Simulacro Parcial 1.

1. Evalúe

$$\text{a } \int x \tan^{-1} x^2 \, dx = \frac{x^2}{2}\tan^{-1}(x^2) - \int \frac{x^2}{2}\frac{2x}{1+x^4}\,dx \qquad uv - \int v\,du. \qquad \text{IPP.}$$

$u = \tan^{-1}(x^2) \qquad dv = x\,dx$

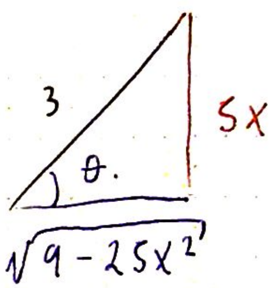
$du = \dfrac{2x}{1+x^4} \qquad V = \dfrac{x^2}{2}.$

$$\frac{x^2}{2}\tan^{-1}(x^2) - \int \frac{x^3}{1+x^4}\,dx \qquad w = 1+x^4, \quad \frac{dw}{4} = x^3\,dx$$

$$\frac{1}{2}x^2\tan^{-1}(x^2) - \frac{1}{4}\int \frac{dw}{w} = \frac{1}{2}x^2\tan^{-1}(x^2) - \frac{1}{4}\ln|w| + C.$$

$$= \frac{1}{2}x^2\tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4| + C.$$

$$\text{b. } \int \frac{x^2}{\sqrt{9-25x^2}}\,dx = \int \frac{(9/25)\sin^2\theta}{\cancel{3}\cos\theta}\,\frac{\cancel{3}}{5}\cos\theta\,d\theta.$$



$\sin\theta = \dfrac{5x}{3} \qquad x = \dfrac{3}{5}\sin\theta., \qquad dx = \dfrac{3}{5}\cos\theta\,d\theta.$

$\cos\theta = \dfrac{\sqrt{9-25x^2}}{3} \qquad \sqrt{9-25x^2} = 3\cdot\cos\theta.$

$$\frac{9}{125}\int \sin^2\theta\,d\theta. = \frac{9}{250}\int (1-\cos 2\theta)\,d\theta. = \frac{9}{250}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

I Doble Ángulo.

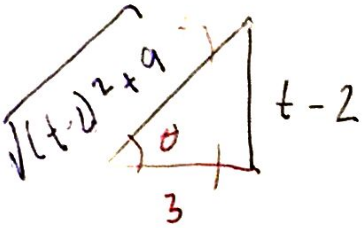
$$= \frac{9}{250}(\theta - \sin\theta\cos\theta) + C.$$

$\theta = \sin^{-1}\left(\dfrac{5x}{3}\right)$

$$\boxed{= \frac{9}{250}\left(\sin^{-1}\left(\frac{5x}{3}\right) - \frac{5x\sqrt{9-25x^2}}{9}\right) + C.}$$

i. $\int \frac{1}{\sqrt{(t-2)^2+9}}\,dt.$

j. $\int \frac{\sqrt{(t-2)^2+9}}{t^3}\,dt.$



$\tan\theta = \frac{t-2}{3}$ $\quad (t-2) = 3\tan\theta.$

$dt = 3\sec^2\theta\, d\theta.$

$\sec\theta = \frac{\sqrt{(t-2)^2+9}}{3}.$

$$\int \frac{1}{\sqrt{(t-2)^2+9}}\,dt = \int \frac{3\sec^2\theta\, d\theta}{3\sec\theta.} = \int \sec\theta\, d\theta.$$

$$= \ln|\sec\theta + \tan\theta| + C.$$

Regrese a la variable t.

