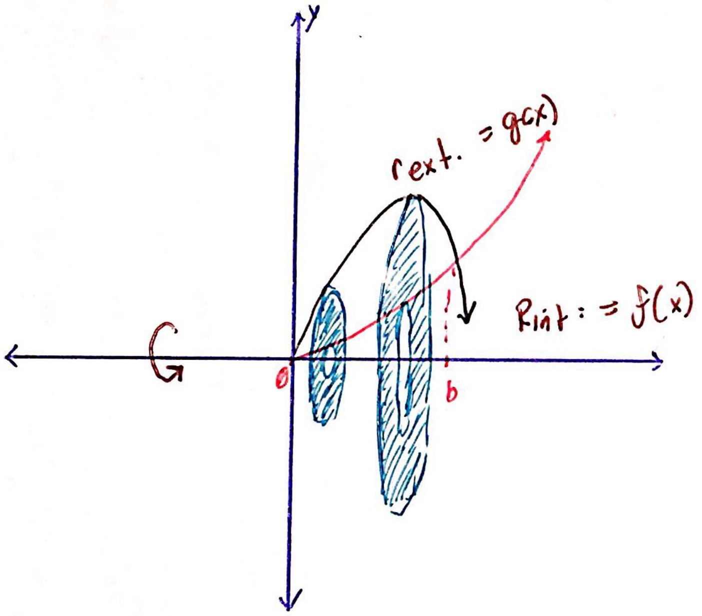
Continuación de Volúmenes

① Región

② Identificar la recta de rotación

③ Identificar las funciones de radio para cada anillo

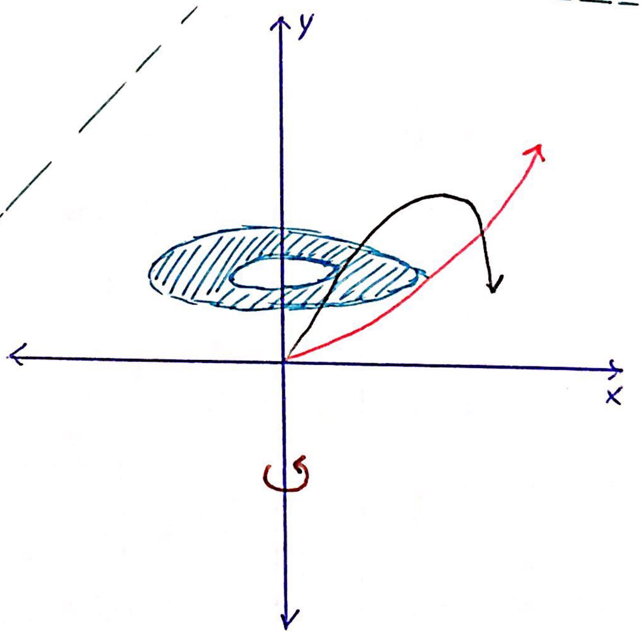
④ Escoger la variable de integración.



El reflejo en eje-x

$$A = \pi r_{ext.}^2 - \pi r_{int.}^2$$

$$A = \pi \int_0^b r^2_{ext} - r^2_{int} \, dx$$



El reflejo en eje-y

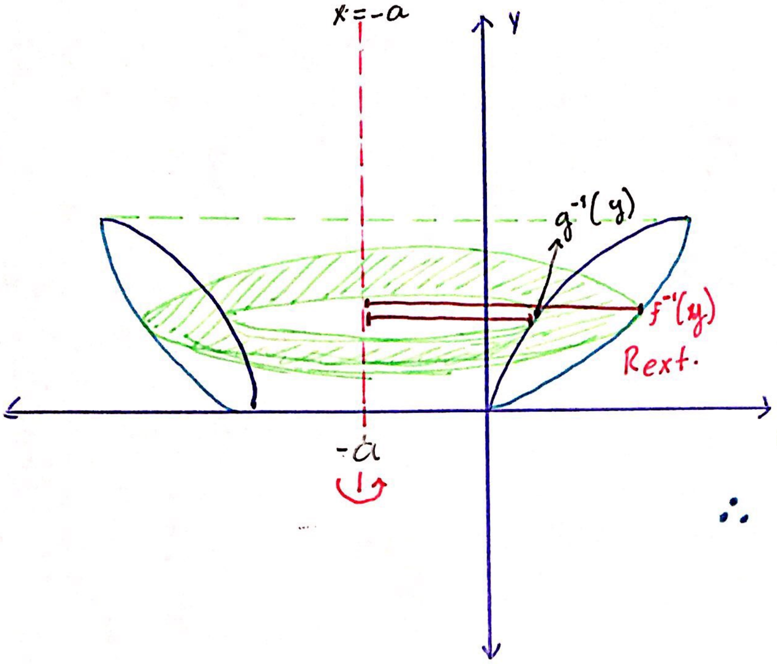
$0 \leq y \leq y_{máximo}$

$R_{int.} = x = g^{-1}(y)$

$R_{ext} = x = f^{-1}(y)$

$$\text{Volumen} = \pi \int_0^{y_{max}} r^2_{max.} - r^2_{int.} \, dy = \pi \int_0^{y_{máx}} (f^{-1}(y))^2 - (g^{-1}(y))^2 \, dy$$

Rectas de rotación



$R_{int} = a + g^{-1}(y)$

$R_{ext} = a + f^{-1}(y)$

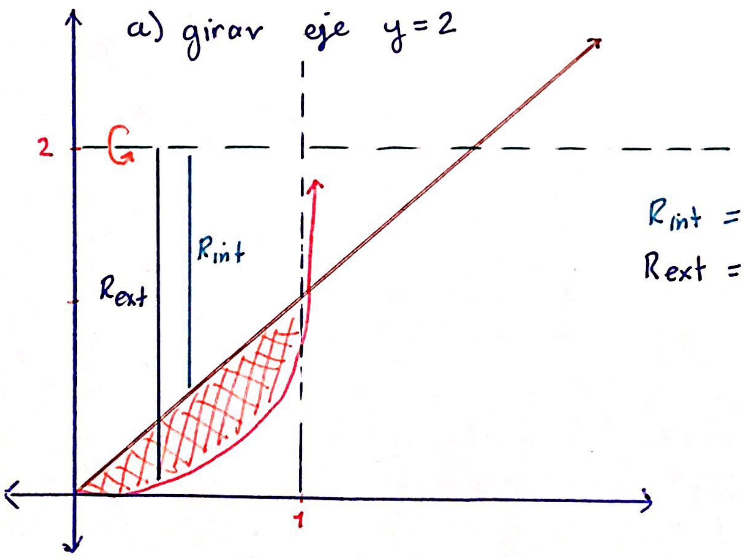
$0 \leq y \leq y_{máx}.$

$$\therefore \; V = \pi \int_0^{y_{max}} (R_{ext.})^2 - (R_{int})^2 \, dy$$

$$\boxed{= \pi \int_0^{y_{max}} (a + f^{-1}(y))^2 - (a + g^{-1}(y)) \, dy}$$

Ej: Considere la región entre $f(x) = x$ & $g(x) = x^3$ en el 1er cuadrante.

a) girar eje $y = 2$



$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0 \quad x = \pm\sqrt{1}$

$x = 0, \; x = 1$

$R_{int} = 2 - x$

$R_{ext} = 2 - x^3$

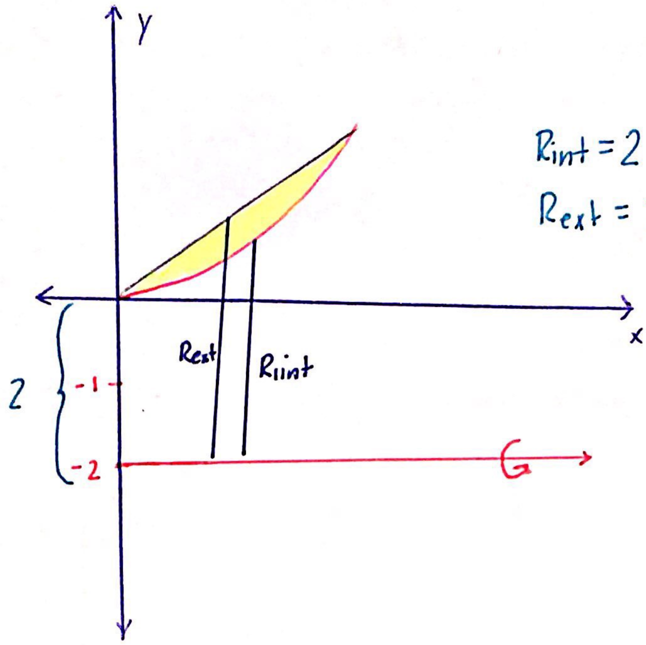
$$A = \pi \left[R_{ext}^2 - R_{int}^2 \right]$$

$$V = \pi \int_0^1 (2 - x^3)^2 - (2 - x)^2 \, dx$$

$$= \pi \int_0^1 x^6 - 4x^3 - x^2 + 4x \, dx$$

$$= \frac{17\pi}{21}$$

b) Plantee la integral de volúmen del sólido que se obtiene al girar la región respecto a $y = -2$



$R_{int} = 2 + x^3$

$R_{ext} = 2 + x$

$$A = (R_{ext})^2 - (R_{int})^2$$

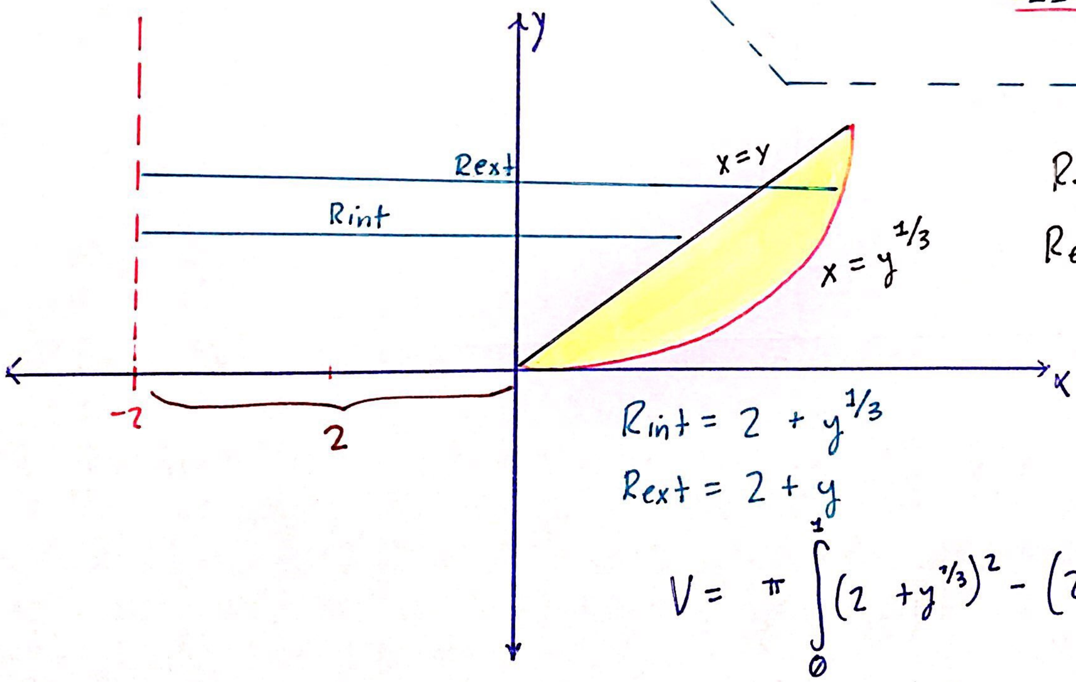
$$V = \pi \int_0^1 R_{ext}^2 - R_{int}^2 \, dx$$

$$V = \pi \int_0^1 2 + x - 2 - x^3 \, dx$$

$$= \pi \int_0^1 x - x^3 \, dx$$

$$= \pi \frac{1}{2} x^2 - \pi \frac{1}{4} x^4 = \frac{32\pi}{21}$$

ⓒ Rote la región respecto a $x = -2$



$R_{int} = 2 + y_1^{-1}$

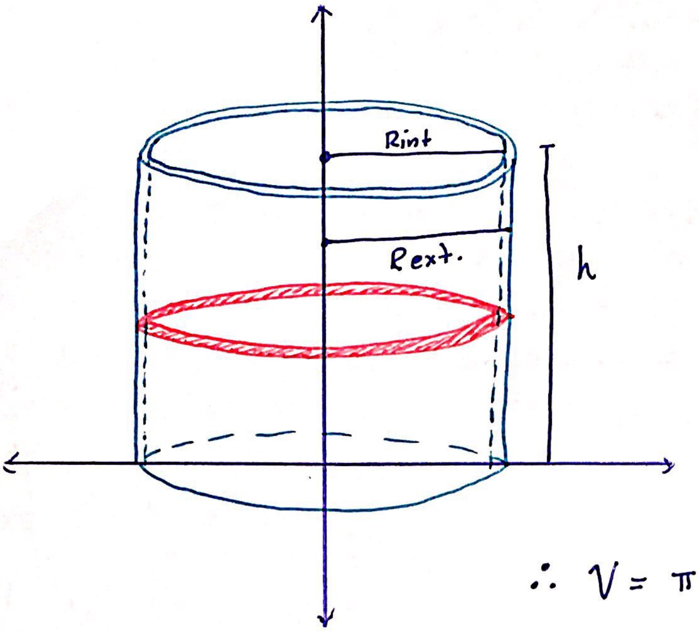
$R_{ext} = 2 + y_2^{-1}$

$R_{int} = 2 + y^{1/3}$

$R_{ext} = 2 + y$

$$V = \pi \int_0^1 (2 + y^{1/3})^2 - (2 + y)^2 \, dy$$

6.3. Volúmenes con un cascarón cilíndrico (latas)



Área anillo $=$

$$\pi r_{ext}^2 - \pi r_{int}^2$$

Volúmen $=$

$$\pi h \left(R_{ext}^2 - R_{int}^2 \right)$$

Grosor $= \Delta r = r_{ext} - r_{int} = dr$

$$\therefore V = \pi h \left(R_{ext} + R_{int} \right) \left(R_{ext} - R_{int} \right) =$$

- Volúmen de la expresión $= \pi h r^2$

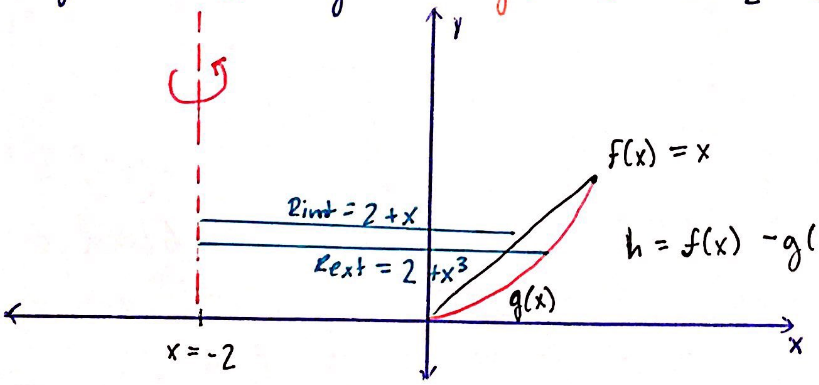
- Derive Respecto a r $\quad dv = 2\pi h\, r\, dr$

$$\boxed{V = 2\pi \int_a^b h\, r\, dr}$$

el volumen de cascarones cilíndricos.

Redo inciso c.

$y = x^3$ & $y = x$ gire a $x = -2$.



$h = f(x) - g(x) = x - x^3$

$h = x - x^3 \quad ; \quad r = 2 - x \qquad 0 \leq x \leq 1$

$$V = 2\pi \int_0^1 (x - x^3)(2 + x)\, dx$$

Ej: encuentre el volumen del sólido que se obtiene al girar la región entre el eje-x y la curva $f(x) = 2x^2 - x^3$ en el 1er cuadrante respecto al eje-y.

Interceptos -x $\quad 2x^2 - x^3 = 0$

$$x^2(2 - x) = 0$$

$$x = 0 \qquad x = 2$$

$$4x - 3x^2 = 0$$

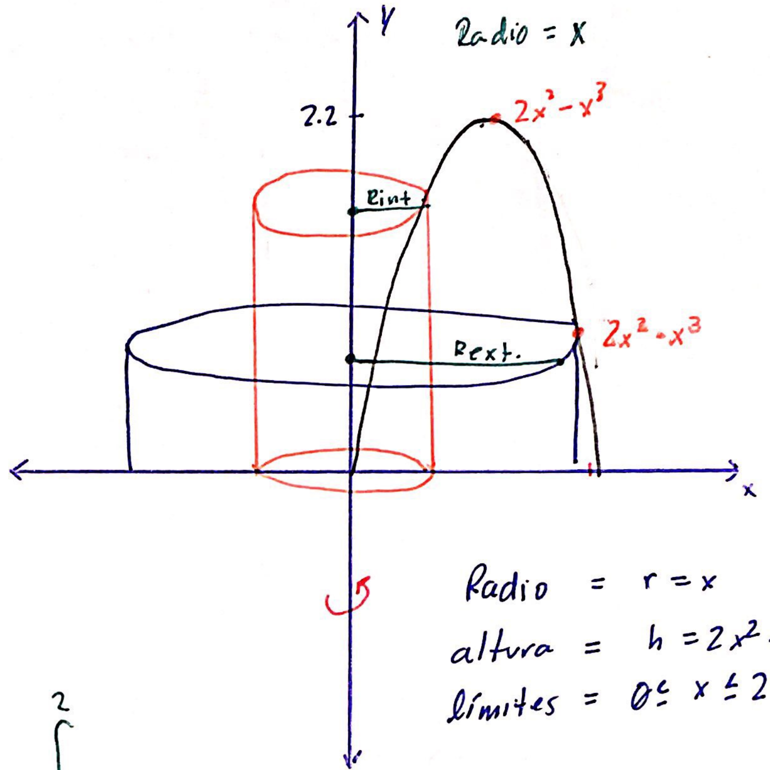
$$x(4 - 3x) = 0$$

$$x = 0 \qquad 4 - 3x = 0$$

$$-3x = -4$$

$$x = \frac{4}{3}$$

$$f(4/3) = 2(16/9) - (4/3)$$



Radio $= r = x$

altura $= h = 2x^2 - x^3$

límites $= 0 \leq x \leq 2$

$$V = 2\pi \int_0^2 hr\,dx =$$

$$= 2\pi \int_0^2 2x^3 - x^4\,dx = 2\pi\left(8 - \frac{32}{5}\right)$$

[!] Si estan rotando con un eje horizontal es recomendable usar anillos

$$V = \int_a^b \pi\left(r_{ext}^2 - r_{int}^2\right)dx$$

eje-x

$y = 0$

$y =$ contante

[!] Si está rotando un eje vertical usar cilíndros:

$$V = 2\pi \int_a^b hr\,dx$$

Ej: encuentre el volumen del sólido obtenido al girar la región entre $y_1 = x^2$ & $y_2 = 6x - 2x^2$ alrededor del eje-y.

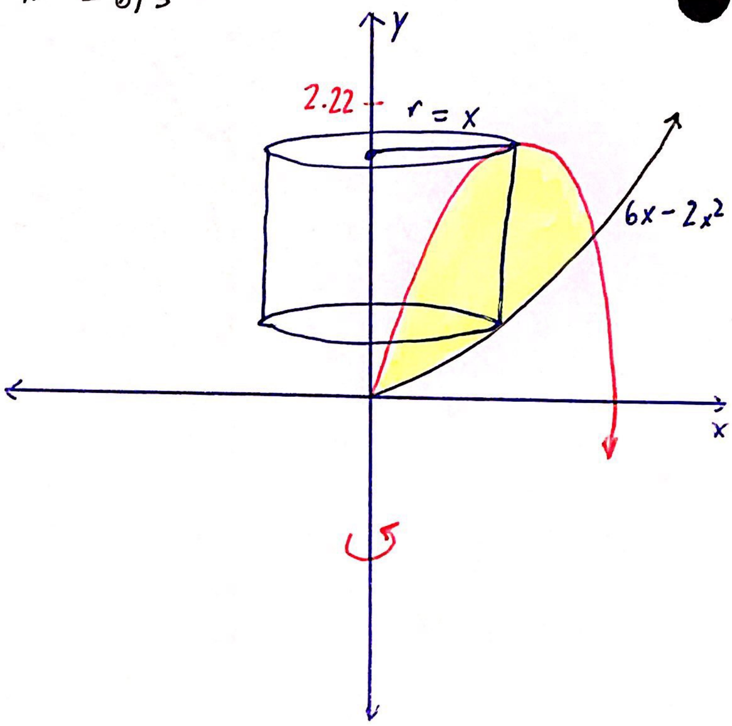
$y_2 = 0 \qquad 2x(3 - x) = 0 \Rightarrow x = 0, 3$

$y_1 = y_2 \qquad x^2 = 6x - 2x^2$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0 \qquad x = 2$



altura $\Rightarrow h = y_2 - y_1$

$h = 6x - 3x^2$

Radio $\Rightarrow r = x$

Límites $\Rightarrow 0 \leq x \leq 2$

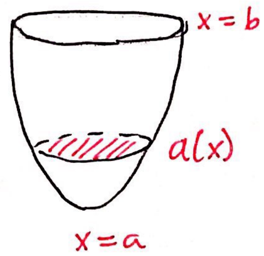
$$V = 2\pi \int_0^2 h\,r\,dx$$

$$V = 2\pi \int_0^2 6x^2 - 3x^3\,dx = 2\pi \left(2x^3 - \frac{3}{4}x^4 \Big]_0^2 \right) = 8\pi$$

Volúmenes

en revolución

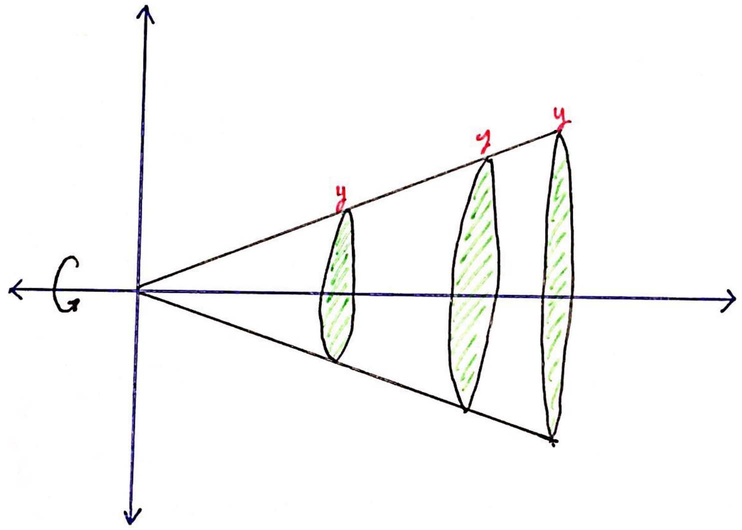
2019-09-12



$$V = \int_a^b A(x)\, dx$$

área de la sección transversal.

Ejemplo: Encuentre el volumen de una curva de altura H y base circular de radio R



Las seccions transversales son circulares del radio

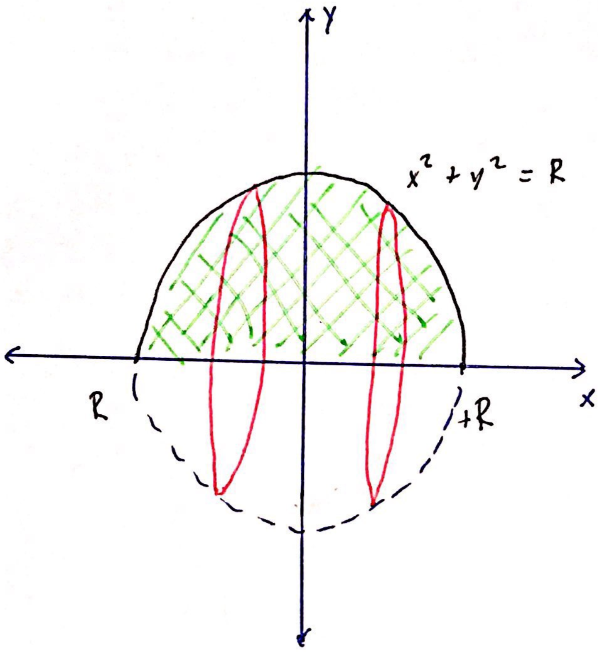
$y(x)$

$$A = \pi y^2$$

$$V = \int_0^4 \pi y^2\, dx$$

Ejercicio 1: pg. 89 volúmen de una esfera

La esfera se obtiene al girar el círculo $x^2 + y^2 \leq R^2$ respecto al eje -x:



dominio =

$y^2 = R^2 - x^2$ $\quad ID \to -R \leq \ \leq R^2$

$y = \sqrt{R^2 - x^2}$

$\underbrace{x^2 = R^2}_{0}$ $\quad x = \pm R$

Sección transversal círculo de radio y

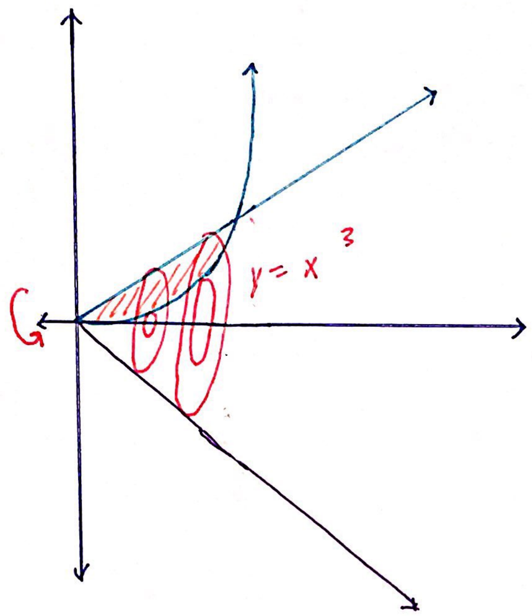
$f(x) = \pi y^2$

$$V = \pi \int_{-R}^{R} A(x)\,dx = 2\pi \int_{0}^{R} (R^2 - x^2) =$$

$$V = 2\pi \left(R^2 x - \frac{x^3}{3} \Big]_0^R \right) = 2\pi \left(R^3 - \frac{R^3}{3} \right) =$$

$$= 2\pi R^3 \left(1 - \frac{1}{3} \right) = \frac{4\pi}{3} R^3$$

Encuentre el volúmen del sólido obtenido al girar la región entre las curvas $y = x$ & $y = x^3$ en el cuadrante respecto al eje-x.



V = Vexterna − Vinterna

Área Anillo

$r_{ext} = x$ $\quad r_{int} = x^3$

$$A = \pi r_{ext.}^2 - \pi r_{int.}^2$$

$$A = \pi x^2 - \pi x^6$$

$$\text{Volumen} \quad V = \int_0^1 A\,dx = \int_0^1 (\pi x^2 - \pi x^6)\,dx$$

$$V = \frac{\pi}{3}x^3 - \frac{\pi}{7}x^7 \Big]_0^1 =$$

$$= \left\{ \frac{\pi}{3} - \frac{\pi}{7} \right\} - \{0\}$$

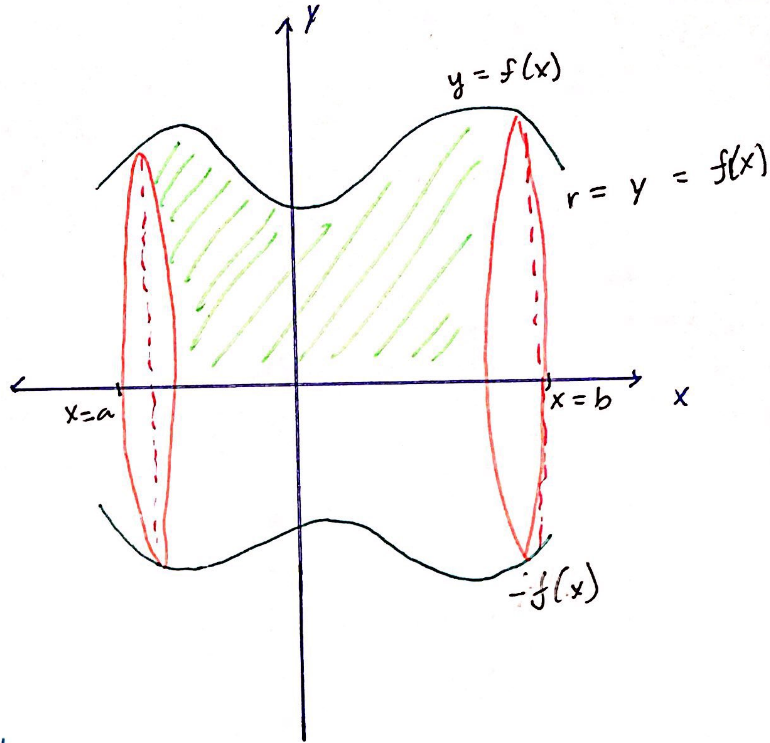
$$= \frac{7\pi - 3\pi}{3 \cdot 7} = \frac{4\pi}{21} \quad \square$$

Sólidos en revolución

$R: a \leq x \leq b$

$0 \leq y \leq f(x)$

- gire respecto a x

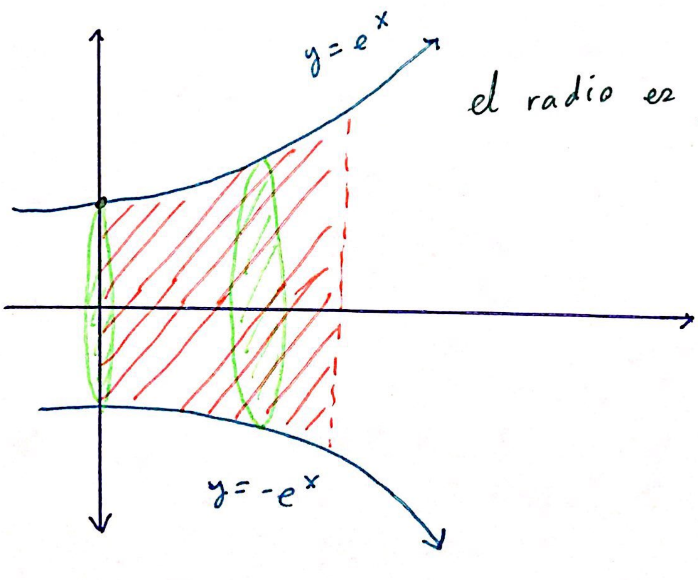


Área transversal:

$A = \pi y^2$

$$V = \int_a^b \pi y^2 \, dx = \int_a^b \pi f^2(x) \, dx$$

Ej 4: encuentra el volumen del sólido que se obtiene al girar la región $R: 0 \leq x \leq \ln(3)$; $0 \leq y \leq e^x$ respecto al eje-x Pg 93.



el radio es y

$A = \pi r^2$

$A = \pi y^2$

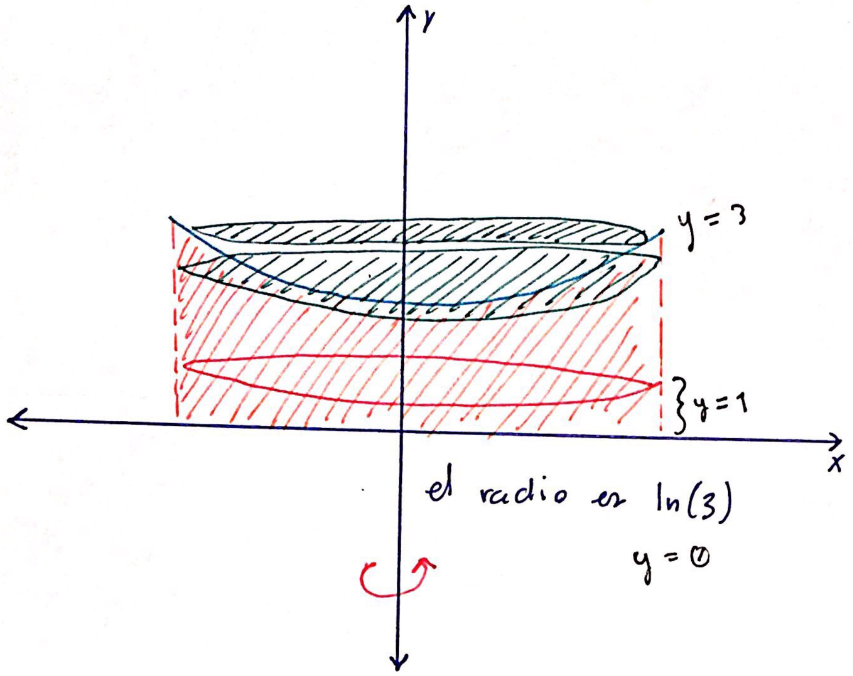
$$\therefore \int A = \int_0^{\ln(3)} \pi e^{2x} \, dx$$

$$V = \pi \int_0^{\ln(3)} e^{2x} \, dx = \frac{\pi}{2} e^{2x} \Big]_0^{\ln(3)}$$

$$= \left\{ \frac{\pi}{2} e^{2\ln(3)} \right\} - \left\{ \frac{\pi}{2} \cancel{e^{2(0)}}^{1} \right\}$$

$$= \frac{\pi}{2} 3^2 - \frac{\pi}{2} = \frac{\pi}{2}(9-1) \quad \square$$

Girando la misma región respecto al eje -x al eje -y.

