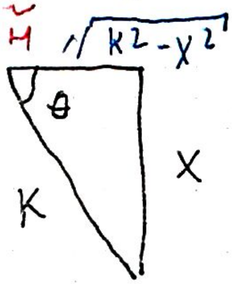
7.3 Sustitución Trigonométrica

Forma $\sqrt{K^2 - x^2}$

$H = K$

$C.O = x$

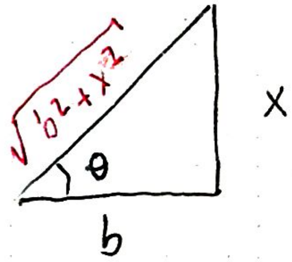


$\frac{C.O}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta.$

$dx = K\cos\theta\, d\theta.$

$\frac{\sqrt{K^2-x^2}}{K} = \cos\theta \Rightarrow \sqrt{K^2-x^2} = K\cdot\cos\theta.$

Forma $\sqrt{b^2 + x^2}$

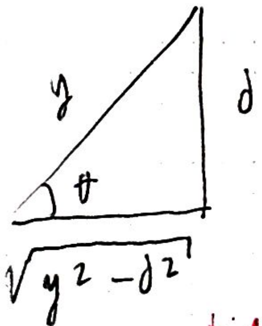


$\frac{x}{b} = \tan\theta. \Rightarrow x = b\cdot\tan\theta.$

$dx = b\cdot\sec^2\theta\, d\theta.$

$\frac{\sqrt{b^2+x^2}}{b.} = \sec\theta \Rightarrow \sqrt{b^2+x^2} = b\sec\theta.$

Forma $\sqrt{y^2 - d^2}.$



$\text{sen}\,\theta = \frac{d}{y}$

$y = d\cdot\csc\theta.$

$dy = -d\cdot\csc\theta\cot\theta.$

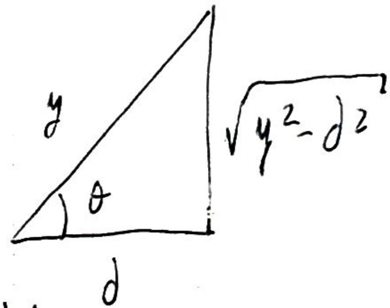
tiene signos negativos.

$\frac{y}{d} = \sec\theta.$

$y = d\sec\theta.$

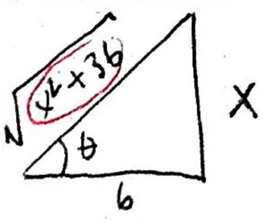
$dy = d\cdot\sec\theta\tan\theta.$

$\sqrt{y^2-d^2} = d\tan\theta.$



Ejercicios 2 y 3 Pág 58 y 59.

20) $\int \frac{1}{x^2+36}dx = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$



$x = 6\cdot\tan\theta.$

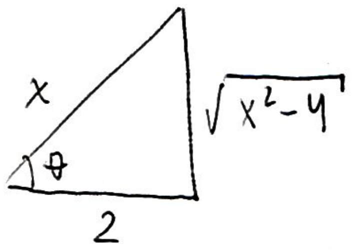
$dx = 6\cdot\sec^2\theta\, d\theta.$

$x^2+36 = 36(\tan^2\theta + 1) = 36\sec^2\theta$

$$\frac{1}{6}\int d\theta = \frac{1}{6}\theta + C. = \boxed{\frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C.}$$

$$\tan^{-1}\left(\frac{x}{6}\right) = \theta.$$

3a. $$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \int \frac{2^3\tan^3\theta.}{2^6\cdot\sec^6\theta.}\, 2\cdot\sec\theta\tan\theta\, d\theta.$$



$$\frac{2}{x} = \cos\theta. \to x = \frac{2}{\cos\theta} = 2\sec\theta.$$

$$dx = 2\sec\theta\tan\theta\, d\theta.$$

$$\sqrt{x^2-4} = 2\tan\theta.$$

$$\left[(x^2-4)^{1/2}\right]^3 = 8\tan^3\theta.$$

$$\frac{1}{2^2}\int \frac{\tan^4\theta.}{\sec^5\theta.}\,d\theta. = \frac{1}{4}\int \frac{\sin^4\theta}{\cos^4\theta.}\cos^5\theta\, d\theta.$$

$$= \frac{1}{4}\int \underbrace{\sin^4\theta}_{u^4}\ \underbrace{\cos\theta\, d\theta}_{du}. = \boxed{\frac{1}{4}\cdot\frac{1}{5}\sin^5\theta + C.}$$

