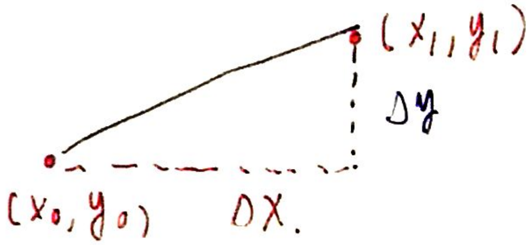
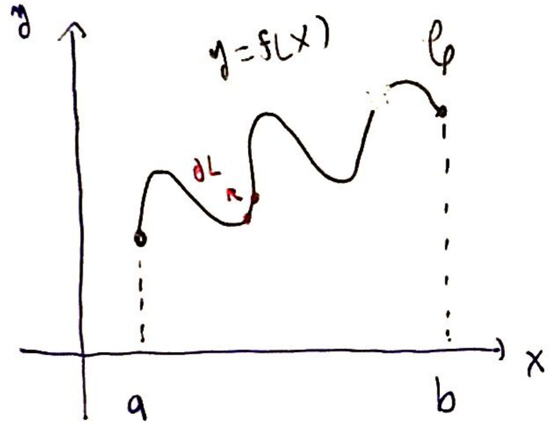
8.1 Longitud de Arco.

Derivación Fórmula.



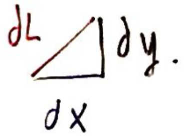
$$L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



largo de la curva $\mathcal{C}$.

$a \leq x \leq b, \quad y = f(x)$



Longitud infinitesimal del segmento.

$$dL = \sqrt{(dy)^2 + (dx)^2}$$

$$dL = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\; dx.$$

Integrando

Longitud de arco de $\mathcal{C}$: $\boxed{L = \int_a^b \sqrt{1 + [y']^2}\, dx}$

derivada.

Valor Promedio: $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx$

No es necesario graficar ninguna curva.

Ejemplo. Encuentre la longitud de la curva

$$y(x)=1+2x^{3/2} \quad \text{en} \quad 0\le x\le 8/9$$

Simplifique $1+(y')^2$ antes de integrar.

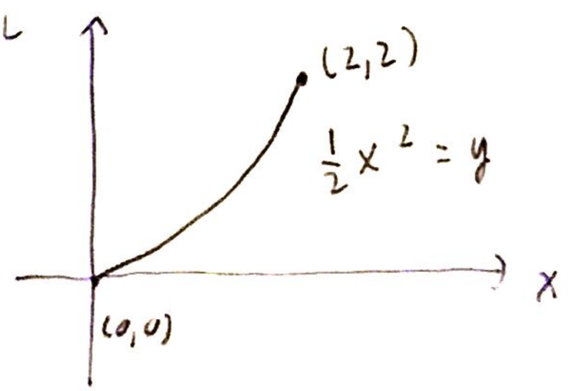
$y'=3x^{1/2}$ $\qquad (y')^2=9x$ $\qquad 1+(y')^2=1+9x$

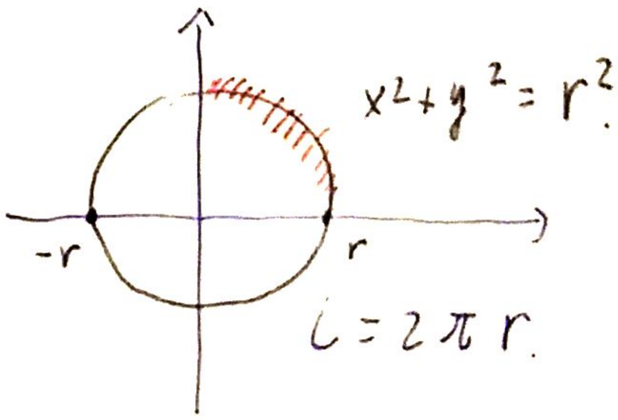
Longitud Arco: $L=\int_0^{8/9}(1+9x)^{1/2}\,dx$ $\qquad u=1+9x$, $\; du=9\cdot dx$

$$L=(1+9x)^{3/2}\cdot\frac{2}{3}\cdot\frac{1}{9}\Big]_0^{8/9}$$

$$L=\frac{2}{27}\Big(\underbrace{9^{3/2}}_{(3^2)^{3/2}}-\underbrace{1^{3/2}}_{1}\Big)=\frac{2}{27}\left(3^3-1\right)=\frac{2}{27}\cdot 26.$$