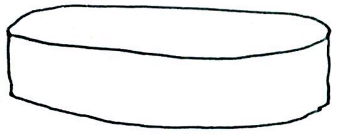


$f(0) = 1$
$f(\ln(3)) = 3$

$V = V_1 - V_2$

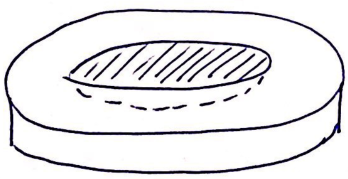
Por casos:

caso 1 el cilíndro

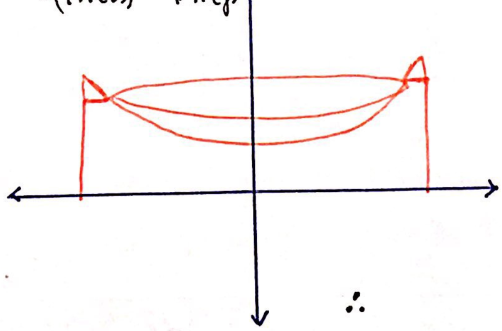


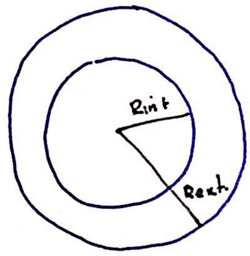
$v_1 =$ cilíndro
$v_1 = \pi r^2 h = \pi \ln^2(3)$

caso 2 : Sólido hueco



$A = \pi r^2_{ext.} - \pi r^2_{int.}$
$A = \pi(\ln(3))^2 - \pi \ln(y)^2$





∴

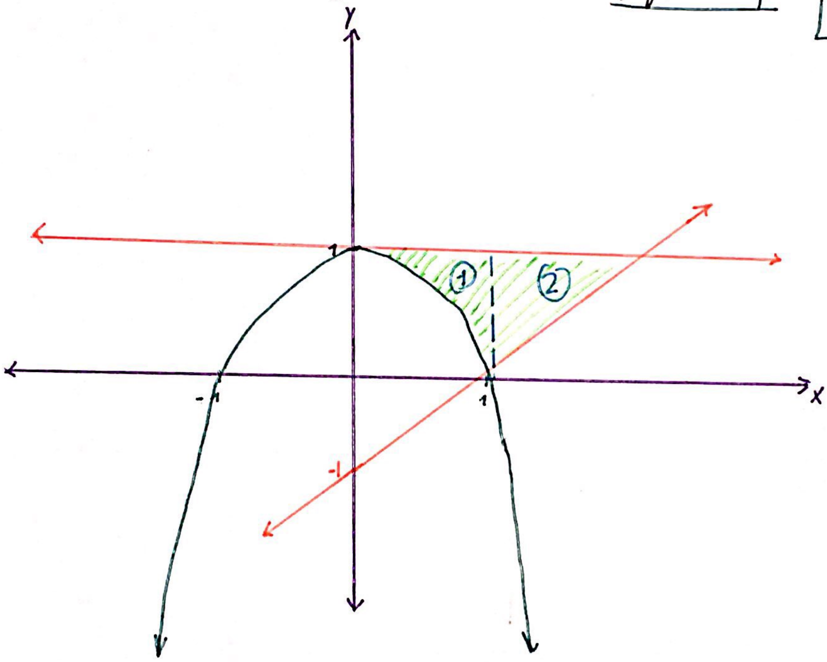
$$V_2 = \int_1^3 \pi \ln(3)^2 - \pi \ln(y)^2 \, dy$$

2019-09-16

Aplicación de la integraciones

- Planteamiento
- Gráfica de una región → área → volúmen sólidos en revolución

Ej: Encuentre la región entre $\boxed{y_1 = 1}$ $\boxed{y_2 = 1 - x^2}$ & $\boxed{y_3 = x - 1}$



$y_1 = y_2$ $(0, 1)$

$y_1 = y_3$

$1 = x - 1$

$x = 2$ $(2, 1)$

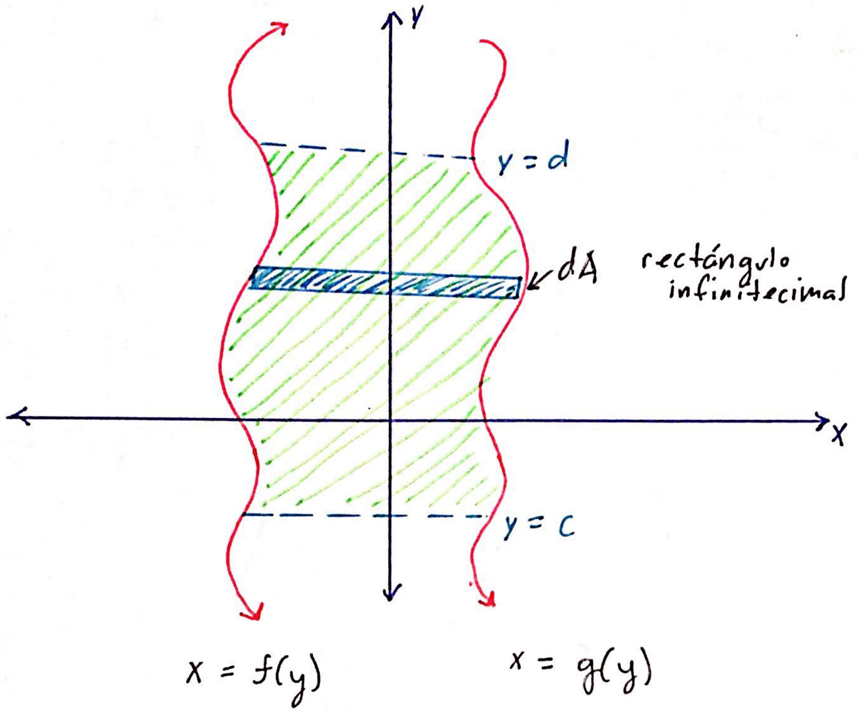
$$A = \int_0^1 1 - (1 - x^2)\, dx + \int_1^2 1 - (x + 1)\, dx$$

$$= \int_0^1 0 + x^2\, dx + \int_1^2 2 - x\, dx$$

$$= \left.\frac{x^3}{3}\right]_0^1 + \left. 2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{3} + 4 - \frac{4}{2} - \left(2 - \frac{1}{2}\right) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

Integración en el eje-y: Franjas horizontales derecha - izquierda



Derecha - izquierda

Región S: $f(y) \le x \le g(y)$

$c \le x \le d$

altura = dy

ancho = $g(y) - f(y)$

$dA = [g(y) - f(y)]\,dy$

$$A = \int_c^d [(g(y) - f(y)]\,dy$$

$$A = \int_c^d x_{der.} - x_{izq.}\,dy$$

$x_{der.} = g(y)$

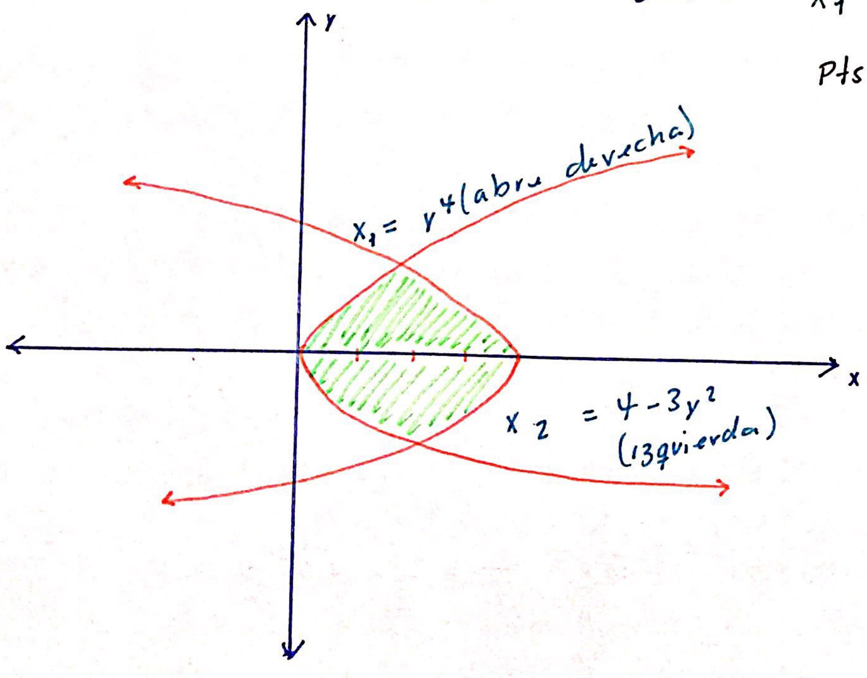
$x_{izq.} = f(y)$

$$A = \int_a^b y_{arriba} - y_{abajo}\,dx$$

$y_{arriba} = f(x)$

$y_{abajo} = g(x)$

Ejercicio: encuentre el área entre $x_1 = y^4$ & $x_2 = 4 - 3y^2$ &



Pts. Intersección $x_1 = x_2$

$y^4 = 4 - 3y^2$

$y^4 + 3y^2 - 4 = 0$

$(y^2 + 4)(y^2 - 1)$

$y = \sqrt{-4}$ imaginario

$y = \sqrt{1}$

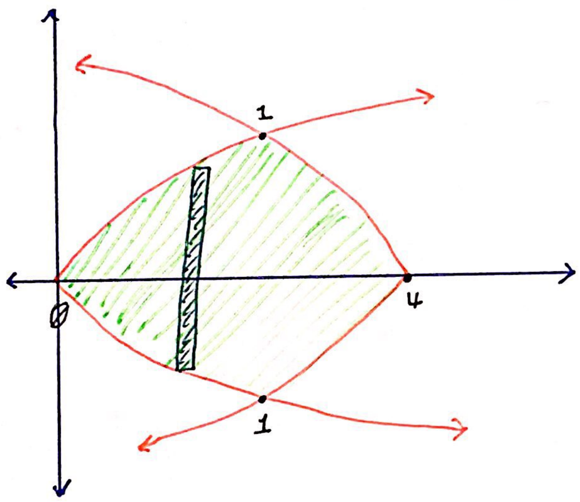
$y = \pm 1$

$$A = \int_{-1}^{1} X_{derecha} - X_{izquierda}\, dy =$$

$$= 2\int_{0}^{1} 4 - 3y^2 - y^4\, dy = 2\left(4y - y^3 - \frac{y^5}{5}\Big]_0^1\right)$$

$$= 2\left(3 - \frac{1}{5}\right) = 2\left(\frac{15}{5} - \frac{1}{5}\right) = \frac{28}{5}$$

ahora lo mismo pero integrando respecto a eje-x.



$$\int f(x) - g(x)\, dx$$

$x = y^4 \qquad x = 4 - 3y^2$

¡Resolver para x!

$a^2 = b \Rightarrow a = \pm\sqrt{b}$

$$2\int_{0}^{1} f - g\, dx + \int_{1}^{4} h - i\, dx$$

Sacar la inversa: $x = y^4$ & $x = 4 - 3y^2$

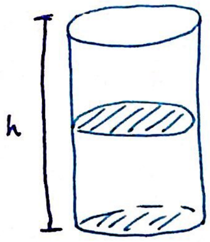
$y = \pm\sqrt[4]{x}$

$x = 4 - 3y^2 \Rightarrow x - 4 = 3y^2$

$\pm\sqrt{\frac{x-4}{3}} = y$

$$A = 2\int_{0}^{1} x^{1/4}\, dx + 2\int_{1}^{4} \left(\frac{4-x}{3}\right)^{1/2} dx = \frac{28}{5}$$

Volúmenes



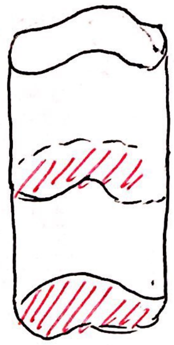
(rebanado)

Sección transversal

$A = \pi r^2$

$V = Ah$

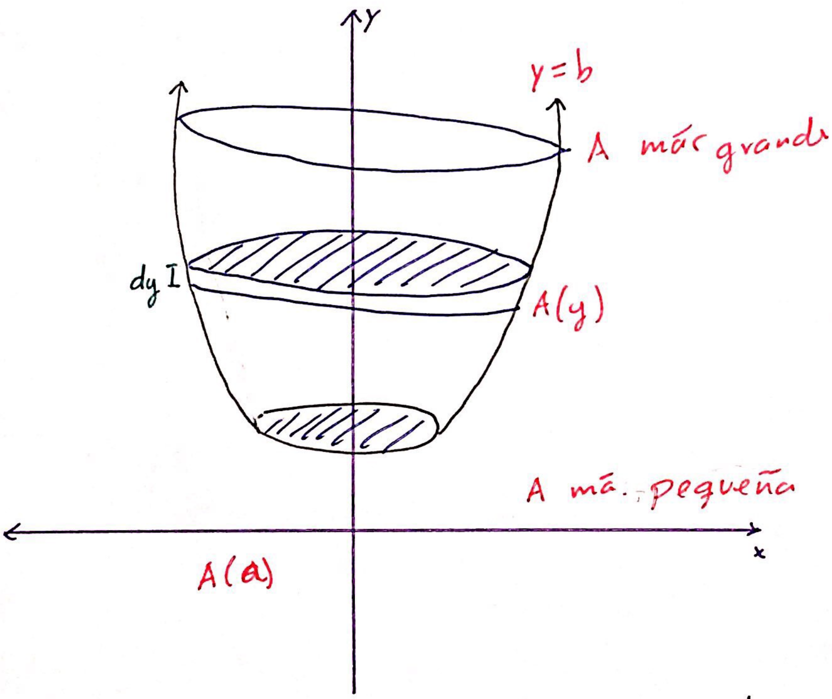
$V = \pi r^2 h$



Área sección transversal

$V = Ah$

Volúmen de un sólido S.



Parte infinitecimal de este sólido

cilíndro

Área transversal $A(y)$

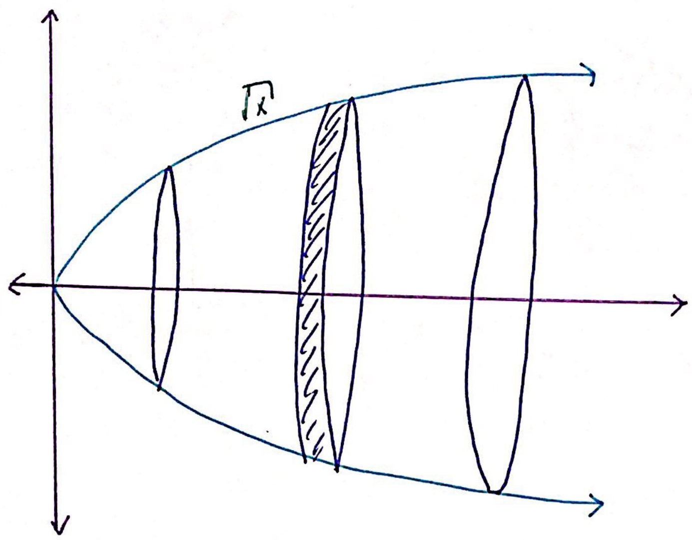
Altura = dy

$dv = A(y)\,dy$

Integrando $V = \int_a^b dv = \int_a^b A(y)\,dy$

¿cuál es el área transversal

Ejemplo: Considere la región $0 \leq y \leq \sqrt{x}$



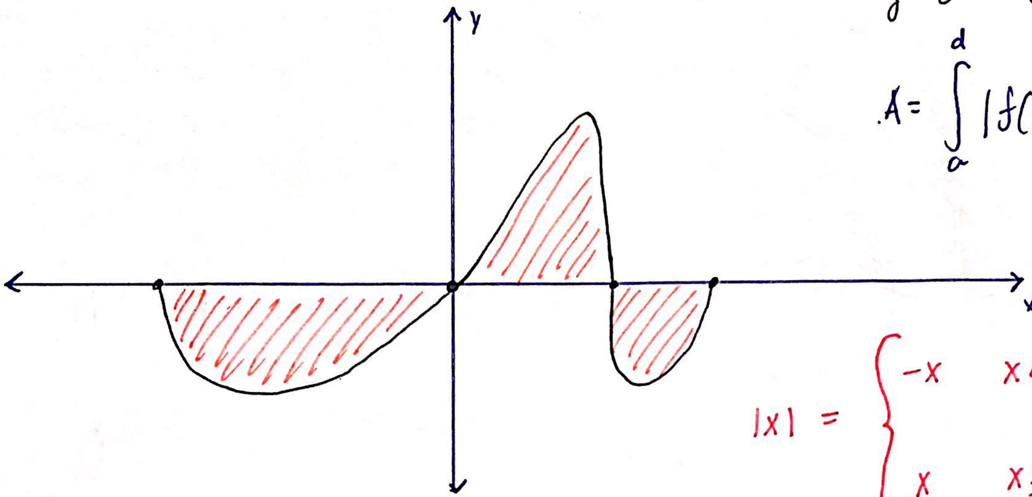
Volumen del solido es la sección transversal es un cilindro disco de radio $r = \sqrt{x}$

$$dv = \pi r^2 \, dx = \pi x \, dx$$

$$V = \int_0^4 \pi r^2 \, dx = \pi \int_0^4 x \, dx = \frac{\pi x^2}{2}\Bigg]_0^4 = \frac{16\pi}{2} = 8\pi$$

6.1. Área Entre Curvas   Pg. 79   2019-09-5

Región entre la curva $y = f(x)$, y el eje-x.



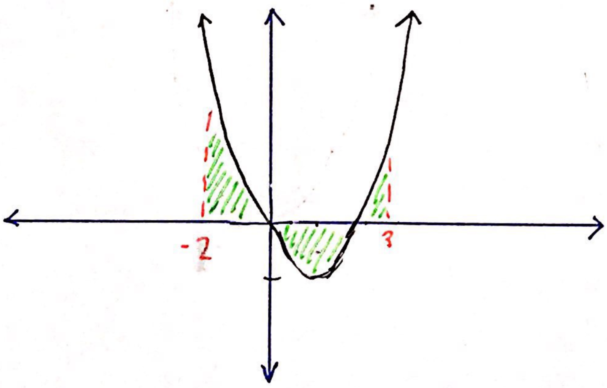
$$A = \int_a^d |f(x)|\,dx$$

$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$A = -\int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx$$

Intersecciones y bosquejos la curva y la región.

Ejercicio I: Bosqueje y encuentre el área de la región limitada por $y = 3x^2 - 6x$, $x = -2$, $x = 3$ & $y = 0$.



Ix

$y = 3x^2 - 6x = 0$

$3x(x-2) = 0 \quad x = 2, \; x = 0$

Sume el área de 3 subregiones:

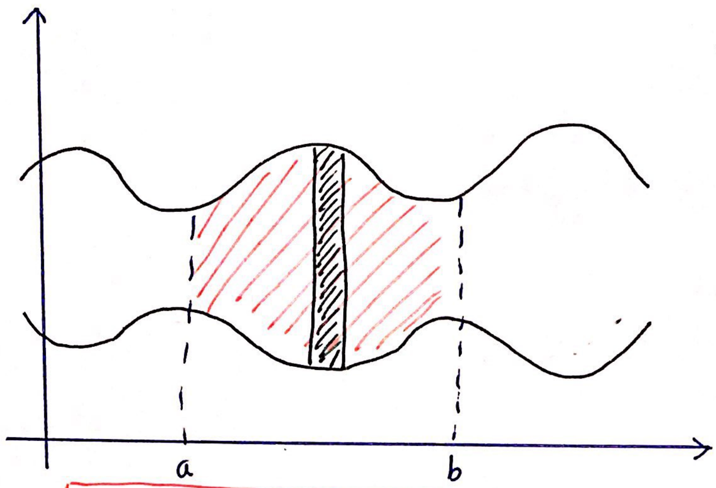
$$A = \int_{-2}^{0} 3x^2 - 6x\,dx - \int_0^2 3x^2 - 6x\,dx + \int_2^3 3x^2 - 6x\,dx$$

$$A = x^3 - 3x^2\Big]_{-2}^{0} + \left(-x^3 + 3x^2\Big]_0^2\right) + x^3 - 3x^2\Big]_2^3$$

$$A = 0 - (-8 - 12) + (-8 + 12) + (27 - 27 - 8$$

$A = 28$ $\square$

¿Cuando hay una curva interior?



$y = f(x)$

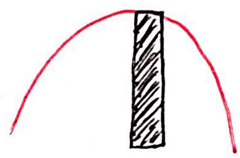
Región $= g(x) \leq y \leq f(x)$

$a \leq x \leq b$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

diferencia de áreas

$$\boxed{A = \int_a^b y_{arriba} - y_{abajo}\,dx}$$



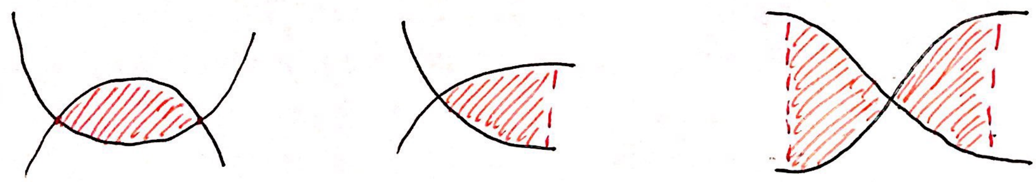
- franja horizontal ó rectángulo infinitesimal
  - def. pt más alto y más bajo
  - ancho dx altura $f(x) - g(x)$

$$dA = (f(x) - g(x))\,dx$$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

Pasos:

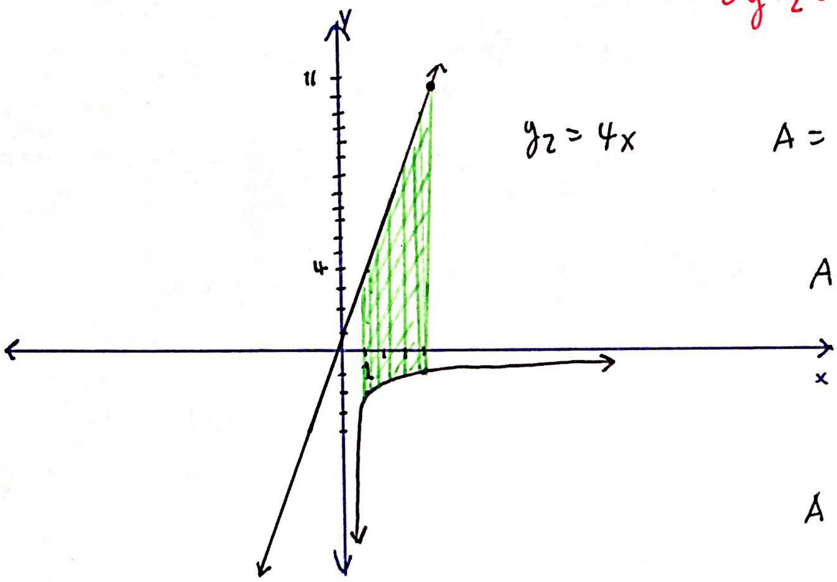
1. Bosqueje $g(x)$ & $f(x)$.
2. Ojo con intersecto ente $f(x)$ & $g(x)$.
3. Bosqueje la región.



Ejemplo: Bosqueje y encuentre el área entre:

$$y_1 = \frac{-2}{\sqrt{x}}, \quad y_2 = 4x \quad \text{en} \quad 1 \leq x \leq 4$$

¿$y_2 > y_1$? ó ¿$y_2 > y_2$?



$y_2 = 4x$

$$A = \int_1^4 y_2 - y_1 \, dx$$

$$A = \int_1^4 4x - (-2x^{-1/2}) \, dx$$

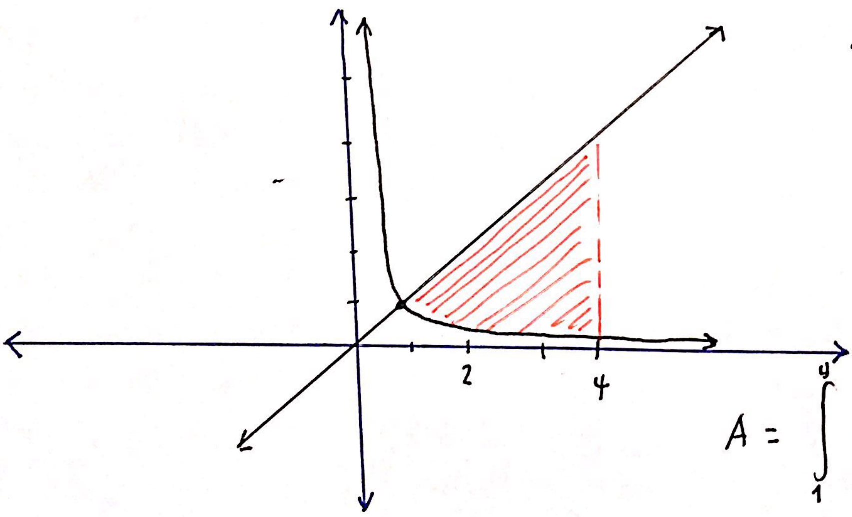
$$A = 2x^2 \Big]_1^4 + 4x^{1/2} \Big]_1^4$$

$$A = \{2 \cdot 16 + 4 \cdot 2\} - \{2 + 4\}$$

$$A = 32 + 8 - 6 = 34 \quad \square$$

Variación $y_1 = \frac{4}{\sqrt{x}}$ & $y_2 = 4x$ y la recta $x = 4$.

$$A = \int^4 y_2 - y_1$$



Intersección y_1 & y_2

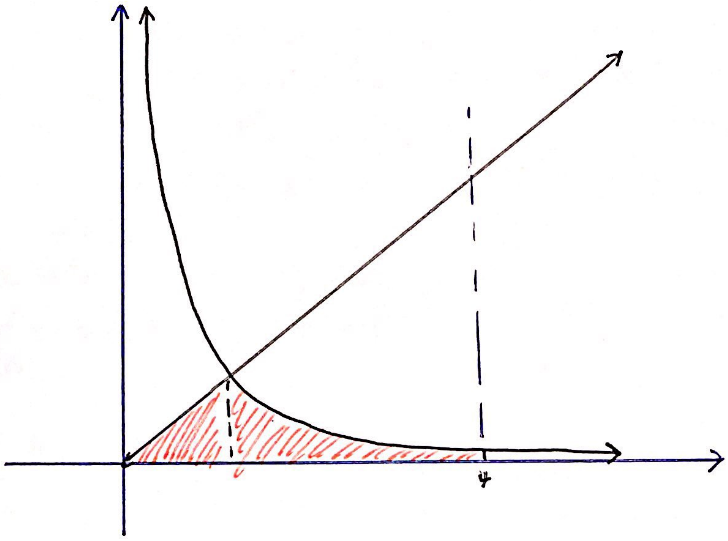
$$\frac{4}{x^{1/2}} = 4x$$

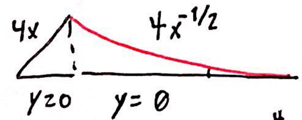
$$1 = x^{3/2} \Rightarrow x = 1$$

$$A = \int_1^4 4x - 4x^{-1/2} \, dx = 2x^2 - 8x^{1/2} \Big]_1^4 =$$

$$= 32 - 16 - (2 - 8) - 16 + 6 = 22$$

Variación C: Área de la región entre $y_1 = 4x^{-1/2}$ $y_2 = 4x$, $x = 4$, $x = 0$



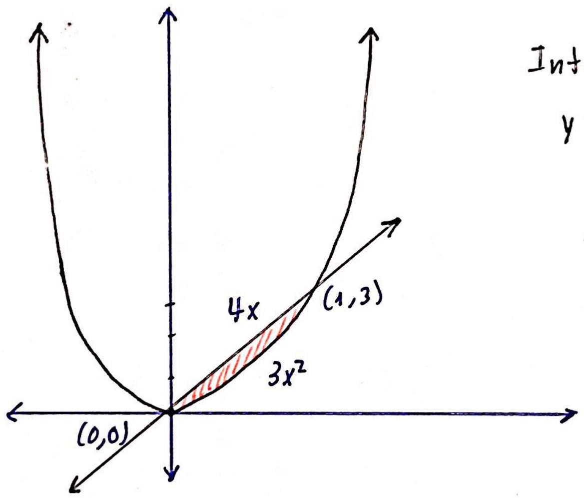


$$A = \int_0^1 4x\,dx + \int_1^4 4x^{-1/2}\,dx$$

$$A = 2x^2\Big]_0^1 + 8x^{1/2}\Big]_1^4$$

$$A = 2 + 16 - 8 = 10$$

Ejemplo: Encuentre el área de la recta de la región entre las curvas $y_1 = 3x$ & $y_2 = 3x^2$.



