Laboratorio # 3

NOMBRE ¿? 2019-08/15

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① $\int \operatorname{sech}^2(\log_2 x)\,\frac{1}{x}\,dx =$ La derivada está presente inclinación por sustitución

$\log_2 x = \frac{\log x}{\log(2)}$

$\int \operatorname{sech}^2\left(\frac{\log x}{\log(2)}\right)\frac{1}{x} = \int \operatorname{sech}^2(u)\,\frac{du}{\log(2)}$

$u = \frac{\log x}{\log(2)}$

$du = \frac{1}{x}\cdot\frac{1}{\log(2)}$

$= \frac{1}{\log(2)}\int \operatorname{sech}^2(u)\,du$

$= \frac{1}{\log(2)}\tanh(u) + C$

$= \frac{1}{\log(2)}\cdot\tanh\left(\frac{\log x}{\log(2)}\right) + C$

② $72\int_1^2 \frac{\ln(x)}{x^4}\,dx = 72\int_1^2 \ln(x)\cdot\frac{1}{x^4}\,dx$

$= -\ln(x)\,\frac{1}{3x^3} - \underbrace{\int -\frac{1}{3x^3}\,\frac{1}{x}\,dx}$

$u = \ln(x) \qquad dv = x^{-4}\,dx$

$du = \frac{1}{x} \qquad v = \frac{x^{-3}}{-3}$

$-\int_1^2 -\frac{1}{3}\cdot x^{-3}\cdot x^{-1}\,dx$

$-1\cdot -\frac{1}{3}\int_1^2 x^{-4}\,dx$

$+\frac{1}{3}\cdot\frac{x^{-3}}{-3}$

$-\frac{1}{3}\cdot\frac{1}{3}\cdot x^{-3}$

$-\frac{1}{9x^3}$

$72\left(-\frac{\ln(x)}{3x^3} - \frac{1}{9x^3}\right]_1^2$

$\left\{-\frac{72\ln(2)}{3(2)^3} - \frac{72}{9(2)^3}\right\} - \left\{-\frac{72\ln(1)}{3(1)^3} - \frac{72}{9(1)^3}\right\}$

$-\frac{24\ln(2)}{8} - \frac{72}{72} - \left(-\frac{24(0)}{3(1)} - 8\right)$

$-3\ln(2) - 1 + 0 + 8$

$-3\ln(2) + 7$

③ $\int \sec^2\theta \tan^3\theta \, d\theta =$

$\tan^3\theta' = 3\tan^2\theta \cdot \sec^2\theta$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$= \int u^3 \, du = \frac{u^4}{4} + C$

$= \frac{\tan^4(\theta)}{4} + C$ #

④ $\int (x-1)\sin(\pi x)\, dx =$

$u = x - 1$ $\quad dv = \sin(\pi x)\, dx$

$du = 1\, dx$ $\quad v = \frac{-\cos(\pi x)}{\pi}$

$uv - \int v\, du$

$$-\frac{(x-1)\cos(\pi x)}{\pi} - \underbrace{\int \frac{-\cos(\pi x)}{\pi} dx}_{\text{sustitución}}$$

$$-\frac{1}{\pi}\int -\cos(\pi x) = \left(-\frac{1}{\pi} \cdot -1\right)\int \cos(w)\, dw$$

$w = \pi x$

$dw = \pi\, dx$

$\frac{dw}{\pi} = dx$

$$= \frac{1}{\pi} \frac{\sin(w)}{\pi} = \frac{1}{\pi^2}\sin(\pi x)$$

$$-\frac{(x-1)\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C$$ #

⑤ $\int e^{-\theta} \cos(2\theta)\, d\theta =$ $\qquad \cos(2u) = 1 - \sin^2 u$

$u = \cos(2\theta) \qquad dv = e^{-\theta}\, d\theta$

$du = -2\sin(2\theta)\, d\theta \qquad v = -e^{-\theta}$

$\underbrace{\ }_{u}\underbrace{\ }_{v} - \int \underbrace{\ }_{v}\underbrace{\ }_{du}$

