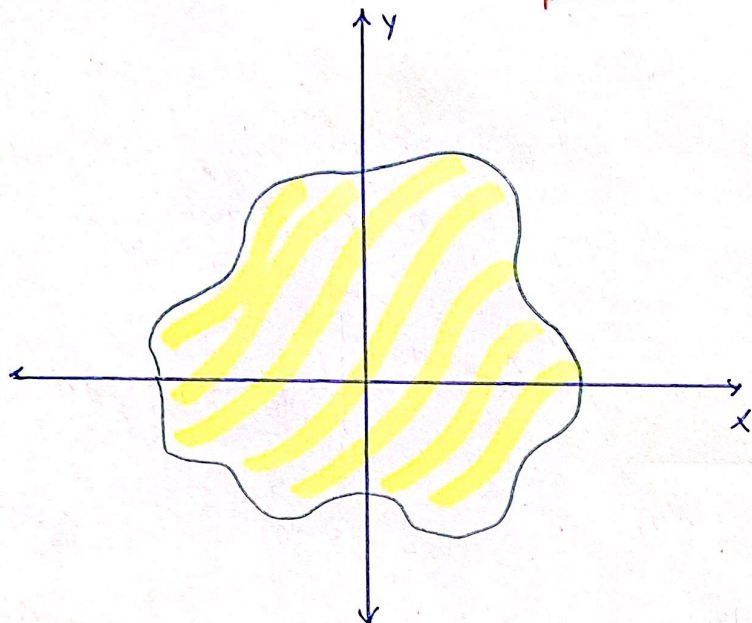
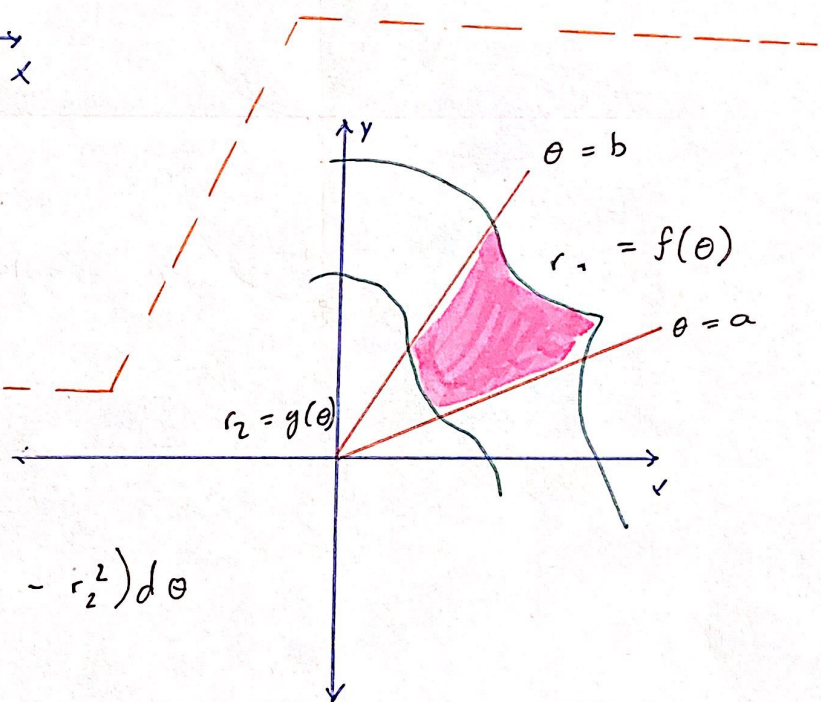


