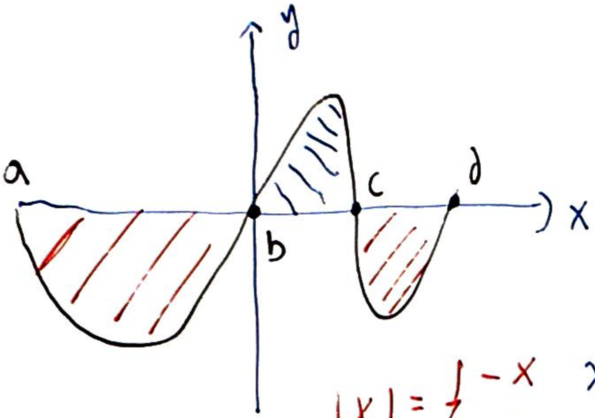
6.1 A'reas entre curvas (p 79)

Región entre la curva $y = f(x)$ y el eje x.



Área de la región

$$A = \int_a^d |f(x)|\,dx$$

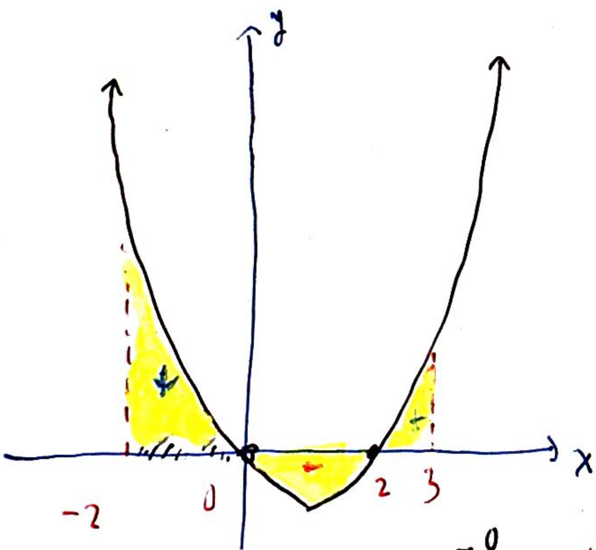
$$A = -\int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx$$

$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

Intersecciones y bosquejar la curva y la región,

Ejercicio 1: Bosqueje y encuentre el área de la región limitada por $y = 3x^2 - 6x$, $x = -2$, $x = 3$ & $y = 0$.

Interceptos - x



$$y = 3x^2 - 6x = 0$$

$$3x(x-2) = 0 \Rightarrow x = 0,\ x = 2.$$

Sume el área de 3 subregiones

$$A = \int_{-2}^{0} 3x^2 - 6x\,dx + \int_0^2 -3x^2 + 6x\,dx$$

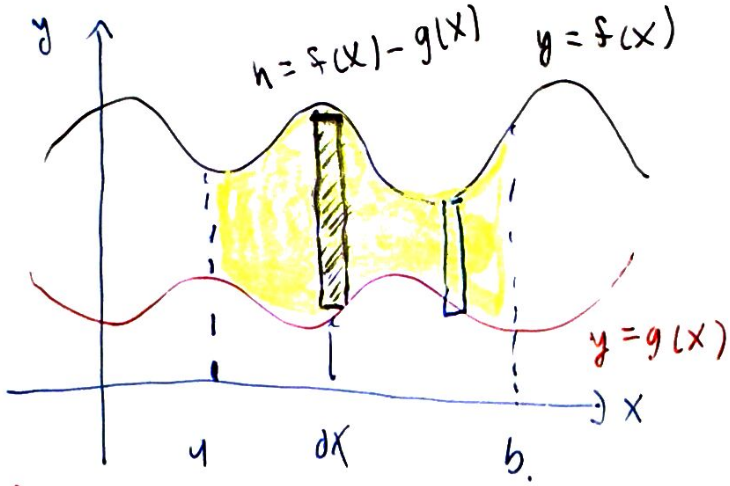
$$+ \int_2^3 3x^2 - 6x\,dx$$

$$A = x^3 - 3x^2\Big]_{-2}^{0} + \left(-x^3 + 3x^2\Big]_0^2\right) + x^3 - 3x^2\Big]_2^3$$

$$A = 0 - (-8 - 12) + (-8 + 12) + (27 - 27 - 8 + 12) = 28$$

$A = 20 + 4 + 4 = 28$

¿Qué sucede cuando hay una curva inferior?



