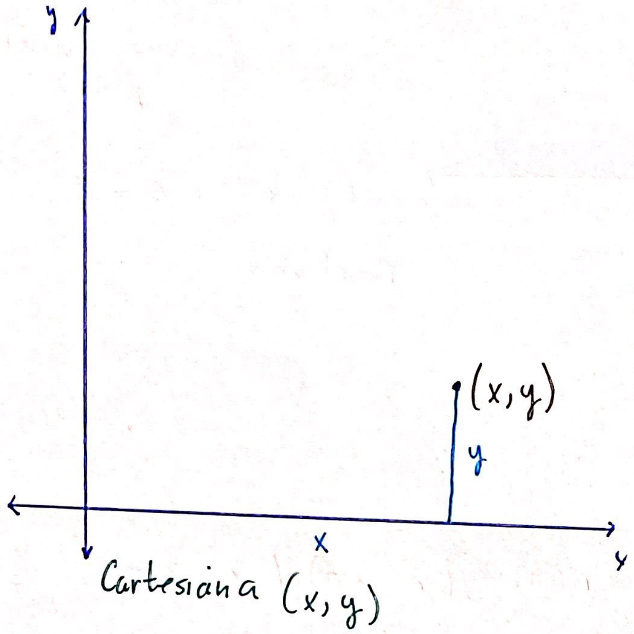
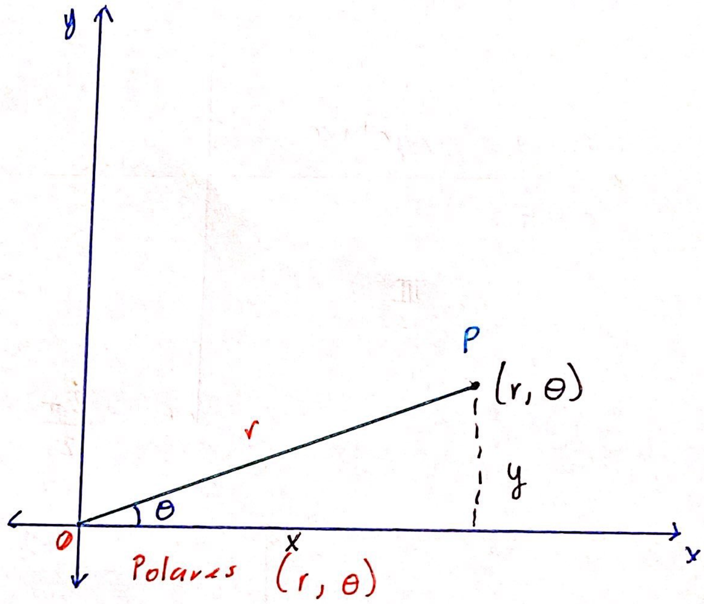
Resolución de corto a priori

10.3 Coordenadas polares p. 147





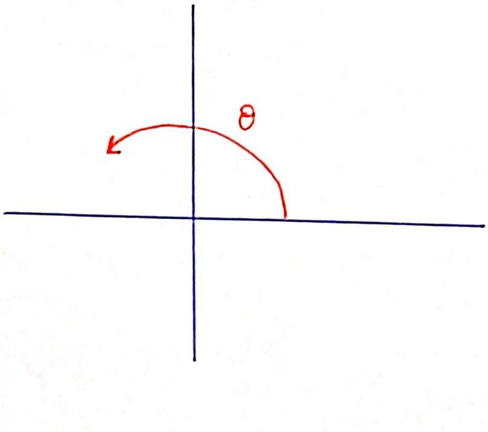
r: radio o la distancia del punto x, y al origen (0,0)

$$\boxed{r = \sqrt{x^2 + y^2}}$$

θ: ángulo entre la recta $\overline{OP}$ y el eje-x.

