2019-10-2[illegible]

Derivadas de funciones paramétricas

tangentes horizontales: $y'(t) = 0$ & $x'(t) \neq 0$

tangentes verticales: $y'(t) \neq 0$ $x'(t) = 0$

c) Segundas derivadas de $y(x)$:

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} \; ; \; \frac{d^2y}{d^2x} = \frac{d}{dx}\left(y'(x)\right)$$

$$\frac{d^2y}{d^2x} = \frac{\frac{d}{dt}(y'(x))}{x'(t)}$$

Ejercicio: encontrar 1era & segunda derivada:

a) $x = 1.5t^2$; $y = t^3 + 1.5t^6$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3t^2 + 9t^5}{3t} = \underbrace{t + 3t^4}_{\frac{dy}{dx}}$$

$$\frac{d^2y}{d^2x} = \frac{\frac{d}{dt}(y'(x))}{x'(t)} = \frac{1 + 12t^3}{3t} = \underbrace{\frac{1}{3t} + 4t^2}_{\frac{d^2y}{d^2x}}$$

$$y''' = \frac{\frac{d}{dt}(y''(x))}{x'(x)} = \underbrace{\frac{1}{3t}\left(\frac{-1}{3t^2} + 8t\right)}_{\frac{d^3y}{d^3x}}$$

b) $x = e^t$; $y = te^{-t} \Rightarrow 1 \cdot e^{-t} - te^{-t}$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{1 \cdot e^{-t} - te^{-t}}{e^t} = \boxed{e^{-2t} - te^{-2t}}$$

$$\frac{d^2y}{d^2x} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{x'(t)} = \frac{1}{e^t}\left(-2e^{-2t} - e^{-2t} + 2te^{-2t}\right)$$

$$= \boxed{-3e^{-3t} - 2e^{-3t}t}$$

c) $x = \cos\theta$; $y = \cos 2\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{-2\sin(2\theta)}{-\sin\theta} = \frac{2 \cdot 2 \cdot \cancel{\sin\theta} \cdot \cos\theta}{\cancel{-\sin\theta}} = 2 \cdot 2\cos\theta = \underbrace{4\cos\theta}_{\frac{dy}{dx}}$$

$$\frac{d^2y}{d^2x} = \frac{\frac{d}{d\theta}(4\cos\theta)}{-\sin\theta} = \frac{-4\sin\theta}{-\sin\theta} = -4$$

Dado a que la $\frac{d^2y}{d^2x}$ es una constante asumimos que es una constante

$$y = \cos(2\theta) = \sin^2\theta - \cos^2\theta$$

$$= 1 - \cos^2 - \cos^2\theta = -1 + 2(\cos\theta)^2$$

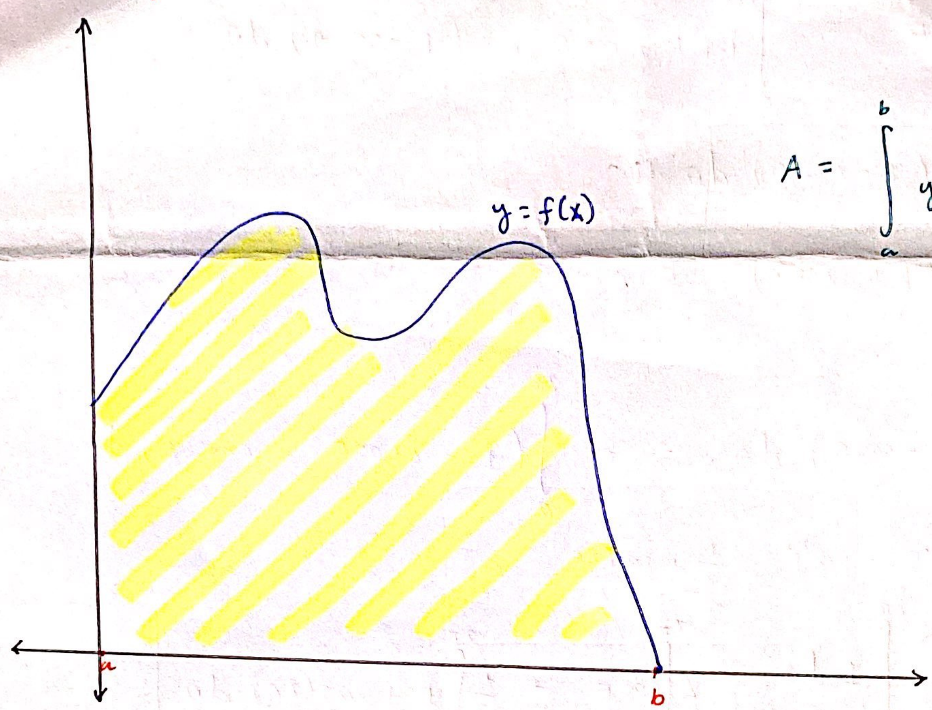
Ec. Curva paramétrica:

$$y = -1 + 2x^2$$

$$\frac{dy}{dx} = 4x$$

$$\underbrace{\frac{d^2y}{d^2x} = 4}_{\text{misma respuesta}}$$

Áreas Pg. 14 1



$$A = \int_a^b y(x)\, dx$$

Si la región está encerrada por la curva

$\mathcal{C}: \quad x = f(t) \quad ; \quad y = g(t) \quad ; \quad t_1 \leq t \leq t_2$

