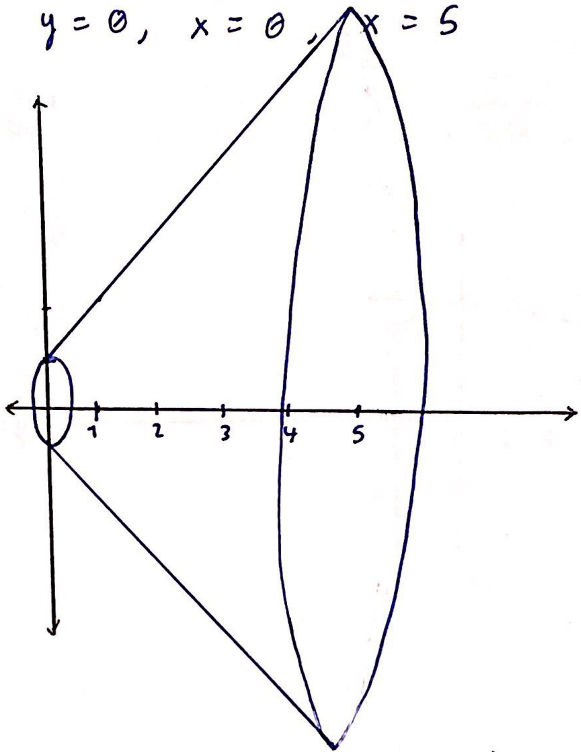
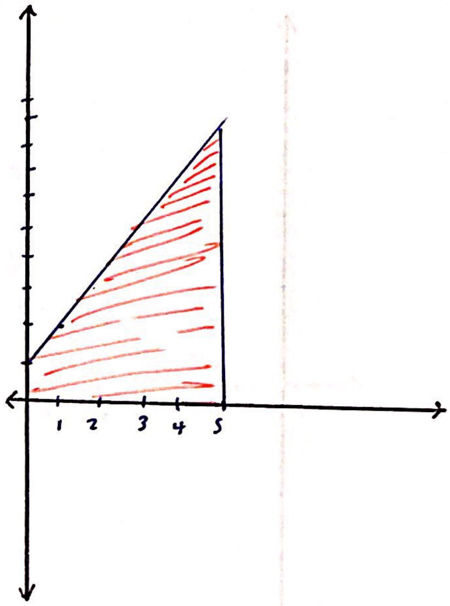
2019-09-27

