# Cálculo Integral

Contenido de clase

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# Índice general

# Parte I

# WebAssign

# Capítulo 1

## Sustitución

Webassign Sustitución

① $$\int \cos(6x)\,dx = \frac{1}{6}\int \cos(u)\,du = \frac{\sin(6x)}{6} + C$$

$u = 6x$

$\frac{du}{6} = dx$

② $$\int x^2\sqrt{x^3+13}\,dx = \int \sqrt{u}\,\frac{du}{3} = \frac{1}{3}\int u^{3/2} = \frac{1}{3}\,\frac{(x^3+13)^{3/2}}{3/2}$$

$u = x^3 + 13$

$du = 3x^2 + 0\ dx$

$\frac{du}{3} = x^2\,dx$

$$= \frac{1}{3}\cdot\left[\frac{\frac{(x^3+13)^{3/2}}{1}}{\frac{3}{2}}\right] =$$

$$= \frac{1}{3}\cdot\frac{2(x^3+13)^{3/2}}{3}$$

$$= \frac{2}{9}(x^3+13)^{3/2} + C$$

③ $$\int \sin^3\theta\cos\theta\,d\theta = u = \sin\theta$$

$u = \sin\theta$

$du = \cos\theta$

$$= \int u^3\,du = \frac{u^4}{4} + C$$

$$= \frac{\sin^4\theta}{4} + C$$

④ $$\int \frac{x^3}{x^4-2}\,dx = \int \frac{du}{u} = \ln|u| + C$$

$$= \frac{\ln|x^4-2|}{-4} + C$$

$u = x^4 - 2$

$du = 4x^3 - 0\ dx$

$\frac{du}{4} = x^3\,dx$

⑤ $$\int (4-4x)^7\,dx = \int u^7 \cdot -\frac{du}{4} = -\frac{1}{4}\int u^7 = -\frac{1}{4}\cdot\frac{u^8}{8}$$

$u = 4-4x$

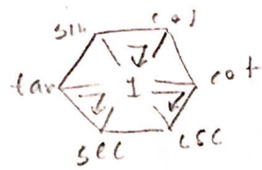
$du = -4\,dx$

$-\frac{du}{4} = dx$

$$= -\frac{(4-4x)^8}{8\cdot 4} + C$$

$$= -\frac{(4-4x)^8}{32} + C$$

⑥ $$\int \sin(t)\sqrt{1+\cos(t)}\;dt = -\int \sqrt{u}\;du$$




$u = 1 + \cos(t)$

$du = (0 - \sin(t))\,dt$

$-du = \sin(t)\,dt$

$$= -\frac{u^{3/2}}{3/2} + C$$

$\sin^2\theta + \cos^2\theta = 1$

$$= -\frac{(1+\cos(t))^{3/2}}{3/2}$$

$$= -\frac{2(1+\cos(t))^{3/2}}{3} + C$$

⑦ $$\int \frac{e^u}{(1-e^u)^2}\,du = -\int \frac{1}{(\upsilon)^2}\,du = -\int \upsilon^{-2}\,du$$

$\upsilon = 1 - e^u$

$d\upsilon = 0 - e^u\,du$

$-d\upsilon = e^u\,du$

$$= -\left\{\frac{\upsilon^{-1}}{-1}\right\}$$

$$= -\left\{-\frac{1}{\upsilon}\right\}$$

$$= \frac{1}{\upsilon} + C$$

$$= \frac{1}{(1-e^u)} + C$$

8

$$\int \frac{a + bx^7}{\sqrt{8ax + bx^8}}\, dx = \frac{1}{8}\int \frac{du}{\sqrt{u}} = \frac{1}{8}\int u^{-1/2}\, du$$

$u = 8ax + bx^8$

$$= \frac{1}{8}\left\{\frac{u^{1/2}}{1/2}\right\}$$

$du = 8a + 8bx^7$

$du = 8(a + bx^7)$

$$= \frac{1}{8}\left\{2u^{1/2}\right\}$$

$\frac{du}{8} = a + bx^7$

$$= \frac{1}{\cancel{2}\cdot 4} \cdot \cancel{2}u^{1/2}$$

$$= \frac{u^{1/2}}{4} + C$$

$$= \frac{\sqrt{8ax + bx^8}}{4} + C$$

9

$$\int \frac{\ln(x)^{16}}{x}\, dx = \int \ln(x)^{16} \cdot \underbrace{\left(\frac{1}{x}\, dx\right)}_{du}$$

$u = \ln(x)$

$du = \frac{1}{x}\, dx$

$$= \int u^{16}\, du = \frac{u^{17}}{17} + C = \frac{\ln^{17}(x)}{17} + C$$

10

$$\int \sec^2\theta \tan^3\theta\, d\theta = \int u^3\, du = \frac{u^4}{4} + C$$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$= \frac{\tan^4\theta}{4} + C$$

11

$$\int e^x \sqrt{7 + e^x}\, dx = \int u\, du$$

$u = 7 + e^x$

$du = e^x\, dx$

$$= \frac{u^2}{2} + c$$

$$= \frac{(7 + e^x)}{2} + C$$

(12) $\displaystyle\int \frac{dx}{tx+g} = \frac{1}{t}\int \frac{1}{u}\,du = \frac{1}{t}\ln|tx+g|+C$

$u = tx+g$

$du = (t+0)\,dx$

$\dfrac{du}{t} = dx$

(18) $\displaystyle\int \tan^8(\theta)\sec^2\theta\,d\theta = \int u^8\,du = \frac{u^9}{9}+C = \frac{\tan^9(\theta)}{9}+C$

$u = \tan(\theta)$

$du = \sec^2\theta\,d\theta$

(17) $\displaystyle\int \frac{x^5\,dx}{1+x^{12}} = \frac{1}{6}\int \frac{du}{1+u^2} = \frac{1}{6}\int \frac{1}{1+u^2}\,du$

$u = x^6$

$du = 6x^5\,dx$

$\dfrac{du}{6} = x^5\,dx$

$= \dfrac{1}{6}\tan^{-1}(u)+C$

$= \dfrac{1}{6}\tan^{-1}(x^6)+C$

(16) $\displaystyle\int \cot(18x)\,dx = \int \frac{\cos(18x)}{\sin(18x)}\,dx$

$u = \sin(18x)$

$du = \cos(18x)\,18\,dx$

$\dfrac{du}{18} = \cos(18x)\,dx$

$\displaystyle= \frac{1}{18}\int \frac{1}{u}\,du = \frac{1}{18}\ln|u|+C$

$= \dfrac{1}{18}\ln|\sin(18x)|+C$

(15)

$$\int \frac{\sin(2x)}{28+\cos^2 x}\,dx = \int \frac{\sin(2x)}{28+\left(\frac{1+\cos(2x)}{2}\right)}\,dx$$

$$= \int \frac{\sin(2x)}{\frac{56}{2}+\frac{1}{2}+\frac{\cos(2x)}{2}}\,dx$$

$$= \int \frac{\frac{\sin(2x)}{1}}{\frac{57+\cos(2x)}{2}}\,dx = \int \frac{2\sin(2x)}{57+\cos(2x)}\,dx = -\frac{2}{2}\int \frac{du}{57+u}$$

$u = \cos(2x)$

$du = -\sin(2x)\cdot 2\,dx$

$-\frac{du}{2} = \sin(2x)\,dx$

$$= -\int \frac{1}{57+u}\,du$$

$$= -\ln|57+\cos(2x)| + C$$

(14)

$$\int (\cot(x))^{\frac{1}{30}} \csc^2(x)\,dx = -\int u^{1/30}\,du = \frac{-(\cot x)^{31/30}}{31/30} + C$$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$\frac{1}{30} + \frac{30}{30} = \frac{31}{30}$

(13) $$\int \frac{\cos\left(\frac{\pi}{x^{11}}\right)}{x^{12}}\,dx = \frac{1}{\pi}\int \cos(u)\,du = -\frac{1}{\pi}\sin(u) + C$$

$$= -\frac{1}{11\pi}\sin\left(\frac{\pi}{x^{12}}\right) + C$$

$u = \dfrac{\pi}{x^{12}} = \pi \cdot x^{-12}$

$du = -11\pi\, x^{-12}$

$du = -\dfrac{\pi}{x^{12}}$

$-\dfrac{du}{11\pi} = \dfrac{1}{x^{12}}$

(11) $$\int e^x\sqrt{7+e^x}\,dx = \int \sqrt{u}\,du = \frac{u^{3/2}}{3/2} + C$$

$u = 7 + e^x$

$du = e^x\,dx$

$$= \frac{2(7+e^x)^{3/2}}{3} + C$$

# Capítulo 2

## Área Entre Curvas

Webassign área entre curvas. 2019-09-21

① $y_1 = \sqrt[3]{x}$ $y_2 = \frac{1}{x}$ $x = 8$

$I_{y_1} = I_{y_2}$ en $(1,1)$

$$A = \int_1^8 \sqrt[3]{x} - \frac{1}{x}dx = \frac{3x^{\frac{4}{3}}}{4} - \ln(x)\Big]_1^8 =$$

$$= \left\{\frac{3}{4}(8)^{\frac{4}{3}} - \ln(8)\right\} - \left\{\frac{3}{4} - 0\right\} = \frac{3}{4}(\sqrt[3]{8})^4 - \ln(8) - \frac{3}{4}$$

$$= \frac{3}{4}(2)^4 - \ln(8) - \frac{3}{4} = \frac{16}{4} \cdot 3 - \ln(8) - \frac{3}{4}$$

$$= 4 \cdot 3 - \ln(8) - \frac{3}{4} = 12 - \ln(8) - \frac{3}{4}$$

② $x = y^2 - 4$ $y = \pm 1$ $x = e^y$

$$A = 2\int_0^1 e^y - y^2 + 4 = e^y - \frac{1}{3}y^3 - 4y\Big]_0^1 =$$

$$= 2\left[\left\{e - \frac{1}{3} - 4\right\} - \left\{0\right\}\right] =$$

③ $y = e^x, \quad y = x^2 - 1, \quad x = -1, \quad x = 1$

$$A = \int_{-1}^{1} e^x - (x^2 - 1)\, dx = e^x - \frac{1}{3}x^3 + x \Big]_{-1}^{1} =$$

$$= \left\{ e - \frac{1}{3} + 1 \right\} - \left\{ e^{-1} + \frac{1}{3} - 1 \right\}$$

$$= e - \frac{1}{3} + 1 - e^{-1} - \frac{1}{3} + 1$$

$$e - e^{-1} - \frac{2}{3} + \frac{2 \cdot 3}{3}$$

$$e - e^{-1} + \frac{8}{3}$$

mcd(5, 2)

$5 = 2 \cdot 2 + 1$

$2 = 2 + 0$

$2 = 2 + 0$

$(2, 0)$

mcd(205, 100)

$205 = 100 \cdot 2 + 5$

$100 = 5 \cdot 20 + 0$

mcd(20, 100)

10 3

$10 = 3 \cdot 3 + 1$

$3 = 3 + 0$

mcd(3, 0)

10, 3

$3 \cdot 3 + 1$

3, 1

$3 = 1 \cdot 3 + 0$

$(3, 0)$

② $x = y^2 - 4 \qquad x = e^y$

$$A = \int_{-1}^{1} e^y - y^2 + 4 \, dy = e^y - \frac{1}{3}y^3 + y \Big]_{-1}^{1} =$$

$$= \left\{ e - \frac{1}{3} + 4 \right\} - \left\{ e^{-1} + \frac{1}{3} - 4 \right\} =$$

$$= e - \frac{1}{3} + 4 - e^{-1} - \frac{1}{3} + 4$$

$$e - e^{-1} - \frac{2}{3} + 8 = \underline{e - e^{-1} + \frac{22}{3}}$$

③ $y_1 = e^x \qquad y_2 = x^2 - 1 \quad , \; x = \pm 1$

$$A = \int_{-1}^{1} e^x - (x^2 - 1) \, dx = \int e^x - x^2 + 1 \, dx = e^x - \frac{1}{3}x^3 + x \Big]_{-1}^{1}$$

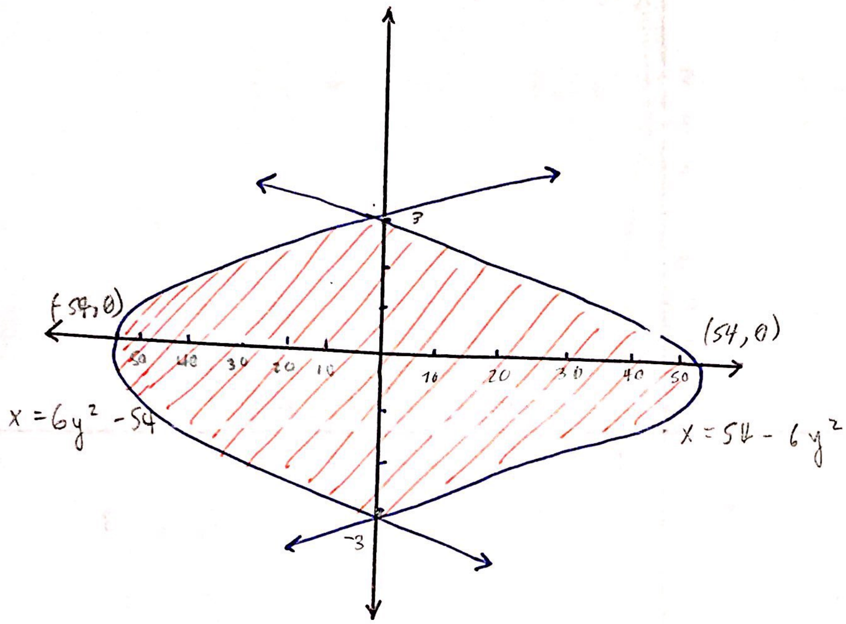
$$= \left\{ e - \frac{1}{3} + 1 \right\} - \left\{ e^{-1} + \frac{1}{3} - 1 \right\} = e - \frac{1}{3} + 1 - e^{-1} - \frac{1}{3} + 1$$

$$= e - e^{-1} - \frac{2}{3} + 2 = \underline{e - e^{-1} + \frac{4}{3}} \; A$$

$$-\frac{2}{3} + \frac{2 \cdot 3}{3} = \frac{-2 + 6}{3} = \frac{4}{3}$$

④ By parts:

$x = 54 - 6y^2 \qquad x = 6y^2 - 54$




$0 = 54 - 6y^2$

$0 = 6(9 - y^2)$

$0 = 9 - y^2$

$-9 = -y^2$

$9 = y^2$

$\pm\sqrt{9} = y$

$\pm 3 = y$

$x = 54 - 0$

$x = 54$

$0 = 6y^2 - 54$

$0 = 6(y^2 - 9)$

$0 = y^2 - 9$

$\pm 9 = y^2$

$\pm\sqrt{9} = y$

$\pm 3 = y$

$x = -54$

• with respect to -y:

$$A = \int_{-3}^{3} (54 - 6y^2) - (6y^2 - 54)\, dy$$

$$= \int_{-3}^{3} 54 - 6y^2 - 6y^2 + 54\, dy$$

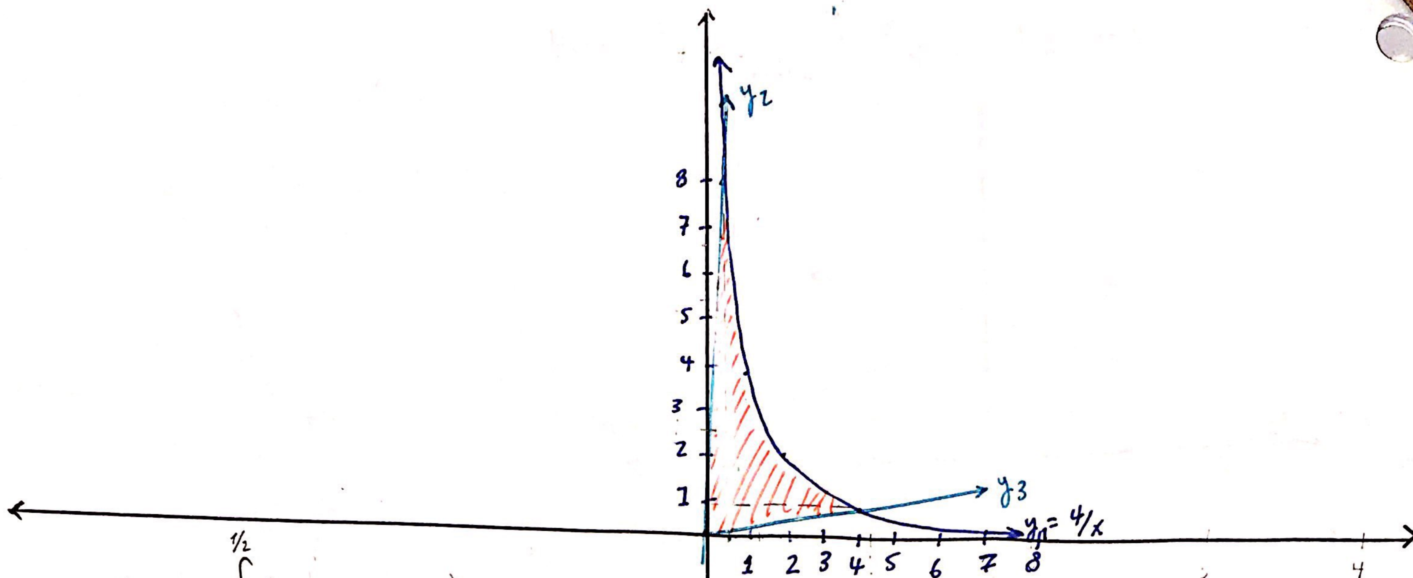
$$= \int_{-3}^{3} 108 - 12y^2\, dy$$

$$= 12\int_{-3}^{3} (9 - y^2)\, dy = 12\left[\left(9y - \frac{1}{3}y^3\right)\right]_{-3}^{3} =$$

$$= 24\left[\left(9(3) - \frac{1}{3}(3)^3\right) - (0)\right]_{0}^{3} = 24\,(27 - 9) = 24 \cdot 18 =$$

$$= \underline{432}$$

$y_1 = \frac{4}{x}$ ; $y_2 = 16x$ ; $y_3 = \frac{1}{4}x$ , $x > 0$




$$A_1 = \int_0^{1/2} \left(16x - \frac{1}{4}x\right) dx$$

$$= \left(\frac{16}{2}x^2 - \frac{1}{8}x^2\right)\Big]_0^{1/2} =$$

$$= \left[\left(8\left(\frac{1}{2}\right)^2 - \frac{1}{8}\left(\frac{1}{2}\right)\right) - (0)\right] =$$

$$= 8\left(\frac{1}{4}\right) - \frac{1}{16} = \frac{8}{4} - \frac{1}{16} =$$

$$= 2 - \frac{1}{16} = \boxed{\frac{31}{16}}$$

$y_1 = y_2$

$\frac{4}{x} = 16x$

$4 = 16x^2$

$\frac{4}{16} = x^2$

$\pm\sqrt{\frac{1}{4}} = x$

$x = \frac{1}{2}$

$y_1 = y_3$

$\frac{4}{x} = \frac{1}{4}x$

$4 = \frac{1}{4}x^2$

$16 = x^2$

$\pm\sqrt{16} = x$

$\pm 4 = x$

$x = 4$

$$A_2 = \int_{1/2}^{4} \left(\frac{4}{x} - \frac{1}{4}x\right)dx = \int_{1/2}^{4} \left(4\cdot\frac{1}{x} - \frac{1}{4}\cdot x\right)dx =$$

$$= \left[\left(4\ln|x| - \frac{1}{4\cdot 2}x^2\right)\right]_{1/2}^{4} =$$

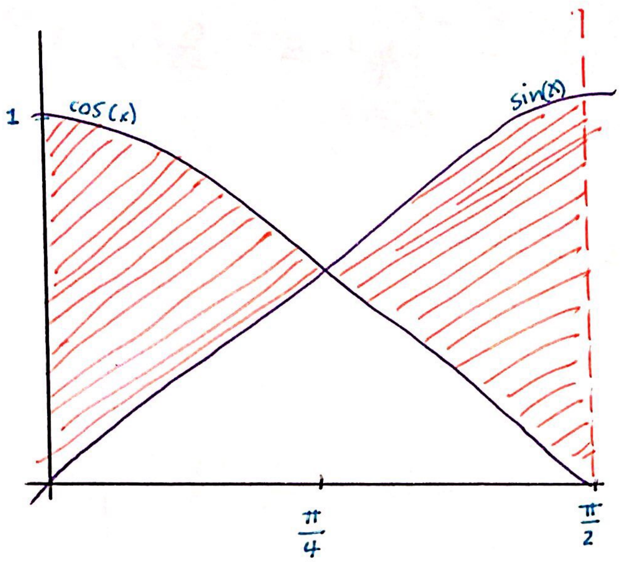
$$= \left[\left(4\ln(4) - \frac{1}{8}(16)\right) - \left(4\ln(1/2) - \frac{1}{8}\left(\frac{1}{2}\right)^2\right)\right] =$$

$$= \left[4\ln(4) - 2 - 4\ln(1/2) + \frac{1}{16}\right] =$$

$$= 4\ln(4) - 4\ln(1/2) \underbrace{-2 + \frac{1}{16} + \frac{31}{16}}_{0}$$

$$4(\ln(4) - \ln(1/2))$$

6




$$A_1 = \int_0^{\pi/4} \left(\cos(x) - \sin(x)\right) dx = \left[\left(\sin x + \cos x\right)\right]_0^{\pi/4} =$$

$$= \left[\left(\underbrace{\underbrace{\sin(\pi/4)} + \underbrace{\cos(\pi/4)}}_{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)}\right) - \left(\underbrace{\underbrace{\sin(0)}_{0} + \underbrace{\cos(0)}_{1}}_{\underbrace{0+1}_{1}}\right)\right]$$

$$= \frac{\sqrt{2} + \sqrt{2}}{2} - 1$$

$$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}} - 1 = \sqrt{2} - 1$$

$$A_2 = \int_{\pi/4}^{\pi/2} \left(\sin(x) - \cos(x)\right) dx =$$

$$= \left[\left(-\cos x - \sin x\right)\right]_{\pi/4}^{\pi/2} = \left[\left(-\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} - \cancel{\sin\left(\frac{\pi}{2}\right)}^{1}\right) - \left(-\cancel{\cos\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}} - \cancel{\sin\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}}\right)\right]$$

$$= \left[\left(-0 - 1\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right] = \left[-1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] =$$

$$= -1 + \sqrt{2} + \sqrt{2} - 1 = \boxed{-2 + 2\sqrt{2}}$$

$\square$

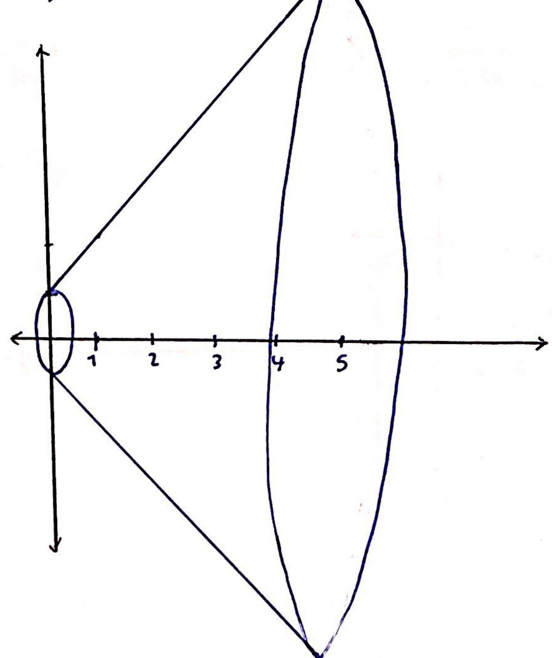
# Capítulo 3

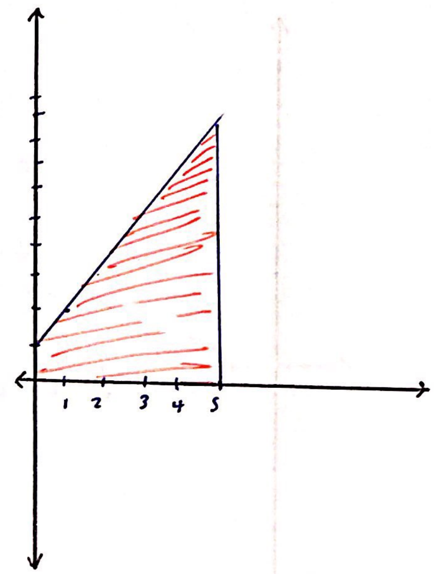
## Volúmenes

2019-09-27

# Webassign 6.2 - 6.3 Volumenes

(1) $y = x + 1$ ; $y = 0$, $x = 0$, $x = 5$




integro respecto de x

$$V = \pi \int_0^5 (x+1)^2 \, dx$$

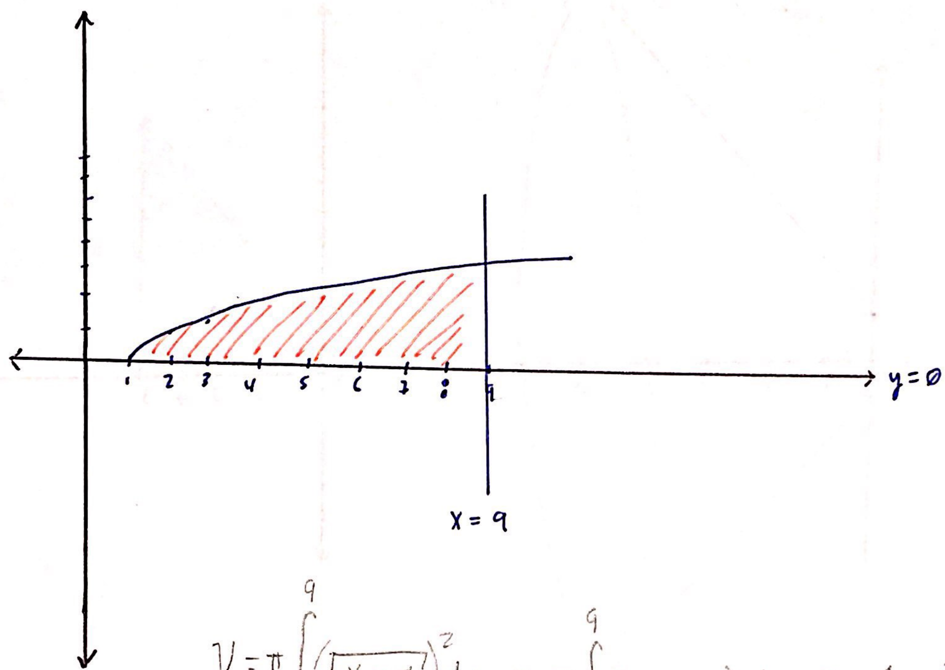
$$= \pi \int_0^5 x^2 + 2x + 1 \, dx$$

$$= \pi \left[ \frac{1}{3}x^3 + x^2 + x \right]_0^5 =$$

$$= \pi \left[ \left( \frac{(5)^3}{3} + (5)^2 + (5) \right] \right.$$

$$= \pi \, \frac{215}{3}$$

② $y = \sqrt{x-1}$, $y = 0$; $x = 9$; about x-axis




$$V = \pi \int_1^9 \left(\sqrt{x-1}\right)^2 dx = \pi \int_1^9 (x-1)\,dx = \frac{1}{2}x^2 - x \Big]_1^9 =$$

$$= \pi\left[\left(\frac{1}{2}(9)^2 - (9)\right) - \left(\frac{1}{2}(1)^2 - 1\right)\right]$$

$$= \pi\left[\left(\frac{63}{2}\right) - \left(-\frac{1}{2}\right)\right] = \pi\,\boxed{32}$$

