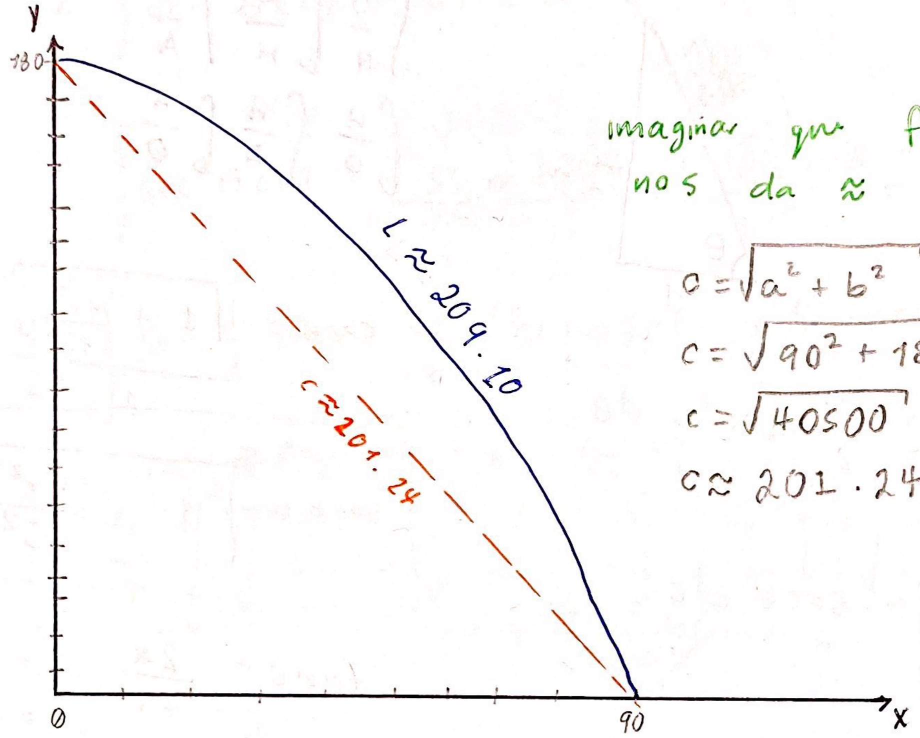
Laboratorio 8 David Corzo 20190432 2019-10-02

85

(5) Alcón vuela a 15 m/s, altitud 180; $y = 180 - \frac{x^2}{45}$



imaginar que fuera un triángulo nos da ≈

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{90^2 + 180^2}$

$c = \sqrt{40500}$

$c \approx 201.24$

$y = 0$

$0 = 180 - \frac{x^2}{45}$

$\frac{x^2}{45} = 180$

$x^2 = 180 * 45$

$x = \sqrt{180 * 45}$

$x = \sqrt{8100}$

$\boxed{x = 90}$

$y' = 0 - \frac{2x}{45}$

$y' = -\frac{2x}{45} \Rightarrow (y')^2 = \boxed{\frac{4x^2}{45^2}} = (y')^2$

$$L = \int_0^{90} \sqrt{1 + (y'(x))^2}\, dx$$

$$= \int_0^{90} \sqrt{1 + \frac{4x^2}{45^2}}\, dx$$

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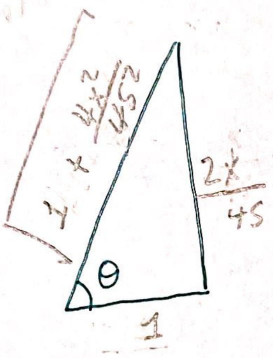


$S\,\frac{O}{H} \quad C\,\frac{A}{H} \quad T\,\frac{O}{A}$

$C\,\frac{H}{O} \quad S\,\frac{H}{A} \quad C\,\frac{A}{O}$

$$\sec\theta = \frac{\sqrt{1 + \frac{4x^2}{45^2}}}{1}$$

$$\boxed{\sec\theta = \sqrt{1 + \frac{4x^2}{45^2}}}$$

$$\tan\theta = \frac{2x}{45}$$

$$45 \tan\theta = 2x$$

$$45 \sec^2\theta = 2dx$$

$$\boxed{\frac{45}{2}\sec^2\theta\, d\theta = dx}$$

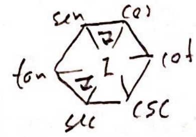
$$L = \int \sec\theta \cdot \frac{45}{2}\sec^2\theta\, d\theta$$