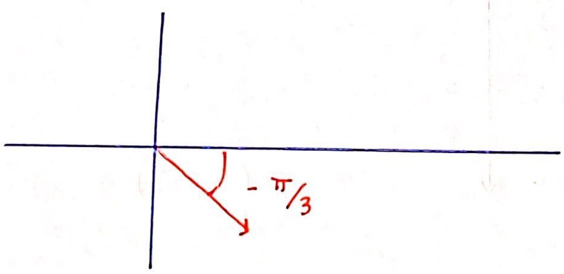
$$\frac{\text{Cateto Opuesto}}{\text{Cateto Adyacente}} = \tan\theta = \frac{y}{x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

Convenciones y observaciones de coordenadas polares

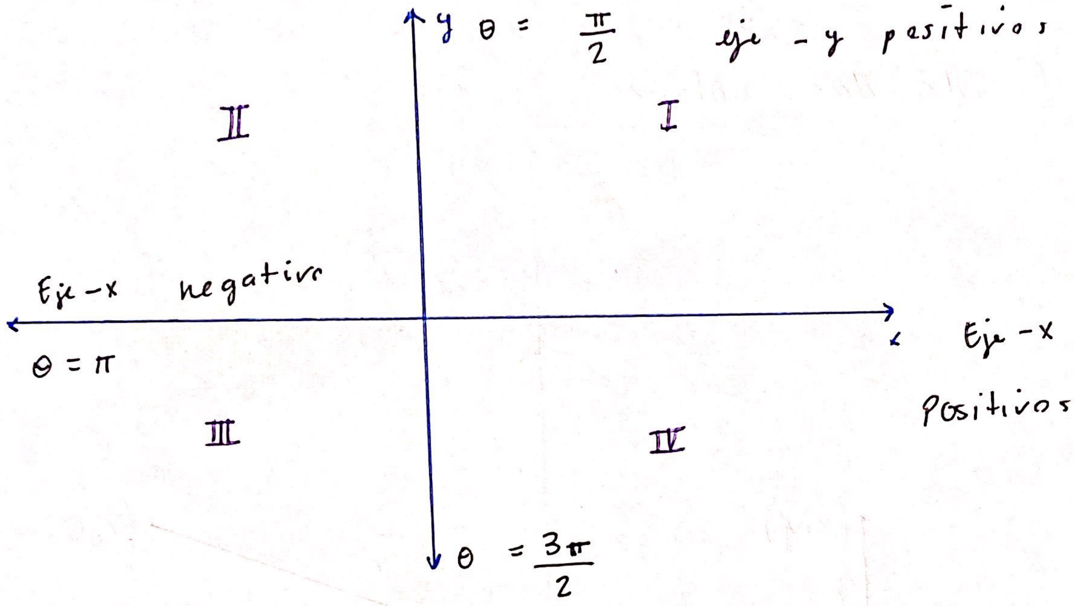


$\theta > 0$ en sentido anti-horario

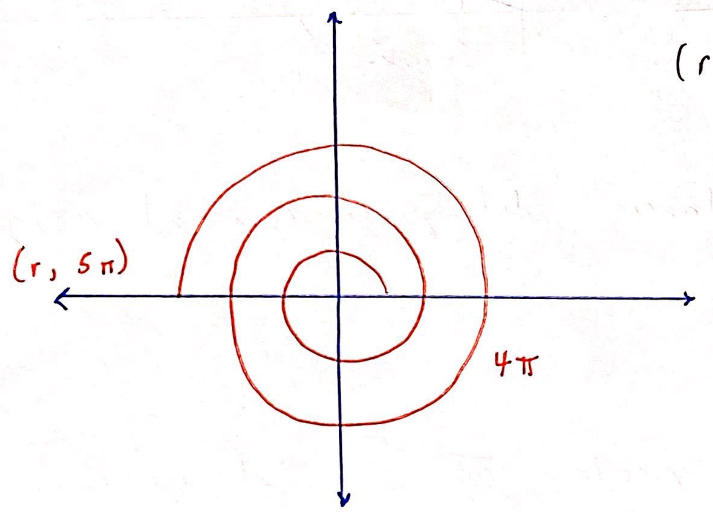
$\theta < 0$ en sentido horario.