③ $x = 2\sqrt{5y}$ , $x = 0$ ; $y = 3$

$$\frac{x}{2} = \sqrt{5y}$$

$$\left(\frac{x}{2}\right)^2 = 5y$$

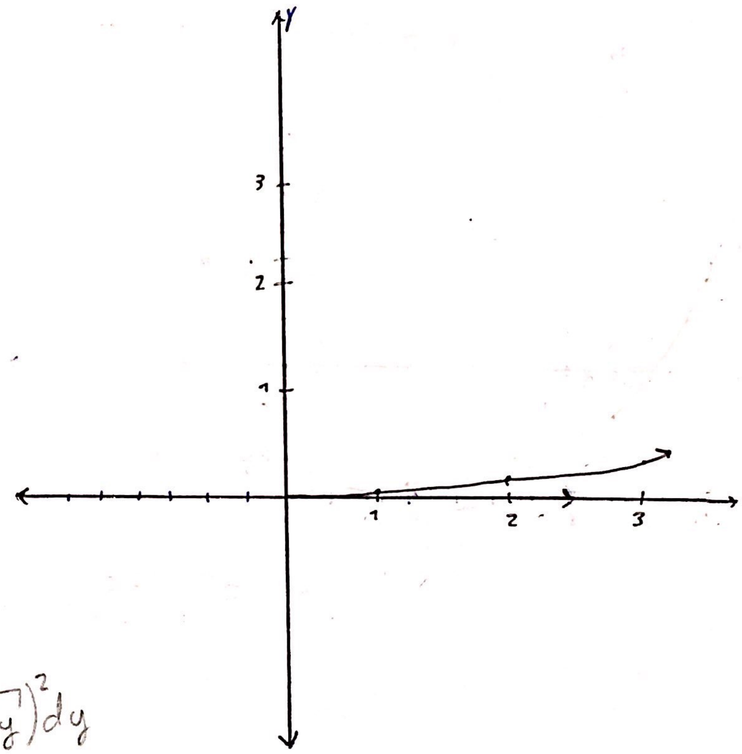
$$\frac{x^2}{4 \cdot 5} = y$$

$$\frac{x^2}{20} = y$$

$$\frac{x^2}{20} = 3$$

$$x^2 = 60$$

$$x = \sqrt{60}$$




$$V = \pi \int_0^3 \left(2\sqrt{5y}\right)^2 dy$$

$$= \pi \int_0^3 4(5y)\, dy = \pi \int_0^3 20y\, dy$$

$$= 20\pi \int_0^3 y\, dy = 20\pi\left(\frac{1}{2}y^2\right)\Big]_0^3$$

$$= 20\pi\left[\left(\frac{1}{2}(3)^2\right) - \cancel{\left(\frac{1}{2}(0)\right)}^{0}\right]$$

$$= 20\pi \, \frac{9}{2} = 10 \cdot 9\pi = \underline{90\pi}$$

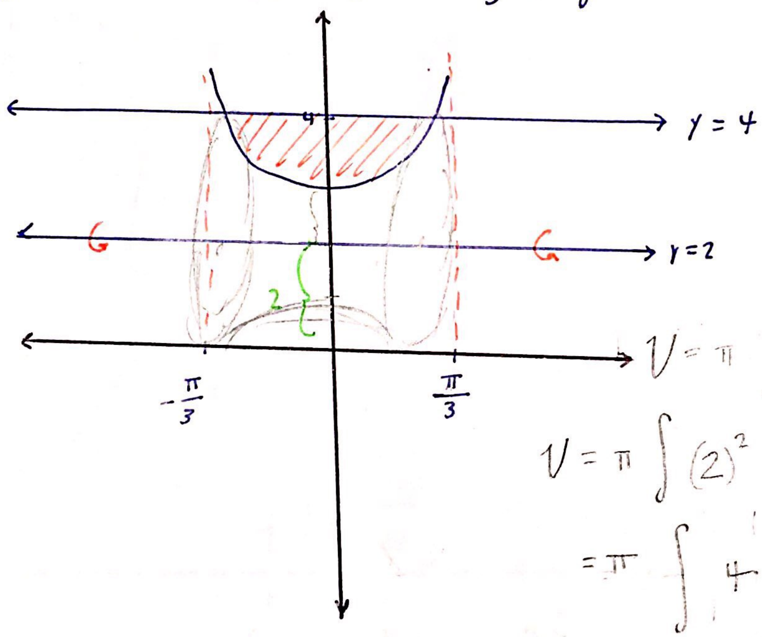
④ Step by step =

$$V = 36\pi \int_0^1 (x^2 - x^{12})\,dx$$

$$= 36\pi \left(\frac{1}{3}x^3 - \frac{1}{13}x^{13}\right)\Big]_0^1$$

$$= 36\pi\left(\frac{1}{3} - \frac{1}{13}\right) = 36\pi\left(\frac{10}{39}\right) = \frac{360}{39}\pi = \frac{120}{13}\pi$$

⑤ $y = 2 + \sec(x)$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$, $y = 4$




$y_{int} = -2 + (2 + \sec x)$

$y_{ext} = -2 + 4$

$$V = \pi \int (-2+4)^2 - (-2 + (2 + \sec x))^2\,dx$$

$$V = \pi \int (2)^2 - (-2 + 2 \sec x)^2\,dx$$

$$= \pi \int 4 - (\sec x)^2\,dx$$

$$= 2\pi \left[(4x) - (\tan x)\right]_0^{\pi/3}$$

$$= 2\pi \left\{\left[\left(\frac{4\pi}{3}\right) - \left(\tan(\pi/3)\right)\right] - [0]\right\}$$

$$= 2\pi\left(\frac{4\pi}{3} - \sqrt{3}\right)$$

⑥

$h = 5e^{-x^2}$ $\qquad r_{int} = x$

$$V = 2\pi \int_0^1 (x)(5e^{-x^2})\,dx =$$

$u = -x^2$

$du = -2x\,dx$

$\frac{-du}{2} = x\,dx$

$$= 2\pi \int_0^1 5e^{u} \cdot -\frac{du}{2} = -\frac{2\cdot 5\pi}{2} \int_0^1 e^{u}\,du$$

$$= -5\pi \int_0^1 e^{u}\,du = -5\pi\, e^{u}\Big]_0^1 =$$

$$= -5\pi\left[\left(e^{-1^2}\right) - \left(e^{-0^2}\right)\right]$$

$$= -5\pi\left[e^{-1} - e^{0}\right]$$

$$= -5\pi\left[e^{-1} - 1\right]$$

$$= \frac{-5\pi}{e} + 5\pi$$

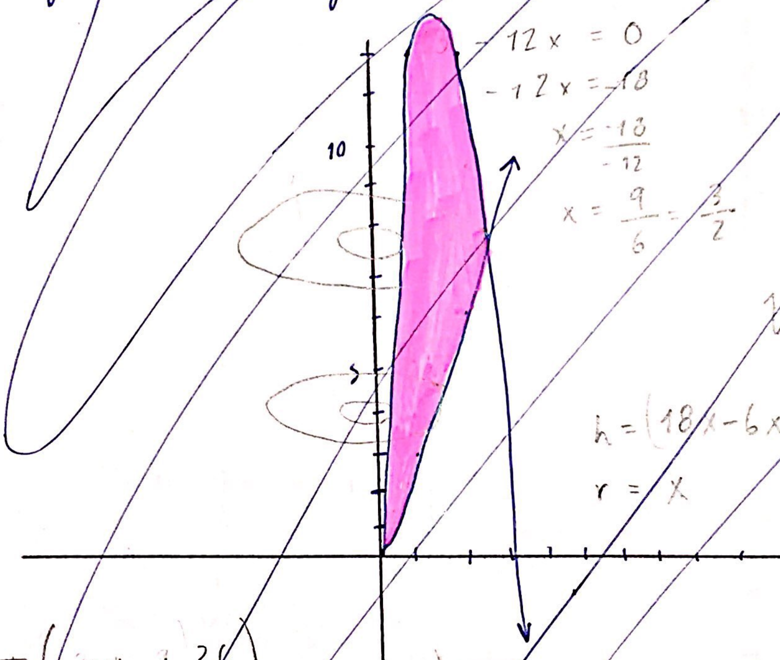
$$2\pi \int 12 e^{-x^2} x\,dx = 2\pi \cdot 12 \cdot -\frac{1}{2} \int e^{u}\,du$$

$u = -x^2$ $\qquad -24\pi \cdot \frac{1}{2}$

$-\frac{du}{2} = x\,dx$ $\qquad -12\pi\, e^{u} + C \Big]_0^1$

$$-12\pi \left[ e^{-1} - e^{0} \right]$$

(8) $y = 3x^2$ ; $y = 18x - 6x^2$



$-12x = 0$

$-12x = -18$

$x = \frac{-18}{-12}$

$x = \frac{9}{6} = \frac{3}{2}$

$18 - 6 = 12$

$36 - 24 = 12$

$$V = \pi \int_0^2 \left( (18x - 6x^2) - (3x^2) \right) x\,dx$$

$h = (18x - 6x^2) - (3x^2)$

$r = x$

$$V = \pi \int_0^2 18x - 6x^2 - 3x^2\,dx$$

$$= \pi \int -9x^2 + 18x$$

$$\pi \left( -\frac{9}{3}x^3 + \frac{18}{2}x^2 \right) \Big]_0^2$$

$= \pi(-24 + 36)$

$= \pi(12) = 12\pi$

$3x^2 = 18x - 6x^2$

$3x^2 = 3x(6 - 2x)$

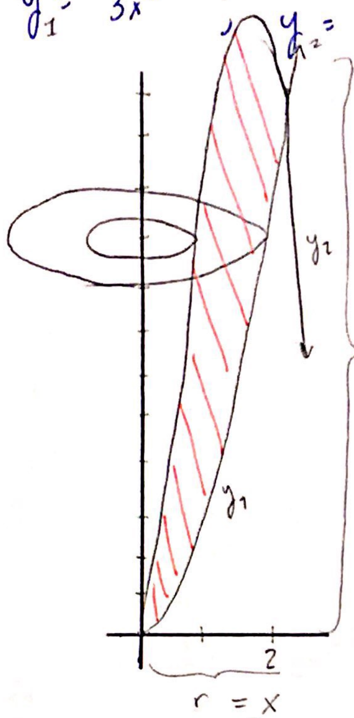
$\frac{3x^2}{3x} = 6 - 2x$ ; $x = 6 - 2x$

$x + 2x = 6$

$3x = 6$

$x = \frac{6}{3} = 2$

$y_1 = 3x^2$ ; $y_2 = 18x - 6x^2$




$h = (18x - 6x^2) - (3x^2)$

$h = 18x - 9x^2$

$3x^2 = 18x - 6x^2$

$3x^2 = 6(3x - x^2)$

$\frac{3x^2}{6} = 3x - x^2$

$\frac{1}{2}x^2 - 3x + x^2 = 0$

$\frac{3}{2}x^2 - 3x = 0$

$3x(\frac{1}{2}x - 1) = 0$

$3x = 0$ $\quad$ $\frac{1}{2}x - 1 = 0$

$\boxed{x = 0}$ $\quad$ $\frac{1}{2}x = 1$

$\boxed{x = 2}$

$3(0)^2$

$(18x - 6x^2)'$

$18 - 12x = 0$

$-12x = -18$

$x = \frac{18}{12} = \frac{9}{6} = \frac{3}{2}$

$$V = 2\pi \int_0^2 x(18x - 9x^2)\,dx$$

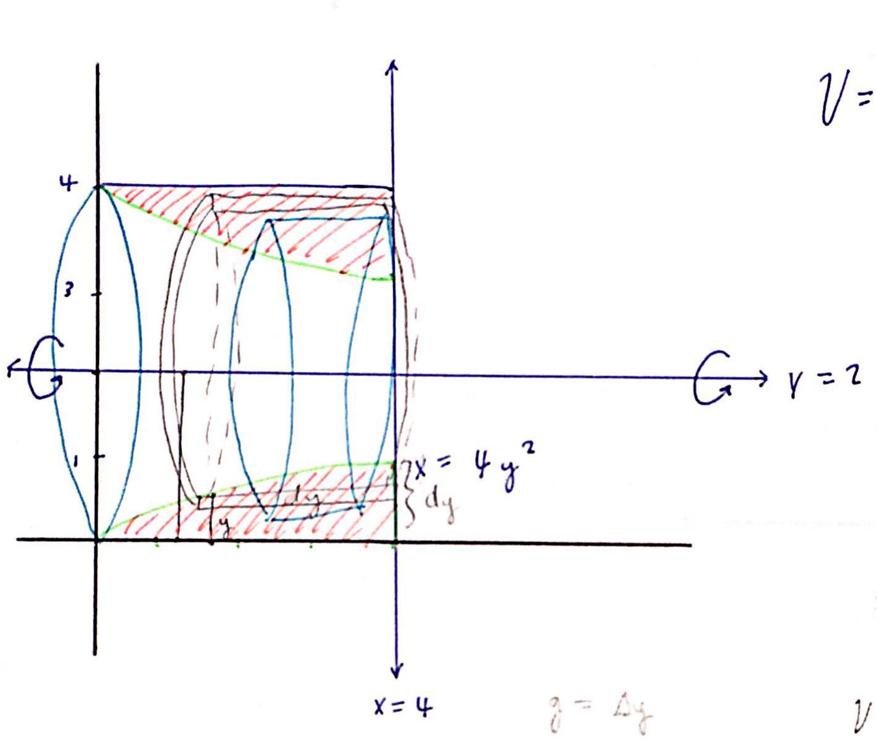
$$= 2\pi \int_0^2 9(2x^2 - x^3)\,dx$$

$$= 18\pi \int_0^2 (2x^2 - x^3)\,dx = 18\pi \left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big]_0^2$$

$$18\pi\left[\left(\frac{2(2)^3}{3} - \frac{(2)^4}{4}\right) - (0)\right] = 18\pi \cdot \frac{4}{3} = \frac{72}{3}\pi = \underline{24\pi}$$

(9)

$$V = 2\pi \int_a^b r\,h\,dx$$




$A = 2\pi r h g$

$g = \Delta y$

$h = 4 - 4y^2$

$r = 2 - y$

$$V = 2\pi \int_0^1 (2-y)(4-4y^2)\,dy$$

$$V = 2\pi \int_0^1 (8 - 8y^2 - 4y + 4y^3)\,dy$$

$$= 2\pi \int_0^1 (4y^3 - 8y^2 - 4y + 8)\,dy$$

$$= \cancel{8}\pi \left[ \left( \frac{4}{4}y^4 - \frac{8}{3}y^3 - \frac{4}{2}y^2 + 8y \right) \right]_0^1$$

$$= 2\pi \left[ \left( 1 - \frac{8}{3} - \frac{4}{2} + 8 \right) - (0) \right] = 2\pi \cdot \frac{13}{3} = \frac{26}{3}\pi$$

(a) $x = 4y^2$, $y \geq 0$, $x = 4$; about $y = 2$

$\pm\sqrt{\frac{1}{4}x} = y$

$V = 2\pi \int_a^b r\,h\,dy$

$4y^2 = 4$

$4(y^2 - 1) = 0$

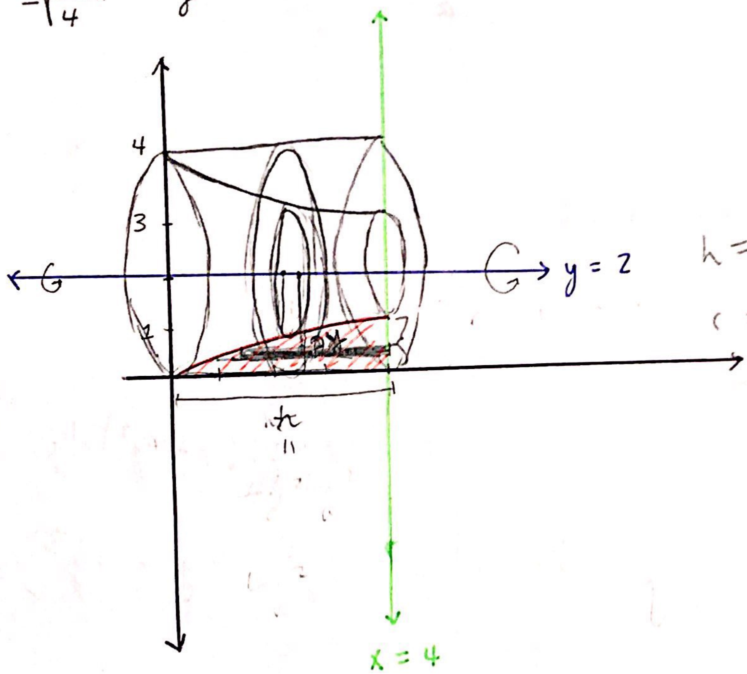
$y^2 = 1$

$y = \pm\sqrt{1}$

$y = \sqrt{1/4\,x}$

$h = 2 - \sqrt{1/4\,x}$

$r = 2 - y$



# Capítulo 4

## Longitud de arco

2019-10-04

# Webassign Longitud de arco

(1) $y = 5x - 1 \; ; \; -1 \le x \le 3 \; ;$

$$y' = 5 = (y')^2 = 25$$

$$L = \int_{-1}^{3} \sqrt{1 + 25}\, dx = \int_{-1}^{3} \sqrt{26}\, dx = \sqrt{26}\, x \Big]_{-1}^{3} =$$

$$= \sqrt{26}\left[(3) - (-1)\right] = \sqrt{26}\left[4\right] = 4\sqrt{26} \;//$$

(2) $y = \sqrt{2 - x^2} \; ; \; 0 \le x \le 1 \; ;$

$$y'(x) = \frac{1}{2}(2 - x^2)^{-1/2} \cdot -2x = -\frac{2x}{\sqrt{2-x^2}} =$$

$$(y'(x))^2 = \left(-\frac{2x}{\sqrt{2-x^2}}\right)^2 = \frac{(-2x)^2}{(\sqrt{2-x^2})^2} = \frac{4x^2}{2-x^2} = 4\left(\frac{x^2}{2-x^2}\right)$$

$$L = 4\int_{0}^{1} \frac{x^2}{2-x^2}\, dx = 4\left[\int_{0}^{1} \frac{2}{2-x^2}\, dx - \int_{0}^{1} 1\, dx\right] =$$

$$\frac{x^2}{2-x^2} = \frac{x^2}{2-x^2} + \frac{2-x^2}{2-x^2} - 1$$

$$= \frac{x^2 + 2 - x^2}{2 - x^2} - 1 = \frac{2}{2-x^2} - 1$$

$$= 4\left[\int_{0}^{1} \frac{2}{2-x^2}\, dx - \int_{0}^{1} 1\, dx\right]$$

③ $y = \ln(\sec x)\ ; \quad 0 \le x \le \frac{\pi}{4}$

$$y' = \frac{\cancel{\sec x}\tan x}{\cancel{\sec x}} = (\tan x)^2 = \tan^2 x$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx =$$

$$= \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x|\Big]_0^{\pi/4} =$$

$$= \ln\left|\sec\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)\right| - \ln|\sec(0) + \tan(0)|$$

$$\boxed{\sec\frac{\pi}{4} = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{2}{\sqrt{2}}} \quad \boxed{\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = 1}$$

$$= \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \cancel{\ln|1|}^{\,0} = \ln\left|\frac{2}{\sqrt{2}} + 1\right|$$

② . $y = \sqrt{2 - x^2} \quad ; \; 0 \leq x \leq 1$

$$y'(x) = \frac{1}{2}(2-x^2)^{-1/2} \cdot -2x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{2-x^2}} = \left(\frac{-x}{\sqrt{2-x^2}}\right)^2 = \frac{x^2}{2-x^2}$$

$$(y'(x))^2 = \frac{x^2}{2-x^2}$$

$$L = \int_0^1 \sqrt{1 + \frac{x^2}{2-x^2}}\, dx =$$

$$\sqrt{\frac{1\,(2-x^2)}{(2-x^2)} + \frac{x^2}{(2-x^2)}}$$

$$\sqrt{\frac{2 - \cancel{x^2} + \cancel{x^2}}{(2-x^2)}} = \int \sqrt{\frac{2}{2-x^2}} = \sqrt{2} \int \frac{1}{\sqrt{2-x^2}}\, dx$$

$$\frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} \arcsin\left(\frac{x}{\sqrt{2}}\right)\Bigg]_0^1$$

$$\arcsin\left(\frac{1}{\sqrt{2}}\right) - \cancel{\arcsin(0)}^{\,0}$$

(4) $y = 8 + \frac{1}{2}\cosh(2x);\ 0 \leq x \leq 2$

$$y'(x) = 0 + \frac{1}{\cancel{2}}\sinh(2x)\cdot\cancel{2} = \left(\sinh(2x)\right)^2 = \underbrace{\sinh^2(2x)}_{(y'(x))^2}$$

$$L = \int_0^2 \sqrt{\sinh^2(2x) + 1}\, dx$$

$\sinh^2(x) - \cosh^2(x) = 1$

$\sinh^2(x) + 1 = \cosh^2(x)$

$w = 2x$

$du = 2dx = \frac{1}{2}du$

$$= \frac{1}{2}\int_0^2 \sqrt{\sinh^2(w) + 1}\, dw = \frac{1}{2}\int_0^2 \sqrt{\cosh^2(w)}\, du = \frac{1}{2}\int_0^2 \cosh(w)\, du$$

$$= \frac{1}{2}\sinh(w) = \frac{1}{2}\sinh(2x)\Big|_0^2 =$$

$$= \frac{1}{2}\left[\left(\sinh(2\cdot2)\right) - \cancel{\left(\sinh(2\cdot0)\right)}^{0}\right] = \frac{1}{2}\left(\sinh(4)\right)$$

5

$$y = \ln(1 - x^2) \; ; \; 0 \leq x \leq \frac{1}{8}$$

$$y'(x) = \frac{2x}{1-x^2} = \left(\frac{2x}{1-x^2}\right)^2 = \frac{4x^2}{(1-x^2)}$$

$$L = \int_0^{1/8} \underbrace{\sqrt{\left(\frac{2x}{1-x^2}\right)^2 + 1^2}} =$$

$$\underbrace{\sqrt{\frac{4x^2}{(1-x^2)^2} + \frac{(1-x^2)^2}{(1-x^2)^2}}}$$

$$\sqrt{\frac{4x^2 + 1 - 2x^2 + x^4}{(1-x^2)^2}} = \sqrt{\frac{x^4 + 2x^2 + 1}{x^4 + 2x}} =$$

$$= \frac{\sqrt{(x^2+1)^2}}{\sqrt{(1-x^2)^2}} = \frac{x^2+1}{1-x^2} = \frac{x^2+1}{1-x^2} + \frac{1-x^2}{1-x^2} - 1$$

$$= \frac{\cancel{x^2} + 1 + 1 - \cancel{x^2}}{1-x^2} - 1 = \frac{2}{1-x^2} - 1 = \frac{2}{-(x+1)(x-1)} - 1$$

$$L = \int_0^{1/8} -\frac{2}{(x+1)(x-1)} - 1$$

# Parte II

# Laboratorios

# Capítulo 5

## Laboratorio #1

Laboratorio # 1

29/07/2019
20190432
David Corzo

100 pts *

① a) $\int (\sqrt{x}+2)(\sqrt{x}-2)(x+4)\,dx$

$\int (x-4)(x+4)\,dx$

$\int (x^2-16)\,dx$

$\int x^2\,dx - \int 16\,dx$

$\frac{x^{2+1}}{2+1} - 16x = \frac{x^3}{3} - 16x + C$

6pts

b) $\int \frac{3x^{3/2}+x+3\sqrt{x}}{x^2}\,dx$

$\int \frac{3x^{3/2}}{x^2} + \frac{x}{x^2} + \frac{3(x)^{1/2}}{x^2}\,dx$

$\frac{3}{2}-\frac{4}{2}=-\frac{1}{2}$

$\int \left[3x^{-1/2} + \frac{1}{x} + 3x^{-3/2}\right] dx$

$\left\{3\int x^{-1/2}\,dx\right\} + \left\{\int x^{-1}\,dx\right\} + \left\{3\int x^{-3/2}\,dx\right\}$

$\frac{3\cdot x^{1/2}}{1/2} + \frac{x^2}{2} + \frac{3\cdot x^{-1/2}}{-1/2} = 6\sqrt{x} + \ln(x) - 6x^{-1/2} + C$

6pts

c) $\int \left( e^{\pi} \sin(x) + \tan(5) \sinh(x) - 5 \cdot \pi^{x} \right) dx$

$= \left\{ e^{\pi} \int \sin(x)\, dx \right\} + \left\{ \tan(5) \int \sinh(x)\, dx \right\} - \left\{ 5 \int \pi^{x}\, dx \right\}$ $\cos x \overset{?}{=} -\sin x$, $\sin x = \cos x$

$= -e^{\pi} \cos(x) + \tan(5) \cosh(x) - 5 \cdot \frac{\pi^{x}}{\ln(\pi)} + C$

6pts

d) $\int_{1/2}^{1} \frac{4u + u^{2}}{u^{4}}\, du$

$= \int_{1/2}^{1} \frac{4u}{u^{4}} + \frac{u^{2}}{u^{4}}\, du$

$= \int_{1/2}^{1} \frac{4u}{u^{4}}\, du + \int \frac{u^{2}}{u^{4}}\, du$ $\quad \frac{1}{u^{2}} = \frac{u^{-2+1}}{-1} = \frac{u^{-1}}{-1} \Big]_{1/2}^{1} = -\frac{1}{u} = \left[ -\frac{1}{(1)^{2}} - \left( -\frac{1}{(1/2)^{2}} \right) \right]$