$$\underbrace{-\cos(2\theta)\, e^{-\theta} - \int -e^{-\theta} \cdot -2\sin(2\theta)}$$

$$-\frac{\cos(2\theta)}{e^{\theta}} - \underbrace{\int 2e^{-\theta}\sin(2\theta)\, d\theta}$$

$$= 2\int e^{-\theta}\sin(2\theta)\, d\theta$$

$u = \sin(2\theta) \qquad dv = e^{-\theta}\, d\theta$

$du = 2\cos(2\theta)\, d\theta \qquad v = -e^{-\theta}$

$$\underbrace{\sin(2\theta)}_{u} \cdot \underbrace{-e^{-\theta}}_{v} - \int \underbrace{-e^{-\theta}}_{v} \cdot \underbrace{2\cos(2\theta)\, d\theta}_{du}$$

$$2\left\{-\frac{\sin(2\theta)}{e^{\theta}} - \int -2e^{-\theta}\cos(2\theta)\, d\theta\right\}$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} - 2 \cdot -2 \int e^{-\theta}\cos(2\theta)\, d\theta$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} + 4\int e^{-\theta}\cos(2\theta)\, d\theta$$

$$w = \int e^{-\theta}\cos(2\theta)\, d\theta$$

variable cíclica

$$w = -\frac{\cos(2\theta)}{e^{\theta}} - \left(-\frac{2\sin(2\theta)}{e^{\theta}} + 4w\right)$$

$$w = -\frac{\cos(2\theta)}{e^{\theta}} + \frac{2\sin(2\theta)}{e^{\theta}} - 4w$$

$$w + 4w = -\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}$$

$$w = \frac{-\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}}{5} + C$$

6 $\int e^{\tan(x + \tan^{-1} x)} \cdot \sec^2 (x + \tan^{-1}(x)) \left(1 + \frac{1}{1+x^2}\right) dx =$

sustitución

$u = \tan(x + \tan^{-1}(x))$

$du = \sec^2(x + \tan^{-1}(x)) \cdot (1 + \frac{1}{1+x^2})$

$= \int e^u \, du$

$= e^u + C$

$= e^{\tan(x + \tan^{-1}(x))} + C$

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(8) $\int 5^t \sin(5^t)\, d\cancel{x} = \frac{1}{\ln(5)} \cdot \int \sin(w)\, dw = \frac{1}{\ln(5)} \left[-\cos(w) \right] = -\frac{\cos(5^t)}{\ln(5)} + C$ A

$w = 5^t$

$dw = 5^t \cdot \ln(5)\, dt$

$\frac{dw}{\ln(5)} = 5^t\, dt$

(9) $\int_0^2 x \sin \pi x \, dx = -\frac{x \cdot \cos(\pi x)}{\pi} - \int -\frac{\cos(\pi x)}{\pi}\, dx = \frac{-x \cos(\pi x)}{\pi} - \frac{1}{\pi}\left[\int -\cos(\pi x)\, dx \right]$

$u = x \qquad dv = \sin(\pi x)\, dx$

$du = 1\, dx \qquad v = -\frac{\cos(\pi x)}{\pi}$

$w = \pi x$

$dw = \pi\, dx$

$\frac{dw}{\pi} = dx$

$= \int -\cos(w) \frac{dw}{\pi}$

$= -\sin(w)$

$= \frac{-\sin(\pi x)}{\pi}$

$\frac{-x \cos(\pi x)}{\pi} - \frac{1}{\pi}\left(-\frac{\sin(\pi x)}{\pi} \right) + C$

$\left. \frac{-x \cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C \right]_0^2$ A

