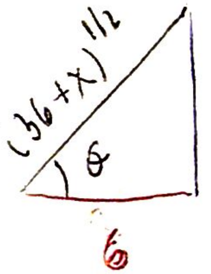
Parcial Lunes 2:30 D-504

Corto 4 | 1 Problema Sustitución Trigonométrica.

Práctica Problemas Integración. Pág 59.

2a) $\displaystyle\int_0^6 \frac{72}{(36+x^2)^{3/2}}\,dx = \int_0^{\pi/4} \frac{72\cdot 6\sec^2\theta\,d\theta}{36\cdot 6\sec^3\theta} = 2\int_0^{\pi/4} \frac{1}{\sec\theta}\,d\theta.$



$x = 6\tan\theta.$ $\quad \frac{x}{6} = \tan\theta$

$dx = 6\sec^2\theta\,d\theta.$ $\quad \theta = \tan^{-1}\left(\frac{x}{6}\right)$

$(36+x)^{1/2} = 6\sec\theta$

$(36+x)^{3/2} = 6^3\sec^3\theta$ $\quad \theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\theta = \tan^{-1}(0) = 0$

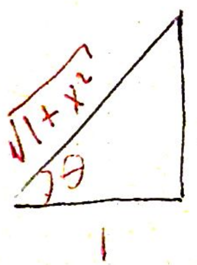
$\displaystyle 2\int_0^{\pi/4} \frac{1}{\sec\theta.}\,d\theta = 2\int_0^{\pi/4} \cos\theta\,d\theta. = 2\sin\theta\Big]_0^{\pi/4} = 2\sin\frac{\pi}{4} = \frac{2\sqrt{2}}{2.} = \sqrt{2}$

2c) $\displaystyle\int_0^1 \frac{1}{(1+x)^2}\,dx = \int_1^2 u^{-2}\,du = -\frac{1}{u}\Big]_1^2 = -\frac{1}{2} + \frac{1}{1} = \frac{1}{2.}$

$u = 1+x$ $\quad u(1) = 2.$

$du = dx$ $\quad u(0) = 1$

2c1) $\displaystyle\int_0^1 \frac{1}{(1+x^2)^2}\,dx = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{(1+\tan^2\theta)^2} = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \int_0^{\pi/4} \frac{1}{\sec^2\theta}\,d\theta.$



$x = \tan\theta.$ $\quad dx = \sec^2\theta\,d\theta.$

$1 = \tan\theta. \Rightarrow \theta = \pi/4$

$0 = \tan\theta$ $\quad \theta = 0$

$\displaystyle\int_0^{\pi/4} \cos^2\theta\,d\theta = \frac{1}{2}\int_0^{\pi/4} (1+\cos 2\theta)\,d\theta. = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\Big]_0^{\pi/4}\right)$

doble ángulo

$$\int_0^{\pi/4} \cos^2\theta \, d\theta = \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{2} - 0\right) = \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$$

3c) $$\int_0^{1/3} 3^6 x^5 \sqrt{1-9x^2}\, dx = \int_0^{\pi/2} \frac{3^6}{3^5} \sin^5\theta \cos\theta \frac{1}{3}\cos\theta \, d\theta.$$

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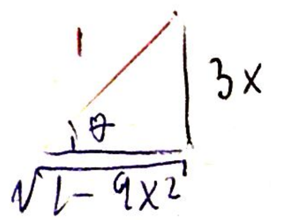


$\sin\theta = 3x$ $\Rightarrow$ $x = \frac{1}{3}\sin\theta.$ $\sin\theta = 1$

$\cos\theta \, d\theta = 3\,dx$ $\quad dx = \frac{1}{3}\cos\theta\, d\theta.$ $\theta = \pi/2$

$\sqrt{1-9x^2} = \cos\theta.$ $\quad x^5 = \frac{1}{3^5}\sin^5\theta.$ $\sin\theta = 0$

$\theta = 0$

$$\frac{3^6}{3^6}\int_0^{\pi/2} \sin^5\theta \cdot \cos^2\theta \, d\theta = \int_0^{\pi/2} \underbrace{\sin^4\theta}_{(\sin^2\theta)^2 = (1-\cos^2\theta)^2} \cos^2\theta \,(\sin\theta\, d\theta)$$

$$\int_0^{\pi/2} (1-\cos^2\theta)^2 \cos^2\theta \,\underbrace{(\sin\theta\, d\theta)}_{-du} = -\int_1^0 (1-u^2)^2 u^2 \, du.$$

$u = \cos\theta$ $\quad u = \cos\pi/2 = 0$

$du = -\sin\theta\, d\theta$ $\quad u = \cos 0 = 1$

$$= \int_0^1 (1-2u^2+u^4)\,u^2\, du.$$

$$-\int_b^a f\,dx = \int_a^b f\,dx$$

$$= \int_0^1 u^2 - 2u^4 + u^6 \, du.$$

$$= \left.\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7\right]_0^1$$

$$= \frac{1}{3} - \frac{2}{5} + \frac{1}{7}.$$

$$\int 3^6 x^5 \sqrt{1-9x^2}\, dx$$

$$\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C.$$

$\cos\theta = \sqrt{1-9x^2}$

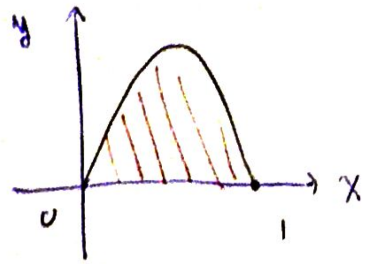
$$\frac{1}{3}\cos^3\theta - \frac{2}{5}\cos^5\theta + \frac{1}{7}\cos^7\theta + C.$$

$$\frac{1}{3}(1-9x^2)^{3/2} - \frac{2}{5}(1-9x^2)^{3/2} + \frac{1}{7}(1-9x^2)^{7/2} + C$$

