2019-09-21

Webassign área entre curvas.

① $y_1 = \sqrt[3]{x}$ $y_2 = \frac{1}{x}$ $x = 8$

$I_{y_1} = I_{y_2}$ en $(1,1)$

$$A = \int_1^8 \sqrt[3]{x} - \frac{1}{x}\,dx = \frac{3x^{\frac{4}{3}}}{4} - \ln(x)\Big]_1^8 =$$

$$= \left\{\frac{3}{4}(8)^{\frac{4}{3}} - \ln(8)\right\} - \left\{\frac{3}{4} - 0\right\} = \frac{3}{4}(\sqrt[3]{8})^4 - \ln(8) - \frac{3}{4}$$

$$= \frac{3}{4}(2)^4 - \ln(8) - \frac{3}{4} = \frac{16 \cdot 3}{4} - \ln(8) - \frac{3}{4}$$

$$= 4 \cdot 3 - \ln(8) - \frac{3}{4} = 12 - \ln(8) - \frac{3}{4}$$

② $x = y^2 - 4$ $y = \pm 1$ $x = e^y$

$$A = 2\int_0^1 e^y - y^2 + 4 = e^y - \frac{1}{3}y^3 - 4y\Big]_0^1 =$$

$$= 2\left[\left\{e - \frac{1}{3} - 4\right\} - \left\{0\right\}\right]$$

③ $y = e^x, \quad y = x^2 - 1, \quad x = -1, \quad x = 1$

$$A = \int_{-1}^{1} e^x - (x^2 - 1)\, dx = e^x - \frac{1}{3}x^3 + x \Big]_{-1}^{1} =$$

$$= \{e - \frac{1}{3} + 1\} - \{e^{-1} + \frac{1}{3} - 1\}$$

$$= e - \frac{1}{3} + 1 - e^{-1} - \frac{1}{3} + 1$$

$$e - e^{-1} - \frac{2}{3} + \frac{2 \cdot 3}{3}$$

$$e - e^{-1} + \frac{8}{3}$$

mcd (5, 2)

$5 = 2 \cdot 2 + 1$

$2 = 2 + 0$

$2 = 2 + 0$

(2, 0)

mcd (205, 100)

$205 = 100 \cdot 2 + 5$

$100 = 5 \cdot 20 + 0$

mcd (20, ~~100~~)

10 3

$10 = 3 \cdot 3 + 1$

$3 = 3 + 0$

mcd (3, 0)

10, 3

$3 \cdot 3 + 1$

3, 1

$3 = 1 \cdot 3 + 0$

(3, 0)

(2) $x = y^2 - 4 \qquad x = e^y$

$$A = \int_{-1}^{1} e^y - y^2 + 4 \, dy = e^y - \frac{1}{3}y^3 + y \Big]_{-1}^{1} =$$

$$= \left\{e - \frac{1}{3} + 4\right\} - \left\{e^{-1} + \frac{1}{3} - 4\right\} =$$

$$= e - \frac{1}{3} + 4 - e^{-1} - \frac{1}{3} + 4$$

$$e - e^{-1} - \frac{2}{3} + 8 = e - e^{-1} + \frac{22}{3}$$

(3) $y_1 = e^x \qquad y_2 = x^2 - 1 \quad , \; x = \pm 1$

$$A = \int_{-1}^{1} e^x - (x^2 - 1)\, dx = \int e^x - x^2 + 1 \, dx = e^x - \frac{1}{3}x^3 + x \Big]_{-1}^{1}$$

