5.5 Regla de la sustitución 2019/08/06

Objetivo: integre $f(g(x))$ funciones compuestas

a $$\int 3(x+2)^2\,dx = \int (3x^2+12x+12)\,dx$$
$$= x^3 + 6x^2 + 12x + C.$$

Conjeturando $\int 3(x+2)^2\,dx = (x+3)^3 + C$

b) $$\int 11\,(x-20)^{10}\,dx = (x-20)^{11} + C$$

Regla de potencia

$$\frac{d}{dx}\left[F(x)\right]^{n+1} \xrightarrow{\text{derivar}} = (n+1)\,(f(x))^n\,f'(x)$$

$\xleftarrow{\text{integrar}}$

Regla de la sustitución

- Regla de la sustitución

Funciones Potencia

$$\int \underbrace{[f(x)]^n}_{u=f(x)}\ \underbrace{f'(x)\,dx}_{du=f'(x)dx} = \frac{f(x)^{n+1}}{n+1} + C$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

si $n \neq 1$

Ejercicio 1: evalúe las sig integrales

a.) $$\int \underbrace{(11x-20)^{10}}_{u}\ \underbrace{11\,dx}_{du} = \int u^{10}\,du = \frac{u^{11}}{11} + C$$
$$= \frac{(11x-20)^{11}}{11} + C$$

a.

$$\int \underbrace{(x^2+x+3)^5}_{u} \underbrace{(2x+1)\,dx}_{du}$$

$$= \int u^5\,du = \frac{u^{5+1}}{5+1} + C = \frac{u^6}{6} + C$$

$$= \frac{(x^2+x+3)^6}{6} + C$$

b.)

$$\int \underbrace{[30w^3-8]}_{u}{}^{19} \underbrace{w^2\,dw}_{du}$$

$u = 30w^3 - 8 \qquad du = 90w^2\,dw$

$$\frac{du}{90} = w^2\,dw$$

$$= \int u^{19}\frac{du}{90} = \frac{1}{90}\int u^{19}\,du = \frac{1}{90}\cdot\frac{u^{20}}{20} = \frac{u^{20}}{1800} = \frac{1}{1800}(30w^3-8) + C$$

c) $\int (30w^3-8)^{19} - 90w^3\,dw \qquad = \int u^{14}\,w\,du$

$u = 30w^3 - 8 \qquad du = 90w^2\,dw$

Solo se puede integrar por fuerza bruta

d. $\int \underbrace{8x^3}_{du} \underbrace{\sqrt{8+x^4}}_{u} \underbrace{dx}_{du}$

$u = 8 + x^4 \qquad du = 4x^3$

$2(du) = 2(4x^3)$

$2\,du = 8x^3$

$= 2\int \sqrt{u}\, du \qquad = 2 \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{4}{3}(8 + x^4)^{3/2} + C$

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e. $\int (10x^2 + 6x)^2\, dx =$

No se usa sustitución por que no hay du.

$\int 100x^4 + 2\cdot 10x^2 \cdot 6x + 36x^2\, dx$

Expanda Luego integre

$= \frac{100x^5}{5} + \frac{120x^4}{4} + \frac{36x^3}{3}$

$= 20x^5 + 30x^4 + 12x^3 + C$

Regla de la cadena derivadas

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Regla de la sustitución

$$\int f'(g(x))\, g'(x)\, dx = \int f'(u)\, du = f(u) + C$$

$u = g(x) \qquad du = g'(x)\,dx$

$$= f(g(x)) + C$$

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0. $\int \frac{(8+16x+48x^2)}{x+x^2+2x^3} dx$

$u = x + x^2 + 2v^3$

$du = 1 + 2x + 6x^2 \, dx$

$8(du) = 8(1+2x+6x^2)\, dx$

$8du = 8 + 16x + 48x^2 \, dx$

$= \int \frac{8du}{u} = 8 \ln|u| + c$

$= 8 \ln|x| + C$

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a.) $\int e^{\overbrace{x^{10}+\sqrt{2}}^{u}} \underbrace{x^9 dx}_{du} =$