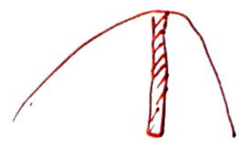
Región : $g(x) \leq y \leq f(x)$

$a \leq x \leq b.$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

diferencia de áreas.

$A = bh.$



$$\boxed{A = \int_a^b y_{arriba} - y_{abajo}\; dx}$$

Franja horizontal ó rectangulo infinitesimal. ancho dx

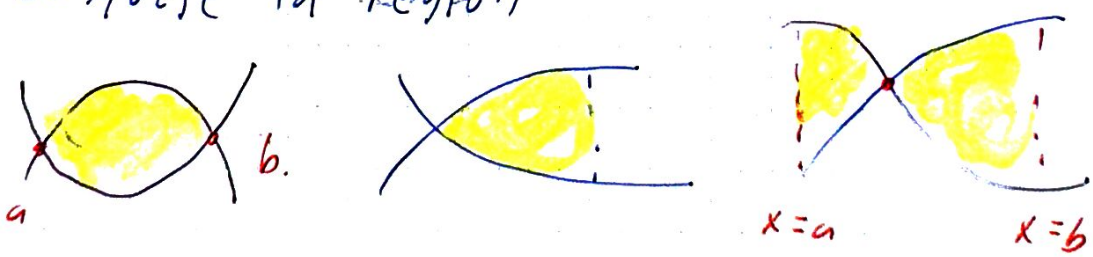
altura $f(x) - g(x)$

$$dA = (f(x) - g(x))\,dx$$

Integrando en $a \leq x \leq b.$ $\int dA = A$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

1. Bosquejar las curvas f & g.
2. Intersectos entre las curvas.
3. Bosqueje la región

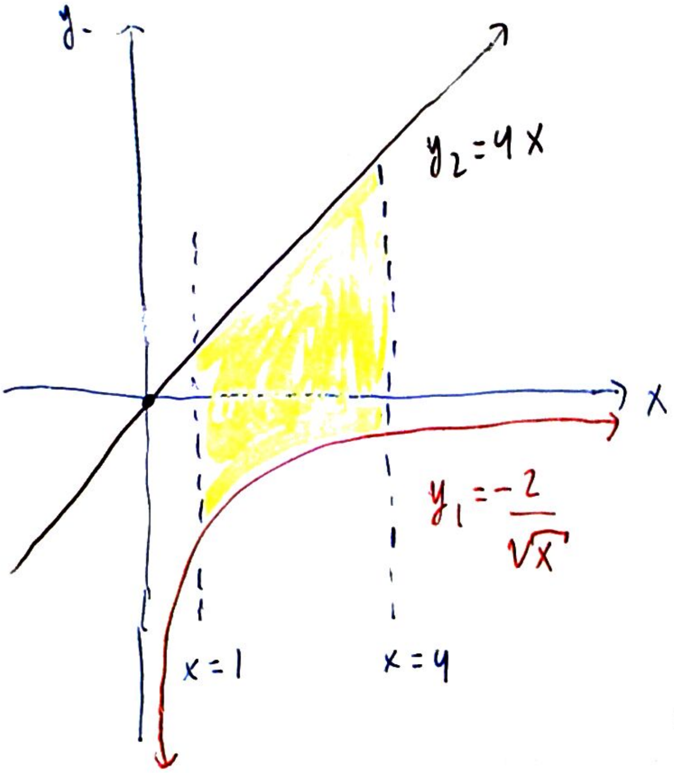


Ejemplo: Bosqueje y encuentre el área entre

$y_1 = \frac{-2}{\sqrt{x}}$ & $y_2 = 4x$ en $1 \leq x \leq 4$.

