5.5 Regla de la Sustitución

Objetivo: Integre $F(g(x))$ funciones compuestas.

a. $\int 3\underbrace{(x+2)^2}_{x^2+4x+4} dx = \int (3x^2+12x+12)\,dx$

$= x^3 + 6x^2 + 12x + C.$

conjeturando $\int 3(x+2)^2 dx = \underbrace{(x+2)^3 + C.}$

derivada $3(x+2)^2 \cdot 1 + 0$

b. $\int 11(x-20)^{10} dx = (x-20)^{11} + C.$

Regla de la Potencia $\quad \frac{d}{dx}[f(x)]^{n+1} \xrightarrow{\text{derivando}} (n+1)[f(x)]^n f'(x)$

$\xleftarrow{\text{integrando}}$

Regla de la sustitución funciones Potencia

$$\int [\underbrace{f(x)}_{u}]^n \underbrace{f'(x)\,dx}_{du} = \frac{f(x)^{n+1}}{n+1} + C$$

$u = f(x) \quad du = f'(x)\,dx$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad \text{si } n \neq -1.$$

Ejercicio 1: Evalúe las sigs. integrales.

a. $\int (\overbrace{11x-20}^{u})^{10} \underbrace{11\,dx}_{du} = \int u^{10}\,du = \frac{u^{11}}{11} + C_1$

$u = 11x - 20 \quad du = 11\,dx \quad = \boxed{\frac{1}{11}(11x-20)^{11} + C_1}$

aa. $\int (\overbrace{x^2+x+3}^{u})^5 \overbrace{(2x+1)\,dx}^{du} = \int u^5\,du = \frac{1}{6}u^6 + C_2$

Pág. 30 $\quad u = x^2+x+3, \quad du = (2x+1)\,dx \quad = \frac{1}{6}(x^2+x+3)^6 + C_2$

Multiplicar/Dividir por una constante.

b. $\int (30w^3-8)^{19} w^2 dw. = \int u^{19} \frac{du}{90}$

$u = 30w^3 - 8$, $du = 90w^2 dw. \Rightarrow w^2 dw = \frac{du}{90.}$

$dw = \frac{du}{90w^2}$

$\int u^{19} \frac{du}{90} = \frac{1}{90} \cdot \frac{1}{20} u^{20} + C_3 = \boxed{\frac{1}{1{,}800}(30w^3-8)^{20} + C_3.}$

c. $\int (30w^3-8)^{19} 90w^3 dw. = \int u^{19} w \, du.$ X

sustitución incompleta.

$\int \cancel{u^{19}\left(\frac{1}{30}(u+8)\right)^{1/3}} du$ sólo se puede integrar. por fuerza bruta.

d. $\int 8x^3\sqrt{8+x^4}\, dx = \int 2u^{1/2} du = 2 \cdot \frac{2}{3} u^{3/2} + C$

$u = 8 + x^4$ $du = 4x^3 dx$ $\Rightarrow$ $2du = 8x^3 dx.$

La integral es $\frac{4}{3}(8+x^4)^{3/2} + C.$

e. $\int (10x^2+6x)^2 dx = \int (100x^4 + 120x^3 + 36x^2) dx$

No se usa la sustitución $= \boxed{20x^5 + 30x^4 + 12x^3 + C.}$

expanda y luego integre

Regla de la Cadena Derivadas. $\frac{d}{dx}\left[F(g(x))\right] = f'(g(x))\, g'(x)$

Regla de la sustitución $\int f'(g(x))\, g'(x)\, dx = \int f'(u)\, du = f(u) + C.$

Cadena a la Inversa.

$u = g(x), \quad du = g'(x)\, dx \qquad = f(g(x)) + C.$

Ejercicio 2: Integre. Pág 32.