Intersectos: $3x^2 = 3x$

$y_2 = y_2$ $x = 1, x = 0$

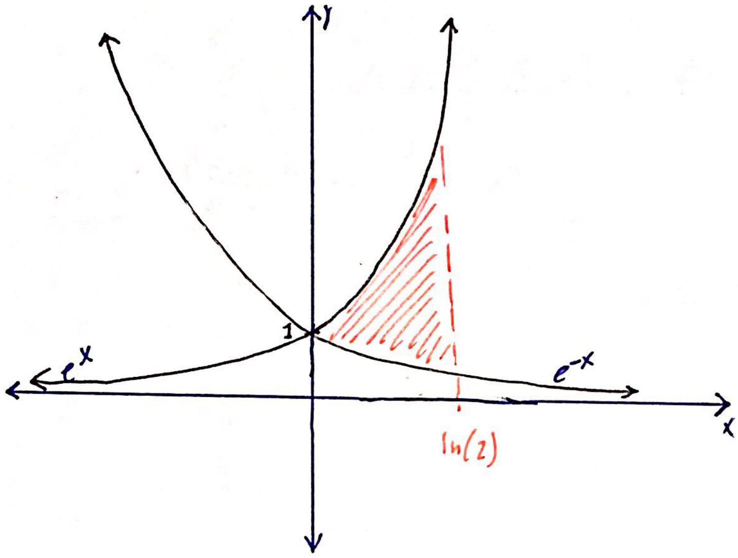
$$A = \int_0^1 (3x - 3x^2)\,dx$$

$$A = \frac{3x^2}{2} - x^3\Big]_0^1 = \left\{\frac{3}{2} - 1\right\} - \{0\} = \frac{1}{2}$$

□

Ejercicio 2: Bosqueje y encuentre el área de la región entre las curvas.

a) $y_1 = e^x$, $y_2 = e^{-x}$, $x = 0$, $x = \ln(2)$



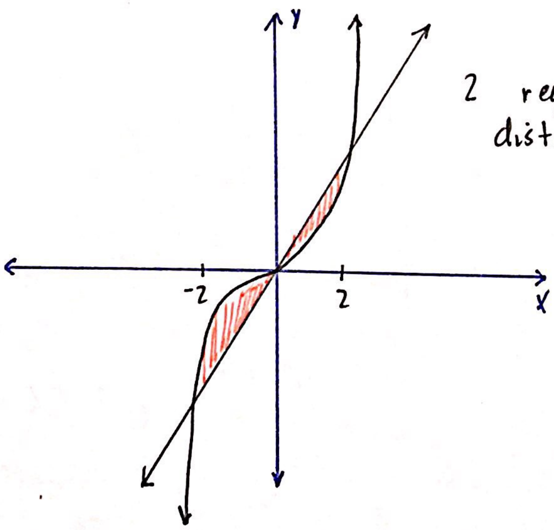
$$A = \int_0^{\ln(2)} (e^x - e^{-x})\,dx$$

$$A = e^x + e^{-x} \Big]_0^{\ln(2)}$$

$$A = \{2 + 2^{-1}\} - \{1 + 1\}$$

$$A = 2 + \frac{1}{2} - 2 = \frac{1}{2}$$

b) $y_1 = x^3$, $y_2 = 4x$



2 regiones distintas

Intersectos

$y_1 = y_2$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

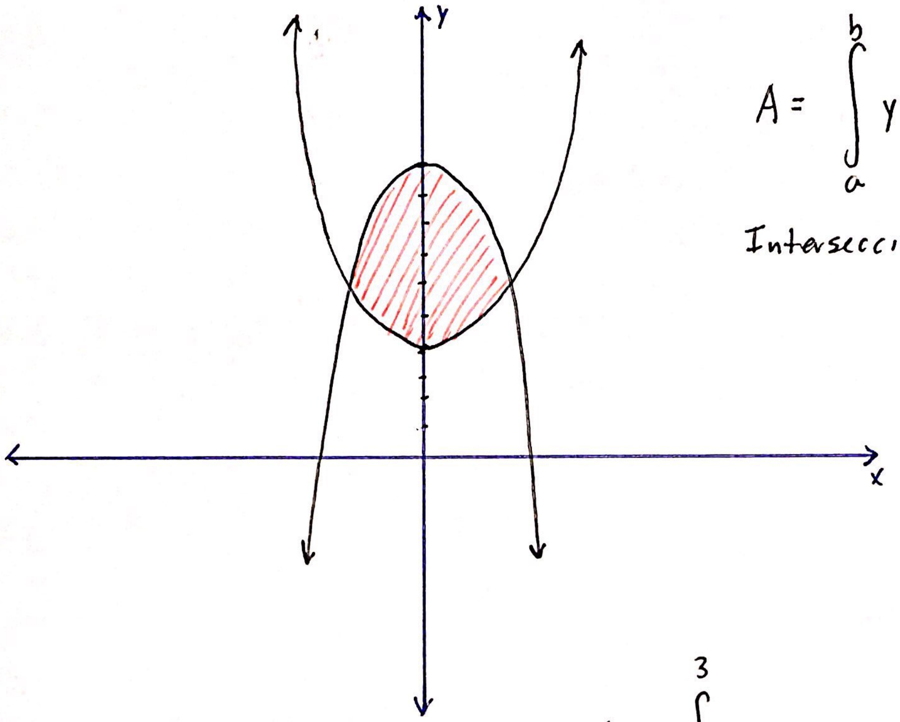
$x = 0$, $x = -2$, $x = 2$

$$A = 2\int_0^2 4x - x^3\,dx = 2\left\{2x^2 - \frac{x^4}{4}\Big]_0^2\right\} =$$

$$= 2\left[\left\{2(2)^2 - \frac{(2)^4}{4}\right\} - \{0\}\right] = 2\left(8 - \frac{16}{4}\right)$$

$$= 8$$

c) $y_1 = x^2 - 4x + 4, \quad y_2 = 10 - x^2$



$$A = \int_a^b y_2 - y_1 \, dx$$

Intersecciones $\quad y_1 = y_2$

$$x^2 - 4x + 4 = 10 - x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x-3)(x+1)$$

$$x = 3 \quad x = -1$$

$$A = \int_{-1}^{3} 10 - x^2 - (x^2 - 4x + 4) \, dx$$

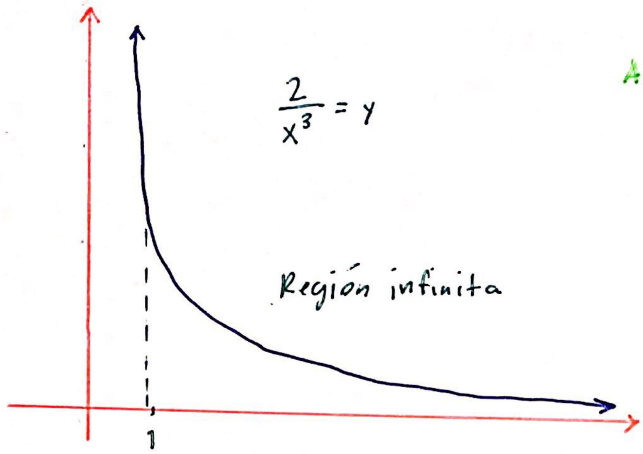
$$A = \int_{-1}^{3} (10 - 2x^2 + 4x) \, dx =$$

$$= 6x - \frac{2}{3}x^3 + 2x^2 \Big]_{-1}^{3} = 18 - 18 + 18 - \left(-6 + \frac{2}{3} + 2\right)$$

$$= 18 + 4 - \frac{2}{3} = 22 - \frac{2}{3}$$

7.8. Integrales impropias 2019-09-03

Considere la región bajo la curva $y = \frac{2}{x^3}$ encima del eje-x y a la derecha de la recta $x = 1$



$$A = \int_1^t 2x^{-3}\, dx =$$

$$A = \frac{2}{-2} x^{-2} \Big]_1^t$$

$$A = -1 \cdot t^{-2} + 1 \cdot 1^{-2} = 1 - \frac{1}{t^2}$$

$$\lim_{t \to \infty} A = \lim \left(1 - \frac{1}{t^2}\right) = 1$$

$$\boxed{\int_1^t \frac{2}{x^3}\, dx = 1}$$

Límites básicos

a) $\lim_{x \to \infty} \left(\frac{1}{x^r}\right) = 0 \quad \left[\frac{1}{\infty}\right]$

b) $\lim_{x \to \infty} \left(e^x\right) = \infty \quad \left[e^{\infty}\right]$

c) $\lim_{x \to 0^+} \left(\ln x\right) = -\infty$

d) $\lim_{x \to \infty} \left(x^r\right) = \infty$

e) $\lim_{x \to -\infty} \left(e^x\right) = 0$

f) $\lim_{x \to \infty} \left(\ln x\right) = \infty$

Integrales impropias:

tipo 1: Intervalos infinitos $\pm\infty$

tipo 2: Funciones discontinuas (AVs, en $x = \pm a$)

Integrales Impropias tipo 1:

- $$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^{t} f(x)\,dx$$

- $$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^{a} f(x)\,dx$$

- $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

- Convergente: se acerca a un número, el límite <u>existe</u>

  Divergente: el límite <u>no existe</u>.

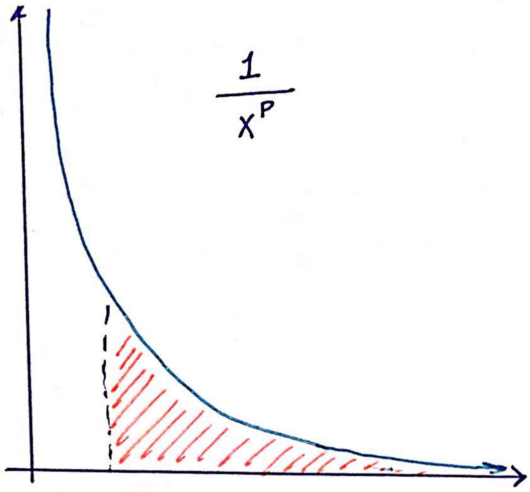
• <u>Ejercicio:</u> Evalúe

a) $$\int_1^{\infty} x^{-1/2}\,dx = 2x^{1/2}\Big]_1^{\infty} = \lim_{x\to\infty}\left(2\sqrt{x} - 2\right) = 2\sqrt{\infty} - 2 = \infty$$

Es una integral <u>divergente</u>.

b) $$\int_1^{\infty} \frac{1}{x}\,dx = \ln x\Big]_1^{\infty} = \lim_{t\to\infty}\left(\ln x - 0\right) = \infty$$

también es <u>divergente</u>.



$$\frac{1}{x^p}$$

$$\int_1^\infty \frac{1}{x^p}\,dx = \text{no necesariamente existe}$$

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} p \le 1 & \text{Diverge} \\ p > 1 & \text{converge} \end{cases}$$

- $p = 0.99$

$$\int_1^\infty \frac{1}{x^{0.99}}\,dx = \left.\frac{x^{0.01}}{0.01}\right]_1^\infty = \lim_{x\to\infty}\left(x^{0.01} - \frac{1}{0.01}\right) = +\infty$$

Diverge

- $p = 1.001$

$$\int_1^\infty x^{-1.001}\,dx = \left.\frac{1}{x^{0.001}} \cdot \frac{1}{0.001}\right]_1^\infty = \lim_{x\to\infty}\left(\frac{1000}{x^{0.001}} + \frac{1}{0.001}\right) =$$

$$= \left.\frac{1000}{x^{0.001}}\right]_\infty^1 = 1000 - \lim_{x\to\infty}\left(\frac{1000}{x^{0.001}}\right) = \underline{1000}$$

Diverge.

$$\int_{-\infty}^0 e^{-x^2}\cdot x\,dx = \int_{-\infty}^0 e^u \frac{du}{-2} = \left.-\frac{1}{2}e^u\right]_{-\infty}^0 = -\frac{1}{2}e^0 + \frac{1}{2}e^{-\infty} = \underline{-\frac{1}{2}}$$

$u = -x^2$

$-\frac{du}{2} = x\,dx$

$u(0) = -0^2$

$u(-\infty) = -(-\infty)^2 = -\infty$

Converge

b.) $\frac{1}{2}\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1}(x)\Big]_{-\infty}^{\infty} = \left\{\frac{1}{2}\tan^{-1}(\infty)\right\} - \left\{\frac{1}{2}\tan^{-1}(-\infty)\right\} = \frac{\pi}{2}$ Diverge

$\tan x \Rightarrow$ ID : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

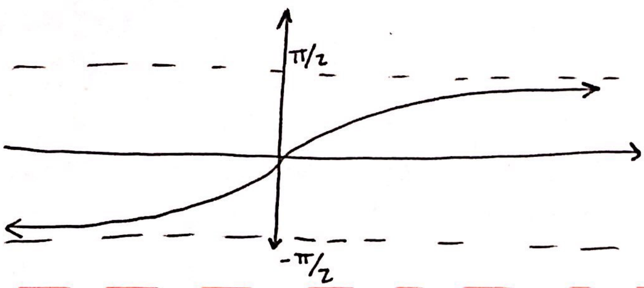
R : $(-\infty, \infty)$

AV: $x = -\pi/2 , \pi/2$

$= \tan^{-1}x$ ID $=(-\infty, \infty)$

R $= (-\pi/2 , \pi/2)$

A.H $= \pm \pi/2$



$= \tan^{-1}(\infty) = \pi/2$

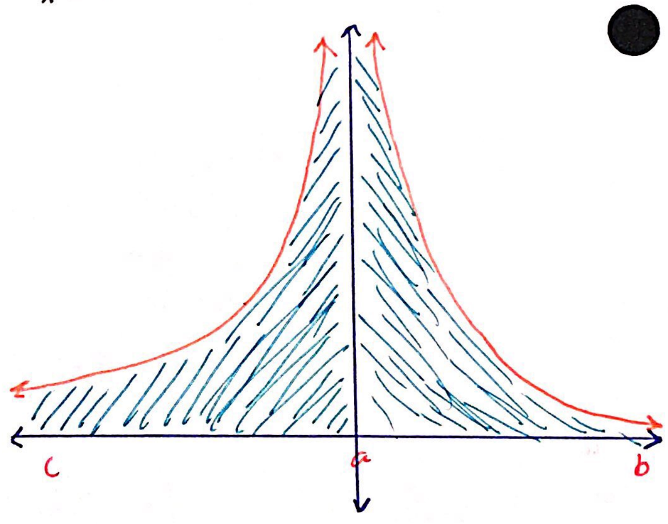
$= \tan^{-1}(-\infty) = -\pi/2$

Integrales impropias: Tipo 2

Hay una asíntota vertical en $x = a$:

$$\int_b^a f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$$

$$\int_C^a f(x)\,dx = \lim_{t\to a^-}\int_c^t f(x)\,dx$$



A.V. $x=a$ $\quad \int_c^b f(x)\,dx = \int_c^a f(x)\,dx + \int_a^b f(x)\,dx$

Ejercicio 4: Evalúe. Indique donde es discontinua

a) $\displaystyle\int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^8 u^{-1/3}\,du = \frac{3}{2}u^{2/3}\Big]_{0^+}^{8} =$

Discontinuidades = $x = 1$ denominador igual a $1/0$

$u = x - 1$ $\quad u(1) = 0$

$du = dx$ $\quad u(9) = 8$

$$= \frac{3}{2}(8^2)^{1/3} - \frac{3}{2}\lim_{u\to0^+} u^{2/3} = \frac{3}{2}\sqrt[3]{64} - 0 = \frac{3}{2}\cdot 4 = \underline{6}$$

$\square$

b) discontinua en 0

$$\int_{-2}^{3}\frac{3}{x^4}\,dx = \underset{(1)}{\int_{-2}^{0} 3x^{-4}\,dx} + \underset{(2)}{\int_0^3 3x^{-4}\,dx} = \underline{\infty} \text{ diverge}$$

$$(1) = \frac{3x^{-3}}{-3}\Big]_{-2}^{0^-} = \underbrace{\lim_{x\to0^-}\left(-\frac{1}{x^3}\right)}_{\infty} + \frac{1}{-2^8} = +\infty$$

$$(2) = \int_0^3 3x^{-4}\,dx = -x^{-3}\Big]_0^3 = -\frac{1}{3^3} + \lim_{x\to0^+}\frac{1}{x^3} = +\infty$$

c.) $$\int_0^1 \ln(x)\,dx = x\ln(x) - \int dx = x\ln x - x + C\Big]_{0^+}^{1} = \overbrace{1\cdot\ln(1)}^{0} - \overbrace{1 - \lim_{x\to0} x\ln x}^{\infty}$$

$$= -1 - \underbrace{\lim_{x\to0}(x\ln x)}$$

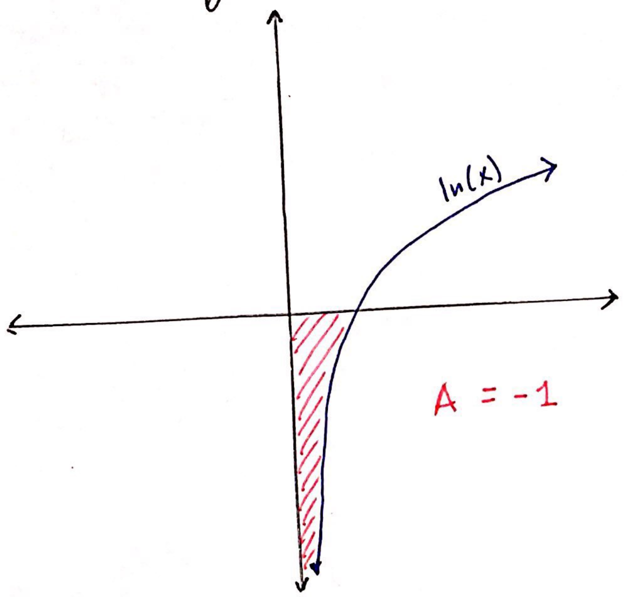
Regla de L'Hopital $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)}$

$$\lim_{x\to 0^+}\left(x\ln(x)\right) = \lim_{x\to 0^+}\left(\frac{\ln(x)}{x^{-1}}\right) \overset{LH}{=} \lim_{x\to 0^+}\left(\frac{1}{x(-x^{-2})}\right) = \lim_{x\to 0^+}\left(\frac{1}{-x^{-1}}\right) = \cdots$$

$$\cdots = \lim_{x\to 0^+}\left(\frac{1}{-x^{-1}}\right) = \lim_{x\to 0^+}\left(-x\right) = \underline{0}$$

$$\therefore \int_0^1 \ln(x)\,dx = -1 - \lim_{x\to 0} x\ln(x) = -1 + 0$$

converge



$\square$

Repaso Simulacro Parcial

2019-08-29
20190432

a) $$\int x \tan^{-1}(x^2)\,dx = \frac{1}{2}\int \tan^{-1}(y)\,dy =$$

$y = x^2$
$dy = 2x\,dx$

IPP: $u = \tan^{-1} y$ $\quad dv = dy$
$du = \dfrac{1}{x^2+1}$ $\quad v = y$

$$= \frac{1}{2}\tan^{-1}(y)\,y - \frac{1}{2}\int y\,\frac{1}{y^2+1}\,dy$$

$w = 1 + y^2$
$dw = 2y\,dy$

$$= \frac{1}{2}\,y\tan^{-1}y - \frac{1}{4}\int \frac{dw}{w}$$

$$= \frac{1}{2}\,y\tan^{-1}y - \frac{1}{4}\ln|w| + C$$

$$= \frac{1}{2}\cdot x^2\cdot \tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4| + C$$

---

b) $$\int \frac{xe^x}{(x+1)^2}\,dx =$$

tres funciones

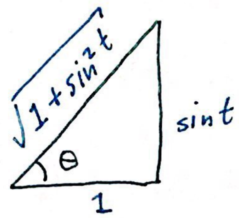
$u = xe^x$ $\quad dv = (x+1)^{-2}$
$du = e^x + xe^x$ $\quad v = \dfrac{(x+1)^{-1}}{-1} = \dfrac{-1}{(x+1)}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = -\frac{xe^x}{(x+1)} + \int \frac{(e^x + xe^x)\,dx}{(x+1)}$$

$$= \frac{-xe^x}{(x+1)} + \int e^x\,dx = \frac{-xe^x}{(x+1)} + e^x + C$$

(2b)

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{1+\sin^2 t}}\,dt = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int_0^{\pi/4} \sec\theta\,d\theta = \ln\left|\sec\theta + \tan\theta\right|\Big]_0^{\pi/4}$$



$\sin t = \tan\theta$

$\cos t\,dt = \sec^2\theta\,d\theta$

$\sqrt{1+\sin^2 t} = \sec\theta$

cambie límites de integración

$\tan\theta = \sin \pi/2 = 1 \rightarrow \theta = \frac{\pi}{4}$

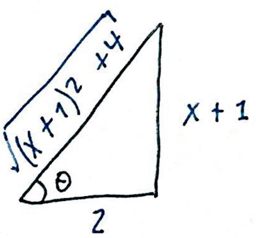
$\tan\theta = \sin 0 = 0 \rightarrow \theta = 0$

$$= \left\{\ln\left|\sec \pi/4 + \tan \pi/4\right|\right\} - \left\{\ln\left|\sec 0 + \tan 0\right|\right\}$$

$$= \ln\left|\sqrt{2}+1\right| - \ln\left|\sqrt{2}+1\right|$$

Corto 4a

$$\int \frac{1}{(x^2+2x+5)^2}\,dx = \int \frac{1}{\left[(x+1)^2+4\right]^2}\,dx = \int \frac{1\cdot 2\sec^2\theta\,d\theta}{16\sec^4\theta}$$



$\tan\theta = \frac{x+1}{2}$

$x+1 = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$$(\sec\theta)^2 = \left(\frac{\sqrt{(x+1)^2+4}}{2}\right)^2$$

$$16\sec^2\theta = \left((x+1)^2+4\right)^2$$

$$= \frac{1}{8}\int \cos^2\theta\,d\theta = \frac{1}{8}\left(\theta + \sin\theta\cos\theta\right) = \frac{\frac{1}{2}\left(\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{16}\left(\frac{(x+1)\cdot 2}{\left(\sqrt{x^2+2x+5}\right)^2}\right)\right)}{16}$$

$$= \frac{1}{16}\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{8}\left(\frac{x+1}{\sqrt{x^2+2x+5}\,\sqrt{x^2+2x+5}}\right) + C$$

IPP:

2 $\displaystyle\int (x-1)\sin \pi x \, dx =$

$u = x - 1 \qquad dv = \sin \pi x \, dx$

$du = dx \qquad v = -\dfrac{1}{\pi}\cos \pi x$

$$= (x-1)\left(-\tfrac{1}{\pi}\cos \pi x\right) - \int \frac{1}{\pi}\cos \pi x \, dx$$

$$= -\frac{(x-1)}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + C$$

Cíclico:

$$\int e^{-\theta}\cos(2\theta)\, d\theta = -\frac{1}{2}e^{-\theta}\sin(2\theta) - \frac{1}{2}\underbrace{\int e^{-\theta}\sin(2\theta)\, d\theta}$$

$u = e^{-\theta} \qquad dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta}\, d\theta \qquad v = \dfrac{\sin 2\theta}{2}$

$$= \int e^{-\theta}\sin(2\theta)$$

$u = e^{-\theta} \qquad dv = \sin(2\theta)\, d\theta$

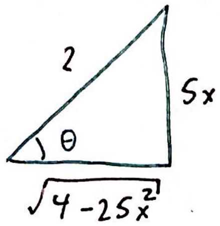
$du = -e^{-\theta}\, d\theta$

•Parcial 1 lunes 2 septiembre 2019-08-27

Repaso De Sustitución Trigonométrica

Lab #5

(1) $$\int 5^8 x^7 \sqrt{4-25x^2}\, dx = 5^8 \int \frac{2^7}{5^7} \sin^7\theta \cdot 2\cos\theta \cdot \frac{2}{5}\cos\theta \, d\theta =$$



$\sin\theta = \frac{5x}{2}$

$x = \frac{2}{5}\sin\theta$

$dx = \frac{2}{5}\cos\theta \, d\theta$

$\sqrt{4-25x^2} = 2\cos\theta$

$$= \frac{5^8}{5^8} \cdot 2^9 \int \sin^7\theta \, d\theta \cdot \cos^2\theta \, d\theta = 512 \int \sin^6\theta \cos^2\theta \sin\theta \, d\theta$$

$\sin^6\theta = (\sin^2\theta)^3 = (1-\cos^2\theta)^3$

$$\therefore = 512 \int (1-\cos^2\theta)^6 \cos^2\theta \cdot \sin\theta \cdot d\theta$$

$u = \cos\theta$

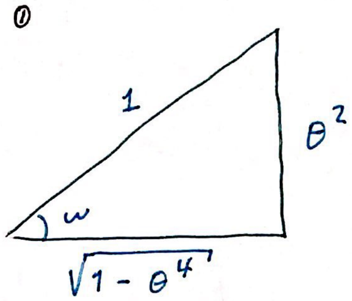
$du = -\sin\theta \, d\theta$

$$= 512 \int (1-u^2)^3 u^2 \, du$$

$(1-u^2)^3 u^2 =$
$u^2 - 3u^4 + 3u^6 - u^8$

$$= 512 \int u^2 - 3u^4 + 3u^6 - u^8 \, du$$

② $$\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta = \frac{2}{\pi}\int_0^1 \sqrt{1-\theta^4}\cdot\underbrace{2\theta\cdot d\theta}_{\text{especulando}} =$$



$$\sin(w) = \theta^2$$

$$\cos(w)\,dw = 2\theta\,d\theta$$

$$\sqrt{1-\theta^4} = \cos(w)$$

encontrar nuevos límites =>

$$\sin(w) = 1^2$$
$$w = \frac{\pi}{2}$$

$$\sin(w) = 0$$
$$w = 0$$

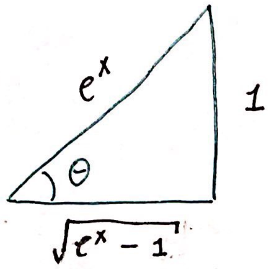
$$= \frac{2}{\pi}\int_0^{\pi/2} \cos^2(w)\,dw = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} (1 + \cos(2w))\,dw$$

$$= \frac{1}{\pi}\left(w + \frac{1}{2}\sin(2w)\right)\Bigg]_0^{\pi/2} \doteq$$

$$\doteq \frac{1}{\pi}\left\{\frac{\pi}{2} + \frac{1}{2}\sin(2\cdot\pi/2)\right\} - \cancel{\frac{1}{\pi}\left\{0 + \frac{1}{2}\sin(2\cdot 0)\right\}}^{\,0}$$

$$\doteq \frac{1}{2}\quad\square$$

⑤ $$\int_0^{\ln(\sqrt{2})} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\, dx = \int_0^{\ln(\sqrt{2})} \frac{e^{3x}}{\sqrt{e^{2x}-1}}\, e^{x} \cdot dx = \int_0^{\ln(\sqrt{2})} \frac{\sec^3\theta}{\tan\theta} \cdot \sec\theta \tan\theta \, d\theta$$



$e^x = \sec\theta \, d\theta$

$\sqrt{e^x - 1} = \tan\theta \, d\theta$

$e^x dx = \sec\theta \tan\theta \, d\theta$

$\sec\theta = e^{\ln(\sqrt{2})} = \sqrt{2}$

nuevo límite

$\sec\theta = e^{0} = 1$

$\theta = 0$

$$= \int_0^{\ln(2)} \sec^4\theta \, d\theta = \int_0^{\pi/4} \sec^2\theta \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} (1 + \tan^2\theta) \sec^2\theta \, d\theta = \int_0^{1} (1 + w^2)\, dw =$$

$w = \tan\theta$

$dw = \sec^2\theta \, d\theta$

$$= w + \frac{w^3}{3} \Big]_0^1 = 1 + \frac{1}{3} = \frac{4}{3}$$

FORMA ALTERNA

⑤ $$\int_0^{\ln(\sqrt{2})} \frac{e^{2x}}{\sqrt{e^{2x}-1}} \cdot e^{2x}\, dx = \int_0^{\ln(\sqrt{2})} \frac{e^{x}}{\sqrt{u}}\, du =$$

$u = e^{2x} - 1$ $\quad u(\ln(\sqrt{2})) = e^{2(\ln(\sqrt{2}))} - 1 = 2 - 1 = 1$

$du = 2e^{2x}\, dx$ $\quad u(0) = e^{0} - 1 = 0$

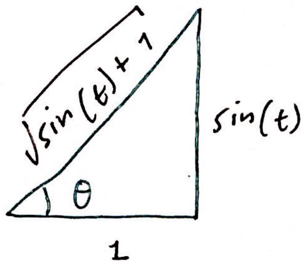
$$= \frac{1}{2}\int_0^1 \frac{u+1}{u^{1/2}}\, du = \frac{1}{2}\int_0^1 \left(u^{1/2} + u^{-1/2}\right) du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right)\Bigg]_0^1$$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{6}{3}\right) = \frac{4}{3} \quad \square$$

$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\, du = \frac{1}{3}\left(e^{2x}-1\right)^{3/2} + \left(e^{2x}-1\right)^{1/2}\Bigg]_0^{\ln(\sqrt{2})}$$

Problema 2b simulacro:

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{\sin^2(t)+1}}\,dt = \int_0^{\pi/2} \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int_0^{\pi/2} \sec\theta\,d\theta$$



$\tan\theta = \sin(t)$

$\sec^2\theta\,d\theta = \cos(t)\,dt$

$\sqrt{\sin^2 t + 1} = \sec\theta$

$\tan\theta = \sin(\pi/2) = 1$

$\theta = \tan^{-1}(1) = \pi/4$

$\tan\theta = \sin(0) = 0$

$\theta = 0$

$$\therefore \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C \Big|_0^{\pi/4} =$$

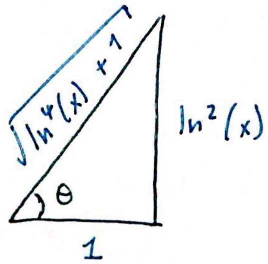
$$= \left\{\ln\left|\overset{\sqrt{2}}{\cancel{\sec(\pi/4)}} + \tan(\pi/4)\right|\right\} - \left\{\ln\left|\sec(0) + \tan(0)\right|\right\}$$

$$= \ln|\sqrt{2}+1| - \overset{0}{\cancel{\ln|1|}} - \ln|\sqrt{2}+1|$$

$$= \ln|\sqrt{2}+1| - \ln|\sqrt{2}+1|$$

$\square$

Problema curioso = $\displaystyle\int \frac{8}{\sqrt{\ln^4 x + 1}}\,\frac{\ln(x)}{x}\,dx = \int \frac{4\cdot 2}{\sqrt{\ln^4(x)+1}}\cdot\frac{\ln(x)}{x}\,dx =$



$$\tan\theta = \frac{\ln^2(x)}{1} = \ln^2(x)$$

$$\sec^2\theta\, d\theta = 2\ln(x)\cdot\frac{1}{x}\,dx = \frac{2\ln(x)}{x}$$

$$\sqrt{\ln^4(x)+1} = \sec\theta$$

$$= 4\int \sec\theta\, d\theta = 4\ln|\sec\theta + \tan\theta| + C$$

$$\therefore\; = 4\cdot\ln\left|\sqrt{\ln^4(x)+1} + \ln^2(x)\right| + C$$

$$\int \frac{(x-2)^3}{\sqrt{x^2-4x+13}}\,dx =$$

completar al cuadrado

$(x^2-4x+4)+13-4$

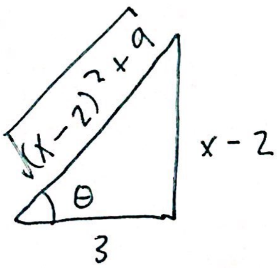
$(x-2)^2+9$

$u = x^2-4x+13$

$du = 2x-4 = 2(x-2)\,dx$

no está

$$\therefore\; = \int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$



$$3\tan\theta = x-2$$

$$3\sec^2\theta\, d\theta = dx$$

$$\sqrt{(x-2)^2+9} = 3\sec\theta$$

$$(x-2)^3 = 3^3\sec^3\theta$$

$$= \int \frac{(x-2)^3}{\sqrt{(x-2)+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\cdot 3\sec^2\theta\, d\theta = 3^3\int \tan^3\theta\sec\theta\, d\theta$$

$$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\, d\theta)$$

$$= 27 \int \tan^2\theta \, (\tan\theta \sec\theta) \, d\theta = 27 \int (\sec^2\theta - 1)(\tan\theta \sec\theta) \, d\theta$$

