Lunes 26 de agosto Simulacro Parcial.
3 de septiembre Parcial 1
capítulos 5 y 7 Págs 11-70.

Integrales de la forma $\int \cot^n x \csc^m x \, dx$

$(\csc x)' = -\csc x \cot x$ $\quad$ $(\cot x)' = -\csc^2 x$

$\cot^2 x = \csc^2 x - 1$ $\quad$ $\csc^2 x = \cot^2 x + 1$

$u = \csc x$ $\quad$ $u = \cot x$

Ejercicio 4: Integre (Pág 50).

a $\int \cot^2 x \csc^4 x \, dx$ $\quad$ $\cot^2 \csc^2 x \csc^2 x$ ✓
$\times$ $\cot x \csc^3 x (\csc x \cot x)$

$$\int \cot^2 x \csc^2 x (\csc^2 x \, dx) = \int \cot^2 x (\cot^2 x + 1) \csc^2 x \, dx$$

$\csc^2 x = \cot^2 x + 1$, $\quad u = \cot x$ $\quad du = -\csc^2 x \, dx$

$$= -\int u^2 (u^2 + 1) \, du$$

$$= -\int (u^4 + u^2) \, du = -\frac{u^5}{5} - \frac{u^3}{3} + C.$$

$$= \boxed{-\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C.}$$

b. $\int \cot^3 x \csc^3 x \, dx = \int \cot^2 x \csc^2 x (\cot x \csc x \, dx)$

$\cot^2 x = \csc^2 x - 1$ $\quad$ $= \int (\csc^2 x - 1) \csc^2 x (\cot x \csc x \, dx)$

$u = \csc x \quad du = -\csc x \cot x \, dx$ $\quad$ $= -\int (u^2 - 1)(u^2) \, du.$

$$-\int (u^4 - u^2)\,du = -\frac{u^5}{5} + \frac{u^3}{3} + C.$$

$$= \boxed{-\frac{\csc^5 x}{5} + \frac{\csc^3 x}{3} + C.}$$

casos especiales $\int \csc x \, dx$ $\quad \int \csc^3 x \, dx$

$$\boxed{\int \sec x \, dx = \ln|\sec x + \tan x| + C.}$$

$$\int \csc x \frac{(\csc x + \cot x)}{\cot x + \csc x} dx = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x} dx$$

$\ast$ "1" especial. $\qquad u = \cot x + \csc x.$

$$-du = (\csc^2 x + \csc x \cot x)\,dx$$

$$= -\int \frac{du}{u} = -\ln|u| + C. = -\ln|\cot x + \csc x| + C.$$

$$\boxed{\int \csc x \, dx = -\ln|\csc x + \cot x| + C.}$$

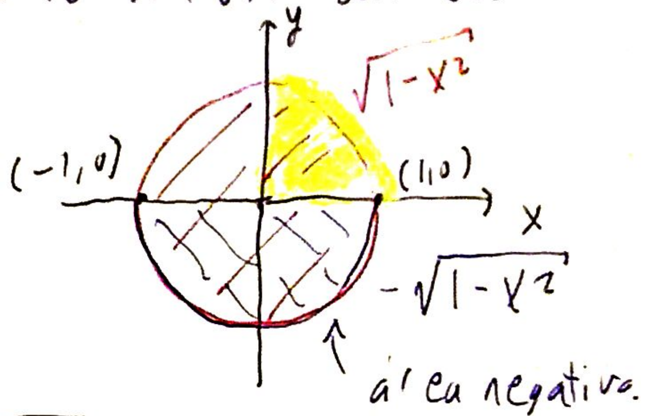
$$\int \sec^3 x \, dx = \frac{1}{2}(\sec x)' + \frac{1}{2}\int \sec x \, dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C.$$

