Continuación Integración Trigonométrica

• Lunes 26 de Agosto Simulacro Parcial 3 de septiembre parcial 1; capítulos 5 y 7 pg 11 - 70

Integrales de la forma $\int \cot^n x \csc^m dx$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\cot^2 x = \csc^2 x - 1$$

Ejercicio 4: Integre (pg. 50)

ⓐ $\int \cot^2(x) \csc^4(x)\, dx =$

$\therefore \cot^2 x \csc^2 x \csc^2 x$

$\therefore \cot x \csc^3 x (\csc x \cot x)$

$$= \int \cot^2 x \csc^2 x (\csc^2 x\, dx) = \int \cot^2 x (\cot^2 x + 1) \csc^2 x\, dx$$

sustitución

$u = \cot x \qquad du = -\csc^2 x\, dx$

$$= -\int u^2 (u^2 + 1)\, du$$

$$= -\int (u^4 + u^2)\, du$$

$$= -\frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= -\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C$$

ⓑ $\int \cot^3 x \csc^3 x\, dx =$

$$= \int \cot^2 x \csc^2 x (\cot x \csc x\, dx)$$

$$= \int (\csc^2 x - 1) \csc^2 x (\cot x \csc x\, dx)$$

$u = \csc x \qquad du = -\csc x \cot x\, dx$

$$= -\int (u^2 - 1)(u^2)\, du$$

$$= -\int (u^4 - u^2)\, du = -\frac{u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{-\csc^5 x}{5} + \frac{\csc^3 x}{3} + C$$

Casos especiales $\int \csc x \, dx \quad \int \csc^3 x \, dx$

- $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

- $\int \csc x \underbrace{\frac{(\csc x + \cot x)}{(\cot x + \csc x)}}_{1 \text{ especial}} = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x} \, dx$

$u = \cot x + \csc x$

$-du = \csc^2 x + \csc x \cot x \, dx$

$= -\int \frac{du}{u} = -\ln|u| + C$

$= -\ln|\cot x + \csc x| + C$

- $\int \sec^3 x \, dx = \frac{1}{2}(\sec x)^9 + \frac{1}{2}\int \sec x \, dx$

$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

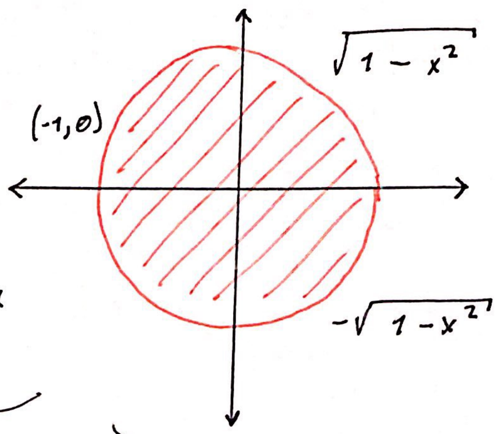
- $\int \csc^3 x \, dx = \frac{1}{2}(\csc x)^9 + \frac{1}{2}\int \csc x \, dx$

$= -\frac{1}{2}\csc x - \frac{1}{2}\ln|\csc x + \cot x| + C$

Área de un círculo unitario sin utilizar Geometría

Ec. $x^2 + y^2 = 1$

Función: $y = \pm\sqrt{1 - x^2}$



área negativa

$$\text{Área} = \int_{-1}^{1} \sqrt{1-x^2}\,dx + \underbrace{\int_{-1}^{1} -\sqrt{1-x^2}\,dx}_{\text{por } -1}$$

$$= 2\int_{-1}^{1} \sqrt{1-x^2}\,dx \qquad * \ 2$$

$$= 4\int_{0}^{1} \sqrt{1-x^2}\,dx \qquad * \ 4$$

ni sustitución, ni integración por partes

$\therefore \quad 1 - \sin^2\theta = \cos^2\theta$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

Para evaluación de la integral
$$\begin{cases} x = \sin\theta = 1 = \therefore \frac{\pi}{2} \\ x = \sin\theta = 0 = \therefore 0 \end{cases}$$

$$\therefore A = 4\int_{0}^{\pi/2} \underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta} \cos\theta \, d\theta$$

$$A = 4\int_{0}^{\pi/2} \cos^2\theta \, d\theta$$

$$A = \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$A = \frac{4}{2}\int_{0}^{\pi/2} (1 + \cos(2\theta))\, d\theta$$

$$A = 2\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Bigg]_{0}^{\pi/2}$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right)$$

$$= \frac{2}{2}\cdot\pi = \therefore \pi \quad \square$$

el área de un círculo de radio 1 es π

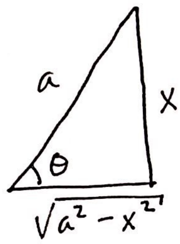
7.3. Sustitución Trigonométrica (pg. 54)