$u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$$= 27 \int (u^2 - 1) \, du = 9u^3 - 27u + C$$

$$= 9\sec^3\theta - 27\sec\theta + C$$

$$= \frac{9}{27}(x^2 - 4x + 13)^{3/2} - \frac{27}{3}(x^2 - 4x + 13)^{1/2} + C$$

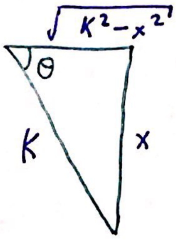
7.3. Sustitución Trigonométrica

2019-08/22

Forma $\sqrt{K^2-x^2}$

$H = K$

$C.O. = x$

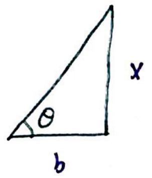


$$\frac{C.O.}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta$$

$$dx = K\cos\theta \, d\theta$$

$$\frac{\sqrt{K^2-x^2}}{K} = \cos\theta$$

Forma $\sqrt{b^2+x^2}$



$$\frac{x}{b} = \tan\theta \Rightarrow x = b\cdot\tan\theta$$

$$dx = b\cdot\sec^2\theta \, d\theta$$

$c^2 = x^2+y^2$

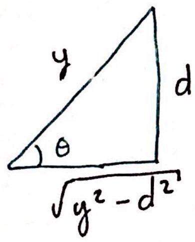
$\sqrt{c^2-y^2} = x$

$\sqrt{c^2-x^2} = y$

$$\frac{b}{\sqrt{b^2+x^2}} = \cos\theta \Rightarrow \sqrt{b^2+x^2} = b\sec\theta$$

$$\frac{\sqrt{b^2+x^2}}{b} = \sec\theta$$

Forma $\sqrt{y^2-d^2}$



$$\sin\theta = \frac{d}{y}$$

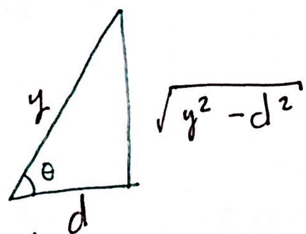
$$y = d\cdot\csc\theta$$

$$dy = -d\csc\theta\cot\theta \, d\theta$$

$$\frac{y}{d} = \sec\theta$$

$$y = d\sec\theta$$

$$dy = d\sec\theta\tan\theta$$



$$\sqrt{y^2-d^2} = d\tan\theta$$

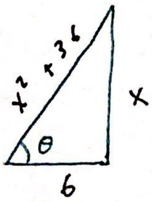
Ejercicio 2 y 6 Pág 58 y 59

20 $$\int \frac{1}{x^2+36}\,dx = \int \frac{6\sec^2\theta \, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$$

$x = 6\tan\theta$

$dx = 6\cdot\sec^2\theta \, d\theta$

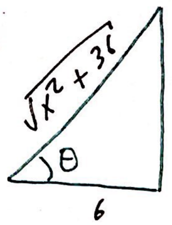
$x^2+36 = 36(\tan^2\theta+1) = 36\sec^2\theta$



$$x = 6\tan\theta \Rightarrow \frac{x}{6}\ \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{6}\right) = \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + c$$

② $$\int \frac{1}{\sqrt{x^2+36}}\,dx = \int \frac{6\sec^2\theta}{6\sec\theta}\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\therefore \ln\left|\frac{\sqrt{x^2+36}}{6} + \frac{x}{6}\right| + C$$

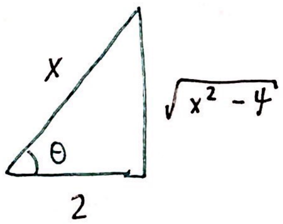


$$\frac{x}{6} = \tan\theta \qquad x = 6\cdot\tan\theta$$

$$dx = 6\sec^2\theta\,d\theta$$

$$\frac{\sqrt{x^2+36}}{6} = \sec\theta \qquad \sqrt{x^2+36} = 6\sec\theta$$

③ $$\int \frac{\left(\sqrt{x^2-4}\right)^3}{x^6}\,dx = \int \frac{2^3\tan^3\theta}{2^6\sec^6\theta}\cdot 2\tan\theta\sec\theta\,d\theta =$$



$$\frac{x}{2} = \sec\theta \qquad x = 2\sec\theta$$

$$dx = 2\sec\theta\tan\theta\,d\theta$$

$$\frac{\sqrt{x^2-4}}{2} = \tan\theta \qquad \sqrt{x^2-4} = 2\tan\theta$$

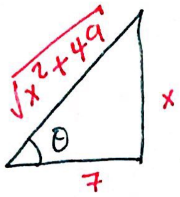
$$= \frac{2^4}{2^6}\int \frac{\tan^4\theta}{\sec^5\theta}\,d\theta = \frac{1}{2^2}\int \tan^4\theta\cos^5\theta\,d\theta = \frac{1}{4}\int \frac{\sin^4\theta}{\cancel{\cos^4\theta}}\cdot \cancel{\cos^5}\theta$$

$$= \frac{1}{4}\int \sin^4\theta\cos\theta\,d\theta \Rightarrow \begin{matrix} w = \sin\theta \\ du = \cos\theta \end{matrix} \Rightarrow \frac{1}{4}\int u^4\,du = \frac{1}{4}\left[\frac{w^5}{5}\right] + C$$

$$= \frac{\sin^5\theta}{20} + C = \frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C$$

$$\sin\theta = \frac{\sqrt{x^2-4}}{x}$$

2a $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49\cdot 7\csc^2\theta}{49\ \cot^2\theta\ 7\csc\theta}\,d\theta = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta$$



$$\cot\theta = \frac{x}{7} \implies x = 7\cot x$$
$$dx = -7\csc^2\theta\,d\theta$$

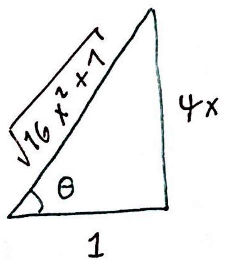
$$x = 7\tan\theta$$

$$\frac{\sqrt{x^2+49}}{7} = \csc\theta \implies \sqrt{x^2+49} = 7\csc\theta$$

$$= -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\,\frac{\sin^2\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$

$$= -\int \tan\theta\,\sec\theta\,d\theta = -\sec\theta + C = \frac{-\sqrt{x^2+49}}{x} + C$$

3b $$\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta}{1/4\ \tan\theta\cdot\sec\theta} = \int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{\sec\theta}{\tan\theta}\,d\theta$$



$$\frac{4x}{1} = \tan\theta \implies x = \frac{\tan\theta}{4}$$

$$\sqrt{16x^2+1} = \sec\theta \qquad dx = \frac{1}{4}\sec^2\theta\,d\theta$$

$$= \int \left[\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right] d\theta = \int \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}\sin\theta}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta =$$

$$= -\ln\left|\csc\theta + \cot\theta\right| + C = -\ln\left|\frac{\sqrt{16x^2-1}}{4x} + \frac{1}{4x}\right| + C$$

Continuación Integración Trigonométrica

- Lunes 26 de Agosto Simulacro Parcial 3 de septiembre parcial 1; capítulos 5 y 7 pg 11 - 70

Integrales de la forma $\int \cot^n x \csc^m dx$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\cot^2 x = \csc^2 x - 1$$

Ejercicio 4: Integre (pg. 50)

ⓐ $\int \cot^2(x)\csc^4(x)\,dx =$

$\therefore \cot^2 x \csc^2 x \csc^2 x$

$\therefore \cot x \csc^3 x (\csc x \cot x)$

$$= \int \cot^2 x \csc^2 x (\csc^2 x\, dx) = \int \cot^2 x (\cot^2 x + 1)\csc^2 x\, dx$$

sustitución

$u = \cot x \qquad du = -\csc^2 x\, dx$

$$= -\int u^2 (u^2+1)\,du$$

$$= -\int (u^4 + u^2)\,du$$

$$= -\frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= -\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C$$

ⓑ $\int \cot^3 x \csc^3 x\, dx =$

$$= \int \cot^2 x \csc^2 x (\cot x \csc x\, dx)$$

$$= \int (\csc^2 x - 1)\csc^2 x (\cot x \csc x\, dx)$$

$u = \csc x \qquad du = -\csc x \cot x\, dx$

$$= -\int (u^2 - 1)(u^2)\,du$$

$$= -\int (u^4 - u^2)\,du = -\frac{u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{-\csc^5 x}{5} + \frac{\csc^3 x}{3} + C$$

Casos especiales $\int \csc x\, dx$ $\int \csc^3 x\, dx$

- $\int \sec x\, dx = \ln|\sec x + \tan x| + C$

- $\int \csc x \underbrace{\frac{(\csc x + \cot x)}{(\cot x + \csc x)}}_{1 \text{ especial}} = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x}\, dx$

$u = \cot x + \csc x$

$-du = \csc^2 x + \csc x \cot x\, dx$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cot x + \csc x| + C$$

- $\int \sec^3 x\, dx = \frac{1}{2}(\sec x)^9 + \frac{1}{2}\int \sec x\, dx$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

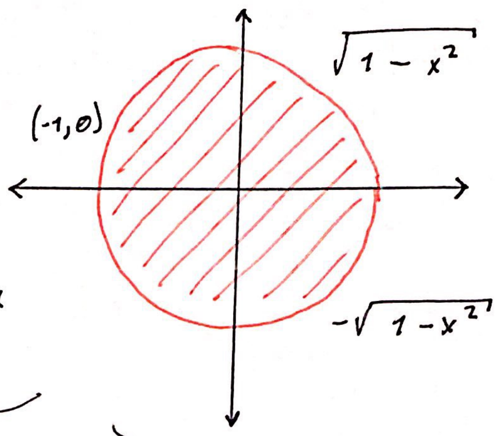
- $\int \csc^3 x\, dx = \frac{1}{2}(\csc x)^9 + \frac{1}{2}\int \csc x\, dx$

$$= -\frac{1}{2}\csc x - \frac{1}{2}\ln|\csc x + \cot x| + C$$

Área de un círculo unitario sin utilizar Geometría

Ec. $x^2 + y^2 = 1$

Función: $y = \pm\sqrt{1 - x^2}$



$$\text{Área} = \int_{-1}^{1} \sqrt{1 - x^2}\,dx + \underbrace{\int_{-1}^{1} -\sqrt{1-x^2}\,dx}_{\text{por } -1}$$

área negativa

$$= 2\int_{-1}^{1} \sqrt{1 - x^2}\,dx \qquad * \, 2$$

$$= 4\int_{0}^{1} \sqrt{1 - x^2}\,dx \qquad * \, 4$$

ni sustitución, ni integración por partes

$\therefore \; 1 - \sin^2\theta = \cos^2\theta$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

Para evaluación de la integral
$$\begin{cases} x = \sin\theta = 1 = \therefore \frac{\pi}{2} \\ x = \sin\theta = 0 = \therefore 0 \end{cases}$$

$$\therefore A = 4\int_0^{\pi/2} \underbrace{\sqrt{1 - \sin^2\theta}}_{\cos\theta} \cos\theta \, d\theta$$

$$A = 4\int_0^{\pi/2} \cos^2\theta \, d\theta$$

$$A = \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$A = \frac{4}{2}\int_0^{\pi/2} (1 + \cos(2\theta))\,d\theta$$

$$A = 2\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big]_0^{\pi/2}$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right)$$

$$= \frac{2}{2}\cdot\pi = \therefore \pi \quad \square$$

el área de un círculo de radio 1 es π

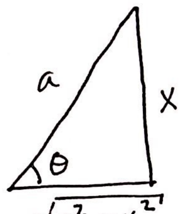
7.3. Sustitución Trigonométrica (pg. 54)

$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}_{\text{simplifique si es posible.}} \; g'(\theta)\,d\theta$$

$$x = g(\theta) \qquad dx = g'(\theta)\,d\theta$$

$\sqrt{1-x^2}$	$\sqrt{1+x^2}$	$\sqrt{x^2-1}$	$\sqrt{\sec^2\theta - 1}$
$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta$	$\sqrt{\tan^2\theta}$
$1 - \sin^2\theta = \cos^2\theta$	$1 + \tan^2\theta = \sec^2\theta$		$\sqrt{x^2-1} = \tan\theta$
$\sqrt{1-x^2} = \cos\theta$	$\sqrt{1+x^2} = \sec\theta$		

forma más general $\sqrt{a^2 - x^2}$

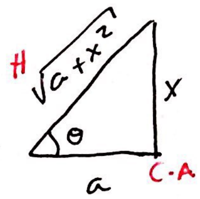


$$\sin(\theta) = \frac{C.O.}{H} = \frac{x}{a} \qquad x = a\sin\theta$$

$$dx = a \cdot \cos\theta\,d\theta$$

$$\cos(\theta) = \frac{C.A.}{H} = \frac{\sqrt{a^2 - x^2}}{a} = \qquad \sqrt{a^2 - x^2} = a\cos\theta$$

forma $\sqrt{a^2 + x^2}$



$$\sin\theta = \frac{x}{\sqrt{a^2 + x^2}} \qquad \tan\theta = \frac{x}{a}$$

$$\frac{H}{C.A} = \sec\theta = \frac{\sqrt{a^2 + x^2}}{a}$$

$$x = a \cdot \tan\theta$$

$$dx = a\sec^2\theta\,d\theta \qquad \sqrt{a^2 + x^2} = a \cdot \sec\theta$$

Ejercicio 1: Evalúe

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\,\frac{du}{2} = \int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

$u = 25 - x^2$ $\qquad$ $= -u^{1/2} + C$

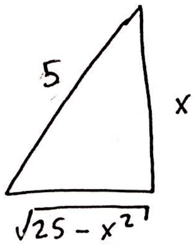
$du = -2x\,dx = \frac{du}{-2x}$ $\qquad$ $= -\sqrt{25-x^2} + C$ ✗

Sustitución Trigonométrica

$H = 5$

$C.O. = x$

$C.A. = \sqrt{25-x^2}$



$x = 5\sin\theta$ ✓

$dx = 5\cos\theta\,d\theta$ ✓

$\sqrt{25-x^2} = 5\cos\theta$ ✓

$\frac{C.A.}{H}$

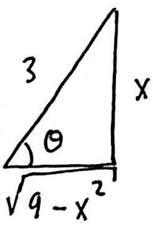
$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{5\cos\theta}\cdot 5\cos\theta\,d\theta = 5\int \sin\theta\,d\theta$$

$= -5\cos\theta + C$

$= \frac{-5}{5}\sqrt{25-x^2} + C$

$= -\sqrt{25-x^2} + C$ ✗

ⓐ $\displaystyle\int \frac{x^3}{\sqrt{9-x^2}}\,dx =$



$S = \frac{x}{3}$ $\qquad$ $C = \frac{\sqrt{9-x^2}}{3}$

$3\sin\theta = x$ $\qquad$ $3\cos\theta = \sqrt{9-x^2}$ $\qquad$ $\rightarrow dx = 3\cos\theta\,d\theta$

$(3\sin\theta)^3 = x^3$

sustituimos

$$= \int \frac{27\sin^3\theta}{3\cancel{\cos\theta}}\cdot 3\cancel{\cos\theta}\,d\theta = \int 27\sin^3\theta\,d\theta$$

$$\longrightarrow \int 27\sin^3\theta\, d\theta = 27\int \sin^3\theta\, d\theta = 27\int \sin\theta\,(1-\cos^2\theta)\, d\theta$$

$$= -27u + 9u^3 + C = -27\cos\theta + 9\cos^3\theta + C$$

$u = \cos\theta \qquad du = -\sin\theta\, d\theta$

sustituyo

$$= -27 \cdot \frac{1}{3}\sqrt{9-x^2} + 9 \cdot \frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C$$

Caso Integrales trigonométricas

- $\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$
- $\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$
- $\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$

Ejercicio 5: Evalúe (pg. 51)

ⓐ $$\int_{-\pi}^{\pi} \sin(8x)\cos(4x)\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(\sin(4x) + \sin(12x)\right)dx = \frac{1}{2}\left[-\frac{\cos(4x)}{4} - \frac{\cos(12x)}{12}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left(-\frac{\cos(4\pi)}{4} + \frac{\cos(-4\pi)}{4} - \frac{\cos(12\pi)}{12} + \frac{\cos(12\pi)}{12}\right)$$

$$= \emptyset$$

7.2. Integrales Trigonométricas

2019-08/13

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x \div \cos^2 x$

$1 + \cot^2 x = \csc^2 x \div \sin^2 x$

Integrales de la forma $\int \sin^n x \cdot \cos^m x \, dx$

$\frac{d}{dx}(\sin x) = \cos x$

$u = \sin x$

$du = \cos x \, dx$

$\frac{d}{dx}(\cos x) = -\sin x$

$u = \cos x$

$du = -\sin x \, dx$

Primer Caso = Se necesita una par y una impar

Evalúe $\int \cos^5 x \, dx = \int \cos^4 x \, (\cos x \, dx)$

Reescribir $\cos^4 x = (\cos^2 x)^2 = (1 - \sin^2 x)^2$

$\cos^2 x = 1 - \sin^2 x$

$\therefore \int \cos^5 x \, dx = \int (1 - \sin^2 x)^2 (\cos x \, dx)$

$u = \sin x \qquad du = \cos x \, dx$

$= \int (1 - u^2)^2 \, du$

$= \int (1 - 2u^2 + u^4) \, du = u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C$

$\therefore = \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$ $\square$

Aparte algún término $\sin x$ o $\cos x$.

a. Potencias impares de seno o coseno.

Evalué

esto es un problema

Preferimos potencias pares

$$\int \cos^3 x \sin^6 x \, dx =$$

$$\int \cos^2 x \sin^6 x \cos x \, dx \quad ó \quad \int \cos^3 x \sin^5 x \sin x \, dx$$

$$= \int \cos^2 x \sin^6 x (\cos x) dx = \int (1-\sin^2 x) \sin^6 x (\cos x \, dx)$$

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$

$du = \cos x \, dx$

$$= \int (1-u^2) u^6 \, du$$

$$= \int u^6 - u^8 \, du$$

$$= \frac{1}{7}u^7 - \frac{1}{8}u^9 + C$$

$$\therefore = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$$

b. $\int \cos^5 x \sin^3 x \, dx =$

$$\int \cos^4 x \sin^3 x \cos x \, dx \quad ó$$

$$\int \cos^5 x \sin^2 x \sin x \, dx$$

$$= \int \cos^5 x (\sin^2 x) \sin x \, dx$$

$$= \int \cos^5 x (1 - \cos^2 x) \sin x \, dx$$

$u = \cos x$ $\quad du = -\sin x \, dx$

$$= -\int u^5 (1 - u^2) du$$

$$= -\frac{1}{6}u^6 + \frac{u^8}{8} + C$$

$$\therefore -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$$

b) Potencias pares de seno y coseno

$$\int \cos^2 x\, dx = \int (1-\sin^2 x)\, dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$$

$$1 = \cos^2 x + \sin^2 x \quad (1)$$

$$+ \cos(x+x) = \cos^2 x - \sin^2 x \quad (2)$$

suma(1 y 2) $1 + \cos(2x) = 2\cos^2 x \Rightarrow \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

Ejercicio potencias pares:

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

a. $$\int_{-\pi}^{\pi} \sin^2 x\, dx = 2\int_0^{\pi} \sin^2 x\, dx = \frac{2}{2}\int_0^{\pi}(1-\cos 2x)\, dx = x - \frac{1}{2}\sin 2x$$

si fuera impar sería 0

$u = 2x \quad du = 2dx$

$$= x - \frac{1}{2}\sin^2 x\Big]_0^{\pi} = \pi - \frac{1}{2}\cancel{\sin 2\pi}^{0} - 0 + \frac{\cancel{\sin 0}^{0}}{2}$$

$$= \pi$$

b. $$\int \sin^2 x \cos^2 x\, dx$$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$= \int \frac{1}{2}(1-\cos 2x)\frac{1}{2}(1+\cos 2x)\, dx$$

diferencia de cuadrados

$$\frac{1}{4}\int (1-\cos^2 2x)\, dx = \frac{1}{4}\int(1-\cos^2 2x)\, dx$$

$$\cos^2(2x) = \frac{1}{2}(1+\cos 4x)$$

$$= \frac{1}{4}\int\left(1 - \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \int \frac{1}{8} + \frac{1}{8}\cos 4x\, dx = \frac{1}{8}x + \frac{1}{8\cdot 4}\sin 4x + C$$

$$\int a\, f\, dx = aF + C$$

Forma $\int \tan^m x \sec^n x\, dx$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$u = \tan x$

$\sec^2 x = \tan^2 x + 1$

$\sec x = \sec x \tan x$

$w = \sec x$

$\tan^2 x = \sec^2 x + 1$

Ejercicio 3 Evalúe Pg. 48

1. $\int \tan^5 x \sec^4 x\, dx$

$\int \tan^5 x \sec^2 x (\sec^2 x\, dx)$ ó $\int \tan^4 x \sec^3 x (\tan x \sec x\, dx)$

$u = \tan x \quad \tan^2 x + 1$

$w = \sec x$

$(\tan^2 x)^2 = (\sec^2 x - 1)^2$

$\int \tan^5 x \sec^2 x (\sec^2 x\, dx)$

$\int \tan^5 x (\tan^2 x + 1)(\sec^2 x\, dx)$

$u = \tan^2 x$

$du = 2 \tan x \sec^2 x$

$$\int u^5 (u^2 + 1)\, du = \int (u^7 + u^5)\, du = \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \frac{1}{8} \tan^8 x + \frac{1}{6} \tan^6 x + C$$

b. $\int \tan^5 x \sec^5 x \, dx =$

$\int \tan^4 x \sec^4 x (\sec x \tan x) dx$ ó $\int \tan^5 x \sec^3 x (\sec^2 x \, dx)$

✓ ✗

$\int (\tan^2 x)^2 \sec^4 x (\sec x \tan x \, dx)$ $\tan^2 x = \sec^2 x - 1$

$\int (\sec^2 x - 1)^2 \sec^4 x (\sec x \tan x \, dx)$

$u = \sec x$ $du = \sec x \tan x \, dx$

$\int (u^2 - 1)^2 u^4 \, du = \int (u^4 - 2u^2 + 1) u^4 \, du$

$= \int u^8 - 2u^6 + u^4$

$= \frac{u^9}{9} - \frac{2u^7}{7} + \frac{u^5}{5} + C$

$\frac{1}{9} \sec^9 x - \frac{2}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C$

c. $\int \tan^4 x \sec^4 x \, dx$

$= \int \tan^4 x \underbrace{\sec^2 x}_{\tan x^?} \sec^2 x \, dx$

$\sec^2 x = \tan^2 x + 1$

$= \int \tan^4 x (\tan^2 x + 1) \sec^2 x dx$

$u = \tan x$ $du = \sec^2 x$

$= \int u^4 (u^2 + 1) \, du$

$= \frac{1}{7} u^7 + \frac{1}{5} u^5 + C = \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$

Casos especiales $\int \tan^m x \, dx$ $\int \sec^n x$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{du}{u} = -\ln|u| + C$$

$w = \cos x$
$dw = -\sin x$

$$= -\ln(\cos x) + C$$

$$\int \sec x \, dx = \int \sec x \frac{(\sec x + \tan x)}{\sec x + \tan x} dx = \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x} dx$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$u = \tan x + \sec x$
$du = \sec^2 x + \sec x \tan x \; dx$

$$= \ln|\tan x + \sec x| + C$$

$$\int \csc x \, dx = -\ln|\csc + \cot x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C$$

$$\int \tan^3 x \, dx = \int \tan^2 x \tan x \, dx = \int (\sec^2 x - 1) \tan x \, dx$$

$u = \tan x$ $du = \sec^2 x$

$$= \int \sec^2 x \tan x - \tan x \, dx$$

$$= \int \underset{u}{\tan x} \, \underset{du}{\sec^2 x \, dx} - \int \tan x \, dx$$

$$= \frac{1}{2} \tan^2 x + \ln|\cos x| + C$$

$$\int \sec^3 x\,dx = \int \sec^2 x \sec x\,dx$$

IPP $\quad u = \sec x \qquad dv = \sec^2 x\,dx$

$\qquad du = \sec x \tan x \qquad v = \tan x$

$$= \sec x \tan x - \int \tan^2 x \sec x\,dx$$

$$= \int \tan^2 x \sec x\,dx = \int \sec^2 x - 1 \sec x\,dx$$

$$= \int \sec^3 x - \sec x\,dx$$

$$= \int \sec^3 x\,dx - \int \sec x\,dx$$

$$= \frac{\sec x \tan x + \ln|\sec x + \tan x| + C}{2}$$

2019-08/13

Identidades Trigonométricas

$$\begin{aligned} \operatorname{sen}^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \\ \cot^2 x + 1 &= \csc^2 x \\ \sec^2 x - 1 &= \tan^2 x \end{aligned}$$

$$\begin{aligned} \operatorname{sen} 2x &= 2 \operatorname{sen} x \cos x \\ \cos 2x &= \cos^2 x - \operatorname{sen}^2 x \\ \operatorname{sen}^2 x &= \tfrac{1}{2}\Big(1 - \cos 2x\Big) \\ \cos^2 x &= \tfrac{1}{2}\Big(1 + \cos 2x\Big) \end{aligned}$$

Técnicas de Integración

- 5.5 Regla de la Sustitución

$$\int f(g(x)\ g'(x)\ dx = \int f(u)\ du$$

- 7.1 Integración por Partes

$$\int u\ dv = uv - \int vdu$$

- 7.2 Integración Trigonométrica

  a. **Potencias Impares de Seno o Coseno:** Aparte un término $\operatorname{sen} x$ o $\cos x$ y utilice la identidad $\operatorname{sen}^2 x + \cos^2 x = 1$.

  b. **Potencias Pares de Seno o Coseno:** Utilice la identidad

  $$\operatorname{sen}^2 x = \tfrac{1}{2}\Big(1 - \cos 2x\Big) \qquad y/o \qquad \cos^2 x = \tfrac{1}{2}\Big(1 + \cos 2x\Big).$$

  c. **Potencia Par de tangente:** Aparte $\sec^2 x$ y use $\sec^2 x = \tan^2 x + 1$.

  d. **Potencia Impar de tangente:** Aparte $\sec x \tan x$ y use $\tan^2 x = \sec^2 x - 1$.

  e. **Potencia Par de cosecante:** Aparte $\csc^2 x$ y use $\csc^2 x = \cot^2 x + 1$.

  f. **Potencia Impar de cotangente:** Aparte $\cot x \tan x$ y use $\cot^2 x = \csc^2 x - 1$.

  g. **Productos** $\operatorname{sen}(mx)$ **y** $\cos(nx)$**:** Utilice la identidad trigonométrica adecuada.

  $$\begin{aligned} \operatorname{sen} A \cos B &= \tfrac{1}{2}\big[\ \operatorname{sen}(A-B) + \operatorname{sen}(A+B)\ \big] \\ \operatorname{sen} A \operatorname{sen} B &= \tfrac{1}{2}\big[\ \cos(A-B) - \cos(A+B)\ \big] \\ \cos A \cos B &= \tfrac{1}{2}\big[\ \cos(A-B) + \cos(A+B)\ \big] \end{aligned}$$

- 7.3 Sustitución Trigonométrica

  a. $x = a \operatorname{sen}\theta$ sustituye $a^2 - u^2$ por $a^2\cos^2\theta$ y $dx = a\cos\theta\ d\theta$.

  b. $x = a \tan\theta$ sustituye $a^2 + u^2$ por $a^2\sec^2\theta$ y $dx = a\sec^2\theta\ d\theta$.

  c. $x = a \sec\theta$ sustituye $u^2 - a^2$ por $a^2\tan^2\theta$ y $dx = a\sec\theta\tan\theta\ d\theta$.

  d. Hay otros casos que requieren el trazo de un triángulo apropiado.

Integrales Indefinidas Básicas

$$\int af(x)\,dx = a\int f(x)\,dx \qquad \int (f(x)\pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C \qquad \int \frac{1}{x}\,dx = \ln|x|+C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a}+C$$

$$\int \operatorname{sen} x\,dx = -\cos x + C \qquad \int \cos x\,dx = \operatorname{sen} x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\operatorname{sen} x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec^3 x\,dx = \frac{\ln|\sec + \tan x| + \sec x\tan x}{2} + C$$

$$\int \csc^3 x\,dx = -\frac{\ln|\csc + \cot x| + \csc x\cot x}{2} + C$$

$$\int \sec x\tan x\,dx = \sec x + C \qquad \int \csc x\cot x\,dx = -\csc x + C$$

$$\int \operatorname{senh} x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \operatorname{senh} x + C$$

$$\int \operatorname{sech}^2 x\,dx = \tanh x + C \qquad \int \operatorname{csch}^2 x\,dx = -\operatorname{coth} x + C$$

$$\int \operatorname{sech} x\ \tanh x\,dx = -\operatorname{sech} x + C \qquad \int \operatorname{csch} x\ \operatorname{coth} x\,dx = -\operatorname{csch} x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{sen}^{-1}(x) + C$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{a}\operatorname{sen}^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1}(x) + C \qquad \int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C$$

100

Corto #5 Cálculo Integral (10 min)

Nombre: David Gabriel Corzo Mcwath **Carnet:** 20190432

Determine si la integral es convergente o divergente. Evalúe si es convergente. Utilice la regla de L'Hospital para evaluar límites con formas indeterminadas.

