Problemas Variados de Integración

Problema 3: Encuentre las siguientes integrales.

a.) $$\int \sqrt{64x} - \frac{1}{\sqrt{64x}}\,dx$$

b.) $$\int \left(\frac{7x^2}{7x^3+8} - \frac{x^3}{(x^4+8)^5}\right)dx$$

c.) $$\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\,dx$$

d.) $$\int 5\frac{(x^{1/3}+2)^4}{x^{2/3}}\,dx$$

e.) $$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$$

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a) $\int \sqrt{64x} - \frac{1}{\sqrt{64x}}\, dx$

$\int \sqrt{64x}\, dx - \int \frac{1}{\sqrt{64x}}\, dx$

$u = 64x$ | $u = 64x$

$du = 64\, dx$ | $du = 64$

$\frac{du}{64} = dx$ | $\frac{du}{64} = dx$

$\int \sqrt{64x}\, dx = \int \sqrt{u}\, \frac{du}{64} = 64 \int \frac{(u)^{1/2+1}}{1/2+1}\, du$

$64 \cdot \frac{u^{3/2}}{3/2}$

$\left[\frac{\frac{64u^{3/2}}{1}}{\frac{3}{2}}\right] = \frac{64 \cdot 2u^{3/2}}{3}$

$\int \frac{1}{\sqrt{64x}}\, dx = \frac{1}{64} \int \frac{1}{\sqrt{u}}\, du$

$u = 64x$

$du = 64dx$

$\frac{du}{64} = dx$

$\frac{1}{64}\sqrt{u} - \frac{64 \cdot 2u^{3/2}}{3}$

$\frac{\sqrt{64x}}{64} - \frac{64 \cdot 2(64x)^{3/2}}{3} + C$

b) $\int \left(\frac{7x^2}{7x^3+8} - \frac{x^3}{(x^4+8)^5}\right) dx$

$u = 7x^3 + 8$ | $u = x^4 + 5$

$du = 7x^2$ | $du = x^3$

$\int \frac{du}{u} - \int \frac{du}{u^5}$

$(\ln(u)) - \left(\frac{u^{-4}}{-4}\right) = \ln(u) + \frac{1}{4u^4} + C$

$= \ln(7x^3+8) + \frac{1}{4(x^4+8)^4} + C$

c. $\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx = \sqrt[3]{x}\, e^{2(x)^{4/3}}\, dx$

$u = 2(x)^{4/3}$

$du = 2x^{1/3}\, dx$

$\frac{du}{2} = x^{1/3}\, dx$

$= \frac{1}{2}\int e^{u}\, du$

$= \frac{1}{2} e^{u} + c$

$= \frac{1}{2} e^{2\sqrt[3]{x^4}} + C$ **

d. $\int 5 \frac{(x^{1/3}+2)^4}{x^{2/3}}\, dx = 5\int (x^{1/3}+2)^4\, x^{-2/3}\, dx$

$u = x^{1/3} + 2$

$du = x^{-2/3}\, dx$

$= 5\int u^4 \cdot du$

$= \frac{\cancel{5}\, u^5}{\cancel{5}} = (x^{1/3}+2)^5 + C$ **

e) $\int \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\, dx = \int \frac{1}{u}\, du = \ln(u)$

$= \ln(e^{x} + e^{-x}) + C$ **

$u = e^{x} + e^{-x}$

$du = e^{x} - e^{-x}\, dx$