6pts

$= 4 * u^{-3} \Big]_{1/2}^{1} + u^{-2} \Big]_{1/2}^{1} =$

$\frac{7}{\cancel{4}}$

$+ \cancel{(1/3)/2 = \frac{3}{4}}$

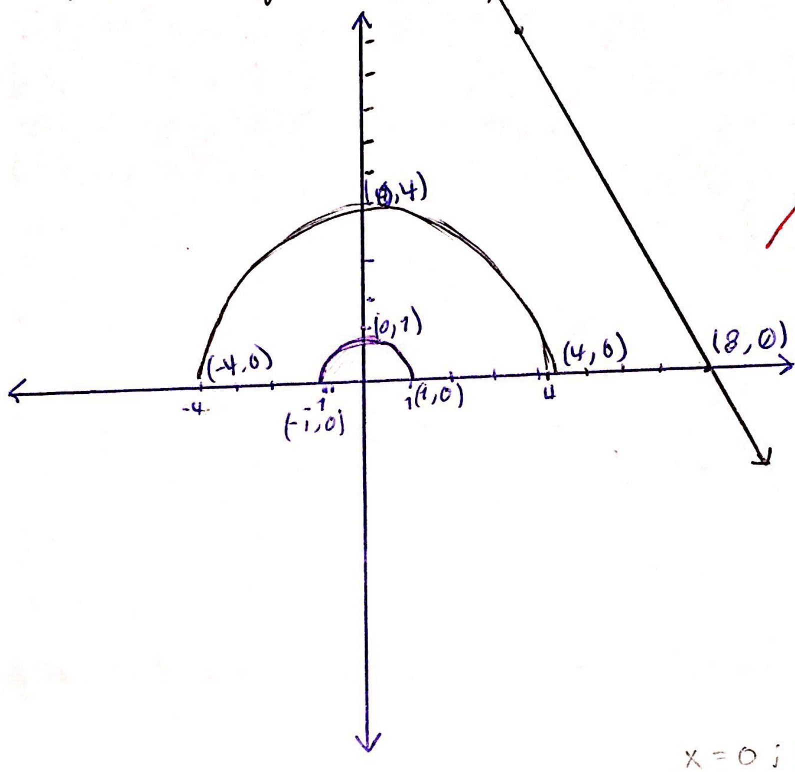
$= \frac{4u^{-2}}{-2} \Big]_{1/2}^{1}$ $\underbrace{\left\{ \left( \frac{4}{(1)^{2} \cdot -2} \right) - \left( \frac{4}{(1/4) \cdot -2} \right) \right\}}_{6} + \underbrace{\left\{ \cancel{\left( \frac{1}{(1)^{2}} \right)} - \cancel{\left( \frac{1}{(1/2)^{2}} \right)} \right\}}_{1}$

$\left( \frac{4}{-2} \right) - \left( \frac{4}{\frac{1}{4} \cdot \frac{-2}{1}} \right)$

$(-2) - \left( \frac{\frac{4}{1}}{\frac{-2}{4}} \right) = -2 - \left( \frac{8}{-2} \right) = -2 + \frac{8}{2}$

2.) $\int_{-1}^{0} \sqrt{1-x^2}\, dx + \int_{0}^{4} \sqrt{16-x^2}\, dx + \int_{6}^{8} (16-2x)\, dx$

ⓐ Realizar la gráfica indicando interceptos




15pts

$y = 0 \;;\; x = 0$

$\sqrt{1-x^2} = y$

$\sqrt{1-0} = y$

$\pm 1 = y$

$\sqrt{1-x^2} = 0$

$1 - x^2 = 0$

$-x^2 = -1$

$x = \sqrt{1}$

$x = \pm 1$

$y = 0 \;;\; x = 0$

$\sqrt{16-x^2} = y$

$\sqrt{16-0^2} = y$

$\pm 4 = y$

$\sqrt{16-x^2} = 0$

$16 - x^2 = 0$

$+x^2 = +16$

$x = \sqrt{16}$

$x = \pm 4$

$x = 0 \;;\; y = 0$

$16 - 2x = y$

$16 = y$

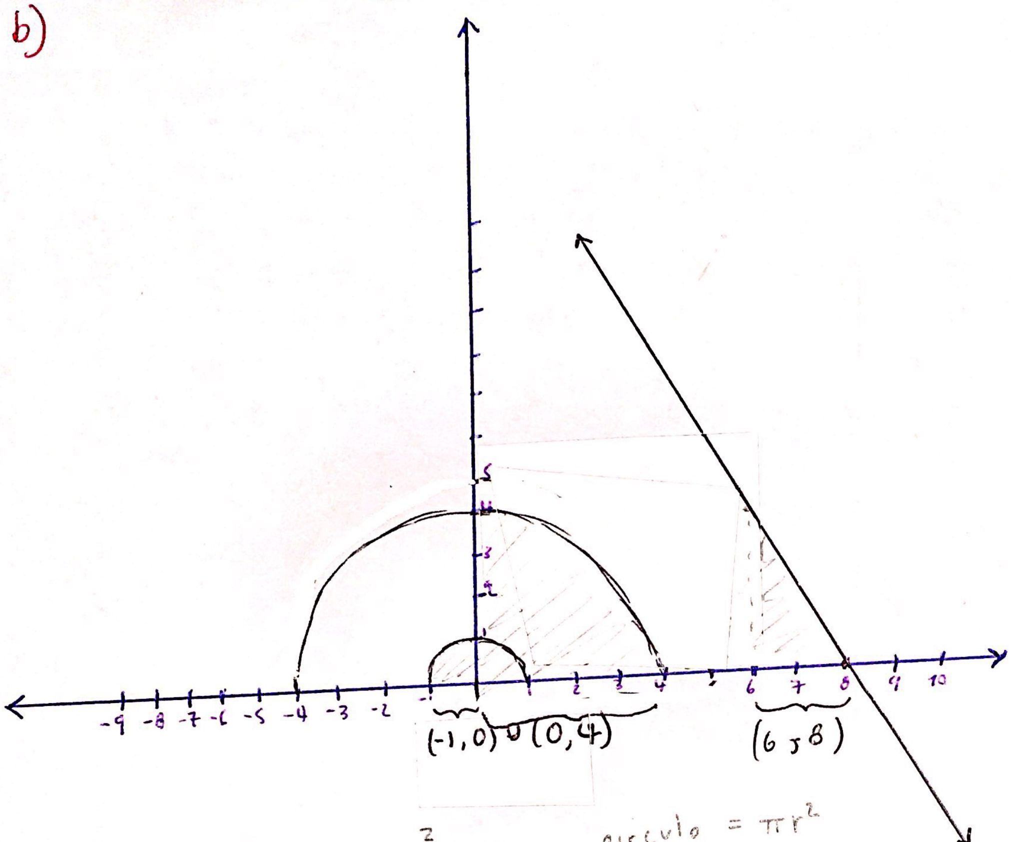
$16 - 2x = 0$

$-2x = -16$

$x = \frac{-16}{-2}$

$x = 8$

b)




circulo $= \pi r^2$

$Ac = \pi(-1)^2$

$A.C. = \pi$

$AC_1 = \frac{1}{4}\pi$

circulo $= \pi r^2$

$Ac_2 = \frac{1}{4}\pi(4)^2$

$Ac_2 = \frac{16}{4}\pi$

$Ac_2 = 4\pi$

triángulo

$T = \frac{1}{2}bh$

$T = \frac{1}{2}(2)(4)$

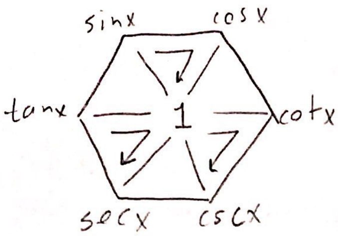
$T = \frac{8}{2}$

$T = 4$

$\frac{1}{4}\pi + 4\pi + 4$

10pts

e) $\displaystyle\int_0^{\pi/4} \frac{\sec\theta \cot\theta}{\sin\theta}\, d\theta$   $\theta = x$




$$\frac{\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}}{\sin\theta} = \left[\frac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{1}}\right] = \frac{1}{\sin^2\theta} = \csc^2\theta$$

$$\int_0^{\pi/4}\csc^2\theta\, d\theta = -\cot\theta\Big]_0^{\pi/4}$$

$$[-\cot(\pi/4)] - [-\cot(0)] = \text{indefinida}$$

8pts

f) $\displaystyle\int_{\pi/4}^{\pi/2} \sec t\,(\sec(t) + \tan(t))\, dt$

$$\int(\sec^2 t)\,dt + \int(\sec t\cdot\tan t)\,dt$$

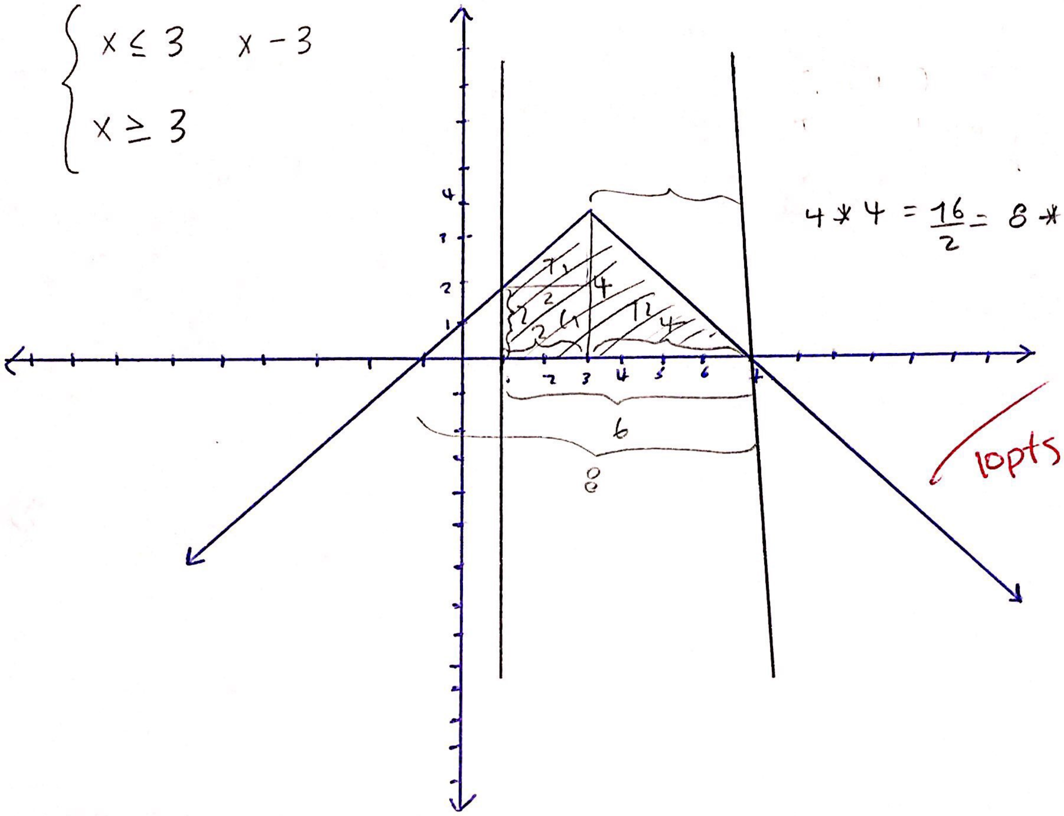
$$(\tan(t)) + (\sec t)\Big]_{\pi/4}^{\pi/2}$$

$$\underbrace{\left[\frac{\sin(\pi/2)}{\cos(\pi/2)} + \frac{1}{\cos(\pi/2)}\right]}_{\text{indefinida}} - \left[\frac{\sin(\pi/4)}{\cos(\pi/4)} + \frac{1}{\cos(\pi/4)}\right] = \text{indefinida}$$

8pts

③ $f(x) = 4 - |x-3|$ ; $x = 1$ & $x = 7$

$$|x-3| = \begin{cases} x \le 3 & x-3 \\ x \ge 3 & \end{cases}$$




$4 * 4 = \frac{16}{2} = 8 * 2 = 16 - 2 = 1$

100pts

10pts

srodas1293@gmail.com

$f(x) = 4 - (-x + 3)$ $\quad$ $f(x) = 4 - (x - 3)$

$4 + x - 3$ $\quad$ $4 - x + 3$

$f(x) = \int (x + 1)\,dx$ $\quad$ $f(x) = \int (-x + 7)\,dx$

$\int -x\,dx + \int 7$

$-\frac{x^2}{2} + 7x\Big]_3^7 = -\frac{7^2}{2} + 7(7) - \left[-\frac{3^2}{2} + 3(3)\right]$

$\int x\,dx + \int 1\,dx$

$\frac{x^2}{2} + x\Big]_1^3 = \left[\frac{3^2}{2} + 3\right] - \left[\frac{1^2}{2} + 1\right] = \frac{9}{2} + 3 - \frac{1}{2} - 1$

$\frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4 + 2 = 6$

$\int_1^3 (x + 1)\,dx = 6$

15pts

# Capítulo 6

## Laboratorio #2

Laboratorio # 2 David Gabriel Corzo Mcmath

20190432 - 2019/08/05

(1) a) $f(x) = \int_{1}^{x} \frac{1}{t^3+1}\,dt = \frac{1}{x^3+1} \cdot 1$ 5

110 pts *

b) $h(r) = \int_{-100}^{r} \sqrt{x^2+4}\,dx = \sqrt{r^2+4} \cdot 1$ 5

c) $i(x) = \int_{0}^{x^4} \cos^2(\theta)\,d\theta = \cos^2(x^4) \cdot 4x^3$ 5

d) $j(x) = \int_{\sec x}^{\tan x} \sqrt{t+\sqrt{t}}\,dt =$

$\sqrt{(\tan x)+\sqrt{(\tan x)}} \cdot \sec^2 x - \sqrt{\sec x + \sqrt{\sec x}} \cdot \sec x \tan x$ 10

e) $k(x) = \int_{x^3-x}^{x^4+x} \frac{u^3}{1+u^2}\,dt =$

$\left\{ \frac{(x^4+x)^3}{1+(x^4+x)^2} \cdot (4x^3+1) \right\} - \left\{ \frac{(x^3-x)^3}{1+(x^3-x)^2} \cdot (3x^2-1) \right\}$ 10

(2) $f(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dx$ ; recta tangente en $x = 1$

ecuación

$f'(x) = \sin\left(\frac{\pi}{2} x^2\right) \cdot 1$

$y = f(1) + f'(1)(x-1)$

$y = 0.4382591 + 1(x-1)$

$y = x - 1 + 0.4382591$

$y = x - 0.5617409$ ✗ 15

(3) $C'(x) = 3000 + 2x + \frac{3}{10} x^2$

$\int_{10}^{20} C'(x) = \int_{10}^{20} 3000 + 2x + \frac{3}{10} x^2 \, dx$

$\int_{10}^{20} C'(x) = \int_{10}^{20} 3000 dx + \int_{10}^{20} 2x \, dx + \int_{10}^{20} \frac{3}{10} x^2 \, dx$

$C(x) = 3000x + \frac{2x^2}{2} + \frac{3}{10} \cdot \frac{x^3}{3}$

$C(x) = 3000x + x^2 + \frac{x^3}{10} + C \Big]_{10}^{20}$

$\left\{3000(20) + (20)^2 + \frac{(20)^3}{10}\right\} - \left\{3000(10) + (10)^2 + \frac{(10)^3}{10}\right\}$ 15

$61\,200 - 30\,200 = \mathbf{31000}$ ✗ de aumento cuando se incrementa de 10 a 20 yardas

④ $P'(t) = 40\sqrt[3]{t}$

$$\int_0^8 P'(t) = \int_0^8 40\sqrt[3]{t}\,dt$$

$$= 40 \int_0^8 \frac{(t)^{1/3+1}}{1/3+1}\,dt$$

$$= 40 \quad \frac{t^{4/3}}{4/3}$$

$$= \frac{40 \cdot 3\,(t^{4/3})}{4}$$

$$= \frac{\cancel{4} \cdot 10 \cdot 3\,(t^{4/3})}{\cancel{4}}$$

$$= 30\,(t^{4/3})\Big]_0^8 = \left\{30\,(8^{4/3})\right\} - \left\{30\,(0^{4/3})\right\}$$

15

$= 480$ macatitas de cemento

⑤ $v(t) = 3t - 6, \quad 0 \leq t \leq 3$

a) $I_x ; y = 0$

$3t - 6 = 0$

$3t = 6$

$t = \frac{6}{3} = 2$

$I_y ; x = 0$

$3(0) - 6 = y$

$-6 = y$

$$\int_0^3 3t - 6\,dt = 3\int t\,dt - \int 6\,dt =$$

$$= \frac{3t^2}{2} - 6t\Big]_0^3 = \left\{\frac{3}{2}(3)^2 - 6(3)\right\} - \left\{\frac{3(0)}{2} - 6(0)\right\}$$

$= -\frac{9}{2}$ m

5

b) $\int 3t-6\ dt$

$d = \frac{3t^2}{2} - 6t$

$d = \left. -\frac{3t^2}{2} + 6t \right]_0^2 + \left. \frac{3t^2}{2} - 6t \right]_2^3$

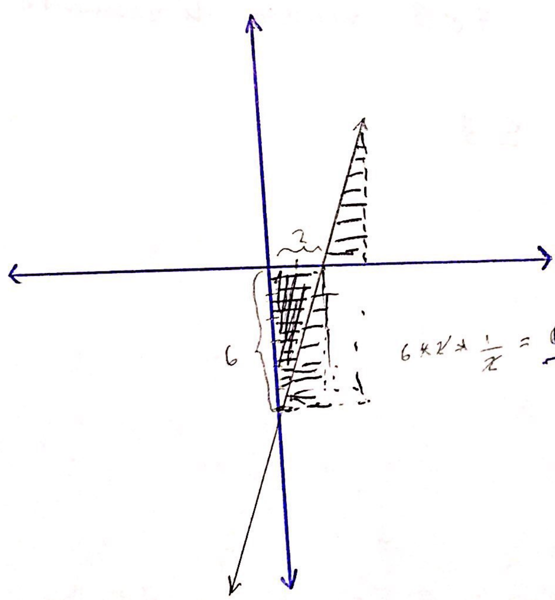
$\left\{\left[-\frac{3(2)^2}{2} + 6(2)\right] - [0]\right\} + \left\{\left[\frac{3(3)^2}{2} - 6(3)\right] - \left[\frac{3(2)^2}{2} - 6(2)\right]\right\}$

$\left\{-\frac{12}{2} + 12\right\} + \left\{\frac{27}{2} - 18 - \frac{12}{2} + 12\right\}$

$6 + \frac{3}{2} = \frac{15}{2}\ m$ x

10

c)




$A = \frac{6(2)}{2} + \frac{3(1)}{2} = \frac{15}{2}$

$6 \times 2 \times \frac{1}{2} = 6\ m^2$ x

5

6 a) $$f(x) = \int_{-\pi/3}^{\pi/3} \left( \frac{3}{5}(x^3 + x)^5 - 2x^4 \sin x \right) dx$$

$= 0$ porque es impar.

$\frac{}{5}$

b) $$g(x) = \frac{1}{\pi} \int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\, dx$$

$$\frac{1}{\pi}\left(\tan^{-1}(x)\right)\Big]_0^{\sqrt{3}} = \left\{\left\{\frac{1}{\pi}\tan^{-1}(\sqrt{3})\right\} - \left\{0\right\}\right\} * 2$$

$$\frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{3} = \frac{1}{3} * \frac{2}{1} = \frac{2}{3}$$

A

$\frac{}{5}$

# Capítulo 7

## Laboratorio #3

# Laboratorio # 3

NOMBRE ¿? 2019-08/15

David Corzo

110

① $\int \operatorname{sech}^2(\log_2 x)\,\frac{1}{x}\,dx =$ La derivada está presente. inclinación por sustitución

$\log_2 x = \frac{\log x}{\log(2)}$

$$\int \operatorname{sech}^2\left(\frac{\log x}{\log(2)}\right)\frac{1}{x} = \int \operatorname{sech}^2(u)\,\frac{du}{\log(2)}$$

$u = \frac{\log x}{\log(2)}$

$du = \frac{1}{x}\cdot\frac{1}{\log(2)}$

$$= \frac{1}{\log(2)}\int \operatorname{sech}^2(u)\,du$$

$$= \frac{1}{\log(2)}\tanh(u) + C$$

$$= \frac{1}{\log(2)}\cdot\tanh\left(\frac{\log x}{\log(2)}\right) + C$$

② $72\int_1^2 \frac{\ln(x)}{x^4}\,dx = 72\int_1^2 \ln(x)\cdot\frac{1}{x^4}\,dx$

$$= -\ln(x)\,\frac{1}{3x^3} - \int -\frac{1}{3x^3}\,\frac{1}{x}\,dx$$

$u = \ln(x)$ $\quad dv = x^{-4}\,dx$

$du = \frac{1}{x}$ $\quad v = \frac{x^{-3}}{-3}$

$$-\int -\frac{1}{3}\cdot x^{-3}\cdot x^{-1}\,dx$$

$$-1\cdot -\frac{1}{3}\int_1^2 x^{-4}\,dx$$

$$+\frac{1}{3}\cdot\frac{x^{-3}}{-3}$$

$$-\frac{1}{3}\cdot\frac{1}{3}\cdot x^{-3}$$

$$-\frac{1}{9x^3}$$

$$72\left(-\frac{\ln(x)}{3x^3} - \frac{1}{9x^3}\right]_1^2$$

$$\left\{-\frac{72\ln(2)}{3(2)^3} - \frac{72}{9(2)^3}\right\} - \left\{-\frac{72\ln(1)}{3(1)^3} - \frac{72}{9(1)^3}\right\}$$

$$-\frac{24\ln(2)}{8} - \frac{72}{72} - \left(-\frac{24(0)}{3(1)} - 8\right)$$

$$-3\ln(2) - 1 + 0 + 8.$$

$$-3\ln(2) + 7$$

③ $\int \sec^2\theta \tan^3\theta \, d\theta =$ $\qquad (\tan^3\theta)' = 3\tan^2\theta \cdot \sec^2\theta$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$= \int u^3 \, du = \dfrac{u^4}{4} + C$

$= \dfrac{\tan^4(\theta)}{4} + C$

④ $\int (x-1)\sin(\pi x)\, dx =$

$u = x - 1 \qquad dv = \sin(\pi x)\, dx \qquad uv - \int v\, du$

$du = 1\, dx \qquad v = \dfrac{-\cos(\pi x)}{\pi}$

$-\dfrac{(x-1)\cos(\pi x)}{\pi} - \underbrace{\int \dfrac{-\cos(\pi x)}{\pi}\, dx}_{\text{sustitución}}$

$-\dfrac{1}{\pi}\int -\cos(\pi x) = \left(-\dfrac{1}{\pi} \cdot -1\right)\int \cos(u)\, du$

$u = \pi x$

$du = \pi\, dx \qquad = \dfrac{1}{\pi}\dfrac{\sin(u)}{\pi} = \dfrac{1}{\pi^2}\sin(\pi x)$

$\dfrac{du}{\pi} = dx$

$-\dfrac{(x-1)\cos(\pi x)}{\pi} + \dfrac{\sin(\pi x)}{\pi^2} + C$

(5) $\int e^{-\theta}\cos(2\theta)\,d\theta =$ $\quad\quad \cos(2u) = 1 - \sin^2 u$

$u = \cos(2\theta) \quad\quad dv = e^{-\theta}\,d\theta$

$du = -2\sin(2\theta)\,d\theta \quad\quad v = -e^{-\theta}$

$\underbrace{\;}_{u}\underbrace{\;}_{v} - \int \underbrace{\;}_{v}\underbrace{\;}_{du}$

$$\underbrace{-\cos(2\theta)\,e^{-\theta} - \int -e^{-\theta}\cdot -2\sin(2\theta)}$$

$$-\frac{\cos(2\theta)}{e^{\theta}} - \underbrace{\int 2e^{-\theta}\sin(2\theta)\,d\theta}$$

$$\equiv 2\int e^{-\theta}\sin(2\theta)\,d\theta$$

$u = \sin(2\theta) \quad\quad dv = e^{-\theta}\,d\theta$

$du = 2\cos(2\theta)\,d\theta \quad\quad v = -e^{-\theta}$

$$\underbrace{\sin(2\theta)}_{u}\cdot\underbrace{-e^{-\theta}}_{v} - \int \underbrace{-e^{-\theta}}_{v}\cdot\underbrace{2\cos(2\theta)\,d\theta}_{du}$$

$$2\left\{-\frac{\sin(2\theta)}{e^{\theta}} - \int -2e^{-\theta}\cos(2\theta)\,d\theta\right\}$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} - 2\cdot -2\int e^{-\theta}\cos(2\theta)\,d\theta$$

$$-\frac{2\sin(2\theta)}{e^{\theta}} + 4\int e^{-\theta}\cos(2\theta)\,d\theta$$

$$w = \int e^{-\theta}\cos(2\theta)\,d\theta$$

variable cíclica

$$w = -\frac{\cos(2\theta)}{e^{\theta}} - \left(-\frac{2\sin(2\theta)}{e^{\theta}} + 4w\right)$$

$$w = -\frac{\cos(2\theta)}{e^{\theta}} + \frac{2\sin(2\theta)}{e^{\theta}} - 4w$$

$$w + 4w = -\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}$$

$$w = \frac{-\cos(2\theta)\cdot e^{-\theta} + 2\sin(2\theta)\cdot e^{-\theta}}{5} + C$$

6

$$\int e^{\tan(x+\tan^{-1}x)} \cdot \sec^2(x+\tan^{-1}(x))\left(1+\frac{1}{1+x^2}\right)dx =$$

sustitución

$u = \tan(x+\tan^{-1}(x))$

$du = \sec^2(x+\tan^{-1}(x)) \cdot \left(1+\frac{1}{1+x^2}\right)$

$$= \int e^{u}\, du$$

$$= e^{u} + C$$

$$= e^{\tan(x+\tan^{-1}(x))} + C$$

4

(8) $\int 5^t \sin(5^t)\, dt = \frac{1}{\ln(5)} \cdot \int \sin(u)\, du = \frac{1}{\ln(5)} \left[ -\cos(u) \right] = -\frac{\cos(5^t)}{\ln(5)} + C$ A

$u = 5^t$

$du = 5^t \cdot \ln(5)\, dt$

$\frac{du}{\ln(5)} = 5^t\, dt$

(9) $\int_0^2 x \sin \pi x\, dx = -\frac{x \cdot \cos(\pi x)}{\pi} - \int -\frac{\cos(\pi x)}{\pi}\, dx = \frac{-x \cos(\pi x)}{\pi} - \frac{1}{\pi}\left[ \int -\cos(\pi x)\, dx \right]$