Ejercicio 1(a): Parábola (Planteado)





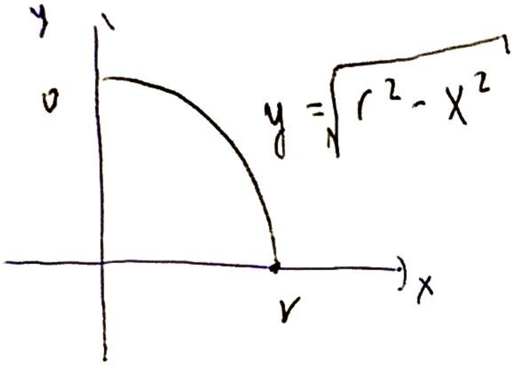
$y'=x$

$1+(y')^2=1+x^2$

$L=\int_0^2\sqrt{1+x^2}\,dx$ $\qquad x=\tan\theta$, $\; dx=\sec^2\theta\cdot d\theta.$

Ejercicio 1 b: Longitud de una circunferencia de radio r. $r = \text{cte.}$

$$Ec: \; x^2 + y^2 = r^2 \qquad L = 4\int_0^r \sqrt{1+(y')^2}\, dx.$$



$$y' = \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x)$$

$$y' = \frac{-x}{\sqrt{r^2 - x^2}}$$

$$1 + (y')^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2 - x^2 + x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}.$$

$$L = 4\int_0^r \sqrt{\frac{r^2}{r^2 - x^2}}\, dx = 4r \int_0^r \frac{dx}{\sqrt{r^2 - x^2}}$$

$\sqrt{r^2} = r$

$$= 4r \sin^{-1}\left(\frac{x}{r}\right)\Big]_0^r$$

$$= 4r\left[\sin^{-1}\left(\frac{r}{r}\right) - \sin^{-1}(0)\right]$$

$\sin\left(\frac{\pi}{2}\right) = 1$

$$= 4r\left[\cancel{\sin^{-1} 1}^{\,\pi/2} - \cancel{\sin^{-1}(0)}^{\,0}\right]$$

$$= 4r\,\frac{\pi}{2} = 2\pi r.$$

Longitud de circunferencia $2\pi r. = L$

A'rea de un círculo $\pi r^2 = A$

Volumen $4\pi r^3/3 = V$

4.

Longitud: Un cable telefónico cuelga entre dos postes con posiciones horizontales en $x = \pm 25$. El cable tiene una curva con ec. $y = -5 + 25\cosh\left(\frac{x}{25}\right)$

Encuentre la longitud del cable.

$$L = \int_{-25}^{25} \sqrt{1 + (y')^2}\, dx \qquad \frac{d}{dx}\cosh x = \sinh x$$

$$y' = 23\sinh\left(\frac{x}{25}\right) \cdot \frac{1}{25} = \sinh\left(\frac{x}{25}\right)$$

se cancelan los 25's.

$$1 + (y')^2 = 1 + \sinh^2\left(\frac{x}{25}\right) = \cosh^2\left(\frac{x}{25}\right)$$

identidad hiperbólica

$$L = \int_{-25}^{25} \sqrt{\cosh^2(x/25)}\, dx = \int_{-25}^{25} \cosh\left(\frac{x}{25}\right) dx$$

$$L = 2\int_{0}^{25} \cosh\left(\frac{x}{25}\right) dx = 2 \cdot 25\sinh\left(\frac{x}{25}\right)\Big]_{0}^{25}$$

$$L = 50\left[\sinh(1) - \cancel{\sinh(0)}^{\,0}\right] \cong 58.7600$$

función tiene diferente variable independiente.

