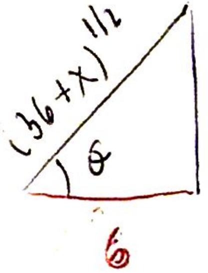
Parcial Lunes 2:30 D-504

Corto 4 | 1 Problema Sustitución Trigonométrica.

Práctica Problemas Integración. Pág 59.

2a) $\int_0^6 \frac{72}{(36+x^2)^{3/2}}\,dx = \int_0^{\pi/4} \frac{72\cdot 6\sec^2\theta\, d\theta}{36\cdot 6\sec^3\theta} = 2\int_0^{\pi/4} \frac{1}{\sec\theta}\, d\theta.$



$x = 6\tan\theta.$ $\qquad \frac{x}{6} = \tan\theta$

$dx = 6\sec^2\theta\, d\theta.$ $\qquad \theta = \tan^{-1}\left(\frac{x}{6}\right)$

$(36+x)^{1/2} = 6\sec\theta$

$(36+x)^{3/2} = 6^3\sec^3\theta$ $\qquad \theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\theta = \tan^{-1}(0) = 0$

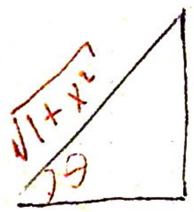
$2\int_0^{\pi/4} \frac{1}{\sec\theta.}\, d\theta = 2\int_0^{\pi/4} \cos\theta\, d\theta. = 2\sin\theta\Big]_0^{\pi/4} = 2\sin\frac{\pi}{4} = \frac{2\sqrt{2}}{2.} = \sqrt{2}$

2c) $\int_0^1 \frac{1}{(1+x)^2}\,dx = \int_1^2 u^{-2}\,du = -\frac{1}{u}\Big]_1^2 = -\frac{1}{2} + \frac{1}{1} = \frac{1}{2.}$

$u = 1+x$ $\qquad u(1) = 2.$

$du = dx$ $\qquad u(0) = 1$

2c1) $\int_0^1 \frac{1}{(1+x^2)^2}\,dx = \int_0^{\pi/4} \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2} = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^4\theta}\, d\theta = \int_0^{\pi/4} \frac{1}{\sec^2\theta}\, d\theta.$



$x = \tan\theta.$ $\qquad dx = \sec^2\theta\, d\theta.$

$1 = \tan\theta. \rightarrow \theta = \pi/4$

$0 = \tan\theta$ $\qquad \theta = 0$

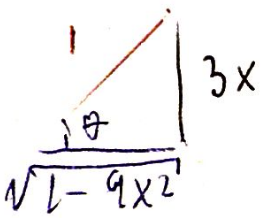
$\int_0^{\pi/4} \cos^2\theta\, d\theta = \frac{1}{2}\int_0^{\pi/4} (1+\cos 2\theta)\, d\theta = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\Big]_0^{\pi/4}\right)$

doble ángulo

$$\int_0^{\pi/4}\cos^2\theta\,d\theta = \frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\overset{1}{\cancel{\sin\frac{\pi}{2}}}-0\right) = \frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right) = \frac{\pi}{8}+\frac{1}{4}$$

3c) $$\int_0^{1/3}3^6x^5\sqrt{\underset{H}{1}-9x^2}\,dX = \int_0^{\pi/2}\frac{3^6}{3^5}\sin^5\theta\cos\theta\,\frac{1}{3}\cos\theta\,d\theta.$$

Pág 61



$\sin\theta = 3X$ $\Rightarrow$ $x=\frac{1}{3}\sin\theta.$ $\sin\theta = 1$, $\theta=\pi/2$

$\cos\theta\,d\theta = 3dX$ $\quad dx = \frac{1}{3}\cos\theta\,d\theta.$

$\sqrt{1-9x^2}=\cos\theta.$ $\quad x^5=\frac{1}{3^5}\sin^5\theta.$ $\quad \sin\theta=0$, $\theta=0$

$$\frac{3^6}{3^6}\int_0^{\pi/2}\sin^5\theta\cdot\cos^2\theta\,d\theta. = \int_0^{\pi/2}\underbrace{\sin^4\theta}\cos^2\theta\,(\sin\theta\,d\theta)$$

$$(\sin^2\theta)^2 = (1-\cos^2\theta)^2\cos^2\theta.$$

$$\int_0^{\pi/2}(1-\cos^2\theta)^2\cos^2\theta\,(\underbrace{\sin\theta\,d\theta}_{-du.}) = -\int_1^0(1-u^2)^2u^2\,du.$$

$u=\cos\theta$ $\quad u=\cos\pi/2=0$ $\quad = \int_0^1(1-2u^2+u^4)u^2\,du.$

$du=-\sin\theta\,d\theta$ $\quad u=\cos 0=1$

$$=\int_0^1 u^2-2u^4+u^6\,du.$$

$$-\int_b^a f\,dx = \int_a^b f\,dx$$

$$=\frac{1}{3}u^3-\frac{2}{5}u^5+\frac{1}{7}u^7\Big]_0^1$$

$$=\frac{1}{3}-\frac{2}{5}+\frac{1}{7}.$$

$$\int 3^6x^5\sqrt{1-9x^2}\,dX$$

$$\frac{1}{3}u^3-\frac{2}{5}u^5+\frac{1}{7}u^7+C.$$

$\cos\theta=\sqrt{1-9x^2}$

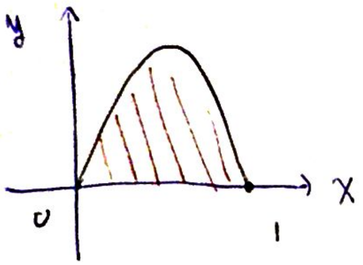
$$\frac{1}{3}\cos^3\theta-\frac{2}{5}\cos^5\theta+\frac{1}{7}\cos^7\theta+C.$$

$$\frac{1}{3}(1-9x^2)^{3/2}-\frac{2}{5}(1-9x^2)^{3/2}+\frac{1}{7}(1-9x^2)^{7/2}+C$$

