

## 10.1 Cálculo de ecuaciones paramétricas

1) Eq. of tangent curve:

$$x = t^9 + 1 \quad ; \quad y = t^{10} - 1 \quad \text{in} \quad t = -1$$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10t^9 + 1}{9t^8} = \frac{10(-1)^9 + 1}{9(-1)^8} = \frac{-10 + 1}{9} = -\frac{1}{9}$$

$$x(-1) = (-1)^9 + 1 = 0$$

$$y(-1) = (-1)^{10} - 1 = 0$$

$$y = m(x - x_0) - y_0$$

$$y = -x$$

$$2) x(t) = 8 \sec t \quad ; \quad y(t) = 8 \tan(t) \quad ; \quad t = \frac{\pi}{4}$$

$$x'(t) = 8 \sec(t) \cdot \tan(t) \quad y'(t) = 8 \sec^2(t)$$

$$\frac{dy}{dx} = \frac{8 \sec(-t) \cdot \sec(t)}{8 \sec(t) \cdot \tan(t)} = \frac{\sec(t)}{\tan(t)} = \frac{\sec(\frac{\pi}{4})}{\tan(\frac{\pi}{4})} = \frac{\frac{2}{\sqrt{2}}}{\frac{1}{1}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$x(\frac{\pi}{4}) = \frac{16}{\sqrt{2}} = 2^4 \cdot 2^{\frac{1}{2}} = 2^{\frac{7}{2}}$$

$$\frac{8}{2} \cdot \frac{1}{2} = \frac{7}{2}$$

$$y(\frac{\pi}{4}) = 8 \cdot 1 = 8$$

$$y = \sqrt{2}(x - 2^{\frac{7}{2}}) + 8$$

$$y = \sqrt{2}x - \sqrt{2} \cdot 2^{\frac{7}{2}} + 8$$

$$y = \sqrt{2}x - 8$$

$$3) \quad x = t^2 + 9 \quad ; \quad y = t^2 + 9t$$

$$\frac{dy}{dx} = \frac{2t + 9}{2t}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{2t+9}{2t}\right)}{(2t)^2} = \frac{\frac{(2t)(2) - (2t+9)(2)}{(2t)^2}}{(2t)} \\ &= \frac{4t - 4t^0 - 18}{2t^2} = -\frac{18}{(2t)^3} \end{aligned}$$

$$-\frac{18}{(2t)^3} = 0$$

$$t = 0$$

$$t = \underbrace{1}_{-} \quad t = \underbrace{2}_{-}$$

$$t = -1$$

$$4) \quad x = t \sin(t) \quad ; \quad y = t^2 + 5t$$

$$\frac{dy}{dx} = \frac{2t + 5}{\sin(t) + t \cos(t)}$$

$$5) \quad x = 3\cos(t) \quad y = 2\sin(t)\cos(t)$$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{2\cos(2t)}{3\sin(t)} \Big|_{\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)} = 0 \\ &= \frac{2\cos(t)\cos(t) - 2\sin(t)\sin(t)}{-3\sin(t)} \\ &= -\frac{2}{3} \end{aligned}$$

$$y = \frac{2}{3}(x - 0) + 0 = \frac{2}{3}x$$

$$y = -\frac{2}{3}(x - 0) + 0 = -\frac{2}{3}x$$

$$6) \quad x(t) = 7 + \ln(t) \quad y(t) = t^2 + 3t \quad (7, 4)$$

$$\frac{dy}{dx} = \frac{2t+3}{\frac{1}{t}} = t(2t+3) = 2t^2 + 3t$$

$$7) \quad x = a \cos \theta ; \quad y = b \sin \theta \quad j \quad 0 \leq \theta \leq 2\pi$$

$$\begin{aligned} A &= \int_{t_1}^{t_2} y \, dx \\ &= \int_0^{2\pi} b \sin \theta \cdot a (-\sin \theta) \, d\theta = -ab \int_0^{2\pi} \sin^2 \theta \, d\theta \\ &= -ab \int_0^{2\pi} \left( \frac{1}{2} - \frac{\cos(2\theta)}{2} \right) \, d\theta \\ &= -ab \left[ \frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right]_0^{2\pi} = \end{aligned}$$

