5.4 La Integral Indefinida. (Pag 9)

Una antiderivada de f. es una función $F(x)$

$$F'(x) = f(x)$$

Por ejemplo, encuentre la antiderivada de $f(x) = 14x^6$.

$F(x) = 2 \cdot x^7$ $\qquad F'(x) = 14x^6 = f(x)$

$F(x) = 2 \cdot x^7 + \sqrt{10}$

$F(x) = 2 \cdot x^7 - 10^{20} + \ln(10)$

Antiderivada más general

$F(x) = 2x^7 + C$

La Integral Indefinida de $f(x)$ respecto a x, es la antiderivada más general de f.

$$\int f(x)\,dx = F(x) + C.$$

C es una constante de integración.

$\int (\;)\,dx$ integre la función es antiderivar.

signa $\int$ $\quad dx$ diferencial, integre respecto a x.

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

$u = g(x) \quad du = g'(x)\,dx$

$$\int 14x^6\,dx = 2x^7 + C.$$

Reglas de Integración Básicas.

$\frac{d}{dx}(\ln(-x)) = \frac{1}{x}$

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$ $n \neq -1$

$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

valor absoluto.

$\int e^x \, dx = e^x + C.$

$\int a^x \, dx = \frac{a^x}{\ln a} + C.$

	0	$\sin x$
	$\sin x$	$\cos x$
	$\cos x$	$-\sin x$
$+C$	$\tan x$	$\sec^2 x$
	$\cot x$	$-\csc^2 x$

$\int dx$

$\int \cos x \, dx = \sin x + C.$

$\int \sin x \, dx = -\cos x + C.$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C.$

$\int \sec x \tan x \, dx = \sec x + C.$

$\int \csc x \cot x \, dx = -\csc x + C.$

Gulio: $\int \tan x \, dx = \ln|\sec x| + C.$ $\longrightarrow$ Compruebe $\frac{\sec x \tan x}{\sec x} + 0.$

$\int \cot x \, dx$ $\int \sec x \, dx$ $\int \csc x \, dx.$

$\int \frac{\cos x}{\sin x} dx$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C.$ $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C.$

$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C.$ $\int \sinh x \, dx = \cosh x + C.$

Suma/Diferencia $\int f(x) \pm g(x) \, dx = \int f(x) dx \pm \int g(x) dx$

Múltiplo Constante $\int a f(x) \, dx = a \int f(x) dx$

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	$\cos x$	$-\sin x$
	$\tan x$	$\sec^2 x$
	$\cot x$	$-\csc^2 x.$

$\int dx$

$+C.$

$\int \cos x \, dx = \sin x + C.$

$\int \sin x \, dx = -\cos x + C.$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C.$

$\int \sec x \tan x \, dx = \sec x + C.$

$\int \csc x \cot x \, dx = -\csc x + C.$

$\longrightarrow$ Compruebe

Gulio: $\int \tan x \, dx = \ln|\sec x| + C.$ $\quad \frac{\sec x \tan x}{\sec x} + 0.$

$\int \cot x \, dx \qquad \int \sec x \, dx \qquad \int \csc x \, dx.$

$\int \frac{\cos x}{\sin x} dx$

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Suma/Diferencia $\int f(x) \pm g(x) \, dx = \int f(x) dx \pm \int g(x) dx$

Múltiplo Constante $\int a f(x) \, dx = a \int f(x) dx$

Ejemplos pág 11.

a) $\int x^{50} + 2x^{6}\, dx = \frac{x^{51}}{51} + \frac{2}{7}x^{7} + C.$

b) $\int \frac{1}{1+x^2} + \frac{1}{x} + \frac{1}{x^2}\, dx = \tan^{-1} x + \ln|x| + \frac{x^{-1}}{-1} + C.$

x^{-2}

c) $\int \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[5]{x^3}}\, dx$

$\int x^{1/2} + x^{-1/2} + x^{-3/5}\, dx = \frac{2}{3}x^{3/2} + 2x^{1/2} + \frac{5}{2}x^{2/5} + C.$

d) $\int x^{\ln 2} + x^{\sqrt{2}} + x^{\sin(2)}\, dx = C + \frac{x^{1+\ln 2}}{1+\ln 2} + \frac{x^{1+\sqrt{2}}}{1+\sqrt{2}} + \frac{x^{1+\sin(2)}}{1+\sin(2)}$

potencia

Ejercicio 1: Evalúe las sigs. integrales.