Ejercicio 3: Pág 112. Encuentre la longitud para las siguientes curvas.

a. $\mathcal{C}_1: x = \frac{y^3}{6} + \frac{1}{2y} \qquad 1 \leq y \leq 2.$

Utilice el eje-y para integrar

$$L = \int_a^b \sqrt{1+(y')^2}\, dx \quad ; \quad \int_a^b \sqrt{1+(\underline{x'})^2}\, dy.$$

Objetivo: SIMPLIFIQUE $1+(x')^2$.

$$x' = \frac{3y^2}{6} - \frac{1}{2}y^{-2} = \frac{1}{2}\left(y^2 - y^{-2}\right) \qquad y^2y^{-2} = y^0$$

$$(x')^2 = \frac{1}{4}\left(y^2-y^{-2}\right)^2 = \frac{1}{4}\left(y^4 - 2\cdot 1 + y^{-4}\right)$$

$$1+(x')^2 = 1 + \frac{1}{4}\left(y^4 - 2 + y^{-4}\right)$$ Simplifique y factorice.

$$= \frac{1}{4}\left(4 + y^4 - 2 + y^{-4}\right) \qquad a^2 + 2a + 1 \quad (a+1)^2$$

$y^2 \cdot y^{-2} = 1$

$$= \frac{1}{4}\left(y^4 + 2 + y^{-4}\right) \qquad 1 + \frac{1}{4}a = \frac{4+a}{4}$$

$$= \frac{1}{4}\left(y^2 + y^{-2}\right)^2 = 1+(x')^2.$$

$$L = \int_1^2 \sqrt{1+(x')^2}\, dy = \int_1^2 \sqrt{\frac{1}{4}\left(y^2+y^{-2}\right)^2}\, dy.$$

$$= \frac{1}{2}\int_1^2 (y^2 + y^{-2})\,dy = \frac{1}{2}\left(\frac{y^3}{3} - \frac{1}{y}\Big]_1^2\right)$$

$$L = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right)$$

b. $\mathcal{C}_2: \quad y = \ln(\sec\theta) \qquad 0 \leq \theta \leq \pi/4.$

$$y'(\theta) = \frac{\sec\theta\tan\theta}{\sec\theta} = \tan\theta.$$

$$1 + (y')^2 = 1 + \tan^2\theta = \sec^2\theta.$$

$$L = \int_0^{\pi/4} \sqrt{1+(y')^2}\,d\theta = \int_0^{\pi/4} \sqrt{\sec^2\theta}\,d\theta.$$

$$L = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}.$$

$$L = \ln(\sec\pi/4 + \tan\pi/4) - \ln(\overset{1}{\sec 0} + \overset{0}{\tan 0})$$

$$= \ln(\sqrt{2} + 1) - \ln 1 = \ln(\sqrt{2}+1)$$

Longitud de Arco: Curva está en $a \leq t \leq x$.

límite superior indefinido.

$$S(x) = \int_a^x \sqrt{1+(y')^2}\,dt \Big\}$$ Función de Longitud de Arco.

Ejercicio 4: Encuentre la función de longitud de arco para la curva $y = \ln(\sin t)$ en $-\frac{\pi}{2} \leq t \leq \underline{X}$.

$$y' = \frac{\cos t}{\sin t} = \cot(t) \qquad \int \csc^2 t \, dt = \cot x$$

$$1 + (y')^2 = 1 + \cot^2(t) = \csc^2(t).$$

$$L(x) = \int_{-\pi/2}^{x} \sqrt{1 + (y')^2}\, dt = \int_{-\pi/2}^{x} \sqrt{\csc^2 t}\, dt.$$

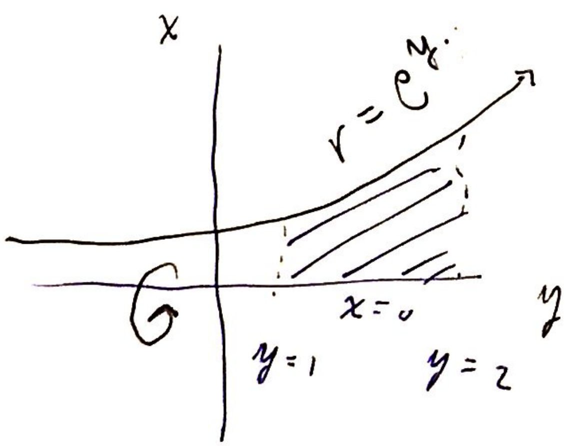
$$L(x) = \int_{-\pi/2}^{x} \csc t \, dt. = -\ln|\csc t + \cot t| \Big]_{-\pi/2}^{x}$$

$$= -\ln|\csc x + \cot x| + \ln|\csc(-\pi/2) + \cot(-\pi/2)|$$

$$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = 0 \qquad \csc(-\pi/2) = \frac{1}{\sin(-\pi/2)} = -1$$