$u = x \qquad dv = \sin(\pi x)\, dx$

$du = 1\, dx \qquad v = -\frac{\cos(\pi x)}{\pi}$

$u = \pi x$

$du = \pi\, dx$

$\frac{du}{\pi} = dx$

$= \int -\cos(u) \frac{du}{\pi}$

$= -\sin(u)$

$= \frac{-\sin(\pi x)}{\pi}$

$\frac{-x \cos(\pi x)}{\pi} - \frac{1}{\pi}\left( -\frac{\sin(\pi x)}{\pi} \right) + C$

$\left. \frac{-x \cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C \right]_0^2$ A

$\left\{ \frac{-2\cos(\pi 2)}{\pi} + \frac{\sin(\pi 2)}{\pi^2} \right\} - \left\{ \cancel{\frac{-0 \cos(\pi(0))}{\pi}} + \cancel{\frac{\sin(\pi 0)}{\pi}} \right\}$

$0$

$\left\{ -\frac{2(1)}{\pi} \right\} = -\frac{2}{\pi}$

7

$$\int_{-13}^{+13} \frac{4x}{\sqrt[3]{(1+2x^2)^2}}\,dx = \int_{-13}^{+13} \frac{du}{\sqrt[3]{(u)^2}} = \int_{-13}^{13} u^{-\frac{2}{3}}\,du$$

sustitución

$u = 1 + 2x^2$

$du = 0 + 4x\ dx$

$du = 4x\,dx$

$$\frac{u^{-\frac{2}{3}+\frac{3}{3}}}{-\frac{2}{3}+\frac{3}{3}} \qquad -\frac{2}{3}+\frac{3}{3} = \frac{-2+3}{3} = \frac{1}{3}$$

$$\frac{u^{\frac{1}{3}}}{1/3} = \frac{\frac{u^{1/3}}{1}}{\frac{1}{3}} = \frac{3u^{1/3}}{1} = 3u^{1/3}$$

$$f(-x) = \frac{4(-x)}{\sqrt[3]{(1+2(-x)^2}} = -\frac{4x}{\sqrt[3]{(1+2x^2)}}$$

impar

$$3u^{1/3} = \left. 3(1+2x^2)^{1/3} + C \right]_{-13}^{13}$$

$$= \left\{3\left(1+2(13)^2\right)^{1/3}\right\} - \left\{3\left(1+2(-13)^2\right)^{1/3}\right\}$$

$$= 3\sqrt[3]{1+2(13)^2} - 3\sqrt{1+2(-13)^2}$$

$$= 3\sqrt[3]{339} - 3\sqrt{339}$$

$$= \emptyset$$

(10) $$\int \frac{e^{y/(y+1)}}{(y+1)^2}\,dy = \int \frac{e^{\frac{y}{(y+1)}}}{(y+1)^2}\,dy =$$

$$\frac{d}{dx}\left(\frac{y}{y+1}\right) = \frac{1\cdot(y+1) - y(1)}{(y+1)^2} = \frac{\cancel{y}+1-\cancel{y}}{(y+1)^2} = \frac{1}{(y+1)^2}$$

$$\frac{f'g - fg'}{(g)^2}$$

$$u = \frac{y}{y+1}$$

$$du = \frac{1}{(y+1)^2}\,dy$$

$$= \int e^u\,du = e^u = e^{y/(y+1)} + C$$

(11) $\displaystyle\int \frac{xe^{2x}}{(1+2x)^2}\,dx = \int xe^{2x}(1+2x)^{-2}$

$u = xe^{2x}$ $\qquad dv = (1+2x)^{-2}$

$du = e^{2x} + xe^{2x}\cdot 2\,dx \qquad v = -\dfrac{1}{2+4x}$

$\displaystyle\int \frac{1}{(1+2x)^2}\,dx \qquad w = 1+2x \qquad dw = 2\,dx \qquad \frac{dw}{2} = dx$

$\displaystyle = \int w^{-2}\,\frac{dw}{2} = \frac{w^{-1}}{-1}\cdot\frac{1}{2} = \frac{1}{-2(1+2x)} = \frac{1}{-2-4x}$

$\underbrace{xe^{2x}}_{u}\cdot\underbrace{-\frac{1}{2+4x}}_{v} - \underbrace{\boxed{\int -\frac{1}{2+4x}\cdot e^{2x}+xe^{2x}\cdot 2\,dx}}_{-\frac{e^{2x}}{4}}$

$\displaystyle -\int \frac{e^{2x}+xe^{2x}\cdot 2}{2+4x}\,dx$

$\displaystyle \frac{e^{2x}\cancel{(1+2x)}}{2\cancel{(1+2x)}}\,dx$

$\displaystyle \frac{e^{2x}}{2}\,dx \longrightarrow \frac{e^{2x}}{4}$

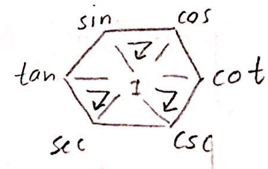
$\displaystyle -\frac{xe^{2x}}{2+4x} + \frac{e^{2x}}{4} + C$

# Capítulo 8

## Laboratorio #4

David Gabriel Corzo Mcmath
20190432

# Laboratorio # 4




$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

100pts

① $$\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin^3\left[(x)^{1/2}\right]\cdot (x)^{-1/2}\,dx$$

$u = \sqrt{x} = (x)^{1/2}$

$du = \frac{1}{2}(x)^{-1/2}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= \int \sin^3(u)\cdot 2\,du$$

$$= 2\int \sin^3(u)\,du$$

$$= 2\int \sin^2(u)\cdot \sin(u)\,du$$

$$= 2\cdot\int \left[1-\cos^2(u)\right]\cdot \sin(u)\,du$$

$w = \cos(u)$

$dw = -\sin(u)$ $\quad -dw = \sin(u)$

$$= 2\int \left[1 - w^2\right]\cdot -dw$$

$$= 2\cdot -1\int \left[1 - w^2\right]\cdot dw$$

$$= -2\left\{\int 1\,dw - \int w^2\,dw\right\}$$

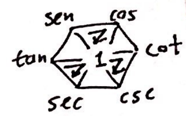
$$= -2\left\{w - \frac{w^3}{3}\right\}$$

$$= -2w + \frac{2w^3}{3}$$ sustituir $w$ por correspondiente

$$= -2\cos(u) + \frac{2\cos^3(u)}{3}$$

$$= -2\cos(\sqrt{x}) + \frac{2\cos^3(\sqrt{x})}{3} + C$$

② $\int \cos^4(\theta)\tan^2(\theta)\,d\theta = \int (\cos^2\theta)^2 \tan^2\theta\,d\theta$




$\sin^2 + \cos^2 = 1$

$1 + \cot^2 = \csc^2$

$\tan^2 + 1 = \sec^2$

$\rightarrow \tan^2 = \sec^2 - 1$

or

$\rightarrow \cos^2 = 1 - \sin^2$

$$= \int \cos^4(\theta)\,\frac{\sin^2(\theta)}{\cos^2(\theta)}\,d\theta = \int \cancel{\cos^2\theta}\,\cos^2\theta\,\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\,d\theta$$

$$= \int \cos^2\theta \cdot \sin^2\theta\,d\theta = \int \cos^2\theta\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$

$$= \int \left[\frac{1+\cos(2\theta)}{2}\right]\left[\frac{1-\cos(2\theta)}{2}\right]d\theta$$

$$= \int \left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right)\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$

simplificar $(a-b)(a+b) = a^2 - b^2$

$$= \int \left(\frac{1}{2}\right)^2 - \left(\frac{\cos(2\theta)}{2}\right)^2 d\theta = \int \frac{1}{4}\,d\theta - \int \frac{\cos^2(2\theta)}{4}\,d\theta$$

$$= \frac{\theta}{4} - \frac{1}{4}\underbrace{\int \cos^2(2\theta)\,d\theta}_{} = \frac{\theta}{4} - \frac{1}{4}\int \cos^2(\alpha)\,d\alpha$$

$\alpha = 2\theta$

$\frac{d\alpha}{2} = d\theta$

$$= \frac{\theta}{4} - \frac{1}{4}\left\{\int \frac{1}{2} + \frac{\cos^2(2\alpha)}{2}\,d\alpha\right\}$$

$$= \frac{\theta}{4} - \frac{1}{4}\left\{ \int \frac{1}{2}\, d\alpha + \frac{1}{2}\int \cos^2(2\alpha)\, d\alpha \right.$$

$$= \frac{\theta}{4} - \frac{1}{4} \cdot \int \frac{1}{2}\, d\alpha + \frac{1}{4} \cdot \frac{1}{2} \int \cos(4\theta)\, d\alpha$$

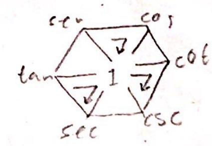
$$= \frac{\theta}{4} - \frac{\theta}{8} + \frac{1}{8}\sin(4\theta) \cdot \frac{1}{4} + C$$

$$= \frac{2\theta - \theta}{8} + \frac{1}{32}\sin(4\theta) + C$$

$$= \frac{\theta}{8} + \frac{\sin(4\theta)}{32} + C$$

$$= \frac{1}{8}\left(\theta - \frac{\sin(4\theta)}{4}\right) + C$$

③ $$\int \cos^3(\sin\theta)\, \sin^4(\sin\theta)\cos\theta\, d\theta$$




$\operatorname{sen}^2 + \cos^2 = 1$

$\cos^2 = 1 - \operatorname{sen}^2$

$$\left.\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\, d\theta \end{aligned}\right\} \therefore \int \cos^3(u)\, \sin^4(u)\, du$$

$$= \int \cos^3(u)\, \sin^4(u)\, du$$

$$= \int \left(1 - \operatorname{sen}^2(u)\right) \sin^4(u)\cos(u)\, du$$

$$= \int \sin^4(u)\cos(u) - \sin^6(u)\cos(u)\, du$$

$$= \int \sin^4(u)\cos(u)\, du - \int \sin^6(u)\cos(u)\, du$$

$$\underbrace{\begin{aligned} u &= \sin(u) \\ du &= \cos(u)\, du \end{aligned}}_{\int u^4\, du} \qquad - \qquad \underbrace{\begin{aligned} u &= \sin(u) \\ du &= \cos(u) \end{aligned}}_{\int u^6\, du}$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

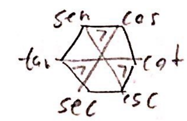
$$= \frac{\sin^5(u)}{5} - \frac{\sin^7(u)}{7} + C$$

$$= \frac{\sin^5(\sin\theta)}{5} - \frac{\sin^7(\sin\theta)}{7} + C$$

#

4

$$\int \tan^5(x)\sec^4(x)\,dx =$$




$\tan^2 + 1 = \sec^2$

$$= \int \tan^5(x)\sec^2(x)\sec^2(x)\,dx$$

$$= \int \tan^5(x)\left(\tan^2 x + 1\right)\sec^2(x)\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int u^5(u^2+1)\,du = \int u^7 + u^5\,du$$

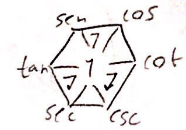
$$= \int u^7\,du + \int u^5\,du$$

$$= \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \frac{\tan^8(x)}{8} + \cancel{\frac{\tan^6(x)}{6}} + C \quad X$$

5

$$\int \sec^4(x)\,dx =$$




$\tan^2 + 1 = \sec^2$

$$= \int \sec^2(x)\left[\tan^2 + 1\right] dx$$

$$= \int \sec^2(x)\tan^2(x) + \sec^2(x)\,dx$$

$$= \int \sec^2(x)\tan^2(x)\,dx + \int \sec^2(x)\,dx$$

$$\underbrace{\begin{aligned} u &= \tan(x) \\ du &= \sec^2(x) \end{aligned}} \qquad + \tan(x)$$

$$= \int u^2\,du + \tan(x)$$

$$= \frac{u^3}{3} + \tan(x) + C$$

$$= \frac{\tan^3(x)}{3} + \tan(x) + C$$

# Capítulo 9

## Laboratorio #5

Laboratorio # 5

David Corzo
20190432

2019-08-2

100

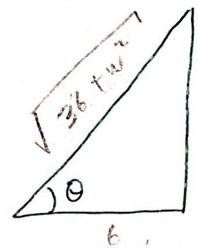
(3) $\displaystyle\int \frac{72}{(36+x^6)^{3/2}} 3x^2\,dx = \int \frac{72 \cdot 3x^2}{\sqrt[2]{(36+x^6)^3}}$

$\left(\sqrt{(36+x^6)}\right)^3$

$\displaystyle = 72 \int \frac{3x^2}{\sqrt{(36+x^6)^3}}\,dx = 72 \int \frac{du}{\sqrt{(36+u^2)^3}} =$

$c = \frac{A}{H}$

$u = x^3$
$du = 3x^2\,dx$




$\tan\theta = \frac{u}{6}$

$6\tan\theta = u$

$6\sec^2\theta \cdot d\theta = du$

$\sin(\theta) = \frac{u}{\sqrt{36+u^2}}$

$\displaystyle = 72 \int \frac{6\sec^2\theta\,d\theta}{\left(\sqrt{36+36\tan^2\theta}\right)^3} = 72 \int \frac{6\sec^2\theta\,d\theta}{216\sec^3\theta} = \frac{72 \cdot 6}{216} \int \frac{\sec^2\theta\,d\theta}{\sec^3\theta}$

$36(1+\tan^2\theta)$

$\left(\sqrt{36\sec^2\theta}\right)^3$

$(6\sec\theta)^3$

$216\sec^3\theta$

$\displaystyle = 2 \int \frac{1}{\sec\theta}\,d\theta = 2 \int \cos\theta\,d\theta$

$= 2\sin\theta\,d\theta + C$

$= 2 \cdot \frac{u}{\sqrt{36+u^2}} + C$

$= \frac{2x^3}{\sqrt{36+x^6}} + C$

4 $$\int \frac{1}{\sqrt{x^2+6x+10}}\,dx = \int \frac{1}{\sqrt{(x+3)^2+1}}\,dx$$

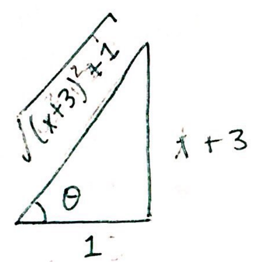
$$\left(\frac{6}{2}\right)^2 = \frac{36}{2} = 18$$

$$\underbrace{x^2+6x+9}-9+10 = 0$$

$$(x+3)^2+1$$

$$= \int \frac{1}{\sqrt{\underbrace{\tan^2\theta+1}_{\sec^2\theta}}}\sec^2\theta\,d\theta$$

$$= \int \frac{1}{\sec\theta}\sec^2\theta\,d\theta$$




$\tan\theta = x+3$

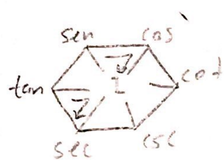
$\sec^2\theta\,d\theta = 1\,dx$

$\sec\theta = \frac{1}{\frac{1}{\frac{A}{H}}} = \frac{H}{A} = \frac{\sqrt{(x+3)^2+1}}{1}$

$$= \int \sec\theta\,d\theta$$

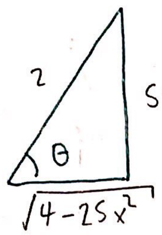
$$= \ln|\sec\theta+\tan\theta|+C$$

$$= \ln\left|\sqrt{(x+3)^2+1}+x+3\right|+C$$



1

$$\int 5^8 x^7 \sqrt{4-25x^2}\, dx =$$




$\sin\theta = \dfrac{5x}{2}$ $\qquad 2\cos\theta = \sqrt{4-25x^2}$

$2\sin\theta = 5x$

$\dfrac{2}{5}\sin\theta = x$

$\cos\theta = \dfrac{\sqrt{4-25x^2}}{2}$

$\dfrac{2}{5}\cos\theta \, d\theta = dx$

$$= 5^8 \int \frac{2^7}{5^7}\sin^7\theta \cdot 2\cos\theta \cdot \frac{2}{5}\cos\theta \, d\theta$$

$$= 5^8 \int \frac{2^9}{5^8}\sin^7\theta \cos^2\theta \, d\theta = \frac{5^8 \cdot 2^9}{5^8} \int \underbrace{\sin^7\theta \cos^2\theta \, d\theta}$$

$$= \int \sin^6\theta \cos^2\theta \sin\theta \, d\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$(\sin^2\theta)^3 = (1-\cos^2\theta)^3$

$$= \int (1-\cos^2\theta)^3 \cdot \cos^2\theta \cdot \sin\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$$= -\int (1-u^2)^3 u^2 \, du$$

$$= -\int -u^8 + 3u^6 - 3u^4 + u^2 \, du$$

$$= 512 \left\{ -\left[ -\frac{u^9}{9} \right] + \left[ \frac{3u^7}{7} \right] - \left[ \frac{3u^5}{5} \right] + \frac{u^3}{3} \right\}$$

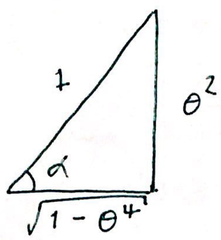
$$= 512 \left\{ \frac{u^9}{9} - \frac{3u^7}{7} + \frac{3u^5}{5} - \frac{u^3}{3} \right\}$$

$$= 512 \left\{ \frac{\cos^9\theta}{9} - \frac{3\cos^7\theta}{7} + \frac{3\cos^5\theta}{5} - \frac{\cos^3\theta}{3} \right\}$$

$$= 512 \left\{ \frac{\left(\frac{\sqrt{4-25x^2}}{2}\right)^9}{9} - \frac{3\left(\frac{\sqrt{4-25x^2}}{2}\right)^7}{7} + \frac{3\left(\frac{\sqrt{4-25x^2}}{2}\right)^5}{5} - \frac{\left(\frac{\sqrt{4-25x^2}}{2}\right)^3}{3} \right\} + C$$

②

$$\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta =$$




$$\cos(\alpha) = \sqrt{1-\theta^4}$$

$$\sin(\alpha) = \theta^2$$

$$\cos(\alpha)\,d\alpha = 2\theta\,d\theta$$

$$\frac{\cos(\alpha)\,d\alpha}{2} = \theta\,d\theta$$

$\sin(\alpha) = 1$

$\alpha = \frac{\pi}{2}$

$\sin(\alpha) = 0$

$\alpha = 0$

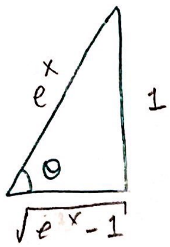
$$= \frac{4}{\pi}\int_0^{\pi/2} \frac{\cos(\alpha)\cos(\alpha)\,d\alpha}{2} = \frac{2}{\pi}\int_0^{\pi/2} \cos^2\alpha\,d\alpha$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \frac{1}{2} + \cos(2\alpha)\,d\alpha = \frac{2}{\pi}\left\{\frac{1}{2}\alpha + \frac{\sin(2\alpha)}{2}\right\}\Bigg|_0^{\pi/2}$$

$$= \frac{2}{\pi}\left[\left\{\frac{1}{2}\cdot\frac{\pi}{2} + \overset{0}{\cancel{\frac{\sin(\pi)}{2}}}\right\} - \cancel{\left\{\frac{1}{2}\cdot 0 + \overset{0}{\frac{\sin(0)}{2}}\right\}}\right]$$

$$= \frac{2}{\pi}\left[\left\{\frac{\pi}{4}\right\}\right] = \frac{2}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{4} = \frac{1}{2}$$

(5) $$\int_0^{\ln(\sqrt{2})} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx = \int_0^{\ln(\sqrt{2})} \frac{\sec^3\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta = \int_0^{\pi/4} \sec^4\theta\,d\theta$$




$\sec\theta = e^x$

$e^x dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{e^x - 1} = \tan\theta\,d\theta$

$e^{\ln(\sqrt{2})} = \sqrt{2}$

$\tan^{-1}(\sqrt{2}) = \frac{\pi}{4}$

$e^{\theta} = 1$

$\theta = 0$

$$= \int_0^{\pi/4} \sec^4\theta\,d\theta = \int_0^{\pi/4} (1+\tan^2\theta)\sec^2\theta\,d\theta = \int_0^1 (1+u^2)\,du = u + \frac{u^3}{3}$$

$u = \tan\theta$

$du = \sec^2\theta$

$$\tan\theta + \frac{\tan^3(\theta)}{3}\Big|_0^{\pi/4}$$

$$\left\{1 + \frac{1}{3}\right\} - \left\{0 + \frac{0^3}{3}\right\}$$

$$\frac{3\cdot 1}{3} + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$$

# Capítulo 10

## Laboratorio #6

Laboratorio # 6

David Carzo 20190432

2019-09-11

100

1a) $$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^{\infty} \frac{du}{9+u^2}\cdot\frac{1}{3} = \int_{-\infty}^{\infty} \frac{3dv}{9+9v^2}\frac{1}{3}$$

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$u = 3v$

$du = 3dv$

$$= \frac{1}{3}\int_{-\infty}^{\infty} \frac{3}{9(1+v^2)}\,dv =$$

$$= \frac{1}{3}\cdot\frac{1}{3}\int_{-\infty}^{\infty} \frac{1}{(1+v^2)}\,dv =$$

$$= \frac{1}{9}\tan^{-1}(v)$$

$$= \frac{1}{9}\tan^{-1}\left(\frac{u}{3}\right) = \frac{1}{9}\tan^{-1}\left(\frac{x^3}{3}\right)\Bigg]_{-\infty}^{\infty}$$

$$= \left\{\lim_{t\to\infty}\left(\frac{1}{9}\underbrace{\tan^{-1}\left(t^3/3\right)}_{\tan^{-1}(\infty)}\right)\right\} - \left\{\lim_{a\to-\infty}\left(1/9\,\underbrace{\tan^{-1}\left(a^3/3\right)}_{-\frac{\pi}{2}}\right)\right\}$$

$-\frac{\pi}{2}$

$$= \left(\frac{1}{9}\cdot\frac{\pi}{2}\right) - \left(-\frac{1}{9}\cdot\frac{\pi}{2}\right)$$

$$= \frac{\pi}{18} + \frac{\pi}{18} = \frac{\pi}{9}$$ A □

(1) • La integral converge por que al ser evaluada en $\infty$ y $-\infty$ que do una constante.

(2) Es impropia por los límtes ser infinitos

1.b $$\int_{-2}^{3} \frac{dx}{\sqrt[4]{x+2}} =$$

① asíntota en $-2$

$x + 2 = 0$

$x = -2$

$$\int_{-2}^{3} (x+2)^{-1/4}\,dx = (x+2)^{-\frac{1}{4}+\frac{4}{4}} \quad \left.\frac{4(x+2)^{3/4}}{3}\right]_{-2}^{3}$$

③ Es impropia por un límite (-2) indefinido la función

$$= \left\{\frac{4}{3}\sqrt[4]{(3+2)^3}\right\} - \left\{\cancel{\frac{4}{3}\sqrt[4]{(-2+2)^3}}\right\}$$

$$\underline{\frac{4}{3}\sqrt[4]{5^3}}$$

② convergente por que existe

1.c $$\int_{0}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -2\int_{0}^{\infty} e^{u}\,du = -2\cdot e^{u}\Big]_{0}^{\infty} = -2e^{-\sqrt{x}}\Big]_{0}^{\infty}$$

$u = -(x)^{1/2}$ $\quad u(0) = 0$

$$= \left\{\underbrace{\lim_{a\to\infty}(-2e^{-\sqrt{a}})}_{0}\right\} - \left\{\underbrace{\lim_{a\to 0}(-2e^{-\sqrt{0}})}\right\}$$

$du = -\frac{1}{2}(x)^{-1/2}\,dx$

$$= \{ -0 - (-2) = 2$$

$du = -\frac{1}{2\sqrt{x}}\,dx$

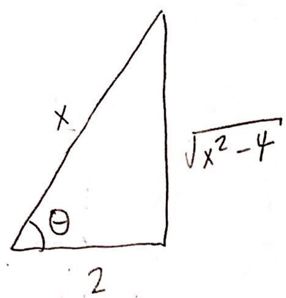
$-2\,du = \frac{1}{\sqrt{x}}\,dx$

① La integral converge

② es impropia por el límite superior ser infinito & A.V. en 0

1 d) $\displaystyle\int_2^{\infty} \frac{1}{x\sqrt{x^2-4}}\,dx =$

① Se indefine en $x = 2$ y está evaluada en infinto por eso es impropia.