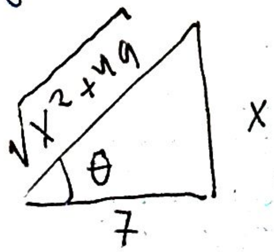
$u = \sin\theta \quad du = \cos\theta\, d\theta$

$\frac{1}{4}\int u^4\, du.$

Regrese a la variable x $\quad \sin\theta = \frac{\sqrt{x^2-4}}{x}$, $\quad \sin^5\theta = \frac{(x^2-4)^{5/2}}{x^5}$

$$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \boxed{\frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C.}$$

2a $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{49\ \cancel{7}\sec^2\theta}{49\tan^2\theta.\ \cancel{7}\sec\theta.}\,d\theta. = \int \frac{\sec\theta}{\underbrace{\tan^2\theta}_{\sec^2\theta - 1}}\,d\theta.$$

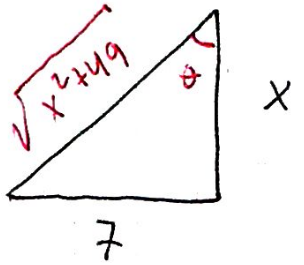


$\frac{x}{7} = \tan\theta$ $\qquad x = 7\tan\theta.$

$dx = 7\sec^2\theta\, d\theta.$

$\frac{H}{C.A.} = \sec\theta.$ $\qquad \sqrt{x^2+49} = 7\sec\theta.$

2a) $\displaystyle\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49 \cdot 7\csc^2\theta\,d\theta}{49\cot^2\theta\; 7\csc\theta} = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta.$



$\dfrac{x}{7} = \cot\theta \Rightarrow x = 7\cot\theta.$

$dx = -7\csc^2\theta\,d\theta.$

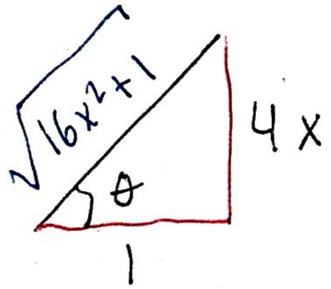
$\dfrac{\sqrt{x^2+49}}{7} = \csc\theta \Rightarrow \sqrt{x^2+49} = 7\csc\theta.$

$$-\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\,\frac{\sin^2\theta}{\cos^2\theta}\,d\theta. = -\int \frac{\sin\theta}{\cos\theta}\,\frac{1}{\cos\theta}\,d\theta.$$

$$= -\int \sec\theta\tan\theta\,d\theta.$$

$$= -\sec\theta + C. = -\frac{\sqrt{x^2+49}}{x} + C.$$

3b) $\displaystyle\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta.}{(1/4)\tan\theta.\sec\theta.} = \int \frac{\sec\theta}{\tan\theta}\,d\theta.$



$\dfrac{4x}{1} = \tan\theta. \Rightarrow x = \frac{1}{4}\tan\theta.$

$dx = \frac{1}{4}\sec^2\theta\,d\theta.$

$\sqrt{16x^2+1} = \sec\theta.$

$\underbrace{\ln|\quad\quad|}$

$$\int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{1}{\cos\theta}\,\frac{\cos\theta}{\sin\theta.}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta.$$

$$= -\ln\left|\csc\theta + \cot\theta\right| + C.$$

$$= -\ln\left|\frac{\sqrt{16x^2+1}}{4x} + \frac{1}{4x}\right| + C.$$