$u = x^{10} + \sqrt{2}$

$du = 10x^9 + 0 \, dx$

$du = 10x^9 \, dx$

$\frac{du}{10} = x^9 \, dx$

$= \int e^u \frac{du}{10} = \frac{1}{10} e^u + c$

$= \frac{1}{2} e^{x^{10}+\sqrt{2}} + C$

b) $\int e^{x^{10}} x^8 \, dx$ $\int e^{x^{10}} dx$

$\neq du$ $\neq du$

no es integrable

c) $\int x^3 (x^4+3)^2 \sin(x^4+3)^3 \, dx = \int u^2 \sin u^3 \frac{du}{4}$

$u = (x^4+3)$ $du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$\frac{1}{4} \int u^2 \sin(u^3) du \; du$

$t = u^3 \quad dt = 3u^2 du$

$\frac{1}{4} \int \sin t \frac{dt}{3} = -\frac{1}{12} \cos u^3 + C$

$= -\frac{1}{12} \cos(x^4+3)^3 + C$

Una sola sustitución

a) $\int \sin(x^4+3)^3 \left[(x^4+3)^2 x^3\right] dx$

$u = (x^4+3)^3 \qquad du = 3(x^4+3)^2 4x^3\, dx$

$\frac{du}{12} = (x^4+3)^2 x^3\, dx$

$= \int \sin(u)\, \frac{du}{12} = \frac{1}{2}$

b) $\int \cot x\, dx = \int \frac{\overbrace{\cos x}^{du}}{\underbrace{\sin x}_{u}} dx = \int \frac{du}{u} = \ln|u| + C$

$= \ln|\sin x| + C$

c) $\int \sec^2(e^x+x)(e^x+1)\, dx = \int \sec^2 u\, du = \tan u + C$

$\tan(e^x+x) + C$

$u = e^x + x$

$du = e^x + 1\, dx$

e) $\int 28x(x+4)^{1/3}\, dx = \int 28x\, u^{1/3}\, du = \int 28(u-4)u^{1/3}\, du$

$u = x+4 \qquad du = dx$

$u - 4 = x$

$= \int 28 \cdot u^{4/3} - 4u^{1/3}\, du$

$28\left[\frac{3(x+4)^{7/3}}{7} - \frac{4\cdot 3(x+4)^{4/3}}{4}\right] + C \qquad = 28\left[\frac{3}{7} u^{7/3} - \frac{4\cdot 3u^{4/3}}{4}\right] + C$

Regla de la sustitución para integrales definidas

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

$u = g(x)$ = cambian también los límites.

$du = g'(x)\,dx$

Ejercicio: Integre

a. $\displaystyle\int_{-4}^{0} \frac{1}{3x-2}\,dx$

$u = 3x - 2$

$du = 3\,dx$

$\frac{du}{3} = dx$

f es continua en este intervalo

$$= \int_{-14}^{-2} \frac{1}{u}\frac{du}{3} = \ln|u|\Big]_{-14}^{-2}$$

$$= (\ln|-2| - \ln|-14|)\frac{1}{3}$$

$$= [\ln|2| - \ln|-14|]\frac{1}{3}$$

$$= (-\ln|7|)\frac{1}{3}$$

b. $\displaystyle\int_0^1 \frac{8}{\pi}\cdot\frac{\sin^{-1}(t)}{\sqrt{1-t^2}}\,dt = \frac{8}{\pi}\int_0^{\pi/2} u\,du = \frac{8}{\pi}\frac{u^2}{2} = \frac{4}{\pi}u^2\Big]_0^{\pi/2} =$

$u = \sin^{-1}(t)$

$du = \frac{1}{\sqrt{1-t^2}}\,dt$

$u = \pi/2 \quad u = 0$

$u(0) = \sin^{-1}(0) = 0$

$u(1) = \sin^{-1}(1) = \frac{\pi}{2}$

$$\frac{4\left(\frac{\pi}{2}\right)^2}{\pi} - 0 = \pi$$