

① Paramétricas

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$$x = \sin^3 \theta \quad ; \quad y = \cos^3 \theta$$

a) derivada $\frac{dy}{dx}$

$$\frac{y'(\theta)}{x'(\theta)}$$

$$\Rightarrow y(\theta) = (\cos(\theta))^3$$

$$y'(\theta) = -3(\cos(\theta))^2 \cdot \sin \theta$$

$$\Rightarrow x'(\theta) = (\sin(\theta))^3$$

$$= 3(\sin \theta)^2 \cdot \cos \theta$$

$$= \frac{-3 \cos^2 \theta \cdot \sin \theta}{3 \sin^2 \theta \cdot \cos \theta} =$$

$$= - \frac{\cancel{3} \cos \theta \cdot \cancel{\cos \theta} \cdot \sin \theta}{\cancel{3} \sin \theta \cdot \cancel{\sin \theta} \cdot \cos \theta} = - \frac{\cos \theta}{\sin \theta} = \boxed{-\cot \theta}$$

b) Ec. recta tangente

$$y - y_0 = m(x - x_0)$$

$$y\left(\frac{\pi}{6}\right) = \left(\cos \frac{\pi}{6}\right)^3 = \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{\left(\frac{3}{2}\right)^{\frac{3}{2}}}{(2)^3} = \boxed{\frac{\sqrt{27}}{8}}$$

$$x\left(\frac{\pi}{6}\right) = \left(\sin \frac{\pi}{6}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \boxed{\frac{1}{8}}$$

$$m\left]\frac{\pi}{6}\right. = -\cot \theta = -\frac{\cos \theta}{\sin \theta}$$

$$= -\frac{\left[\frac{\sqrt{3}}{2}\right]}{\left[\frac{1}{2}\right]} = -\frac{2\sqrt{3}}{2} = -\sqrt{3}$$

$$\boxed{y = -\sqrt{3}\left(x - \frac{1}{8}\right) + \frac{\sqrt{27}}{8}}$$

~~Quitar~~

c)

$$y'(\theta) = -3 \cos^2 \theta \cdot \sin \theta = 0$$

$$-3 (1 - \sin^2 \theta) \sin \theta = 0$$

$$(1 - \sin^2 \theta) \sin \theta = 0$$

$$\sin \theta - \sin^3 \theta = 0$$

$$\frac{\pi}{2}, 0, \pi$$

$$x'(\theta) = 3 \sin^2 \theta \cdot \cos \theta = 0$$

$$3 (1 - \cos^2 \theta) \cos \theta = 0$$

$$(1 - \cos^2 \theta) \cos \theta = 0$$

$$\cos \theta - \cos^3 \theta = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}, 0$$

$$\frac{dy}{dx} = -\cot \theta = -\frac{\cos \theta}{\sin \theta}$$

no se define en $0, \pi$

entonces tiene tangentes

en $0, \pi$ dado

también al denominador

tampoco volverse 0.

tangente vertical

$$y'(\theta) = 0$$

$$x'(\theta) \neq 0$$

② Longitud paramétricas:

$$x = t^3 - 3t \quad ; \quad y = 3t^2$$

$$-1 \leq t \leq 3$$

$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$x'(t) = 3t^2 - 3$$

$$y'(t) = 3 \cdot 2t = 6t$$

$$= \int_{-1}^3 \sqrt{(3t^2 - 3)^2 + (6t)^2} dt$$

$$9t^4 - 2 \cdot 3 \cdot 3t^2 + 9 + 36t^2$$

$$9t^4 - 18t^2 + 9 + 36t^2$$

$$9t^4 + 18t^2 + 9$$

$$9(t^4 + 2t^2 + 1)$$

$$9(t^2 + 1)^2 \Rightarrow \sqrt{9(t^2 + 1)^2} = \sqrt{9} \cdot \sqrt{(t^2 + 1)^2}$$

$$= \sqrt{3^2} \cdot \sqrt{(t^2 + 1)^2}$$

$$= 3 \cdot (t^2 + 1)$$

$$L = \int_{-1}^3 3t^2 + 3 dt = \left[\frac{3}{3} t^3 + 3t \right]_{-1}^3 = t^3 + 3t$$

$$L = \left[\left(\underbrace{3^3}_{27} + 3(3) \right) - \left(\underbrace{(-1)^3}_{-1} + 3(-1) \right) \right] =$$

$$27 + 9 - (-4) = 27 + 9 + 4 = 31 + 9 = 40$$

25pts

③ Paramétricas

$$x = 4t - t^3 \quad ; \quad y = 7.5t^2$$

Interceptos

$$4t - t^3 = 0 \quad ; \quad 7.5t^2 = 0$$

$$t(4 - t^2) = 0 \quad ; \quad \boxed{t^2 = 0}$$

$$\boxed{t = 0}$$

$$4 - t^2 = 0$$

$$4 = t^2$$

$$\pm \sqrt{4} = t$$

$$\boxed{\pm 2 = t}$$

$$x^2(t) = 4 - 3t^2$$

$$A = \int_{t_1}^{t_2} 7.5t^2 (4 - 3t^2) dt$$

$$A = 2 * 7.5 \int_2^0 t^2 (4 - 3t^2) dt$$

$$= 15 \int_2^0 4t^2 - 3t^4 = 15 \left[\frac{4}{3}t^3 - \frac{3}{5}t^5 \right]_2^0$$

