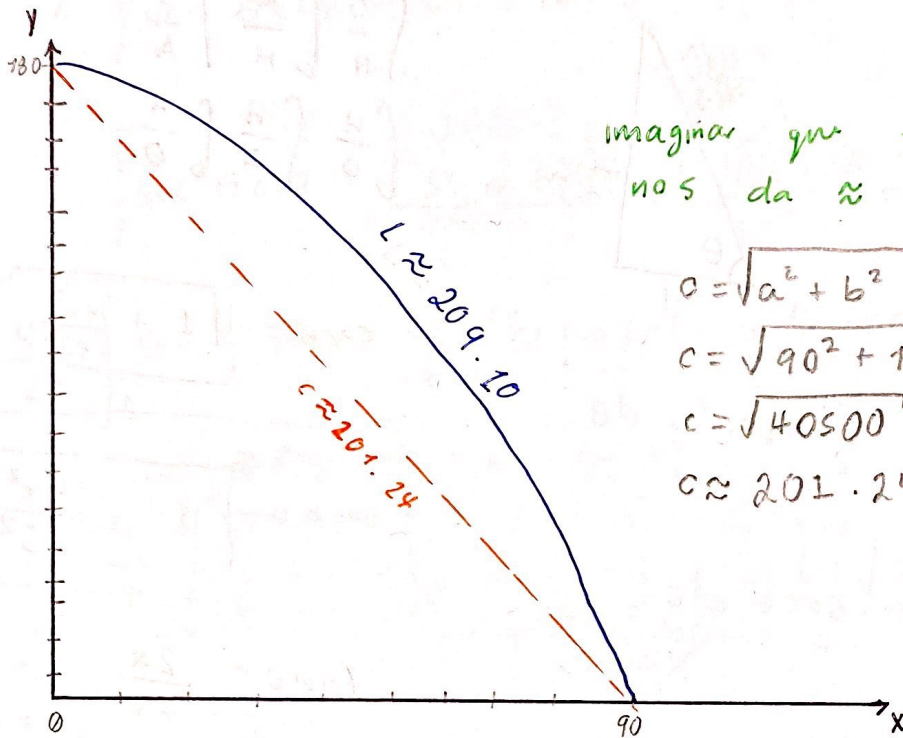


- 5) Alcañ vuela a 15 m/s, altitud 180;  $y = 180 - \frac{x^2}{45}$



imaginar que fuera un triángulo  
nos da  $\approx$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{90^2 + 180^2}$$

$$c = \sqrt{40500}$$

$$c \approx 201.24$$

$$y = 0$$

$$y' = 0 - \frac{2x}{45}$$

$$0 = 180 - \frac{x^2}{45}$$

$$y' = -\frac{2x}{45} \Rightarrow (y')^2 = \left[ \frac{4x^2}{45^2} \right] = \left( \frac{y'}{45} \right)^2$$

$$\frac{x^2}{45} = 180$$

$$x^2 = 180 \times 45$$

$$x = \sqrt{180 \times 45}$$

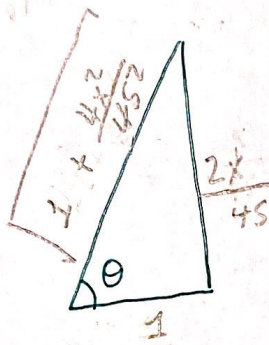
$$x = \sqrt{8100}$$

$$x = 90$$

$$L = \int_0^{90} \sqrt{1 + (y'(x))^2} dx$$

$$= \int_0^{90} \sqrt{1 + \frac{4x^2}{45^2}} dx$$

$$L = \int_0^{90} \sqrt{1 + \frac{4x^2}{45^2}} dx$$



$$\int_0^{\frac{O}{H}} \left[ \frac{A}{H} \right] \frac{O}{A}$$

$$\left( \frac{H}{O} \right) \int \frac{H}{A} \left( \frac{A}{O} \right)$$

$$\sec \theta = \sqrt{1 + \frac{4x^2}{45^2}}$$

$$\sec \theta = \sqrt{1 + \frac{4x^2}{45^2}}$$

$$\tan \theta = \frac{2x}{45}$$

$$45 \tan \theta = 2x \quad \frac{d}{dx}$$

$$45 \sec^2 \theta = 2 dx = dx$$

$$\frac{45}{2} \sec^2 \theta d\theta = dx$$

$$u = \sec \theta \quad dv = \sec^2 \theta d\theta$$

$$du = \sec \theta \tan \theta d\theta \quad v = \tan \theta$$

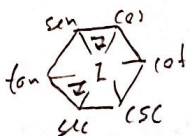
$$\sec \theta \tan \theta - \int \sec \theta \tan^2 \theta d\theta$$

