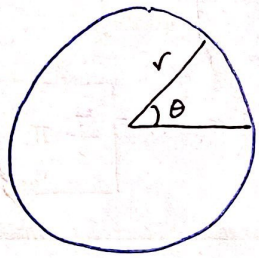
2019-10-31

Resolución de corto a priori

Áreas de regiones polares

- Área de una "rebanada de pizza".

* el círculo tiene ángulo 2π



La rebanada o sector circular tiene un ángulo central θ.

$$A_{rebanada} = \pi r^2 \left(\frac{\theta}{2\pi}\right) = \frac{r^2}{2}\theta$$

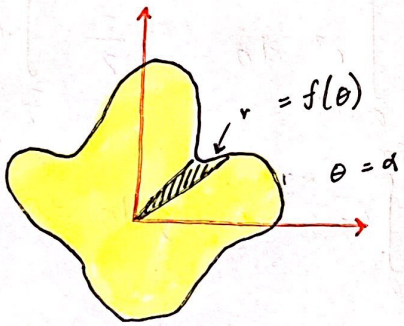
Si la pizza tiene 8 pedazos

$$\hookrightarrow \frac{2\pi}{8} = \frac{\pi}{4} \text{ ó } 45^\circ$$

$$A = \frac{r^2}{2}\cdot\frac{\pi}{4} = \frac{\pi r^2}{8} = \pi\,\frac{144}{8}$$

Área de una Región polar

$r = f(\theta) \quad \alpha \le \theta \le \beta$



Considere una rebanada muy delgada "infinitecimal".

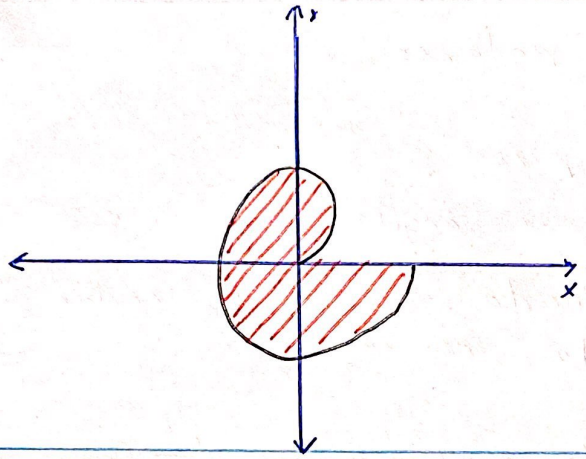
$r = f(\theta) \quad d\theta$

Integre dA en $\alpha \le \theta \le \beta$

$$\underbrace{dA}_{\text{área infinitesimal}} = \frac{r^2}{2}\,d\theta = \frac{f^2(\theta)}{2}\,d\theta$$

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2(\theta)\,d\theta$$

Ej: Encuentre el área dentro del espiral $r = \theta$ en $0 \leq \theta \leq 2\pi$



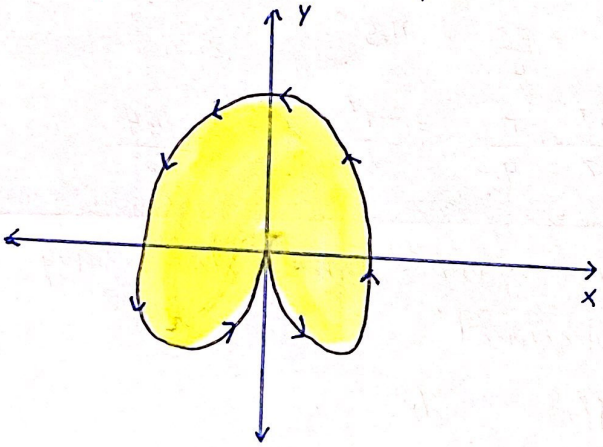
$$A = \frac{1}{2}\int_0^{2\pi} 1\,\theta^2\,d\theta$$

$$= \frac{1}{6}\theta^3\Big]_0^{2\pi}$$

$$= \frac{1}{6}8\pi^3 = \boxed{\frac{4}{3}\pi^3}$$

Ej 1: Encuentre el área de las siguientes regiones:

a) Encervada por el cardioide $r = 1 - \sin\theta$



Límites $0 \leq \theta \leq 2\pi$

$$A = \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \frac{2}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} r^2\,d\theta =$$

$$= \frac{1}{2}\int_0^{2\pi}(1 - \sin\theta)\,d\theta =$$

$$A = \frac{1}{2}\int_0^{2\pi}(1 - 2\sin\theta + \sin^2\theta)\,d\theta$$

$$A = \frac{1}{2}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{1}{2}\cdot\frac{1}{2}\sin(2\theta)\right] =$$

$$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin(2\theta)\right]_0^{2\pi} =$$

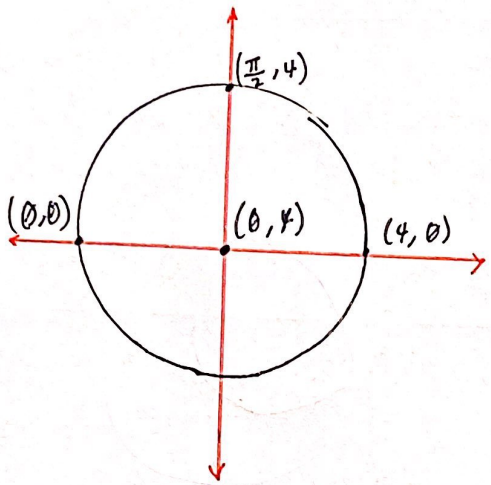
$$= \frac{1}{2}\left(\frac{3}{2}2\pi + 2\cos(2\pi) - \frac{1}{4}\sin(4\pi) - 0 - 2\cos(0) - \frac{1}{4}\sin 0\right)$$

$$\therefore = \boxed{\frac{3\pi}{2}}$$

Hay tangentes verticales en $\theta = 0$ y en $\theta = \pi$

Hay tangentes horizontales en $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

b) Dentro del círculo $r = 4\sin\theta$ en $0 \le \theta \le \pi$



$$A = \frac{1}{2}\int_0^{\pi} r^2\, d\theta =$$

$$A = \frac{1}{2}\int_0^{\pi} 16\sin^2(\theta)\, d\theta =$$

$$= \int_0^{\pi} 8\sin^2(\theta)\, d\theta =$$

$$= \int_0^{\pi} 4\left(1 - \cos(2\theta)\right) d\theta$$

$$= 4\left[\theta - 2\sin(2\theta)\right]_0^{\pi}$$

$$= 4\pi - 2\sin(2\pi) - 0 + 0 = \boxed{4\pi}$$

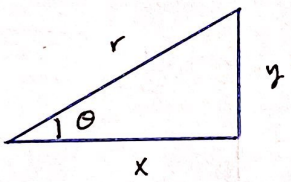
Ejercicio 2: Rosa de 4 pétalos.

a) Encuentre la derivada $\frac{dy}{dx}$ de $r = \cos(2\theta)$

Ecs. Paramétricas de la curva polar

$$y = r\sin(\theta)$$

$$x = r\cos(\theta)$$

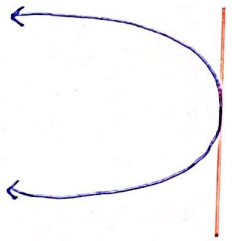


$$y = \cos(2\theta)\sin(\theta)$$

$$x = \cos(2\theta)\cos(\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2\sin 2\theta \sin(\theta) + \cos(2\theta)\cos(\theta)}{-2\sin(2\theta)\cos(\theta) - \cos(2\theta)\sin(\theta)}$$

b) Comprueba que la rosa tiene tangentes verticales en $\theta = 0$ y en $\theta = \pi$



$\frac{dy}{dx}$ no existe $\Rightarrow$ $\frac{dy}{dx} = \frac{-2\cdot 0 + 1}{-2\cdot 0 - 0} = \frac{1}{0}$ no existe

$$\Rightarrow \frac{dy}{dx} = \frac{-2\cdot 0 - 1}{2\cdot 0 - 0} = -\frac{1}{0} \text{ No existe}$$