Class Key ufm.edu. 1901 6665. Pág 15.

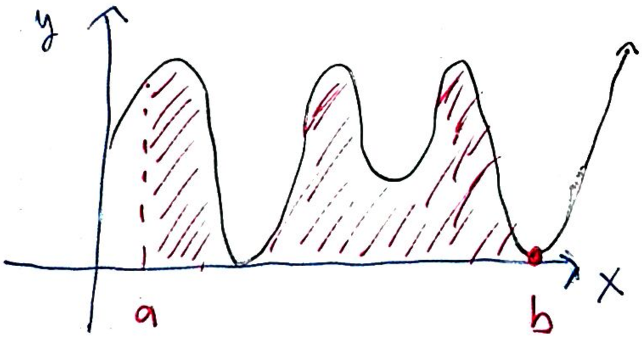
5.4 A'rea y Propiedades. Integral Definida.

El A'rea de una región $\underline{A} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Integral definida de f en [a,b] f es continua $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Interpretación Integral Definida.

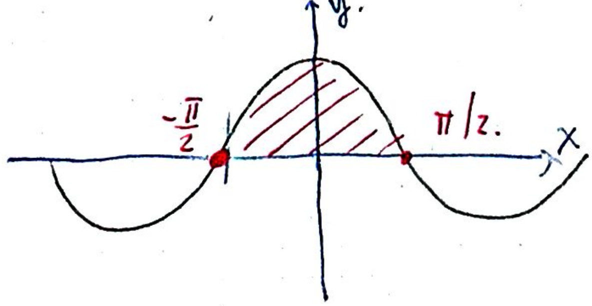
El área de la región bajo la curva $y = f(x)$, encima del eje-x, y entre las rectas verticales $x = a$ y $x = b$. es la integral definida de f en [a,b] ($f \geq 0$)



$A = \int_a^b f(x)\,dx.$

$F(b) - F(a)$

Considere la región debajo de $y = \cos x$ en $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



$A = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2.$

$A = \sin x \Big]_{-\pi/2}^{\pi/2} = \sin\frac{\pi}{2} - (-1)$
$= 1 + 1 = 2.$

$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_{0}^{3} = -0-(-(-2)^2)+9-0$$
$$= 0+4+9 = 13.$$

Regla Integral Definida. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

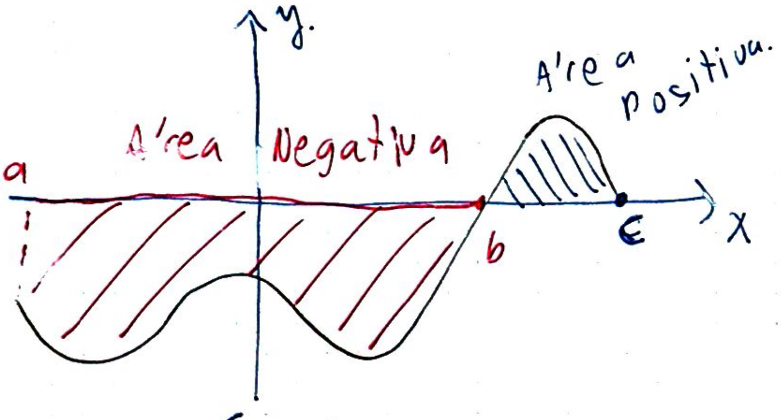
Invertir el orden

Ej: $\int_{-2}^{0} -2x\,dx = \int_{0}^{-2} 2x\,dx = x^2\Big]_{0}^{-2} = 4-0 = 4$

b) $\int_0^{\pi} \sin x\,dx = -\int_{\pi}^{0} \sin x\,dx = \cos x\Big]_{\pi}^{0} = 1-(-1) = 2.$

ó $-\cos x\Big]_0^{\pi} = -\underbrace{\cos(\pi)}_{-1} - (-\cos 0) = 1+1 = 2.$

¿Qué sucede cuando f(x) es negativa?