Webassign 6.2 - 6.3 Volumenes

1) $y = x + 1$; $y = 0$, $x = 0$, $x = 5$





integro respecto de x

$$V = \pi \int_0^5 (x+1)^2 \, dx$$

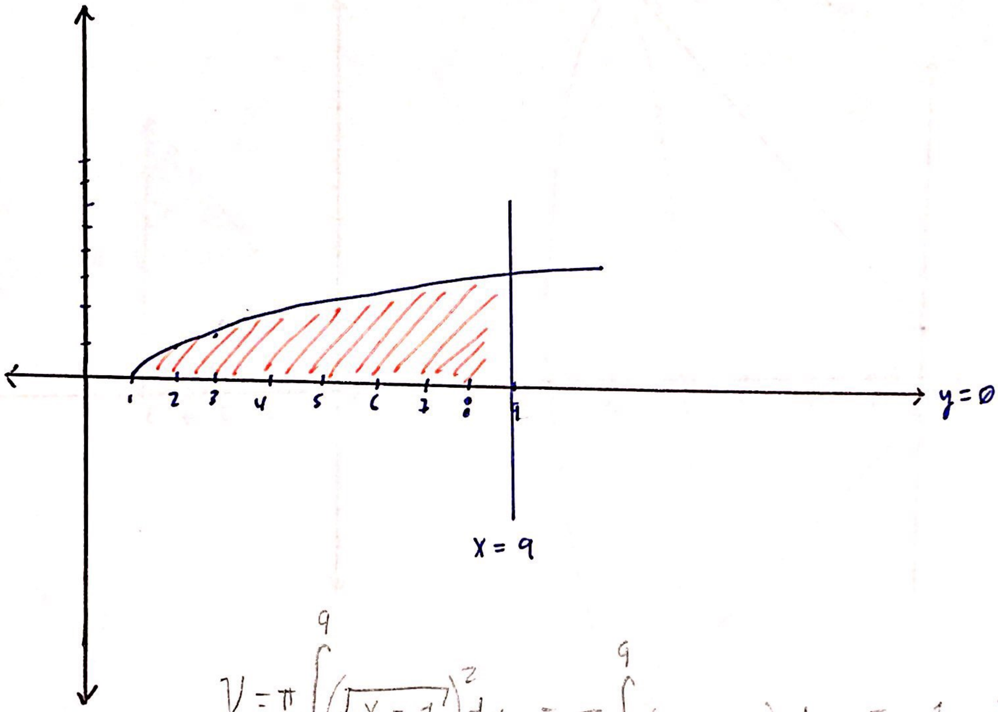
$$= \pi \int_0^5 x^2 + 2x + 1 \, dx$$

$$= \pi \left[\frac{1}{3}x^3 + x^2 + x \right]_0^5 =$$

$$= \pi \left[\left(\frac{(5)^3}{3} + (5)^2 + (5) \right) \right]$$

$$= \pi \, \frac{215}{3}$$

② $y = \sqrt{x-1}$, $y = 0$; $x = 9$; about x-axis



$$V = \pi \int_1^9 \left(\sqrt{x-1}\right)^2 dx = \pi \int_1^9 (x-1)\,dx = \frac{1}{2}x^2 - x \Big]_1^9 =$$

$$= \pi\left[\left(\frac{1}{2}(9)^2 - (9)\right) - \left(\frac{1}{2}(1)^2 - 1\right)\right]$$

$$= \pi\left[\left(\frac{63}{2}\right) - \left(-\frac{1}{2}\right)\right] = \pi\,\boxed{32}$$

