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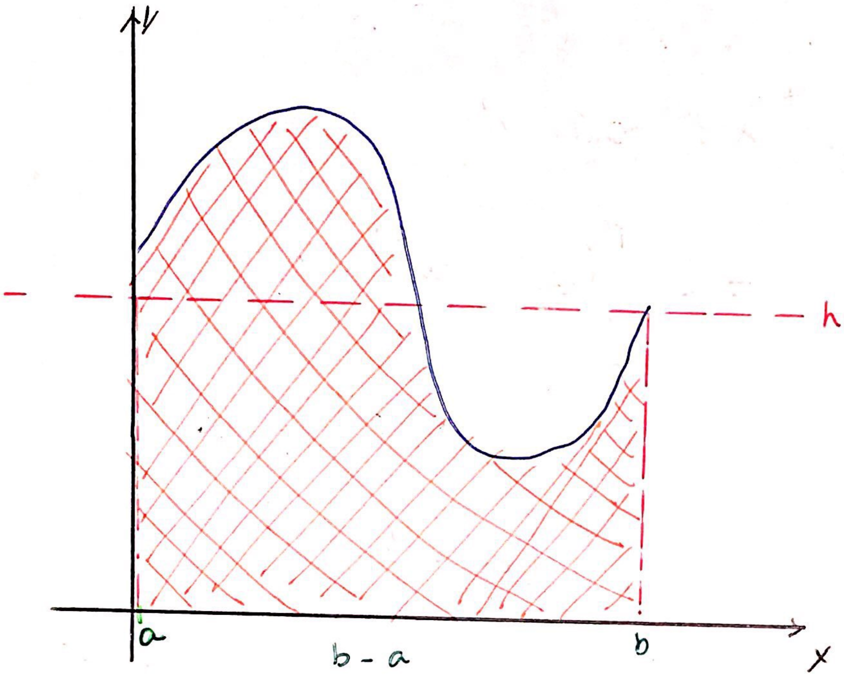
6.5 Valor promedio de una función

- al inicio revolución de corte

- Promedio altura y Iguale el área del rectángulo con el área de la región



$h = f_{prom}$

$f(x)$ sea continua

ancho $b-a$

altura f promedio

Área $f_{promedio}(b-a) = \int_a^b f(x)\,dx$

$$\therefore \boxed{f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Ejemplo: Encuentra el valor promedio de $f(x) = \csc^4(x)$ en $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$$b - a = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \qquad \frac{4}{\pi} = \frac{1}{b-a}$$

$$f_{prom} = \frac{4}{\pi}\int_{\pi/4}^{\pi/2} \csc^2(x)\,dx = -\frac{4}{\pi}\cot(x)\Big]_{\pi/4}^{\pi/2} = \frac{4}{\pi}$$

Ej:

a) $f(t) = \cos^4(t)\sin(t)\,dt$ en $[0,\pi]$

$$f_{prom} = \int_0^{\pi} \cos^4 t \sin t = \int_0^{\pi} u^4\,du = \left.\frac{u^5}{5}\right]_0^{\pi} =$$

$$= \left.\frac{-\cos^5(t)}{5}\right]_0^{\pi} = \left\{\frac{1}{5}\right\} - \left\{-\frac{1}{5}\right\} = \frac{1}{\pi}\cdot\frac{2}{5}$$

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b) $g(x) = \frac{1}{x}$ en $[e^4, e^{10}]$

$e^{10} - e^4 = \text{ancho}$

$$P = \frac{1}{e^{10}-e^4}\int_{e^4}^{e^{10}} \frac{1}{x}\,dx$$

$$P = \frac{1}{e^{10}-e^4} \left.\ln|x|\right]_{e^4}^{e^{10}}$$

$$P = \left\{\frac{10}{e^{10}-e^4}\right\} - \left\{\frac{4}{e^{10}-e^4}\right\}$$

$$P = \frac{6}{e^{10}-e^4}$$

observación:

Valor promedio de F en $0 \leq x \leq 1$

$$\left.\ln x\right]_0^1 = -\lim_{x\to 0^+} \ln(x) =$$

$= +\infty$

en este caso no hay valor promedio

c) $h(x) = \frac{3}{(4+x)^{1/2}}$

en $[-4, 5]$

$$h_{prom} = \frac{1}{5-(-4)}\int_{-4}^{5} 3(4+x)^{-1/2}\,dx$$

$$= \left.\frac{2\cdot 3(4+x)^{1/2}}{5+4}\right]_{-4}^{5} = \left\{\frac{2}{3}(4+5)^{1/2}\right\} - \{0\}$$

$$= \frac{2}{3}(4+5)^{1/2} = 2$$

Ej: 2: Densidad lineal $p = 12(x+1)^{-1/2}$ la varilla tiene 8 m de longitud.

a) encuentre la densidad promedio de la varilla.

$$p_{prom} = \frac{1}{8-0}\int_0^8 12(x+1)^{-1/2}\,dx = \frac{12}{8}\int_0^8 (x+1)^{-1/2}\,dx =$$

$$= \frac{12}{8}(x+1)^{1/2}\Big]_0^8 = 3\left(9^{1/2} - 1^{1/2}\right) = 3(3-1) = 6 \text{ Kg/m}$$