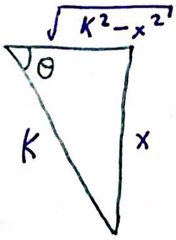
7.3. Sustitución Trigonométrica

2019-08/22

Forma $\sqrt{K^2 - x^2}$

$H = K$

$C.O. = x$

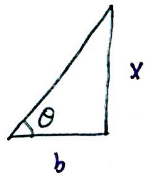


$$\frac{C.O.}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta$$

$$dx = K\cos\theta \, d\theta$$

$$\frac{\sqrt{K^2 - x^2}}{K} = \cos\theta$$

Forma $\sqrt{b^2 + x^2}$



$$\frac{x}{b} = \tan\theta \Rightarrow x = b \cdot \tan\theta$$

$$dx = b \cdot \sec^2\theta \, d\theta$$

$c^2 = x^2 + y^2$

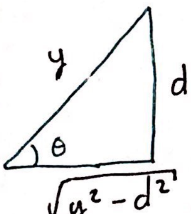
$\sqrt{c^2 - y^2} = x$

$\sqrt{c^2 - x^2} = y$

$$\frac{b}{\sqrt{b^2+x^2}} = \cos\theta \Rightarrow \sqrt{b^2 + x^2} = b\sec\theta$$

$$\frac{\sqrt{b^2+x^2}}{b} = \sec\theta$$

Forma $\sqrt{y^2 - d^2}$



$$\sin\theta = \frac{d}{y}$$

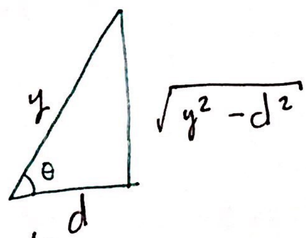
$$y = d \cdot \csc\theta$$

$$dy = -d \csc\theta \cot\theta \, d\theta$$

$$\frac{y}{d} = \sec\theta$$

$$y = d\sec\theta$$

$$dy = d\sec\theta\tan\theta$$



$$\sqrt{y^2 - d^2} = d\tan\theta$$

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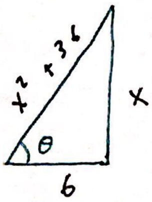
20 $$\int \frac{1}{x^2+36}\,dx = \int \frac{6\sec^2\theta\,d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$$

$x = 6\tan\theta$

$dx = 6 \cdot \sec^2\theta \, d\theta$

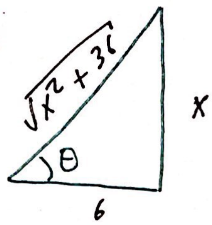
$x^2 + 36 = 36(\tan^2\theta + 1) = 36\sec^2\theta$



$x = 6\tan\theta \Rightarrow \frac{x}{6}\ \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{6}\right) = \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C$

② $$\int \frac{1}{\sqrt{x^2+36}}\,dx = \int \frac{6\sec^2\theta}{6\sec\theta}\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\therefore \ln\left|\frac{\sqrt{x^2+36}}{6} + \frac{x}{6}\right| + C$$

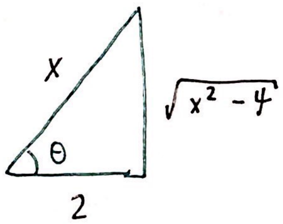


$\frac{x}{6} = \tan\theta$ $\qquad x = 6 \cdot \tan\theta$

$dx = 6\sec^2\theta\,d\theta$

$\frac{\sqrt{x^2+36}}{6} = \sec\theta$ $\qquad \sqrt{x^2+36} = 6\sec\theta$

③ $$\int \frac{\left(\sqrt{x^2-4}\right)^3}{x^6}\,dx = \int \frac{2^3\tan^3\theta}{2^6\sec^6\theta} \cdot 2\tan\theta\sec\theta\,d\theta =$$



$\frac{x}{2} = \sec\theta$ $\qquad x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\frac{\sqrt{x^2-4}}{2} = \tan\theta$ $\qquad \sqrt{x^2-4} = 2\tan\theta$

$$= \frac{2^4}{2^6}\int \frac{\tan^4\theta}{\sec^5\theta}\,d\theta = \frac{1}{2^2}\int \tan^4\theta\cos^5\theta\,d\theta = \frac{1}{4}\int \frac{\sin^4\theta}{\cancel{\cos^4\theta}} \cdot \cancel{\cos^5}\theta$$

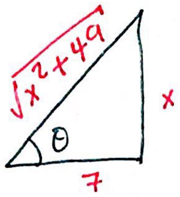
$$= \frac{1}{4}\int \sin^4\theta\cos\theta\,d\theta \Rightarrow \begin{matrix} w = \sin\theta \\ du = \cos\theta \end{matrix} \Rightarrow \frac{1}{4}\int u^4\,du = \frac{1}{4}\left[\frac{w^5}{5}\right] + C$$

$$= \frac{\sin^5\theta}{20} + C = \frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C$$

$\sin\theta = \frac{\sqrt{x^2-4}}{x}$

2a

$$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49 \cdot 7\csc^2\theta}{49\,\cot^2\theta\, 7\csc\theta}\,d\theta = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta$$



$$\cot\theta = \frac{x}{7} \implies x = 7\cot x$$

$$dx = -7\csc^2\theta\,d\theta$$

$$x = 7\tan\theta$$

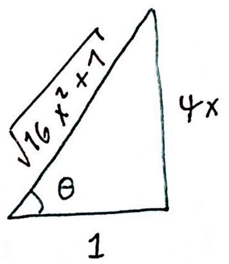
$$\frac{\sqrt{x^2+49}}{7} = \csc\theta \implies \sqrt{x^2+49} = 7\csc\theta$$

$$= -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\,\frac{\sin^2\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos^2\theta}\,d\theta = -\int \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$

$$= -\int \tan\theta\sec\theta\,d\theta = -\sec\theta + C = \frac{-\sqrt{x^2+49}}{x} + C$$

3b

$$\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta}{1/4\,\tan\theta\cdot\sec\theta} = \int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{\sec\theta}{\tan\theta}\,d\theta$$



$$\frac{4x}{1} = \tan\theta \implies x = \frac{\tan\theta}{4}$$

$$\sqrt{16x^2+1} = \sec\theta \qquad dx = \frac{1}{4}\sec^2\theta\,d\theta$$

$$= \int \left[\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right] d\theta = \int \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}\sin\theta}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta =$$

$$= -\ln\left|\csc\theta + \cot\theta\right| + C = -\ln\left|\frac{\sqrt{16x^2-1}}{4x} + \frac{1}{4x}\right| + C$$