$$8) \quad x = t^2 - 2t \quad y = \sqrt{t} \quad \xrightarrow{ab\pi}$$

$$A = \int t^{\frac{1}{2}} (2t - 2) \, dt = \int_0^2 2t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \, dt$$

$$= 2 \left[ \frac{2}{5} t^{\frac{5}{2}} - \frac{2}{3} t^{\frac{3}{2}} \right]_0^2$$

$$\begin{array}{l|l} 0 = t^2 - 2t & = 2 \left[ \left( \frac{2}{5} (2)^{\frac{5}{2}} - \frac{2}{3} (2)^{\frac{3}{2}} \right) - (0) \right] \\ 0 = t(t - 2) & \end{array}$$

$$t = 0$$

$$t = 2$$

$$= 2 \left[ \frac{2}{5} \sqrt{32} - \frac{2\sqrt{8}}{3} \right] = \frac{2^2}{5} \sqrt{32} - \frac{2\sqrt{8}}{3}$$

$$= \frac{2^4 \sqrt{2}}{5} - \frac{2^2 \sqrt{2}}{3}$$

$$= \sqrt{2} \left( \frac{2^4 \cdot 3 - 2^2 \cdot 5}{5 \cdot 3} \right) = \frac{8\sqrt{2}}{15}$$

$$9) \quad x = 4t + 2 ; \quad y = 3t - 4 \quad 0 \leq t \leq 4$$

$$\begin{aligned} L &= \int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= \int_0^4 \sqrt{(4)^2 + (3)^2} dt \\ &= \int_0^4 \sqrt{25} dt = [5t]_0^4 = 5[(4) - (0)] = \underline{\underline{20}} \end{aligned}$$

$$10) \quad x = 8 + 12t^2 \quad y = 8 + 3t^3 \quad 0 \leq t \leq 5$$

$$\begin{aligned} L &= \int \sqrt{(24t)^2 + (24t^2)^2} dt \\ &= \int \sqrt{24^2 t^2 + 24^2 t^4} dt = \int \sqrt{24^2 t^2 (1 + t^2)} dt \\ &= \int 24t \sqrt{1 + t^2} dt = 12 \int \sqrt{u} du \\ &\quad \left| \begin{array}{l} u = 1 + t^2 \\ du = 2t \\ 12du = 24t \end{array} \right| \\ &\quad = 12 \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_{u(0)}^{u(s)} = 3 \cdot 4 \cdot \frac{2}{3} \\ &\quad = 8 u^{\frac{3}{2}} \Big|_{u(0)}^{u(s)} \\ &\quad u(s) = 26 \\ &\quad u(0) = 1 \end{aligned}$$

$$= 8 \left[ (26)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] = 8 \left[ 26^{\frac{3}{2}} - 1 \right]$$

$$11) \quad x = e^t - t \quad ; \quad y = 4e^{\frac{t}{2}} \quad ; \quad 0 \leq t \leq 4$$

$$\begin{aligned} L &= \int \sqrt{(e^t - 1)^2 + \left(4e^{\frac{t}{2}} \cdot \frac{1}{2}\right)^2} dt \\ &= \int \sqrt{e^{2t} - 2e^t + 1 + \left(\frac{16}{4}e^{t+1}\right)} dt \\ &= \int \sqrt{e^{2t} - 2e^t + 4e^t + 1} dt \\ &= \int \sqrt{e^{2t} + 2e^t + 1} dt \\ &= \int \sqrt{(e^t + 1)(e^t + 1)} dt \\ &= \int \sqrt{(e^t + 1)^2} dt = \int e^t + 1 dt = \left[ e^t + t \right]_0^4 \\ &= \left[ (e^4 + 4) - (1 + 0) \right] = e^4 + 3 \end{aligned}$$

$$12) \quad x = 5\cos(t) - \cos(st) \quad ; \quad y = 5\sin(t) - \sin(st) \quad 0 \leq t \leq \pi$$

$$L = \sqrt{\underbrace{(-5\sin(t) + \sin(st) \cdot 5)^2}_{(1)} + \underbrace{(5\cos(t) - \cos(st) \cdot 5)^2}_{(2)}} dt$$