$$\int \csc^3 x \, dx = \frac{1}{2}(\csc x)' + \frac{1}{2}\int \csc x \, dx$$

$$= -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + C.$$

A'rea de un círculo unitario sin utilizar Geometría



Ec: $x^2 + y^2 = 1$

función: $y^2 = 1 - x^2$

2 Funciones $y = \pm\sqrt{1-x^2}$

$$A = \int_{-1}^{1} \sqrt{1-x^2}\, dx + \int_{-1}^{1} \sqrt{1-x^2}\, dx \qquad \int_{1}^{-1} \sqrt{1-x^2}\, dx$$


$$A = 2\int_{-1}^{1} \sqrt{1-x^2}\, dx = 4\int_{0}^{1} \sqrt{1-x^2}\, dx$$

función par

$u = 1 - x^2$

$du \neq -2x\,dx$

Ni sustitución ni integración por partes.

$1 - \sin^2\theta = \cos^2\theta.$

$$A = 4\int_0^{\pi/2} \underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta.} \cos\theta\, d\theta.$$

$x = \sin\theta.$

$dx = \cos\theta\, d\theta.$

$$A = 4\int_0^{\pi/2} \cos^2\theta\, d\theta.$$

$x = 1 = \sin\theta \Rightarrow \theta = \pi/2$

$x = 0 = \sin\theta \Rightarrow \theta = 0$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$A = \frac{4}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta. = 2\left(\theta + \frac{1}{2}\sin 2\theta\Big]_0^{\pi/2}\right)$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\sin\pi}^{\,0} - 0 - \frac{1}{2}\cancel{\sin 0}^{\,0}\right) = \frac{2\pi}{2} = \pi.$$

A'rea de un círculo de radio (1) $\quad \pi(1)^2.$

7.3 Sustitución Trigonométrica (Pág 54).

$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}\, g'(\theta)\,d\theta.$$

simplifique si es posible.

$x = g(\theta)$ $\quad dx = g'(\theta)\,d\theta$

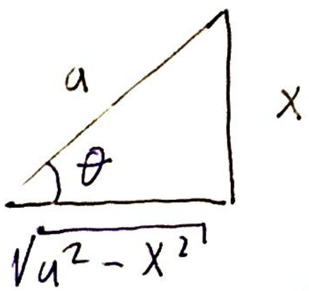
$\sqrt{1-x^2}$	$\sqrt{1+x^2}$	$\sqrt{x^2-1}$
$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta.$
$1-\sin^2\theta = \cos^2\theta.$	$1+\tan^2\theta = \sec^2\theta.$	
$\sqrt{1-x^2} = \cos\theta.$	$\sqrt{1+x^2} = \sec\theta.$	

$\sqrt{\sec^2\theta - 1}$

$\sqrt{\tan^2\theta}$

$\sqrt{x^2-1} = \tan\theta.$

forma $\sqrt{a^2-x^2}$



$\sin\theta = \dfrac{C.O.}{H} = \dfrac{x}{a}.$

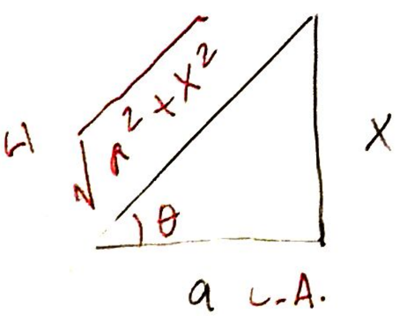
$\cos\theta = \dfrac{C.A.}{H} = \dfrac{\sqrt{a^2-x^2}}{a.}$

$x = a\cdot\sin\theta$

$dx = a\cdot\cos\theta\,d\theta.$

$\sqrt{a^2-x^2} = a\cdot\cos\theta.$

Forma $\sqrt{a^2+x^2}$



$\sin\theta = \dfrac{x}{\sqrt{a^2+x^2}}$ $\qquad \tan\theta = \dfrac{x}{a.}$

$\dfrac{H}{C.A.} = \sec\theta = \dfrac{\sqrt{a^2+x^2}}{a.}$ $\qquad \dfrac{C.A.}{H}$

$x = a\cdot\tan\theta.$

$dx = a\cdot\sec^2\theta\,d\theta.$ $\qquad \sqrt{a^2+x^2} = a\cdot\sec\theta.$

Ejercicio 1: Evalúe.