$x(t_1) = a \quad ; \quad g(t_2) = b$

$dx = f'(t)\, dt \qquad y = g(t) \qquad$ integra en t $\quad t_1 \leq t \leq t_2$

Por lo tanto

$$\boxed{A = \int_a^b y\, dx = \int_{t_1}^{t_2} g(t)\, g'(t)\, dt}$$

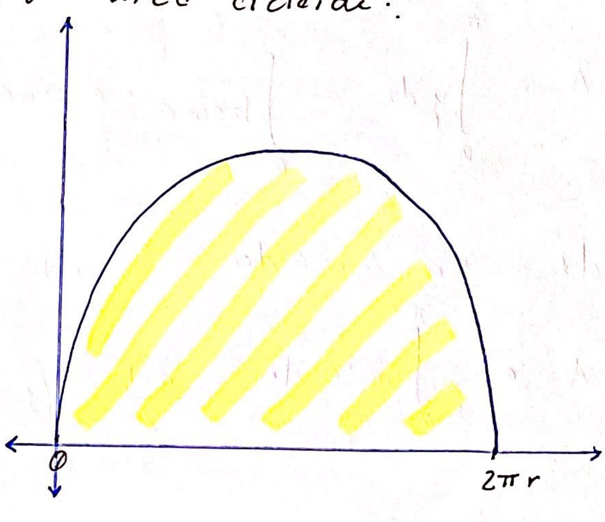
Ejercicio 6 : encuentra el área de la región dada.

a. La región de bajo de un arco cicloide.

$x = r(\theta - \sin\theta)$

$y = r(1 - \cos\theta)$

$0 \leq \theta \leq 2\pi$



$$A = \int_a^b y\, dx = \int_0^{2\pi} r(1-\cos\theta)\, r\, (1-\cos\theta)\, d\theta$$

$$dx = r(1-\cos\theta)\, d\theta$$

$$y = r(1-\cos\theta)$$

$$A = r^2 \int_0^{2\pi} (1-\cos\theta)^2\, d\theta = r^2 \int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\, d\theta$$

usar $\cos^2\theta = \frac{1}{2} + \frac{\cos(2\theta)}{2}$

$$= r^2 \left[\int_0^{2\pi} 1\, d\theta - 2\int_0^{2\pi} \cos\theta + \frac{1}{2}\int_0^{2\pi} 1 + \cos(2\theta)\, d\theta \right]$$

$$= r^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin(2\theta) \right]_0^{2\pi} =$$

$$= r^2 \left[\frac{3}{2} 2\pi - 2\sin^2\pi + \frac{1}{4}\sin(4\pi) \right] = r^2 \frac{6\pi}{2}$$

$A = 3\pi r^2$; Interesante, es 3 veces su área del círculo.

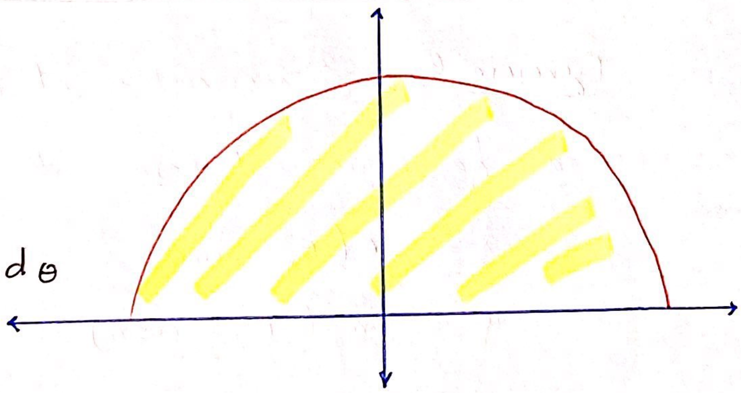
b) · Media elipse

$x = -a\cos\theta$ $\qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$y = b \cdot \sin\theta$

$0 \leq \theta \leq \pi$

$$A = \int_0^{\pi} y\, dx = \int_0^{\pi} b\sin\theta \cdot a\sin\theta\, d\theta$$



$dx = +a \sin\theta\, d\theta$ $\qquad a, b$ son constantes

$$A = ab \int_0^{\pi} \sin^2\theta\, d\theta = \frac{a \cdot b}{2} \int_0^{\pi} (1 - \cos 2\theta)\, d\theta$$

$$A = \frac{a \cdot b}{2} \left[\theta - \frac{1}{2} \sin(2\theta) \right]_0^{\pi} = \frac{a \cdot b}{2} \left[\left(\pi - \cancel{\frac{1}{2} \sin(2\pi)}^{0} \right) - \cancel{\left(0 - \frac{1}{2} \sin 0 \right)}^{0} \right]$$

$$\boxed{A = \frac{a \cdot b}{2} \cdot \pi}$$

Por lo tanto ... la media elipse es $0{\cdot}5\, a \cdot b\, \pi$