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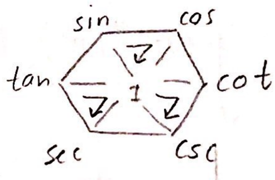
Laboratorio # 4

① $$\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin^3\left[(x)^{1/2}\right]\cdot(x)^{-1/2}\,dx$$



$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

100 pts

$u = \sqrt{x} = (x)^{1/2}$

$du = \frac{1}{2}(x)^{-1/2}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= \int \sin^3(u)\cdot 2\,du$$

$$= 2\int \sin^3(u)\,du$$

$$= 2\int \sin^2(u)\cdot\sin(u)\,du$$

$$= 2\cdot\int\left[1-\cos^2(u)\right]\cdot\sin(u)\,du$$

$w = \cos(u)$

$dw = -\sin(u)$ $\quad -dw = \sin(u)$

$$= 2\int\left[1-w^2\right]\cdot -dw$$

$$= 2\cdot -1\int\left[1-w^2\right]\cdot dw$$

$$= -2\left\{\int 1\,dw - \int w^2\,dw\right\}$$

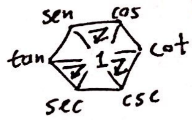
$$= -2\left\{w - \frac{w^3}{3}\right\}$$

$$= -2w + \frac{2w^3}{3}$$ sustituir w por correspondiente

$$= -2\cos(u) + \frac{2\cos^3(u)}{3}$$

$$= -2\cos(\sqrt{x}) + \frac{2\cos^3(\sqrt{x})}{3} + C$$

② $\int \cos^4(\theta)\tan^2(\theta)\,d\theta = \int (\cos^2\theta)^2 \tan^2\theta\, d\theta$



$\sin^2 + \cos^2 = 1$

$1 + \cot^2 = \csc^2$

$\tan^2 + 1 = \sec^2$

$\longrightarrow \tan^2 = \sec^2 - 1$

or

$\longrightarrow \cos^2 = 1 - \sin^2$

$= \int \cos^4(\theta) \dfrac{\sin^2(\theta)}{\cos^2(\theta)}\, d\theta = \int \cancel{\cos^2\theta}\, \cos^2\theta \dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\, d\theta$

$= \int \cos^2\theta \cdot \sin^2\theta\, d\theta = \int \cos^2\theta \left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right) d\theta$

$= \int \left[\dfrac{1 + \cos(2\theta)}{2}\right]\left[\dfrac{1 - \cos(2\theta)}{2}\right] d\theta$

$= \int \left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right)\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right) d\theta$ simplificar $(a-b)(a+b) = a^2 - b^2$

$= \int \left(\frac{1}{2}\right)^2 - \left(\frac{\cos(2\theta)}{2}\right)^2 d\theta = \int \frac{1}{4}\, d\theta - \int \frac{\cos^2(2\theta)}{4}\, d\theta$

$= \frac{\theta}{4} - \frac{1}{4}\underbrace{\int \cos^2(2\theta)\, d\theta}$ $= \frac{\theta}{4} - \frac{1}{4}\int \cos^2(\alpha)\, d\alpha$

$\alpha = 2\theta$

$\dfrac{d\alpha}{2} = d\theta$

$= \frac{\theta}{4} - \frac{1}{4}\left\{\int \frac{1}{2} + \frac{\cos^2(2\alpha)}{2}\, d\alpha\right\}$

$$= \frac{\theta}{4} - \frac{1}{4}\left\{ \int \frac{1}{2}\,d\alpha + \frac{1}{2}\int \cos^2(2\alpha)\,d\alpha \right.$$

$$= \frac{\theta}{4} - \frac{1}{4} \cdot \int \frac{1}{2}\,d\alpha + \frac{1}{4} \cdot \frac{1}{2} \int \cos(4\theta)\,d\alpha$$

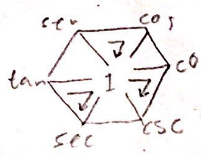
$$= \frac{\theta}{4} - \frac{\theta}{8} + \frac{1}{8}\sin(4\theta) \cdot \frac{1}{4} + C$$

$$= \frac{2\theta - \theta}{8} + \frac{1}{32}\sin(4\theta) + C$$

$$= \frac{\theta}{8} + \frac{\sin(4\theta)}{32} + C$$

$$= \frac{1}{8}\left(\theta - \frac{\sin(4\theta)}{4}\right) + C$$

③ $$\int \cos^3(\sin\theta)\,\sin^4(\sin\theta)\cos\theta\,d\theta$$



$$\left.\begin{array}{l} u = \sin\theta \\ du = \cos\theta\,d\theta \end{array}\right\} \therefore \int \cos^3(u)\,\sin^4(u)\,du$$

$\sin^2 + \cos^2 = 1$

$\cos^2 = 1 - \sin^2$

$$= \int \cos^3(u)\,\sin^4(u)\,du$$

$$= \int (1-\sin^2(u))\sin^4(u)\cos(u)\,du$$

$$= \int \sin^4(u)\cos(u) - \sin^6(u)\cos(u)\,du$$

$$= \int \sin^4(u)\cos(u)\,du - \int \sin^6(u)\cos(u)\,du$$

$$\underbrace{\begin{array}{l} w = \sin(u) \\ dw = \cos(u)\,du \end{array}}_{\int w^4\,dw} \qquad - \qquad \underbrace{\begin{array}{l} w = \sin(u) \\ dw = \cos(u) \end{array}}_{\int w^6\,dw}$$

$$= \frac{w^5}{5} - \frac{w^7}{7} + C$$

$$= \frac{\sin^5(u)}{5} - \frac{\sin^7(u)}{7} + C$$

$$= \frac{\sin^5(\sin\theta)}{5} - \frac{\sin^7(\sin\theta)}{7} + C$$

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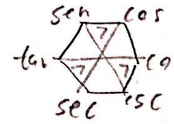
④ $$\int \tan^5(x)\sec^4(x)\,dx =$$



$\tan^2 + 1 = \sec^2$

$$= \int \tan^5(x)\sec^2(x)\sec^2(x)\,dx$$

$$= \int \tan^5(x)(\tan^2 x + 1)\sec^2(x)\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int u^5(u^2+1)\,du = \int u^7 + u^5\,du$$

$$= \int u^7\,du + \int u^5\,du$$

$$= \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \frac{\tan^8(x)}{8} + \frac{\tan^6(x)}{6} + C$$

X

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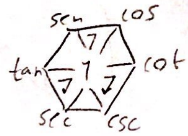
$$\int \sec^4(x)\,dx =$$



$\tan^2 + 1 = \sec^2$

$$= \int \sec^2(x)\left[\tan^2 + 1\right] dx$$

$$= \int \sec^2(x)\tan^2(x) + \sec^2(x)\,dx$$

$$= \int \sec^2(x)\tan^2(x)\,dx + \int \sec^2(x)\,dx$$

$$\underbrace{\begin{aligned} u &= \tan(x) \\ du &= \sec^2(x) \end{aligned}} \qquad + \tan(x)$$

$$= \int u^2\,du + \tan(x)$$

$$= \frac{u^3}{3} + \tan(x) + C$$

$$= \frac{\tan^3(x)}{3} + \tan(x) + C$$