7.1 Integración por partes

$\int \ln x \, dx \qquad \int \tan^{-1} x \, dx \qquad \int x^3 e^x \, dx \qquad \int e^x \cos x \, dx$

IPP: Integre productos de funciones "disimilares".

$$\int \underbrace{f(x)}\, \underbrace{g(x)\, dx} = ?$$

Regla del Producto para Derivadas.

$$(fg)' = f'g + fg'$$

$(fg)' - f'g = fg'$ Integre esta expresión.

IPP $$\boxed{\int fg' = fg - \int f'g}$$

f deriva.
g' integra.

$\int fg'$

$\int f'g$ más simple que la integral original.

$$\int \underbrace{f(x)}_{u} \underbrace{g(x)\, dx}_{dv} = uv - \int v\, du$$

$u = f(x) \qquad dv = g(x)\, dx$

$du = f'(x)\, dx \qquad v = G(x)$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ejercicio 1: Pág 39 Integre $\int x e^x \, dx$

Opción 1: $u = x \qquad dv = e^x \, dx$

$du = 1 \cdot dx \qquad v = e^x$

Opción 2: $\cancel{u = e^x \qquad dv = x\, dx}$

$\cancel{du = e^x \, dx \qquad v = \frac{1}{2} x^2}$

$$\int x e^x \, dx = \underbrace{x e^x}_{u \cdot v} - \underbrace{\int e^x \, dx}_{v\, du} = x e^x - e^x + C.$$

Derive y compruebe su respuesta.

Ejercicio 2: Integre $\int f g' = fg - \int f' g.$

ó $\int u\,dv = uv - \int v\,du$

a. $\int 6x^2 \overbrace{\ln x}\, dx = (\ln x)\, 2x^3 - \int \frac{1}{x} 2x^3\, dx$ $\qquad \frac{x^3}{x} = x^2.$

$u = \ln x \qquad dv = 6x^2 \qquad 2x^3 \ln x - \int 2x^2\, dx$

$du = \frac{1}{x} \qquad v = 2x^3 \qquad \boxed{2x^3 \ln x - \frac{2}{3} x^3 + C.}$

b. $\int \ln x \, dx = \underset{v\ u}{x \ln x} - \int x \frac{dx}{x} = x\ln x - \int dx$

$\checkmark\, du \qquad = \boxed{x \ln x - x + C.}$

$u = \ln x \qquad dv = 1 \cdot dx$

$du = \frac{dx}{x} \qquad v = x$

c. $\int \tan^{-1} x\, dx = \overset{v\ \ u}{x \tan^{-1} x} - \int \overset{\checkmark\, du}{\frac{x}{1+x^2}}\, dx$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$\int \frac{1}{1+x^2} \overbrace{x\,dx}^{du/2} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+x^2| + C.$

Sustitución. $u = 1 + x^2 \quad du = 2x\, dx$

$\int \tan^{-1} x\, dx = \boxed{x \tan^{-1} x - \frac{1}{2} \ln|1+x^2| - C}$

J. $\int x^2\cos x\,dx = \underbrace{x^2\sin x}_{uv} - \int \underbrace{2x\sin x\,dx}_{v\,du.}$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

Otra vez IPP.

$\int 2x\sin x\,dx = -2x\cos x + \int 2\cos x\,dx = -2x\cos x + 2\sin x + C$

$u = 2x \qquad dv = \sin x\,dx$

$du = 2\,dx \qquad v = -\cos x$

$\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x + C.$

ILATE. Mnemotecnia.

Inversas trigonométricas $\sin^{-1}x$, $\tan^{-1}x$

Logaritmicas $\ln x$, $\log_a x$

Algebraicas x^n, $\sqrt{x}$, $1/x^r$

Trigonométricas $\sin x$, $\cos x$, $\tan x$

Exponenciales. e^x, a^x

IPP:

Integrales Definidas:

no cambian los límites

$$\boxed{\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du}$$

Ejercicio 3: Evalúe. u) Interesante.

b. $72\int_1^2 \frac{\ln x}{x^4}\,dx. = 72\ \ln x\left(\frac{-1}{3x^3}\right)\Big]_1^2 - \int_1^2 \frac{x^{-3}}{-3}\,x^{-1}\,dx$

$u = \ln x \qquad dv = x^{-4}\,dx$

$du = x^{-1}\,dx \qquad v = \frac{x^{-3}}{-3}$

$$-\frac{24}{1 \times 3} \ln x \Big]_1^2 - \int_2^1 \frac{x^{-4}}{3} dx \qquad -x^{-4} \to \frac{-x^{-3}}{-3}$$

$$-\frac{24}{8} \ln 2 + \frac{24}{1} \cancel{\ln 1}^{\,0} + \frac{1}{3x^3} \Big]_2^1$$

$$-3 \ln 2 + \frac{1}{3} - \frac{1}{3 \cdot 8}$$

Resuelto con detalles pág 42.

incógnita

$$\boxed{\int e^x \cos x \, dx} = e^x \cos x + \int e^x \sin x \, dx \quad (1)$$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \boxed{\int e^x \cos x \, dx} \quad (2)$$

incógnita.

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C.$$

$$\int e^x \sin x \, dx = \frac{1}{2} e^x \cos x - \frac{1}{2} e^x \sin x + C.$$