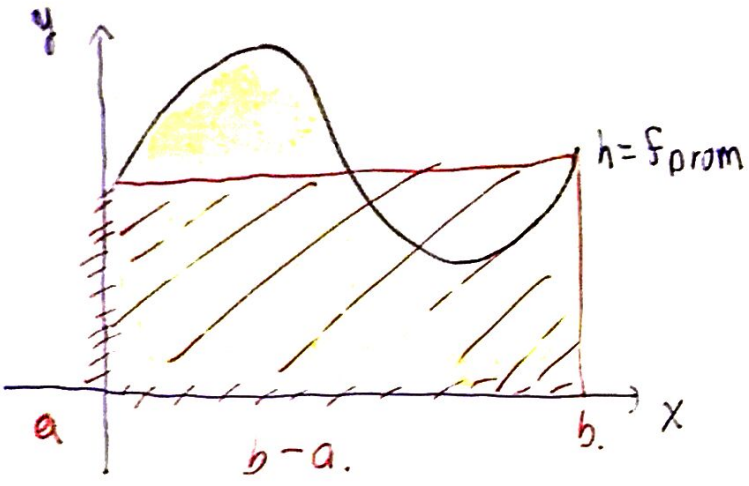
6.5 Valor Promedio de una función



$f_{prom.}$

¿Promedio altura y.?

Iguale el área del rectángulo con el área de la región amarilla

misma área

Ancho $b-a$ Área Región Amarilla

altura f_{prom}

Área $f_{prom}(b-a) = \int_a^b f(x)\,dx.$

$$f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$f(x)$ sea continua en $[a,b]$.

Ejemplo: Encuentre el valor promedio de $f(x) = \csc^2 x$ en $[\frac{\pi}{4}, \frac{\pi}{2}]$.

$b-a = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$. $\quad \frac{1}{b-a} = \frac{4}{\pi}$

$$f_{prom} = \frac{4}{\pi}\int_{\pi/4}^{\pi/2} \csc^2 x\,dx = -\frac{4}{\pi}\cot x\Big]_{\pi/4}^{\pi/2}$$

$$f_{prom} = \frac{-4}{\pi}\cancel{\cot \pi/2}^{\,0} + \frac{4}{\pi}\cot \pi/4. = \frac{4}{\pi}.$$

$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1}$ $\quad \tan \frac{\pi}{4} = 1$

Ejercicio 1: Encuentre el valor promedio de $F(x)$.

a. $f(t) = \cos^4 t \sin t$ en $[0, \pi]$.

$$f_{prom} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{\pi - 0}\int_0^{\pi} \underbrace{\cos^4 t}_{u^4} \underbrace{\sin t\,dt}_{-du}.$$

$u = \cos t \quad du = -\sin t\,dt. \quad u(\pi) = \cos\pi = -1$

$u(0) = \cos 0 = 1$

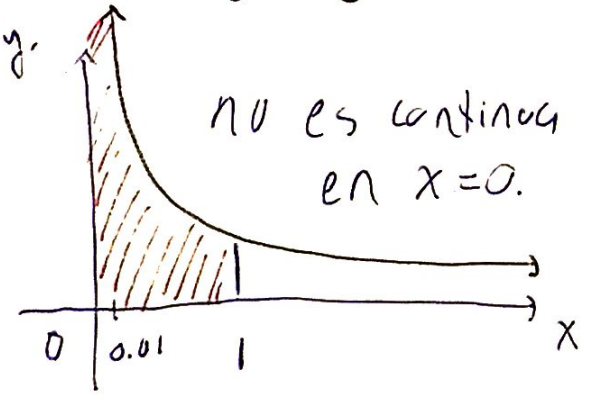
$$f_{prom} = \frac{-1}{\pi}\int_1^{-1} u^4\,du = \frac{1}{\pi}\int_{-1}^{1} u^4\,du \overset{\text{par.}}{=} \frac{2}{\pi}\int_0^1 u^4\,du.$$

$$f_{prom} = \frac{2}{\pi}\,\frac{u^5}{5}\Big]_0^1 = \frac{2}{5}\cdot\frac{1}{\pi}.$$

b. $g(x) = \frac{1}{x}$ en $[e^4, e^{10}]$.

$$y_{prom} = \frac{1}{e^{10}-e^4}\int_{e^4}^{e^{10}} \frac{1}{x}\,dx = \frac{1}{e^{10}-e^4}\ln|x|\Big]_{e^4}^{e^{10}}$$

$$y_{prom} = \frac{1}{e^{10}-e^4}\left(\ln e^{10} - \ln e^4\right) = \frac{6}{e^{10}-e^4}$$



no es continua en $x = 0$.

Valor promedio de f en $0 \le x \le 1$

$$y_{prom} = \frac{1}{1}\int_0^1 \frac{1}{x}\,dx$$

$$= \ln x\Big]_0^1 = -\lim_{x\to 0^+} \ln x$$

$y_{prom} = -\lim_{x \to 0^+} \ln x = +\infty$ no existe.

La función no tiene valor promedio

$u^{-1/2}$
$2u^{1/2}$.

c. $h(x) = \dfrac{3}{(4+x)^{1/2}}$ en $[-4, 5]$.

no es continua en $x = -4$.

$$h_{prom} = \frac{1}{5-(-4)} \int_{(-4)}^{5} 3(4+x)^{-1/2}\, dx \qquad (x)' = 1.$$

$$h_{prom} = \frac{3}{9}\, 2(4+x)^{1/2} \Big]_{(-4)}^{5} \qquad (4-4)^{1/2} = 0^{1/2}$$

$$h_{prom} = \frac{2}{3}\left(9^{1/2} - \lim_{x \to -4^+} (4+x)^{1/2} \right) = \frac{2}{3} \cdot 3 = 2.$$

Ejercicio 2: Densidad Lineal $\rho = 12(x+1)^{-1/2}$.

La varilla tiene 8 m. de longitud.

a. Encuentre la densidad promedio de la varilla.

$$\rho_{prom} = \frac{1}{8} \int_0^8 12(x+1)^{-1/2}\, dx$$

$$\rho_{prom} = \frac{24}{8} (x+1)^{1/2} \Big]_0^8 = 3\left(9^{1/2} - 1^{1/2} \right)$$

$3(3-1) = 6$ Kg/m.