2019-10-09

Laboratorio # 10
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100

5

media de 4 minutos

¿Que sea atendida antes de 3 minutos?

Distribución exponencial:

$$f(x) = \int_0^3 \frac{e^{-\frac{x}{u}}}{u} \, dx \qquad u = 4$$

$$= \int_0^3 \frac{1}{4} e^{-\frac{x}{4}} \, dx = -\int_0^3 e^{u} \cdot du = -\int_0^3 e^{u} \cdot du =$$

$$u = -\frac{x}{4}$$

$$-du = \frac{dx}{4}$$

$$= -\left(e^{-\frac{x}{4}}\right)\Big]_0^3 =$$

$$-\left[\left(e^{-\frac{3}{4}}\right) - \left(e^{0}\right)\right] = -\left[e^{-\frac{3}{4}} - 1\right]$$

$$= -e^{-\frac{3}{4}} + 1 \approx 0.52$$

(4) Encuentre la media, mediana, varianza & desviación estándar

$$f(x) = \frac{1}{18}(6-x) \quad ; \quad 0 \leq x \leq 6$$

Media ;

$$\mu = \frac{1}{18}\int_0^6 x(6-x)\,dx$$

$$= \frac{1}{18}\int_0^6 (6x - x^2)\,dx = \frac{1}{18}\left(\frac{6}{2}x^2 - \frac{1}{3}x^3\right)\Bigg]_0^6 =$$

$$= \frac{1}{18}\left[\left(3x^2 - \frac{1}{3}x^3\right)\right]_0^6 = \frac{1}{18}\left[\left(3(6)^2 - \frac{1}{3}(6)^3\right) - (0)\right] =$$

$$= \frac{1}{18}\left[108 - 72\right] = \frac{1}{18}\left[36\right] = \underline{2}$$

mediana =

$$\underbrace{\frac{1}{18}\int_{0}^{m} (6-x)\,dx = 0.5}$$

$$\frac{1}{18}\left(6x - \frac{1}{2}x^2\right)\Big]_0^m = 0.5$$

$$\frac{1}{18}\left[\left(6m - \frac{1}{2}m^2\right) - (0)\right] = 0.5$$

$$\frac{1}{18}\left[6m - \frac{1}{2}m^2\right] = 0.5$$

$$\frac{1}{18}\left[m\left(6 - \frac{1}{2}m\right)\right] = 0.5$$

$$\frac{m}{18}\left(6 - \frac{1}{2}m\right) = 0.5$$

$$m\left(6 - \frac{1}{2}m\right) = \frac{1}{2}\cdot 18$$

$$6m - \frac{1}{2}m^2 = 9$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$2\left(6m - \frac{1}{2}m^2\right) = 18$$

$$-m^2 + 12m - 18 = 0$$

$$a = -1 \quad b = 12 \quad c = -18$$

$$x_1, x_2 = \frac{-12 \pm \sqrt{12^2 - 4(-1)(-18)}}{2(-1)} = \frac{-12 \pm \sqrt{12^2 - 72}}{-2} = \frac{-12 \pm \sqrt{72}}{-2}$$

$$\underline{x_1 = 6 - 3\sqrt{2}} \quad x_2 = 6 + 3\sqrt{2}$$

$$x_1 \approx 1.75 \quad x_2 \approx 10.24$$

Dado el intervalo en cuestión 0,6 $\underline{x_1}$ es $\underline{m}$.

Varianza $\Rightarrow$

$$\sigma^2 = \frac{1}{18}\int_0^6 \underbrace{(x-\mu)^2 \cdot (6-x)}\,dx \qquad \mu = 2$$

$$\frac{1}{18}\int_0^6 (x^2 - 2\cdot x\cdot 2 + 2^2)(6-x)\,dx$$

$$\frac{1}{18}\int_0^6 (x^2 - 4x + 4)(6-x)\,dx =$$

$$= \frac{1}{18}\int_0^6 \underbrace{(6x^2 - 24x + 24 - x^3 + 4x^2 - 4x)}\,dx$$

$$-x^3 + 6x^2 + 4x^2 - 24x - 4x + 24$$

$$-x^3 + 10x^2 - 28x + 24$$

$$= \frac{1}{18}\int_0^6 (-x^3 + 10x^2 - 28x + 24)\,dx =$$

$$= \frac{1}{18}\left[-\frac{1}{4}x^4 + \frac{10}{3}x^3 - \frac{28}{2}x^2 + 24x\right] =$$

$$= \frac{1}{18}\left[-\frac{1}{4}x^4 + \frac{10}{3}x^3 - 14x^2 + 24x\right]_0^6 =$$

$$= \frac{1}{18}\left[\left(-\frac{1}{4}(6)^4 + \frac{10}{3}(6)^3 - 14(6)^2 + 24(6)\right) - (0)\right] =$$

$$= \frac{1}{18}\left[-324 + 720 - 504 + 144\right] = \frac{1}{18}\cdot 36 = \frac{\cancel{18}\cdot 2}{\cancel{18}} = \underline{2}$$

desviación estándar $= \underline{\sqrt{2}}$

③ $\mu = 2$ minutos

a) más de 4 minutos

$\boxed{!}\ \mu = 2$

$$f(x) = \int_4^\infty \frac{e^{-\frac{x}{\mu}}}{\mu}\,dx = \int_4^\infty e^{-\frac{x}{2}} \cdot \frac{1}{2} \cdot dx$$