$$= -\ln|\csc x + \cot x| + \overset{0.}{\cancel{\ln|-1|}}$$

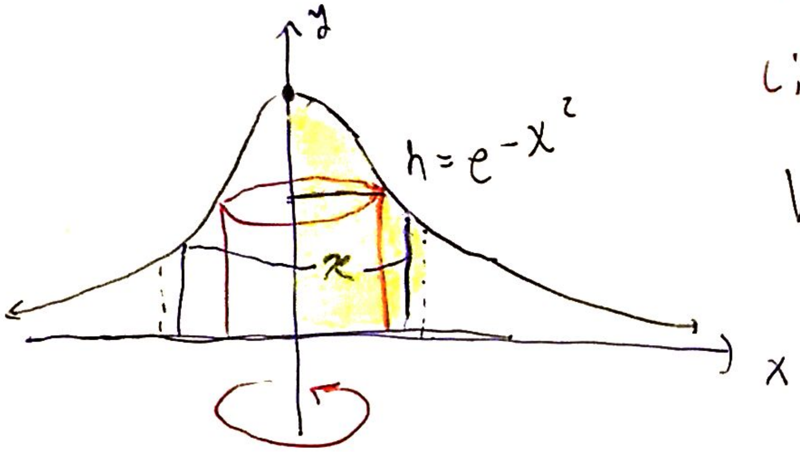
Ej: 3 Lab 8.



Ej: 4 $y = e^{-x^2}$ $y = 0,$ $x = -1$ $x = 1$

Lab 8.

a) alrededor del eje-y.



Cilindros.

$$V = 2\pi \int_0^1 h r \, dx$$

Discos

$$V = \pi \int_0^1 r^2(y)\, dy.$$

$$V = \pi \int_0^1 e^{-x^2} \overbrace{2x\,dx}^{-du} = \pi \int_0^{-1} -e^u \, du = -\pi e^u \Big]_0^{-1}$$

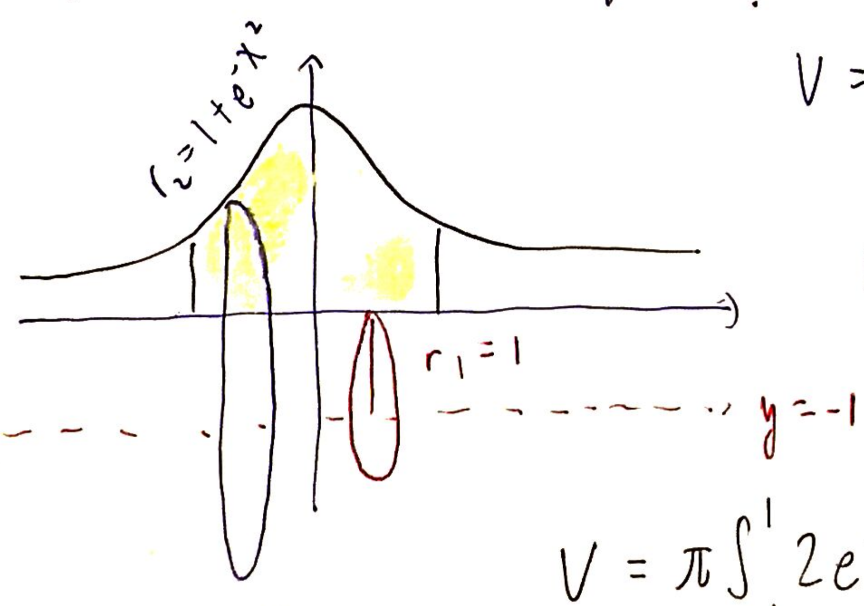
$u = -x^2$ $du = -2x\,dx$

$u(0) = 0$ $u(1) = -1$

$$-\pi\left(e^{-1} - e^0\right)$$

$$\pi\left(e^0 - e^{-1}\right) = \pi\left(1 - \frac{1}{e}\right)$$

b) alrededor de $y = -1$.



$$V = \pi \int_{-1}^1 r_2^2 - r_1^2 \, dx.$$

$$(1 + e^{-x^2})^2 - 1$$

$$1 + 2e^{-x^2} + e^{-x^4} - 1$$

$$2e^{-x^2} + e^{-x^4}$$

$$V = \pi \int_{-1}^1 2e^{-x^2} + e^{-x^4} \, dx$$