$$= \frac{45}{2}\int \sec^2\theta \cdot \sec\theta\, d\theta$$

$$= \frac{45}{2}\int \sec\theta \cdot \sec^2 x$$

$$\underbrace{\begin{array}{ll} u = \sec\theta & dv = \sec^2\theta\, d\theta \\ du = \sec\theta\tan\theta\, d\theta & v = \tan\theta \end{array}}$$

$$\sec\theta\tan\theta - \int \sec\theta\tan^2\theta\, d\theta$$



$$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \left(\int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta\right)$$

$$\underbrace{\underbrace{\int \sec^3\theta\, d\theta} = \sec\theta\tan\theta - \underbrace{\int \sec^3\theta\, d\theta}}_{\text{cíclica}} + \int \sec\theta\, d\theta$$

$$\int \sec^3\theta\, d\theta + \int \sec^3\theta\, d\theta = \sec\theta\tan\theta + \int \sec\theta\, d\theta$$

$$2\int \sec^3\theta\, d\theta = \sec\theta\tan\theta + \ln(|\sec\theta + \tan\theta|)$$

$$\int \sec^3\theta\, d\theta = \frac{\sec\theta\tan\theta + \ln(|\sec\theta + \tan\theta|)}{2}$$

$$\frac{45}{2}\int \sec^3\theta\, d\theta = \frac{45}{2}\left(\frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2}\right)\Bigg]_0^{90}$$

#Regresar a la variable original

$$\frac{45}{4}\left[\left(\sqrt{1+\frac{4x^2}{45^2}}\right)\left(\frac{2x}{45}\right) + \ln\left|\left(\sqrt{1+\frac{4x^2}{45^2}} + \frac{2x}{45}\right)\right]$$

evaluación =

$$\frac{45}{4}\left[\left\{\left(\underbrace{\sqrt{1+\frac{4(90)^2}{45^2}}}_{17}\right)\underbrace{\left(\frac{2(90)}{45}\right)}_{4} + \ln\left(\underbrace{\sqrt{1+\frac{4(9)^2}{45^2}}}_{17} + \underbrace{\frac{2(9)}{45}}_{4}\right)\right\} - \left\{\ldots\right\}\right.$$

$$\frac{45}{4}\left[\left\{\sqrt{17}\cdot 4 + \ln|\sqrt{17}+4|\right\} - \left\{\underbrace{\cancel{\left(\sqrt{1+\frac{4(0)^2}{45^2}}\right)\left(\frac{2(0)}{45}\right)}}_{0} + \underbrace{\ln\left|\sqrt{1+\frac{4(0)^2}{45^2}} + \frac{2(0)}{45}\right|}_{\ln|\sqrt{1}|}\right.\right.$$

$$\frac{45}{4}\left[\sqrt{17}\cdot 4 + \ln|\sqrt{17}+4| - \cancel{\ln|\sqrt{1}|}^{\,0}\right]$$

$$\frac{45}{4}\left[4\sqrt{17} + \ln|\sqrt{17}+4|\right] \approx 209.10$$

① Valor promedio

$$f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

ⓐ $f(\theta) = e^{\tan \pi t} \cdot \sec^2 \pi t$ en $[0, \pi/4]$

$$f_{prom} = \frac{1}{\frac{\pi}{4} - 0}\int_0^{\pi/4} e^{\tan \pi t} \cdot \sec^2 \pi t \, dt$$

$u = \tan(\pi t)$ $\quad du = \sec^2(\pi t)\,\pi \cdot dx$

$$f_{prom} = \frac{4}{\pi^2}\int_0^{\pi/4} e^u \, du$$

$$= \frac{4}{\pi^2}\left(e^u\right)\Big]_a^b = \frac{4}{\pi}\left(e^{\tan(\pi t)}\right)\Big]_0^{\pi/4} =$$

$$= \frac{4}{\pi^2}\left[\left(e^{\tan\left(\pi \cdot \frac{\pi}{4}\right)}\right) - \left(e^{\tan(\pi \cdot 0)}\right)\right] =$$

$$= \frac{4}{\pi^2}\left[e^{\tan\left(\frac{\pi^2}{4}\right)} - e^0\right] = \frac{4}{\pi}\left[e^{\tan\left(\frac{\pi^2}{4}\right)} - 1\right] =$$