3. $x = 2\sqrt{5y}$, $x = 0$; $y = 3$

$$\frac{x}{2} = \sqrt{5y}$$

$$\left(\frac{x}{2}\right)^2 = 5y$$

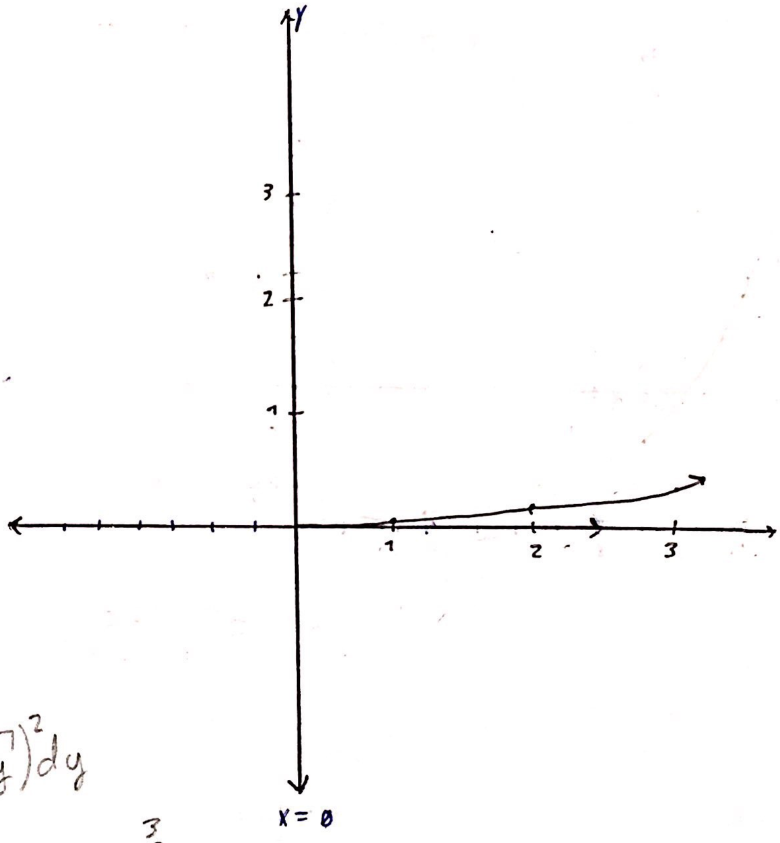
$$\frac{x^2}{4 \cdot 5} = y$$

$$\frac{x^2}{20} = y$$

$$\frac{x^2}{20} = 3$$

$$x^2 = 60$$

$$x = \sqrt{66}$$



$$V = \pi \int_0^3 \left(2\sqrt{5y}\right)^2 dy$$

$$= \pi \int_0^3 4(5y)\, dy = \pi \int_0^3 10y\, dy$$

$$= 20\pi \int_0^3 y\, dy = 20\pi \left(\frac{1}{2}y^2\right)\Big]_0^3$$

$$= 20\pi \left[\left(\frac{1}{2}(3)^2\right) - \cancel{\left(\frac{1}{2}(0)\right)}^{0}\right]$$

$$= 20\pi \, \frac{9}{2} = 10 \cdot 9\pi = \underline{90\pi}$$

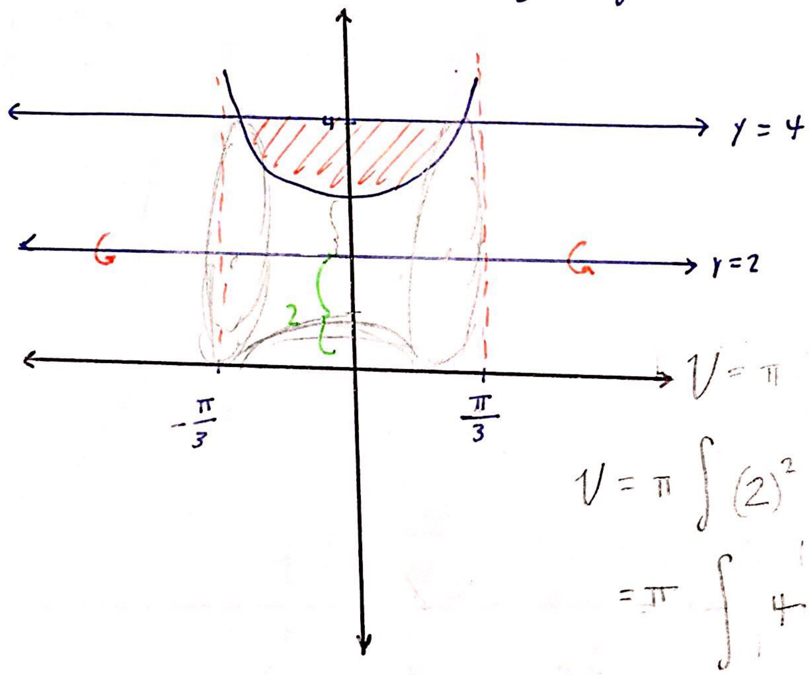
④ Step by step =

$$V = 36\pi \int_0^1 (x^2 - x^{12})\,dx$$

$$= 36\pi \left(\frac{1}{3}x^3 - \frac{1}{13}x^{13} \right)\Bigg]_0^1$$

$$= 36\pi \left(\frac{1}{3} - \frac{1}{13} \right) = 36\pi \left(\frac{10}{39} \right) = \frac{360}{39}\pi = \frac{120}{13}\pi$$

⑤ $y = 2 + \sec(x)$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$, $y = 4$



$y_{int} = -2 + (2 + \sec x)$

$y_{ext} = -2 + 4$

$$V = \pi \int (-2+4)^2 - (-2 + (2 + \sec x))^2\,dx$$

$$V = \pi \int (2)^2 - (-2 + 2\sec x)^2\,dx$$

$$= \pi \int 4 - (\sec x)^2\,dx$$

$$= 2\pi \left[(4x) - (\tan x) \right]_0^{\pi/3}$$

$$= 2\pi \left\{ \left[\left(\frac{4\pi}{3} \right) - \left(\tan(\pi/3) \right) \right] - [0] \right\}$$

