Laboratorio 8 David Gabriel Corzo Mcmath

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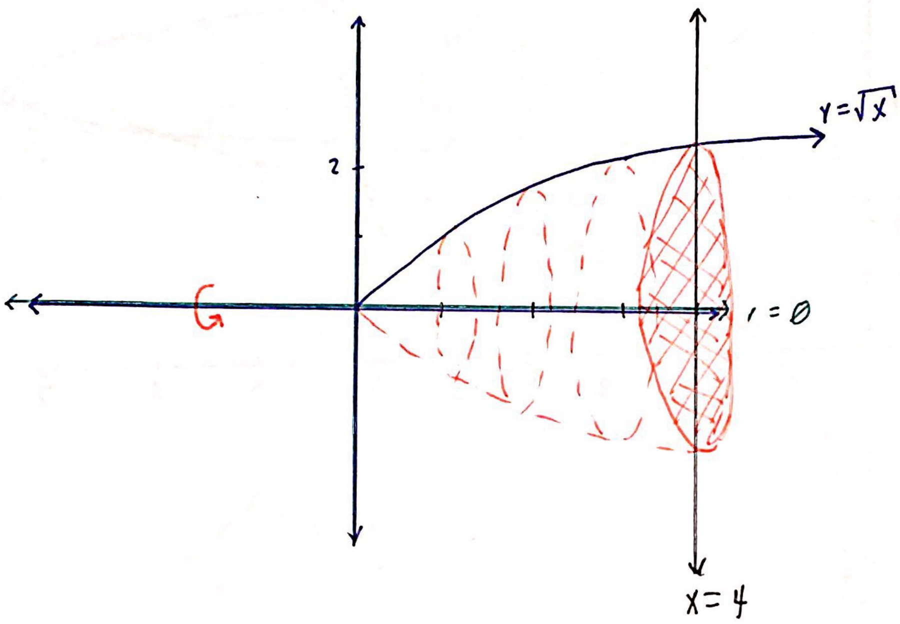
① rotar eje-x ; $y = \sqrt{x}$; $y = 0$; $x = 4$



Primero encontrar el área de la curva $\sqrt{x}$

$\boxed{A = \pi r^2}$ como r es en este caso y la fórmula queda así:

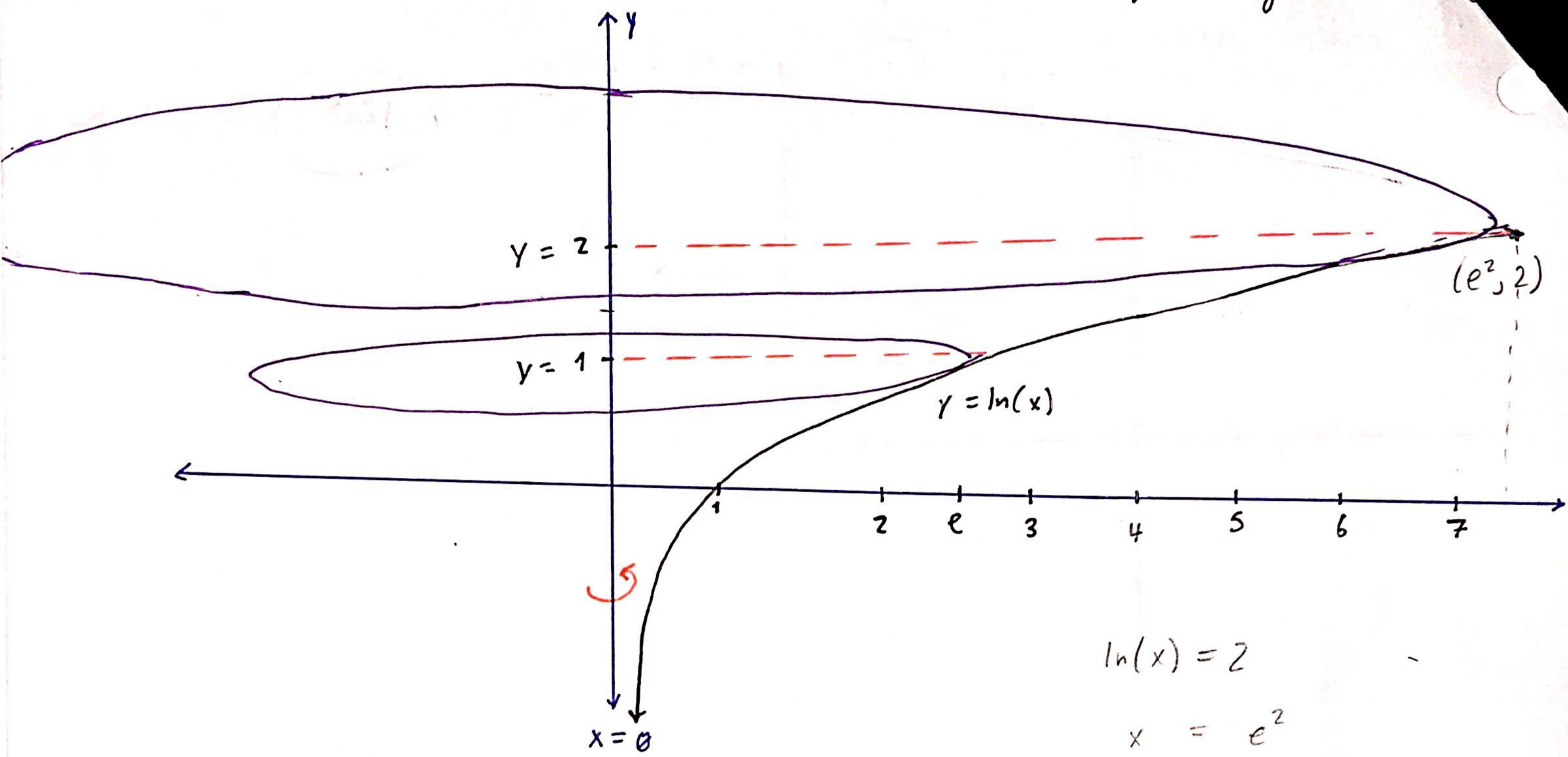
$$\boxed{A = \pi y^2}$$

$$V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x \, dx = \pi \frac{x^2}{2}\Big]_0^4 =$$

$$= \pi \left[\left(\frac{4^2}{2}\right) - \left(\frac{0^2}{2}\right) \right] = \pi \left[\left(\frac{16}{2}\right) - (0) \right] \cancel{= \pi 8}$$

u^3+5

② eje - y es el eje de rotación $y = \ln(x)$; $y = 1$; $y = 2$; $x =$



$\ln(x) = 2$

$x = e^2$

el radio siempre será $\ln(x)$

$A = \pi r^2$

$\therefore A = \pi(\ln^2(x))$

hacerlo respecto a y:

$y = \ln(x)$

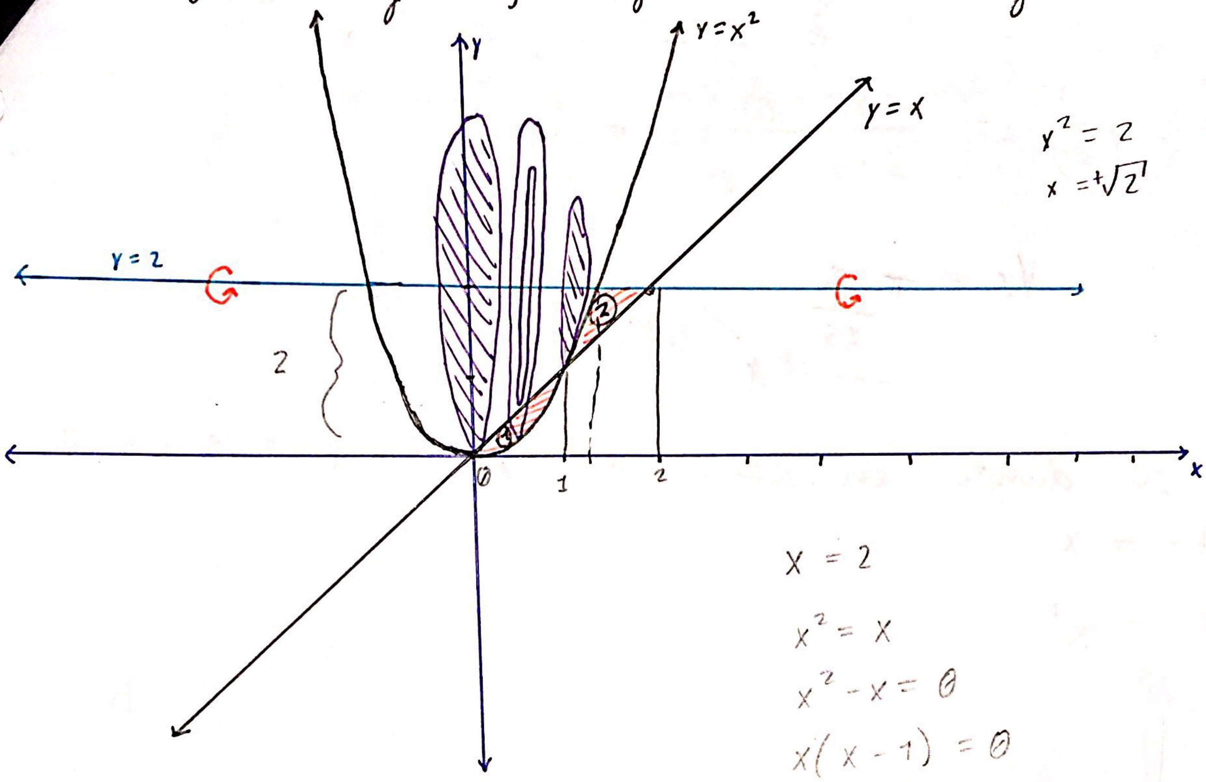
$e^y = x$

$$V = \pi \int_1^2 e^{2y}\, dy = \pi\left(\frac{1}{2} e^{2y}\right)\Big]_1^2 =$$

$$= \frac{\pi}{2}\left[\left(e^{2\cdot 2}\right) - \left(e^{2\cdot 1}\right)\right] = \frac{\pi}{2}\left(e^4 - e^2\right)$$

✓ 20

$y = x$; $y = x^2$; eje de rotación: $y = 2$



$x^2 = 2$

$x = \pm\sqrt{2}$

$x = 2$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad x = 1$

Región 1:

Radio Externo = x^2

Radio Interno = x

haremos anillos de $x = 0$ a $x = 1$ y posteriormente de $x = 1$ a $x = 2$

$$V_1 = \pi \int_0^1 \left[(2 - x^2)^2 - (2 - x)^2 \right] dx$$

$$V_1 = \pi \int_0^1 \left[(2)^2 - 2(2)(x^2) + (x^2)^2 \right] - \left[(2)^2 - 2(2)(x) + (x)^2 \right] dx$$

$$= \pi \int_0^1 4 - 4x^2 + x^4 - \left[4 - 4x + x^2 \right] dx$$

$$= \pi \int_0^1 (\cancel{4} - 4x^2 + x^4 \cancel{- 4} + 4x - x^2) dx$$

$$= \pi \int_0^1 (x^4 - 5x^2 + 4x) dx = \pi \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2 \right) \Bigg]_0^1$$

$$= \pi \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2 \right)\Big]_0^1 = \pi \left[\left(\frac{1}{5} - \frac{5}{3} + 2 \right) - (0) \right]$$