$$= \frac{4}{\pi^2}\left[e^{\tan\left(\frac{\pi^2}{4}\right)} - 1\right]$$

✓

$$f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

ⓑ $f(t) = \dfrac{120x^2}{(2+x^3)^2}$ en $[0,2]$

$$f_{prom} = \frac{1}{2}\int_0^2 \frac{120x^2}{(2+x^3)^2}\,dx$$

$u = 2 + x^3$

$du = 2x^2\,dx$

$60\,du = 120x^2\,dx$

$$= \frac{1}{2}\int_0^2 \frac{60\,du}{u^2} = \frac{60}{2}\int_0^2 u^{-2}\,du =$$

$$= 30\left(u^{-1}\right)\Big]_0^2 = 30\left(2+x^3\right)\Big]_0^2$$

$$= 30\left[\left(2+(2)^3\right) - \left(2+(0)^3\right)\right] = 30\left[(10) - (2)\right] = 30\left[8\right]$$

$$= 30 \cdot 8 = \underline{240} \quad \times \quad 5$$

(2) $f(x) = \frac{9}{x^2}$ en $[1, 3]$

(a) Calcular el valor promedio:

$$f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{prom} = \frac{1}{3-1}\int_1^3 \frac{9}{x^2}\,dx$$

$$= \frac{1}{2}\left(9\int_1^3 x^{-2}\,dx\right) = \frac{1}{2}\left(9\left(\frac{1}{-1}x^{-1}\right)\right]_1^3 =$$

$$= \frac{9}{2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{1}\right)\right] =$$

$$= \frac{9}{2}\left[-\frac{1}{3} + \frac{3}{3}\right] = \frac{9}{2}\left(\frac{-1+3}{3}\right) = \frac{9}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right) =$$

$$= \frac{9}{3} = 3$$

(b) tal c que cumpla $f(c) = f_{prom}$

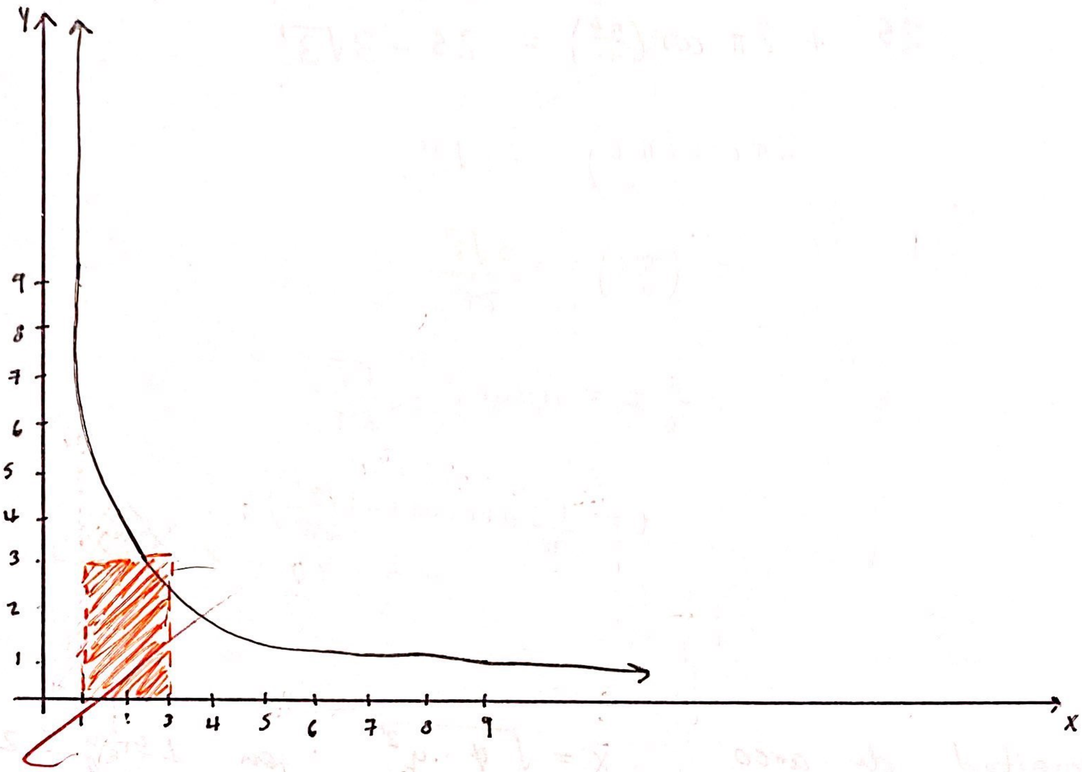
$$\frac{9}{x^2} = 3$$

$$9 = 3x^2$$

$$\frac{9}{3} = x^2$$

$$\pm\sqrt{3} = x$$

$$c = \pm\sqrt{3}$$



3 $T(t) = 25 + 2\pi \cos\left(\frac{\pi t}{6}\right)$ en $0 \le t \le 24$

a Promedio en $[6, 8]$

$$f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{prom} = \frac{1}{8-6}\int_6^8 25 + 2\pi \cos\left(\frac{\pi t}{6}\right)$$

