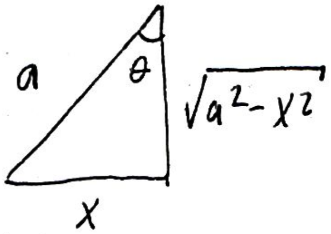
Sustitución trigonométrica.

Forma $\sqrt{a^2-x^2}$

$H = a$

$C.O. = x$



$\sin\theta = \dfrac{x}{a}$

$\cos\theta = \dfrac{\sqrt{a^2-x^2}}{a}$

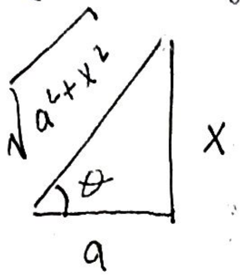
$x = a\cdot\sin\theta \qquad dx = a\cdot\cos\theta\, d\theta \qquad \sqrt{a^2-x^2} = a\cdot\cos\theta.$

Forma $\sqrt{a^2+x^2}$ Pág. 58.

$H: \sqrt{a^2+x^2}$

$C.O. = x$

$C.A. = a.$



$\tan\theta = \dfrac{x}{a} \Rightarrow x = a\cdot\tan\theta.$

$dx = a\cdot\sec^2\theta\, d\theta.$

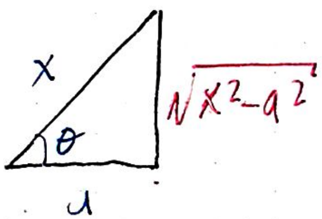
$a\cdot\sec\theta = \sqrt{a^2+x^2}$ $\quad \sec\theta = \dfrac{H}{C.A.}$

Forma $\sqrt{x^2-a^2}$

$H = x$

$C.O. =$

$C.A. = a$



$\dfrac{x}{a} = \sec\theta.$

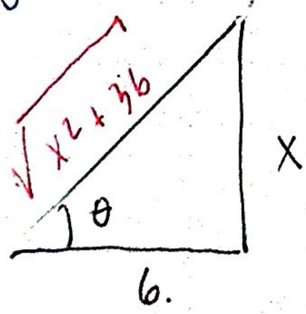
$x = a\cdot\sec\theta.$

$dx = a\cdot\sec\theta\tan\theta\, d\theta.$

$\sqrt{x^2-a^2} = a\cdot\tan\theta.$

Ejercicio 2 y 3.

a. $\displaystyle\int \frac{1}{x^2+36}\,dx. = \int \frac{6\sec^2\theta\, d\theta}{36\tan^2\theta+36} = \frac{6}{36}\int \frac{\sec^2\theta}{\sec^2\theta}\, d\theta.$



$\dfrac{x}{6} = \tan\theta. \qquad x = 6\cdot\tan\theta.$

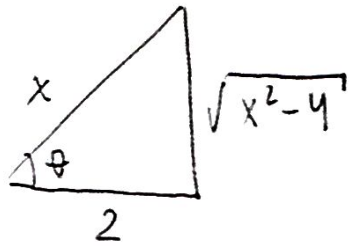
$dx = 6\sec^2\theta\, d\theta.$

$\dfrac{\sqrt{x^2+36}}{6} = \sec\theta \Rightarrow x^2+36 = 36\sec^2\theta.$

$$\frac{1}{6}\int d\theta = \frac{1}{6}\theta + C. = \boxed{\frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C.}$$

$$\tan^{-1}\left(\frac{x}{6}\right) = \theta.$$

3a. $$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \int \frac{2^3\tan^3\theta.}{2^6\cdot\sec^6\theta.}\, 2\cdot\sec\theta\tan\theta\, d\theta.$$



$$\frac{2}{x} = \cos\theta. \rightarrow x = \frac{2}{\cos\theta} = 2\sec\theta.$$

$$dx = 2\sec\theta\tan\theta\, d\theta.$$

$$\sqrt{x^2-4} = 2\tan\theta.$$

$$\left[(x^2-4)^{1/2}\right]^3 = 8\tan^3\theta.$$

$$\frac{1}{2^2}\int \frac{\tan^4\theta.}{\sec^5\theta.}\, d\theta. = \frac{1}{4}\int \frac{\sin^4\theta}{\cos^4\theta.}\cos^5\theta\, d\theta.$$

$u = \sin\theta \quad du = \cos\theta\, d\theta$

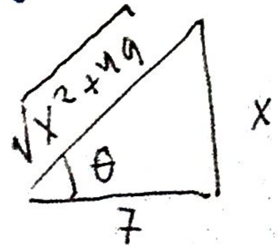
$\frac{1}{4}\int u^4 du.$

$$= \frac{1}{4}\int \underbrace{\sin^4\theta}_{u^4}\, \underbrace{\cos\theta\, d\theta.}_{du} = \boxed{\frac{1}{4}\cdot\frac{1}{5}\sin^5\theta + C.}$$

Regrese a la variable x $\quad \sin\theta = \frac{\sqrt{x^2-4}}{x}$, $\sin^5\theta = \frac{(x^2-4)^{5/2}}{x^5}$

$$\int \frac{(x^2-4)^{3/2}}{x^6}\, dx = \boxed{\frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C.}$$

2a $$\int \frac{49}{x^2\sqrt{x^2+49}}\, dx = \int \frac{\cancel{49}\ \cancel{7}\sec^2\theta}{\cancel{49}\tan^2\theta.\ \cancel{7}\sec\theta.}\, d\theta. = \int \frac{\sec\theta}{\underbrace{\tan^2\theta}_{\sec^2\theta - 1}}\, d\theta.$$



$$\frac{x}{7} = \tan\theta \qquad x = 7\tan\theta.$$

$$dx = 7\sec^2\theta\, d\theta.$$

$$\frac{H}{C.A.} = \sec\theta. \qquad \sqrt{x^2+49} = 7\sec\theta.$$

3. $\int \sec^m\theta \tan\theta^n x \, dx$ No está disponible.

