2019 - 10 - 06

Webassign: Integrales impropias

(1) $$\int_6^\infty \frac{1}{(x-5)^{3/2}}\,dx = \int_6^\infty \frac{1}{u^{3/2}}\,du = \int_6^\infty u^{-\frac{3}{2}}\,du = 2u^{-1/2}$$

$u = (x-5)$

$du = dx$

$$= -2\cdot(x-5)^{-\frac{1}{2}}\Big]_6^\infty =$$

$$= \left[\underbrace{\left(\lim_{t\to\infty}\left(\frac{-2}{\sqrt{t-5}}\right)\right)}_{\frac{2}{\infty}\to 0} - \left(\frac{-2}{\sqrt{1}}\right)\right] = 2$$

(2) $$\int_0^\infty \frac{1}{\sqrt[8]{1+x}}\,dx = \int_0^\infty (u)^{-\frac{1}{8}}\,du = \frac{1}{-\frac{1}{8}+\frac{8}{8}}\,u^{-\frac{1}{8}+\frac{8}{8}} = \frac{8}{7}u^{\frac{7}{8}}\Big]_0^\infty =$$

$u = 1 + x$

$du = dx$

$$= \left[\lim_{t\to\infty}\overbrace{\left(\frac{8}{7}(1+\infty)^{\frac{7}{8}}\right)}^{\infty} - \left(\frac{8}{7}(1+0)^{\frac{7}{8}}\right)\right] = \infty$$

③ $$\int_2^\infty e^{-7p}\,dp = -\frac{1}{7}\int_2^\infty e^u\,du = -\frac{1}{7}\left(e^u\right)\Big]_2^\infty = -\frac{1}{7}\left(e^{-7p}\right)\Big]_2^\infty$$

$u = -7p$

$du = -7\,dp$

$-\frac{du}{7} = dp$

$$= \left[\underbrace{\lim_{t\to\infty}\left(-\frac{1}{7e^{7\infty}}\right)}_{0} - \left(-\frac{1}{7e^{7(2)}}\right)\right] = \frac{1}{7e^{14}}$$

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④ $$\int_1^\infty \frac{e^{-\frac{1}{x}}}{x^2}\,dx = \int_1^\infty e^{-\frac{1}{x}} \cdot x^{-2}\,dx = \int_1^\infty e^u\,du = e^u\Big]_1^\infty = e^{-\frac{1}{x}}\Big]_1^\infty =$$

$u = -\frac{1}{x} = -x^{-1}$

$du = x^{-2}\,dx$

$$= \left[\lim_{t\to\infty}\left(\frac{1}{e^{\frac{1}{\infty}}}\right) - \left(e^{-\frac{1}{1}}\right)\right] =$$

$$= \left[1 - e^{-1}\right]$$

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⑤ $$\int_{-\infty}^{0} \frac{z}{z^4+4}\,dz =$$

$u = z^2$

$\frac{1}{2}du = z\,dz$

$$= \frac{1}{2}\int \frac{du}{u^2+4} =$$

$u = 2v$

$du = 2dv$

$$\int \frac{2dv}{4v^2+4} = \frac{1}{2}\int \frac{2\ dv}{4(v^2+1)} =$$

$v = \frac{u}{2}$

$$= \frac{1}{2}\cdot\frac{1}{2}\int \frac{1}{v^2+1}\,dv = \frac{1}{4}\arctan(v)$$

$$= \frac{1}{4}\arctan\left(\frac{z^2}{2}\right)\Bigg]_{-\infty}^{0} = \xrightarrow{\text{eval}}$$

eval

$$= \frac{1}{4}\left[\underbrace{\left(\arctan\left(\frac{0^2}{2}\right)\right)}_{0} - \left(\arctan\left(\frac{(-\infty)^2}{2}\right)\right)\right] =$$

$- \arctan(\infty)$

$-\frac{\pi}{2} = \frac{1}{4}\cdot -\frac{\pi}{2} = -\frac{\pi}{8}$

$-\frac{\pi}{8}$

⑥ $S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq e^{-x}\}$

$$S = \int_1^\infty e^{-x}\,dx = -\int_1^\infty e^{u}\,du = -e^{-x}\Big]_1^\infty =$$

$u = -x$

$-du = dx$

$$= \left[\underbrace{\lim_{t\to\infty}\left(-\frac{1}{e^\infty}\right)}^{0} - \left(-e^{-1}\right)\right] = \underline{e^{-1}}$$

⑦ $$\int_0^\infty \frac{4}{\sqrt{x}\,(1+x)}\,dx = \int_0^1 \frac{4}{\sqrt{x}(1+x)}\,dx + \int_1^\infty \frac{4}{\sqrt{x}(1+x)}\,dx = 2\pi + 2\pi = 4\pi$$

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$$= 8\left[\underbrace{\left(\arctan(\sqrt{1})\right)}_{1} - \cancel{\arctan(\sqrt{0})}^{0}\right]$$

$$\frac{\pi}{4} = \underline{2\pi}$$

$$2\cdot 4\int_0^\infty \frac{u\,du}{u(1+u^2)} = 8\int \frac{\cancel{u}\,du}{\cancel{u}(1+u^2)} = 8\arctan(u)$$

$$8\left[\underbrace{\left(\arctan(\infty)\right)}_{\frac{2\pi}{2\cdot 2}} - \underbrace{\left(\arctan(1)\right)}_{\frac{\pi}{4}}\right] = 8\left(\frac{2\pi}{4} - \frac{\pi}{4}\right) = 8\left(\frac{\pi}{4}\right) = 2\pi$$

8

$$23\int_{1}^{\infty}\frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -23\cdot -2\int_{1}^{\infty} e^{u}\,du = -23\left(e^{-\sqrt{x}}\right)\Big]_{1}^{\infty} =$$

$u = -\sqrt{x}$

$du = -\frac{1}{2\sqrt{x}}\,dx$

$\cdot 2du = -\frac{1}{\sqrt{x}}\,dx$

$$+23\cdot 2\left[\underbrace{\left(\lim_{x\to\infty}\left(\frac{1}{e^{\sqrt{x}}}\right)\right)}_{0} - \left(e^{-1}\right)\right] = +23\cdot 2\left[-e^{-1}\right] =$$

$$= 46\, e^{-1}$$