1. (100 pts.) $\int_{-\infty}^{0} xe^x \, dx$

$u = x \qquad dv = e^x$

$du = dx \qquad v = e^x$

$$= xe^x - \int_{-\infty}^{0} e^x \, dx = xe^x - e^x \Big]_{-\infty}^{0}$$

$$= \left\{ \underbrace{\lim_{a \to 0} (xe^x - e^x)}_{0 \cdot e^0 - 1} \right\} - \left\{ \lim_{a \to -\infty} \underbrace{(xe^x - e^x)}_{-\infty e^{-\infty} - \infty} \right\}$$

forma indeterminada

$$\underbrace{\lim_{a \to -\infty} (xe^x)}_{} - \lim_{a \to -\infty} (e^x)$$

$$\underbrace{\lim_{a \to -\infty} \left(\frac{x}{e^x} \right)}_{\text{L'Hopital}} - \underbrace{\lim_{a \to -\infty} \left(e^{-\infty} \right)}_{}$$

$$\lim_{a \to \infty} \left(\frac{1}{e^{\infty}} \right) = 0$$

$$\lim_{a \to -\infty} \left(\frac{e^x}{x} \right) = \lim_{a \to -\infty} \left(\frac{e^x}{1} \right) =$$

$$= \lim_{a \to -\infty} \left(e^{-\infty} \right) = \lim_{a \to -\infty} \left(\frac{1}{e^{\infty}} \right) = 0 \qquad \therefore \left\{ \lim_{a \to -\infty} (xe^x - e^x) \right\} = 0$$

en conclusión

$\therefore$

$$= \{ 0 \cdot e^0 - e^0 \} - \{0\} = \{ \cancel{0 \cdot 1} - 1 \} - \{0\}$$

$$= \{-1\} - 0 = -1 - 0$$

$$= \underline{-1}$$

Convergente por la respuesta ser -1

Simulacro de Parcial #1,
Cálculo Integral

Lunes, 26 de Agosto

Nombre y Apellidos: ______________________________

Tema:	1	2	3	4	5	6	Total
Puntos:	40	20	15	15	20	10	120
Nota:							

1. Evalúa las siguientes integrales indefinidas.

   (a) (10 pts.) $\int x \tan^{-1} x^2 \, dx$

   (b) (10 pts.) $\int \frac{x^2}{\sqrt{9-25x^2}} \, dx$

   (c) (10 pts.) $\int \frac{dt}{\sqrt{(t-2)^2+9}}$ dt

   (d) (10 pts.) $\int \frac{xe^x}{(x+1)^2} \, dx$

2. Evalúa las siguientes integrales definidas.

   (a) (10 pts.) $\int_{\pi/4}^{0} \tan^5 \theta \sec^3 \theta \, d\theta$

   $\int_0^{\pi/4}$

   (b) (10 pts.) $\int_{\pi/2}^{0} \frac{\cos t}{\sqrt{1+\sin^2 t}} \, dt$

3. Considere la región entre $f(x) = |x-8|$, el eje-x, y las rectas verticales $x = 0$, $x = 8$.

   (a) (5 pts.) Trace la gráfica de la región.

   (b) (5 pts.) Encuentre el área de la región utilizando geometría.

   (c) (5 pts.) Planteé la integral para encontrar el área de la región.

4. La función de aceleración (en m/s^2) para una partícula moviéndose en una recta es $a(t) = \frac{3}{\sqrt{2t+1}}$.

   (a) (5 pts.) Encuentra la velocidad de la partícula si su velocidad inicial es de -3 m/s.

   (b) (5 pts.) Encuentra la función de desplazamiento de la partícula si su posición a los 2 segundos es de 8 m en la dirección positiva.

   (c) (5 pts.) ¿Cuál es la posición de la partícula a los 7 segs?

5. La velocidad de una partícula (en metros por segundo) sobre una línea recta es $v(t) = 1 - (t-2)^2$ para $0 \leq t \leq 2$. Encuentra:

   (a) (10 pts.) El desplazamiento de la partícula en el intervalo de tiempo dado

   (b) (10 pts.) La distancia recorrida de la partícula en el mismo intervalo de tiempo

6. (10 pts.) Calcule la ec. de la recta tangente a la curva de $f(x) = \int_{\sin x}^{2e^x - 2} \sqrt{t^2+2t+4} \, dt$ en $x = 0$.

David Corzo
20190432

Actividad especial:

$$A = 2\int_0^1 x^{1/4}\,dx + 2\int_1^4 \left(\frac{4-x}{3}\right)^{1/2} dx = \frac{28}{5}$$

$$A = 2\left\{\left\{\frac{x^{5/4}}{5/4}\right\}\Big]_0^1\right\} + 2\int_1^4 \left(\frac{4-x}{3}\right)^{1/2} dx$$

$$2\left(\left\{\frac{4}{5}(1)^{5/4}\right\} - \left\{\cancel{\frac{0^{5/4}}{5/4}}\right\}\right) =$$

$$= 2\left(\left\{\frac{4}{5} - 0\right\}\right) = 2\left(\frac{4}{5}\right)$$

$$= \frac{8}{5}$$

$u(4) = 0$

$$u = \frac{4-x}{3} = \frac{4}{3} - \frac{x}{3} \qquad u(1) = 1$$

$$du = 0 - \frac{1}{3}dx \qquad du = -\frac{1}{3}dx$$

$$-3\,du = dx$$

$$-3\,du = dx$$

$$= 2\int_1^0 \sqrt{u} \cdot -3\,du =$$

$$= -6\int_1^0 \sqrt{u}\,du$$

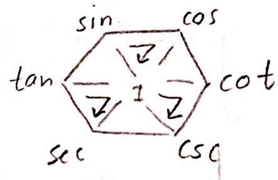
$$= 6\int_0^1 \sqrt{u}\,du = 6\left(\frac{2u^{3/2}}{3}\right)\Big]_0^1 =$$

$$= 6\left(\left\{\frac{2}{3} - 0\right\}\right) = 6\cdot\frac{2}{3} = \frac{12}{3} = 4$$

$$= \frac{8}{5} + 4 = \frac{28}{5}$$

David Gabriel Corzo Mcmath
20190432

Laboratorio # 4



$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

100 pts

(1) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin^3\left[(x)^{1/2}\right] \cdot (x)^{-1/2}\,dx$

$u = \sqrt{x} = (x)^{1/2}$

$du = \frac{1}{2}(x)^{-1/2}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$= \displaystyle\int \sin^3(u) \cdot 2\,du$

$= 2\displaystyle\int \sin^3(u)\,du$

$= 2\displaystyle\int \sin^2(u) \cdot \sin(u)\,du$

$= 2 \cdot \displaystyle\int \left[1 - \cos^2(u)\right] \cdot \sin(u)\,du$

$w = \cos(u)$

$dw = -\sin(u) \quad -dw = \sin(u)$

$= 2 \displaystyle\int \left[1 - w^2\right] \cdot -dw$

$= 2 \cdot -1 \displaystyle\int \left[1 - w^2\right] \cdot dw$

$= -2\left\{\displaystyle\int 1\,dw - \int w^2\,dw\right\}$

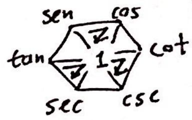
$= -2\left\{w - \frac{w^3}{3}\right\}$

$= -2w + \frac{2w^3}{3}$ sustituir w por correspondiente

$= -2\cos(u) + \frac{2\cos^3(u)}{3}$

$= -2\cos(\sqrt{x}) + \frac{2\cos^3(\sqrt{x})}{3} + C$

② $$\int \cos^4(\theta)\tan^2(\theta)\,d\theta = \int (\cos^2\theta)^2 \tan^2\theta\, d\theta$$



$\sin^2 + \cos^2 = 1$

$1 + \cot^2 = \csc^2$

$\tan^2 + 1 = \sec^2$

$\longrightarrow \tan^2 = \sec^2 - 1$

or

$\longrightarrow \cos^2 = 1 - \sin^2$

$$= \int \cos^4(\theta)\frac{\sin^2(\theta)}{\cos^2(\theta)}\,d\theta = \int \cancel{\cos^2\theta}\,\cos^2\theta\,\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\,d\theta$$

$$= \int \cos^2\theta\cdot\sin^2\theta\,d\theta = \int \cos^2\theta\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$

$$= \int\left[\frac{1+\cos(2\theta)}{2}\right]\left[\frac{1-\cos(2\theta)}{2}\right]d\theta$$

$$= \int\left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right)\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$ simplificar $(a-b)(a+b) = a^2 - b^2$

$$= \int\left(\frac{1}{2}\right)^2 - \left(\frac{\cos(2\theta)}{2}\right)^2 d\theta = \int \frac{1}{4}\,d\theta - \int \frac{\cos^2(2\theta)}{4}\,d\theta$$

$$= \frac{\theta}{4} - \frac{1}{4}\underbrace{\int \cos^2(2\theta)\,d\theta}_{\substack{\alpha = 2\theta \\ \frac{d\alpha}{2} = d\theta}} = \frac{\theta}{4} - \frac{1}{4}\int \cos^2(\alpha)\,d\alpha$$

$$= \frac{\theta}{4} - \frac{1}{4}\left\{\int \frac{1}{2} + \frac{\cos^2(2\alpha)}{2}\,d\alpha\right\}$$

$$= \frac{\theta}{4} - \frac{1}{4}\left\{\int \frac{1}{2}\, d\alpha + \frac{1}{2}\int \cos^2(2\alpha)\, d\alpha\right.$$

$$= \frac{\theta}{4} - \frac{1}{4} \cdot \int \frac{1}{2}\, d\alpha + \frac{1}{4} \cdot \frac{1}{2} \int \cos(4\theta)\, d\alpha$$

$$= \frac{\theta}{4} - \frac{\theta}{8} + \frac{1}{8} \sin(4\theta) \cdot \frac{1}{4} + C$$

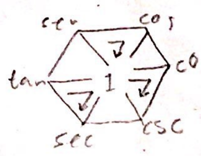
$$= \frac{2\theta - \theta}{8} + \frac{1}{32} \sin(4\theta) + C$$

$$= \frac{\theta}{8} + \frac{\sin(4\theta)}{32} + C$$

$$= \frac{1}{8}\left(\theta - \frac{\sin(4\theta)}{4}\right) + C$$

③ $$\int \cos^3(\sin\theta)\,\sin^4(\sin\theta)\cos\theta\, d\theta$$

$$\left.\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\, d\theta \end{aligned}\right\} \therefore \int \cos^3(u)\,\sin^4(u)\, du$$



$$\operatorname{sen}^2 + \cos^2 = 1$$
$$\cos^2 = 1 - \operatorname{sen}^2$$

$$= \int \cos^3(u)\,\sin^4(u)\, du$$

$$= \int (1-\operatorname{sen}^2(u))\sin^4(u)\cos(u)\, du$$

$$= \int \sin^4(u)\cos(u) - \sin^6(u)\cos(u)\, du$$

$$= \int \sin^4(u)\cos(u)\, du - \int \sin^6(u)\cos(u)\, du$$

$$\underbrace{\begin{aligned} u &= \sin(u) \\ du &= \cos(u)\, du \end{aligned}}_{} \qquad \underbrace{\begin{aligned} u &= \sin(u) \\ du &= \cos(u) \end{aligned}}_{}$$

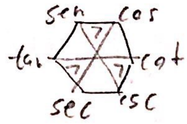
$$\int u^4\, du - \int u^6\, du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= \frac{\sin^5(u)}{5} - \frac{\sin^7(u)}{7} + C$$

$$= \frac{\sin^5(\sin\theta)}{5} - \frac{\sin^7(\sin\theta)}{7} + C$$

④ $\int \tan^5(x)\sec^4(x)\,dx =$



$\tan^2 + 1 = \sec^2$

$= \int \tan^5(x)\sec^2(x)\sec^2(x)\,dx$

$= \int \tan^5(x)(\tan^2 x + 1)\sec^2(x)\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

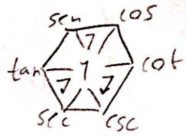
$= \int u^5(u^2+1)\,du = \int u^7 + u^5\,du$

$= \int u^7\,du + \int u^5\,du$

$= \frac{u^8}{8} + \frac{u^6}{6} + C$

$= \frac{\tan^8(x)}{8} + \frac{\tan^6(x)}{6} + C$ ✗

⑤ $\int \sec^4(x)\,dx =$



$\tan^2 + 1 = \sec^2$

$= \int \sec^2(x) \left[\tan^2 + 1\right] dx$

$= \int \sec^2(x) \tan^2(x) + \sec^2(x)\,dx$

$= \int \sec^2(x) \tan^2(x)\,dx + \int \sec^2(x)\,dx$

$\underbrace{\begin{array}{l} u = \tan(x) \\ du = \sec^2(x) \end{array}}$ $\quad + \tan(x)$

$= \int u^2\,du + \tan(x)$

$= \frac{u^3}{3} + \tan(x) + C$

$= \frac{\tan^3(x)}{3} + \tan(x) + C$

Problemas Variados de Integración

Problema 3: Encuentre las siguientes integrales.

a.) $\displaystyle\int \sqrt{64x} - \frac{1}{\sqrt{64x}}\, dx$

b.) $\displaystyle\int \left(\frac{7x^2}{7x^3 + 8} - \frac{x^3}{(x^4 + 8)^5} \right) dx$

c.) $\displaystyle\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx$

d.) $\displaystyle\int 5 \frac{(x^{1/3} + 2)^4}{x^{2/3}}\, dx$

e.) $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

David Corzo
20190432

a) $\int \sqrt{64x} - \frac{1}{\sqrt{64x}}\, dx$

+10

$\int \sqrt{64x}\, dx - \int \frac{1}{\sqrt{64x}}\, dx$

$u = 64x$ $\quad$ $u = 64x$

$du = 64\, dx$ $\quad$ $du = 64$

$\frac{du}{64} = dx$ $\quad$ $\frac{du}{64} = dx$

$\int \sqrt{64x}\, dx = \int \sqrt{u}\, \frac{du}{64} = 64 \int \frac{(u)^{1/2+1}}{1/2+1}\, du$

$64 \cdot \frac{u^{3/2}}{3/2}$

$\frac{\frac{64u^{3/2}}{1}}{\frac{3}{2}} = \frac{64 \cdot 2u^{3/2}}{3}$

$\int \frac{1}{\sqrt{64x}}\, dx = \frac{1}{64} \int \frac{1}{\sqrt{u}}\, du$

$u = 64x$

$du = 64dx$

$\frac{du}{64} = dx$

$\frac{1}{64}\sqrt{u} - \frac{64 \cdot 2u^{3/2}}{3}$

$\frac{\sqrt{64x}}{64} - \frac{64 \cdot 2(64x)^{3/2}}{3} + C$

b) $\int \left(\frac{7x^2}{7x^3+8} - \frac{x^3}{(x^4+8)^5} \right) dx$

$u = 7x^3 + 8$ $\quad$ $u = x^4 + 5$

$du = 7x^2$ $\quad$ $du = x^3$

$\int \frac{du}{u} - \int \frac{du}{u^5}$

$(\ln(u)) - \left(\frac{u^{-4}}{-4}\right) = \ln(u) + \frac{1}{4u^4} + C$

$= \ln(7x^3+8) + \frac{1}{4(x^4+8)^4} + C$

c. $\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx = \sqrt[3]{x}\, e^{2(x)^{4/3}}\, dx$

$u = 2(x)^{4/3}$

$du = 2x^{1/3}\, dx$

$\frac{du}{2} = x^{1/3}\, dx$

$= \frac{1}{2}\int e^{u}\, du$

$= \frac{1}{2} e^{u} + c$

$= \frac{1}{2} e^{2\sqrt[3]{x^4}} + C$

d. $\int 5 \frac{(x^{1/3}+2)^4}{x^{2/3}}\, dx = 5\int (x^{1/3}+2)^4\, x^{-2/3}\, dx$

$u = x^{1/3} + 2$

$du = x^{-2/3}\, dx$

$= 5\int u^4 \cdot du$

$= \frac{\cancel{5}\, u^5}{\cancel{5}} = (x^{1/3}+2)^5 + C$

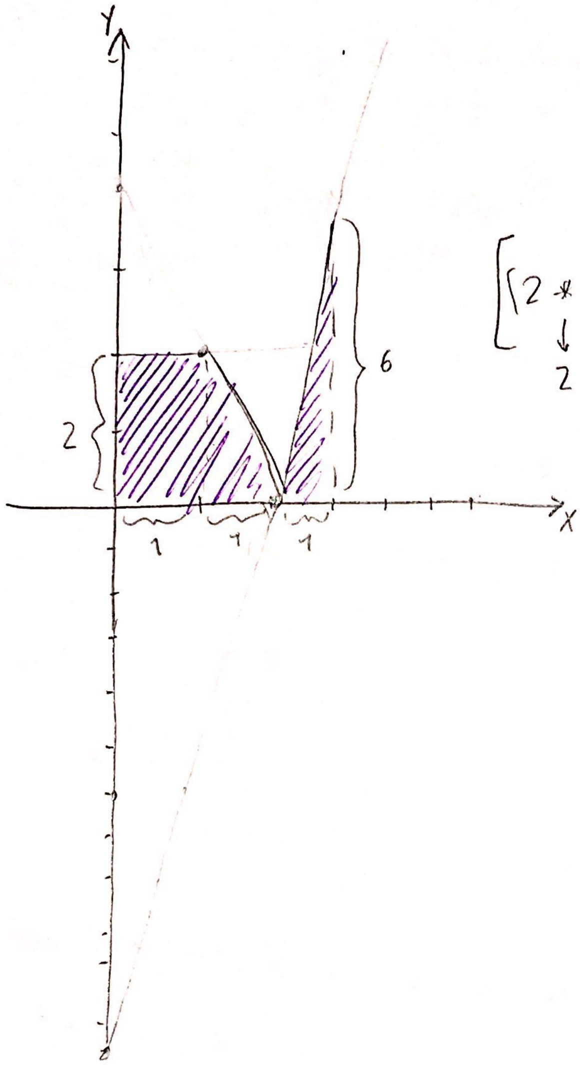
e) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx = \int \frac{1}{u}\, du = \ln(u)$

$= \ln(e^x + e^{-x}) + C$

$u = e^x + e^{-x}$

$du = e^x - e^{-x}\, dx$

David Gabriel Corzo Mcmath



$$\left[(2 * 1)\right] + \left[(2 * 1)\tfrac{1}{2}\right] + \left[(6 * 1)\tfrac{1}{2}\right]$$

$$2 + 1 + 3 = \underline{6}$$

10

Laboratorio # 2 David Gabriel Corzo Mcmath

20190432 - 2019/08/05

(1) a) $f(x) = \int_{1}^{x} \frac{1}{t^3+1}\,dt = \frac{1}{x^3+1} \cdot 1$ 5

110 pts *

b) $h(r) = \int_{-100}^{r} \sqrt{x^2+4}\,dx = \sqrt{r^2+4} \cdot 1$ 5

c) $i(x) = \int_{0}^{x^4} \cos^2(\theta)\,d\theta = \cos^2(x^4) \cdot 4x^3$ 5

d) $j(x) = \int_{\sec x}^{\tan x} \sqrt{t+\sqrt{t}}\,dt =$

$\sqrt{(\tan x)+\sqrt{(\tan x)}} \cdot \sec^2 x - \sqrt{\sec x + \sqrt{\sec x}} \cdot \sec x \tan x$ 10

e) $K(x) = \int_{x^3-x}^{x^4+x} \frac{u^3}{1+u^2}\,dt =$

$\left\{ \frac{(x^4+x)^3}{1+(x^4+x)^2} \cdot (4x^3+1) \right\} - \left\{ \frac{(x^3-x)^3}{1+(x^3-x)^2} \cdot (3x^2-1) \right\}$ 10

② $f(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dx$; recta tangente en $x = 1$

ecuación

$f'(x) = \sin\left(\frac{\pi}{2} x^2\right) \cdot 1$

$y = f(1) + f'(1)(x-1)$

$y = 0.4382591 + 1(x-1)$

$y = x - 1 + 0.4382591$

$\underline{y = x - 0.5617409}$

15

③ $C'(x) = 3000 + 2x + \frac{3}{10} x^2$

$\int_{10}^{20} C'(x) = \int_{10}^{20} 3000 + 2x + \frac{3}{10} x^2 \, dx$

$\int_{10}^{20} C'(x) = \int_{10}^{20} 3000 dx + \int_{10}^{20} 2x \, dx + \int_{10}^{20} \frac{3}{10} x^2 \, dx$

$C(x) = 3000x + \frac{2x^2}{2} + \frac{3}{10} \cdot \frac{x^3}{3}$

$C(x) = 3000x + x^2 + \frac{x^3}{10} + C \Big]_{10}^{20}$

$\left\{ 3000(20) + (20)^2 + \frac{(20)^3}{10} \right\} - \left\{ 3000(10) + (10)^2 + \frac{(10)^3}{10} \right\}$

15

$61200 - 30200 = \underline{\underline{31000}}$ de aumento cuando se incrementa de 10 a 20 yardas

④ $P'(t) = 40\sqrt[3]{t}$

$$\int_0^8 P'(t) = \int_0^8 40\sqrt[3]{t}\,dt$$

$$= 40\int_0^8 \frac{(t)^{1/3+1}}{1/3+1}\,dt$$

$$= 40\ \frac{t^{4/3}}{4/3}$$

$$= \frac{40\cdot 3\,(t^{4/3})}{4}$$

$$= \frac{\cancel{4}\cdot 10\cdot 3\,(t^{4/3})}{\cancel{4}}$$

$$= 30\,(t^{4/3})\Big]_0^8 = \{30\,(8^{4/3})\} - \{30\,(0^{4/3})\}$$

15

$= 480$ macetitas de cemento

⑤ $v(t) = 3t - 6 \quad , \quad 0 \le t \le 3$

a) $I_x ;\ y = 0$

$3t - 6 = 0$

$3t = 6$

$t = \frac{6}{3} = 2$

$I_y ;\ x = 0$

$3(0) - 6 = y$

$-6 = y$

$$\int_0^3 3t - 6\ dt = 3\int t\,dt - \int 6\,dt =$$

$$= \frac{3t^2}{2} - 6t\Big]_0^3 = \left\{\frac{3}{2}(3)^2 - 6(3)\right\} - \left\{\frac{3(0)}{2} - 6(0)\right\}$$

$$= -\frac{9}{2}\ \text{m}$$

5

b) $\int 3t-6\ dt$

$d = \frac{3t^2}{2} - 6t$

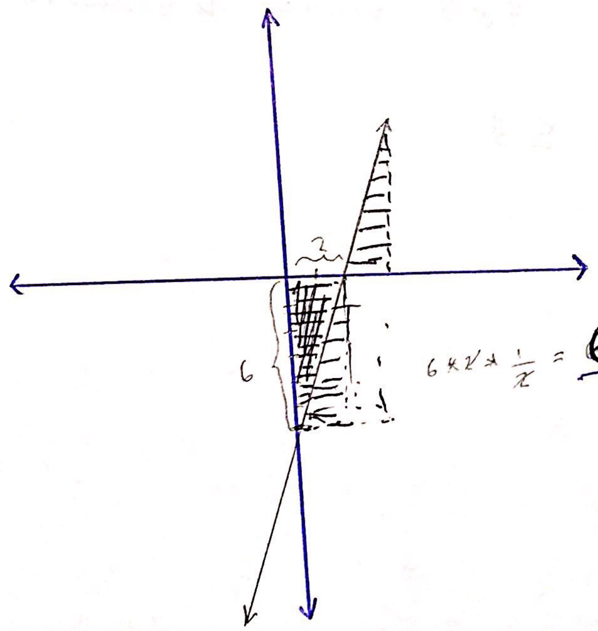
$d = -\frac{3t^2}{2} + 6t \Big]_0^2 + \frac{3t^2}{2} - 6t \Big]_2^3$

$\left\{\left[-\frac{3(2)^2}{2} + 6(2)\right] - [0]\right\} + \left\{\left[\frac{3(3)^2}{2} - 6(3)\right] - \left[\frac{3(2)^2}{2} - 6(2)\right]\right\}$

$\left\{-\frac{12}{2} + 12\right\} + \left\{\frac{27}{2} - 18 - \frac{12}{2} + 12\right\}$

$6 + \frac{3}{2} = \frac{15}{2}\,m$ 10

c)



$A = \frac{6(2)}{2} + \frac{3(1)}{2} = \frac{15}{2}$

$6 \times 2 \times \frac{1}{2} = 6\ m^2$ 5

⑥ a) $f(x) = \int_{-\pi/3}^{\pi/3} \left(\frac{3}{5}(x^3 + x)^5 - 2x^4 \sin x \right) dx$

$= 0$ porque es impar.

5

b) $g(x) = \frac{1}{\pi} \int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\, dx$

$\frac{1}{\pi} \left(\tan^{-1}(x) \right) \Big]_0^{\sqrt{3}} = \left\{ \left\{ \frac{1}{\pi} \tan^{-1}(\sqrt{3}) \right\} - \left\{ 0 \right\} \right\} * 2$

$\frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{3} = \frac{1}{3} * \frac{2}{1} = \frac{2}{3}$ A

5

Laboratorio # 1

29/07/2019

20190432
David Corzo

100 pts *

① a) $\int (\sqrt{x} + 2)(\sqrt{x} - 2)(x + 4)\, dx$

$\int (x - 4)(x + 4)\, dx$

$\int (x^2 - 16)\, dx$

$\int x^2\, dx - \int 16\, dx$

$\frac{x^{2+1}}{2+1} - 16x = \frac{x^3}{3} - 16x + C$

6pts

b) $\int \frac{3x^{3/2} + x + 3\sqrt{x}}{x^2}\, dx$

$\int \frac{3x^{3/2}}{x^2} + \frac{x}{x^2} + \frac{3(x)^{1/2}}{x^2}\, dx$

$\frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$

$\int \left[3x^{-1/2} + \frac{1}{x} + 3x^{-3/2}\right] dx$

$\left\{3\int x^{-1/2}\, dx\right\} + \left\{\int x^{-1}\, dx\right\} + \left\{3\int x^{-3/2}\, dx\right\}$

$\frac{3 \cdot x^{1/2}}{1/2} + \frac{x^2}{2} + \frac{3 \cdot x^{-1/2}}{-1/2} = 6\sqrt{x} + \ln(x) - 6x^{-1/2} + C$

6pts

c) $\int \left(e^{\pi}\sin(x) + \tan(5)\sinh(x) - 5\cdot\pi^{x}\right) dx$

$= \left\{e^{\pi}\int \sin(x)\,dx\right\} + \left\{\tan(5)\int \sinh(x)\,dx\right\} - \left\{5\int \pi^{x}\,dx\right\}$

$\cos x \overset{?}{=} -\sin x$
$\sin x = \cos x$

$= -e^{\pi}\cos(x) + \tan(5)\cosh(x) - 5\cdot\frac{\pi^{x}}{\ln(\pi)} + C$

6pts

7

d) $\int_{1/2}^{1} \frac{4u + u^{2}}{u^{4}}\,du$

$= \int_{1/2}^{1} \frac{4u}{u^{4}} + \frac{u^{2}}{u^{4}}\,du$

$= \int_{1/2}^{1} \frac{4u}{u^{4}}\,du + \int \frac{u^{2}}{u^{4}}\,du$

$\frac{1}{u^{2}} = \frac{u^{-2+1}}{-1} = \frac{u^{-1}}{-1}\Big]_{1/2}^{1} = -\frac{1}{u} = \left[-\frac{1}{(1)^{2}} - \left(-\frac{1}{(1/2)^{2}}\right)\right]$

6pts

$= 4 * u^{-3}\Big]_{1/2}^{1} + u^{-2}\Big]_{1/2}^{1} =$

$\cancel{7}$

$+ \cancel{(1/3)} = \cancel{3}$

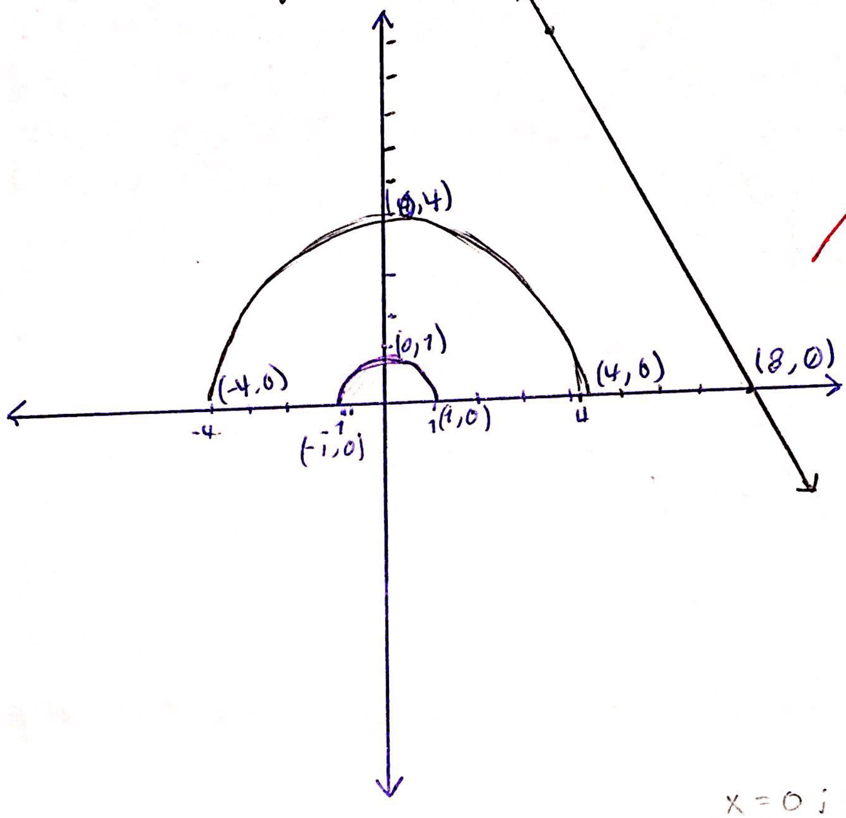
$= \frac{4u^{-2}}{-2}\Big]_{1/2}^{1}\ \underbrace{\left\{\left(\frac{4}{(1)^{2}\cdot -2}\right) - \left(\frac{4}{(1/4)\cdot -2}\right)\right\}}_{6} + \underbrace{\left\{\cancel{\left(\frac{1}{(1)^{2}}\right)} - \cancel{\left(\frac{1}{(1/2)^{2}}\right)}\right\}}_{1}$

$\left(\frac{4}{-2}\right) - \left(\frac{4}{\frac{1}{4}\cdot\frac{-2}{1}}\right)$

$(-2) - \left(\frac{\frac{4}{1}}{\frac{-2}{4}}\right) = -2 - \left(\frac{8}{-2}\right) = -2 + \frac{8}{2}$

2.) $\int_{-1}^{0}\sqrt{1-x^2}\,dx + \int_{0}^{4}\sqrt{16-x^2}\,dx + \int_{6}^{8}(16-2x)\,dx$

