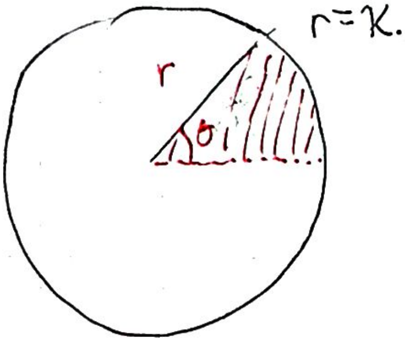
A'reas Regiones Polares.

A'rea de una "Rebanada de Pizza"



$A = \pi r^2.$ Circulo 2π radianes.

Rebanada o sector circular tiene un ángulo central θ.

$$A_{Rebanada} = \pi r^2 \left(\frac{\theta}{2\pi}\right) = \frac{r^2}{2}\theta.$$

Pizza 8 pedazos

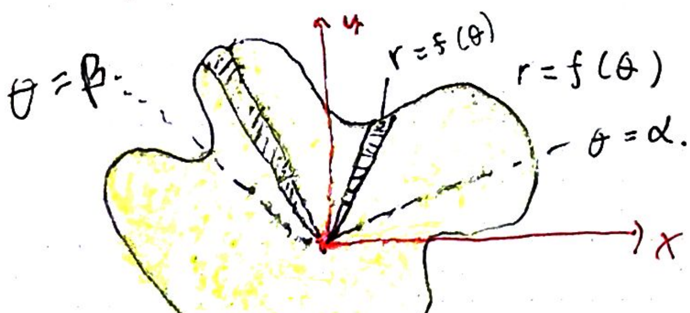
↳ $\frac{2\pi}{8} = \frac{\pi}{4}$ ó $45°$.

$r = 12''$

$$A = \frac{r^2}{2}\frac{\pi}{4} = \frac{\pi r^2}{8} = \pi\frac{144}{8}.$$

A'rea de una Región Polar.

$r = f(\theta)$ $\quad \alpha \leq \theta \leq \beta.$



Considere una rebanada muy delgada "infinitesimal" $r = f(\theta)$ $\quad d\theta.$

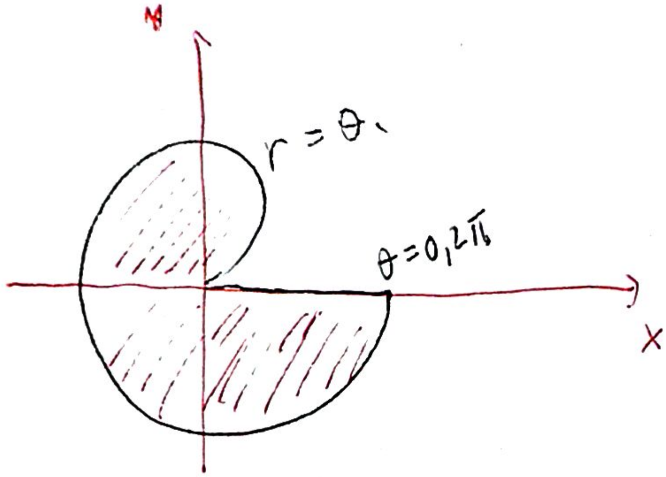
$$\underbrace{dA}_{\text{área infinitesimal}} = \frac{r^2}{2}d\theta. = \frac{f^2(\theta)}{2}d\theta.$$

Integre dA en $\alpha \leq \theta \leq \beta.$

$$\boxed{A = \frac{1}{2}\int_{\alpha}^{\beta} r^2(\theta)\, d\theta.}$$

Ejemplo: Encuentre el área dentro de la espiral $r = \theta$. theta.

en $0 \le \theta \le 2\pi$.



$$A = \frac{1}{2}\int_0^{2\pi} \theta^2 \, d\theta.$$

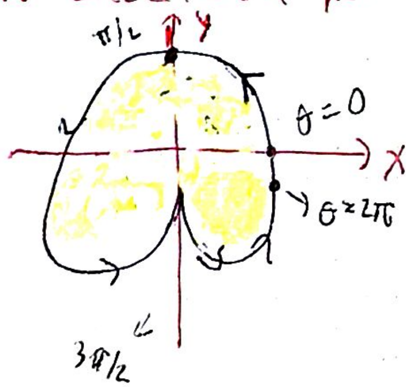
$$A = \frac{1}{6}\, \theta^3 \Big]_0^{2\pi}$$

$$A = \frac{1}{6}\, 8\pi^3 = \frac{4}{3}\pi^3.$$

p. 159 $\quad r = \sqrt{\theta}$

Ejercicio 1: Encuentre el área de las siguientes regiones

a. Encerrada por el cardioide $r = 1 - \sin\theta. = f(\theta)$



Limites $0 \le \theta \le 2\pi$.