$$= 2\pi \left(\frac{4\pi}{3} - \sqrt{3} \right)$$

⑥

$$h = 5e^{-x^2} \qquad r_{int} = x$$

$$V = 2\pi \int_0^1 (x)(5e^{-x^2})\,dx =$$

$$u = -x^2$$

$$du = -2x\,dx$$

$$\frac{-du}{2} = x\,dx$$

$$= 2\pi \int_0^1 5e^{u} \cdot -\frac{du}{2} = -\frac{2 \cdot 5\pi}{2} \int_0^1 e^{u}\,du$$

$$= -5\pi \int_0^1 e^{u}\,du = -5\pi\, e^{u}\Big]_0^1 =$$

$$= -5\pi\left[\left(e^{-1^2}\right) - \left(e^{-0^2}\right)\right]$$

$$= -5\pi\left[e^{-1} - e^{0}\right]$$

$$= -5\pi\left[e^{-1} - 1\right]$$

$$= \frac{-5\pi}{e} + 5\pi$$

A

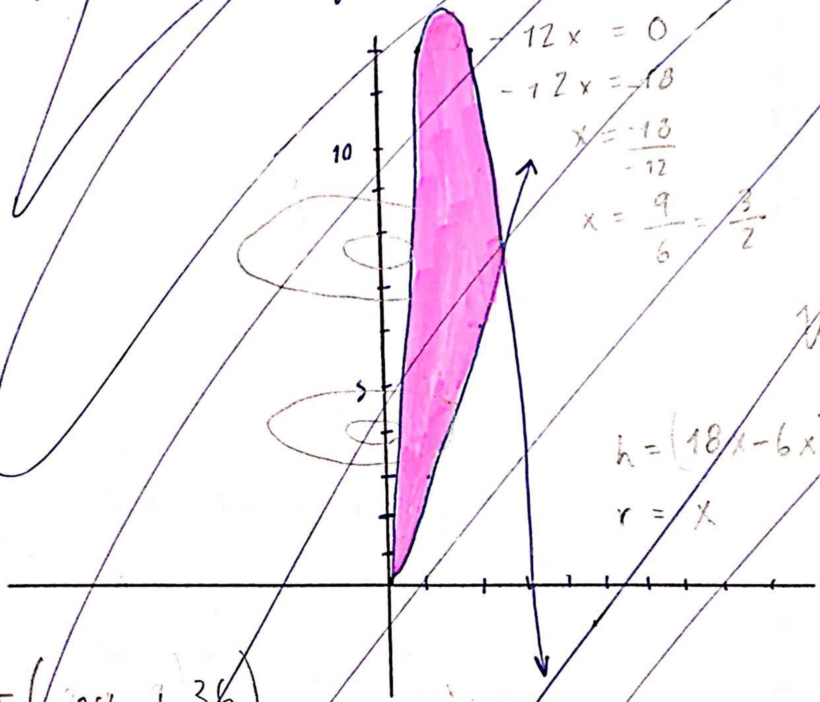
$$-2\pi \int 12 e^{-x^2} x\,dx = 2\pi \cdot 12 \cdot -\frac{1}{2} \int e^u\,du$$

