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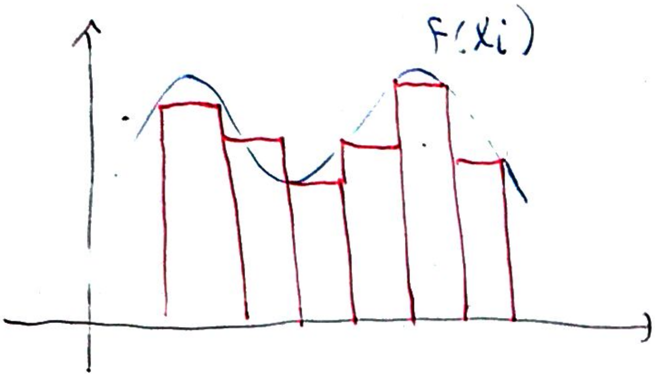
5.4 A'reas y Propiedades de la Integral Definida



A'rea

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

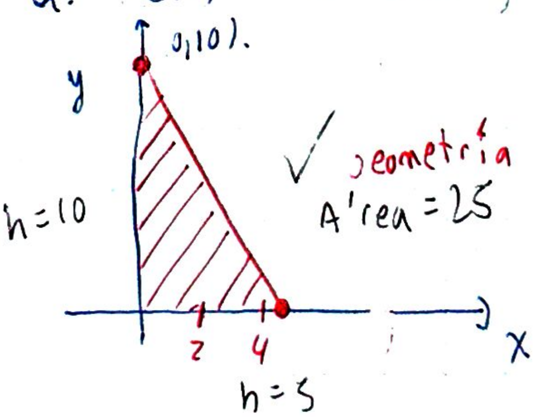
Si f es continua en $[a,b]$, la integral definida es

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

La integral definida es el área de la región bajo la curva $y = f(x)$ en $[a,b]$. $f(x) \geq 0$.

Ejercicio 2: Encuentre el área de las sigs. regiones. Bosqueje cada región.

a. $f(x) = 10 - 2x$, $f(x) \geq 0$ en $0 \leq x \leq 5$.



$$A = \int_0^5 (10 - 2x)\,dx$$

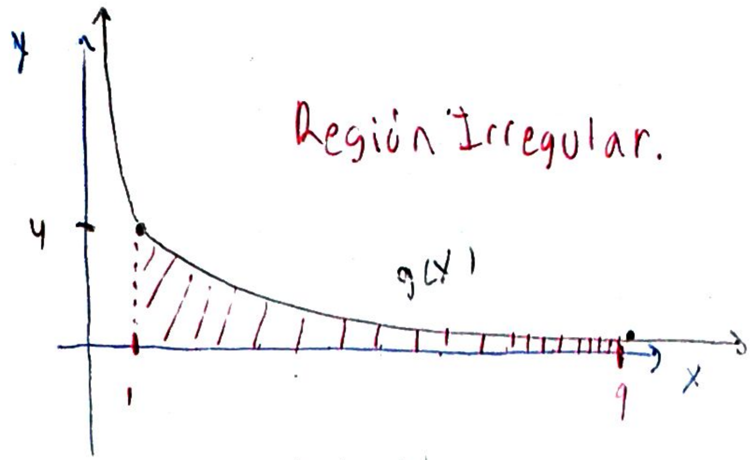
$$A = 10x - x^2 \Big]_0^5 = 50 - 25 - 0$$

$$A = 25.$$

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a).$$

b. $g(x) = \dfrac{4}{\sqrt{x}}$ entre $1 \le x \le 9$

AV en $x=0$
AH en $y=0$.