0. $\int \frac{(8+16x+48x^2)}{x+x^2+2x^3}\, dx = \int \frac{8\,du}{u} = 8\ln|u| + C$
$= \boxed{8\ln|x+x^2+2x^3| + C.}$

$u = x + x^2 + 2x^3 \qquad du = (1+2x+6x^2)\, dx$

$8\,du = (8+16x+48x^2)\, dx$

a. $\int e^{\overbrace{x^{10}+\sqrt{2}}^{u}}\, x^9\, dx = \int e^u \frac{du}{10} = \frac{1}{10} e^u + C$
$= \frac{1}{10} e^{x^{10}+\sqrt{2}} + C.$

$u = x^{10} + \sqrt{2} \qquad \frac{du}{10} = \frac{10x^9}{10}\, dx.$

a2 $\int e^{x^{10}} \underbrace{x^8\, dx}_{\neq du} \qquad \int e^{x^{10}} \underbrace{dx}_{\neq du.}$ no se puede integrar.

b. $\int x^3 (x^4+3)^2 \sin(x^4+3)^3\, dx = \int u^2 \sin u^3 \frac{du}{4.}$

$u = (x^4+3) \qquad du = 4x^3\, dx \qquad \frac{du}{4} = x^3\, dx$

$u = (x^4+3)^3$ ✓

$\frac{1}{4}\int \sin(u^3)\, u^2\, du = \frac{1}{4}\int \sin t \frac{dt}{3} = -\frac{1}{4}\cdot\frac{1}{3}\cos t + C$
$= -\frac{1}{12}\cos u^3 + C$
$= -\frac{1}{12}\cos(x^4+3)^3 + C$

$t = u^3 \qquad dt = 3u^2\, du.$

Una sóla sustitución.

$$\int \sin(x^4+3)^3 \underbrace{[(x^4+3)^2 x^3]\,dx}_{du/12} = \frac{1}{12}\int \sin u\,du.$$

$u = (x^4+3)^3$, $du = 3(x^4+3)^2 4x^3\,dx$

$$\frac{du}{12} = (x^4+3)^2 x^3\,dx$$

c. $\int \cot x\,dx = \int \underbrace{\frac{\cos x}{\sin x}\,dx}_{du} = \int \frac{du}{u} = \ln|u| + C.$

$= \ln|\sin x| + C.$

$u = \sin x \quad du = \cos x\,dx$

d $\int \sec^2(\underbrace{e^x+x}_{u})\underbrace{(e^x+1)\,dx}_{du} = \int \sec^2 u\,du = \tan u + C.$

$u = e^x + x \quad du = (e^x+1)dx \qquad = \tan(e^x+x) + C.$

Sustitución Incompleta. $x = u-4.$

e. $\int 28x(x+4)^{1/3}\,dx = \int 28x\,u^{1/3}\,du.$

$u = x+4 \quad du = 1\cdot dx \quad = \int 28(u-4)\,u^{1/3}\,du.$

$28\int u^{4/3} - 4u^{1/3}\,du = 28\left[\frac{3}{7}u^{7/3} - 4\cdot\frac{3}{4}u^{4/3}\right] + C$

$= 12(x+4)^{7/3} - 84(x+4)^{4/3} + C.$

Regla de la sustitución para Integrales Definidas

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$u = g(x) \qquad u = g(a)$

$du = g'(x)dx \qquad u = g(b)$ } Cambie también los límites.

Ejercicio 3: Integre.

a. $\displaystyle\int_{-4}^{0} \frac{1}{3x-2}\,dx = \int_{-14}^{-2} \frac{1}{u}\,\frac{du}{3} = \frac{1}{3}\ln|u|\Big]_{-14}^{-2} = (\ln 2 - \ln 14)\frac{1}{3}$

$= \boxed{-\dfrac{\ln 7}{3}}$

f es continua en $x \neq \frac{2}{3}$

$u = 3x-2 \qquad du = 3\cdot dx$

$u(0) = 0-2 = -2, \qquad u(-4) = -12-2 = -14$

b. $\displaystyle\int_{0}^{1} \frac{8}{\pi}\,\frac{\sin^{-1} t}{\sqrt{1-t^2}}\,dt = \frac{8}{\pi}\int_{0}^{\pi/2} u\,du = \frac{8}{\pi}\,\frac{u^2}{2}\Big]_{0}^{\pi/2} = \frac{4}{\pi}\,\frac{\pi^2}{4}$

$= \pi$

¿du? $\quad u = \sin^{-1} t \qquad u(1) = \sin^{-1}(1) = \pi/2$

$du = \dfrac{dt}{\sqrt{1-t^2}} \qquad u(0) = \sin^{-1}(0) = 0$