a) $\int \underbrace{x^{e}}_{\text{potencia}} + \underbrace{e^{x}}_{\text{exponencial}}\, dx. = \frac{x^{e+1}}{e+1} + e^{x} + C.$

b) $\int \left(8\cdot 10^{x} - \frac{2}{x}\right) dx = \frac{8\cdot 10^{x}}{\ln(10)} - 2\ln|x| + C.$

A'lgebra.

c) $\int (x-2)(x+2)(x^2+4)\, dx = \int (x^2-4)(x^2+4)\, dx$

$\int (x^4-16)\, dx = \frac{1}{5}x^{5} - 16x + C.$

d) $\int e^{-4x}(e^{4x}+e^{5x})\, dx = \int (1+e^{x})\, dx = x + e^{x} + C.$

Integrales Definidas

Son integrales con limites de integración en $x = a$ y $x = b$.

$$\int_a^b f(x)\,dx \qquad \int f(x)\,dx = F(x) + C$$

Teorema de Evaluación (TFC parte 2).

Si $f(x)$ es continua en $[a,b]$ entonces.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Utilizando la notación de corchete.

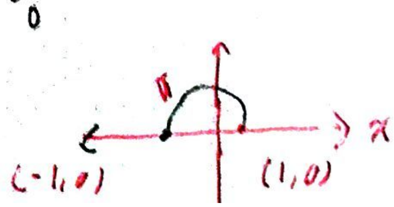
$$\int_a^b f(x)\,dx = \underbrace{F(x)}_{\text{primero antiderive}}\Big]_{x=a}^{x=b} \quad \text{luego evalúe.}$$

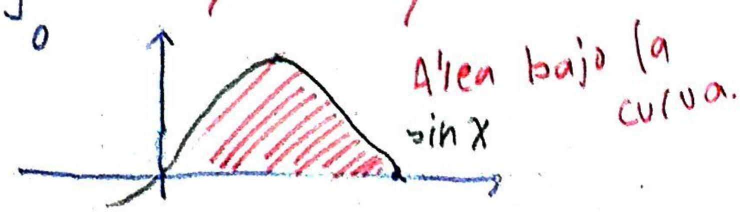
$$F(x) + C\Big]_{x=a}^{x=b} = F(b) + C - (F(a) + C) = F(b) - F(a)$$

Función es Integrable si $\int_a^b f(x)\,dx$ existe.

Ejercicio 1: Evalúe las sigs. integrales.

0. $\int_0^\pi \sin x\,dx = -\cos x\Big]_0^\pi = -\overset{-1}{\cos\pi} + \overset{1}{\cos 0} = 1 + 1 = 2.$





a. $\int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 = \frac{27}{3} - 0 = 9.$

b. $\int_9^{36} \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big]_9^{36.} = \frac{2}{3}\left((6^2)^{3/2} - \underbrace{9^{3/2}}_{3^3}\right)$

$x^{1/2}$

$= \frac{2}{3}(216-27) = 144-18$

$= 126.$

c. $\int_0^2 \frac{1}{1-x^2}\,dx$ no existe discontinua en $[0,2]$

se indefine en -1 y 1.

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C.$ $\quad \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$

d) $\int_1^4 \left(\frac{1}{\sqrt{x}} + 3\sqrt{x}\right)dx = 2\cdot x^{1/2} + \frac{3\cdot 2}{3}x^{3/2}\Big]_1^4$

$x^{-1/2}$ $\quad x^{1/2}$

$= 2\sqrt{4} + 2(2^2)^{3/2} - (2\cdot 1^{1/2} + 2\cdot 1^{3/2})$

$= 4 + 16 - (2+2) = 16.$

3. $\frac{x^{3/2} * 2/3}{3/2 * 2/3} = \frac{3\cdot 2}{3}x^{3/2} = 2x^{3/2}.$ $\quad \int (\ \ )\,dx.$

$\frac{d}{dx}(\ \ )$