$$\int \sec^3 \theta d\theta = \sec \theta \tan \theta - \int \sec \theta (\sec^2 \theta - 1) d\theta$$

$$\int \sec^3 \theta d\theta = \sec \theta \tan \theta - \left( \int \sec^3 \theta d\theta - \int \sec \theta d\theta \right)$$

$$\int \sec^3 \theta d\theta = \sec \theta \tan \theta - \int \sec^3 \theta d\theta + \int \sec \theta d\theta$$

cíclica



$$\int \sec^3 \theta d\theta + \int \sec^3 \theta d\theta = \sec \theta \tan \theta + \int \sec \theta d\theta$$

$$2 \int \sec^3 \theta d\theta = \sec \theta \tan \theta + \ln(|\sec \theta + \tan \theta|)$$

$$\int \sec^3 \theta d\theta = \frac{\sec \theta \tan \theta + \ln(|\sec \theta + \tan \theta|)}{2}$$

$$\frac{45}{2} \int \sec^3 \theta d\theta = \frac{45}{2} \left( \frac{\sec \theta \tan \theta + \ln(|\sec \theta + \tan \theta|)}{2} \right) \Bigg|_0^{90}$$

# Regresar a la variable original

$$\frac{45}{4} \left[ \left( \sqrt{1 + \frac{4x^2}{45^2}} \right) \left( \frac{2x}{45} \right) + \ln \left( \sqrt{1 + \frac{4x^2}{45^2}} + \frac{2x}{45} \right) \right]$$

evaluación =

$$\frac{45}{4} \left[ \left\{ \left( \sqrt{1 + \frac{4(90)^2}{45^2}} \right) \left( \frac{2(90)}{45} \right) + \ln \left( \sqrt{1 + \frac{4(9)^2}{45^2}} + \frac{2(9)}{45} \right) \right\} - \{ \dots \} \right]$$

$$\frac{45}{4} \left[ \left\{ \sqrt{17} \cdot 4 + \ln |\sqrt{17} + 4| \right\} - \left\{ \sqrt{1 + \frac{4(0)^2}{45^2}} \left( \frac{2(0)}{45} \right) + \ln \left| \sqrt{1 + \frac{4(0)^2}{45^2}} + \frac{2(0)}{45} \right| \right\} \right]$$

$$\frac{45}{4} \left[ \sqrt{17} \cdot 4 + \ln |\sqrt{17} + 4| - \ln |\sqrt{1} + 0| \right]$$

$$\frac{45}{4} \left[ 4\sqrt{17} + \ln |\sqrt{17} + 4| \right] \approx 209.10$$



① Valor promedio

$$f_{prom} = \frac{1}{b-a} \int_a^b f(x) dx$$

②  $f(t) = e^{\tan \pi t} \cdot \sec^2 \pi t$  en  $[0, \pi/4]$

$$f_{prom} = \frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} e^{\tan \pi t} \cdot \sec^2 \pi t dt$$

$$u = \tan(\pi t) \quad du = \sec^2(\pi t) \pi \cdot dt$$

$$f_{prom} = \frac{4}{\pi^2} \int_0^{\pi/4} e^u du$$

$$= \frac{4}{\pi^2} \left( e^u \right) \Big|_0^{\pi/4} = \frac{4}{\pi^2} \left( e^{\tan(\pi t)} \right) \Big|_0^{\pi/4} =$$

$$= \frac{4}{\pi^2} \left[ \left( e^{\tan(\pi \cdot \frac{\pi}{4})} \right) - \left( e^{\tan(\pi \cdot 0)} \right) \right] =$$

$$= \frac{4}{\pi^2} \left[ e^{\tan(\frac{\pi^2}{4})} - e^0 \right] = \frac{4}{\pi^2} \left[ e^{\tan(\frac{\pi^2}{4})} - 1 \right] =$$

$$= \frac{4}{\pi^2} \left[ e^{\tan(\frac{\pi^2}{4})} - 1 \right]$$



$$\textcircled{b} f(x) = \frac{120 x^2}{(2 + x^3)^2} \text{ en } [0, 2]$$

$$f_{\text{prom}} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$f_{\text{prom}} = \frac{1}{2} \int_0^2 \frac{120 x^2}{(2 + x^3)^2} dx$$

