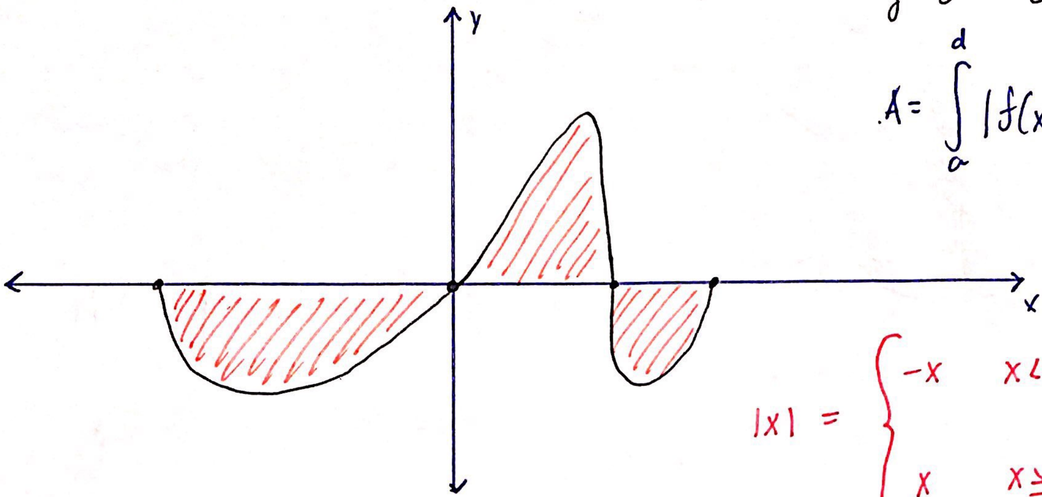
6.1. Área Entre Curvas   Pg. 79   2019-09-5

Región entre la curva $y = f(x)$, y el eje-x.

$$A = \int_a^d |f(x)|\,dx$$

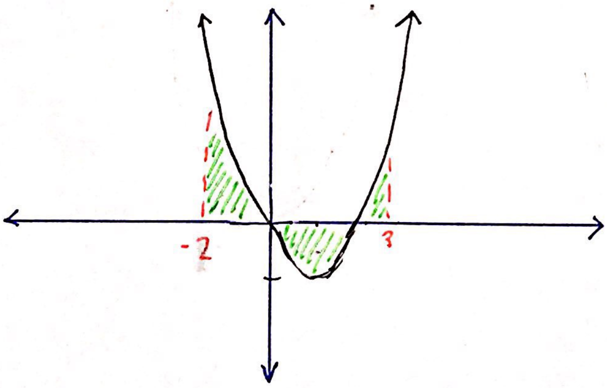


$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$A = -\int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx$$

Intersecciones y bosquejos la curva y la región.

Ejercicio I: Bosqueje y encuentre el área de la región limitada por $y = 3x^2 - 6x$, $x = -2$, $x = 3$ & $y = 0$.



Ix

$y = 3x^2 - 6x = 0$

$3x(x-2) = 0 \quad x = 2,\ x = 0$

Sume el área de 3 subregiones:

$$A = \int_{-2}^{0} 3x^2 - 6x\,dx - \int_{0}^{2} 3x^2 - 6x\,dx + \int_{2}^{3} 3x^2 - 6x\,dx$$

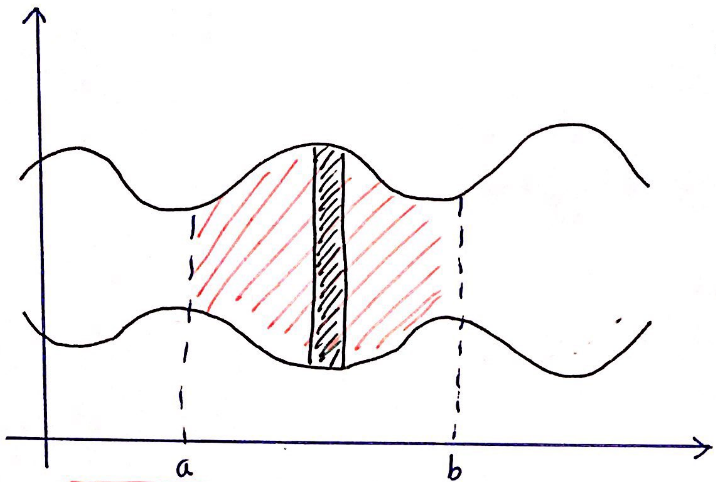
$$A = x^3 - 3x^2\Big]_{-2}^{0} + \left(-x^3 + 3x^2\Big]_0^2\right) + x^3 - 3x^2\Big]_2^3$$