$$A = \frac{1}{2}\int_0^{2\pi} r^2 \, d\theta. = \frac{2}{2}\int_{\pi/2}^{3\pi/2} r^2 \, d\theta.$$

$$A = \frac{1}{2}\int_0^{2\pi} (1 - \sin\theta)^2 \, d\theta.$$

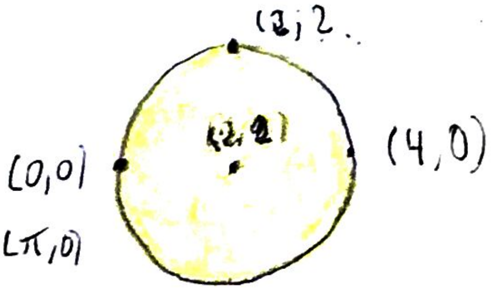
$$A = \frac{1}{2}\int_0^{2\pi} \left(1 - 2\sin\theta. + \underline{\sin^2\theta}\right) d\theta. \qquad \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta.$$

$$A = \frac{1}{2}\int_0^{2\pi} \frac{3}{2} - 2\sin\theta - \frac{1}{2}\cos 2\theta \, d\theta. = \frac{1}{2}\left(\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\Big]_0^{2\pi}\right)$$

$$A = \frac{1}{2}\left(\frac{3}{2}2\pi + 2\cos 2\pi - \frac{1}{4}\sin 4\pi - 0 - 2\cos 0 - \frac{1}{4}\sin 0\right).$$

$$A = \frac{1}{2}\left(3\pi + 2 - 2\right) = \frac{3\pi}{2}.$$

b. Dentro del círculo $r = 4\sin\theta$. en $0 \leq \theta \leq \pi$.



$$A = \frac{1}{2}\int_0^{\pi} r^2 \, d\theta.$$

$$A = \frac{1}{2}\int_0^{\pi} 16\sin^2\theta \, d\theta.$$

$$A = \int_0^{\pi} 8\sin^2\theta \, d\theta. = \int_0^{\pi} 4(1-\cos 2\theta)\, d\theta.$$

$$A = \int_0^{\pi} (4 - 4\cos 2\theta)\, d\theta. = 4\theta - 2\sin 2\theta \Big]_0^{\pi}$$

$$A = 4\pi - 2\sin 2\pi - 0 + 0 = 4\pi.$$

círculo de radio 2. $\longrightarrow$ $\pi(2)^2 = 4\pi$.

$$y = r\sin\theta = 4\sin^2\theta$$
$$x = r\cos\theta = 4\sin\theta\cos\theta.$$

$$(x-2)^2 + y^2 = 4$$
$$y = \sqrt{4 - (x-2)^2}$$

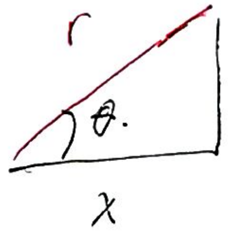
$$A = 2\int_0^4 \sqrt{4-(x-2)^2}\, dx$$

Cartesianas.

más complicado.

Ejercicio 2: Rosa de 4 pétalos $\underline{\underline{r = \cos 2\theta.}}$

a. Encuentre la derivada dy/dx $\quad r = \sqrt{x^2+y^2}$



Ecs. Paramétricas de la curva polar $\quad y = r\sin\theta.$ $\quad x = r\cos\theta.$

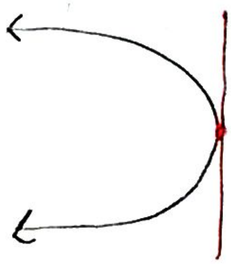
$y = \cos 2\theta \sin\theta$

$x = \cos 2\theta \cos\theta.$

$$\frac{dy}{dx} = \frac{-2\sin 2\theta \sin\theta + \cos 2\theta \cos\theta}{-2\sin 2\theta \cos\theta - \cos 2\theta \sin\theta.}$$

use la regla del producto.

b. Compruebe que la rosa tiene tangentes verticales en $\theta = 0$ y en $\theta = \pi$.



