2019-10-04

Webassign Longitud de arco

1. $y = 5x - 1 \; ; \; -1 \leq x \leq 3 \; ;$

$y' = 5 = (y')^2 = 25$

$$L = \int_{-1}^{3} \sqrt{1 + 25}\, dx = \int_{-1}^{3} \sqrt{26}\, dx = \sqrt{26}x \Big]_{-1}^{3} =$$

$$= \sqrt{26}\left[(3) - (-1)\right] = \sqrt{26}\left[4\right] = \underline{4\sqrt{26}}$$

2. $y = \sqrt{2 - x^2} \; , \; 0 \leq x \leq 1 \; ;$

$$y'(x) = \frac{1}{2}(2 - x^2)^{-1/2} \cdot -2x = -\frac{2x}{\sqrt{2-x^2}} =$$

$$(y'(x))^2 = \left(-\frac{2x}{\sqrt{2-x^2}}\right)^2 = \frac{(-2x)^2}{(\sqrt{2-x^2})^2} = \frac{4x^2}{2-x^2} = 4\left(\frac{x^2}{2-x^2}\right)$$

$$L = 4\int_0^1 \underbrace{\frac{x^2}{2-x^2}}\, dx = 4\left[\int_0^1 \frac{2}{2-x^2}\, dx - \int_0^1 1\, dx\right] =$$

$$\frac{x^2}{2-x^2} = \frac{x^2}{2-x^2} + \frac{2-x^2}{2-x^2} - 1$$

$$= \frac{x^2 + 2 - x^2}{2-x^2} - 1 = \frac{2}{2-x^2} - 1$$

$$= 4\left[\int_0^1 \frac{2}{2-x^2}\, dx - \int_0^1 1\, dx\right]$$

3 $y = \ln(\sec x)$; $0 \le x \le \frac{\pi}{4}$

$$y' = \frac{\cancel{\sec x}\tan x}{\cancel{\sec x}} = (\tan x)^2 = \tan^2 x$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx =$$

$$= \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x| \Big]_0^{\pi/4} =$$

$$= \ln\left|\sec\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)\right| - \ln|\sec(0) + \tan(0)|$$

$$\boxed{\sec\frac{\pi}{4} = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{2}{\sqrt{2}}} \quad \boxed{\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = 1}$$

$$= \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \cancel{\ln|1|}^{\,0} = \ln\left|\frac{2}{\sqrt{2}} + 1\right|$$

2 . $y = \sqrt{2 - x^2} \; ; \; 0 \leq x \leq 1$

$$y'(x) = \frac{1}{2}(2-x^2)^{-1/2} \cdot -2x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{2-x^2}} = \left(\frac{-x}{\sqrt{2-x^2}}\right)^2 = \frac{x^2}{2-x^2}$$

$$(y'(x))^2 = \frac{x^2}{2-x^2}$$

$$L = \int_0^1 \sqrt{1 + \frac{x^2}{2-x^2}}\, dx =$$

$$\sqrt{\frac{1\,(2-x^2)}{(2-x^2)} + \frac{x^2}{(2-x^2)}}$$

$$\sqrt{\frac{2 - \cancel{x^2} + \cancel{x^2}}{(2-x^2)}} = \int \sqrt{\frac{2}{2-x^2}} = \sqrt{2} \int \frac{1}{\sqrt{2-x^2}}\, dx$$

$$\frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} \arcsin\left(\frac{x}{\sqrt{2}}\right)\Big]_0^1$$

$$\arcsin\left(\frac{1}{\sqrt{2}}\right) - \cancel{\arcsin(0)}^{\,0}$$

(4) $y = 8 + \frac{1}{2}\cosh(2x)$; $0 \leq x \leq 2$

$$y'(x) = 0 + \frac{1}{\cancel{2}}\sinh(2x)\cdot\cancel{2} = (\sinh(2x))^2 = \underbrace{\sinh^2(2x)}_{(y'(x))^2}$$

$$L = \int_0^2 \sqrt{\sinh^2(2x) + 1}\,dx$$

$\sinh^2(x) - \cosh^2(x) = 1$

$\sinh^2(x) + 1 = \cosh^2(x)$

$u = 2x$

$du = 2dx = \frac{1}{2}du$

$$= \frac{1}{2}\int_0^2 \sqrt{\sinh^2(u) + 1}\,du = \frac{1}{2}\int_0^2 \sqrt{\cosh^2(u)}\,du = \frac{1}{2}\int_0^2 \cosh(u)\,du$$

$$= \frac{1}{2}\sinh(u) = \frac{1}{2}\sinh(2x)\Big|_0^2 =$$

$$= \frac{1}{2}\left[(\sinh(2\cdot 2)) - \cancel{(\sinh(2\cdot 0))}\right] = \frac{1}{2}(\sinh(4))$$

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$y = \ln(1 - x^2)$; $0 \leq x \leq \frac{1}{8}$

$y'(x) = \frac{2x}{1 - x^2} = \left(\frac{2x}{1-x^2}\right)^2 = \frac{4x^2}{(1-x^2)}$

$L = \int_0^{1/8} \underbrace{\sqrt{\left(\frac{2x}{1-x^2}\right)^2 + 1^2}} =$

$\underbrace{\sqrt{\frac{4x^2}{(1-x^2)^2} + \frac{(1-x^2)^2}{(1-x^2)^2}}}$

$\sqrt{\frac{4x^2 + 1 - 2x^2 + x^4}{(1-x^2)^2}} = \sqrt{\frac{x^4 + 2x^2 + 1}{x^4 + 2x}} =$

$= \frac{\sqrt{(x^2+1)^2}}{\sqrt{(1-x^2)^2}} = \frac{x^2+1}{1-x^2} = \frac{x^2+1}{1-x^2} + \frac{1-x^2}{1-x^2} - 1$

$= \frac{\cancel{x^2} + 1 + 1 - \cancel{x^2}}{1-x^2} - 1 = \frac{2}{1-x^2} - 1 = \frac{2}{-(x+1)(x-1)} - 1$

$L = \int_0^{1/8} - \frac{2}{(x+1)(x-1)} - 1$