Laboratorio # 5

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③ $$\int \frac{72}{(36+x^6)^{3/2}} 3x^2\, dx = \int \frac{72 \cdot 3x^2}{\sqrt[2]{(36+x^6)^3}}$$

$$\left(\sqrt{(36+x^6)}\right)^3$$

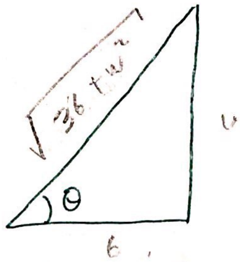
100

$$= 72 \int \frac{3x^2}{\sqrt{(36+x^6)^3}} dx = 72 \int \frac{du}{\sqrt{(36+u^2)^3}} =$$

$c = \frac{A}{H}$

$u = x^3$
$du = 3x^2\, dx$

$\sin(\theta) = \frac{u}{\sqrt{36+u^2}}$



$\tan\theta = \frac{u}{6}$

$6\tan\theta = u$

$6\sec^2\theta \cdot d\theta = du$

$$= 72 \int \frac{6\sec^2\theta\, d\theta}{\left(\sqrt{36+36\tan^2\theta}\right)^3} = 72 \int \frac{6\sec^2\theta\, d\theta}{216\sec^3\theta} = \frac{72 \cdot 6}{216} \int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$36(1+\tan^2\theta)$
$\left(\sqrt{36\sec^2\theta}\right)^3$
$(6\sec\theta)^3$
$216\sec^3\theta$

$$= 2 \int \frac{1}{\sec\theta} d\theta = 2 \int \cos\theta\, d\theta$$

$$= 2\sin\theta\, d\theta + C$$

$$= 2 \cdot \frac{u}{\sqrt{36+u^2}} + C$$

$$= \frac{2x^3}{\sqrt{36+x^6}} + C$$

④ $$\int \frac{1}{\sqrt{x^2+6x+10}}\,dx = \int \frac{1}{\sqrt{(x+3)^2+1}}\,dx$$

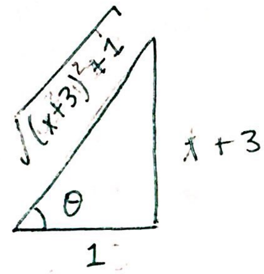
$$\left(\frac{6}{2}\right)^2 = \frac{36}{2} = 18$$

$$\underbrace{x^2+6x+9}-9+10 = 0$$

$$(x+3)^2+1$$

$$= \int \frac{1}{\sqrt{\underbrace{\tan^2\theta+1}_{\sec^2\theta}}}\sec^2\theta\,d\theta$$

$$= \int \frac{1}{\sec\theta}\sec^2\theta\,d\theta$$



$\tan\theta = x+3$

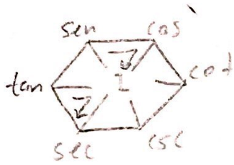
$\sec^2\theta\,d\theta = 1\,dx$

$$\sec\theta = \frac{1}{\frac{1}{\frac{A}{H}}} = \frac{H}{A} = \frac{\sqrt{(x+3)^2+1}}{1}$$

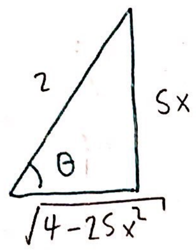
$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\sqrt{(x+3)^2+1} + x+3\right| + C$$



1. $$\int 5^8 x^7 \sqrt{4-25x^2}\, dx =$$



$$\sin\theta = \frac{5x}{2} \qquad 2\cos\theta = \sqrt{4-25x^2}$$

$$2\sin\theta = 5x$$

$$\frac{2}{5}\sin\theta = x$$

$$\cos\theta = \frac{\sqrt{4-25x^2}}{2}$$

$$\frac{2}{5}\cos\theta\, d\theta = dx$$

$$= 5^8 \int \frac{2^7}{5^7}\sin^7\theta \cdot 2\cos\theta \cdot \frac{2}{5}\cos\theta\, d\theta$$

$$= 5^8 \int \frac{2^9}{5^8}\sin^7\theta \cos^2\theta\, d\theta = \frac{5^8 \cdot 2^9}{5^8} \int \underbrace{\sin^7\theta \cos^2\theta\, d\theta}$$

$$= \int \sin^6\theta \cos^2\theta \sin\theta\, d\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$(\sin^2\theta)^3 = (1-\cos^2\theta)^3$$

