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Laboratorio # 6

David Carzo 20190432

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① ⓐ $$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^{\infty} \frac{du}{9+u^2}\cdot\frac{1}{3} = \int_{-\infty}^{\infty} \frac{3dv}{9+9v^2}\,\frac{1}{3} =$$

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$u = 3v$

$du = 3dv$

$$= \frac{1}{3}\int_{-\infty}^{\infty} \frac{3}{9(1+v^2)}\,dv =$$

$$= \frac{1}{3}\cdot\frac{1}{3}\int_{-\infty}^{\infty} \frac{1}{(1+v^2)}\,dv =$$

$$= \frac{1}{9}\tan^{-1}(v)$$

$$= \frac{1}{9}\tan^{-1}\left(\frac{u}{3}\right) = \frac{1}{9}\tan^{-1}\left(\frac{x^3}{3}\right)\Bigg]_{-\infty}^{\infty}$$

$$= \left\{\lim_{t\to\infty}\left(\frac{1}{9}\underbrace{\tan^{-1}(t^3/3)}_{\tan^{-1}(\infty)}\right)\right\} - \left\{\lim_{a\to-\infty}\left(1/9\,\underbrace{\tan^{-1}(a^3/3)}_{-\frac{\pi}{2}}\right)\right\}$$

$-\frac{\pi}{2}$

$$= \left(\frac{1}{9}\cdot\frac{\pi}{2}\right) - \left(-\frac{1}{9}\cdot\frac{\pi}{2}\right)$$

$$= \frac{\pi}{18} + \frac{\pi}{18} = \frac{\pi}{9}$$ A □

① • La integral converge por que al ser evaluada en ∞ y $-\infty$ queda una constante.

② Es impropia por los límites ser infinitos

1\. b) $\displaystyle\int_{-2}^{3} \frac{dx}{\sqrt[4]{x+2}} =$

① asíntota en -2

$x + 2 = 0$

$x = -2$

$$\int_{-2}^{3} (x+2)^{-1/4}\,dx = (x+2)^{-\frac{1}{4}+\frac{4}{4}} \quad \left.\frac{4(x+2)^{3/4}}{3}\right]_{-2}^{3}$$

③ Es impropia por un límite (-2) indefinir la función

$$= \left\{\frac{4}{3}\sqrt[4]{(3+2)^3}\right\} - \left\{\cancel{\frac{4}{3}\sqrt[4]{(-2+2)^3}}\right\}$$

$$\underline{\frac{4}{3}\sqrt[4]{5^3}}$$

② convergente porque existe

1\. c. $\displaystyle\int_{0}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -2\int_{0}^{\infty} e^{u}\,du = -2 \cdot e^{u}\Big]_{0}^{\infty} = -2e^{-\sqrt{x}}\Big]_{0}^{\infty}$

$u = -(x)^{1/2} \qquad u(0) = 0$

$$= \underbrace{\left\{\lim_{a\to\infty}(-2e^{-\sqrt{a}})\right\}}_{0} - \underbrace{\left\{\lim_{a\to 0}(-2e^{-\sqrt{0}})\right\}}$$

$du = -\frac{1}{2}(x)^{-1/2}\,dx \qquad = 0 - (-2) = \underline{2}$

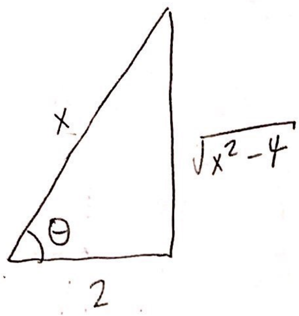
$du = -\frac{1}{2\sqrt{x}}\,dx$

$-2\,du = \frac{1}{\sqrt{x}}\,dx$

① La integral converge
② es impropia por el límite superior ser infinito & A.V. en 0

1 d) $$\int_2^{\infty} \frac{1}{x\sqrt{x^2-4}}\,dx =$$

① Se indefine en $x = 2$ y está evaluada en infinito por eso es impropia.



$$c^2 = a^2 + b^2$$
$$-a^2 = -c^2 + b^2$$
$$a^2 = c^2 - b^2$$

$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

$$\tan\theta = \frac{\sqrt{x^2-4}}{2} \qquad \sec\theta = \frac{x}{2} \qquad \theta = \operatorname{arcsec}\left(\frac{x}{2}\right)$$

$$2\tan\theta = \sqrt{x^2-4} \qquad 2\sec\theta = x$$

$$2\sec\theta\tan\theta\,d\theta = dx$$

$$= \int_2^{\infty} \frac{2\cancel{\sec\theta\tan\theta}\,d\theta}{2\cancel{\sec\theta}\,2\cancel{\tan\theta}} = \int_2^{\infty} \frac{2}{4}\,d\theta = \int_2^{\infty} \frac{1}{2}\,d\theta = \frac{1}{2}\int_1^{\infty} 1\,d\theta$$