A'rea de la región entre f(x) & y=0

$\int_{-1}^{0} x^3\,dx = \frac{x^4}{4}\Big]_{-1}^{0}$
$= 0 - \frac{1}{4} = -\frac{1}{4}.$

$A \neq \int_a^c f(x)\,dx$

$A = -\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$ ✓ valor absoluta

Definición Más compacta $A = \int_a^b |f(x)|\,dx$

Ejercicio 3: (Pág 16) Considere $f(x)=4x^3-4$ en
$-2 \le x \le 2.$

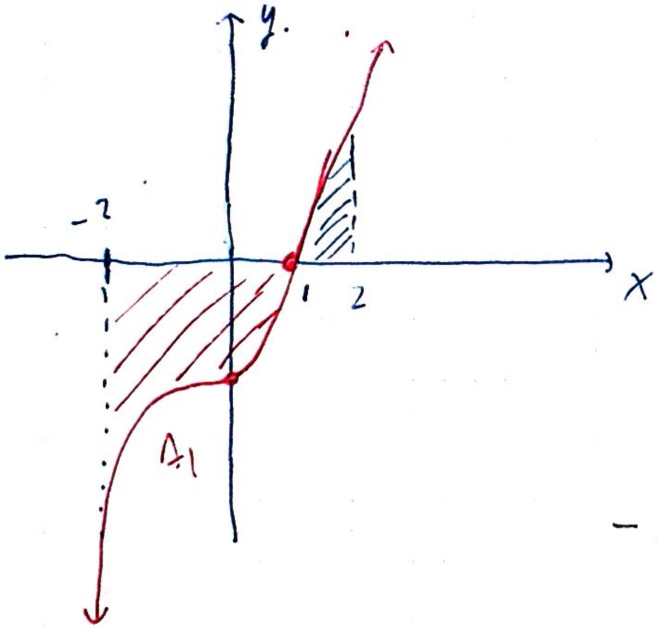
a. Evalúe $\int_{-2}^{2}(4x^3-4)\,dx = x^4-4x\Big]_{-2}^{2}$

$x^4\Big]_{-2}^{2} - 4x\Big]_{-2}^{2}$

$= (16-8)-(16+8)$

$= 8-24 = -16.$

b. Bosqueje la región y explique si la integral definida es igual al área de la región.



Intercepto-x: $4(x^3-1)=0$

$x = 1.$

Intercepto-y: $0-4=-4$

$A \neq \int_{-2}^{2} f(x)\,dx$

$-f(x) = -4x^3+4$

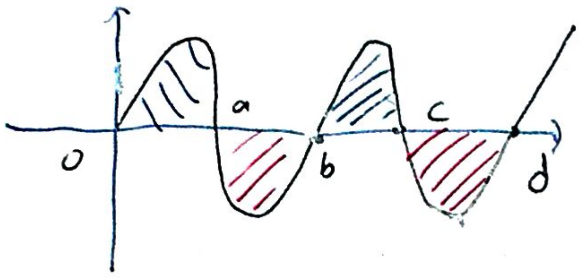
c. Encuentre el área de la región.

$A = \int_{-2}^{1}\overbrace{(4-4x^3)}^{A_1}\,dx + \int_{1}^{2}(4x^3-4)\,dx.$

$A = 4x-x^4\Big]_{-2}^{1} + x^4-4x\Big]_{1}^{2} = \overbrace{(4-1)-(-8-16)}^{A_1}$

$+(16-8)-(1-4)$

$A = 3+24+8+3 = 27+11 = 38.$



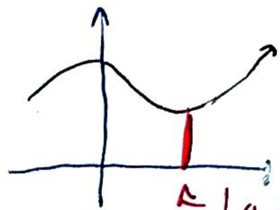
$$A = \int_0^a f\,dx - \int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx.$$

Propiedades Integrales Definidas.