$$= \frac{1}{2}\left(25t + 2\cancel{\pi}\left(\frac{6}{\cancel{\pi}}\sin\left(\frac{\pi}{6}t\right)\right)\right)\Bigg]_6^8 = \frac{1}{2}\left(25t + 12\sin\left(\frac{\pi t}{6}\right)\right)\Bigg]_6^8$$

$$= \frac{1}{2}\left[\left(25(8) + 12\underbrace{\sin\left(\frac{\pi}{6}(8)\right)}_{\sin\left(\pi\frac{4}{3}\right)}\right) - \left(25(6) + 12\sin\left(\frac{\pi\cancel{6}}{\cancel{6}}\right)\right)\right]$$

$$-\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2}\left[\left(200 - \frac{12\sqrt{3}}{2}\right) - \left(150 + \cancel{12 \cdot 0}\right)\right] = \frac{1}{2}\left(50 - \frac{12\sqrt{3}}{2}\right)$$

$$\frac{1}{2}\left(50 - 6\sqrt{3}\right) = 25 - 3\sqrt{3}$$

5

6

$$25 + 2\pi \cos\left(\frac{\pi t}{6}\right) = 25 - 3\sqrt{3}$$

$$2\pi \cos\left(\frac{\pi t}{6}\right) = -3\sqrt{3}$$

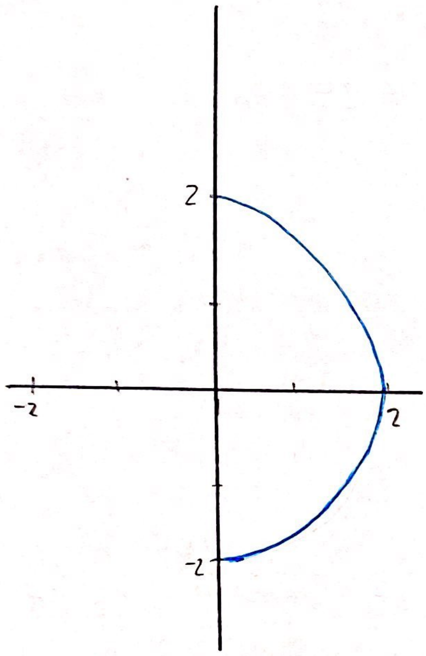
$$\cos\left(\frac{\pi}{6} t\right) = -\frac{3\sqrt{3}}{2\pi}$$

$$\frac{\pi}{6} t = \arccos\left(-\frac{3\sqrt{3}}{2\pi}\right)$$

$$t = \frac{6}{\pi} \arccos\left(-\frac{3\sqrt{3}}{2\pi}\right)$$

X 5

4 Longitud de arco $\quad x = \sqrt{4-y^2}$ en $1 \le y \le 2$



$$\frac{dx}{dy} = \frac{1}{2}(4-y^2)^{-1/2} \cdot -2y$$

$$= \frac{-\cancel{2}y}{\cancel{2}\sqrt{4-y^2}} = \left(-\frac{y}{\sqrt{4-y^2}}\right)^2 = \frac{y^2}{4-y^2}$$

$$L = \int_1^2 \underbrace{\sqrt{1 + \frac{y^2}{4-y^2}}}\, dy$$

$$\underbrace{\sqrt{\frac{4 \cancel{-y^2} \cancel{+y^2}}{4-y^2}}}$$

$$\sqrt{\frac{4}{4-y^2}} = \frac{\sqrt{4}}{\sqrt{4-y^2}} = \frac{2}{\sqrt{4-y^2}}$$

$$L = 2 \int_1^2 \frac{1}{\sqrt{4-y^2}}\, dy = 2\left[\sin^{-1}\left(\frac{y}{2}\right)\right]_1^2$$

$$= 2\left[\left(\sin^{-1}\left(\frac{2}{2}\right)\right) - \left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right] =$$

$$= 2\left[\underbrace{\sin^{-1}(1)} - \underbrace{\sin^{-1}\left(\frac{1}{2}\right)}\right] = \frac{2\pi}{3}$$