Repaso Simulacro Parcial

2019-08-29
20190432

a) $$\int x \tan^{-1}(x^2)\,dx = \frac{1}{2}\int \tan^{-1}(y)\,dy =$$

$y = x^2$
$dy = 2x\,dx$

IPP: $u = \tan^{-1} y$ $\quad dv = dy$
$du = \dfrac{1}{x^2+1}$ $\quad v = y$

$$= \frac{1}{2}\tan^{-1}(y)\,y - \frac{1}{2}\int y\,\frac{1}{y^2+1}\,dy$$

$w = 1 + y^2$
$dw = 2y\,dy$

$$= \frac{1}{2}\,y\tan^{-1}y - \frac{1}{4}\int \frac{dw}{w}$$

$$= \frac{1}{2}\,y\tan^{-1}y - \frac{1}{4}\ln|w| + C$$

$$= \frac{1}{2}\cdot x^2\cdot\tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4| + C$$

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b) $$\int \frac{xe^x}{(x+1)^2}\,dx =$$

tres funciones

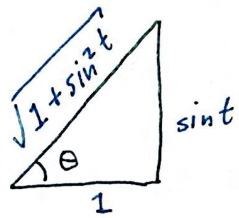
$u = xe^x$ $\quad dv = (x+1)^{-2}$
$du = e^x + xe^x$ $\quad v = \dfrac{(x+1)^{-1}}{-1} = \dfrac{-1}{(x+1)}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = -\frac{xe^x}{(x+1)} + \int \frac{(e^x + xe^x)\,dx}{(x+1)}$$

$$= \frac{-xe^x}{(x+1)} + \int e^x\,dx = \frac{-xe^x}{(x+1)} + e^x + C$$

2b

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{1+\sin^2 t}}\,dt = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}$$



$\sin t = \tan\theta$

$\cos t\,dt = \sec^2\theta\,d\theta$

$\sqrt{1+\sin^2 t} = \sec\theta$

cambie límites de integración

$\tan\theta = \sin \pi/2 = 1 \rightarrow \theta = \frac{\pi}{4}$

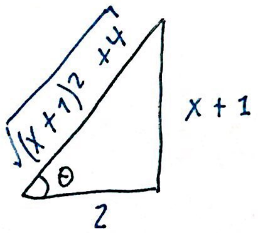
$\tan\theta = \sin 0 = 0 \rightarrow \theta = 0$

$$= \{\ln|\sec\pi/4 + \tan\pi/4|\} - \{\ln|\sec 0 + \tan 0|\}$$

$$= \ln|\sqrt{2} + 1| - \ln|\sqrt{2} + 1|$$

Corto 4a

$$\int \frac{1}{(x^2+2x+5)^2}\,dx = \int \frac{1}{[(x+1)^2+4]^2}\,dx = \int \frac{1 \cdot 2\sec^2\theta\,d\theta}{16\sec^4\theta}$$



$\tan\theta = \frac{x+1}{2}$

$x+1 = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$$(\sec\theta)^2 = \left(\frac{\sqrt{(x+1)^2+4}}{2}\right)^2$$

$$16\sec^2\theta = ((x+1)^2+4)^2$$

$$= \frac{1}{8}\int \cos^2\theta\,d\theta = \frac{1}{8}(\theta + \sin\theta\cos\theta) = \frac{\frac{1}{2}\left(\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{16}\left(\frac{(x+1)\cdot 2}{(\sqrt{x^2+2x+5})^2}\right)\right)}{16}$$

$$= \frac{1}{16}\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{8}\left(\frac{x+1}{\sqrt{x^2+2x+5}\sqrt{x^2+2x+5}}\right) + C$$

IPP:

2 $\displaystyle\int (x-1)\sin \pi x \, dx =$

$u = x - 1 \qquad dv = \sin \pi x \, dx$

$du = dx \qquad v = -\frac{1}{\pi}\cos \pi x$

$= (x-1)\left(-1/\pi \cos \pi x\right) - \displaystyle\int \frac{1}{\pi}\cos \pi x \, dx$

$= -\frac{(x-1)}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + C$

Cíclico:

$\displaystyle\int e^{-\theta}\cos(2\theta)\, d\theta = -\frac{1}{2}e^{-\theta}\sin(2\theta) - \frac{1}{2}\underbrace{\int e^{-\theta}\sin(2\theta)\, d\theta}$

$u = e^{-\theta} \qquad dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta}\, d\theta \qquad v = \frac{\sin 2\theta}{2}$

$= \displaystyle\int e^{-\theta}\sin(2\theta)$

$u = e^{-\theta} \qquad dv = \sin(2\theta)\, d\theta$

$du = -e^{-\theta}\, d\theta$