$c^2 = a^2 + b^2$

$-a^2 = -c^2 + b^2$

$a^2 = c^2 - b^2$

$S\frac{O}{H}\,C\frac{A}{H}\,T\frac{O}{A}$

$\tan\theta = \dfrac{\sqrt{x^2-4}}{2}$ $\qquad \sec\theta = \dfrac{x}{2}$ $\qquad \theta = \text{arcsec}\left(\frac{x}{2}\right)$

$2\tan\theta = \sqrt{x^2-4}$ $\qquad 2\sec\theta = x$

$2\sec\theta\tan\theta\,d\theta = dx$

$$= \int_2^{\infty} \frac{2\,\cancel{\sec\theta\tan\theta}\,d\theta}{2\cancel{\sec\theta}\,2\,\cancel{\tan\theta}} = \int_2^{\infty} \frac{2}{4}\,d\theta = \int_2^{\infty} \frac{1}{2}\,d\theta = \frac{1}{2}\int_1 1\,d\theta$$

$$= \frac{1}{2}\theta = \frac{1}{2}\text{arcsec}\left(\frac{x}{2}\right)\Bigg]_1^{\infty} = \left\{\underbrace{\lim_{a\to\infty}\left(\frac{1}{2}\sec^{-1}\left(\frac{\infty}{2}\right)\right)}\right\} - \left\{\cancel{\lim_{b\to 2^+}\left(\frac{1}{2}\sec^{-1}(1)\right)}^{0}\right\}$$

$$= \left\{\frac{1}{2}\cdot\frac{\pi}{2}\right\} - \{0\}$$

$\dfrac{1}{\cos(\theta)} = 1$

$$= \frac{\pi}{4} \;\square$$

② Convergente

(e) $$\int_0^2 z^2 \ln(z)\,dz =$$

① es impropia por $\ln(0)$ esta indefinido

$u = \ln(z)$ $\quad dv = z^2$

$du = \frac{1}{z}dz$ $\quad v = \frac{z^3}{3}$

$$= \ln(z)\frac{z^3}{3} - \int_0^2 \frac{z^3}{3}\cdot\frac{1}{z}\,dz$$

$$= \ln(z)\frac{z^3}{3} - \int_0^2 \frac{z^2}{3}\,dz = \ln(z)\frac{z^3}{3} - \frac{1}{3}\frac{z^3}{3} = \ln(z)\frac{z^3}{3} - \frac{z^3}{9}\Bigg]_0^2 =$$

$$\overset{\text{con } 0}{=} \underbrace{\left\{\lim_{a\to 0^+}\left(\overset{-\infty}{\ln(z)}\,\overset{0}{\frac{z^3}{3}}\right)\right\}}_{\frac{1}{3}\frac{\ln(z)}{z^{-3}} \overset{LH}{=} \frac{1}{3}\frac{\left(\frac{1}{z}\right)}{\left(\frac{1}{z^4}\right)} = \frac{1}{3}\frac{z^4}{z} = \frac{z^3}{3} = 0} - \left\{\lim_{a\to 0^+}\left(\frac{z^3}{9}\right)\right\}$$

$$\overset{\text{con } 0}{=} \{0 - 0\}$$

$$\overset{\text{con } 2}{=} \left\{\ln(2)\frac{8}{3} - \frac{8}{9}\right\} - \{0\}$$

$\therefore \quad \ln(2)\frac{8}{3} - \frac{8}{9}$ ② La integral es convergente

(2)

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

(a) $f(t) = e^t$

$$F(s) = \int_0^\infty e^t\, e^{-st}\, dt$$

$$F(s) = \int_0^\infty e^t\, e^{-st}\, dt$$

$$F(s) = \int_0^\infty e^{t-st}\, dt = \int e^{t(1-s)} = \left.\frac{e^{t(1-s)}}{1-s}\right]_0^\infty$$

$s > 1$

como ser mayor a uno

$$\left\{ \lim_{a\to\infty} \underbrace{\frac{e^{\overbrace{\infty(1-s)}}}{1-s}}_{0} \right\} - \left\{ \frac{e^{\cancel{0}}{}^{1}}{1-s} \right\} = \frac{1}{s-1}$$

(b) $f(t) = t$

$$F(s) = \int_0^\infty t\, e^{-st}\, dt = \int_0^\infty -\frac{u}{s} \cdot e^u \cdot -\frac{du}{s} = \int_0^\infty \frac{+u e^u\, du}{s^2}$$

$-\frac{u}{s} = t$ $\quad u = -st$

$du = -s\, dt$

$-\frac{du}{s} = dt$

$$= \frac{1}{s^2}\int_0^\infty u e^u\, du$$

$v = u \quad d\beta = e^u$

$dv = du \quad \beta = e^u$

$$u e^u - \int_0^\infty e^u\, du$$

$$\left. u e^u - e^u \right]_0 =$$

$$= \frac{1}{s^2}\left.\left(-st e^{-st} - e^{-st}\right)\right]_0^\infty$$

$$\frac{1}{s^2}\left\{ \lim_{a\to\infty} \left(\frac{-st}{e^{st}}\right) - \underbrace{\lim_{a\to\infty}\left(e^{-st}\right)}_{0}\right.$$

$$\left. \lim_{a\to\infty} \frac{-s}{e^{st} \cdot s} \right\} 0$$

$$\frac{1}{s^2}(0) - \frac{1}{s^2}\left(\cancel{-s(0) e^{-s(0)}}\, 0 - \cancel{e^{-s(0)}}\, 1\right) = -\frac{1}{s^2} \cdot -1 = \frac{1}{s^2}$$

# Capítulo 11

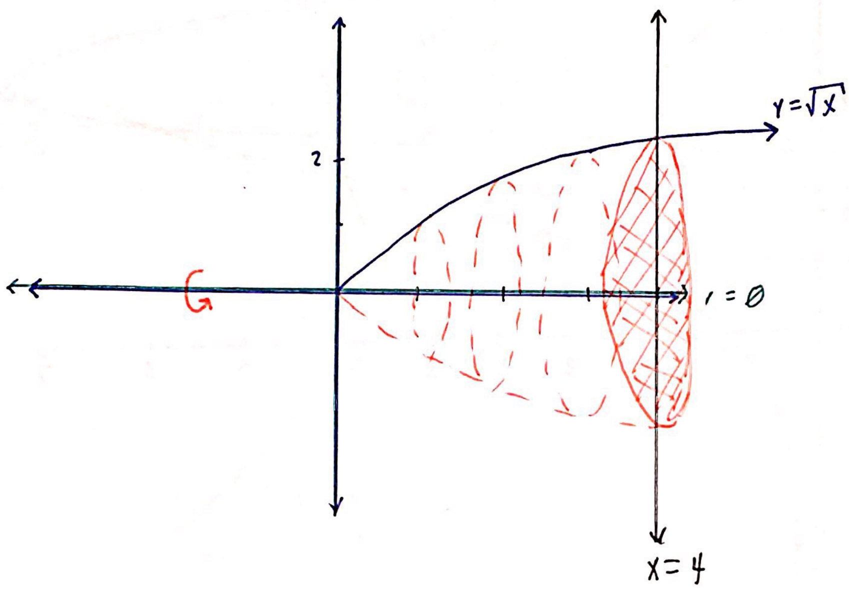
## Laboratorio #8

Laboratorio 8 David Gabriel Corzo Mcmath

20190432 2019-09-24

(1) rotar eje-x ; $y = \sqrt{x}$ ; $y = 0$ ; $x = 4$

120




Primero encontrar el área de la curva $\sqrt{x}$

$\boxed{A = \pi r^2}$ como r es en este caso y la fórmula queda así:

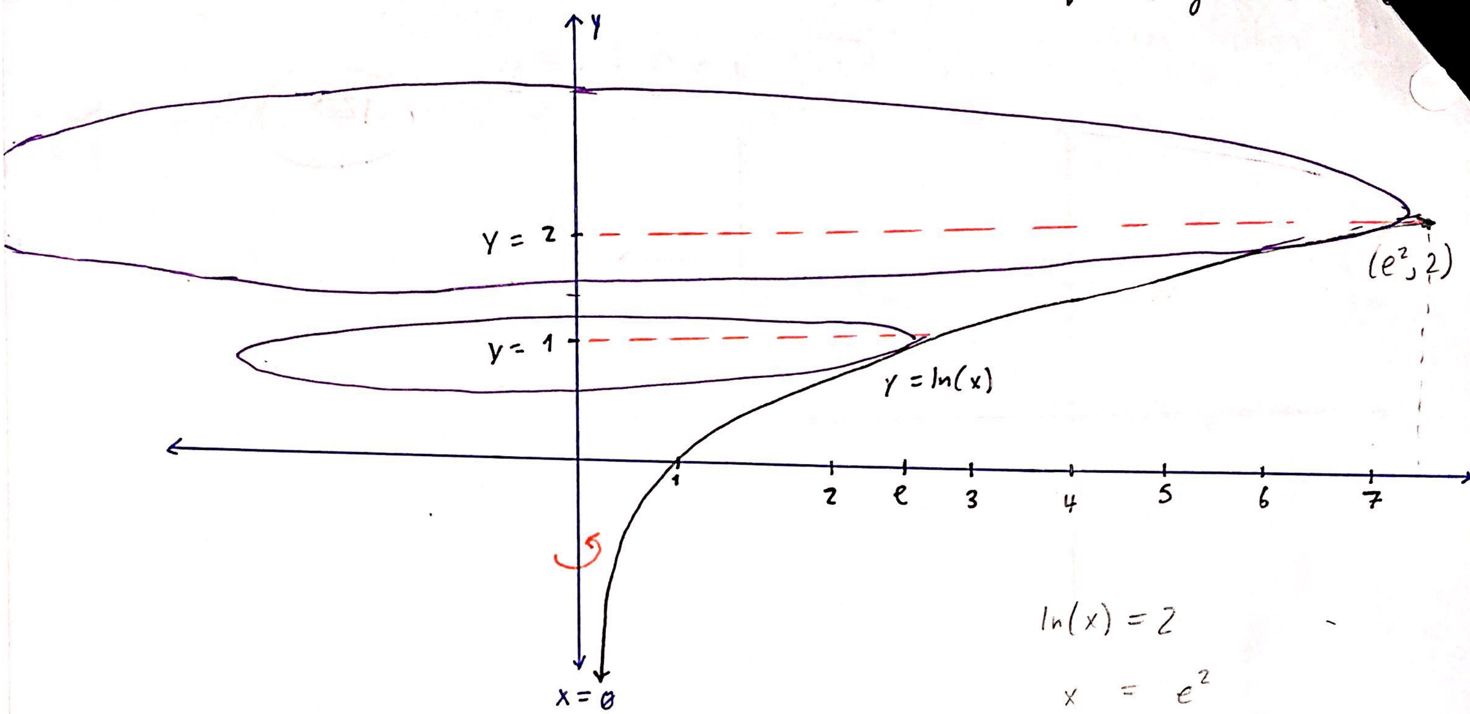
$$\boxed{A = \pi y^2}$$

$$V = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx = \pi \int_0^4 x \, dx = \pi \left. \frac{x^2}{2} \right]_0^4 =$$

$$= \pi \left[ \left(\frac{4^2}{2}\right) - \left(\frac{0^2}{2}\right) \right] = \pi \left[ \left(\frac{16}{2}\right) - \left(0\right) \right] = \cancel{\pi 8}$$

25+5

② eje-y es el eje de rotación $\quad y = \ln(x)$ ; $y = 1$ ; $y = 2$ ; $x =$




$$\ln(x) = 2$$
$$x = e^2$$

el radio siempre será $\ln(x)$

$A = \pi r^2$

$\therefore\ A = \pi\left(\ln^2(x)\right)$

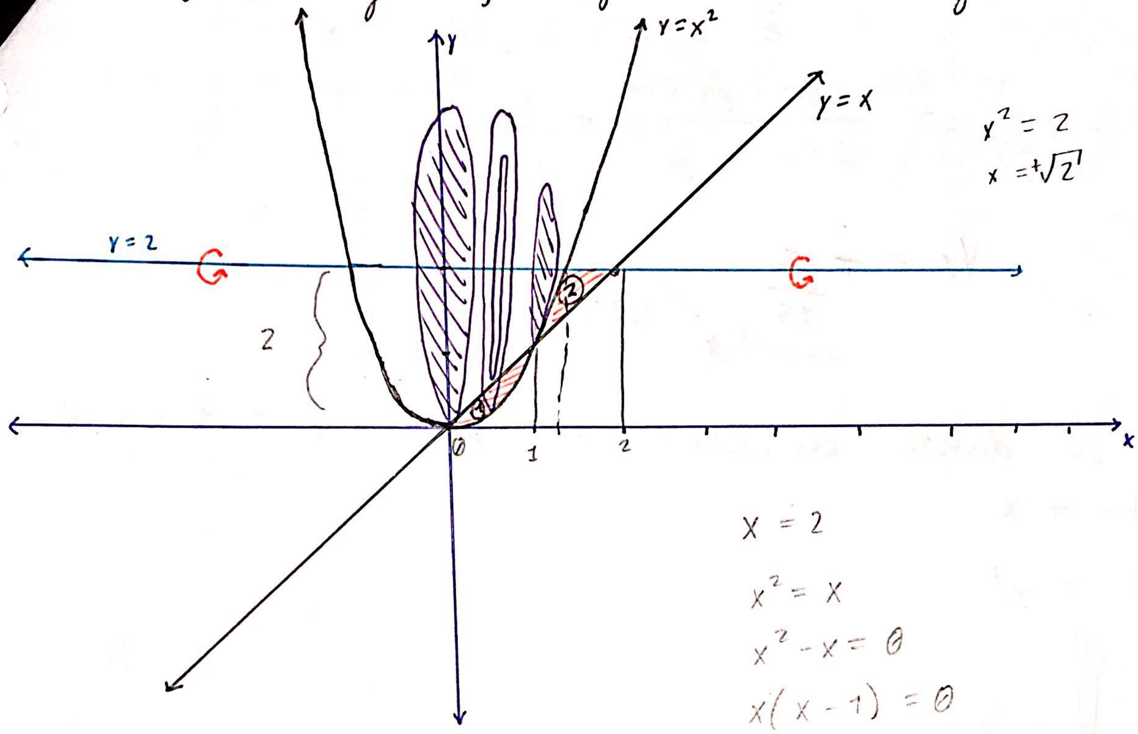
hacerlo respecto a y:

$$y = \ln(x)$$
$$e^y = x$$

$$V = \pi \int_1^2 e^{2y}\, dy = \pi\left(\frac{1}{2} e^{2y}\right)\Bigg|_1^2 =$$

$$= \frac{\pi}{2}\left[\left(e^{2\cdot 2}\right) - \left(e^{2\cdot 1}\right)\right] = \frac{\pi}{2}\left(e^4 - e^2\right)$$

$y = x$ ; $y = x^2$ ; eje de rotación : $y = 2$




$x^2 = 2$

$x = \pm\sqrt{2}$

$x = 2$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad x = 1$

Región 1:

Radio Externo = $x^2$

Radio Interno = $x$

haremos anillos de $x = 0$ a $x = 1$ y posteriormente de $x = 1$ a $x = 2$

$$V_1 = \pi \int_0^1 \left[ (2 - x^2)^2 - (2 - x)^2 \right] dx$$

$$V_1 = \pi \int_0^1 \left[ (2)^2 - 2(2)(x^2) + (x^2)^2 \right] - \left[ (2)^2 - 2(2)(x) + (x)^2 \right] dx$$

$$= \pi \int_0^1 4 - 4x^2 + x^4 - \left[ 4 - 4x + x^2 \right] dx$$

$$= \pi \int_0^1 \left( \cancel{4} - 4x^2 + x^4 \cancel{- 4} + 4x - x^2 \right) dx$$

$$= \pi \int_0^1 \left( x^4 - 5x^2 + 4x \right) dx = \pi \left( \frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2 \right) \Bigg]_0^1$$

$$= \pi \left( \frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2 \right]_0^1 = \pi \left[ \left( \frac{1}{5} - \frac{5}{3} + 2 \right) - (0) \right]$$

$$= \pi \left( \frac{1}{5} - \frac{5}{3} + \frac{2}{1} \right) = \pi \left( \frac{3 - 25 + 30}{5 \cdot 3} \right) = \pi \, \frac{8}{15}$$

$$V_1 = \pi \, \frac{8}{15}$$

2/4

<u>Región 2:</u> se divide en dos. uno para $x = 1$ & $x = \sqrt{2}$

rad. ext. $= x$

rad. int $= x^2$

$$V_{2.1} = \pi \int_1^{\sqrt{2}} \left[ (2-x)^2 - (2-x^2)^2 \right] dx \qquad V_{2.2} = \pi \int_{\sqrt{2}}^{2} \left[ (2-x)^2 \right] dx$$

$$V_{2.1} = \pi \int_1^{\sqrt{2}} \underbrace{\left[ (2)^2 - 2(2)(x) + (x)^2 \right] - \left[ (2)^2 - 2(2)(x^2) + (x^2)^2 \right]}_{\underbrace{4 - 4x + x^2 - \left[ 4 - 4x^2 + x^4 \right]}_{(\cancel{4} - 4x + x^2 - \cancel{4} + 4x^2 - x^4)}} dx$$

$$V_{2.1} = \pi \int_1^{\sqrt{2}} -x^4 + 5x^2 - 4x \; dx$$

$$= \pi \left( -\frac{1}{5}x^5 + \frac{5}{3}x^3 - 2x \right]_1^{\sqrt{2}} =$$

$$= \pi \left[ \left( -\frac{1}{5}(2)^{5/2} + \frac{5}{3}(2)^{3/2} - 2(\sqrt{2}) \right) - \left( -\frac{1}{5} + \frac{5}{3} - 2 \right) \right]$$

$$= \pi \left( -\frac{\sqrt{32}}{5} + \frac{5\sqrt{8}}{3} - 2\sqrt{2} + \frac{1}{5} - \frac{5}{3} + 2 \right) = \frac{\pi(38\sqrt{2} - 52)}{15}$$

$$= \pi \int_{\sqrt{2}}^{2} \left((2)^2 - 2(2)(x) + (x)^2\right) dx =$$

$$= \pi \int_{\sqrt{2}}^{2} (4 - 4x + x^2)\, dx = \pi \int_{\sqrt{2}}^{2} (x^2 - 4x + 4)\, dx$$

$$= \pi \left( \frac{1}{3} x^3 - 2x^2 + 4x \right) \Bigg]_{\sqrt{2}}^{2} =$$

$$= \pi \left[ \left( \frac{1}{3}(2)^3 - 2(2)^2 + 4(2) \right) - \left( \frac{1}{3}(2)^{3/2} - 2(2)^{2/2} + 4(2)^{1/2} \right) \right]$$

$$= \pi \left[ \left( \frac{8}{3} - \cancel{8} + \cancel{8} \right) - \left( \frac{\sqrt{8}}{3} - 4 + 4\sqrt{2} \right) \right]$$

$$= \pi \left[ \frac{8}{3} - \frac{\sqrt{8}}{3} + 4 - 4\sqrt{2} \right] = \pi \left( \frac{20 - 14\sqrt{2}}{3} \right)$$

Para sacar el volúmen neto se suman $V_1 + V_{2.2} + V_{2.1} = V_m$

$$V_m = \left[ \left( \pi \frac{8}{15} \right) + \left( \frac{\pi (38\sqrt{2} - 52)}{15} \right) + \left( \frac{\pi (20 - 14\sqrt{2})}{3} \right) \right]$$

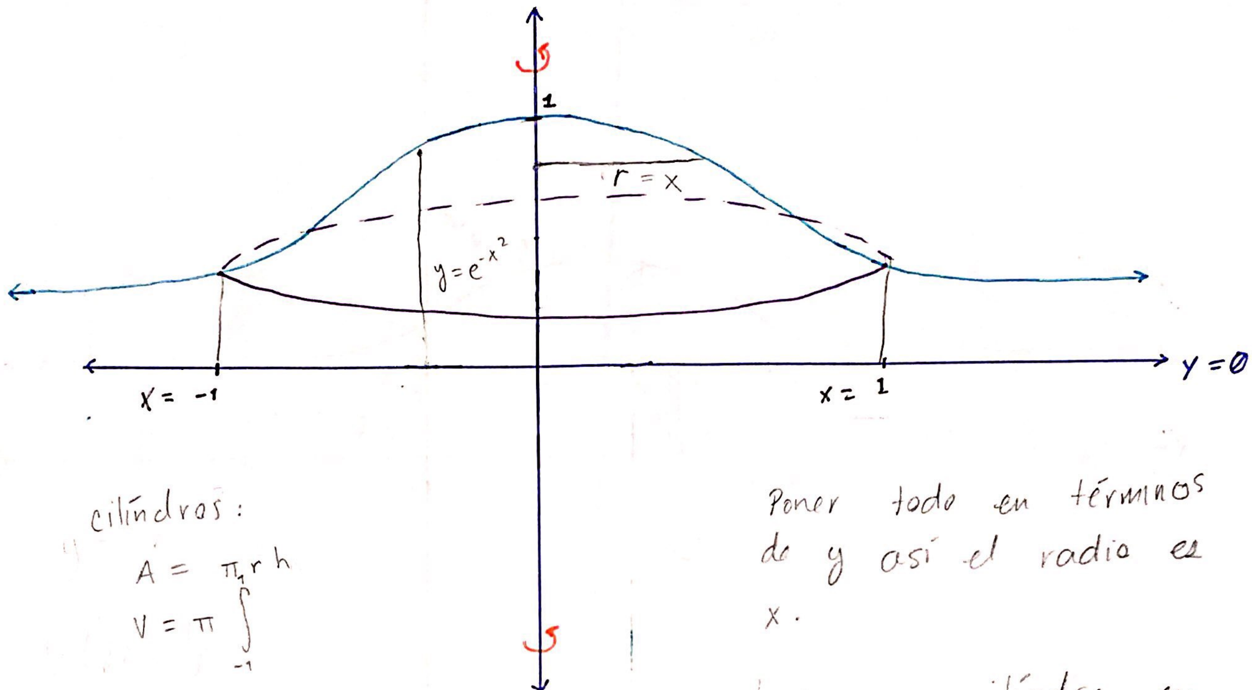
$$= \frac{8}{15}\pi + \frac{38\sqrt{2} - 52}{15}\pi + \frac{20 - 14\sqrt{2}}{3}\pi$$

$$= \pi \left( \frac{8}{15} + \frac{38\sqrt{2} - 52}{15} + \frac{20 - 14\sqrt{2}}{3} \right)$$

$$= \pi \left( \frac{56 - 32\sqrt{2}}{15} \right)$$

$y = e^{-x^2}$ ; $y = 0$ ; $x = -1$ ; $x = 1$

a) eje de rotación eje - y :




cilindros :

$A = \pi r h$

$V = \pi \int_{-1}$

Poner todo en términos de y así el radio es x.

hay un cilindro en $x = -1$ ; $x = 1$

$(1, e^{-1})$

$r = x$

$h = y = e^{-x^2}$

$$V = 2\pi \int_0^1 x \, e^{-x^2} dx$$

$u = x^2$

$du = 2x dx$

$\frac{du}{2} = x dx$

$$= 2\pi \int_0^1 e^{-u} \cdot \frac{du}{2} = \frac{\cancel{2}\pi}{\cancel{2}} \int_0^1 e^{-u} du$$

$$= \pi \int_0^1 e^{-u} = \pi - e^{-u} \Big]_0^1$$

$$= -\pi e^{-x^2} \Big]_0^1 =$$

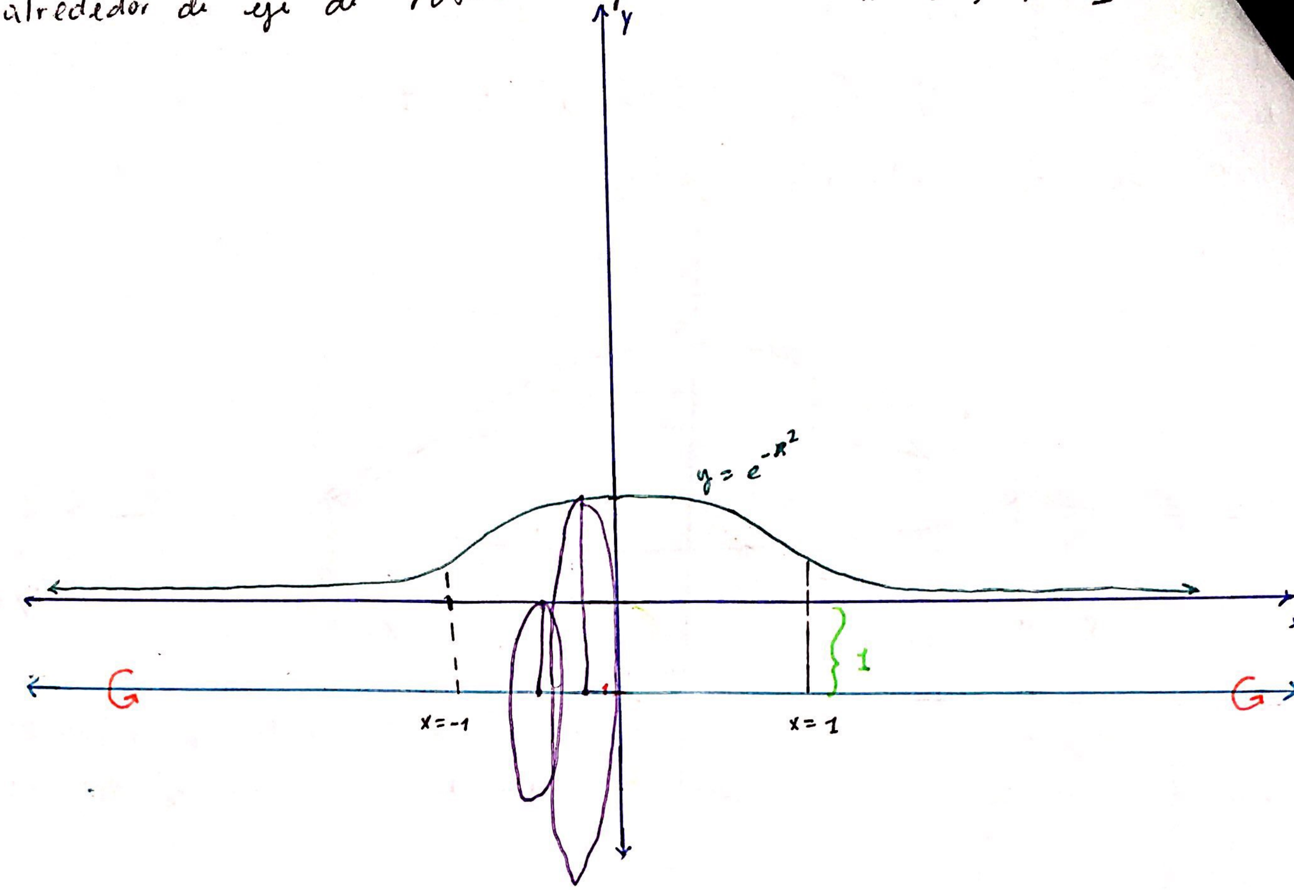
$$= -\pi \left( \left[ e^{-(1)^2} \right] - \left[ e^{-(0)^2} \right] \right) = -\pi \left( e^{-1} - e^{0} \right)$$

$$= -\pi \left( e^{-1} - 1 \right) \checkmark$$

, 20 ✓

$$= \pi \underline{\left( 1 - e^{-1} \right)}$$

(1) alrededor de eje de rotación $y = -1$ $x = -1$ ; $x = 1$




radio interior: $r_{int} = 1$

radio exterior: $r_{ext} = 1 + e^{-x^2}$

$A = \pi r^2$

$$V = \pi \int_{-1}^{1} \left(1 + e^{-x^2}\right)^2 - (1)^2 \, dx =$$

$$= 2\pi \int_{0}^{1} \cancel{1^2} + 2e^{-x^2} + \left(e^{-x^2}\right)^2 - \cancel{1^2}$$

$$= 2\pi \int_{0}^{1} 2e^{-x^2} + \left(e^{-x^2}\right)^2 \, dx$$

$$= 2\pi \int_{0}^{1} e^{-x^2}\left(2 + e^{-x^2}\right) dx$$

20

# Parte III

# Cortos

# Capítulo 12

## Examen Corto # 1

100pts

# Corto #1 Cálculo Integral (20 min)

**Nombre:** David Gabriel Corzo Mcmath **Carnet:** 20190432

1. Evalúe las siguientes integrales

   (a) (30 pts.) $\int \left( x^2 - \frac{1}{2}x + \frac{1}{4x} - \frac{2}{x^2} \right) dx$ $= \int [x^2] dx - \frac{1}{2}\int [x] dx + \frac{1}{4}\int\left[\frac{1}{x}\right] dx - 2\int\left[\frac{1}{x^2}\right] dx$

   $-\frac{1}{x}$; $\frac{0 * x - 1x}{x^2} = -\frac{x}{x^2}$

   $= \frac{x^{2+1}}{2+1} - \left(\frac{1}{2}\right)\frac{x^{1+1}}{1+1} + \left(\frac{1}{4}\right)\ln(x) - (2)\frac{x^{-2+1}}{-2+1} + C$

   $+\left(\frac{2x^{-1}}{-1}\right)$

   $= \frac{x^3}{3} - \frac{x^2}{4} + \frac{\ln(x)}{4} + \frac{2}{x} + C$

   30pts

   $\sin x = \cos x$

   $\cos x = -\sin x$

   (b) (30 pts.) $\int_0^{\pi/4} (2\sin x - 4\cos(x))dx$

   $2\int \sin x\, dx - 4\int \cos x\, dx = 2(-\cos x) - 4(\sin x)\Big]_0^{\pi/4}$

   $\{-2\cos(\pi/4) - 4\sin(\pi/4)\} - \{-2\cos(0) - 4\sin(0)\}$

   $= \{-\cancel{2}\frac{\sqrt{2}}{\cancel{2}} - \cancel{4}^{2}\frac{\sqrt{2}}{\cancel{2}}\} - \{-2\} = -\sqrt{2} - 2\sqrt{2} + 2$

   30pts

2. Considere la región entre $f(x) = |x-5|$, el eje-x, y las rectas verticales $x = 0$, $x = 10$.

   (a) (20 pts.) Trace la gráfica de la región.