$=0$

$\frac{dy}{dx}$ no existe

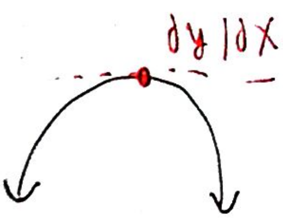
$x'(\theta) = 0$

$$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{-2\cdot 0 + 1}{-2\cdot 0 - 0} = \frac{1}{0} \text{ no existe.}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{-2\cdot 0 - 1}{-2\cdot 0 - 0} = \frac{-1}{0} \text{ no existe.}$$

Hay tangentes verticales en $\theta = 0$ y en $\theta = \pi$.

Hay tangentes horizontales en $\theta = \pi/2, 3\pi/2$



$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2.} = \frac{0}{1} = 0.$$

$$\left.\frac{dy}{dx}\right|_{\theta=3\pi/2} = \frac{0}{-1} = 0$$

Hay tangentes horizontales

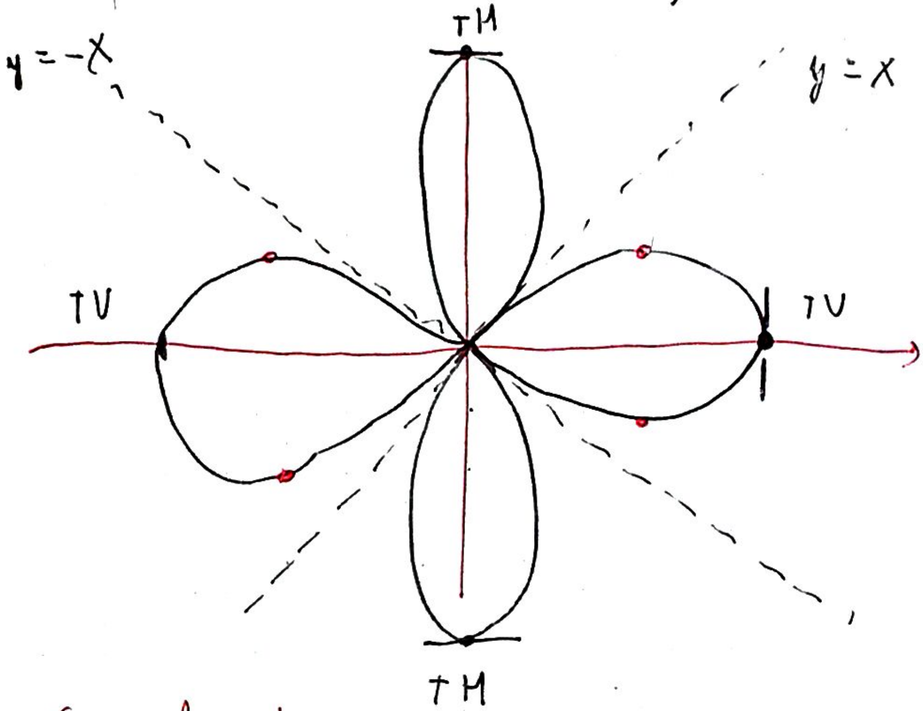
∴ Encuentre la ec. de la recta tangente en $\pi/4$.

$$x(\pi/4) = \cos\pi/2 \sin\pi/4 = 0$$

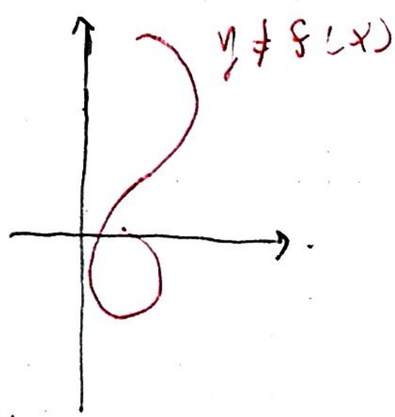
$$y(\pi/4) = \cos\pi/2 \sin\pi/4 = 0.$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{-2(1)\frac{\sqrt{2}}{2} + 0}{-2(1)\frac{\sqrt{2}}{2} - 0} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1$$

$$y = y(\pi/4) + m(x - x(\pi/4)) = x \qquad \boxed{y = x}$$



$y = f(t)$
$x = g(t)$



Área Superficial

$$A_S = 2\pi \int_a^b y\sqrt{1 + (y')^2}\, dx$$

$$A_S = 2\pi \int_a^b y(t)\sqrt{(x')^2 + (y')^2}\, dt.$$