1 y 2. $$\int_a^b k_1 f(x) \pm k_2 g(x)\,dx = k_1 \int_a^b f(x)\,dx \pm k_2 \int_a^b g\,dx.$$

$$\int_{\sqrt{2}}^{\sqrt{2}} e^{x^2 + \ln x + \sinh x}\,dx. = F(\sqrt{2}) - F(\sqrt{2}) = 0$$

3. $$\int_a^a f(x)\,dx. = 0$$

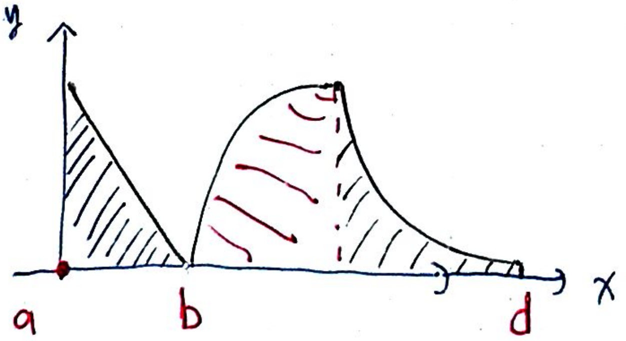


la región es sólo una línea.

4. $$\int_a^b h\,dx = hx\Big]_a^b = h(b-a)$$ rectángulo altura h largo b-a.

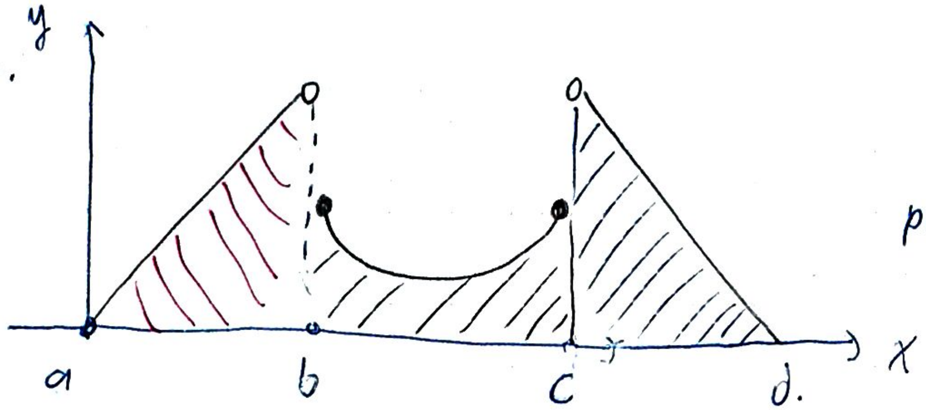
$$\int_e^{\sqrt{10}} \ln(10)\,dx = \ln(10)\left[\sqrt{10} - e\right]$$ $\int \ln x\,dx$

5.



$$\int_a^d f(x)\,dx = \int_a^b f\,dx. + \int_b^c f\,dx + \int_c^d f\,dx$$

6.



continua por tramos

piecewise continuous.

$$\int_a^d f\,dx = \int_a^b f\,dx + \int_b^c f\,dx + \int_c^d f\,dx$$

$$2x\Big]_0^1 = 2-0$$

Ejercicio 5: Evalúe la sig. integral definida.

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2 & \text{si } 0 \leq x \leq 1 \\ 4-2x & \text{si } 1 \leq x \leq 2 \\ 6x-12 & \text{si } 2 \leq x \leq 3 \end{cases}$$

$$\int_0^3 f\,dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx$$

$$= 2 + \left(4x - x^2\Big]_1^2\right) + \left(3x^2 - 12x\Big]_2^3\right)$$

$$= 2 + (4-3) + (-9-(-12))$$

$$= 2 + 1 + 3 = 6.$$

Derivación Logarítmica $\quad y = a^x$

$$y = \frac{a^x}{\ln a}, \quad y' = a^x \frac{\cancel{\ln a}}{\cancel{\ln a}}{}' \qquad \ln y = x \ln a.$$

$$\frac{y'}{y} = \ln a. \Rightarrow y' = a^x \ln a.$$