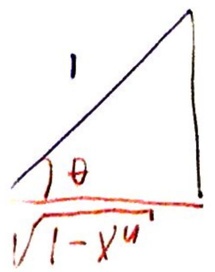
Triángulos "Interesantes"

Ejercicio 4.

Encuentre el área entre $f(x) = 4\pi x\sqrt{1-x^4}$, el eje x, y las rectas $x=0$ & $x=1$.



$$A = \int_0^1 f(x)\,dx = 2\pi\int_0^1 \sqrt{1-x^4}\;\underline{2x\,dx}$$



$x^2 = \sin\theta.$

$2x\,dx = \cos\theta\,d\theta.$

$\sqrt{1-x^4} = \cos\theta.$

$\sin\theta = 1$

$\theta = \pi/2$

$\sin\theta = 0$

$\theta = 0$

$$A = 2\pi\int_0^{\pi/2} \cos\theta\cos\theta\,d\theta. = \frac{2\pi}{2}\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta.$$

$$A = \pi\left(\theta + \frac{\sin 2\theta}{2}\Big]_0^{\pi/2}\right) = \pi\left(\frac{\pi}{2} + \cancel{\frac{\sin\pi}{2}}^{\,0} - 0 - \cancel{\frac{\sin 0}{2}}^{\,0}\right)$$

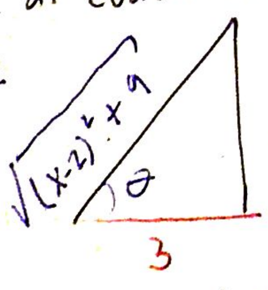
$$A = \pi\cdot\frac{\pi}{2} = \frac{\pi^2}{2}$$

$$\int \frac{(x-2)^3}{(x^2-4x+13)^{1/2}}\,dx. = \int\frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int\frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\,d\theta.$$

Complete al cuadrado $(x^2-4x+4)+13-4 = (x-2)^2+9$

c. $x-2$

c. 3



$3.\tan\theta = x-2$

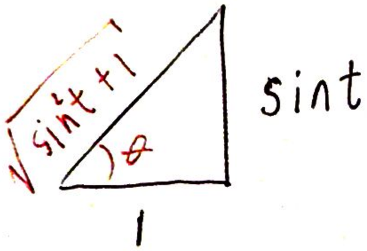
$3\sec^2\theta\,d\theta = dx.$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$$3^3\int\tan^3\theta.\sec\theta\,d\theta. = 27\int\tan^2\theta\,(\tan\theta\sec\theta\,d\theta)$$

Problema 2 b) Simulacro.

$$\int_0^{\pi/2} \frac{\cos t}{\sqrt{\sin^2 t + 1}}\, dt = \int \frac{\sec^2\theta \, d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta \, d\theta.$$



$\sin\theta$

$\tan\theta = \sin t.$

$\sec^2\theta \, d\theta = \cos t \, dt.$

$\sqrt{\sin^2 t + 1} = \sec\theta.$

$\tan\theta = \sin\frac{\pi}{2} = 1$

$\theta = \tan^{-1}(1) = \pi/4$

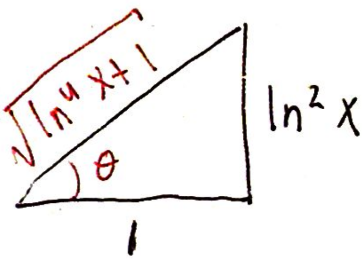
$\tan\theta = \sin 0 = 0$

$\theta = 0$

$$\int_0^{\pi/4} \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| \Big]_0^{\pi/4} = \ln|\sec\pi/4 + \tan\pi/4) - \ln(\sec 0 + \tan 0)$$

$$= \ln(\sqrt{2} + 1) - \cancel{\ln(1)}^{0} = \underline{\ln(\sqrt{2} + 1)}$$

$$\int \frac{4}{\sqrt{\ln^4 x + 1}} \frac{2(\ln x)}{x} dx = 4\int \frac{\sec^2\theta \, d\theta}{\sec\theta} = 4\int \sec\theta \, d\theta.$$



$\tan\theta = [\ln x]^2$

$\sec^2\theta \, d\theta = 2\ln x \frac{1}{x} dx.$

$\sqrt{\ln^4 x + 1} = \sec\theta.$

$$4\int \sec\theta \, d\theta = 4\ln|\sec\theta + \tan\theta| + C.$$

$$4\ln\left|\sqrt{\ln^4 x + 1} + \ln^2 x\right| + C.$$