   (b) (20 pts.) Encuentre el área de la región utilizando geometría.

$f(x) = |x-5| = \begin{cases} x - 5 & x > 5 \\ -x + 5 & x < 5 \end{cases}$

$f(x) = 0$

$-x + 5 = y$

$-0 + 5 = y$

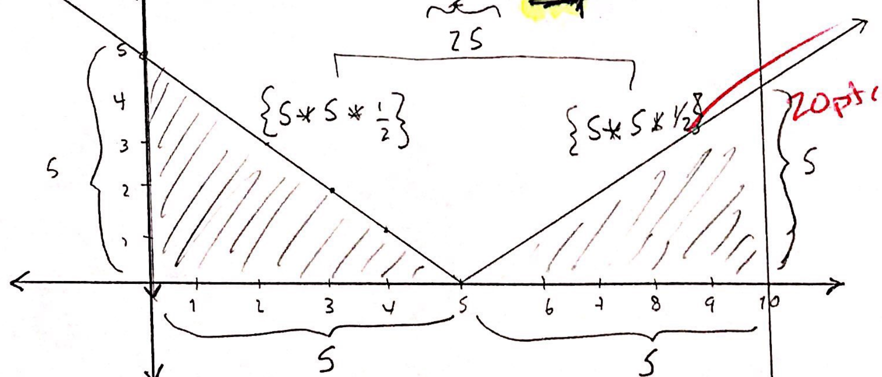
$y = 5$

$x = 1$

$10 - 5 = 5$

$\frac{25 * 2}{2} = 25$

20pts



# Capítulo 13

## Examen Corto # 2

55pts

# Corto #2 Cálculo Integral (15 min)

**Nombre:** David Gabriel Corzo Mcmath **Carnet:** 20190432

1. (50 pts.) ¿Cuál es la ecuación de la recta tangente a $h(x) = 2 + \int_1^{e^x} \cos^3\theta d\theta$ en $x = 0$?

$y = f(a) + f'(a)(x-a)$

$h(x) = 2 + \underbrace{\int_1^{e^x} \cos^3\theta \, d\theta}_{\text{derivo}}$ $x = 0$

$y = 3 + 1(x-0)$

20pts

$h'(x) = 0 + \cos^3(e^x) \cdot e^x$

$h'(0) = 0 + \cos^3(e^0) e^0$

$= 1$

$h(0) = 2 + \cos^3(0)$

$= 2 + 1 = 3$

2. (50 pts.) Un resporte sin amortiguamiento y sujeto a una fuerza externa tiene aceleracion $a(t) = 2t - 4\cos t + 5\sin t$, velocidad y posición iniciales de 2 m/s y 0 m.

(a) (25 pts.) Encuentra la función de velocidad del resorte

$a''(t) = 2t - 4\cos t - 5\sin t$

$a'(t) = 2\int t - 4\int \cos t - 5\int \sin t$

$a'(t) = \frac{2t^2}{2} - 4\sin t - 5\cos t$

$a'(t) = t^2 - 4\sin t - 5\cos t + C_1$

$a'(0) = 0^2 - 4\sin 0 - 5\cos 0 + C_1$

$y = 0 - 0 - 5 + C_1$

$y = -5 + C_1$

$C_1 = -5$

15pts

(b) (25 pts.) Encuentra la función de desplazamiento del resorte.

$a'(t) = t^2 - 4\sin t - 5\cos t$

$a(t) = \int t^2 - 4\int \sin t - 5\int \cos t$

$\sin' = \cos$

$\cos' = -\sin$

$a(t) = \frac{t^3}{3} + 4\cos t - 5\sin t + C_1 + C_2$

20pts

$a(0) = \frac{0^3}{3} + 4\cos 0 - 5\sin 0 + (-5) + C_2$

$a(0) = 0 + 4 - 0 - 5 + C_2$

$a(0) = 4 - 5 + C_2$

$a(0) = -4 + C$

$C_2 = -4$

# Capítulo 14

## Examen Corto # 3

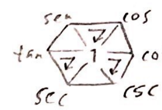
90pts

# Corto #3 Cálculo Integral (20 min)

**Nombre:** David Corzo **Carnet:** 20190432

Resuelva las siguientes integrales:

1. (50 pts.) $\int \tan^5\theta \sec^4\theta \, d\theta$




$= \int \tan^5\theta \cdot \sec^2\theta \left(\tan^2\theta + 1\right) d\theta$

$= \int \tan^7\theta \sec^2\theta + \int \tan^5 \sec^2\theta$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\int d\theta \quad \tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right)$

$\tan^2 x = \sec^2 x - 1$

$u = \tan\theta$ $\qquad$ $u = \tan\theta$

$du = \sec^2\theta$ $\qquad$ $du = \sec^2\theta$

$\therefore = \int u^7 \, du + \int u^5 \, du$

$= \frac{u^8}{8} + \frac{u^5}{5} + C$

$= \frac{\tan^8\theta}{8} + \frac{\tan^5\theta}{5} + C$

40pts

2. (50 pts.) $\int e^{x^8+1}(x^8+1)16x^7 \, dx = 2\int e^u (u) \, du$

$u = x^8 + 1$

$du = 8x^7 \, dx$

$2(du) = 16x^7 \, dx$

$\upsilon = u \qquad d\beta = e^u$

$d\upsilon = 1 \, du \qquad \beta = e^u$

$\upsilon\beta - \int \beta \, d\upsilon$

$u \cdot e^u - \int e^u \, du$

$u \cdot e^u - e^u + C$

~~$\frac{d}{dx}\left(2(x^8+1)(e^{x^8+1}) - (2e^{x^8+1})\right)$~~

~~$2x^8 + 2 + 2e^{x^8+1} - 2e^{x^8+1}$~~

$2x^8 + 2$

$2\left[(x^8+1)(e^{x^8+1}) - (e^{x^8+1}) + C\right]$

$2\left[(x^8+1)\left(e^{x^8+1}\right) - \left(e^{x^8+1}\right) + C\right]$

50pts

# Capítulo 15

## Examen Corto # 4

# Capítulo 16

## Examen Corto # 5

# Corto #5 Cálculo Integral (10 min)

**Nombre:** David Gabriel Corzo Mcwrath **Carnet:** 20190432

100

Determine si la integral es convergente o divergente. Evalúe si es convergente. Utilice la regla de L'Hospital para evaluar límites con formas indeterminadas.

1. (100 pts.) $\int_{-\infty}^{0} xe^x \, dx$

$u = x \qquad dv = e^x$

$du = dx \qquad v = e^x$

$$= xe^x - \int_{-\infty}^{0} e^x \, dx = xe^x - e^x \Big]_{-\infty}^{0}$$

$$= \Big\{ \underbrace{\lim_{a \to 0} (xe^x - e^x)}_{0 \cdot e^0 - 1} \Big\} - \Big\{ \lim_{a \to -\infty} \underbrace{(xe^x - e^x)}_{-\infty e^{-\infty} - \infty} \Big\}$$

forma indeterminada

$$\underbrace{\lim_{a \to -\infty} (xe^x)}_{} - \lim_{a \to -\infty} (e^x)$$

$$\underbrace{\lim_{a \to -\infty} \left( \frac{x}{e^x} \right)}_{\text{L'Hopital}} - \underbrace{\lim_{a \to -\infty} \left( e^{-\infty} \right)}_{\lim_{a \to \infty} \left( \frac{1}{e^{\infty}} \right) = 0}$$

$$\lim_{a \to -\infty} \left( \frac{e^x}{x} \right) = \lim_{a \to -\infty} \left( \frac{e^x}{1} \right) =$$

$$= \lim_{a \to -\infty} \left( e^{-\infty} \right) = \lim_{a \to -\infty} \left( \frac{1}{e^{\infty}} \right) = 0 \qquad \therefore \Big\{ \lim_{a \to -\infty} (xe^x - e^x) \Big\} = 0$$

en conclusión

$\therefore$

$$= \{ 0 \cdot e^0 - e^0 \} - \{0\} = \{ \cancel{0 \cdot 1} - 1 \} - \{0\}$$

$$= \{-1\} - 0 = -1 - 0$$

$$= \underline{-1}_{\#}$$

Convergente por la respuesta ser $-1$

# Capítulo 17

## Examen Corto # 6

110

# Corto #6 Cálculo Integral (20 min)

**Nombre:** David Gabriel Corzo **Carnet:** 20190432

1. Considera la región limitada por las curvas $y_1 = x^2 - 4x + 4$ y $y_2 = 10 - x^2$.

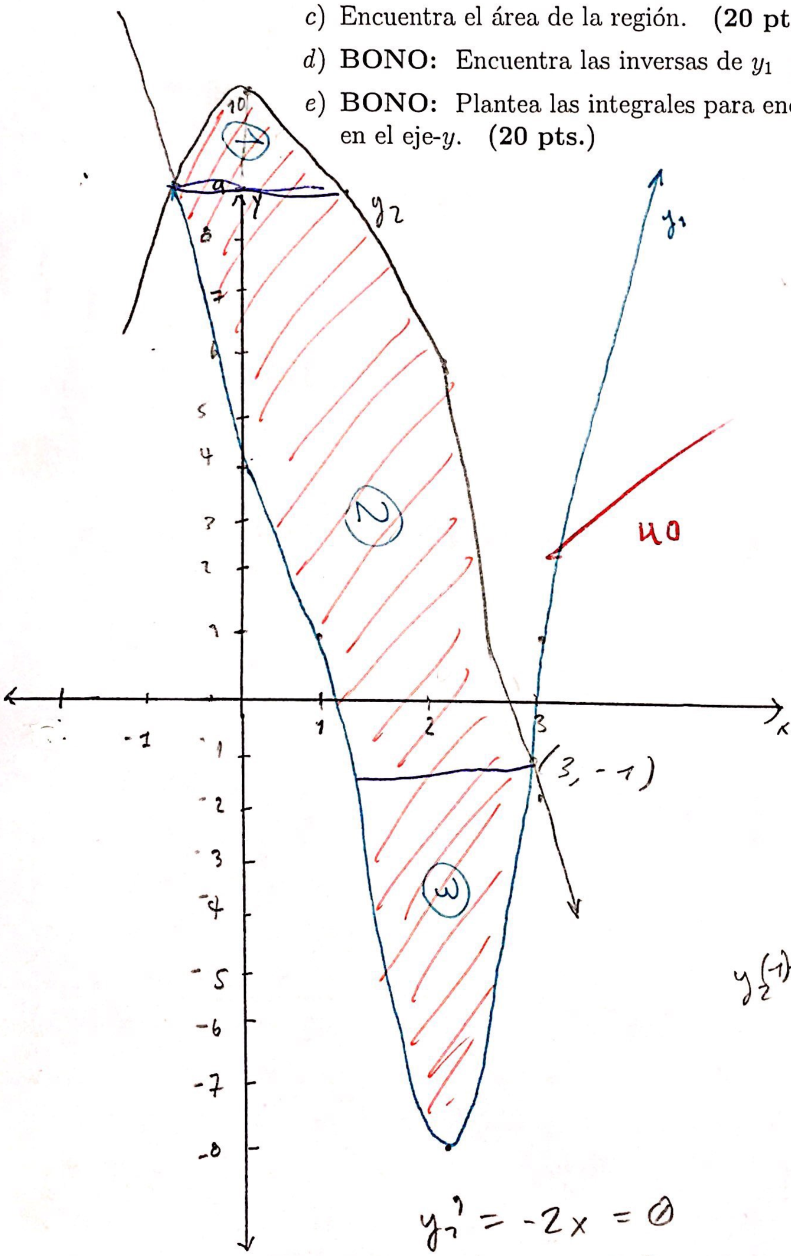
   *a*) Dibuja las regiones acotadas por cada una de las curvas dadas. **(40 pts.)** Para su información $\sqrt{10} \approx 3.2$ .

   *b*) Plantea la integral para encontrar el área de la región. **(40 pts.)**

   *c*) Encuentra el área de la región. **(20 pts.)**

   *d*) **BONO:** Encuentra las inversas de $y_1$ y $y_2$. **(10 pts.)**

   *e*) **BONO:** Plantea las integrales para encontrar el área de la región integrando en el eje-$y$. **(20 pts.)**




$2x - 4 = 0$

$x = \frac{4}{2} = 2$

$x^2 - 4x + 4 = 10 - x^2$

$x^2 + x^2 - 4x + 4 - 10$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x-3)(x+1) = 0$

interceptos:

$x = 3 \quad x = -1$

$y_1(3) = 3^2 - 4(3) + 4$

$= 9 - 12 + 4$

$= -3 + 4$

$= 1$

$y_2(-1) = 10 - (-1)^2$

$10 - 1$

$9$

$y_1(-1) = 1 + 4 + 4$

$y_1(-1) = 9$

$y_1(2) = 4 - 8 + 4$

$y_1(1) = 1 - 4 + 4 \quad = -8$

$= 1$

$y_2' = -2x = 0$

$x = 0$

David Corzo

$$A = \int_{-1}^{3} (10 - x^2) - (x^2 - 4x + 4)\,dx$$

40

$$= \int_{-1}^{3} 10 - \cancel{x^2} - \cancel{x^2} + 4x - 4\,dx$$

$$= \int_{-1}^{3} -2x^2 + 4x + 6\,dx$$

$$= \left. -\frac{2}{3}x^3 + \frac{4}{2}x^2 + 6x \right]_{-1}^{3} = \left. -\frac{2}{3}x^3 + 2x^2 + 6x \right]_{1}^{3}$$

$$= \left\{ -\frac{2}{3}(3)^3 + 2(3)^2 + 6(3) \right\} - \left\{ -\frac{2}{3}(1)^3 + 2(1)^2 + 6(1) \right\}$$

$$= -\frac{2}{\cancel{3}} \cdot 3^2 \cdot \cancel{3} + 18 + 18 - \left\{ -\frac{2}{3} + 2 + 6 \right\}$$

$$= -\cancel{18} + \cancel{18} + 18 + \frac{2}{3} - 2 - 6$$

$$= 18 + \frac{2}{3} - 8 = \frac{18 \cdot 3}{3} + \frac{2}{3} - \frac{8 \cdot 3}{3} = \frac{18 \cdot 3 + 2 - 8 \cdot 3}{3}$$

$$= \frac{54 + 2 - 24}{3} = \frac{56 - 24}{3} = \frac{32}{3}$$

A

10

$$\begin{array}{r} \overset{2}{1}8 \\ 3 \\ \hline 54 \end{array}$$

BONO inversos

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$$y_1 = x^2 - 4x + 4$$

$$y = (x-2)(x-2)$$

$$y = (x-2)^2$$

$$\pm\sqrt{y} = x - 2$$

$$\pm\sqrt{y} - 2 = x$$

$$\boxed{x_{1,1} = -\sqrt{y} - 2}$$

$$\boxed{x_{1,2} = \sqrt{y} - 2}$$

+10

$$y_2 = 10 - x^2$$

$$y - 10 = -x^2$$

$$-y + 10 = x^2$$

$$\pm\sqrt{-y+10} = x_2$$

$$A = \underbrace{\int_{10}^{9} \sqrt{-y+10} + \sqrt{-y+10}\, dy}_{R①} + \int_{9}^{-1} \sqrt{-y+10} - (\sqrt{y} - 2)\, dy + \ldots$$

$$\ldots + \int_{-1}^{-8} (\sqrt{y} - 2) - (-\sqrt{y} - 2)\, dy$$

$$A = \int_{10}^{9} \sqrt{-y+10} + \sqrt{-y+10}\, dy \quad \int_{9}^{-1} \sqrt{-y+10} - (\sqrt{y} - 2)\, dy + \ldots$$

$$+ \int_{-1}^{-8} (\sqrt{y} - 2) - (-\sqrt{y} - 2)$$

+10

# Capítulo 18

## Examen Corto # 7

60

# Corto #7 Cálculo Integral (20 min)

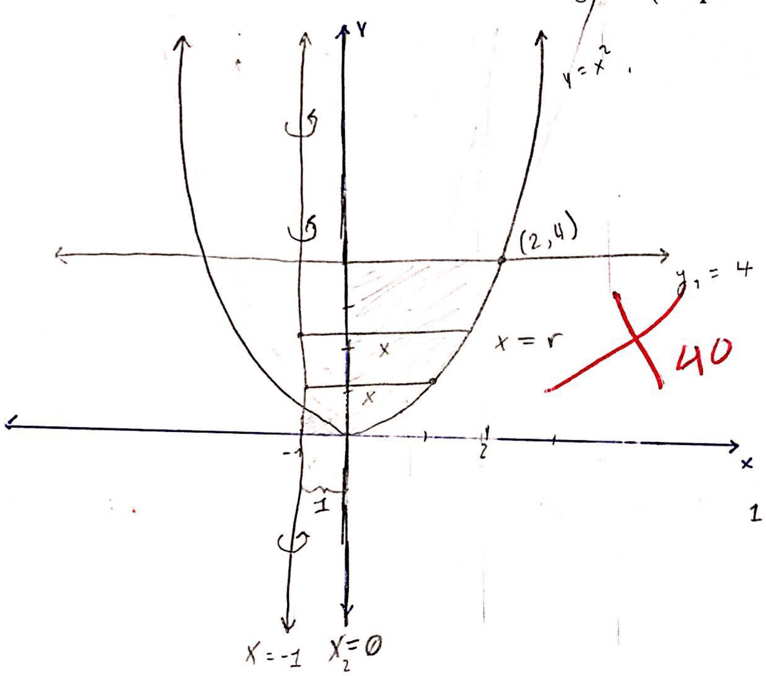
**Nombre:** David Corzo **Carnet:** 20190432

Un sólido se obtiene al girar $x_1 = \sqrt{y}$, $x_2 = 0$ y $y_1 = 4$, alrededor de la recta $x = -1$.

a. Grafique la región, indicando curvas, intersecciones y el eje de rotación. **(50 pts.)**

b. Planteé la integral de volúmen. **(50 pts.)**

c. Calcule el volúmen resolviendo la integral. **(20 pts. extra)**




X 40

$x_1 = \sqrt{y}$

$x^2 = y$

$x^2 = 16$

$x = 4$

intercepto $x_1$ con $y_1$:

$x^2 = 4$

$x = \pm 2$

intercepto en (2, 4)

Radio =

$A = \pi r^2$

$r = 1 + \sqrt{y}$

integro respecto a y

$A = \pi r^2$

$V = \pi \int_{-1}^{2} (1 + \sqrt{y})^2 dy$ X 20

$= \pi \int_{-1}^{2} (1 + 2\sqrt{y} + y) dy$

$= \pi \left( y + \frac{4}{3} y^{3/2} + \frac{1}{2} y^2 \right) \Big]_{-1}^{2}$

$= \pi \left[ \left(2 + \frac{4}{3}(2)^{3/2} + \frac{1}{2}(2)^2\right) - \left(-1 + \frac{4}{3}(-1)^{3/2} + \frac{1}{2}(-1)^2\right) \right]$

# Capítulo 19

## Examen Corto # 8

100

# Corto #8 Cálculo Integral (20 min)

**Nombre:** David Gabriel Garza Mcmath **Carnet:** 20190432

a. Calcule el valor promedio de $f(x) = (x-4)^2$ en el intervalo $[0,6]$ **(50 pts.)**

b. Encuentra las $c's$ tal que $f(c) = f_{prom}$ . **(25 pts.)**

c. Gráfica $f$ y el rectángulo cuya área es igual al área bajo la gráfica de $f$. **(25 pts)**

a) $\frac{1}{b-a}\int_a^b f(x)\,dx$

$$f_{prom} = \frac{1}{6-0}\int_0^6 (x-4)^2\,dx = \frac{1}{6}\int_0^6 \left(x^2 - 2\cdot 4\cdot x + 4^2\right)dx$$

$$= \frac{1}{6}\int_0^6 \left(x^2 - 8x + 16\right)dx = \frac{1}{6}\left[\left(\frac{1}{3}x^3 - \frac{8}{2}x^2 + 16x\right)\right]_0^6 =$$

$$= \frac{1}{6}\left[\left(\frac{1}{3}(6)^3 - 4(6)^2 + 16(6)\right) - (0)\right] =$$

$$= \frac{1}{6}\left[\frac{216}{3} - 4(36) + 96\right] = \frac{1}{6}\left[72 - 144 + 96\right] =$$

$$= \frac{24}{6} = \boxed{4}$$

36 × 6 = 216

16 × 6 = 96

216 ÷ 3 = 72

36 × 4 = 144

96 + 72 = 168

168 − 144 = 24

24 ÷ 6 = 4

b)

$$(x-4)^2 = 4$$

$$x - 4 = \pm\sqrt{4}$$

$$x = \pm 2 + 4$$

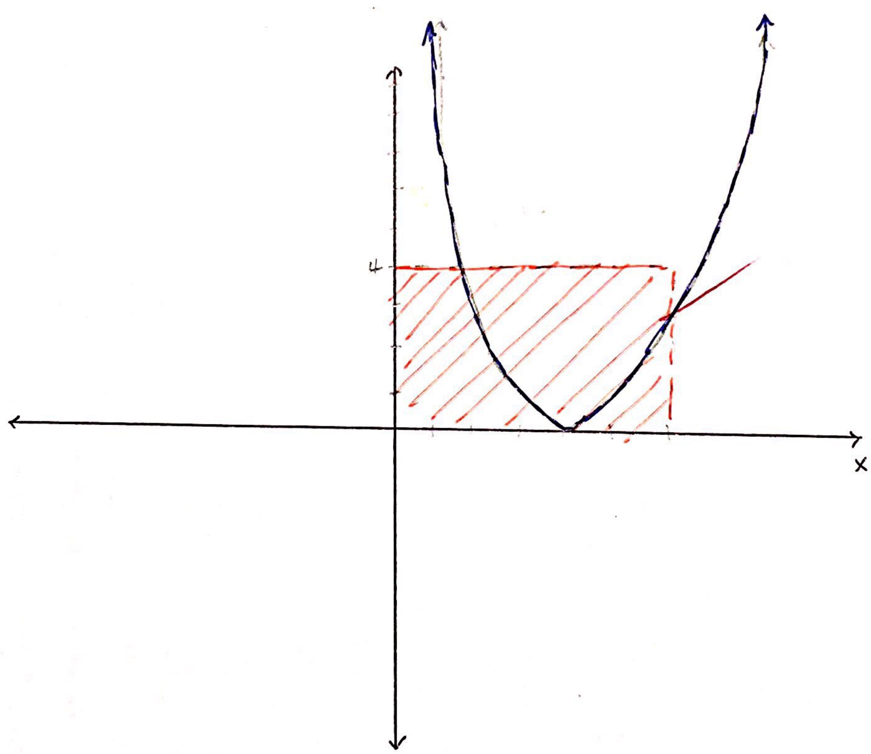
$$x_1 = 2 + 4 = \boxed{6}$$

$$x_2 = -2 + 4 = \boxed{2}$$

$c = 6$

$c = 2$

c)



# Capítulo 20

## Examen Corto # 9

100

# CORTO #9 Cálculo Integral (15 min)

**Nombre:** David Carzo **Carnet:** 20190432

1. Resuelva la siguiente integral $\displaystyle\int \frac{x^2+2x-4}{x^3+4x}\,dx$

$$\int \frac{x^2+2x-4}{x(x^2+4)}\,dx = \underbrace{\int -\frac{1}{x}\,dx}_{①} + \underbrace{\int \frac{2x+2}{x^2+4}\,dx}_{②}$$

$$\frac{x^2+2x-4}{\cancel{x(x^2+4)}} = \frac{A}{\cancel{x}} + \frac{Bx+C}{\cancel{x^2+4}}$$

$$x^2+2x-4 = A(x^2+4)+(Bx+C)(x)$$

$$x^2+2x-4 = Ax^2+4A+Bx^2+Cx$$

$x=0$ ;

$$-4 = \cancel{A(0)^2}+4A+\cancel{B(0)^2}+\cancel{C(0)}$$

$$-4 = 4A$$

$$4A=-4$$

$$A=-1$$

$A=-1$
$B=2$
$C=2$

$$x^2+2x^1-4x^0 = x^2(-1)+4(-1)+Bx^2+Cx$$

$$x^2+2x^1-4x^0 = -x^2-4+Cx+Bx^2$$

$$x^2+2x^1-4x^0 = -x^2+Bx^2+Cx-4$$

$$1x^2+2x^1-4x^0 = (B-1)x^2+Cx-4 \quad ; \quad C=2$$

$$B-1=1$$

$$B=1+1$$

$$B=2$$

① $-\displaystyle\int \frac{1}{x}\,dx = \boxed{-\ln|x|}$ ①

② $\displaystyle\int \underbrace{\frac{2x}{x^2+4}}_{2.1} + \underbrace{\frac{2}{x^2+4}}_{2.2}\,dx$

2.1 $\displaystyle\int \frac{du}{u} = \ln|u| = \boxed{\ln|x^2+4|}$ 2.1

$u=x^2+4$

$du=2x\,dx$

2.2 $2\displaystyle\int \frac{1}{x^2+4}\,dx$

usar fórmula de:

$$\int \frac{1}{x^2+a}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

$a=4$

$$\cancel{2}\left[\frac{1}{\cancel{2}}\arctan\left(\frac{x}{2}\right)\right] = \boxed{\arctan\left(\frac{x}{2}\right)}$$ 2.2

R/ $-\ln|x| + \ln|x^2+4| + \arctan\left(\frac{x}{2}\right) + C$

# Parte IV

# Simulacros

# Capítulo 21

## Parcial Simulacro I

# Simulacro de Parcial #1, Cálculo Integral

Lunes, 26 de Agosto

Nombre y Apellidos: ______________________________

| Tema: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Puntos: | 40 | 20 | 15 | 15 | 20 | 10 | 120 |
| Nota: | | | | | | | |

1. Evalúa las siguientes integrales indefinidas.

   (a) (10 pts.) $\int x \tan^{-1} x^2 \, dx$

   (b) (10 pts.) $\int \frac{x^2}{\sqrt{9 - 25x^2}} \, dx$

   (c) (10 pts.) $\int \frac{dt}{\sqrt{(t-2)^2 + 9}} \cancel{dt}$

   (d) (10 pts.) $\int \frac{xe^x}{(x+1)^2} \, dx$

2. Evalúa las siguientes integrales definidas.

   (a) (10 pts.) $\int_{\pi/4}^{0} \tan^5 \theta \sec^3 \theta \, d\theta$

   $\int_0^{\pi/4}$

   (b) (10 pts.) $\int_{\pi/2}^{0} \frac{\cos t}{\sqrt{1 + \sin^2 t}} \, dt$

3. Considere la región entre $f(x) = |x - 8|$, el eje-x, y las rectas verticales $x = 0$, $x = 8$.

   (a) (5 pts.) Trace la gráfica de la región.

