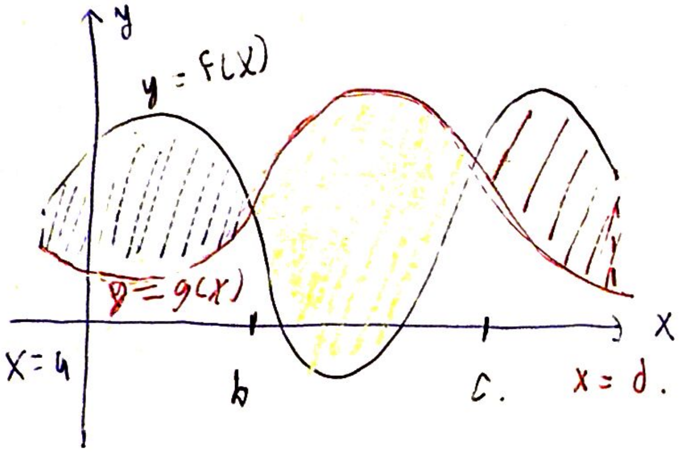
Áreas entre Curvas.

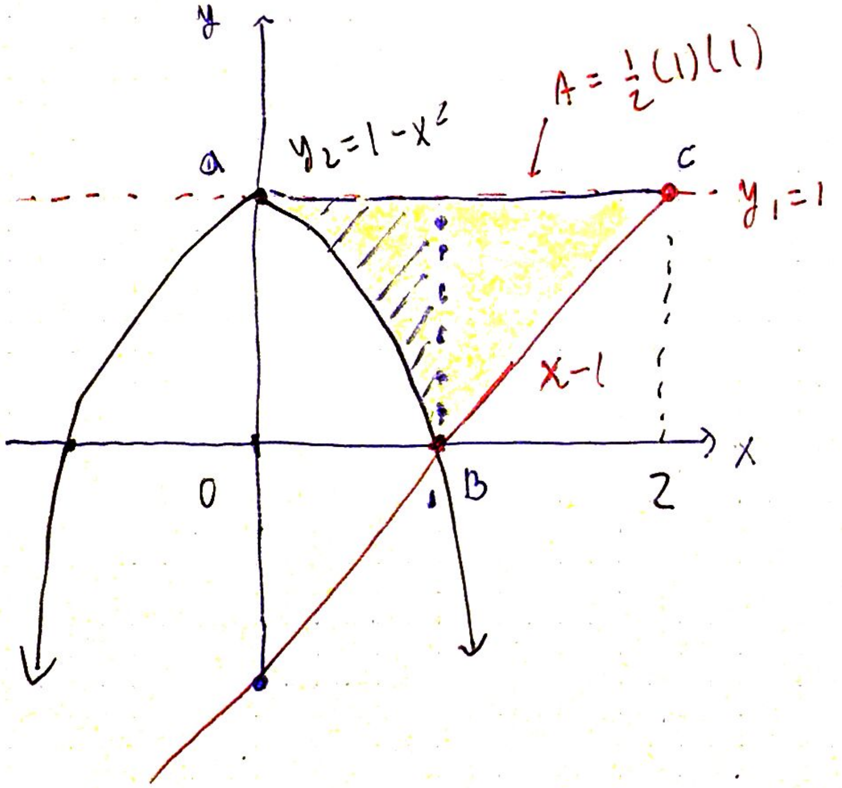


$$A = \int_a^b f-g\,dx + \int_b^c g-f\,dx + \int_c^d f-g\,dx$$

$$A = \int_a^b |f-g|\,dx$$ ✓

Ejercicio 3: Encuentre el área de la región entre las curvas dadas.

b. $y_1 = 1$, $y_2 = 1-x^2$, $y_3 = x-1$. P(83)



a. $y_1 = y_2$

$1 = 1 - x^2$

$x^2 = 0 \Rightarrow x = 0$

c. $y_1 = y_3$.

$1 = x - 1 \Rightarrow x = 2$.

B. $y_2 = y_3 \Rightarrow x = 1$

$1 - x^2 = x - 1$

$-x^2 - x + 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, \quad x = 1$

$$A = \left.\int_0^1 1-(1-x^2)\,dx + \int_1^2 1-(x-1)\,dx\right\} 1/2.$$

$$A = \int_0^1 x^2 \, dx + \int_1^2 2 - x \; dx$$

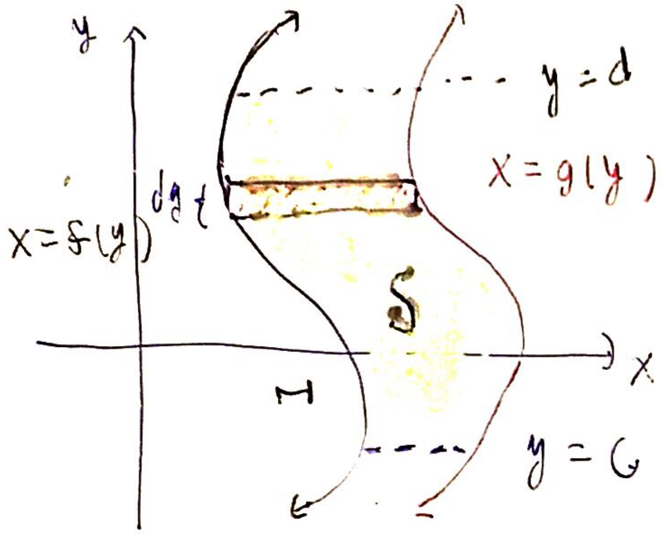
$$A = \frac{1}{3}x^3 \Big]_0^1 + 2x - \frac{x^2}{2} \Big]_1^2$$

$$A = \frac{1}{3} + \underbrace{4 - 2 - \left(2 - \frac{1}{2}\right)}_{1/2}$$

$$A = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}.$$

P(83)