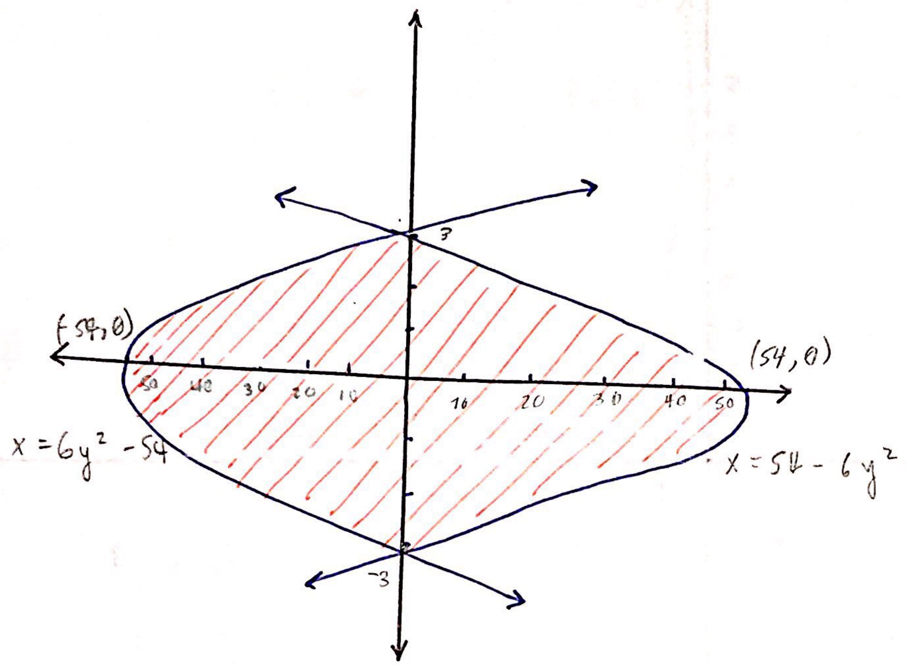
$$= \left\{e - \frac{1}{3} + 1\right\} - \left\{e^{-1} + \frac{1}{3} - 1\right\} = e - \frac{1}{3} + 1 - e^{-1} - \frac{1}{3} + 1$$

$$= e - e^{-1} - \frac{2}{3} + 2 = e - e^{-1} + \frac{4}{3}$$

$$-\frac{2}{3} + \frac{2 \cdot 3}{3} = \frac{-2 + 6}{3} = \frac{4}{3}$$

④ By parts:

$x = 54 - 6y^2 \qquad x = 6y^2 - 54$



$0 = 54 - 6y^2$

$0 = 6(9 - y^2)$

$0 = 9 - y^2$

$-9 = -y^2$

$9 = y^2$

$\pm\sqrt{9} = y$

$\pm 3 = y$

$x = 54 - 0$

$x = 54$

$0 = 6y^2 - 54$

$0 = 6(y^2 - 9)$

$0 = y^2 - 9$

$\pm 9 = y^2$

$\pm\sqrt{9} = y$

$\pm 3 = y$

$x = -54$

- with respect to - y:

$$A = \int_{-3}^{3} (54 - 6y^2) - (6y^2 - 54)\, dy$$

$$= \int_{-3}^{3} 54 - 6y^2 - 6y^2 + 54\, dy$$

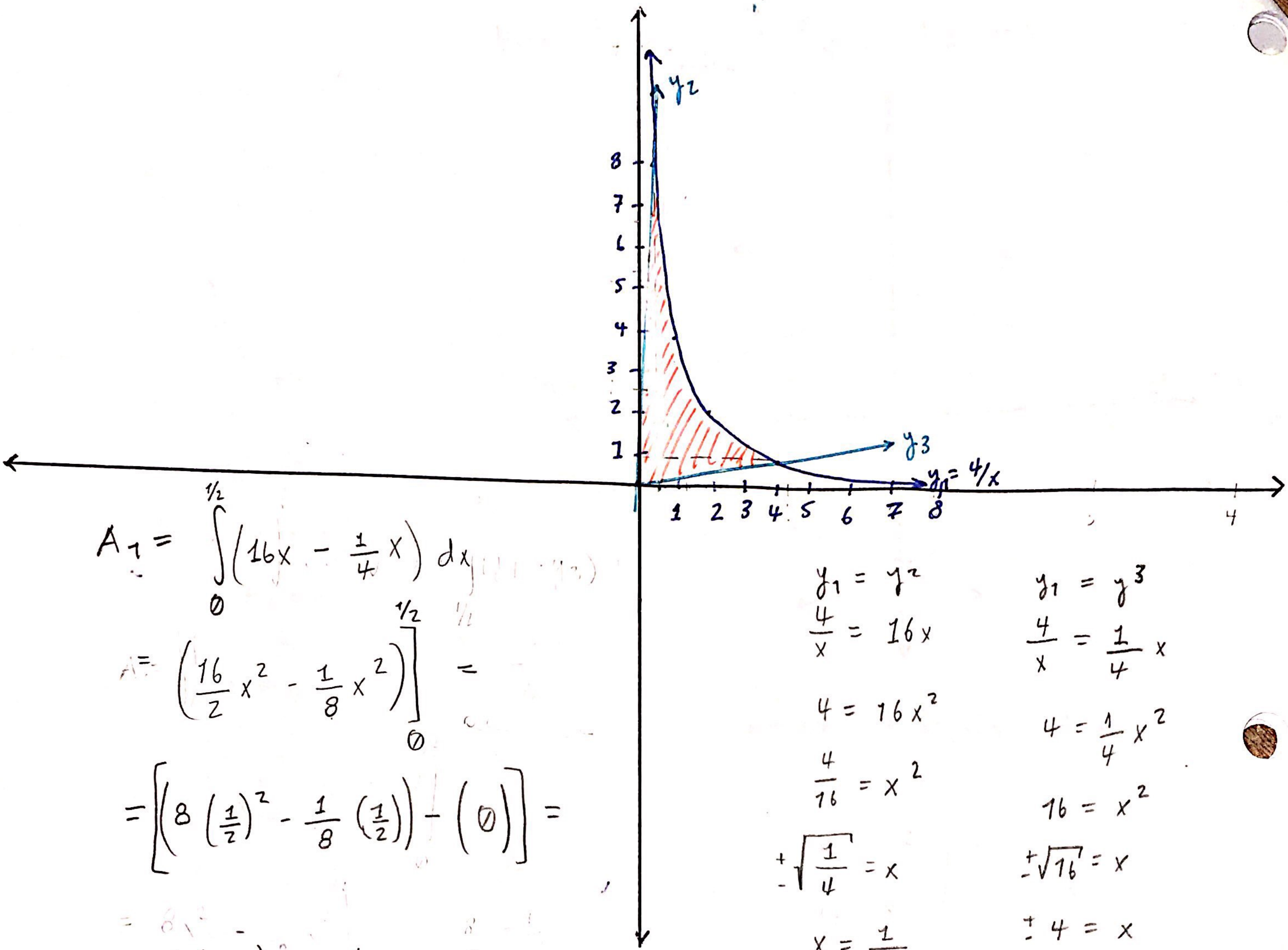
$$= \int_{-3}^{3} 108 - 12y^2\, dy$$

$$= 12\int_{-3}^{3} (9 - y^2)\, dy = 12\left[\left(9y - \frac{1}{3}y^3\right)\right]_{-3}^{3} =$$

$$= 24\left[\left(9(3) - \frac{1}{3}(3)^3\right) - (0)\right]_{0}^{3} = 24\,(27 - 9) = 24 \cdot 18 =$$

$$= \underline{432}$$

$y_1 = \frac{4}{x}$; $y_2 = 16x$; $y_3 = \frac{1}{4}x$, $x > 0$



$$A_1 = \int_0^{1/2} \left(16x - \frac{1}{4}x\right) dx$$

$$= \left(\frac{16}{2}x^2 - \frac{1}{8}x^2\right)\Big]_0^{1/2} =$$

$$= \left[\left(8\left(\frac{1}{2}\right)^2 - \frac{1}{8}\left(\frac{1}{2}\right)\right) - (0)\right] =$$

