1. Evaluate the integral (a) $\int_1^9 \frac{x-1}{\sqrt{x}}dx$, (b) $\int_0^{\pi/4} \sec\theta\tan\theta d\theta$.

**Solution:**

(a)

$$\int_1^9 \frac{x-1}{\sqrt{x}}dx = \int_1^9 \left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right)dx = \int_1^9 \left(x^{1/2} - x^{-1/2}\right)dx$$

$$= \left[\frac{2}{3}x^{3/2} - 2x^{1/2}\right]_1^9 = \left(\frac{2}{3}\cdot 27 - 2\cdot 3\right) - \left(\frac{2}{3} - 2\right) = 12 + \frac{4}{3} = \frac{40}{3}$$

(b)

$$\int_0^{\pi/4} \sec\theta\tan\theta d\theta = \Big[\sec\theta\Big]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$$

2. What is wrong with the following calculation?

$$\int_{-1}^3 \frac{1}{x^2}dx = \left[\frac{x^{-1}}{-1}\right]_{-1}^3 = -\frac{1}{3} - 1 = -\frac{4}{3}$$

**Solution:**

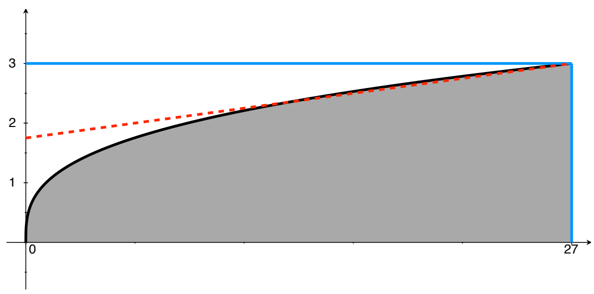
We know from the property of integrals that if $f(x) \geq 0$ for $x \in [a,b]$, then $\int_a^b f(x)dx \geq 0$. In the above calculation though $f(x) = \frac{1}{x^2} \geq 0$ for $x \in [-1,3]$, but we have negative value for the integral. Therefore, this calculation must be wrong.

The *Fundamental Theorem of Calculus* applies to continuous functions. But $f(x) = \frac{1}{x^2} \geq 0$ is not continuous on $[-1,3]$ ($f(x)$ has an infinite discontinuity at $x = 0$). Therefore,

$$\boxed{\int_{-1}^3 \frac{1}{x^2}dx \quad \text{does not exist}}$$

3. Use a graph to give a rough estimate of the area of the region that lies beneath the curve $y = \sqrt[3]{x}, \quad 0 \leq x \leq 27$ (feel free to use your graphing calculator for this part of the problem). Then find the exact area.

**Solution:**



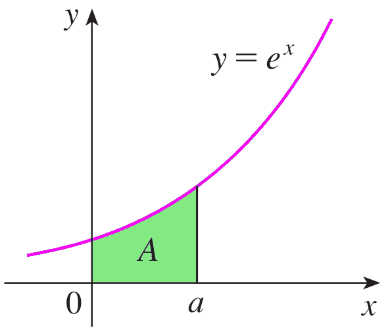

The area of the viewing rectangle is $3 \times 27 = 81$. The area of the right triangle between the red and blue lines includes majority of the non-shaded region. The red line intersects the $y-$axis at $y = 1.75$. Therefore, the approximate area of this triangle (non-shaded region) is $1/2 \times 27 \times (3 - 1.75) = 16.875$, therefore the approximate area of shaded region is $81 - 16.875 \approx 64$. This must be an overestimate.

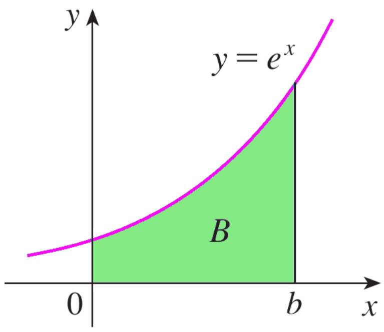
Let us next estimate the exact area, the area under $y = \sqrt[3]{x}$, for $0 \leq x \leq 27$ will be given by

$$\int_0^{27} \sqrt[3]{x} dx = \int_0^{27} x^{1/3} dx = \frac{3}{4} x^{4/3} \Big|_0^{27} = \frac{3}{4} \cdot (27)^{4/3} - 0 = \frac{3}{4} \cdot 81 = \frac{243}{4} = 60.75$$

Our approximation was off by $1 - \dfrac{60.75}{64} \times 100 \approx 5\%$.

4. The area labeled $B$ is three times the area labeled $A$. Express $b$ in terms of $a$.




**Solution:**

$$A = \int_0^a e^x dx = e^a - 1$$

$$B = \int_0^b e^x dx = e^b - 1$$

Now, $B = 3A \implies e^b - 1 = 3(e^a - 1) \implies e^b = 3e^a - 2 \implies \boxed{b = \ln(3e^a - 2)}$