$\left\{ -\frac{2\cos(\pi 2)}{\pi} + \frac{\sin(\pi 2)}{\pi^2} \right\} - \left\{ \cancel{\frac{-0 \cos(\pi 0)}{\pi}} + \cancel{\frac{\sin(\pi 0)}{\pi}} \right\}$

0

$\left\{ -\frac{2(1)}{\pi} \right\} = -\frac{2}{\pi}$ A

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$$\int_{-13}^{+13} \frac{4x}{\sqrt[3]{(1+2x^2)^2}}\,dx = \int_{-13}^{+13} \frac{du}{\sqrt[3]{(u)^2}} = \int_{-13}^{13} u^{-\frac{2}{3}}\,du$$

sustitución

$u = 1 + 2x^2$

$du = 0 + 4x\ dx$

$du = 4x\,dx$

$$\frac{u^{-\frac{2}{3}+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}} \qquad -\frac{2}{3}+\frac{3}{3} = \frac{-2+3}{3} = \frac{1}{3}$$

$$\frac{u^{\frac{1}{3}}}{1/3} = \left[\frac{\frac{u^{1/3}}{1}}{\frac{1}{3}}\right] = \frac{3u^{1/3}}{1} = 3u^{1/3}$$

$$f(-x) = \frac{4(-x)}{\sqrt[3]{(1+2(-x)^2}}$$

$$= -\frac{4x}{\sqrt[3]{(1+2x^2)}}$$

impar

$$3u^{1/3} = 3(1+2x^2)^{1/3} + C\Big]_{-13}^{13}$$

$$= \left\{3(1+2(13)^2)^{1/3}\right\} - \left\{3(1+2(-13)^2)^{1/3}\right\}$$

$$= 3\sqrt[3]{1+2(13)^2} - 3\sqrt{1+2(-13)^2}$$

$$= 3\sqrt[3]{339} - 3\sqrt{339}$$

$$= 0$$

A

(10) $\int \frac{e^{y/(y+1)}}{(y+1)^2} dy = \int \frac{e^{\frac{y}{(y+1)}}}{(y+1)^2} dy =$

$\frac{d}{dx}\left(\frac{y}{y+1}\right) = \frac{1 \cdot (y+1) - y(1)}{(y+1)^2} = \frac{\cancel{y} + 1 - \cancel{y}}{(y+1)^2} = \frac{1}{(y+1)^2}$

$\frac{f'g - fg'}{(g)^2}$

$u = \frac{y}{y+1}$

$du = \frac{1}{(y+1)^2} dy$

$= \int e^u \, du = e^u = e^{y/(y+1)} + C$

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$$\int \frac{xe^{2x}}{(1+2x)^2}\,dx = \int xe^{2x}(1+2x)^{-2}$$

$u = xe^{2x}$ $\qquad dv = (1+2x)^{-2}$

$du = e^{2x} + xe^{2x}\cdot 2\,dx \quad v = -\frac{1}{2+4x}$

$$\int \frac{1}{(1+2x)^2}\,dx$$

$u = 1+2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$= \int u^{-2}\frac{du}{2}$$

$$= \frac{u^{-1}}{-1}\cdot\frac{1}{2}$$

$$= \frac{1}{-2(1+2x)}$$

$$= \frac{1}{-2-4x}$$

$$\underbrace{xe^{2x}}_{u}\cdot\underbrace{-\frac{1}{2+4x}}_{v} - \underbrace{\boxed{\int -\frac{1}{2+4x}\cdot e^{2x}+xe^{2x}\cdot 2\,dx}}_{-\frac{e^{2x}}{4}}$$

$$\int \frac{e^{2x}+xe^{2x}\cdot 2}{2+4x}\,dx$$

$$\int \frac{e^{2x}(\cancel{1+2x})}{2(\cancel{1+2x})}\,dx$$

$$\int \frac{e^{2x}}{2}\,dx \longrightarrow \frac{e^{2x}}{4}$$

$$-\frac{xe^{2x}}{2+4x} + \frac{e^{2x}}{4} + C$$