$$A = 0 - (-8 - 12) + (-8 + 12) + (27 - 27 - 8$$

$$A = 28$$

$\square$

¿Cuando hay una curva interior?



$y = f(x)$

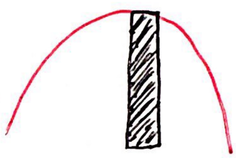
Región $= g(x) \leq y \leq f(x)$

$a \leq x \leq b$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

diferencia de áreas

$$\boxed{A = \int_a^b y_{\text{arriba}} - y_{\text{abajo}}\,dx}$$



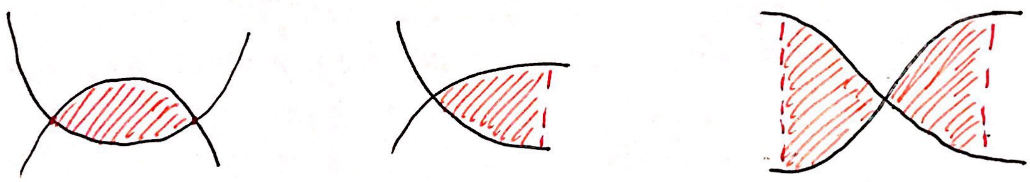
- franja horizontal ó rectángulo infinitesimal
  - def. pt más alto y más bajo
  - ancho dx altura $f(x) - g(x)$

$$dA = (f(x) - g(x))\,dx$$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

Pasos:

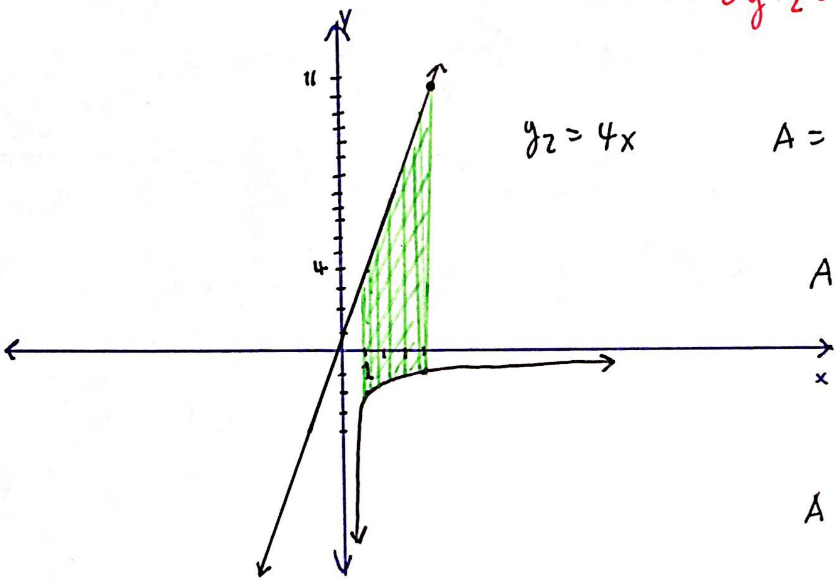
1. Bosqueje $g(x)$ & $f(x)$.
2. Ojo con intersecto entre $f(x)$ & $g(x)$.
3. Bosqueje la región.



Ejemplo: Bosqueje y encuentre el área entre:

$$y_1 = \frac{-2}{\sqrt{x}} \quad , \quad y_2 = 4x \quad \text{en} \quad 1 \leq x \leq 4$$

¿$y_2 \geq y_1$? ó ¿$y_2 \geq y_2$?