$$= 15 \left[(0) - \left(\frac{4}{3}(2^3) - \frac{3}{5}(2^5) \right) \right]$$

$$= 15 \left[- \left(\frac{5 \cdot 32}{5 \cdot 3} - \frac{3 \cdot 32 \cdot 3}{3 \cdot 5} \right) \right] =$$

$$15 \cdot \frac{128}{15} = \boxed{128}$$

$$32(5-9)$$

$$32 \cdot 4$$

$$32$$

$$4$$

$$-128$$

$$15$$

$$640$$

$$1280$$

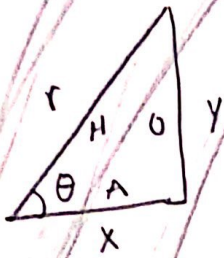
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25 pts

④ Longitud polares.

$$r = 2 - 2\cos(\theta)$$

Polares a paramétricas



$$\begin{pmatrix} \frac{0}{H} \\ \frac{H}{0} \end{pmatrix} \begin{pmatrix} \frac{A}{H} \\ \frac{H}{A} \end{pmatrix} \begin{pmatrix} \frac{0}{A} \\ \frac{A}{0} \end{pmatrix}$$

$$\sin \theta = \frac{y}{r} \Rightarrow y = \sin \theta \cdot r \Rightarrow y = \sin \theta (2 - 2\cos(\theta))$$

$$\cos \theta = \frac{x}{r} \Rightarrow x = \cos \theta \cdot r \Rightarrow x = \cos \theta (2 - 2\cos(\theta))$$

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} =$$

$$\begin{aligned} y &= 2\sin \theta - 2\cos(\theta)\sin(\theta) \\ &= 2\sin \theta - \sin(2\theta) \\ y'(\theta) &= 2\cos \theta - \cos(2\theta) \cdot 2 \end{aligned}$$

$$x = 2\cos \theta - 2\cos^2 \theta$$



(4)

continuidad 4

$$L = \int_0^{2\pi} \sqrt{(r')^2 + r^2} d\theta$$

$$r = 2 - 2 \cos \theta$$

$$r' = 0 + 2 \sin \theta$$

$$(r')^2 = (2 \sin \theta)^2 = 4 \sin^2 \theta$$

$$\begin{aligned} (r)^2 &= (2 - 2 \cos \theta)^2 = 4 - 2 \cdot 2 \cdot 2 \cos \theta + 4 \cos^2 \theta \\ &= 4 - 8 \cos \theta + 4 \cos^2 \theta \end{aligned}$$

$$(r')^2 + r = 4 - 8 \cos \theta + \underbrace{4 \cos^2 \theta + 4 \sin^2 \theta}_4$$

$$= 4 + 4 - 8 \cos \theta$$

$$= 8 - 8 \cos \theta = 8 (1 - \cos \theta)$$

$$= 8 \left(2 \sin^2 \left(\frac{\theta}{2} \right) \right)$$

$$= 16 \sin^2 \left(\frac{\theta}{2} \right)$$

$$\sqrt{(r')^2 + r} = \sqrt{16 \sin^2 \left(\frac{\theta}{2} \right)}$$

$$= \sqrt{4^2} \cdot \sqrt{\sin^2 \left(\frac{\theta}{2} \right)}$$

$$= 4 \sin \left(\frac{\theta}{2} \right)$$

$$L = \int_0^{2\pi} 4 \sin \left(\frac{\theta}{2} \right) d\theta$$

$$u = \frac{\theta}{2}$$

$$du = \frac{1}{2} d\theta = 2 du$$

$$= \int_0^{\pi} 8 \sin(u) du$$

$$\begin{aligned} &= -8 \cos \theta \Big|_0^{2\pi} = \left(-8 \left[(\cos \pi) - (\cos 0) \right] \right) = 2(-8[-1 - 1]) \\ &= 2(-8(-2)) = \boxed{32} \end{aligned}$$

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Anexos

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

Paramétricas

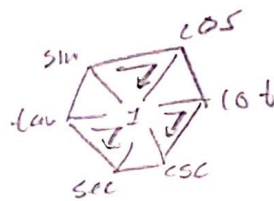
$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

Polares

$$A = \frac{1}{2} \int_a^b r^2 d\theta$$

$$A = \frac{1}{2} \int_a^b \underbrace{r_1^2 - r_2^2}_{\text{alejado del origen}} d\theta$$

$$L = \int_a^b \sqrt{(r')^2 + r^2} d\theta$$



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$A = \int_{t_1}^{t_2} y \underbrace{dx}_{\text{derivada}}$$

Horizontales

$$x' = 0$$

$$y' \neq 0$$

Verticales

$$y' = 0$$

$$x' \neq 0$$