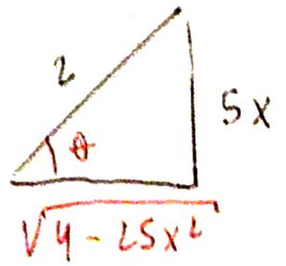
Parcial Lunes 2 septiembre 1:00 PM. D-503 B.
2:30 PM D-505 A

Sub S.

$$\int 5^8 x^7 \sqrt{4-25x^2}\, dx = 5^8 \int \frac{2^7}{5^7} \sin^7\theta \, 2\cdot\cos\theta \cdot \frac{2}{5}\cos\theta \, d\theta.$$



$\frac{5x}{2} = \sin\theta$ $\quad x = \frac{2}{5}\sin\theta.$

$dx = \frac{2}{5}\cos\theta \, d\theta$

$\sqrt{4-25x^2} = 2\cos\theta.$

$x^7 = \frac{2^7}{5^7}\sin^7\theta.$

$$\frac{5^8}{5^8} 2^9 \int \sin^7\theta \cdot \cos^2\theta \, d\theta = 512 \int \sin^6\theta \cos^2\theta \sin\theta \, d\theta.$$

$\sin^6\theta = (\sin^2\theta)^3 = (1-\cos^2\theta)^3$ $\quad = 512\int (1-\cos^2\theta)^3 \cos^2\theta \sin\theta \, d\theta.$

$u = \cos\theta \quad du = -\sin\theta \, d\theta.$ $\quad - 512 \int (1-u^2)^3 u^2 \, du.$

$(1-u^2)^3 = 1 - 3u^2 + 3u^4 - u^6$

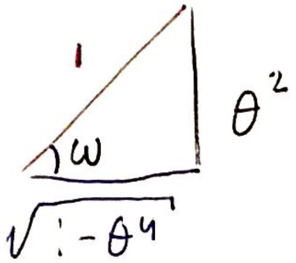
$(1-u^2)^3 u^2 = u^2 - 3u^4 + 3u^6 - u^8$

$\cos\theta.$

$$\cos\theta = \frac{(4-25x^2)^{1/2}}{2}.$$

2. $\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta = \frac{4}{\pi}\int_0^1 \underbrace{\sqrt{1-\theta^4}}_{\cos w}\ \underbrace{\theta\,d\theta}_{\frac{\cos w\,dw}{2}}$

Lab. 5.



$\sin w = \theta^2$

$\cos w\,dw = 2\theta\,d\theta$

$\sqrt{1-\theta^4} = \cos w$

$\sin w = 1^2$

$w = \pi/2$

$\sin w = 0$

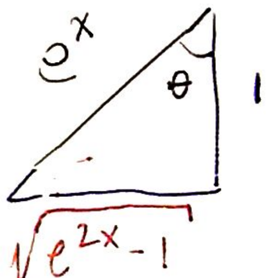
$w = 0$

$\frac{2}{\pi}\int_0^{\pi/2} \cos^2 w\,dw = \frac{1}{\pi}\int_0^{\pi/2}(1+\cos 2w)\,dw$ $\qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$= \frac{1}{\pi}\left(w + \frac{1}{2}\sin 2w\Big]_0^{\pi/2}\right)$

$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right) = \boxed{\frac{1}{2}}$

5. $\int_0^{\ln\sqrt{2}} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx = \int \frac{e^{3x}}{\sqrt{e^{2x}-1}} e^x\,dx = \int \frac{\sec^3\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta$



$e^x = \sec\theta$

$\sqrt{e^{2x}-1} = \tan\theta\,d\theta$

$e^{3x} = \sec^3\theta$

$e^x\,dx = \sec\theta\tan\theta\,d\theta$

$\sec\theta = \cancel{e^{\ln\sqrt{2}}} = \sqrt{2}$

$\sec\theta = e^0 = 1$

$\theta = 0$

$\int_0^{\pi/4} \sec^4\theta\,d\theta = \int_0^{\pi/4} \sec^2\theta\,\sec^2\theta\,d\theta$ $\qquad \sec 0 = \frac{1}{\cos 0} = 1$

$\int_0^{\pi/4}(1+\tan^2\theta)\sec^2\theta\,d\theta = \int_0^1 (1+u^2)\,du$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta \quad u(\pi/4) = \tan\pi/4 = 1$

$u(0) = \tan 0 = 0$

$$\int_0^1 (1+u^2)\,du = u + \frac{1}{3}u^3\Big]_0^1 = 1 + \frac{1}{3} = \frac{4}{3}.$$

5. $$\int_0^{\ln\sqrt{2}} \frac{e^{2x}}{\sqrt{e^{2x}-1}}\, e^{2x}\,dx. = \int_0^1 \frac{e^{2x}}{\sqrt{u}}\,\frac{du}{2.} \qquad e^{2x} = u+1$$

$$u = e^{2x} - 1 \qquad u(\ln\sqrt{2}) = e^{2\cdot\ln 2^{1/2}} - 1 = 2 - 1 = 1$$

$$\frac{du}{2} = e^{2x}\,dx \qquad u(0) = e^0 - 1 = 0$$

$$\frac{1}{2}\int_0^1 \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\int_0^1 u^{1/2} + u^{-1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\Big]_0^1\right)$$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{6}{3}\right) = \frac{4}{3}.$$

$$\int \frac{\overbrace{e^{2x}}^{u+1}}{\sqrt{e^{2x}-1}}\,\overbrace{e^{2x}\,dx}^{du/2.} \qquad \int \frac{u+1}{\sqrt{u}}\,\frac{du}{2}$$