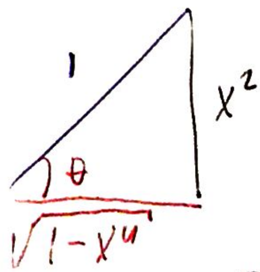
Triángulos "Interesantes"

Ejercicio 4.

Encuentre el área entre $f(x) = 4\pi x\sqrt{1-x^4}$, el eje x, y las rectas $x=0$ & $x=1$.



$$A = \int_0^1 f(x)\,dx = 2\pi \int_0^1 \sqrt{1-x^4}\; \underbrace{2x\,dx}$$



$x^2 = \sin\theta.$

$2x\,dx = \cos\theta\, d\theta.$

$\sqrt{1-x^4} = \cos\theta.$

$\sin\theta = 1$

$\theta = \pi/2$

$\sin\theta = 0$

$\theta = 0$

$$A = 2\pi \int_0^{\pi/2} \cos\theta \cos\theta\, d\theta. = \frac{2\pi}{2}\int_0^{\pi/2} (1+\cos 2\theta)\, d\theta.$$

$$A = \pi\left(\theta + \frac{\sin 2\theta}{2}\Big]_0^{\pi/2}\right) = \pi\left(\frac{\pi}{2} + \cancel{\frac{\sin\pi}{2}}^{0} - 0 - \cancel{\frac{\sin 0}{2}}^{0}\right)$$

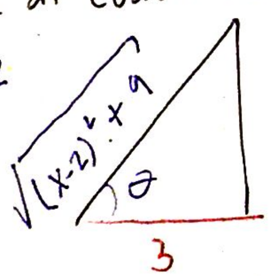
$$A = \pi\cdot\frac{\pi}{2} = \frac{\pi^2}{2}$$

$$\int \frac{(x-2)^3}{(x^2-4x+13)^{1/2}}\,dx. = \int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta\; 3\sec^2\theta\, d\theta.}{3\sec\theta}$$

complete al cuadrado $(x^2-4x+4)+13-4 = (x-2)^2+9$

c. $x-2$

c. 3



$3.\tan\theta = x-2$

$3\sec^2\theta\, d\theta = dx.$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$$3^3\int \tan^3\theta.\sec\theta\, d\theta. = 27\int \tan^2\theta\,(\tan\theta\sec\theta\, d\theta)$$

$$27\int(\sec^2\theta - 1)(\tan\theta\sec\theta\, d\theta) \leftarrow du$$

$u = \sec\theta \qquad du = \sec\theta\tan\theta\, d\theta.$

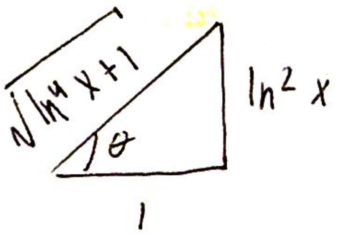
$$\int(27u^2 - 27)\,du = 9u^3 - 27u + C.$$

$$= 9\sec^3\theta - 27\sec\theta + C.$$

$\sec\theta = \dfrac{\sqrt{x^2-4x+13}}{3}$

$$\boxed{= \frac{9}{27}(x^2-4x+13)^{3/2} - 9(x^2-4x+13)^{1/2} + C.}$$

$$\int\sqrt{\ln^4 x + 1}\,\ln x\,\frac{dx}{x} = \int \sec\theta\cdot\frac{\sec^2\theta\, d\theta}{2}$$



$\ln^2 x = \tan\theta. \qquad \sqrt{\ln^4 x+1} = \sec\theta.$

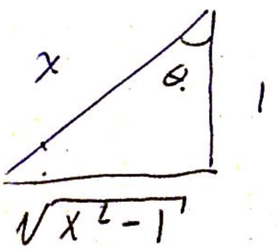
$\ln x\,\dfrac{dx}{x} = \dfrac{\sec^2\theta\, d\theta.}{2}$

$$\frac{1}{2}\int\sec^3\theta\, d\theta. = \frac{1}{4}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + c\right)$$

$$\frac{1}{2}(\text{Der} + \text{Int}) = \frac{1}{4}\left(\ln^2 x\sqrt{\ln^4 x+1} + \ln\left|\sqrt{\ln^4 x+1} + \ln^2 x\right| + c\right).$$

$$\int\sqrt{x^2-1}\,dx = \int\tan^2\theta\cdot\sec\theta\, d\theta. = \int(\sec^2\theta - 1)\sec\theta\, d\theta.$$

$$= \int\sec^3\theta - \sec\theta\, d\theta.$$



$\dfrac{H}{C.A.} = x = \sec\theta.$

$dx = \sec\theta\tan\theta.d\theta.$

$\sqrt{x^2-1} = \tan\theta.$