Usualmente están entre $0 \leq \theta \leq 2\pi$



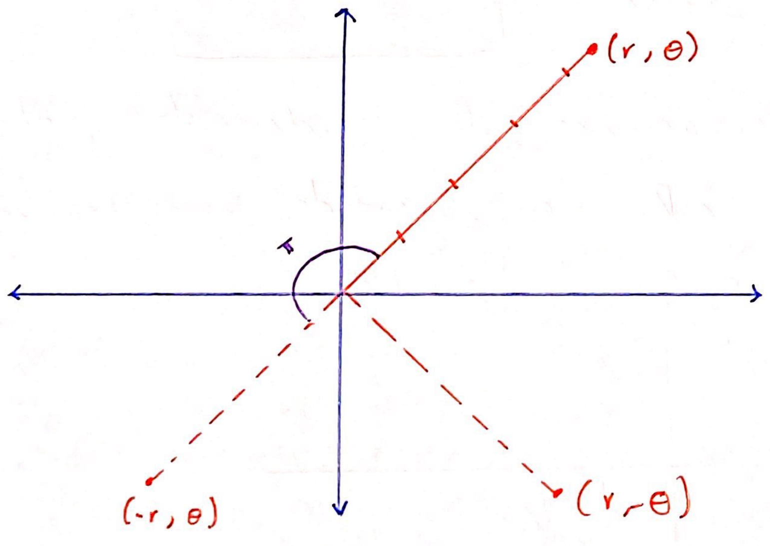
Ángulos mayores 2π se da una vuelta al plano.



(r, π), $(r, -\pi)$, $(r, 3\pi)$ representa al mismo punto

Coordenadas cartesianas $(-r, \theta)$

Reescribir radios negativos: $r > 0$



$(-r, \theta)$ es diametralmente opuesto a (r, θ).

Más conversiones:

origen $(0,0)$ $\quad r = 0$

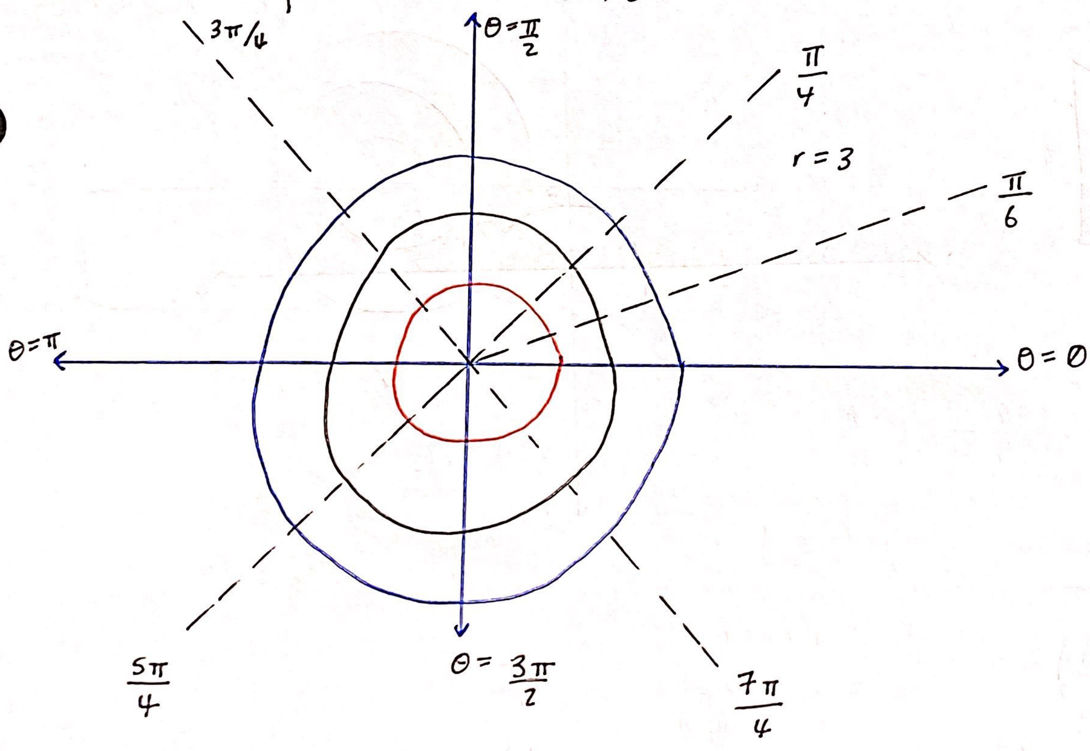
cualquier punto de la $(0, \theta)$ representa al origen.

