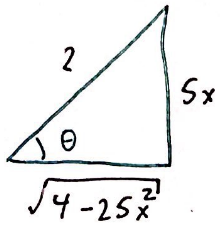
•Parcial 1 lunes 2 septiembre

2019-08-27

Repaso De Sustitución Trigonométrica

Lab # 5

① $$\int 5^8 x^7 \sqrt{4-25x^2}\, dx = 5^8 \int \frac{2^7}{5^7} \sin^7\theta \cdot 2\cos\theta \cdot \frac{2}{5}\cos\theta\, d\theta =$$



$\sin\theta = \frac{5x}{2}$

$x = \frac{2}{5}\sin\theta$

$dx = \frac{2}{5}\cos\theta\, d\theta$

$\sqrt{4-25x^2} = 2\cos\theta$

$$= \frac{5^8}{5^8} \cdot 2^9 \int \sin^7\theta\, d\theta \cdot \cos^2\theta\, d\theta = 512 \int \sin^6\theta \cos^2\theta \sin\theta\, d\theta$$

$\sin^6\theta = (\sin^2\theta)^3 = (1-\cos^2\theta)^3$

$$\therefore = 512 \int (1-\cos^2\theta)^6 \cos^2\theta \cdot \sin\theta \cdot d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

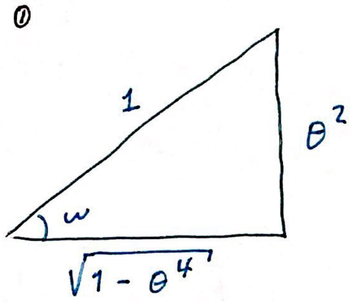
$$= 512 \int (1-u^2)^3 u^2\, du$$

$(1-u^2)^3 u^2 =$

$u^2 - 3u^4 + 3u^6 - u^8$

$$= 512 \int u^2 - 3u^4 + 3u^6 - u^8\, du$$

② $$\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta = \frac{2}{\pi}\int_0^1 \sqrt{1-\theta^4}\cdot\underbrace{2\theta\cdot d\theta}_{\text{especulando}} =$$



$$\sin(w) = \theta^2$$
$$\cos(w)\,dw = 2\theta\,d\theta$$
$$\sqrt{1-\theta^4} = \cos(w)$$

encontrar nuevos límites =>

$\sin(w) = 1^2$

$w = \frac{\pi}{2}$

$\sin(w) = 0$

$w = 0$

$$= \frac{2}{\pi}\int_0^{\pi/2} \cos^2(w)\,dw = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} (1 + \cos(2w))\,dw$$

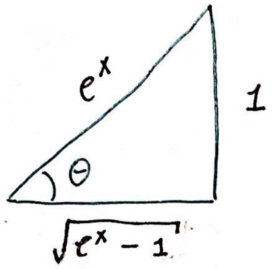
$$= \frac{1}{\pi}\left(w + \frac{1}{2}\sin(2w)\right)\Bigg|_0^{\pi/2} \doteq$$

$$\doteq \frac{1}{\pi}\left\{\frac{\pi}{2} + \frac{1}{2}\sin(2\cdot\pi/2)\right\} - \cancel{\frac{1}{\pi}\left\{0 + \frac{1}{2}\sin(2\cdot 0)\right\}}^{\,0}$$

$$\doteq \frac{1}{2} \quad \square$$

5

$$\int_0^{\ln(\sqrt{2})} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx = \int_0^{\ln(\sqrt{2})} \frac{e^{3x}}{\sqrt{e^{2x}-1}}\,e^x \cdot dx = \int_0^{\ln(\sqrt{2})} \frac{\sec^3\theta}{\tan\theta} \cdot \sec\theta \tan\theta \, d\theta$$



$e^x = \sec\theta \, d\theta$

$\sqrt{e^x - 1} = \tan\theta \, d\theta$

$e^x dx = \sec\theta \tan\theta \, d\theta$

$\sec\theta = e^{\ln(\sqrt{2})} = \sqrt{2}$

nuevo límite

$\sec\theta = e^0 = 1$

$\theta = 0$

$$= \int_0^{\ln(2)} \sec^4\theta \, d\theta = \int_0^{\pi/4} \sec^2\theta \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} (1 + \tan^2\theta)\sec^2\theta \, d\theta = \int_0^1 (1 + w^2)\,dw =$$

$w = \tan\theta$

$dw = \sec^2\theta \, d\theta$

$$= w + \frac{w^3}{3}\Big]_0^1 = 1 + \frac{1}{3} = \frac{4}{3}$$

FORMA ALTERNA

⑤ $$\int_0^{\ln(\sqrt{2})} \frac{e^{2x}}{\sqrt{e^{2x}-1}} \cdot e^{2x}\, dx = \int_0^{\ln(\sqrt{2})} \frac{e^{x}}{\sqrt{u}}\, du =$$

$u = e^{2x} - 1$ $\qquad u(\ln(\sqrt{2})) = e^{2(\ln(\sqrt{2}))} - 1 = 2 - 1 = 1$