$u = -x^2$ $\quad -24\pi \cdot \frac{1}{2}$

$-\frac{du}{2} = x\,dx$ $\quad -12\pi\, e^u + C \Big]_0^1$

$$-12\pi \left[e^{-1} - e^0\right]$$

8) $y = 3x^2$; $y = 18x - 6x^2$



$-12x = 0$

$-12x = -18$

$x = \frac{-18}{-12}$

$x = \frac{9}{6} = \frac{3}{2}$

$18 - 6 = 12$

$36 - 24 = 12$

$$V = \pi \int_0^2 \left((18x - 6x^2) - (3x^2)\right)x\,dx$$

$h = (18x - 6x^2) - (3x^2)$

$r = x$

$$V = \pi \int_0^2 18x - 6x^2 - 3x^2\,dx$$

$$= \pi \int -9x^2 + 18x$$

$$\pi\left(-\frac{9}{3}x^3 + \frac{18}{2}x^2\right)\Big]_0^2$$

$= \pi(-24 + 36)$

$= \pi(12) = 12\pi$

$3x^2 = 18x - 6x^2$

$3x^2 = 3x(6 - 2x)$

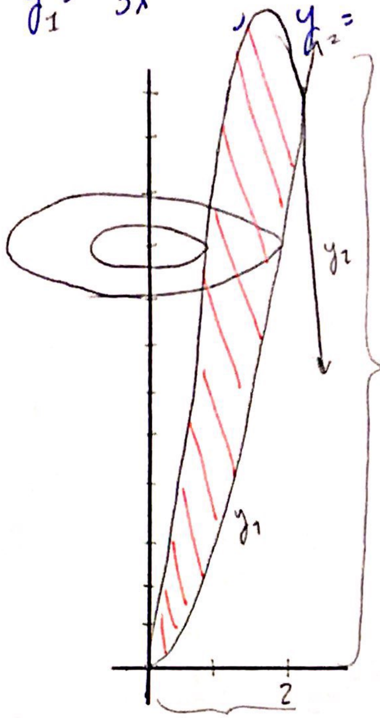
$\frac{3x^2}{3x} = 6 - 2x$; $x = 6 - 2x$

$x + 2x = 6$

$3x = 6$

$x = \frac{6}{3} = 2$

$y_1 = 3x^2$; $y_2 = 18x - 6x^2$



$h = (18x - 6x^2) - (3x^2)$

$h = 18x - 9x^2$

$3x^2 = 18x - 6x^2$

$3x^2 = 6(3x - x^2)$

$\frac{3x^2}{6} = 3x - x^2$

$\frac{1}{2}x^2 - 3x + x^2 = 0$

$\frac{3}{2}x^2 - 3x = 0$

$3x\left(\frac{1}{2}x - 1\right) = 0$

$3x = 0$ $\qquad$ $\frac{1}{2}x - 1 = 0$

$\boxed{x = 0}$ $\qquad$ $\frac{1}{2}x = 1$

$\boxed{x = 2}$

$3(0)^2$

$(18x - 6x^2)'$

$18 - 12x = 0$

$-12x = -18$

$x = \frac{18}{12} = \frac{9}{6} = \frac{3}{2}$

$$V = 2\pi \int_0^2 x(18x - 9x^2)\,dx$$

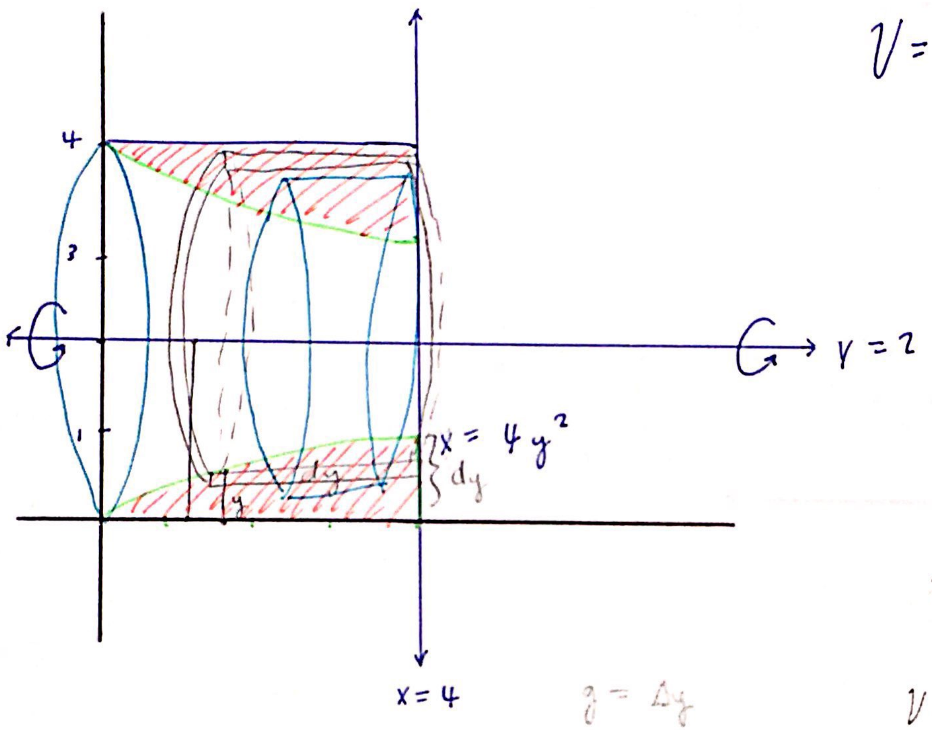
$$= 2\pi \int_0^2 9(2x^2 - x^3)\,dx$$

$$= 18\pi \int_0^2 (2x^2 - x^3)\,dx = 18\pi \left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big]_0^2$$