$$= \frac{1}{2}\theta = \frac{1}{2}\operatorname{arcsec}\left(\frac{x}{2}\right)\Big]_1^{\infty} = \left\{\lim_{a\to\infty}\left(\frac{1}{2}\sec^{-1}\left(\frac{\infty}{2}\right)\right)\right\} - \left\{\cancel{\lim_{b\to 2^+}\left(\frac{1}{2}\sec^{-1}(1)\right)}^{0}\right\}$$

$$= \left\{\frac{1}{2}\cdot\frac{\pi}{2}\right\} - \{0\}$$

$$\frac{1}{\cos(\theta)} = 1 \qquad = \frac{\pi}{4} \ \square$$

② Convergente

e) $\int_0^2 z^2 \ln(z)\,dz =$

① es impropia por $\ln(0)$ esta indefinido

$u = \ln(z) \qquad dv = z^2$

$du = \frac{1}{z}\,dz \qquad v = \frac{z^3}{3}$

$= \ln(z)\frac{z^3}{3} - \int_0^2 \frac{z^3}{3}\cdot\frac{1}{z}\,dz$

$= \ln(z)\frac{z^3}{3} - \int_0^2 \frac{z^2}{3}\,dz = \ln(z)\frac{z^3}{3} - \frac{1}{3}\frac{z^3}{3} = \ln(z)\frac{z^3}{3} - \frac{z^3}{9}\Big]_0^2 =$

$\overset{\text{con } 0}{=} \underbrace{\left\{\lim_{a\to 0^+}\left(\overbrace{\ln(z)}^{-\infty}\overbrace{\frac{z^3}{3}}^{0}\right)\right\}}_{\frac{1}{3}\frac{\ln(z)}{z^{-3}} \overset{LH}{=} \frac{1}{3}\frac{\left(\frac{1}{z}\right)}{\left(\frac{1}{z^4}\right)} = \frac{1}{3}\frac{z^4}{z} = \frac{z^3}{3} = 0} - \left\{\lim_{a\to 0^+}\left(\frac{z^3}{9}\right)\right\}$

$\overset{\text{con } 0}{=} \{0 - \overset{0}{0}\}$

$\overset{\text{con } 2}{=} \left\{\ln(2)\frac{8}{3} - \frac{8}{9}\right\} - \{0\}$

$\therefore \ \underline{\ln(2)\frac{8}{3} - \frac{8}{9}}$ ② La integral es convergante

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$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

a) $f(t) = e^t$

$$F(s) = \int_0^\infty e^t\, e^{-st}\, dt$$

$$F(s) = \int_0^\infty e^t\, e^{-st}\, dt$$

$$F(s) = \int_0^\infty e^{t-st}\, dt = \int e^{t(1-s)} = \frac{e^{t(1-s)}}{1-s}\Bigg]_0^\infty$$

s > 1
como ser mayor a uno

$$\left\{\lim_{a\to\infty} \underbrace{\cancel{\frac{e^{\overbrace{\infty(1-s)}}}{1-s}}}_{0}\right\} - \left\{\frac{\cancel{e^{0}}\,1}{1-s}\right\} = \frac{1}{s-1}$$

b) $f(t) = t$

$$F(s) = \int_0^\infty t\, e^{-st}\, dt = \int_0^\infty -\frac{u}{s}\cdot e^{u}\cdot -\frac{du}{s} = \int_0^\infty \frac{+\, u e^{u}\, du}{s^2}$$

$-\frac{u}{s} = t$ $\quad u = -st$

$du = -s\, dt$

$-\frac{du}{s} = dt$

$$= \frac{1}{s^2}\int_0^\infty u e^u\, du$$

$\sigma = u \quad d\beta = e^u$

$d\sigma = du \quad \beta = e^u$

$$u e^u - \int e^u\, du$$

$$u e^u - e^u\Big] =$$

$$= \frac{1}{s^2}\left(-st e^{-st} - e^{-st}\right)\Bigg]_0^\infty$$

$$\frac{1}{s^2}\left\{\lim_{a\to\infty}\left(\frac{-st}{e^{st}}\right) - \underbrace{\lim_{a\to\infty}\left(e^{-st}\right)}_{0}\right.$$

$$\left.\lim_{a\to\infty}\frac{-s}{e^{st}\cdot s}\right\} 0$$

$$\frac{1}{s^2}(0) - \frac{1}{s^2}\left(\cancel{-s(0)e^{s(0)}}\,0 - \cancel{e^{-s(0)}}\,1\right) = -\frac{1}{s^2}\cdot -1 = \frac{1}{s^2}$$