Lunes 26 de agosto Simulacro Parcial
Lunes 3 de septiembre Parcial 1, capítulos 5 y 7.
Libro Págs. 9-70.

Integrales Trigonométricas.

Forma $\int \operatorname{sen}^n x \cos^m x \, dx$

$\int \sec^n x \tan^m x \, dx$

$\int \csc^n x \cot^m x \, dx$

$d(\csc x) = -\csc x \cot x$ aparte $\csc x \cot x$. Pág 50.

$\cot^2 x = \csc^2 x - 1$ $u = \csc x$ $du = -\csc x \cot x$.

$d(\cot x) = -\csc^2 x$

$\csc^2 x = \cot^2 x + 1$ $u = \cot x$, $du = -\csc^2 x$.

$\sec^2 x = \tan^2 x + 1$

Ejercicio 4: Integre. $\cot x \csc x$ ó $\csc^2 x$

a $\int \cot^2 x \csc^4 x \, dx = \int \cot^2 x \csc^2 x \csc^2 x \, dx.$

$\csc^2 x = \cot^2 x + 1$ $= \int \cot^2 x (\cot^2 x + 1) \csc^2 x \, dx$

$u = \cot x$, $du = -\csc^2 x \, dx$ $= -\int u^2 (u^2 + 1) \, du$

$= \int (-u^4 - u^2) \, du.$

$= -\frac{1}{5} u^5 - \frac{1}{3} u^3 + C.$

$= -\frac{1}{5} \cot^5 x - \frac{1}{3} \cot^3 x + C.$

b. $\int \csc^3 x \cot^3 x \, dx = \int \csc^2 x \cot^2 x (\csc x \cot x \, dx)$

$\cot^2 x = \csc^2 x - 1$ $\quad = \int \csc^2 x (\csc^2 x - 1)(\csc x \cot x \, dx)$

$u = \csc x, \ du = -\csc x \cot x \, dx$ $\quad = -\int u^2 (u^2 - 1) \, du.$

$$= -\int (u^4 - u^2) \, du.$$

$$= -\left(\frac{u^5}{5} - \frac{u^3}{3} + c \right)$$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C.$

$$\boxed{= -\frac{\csc^5 x}{5} + \frac{\csc^3 x}{3} - C.}$$

c. $\int \csc x \, dx = \int \csc x \frac{(\csc x + \cot x)}{\cot x + \csc x} \, dx$

$$= \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x} \, dx$$

$u = \cot x + \csc x$ $\quad du = -(\csc^2 x + \csc x \cot x) \, dx$

$$= -\int \frac{du}{u} = -\ln|u| + C.$$

$$= -\ln|\cot x + \csc x| + C.$$

$\int \sec^3 x \, dx = \frac{1}{2} d(\sec x) + \frac{1}{2} \int \sec x \, dx$

$$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C.$$

$\int \csc^3 x \, dx = \frac{1}{2} d(\csc x) + \frac{1}{2} \int \csc x \, dx$

$$= -\frac{1}{2} \csc x \cot x - \frac{1}{2} \ln|\cot x + \csc x| + C.$$

Integrales de la forma $\int \sin(mx)\cos(px)\,dx$ utilice la identidad trigonométrica apropiada.

$$\sin(mx)\cos(px) = \frac{1}{2}[\sin(m-p)x + \sin(m+p)x].$$

$$\sin(mx)\sin(px) = \frac{1}{2}[\cos(m-p)x - \cos(m+p)x]$$

$$\cos(mx)\cos(px) = \frac{1}{2}[\cos(m-p)x + \cos(m+p)x].$$

se pueden integrar.