$$= \pi \left(\frac{1}{5} - \frac{5}{3} + \frac{2}{1} \right) = \pi \left(\frac{3 - 25 + 30}{5 \cdot 3} \right) = \pi \frac{8}{15}$$

$$V_1 = \pi \frac{8}{15}$$

2.0/4

Región 2: se divide en dos. uno para $x = 1$ & $x = \sqrt{2}$

rad. ext. = x

rad. int = x^2

$$V_{2.1} = \pi \int_1^{\sqrt{2}} \left[(2-x)^2 - (2-x^2)^2 \right] dx \qquad V_{2.2} = \pi \int_{\sqrt{2}}^{2} \left[(2-x)^2 \right] dx$$

$$V_{2.1} = \pi \int_1^{\sqrt{2}} \underbrace{\left[(2)^2 - 2(2)(x) + (x)^2 \right] - \left[(2)^2 - 2(2)(x^2) + (x^2)^2 \right]}_{\underbrace{4 - 4x + x^2 - [4 - 4x^2 + x^4]}_{(\cancel{4} - 4x + x^2 - \cancel{4} + 4x^2 - x^4)}} dx$$

$$V_{2.1} = \pi \int_1^{\sqrt{2}} -x^4 + 5x^2 - 4x \; dx$$

$$= \pi \left(-\frac{1}{5}x^5 + \frac{5}{3}x^3 - 2x \right)\Big]_1^{\sqrt{2}} =$$

$$= \pi \left[\left(-\frac{1}{5}(2)^{5/2} + \frac{5}{3}(2)^{3/2} - 2(\sqrt{2}) \right) - \left(-\frac{1}{5} + \frac{5}{3} - 2 \right) \right]$$

$$= \pi \left(-\frac{\sqrt{32}}{5} + \frac{5\sqrt{8}}{3} - 2\sqrt{2} + \frac{1}{5} - \frac{5}{3} + 2 \right) = \frac{\pi(38\sqrt{2} - 52)}{15}$$

$$= \pi \int_{\sqrt{2}}^{2} \left((2)^2 - 2(2)(x) + (x)^2 \right) dx =$$

$$= \pi \int_{\sqrt{2}}^{2} (4 - 4x + x^2)\, dx = \pi \int_{\sqrt{2}}^{2} (x^2 - 4x + 4)\, dx$$

$$= \pi \left(\frac{1}{3}x^3 - 2x^2 + 4x \right) \Big]_{\sqrt{2}}^{2} =$$

$$= \pi \left[\left(\frac{1}{3}(2)^3 - 2(2)^2 + 4(2) \right) - \left(\frac{1}{3}(2)^{3/2} - 2(2)^{2/2} + 4(2)^{1/2} \right) \right]$$

$$= \pi \left[\left(\frac{8}{3} - \cancel{8} + \cancel{8} \right) - \left(\frac{\sqrt{8}}{3} - 4 + 4\sqrt{2} \right) \right]$$

$$= \pi \left[\frac{8}{3} - \frac{\sqrt{8}}{3} + 4 - 4\sqrt{2} \right] = \pi \left(\frac{20 - 14\sqrt{2}}{3} \right)$$

Para sacar el volúmen neto se suman $V_1 + V_{2.1} + V_{2.2} = V_m$

$$V_m = \left[\left(\pi \frac{8}{15} \right) + \left(\frac{\pi (38\sqrt{2} - 52)}{15} \right) + \left(\frac{\pi (20 - 14\sqrt{2})}{3} \right) \right]$$