Integración en el eje-y, franjas Horizontales.



Región S: $c \leq y \leq d$.
$f(y) \leq x \leq g(y)$

rectángulo. altura dy.
ancho $g(y) - f(y)$

$$dA = [g(y) - f(y)] \, dy.$$

$$\boxed{A = \int_c^d [g(y) - f(y)] \, dy.}$$

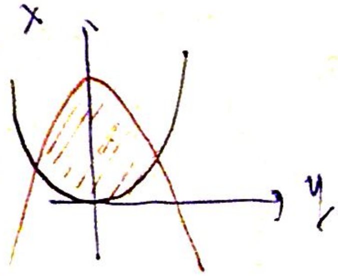
$$A = \int_a^b y_{arr} - y_{aba} \, dx$$

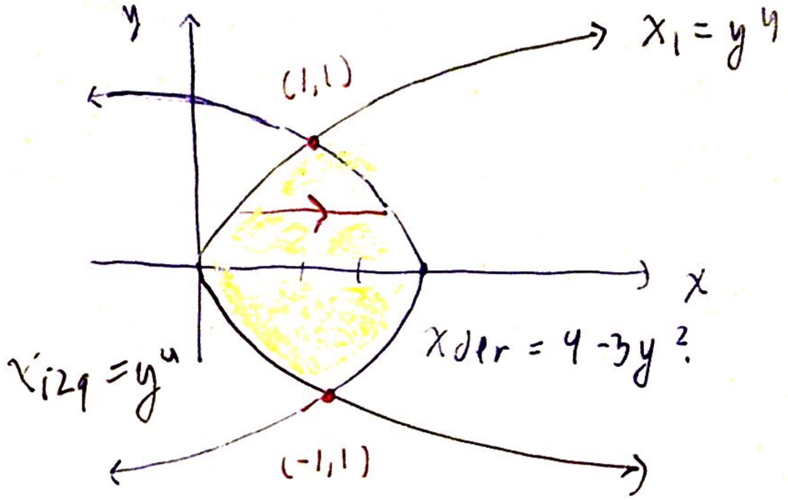
$$A = \int_c^d X_{der} - X_{izq} \; dy.$$

$$x = f(y) \Rightarrow y = f^{-1}(x)$$

evite las inversas.

Ejemplo: Encuentre el área entre $x_1 = y^4$ & $x_2 = 4 - 3y^2$.





$$A = \int_{-1}^{1} x_{der} - x_{izq}\, dy = \int_{-1}^{1} 4 - 3y^2 - y^4 \, dy.$$

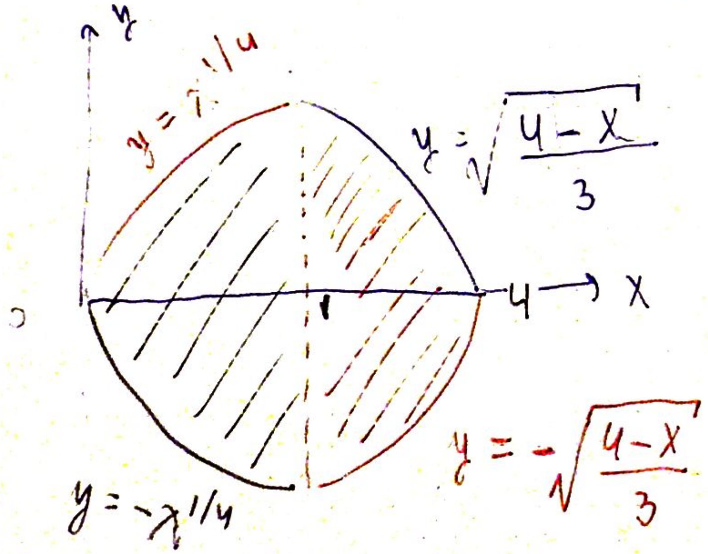
Intersecciones $\quad y^4 = 4 - 3y^2.$ $\quad y^4 + 3y^2 - 4 = 0$

$$\begin{aligned} y^2 &\quad +4 = 0 \\ y^2 &\quad -1 = 0 \end{aligned}$$

$$\int_{-\infty}^{\infty} \frac{1}{9 + x^2}\, dx$$

$$y^2 \neq -4$$
$$y^2 = 1$$
$$y = \pm 1$$

$$A = 2\int_0^1 4 - 3y^2 - y^4 \, dy. = 2\left(4y - y^3 - \frac{1}{5}y^5\Big]_0^1\right)$$
$$= 2\left(4 - 1 - \frac{1}{5}\right) = 2\left(\frac{15}{5} - \frac{1}{5}\right) = \frac{28}{5}.$$