Ejercicio 5: Evalúe. Pág 51.

a. $$\int_0^{\pi/4} \sin 8x \cos 4x\,dx = \frac{1}{2}\int^{\pi/4} (\sin 4x + \sin 12x)\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{4}\cos 4x\Big]_{\pi/4}^{0} + \frac{1}{2}\cdot\frac{1}{12}\cos(12x)\Big]_{\pi/4}^{0}$$

$$= \frac{1}{8}(\cos 0 - \cos\pi) + \frac{1}{24}(\cos 0 - \cos 3\pi)$$

$$= \frac{2}{8} + \frac{2}{24} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}.$$

b. $$\int_{-\pi}^{\pi} \underbrace{\cos mx}_{\text{PAR}}\ \underbrace{\cos nx}_{\text{PAR.}}\,dx = 2\int_0^{\pi} \cos mx \cos nx\,dx$$

$$= \int_0^{\pi} \cos(m-n)x + \cos(m+n)x\,dx$$

m, n son enteros diferentes

$$= \frac{1}{m-n}\sin(m-n)x + \frac{\sin(m+n)x}{m+n}\Big]_0^{\pi}$$

$$= \frac{1}{m-n}(\sin(m-n)\pi - \sin 0) + \frac{1}{m+n}(\sin(m+n)\pi - \sin 0)$$

$\sin K\pi = 0$ múltiplos de $180°$ son iguales a cero

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = 0.$$

si $m=n$ $\displaystyle\int_{-\pi}^{\pi} \cos mx \cos mx \, dx = 2\int_0^{\pi} \cos^2 mx \, dx$

$$= \int_0^{\pi} 1 + \cos(2mx) \, dx$$

$$x + \frac{\sin(2mx)}{2m}\Big]_0^{\pi} = \pi + \frac{\sin(2m\pi)}{2m} - 0 - 0$$

$360°$

$$\int_{-\pi}^{\pi} \cos^2 mx \, dx = \pi.$$

Pág 51. $\displaystyle\int_{-\pi}^{\pi} \sin 8x \cos 4x \, dx = 0$

impar.

impar * par

Área de un Círculo de Unitario sin utilizar Geometría.

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2.$$

$$y = \pm\sqrt{1-x^2} \quad \text{en } [-1,1]$$

$$A = \int_{-1}^{1} \sqrt{1-x^2} \, dx - \int_{-1}^{1} -\sqrt{1-x^2} \, dx$$

$$A = 2\int_{-1}^{1} \sqrt{1-x^2} \, dx = 4\int_0^{1} \sqrt{1-x^2} \, dx$$

$u = 1 - x^2 \quad du = -2x\,dx$ } No Sustitución

$u = \sqrt{1-x^2} \quad dv = dx$

$du = \dfrac{-x}{\sqrt{1-x^2}} \quad v = x$ } No IPP

$A = 4\int_0^1 \sqrt{1-x^2}\,dx \qquad x = \sin\theta \qquad dx = \cos\theta\,d\theta.$

$1 - x^2 = 1 - \sin^2\theta = \cos^2\theta.$

$A = 4\int_0^{\pi/2} \underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta.}\cos\theta\,d\theta = \boxed{4\int_0^{\pi/2} \cos^2\theta\,d\theta.}$

$\sin\theta = x \qquad \sin\theta = 1 \Rightarrow \theta = \sin^{-1}(1) = \pi/2.$

$\sin\theta = 0 \qquad \theta = 0$

$A = 4\int_0^{\pi/2} \cos^2\theta\,d\theta = \frac{4}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta.$

$A = 2\left(\theta + \frac{1}{2}\sin 2\theta\Big]_0^{\pi/2}\right) = 2\left(\frac{\pi}{2} - \frac{1}{2}\cancel{\sin\pi} - 0 - 0\right)$

$A = \frac{2\pi}{2} = \pi.$ ✓ consistente con la geometría $A = \pi(1)^2.$

7.3 Sustitución Trigonométrica o Sustitución Inversa.

$\int f(x)\,dx = \int \overbrace{f(u(\theta))}\,u'(\theta)\,d\theta.$

$x = u(\theta) \qquad dx = u'(\theta)\,d\theta$

utilice identidades o triángulos para simp. $f(u(\theta))$.