$y_2 = 4x$

$$A = \int_1^4 y_2 - y_1 \, dx$$

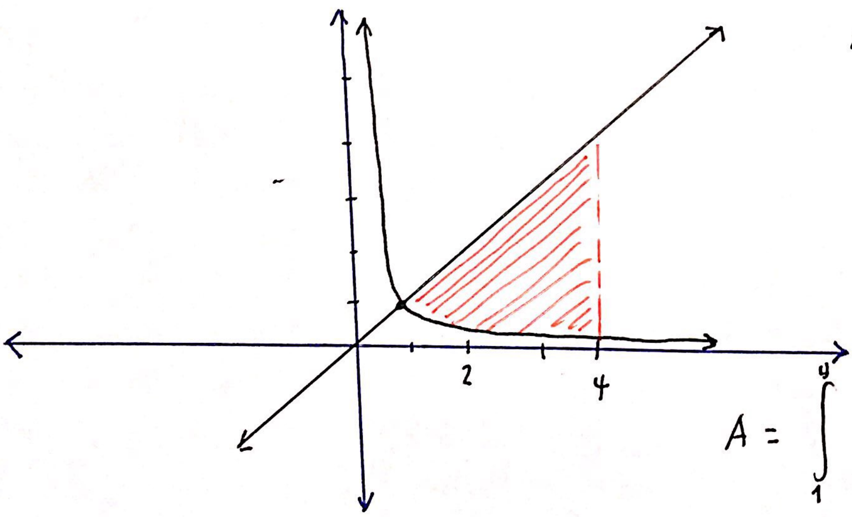
$$A = \int_1^4 4x - \left(-2x^{-1/2}\right) dx$$

$$A = 2x^2\Big]_1^4 + 4x^{1/2}\Big]_1^4$$

$$A = \{2 \cdot 16 + 4 \cdot 2\} - \{2 + 4\}$$

$$A = 32 + 8 - 6 = 34 \quad \square$$

Variación $y_1 = \frac{4}{\sqrt{x}}$ & $y_2 = 4x$ y la recta $x = 4$.



$$A = \int^4 y_2 - y_1$$

Intersección y_1 & y_2

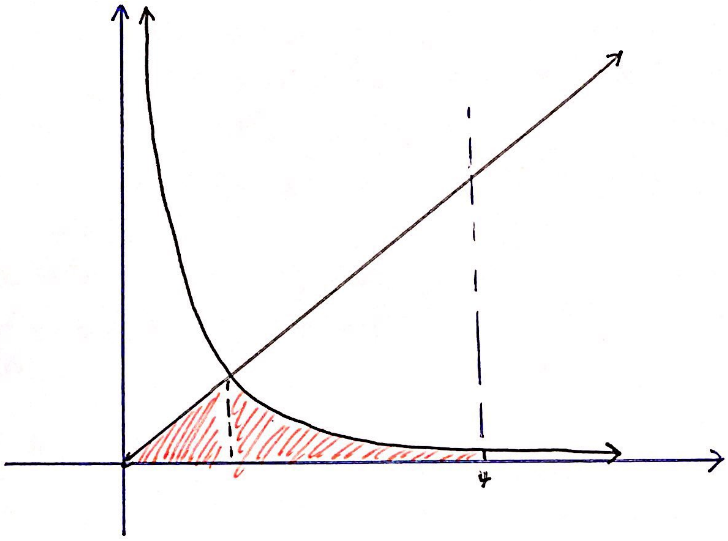
$$\frac{4}{x^{1/2}} = 4x$$

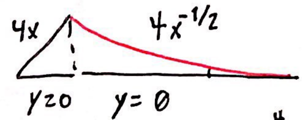
$$1 = x^{3/2} \Rightarrow x = 1$$

$$A = \int_1^4 4x - 4x^{-1/2} \, dx = 2x^2 - 8x^{1/2}\Big]_1^4 =$$

$$= 32 - 16 - (2 - 8) - 16 + 6 = 22$$

Variación C: Área de la región entre $y_1 = 4x^{-1/2}$ $y_2 = 4x$, $x = 4$, $x = 0$



