sesión Resolución Dudas Lun 1-2:30 PM D-503.

Parcial 2 Lun 2:30 CES.

Conto 9 1 Fracciones Parciales.

Lab 11 Mux Entrega Jueves 17 octubre.

Fracciones Parciales.

Caso 1 y 2: Factores Lineales.

$$\frac{P(x)}{x(x+1)^2(x+2)^3} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+2)} + \frac{E}{(x+2)^2} + \frac{F}{(x+2)^3}$$

$$\int \frac{A\,dx}{x+a} = A\ln|x+a|+C \qquad \int B(x+a)^{-n}\,dx \; \frac{-B}{-n+1}\frac{1}{(x+a)^{n-1}} + C.$$

Ejemplo 1: $\int \frac{x^2+x}{(x+2)^2(x-1)}\,dx$ $\qquad (x^2+4x+4)(x-1)$

$$\frac{x^2+x}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)} \quad *(x+2)^2(x-1)$$

$$x^2+x = A(x+2)(x-1) + B(x-1) + C(x+2)^2.$$

cero en $x=-2$ y en $x=1$

$x=-2$: $2 = 0 - 3B + 0$ $\qquad B = -2/3. = -6/9$

$x=1$: $1^2+1 = 0 + 0 + 9C$ $\qquad C = 2/9$

$x=0$: $0 = -2A - B + 4C.$ $\qquad 2A = -B - 4C = -2/9$

$A = -1/9, \quad B = -2/3 \quad C = 2/9. \quad \int (x+2)^{-2}\,dx$

$$\int \frac{x^2+x}{(x+2)^2(x-1)}\,dx = -\frac{1}{9}\int \frac{dx}{x+2} - \frac{2}{3}\int \frac{dx}{(x+2)^2} + \frac{2}{9}\int \frac{dx}{x-1}$$

$$= C \quad -\frac{1}{9}\ln|x+2| + \frac{2}{3}\,\frac{1}{x+2} + \frac{2}{9}\ln|x-1|$$

Caso 3: Factores Cuadráticos Irreducibles.

no tienen raíces reales.

x^2+36 $\qquad$ x^2+x+1

$b^2-4ac<0$ imaginario

no se pueden factorizar.

$x^2 = -36.$ $\qquad$ $x^2+x+1=0$

cuadrática: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-4}}{2}$ no tiene soln reales

$$\frac{P(x)}{(x^2+36)(x^2+x+1)} = \frac{Ax+B}{x^2+36} + \frac{Cx+D}{x^2+x+1}$$

$$\int \frac{x}{\underbrace{x^2+k^2}_{u}}\,\overbrace{dx}^{du/2} \qquad \int \frac{du}{2u} = \frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|x^2+k^2|+C.$$

$$\int \frac{1}{x^2+k^2}\,dx = \int \frac{k\sec^2\theta\,d\theta}{k^2\sec^2\theta} = \frac{1}{k}\int d\theta = \frac{1}{k}\theta + C.$$

$x = k\tan\theta.$

$dx = k\sec^2\theta\,d\theta$

$$\frac{1}{k}\tan^{-1}\left(\frac{x}{k}\right) + C.$$

Ejercicio 6: Integre P. 69.

b. $\int \frac{2x^2-x-4}{x^3+4x}\,dx$ $\qquad x(x^2+4) = x^3+4x$

$$\frac{2x^2-x-4}{x^3+4x} = \frac{A}{x} + \frac{Bx}{x^2+4} + \frac{C}{x^2+4} \quad * \; x^3+4x.$$

$$2x^2-x-4 = Ax^2+4A+Bx^2+Cx$$

$$2x^2-x-4 = (A+B)x^2 + Cx + 4A.$$

Igualando los coeficientes.

