

2019-10-29

Lab 13

Jueves 4 noviembre Lab 13

Corto II Jueves 31 10.2 Longitud de arco y Área
Jueves 6 10.4 Áreas coordenadas polares.

Simulacro 3: 11 de noviembre

Parcial 3: Martes 12 de noviembre.

Curvas Polares

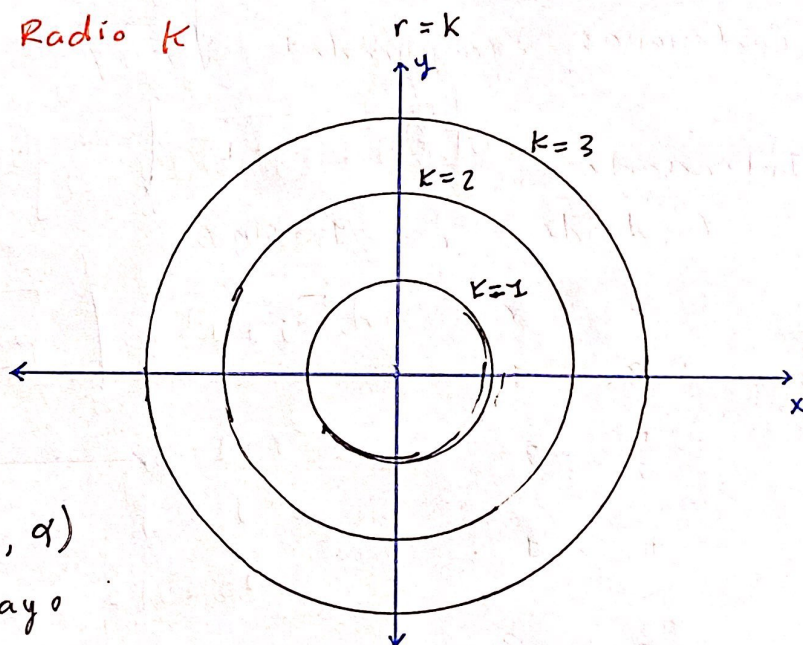
Función polar $r = f(\theta)$
 θ variable independiente
 r es dependiente

a) Circunferencia de Radio k

$$x^2 + y^2 = k^2$$

$$r^2 = k^2$$

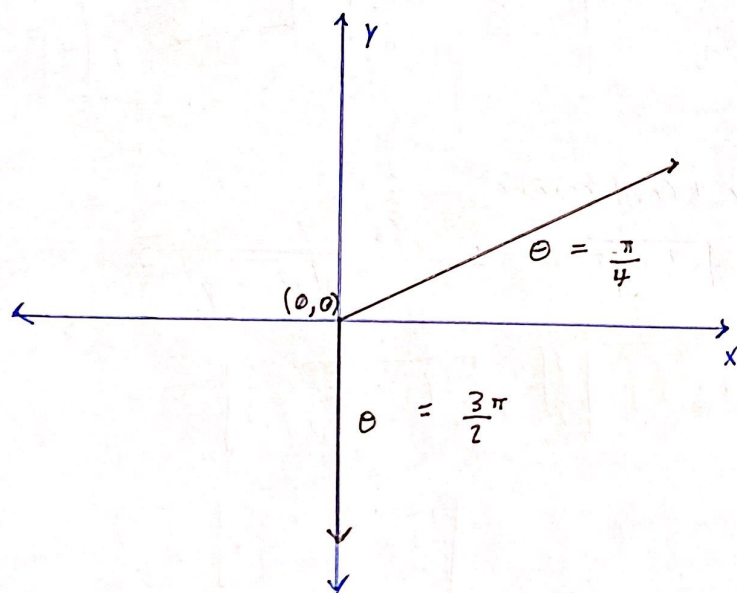
$$r = k$$



b) Línea o rayo $\theta = \alpha$

$(0, \alpha)$ $(1, \alpha)$ $(2, \alpha)$

son parte del rayo



Recordar:

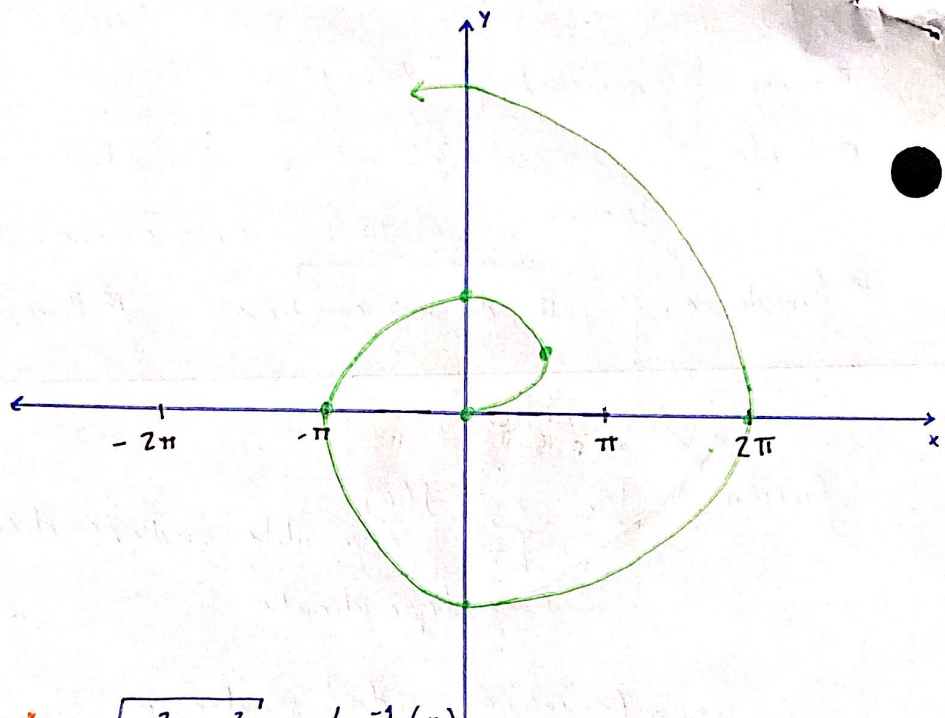
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \alpha$$

$$y = x(\tan \alpha)$$

cartesianas

Espiral $r = \theta$

θ	r
0	0
$\frac{\pi}{2}$	$\frac{\pi}{2}$
π	π
$\frac{3\pi}{2}$	$\frac{3\pi}{2}$
2π	2π

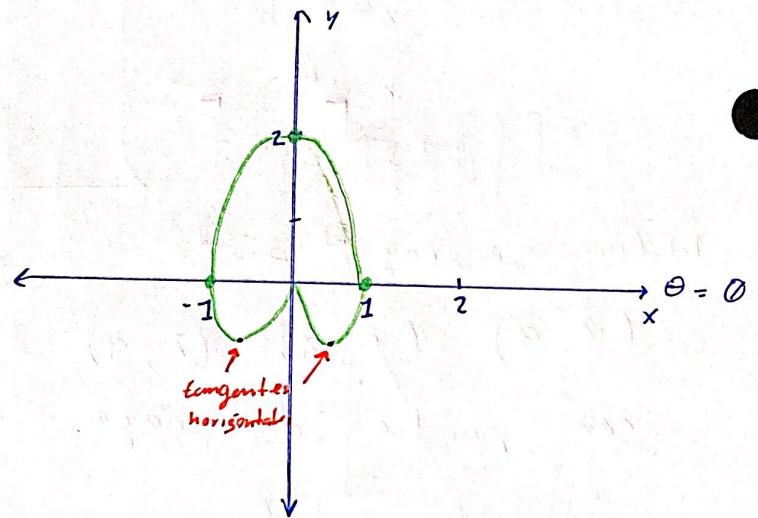


Cartesianas *complicado* $\sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$

Interesante:

- Cardioides $r = 1 + \sin \theta$

θ	r
0	1
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	0
2π	1
$\frac{7\pi}{6}$	$\frac{1}{2}$
$\frac{11\pi}{6}$	$\frac{1}{2}$



Ec. Cartesiana

$$\sqrt{x^2 + y^2} = 1 + \frac{y}{\sqrt{x^2 + y^2}}$$

Rosa o pétalos de 4 hojas:

$$r = \cos(2\theta) = 0$$

$$\cos(u) = 0 \text{ cuando } 2\theta = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi$$

$$r=0 \text{ en } \left\{ \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$r = \cos(2\theta) = 1$$