$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}_{\text{simplifique si es posible.}} g'(\theta)\,d\theta$$

$x = g(\theta) \qquad dx = g'(\theta)\,d\theta$

$\sqrt{1-x^2}$	$\sqrt{1+x^2}$	$\sqrt{x^2-1}$	$\sqrt{\sec^2\theta - 1}$
$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta$	$\sqrt{\tan^2\theta}$
$1 - \sin^2\theta = \cos^2\theta$	$1 + \tan^2\theta = \sec^2\theta$		$\sqrt{x^2-1} = \tan\theta$
$\sqrt{1-x^2} = \cos\theta$	$\sqrt{1+x^2} = \sec\theta$		

forma más general $\sqrt{a^2-x^2}$

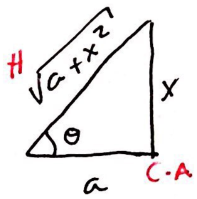


$\sin(\theta) = \dfrac{C.O.}{H} = \dfrac{x}{a} \qquad x = a\sin\theta$

$dx = a\cdot\cos\theta\, d\theta$

$\cos(\theta) = \dfrac{C.A.}{H} = \dfrac{\sqrt{a^2-x^2}}{a} = \sqrt{a^2-x^2} = a\cos\theta$

forma $\sqrt{a^2+x^2}$



$\sin\theta = \dfrac{x}{\sqrt{a^2+x^2}} \qquad \tan\theta = \dfrac{x}{a}$

$\dfrac{H}{C.A} = \sec\theta = \dfrac{\sqrt{a^2+x^2}}{a}$

$x = a\cdot\tan\theta$

$dx = a\sec^2\theta\, d\theta \qquad \sqrt{a^2+x^2} = a\cdot\sec\theta$

Ejercicio 1: Evalúe

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\,\frac{du}{2} = \int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

$$= -u^{1/2} + C$$

$$= -\sqrt{25-x^2} + C \ \text{✗}$$

$u = 25 - x^2$

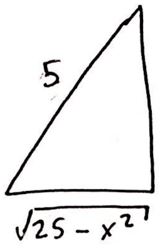
$du = -2x\ dx = \frac{du}{-2x}$

Sustitución Trigonométrica

$H = 5$

$C.O. = x$

$C.A. = \sqrt{25-x^2}$



$x = 5\sin\theta$ ✓

$dx = 5\cos\theta\, d\theta$ ✓

$\sqrt{25-x^2} = 5\cos\theta$ ✓

$\frac{C.A.}{H}$

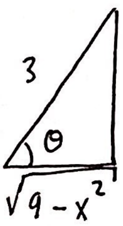
$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{5\cos\theta}\cdot 5\cos\theta\ d\theta = 5\int \sin\theta\ d\theta$$

$$= -5\cos\theta + C$$

$$= \frac{-5}{5}\sqrt{25-x^2} + C$$

$$= -\sqrt{25-x^2} + C \ \text{✗}$$

ⓐ $$\int \frac{x^3}{\sqrt{9-x^2}}\,dx =$$



$S = \frac{x}{3}$ $C = \frac{\sqrt{9-x^2}}{3}$

$3\sin\theta = x$ $3\cos\theta = \sqrt{9-x^2}$ → $dx = 3\cos\theta\ d\theta$

$(3\sin\theta)^3 = x^3$

sustituimos

$$= \int \frac{27\sin^3\theta}{\cancel{3\cos\theta}}\cdot \cancel{3\cos\theta}\, d\theta = \int 27\sin^3\theta\ d\theta$$

$$\longrightarrow \int 27\sin^3\theta\, d\theta = 27\int \sin^3\theta\, d\theta = 27\int \sin\theta\,(1-\cos^2\theta)\,d\theta$$

$$= -27u + 9u^3 + C = -27\cos\theta + 9\cos^3\theta + C$$

$u = \cos\theta \qquad du = -\sin\theta\, d\theta$

sustituyo

$$= -27 \cdot \frac{1}{3}\sqrt{9-x^2} + 9 \cdot \frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C$$

Caso Integrales trigonométricas

- $\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$
- $\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$
- $\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$

Ejercicio 5: Evalúe (pg. 51)

ⓐ $$\int_{-\pi}^{\pi} \sin(8x)\cos(4x)\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(\sin(4x) + \sin(12x)\right)dx = \frac{1}{2}\left[-\frac{\cos(4x)}{4} - \frac{\cos(12x)}{12}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left(-\frac{\cos(4\pi)}{4} + \frac{\cos(-4\pi)}{4} - \frac{\cos(12\pi)}{12} + \frac{\cos(12\pi)}{12}\right)$$

$$= \emptyset$$