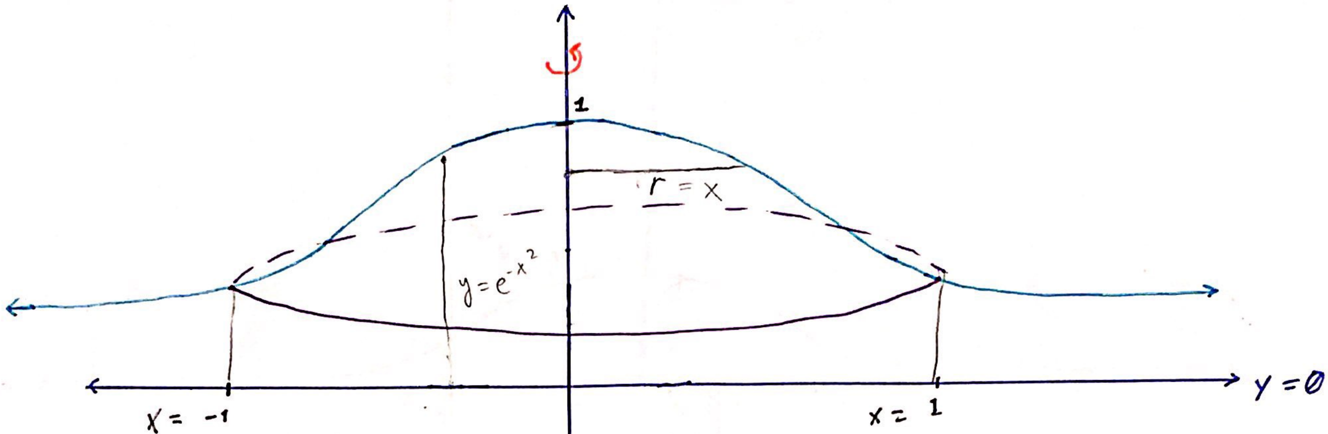
$$= \frac{8}{15}\pi + \frac{38\sqrt{2} - 52}{15}\pi + \frac{20 - 14\sqrt{2}}{3}\pi$$

$$= \pi \left(\frac{8}{15} + \frac{38\sqrt{2} - 52}{15} + \frac{20 - 14\sqrt{2}}{3} \right)$$

$$= \pi \left(\frac{56 - 32\sqrt{2}}{15} \right)$$

$y = e^{-x^2}$; $y = 0$; $x = -1$; $x = 1$

a) eje de rotación eje-y:



cilíndros:

$A = \pi r h$

$V = \pi \int_{-1}^{1}$

$r = x$

$h = y = e^{-x^2}$

Poner todo en términos de y así el radio es x.

hay un cilíndro en $x = -1$; $x = 1$

$(1, e^{-1})$

$$V = 2\pi \int_0^1 x \, e^{-x^2} \, dx$$

$u = x^2$

$du = 2x dx$

$\frac{du}{2} = x dx$

$$= 2\pi \int_0^1 e^{-u} \cdot \frac{du}{2} = \frac{\cancel{2}\pi}{\cancel{2}} \int_0^1 e^{-u} \, du$$