Área de una región
acotada por dos curvas
polares



$$A = \frac{1}{2} \int_a^b r^2 d\theta$$

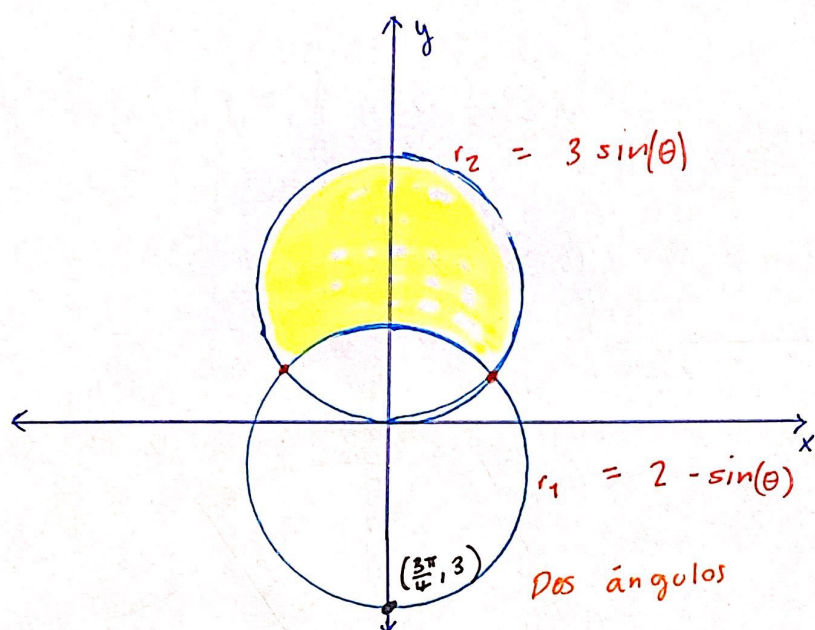


$$A = \frac{1}{2} \int_a^b (r_1^2 - r_2^2) d\theta$$

r_1 es la que está
más alejada del origen

$$R: r_2 \leq r \leq r_1 \quad a \leq \theta \leq b$$

Ejemplo : Encuentra el área de la región fuera del limazón $r_1 = 2 - \sin \theta$ & entre el círculo



r_2 está más alejada del origen que r_1

$$A = \frac{1}{2} \int_a^b (r_2^2 - r_1^2) d\theta$$

$$2 - \sin(\theta) = 3 \sin(\theta)$$

$$2 = 4 \sin(\theta)$$

$$\frac{1}{2} = \sin(\theta)$$

$$\theta = \frac{\pi}{6} ; \theta = \frac{5\pi}{6}$$

$$A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} (3 \sin(\theta))^2 - (2 - \sin(\theta))^2 d\theta$$

Duplicar resultado

$$\dots = \frac{2}{2} \int_{\pi/6}^{\pi/2} 9 \sin^2 \theta - 1 + 2 \sin \theta - \sin^2 \theta d\theta$$

$$\dots = \int_{\pi/6}^{\pi/2} (2 \sin \theta - 4 \cos(2\theta)) d\theta = -2 \cos \theta - 2 \sin 2\theta \Big|_{\pi/6}^{\pi/2}$$

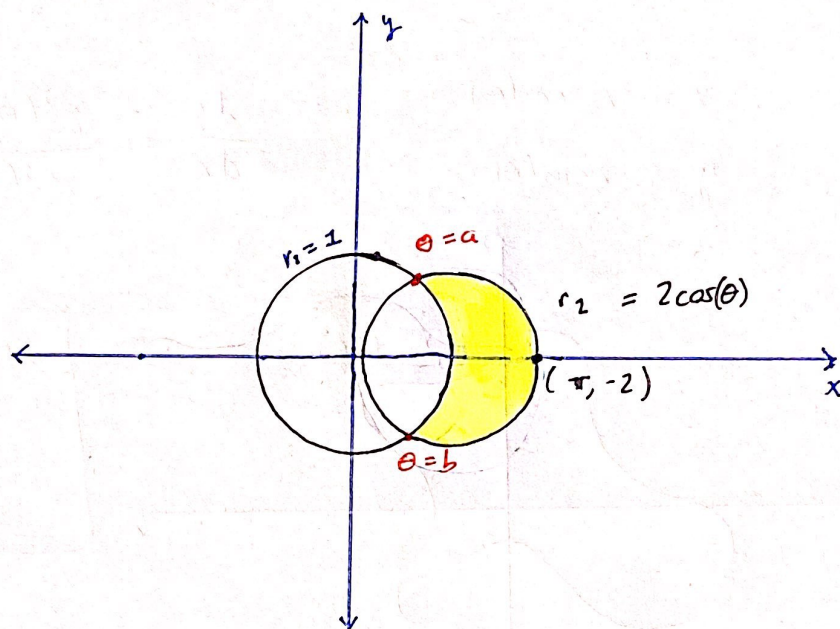
$$\dots = -2 \cos\left(\frac{\pi}{2}\right) + 2 \cos\left(\frac{\pi}{6}\right) - \cancel{2 \sin(\pi)} + 2 \sin\left(\frac{\pi}{3}\right) = \dots$$

$$\dots = 2 \frac{\sqrt{3}}{2} + 2 \frac{\sqrt{3}}{2} = \sqrt{3} + \sqrt{3} = \boxed{2\sqrt{3}}$$

Ejercicio 3: Considerar curvas $r_1 = 1$ & $r_2 = 2\cos(\theta)$.