¿ $y_2 > y_1$? ó ¿ $y_1 > y_2$?

$\frac{1}{x^r}$



$$A = \int_1^4 y_2 - y_1 \, dx$$

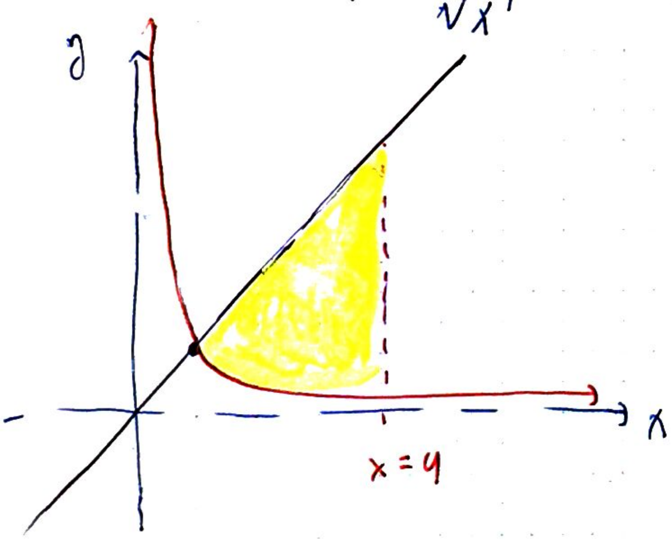
$$A = \int_1^4 4x + 2x^{-1/2}) \, dx$$

$$A = 2x^2 + 4x^{1/2} \Big]_1^4$$

$$A = 2 \cdot 16 + 4 \cdot 2 - (2 + 4).$$

$$A = 32 + 8 - 6 = 34.$$

Variación B: $y_1 = \frac{+4}{\sqrt{x}}$ & $y_2 = 4x$ y la recta $x = 4$.



$$A = \int_1^4 y_2 - y_1 \, dx$$

intersección entre y_1 & y_2.

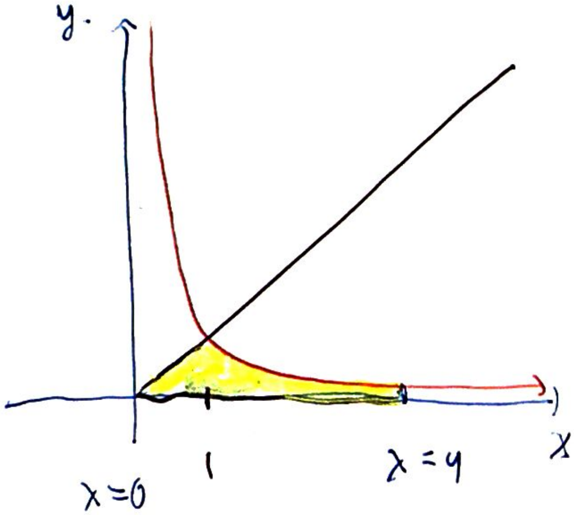
$$\frac{4}{x^{1/2}} = 4x$$

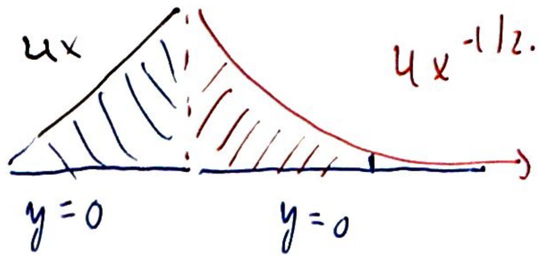
$$1 = x^{3/2} \Rightarrow x = 1$$

$$A = \int_1^4 4x - 4x^{-1/2}\,dx = 2x^2 - 8x^{1/2}\Big]_1^4$$

$$A = 32 - 16 - (2 - 8) = 16 + 6 = 22.$$

Variación C: Área de la región entre $y_1 = 4x^{-1/2}$, $y_2 = 4x$, $x = 4$ & $y = 0$.