$u = -\frac{x}{2}$

$-du = \frac{dx}{2}$

$$= -\int_4^\infty e^{u}\,du = -e^{u}\Big]_{u(4)}^{u(\infty)} =$$

$u(\infty) = -\infty$

$u(4) = -2$

$$= -e^{u}\Big]_{-2}^{-\infty} = -\left[\left(\underbrace{\lim_{t\to-\infty}\left(e^{t}\right)}_{e^{-\infty}\to 0}\right) - \left(e^{-2}\right)\right] = e^{-2}$$

(b) en menos de dos minutos.

$$f(x) = \int_0^2 \frac{1}{2} e^{-\frac{x}{2}} dx = -\int_0^{-1} e^u du = -e^u \Big]_0^{-1} =$$

$u = -\frac{x}{2}$ $\quad u(2) = -1$

$-du = \frac{dx}{2}$ $\quad u(0) = 0$

$$= -1\left[\left(e^{-1}\right) - \left(e^{0}\right)\right] = -1\left[e^{-1} - 1\right] =$$

$$= -e^{-1} + 1$$

(c) Que solo 0.02 sirvan gratis.

$$\underbrace{\int_m^\infty f(x)\, dx}_{} = \frac{1}{50}$$

$$\int_m^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = -\int_m^\infty e^u du = -e^u \Big]_{u(m)}^{u(\infty)} = -e^u \Big]_{-\frac{m}{2}}^{-\infty} =$$

$u = -\frac{x}{2}$

$du = -\frac{dx}{2} = -du$ $\quad u(\infty) = -\frac{\infty}{2} = -\infty$

$u(m) = -\frac{m}{2} = -\frac{m}{2}$

$$-\left[\underbrace{\left(\lim_{t \to -\infty}(e^t)\right)}_{0} - \left(e^{-\frac{m}{2}}\right)\right] = \left[-e^{-\frac{m}{2}}\right] = +e^{-\frac{m}{2}}$$

$$e^{-\frac{m}{2}} = \frac{1}{50}$$

$$-\frac{m}{2} = \ln\left(\frac{1}{50}\right) \rightarrow -m = 2\ln(1/50)$$

$$m = -2\ln(1/50)$$

$$m \approx 7.824$$

②

$$f(x) = \begin{cases} K(8 - x^3) & 0 \leq x \leq 2 \\ 0 & x < 0 \text{ ó } x > 3 \end{cases}$$

a) Hallar el valor de K.

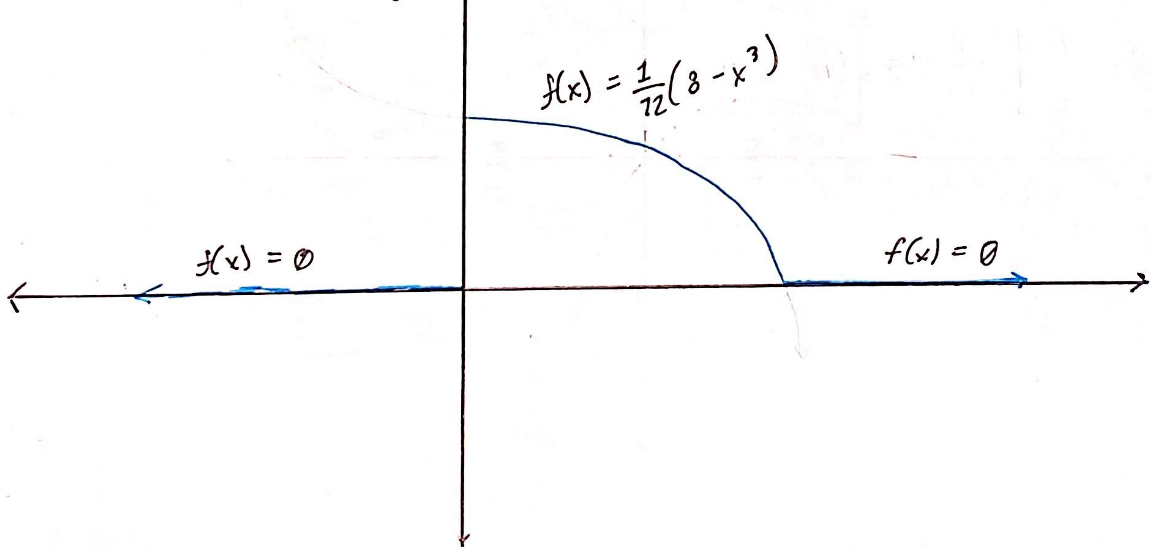
$$\int_0^2 K(8 - x^3)\,dx = K\int_0^2 (8 - x^3)\,dx = K\left[\left(8x - \frac{1}{4}x^4\right)\right]_0^2 =$$

