7.2 Integrales Trigonométricas.

$$\sin^2 x + \cos^2 x = 1$$

 $\tan^2 x + 1 = \sec^2 x \div \cos^2 x$.
 $1 + \cot^2 x = \csc^2 x \div \sin^2 x$.

I. Integrales de la forma J'sin'x cos" x dx

$$\frac{\partial}{\partial x} (\sin x) = \cos x$$
 $\frac{\partial}{\partial x} (\cos x) = -\sin x$

$$\frac{\partial}{\partial x} (\cos x) = -\sin x$$

 $u = \sin x$, $du = \cos x dx$ $u = \cos x$, $du = -\sin x dx$

Reesciba 654 x = (cos2x)2 = (1-sin2x)2

$$\cos^2 x = |-\sin^2 x|$$

$$\int \cos^5 x \, dx = \int \left(1 - \sin 2x\right)^2 \left(\cos x \, dx\right)$$

$$u = \sin x$$
 $\partial u = \cos x \, \partial x$

$$= \int (1-4^2)^2 d4$$

$$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C.$$

sin X + Sin X3 (sinx)3

$$= \frac{3 \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^3 x + C}{3}$$

Aparte algun término sinx à cosx n. Potencias impares de sero o coseno.

Ejercicio 1: Evalue.

 $u. \int cos^3 x sinb x dx = \int cos^2 x sinb x (cos x dx)$

J cos2x sin 6 x cosxdx o Scos3x sin 5 x sin xdx 1-(052 X X 1-Sin2x

L052X= 1-5in2X

= \((1-sin^2x)sin^6x (cosxdx)

= J (1 - 42) 46 du u=sinx

= \((ub - u8) du Ju = c05 x 0x

 $=\frac{1}{7}u^{7}-\frac{1}{9}u^{9}+C.$

= 1 sin3x - 1 sin9x + C.

(a+b+c) d ad+6d+cd.

b. Scossxsin3x dx = Scossx(sin2x) sinxdx

Jeosyxsin3x cosxdx of Jeos5x sin2x sin xJx

(1-5in2x) $= \int \cos^5 x \left(1 - \cos^2 x \right) \sin x \, dx$

4=65X

 $= - \int u^{5} (1 - u^{2}) du = - \int (-u^{5} + u^{7}) du$

= - 1 46 + 1 48 + C.

= - 1 cosh x + 1 cosh x + C.

dn=-sinxdx

b) Potencias pares de seno y cuseno

$$\int \cos^2 x \, dx - \int \frac{1}{2} + \frac{1}{2} \cos 2x \, dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C.$$

= cos2 x + sin2 x. (1)

$$\cos(X+X) = \cos^2 X - \sin^2 X \quad (Z)$$

 $1 + \cos(2x) = 2\cos^2 x = 1 + 1\cos 2x$ (1) + (2) $= -\sin^2 x = 1/1 \cdot (--2)$ $-i\eta^2 x = \frac{1}{2} \left(1 - \cos 2x \right)$

Ejercicio 2: Evalue.

$$a \int_{-\pi}^{\pi} \sin^2 x \, dx = 2 \int_{0}^{\pi} \sin^2 x \, dx = \frac{2}{2} \int_{0}^{\pi} (1 - \cos 2x) \, dx$$

Ejercicio 2. Evalue.
a
$$\int_{-\pi}^{\pi} \sin^2 x \, dx = 2 \int_{0}^{\pi} \sin^2 x \, dx = \frac{2}{2} \int_{0}^{\pi} (1 - \cos 2x) \, dx$$