ⓐ Realizar la gráfica indicando interceptos



15pts

$y=0 \; ; \; x=0$

$\sqrt{1-x^2} = y$

$\sqrt{1-0} = y$

$\pm 1 = y$

$\sqrt{1-x^2} = 0$

$1-x^2 = 0$

$-x^2 = -1$

$x = \sqrt{1}$

$x = \pm 1$

$y=0 \; ; \; x=0$

$\sqrt{16-x^2} = y$

$\sqrt{16-0^2} = y$

$\pm 4 = y$

$\sqrt{16-x^2} = 0$

$16-x^2 = 0$

$x^2 = 16$

$x = \sqrt{16}$

$x = \pm 4$

$x=0 \; ; \; y=0$

$16-2x = y$

$\cancel{16 = y}$

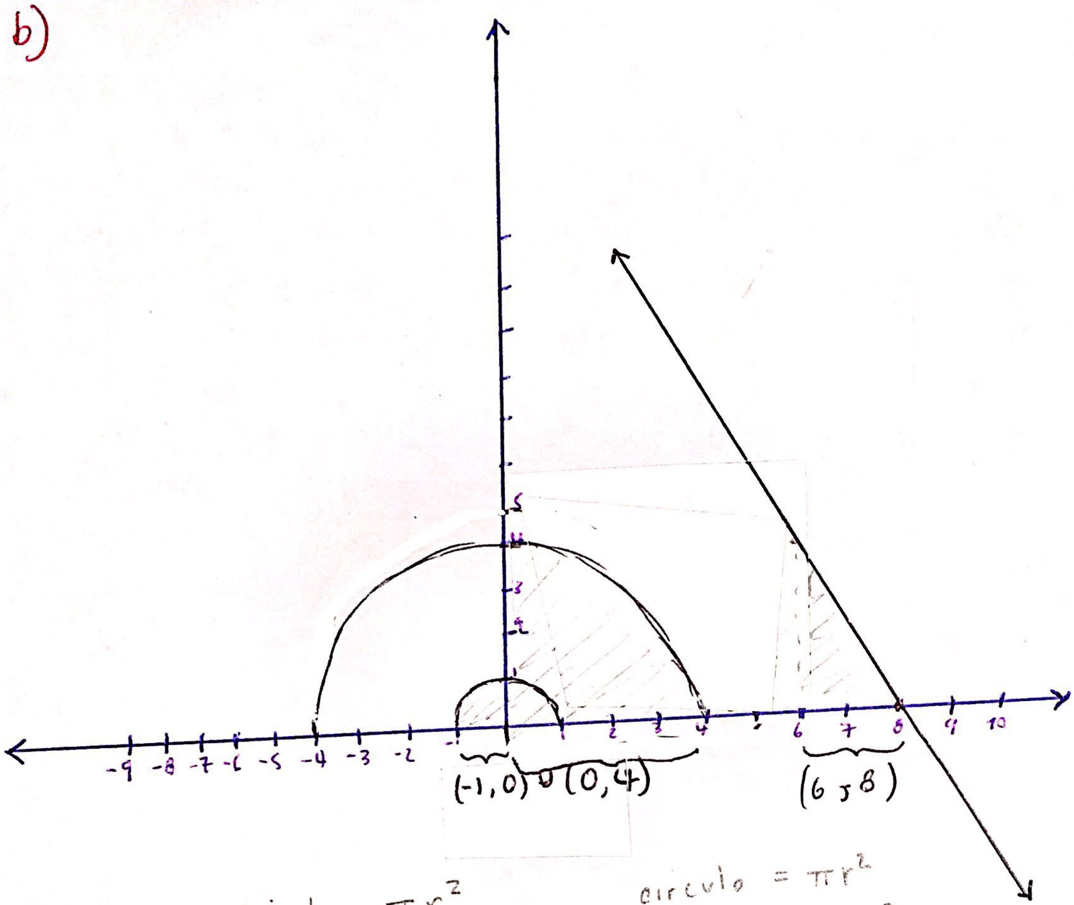
$16-2x = 0$

$-2x = -16$

$x = \frac{-16}{-2}$

$x = 8$

b)



circulo $= \pi r^2$

$AC = \pi(-1)^2$

$A.C. = \pi$

$AC_1 = \frac{1}{4}\pi$

circulo $= \pi r^2$

$AC_2 = \frac{1}{4}\pi(4)^2$

$AC_2 = \frac{16}{4}\pi$

$AC_2 = 4\pi$

triángulo

$T = \frac{1}{2}bh$

$T = \frac{1}{2}(2)(4)$

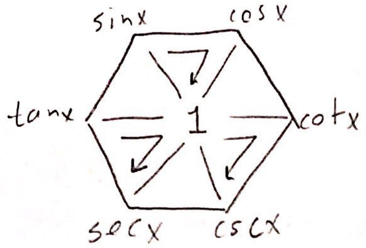
$T = \frac{8}{2}$

$T = 4$

$\frac{1}{4}\pi + 4\pi + 4$

10pts

e) $$\int_0^{\pi/4} \frac{\sec\theta \cot\theta}{\sin\theta}\, d\theta \qquad \theta = x$$



$$\frac{\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}}{\sin\theta} = \left[\frac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{1}}\right] = \frac{1}{\sin^2\theta} = \csc^2\theta$$

$$\int_0^{\pi/4} \csc^2\theta\, d\theta = -\cot\theta\Big]_0^{\pi/4}$$

$[-\cot(\pi/4)] - [-\cot(0)]$ = indefinida

8pts

f) $$\int_{\pi/4}^{\pi/2} \sec t(\sec(t) + \tan(t))\, dt$$

$$\int (\sec^2 t)\,dt + \int (\sec t \cdot \tan t)\, dt$$

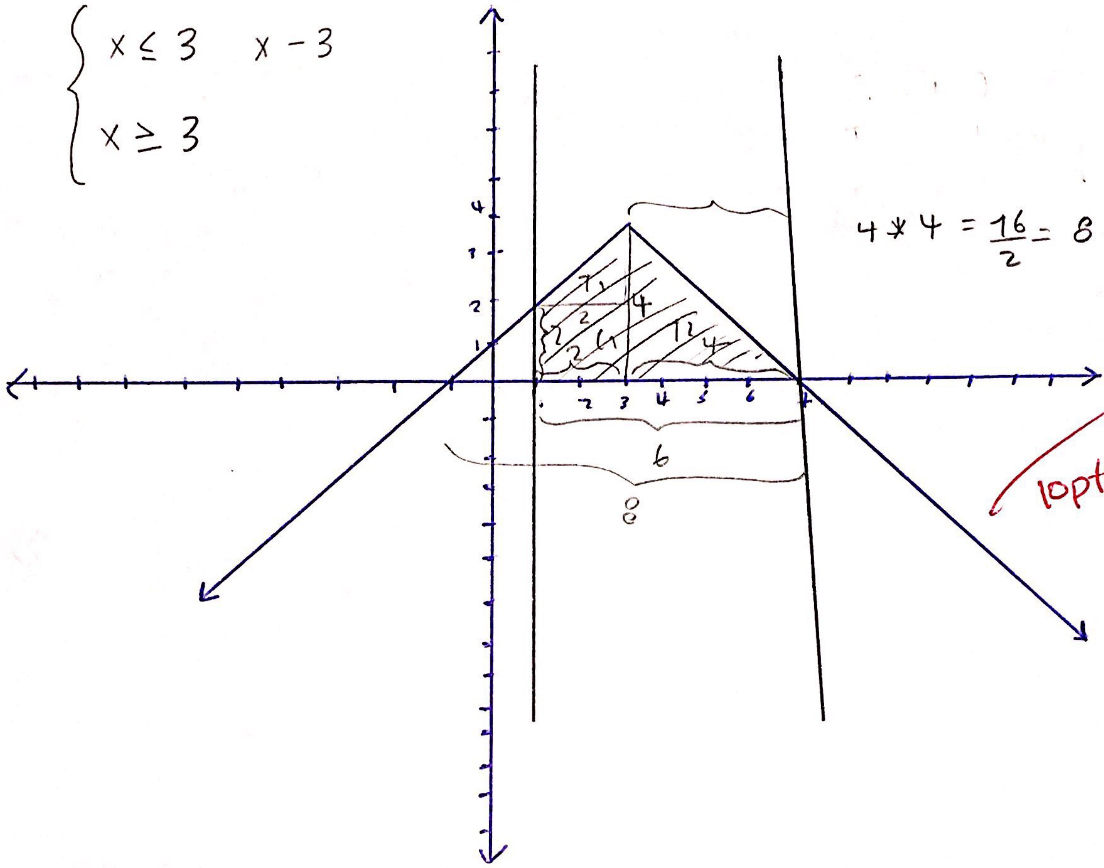
$$(\tan(t)) + (\sec t)\Big]_{\pi/4}^{\pi/2}$$

$$\underbrace{\left[\frac{\sin(\pi/2)}{\cos(\pi/2)} + \frac{1}{\cos(\pi/2)}\right]}_{\text{indefinida}} - \left[\frac{\sin(\pi/4)}{\cos(\pi/4)} + \frac{1}{\cos(\pi/4)}\right] = \text{indefinida}$$

8pts

3 $f(x) = 4 - |x-3|$; $x = 1$ & $x = 7$

$$|x-3| = \begin{cases} x \le 3 & x-3 \\ x \ge 3 & \end{cases}$$



$4 * 4 = \frac{16}{2} = 8 * 2 = \cancel{16 - 2} = 14$

100pts

10pts

srodas1293@gmail.com

$f(x) = 4 - (-x + 3)$

$4 + x - 3$

$f(x) = \int (x + 1)\, dx$

$\int x\, dx + \int 1\, dx$

$$\left.\frac{x^2}{2} + x\right]_1^3 = \left[\frac{3^2}{2} + 3\right] - \left[\frac{1^2}{2} + 1\right] = \frac{9}{2} + 3 - \frac{1}{2} - 1$$

$$\frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4 + 2 = 6$$

$$\int_1^3 (x+1)\, dx = 6$$

15pts

$f(x) = 4 - (x - 3)$

$4 - x + 3$

$f(x) = \int (-x + 7)\, dx$

$\int -x\, dx + \int 7$

$$\left. -\frac{x^2}{2} + 7x \right]_3^7 = -\frac{7^2}{2} + 7(7) - \left[-\frac{3^2}{2} + 3(3)\right]$$

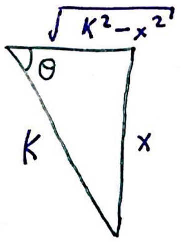
7.3. Sustitución Trigonométrica

2019-08/22

Forma $\sqrt{K^2 - x^2}$

$H = K$

$C.O. = x$

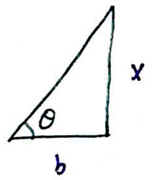


$$\frac{C.O.}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta$$

$$dx = K\cos\theta \, d\theta$$

$$\frac{\sqrt{K^2 - x^2}}{K} = \cos\theta$$

Forma $\sqrt{b^2 + x^2}$



$$\frac{x}{b} = \tan\theta \Rightarrow x = b \cdot \tan\theta$$

$$dx = b \cdot \sec^2\theta \, d\theta$$

$c^2 = x^2 + y^2$

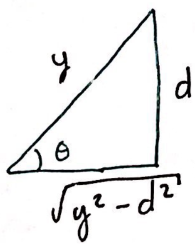
$\sqrt{c^2 - y^2} = x$

$\sqrt{c^2 - x^2} = y$

$$\frac{b}{\sqrt{b^2+x^2}} = \cos\theta \Rightarrow \sqrt{b^2 + x^2} = b\sec\theta$$

$$\frac{\sqrt{b^2+x^2}}{b} = \sec\theta$$

Forma $\sqrt{y^2 - d^2}$



$$\sin\theta = \frac{d}{y}$$

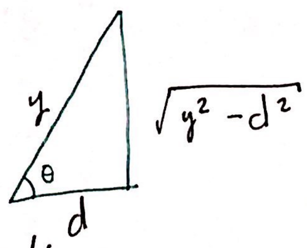
$$y = d \cdot \csc\theta$$

$$dy = -d \csc\theta \cot\theta \, d\theta$$

$$\frac{y}{d} = \sec\theta$$

$$y = d\sec\theta$$

$$dy = d\sec\theta\tan\theta$$



$$\sqrt{y^2 - d^2} = d\tan\theta$$

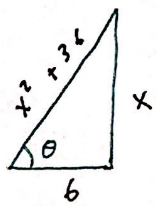
Ejercicio 2 y 6 Pág 58 y 59

(20) $$\int \frac{1}{x^2+36} dx = \int \frac{6\sec^2\theta \, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$$

$$x = 6\tan\theta$$

$$dx = 6 \cdot \sec^2\theta \, d\theta$$

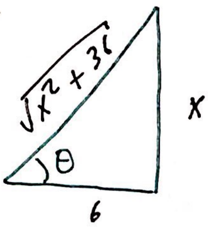
$$x^2 + 36 = 36(\tan^2\theta + 1) = 36\sec^2\theta$$



$x = 6\tan\theta \Rightarrow \frac{x}{6} \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{6}\right) = \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C$

② $$\int \frac{1}{\sqrt{x^2+36}}\,dx = \int \frac{6\sec^2\theta}{6\sec\theta}\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\therefore \ln\left|\frac{\sqrt{x^2+36}}{6} + \frac{x}{6}\right| + C$$



$\frac{x}{6} = \tan\theta$

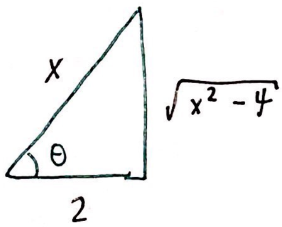
$\frac{\sqrt{x^2+36}}{6} = \sec\theta$

$x = 6 \cdot \tan\theta$

$dx = 6\sec^2\theta\,d\theta$

$\sqrt{x^2+36} = 6\sec\theta$

③ $$\int \frac{\left(\sqrt{x^2-4}\right)^3}{x^6}\,dx = \int \frac{2^3\tan^3\theta}{2^6\sec^6\theta} \cdot 2\tan\theta\sec\theta\,d\theta =$$



$\frac{x}{2} = \sec\theta$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\frac{\sqrt{x^2-4}}{2} = \tan\theta$

$\sqrt{x^2-4} = 2\tan\theta$

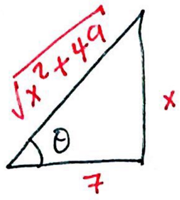
$$= \frac{2^4}{2^6}\int \frac{\tan^4\theta}{\sec^5\theta}\,d\theta = \frac{1}{2^2}\int \tan^4\theta\cos^5\theta\,d\theta = \frac{1}{4}\int \frac{\sin^4\theta}{\cancel{\cos^4\theta}} \cdot \cancel{\cos^5}\theta$$

$$= \frac{1}{4}\int \sin^4\theta\cos\theta\,d\theta \Rightarrow \begin{matrix} w = \sin\theta \\ du = \cos\theta \end{matrix} \Rightarrow \frac{1}{4}\int u^4\,du = \frac{1}{4}\left[\frac{w^5}{5}\right] + C$$

$$= \frac{\sin^5\theta}{20} + C = \frac{1}{20}\frac{(x^2-4)^{5/2}}{x^5} + C$$

$\sin\theta = \frac{\sqrt{x^2-4}}{x}$

(2a) $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49 \cdot 7\csc^2\theta}{49\ \cot^2\theta\ 7\csc\theta}\,d\theta = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta$$



$$\cot\theta = \frac{x}{7} \implies x = 7\cot x$$
$$dx = -7\csc^2\theta\,d\theta$$

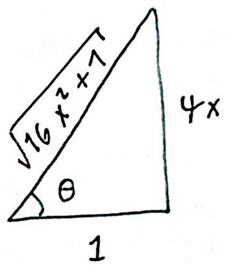
$$x = 7\tan\theta$$

$$\frac{\sqrt{x^2+49}}{7} = \csc\theta \implies \sqrt{x^2+49} = 7\csc\theta$$

$$= -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\,\frac{\sin^2\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$

$$= -\int \tan\theta\sec\theta\,d\theta = -\sec\theta + C = \frac{-\sqrt{x^2+49}}{x} + C$$

(3b) $$\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta}{1/4\tan\theta\cdot\sec\theta} = \int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{\sec\theta}{\tan\theta}\,d\theta$$



$$\frac{4x}{1} = \tan\theta \implies x = \frac{\tan\theta}{4}$$

$$\sqrt{16x^2+1} = \sec\theta \qquad dx = \frac{1}{4}\sec^2\theta\,d\theta$$

$$= \int \left[\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right] d\theta = \int \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}\sin\theta}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta =$$

$$= -\ln|\csc\theta + \cot\theta| + C = -\ln\left|\frac{\sqrt{16x^2-1}}{4x} + \frac{1}{4x}\right| + C$$

Continuación Integración Trigonométrica

- Lunes 26 de Agosto Simulacro Parcial 3 de septiembre parcial 1; capítulos 5 y 7 pg 11 - 70

Integrales de la forma $\int \cot^n x \csc^m dx$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\cot^2 x = \csc^2 x - 1$$

Ejercicio 4: Integre (pg. 50)

ⓐ $\int \cot^2(x) \csc^4(x)\, dx =$

$\therefore \cot^2 x \csc^2 x \csc^2 x$

$\therefore \cot x \csc^3 x (\csc x \cot x)$

$$= \int \cot^2 x \csc^2 x (\csc^2 x\, dx) = \int \cot^2 x (\cot^2 x + 1) \csc^2 x\, dx$$

sustitución

$u = \cot x \qquad du = -\csc^2 x\, dx$

$$= -\int u^2 (u^2 + 1)\, du$$

$$= -\int (u^4 + u^2)\, du$$

$$= -\frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= -\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C$$

ⓑ $\int \cot^3 x \csc^3 x\, dx =$

$$= \int \cot^2 x \csc^2 x (\cot x \csc x\, dx)$$

$$= \int (\csc^2 x - 1) \csc^2 x (\cot x \csc x\, dx)$$

$u = \csc x \qquad du = -\csc x \cot x\, dx$

$$= -\int (u^2 - 1)(u^2)\, du$$

$$= -\int (u^4 - u^2)\, du = -\frac{u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{-\csc^5 x}{5} + \frac{\csc^3 x}{3} + C$$

Casos especiales $\int \csc x \, dx$ $\int \csc^3 x \, dx$

- $$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

- $$\int \csc x \underbrace{\frac{(\csc x + \cot x)}{(\cot x + \csc x)}}_{1 \text{ especial}} = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x} \, dx$$

$u = \cot x + \csc x$

$-du = \csc^2 x + \csc x \cot x \, dx$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cot x + \csc x| + C$$

- $$\int \sec^3 x \, dx = \frac{1}{2}(\sec x)^9 + \frac{1}{2}\int \sec x \, dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

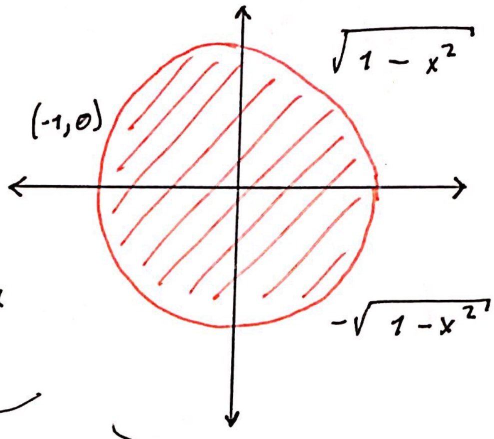
- $$\int \csc^3 x \, dx = \frac{1}{2}(\csc x)^9 + \frac{1}{2}\int \csc x \, dx$$

$$= -\frac{1}{2}\csc x - \frac{1}{2}\ln|\csc x + \cot x| + C$$

Área de un círculo unitario sin utilizar Geometría

Ec. $x^2 + y^2 = 1$

Función: $y = \pm\sqrt{1 - x^2}$



$$\text{Área} = \int_{-1}^{1} \sqrt{1 - x^2}\,dx + \underbrace{\int_{-1}^{1} -\sqrt{1 - x^2}\,dx}_{\text{por } -1}$$

$$= 2\int_{-1}^{1} \sqrt{1 - x^2}\,dx$$  * 2

$$= 4\int_{0}^{1} \sqrt{1 - x^2}\,dx$$  * 4

ni sustitución, ni integración por partes

$\therefore\; 1 - \sin^2\theta = \cos^2\theta$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

Para evaluación de la integral
$$\begin{cases} x = \sin\theta = 1 = \therefore \frac{\pi}{2} \\ x = \sin\theta = 0 = \therefore 0 \end{cases}$$

$$\therefore A = 4\int_{0}^{\pi/2} \underbrace{\sqrt{1 - \sin^2\theta}}_{\cos\theta}\,\cos\theta\, d\theta$$

$$A = 4\int_{0}^{\pi/2} \cos^2\theta\, d\theta$$

$$A = \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$A = \frac{4}{2}\int_{0}^{\pi/2} \left(1 + \cos(2\theta)\right) d\theta$$

$$A = 2\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big]_{0}^{\pi/2}$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right)$$

$$= \frac{2}{2}\cdot\pi = \therefore \pi \quad \square$$

el área de un círculo de radio 1 es π

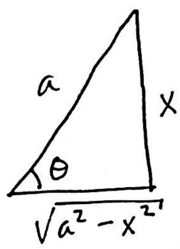
7.3. Sustitución Trigonométrica (pg. 54)

$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}_{\text{simplifique si es posible.}} g'(\theta)\,d\theta$$

$x = g(\theta)$ $\quad dx = g'(\theta)\,d\theta$

$\sqrt{1-x^2}$	$\sqrt{1+x^2}$	$\sqrt{x^2-1}$	$\sqrt{\sec^2\theta - 1}$
$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta$	$\sqrt{\tan^2\theta}$
$1 - \sin^2\theta = \cos^2\theta$	$1+\tan^2\theta = \sec^2\theta$		$\sqrt{x^2-1} = \tan\theta$
$\sqrt{1-x^2} = \cos\theta$	$\sqrt{1+x^2} = \sec\theta$		

forma más general $\sqrt{a^2-x^2}$

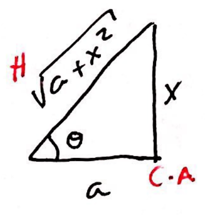


$\sin(\theta) = \frac{C.O.}{H} = \frac{x}{a}$ $\quad x = a\sin\theta$

$dx = a \cdot \cos\theta\, d\theta$

$\cos(\theta) = \frac{C.A.}{H} = \frac{\sqrt{a^2-x^2}}{a} = \quad \sqrt{a^2-x^2} = a\cos\theta$

forma $\sqrt{a^2+x^2}$



$\sin\theta = \frac{x}{\sqrt{a^2+x^2}}$ $\quad \tan\theta = \frac{x}{a}$

$\frac{H}{C.A} = \sec\theta = \frac{\sqrt{a^2+x^2}}{a}$

$x = a \cdot \tan\theta$

$dx = a\sec^2\theta\, d\theta$ $\quad \sqrt{a^2+x^2} = a \cdot \sec\theta$

Ejercicio 1: Evalúe

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\,\frac{du}{2} = \int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

$u = 25 - x^2$

$du = -2x\,dx = \frac{du}{-2x}$

$$= -u^{1/2} + C$$

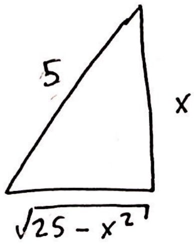
$$= -\sqrt{25-x^2} + C$$

Sustitución Trigonométrica

$H = 5$

$C.O. = x$

$C.A. = \sqrt{25-x^2}$



$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$

$\sqrt{25-x^2} = 5\cos\theta$

$\frac{C.A.}{H}$

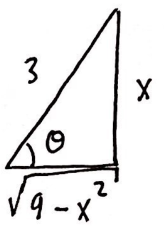
$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{5\cos\theta}\cdot 5\cos\theta\,d\theta = 5\int \sin\theta\,d\theta$$

$$= -5\cos\theta + C$$

$$= \frac{-5}{5}\sqrt{25-x^2} + C$$

$$= -\sqrt{25-x^2} + C$$

ⓐ $$\int \frac{x^3}{\sqrt{9-x^2}}\,dx =$$



$S = \frac{x}{3}$ $\quad C = \frac{\sqrt{9-x^2}}{3}$

$3\sin\theta = x$ $\quad 3\cos\theta = \sqrt{9-x^2}$

$(3\sin\theta)^3 = x^3$

$\rightarrow dx = 3\cos\theta\,d\theta$

sustituimos

$$= \int \frac{27\sin^3\theta}{3\cancel{\cos\theta}}\cdot 3\cancel{\cos\theta}\,d\theta = \int 27\sin^3\theta\,d\theta$$

$$\longrightarrow \int 27\sin^3\theta\, d\theta = 27\int \sin^3\theta\, d\theta = 27\int \sin\theta\,(1-\cos^2\theta)\, d\theta$$

$$= -27u + 9u^3 + C = -27\cos\theta + 9\cos^3\theta + C$$

$u = \cos\theta \quad du = -\sin\theta\, d\theta$

sustituyo

$$= -27\cdot\frac{1}{3}\sqrt{9-x^2} + 9\cdot\frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C$$

Caso Integrales trigonométricas

- $\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$
- $\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$
- $\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$

Ejercicio 5: Evalúe (pg. 51)

a) $$\int_{-\pi}^{\pi} \sin(8x)\cos(4x)\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(\sin(4x) + \sin(12x)\right)dx = \frac{1}{2}\left[-\frac{\cos(4x)}{4} - \frac{\cos(12x)}{12}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left(-\frac{\cos(4\pi)}{4} + \frac{\cos(-4\pi)}{4} - \frac{\cos(12\pi)}{12} + \frac{\cos(12\pi)}{12}\right)$$

$$= 0$$

7.2. Integrales Trigonométricas

2019-08/13

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x \quad \div \cos^2 x$$

$$1 + \cot^2 x = \csc^2 x \quad \div \sin^2 x$$

Integrales de la forma $\int \sin^n x \cos^m x \, dx$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$u = \sin x \qquad u = \cos x$

$du = \cos x \, dx \qquad du = -\sin x \, dx$

Primer Caso = Se necesita una par y una impar

Evalúe $\int \cos^5 x \, dx = \int \cos^4 x \, (\cos x \, dx)$

Reescribir $\cos^4 x = (\cos^2 x)^2 = (1 - \sin^2 x)^2$

$\cos^2 x = 1 - \sin^2 x$

$$\therefore \int \cos^5 x \, dx = \int (1 - \sin^2 x)^2 (\cos x \, dx)$$

$u = \sin x \qquad du = \cos x \, dx$

$$= \int (1 - u^2)^2 \, du$$

$$= \int (1 - 2u^2 + u^4) \, du = u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$$

$$\therefore = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C \quad \square$$

Aparte algún término $\sin x$ o $\cos x$.

a. Potencias impares de seno o coseno.

Evalúe

esto es un problema — Preferimos potencias pares

$$\int \cos^3 x \sin^6 x \, dx =$$

$$\int \cos^2 x \cdot \sin^6 x \cos x \, dx \quad ó \quad \int \cos^3 x \sin^5 x \sin x \, dx$$

$$= \int \cos^2 x \sin^6 x (\cos x) dx = \int (1-\sin^2 x) \sin^6 x \, (\cos x \, dx)$$

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$

$du = \cos x \, dx$

$$= \int (1-u^2) u^6 \, du$$

$$= \int u^6 - u^8 \, du$$

$$= \frac{1}{7}u^7 - \frac{1}{8}u^9 + C$$

$$\doteq \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$$

b. $\int \cos^5 x \sin^3 x \, dx =$

$$\int \cos^4 x \sin^3 x \cos x \, dx \quad ó$$

$$\int \cos^5 x \sin^2 x \sin x \, dx$$

$$= \int \cos^5 x (\sin^2 x) \sin x \, dx$$

$$= \int \cos^5 x (1 - \cos^2 x) \sin x \, dx$$

$u = \cos x \qquad du = -\sin x \, dx$

$$= -\int u^5 (1 - u^2) \, du$$

$$= -\frac{1}{6}u^6 + \frac{u^8}{8} + C$$

$$\therefore -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$$

b) Potencias pares de seno y coseno

$$\int \cos^2 x \, dx = \int (1-\sin^2 x)\, dx = \underline{\frac{x}{2} + \frac{1}{4}\sin 2x + C}$$

$$1 = \cos^2 x + \sin^2 x \quad (1)$$

$$+ \ \underline{\cos(x+x) = \cos^2 x - \sin^2 x} \quad (2)$$

suma(1 y 2) $1 + \cos(2x) = 2\cos^2 x \Rightarrow \cos^2 y = \frac{1}{2} + \frac{1}{2}\cos 2x$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

Ejercicio potencias pares:

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

a. $$\int_{-\pi}^{\pi} \sin^2 x \, dx = 2\int_0^{\pi} \sin^2 x \, dx = \frac{2}{2}\int_0^{\pi}(1-\cos 2x)\, dx = x - \frac{1}{2}\sin 2x$$

si fuera impar sería 0

$u = 2x \quad du = 2dx$

$$= x - \frac{1}{2}\sin^2 x\Big]_0^{\pi} = \pi - \cancel{\frac{1}{2}\sin 2\pi}^{0} - 0 + \cancel{\frac{\sin 0}{2}}^{0}$$

$$\underline{= \pi}$$

---

b. $$\int \sin^2 x \cos^2 x \, dx$$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$= \int \frac{1}{2}(1-\cos 2x)\frac{1}{2}(1+\cos 2x)\, dx$$

diferencia de cuadrados

$$\frac{1}{4}\int (1 - \underline{\cos^2 2x})\, dx = \frac{1}{4}\int (1-\cos^2 2x)\, dx$$

$$\cos^2(2x) = \frac{1}{2}(1+\cos 4x)$$

$$= \frac{1}{4}\int \left(1 - \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \int \frac{1}{8} + \frac{1}{8}\cos 4x \, dx = \underline{\frac{1}{8}x + \frac{1}{8\cdot 4}\sin 4x + C}$$

$\int a f \, dx$

$= aF + C$

Forma $\int \tan^m x \sec^n x \, dx$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$u = \tan x$

$\sec^2 x = \tan^2 x + 1$

$\sec x = \sec x \tan x$

$w = \sec x$

$\tan^2 x = \sec^2 x + 1$

Ejercicio 3 Evalúe Pg. 48

1. $\int \tan^5 x \sec^4 x \, dx$

$\int \tan^5 x \sec^2 x (\sec^2 x \, dx)$ ó $\int \tan^4 x \sec^3 x (\tan x \sec x \, dx)$

$u = \tan x \quad \tan^2 x + 1$

$w = \sec x$

$(\tan^2 x)^2 = (\sec^2 x - 1)^2$

$\int \tan^5 x \sec^2 x (\sec^2 x \, dx)$

$\int \tan^5 x (\tan^2 x + 1)(\sec^2 x \, dx)$

$u = \tan^2 x$

$du = 2 \tan x \sec^2 x$

$\int u^5 (u^2 + 1) \, du = \int (u^7 + u^5) \, du = \frac{u^8}{8} + \frac{u^6}{6} + C$

$= \frac{1}{8} \tan^8 x + \frac{1}{6} \tan^6 x + C$

b. $\int \tan^5 x \sec^5 x \, dx =$

$\int \tan^4 x \sec^4 x (\sec x \tan x) dx$ ó $\int \tan^5 x \sec^3 x (\sec^2 x \, dx)$

$\int (\tan^2 x)^2 \sec^4 x (\sec x \tan x \, dx)$ $\qquad \tan^2 x = \sec^2 x - 1$

$\int (\sec^2 x - 1)^2 \sec^4 x (\sec x \tan x \, dx)$

$u = \sec x \qquad du = \sec x \tan x \, dx$

$\int (u^2 - 1)^2 u^4 \, du = \int (u^4 - 2u^2 + 1) u^4 \, du$

$= \int u^8 - 2u^6 + u^4$

$= \frac{u^9}{9} - \frac{2u^7}{7} + \frac{u^5}{5} + C$

$\frac{1}{9} \sec^9 x - \frac{2}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C$

c. $\int \tan^4 x \sec^4 x \, dx$

$= \int \tan^4 x \underbrace{\sec^2 x}_{\tan x^?} \sec^2 x \, dx$

$\sec^2 x = \tan^2 x + 1$

$= \int \tan^4 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x \qquad du = \sec^2 x$

$= \int u^4 (u^2 + 1) \, du$

$= \frac{1}{7} u^7 + \frac{1}{5} u^5 + C = \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$

Casos especiales $\int \tan^m x\,dx \qquad \int \sec^n x$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{du}{u} = -\ln|u| + C$$