$$A = \int_1^9 4x^{-1/2}\,dx$$

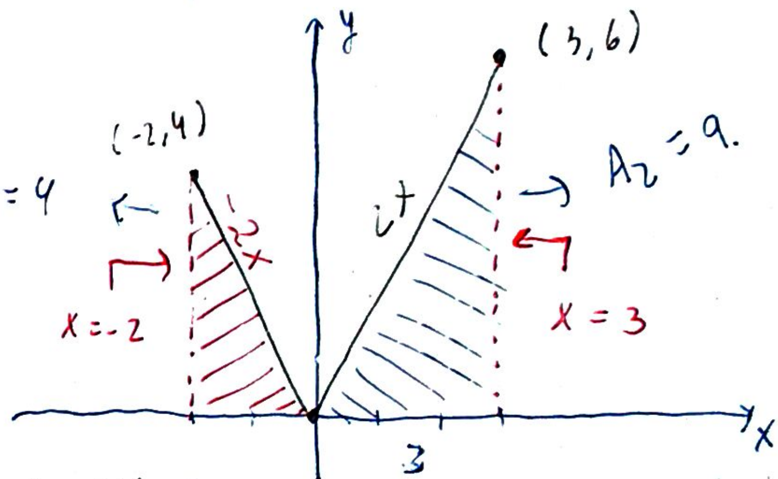
$$A = 8x^{1/2}\Big]_1^9$$

$$A = 8(\sqrt{9} - \sqrt{1}) = 8 \cdot 2 = 16.$$

c. $h(x) = 2|x|$ entre $x=-2$ y $x=3$.

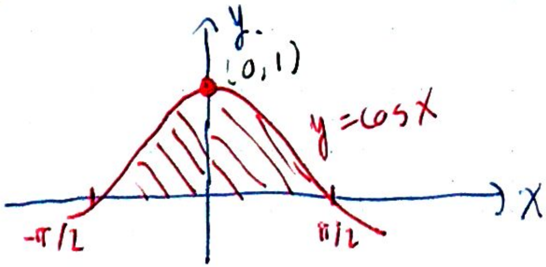
$$A = \int_{-2}^{3} 2|x|\,dx$$

$$A = \int_{-2}^{0} -2x\,dx + \int_0^3 2x\,dx$$



$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_0^3 = -0 - (-4) + 9 - 0 = 4 + 9 = 13.$$

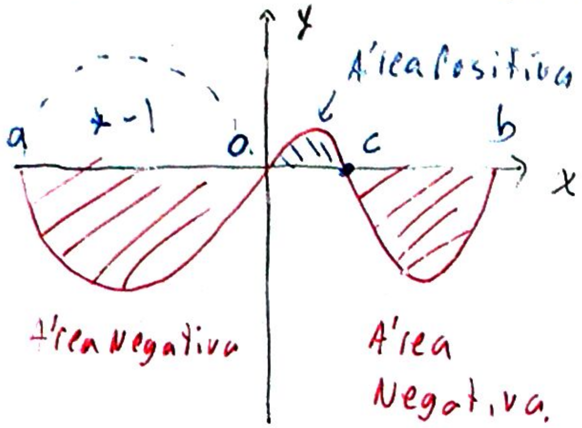
d. $y = \cos x$ entre $x = -\pi/2$ y $x = \pi/2$.



$$A = \int_{-\pi/2}^{\pi/2} \cos x\,dx = \sin x\Big]_{-\pi/2}^{\pi/2}$$

$$A = 1 - (-1) = 2.$$

¿Qué sucede cuando $f(x) < 0$ en partes del intervalo?



Área de la región entre $f(x)$ y el eje-x.