$$u = 2 + x^3$$

$$du = 3x^2 dx$$

$$60 du = 120 x^2 dx$$

$$= \frac{1}{2} \int_0^2 \frac{60 du}{u^2} = \frac{60}{2} \int_0^2 u^{-2} du =$$

$$= 30 \left( u^{-1} \right) \Big|_0^2 = 30 \left( 2 + x^3 \right) \Big|_0^2$$

$$= 30 \left[ \left( 2 + (2)^3 \right) - \left( 2 + (0)^3 \right) \right] = 30 \left[ (10) - (2) \right] = 30 [8]$$

$$= 30 \cdot 8 = 240$$

~~5~~

②  $f(x) = \frac{9}{x^2}$  en  $[1, 3]$

① Calcular el valor promedio:

$$f_{\text{prom}} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$f_{\text{prom}} = \frac{1}{3-1} \int_1^3 \frac{9}{x^2} dx$$

$$= \frac{1}{2} \left( 9 \int_1^3 x^{-2} dx \right) = \frac{1}{2} \left( 9 \left( \frac{1}{-1} x^{-1} \right) \right) \Bigg|_1^3 =$$

$$= \frac{9}{2} \left[ \left( -\frac{1}{3} \right) - \left( -\frac{1}{1} \right) \right] =$$

$$= \frac{9}{2} \left[ -\frac{1}{3} + \frac{3}{3} \right] = \frac{9}{2} \left( \frac{-1+3}{3} \right) = \frac{9}{2} \left( \frac{2}{3} \right) =$$

$$= \frac{9}{3} = 3$$

② tal  $c$  que cumpla  $f(c) = f_{\text{prom}}$

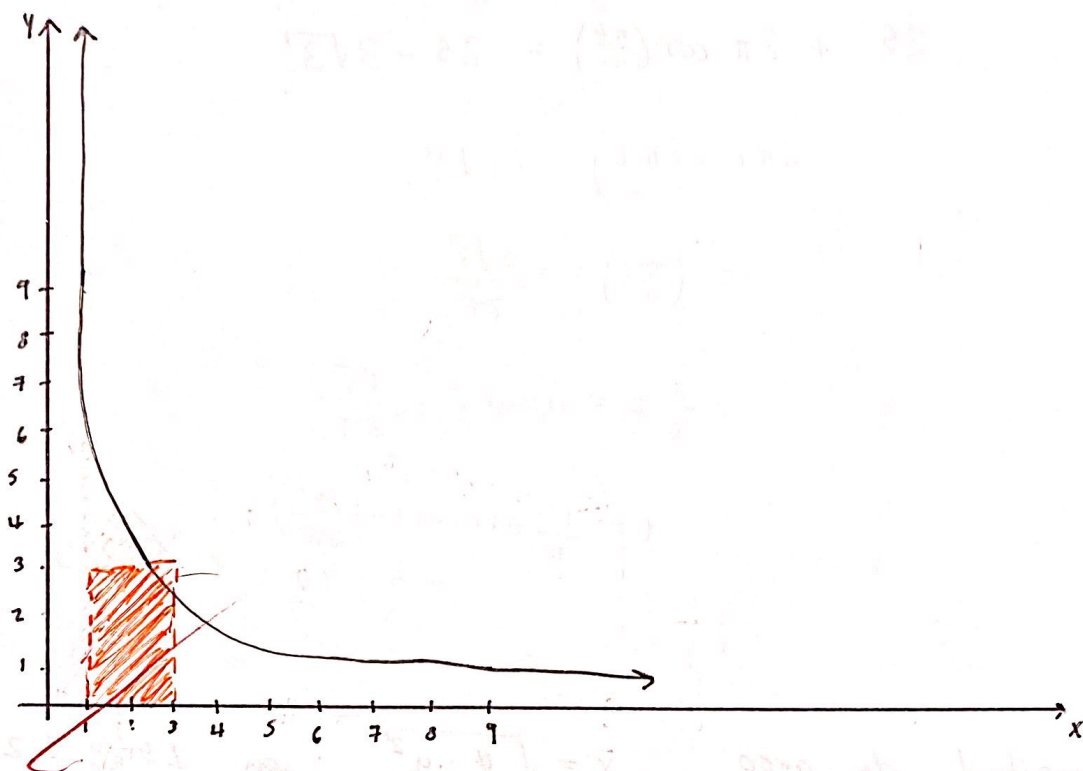
$$\frac{9}{x^2} = 3$$

$$9 = 3x^2$$

$$\frac{9}{3} = x^2$$

$$\pm \sqrt{3} = x$$

$$c = \pm \sqrt{3}$$



③  $T(t) = 25 + 2\pi \cos\left(\frac{\pi t}{6}\right)$  en  $0 \leq t \leq 24$

① Promedio en  $[6, 8]$

$$f_{\text{prom}} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$f_{\text{prom}} = \frac{1}{8-6} \int_6^8 25 + 2\pi \cos\left(\frac{\pi t}{6}\right)$$