   (b) (5 pts.) Encuentre el área de la región utilizando geometría.

   (c) (5 pts.) Planteé la integral para encontrar el área de la región.

4. La función de aceleración (en $m/s^2$) para una partícula moviéndose en una recta es $a(t) = \frac{3}{\sqrt{2t+1}}$.

   (a) (5 pts.) Encuentra la velocidad de la partícula si su velocidad inicial es de -3 m/s.

   (b) (5 pts.) Encuentra la función de desplazamiento de la partícula si su posición a los 2 segundos es de 8 m en la dirección positiva.

   (c) (5 pts.) ¿Cuál es la posición de la partícula a los 7 segs?

5. La velocidad de una partícula (en metros por segundo) sobre una línea recta es $v(t) = 1 - (t-2)^2$ para $0 \le t \le 2$. Encuentra:

   (a) (10 pts.) El desplazamiento de la partícula en el intervalo de tiempo dado

   (b) (10 pts.) La distancia recorrida de la partícula en el mismo intervalo de tiempo

6. (10 pts.) Calcule la ec. de la recta tangente a la curva de $f(x) = \int_{\sin x}^{2e^x - 2} \sqrt{t^2 + 2t + 4} \, dt$ en $x = 0$.

# Capítulo 22

## Simulacro primer intento de terminarlo

DAVID CORRO 20190432

# Parcial Simulacro # 1 - Calculo Integral 2019-08-26

1. $$\int x \tan^{-1}(x^2)\,dx = \frac{1}{2}\int \underbrace{\tan^{-1}(u)\,du}\ =$$

$u = x^2$

$du = 2x\ dx$

$\frac{du}{2} = x\ dx$

$\upsilon = \tan^{-1}(u) \qquad d\beta = du$

$d\upsilon = \frac{1}{u^2+1}\,du \qquad \beta = u$

$\upsilon\beta - \int \beta\, d\upsilon$

$= \tan^{-1}(u)\,u - \int u \cdot \frac{1}{u^2+1}\,du$

$= \tan^{-1}(u)\cdot u - \underbrace{\int \frac{u}{u^2+1}\,du}$

$w = u^2+1$

$\frac{dw}{2} = u\,du$

$\frac{1}{2}\int \frac{dw}{w}$

$\frac{1}{2}\ln|w| = \frac{1}{2}\ln|u^2+1|$

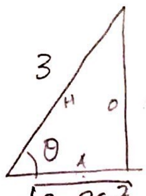
$$\frac{1}{2}\left[\tan^{-1}(u)\cdot u - \frac{1}{2}\ln|u^2+1|\right]$$

$$\frac{1}{2}\left[\tan^{-1}(x^2)\,x^2 - \frac{1}{2}\ln|x^4+1|\right] + C$$

$\square$

① ⓑ $\displaystyle\int \frac{x^2}{\sqrt{9-25x^2}}\,dx$

$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{5x}{3}\right)$




$S\ \frac{O}{H} \quad C\ \frac{A}{H} \quad T\ \frac{O}{A}$

$\csc\theta = \frac{H}{O} \quad \sec\theta = \frac{H}{A} \quad \cot\theta = \frac{A}{O}$

$x = \frac{3}{5}\sin\theta$

$dx = \frac{3}{5}\cos\theta\, d\theta$

$\sin\theta = \frac{5x}{3}$

$3\sin\theta = 5x$

Para $x$: $\boxed{\frac{3}{5}\sin\theta = x}$

$c^2 = a^2 + b^2$

$9 = 25x^2 + b^2$

$\sqrt{9-25x^2} = b$

Para $\sqrt{9-25x^2}$: $\boxed{3\cos\theta = \sqrt{9-25x^2}}$

$\cos\theta = \frac{\sqrt{9-25x^2}}{3}$

$3\cos\theta = \sqrt{9-25x^2}$

$$\therefore \int \frac{\left[\frac{3}{5}\sin\theta\right]^2}{3\cos\theta}\cdot\frac{3}{5}\cos\theta\, d\theta = \int \frac{\frac{9\sin^2\theta}{25}}{\frac{3\cos\theta}{1}}\cdot\frac{3}{5}\cos\theta\, d\theta$$

$$= \int \frac{\cancel{3}\cdot 3\sin^2\theta\cdot 1}{25\cdot\cancel{3}\cos\theta}\cdot\frac{3}{5}\cos\theta\, d\theta = \int \frac{3\cdot 3\cdot\sin^2\theta\cdot\cancel{\cos\theta}}{25\cdot 5\cdot\cancel{\cos\theta}}\, d\theta =$$

$$= \frac{9}{125}\int \sin^2\theta\, d\theta = \frac{9}{125}\int\left(\frac{1}{2} - \frac{\sin(2\theta)}{2}\right)d\theta = \frac{9}{125}\left\{\int\frac{1}{2}d\theta - \frac{1}{2}\int\sin(2\theta)d\theta\right\}$$

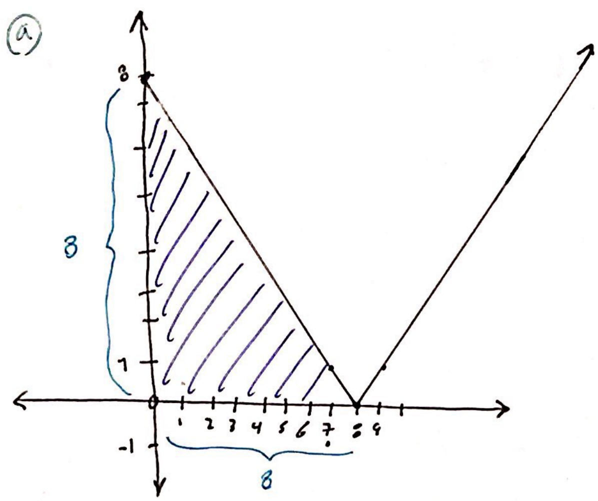
$$= \frac{9}{125}\left\{\frac{\theta}{2} - \frac{1}{2}\left(\frac{-\cos(2\theta)}{2}\right)\right\} = \frac{9}{125}\left[\frac{\theta}{2} + \frac{\cos(2\theta)}{4}\right]$$

$$\frac{9}{125}\left[\frac{\sin^{-1}(5x/3)}{2} + \frac{2\cdot\frac{5x}{3}\cdot\frac{\sqrt{9-25x^3}}{3}}{4}\right] + C$$

□

③

ⓐ




$A = 8 * 8 * \frac{1}{2}$

ⓑ $\underline{\underline{A = 32}}$

ⓒ $A = \int_0^8 x - 8 \quad = \left.\frac{x^2}{2} - 8x\right]_0^8$

$-32$

⑤ $v(t) = 1 - (t^2 - 4t + 4)$

$v(t) = 1 - t^2 + 4t + 4$

$v(t) = -t^2 + 4t + 5 \qquad 0 \leq t \leq 2$

$\int v(t)\,dt = -\int t^2\,dt + 4\int t\,dt + 5\int 1\,dt$

$f(x) = -\frac{t^3}{3} + \frac{4t^2}{2} + 5t$

$f(x) = -\frac{t^3}{3} + 2t^2 + 5t\Big]_0^2 = \left\{-\frac{2^3}{3} + 2(2)^2 + 5(2)\right\} - \{0\}$

$-\frac{8}{3} + 8 + 10$

$-\frac{8}{3} + \frac{18 \cdot 3}{3} = \frac{-8 + 18 \cdot 3}{3} = \frac{46}{3}$

(6) $f(x) = \int_{\sin x}^{2e^x - 2} \sqrt{t^2 + 2t + 4}\, dt$

$= \{\sqrt{(2e^x - 2)^2 + 2(2e^x - 2) + 4} \cdot 2e^x\} - \{\sqrt{\sin^2(x) - 2\sin(x) + 4} \cdot \cos x\}$

$m = \sqrt{\cancel{(2e^0 - 2)^2}_{0} + \cancel{2(0)} + 4} \cdot 2 - \sqrt{4}$

$\sqrt{4} \cdot 2 - \sqrt{4}$

$2 \cdot 2 - 2 = \underline{2}$

$y - y_1 = m(x - x_1)$

$y = f(a) + f'(a)(x - a)$

$f(a) = 0 + 2(x - 0)$

$\square$

① ©

$$\int \frac{x \cdot e^x}{(x+1)^2} \cdot dx$$

# Capítulo 23

## Simulacro segundo intento para estudiar

Resolución De Simulacro

$$\int \tan^{-1}(x^2)\, x\, dx = \frac{1}{2}\int \tan^{-1}(u)\, du =$$

$u = x^2$

$\frac{du}{2} = x\,dx$

$uv - \int v\,du$

$v = \tan^{-1}(u) \qquad dv = du$

$dv = \frac{1}{u^2+1}du \qquad v = u$

$$= \frac{1}{2}\left\{ \tan^{-1}(u)\, u - \underbrace{\int u \frac{1}{u^2+1}}\right\}$$

$$= \int \frac{u}{u^2+1}du = \frac{1}{2}\int \frac{1}{\varepsilon}\,d\varepsilon = \boxed{\frac{1}{2}\ln|\varepsilon|}$$

$\varepsilon = u^2+1$

$d\varepsilon = 2u\,du$

$\frac{d\varepsilon}{2} = u\,du$

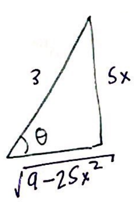
$$= \frac{1}{2}\left\{ \tan^{-1}(u)\, u - \frac{1}{2}\ln|u^2+1| \right\}$$

$$= \frac{1}{2}\tan^{-1}(x^2)\cdot x^2 - \frac{1}{4}\ln|x^4+1| + C \quad \square$$

---

$$\int \frac{x^2}{\sqrt{9-25x^2}}dx = \int \frac{\frac{9\sin^2\theta}{25}}{\frac{3\cos\theta}{1}}\cdot \frac{3}{5}\cos\theta = \int \frac{9\sin^2\theta}{\cancel{3}\cdot 25\,\cancel{\cos\theta}}\cdot\frac{\cancel{3}\,\cancel{\cos\theta}}{5}\,d\theta =$$

$$= \int \frac{9}{125}\sin^2\theta\, d\theta = \frac{9}{125}\int\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$



$c^2 = a^2 + b^2$

$0 = -c^2 + a^2 + b^2$

$-1(-b^2) = (-c^2 + a^2)$

$b^2 = c^2 - a^2$

$c^2 - a^2 = b^2$

$$= \frac{9}{125}\left[\frac{\theta}{2} - \frac{1}{2}\int \cos(2\theta)\,d\theta\right]$$

$dx$

$\boxed{\frac{3}{5}\cos\theta\, d\theta = dx}$

$$= \frac{9}{125}\left[\frac{\theta}{2} - \frac{1}{2}\frac{\sin(2\theta)}{2}\right]$$

S $\frac{O}{H}$ C $\frac{A}{H}$ T $\frac{O}{A}$

$\sin\theta = \frac{5x}{3}$

$\left(\frac{3\sin\theta}{5}\right)^2 = (x)^2 \rightarrow$

$\cos\theta = \frac{\sqrt{9-25x^2}}{3}$

$\boxed{3\cos\theta = \sqrt{9-25x^2}}$

$\sqrt{9-25x^2}$

$\boxed{\frac{9}{25}\sin^2\theta = x^2}$

$x^2$

$$= \frac{9}{125}\cdot\frac{\theta}{2} - \frac{9}{125}\cdot\frac{1}{2}\cdot\frac{1}{2}\sin(2\theta)$$

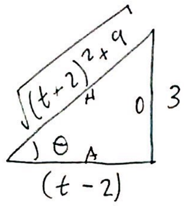
$$= \frac{9}{125\cdot 2}\theta - \frac{9}{125\cdot 4}\sin(2\theta) + C$$

$$= \frac{9}{125\cdot 2}\sin^{-1}\left(\frac{5x}{3}\right) - \frac{9}{125\cdot 4}2\sin\theta\cos\theta + C$$

$$= \frac{9}{125\cdot 2}\sin^{-1}\left(\frac{5x}{3}\right) - \frac{9}{125\cdot 2}\cdot\left(\frac{5x}{3}\right)\left(\frac{\sqrt{9-25x^2}}{3}\right) + C$$

③ $$\int \frac{1}{\sqrt{(t-2)^2+9}}\,dt = \int \frac{-3\csc^2\theta\,d\theta}{3\csc\theta} = -\int \frac{\cancel{\csc\theta}\cdot\csc\theta}{\cancel{\csc\theta}}\,d\theta = -\int \csc\theta\,d\theta = +\ln|\csc\theta+\cot\theta| + C$$

$$= \ln\left|\frac{\sqrt{(t-2)^2+9}}{3} + \frac{t-2}{3}\right| + C$$




$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$\csc\theta$ $\frac{H}{O}$ $\sec\theta$ $\frac{H}{A}$

$$\boxed{3\csc\theta = \sqrt{(t-2)^2+9}}$$

$$\csc\theta = \frac{\sqrt{(t-2)^2+9}}{3}$$

$$\cot\theta = \left(\frac{t-2}{3}\right)$$

$$\cot\theta = \frac{t}{3} - \frac{2}{3}$$

$$-\csc^2\theta = \frac{1}{3} - 0\ dt$$

$$-\csc^2\theta = \frac{1}{3}dt$$

$$\boxed{-3\csc^2\theta\,d\theta = dt}$$

④ $$\int \frac{xe^x}{(x+1)^2}\,dx \qquad f'g + fg'$$

LIPET
log
Inverse trig
Powers
expon
trigg

$u = xe^x \qquad dv = \frac{1}{(x+1)^2}dx$

$du = e^x + xe^x\,dx \qquad v = -\frac{1}{(x+1)}$

$$\frac{1}{(x+2)^2} = \int \underbrace{(x+2)^{-2}}\,dx$$

$u = x+1$

$du = dx$

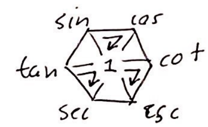
$$= \int u^{-2}\,du = \frac{u^{-1}}{-1} = -\frac{1}{(x+1)}$$

$$= -\frac{xe^x\cdot 1}{(x+1)} - \underbrace{\int -\frac{1}{(x+1)}\,e^x + xe^x\,dx}$$

$$\int -\frac{e^x\cancel{(x+1)}}{\cancel{(x+1)}}\,dx$$

$$= -\frac{xe^x}{x+1} - \left\{-\left[e^x\right]\right\} + C$$

$$= -\frac{xe^x}{x+1} + e^x + C \quad \square$$




$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1^2 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin = \frac{1}{\csc}$

$\tan = \frac{\sin}{\cos}$

$\cos\tan = \sin$

$\cos\frac{\sin}{\cos} = \sin$

$$\int \ln\left(\frac{1}{\ln\left(\frac{1}{\ln\left(\frac{1}{\ln\left(\frac{1}{\ln x}\right)}\right)}\right)}\right)\,dx$$

6

$y = m(x - x_1) + y_1$ $\qquad\qquad$ $y = f(a) + f'(a)(x - x_1)$

$$y = f'(a)(x - a) + f(a)$$

$$f(x) = \int_{\sin(x)}^{2e^x - 2} \sqrt{t^2 + 2t + 4}\, dt \quad \text{en } x = 0$$

$$f(x) = \left((2e^x - 2)^2 + 2(2e^x - 2) + 4\right)^{1/2} - \left(\sin^2(x) + 2\sin x + 4\right)^{1/2}$$

$$\cancel{f'(x) = \frac{1}{2}\left((2e^x - 2)^2 + 4e^x\right)^{-1/2} \cdot \left[2(2e^x - 2) + 4e^x\right] \cdot (2e^x) -}$$

$$\int_a^{x^2} \underbrace{\alpha^4 - \alpha^2 + 16}\, dx = \left(x^8 - x^4 + 16\right) \cdot \left(8x^7 - 4x^3\right)$$

$$\int_{x^2}^{a} \text{algo} = -\int_{a}^{x^2} \text{algo}$$

# Capítulo 24

## Simulacro bota parcial

# Simulacro de Parcial #2,
# Cálculo Integral

Lunes, 7 de octubre

Nombre y Apellidos: ______________________________

| Tema: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Puntos: | 16 | 15 | 15 | 15 | 15 | 24 | 100 |
| Nota: | | | | | | | |

1. Determine si la integral dada es convergente o divergente. Evalúe las que sean convergentes.

   (a) (8 pts.) $\displaystyle\int_0^{\infty} \frac{x^2}{\sqrt{(1+x^3)^3}}\,dx$ (b) (8 pts.) $\displaystyle\int_0^1 \frac{12x^2+4x}{2x^3+x^2}\,dx$

2. Considere la región acotada por las gráficas $x = y^2$ y $x = 4y - y^2$ .
   (a) (5 pts.) Dibuja la región entre las curvas dadas.
   (b) (5 pts.) Plantea la integral para encontrar el área de la región.
   (c) (5 pts.) Encuentra el área de la región.

3. Un sólido se obtiene al girar la región entre $y = e^{-x}$, $y = 1$ y $x = 2$ alrededor de $y = 2$ .
   (a) (5 pts.) Dibuja la región entre la curvas y el eje de rotación.
   (b) (10 pts.) Plantea la integral para encontrar el volumen del sólido.

4. Considere la función $f(x) = \pi + \pi\cos(\pi t)$ en el intervalo $[0, 1]$ .
   (a) (5 pts.) Calcula el valor promedio $f_{prom}$.
   (b) (5 pts.) Encuentra $c$ tal que $f(c) = f_{prom}$.
   (c) (5 pts.) Grafica $f$ y el rectángulo cuya área es la misma que el área bajo la gráfica de $f$.

5. (15 pts.) Encuentra la longitud exacta de la curva $y = \frac{1}{3}(x^2+2)^{3/2}$ en $0 \leqslant x \leqslant 3$ .

6. Un foco LED Luminance de 60W tiene una vida media de 20,000 horas.
   Su tiempo de vida se modela por medio de la función de densidad exponencial.

$$f(x) = \frac{1}{20,000} e^{-x/20,000}, \qquad x \geqslant 0$$

   (a) (8 pts.) ¿Cuál es la probabilidad de que un foco dure menos de 6,000 horas?
   (b) (8 pts.) ¿Cuál es la probabilidad de que un foco dure más de 12,000 horas?
   (c) (8 pts.) ¿Cuál es la mediana de esta distribución? Para su información $\ln 0.5 \approx -0.7$ .

# Parte V

# Parciales

# Capítulo 25

## Parcial I

~~/100~~
74

# Universidad Francisco Marroquín

## Facultad de Ciencias Económicas

## Cálculo Integral

## Examen Parcial 1

## 2do Semestre 2019

Nombre: David Corzo Carnet: 20190432

**Instrucciones:**

- Para tener calificación toda respuesta requiere de procedimiento correcto.
- La duración del examen es de 90 minutos.
- No es permitido utilizar diccionarios, notas, calculadora ni cualquier otro tipo de ayuda.
- Escriba la respuesta final de cada inciso con lapicero o utilice un resaltador.
- Sanciones Académicas. Reglamento General, inciso XV.2 ufm.edu/reglamento-general

| Tema: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Puntos: | 20 | 12 | 18 | 19 | 14 | 17 | 100 |
| Nota: | | | | | | | |

1. Considere la función $v(t) = |t| - 8$ km/h en el intervalo ~~$-12 \leqslant t \leqslant 12$~~ $-10 \leq t \leq 10$.

   (a) (5 **pts.**) Evalúe $\displaystyle\int_{-10}^{10} v(t)\, dt$.

   (b) (8 **pts.**) Bosqueje las regiones entre $v(t)$ y el eje-$t$ en el intervalo dado. Encuentre el área de las regiones.

   (c) (3 **pts.**) Encuentre el desplazamiento de la partícula en $-10 \leqslant t \leqslant 10$ .

   (d) (4 **pts.**) Encuentre la distancia de la partícula en $-10 \leqslant t \leqslant 10$ .

ⓐ
$$\int_{-10}^{10} [\,|t| - 8\,]\, dt = \left.\frac{t^2}{2} - 8t\right]_{-10}^{10} =$$

$$= \left\{\frac{10^2}{2} - 8(10)\right\} - \left\{\frac{(-10)^2}{2} - 8(-10)\right\}$$

$$= \left\{\frac{100}{2} - 80\right\} - \left\{\frac{100}{2} + 80\right\}$$

$$= \{50 - 80\} - \{50 + 80\}$$

$$= \{-30\} - \{130\}$$

$$= -30 - 130$$

$$= \underline{-160}$$

0 pts

b) el resto en hoja

b, c, d en hoja

6 pts

2. Considere la función $g(x) = \int_0^{\pi x} \sin x \, dx + \int_0^{2\ln x} e^{x^4 - x^2} \, dx$ .

(a) (4 **pts.**) Encuentre $g(1)$.

(b) (5 **pts.**) Encuentre $g'(1)$. Para su información $\sin 1 \approx 0.84$.

(c) (3 **pts.**) Encuentre la ecuación de la recta tangente a $g$ en $x = 1$.

$$g(1) = \int_0^{\pi} \sin(t)\, dt + \cancel{\int_0^{0} e^{t^4 - t^2}\, dt} \quad \emptyset$$

$$= -\cos(t)\Big]_0^{\pi} = \{-\cos(\pi)\} - \{-\cos(0)\}$$

$$= 1 + 1$$

(a) $= \underline{2}$ ✓ 4 pts

$$g'(x) = \sin(\pi x)\cdot\pi + e^{(2\ln x)^4 - (2\ln x)^2} \cdot \frac{2}{x}$$

$$g'(1) = \sin(\pi)\cdot\pi + \cancel{e^{2\ln(1)^4 - (2\ln 1)^2}}$$

$$g'(1) = \emptyset + 1$$

✗ 3pts

(b) $g'(1) = \underline{1}$

$$y = f(a) + f'(a)(x - a)$$

(c) $y = 2 + 1(x - 1)$ ✗ 2 pts

9 pts

3. **Resonancia:** Un resorte sujeto a una fuerza oscilatoria tiene la función de aceleración:

$$a(t) = 10t\cos\left(\frac{t}{2}\right) \qquad cm/min^2$$

(a) (8 **pts.**) Encuentre la función de velocidad si el resorte se encuentra en reposo.

(b) (10 **pts.**) Encuentre la función de posición si la posición inicial es de 5 cm.

en hoja las dos

13 pts.

4. Evalúe las siguientes integrales indefinidas.

~~(a)~~ (14 **pts.**) $\displaystyle\int \frac{x^2}{\sqrt{x^6-4}}\,dx$

~~(b)~~ (5 **pts.**) $\displaystyle\int \frac{\cos x - \sin x + \sec^2 x}{\sin x + \cos x + \tan x}\,dx$

en hoja las dos.