$$= \pi \int_0^1 e^{-u} = \pi - e^{-u} \Big]_0^1$$

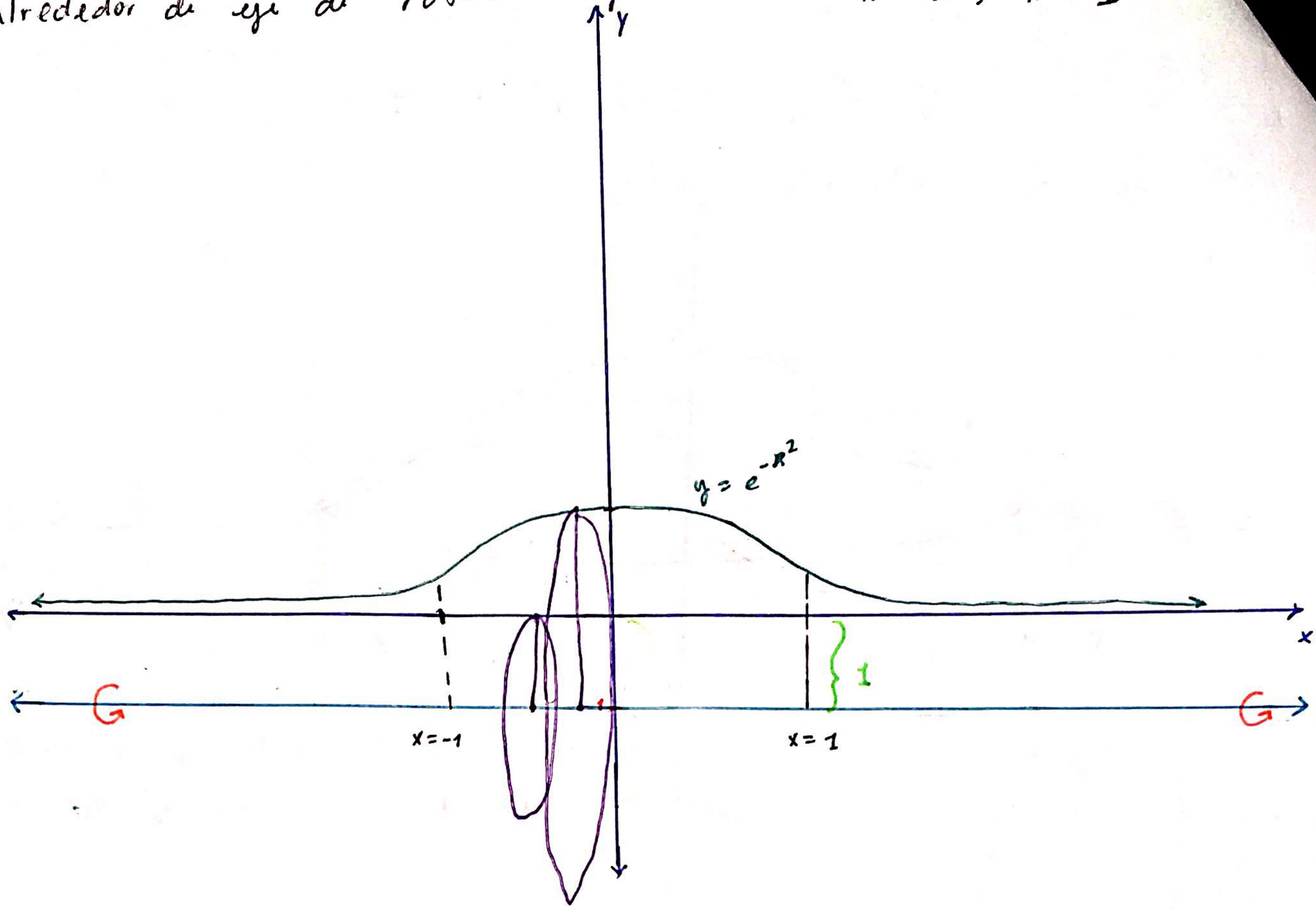
$$= -\pi e^{-x^2} \Big]_0^1 =$$

$$= -\pi \left(\left[e^{-(1)^2} \right] - \left[e^{-(0)^2} \right] \right) = -\pi \left(e^{-1} - e^{0} \right)$$

$$= -\pi \left(e^{-1} - 1 \right) \text{, 20 rs}$$

$$= \pi \underline{\left(1 - e^{-1} \right)}$$

(7) alrededor de eje de rotación $y = -1$ $\qquad x = -1\ ;\ x = 1$



radio interior: $\qquad$ radio exterior:

$r_{int} = 1$ $\qquad r_{ext} = 1 + e^{-x^2}$ $\qquad A = \pi r^2$

$$V = \pi \int_{-1}^{1} (1 + e^{-x^2})^2 - (1)^2 \, dx =$$

$$= 2\pi \int_0^1 \cancel{1^2} + 2e^{-x^2} + (e^{-x^2})^2 - \cancel{1^2}$$

$$= 2\pi \int_0^1 2e^{-x^2} + (e^{-x^2})^2 \, dx$$

$$= 2\pi \int_0^1 e^{-x^2}(2 + e^{-x^2}) \, dx$$

20