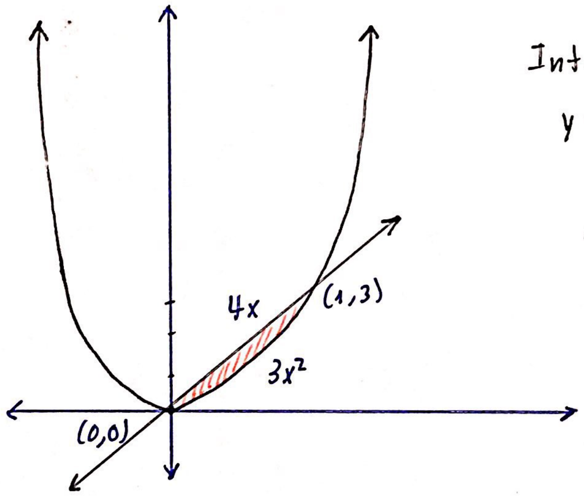


$$A = \int_0^1 4x\,dx + \int_1^4 4x^{-1/2}\,dx$$

$$A = 2x^2\Big]_0^1 + 8x^{1/2}\Big]_1^4$$

$$A = 2 + 16 - 8 = 10$$

Ejemplo: Encuentre el área de la recta de la región entre las curvas $y_1 = 3x$ & $y_2 = 3x^2$.



Intersectos: $3x^2 = 3x$

$y_2 = y_2$ $x = 1$, $x = 0$

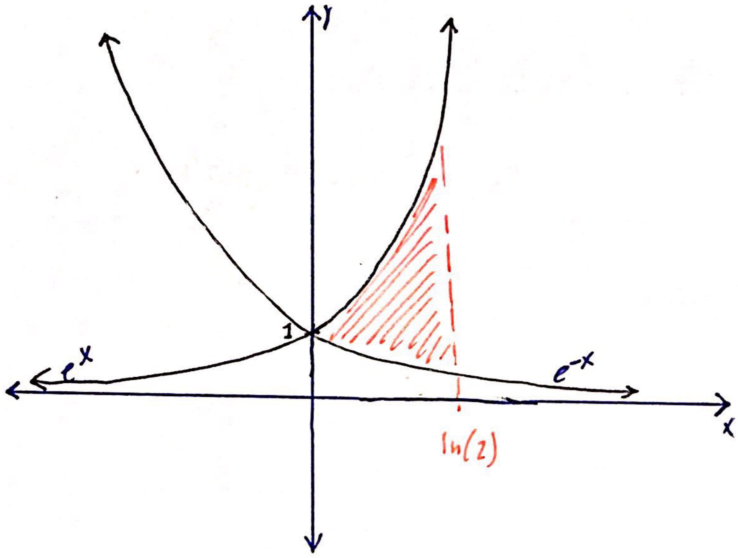
$$A = \int_0^1 (3x - 3x^2)\,dx$$

$$A = \frac{3x^2}{2} - x^3\Big]_0^1 = \left\{\frac{3}{2} - 1\right\} - \left\{0\right\} = \frac{1}{2}$$

☐

Ejercicio 2: Bosqueje y encuentre el área de la región entre las curvas.

a) $y_1 = e^x, \quad y_2 = e^{-x}, \quad x = 0, \quad x = \ln(2)$



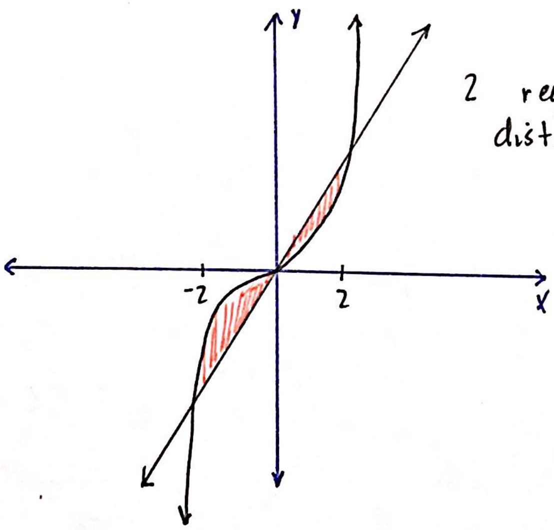
$$A = \int_0^{\ln(2)} (e^x - e^{-x})\, dx$$