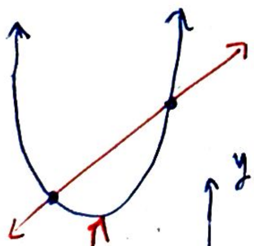


$$A = \int_0^1 4x\,dx + \int_1^4 4x^{-1/2}\,dx$$

$$A = 2x^2\Big]_0^1 + 8x^{1/2}\Big]_1^4$$

$$A = 2 + 16 - 8 = 10$$

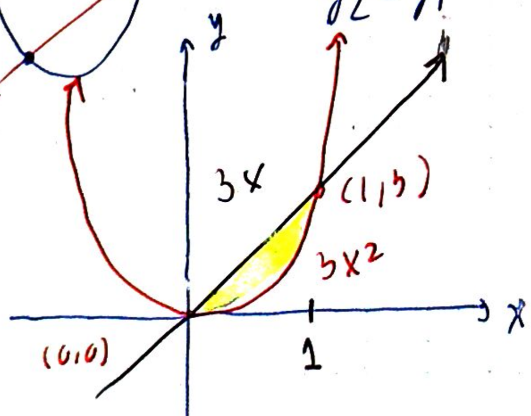
Ejemplo: Encuentre el área de la región entre las curvas $y_1 = 3x$ & $y_2 = 3x^2$?



Intersectos $y_2 = y_1$

$$3x^2 = 3x \qquad x = 0 \ \& \ x = 1$$

$$3x^2 - 3x = 3x(x-1) = 0$$



$$A = \int_0^1 (3x - 3x^2)\,dx$$

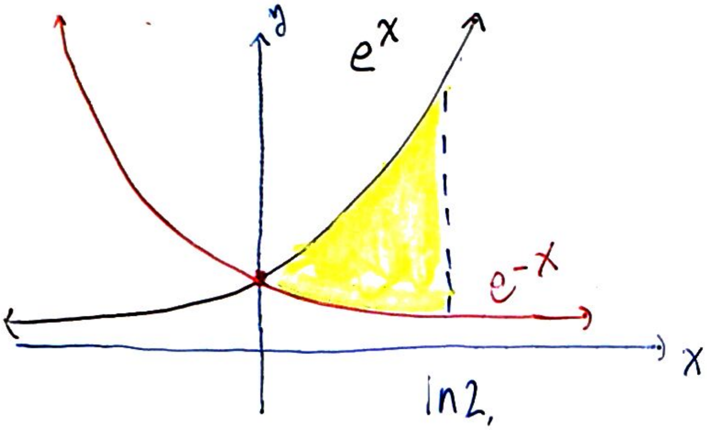
$$A = \frac{3}{2}x^2 - x^3\Big]_0^1 = \frac{3}{2} - 1 - 0$$

$$A = \frac{1}{2}.$$

Ejercicio 2: Bosqueje y encuentre el área de la región entre las curvas.