$u = \cos x$

$du = -\sin x$

$$= -\ln(\cos x) + C$$

$$\int \sec x\,dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}dx = \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x}\,dx$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$u = \tan x + \sec x$

$du = \sec^2 x + \sec x \tan x\,dx$

$$= \ln|\tan x + \sec x| + C$$

$$\int \csc x\,dx = -\ln|\csc + \cot x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C$$

$$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx = \int (\sec^2 x - 1)\tan x\,dx$$

$u = \tan x \quad du = \sec^2 x$

$$= \int \sec^2 x \tan x - \tan x\,dx$$

$$= \int \underset{u}{\tan x}\ \underset{du}{\sec^2 x\,dx} - \int \tan x\,dx$$

$$= \frac{1}{2}\tan^2 x + \ln|\cos x| + C$$

$$\int \sec^3 x\,dx = \int \sec^2 x \sec x\,dx$$

IPP $\quad u = \sec x \qquad dv = \sec^2 x\,dx$

$\quad du = \sec x \tan x \qquad v = \tan x$

$$= \sec x \tan x - \int \tan^2 x \sec x\,dx$$

$$= \int \tan^2 x \sec x\,dx = \int \sec^2 x - 1 \sec x\,dx$$

$$= \int \sec^3 x - \sec x\,dx$$

$$= \int \sec^3 x\,dx - \int \sec x\,dx$$

$$= \frac{\sec x \tan x + \ln|\sec x + \tan x| + C}{2}$$

Integración por partes

2019-08/08

IPP: Generalmente se utiliza para integrar productos

$$\int f(x)\,g(x) = ?$$

Regla del producto para derivadas

$$\frac{d}{dx}(fg) = f'g + fg'$$

$$\frac{d}{dx}(fg) - f'g = fg'$$ integre ésta expresión

$$\underbrace{\int \frac{d}{dx}(fg)}_{\text{función original}} - \int f'g = \int fg'$$

$$\boxed{\int fg' = fg - \int f'g}$$

Integre esta expresión f deriva y g' integra

$\equiv$

f deriva
g integra

$$\int \underbrace{f(x)}_{u}\,\underbrace{g(x)\,dx}_{dv} = uv - \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u = f(x)$ $\quad dv = g(x)\,dx$

$du = f'(x)\,dx$ $\quad v = G(x)$

Ejercicio 1 pag 39 integre $\int xe^x\,dx$

Opción 1:

$u = x$	$dv = e^x\,dx$
$u' = dx$	$v = e^x$

opción 2:

~~$u = e^x$~~	~~$dv = x$~~
~~$du = e^x\,dx$~~	~~$v = \frac{x^2}{2}$~~

$$\int xe^x\,dx = \underbrace{xe^x}_{uv} - \int \underbrace{e^x\,dx}_{v\,du} = \underline{xe^x - e^x + C}$$

derivar:

$$\cancel{e^x} + xe^x - \cancel{e^x} + 0'$$

$$\underline{xe^x}$$

a) $\int 6x^2 \ln x \, dx$

$f = \ln(x)$ $\quad g' = 6x^2 dx$

$f' = \frac{1}{x} dx$ $\quad g = 2x^3$

Integre $\int fg' = fg - \int f'g$

$$\therefore \int 6x^2 \ln x \, dx = \underbrace{(\ln x)}_{f}\, \underbrace{2x^3}_{g} - \int \underbrace{\frac{1}{x}}_{f'}\, \underbrace{2x^3}_{g} dx$$

$$= 2x^3 \ln x - \int 2x^2 \, dx$$

$$= 2x^3 \ln x - 2\int \frac{x^{2+1}}{2+1} dx$$

$$= 2x^3 \ln x - \frac{2x^3}{3} + C$$

b) $\int \ln x \, dx$

$u = \ln x$ $\quad dv = 1\, dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$$\ln x \cdot x - \int x \cdot \frac{1}{x} dx$$

$$\ln x \cdot x - x + C$$

comprobemos

$$(x\ln x - x)\frac{d}{dx} = \ln x + 1 - 1 = \ln x$$

c) $\int \tan^{-1} x \, dx$ $\quad = \tan^{-1} x \cdot x - \int x \cdot \frac{1}{1+x^2} dx$

$u = \tan^{-1} x$ $\quad dv = 1\, dx$

$du = \frac{1}{1+x^2} dx$ $\quad v = x$

sustitución de

$$\int \frac{x}{1+x^2} dx = \int \frac{1}{u} \frac{du}{2} = \frac{1}{2}\int \frac{1}{u}$$

$u = 1 + x^2$

$du = (0 + 2x)\, dx$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

$$= \frac{\ln(u)}{2} + C$$

$$= \frac{\ln|1+x^2|}{2} + C$$

$$\therefore \int \tan^{-1} x \, dx = \tan^{-1}(x) \cdot x - \frac{\ln|1+x^2|}{2} - C$$

d. $\int x^2 \cos x \, dx = x^2 \sin x - \int \underbrace{\sin x \, 2x \, dx}_{2\ IPP}$

IPP 1 $\begin{cases} u = x^2 & dv = \cos x \, dx. \\ du = 2x \, dx & v = \sin x \end{cases}$

$u = 2x \quad dv = \sin x$

$du = 2\,dx \quad v = -\cos x$

$\int \sin x \, 2x \, dx = -2x\cos x + \int \cos x \, 2dx$

$= x^2 \sin x - (-2x\cos x + 2\sin x) + C$

$= x^2 \sin x + 2x\cos x - 2\sin x + C$

Siempre dar prioridad de derivación a:

- **I**nversas trigonométricas
- **L**ogarítmicas
- **A**lgebraicas
- **T**rigonométricas
- **E**xponensiales

Mnemotecnia

IPP: Integrales Definidas

$$\boxed{\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du}$$

no cambian los límites de integración

Ejercicio 3: Evalúe pg. 41

b) $72 \int_1^2 \frac{\ln x}{x^4} dx = 72 \ln x \left(\frac{-1}{3x^3}\right)\Big]_1^2 - \int_1^2 \frac{x^{-3}}{-3} x^{-1} dx$

$u = \ln(x) \qquad dv = x^{-4} dx$

$du = x^{-1} dx \qquad v = \frac{x^{-3}}{-3}$

evaluación $\longrightarrow$

$$72\ln x\left(\frac{-1}{3x^3}\right)\Big]_1^2 - \int_1^2 \underbrace{\frac{x^{-3}}{-3}\,x^{-1}\,dx} \rightarrow -\int_1^2 \frac{1}{-3x^3\cdot x}\,dx \rightarrow -\int_1^2 \frac{1}{-3x^4} \rightarrow -\int_1^2 -\frac{x^{-4}}{3}$$

$$= -\frac{72\ln(x)}{3x^3} - \frac{72}{9x^3}\Big]_1^2 \qquad \rightarrow -1\cdot -\frac{1}{3}\int_1^2 x^{-4}$$

$$= -\frac{24\ln(x)}{x^3} - \frac{3}{x^3}\Big]_1^2 = \left\{\frac{24\ln(2)}{2^3} - \frac{8}{2^3}\right\} - \left\{\frac{\overset{0}{24\cancel{\ln(1)}}}{\cancel{1}^3} - \frac{8}{1^3}\right\} \rightarrow \frac{1}{3}\int_1^2 x^{-4}\,dx$$

$$= \{-3\ln(2) - 1\} - \{-8\} = \qquad \rightarrow \frac{1}{3}\,\frac{x^{-3}}{3} = \frac{1}{9x^3}$$

$$-3\ln(2)+7$$

$$\int e^x \cos x\,dx = e^x \cos x + \int \underbrace{e^x \sin x}_{\text{IPP 2}}\,dx$$

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x \qquad v = e^x$

$u = \sin x \qquad dv = e^x\,dx$

$du = -\cos x\,dx \qquad v = e^x$

$$\int e^x \sin x\,dx = \sin x\, e^x - \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx = -e^x \cos x + \sin x\, e^x - \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx + \int e^x \cos x\,dx = -e^x \cos x + \sin x\, e^x$$

$$2\int e^x \cos x\,dx = -e^x \cos x + \sin x\, e^x$$

$$\int e^x \cos x\,dx = \frac{-e^x \cos x + \sin x\, e^x}{2} + C$$

5.5 **Regla de la sustitución** 2019/08/06

Objetivo: integre $f(g(x))$ funciones compuestas

a $$\int 3(x+2)^2\,dx = \int (3x^2+12x+12)\,dx$$

$$= x^3 + 6x^2 + 12x + C.$$

Conjeturando $\int 3(x+2)^2\,dx = (x+3)^3 + C$

b) $$\int 11\,(x-20)^{10}\,dx = (x-20)^{11} + C$$

Regla de potencia

$$\frac{d}{dx}\left[F(x)\right]^{n+1} \xrightarrow{\text{derivar}} (n+1)\left(f(x)\right)^{n} f'(x)$$

$\xleftarrow{\text{integrar}}$

Regla de la sustitución

- Regla de la sustitución

Funciones Potencia

$$\int \underbrace{[f(x)]^n}_{u=f(x)}\ \underbrace{f'(x)\,dx}_{du=f'(x)dx} = \frac{f(x)^{n+1}}{n+1} + C$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

si $n \neq 1$

Ejercicio 1: evalúe las sig integrales

a) $$\int \underbrace{(11x-20)^{10}}_{u}\ \underbrace{11\,dx}_{du} = \int u^{10}\,du = \frac{u^{11}}{11} + C$$

$$= \frac{(11x-20)^{11}}{11} + C$$

00. $$\int \underbrace{(x^2+x+3)^5}_{u} \underbrace{(2x+1)\,dx}_{du}$$

$$= \int u^5\,du = \frac{u^{5+1}}{5+1} + C = \frac{u^6}{6} + C$$

$$= \frac{(x^2+x+3)^6}{6} + C$$

---

b.) $$\int \underbrace{\left[30w^3-8\right]^{19}}_{u} \underbrace{w^2\,dw}_{du}$$

$u = 30w^3 - 8 \qquad du = 90w^2\,dw$

$$\frac{du}{90} = w^2\,dw$$

$$= \int u^{19}\,\frac{du}{90} = \frac{1}{90}\int u^{19}\,du = \frac{1}{90}\cdot\frac{u^{20}}{20} = \frac{u^{20}}{1800} = \frac{1}{1800}(30w^3-8)+C$$

c) $$\int (30w^3-8)^{19} - 90w^3\,dw \qquad = \int u^{14}\, w\,du$$

$u = 30w^3 - 8 \qquad du = 90w^2\,dw$

Solo se puede integrar por fuerza bruta

d. $\int \underbrace{8x^3}_{du} \underbrace{\sqrt{8+x^4}}_{u} \underbrace{dx}_{du}$

$u = 8 + x^4 \qquad du = 4x^3$

$2(du) = 2(4x^3)$

$2\,du = 8x^3$

$= 2\int \sqrt{u}\,du \qquad = 2 \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{4}{3}(8 + x^4)^{3/2} + C$

---

e. $\int (10x^2 + 6x)^2\,dx =$

No se usa sustitución por que no hay du.

$\int 100x^4 + 2 \cdot 10x^2 \cdot 6x + 36x^2\,dx$

Expanda Luego integre

$= \frac{100x^5}{5} + \frac{120x^4}{4} + \frac{36x^3}{3}$

$= 20x^5 + 30x^4 + 12x^3 + C$

Regla de la cadena derivadas

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Regla de la sustitución

$$\int f'(g(x))\,g'(x)\,dx = \int f'(u)\,du = f(u) + C$$

$u = g(x) \qquad du = g'(x)\,dx$

$$= f(g(x)) + C$$

Ejercicio 2 : Integre pg 32

0. $\int \frac{(8+16x+48x^2)}{x+x^2+2x^3} dx$

$u = x + x^2 + 2v^3$

$du = 1 + 2x + 6x^2 \, dx$

$8(du) = 8(1+2x+6x^2)\, dx$

$8du = 8 + 16x + 48x^2 \, dx$

$= \int \frac{8du}{u} = 8 \ln|u| + C$

$= 8 \ln|x| + C$

---

a.) $\int e^{\overbrace{x^{10}+\sqrt{2}}^{u}} \underbrace{x^9 dx}_{du} =$

$u = x^{10} + \sqrt{2}$

$du = 10x^9 + 0 \, dx$

$du = 10x^9 \, dx$

$\frac{du}{10} = x^9 \, dx$

$= \int e^u \frac{du}{10} = \frac{1}{10} e^u + C$

$= \frac{1}{2} e^{x^{10}+\sqrt{2}} + C$

b) $\int e^{x^{10}} x^8 \, dx$ $\int e^{x^{10}} dx$ no es integrable

$\neq du$ $\neq du$

c) $\int x^3 (x^4+3)^2 \sin(x^4+3)^3 \, dx = \int u^2 \sin u^3 \frac{du}{4}$

$u = (x^4+3)$ $du = 4x^3 \, dx$

$\frac{du}{4} = x^3 \, dx$

$\frac{1}{4}\int u^2 \sin(u^3) du \, du$

$t = u^3$ $dt = 3u^2 du$

$\frac{1}{4}\int \sin t \frac{dt}{3} = -\frac{1}{12} \cos u^3 + C$

$= -\frac{1}{12} \cos(x^4+3)^3 + C$

Una sola sustitución

a) $\int \sin(x^4+3)^3 \left[(x^4+3)^2 x^3\right] dx$

$u = (x^4+3)^3 \qquad du = 3(x^4+3)^2 \, 4x^3 \, dx$

$\frac{du}{12} = (x^4+3)^2 x^3 \, dx$

$= \int \sin(u) \frac{du}{12} = \frac{1}{2}$

b) $\int \cot x \, dx = \int \frac{\overbrace{\cos x}^{du}}{\underbrace{\sin x}_{u}} dx = \int \frac{du}{u} = \ln|u| + C$

$= \ln|\sin x| + C$

c) $\int \sec^2(e^x+x)(e^x+1)\,dx = \int \sec^2 u \, du = \tan u + C$

$\tan(e^x+x) + C$

$u = e^x + x$

$du = e^x + 1 \, dx$

e) $\int 28x(x+4)^{1/3} dx = \int 28x \, u^{1/3} du = \int 28(u-4)\, u^{1/3} du$

$u = x+4 \qquad du = dx$

$u - 4 = x$

$= \int 28 \cdot u^{4/3} - 4u^{1/3} du$

$28\left[\frac{3(x+4)^{7/3}}{7} - \frac{4 \cdot 3(x+4)^{4/3}}{4}\right] + C$

$= 28\left[\frac{3}{7} u^{7/3} - \frac{4 \cdot 3 u^{4/3}}{4}\right] + C$

Regla de la sustitución para integrales definidas

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

$u = g(x)$ = cambian también los límites.

$du = g'(x)\, dx$

Ejercicio: Integre

a. $$\int_{-4}^{0} \frac{1}{3x-2}\, dx$$

$u = 3x - 2$

$du = 3\, dx$

$\frac{du}{3} = dx$

f es continua en este intervalo

$$= \int_{-14}^{-2} \frac{1}{u} \frac{du}{3} = \ln|u| \Big]_{-14}^{-2}$$

$$= \left(\ln|-2| - \ln|-14|\right)\frac{1}{3}$$

$$= \left[\ln|2| - \ln|-14|\right]\frac{1}{3}$$

$$= \left(-\ln|7|\right)\frac{1}{3}$$

b. $$\int_0^1 \frac{8}{\pi} \cdot \frac{\sin^{-1}(t)}{\sqrt{1-t^2}}\, dt = \frac{8}{\pi}\int_0^{\pi/2} u\, du = \frac{8}{\pi}\frac{u^2}{2} = \frac{4}{\pi}u^2 \Big]_0^{\pi/2} =$$

$u = \sin^{-1}(t)$

$du = \frac{1}{\sqrt{1-t^2}}\, dt$

$u = \pi/2 \quad u = 0$

$u(0) = \sin^{-1}(0) = 0$

$u(1) = \sin^{-1}(1) = \frac{\pi}{2}$

$$\frac{4\left(\frac{\pi}{2}\right)^2}{\pi} - 0 = \pi$$

Teorema Fundamental del Cálculo Generalizado

Evalúe la siguiente expresión, integrando y luego derivando:

$$\frac{d}{dx}\left(\int_{x^3}^{x^5} e^y\,dy\right) = \frac{d}{dx}\left(e^{x^5} - e^{x^3}\right) = \underbrace{5x^4}_{b'(x)}\,\underbrace{e^{x^5}}_{e^{b(x)}} - \underbrace{3x^2}_{a'(x)}\,\underbrace{e^{x^3}}_{e^{a(x)}}$$

En este problema ambos límites de integración dependen de x y en la respuesta final se utilizaron dos reglas de la cadena por separado.

El último ejemplo nos indica como el uso del TFC y la regla de la cadena se puede extender para funciones donde los dos límites de integración dependen de x.

> Use regla de la cadena para el límite superior $b(x)$ y para el límite inferior $a(x)$
>
> $$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} F(t)\,dt\right) = F(\,b(x)\,)\,b'(x) - F(\,a(x)\,)\,a'(x)$$

Ejercicio 3: Evalúe las siguientes expresiones

a. $\displaystyle \frac{d}{dx}\left(\int_{\sin x}^{e^x} \sqrt[4]{10+4t^4}\,dt\right) = \sqrt[4]{10+4e^{4x}}\,e^x - \sqrt[4]{10+4\sin^4(x)}\,\cos x$

b. $\displaystyle \frac{d}{dx}\left(\int_{1/x}^{\ln x} \cosh\theta^3\,d\theta\right) = \cosh(\ln^3 x)\frac{1}{x} + \cosh(x^{-3})\frac{1}{x^2}$

Ejercicio 4: Encuentre la ecuación de la recta tangente a $y = f(x)$ *en* $x = 0$.

a. $\displaystyle f(x) = \int_0^x e^{-t^2/2}\,dt.$

Coordenada-y:	$f(0) = \int_0^0 e^{-t^2/2}\,dt = 0$
Derivada:	$f'(x) = e^{-x^2/2}$
Pendiente:	$f'(0) = e^{-0/2} = 1$
Recta Tangente:	$y = f(0) + f'(0)(x-0) = x$

b. $\displaystyle f(x) = \int_0^x \cosh^2 t\,dt.$

Coordenada-y:	$f(0) = \int_0^0 \cosh^2 t\,dt = 0$
Derivada:	$f'(x) = \cosh^2 x$
Pendiente:	$f'(0) = (\cosh 0)^2 = 1$
Recta Tangente:	$y = f(0) + f'(0)(x-0) = x$

Resolución de Corto 1/08/2019

5.3 Teorema Fundamental del Cálculo

Si $f(x)$ es continua en $[a, x]$ entonces

PARTE I: $$\frac{d}{dx}\int_a^x f(t)\,dt = f(t)$$

- Integral y la derivada se cancelan entre sí como la derivada de una antiderivada es la función original, entonces la $\int_a^x f(t)\,dt$ es la antiderivada de x.

- Variable temporal de integración

$f(t)\,dt$

① Se integra respecto a t

② Se deriva respecto a x

ej. $$f(x) = \int_{10}^{x} 4t^3\,dt = t^4\Big]_{t=10}^{t=x} = \underbrace{x^4 - 10^4}_{d/dx}$$

$\downarrow$

$f'(x) = 4x^3$

$f'(x) = \frac{d}{dx}\int_{10}^{x} 4t^3\,dt = 4x^3$ Atajo

PARTE II

TFC2: $$\int_a^b f(x)\,dx = F(x)\Big]_{x=a}^{x=b} \Rightarrow F(b) - F(a)$$

$$\int_a^b f(w)\,dw = F(w)\Big]_{w=a}^{w=b} \Rightarrow F(b) - F(a)$$

- No importa qué variable use el resultado de una integral definida siempre es el mismo.

- Puedo encontrar variables definidas cambiando el nombre de la variable

Se pueden definir funciones por medio de integrales

▲ Distribución normal $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$$P(x) = \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$$ no se puede integrar de manera explicita

$$\int e^t dt = e^t + C \qquad \int e^{t^2} t \, dt = \frac{1}{2} e^{t^2} + C$$

$$P'(x) = \frac{d}{dx} \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}} = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

Ejercicio 1 : Deriva las siguientes funciones

a) $h(x) = \int_a^x 3\sqrt{t+1}\, dt$ $\qquad h'(x) = 3\sqrt{x+1}$

$t \to x$

b) $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right)$ $\qquad S'(x) = \sin\left(\frac{\pi}{2} x^2\right)$

c) $H(w) = \int_{-5}^w \frac{t+4}{t^4+t^2+2}$ $\qquad H'(w) = \frac{w+4}{w^4+w^2+2}$

TFC parte 1 y la regla de la cadena

$$g(x) = \int_{100}^{x^5} e^t dt = e^t \Big]_{t=100}^{t=x^5} \Rightarrow e^{x^5} - e^{100} \overset{?}{=} \underbrace{e^{x^5} \cdot 5x^4}_{\text{Regla de cadena}} - 0$$

$h(x) = \sin(x^5 + x^2)$ $\qquad h'(x) = \cos(x^5 + x^2)(5x^4 + 2x)$

$$f(x) = \int_a^{b(x)} g(t)\, dt \qquad f'(x) = g(b(x))\, b'(x)$$

$t \to b(x)$

Ejercicio 2 : Deriva las siguientes funciones

a.) $g(x) = \int_{5}^{\ln x} \sqrt{t^2+1}\, dt$ $\qquad h(x) = \int_{5}^{x} \sqrt{t^2+1}\, dt$

$\stackrel{?}{=} \sqrt{x^2+1}$

$g'(x) = \sqrt{\ln(x)^2+1} \cdot \frac{1}{x}$

$t \Rightarrow \ln x$

b) $h(x) = \int_{\sec x}^{8} \tan^{-1}(t)\, dt = -\int_{8}^{\sec x} \tan^{-1}(t)\, dt$

$h'(x) = \underbrace{-\tan^{-1}(\sec x)}_{\text{remplaza } t \text{ por lim sup.}} \underbrace{\sec x \tan x}_{\text{derivada de } \sec x}$

c) $\frac{d}{dx}\left(\int_{1000}^{x^5+x^3} \ln(t)\, dt \right) = \ln(x^5+x^3)(5x^4+3x^2)$

$t \to x^5+x^3$

$t \to x^5+x^3 \cdot$ derivada de x^5+x^3

Funciones con ambos límites dependientes de x

$f(x) = \int_{\sinh x}^{\cosh x} \sec^2 t\, dt \Rightarrow f(x) = \tan(t)\Big]_{\sinh x}^{\cosh x} \longrightarrow \tan(\cosh x) - \tan(\sinh x)$

Derive = $f'(x) = \sec^2(\cosh x) \cdot \sinh x - \sec^2(\sinh x)\cosh x$

$f'(x) = \frac{d}{dx}\int_{\tan x}^{\csc x} \sec^2 t\, dt = \sec^2(\csc x)(-\csc x \cot x) - \sec^2(\tan x)(\sec^2 x)$

entonces .. TFC Parte 1 y la regla de la cadena

$$\boxed{\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\, dt = f(b)b' - f(a)a'}$$

Ejercicio 3: Derive

a) $\frac{d}{dx}\int_{\sin x}^{e^x}\sqrt{10+4t^4}\,dt = \sqrt{10+4e^{4x}}\,e^x - \sqrt{10-4\sin^4 x}\cdot\cos x$

b) $\frac{d}{dx}\int_{\ln x}^{\sin^{-1}x}\cosh\theta^3\,d\theta = \cosh(\sin^{-1}x)^3\cdot\frac{1}{\sqrt{1-x^2}} - \cosh(\ln^3 x)\cdot\frac{1}{x}$

Ejercicio 4: Encuentre la ecuación de la recta tangente a $f(x) = \int_0^x \cosh^2 t\,dt$ en $t=0$

Ec. Recta tangente $y = f(0) + f'(0)(x-0)$

$f(0) = \int_0^0 \cos^2 t\,dt = 0$

$f'(x) = \frac{d}{dx}\int_0^x \cosh^2 t\,dt = \cosh^2 x \cdot 1$

$f'(0) = (\cosh 0)^2 = 1^2 = 1$

entonces... La ecuación de la recta tangente

$y = 0 + 1(x-0)$

$\boxed{y = x}$

Desplazamientos y distancias

30/07/2019

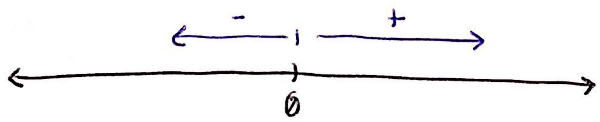
- El dezplazamiento puede ser 0 pero la distancia siempre será positiva.

▲ La integral de la derivada $f'(x)$ es la función original

$$\int_a^b f'(x)\,dx = f(x)\Big]_a^b = \underbrace{f(b) - f(a)}_{\text{cambio neto}}$$

- Si se conoce el la razón de cambio de una función el cambio neto se obtiene integrando la razón de cambio.

– dezplazamiento en <u>una</u> dimensión



$$s = \int_a^b v(t)\,dt$$

$$= \int_a^b s'(t)\,dt$$

Aplicaciones de economía

– Costo marginal $c'(x)$ costo neto $= \int_a^b c'(x)\,dx$

– población

Población neta $\int_a^b f'(x)\,dx$

Ej.: una partícula tiene una velocidad de $v(t) = \dfrac{2}{t^{4/3}}$ cm/s encuentre el desplazamiento entre $t = 1$ y 8

$$\int_1^8 v(t)\,dt = 2\int_1^8 t^{-4/3}\,dt = -6t^{-1/3}\Big]_1^8 = 6t^{-1/3}\Big]_8^1$$

$$s = 6\left(\frac{1}{\sqrt[3]{1}} - \frac{1}{\sqrt[3]{8}}\right) = 6\left(\frac{1}{2}\right) = \underline{3}\ \text{cm/s}$$ Desplazamiento neto

Ejercicio 1: Se lanza una pelota con una velocidad inicial de 64 pies/s, a nivel del suelo. Encuentre el desplazamiento de la pelota entre 1 y 3 s.

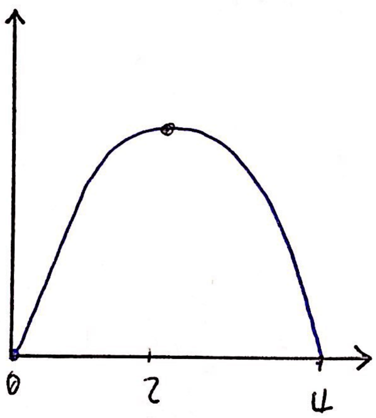
$$v(t) = 64 - 32t$$

$g = -32$ pies/s

$$s = \int_1^3 v(t)\,dt = 64t\Big]_1^3 - 16t^2\Big]_1^3$$

$$= 64(3-1) - 16(9-1) = 128 - 128 = 0$$

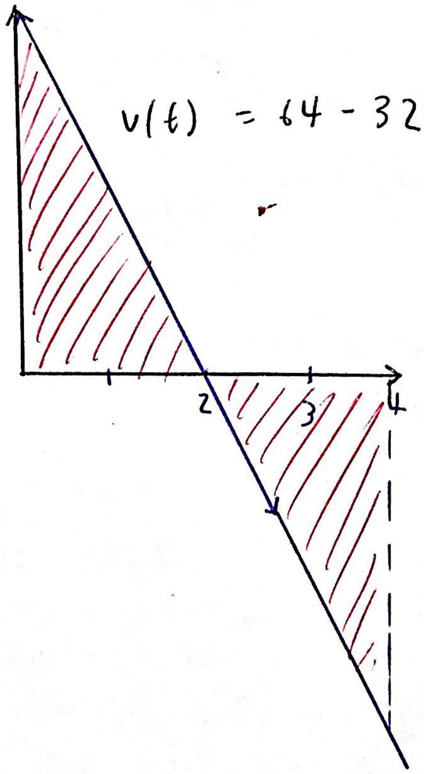
no hay cambio Neto en la posición



$$64 - 32t = 0$$

$$-32t = -64$$

$$t = \frac{-64}{-32} = 2 \text{ Pt. crit.}$$



$v(t) = 64 - 32t$

$$\boxed{\text{Desplazamiento} \quad s = \int_a^b v(t)\,dt}$$

* Por eso se hace 0 entonces..

$$\boxed{\text{Distancia} \quad d = \int_a^b |v(t)|\,dt}$$

Rapidez $= |v(t)|$ número o escalar

velocidad es un vector

Para encontrar la distancia

$$\overbrace{\text{Desplazamiento} \quad s = A_1 + A_2}^{\cong 0 \quad A_2 \text{ es negativa}}$$

$$\text{Distancia,} \quad s = A_1 - A_2$$

Prejurar siempre ser positiva

Para el ejercicio 1, encuentre la distancia recorrida por la pelota 1, 3 s.

$$d = \int_1^3 |v(t)|\, dt = \int_1^3 (64 - 32t)\, dt$$

Dist.

$$d = \underbrace{\int_1^2 v(t)\, dt}_{A_1} - \underbrace{\int_2^3 v(t)\, dt}_{A_2}$$

*Se hace 0 en dos:
- Pts críticos

	2	
$64-32t$	+	−

$$d = \int_1^2 64 - 32t\, dt + \int_2^3 32t - 64\, dt$$

$$d = 64t - 16t^2 \Big]_1^2 + 16t^2 - 64t \Big]_2^3$$

$$d = 64t - 16t^2 \Big]_1^2 + 16t^2 - 64 \Big]_2^3$$

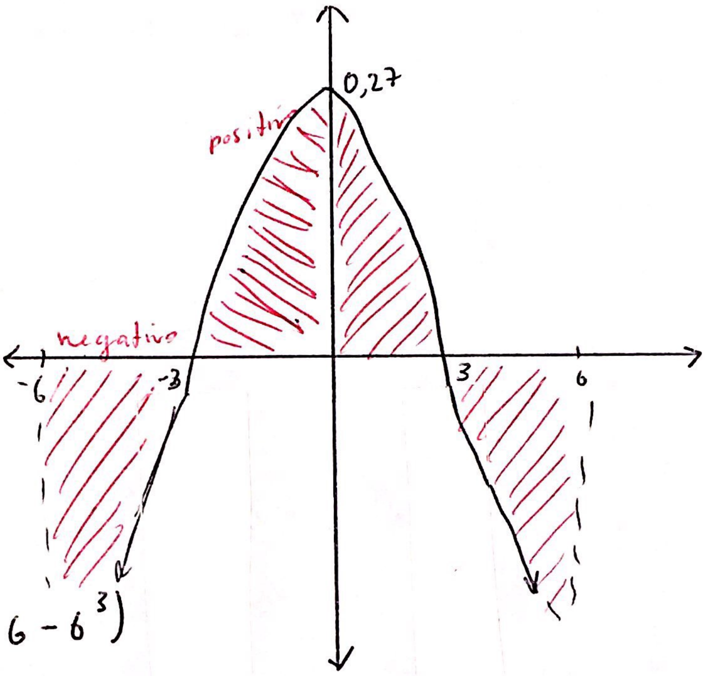
$$d = 128 - 64 - (64 - 16) + 16(9-4) - 64(3-2)$$

$$d = 64 - 48 + 80 - 64$$

Ejercicio 2: Un vehículo da vueltas alrededor de un circuito a una velocidad $v(t) = 27 - 3t^2$ millas/h.