$du = 2e^{2x}\, dx$ $\qquad u(0) = e^{0} - 1 = 0$

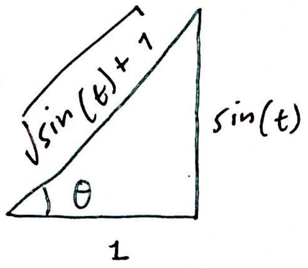
$$= \frac{1}{2}\int_0^1 \frac{u+1}{u^{1/2}}\, du = \frac{1}{2}\int_0^1 \left(u^{1/2} + u^{-1/2}\right) du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right)\Bigg]_0^1$$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{6}{3}\right) = \frac{4}{3} \quad \square$$

$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\, du = \frac{1}{3}\left(e^{2x}-1\right)^{3/2} + \left(e^{2x}-1\right)^{1/2}\Bigg]_0^{\ln(\sqrt{2})}$$

Problema 2b simulacro:

$$\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{\sin^2(t)+1}}\,dt = \int_0^{\pi/2} \frac{\sec^2\theta}{\sec\theta}\,d\theta = \int_0^{\pi/2} \sec\theta\,d\theta$$



$\tan\theta = \sin(t)$

$\sec^2\theta\,d\theta = \cos(t)\,dt$

$\sqrt{\sin^2 t + 1} = \sec\theta$

$\tan\theta = \sin(\pi/2) = 1$

$\theta = \tan^{-1}(1) = \pi/4$

$\tan\theta = \sin(0) = 0$

$\theta = 0$

$$\therefore \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C \Big|_0^{\pi/4} =$$

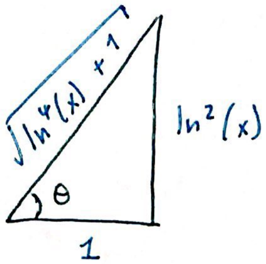
$$= \{\ln|\overset{\sqrt{2}}{\cancel{\sec(\pi/4)}} + \tan(\pi/4)|\} - \{\ln|\sec(0) + \tan(0)|\}$$

$$= \ln|\sqrt{2}+1| - \overset{0}{\cancel{\ln|1|}} - \ln|\sqrt{2}+1|$$

$$= \ln|\sqrt{2}+1| - \ln|\sqrt{2}+1|$$

$\square$

Problema curioso = $\displaystyle\int \frac{8}{\sqrt{\ln^4 x + 1}} \frac{\ln(x)}{x} dx = \int \frac{4 \cdot 2}{\sqrt{\ln^4(x)+1}} \cdot \frac{\ln(x)}{x} dx =$



$\tan\theta = \dfrac{\ln^2(x)}{1} = \ln^2(x)$

$\sec^2\theta \, d\theta = 2\ln(x) \cdot \dfrac{1}{x} dx = \dfrac{2\ln(x)}{x}$

$\sqrt{\ln^4(x)+1} = \sec\theta$

$= 4\displaystyle\int \sec\theta \, d\theta = 4 \ln|\sec\theta + \tan\theta| + C$

$\therefore = 4 \cdot \ln\left|\sqrt{\ln^4(x)+1} + \ln^2(x)\right| + C$ $\square$

$\displaystyle\int \frac{(x-2)^3}{\sqrt{x^2 - 4x + 13}} dx =$ completar al cuadrado

$(x^2 - 4x + 4) + 13 - 4$

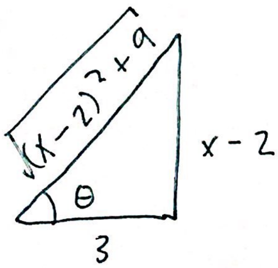
$(x-2)^2 + 9$

$u = x^2 - 4x + 13$

$du = 2x - 4 = 2(x-2) dx$

no está

$\therefore \displaystyle\int \frac{(x-2)^3}{\sqrt{(x-2)^2 + 9}} dx$



$3\tan\theta = x - 2$

$3\sec^2\theta \, d\theta = dx$

$\sqrt{(x-2)^2 + 9} = 3\sec\theta$

$(x-2)^3 = 3^3 \sec^3\theta$

$= \displaystyle\int \frac{(x-2)^3}{\sqrt{(x-2)+9}} dx = \int \frac{3^3 \tan^3\theta}{3\sec\theta} \cdot 3\sec^2\theta \, d\theta = 3^3 \int \tan^3\theta \sec\theta \, d\theta$

$= 27 \displaystyle\int \tan^2\theta \, (\tan\theta \sec\theta \, d\theta)$

$$= 27 \int \tan^2\theta \, (\tan\theta \sec\theta) \, d\theta = 27 \int (\sec^2\theta - 1)(\tan\theta \sec\theta) \, d\theta$$

$u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$$= 27 \int (u^2 - 1) \, du = 9u^3 - 27u + C$$

$$= 9\sec^3\theta - 27\sec\theta + C$$

$$= \frac{9}{27}(x^2 - 4x + 13)^{3/2} - \frac{27}{3}(x^2 - 4x + 13)^{1/2} + C$$