$$A \neq \int_a^b f(x)\,dx$$

$$* \int x f(x)\,dx = x \int f(x)\,dx$$

$$A = \int_a^0 -f(x)\,dx + \int_0^c f(x)\,dx - \int_c^b f(x)\,dx$$

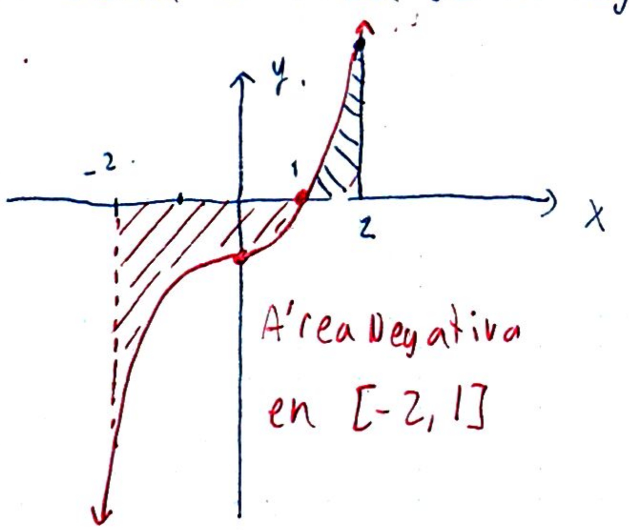
Pag 16. Ejercicio 3: Considere $f(x) = 4x^3 - 4$ en $-2 \leq x \leq 2$.

a. $\displaystyle\int_{-2}^{2} (4x^3 - 4)\,dx = x^4 - 4x \Big]_{-2}^{2} = (16 - 8) - (16 + 8)$

en $x=2$ $\quad 8 - 24 = -16$

$4(x^3 - 1) = 0$

b. Bosqueje la región y explique si la integral definida representa el área de la región.



La integral definida no es el área de la región.

c. Encuentre el área de la región

$$A = \int_{-2}^{1} 4 - 4x^3 \, dx + \int_{1}^{2} (4x^3 - 4) \, dx.$$

$$A = \underbrace{4x - x^4 \Big]_{-2}^{1}}_{A_1} + \underbrace{x^4 - 4x \Big]_{1}^{2}}_{A_2} = \overbrace{(4-1) - (-8-16)}^{A_1} + \underbrace{(16-8) - (1-4)}_{A_2}$$

$$A = (3 + 24) + (8 + 3) = 27 + 11 = 38$$

Propiedades Integral Definida.

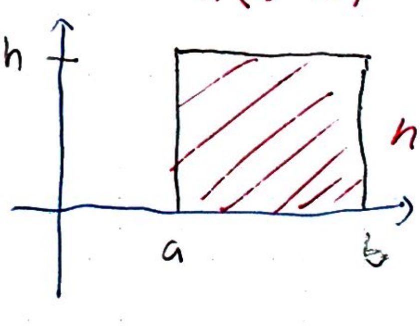
$$\int_a^b k_1 f(x) \pm k_2 g(x) \, dx = k_1 \int_a^b f(x) \, dx \pm k_2 \int_a^b g(x) \, dx$$

