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# Corto #12 Cálculo Integral (1 hora)

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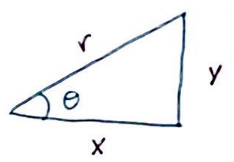
1. (50 pts.) ¿Cuál es la ec. de la recta tangente a la curva polar $r = 2 - \sin\theta$ en $t = \pi$?



$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$C\frac{H}{O}\ S\frac{H}{A}\ C\frac{A}{O}$

$r = 2 - \sin\theta$

$\sin\theta = \frac{y}{r} \Rightarrow \sin\theta \cdot r = y \Rightarrow \sin\theta(2 - \sin\theta) = y$

$\cos\theta = \frac{x}{r} \Rightarrow \cos\theta \cdot r = x \Rightarrow \cos\theta(2 - \sin\theta) = x$

$y'(\theta) = \frac{d}{d\theta}\left(2\sin\theta - \sin^2\theta\right)$

$= 2\cos\theta - 2\sin\theta \cdot \cos\theta$

$= 2\cos\theta - \sin(2\theta)$

$x'(\theta) = \frac{d}{d\theta}\left(2\cos\theta - \cos\theta\sin\theta\right)$

$\sin(2\theta) = 2\sin\theta \cdot \cos\theta$

$\frac{1}{2}\sin(2\theta) = \sin\theta\cos\theta$

$= \frac{d}{d\theta}\left(2\cos\theta - \frac{1}{2}\sin(2\theta)\right)$

$= -2\sin\theta - \frac{1}{\cancel{2}}\cos(2\theta) \cdot \cancel{2}$

$= -2\sin\theta - \cos(2\theta)$

$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{2\cos\theta - \sin(2\theta)}{-2\sin\theta - \cos(2\theta)}$

$= \frac{y'(\pi)}{x'(\pi)} = \frac{2\cos(\pi) - \sin(2\pi)}{-2\sin(\pi) - \cos(2\pi)}$

$= \frac{2(-1) - \cancel{(0)}}{-\cancel{2(0)} - 1} = \frac{-2}{-1} = 2 = m$

$x(\pi) = 2\cancel{\cos(\pi)}^{-1} - \frac{1}{\cancel{2}}\cancel{\sin(2\pi)}^{0}$

$= -2$

$y(\pi) = 2\cancel{\sin(\pi)}^{0} - \cancel{\sin^2(\pi)}^{0}$

$= 0$

$y = 2(x + 2) + 0$

$= \underline{2x + 4}$

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2. (50 pts.) Encuentre la longitud de medio cardioide $r = \sin^2\left(\frac{\theta}{2}\right)$ en $0 \leqslant \theta \leqslant \pi$ .

$$\mathcal{L} = \int_a^b \sqrt{r^2 + (r')^2}\, d\theta$$

$$r'(\theta) = \frac{d}{d\theta}\left(\sin^2\left(\frac{\theta}{2}\right)\right)$$

$$= \cancel{2}\left(\sin\left(\frac{\theta}{2}\right)\right)\cdot \cos\left(\frac{\theta}{2}\right)\cdot \frac{1}{\cancel{2}}$$

$$= \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$(r'(\theta))^2 \Rightarrow \sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)$$

$$r^2 = \left(\sin^2\left(\frac{\theta}{2}\right)\right)^2$$

$$\boxed{= \sin^4\left(\frac{\theta}{2}\right)}$$

$$\mathcal{L} = \int_0^\pi \sqrt{\underbrace{\sin^4\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)}_{\sin^2\left(\frac{\theta}{2}\right)\underbrace{\left[\sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right)\right]}_{1}}}$$

$$\sqrt{\sin^2\left(\frac{\theta}{2}\right)}$$

$$= \int_0^\pi \sin\left(\frac{\theta}{2}\right)\cdot 1\, d\theta$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$u = \frac{\theta}{2} \qquad du = \frac{1}{2}\, d\theta$$

$$= 2\int_0^\pi \sin(u)\, du$$

$$= -\cos(u)\cdot 2\Big]_{u(0)=0}^{u(\pi)=\frac{\pi}{2}}$$

$$= 2\left[\left(-\cos\left(\frac{\pi}{2}\right)\right) - \left(-\cos(0)\right)\right] = 2\left[0 + 1\right] = 2[1] = \boxed{2}$$

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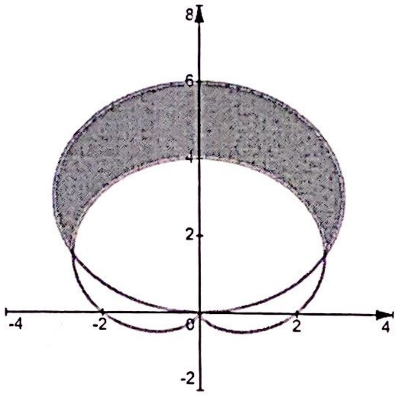
3. (50 pts.) Encuentre el área de la región que está dentro del círculo $r = 6\sin\theta$ y fuera del cardioide $r = 2 + 2\sin\theta$.




$$A = \frac{1}{2}\int_a^b r_{ext}^2 - r_{int}^2 \, d\theta$$

$$6\sin\theta = 2 + 2\sin\theta$$

$$6\sin\theta - 2\sin\theta = 2$$

$$\sin\theta(6-2) = 2$$

$$\sin\theta = \frac{2}{4}$$

$$\sin\theta = \frac{1}{2}$$

$$\frac{\pi}{6}, \quad \frac{5\pi}{6}$$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \underbrace{(6\sin\theta)^2 - (2+2\sin\theta)^2}_{36\sin^2(\theta) - (4 + 8\sin\theta + 4\sin^2\theta)} d\theta$$

$$\underbrace{36\sin^2(\theta) - 4 - 8\sin\theta - 4\sin^2\theta}_{32\sin^2(\theta) - 8\sin\theta - 4}$$

$$= \frac{1}{2}\int 32\cdot\frac{1}{2}(1 - \cos(2\theta)) - 8\sin\theta - 4 \, d\theta$$

$$= \frac{1}{2}\int 12 - 32\cos(2\theta) - 8\sin\theta \, d\theta$$

$$= \frac{1}{2}\left[12\theta - 16\sin(2\theta) + 8\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{2}\left[\left(12\left(\frac{\pi}{2}\right) \cancel{- 16(0) + 8(0)}\right) - \left(12\frac{\pi}{6} - 16\left(\frac{\sqrt{3}}{2}\right) + 8\,\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{2}\left[6\pi - 2\pi + 8\sqrt{3} - 4\sqrt{3}\right]$$

$$= \frac{1}{2}\left[4\pi + 4\sqrt{3}\right]$$

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