$$18\pi\left[\left(\frac{2(2)^3}{3} - \frac{(2)^4}{4}\right) - (0)\right] = 18\pi \cdot \frac{4}{3} = \frac{72}{3}\pi = \underline{24\pi}$$

(9)

$$V = 2\pi \int_{a}^{b} r\,h\,dx$$



$A = 2\pi r h g$

$g = \Delta y$
$h = 4 - 4y^2$
$r = 2 - y$

$$V = 2\pi \int_{0}^{1} (2-y)(4-4y^2)\,dy$$

$$V = 2\pi \int_{0}^{1} (8 - 8y^2 - 4y + 4y^3)\,dy$$

$$= 2\pi \int_{0}^{1} (4y^3 - 8y^2 - 4y + 8)\,dy$$

$$= 8\pi \left[\left(\frac{4}{4}y^4 - \frac{8}{3}y^3 - \frac{4}{2}y^2 + 8y \right) \right]_{0}^{1}$$

$$= 2\pi \left[\left(1 - \frac{8}{3} - \frac{4}{2} + 8 \right) - (0) \right] = 2\pi \cdot \frac{13}{3} = \frac{26}{3}\pi$$

(a) $x = 4y^2$, $y \geq 0$, $x = 4$; about $y = 2$

$\pm\sqrt{\frac{1}{4}x} = y$

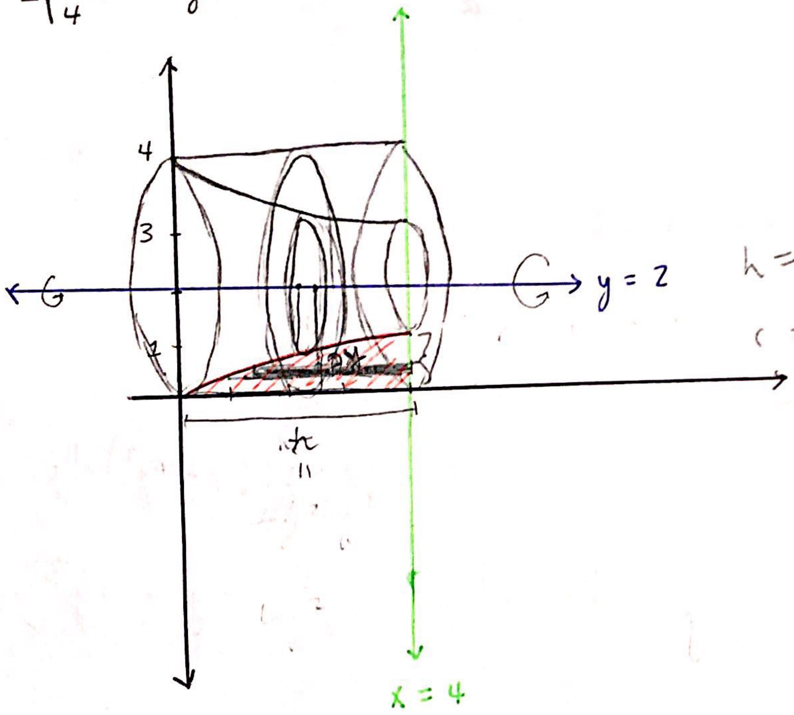
$V = 2\pi \int_a^b r\,h\,dy$

$4y^2 = 4$

$4(y^2 - 1) = 0$

$y^2 = 1$

$y = \pm\sqrt{1}$



$y = \sqrt{1/4\,x}$

$h = 2 - \sqrt{1/4\,x}$

$r = 2 - y$