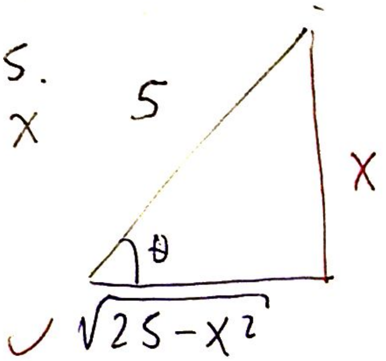
$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\cdot\frac{du}{2} = -\int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

$u = 25 - x^2$ $\quad du = -2x\,dx \Rightarrow dx = \frac{du}{-2x}$ $\quad = -u^{1/2} + C.$

$$= -\sqrt{25-x^2} + C.$$

Sustitución Trigonométrica.

$H = 5.$
$C.O = x$

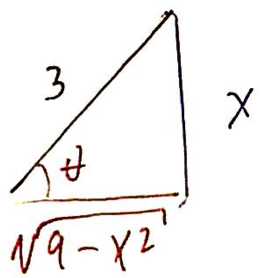


$x = 5\sin\theta.$
$dx = 5\cos\theta\,d\theta.$
$\sqrt{25-x^2} = 5\cos\theta.$ $\quad \frac{C.A.}{H.}$

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta\cdot 5\cos\theta\,d\theta}{5\cos\theta} = \int 5\sin\theta\,d\theta.$$

$$= -5\cos\theta + C. = -\sqrt{25-x^2} + C.$$

a $$\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{27\sin^3\theta\cdot 3\cancel{\cos\theta}\,d\theta}{3\cancel{\cos\theta}} = 27\int \sin^3\theta\,d\theta.$$



$\sin\theta = \frac{x}{3}$
$\cos\theta = \frac{1}{3}\sqrt{9-x^2}$

$x^3 = 27\sin^3\theta$
$x = 3\sin\theta$
$dx = 3\cos\theta\,d\theta$
$\sqrt{9-x^2} = 3\cos\theta.$

$$27\int \sin^2\theta\,\sin\theta\,d\theta = 27\int (1-\cos^2\theta)\sin\theta\,d\theta.$$

$u = \cos\theta \qquad du = -\sin\theta\,d\theta.$

$$= -27\int(1-u^2)\,du = -27\left(u - \frac{u^3}{3}\right) + C.$$

regrese $\quad = -27u + 9u^3 + C.$

a var. θ. $\quad = -27\cos\theta + 9\cos^3\theta + C$

regrese $\quad = -27\,\frac{1}{3}\sqrt{9-x^2} + 9\,\frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C.$

a var x. $\quad = -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)^{3/2} + C.$

Caso Integrales Trigonométricas.

$$\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$$

Ejercicio 5: Evalúe. (Pág 51)

a $\displaystyle\int_{-\pi}^{\pi} \sin \underset{m}{8}x \cos \underset{n}{4}x\,dx = 0.$

$$\frac{1}{2}\int_{-\pi}^{\pi}(\sin 4x + \sin 12x)\,dx = \frac{1}{2}\left(\underset{0}{-\frac{\cos 4x}{4}\Bigg]_{-\pi}^{\pi}} - \frac{\cos 12x}{12}\Bigg]_{-\pi}^{\pi}\right)$$

$$\frac{1}{2}\left(\underset{-\frac{1}{4}}{-\frac{\cos 4\pi}{4}} + \underset{+\frac{1}{4}}{\frac{\cos(4\pi)}{4}} - \frac{\cos 12\pi}{12} + \frac{\cos 12\pi}{12}\right) = \frac{1}{2}(0+0) = 0.$$