$$K\left[\left(8(2) - \frac{1}{4}(2)^4\right) - (0)\right] = K\left[16 - 4\right] = 12k = \ldots 1$$

$$\ldots\ K = \frac{1}{12}\ \cancel{//}$$

b) Graficar la función de densidad:

$$f(x) = \begin{cases} \frac{1}{12}(8 - x^3); & 0 \leq x \leq 2 \\ 0; & x < 0 \text{ ó } x > 3 \end{cases}$$



c) Para que valor de K $P(X < 1)$

$$\frac{1}{12}\int_0^1 (8 - x^3)\,dx = \frac{1}{12}\left[\left(8x - \frac{1}{4}(x)^4\right)\right]_0^1 = \frac{1}{12}\left[\left(8 - \frac{1}{4}\right) - (0)\right] = \ldots$$

$$\ldots = \frac{1}{12}\left(\frac{31}{4}\right) = \frac{31}{48}$$

d) La media μ.

$$\mu = \int_0^2 x\,f(x)\,dx = \frac{1}{12}\int_0^2 x(8 - x^3)\,dx = \frac{1}{12}\int_0^2 (8x - x^4)\,dx = \ldots$$

$$\ldots = \frac{1}{12}\left[4x^2 - \frac{1}{5}x^5\right]_0^2 = \frac{1}{12}\left[\left(4(2)^2 - \frac{1}{5}(2)^5\right) - (0)\right] = \ldots$$

$$= \frac{1}{12}\left[16 - \frac{32}{5}\right] = \frac{48}{12 \cdot 5} = \frac{48}{60} = \frac{24}{30} = \frac{12}{15} = \frac{4}{5} \approx 0.8$$

1 función de densidad standard de Cauchy:

a) ¿Para que valor de c, f es una función de densidad de densidad?

$$\int_{-\infty}^{\infty} \frac{c}{1+x^2}\,dx = c\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = c\arctan(x)\Big]_{-\infty}^{\infty} =$$

$$= c\left[\underbrace{\arctan(\infty)}_{\frac{\pi}{2}} - \underbrace{\arctan(-\infty)}_{-\left(\frac{-\pi}{2}\right)}\right] = c\left[\frac{\pi}{2}+\frac{\pi}{2}\right] =$$

$$= c\left[\pi\right] = \quad c = \frac{1}{\pi}$$

b) Hallar $P(-1 \leq x \leq 1)$ en $f(x) = \dfrac{1}{\pi(1+x^2)}$

$$f(x) = \frac{1}{\pi}\int_{-1}^{1} \frac{1}{(1+x^2)}\,dx = \frac{\arctan(x)}{\pi}\Big]_{-1}^{1} =$$

$$= \frac{1}{\pi}\left[\underbrace{\arctan(1)}_{\frac{\pi}{4}} - \underbrace{\arctan(-1)}_{-\left(\frac{-\pi}{4}\right)}\right] = \frac{1}{\pi}\left[\frac{\pi}{4}+\frac{\pi}{4}\right] = \frac{1}{\pi}\left[\frac{\pi}{2}\right]$$

$$= \frac{1}{2}$$

La probabilidad es de $\frac{1}{2}$ ó 50%.

c) Encontrar la media de la función.

$$\int_{-\infty}^{\infty} x\, f(x)dx = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x}{1+x^2}dx = \frac{1}{\pi}\cdot\frac{1}{2}\int_{-\infty}^{\infty}\frac{du}{u} =$$

$u = x^2 + 1$

$du = 2x\,dx$ $\qquad u(\infty) = (\infty)^2 + 1 = \infty$

$\frac{du}{2} = x\,dx$ $\qquad u(-\infty) = (-\infty)^2 + 1 = \infty$

$$= \frac{1}{2\pi} \ln|u| \Big]_{u(-\infty)}^{u(\infty)} = \frac{1}{2\pi}\left[\left(\lim_{t_1\to\infty}(\ln|t_1|)\right) - \left(\lim_{t_2\to\infty}(\ln|t_2|)\right)\right] =$$

Reescribir los logaritimos naturales

$$= \frac{1}{2\pi}\left[\lim_{t\to\infty}\underbrace{\left(\ln t - \ln t\right)}_{\Downarrow\ \ln\left(\frac{t}{t}\right) \equiv \ln(1) = 0}\right] = \frac{1}{2\pi}(0) = 0$$

Por la imparidad de la función se cancelan las magnitudes.