a) Encontrar el área de la región que está adentro de $r_2 = 2\cos\theta$ & fuera de $r_1 = 1$

θ	r_2
0	2
$\frac{\pi}{2}$	0
π	-2
$\frac{3\pi}{2}$	0
2π	2



$$x = r \cos(\theta)$$

$$= -2 \cos \pi = 2$$

Pts de intersección

$$2 \cos(\theta) = 1 \Rightarrow \cos(\theta) = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}; -\frac{\pi}{3}; \frac{5\pi}{3}$$



$$A = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (r_2^2 - r_1^2) d\theta$$

Duplicamos & cambiamos

$$A = \int_0^{\frac{\pi}{3}} (1 \cdot \cos^2 \theta - 1) d\theta = \theta + \sin(2\theta) \Big|_0^{\frac{\pi}{3}} = \dots$$

$$\dots = \left[\left(\frac{\pi}{3} + \underbrace{\sin\left(2\frac{\pi}{3}\right)}_{\frac{\sqrt{3}}{2}} \right) - (0 + 0) \right] = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$$

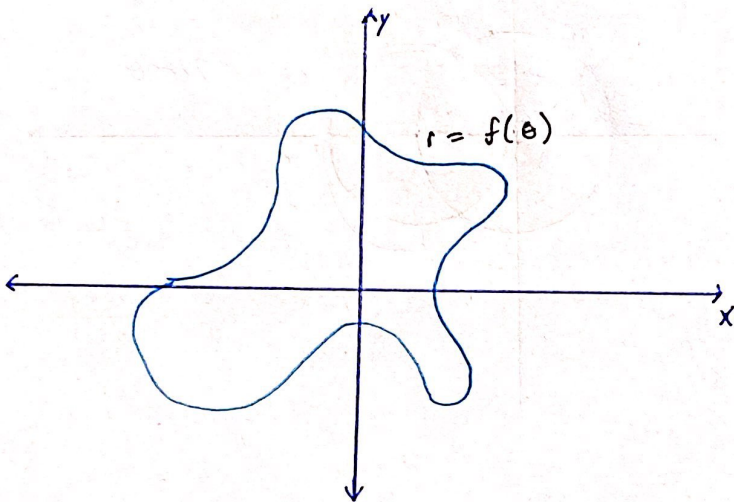
Longitud de arco de curvas polares

La curva polar $r = f(\theta)$ se escribe en términos de sus ecuaciones paramétricas

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)}$$



Longitud de arco paramétrica

$$L = \int_a^b \sqrt{[x'(\theta)]^2 + [y'(\theta)]^2} d\theta$$

$$x'(\theta) = r' \cos \theta - r \sin \theta$$

$$y'(\theta) = r' \sin \theta + r \cos \theta$$

$$\begin{aligned} [x'(\theta)]^2 &= (r')^2 \cos^2 \theta - 2r \cdot r' \cos \theta \sin \theta + r^2 \sin^2 \theta \\ + [y'(\theta)]^2 &= (r')^2 \sin^2 \theta + 2r \cdot r' \cos \theta \sin \theta + r^2 \cos^2 \theta \end{aligned}$$

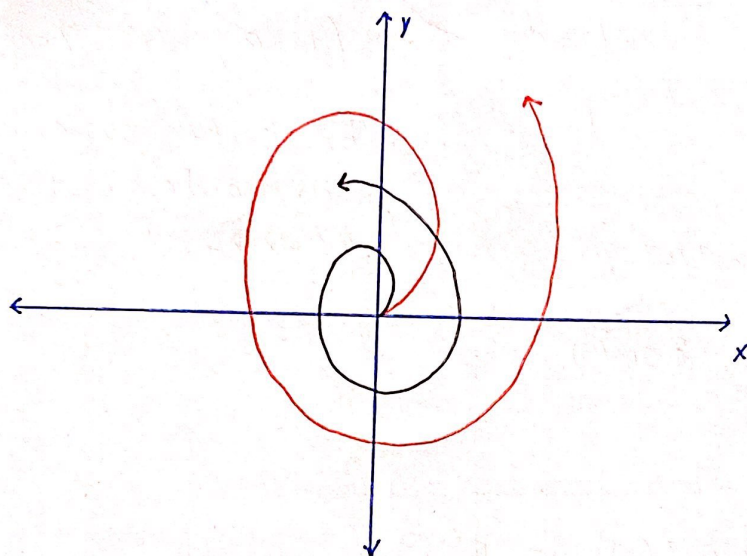
$$(x'(\theta))^2 + (y'(\theta))^2 = (r'(\theta))^2 + 0 + r^2$$