 $\int_{-\pi}^{\pi} \sin x \, dx = 0 \quad \text{im par.}$

$$\int_{0}^{\pi} (1 - \cos 2x) \, dx = x - \frac{1}{2} \sin 2x \int_{0}^{\pi} = \pi - \frac{1}{2} \sin^2 \pi - 0 + \frac{\sin 0}{2}$$

$$u = 2x \quad \text{in } = 2dx$$

 $\int_{-\pi}^{\pi} \sin^2 x \, dx = \pi.$ $\cos^2 D = \frac{1}{2} (1 - \cos 2D)$ b. J Sin2x CO52x dx = [1/211 - CO52x) 1/2 [1+ CO52x] dx $sin^{L}X = \frac{1}{2}(1-cos2x)$ $cos2x = \frac{1}{2}(1+cos2x)$

$$\frac{1}{2}(1-\cos 2x)$$

$$\frac{1}{2}(1-\cos 2x)$$

$$\cos^{2}(2x) = \frac{1}{2}(1+\cos 4x) = \frac{1}{4}\int(1-\frac{1}{2}+\frac{1}{2}\cos 4x)dx$$

$$= \frac{1}{4} \int_{\frac{1}{2}}^{\frac{1}{2}} + \frac{1}{2} \cos 4x \right) dx$$

$$= \int_{\frac{1}{8}}^{\frac{1}{4}} + \frac{1}{8} \cos 4x dx.$$

$$= \int_{\frac{1}{8}}^{\frac{1}{4}} + \frac{1}{8} \cos 4x dx.$$

$$= \frac{x}{8} + \frac{1}{8 \cdot 4} \sin 4x + C.$$

Il Forma Stan^m x secⁿ x
$$\partial x$$
 $(tan x)^{2} = sec^{2} x$
 $u = tan x$
 $u = tan^{2} x + 1$
 $tan^{2} x = sec^{2} x - 1$

Ejercicio 3: Evalde Pág. 48,

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y 10ego
Sharin: Resumen Identidades
          oustitución.
b. Alejandro J tans x secs x dx
 Stany x secy x (secx tanx dx)
 I tans x sec3 x (sec2x dx) X
                                     tan^2 X = \mathcal{L}L^2 X - 1
S(tan2x) sec4x (secx tanx dx)
[ (sec2 x - 1) sec4 x (secx tanxdx)
 u = \sec x  \int u = \sec x \tan x \, dx.
 \int (u^2 - 1)^2 u^4 du = \int (u^4 - 2u^2 + 1)u^4 du.
 J[u8-2u6+44)du= - 4u9- = 47+ + 4545+ C.
                   = = 3ec4x - 2 sec7x + = sec5x + c.
 L. J'tany x sec4 x dx. = Stany x sec2x sec2xdx
                                   solo ton's
                         = Jtan 4x (tan 2x+1) (sec2x 0x)
Sec2x = tan2x + 1
u=tanx du=seczxdx
                         = Juy(u2+1) du.
                        = Ju6 + 44 du = = = 4 47 + = 4 4 + C
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= = tan x + = tan sx + c.

Casus especiales Stunxox Secoxdx $\int tanx dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{dn}{n} = -\ln |u| + C.$ $u = \cos x \qquad dn = -\sin x dx = -\ln |\cos x| + C.$ JSELX dx = [Selx (Selx + tanx)* dx) Brillante. tanx + Secx * = Sec2x + secxtanx dx tanx + secx $du = (Sec^2 x + Sec x tan x) dx$ n = tan X + 52LX= \(\frac{\rm u}{4} - \ln \lu \l + C = \ln \lan x + secx \l + C. JCSCX OX = - In [CSCX + cotx] + C. SCLLX OX = Tanx + C. tanix=secix-1 $\int \tan^2 x \, dx = \int (\sec^2 x - 1) dx = \tan x - x + C.$ 1 tan3 x dx = Stan2x tanx dx $u = tq \eta x$ = $\int [sec^2x - 1] \tan x dx$ Un= Sec2 Xdx = Slsec Lx tanx - tanx) dx = Jtan x seczxdx - Stan x dx = = tan2 X + In/COSX/+C.

Mis Dificil SEC3X dx. SSECX SECZX dX - SELX tanx - tan2x SELX dx) IPP u=secx $dv = 5CC^2 \times dX$ dn = secx tanxdx V = tanx I tan 2 x secx dx = [(sec2 x-1) secx dx = 1(sec3x - secx) dx I secixox = secxtanx - Secixdx + Secxdx 2 | sec3xdx = secxtanx + In|secx+tanx | + C. Sec3 x0x = 1 derivada(sec) + 1 integral (sec) 2 secxtanx + 1 Inlsecx +tanx 1+C.