Otras Propiedades. $F(x)\Big]_a^a = F(a) - F(a) = 0$

Ejemplos: $\int_{\pi}^{\pi} \arctan(e^{x^2} + \log|x|) \, dx = 0$

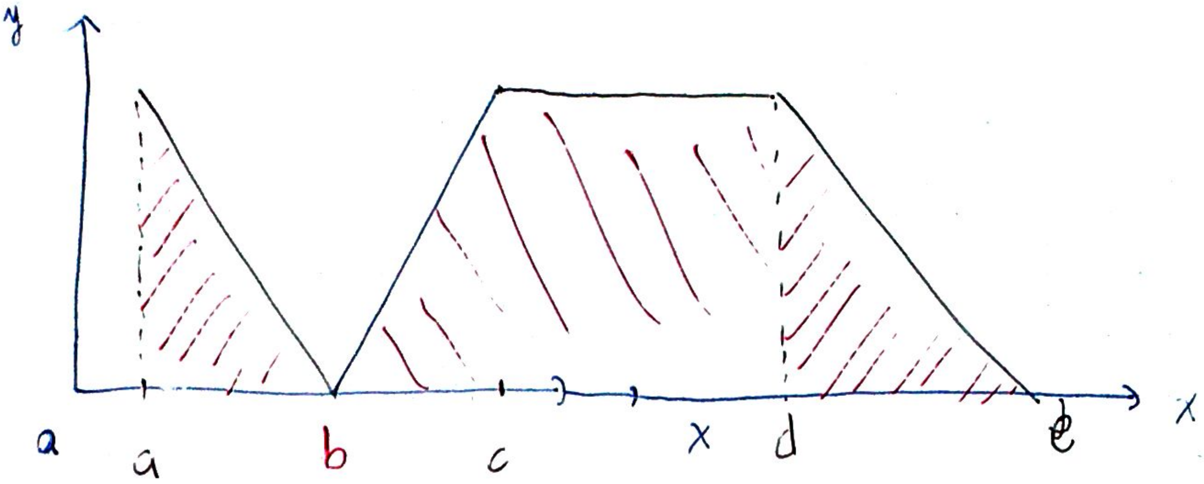
$$\int_a^a f(x) \, dx = 0$$

$h(b-a) = A.$



$$\int_a^b h \, dx = hx\Big]_a^b = h(b-a)$$

p.e. $\int_5^{1005} \sqrt{2} \, dx = \sqrt{2}(1005 - 5)$



7. $$\int_a^e f(x)\,dx = \int_a^b f\,dx + \int_b^c f\,dx + \int_c^d f\,dx + \int_c^e f\,dx.$$

8. Invertir el orden de integración.

$$\int_a^b f(x)\,dx = \int_b^a -f(x)\,dx.$$

$$F(b) - F(a) = (-F(b) + F(a))(-1) = \int_b^a -f(x)\,dx.$$

p.e. $$\int_0^{\pi} \sin x\,dx = -\cos x \Big]_0^{\pi} = -\overbrace{\cos\pi}^{-1} - (-\overbrace{\cos 0}^{1})$$
$$= 1 + 1 = 2.$$

$$\int_{\pi}^{0} -\sin x\,dx = \cos x \Big]_{\pi}^{0} = \cos 0 - \cos\pi = 1+1 = 2.$$

Ejercicio 5: Evalúe la sig. integral definida.

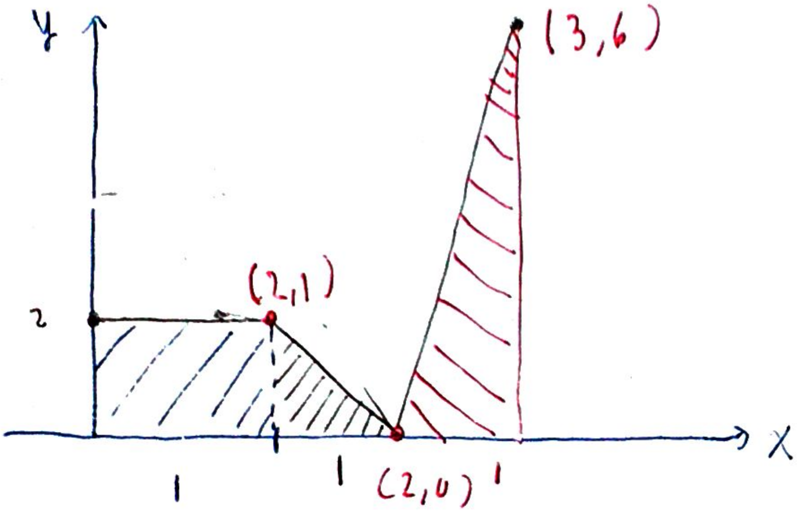
$$\int_0^3 f(x)\,dx \qquad \text{donde} \qquad f(x) = \begin{cases} 2 & \text{si } 0 \leq x \leq 1 \\ 4-2x & \text{si } 1 < x \leq 2 \\ 6x-12 & \text{si } 2 < x \leq 3. \end{cases}$$

$$\int_0^3 f\,dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx.$$

$$= 2x\Big]_0^1 + 4x - x^2\Big]_1^2 + 3x^2 - 12x\Big]_2^3$$

$$= 2. + (8-4) - (4-1) + (27-36) - (12-24)$$

$$= 2 + 1 + 3 = 6.$$



$A_1 = 2 \cdot 1$ Rectángulo

$A_2 = \frac{1}{2} 2 \cdot 1$ triángulo

$A_3 = \frac{1}{2} 6 \cdot 1$ Triángulo

$A = 1 + 2 + 3 = 6$