$$\therefore L = \int_a^b \sqrt{r^2 + (r'(\theta))^2} d\theta$$

* Para polares

c) la espiral $r = \theta^2$

$$0 \leq \theta \leq \sqrt{\pi}$$



$$L = \int_0^{\sqrt{\pi}} \sqrt{r^2 + (r'(\theta))^2} d\theta$$

$$r = \theta^2 \Rightarrow r' = 2\theta$$

$$(r')^2 = 4\theta^2$$

$$L = \int_0^{\sqrt{\pi}} \sqrt{\theta^4 + 4\theta^2} d\theta = \dots$$

$$\dots = \int_0^{\sqrt{\pi}} \sqrt{\theta^2 + 4} \theta d\theta = \dots$$

$$\dots = \int_{u(0)}^{u(\sqrt{\pi})} u^{\frac{1}{2}} \frac{du}{2} = \frac{1}{3} u^{\frac{3}{2}} \Big|_{\pi}^{\pi+4} =$$

derivadas paramétricas
& polares, áreas,
longitud de arco.

Jueves súper corto

$$\begin{aligned} u &= \theta^2 + 4 \\ u(\sqrt{\pi}) &= \pi + 4 \\ du &= 2\theta d\theta \\ u(0) &= \pi \end{aligned}$$

$$= \frac{1}{3} \left[(\pi + 4)^{\frac{3}{2}} - (\pi^{\frac{3}{2}}) \right]$$

Ej 4: Encuentra la longitud de arco de los sigs. problemas:

a) Circunferencia radio 1 centrada en $(1, 0)$; $r = 2 \cos \theta$;
 $0 \leq \theta \leq \pi$

$$r = 2 \cos \theta \Rightarrow r^2 = 4 \cos^2(\theta)$$

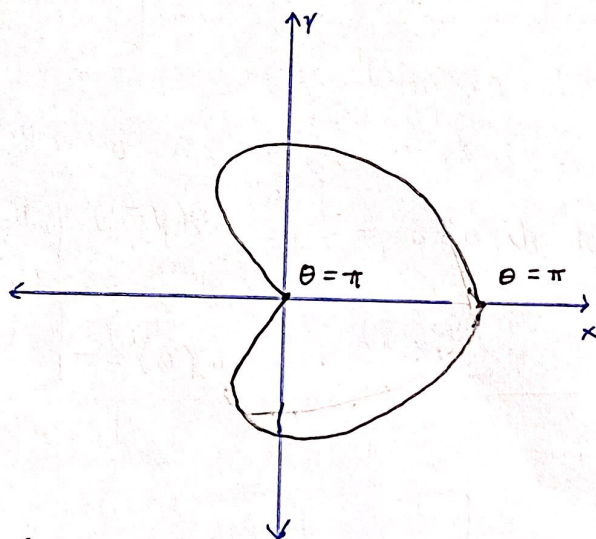
$$r'(\theta) = -2 \sin \theta \Rightarrow (r'(\theta))^2 = 4 \sin^2(\theta)$$

Simplificar:

$$r^2 + (r'(\theta))^2 = 4 \underbrace{(\cos^2(\theta) + \sin^2(\theta))}_1 = 4$$

Longitud de arco:

$$\begin{aligned} L &= \int_0^{\pi} \sqrt{r^2 + (r'(\theta))^2} d\theta \\ &= \int_0^{\pi} 2 d\theta = 2\theta \Big|_0^{\pi} = \boxed{2\pi} \end{aligned}$$



$$\begin{aligned} r^2 &= (1 + \cos \theta)^2 \\ &= 1 + 2 \cos(\theta) + \cos^2 \theta \end{aligned}$$

$$r'(\theta) = -\sin(\theta) \Rightarrow (r'(\theta))^2 = \sin^2 \theta$$

$$r^2(\theta) + (r'(\theta))^2 = 2 + 2 \cos(\theta)$$

$$L = \int_0^{\pi} \sqrt{2 + 2 \cos(\theta)} d\theta = \int_0^{\pi} \sqrt{4 \cos^2 \frac{\theta}{2}} d\theta = 4 \sin\left(\frac{\theta}{2}\right) \Big|_0^{\pi} = 4 - 0 = \boxed{4}$$

recordar: $\cos^2 \frac{\theta}{2} = \frac{1}{2} + \frac{1}{2} \cos(\theta)$