$$= \frac{1}{2} \left( 25t + 2\pi \left( \frac{6}{\pi} \sin\left(\frac{\pi t}{6}\right) \right) \right) \Bigg|_6^8 = \frac{1}{2} \left( 25t + 12 \sin\left(\frac{\pi t}{6}\right) \right) \Bigg|_6^8$$

$$= \frac{1}{2} \left[ \left( 25(8) + 12 \sin\left(\frac{\pi(8)}{6}\right) \right) - \left( 25(6) + 12 \sin\left(\frac{\pi(6)}{6}\right) \right) \right]$$

$$= \frac{1}{2} \left[ \left( 200 - \frac{12\sqrt{3}}{2} \right) - \left( 150 + 12 \right) \right] = \frac{1}{2} \left( 50 - \frac{12\sqrt{3}}{2} \right)$$

$\frac{1}{2} (50 - 6\sqrt{3}) = 25 - 3\sqrt{3}$



⑥

$$25 + 2\pi \cos\left(\frac{\pi t}{6}\right) = 25 - 3\sqrt{3}$$

$$2\pi \cos\left(\frac{\pi t}{6}\right) = -3\sqrt{3}$$

$$\cos\left(\frac{\pi t}{6}\right) = -\frac{3\sqrt{3}}{2\pi}$$

$$\frac{\pi t}{6} = \arccos\left(-\frac{3\sqrt{3}}{2\pi}\right)$$

$$t = \frac{6}{\pi} \arccos\left(-\frac{3\sqrt{3}}{2\pi}\right) \quad \text{X}$$

④ Longitud de arco

$$x = \sqrt{4 - y^2} \quad \text{en } 1 \leq y \leq 2$$

$$\frac{dx}{dy} = \frac{1}{2} (4 - y^2)^{-1/2} \cdot -2y$$

$$= \frac{-2y}{2\sqrt{4 - y^2}} = \left(-\frac{y}{\sqrt{4 - y^2}}\right)^2 = \frac{y^2}{4 - y^2}$$

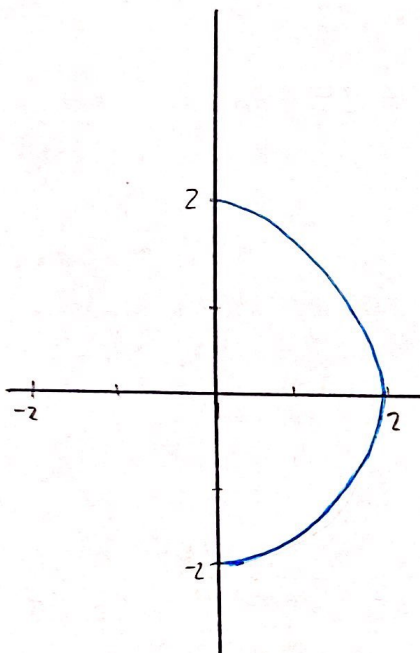
$$L = \int_1^2 \sqrt{1 + \frac{y^2}{4 - y^2}} dy$$

$$\sqrt{\frac{4 - y^2 + y^2}{4 - y^2}}$$


$$\sqrt{\frac{4}{4 - y^2}} = \frac{\sqrt{4}}{\sqrt{4 - y^2}} = \frac{2}{\sqrt{4 - y^2}}$$

$$L = 2 \int_1^2 \frac{1}{\sqrt{4 - y^2}} dy = 2 \left( 2 \left( \sin^{-1}\left(\frac{y}{2}\right) \right) \right) \Big|_1^2$$

$$= 2 \left[ \left( \sin^{-1}\left(\frac{2}{2}\right) \right) - \left( \sin^{-1}\left(\frac{1}{2}\right) \right) \right] =$$






$$= 2 \left[ \underbrace{\sin^{-1}(1)} - \underbrace{\sin^{-1}\left(\frac{1}{2}\right)} \right] = \frac{2\pi}{3}$$