$$= \int (1-\cos^2\theta)^3 \cdot \cos^2\theta \cdot \sin\theta\, d\theta$$

$$u = \cos\theta$$
$$du = -\sin\theta\, d\theta$$
$$-du = \sin\theta\, d\theta$$

$$= -\int (1-u^2)^3 u^2\, du$$

$$= -\int -u^8 + 3u^6 - 3u^4 + u^2\, du$$

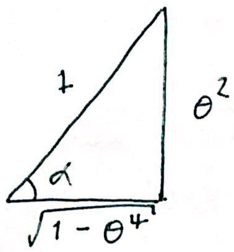
$$= 512\left\{-\left[-\frac{u^9}{9}\right] + \left[\frac{3u^7}{7}\right] - \left[\frac{3u^5}{5}\right] + \frac{u^3}{3}\right\}$$

$$= 512\left\{\frac{u^9}{9} - \frac{3u^7}{7} + \frac{3u^5}{5} - \frac{u^3}{3}\right\}$$

$$= 512\left\{\frac{\cos^9\theta}{9} - \frac{3\cos^7\theta}{7} + \frac{3\cos^5\theta}{5} - \frac{\cos^3\theta}{3}\right\}$$

$$= 512\left\{\frac{\left(\frac{\sqrt{4-25x^2}}{2}\right)^9}{9} - \frac{3\left(\frac{\sqrt{4-25x^2}}{2}\right)^7}{7} + \frac{3\left(\frac{\sqrt{4-25x^2}}{2}\right)^5}{5} - \frac{\left(\frac{\sqrt{4-25x^2}}{2}\right)^3}{3}\right\} + C$$

② $\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\, d\theta =$



$\cos(\alpha) = \sqrt{1-\theta^4}$

$\sin(\alpha) = \theta^2$

$\cos(\alpha)\, d\alpha = 2\theta\, d\theta$

$\frac{\cos(\alpha)\, d\alpha}{2} = \theta . d\theta$

$\sin(\alpha) = 1$
$\alpha = \frac{\pi}{2}$
$\sin(\alpha) = \theta$
$\alpha = 0$

$$= \frac{4}{\pi}\int_0^{\pi/2} \frac{\cos(\alpha)\cos(\alpha)\, d\alpha}{2} = \frac{2}{\pi}\int_0^{\pi/2} \cos^2\alpha\, d\alpha$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \frac{1}{2} + \cos(2\alpha)\, d\alpha = \frac{2}{\pi}\left\{\frac{1}{2}\alpha + \frac{\sin(2\alpha)}{2}\right\}\Bigg|_0^{\pi/2}$$

$$= \frac{2}{\pi}\left[\left\{\frac{1}{2}\cdot\frac{\pi}{2} + \frac{\cancelto{0}{\sin(\pi)}}{2}\right\} - \cancel{\left\{\frac{1}{2}\cdot 0 + \frac{\overset{0}{\sin(0)}}{2}\right\}}\right]$$

$$= \frac{2}{\pi}\left[\left\{\frac{\pi}{4}\right\}\right] = \frac{2}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{4} = \frac{1}{2}$$

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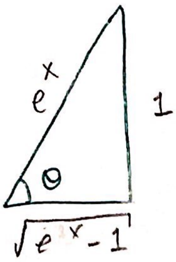
$$\int_{0}^{\ln(\sqrt{2})} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx = \int_{0}^{\ln(\sqrt{2})} \frac{\sec^3\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta = \int_{0}^{\pi/4} \sec^4\theta\,d\theta$$



$\sec\theta = e^x$

$\sqrt{e^x - 1} = \tan\theta\,d\theta$

$e^x dx = \sec\theta\tan\theta\,d\theta$

$e^{\ln(\sqrt{2})} = \sqrt{2}$

$\tan^{-1}(\sqrt{2}) = \frac{\pi}{4}$

$e^{\theta} = 1$

$\theta = 0$

$$= \int_{0}^{\pi/4} \sec^4\theta\,d\theta = \int_{0}^{\pi/4} (1+\tan^2\theta)\sec^2\theta\,d\theta = \int_{0}^{1} (1+u^2)\,du = u + \frac{u^3}{3}$$

$u = \tan\theta$

$du = \sec^2\theta$

$$\tan\theta + \frac{\tan^3(\theta)}{3}\Big|_{0}^{\pi/4}$$

$$\left\{1 + \frac{1}{3}\right\} - \left\{0 + \frac{0^3}{3}\right\}$$

$$\frac{3\cdot 1}{3} + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$$