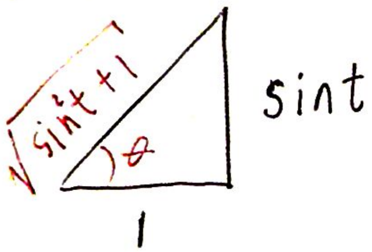
$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C.$$

$$= \frac{1}{3}(e^{2x}-1)^{3/2} + (e^{2x}-1)^{1/2}\Big]_0^{\ln\sqrt{2}}$$

Problema 2 b) Simulacro.

$$\int_0^{\pi/2} \frac{\cos t}{\sqrt{\sin^2 t + 1}}\, dt = \int \frac{\sec^2\theta \, d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta \, d\theta.$$



$\sin\theta$

$\tan\theta = \sin t.$

$\sec^2\theta \, d\theta = \cos t \, dt.$

$\sqrt{\sin^2 t + 1} = \sec\theta.$

$\tan\theta = \sin\frac{\pi}{2} = 1$

$\theta = \tan^{-1}(1) = \pi/4$

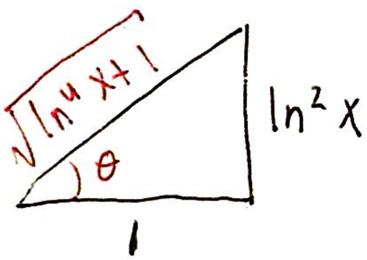
$\tan\theta = \sin 0 = 0$

$\theta = 0$

$$\int_0^{\pi/4} \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| \Big]_0^{\pi/4} = \ln|\sec\pi/4 + \tan\pi/4) - \ln(\sec 0 + \tan 0)$$

$$= \ln(\sqrt{2} + 1) - \cancel{\ln(1)}^{\,0} = \underline{\ln(\sqrt{2} + 1)}$$

$$\int \frac{4}{\sqrt{\ln^4 x + 1}} \frac{2(\ln x)}{x} dx = 4\int \frac{\sec^2\theta \, d\theta}{\sec\theta} = 4\int \sec\theta \, d\theta.$$



$\tan\theta = [\ln x]^2$

$\sec^2\theta \, d\theta = 2\ln x \frac{1}{x} dx.$

$\sqrt{\ln^4 x + 1} = \sec\theta.$

$$4\int \sec\theta \, d\theta = 4\ln|\sec\theta + \tan\theta| + C.$$

$$4\ln\left|\sqrt{\ln^4 x + 1} + \ln^2 x\right| + C.$$

$$\int \frac{(x-2)^3}{\sqrt{x^2-4x+13}}\,dx.$$

$u \cancel{=} x^2-4x+13$

$du \cancel{=} 2x-4 = 2(x-2)\,dx$

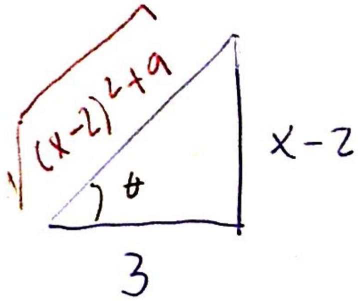
complete el cuadrado. $(x^2-4x+4)+13-4$

$(x-2)^2+9.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = a\cdot \operatorname{sen}\theta.$

$1\cdot dx = a\cdot \cos\theta\, d\theta.$



$3\cdot\tan\theta = x-2$

$3\sec^2\theta\, d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$(x-2)^3 = 3^3\tan^3\theta.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\, d\theta.$$

$9\tan^2\theta+9$

$9\sec^2\theta.$

$= 3^3\int \tan^3\theta\sec\theta\, d\theta.$

$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\, d\theta)$

$= 27\int (\sec^2\theta-1)\,(\tan\theta\sec\theta\, d\theta).$

$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta.$

$= 27\int (u^2-1)\,du = 9u^3 - 27u + C.$

$\sec\theta = \frac{\sqrt{x^2-4x+13}}{3}.$

$9\sec^3\theta - 27\sec\theta + C.$

$$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C.$$

$$\int \frac{[illegible]}{\sqrt{x^2-4x+13}}\,dx$$

$u = x^2 - 4x + 13$

$du = (2x - 4) = 2(x-2)\,dx$

completa el cuadrado $\quad (x^2 - 4x + 4) + 13 - 4$

$(x-2)^2 + 9$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = a \operatorname{sen}\theta$

$dx = a\cos\theta\,d\theta$

$3\tan\theta = x - 2$

$3\sec^2\theta\,d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta$

$(x-2)^3 = 3^3\tan^3\theta$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\,d\theta$$

$9\tan^2\theta + 9$

$9\sec^2\theta$

$= 3^3 \int \tan^3\theta\sec\theta\,d\theta$

$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\,d\theta)$

$= 27\int(\sec^2\theta - 1)(\tan\theta\sec\theta\,d\theta)$

$u = \sec\theta \qquad du = \sec\theta\tan\theta\,d\theta$

$= 27\int(u^2-1)\,du = 9u^3 - 27u + C$

$\sec\theta = \dfrac{\sqrt{x^2-4x+13}}{3}$

$= 9\sec^3\theta - 27\sec\theta + C$

$$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C$$