Infinitas representaciones de un punto en coordenadas polares: Como 2π en una vuelta

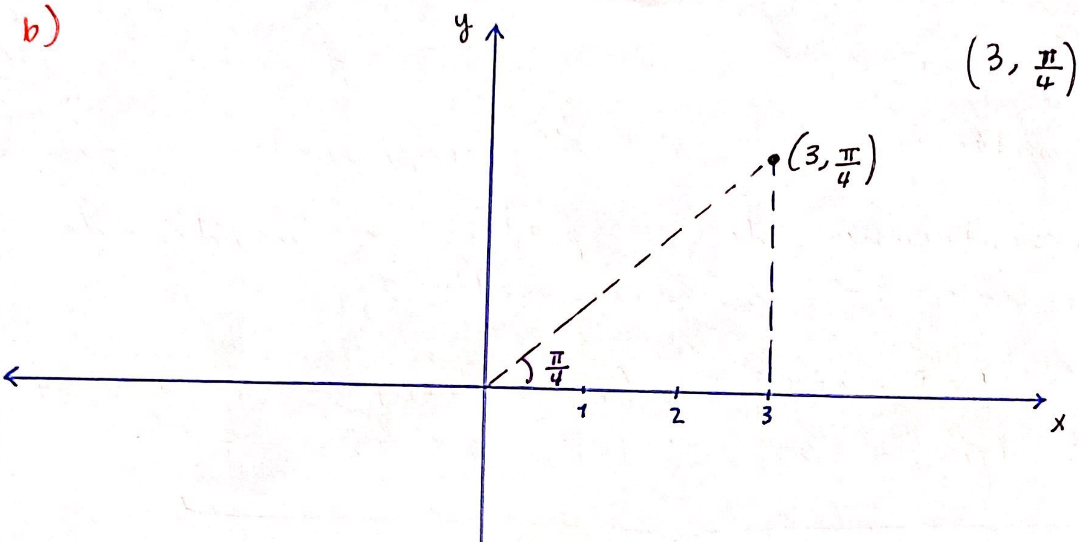
$$\underbrace{\begin{matrix} (r, \theta) & (r, \theta + 2\pi) & (r, \theta \pm 2\pi n) & \pi \in \mathbb{N} \\ (-r, \theta + \pi) & (-r, \theta + 3\pi) & (-r, \theta \pm 2n\pi + \pi & \end{matrix}}_{\text{todas estas son equivalentes}}$$

Representan al mismo punto en coordenadas polares.

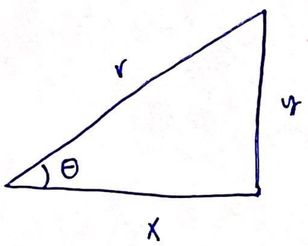
Ej 1: Grafique las coordenadas dadas.

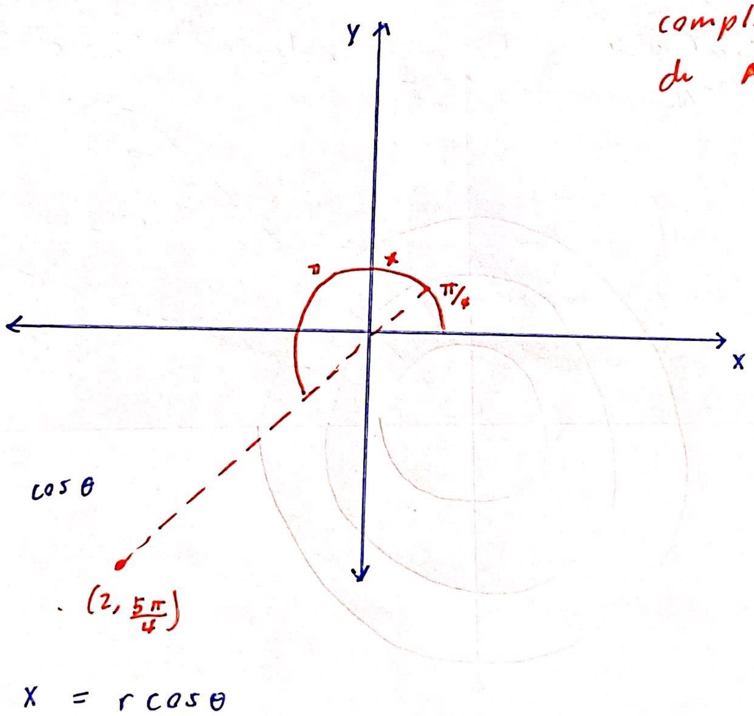


b) $(3, \frac{\pi}{4})$



c) $(2, \underbrace{\frac{5\pi}{4}}_{225°})$ $\quad \frac{5\pi}{4} = \underbrace{\pi + \frac{\pi}{4}}_{\text{complementos de ángulos.}}$





