David Corzo 2019-11-11

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1. Paramétricas

$X = \sin^3\theta$; $y = \cos^3\theta$

a) derivada $\frac{dy}{dx}$

$\frac{y'(\theta)}{x'(\theta)}$

$\Rightarrow y(\theta) = (\cos(\theta))^3$

$y'(\theta) = -3(\cos(\theta))^2 \cdot \sin\theta$

$\Rightarrow x'(\theta) = (\sin(\theta))^3$

$= 3(\sin\theta)^2 \cdot \cos\theta$

$$= \frac{-3\cos^2\theta \cdot \sin\theta}{3\sin^2\theta \cdot \cos\theta} =$$

$$= -\frac{\cancel{3}\cos\theta \cdot \cancel{\cos\theta} \cdot \cancel{\sin\theta}}{\cancel{3}\sin\theta \cdot \cancel{\sin\theta} \cdot \cancel{\cos\theta}} = -\frac{\cos\theta}{\sin\theta} = \boxed{-\cot\theta}$$

b) Ec. recta tangente

$y - y_0 = m(x - x_0)$

$$y\left(\frac{\pi}{6}\right) = \left(\cos\frac{\pi}{6}\right)^3 = \left(\frac{\sqrt{3}}{2}\right)^3 = \left(\frac{(3)^{\frac{3}{2}}}{(2)^3}\right) = \boxed{\frac{\sqrt{27}}{8}}$$

$$x\left(\frac{\pi}{6}\right) = \left(\sin\frac{\pi}{6}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \boxed{\frac{1}{8}}$$

$$\boxed{y = -\sqrt{3}\left(x - \frac{1}{8}\right) + \frac{\sqrt{27}}{8}}$$

$$m\Big]_{\frac{\pi}{6}} = -\cot\theta = -\frac{\cos\theta}{\sin\theta}$$

$$= -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\cancel{2}\sqrt{3}}{\cancel{2}} = -\sqrt{3}$$

c)

$$y'(\theta) = -3\cos^2\theta \cdot \sin\theta = 0$$

$$-3\left(1 - \sin^2\theta\right)\sin\theta = 0$$

$$(1 - \sin^2\theta)\sin\theta = 0$$

$$\sin\theta - \sin^3\theta = 0$$

$$\cancel{\tfrac{\pi}{2}, 0, \tfrac{3\pi}{2}, \pi}$$

$$x'(\theta) = 3\sin^2\theta \cdot \cos\theta = 0$$

$$3(1 - \cos^2\theta)\cos\theta = 0$$

$$(1 - \cos^2\theta)\cos\theta = 0$$

$$\cos\theta - \cos^3\theta = 0$$

$$\cancel{\tfrac{\pi}{2}, \tfrac{3\pi}{2}}, 0$$

$$\frac{dy}{dx} = -\cot\theta = -\frac{\cos\theta}{\sin\theta}$$

sino se indefine en $0, \pi$ entonces tiene tangentes en $0, \pi$ dado tambien al denominador tampoco volverse 0.

tangente vertical

$$y'(\theta) = 0$$

$$x'(\theta) \neq 0$$

21 pts

② Longitud paramétricos:

$$x = t^3 - 3t \quad ; \quad y = 3t^2$$

$$-1 \leq t \leq 3$$

$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$x'(t) = 3t^2 - 3 \qquad y'(t) = 3 \cdot 2t = 6t$$

$$= \int_{t_1}^{t_2} \sqrt{\underbrace{(3t^2-3)^2}_{9t^4 - 2\cdot 3\cdot 3t^2 + 9} + (6t)^2}\, dt \quad \to +36t^2$$

$$9t^4 - 2\cdot3\cdot3t^2 + 9 + 36t^2$$

$$9t^4 - 18t^2 + 9 + 36t^2$$

$$9t^4 + 18t^2 + 9$$

$$9(t^4 + 2t^2 + 1)$$

$$9(t^2+1)^2 \Rightarrow \sqrt{9(t^2+1)^2} = \sqrt{9}\cdot\sqrt{(t^2+1)^2}$$

$$= \sqrt{3^2}\cdot\sqrt{(t^2+1)^2}$$

$$= 3\cdot(t^2+1)$$

$$\begin{array}{r} 36 \\ -18 \\ \hline \end{array}$$

$$\begin{array}{c} 30+6 \\ -(10+8) \\ 20-2 \\ \underbrace{\quad}_{18} \end{array}$$

$$L = \int_{-1}^{3} 3t^2 + 3\, dt = \left[\frac{3}{3}t^3 + 3t\right]_{-1}^{3} = t^3 + 3t$$

$$L = \left[\left(\underset{27}{3^3} + \underset{9}{3(3)}\right) - \left(\underbrace{\underbrace{(-1)^3}_{-1} + \underbrace{3(-1)}_{-3}}_{-(-4)}\right)\right] =$$