a) $y_1 = e^{x}, \quad y_2 = e^{-x}, \quad x = 0, \quad x = \ln(2).$

crece $\to +\infty$ decrece $\to 0$ $x=0 \quad y_1 = y_2 = 1$



$$A = \int_0^{\ln 2} (e^{x} - e^{-x})\,dx$$

$$A = e^{x} + e^{-x} \Big|_0^{\ln 2}$$

$$A = e^{\ln 2} + e^{-\ln 2} - (e^{0} + e^{-0})$$

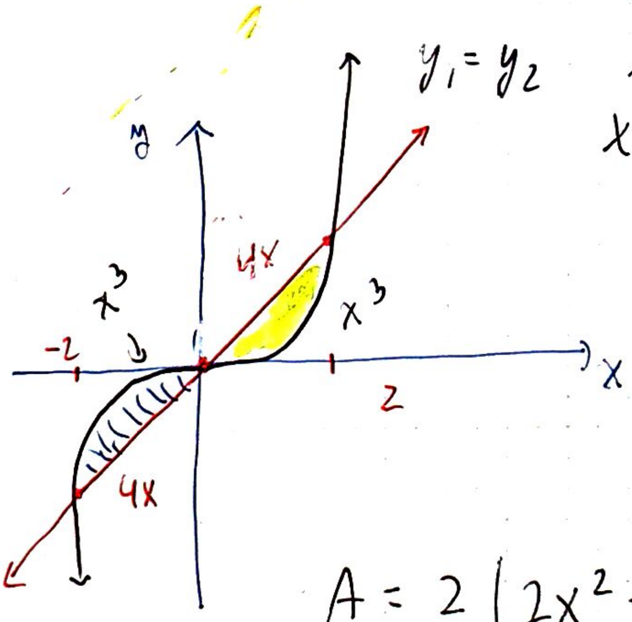
$$A = 2 + \tfrac{1}{2} - 2 = \tfrac{1}{2}$$

$e^{\ln 2} = 2, \quad e^{-\ln 2} = \dfrac{1}{e^{\ln 2}} = \dfrac{1}{2}$

$e^{\ln 2^{-1}}$

b) $y_1 = x^3$ & $y_2 = 4x$

2 regiones distintas y 3 interseclos.



$y_1 = y_2 \qquad x^3 - 4x = 0$

$x(x^2 - 4) = 0 \Rightarrow x = 0, \pm 2.$

$y_1 = y_2(2) = 8.$

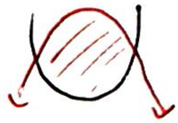
$$A = \int_{-2}^{0} (x^3 - 4x)\,dx + \int_0^2 (4x - x^3)\,dx$$

$$A = 2\int_0^2 (4x - x^3)\,dx$$

$$A = 2\left(2x^2 - \frac{x^4}{4}\Big]_0^2\right) = 2\left(8 - \frac{16}{4}\right) = 2 \cdot 4 = 8$$

c) $y_1 = x^2 - 4x + 4$, $y_2 = 10 - x^2$.

Dos parábolas.



$$A = \int_a^b y_2 - y_1 \, dx$$

Intersecciones $y_1 = y_2$. $x^2 - 4x + 4 = 10 - x^2$.

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 2(x-3)(x+1) =$$

intersectos en $x = -1$ y $x = 3$.

$$A = \int_{-1}^{3} 10 - x^2 - (x^2 - 4x + 4)\, dx$$

$$A = \int_{-1}^{3} (6 - 2x^2 + 4x)\, dx$$

$$A = 6x - \frac{2}{3}x^3 + 2x^2 \Big]_{-1}^{3} = 18 - 18 + 18 - \left(-6 + \frac{2}{3} + 2\right)$$

$$18 + 4 - 2/3 = 22 - \frac{2}{3}.$$

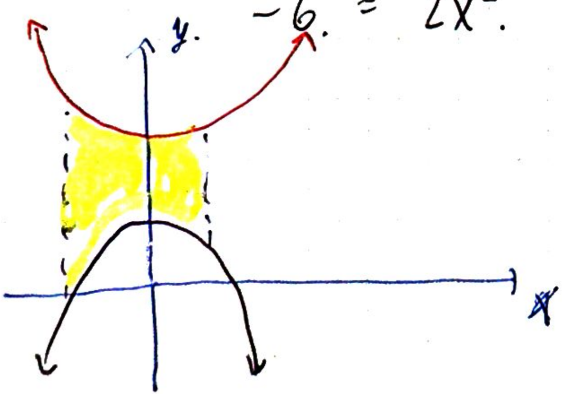
Área entre

Alejandro $y_1 = 4 - x^2$ & $y_2 = 10 + x^2$, $x = a$, $x = b$

$y_1 = y_2$: $4 - x^2 = 10 + x^2$

$-6 = 2x^2$.

No hay intersectos.



$$A = \int_a^b y_2 - y_1 \, dx.$$