$$= 8\left(\frac{1}{4}\right) - \frac{1}{16} = \frac{8}{4} - \frac{1}{16} =$$

$$= 2 - \frac{1}{16} = \boxed{\frac{31}{16}}$$

$y_1 = y_2$

$\frac{4}{x} = 16x$

$4 = 16x^2$

$\frac{4}{16} = x^2$

$\pm\sqrt{\frac{1}{4}} = x$

$x = \frac{1}{2}$

$y_1 = y_3$

$\frac{4}{x} = \frac{1}{4}x$

$4 = \frac{1}{4}x^2$

$16 = x^2$

$\pm\sqrt{16} = x$

$\pm 4 = x$

$x = 4$

$$A_2 = \int_{1/2}^{4} \left(\frac{4}{x} - \frac{1}{4}x \right) dx = \int_{1/2}^{4} \left(4 \cdot \frac{1}{x} - \frac{1}{4} \cdot x \right) dx =$$

$$= \left[\left(4 \ln|x| - \frac{1}{4 \cdot 2} x^2 \right) \right]_{1/2}^{4} =$$

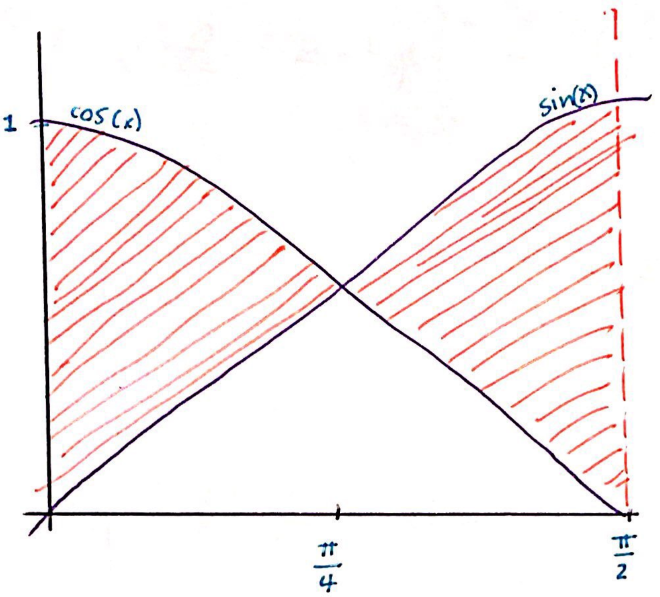
$$= \left[\left(4 \ln(4) - \frac{1}{8}(16) \right) - \left(4 \ln(1/2) - \frac{1}{8} \left(\frac{1}{2} \right)^2 \right) \right] =$$

$$= \left[4 \ln(4) - 2 - 4 \ln(1/2) + \frac{1}{16} \right] =$$

$$= 4 \ln(4) - 4 \ln(1/2) \underbrace{- 2 + \frac{1}{16} + \frac{31}{16}}_{0}$$

$$4(\ln(4) - \ln(1/2))$$

6



$$A_1 = \int_0^{\pi/4} (\cos(x) - \sin(x))\,dx = \Big[(\sin x + \cos x)\Big]_0^{\pi/4} =$$

$$= \Big[\Big(\underbrace{\underbrace{\sin(\pi/4)} + \underbrace{\cos(\pi/4)}}_{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)}\Big) - \Big(\underbrace{\underbrace{\sin(0)}_{0} + \underbrace{\cos(0)}_{1}}_{\underbrace{0 + 1}_{1}}\Big)\Big]$$

$$= \frac{\sqrt{2} + \sqrt{2}}{2} - 1$$

$$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}} - 1 = \sqrt{2} - 1$$

$$A_2 = \int_{\pi/4}^{\pi/2} (\sin(x) - \cos(x))\,dx =$$

$$= \Big[(-\cos x - \sin x)\Big]_{\pi/4}^{\pi/2} = \Big[\Big(-\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{0} - \underbrace{\sin\left(\tfrac{\pi}{2}\right)}_{1}\Big) - \Big(-\underbrace{\cos\left(\tfrac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}} - \underbrace{\sin\left(\tfrac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}\Big)\Big]$$

$$= \Big[(-0 - 1) - \Big(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\Big)\Big] = \Big[-1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\Big] =$$

$$= -1 + \sqrt{2} + \sqrt{2} - 1 = \boxed{-2 + 2\sqrt{2}}$$