$$A = e^x + e^{-x} \Big]_0^{\ln(2)}$$

$$A = \{2 + 2^{-1}\} - \{1 + 1\}$$

$$A = 2 + \frac{1}{2} - 2 = \frac{1}{2}$$

b) $y_1 = x^3, \quad y_2 = 4x$



2 regiones distintas

Intersectos

$y_1 = y_2$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

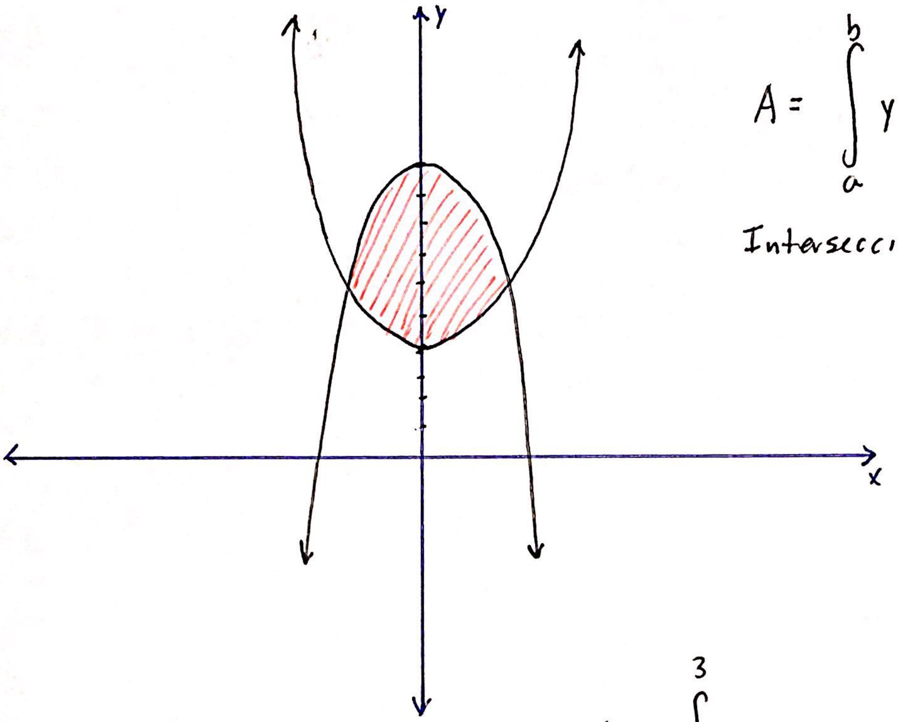
$x = 0, \ x = -2, \ x = 2$

$$A = 2\int_0^2 4x - x^3\, dx = 2\left\{2x^2 - \frac{x^4}{4}\Big]_0^2\right\} =$$

$$= 2\left[\left\{2(2)^2 - \frac{(2)^4}{4}\right\} - \{0\}\right] = 2\left(8 - \frac{16}{4}\right)$$

$$= 8$$

c) $y_1 = x^2 - 4x + 4, \quad y_2 = 10 - x^2$



$$A = \int_a^b y_2 - y_1 \, dx$$

Intersecciónes $\quad y_1 = y_2$

$$x^2 - 4x + 4 = 10 - x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x-3)(x+1)$$

$$x = 3 \quad x = -1$$

$$A = \int_{-1}^{3} 10 - x^2 - (x^2 - 4x + 4) \, dx$$

$$A = \int_{-1}^{3} (10 - 2x^2 + 4x) \, dx =$$

$$= 6x - \frac{2}{3}x^3 + 2x^2 \Big]_{-1}^{3} = 18 - 18 + 18 - (-6 + \frac{2}{3} + 2)$$

$$= 18 + 4 - \frac{2}{3} = 22 - \frac{2}{3}$$