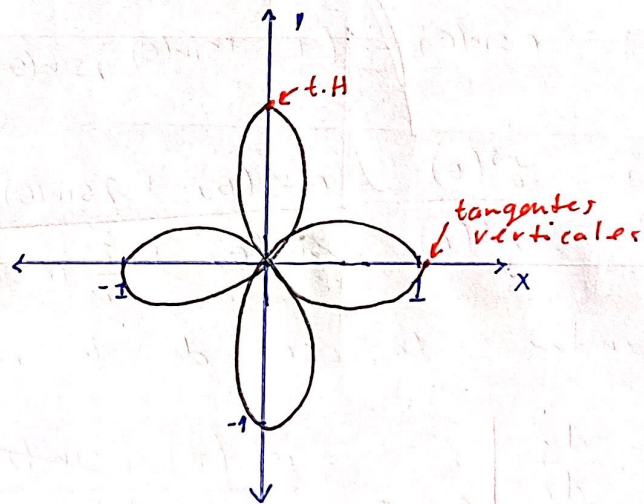
$$2\theta = 0, 2\pi, 4\pi$$

$$\theta = 0, \pi, 2\pi$$

$$r = \cos(2\theta) = -1 \quad 2\theta = \pi, 3\pi$$

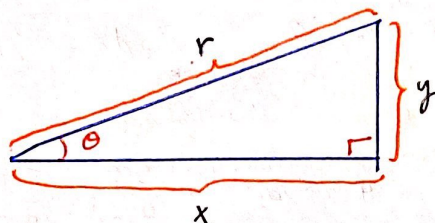
$$\text{cuando } \theta = \frac{\pi}{2}, \frac{3}{2}\pi$$

θ	r
0	1
45	0
90	-1
135	0
180	1
225	0
270	-1
315	0
360	1



Derivada $\frac{dy}{dx}$ de una curva polar $r = f(\theta)$

$$\begin{aligned} x &= r \cos(\theta) \\ y &= r \sin(\theta) \end{aligned}$$



Ecs. Paramétricas

$$x = f(\theta) \cos(\theta)$$

$$y = f(\theta) \sin(\theta)$$

θ es el parámetro

$$r = f(a) \quad (a, f(a))$$

Ejercicio 5: Considere el cardiode $r = 1 + \sin(\theta)$

a) Encuentre $\frac{dy}{dx}$

$$x = r \cos(\theta) = (1 + \sin(\theta)) \cos \theta = \cos \theta + \underbrace{\sin \theta \cos \theta}_{\frac{1}{2} \sin(2\theta)}$$

$$y = r \sin(\theta) = (1 + \sin(\theta)) \sin(\theta) = \sin(\theta) + \sin^2(\theta)$$

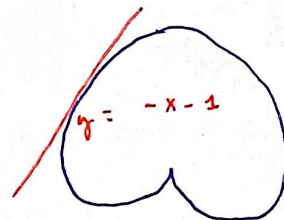
$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{\cos(\theta) + 2 \sin(\theta) \cos(\theta)}{-\sin(\theta) + \frac{2}{2} \cos(2\theta)} =$$

Encuentra la ec. de la recta tangente en $\theta = \pi$

$$m = \left. \frac{dy}{dx} \right|_{\theta = \pi} = \frac{\cos(\pi) + \cancel{\sin(2\pi)}^0}{-\cancel{\sin(\pi)}^0 + \cos(2\pi)} = \frac{-1}{1} = -1$$

$$x(\pi) = \cancel{\cos(\pi)}^{-1} + \frac{1}{2} \cancel{\sin(2\pi)}^0 = -1$$

$$y(\pi) = (1 + \sin(\pi)) \sin(\pi) = 1 \cdot 0 = 0$$



Ec. Recta tangente:

$$y = y(\pi) + m(x - x(\pi)) = 0 + -1(x + 1) = -x - 1$$

Ej 4: Circunferencias $r = A \cos(\theta) \pm B \sin(\theta)$ con centro fuera del origen.

Sea $r = 2 \sin \theta$

a) Encuentre una ecuación cartesiana para la curva

$$r = \sqrt{x^2 + y^2} \quad \text{Elimine } \theta \text{ \& } r$$

$$y = r \sin(\theta) \Rightarrow \sin(\theta) = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\underbrace{r = 2 \sin(\theta)}_{\text{dado}} \Rightarrow \sqrt{x^2 + y^2} = \frac{2y}{\sqrt{x^2 + y^2}}$$

$$x^2 + y^2 = 2y$$

Ec. circunferencia

$$x^2 + y^2 - 2y + 1 = +1$$

completación
al cuadrado

$$(x - a)^2 + (y - b)^2 = R^2$$

$$\underbrace{x^2 + (y - 1)^2}_{\text{Circunferencia de}} = 1$$

Radio 1 centrada

$(0, 1)$.

b) Grafique la función polar $r = 2 \sin(\theta)$