$x = y^4$ Resuelva para x:

$$y = \pm \sqrt[4]{x}$$
$$x = 4 - 3y^2.$$
$$3y^2 = 4 - x$$
$$y^2 = \frac{4-x}{3} \qquad y = \pm\sqrt{\frac{4-x}{3}}$$

A'rea de la región

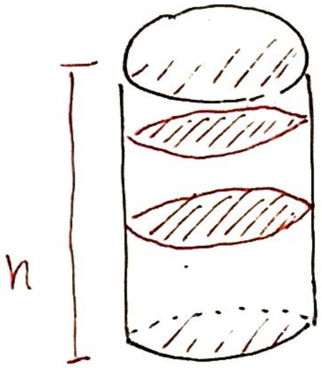
$$A = \int_0^1 x^{1/4} - (-x^{1/4})\, dx + \int_1^4 \sqrt{\frac{4-x}{3}} - \left(-\sqrt{\frac{4-x}{3}}\right) dx$$

$$A = 2\int_0^1 x^{1/4}\, dx + 2\int_1^4 \left(\frac{4-x}{3}\right)^{1/2} dx \Big\} = 28/5$$

< 40 2pts. P1   > 60 1pt. neto.

Volúmenes. de Sólido   Encuentre el área, utilizando integrales.

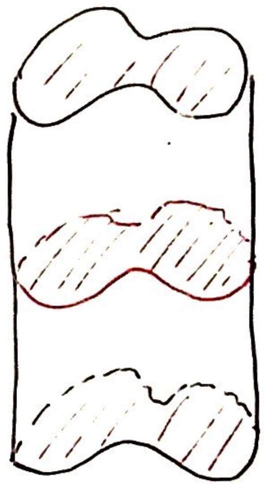
Volumen de un cilindro.



A'rea transversal πr^2.

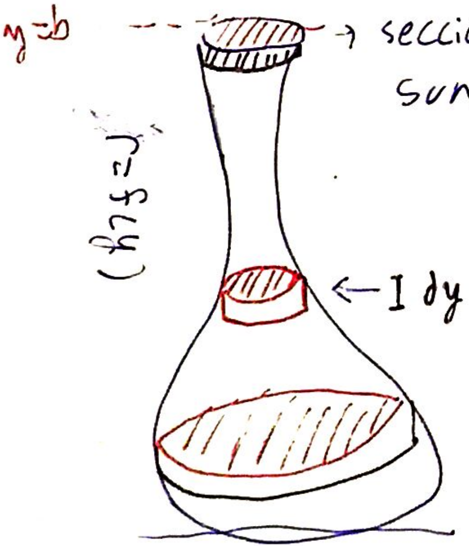
$V = Ah = \pi r^2 h.$

cilindro Circular



sección transversal

$V = Ah.$



→ secciones transversales son diferentes

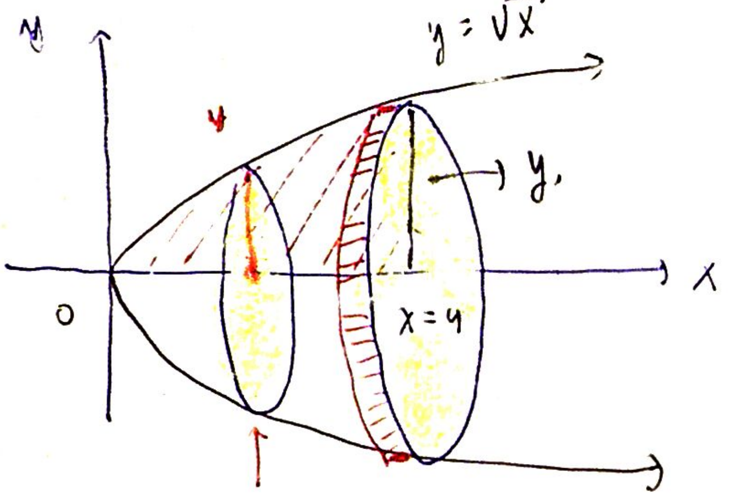
volumen de una sección infinitesimal, el cual es un cilindro con altura dy. y área A(y).

$dV = A(y)\, dy$

Integrando en $a \leq y \leq b$.

Volumen total del sólido $\boxed{V = \int_a^b A(y)\, dy.}$

Ejemplo: Considere la región $0 \leq y \leq \sqrt{x}$
$0 \leq x \leq 4$.



rote la región respecto al eje-x obtiene un sólido de revolución.

sección transversal círculo de radio y.

$A(y) = \pi y^2$. grosor/altura dx.

$$V = \int_0^4 \pi y^2\, dx = \int_0^4 \pi (\sqrt{x})^2\, dx = \pi \int_0^4 x\, dx$$

$$V = \frac{\pi}{2} x^2 \Big]_0^4 = \frac{\pi}{2} 16 = 8\pi.$$