a) Plantee la integral para encontrar el desplazamiento del vehículo entre -6 y 6 horas.

$$S = \int_{-6}^{6} (27 - 3t^2)\,dt$$



Es par entonces solo integremos un solo lado

$$\underbrace{f(-x) = f(x)}_{\text{par}}$$

$$S = 2\left(27t - t^3\right]_0^6) = 2(27 \cdot 6 - 6^3)$$

$$= 2(-54) = \underbrace{-108 \text{ millas}}_{\text{es una magnitud negativa}}$$

---

b) Plantee para encontrar la distancia del vehículo entre -6 y 6 horas

$$d = -A_1 + A_2 - A_3$$

$$d = \int_{-6}^{6} v(t)\,dt$$

$$d = -\int_{-6}^{-3} v(t)\,dt$$

$$d = 2\int_{0}^{3} v(t)\,dt - 2\int_{3}^{6} v(t)\,dt$$

Por paridad

$$d = -\int_{-6}^{-3} (27 - 3t^2)\,dt + \int_{-3}^{3} (27 - 3t^2)\,dt + \int_{3}^{6} (3t^2 - 27)\,dt$$

Integrales indefinidas: desplazamiento, velocidad y aceleración

$$\int f(t)\,dt = F(t) + C \text{ , función}$$

- Dada la aceleración del objeto $a(t) = v'(t)$
- velocidad $= v(t) = \int a(t)\,dt + C_1$
- Velocidad inicial $v(0) = V_0$ reposo es que $V_0 = 0$

Posición $s(t) = \int v(t)\,dt + C_2$

Posición inicial $s(0) = S_0$ posición equilibrio $s(0) = 0$

Ejercicio 3: Un cohete despega con una aceleración vertical de $a(t) = t^2\left(\frac{72}{t} - 36\right) ft/s^2$

- La posición inicial es 0 pies metros sobre el nivel del mar y la velocidad inicial es de 400 ft/s.

a) Encuentre la posición vertical del cohete. sobre el nivel del mar 5 km

$$a(t) = 72t - 36t^2$$

velocidad $= \int (72t - 36t^2)\,dt$

$$v(t) = 36t^2 - 12t^3 + C_1$$

$$v(0) = C_1 = 400$$

Posición $s(t) = \int v(t)\,dt = 12t^3 - 3t^4 + 400t + C_2$

$$s(0) = 0 + 0 + C_2 = 0$$

b) Encuentra la rapidez y la velocidad a los 10 s.

$$v(0) = 36(100) - 12(1000) + 400$$
$$= 4800 - 12{,}000 = -8000 \text{ ps/s}$$

$$\text{rapidez} = |v(0)| = 8000 \text{ ps/s}$$

Ejercicio 4: Un resorte en reposo y en su punto de equilibrio tiene una aceleración de:

$$a(t) = 4\cos(t) - 3\sin(t)$$

velocidad y posición del resorte

$$v(t) = \int 4\cos t - 3\sin t \, dt = 4\sin t + 3\cos t + C_1$$

$$v(0) = 0 + 3 + C_1 = 0$$
$$C_1 = -3$$

$$v(t) = 4\sin t + 3\cos t - 3$$

Posición

$$s(t) = \int (4\sin t + 3\cos t - 3)\,dt$$

$$s(t) = -4\cos t + 3\sin t - 3t + C_2$$

$$s(0) = -4 + 0 + 0 + C_2 = 0 \Rightarrow C_2 = 4$$

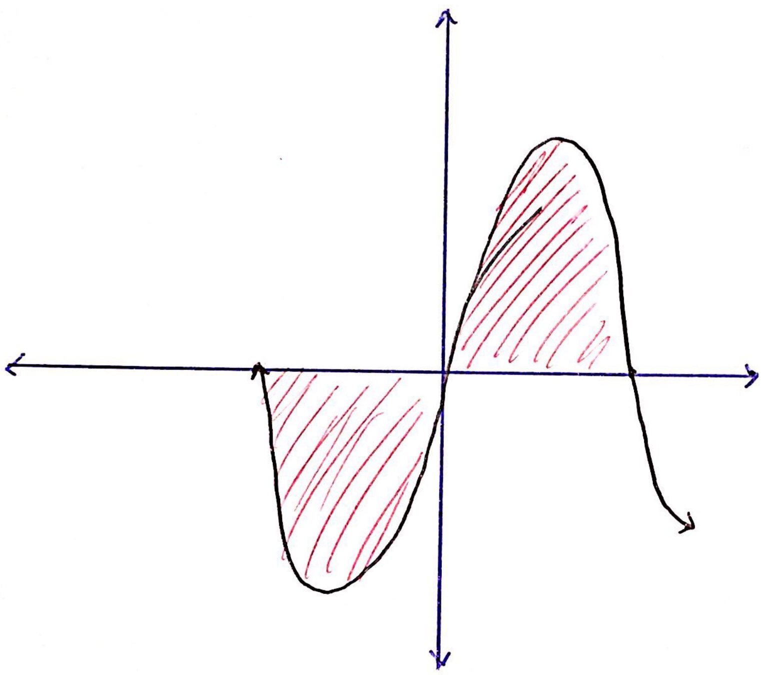
$$s(t) = -4\cos t + 3\sin t - 3t + 4$$

Funciones pares e impares

$$\int_{-100}^{100} \underbrace{(\sin x + x^3 + \tanh x)}_{\text{tres funciones impares}} dx$$

$\sin(-x) = -\sin x$

$(-x)^3 = -x^3$



Las áreas se cancelan entre sí

$$\int_{-a}^{a} f\,par(x)\,dx = 2\int_{0}^{a} f\,par(x)\,dx$$

$= \emptyset$

$$\int_{-a}^{a} f\,par(x)\,dx = 2\int_{0}^{a} f\,par(x)\,dx$$

Calculo Integral

25/07/2019

5.4 Área, Desplazamiento y Propiedades

Cómo se encontraba el área de una región

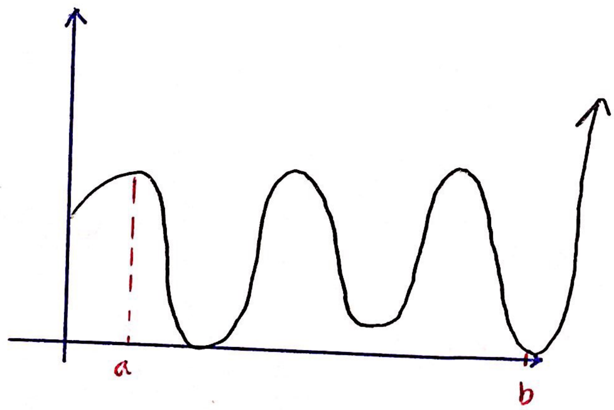
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,dx$$

La integral definida de f en $[a,b]$ f es continua

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,dx$$

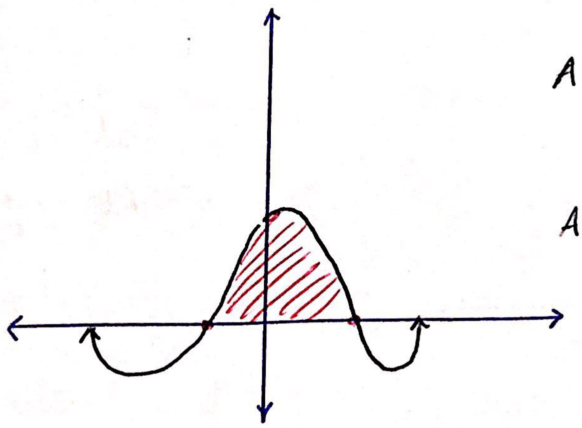
Interpretacion de integral definida

El area de la región bajo la curva $y = f(x)$, encima del eje $-x$ y entre las rectas verticales $x = a$ y $x = b$ en la integral definida de f en $[a,b]$ $f > 0$



$$A = \int_a^b f(x)\,dx$$

Considere el área bajo $y = \cos x$ en $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



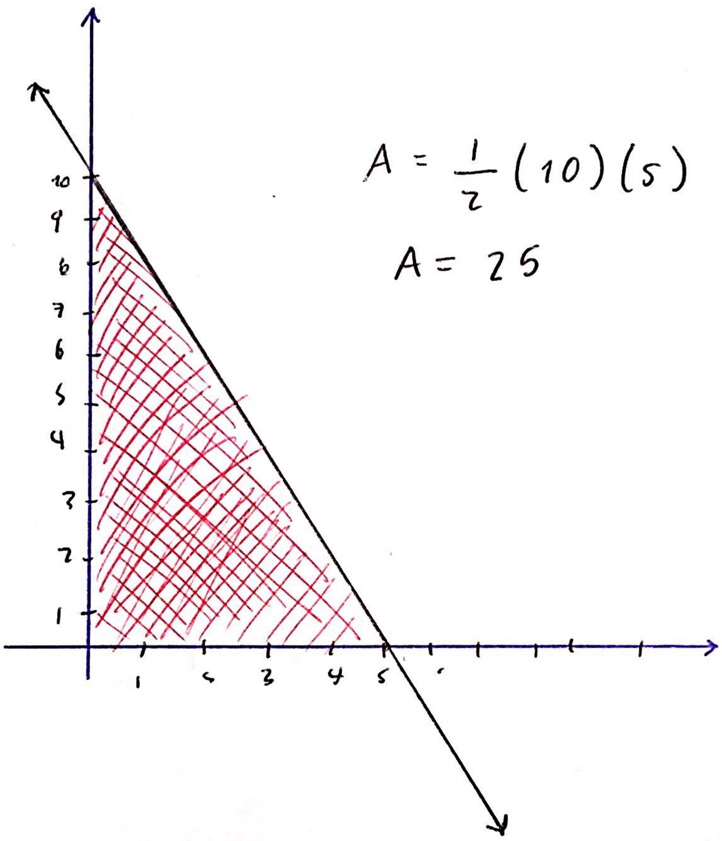
$$A = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2$$

$$A = \sin x \Big]_{-\pi/2}^{\pi/2} = \sin \pi/2 - (-1) =$$

$$= 1 + 1 = 2$$

Ejercicio 2: Encuentre el área de las sigs. funciones bosqueje cada región

a) $f(x) = 10 - 2x$ $\quad f(x) \geq 0$ en $0 \leq x \leq 5$



$$A = \frac{1}{2}(10)(5)$$

$$A = 25$$

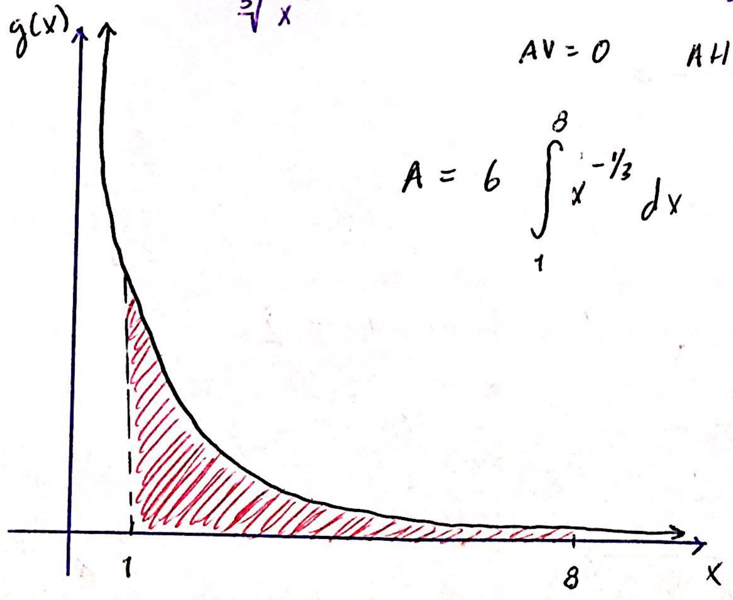
$$A = \int_0^5 (10 - 2x)\,dx$$

$$A = 10x - x^2 \Big]_0^5$$

$$A = 10 \cdot 5^2 - (0 - 0)$$

$$A = 25$$

b) $g(x) = \dfrac{6}{\sqrt[3]{x}}$ entre $1 \leq x \leq 8$



AV = 0 $\quad$ AH = 0

$$A = 6\int_1^8 x^{-1/3}\,dx = \frac{6 \cdot 3}{2} x^{2/3} \Big]_1^8$$

$$= 9\left(2^{3 \cdot 2/3} - 1^{2/3}\right)$$

$$= 9(4 - 1) = 27$$

c) $h(x) = 2|x|$ entre $x=-2$ y $x=3$

$$A = 2\int_{-2}^{3} |x|\,dx$$

$$A = \int_{-2}^{0} -2x\,dx + \int_{0}^{3} 2x$$

$$A = -2\int_{-2}^{0} x\,dx + 2\int_{0}^{3} x$$

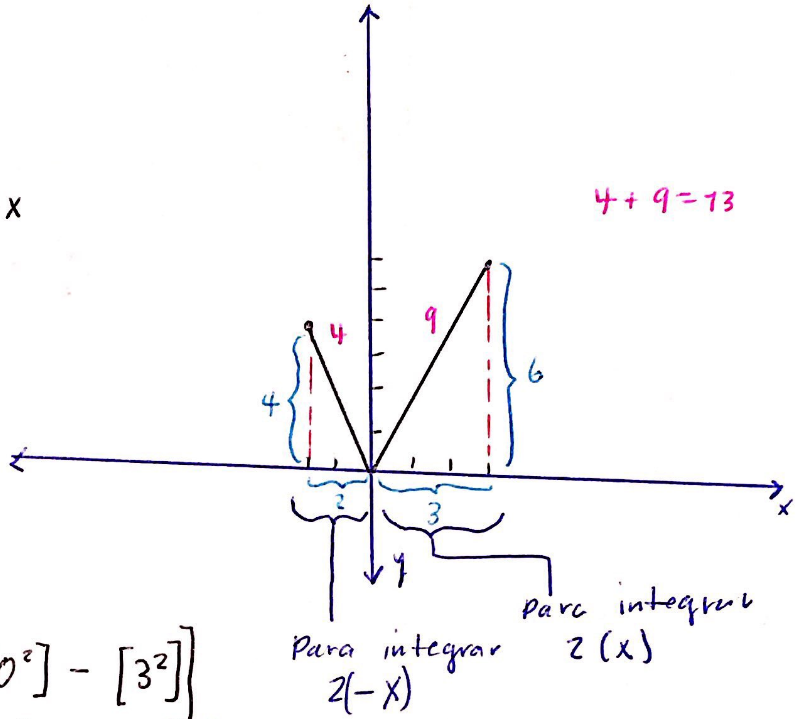
$$A = -2\left(\frac{x^2}{2}\right) + 2\left(\frac{x^2}{2}\right)$$

$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_{0}^{3}$$

$$A = \Big[[-(-2^2)] - [-0]\Big] + \Big[[0^2] - [3^2]\Big]$$

$$2^2 - 0 + 0 - 3^2$$

$$2^2 - 3^2 = 4 - 9 = \underline{13}$$



4 + 9 = 13

Regla de Integrales definidas

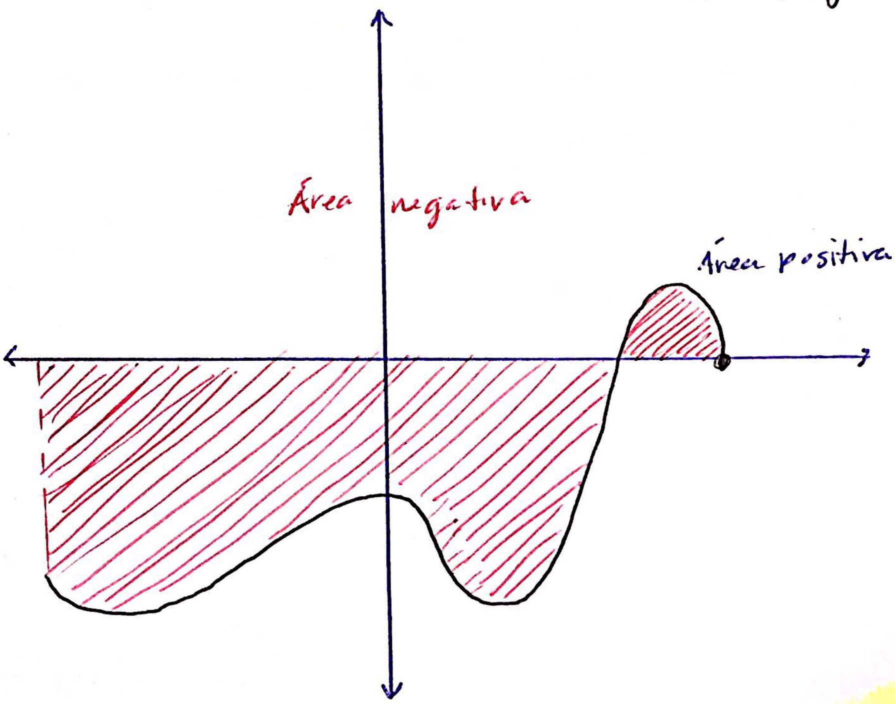
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

ej. $$\int_{-2}^{0} -2x\,dx = \int_{0}^{-2} 2x\,dx = x^2\Big]_{0}^{-2} = 4 - 0 = 4$$

b) $$\int_0^{\pi} \sin x\,dx = -\int_{\pi}^{0} \sin x\,dx = \cos x\Big]_{\pi}^{0} = 1 - (-1) = \underline{2}$$

ó $$-\cos x\Big]_0^{\pi} = -\cos(\pi) - (-\cos 0) = 1 + 1 = \underline{2}$$

¿Qué sucede cuando $f(x)$ es negativa?



Área de la región entre $f(x)$ a $y = 0$

$$\int_{-1}^{0} x^3\, dx = \frac{x^4}{4}\Big]_{-1}^{0} = 0 - 1/4$$
$$= -1/4$$

$$A \neq \int_a^c f(x)\, dx$$

$$A = -\int_a^b f(x)\, dx + \int_b^c f(x)\, dx$$

Ejercicio 3 : Pg 16 | considere

$f(x) = 4x^3 - 4$ en...

Definición más completa

$$A = \int_a^b |f(x)|\, dx$$

ⓐ Evalúe $\int_{-2}^{2} (4x^3 - 4)\, dx =$

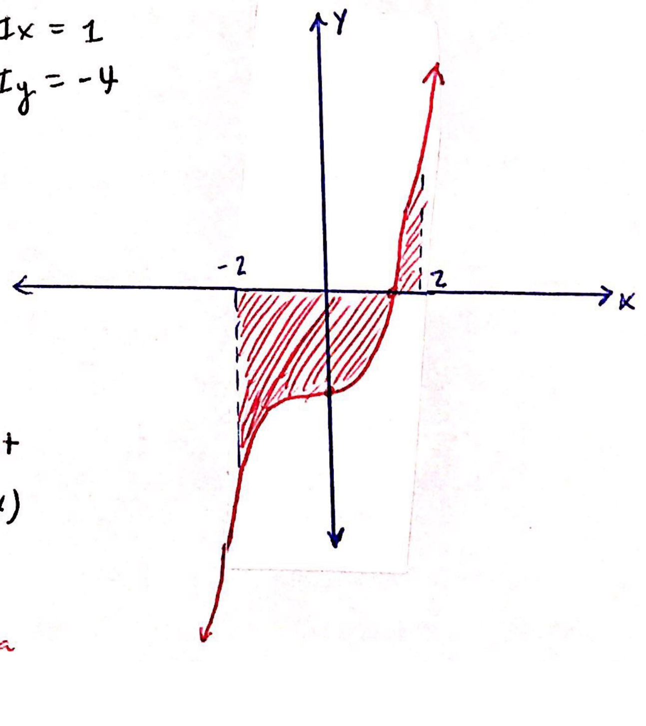
$$= x^4 - 4x\Big]_{-2}^{2}$$
$$= (16 - 8) - (16 + 8)$$
$$= 8 - 24 = -16$$

esto no está correcto.

ⓑ Bosqueje la región y explique si la integral definida es igual al area de la región

$I_x = 1$

$I_y = -4$



$$A \neq \int_{-2}^{2} f(x)\, dx$$

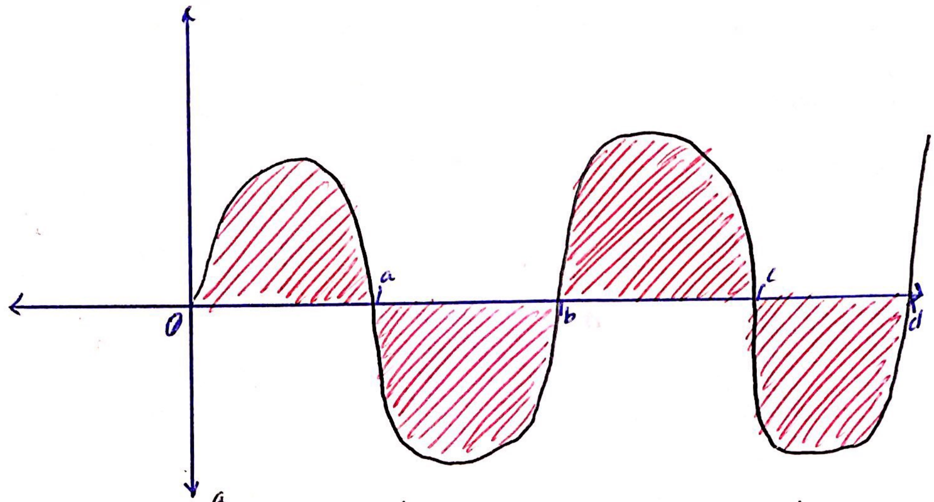
ⓒ encuentre el área de la región

$$A = \int_{2}^{1} (4 - 4x^3)\, dx + \int_1^2 (4x^3 - 4)\, dx$$

$$A = 4x - x^4\Big]_{-2}^{1} + x^4 - 4x\Big]_1^2 = (4-1) - (-8-16) + (16-8) - (1-4)$$

$$A = 3 + 24 + 8 + 3 = 27 + 11 = 38$$

esta es la respuesta



$$A = \int_0^a f(x)\,dx - \int_a^b f(x)\,dx - \int_b^c f(x)\,dx - \int_c^d f(x)\,dx$$

Propiedades Integrales definidas

①,② $\displaystyle\int_a^b K_1 f(x) \pm K_2\, g(x)\,dx = K_1 \int_a^b f(x)\,dx \pm K_2 \int_a^b g(x)\,dx$

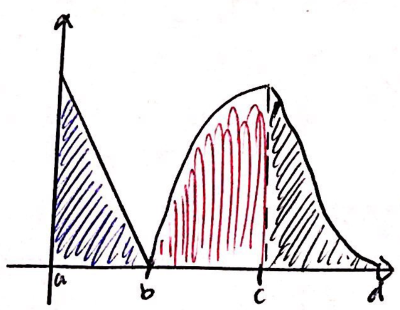
③ $\displaystyle\int_a^a f(x)\,dx = 0$

$$\int_{\sqrt{2}}^{\sqrt{2}} e^{y^2 + \ln x + \sinh x}\,dx = 0$$

④ $\displaystyle\int_a^b h\,dx = hx\Big]_a^b = h(b-a)$

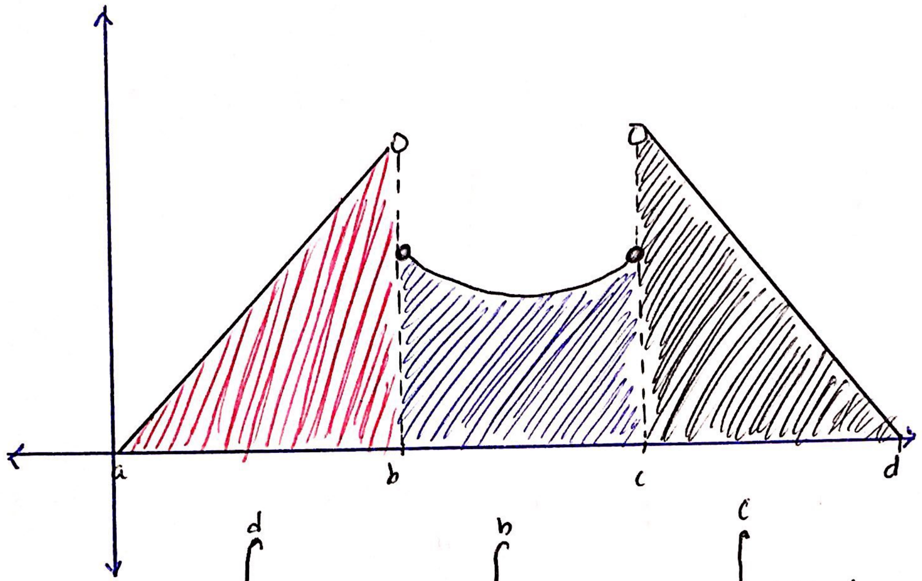
$$\int_e^{\sqrt{10}} \ln(10)\,dx = \ln(10)\left[\sqrt{10} - e\right]$$

⑤



$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$$

⑥ Continuidad por tramos, piecewise continuos



$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$$

Ejercicio 5: Evalúe las sig. integrales definida.

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2 & \text{si} \quad 0 \le x \le 1 \\ 4 - 2x & \text{si} \quad 1 \le x \le 2 \\ 6x - 12 & \text{si} \quad 2 \le x \le 3 \end{cases}$$

$$\int_0^3 f(x)\,dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx$$

$$= 2 + \left(4x - x^2\right)\Big]_1^2 + \left(3x^4 - 12x\right)\Big]_2^3$$

$$= 2 + (4-3) - (-9 - (-12))$$

$$= 2 + 1 + 3 = 6$$

Calculo Integral: Anti derivadas

23/07/2017

encontrar la integral de secante, cosecante, tan

Integral indefinida: es una función $F(x)$ cuya derivada es $f(x)$.

- La notación de integrales $\int \square \, dx$ es más cómodo ya que es más variable, se pueden integrar con respecto ~~a las variables~~ otras variables.

$$\boxed{F'(x) = f(x)}$$

Ejemplo = antiderive $f(x) = 14x^6$

$$F(x) = 2x^7 \longrightarrow F'(x) = 14x^6$$

$$F(x) = \underbrace{2 \cdot x^7 + \pi + \sqrt{12}}$$

por que se puede cancelar cualquier constante

por esto se agrega la constante de integración.

Lo más general sería $F(x) = 2x^7 + C$

$$\underbrace{\int \square \, dx} = F(x) + C$$

esto significa INTEGRE

Sinónimos:

Antiderivada = Integrar

$\int \_\_\_ \, dx$ diferencial integre respecto a x.

Reglas de Integración Básicas ⇒

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$n \neq -1$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \underbrace{\ln|x| + C}$$

valor absoluto por que

$$\frac{d}{dx}\left(\ln(-x)\right) = \frac{1}{x}$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

Trigonometría

$\sin x \longleftrightarrow \cos x$

$\cos x \longleftrightarrow -\sin x$

$\tan x \longleftrightarrow \sec^2 x$

$\cot x \longleftrightarrow -\csc^2 x$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = \csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

Trigonometría Inversa

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

Suma o diferencia

$$\int f(x) \pm g(x) \, dx = \int f(x) \pm \int g(x)$$

Multiplo constante

$$\int a f(x) \, dx = a \int f(x) dx$$

Ejemplos pág 11

(a) $\int x^{50} + 2x^6 \, dx = \frac{x^{51}}{51} + \frac{2}{7}x^7 + C$

(b) $\int \frac{1}{1+x^2} + \frac{1}{x} + \frac{1}{x^2} \, dx = \tan^{-1}x + \ln|x| + x^{-1} + C$

(c) $\int \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[5]{x^3}} \, dx = \int x^{1/2} + x^{-1/2} + x^{-3/5} \, dx$

$= \frac{2}{3}x^{3/2} + 2x^{1/2} + \frac{5}{2}x^{2/5} + C$

(d) $\int x^{\ln(2)} + x^{\sqrt{2}} + x^{\sin(2)} \, dx = \frac{x^{1+\ln(2)}}{1+\ln(2)} + \frac{x^{1+\sqrt{2}}}{1+\sqrt{2}} + \frac{x^{1+\sin(2)}}{1+\sin(2)}$

Ejercicios de libro de trabajo:

(a) $\int (\underbrace{x^e}_{\text{potencia}} + \underbrace{e^x}_{\text{exponencial}}) \, dx = \frac{x^{e+1}}{e+1} + e^x + C$

(b) $\int \left(8 \cdot 10^x - \frac{2}{x}\right) dx = \frac{8 \cdot 10^x}{\ln(10)} - 2\ln|x| + C$

(c) $\int (x-2)(x+2)(x^2+4) \, dx = \int (x^2-4)(x^2+4) \, dx =$

$\int (x^4 - 16) \, dx = \frac{1}{5}x^5 - 16x + C$

(d) $\int e^{-4x}(e^{4x} + e^{5x}) \, dx = \int (1 + e^x) \, dx = x + e^x + C$

Integrales Definidas

Son integrales con límites de integración en $[x=a]$ y $[x=b]$

$$\boxed{\int_a^b f(x)\,dx}$$

$$\int f(x)\,dx = F(x)$$

■ El teorema fundamental del calculo:

Si $f(x)$ es continua en $[a,b]$ entonces

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

- Se utiliza la notación corchete

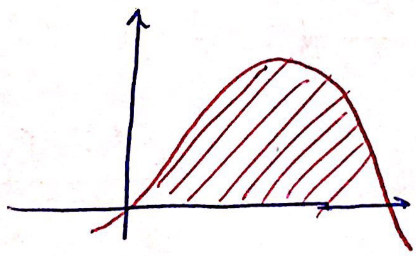
$$\int_a^b f(x)\,dx = F(x)\Big]_{x=a}^{x=b} \text{ luego evalúa}$$

$$F(x) + c\Big]_{x=a}^{x=b} = F(b) + c - (F(a) + c) = F(b) - F(a)$$

Funciones Integrables si $\int_a^b f(x)\,dx$ existe.

Ejercicio 1 : Evalúe

a) $\int_0^{\pi} \sin x \, dx = \cos x\Big]_0^{\pi} = -\cos\pi + \cos 0 = 1 + 1 = 2$



a) $\int_0^3 x^2\,dx = \left.\frac{x^3}{3}\right]_0^3 = \frac{27}{3} - 0 = 9$

b) $\int_1^{36} \sqrt{x}\,dx = \left.\frac{2}{3}\,x^{3/2}\right]_9^{36} = \frac{2}{3}\left(\underbrace{36}_{(6^2)^{3/2}}{}^{3/2} - 9^{3/2}\right)$

$= \frac{2}{3}\,(216 - 27) = 144 - 18$

$= 126$

c) $\int_0^2 \frac{1}{1-x^2}$ * esta función no existe en 1 y -1 es discontinua en el intervalo de evaluación se haría por fracciones parciales

d) $\int_1^4 \left(\underbrace{\frac{1}{\sqrt{x}}}_{x^{-1/2}} + \underbrace{3\sqrt{x}}_{3(x^{1/2})}\right)dx = \left. 2\cdot x^{1/2} + \frac{3\cdot 2x^{3/2}}{3}\right]_1^4$

$= 2\sqrt{4} + 2(2^2)^{3/2} - (2\cdot\sqrt{1} + 2\cdot 1^{3/2})$

$4 + 16 - (2+2) = 16$