Discrete Mathematics Aplied

Sumary

2.1 Book summary 2.1 - 2.4

- Rules of logic validate arguments

If $\underbrace{p}_{\text{true}}$ then $\underbrace{q}_{\text{true}}$
similar to boolean

- establishing this we validate arguments and also validate arguments using the same format

- Proposition - (statement or asertion) is a sentence that is either true or false but not both. (questions and exclamations are not propositions)
- we can turn not statements into statements by adding conditions

- cannot decide Truth of false $\neq$ we don't know how to verify it.

- A proposition can only have the posibility of being true or false, even if we don't know how to prove it if we know it is true or false, it is a proposition.

- p, q, r used as propositional variables, (truth tables vary)
- negation of p for example, $\boxed{\bar{p}}$ or $\boxed{\neg p}$ or $\boxed{\sim p}$

p = True $\quad \bar{p}$ = False

- Notations to facilitate discussion

$\mathbb{N}$ = Natural numpers (positive intigers)

$\mathbb{Z}$ = intigers

$\mathbb{R}$ = Real numbers

$\mathbb{Q}$ = Rational numbers

$\underbrace{A, B, C, S, T}_{\text{sets}}$

$\underbrace{a, b, c, s, t}_{\text{elements of sets}}$

$\underbrace{b \in B}_{\text{b belongs to B}}$

$\underbrace{B \ni b}_{\text{B contains b}}$

- Superscript notation

$\mathbb{R}^+$ = positive real numbers

$\mathbb{R}^-$ = negative real numbers

$\mathbb{R}^*$ = set of all nonzero real numbers

- Same convention applies to $\mathbb{Z}, \mathbb{Q}$;

- Notation as multiples

kS = means in a set S obtained by multiplying k to every number in S.

2.2

- Binary operators = $+, -, \cdot, \div$
- unary operators = $-(x)$

- compound statements = joined statements using operands or logical conectives

- Conjunctions and disjunction

Conjuction AND ↳ $\land$	Disjuction OR ↳ $\lor$
true if both are true	False if both false true otherwise

* Don't use logical operators in math

$$\underbrace{x \land y \in \mathbb{R}}_{\text{incorrect}}$$

maybe like this

$$\underbrace{\underbrace{(x \in \mathbb{R})}_{\text{T or F}} \land \underbrace{(y \in \mathbb{R})}_{\text{T or F}}}_{\text{and result}}$$

short circuit evaluation = only asses the first to know the result and skip the second operator

2.3 Implications

- condition statements are also called implications
  "$P \Rightarrow$ implies q" • if P is True and q false it is false otherwise it is true
- "p" is considered a hypothesis, premise, antecedent and "q" is the conclusion or consequence
- if an implication is True, the hypothesis when is met, the consequence must be true as well, this is why conditional statement.
- takes the form "if p ______, then ______."
  (then: implies)
- "P unless q" means $\bar{p} \Rightarrow q$, q is a necesary condition that prevents p from happening.

converse $q \Rightarrow P$

inverse $\bar{p} \Rightarrow \bar{q}$

contrapositive $\bar{q} = \bar{p}$

P	Q	$P \Rightarrow q$	$q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$\bar{q}$	$\bar{P}$	$\bar{q} \Rightarrow \bar{P}$
F	F	T
T	F	F
F	T	T
T	T	T

- given "$x > 2 \Rightarrow x^2 > 4$"
  - ↳ converse $[x^2 > 4 \Rightarrow x > 2]$
  - ↳ inverse $[x \leq 2 \Rightarrow x^2 \leq 4]$
  - ↳ contrapositive $[x^2 \leq 4 \Rightarrow x \leq 2]$

- The converse of a theorem in the form of an implication

$$(P \Rightarrow q) \neq (q \Rightarrow P)$$

P is a sufficient condition for q.

q is a necesarry condition for p.

- for q to be true, it's enough to say that p is true
- for p to be true it's necesary for q to be true as well.

or ? = what implies, pg. 22

2.4 Biconditional statements. "p if and only if q"

T	T	=	T
T	F	=	F
F	T	=	F
F	F	=	T

- also a compound statement
- Both are true or false simultaneously

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$

- the "exclusive or"

- Order of operations

not	Highest
and	⋮
or	
implies	
Biconditional	Lowest