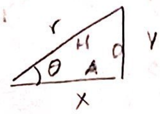
Laboratorio # 13 2019-11-11

1 a) $r = 2\sin\theta$ $\theta = \frac{\pi}{6}$



$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$$\frac{dy}{dx} = \frac{\sin(2\theta)\cdot\cancel{2}}{\cos(2\theta)\cdot\cancel{2}}$$

$$= \tan(2\theta) \quad ; \quad \theta = \frac{\pi}{6}$$

$$m = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$x\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$y\left(\frac{\pi}{6}\right) = 2\sin^2\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\boxed{m = \sqrt{3}}$$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$r\cdot\sin\theta = y$ $r\cdot\cos\theta = x$

$2\sin^2\theta = y$ $2\sin\theta\cos\theta = x$

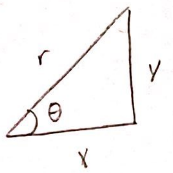
$2\cdot\frac{1}{2}\left(1 - \cos(2\theta)\right) = y$ $\sin(2\theta) = x$

$1 - \cos(2\theta) = y$ $x'(\theta) = \cos(2\theta)\cdot 2$

$$\boxed{y'(\theta) = \sin(2\theta)\cdot 2}$$

b) $r = \frac{1}{\theta}$ $\theta = \pi$

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$



$r\cdot\sin\theta = y$ $r\cos\theta = x$

$\frac{1}{\theta}\sin\theta = y$ $\frac{1}{\theta}\cos\theta = x$

$$y'(\theta) = -\frac{1}{\theta^2}\sin\theta + \frac{1}{\theta}\cos\theta$$

$$x'(\theta) = -\frac{1}{\theta^2}\cos\theta - \frac{1}{\theta}\sin\theta$$

$$\frac{dy}{dx} = \left.\frac{-\frac{1}{\theta^2}\sin\theta + \frac{1}{\theta}\cos\theta}{-\frac{1}{\theta^2}\cos\theta - \frac{1}{\theta}\sin\theta}\right|_{\theta=\pi}$$

$$= \frac{-\frac{1}{\pi^2}\cancel{\sin\pi}^{0} + \frac{1}{\pi}\cos\pi}{-\frac{1}{\pi^2}\cos\pi - \frac{1}{\pi}\cancel{\sin\pi}^{0}} = \frac{-\frac{1}{\pi}}{\frac{1}{\pi^2}} = -\frac{\pi^2}{\pi} = \boxed{-\pi}$$

② a) $r = e^{-\frac{\theta}{4}}$ $\qquad \frac{\pi}{2} \leq \theta \leq \pi$

$$A = \frac{1}{2}\int_a^b r^2 \, d\theta$$

$$= \frac{1}{2}\int \underbrace{\left(e^{-\frac{\theta}{4}}\right)^2}_{\underbrace{e^{-\frac{\cancel{2}\theta}{\cancel{2}\cdot 2}}}_{e^{-\frac{\theta}{2}}}} d\theta \qquad = -\frac{2}{2}\int e^{u} \, du$$

$u = -\frac{\theta}{2}$ $\qquad u\left(\frac{\pi}{2}\right) = -\left[\frac{\frac{\pi}{2}}{\frac{2}{1}}\right] = -\frac{\pi}{4}$

$du = -\frac{1}{2}\, d\theta$ $\qquad u(\pi) = -\frac{\pi}{2}$

$-2\,du = d\theta$

$$= -1 \, e^{u}\Big]_{u\left(\frac{\pi}{2}\right)}^{u(\pi)} = -\left[\left(e^{-\frac{\pi}{4}}\right) - \left(e^{-\frac{\pi}{2}}\right)\right] = \boxed{-e^{-\frac{\pi}{4}} + e^{-\frac{\pi}{2}}}$$

b) $r = \cos\theta$, $\qquad 0 \leq \theta \leq \frac{\pi}{6}$

$$A = \frac{1}{2}\int \cos\theta \, d\theta = \frac{1}{2}\sin\theta\Big]_0^{\frac{\pi}{6}} = \frac{1}{2}\left[\left(\sin\frac{\pi}{6}\right) - \cancel{\left(\sin 0\right)}^{0}\right] = \frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{1}{4}}$$