$$\int \frac{\sec\theta}{\tan^2\theta}\, d\theta = \int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta} = \int \frac{\cos\theta}{\sin\theta}\,\frac{1}{\sin\theta}\, d\theta$$

$$= \int \cot\theta \csc\theta \, d\theta$$

$$= -\csc\theta + C$$

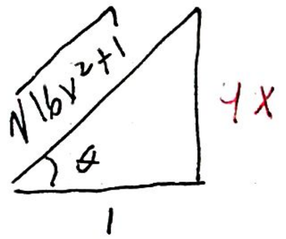
$$= -\frac{\sqrt{x^2+49}}{x} + C$$

$$\csc\theta = \frac{H}{C.O.} = \frac{\sqrt{x^2+49}}{x}$$

$$\int \frac{\cos\theta}{\sin^2\theta}\, d\theta = \int \frac{du}{u^2} = \int u^{-2}du = \frac{u^{-1}}{-1} + C = -\frac{1}{u} = -\frac{1}{\sin\theta} + C$$

$u = \sin\theta$, $du = \cos\theta \, d\theta$.

b. $$\int \frac{1}{x\sqrt{16x^2+1}}\, dx = \int \frac{0.25\sec^2\theta \, d\theta}{0.25\tan\theta \cdot \sec\theta} = \int \frac{\sec\theta}{\tan\theta}\, d\theta$$



$\tan\theta = 4x \rightarrow x = \frac{1}{4}\tan\theta \, d\theta$.

$dx = \frac{1}{4}\sec^2\theta \, d\theta$.

$$\frac{\sqrt{16x^2+1}}{1} = \sec\theta$$

Reescriba la integral.

$$\int \frac{\sec\theta}{\tan\theta}\, d\theta = \int \frac{1}{\cos\theta}\,\frac{\cos\theta}{\sin\theta}\, d\theta = \int \frac{d\theta}{\sin\theta} = \int \csc\theta \, d\theta$$

$$= -\ln|\csc\theta + \cot\theta| + C$$

$$\csc\theta = \frac{H}{C.O.} = \frac{\sqrt{16x^2+1}}{4x} \qquad \cot\theta = \frac{1}{4x}$$

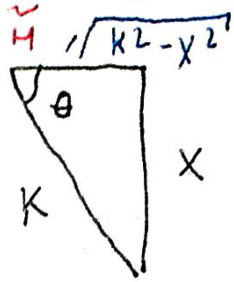
$$= -\ln\left|\frac{\sqrt{16x^2+1}}{4x} + \frac{1}{4x}\right| + C$$

7.3 Sustitución Trigonométrica

Forma $\sqrt{K^2 - x^2}$

$H = K$

$C.O = x$

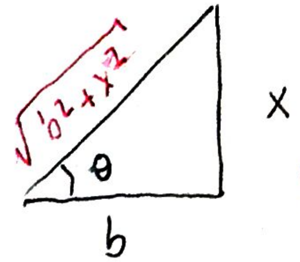


$\frac{C.O}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta.$

$dx = K\cos\theta\, d\theta.$

$\frac{\sqrt{K^2-x^2}}{K} = \cos\theta \Rightarrow \sqrt{K^2-x^2} = K\cdot\cos\theta.$

Forma $\sqrt{b^2 + x^2}$

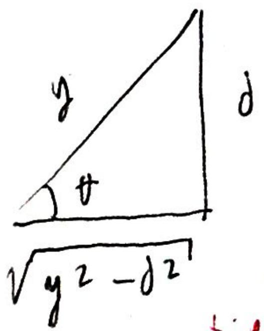


$\frac{x}{b} = \tan\theta. \Rightarrow x = b\cdot\tan\theta.$

$dx = b\cdot\sec^2\theta\, d\theta.$

$\frac{\sqrt{b^2+x^2}}{b.} = \sec\theta \Rightarrow \sqrt{b^2+x^2} = b\sec\theta.$

Forma $\sqrt{y^2 - d^2}.$



$\operatorname{sen}\theta = \frac{d}{y}$

$y = d\cdot\csc\theta.$

$dy = -d\cdot\csc\theta\cot\theta.$

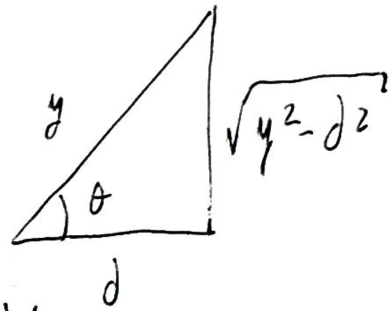
tiene signos negativos.

$\frac{y}{d} = \sec\theta.$

$y = d\sec\theta.$

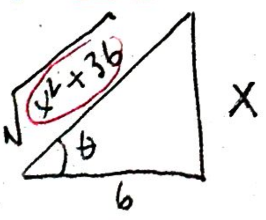
$dy = d\cdot\sec\theta\tan\theta.$

$\sqrt{y^2-d^2} = d\tan\theta.$



Ejercicios 2 y 3 Pág 58 y 59.

20) $\int \frac{1}{x^2+36}\,dx = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$



$x = 6\cdot\tan\theta.$

$dx = 6\cdot\sec^2\theta\, d\theta.$

$x^2+36 = 36\tan^2\theta + 36) = 36\sec^2\theta$