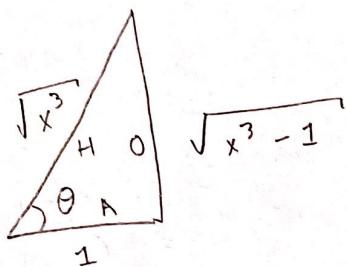
$$\begin{aligned} (1) \quad & (-5\sin(t) + \sin(st) \cdot 5)^2 = \\ & = (5(\sin(st) - \sin(t)))^2 = \\ & = 25(\sin(st) - \sin(t))^2 = \\ & = 25 \left[ \sin^2(st) - 2\sin(t)\sin(st) + \cancel{\sin^2(t)} \right] \end{aligned}$$

$$\begin{aligned} (2) \quad & (5(\cos(t) - \cos(st)))^2 = \\ & = 25(\cos(t) - \cos(st))^2 \\ & = 25 \left[ \cos^2(t) - 2\cos(t)\cos(st) + \cancel{\cos^2(st)} \right] \end{aligned}$$

$$\underbrace{25\sin^2(st) + 25\cos^2(st)}_{25} + \underbrace{25\cos^2(t) + 25\sin^2(t)}_{25}$$

$$\begin{aligned} & 25 + 25 - 2 \cdot 25 \sin(t) \sin(st) - 2 \cdot 25 \sin(t) \sin(st) \\ & 50 - 100 \sin(t) \sin(st) \end{aligned}$$

$$y = \int_1^4 \sqrt{x^3 - 1} dx$$



$$\sin \frac{\theta}{h} \left( \frac{A}{h} \right) T \frac{\theta}{A}$$

$$C \frac{H}{O} S \frac{H}{A} C \frac{A}{O}$$

$$\tan \theta = \sqrt{x^3 - 1}$$

$$\sec \theta = x^{\frac{3}{2}}$$

$$(\sec \theta)^{\frac{2}{3}} = x$$

$$\therefore \frac{2}{3} (\sec \theta)^{\frac{1}{2}} \cdot \sec \tan \theta d\theta dx$$

$$\frac{2}{3} \int \tan^2 \theta (\sec^{\frac{3}{2}} \theta)$$

Ver 8

$$P(x) = \int e^{-\frac{x}{\mu}} \frac{1}{\mu} dx$$

$$0 \leq x \leq 1 \quad \mu = 1.6$$

$$P(0 \leq x \leq 1) = \int_0^1 e^{-\frac{x}{1.6}} \cdot \frac{1}{1.6} dx = - \left[ e^{-\frac{u}{1.6}} \right]_{u(0)}^{u(1)} = - \left[ e^{-\frac{1}{1.6}} - e^0 \right] = -e^{-\frac{1}{1.6}}$$

$$u = \frac{-x}{1.6}$$

$$du = -\frac{1}{1.6} dx = -$$

$$-du = \frac{1}{1.6} dx$$

$$= - \left[ \left( e^{-\frac{1}{1.6}} \right) - \left( e^0 \right) \right] = -e^{-\frac{1}{1.6}} + 1 \approx \underline{0.465}$$

$$P(3 \leq x \leq \infty) = \left[ -e^{-\frac{x}{1.6}} \right]_3^\infty =$$

$$= - \left[ \underbrace{\left( \lim_{\alpha \rightarrow \infty} \left( e^{-\frac{\alpha}{1.6}} \right) \right)}_{\frac{1}{e^\infty} \rightarrow 0} - \left( e^{-\frac{3}{1.6}} \right) \right] = - \left( -e^{-\frac{3}{1.6}} \right) = e^{-\frac{3}{1.6}} \approx 0.153$$