2019-11-04

Laboratorio # 13

① a) $r = 2\sin\theta \quad ; \quad \theta = \frac{\pi}{6}$

$r' = 2\cos\theta$

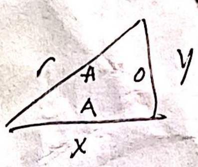
$= 2\cos\left(\frac{\pi}{6}\right)$

$= \cancel{2}\frac{\sqrt{3}}{\cancel{2}} = \sqrt{3} = m$

$r\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{6}\right) = 1$

$\theta = \sqrt{3}\left(r - \frac{\pi}{6}\right) + 1$

$y = y(\pi/6) + \frac{dy}{dx}\left(x - x(\pi/6)\right)$



$\cos\theta = \frac{x}{r}$

$r\cos\theta$

$2\sin\theta\cos\theta = x$

$\sin(2\theta) = x$

b) $r = \frac{1}{\theta} \quad ; \quad \theta = \pi$

$r'(\theta) = -1\theta^{-1-1} = -1\theta^{-2} = -\frac{1}{\theta^2}$

$r'(\pi) = -\frac{1}{\pi^2}$

② a) $r = e^{-\theta/4} \qquad \frac{\pi}{2} \leq \theta \leq \pi$

$A = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} e^{-\frac{\theta}{2}}\, d\theta$

$u = \frac{\theta}{2} \quad du = \frac{1}{2}dx$

$= \int_{\frac{\pi}{2}}^{\pi} e^{-u}\, du = -e^{-u}\Big]_{\frac{\pi}{2}}^{\pi} = -e^{-\frac{\theta}{2}}\Big]_{\frac{\pi}{2}}^{\pi}$

$= -\left[\left(e^{-\frac{\pi}{2}}\right) - \left(e^{-\frac{\pi}{4}}\right)\right] = -e^{-\frac{\pi}{2}} + e^{\frac{\pi}{4}}$

2 b) $r = \cos\theta \quad ; \quad 0 \leq \theta \leq \pi/6$

$$A = \int_a^b (\cos(\theta))^2 \cdot \frac{1}{2} \, d\theta$$

$$= \int_a^b \frac{1}{4}\left(1 + \cos(2\theta)\right) d\theta$$

$$= \frac{\theta}{4} + \frac{1}{8}\sin(2\theta)\Big]_0^{\frac{\pi}{6}}$$

$$= \left[\left(\frac{\pi}{24} + \frac{1}{8}\sin\left(\frac{\pi}{3}\right)\right) - \left(0 + 0\right)\right]$$

$$= \left[\frac{\pi}{24} + \frac{\sqrt{3}}{16}\right]$$

④ $r = 2\cos\theta \quad ; \quad r = 1$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\frac{\pi}{3}, \ \frac{5\pi}{3}$

$A = \frac{1}{2}\int_a^b r_1^2 - r_2^2 \, d\theta$

$A = \frac{1}{2}\int 4\cos^2\theta - 1$

$A = \frac{1}{2}\int \frac{4}{2}(1 + \cos(2\theta)) - 1$

$= \frac{1}{2}\int 2 + \cos(2\theta) - 1$

$= \frac{1}{2}\int \cos(2\theta) + 1 = \frac{1}{4}\sin(2\theta) + \frac{\theta}{2}\Big]_{\frac{\pi}{3}}^{\frac{5}{3}\pi}$

$= \frac{1}{2}\left[\left(\frac{1}{2}\sin\left(\frac{10}{3}\pi\right) + \frac{5}{3}\pi\right) - \left(\frac{1}{2}\sin\left(\frac{\pi}{3}\right) + \frac{\pi}{3}\right)\right] = \boxed{-\sqrt{3} + \frac{4}{3}\pi}$

⑤ $r = 2\cos\theta \quad ; \quad 0 \leq \theta \leq \pi$

$r'(\theta) = -2\sin(\theta)$

$(r'(\theta))^2 = 4\sin^2(\theta)$

$L = \frac{1}{2}\int_a^b \sqrt{(r'(\theta))^2 + r^2} \, d\theta$

$= \frac{1}{2}\int \sqrt{\underbrace{4\sin^2\theta + 4\cos^2\theta}_{4}} \, d\theta$

$= \frac{1}{2}\int \sqrt{4} = \frac{1}{2} \, 2\theta\Big]_0^{\pi} = \theta\Big]_0^{\pi} = [(\pi) - (0)] = \boxed{\pi}$