$$\frac{y}{r} = \sin\theta \qquad \frac{x}{r} = \cos\theta$$

$$y = r\sin\theta$$

$$y = 2\sin\left(\frac{5\pi}{4}\right)$$

$$= -2\frac{\sqrt{2}}{2} = -\sqrt{2}$$

$$x = r\cos\theta$$

$$= 2\cos\left(\frac{5\pi}{4}\right)$$

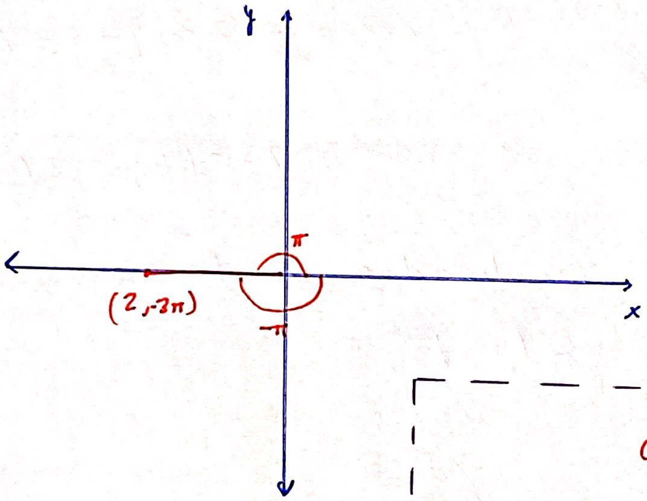
$$= \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

∴ Las coordenadas cartesianas por el método de triángulos va a ser igual a:

$$(-\sqrt{2}, -\sqrt{2})$$

b) $(2, -3\pi)$ 1 vuelta en sentido horario.

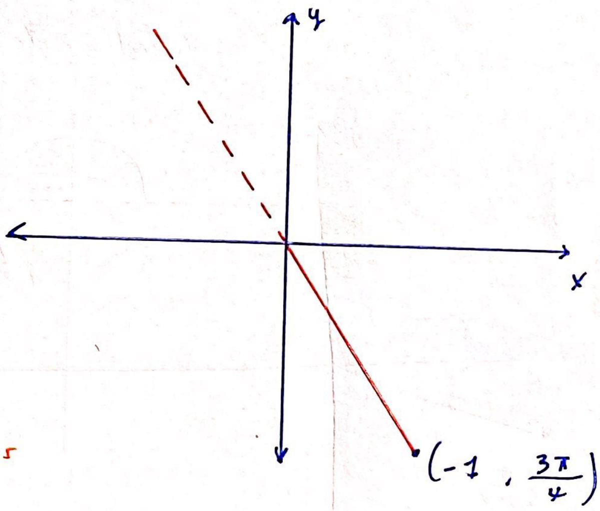
$$(2, -3\pi) \equiv (2, -3\pi + 2\pi) \equiv (2, -\pi)$$



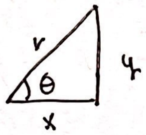
c) $\left(-1, \frac{3\pi}{4}\right)$

Está diametralmente opuesto a:

$\left(1, \frac{3\pi}{4}\right)$



Cambio de coordenadas
Polares a cartesianas



Polares (r, θ) a cartesianas (x, y) ; Exprese x & y en términos de r, θ

$$\left.\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}\right\}\text{ ec. paramétricas}$$

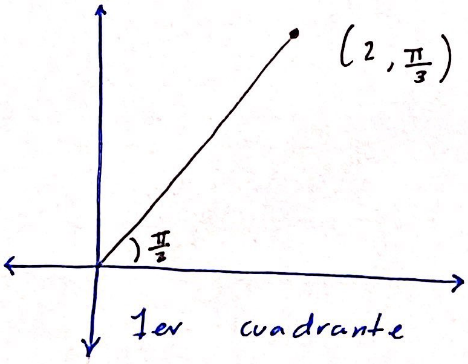
Cartesianas (x, y) a polares (r, θ)

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

tiene que estar en el cuadrante correcto

Ej: 2 : Convierta los sigs. pts. de coordenadas polares a cartesianas

a) $\left(2, \frac{\pi}{3}\right)$



$x = r\cos\theta = 2\cos\left(\frac{\pi}{3}\right) = 2 \cdot \frac{1}{2} = 1$

$y = r\sin\theta = 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$

Coordenadas cartesianas.