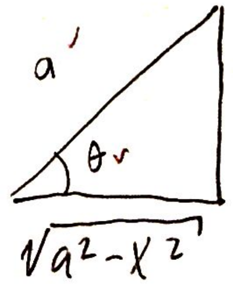
$u = \sin\theta,\ \tan\theta,\ \sec\theta.$

Forma $\sqrt{a^2-x^2}$

Hipotenusa: a

Cateto O: $x = a\sin\theta.$

Cateto A: $\sqrt{a^2-x^2} = a\cdot\cos\theta.$



$\sin\theta = \dfrac{C.O}{H}$

$\sin\theta = \dfrac{x}{a}.$

$\cos\theta = \dfrac{\sqrt{a^2-x^2}}{a}.$

Sustitución $x = a\sin\theta.$

Diferencial $dx = a\cos\theta\, d\theta.$

Sustituya. $\sqrt{a^2-x^2} = a\cdot\cos\theta.$

$\sqrt{a^2-a^2\sin^2\theta} - \sqrt{a^2\cos^2\theta} = a\cdot\cos\theta.$

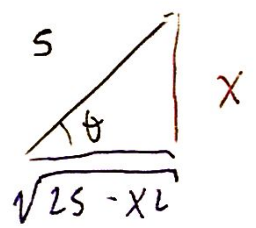
Ejercicio 1: Evalúe.

a. $\displaystyle\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{\cancel{5\cos\theta}}\, \cancel{5\cos\theta}\, d\theta. = 5\int \sin\theta\, d\theta.$

$= -5\cos\theta + C.$

$= -\sqrt{25-x^2} + C.$

Método 1: Sustitución. Trig.



$x = 5\sin\theta. \quad dx = 5\cos\theta\, d\theta$

$\sqrt{25-x^2} = 5\cos\theta.$

Método 2: Regla de la sustitución.

$u = 25 - x^2$

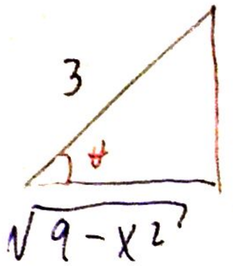
$du = -2x\,dx$

$\displaystyle\int \frac{-du/2}{u^{1/2}} = -\frac{1}{2}\int u^{-1/2}\,du.$

$= -\frac{2}{2}u^{1/2} + C.$

$= -\sqrt{25-x^2} + C.$

a. $\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{27\sin^3\theta}{\cancel{3\cos\theta}}\,\cancel{3\cos\theta}\,d\theta = \int 27\sin^3\theta\,d\theta.$



$\sin\theta = \frac{x}{3} \Rightarrow x = 3\sin\theta,\quad dx = 3\cos\theta\,d\theta$

$3\cos\theta = \sqrt{9-x^2}$

$\int 27\sin^2\theta\,\sin\theta\,d\theta = 27\int(1-\cos^2\theta)\sin\theta\,d\theta.$

$1-\cos^2\theta = \sin^2\theta,$ $\quad \boxed{u=\cos\theta},\quad du = -\sin\theta\,d\theta.$

$= -27\int 1-u^2\,du$

$= -27u + \frac{27}{3}u^3 + C$

u

$\downarrow$

$\theta \quad \cos\theta = \frac{\sqrt{9-x^2}}{3}$

x

$= -27\cos\theta + 9\cos^3\theta + C.$

$= -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)^{3/2} + C.$

b. $\int \frac{u}{\sqrt{4-u^2}}\,du = -\frac{1}{2}\int w^{-1/2}\,dw = -\frac{2}{2}w^{1/2} + C$

$= -\sqrt{4-u^2} + C.$

sólo sustitución $\quad w = 4-u^2$

$dw = -2u\,du$

PRACTIQUE $\quad u = 2\sin\theta.$