$$27 + 9 \quad + 4 = 27 + 4 = 31 + 9 = \boxed{40}$$

25pts

③ Paramétricos

$x = 4t - t^3 \quad ; \quad y = 7.5t^2$

\# Interceptos

$4t - t^3 = 0 \qquad\qquad 7.5t^2 = 0$

$t(4-t^2) = 0 \qquad\qquad t^2 = 0$

$\boxed{t = 0} \qquad\qquad \boxed{t = 0}$

$4 - t^2 = 0$

$4 = t^2$

$\pm\sqrt{4} = t \qquad\qquad x'(t) = 4 - 3t^2$

$\boxed{\pm 2 = t}$

$$A = \int_{t_1}^{t_2} \underbrace{7.5t^2(4-3t^2)}\,dt$$

$$A = 2 * 7.5 \int_2^0 t^2(4-3t^2)\,dt$$

$$= 15 \int_2^0 4t^2 - 3t^4 = 15\left[\frac{4}{3}t^3 - \frac{3}{5}t^5\right]_2^0$$

$$= 15\left[(0) - \underbrace{\left(\frac{4}{3}(2^3) - \frac{3}{5}(2)^5\right)}\right]$$

$$\left(\frac{5\cdot 32}{5\cdot 3} - \frac{3\cdot 32\cdot 3}{3\cdot 5}\right)$$

$$= 15\left[-\left(\frac{5\cdot 32 - 9\cdot 32}{5\cdot 3}\right)\right] =$$

$32(5-9)$

$32\cdot 4$

32
4
-128
15
640
1280
1,920

$$\frac{15\cdot 128}{15} = \boxed{128}$$

25 pts

④ Longitud polares.

$r = 2 - 2\cos(\theta)$

Polares a paramétricos

$S\frac{O}{H} \quad C\frac{A}{H} \quad T\frac{O}{A}$

$C\frac{H}{O} \quad S\frac{H}{A} \quad C\frac{A}{O}$

$\sin\theta = \frac{y}{r} \Rightarrow y = \sin\theta \cdot r \Rightarrow y = \sin\theta(2 - 2\cos(\theta))$

$\cos\theta = \frac{x}{r} \Rightarrow x = \cos\theta \cdot r \Rightarrow x = \cos\theta(2 - 2\cos(\theta))$

$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} =$

$x = 2\cos\theta - 2\cos^2\theta$

$y = 2\sin\theta - 2\cos(\theta)\sin(\theta)$

$= 2\sin\theta - \sin(2\theta)$

$y'(\theta) = 2\cos\theta - \cos(2\theta) \cdot 2$

(4)

Continuación 4

$$\mathcal{L} = \int_0^{2\pi} \sqrt{(r')^2 + r^2}\, d\theta$$

$r = 2 - 2\cos\theta$

$r' = \emptyset + 2\sin\theta$

$(r')^2 = (2\sin\theta)^2 = 4\sin^2\theta$

$(r)^2 = (2 - 2\cos\theta)^2 = 4 - 2\cdot 2\cdot 2\cos\theta + 4\cos^2\theta$

$= 4 - 8\cos\theta + 4\cos^2\theta$

$(r')^2 + r = 4 - 8\cos\theta + \underbrace{4\cos^2\theta + 4\sin^2\theta}_{4}$

$= 4 + 4 - 8\cos\theta$

$= 8 - 8\cos\theta = 8\underbrace{(1 - \cos\theta)}$

$= 8\left(2\sin^2\left(\frac{\theta}{2}\right)\right)$

$= 16\sin^2\left(\frac{\theta}{2}\right)$

$\sqrt{(r')^2 + r} = \sqrt{16\sin^2\left(\frac{\theta}{2}\right)}$

$= \sqrt{4^2}\cdot\sqrt{\sin^2\left(\frac{\theta}{2}\right)}$

$= 4\sin\left(\frac{\theta}{2}\right)$

$$\mathcal{L} = \int_0^{2\pi} 4\sin\left(\frac{\theta}{2}\right) d\theta$$

$w = \frac{\theta}{2}$

$dw = \frac{1}{2}d\theta = 2dw$

$$= \int_0^{\pi} 8\sin(w)\, dw$$

$= -8\cos\theta\Big]_{\emptyset}^{2\pi} = 2\left(-8\left[(\cos\pi) - (\cos 0)\right]\right) = 2(-8[-1 - 1])$

$= 2(-8(-2)) = \boxed{32}$

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Anexos

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

Paramitricas

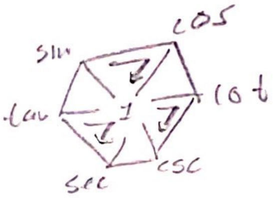
$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

Polares

$$A = \frac{1}{2}\int_a^b r^2 \, d\theta$$

$$A = \frac{1}{2}\int_a^b \underbrace{r_1^2}_{\text{alejada del origen}} - r_2^2 \, d\theta$$

$$L = \int_a^b \sqrt{(r')^2 + r^2}\, d\theta$$



$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$A = \int_{t_1}^{t_2} y \underbrace{dx}_{\text{derivada}}$$

Horizontales

$x' = 0$

$y' \neq 0$

Verticales

$y' = 0$

$x' \neq 0$