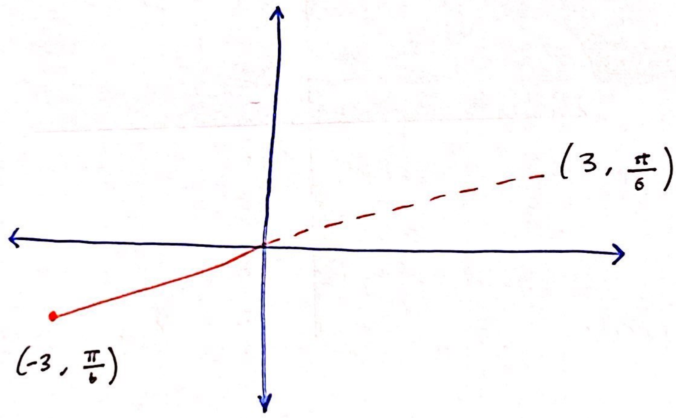
$\left(1, \sqrt{3}\right)$

b) $\left(-3, \frac{\pi}{6}\right)$

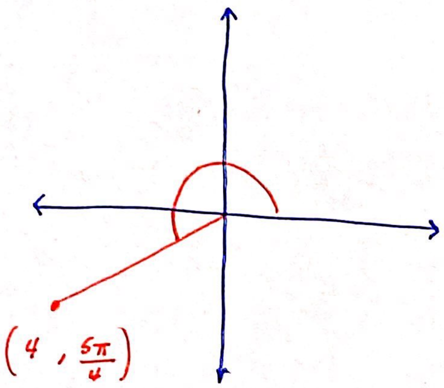
$x = r\cos\theta = -3\cos\left(\frac{\pi}{6}\right) = \frac{-3\sqrt{3}}{2}$

$y = r\sin\theta = -3\sin\left(\frac{\pi}{6}\right) = -3 \cdot \frac{1}{2}$

Coordenadas cartesianas $\left(\frac{-3\sqrt{3}}{2}, -\frac{3}{2}\right)$



b) $\left(4, \frac{5\pi}{4}\right)$



3 er cuadrante

$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

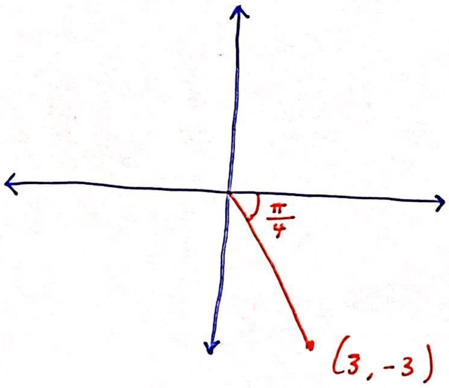
$x = 4\cos\left(\frac{5\pi}{4}\right) = -2\sqrt{2}$

$y = 4\sin\left(\frac{5\pi}{4}\right) = -2\sqrt{2}$

Cartesianas $(-2\sqrt{2}, -2\sqrt{2})$

Ej 3: encuentre las coordenadas polares del punto (x, y)

a) $(3, -3)$



$r = \sqrt{x^2 + y^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

$\therefore \left(3\sqrt{2}, -\pi/4\right)$

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\therefore \left(3\sqrt{2}, -\pi/4\right) \equiv \left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$$f(x) = xe^{-x} \begin{cases} x \geq 0 \\ f(x) = 0 \,; \quad x < 0 \end{cases}$$

$$\int_4^5 xe^{-x}\,dx = \int$$

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$$\left.\begin{matrix} -xe^{-x} + \underbrace{\int e^{-x}}_{-e^{-x}} \end{matrix}\right\} \left. -xe^{-x} - e^{-x} \right]_4^5 =$$

$$= \left[\left(-5e^{-5} - \quad \cdots \right] \approx 0.051$$

$$f(x) = \int_{\emptyset}^{\infty} A e^{-ct} dt = A \int_{\emptyset}^{\infty} e^{-ct} dt = -Ac \int_{\emptyset}^{\infty} e^{u} du = -Ace^{u} =$$

$u = -ct$

$du = -c\,dt$

$= -Ace^{-ct}$