Simulacro Parcial

a) $\int x \tan^{-1} x^2 \, dx$ $\qquad \int x f(x)\, dx$ IPP.

ILATE

$y = x^2 \qquad dy = 2x\,dx$

$$\int \tan^{-1}(x^2)\,(x\,dx) = \frac{1}{2}\int \tan^{-1}(y)\,dy.$$

IPP: $u = \tan^{-1} y \qquad dv = \frac{dy}{2}$

$du = \frac{1}{1+y^2} \qquad v = \frac{y}{2}.$

$w = 1+y^2$

$dw = 2y\,dy.$

$$uv - \int v\,du = \frac{1}{2} y \tan^{-1} y - \frac{1}{2}\int \frac{y}{1+y^2}\,dy.$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\int \frac{dw}{w}$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\ln|w| + C.$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\ln|1+y^2| + C.$$

$$= \boxed{\frac{1}{2} x^2 \tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4| + C.}$$

j) $\int \frac{xe^x}{(x+1)^2}\,dx$

Tres funciones

Der.		1		
x Int.	xe^x	ó	$\frac{x}{(x+1)^2}$	x Int
				x der
Der. x Int ✓	$\frac{1}{(x+1)^2}$	|	e^x	✓

$u = xe^x \qquad dv = (x+1)^{-2}\,dx$

$du = (e^x + xe^x)\,dx, \quad v = \dfrac{(x+1)^{-1}}{-1} = \dfrac{-1}{(x+1)}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = \frac{-xe^x}{(x+1)} + \int \frac{(e^x + xe^x)\,dx}{(x+1)}$$

inspiración,

$$= \frac{-xe^x}{(x+1)} + \int e^x\,dx$$

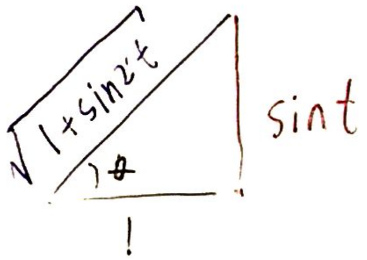
$$= \frac{-xe^x}{(x+1)} + e^x + C$$

Otras sugerencias: $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx \qquad \begin{aligned} u &= x+1 \\ du &= dx \\ x &= u-1. \end{aligned}$

$\displaystyle\int \frac{(u-1)}{u^2} e^{u-1}\,dx$ NO AYUDA. "MUCHO"

2b) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt. = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta\,d\theta.$

$\sin t = \tan\theta. \qquad \cos t\,dt = \sec^2\theta\,d\theta.$



$\sqrt{1+\sin^2 t} = \sec\theta.$

cambie los límites.

$\tan\theta = \sin \pi/2 = 1 \Rightarrow \theta_b = \pi/4.$

$\tan\theta = \sin 0 = 0 \Rightarrow \theta_a = 0$

$$\int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}.$$

$\sec 0 = 1 \quad \tan 0 = 0$

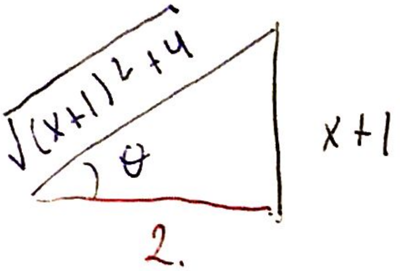
$\sec\frac{\pi}{4} = \sqrt{2} \quad \tan\frac{\pi}{4} = 1$

$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$

$= \ln|\sqrt{2} + 1| - \ln(1) = \ln(\sqrt{2}+1).$

Conto 4A. $\displaystyle\int \frac{1}{(x^2+2x+5)^2}\,dx = \int \frac{1}{[(x+1)^2+4]^{4/2}}\,dx$

$x^2+2x+1 \quad +5-1 = (x+1)^2+4$



$\tan\theta = \dfrac{x+1}{2} \qquad x+1 = 2\tan\theta.$

$dx = 2\sec^2\theta\, d\theta.$

$$\frac{((x+1)^2+4)^{1/2}}{2} = \sec\theta.$$

$$[((x+1)^2+4)]^2 = 16\sec^4\theta.$$

$$\int \frac{1}{[(x+1)^2+4]^2}\,dx = \int \frac{2\sec^2\theta\, d\theta}{16\sec^4\theta} = \frac{1}{8}\int \frac{1}{\sec^2\theta}\, d\theta$$

$$= \frac{1}{8}\int \cos^2\theta\, d\theta. = \frac{1}{16}\int 1+\cos(2\theta)\, d\theta.$$

$$= \frac{1}{16}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C.$$

$2\sin\theta\cos\theta$

$$= \frac{1}{16}(\theta + \sin\theta\cos\theta) + C.$$

$\tan\theta = \dfrac{x+1}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{x+1}{2}\right).\quad \sin\theta = \dfrac{x+1}{\sqrt{x^2+2x+5}}$

$\cos\theta = \dfrac{2}{\sqrt{x^2+2x+5}}$

$$\int \frac{1}{(x^2+2x+5)^2}\,dx = \frac{\tan^{-1}}{16}\left(\frac{x+1}{2}\right) + \frac{1}{16}\,\frac{(x+1)}{\sqrt{x^2+2x+5}}\,\frac{2}{\sqrt{x^2+2x+5}} + C.$$

$$\frac{1}{16}\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{8}\,\frac{x+1}{x^2+2x+5} + C.$$

IPP $\int (x-1)\sin\pi x \, dx$ $\int e^{-\theta}\cos 2\theta \, d\theta.$

$u = x-1$ $dv = \sin\pi x$

$du = dx$ $v = \frac{-1}{\pi}\cos\pi x$

$$\int (x-1)\sin\pi x \, dx = \frac{-(x-1)}{\pi}\cos\pi x + \int \frac{1}{\pi}\cos\pi x \, dx$$

$$= \frac{(1-x)}{\pi}\cos\pi x + \frac{1}{\pi^2}\sin\pi x + C.$$

Ciclico

$$* \int e^{-\theta}\cos 2\theta \, d\theta. = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta \, d\theta$$

$u = e^{-\theta}$ $dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta}d\theta.$ $v = \frac{1}{2}\sin 2\theta.$

$$\int e^{-\theta}\sin 2\theta \, d\theta. = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta \, d\theta. \; *$$

$u = e^{-\theta}$ $dv = \sin 2\theta \, d\theta.$

$du = -e^{-\theta}d\theta$ $v = -\frac{1}{2}\cos 2\theta$

$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta. - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}\int e^{-\theta}\cos 2\theta \, d\theta.$$

$$\frac{5}{4}\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta.$$

$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C_0$$