$$\boxed{= \ln\left|\frac{\sqrt{(t-2)^2+9}}{3} + \frac{t-2}{3}\right| + C.}$$

J. $\int \frac{x e^x}{(x+1)^2}\,dx$

3 funciones. $\int \frac{x}{(x+1)^2}\,dx.$ Jerivada $\frac{x}{(x+1)^2}$ es largu

sigue siendo dificil

Derivar xe^x $\quad$ Integre $(x+1)^{-2}. \rightarrow -(x+1)^{-1}$

$$\int \frac{x e^x\, dx}{(x+1)^2} = -\frac{x}{x+1}e^x + \int \frac{e^x + xe^x}{x+1}\,dx$$

$u = xe^x$ $\quad dv = (x+1)^{-2}dx$

$du = (e^x + xe^x)dx$ $\quad v = \frac{-1}{(x+1)}$

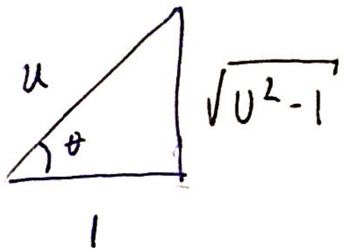
$$= -\frac{x}{x+1}e^x + \int \frac{e^x\cancel{(1+x)}}{\cancel{(x+1)}}\,dx$$

$$\boxed{= -\frac{x}{x+1}e^x + e^x + C.}$$

Miranda $\displaystyle\int_0^{\ln 2} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx = \int_1^2 \frac{u^3}{\sqrt{u^2-1}}\,du.$

$u = e^x \qquad u(\ln 2) = e^{\ln 2} = 2. \qquad u^3 = e^{3x}$

$du = e^x\,dx \qquad u(0) = e^0 = 1$



$u = \sec\theta.$

$du = \sec\theta\tan\theta\,d\theta.$

$\sqrt{u^2-1} = \tan\theta.$

$$\int \frac{u^3}{\sqrt{u^2-1}}\,du = \int \frac{\sec^3\theta}{\tan\theta}\cdot\sec\theta\tan\theta\,d\theta. = \int \sec^4\theta\,d\theta.$$

$\int \tan^m x \sec^n x\,dx$ aparte $\sec^2 x$ o $\sec x\tan x$.

$u = \tan\theta \qquad du = \sec^2\theta\,d\theta.$

$$\int \sec^2\theta\,\sec^2\theta\,d\theta. = \int (\tan^2\theta + 1)\sec^2\theta\,d\theta.$$

$\sec^2\theta = \tan^2\theta + 1$

$$= \frac{1}{3}\tan^3\theta. + \theta + C.$$

$$= \frac{1}{3}(u^2-1)^{3/2} + \sec^{-1}(u). + C.$$

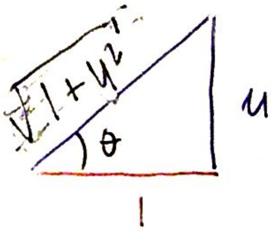
$$\int_1^2 \frac{u^3}{\sqrt{u^2-1}}\,du = \frac{1}{3}\left(3^{3/2} + \sec^{-1}(2) - \cancel{\frac{1}{3}}\overset{0}{\cancel{0}^{3/2}} + \overset{0}{\sec^{-1}(1)}\right)$$

$$= 3^{1/2} + \sec^{-1}(2),$$

2 b) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt = \int_0^{1} \frac{du}{\sqrt{1+u^2}} = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec\theta}.$

$u = \sin t \qquad u(\pi/2) = \sin(\pi/2) = 1$

$du = \cos t\,dt \qquad u(0) = \sin(0) = 0$



$u = \tan\theta \qquad du = \sec^2\theta\,d\theta.$

$\tan\theta = 1 \rightarrow \theta = \pi/4. \qquad \sqrt{1+u^2} = \sec\theta.$

$\tan\theta = 0 \rightarrow \theta = 0$

$$\int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}$$

$$= \ln|\sec\pi/4 + \tan\pi/4| - \ln|\cancelto{1}{\sec 0} + \cancelto{0}{\tan 0}|$$

$$= \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \ln(1) = \ln(\sqrt{2}+1).$$