Grado 2: $A+B=2. \Rightarrow B=2-A. = 3.$

Grado 1: $C=-1 \Rightarrow C=-1$

Grado 0: $4A=-4 \Rightarrow A=-1$

$A=-1,\ B=3,\ C=-1$

$$\int \frac{2x^2-x-4}{x^3+4x}\,dx = -\int \frac{dx}{x} + 3\int \frac{x\,dx}{x^2+4} - \int \frac{dx}{x^2+4}$$

$$= -\ln|x| + \frac{3}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C.$$

b. $\int \frac{1}{x^4-1}\, dx$

$$x^4-1 = (x^2+1)(x^2-1) = (x^2+1)(x-1)(x+1)$$

$$\frac{1}{x^4-1} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+1}$$

$$1 = (Ax+B)(x^2-1) + C(x^2+1)(x+1) + D(x^2+1)(x-1)$$

$$0x^3+0x^2+1 = Ax^3+Bx^2-Ax-B+Cx^3+Cx^2+Cx+C.$$
$$+0x \qquad +Dx^3-Dx^2+Dx-D$$

Igualando coeficientes.

(1) $A+C+D=0$ (1)+(3) $2C+2D=0$ (5)

(2) $B+C-D=0$ (2)+(4) $2C-2D=1$ (6)

(3) $-A+C+D=0$

(4) $-B+C-D=1$ $4C=1$

$C=1/4$ $D=-C=-1/4.$

$A=-C-D=-1/4+1/4=0$ $B=D-C=-\frac{1}{4}-\frac{1}{4}=-\frac{1}{2}.$

$$\int \frac{dx}{x^4-1} = \int \frac{-1/2}{x^2+1} + \frac{1/4}{x-1} - \frac{1/4}{x+1}\, dx$$

$$= -\frac{1}{2}\tan^{-1}x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + C.$$

Factores Cuadráticos repetidos (Caso 4) P. 70.

$$\frac{P(x)}{(x^2+25)^2(x^2+36)^3} = \frac{Ax+B}{(x^2+25)} + \frac{Cx+D}{(x^2+25)^2} + \frac{Ex+F}{(x^2+36)} + \frac{Gx+H}{(x^2+36)^2} + \frac{Ix+J}{(x^2+36)^3}$$

Ejercicio 7: $\int \frac{x^2+2}{x^5+8x^3+16x}\,dx$

Cuadrado perfecto

$$\int \frac{x^2+2}{x^5+8x^3+16x}\,dx \qquad x(x^4+8x^2+16)$$

$$x(x^2+4)^2.$$

$$\frac{x^2+2}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}.$$

Multiplique por $x(x^2+4)^2$.

$$x^2+2 = A(x^4+8x^2+16) + (Bx+C)(x^3+4x) + Dx^2 + Ex$$

$$x^2+2 = Ax^4 + 8Ax^2 + 16A + Bx^4 + Cx^3 + 4Bx^2 + 4Cx + Dx^2 + Ex.$$

Grado Cuatro: $A + B = 0 \Rightarrow B = -A = -1/8.$

Grado tres: $C = 0 \Rightarrow C = 0$

Grado Dos: $8A + 4B + D = 1$ $\quad D = 1 - 1 + 4/8 = 1/2.$

Grado Uno: $4C + E = 0 \Rightarrow E = -4C = 0$

Grado cero. $16A = 2 \Rightarrow A = 1/8$

$A = 1/8, \quad B = -1/8, \quad C = 0, \quad D = 1/2, \quad E = 0.$

$$\int \frac{x^2+2}{x(x^2+4)^2}\,dx = \frac{1}{8}\int \frac{dx}{x} - \frac{1}{8}\int \frac{x}{x^2+4}\,dx + \frac{1}{2}\int \frac{x}{(x^2+4)^2}\,dx$$

$$= \frac{1}{8}\ln|x| - \frac{1}{16}\ln|x^2+4| - \frac{1}{2}\frac{1}{2}\frac{1}{(x^2+4)} + C$$

$$8A + 4B + D = 1$$

$$1 - \frac{4}{8} + 0 = 1 \quad \Rightarrow \quad 0 = 1 - 1 + \frac{4}{8} = \frac{4}{8}$$

$$\frac{P(x)}{(x+1)(x+2)^2(x^2+4)(x^2+9)^2}$$