18 pts +2

5. Evalúe las siguientes integrales indefinidas.

~~(a)~~ (10 **pts.**) $\displaystyle\int_0^{\pi/2} \sin^9 x \cos^3 x \, dx$ ✓

~~(b)~~ (4 **pts.**) $\displaystyle\int_{-495}^{495} \left( \frac{\sin^9}{x^4 + \cos x} + \frac{\tan x}{x^2 + 1} \right)^{111} dx$ ✓

en hoja las dos

14pts +1

6. (17 **pts.**) Evalúe $\displaystyle\int_1^e \frac{24\ln^2 x}{(\ln^6 x + 1)^2}\,\frac{1}{x}\,dx$

en hoja

6pts

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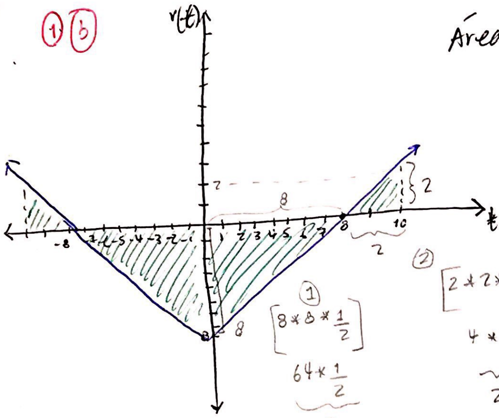
1 b




Área = 32

① $\left[8 * 8 * \frac{1}{2}\right]$

$\underbrace{64 * \frac{1}{2}}_{32}$

② $\left[2 * 2 * \frac{1}{2}\right]$ ✗ 4 pts

$\underbrace{4 * \frac{1}{2}}_{2}$

1 c

$$\int_{-10}^{10} |t|-8 \, dt = -160$$ ✗ 1 pts

1 d

dist = $\left|\left[\int_{-10}^{-8} |t|-8 \, dt\right]\right| + \left|\left[\int_{-8}^{0} |t|-8 \, dt\right]\right| + \left|\left[\int_{0}^{8} |t|-8 \, dt\right]\right| + \left|\int_{8}^{10} |t|-8 \, dt\right|$

distancia = 32 ✗ 0 pts

2

$$g(x) = \int_{0}^{\pi x} \sin(t)\, dt + \int_{0}^{2/...(x)} e^{t^4} - t^2 \, d$$

g(x) = -cos(πx)

(a)

(b)

③ $a(t) = 10t\cos\left(\frac{t}{2}\right)$

$v(t) = \int 10t\cos\left(\frac{t}{2}\right)dt$

LIPET

$w = \frac{t}{2}$

$du = \frac{1}{2}dt = 2du$

$= 2\int \cos(u)\,du$

$= 2\sin\left(\frac{t}{2}\right)$

IPP: $u = 10t$ $\quad v = \cos\left(\frac{t}{2}\right)dt$

$du = 10\,dt$ $\quad dv = 2\sin\left(\frac{t}{2}\right)$

ⓐ $v(t) = 10t\cos(t/2) - 10\int \cos(t/2)\,dt$

$v(t) = 10t\cos\left(\frac{t}{2}\right) - 10\left(2\sin(t/2)\right) + C$

X 6 pts

$v(0) = 10(0)\cos\left(\frac{0}{2}\right) - 10\left(2\sin(0/2)\right) + C$

$\int \sin(u)\,dt = -\cos(u)$

$u = t/2$

$2du = dt$

$2\boxed{-\cos(t/2)}$

ⓑ $f(t) = \int v(t)\,dt$

$= \int 10t\cos(t/2) - 10\left(2\sin(t/2)\right)dt$

$= 10t\cos(t/2) - 10\cdot 2\sin(t/2) + C_0 - 10\cdot 2\cdot 2\cdot -\cos(t/2) + C_1$

$f(t) = 10t\cos(t/2) - 20\sin(t/2) + 40\cos(t/2) + C_0 + C_1$

X 7 pts

$f(5) = 50\cos(5/2) - 20\sin(5/2) + 40\cos(5/2) + C_0 + C_1$

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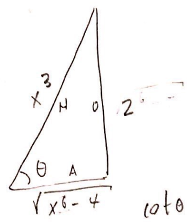
4. (a) $\int \frac{x^2}{\sqrt{x^6-4}}\,dx = \int \underline{\qquad}$




$c^2 = a^2 + b^2$
$-a^2 = -c^2 + b^2$
$a^2 = c^2 - b^2$

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$\csc = \frac{H}{O}$ $\sec = \frac{H}{A}$ $\cot = \frac{A}{O}$

$\cot\theta = \frac{\sqrt{x^6-4}}{2}$

$\cot\theta = \frac{\sqrt{x^6-4}}{2}$

$\boxed{2\cot\theta = \sqrt{x^6-4}}$

$\csc\theta = \frac{x^3}{2}$

$\theta = \csc^{-1}(x^2)$

$2\csc\theta = x^3$

$-\csc\theta\cot\theta\,d\theta = \frac{3x^2}{2}\,dx$

$-\frac{2}{3}\csc\theta\cot\theta\,d\theta = x^2\,dx$

$\int \csc\theta\,d\theta = -\ln|\csc\theta + \cot|$

$= \int \frac{\left[-\frac{2}{3}\csc\theta\cot\theta\right]d\theta}{2\cot\theta}$

$= \int \frac{-\frac{2\csc\theta\cot\theta}{3}}{\frac{2\cot\theta}{1}}\,d\theta = \int -\frac{2\csc\theta\,\cancel{\cot\theta}}{6\,\cancel{\cot\theta}}\,d\theta = -\frac{2}{6}\int \csc\theta\,d\theta = +\frac{2}{6}\ln|\csc\theta + \cot| + C$

$= +\frac{2}{6}\ln\left|\frac{x^3}{2} + \frac{\sqrt{x^6 + 4}}{2}\right| + C$ □

13 pts +1

(b) $\int \frac{\cos x - \sin x + \sec^2 x}{\sin x + \cos x + \tan x}\,dx = \int \frac{du}{u} = \ln|\sin x + \cos x + \tan x| + C$ □

5 pts +1

$u = \sin x + \cos x + \tan x$
$du = \cos x - \sin x + \sec^2 x\,dx$

3 a

$$\int_0^{\pi/2} \sin^9 x \cos^3 x \, dx =$$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

$$\int \sin^9 x \cos^2(x) \cos(x)\, dx$$
$$\sin^9 x \,(1 - \sin^2 x) \cos(x)\, dx$$
$$(\sin^9 x - \sin^{11} x) \cos(x)\, dx$$

$u = \sin x$ $\quad u(\pi/2) = \sin(\pi/2) = 1$

$du = \cos x \, dx$ $\quad u(0) = 0$

$$= \int_0^1 u^9 - u^{11}\, du = \frac{u^{10}}{10} - \frac{u^{12}}{12} = \left.\frac{\sin^{10} x}{10} - \frac{\sin^{12} x}{12}\right]_0^{\pi/2}$$

$$= \left\{\frac{\sin^{10}(\pi/2)}{10} - \frac{\sin^{12}(\pi/2)}{12}\right\} - \left\{\cancel{\frac{\sin^{10}(0)}{10}} - \cancel{\frac{\sin^{12}(0)}{12}}\right\}$$

$$= \left\{\frac{1}{10} - \frac{1}{12}\right\} - \{0\} = \left\{\frac{12-10}{10 \cdot 12}\right\} = \frac{12-10}{1200} \rightarrow \frac{2}{1200}$$

10 pts

b

$$\int_{-495}^{495} \left(\frac{\sin^a x}{x^4 + \cos x} + \frac{\tan x}{x^2+1}\right)^{111} = 0$$

por ser impar y tener un limite negativo y el mismo positivo

4 pts + 1

⑥ $\displaystyle\int_1^e \frac{24\ln^2 x}{(\ln^6 x+1)^2}\,\frac{1}{x}\,dx$

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$u = \ln^3 x$

$du = 3(\ln x)^2 \cdot \frac{1}{x}\,dx$

$\boxed{8\,du = 24\ln^2 x \cdot \frac{1}{x}}$

$(\ln^3(x))^3 = 2(\ln x)\cdot\frac{1}{x}$

$(\ln x)^3 = \frac{3(\ln x)^2}{x}$

$= \displaystyle\int_1^e \frac{8\,du}{(u^2+1)^2} = 8\int_1^e \frac{du}{(u^2+1)^2}$

$u = (u^2+1)^{-2}$ $\quad dv = du$

$du = -2(u^2+1)^{-1}\cdot 2u\,du$ $\quad v = u$

$= \displaystyle\frac{u}{(u^2+1)^2} - \left[-4\int_1^e \frac{u}{u^2+1}\,du\right]$ $\quad \frac{1}{2}\int \frac{1}{u}\,du$

$v = u^2+1$

$dv = 2u$ $\quad -\left\{-4\left[\frac{1}{2}\ln|u|\right]\right\}$

$\cancel{= \frac{u}{(u^2+1)^2}}$ $\quad \frac{dv}{2} = u$ $\quad +\frac{4}{2}\ln|u|$

$= \displaystyle\frac{u}{(u^2+1)^2} + 2\ln\left|\frac{1}{(u^2+1)^2}\right|$

$= \displaystyle\left[\frac{\ln^3 x}{(\ln^3 x+1)^2} + 2\ln\left|\frac{1}{(\ln^6 x+1)^2}\right| + C\right]_1^e$

$= \left\{\frac{1}{4} + 2\ln\left|\frac{1}{4}\right|\right\} - \left\{\frac{0}{1} + 2\ln\left|\cancel{\frac{1}{1}}\right|\right\}$ 0

$= \frac{1}{4} + 2\ln|1/4|$

8pts

# Parte VI

# Actividades especiales

# Capítulo 26

## Actividad de pts. Extra #1

Actividad especial:

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$$A = 2\int_0^1 x^{1/4}\,dx + 2\int_1^4 \left(\frac{4-x}{3}\right)^{1/2} dx = \frac{28}{5}$$

$$A = 2\left\{\left\{\frac{x^{5/4}}{5/4}\right\}\Big]_0^1\right\} + 2\int_1^4\left(\frac{4-x}{3}\right)^{1/2} dx$$

$$2\left(\left\{\frac{4}{5}(1)^{5/4}\right\} - \left\{\cancel{\frac{0^{5/4}}{5/4}}\right\}\right) =$$

$$= 2\left(\left\{\frac{4}{5} - 0\right\}\right) = 2\left(\frac{4}{5}\right)$$

$$= \frac{8}{5}$$

$u(4) = 0$

$u = \frac{4-x}{3} = \frac{4}{3} - \frac{x}{3}$ $\quad u(1) = 1$

$du = 0 - \frac{1}{3}dx \qquad du = -\frac{1}{3}dx$

$-3\,du = dx \qquad -3\,du = dx$

$$= 2\int_1^0 \sqrt{u}\cdot -3\,du =$$

$$= -6\int_1^0 \sqrt{u}\,du$$

$$= 6\int_0^1 \sqrt{u}\,du = 6\left(\frac{2u^{3/2}}{3}\right)\Big]_0^1 =$$

$$= 6\left(\left\{\frac{2}{3} - 0\right\}\right) = 6\cdot\frac{2}{3} = \frac{12}{3} = 4$$

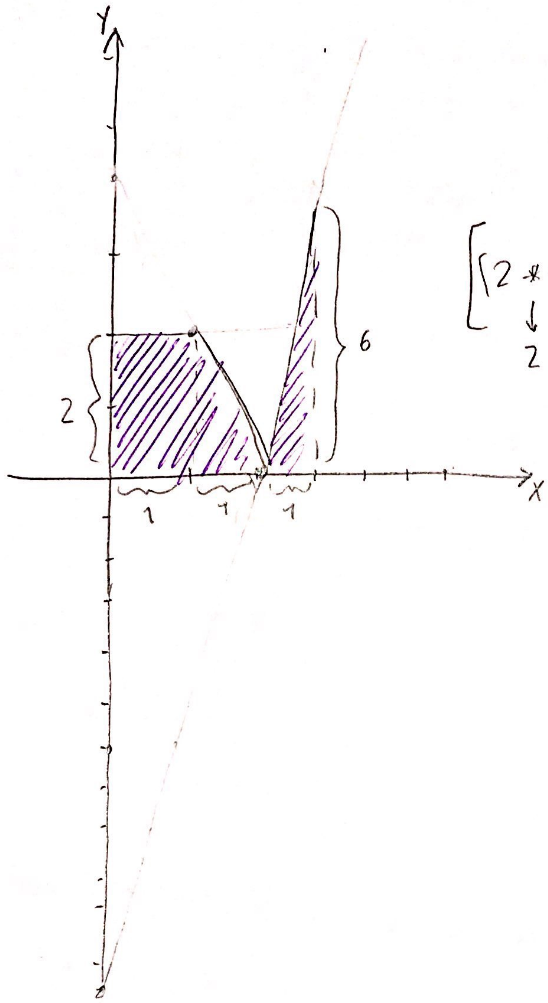
$$= \frac{8}{5} + 4 = \frac{28}{5}$$

$\square$

# Capítulo 27

## Actividad de pts. Extra #2

David Gabriel Corzo Mcmath




$$\left[(2 * 1)\right] + \left[(2 * 1)\tfrac{1}{2}\right] + \left[(6 * 1)\tfrac{1}{2}\right]$$

$$2 + 1 + 3 = \underline{6}$$

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# Capítulo 28

## Actividad de pts. Extra #3, problemas varios de integración

## Problemas Variados de Integración

**Problema 3:** Encuentre las siguientes integrales.

a.) $\displaystyle\int \sqrt{64x} \;-\; \frac{1}{\sqrt{64x}}\,dx$

b.) $\displaystyle\int \left(\frac{7x^2}{7x^3+8} \;-\; \frac{x^3}{(x^4+8)^5}\right) dx$

c.) $\displaystyle\int \sqrt[3]{x}\; e^{\sqrt[3]{8x^4}}\,dx$

d.) $\displaystyle\int 5\frac{(x^{1/3}+2)^4}{x^{2/3}}\,dx$

e.) $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

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a) $\int \sqrt{64x} - \frac{1}{\sqrt{64x}}\, dx$

+10

$\int \sqrt{64x}\, dx - \int \frac{1}{\sqrt{64x}}\, dx$

$u = 64x$ $\quad$ $u = 64x$

$du = 64\, dx$ $\quad$ $du = 64$

$\frac{du}{64} = dx$ $\quad$ $\frac{du}{64} = dx$

$\int \sqrt{64x}\, dx = \int \sqrt{u}\, \frac{du}{64} = 64 \int \frac{(u)^{1/2+1}}{1/2+1}\, du$

$64 \cdot \frac{u^{3/2}}{3/2}$

$\frac{\frac{64u^{3/2}}{1}}{\frac{3}{2}} = \frac{64 \cdot 2u^{3/2}}{3}$

$\int \frac{1}{\sqrt{64x}}\, dx = \frac{1}{64} \int \frac{1}{\sqrt{u}}\, du$

$u = 64x$

$du = 64\, dx$

$\frac{du}{64} = dx$

$\frac{1}{64}\sqrt{u} - \frac{64 \cdot 2u^{3/2}}{3}$

$\frac{\sqrt{64x}}{64} - \frac{64 \cdot 2(64x)^{3/2}}{3} + C$

b) $\int \left( \frac{7x^2}{7x^3+8} - \frac{x^3}{(x^4+8)^5} \right) dx$

$u = 7x^3 + 8$ $\quad$ $u = x^4 + 5$

$du = 7x^2$ $\quad$ $du = x^3$

$\int \frac{du}{u} - \int \frac{du}{u^5}$

$(\ln(u)) - \left(\frac{u^{-4}}{-4}\right) = \ln(u) + \frac{1}{4u^4} + C$

$= \ln(7x^3+8) + \frac{1}{4(x^4+8)^4} + C$

c. $\int \sqrt[3]{x}\, e^{\sqrt[3]{8x^4}}\, dx = \int \sqrt[3]{x}\, e^{2(x)^{4/3}}\, dx$

$u = 2(x)^{4/3}$

$du = 2x^{1/3}\, dx$

$\frac{du}{2} = x^{1/3}\, dx$

$= \frac{1}{2}\int e^{u}\, du$

$= \frac{1}{2} e^{u} + C$

$= \frac{1}{2} e^{2\sqrt[3]{x^4}} + C$

d. $\int 5\, \frac{(x^{1/3}+2)^4}{x^{2/3}}\, dx = 5 \int (x^{1/3}+2)^4\, x^{-2/3}\, dx$

$u = x^{1/3} + 2$

$du = x^{-2/3}\, dx$

$= 5 \int u^4\, du$

$= \frac{\cancel{5}\, u^5}{\cancel{5}} = (x^{1/3}+2)^5 + C$

e) $\int \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\, dx = \int \frac{1}{u}\, du = \ln(u)$

$= \ln(e^{x} + e^{-x}) + C$

$u = e^{x} + e^{-x}$

$du = e^{x} - e^{-x}\, dx$

# Parte VII

# Documentos de Apoyo

# Capítulo 29

## Identidades trigonométricas

2019-08/13

# Identidades Trigonométricas

$$\operatorname{sen}^2 x + \cos^2 x = 1 \qquad \operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos 2x = \cos^2 x - \operatorname{sen}^2 x$$

$$\cot^2 x + 1 = \csc^2 x \qquad \operatorname{sen}^2 x = \tfrac{1}{2}\Big(1 - \cos 2x\Big)$$

$$\sec^2 x - 1 = \tan^2 x \qquad \cos^2 x = \tfrac{1}{2}\Big(1 + \cos 2x\Big)$$

# Técnicas de Integración

- 5.5 Regla de la Sustitución

$$\int f(g(x)\ g'(x)\ dx = \int f(u)\ du$$

- 7.1 Integración por Partes

$$\int u\ dv = uv - \int vdu$$

- 7.2 Integración Trigonométrica

a. **Potencias Impares de Seno o Coseno:** Aparte un término $\operatorname{sen} x$ o $\cos x$ y utilice la identidad $\operatorname{sen}^2 x + \cos^2 x = 1$ .

b. **Potencias Pares de Seno o Coseno:** Utilice la identidad

$$\operatorname{sen}^2 x = \tfrac{1}{2}\Big(1 - \cos 2x\Big) \qquad y/o \qquad \cos^2 x = \tfrac{1}{2}\Big(1 + \cos 2x\Big).$$

c. **Potencia Par de tangente:** Aparte $\sec^2 x$ y use $\sec^2 x = \tan^2 x + 1$.

d. **Potencia Impar de tangente:** Aparte $\sec x \tan x$ y use $\tan^2 x = \sec^2 x - 1$.

e. **Potencia Par de cosecante:** Aparte $\csc^2 x$ y use $\csc^2 x = \cot^2 x + 1$.

f. **Potencia Impar de cotangente:** Aparte $\cot x \tan x$ y use $\cot^2 x = \csc^2 x - 1$ .

g. **Productos** $\operatorname{sen}(mx)$ **y** $\cos(nx)$**:** Utilice la identidad trigonométrica adecuada.

$$\begin{aligned} \operatorname{sen} A \cos B &= \tfrac{1}{2}\big[\ \operatorname{sen}(A-B) + \operatorname{sen}(A+B)\ \big] \\ \operatorname{sen} A \operatorname{sen} B &= \tfrac{1}{2}\big[\ \cos(A-B) - \cos(A+B)\ \big] \\ \cos A \cos B &= \tfrac{1}{2}\big[\ \cos(A-B) + \cos(A+B)\ \big] \end{aligned}$$

- 7.3 Sustitución Trigonométrica

a. $x = a \operatorname{sen} \theta$ sustituye $a^2 - u^2$ por $a^2 \cos^2 \theta$ y $dx = a \cos \theta\ d\theta$ .

b. $x = a \tan \theta$ sustituye $a^2 + u^2$ por $a^2 \sec^2 \theta$ y $dx = a \sec^2 \theta\ d\theta$ .

c. $x = a \sec \theta$ sustituye $u^2 - a^2$ por $a^2 \tan^2 \theta$ y $dx = a \sec \theta \tan \theta\ d\theta$ .

d. Hay otros casos que requieren el trazo de un triángulo apropiado.

# Capítulo 30

## Integrales indefindas básicas

## Integrales Indefinidas Básicas

$$\int af(x)\,dx = a\int f(x)\,dx$$

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \operatorname{sen} x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \operatorname{sen} x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\operatorname{sen} x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec^3 x\,dx = \frac{\ln|\sec + \tan x| + \sec x \tan x}{2} + C$$

$$\int \csc^3 x\,dx = -\frac{\ln|\csc + \cot x| + \csc x \cot x}{2} + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \operatorname{senh} x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \operatorname{senh} x + C$$

$$\int \operatorname{sech}^2 x\,dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\,dx = -\coth x + C$$

$$\int \operatorname{sech} x\ \tanh x\,dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x\ \coth x\,dx = -\operatorname{csch} x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{sen}^{-1}(x) + C$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{a}\operatorname{sen}^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1}(x) + C$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C$$

# Capítulo 31

## Teorema fundamental del cálculo

## Teorema Fundamental del Cálculo Generalizado

Evalúe la siguiente expresión, integrando y luego derivando:

$$\frac{d}{dx}\left(\int_{x^3}^{x^5} e^y\,dy\right) = \frac{d}{dx}\left(e^{x^5}-e^{x^3}\right) = \underbrace{5x^4}_{b'(x)}\underbrace{e^{x^5}}_{e^{b(x)}} - \underbrace{3x^2}_{a'(x)}\underbrace{e^{x^3}}_{e^{a(x)}}$$

En este problema ambos límites de integración dependen de $x$ y en la respuesta final se utilizaron dos reglas de la cadena por separado.

El último ejemplo nos indica como el uso del TFC y la regla de la cadena se puede extender para funciones donde los dos límites de integración dependen de $x$.

> Use regla de la cadena para el límite superior $b(x)$ y para el límite inferior $a(x)$
>
> $$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} F(t)\,dt\right) = F(\,b(x)\,)\,b'(x) - F(\,a(x)\,)\,a'(x)$$

*Ejercicio 3: Evalúe las siguientes expresiones*

a. $\displaystyle\frac{d}{dx}\left(\int_{\sin x}^{e^x} \sqrt[4]{10+4t^4}\,dt\right) = \sqrt[4]{10+4e^{4x}}\,e^x - \sqrt[4]{10+4\sin^4(x)}\,\cos x$

b. $\displaystyle\frac{d}{dx}\left(\int_{1/x}^{\ln x} \cosh\theta^3\,d\theta\right) = \cosh(\ln^3 x)\frac{1}{x} + \cosh(x^{-3})\frac{1}{x^2}$

*Ejercicio 4: Encuentre la ecuación de la recta tangente a $y=f(x)$ en $x=0$.*

a. $\displaystyle f(x) = \int_0^x e^{-t^2/2}\,dt.$

| | |
|---|---|
| Coordenada-y: | $f(0) = \int_0^0 e^{-t^2/2}\,dt = 0$ |
| Derivada: | $f'(x) = e^{-x^2/2}$ |
| Pendiente: | $f'(0) = e^{-0/2} = 1$ |
| Recta Tangente: | $y = f(0) + f'(0)(x-0) = x$ |

b. $\displaystyle f(x) = \int_0^x \cosh^2 t\,dt.$

| | |
|---|---|
| Coordenada-y: | $f(0) = \int_0^0 \cosh^2 t\,dt = 0$ |
| Derivada: | $f'(x) = \cosh^2 x$ |
| Pendiente: | $f'(0) = (\cosh 0)^2 = 1$ |
| Recta Tangente: | $y = f(0) + f'(0)(x-0) = x$ |

# Parte VIII

# Notas de prueba de DM para entrada a la UFM, pts.extra en Cálculo Integral

① $16 - 2 * 4 + 3 * 6 \div 3^2 + 10^2 \div 5^2 - 4 * 3 + 8$

$16 - 2 * 4 + 3 * 6 \div 9 + 100 \div 25 - 4 * 3 + 8$

$16 - 8 + 18 \div 9 + 100 \div 25 - 12 + 8$

$16 - 8 + 9 + 4 - 12 + 8$

$16 - 8 + 13 - 12 + 8$

$8 + 1 + 8$

$16 + 1$

$17$

Parenthesis
Exponents
Multi
Division
Addition
Ssubtraction

② $-2^3 + 6^2 * 3^2 + 8 * 6 + 34 \div 2$

$-8 + 36 * 9 + 8 * 6 + 34 \div 2$

$-8 + 324 + 48 + 34 \div 2$

$-8 + 324 + 48 + 17$

$+316 + 48 + 17$

$316 + 65$

$381$

$36 \times 9 = 324$

$36$

$9 + 9 + 9$

$27 + 27 = 54$

$316 + 65 = 381$

$48 + 17 = 65$

③ $[24 - (3-5)^3] \div [(8-3) \div 5 + 7]$

$[24 - (-2)^3] \div [5 \div 5 + 7]$

$[24 - (-8)] \div [5 \div 5 + 7]$

$[24 + 8] \div [1 + 7]$

$[32] \div [8] = \frac{32}{8} = 4$

$8 + 8 = 16 + 8 = 24 + 8 = 32$

Paren
Expon
Multi
Divis
Add
Sub

④ $(8 + 6\sqrt{25} - 4 * 2^2) - (3 * 6 \div \sqrt{36})$

$(8 + 6\sqrt{25} - 4 * 4) - (3 * 6 \div \sqrt{36}$

$(8 + 6 * 5 - 4 * 4) - (3 * 6 \div \sqrt{36})$

$(8 + 30 - 16) - (18 \div 6)$

$(38 - 16) - (3)$ $38 - 10 = 28 - 6 = 22$

$22 - 3$

$\underline{19}$

⑤ $\left[\dfrac{\frac{1}{3}}{\frac{1}{2}}\right] + \left[\dfrac{\frac{2}{5}}{\frac{1}{4}}\right] - \left[\dfrac{\frac{3}{8}}{\frac{1}{4}}\right] = \dfrac{\frac{1}{3}\cdot\frac{1}{4} + \frac{2}{5}\cdot\frac{1}{2} - \frac{3}{8}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{4}}$

$= \dfrac{\frac{1}{12} + \frac{2}{10} - \frac{3}{16}}{\frac{1}{8}} = \dfrac{\quad}{\frac{1}{8}}$

$\dfrac{\dfrac{10\cdot16 + 2\cdot12\cdot16 - 3\cdot12\cdot10}{12\cdot10\cdot16}}{\frac{1}{8}} = \dfrac{160 + 192 - 360}{1/8} = \left[\dfrac{-8}{\frac{1}{8}}\right] = -64$

$$\begin{array}{r} 10 \\ 16 \\ \hline 60 \\ 10\phantom{0} \\ \hline 160 \end{array} \qquad \begin{array}{r} 12 \\ 16 \\ \hline 72 \\ 12\phantom{0} \\ \hline 192 \end{array} \qquad \begin{array}{r} 120 \\ 3 \\ \hline 360 \end{array} \qquad \begin{array}{r} 160 \\ 192 \\ \hline 352 \end{array}$$

⑥ $\left(21 - \frac{3}{4}\right) \div \left(4 \cdot \frac{2}{3} - 2\right)$

$\frac{4 \cdot 21}{4} - \frac{3}{4}$ $\qquad$ $\frac{8}{3} - \frac{2 \cdot 3}{3}$

$\frac{4 \cdot 21 - 3}{4} = \frac{81}{4}$ $\qquad$ $\frac{8 - 2 \cdot 3}{3} = \frac{8-6}{3} = \frac{2}{3}$

$\left(\frac{81}{4}\right) \div \frac{2}{3} = \frac{\frac{81}{4}}{\frac{2}{3}} = \frac{81 \cdot 3}{8}$

$\begin{array}{r} 21 \\ \times\ 4 \\ \hline 84 \end{array} - 3 = 81$

$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 243 \end{array}$ $\qquad = \frac{243}{8}$

⑧ $(2x^2 + 50x - 5) - (8x^2 - 5x + 6x^3)$

$\cancel{2x^2} + \cancel{50}x - 5 - \cancel{8x^2} + 5x - \cancel{6x^3}$

$-6x^3 - 6x^2 + 55x - 5 =$

⑦ $\cancel{8}x^3 - \cancel{3x^4} + \cancel{5}x + 2 + \cancel{2x} + \cancel{2x^4} - 3\cancel{x^3} + 3$

$= 8x^3 - 3x^3 - 3x^4 + 2x^4 + 5x + 2x + 2 + 3$

$= 5x^3 - 1x^4 + 7x + 5$

$= -x^4 + 5x^3 + 7x + 5$

⑧ $\cancel{2x^2} + \cancel{50}x - 5 \cancel{-3x^2} + \cancel{8x} - 6x^3$

$-6x^2 + 55x - 5 - 6x^3$

$-6x^3 - 6x^2 + 55x - 5$

⑨ $(7a^2 + 8ab - 2b^2) - [(6a^2 - 4ab + 6) + (3a^2 + 8ab - b^2)]$

$\cancel{7a^2} + \cancel{8ab} - \cancel{2b^2} - \cancel{6a^2} + \cancel{4ab} - 6 - \cancel{3a^2} - \cancel{8ab} + \cancel{b^2}$

$(7a^2 - 6a^2 - 3a^2) + (\cancel{8ab} + 4ab - \cancel{8ab}) + (-2b^2 + b^2)$

$a^2 - 3a^2 \quad + -1b^2$

$(-2a^2) + (4ab) - b^2 - 6$

⑩ $(8x - 7x^3 + 3) - (3x^2 - 8x + 2) + (-2x^3 - 4x^2 + 2x)$

$\cancel{8x} - \cancel{7x^3} + 3 \cancel{-3x^2} + \cancel{8x} - 2 \cancel{-2x^3} \cancel{-4x^2} \cancel{+2x}$

$18x - 9x^3 - 9x^2 + 3 - 2$

$-9x^3 - 9x^2 + 18x + 1$

$\cancel{8x} - \cancel{7x^3} + 3 - \cancel{3x^2} + \cancel{8x} - 2 - \cancel{2x^3} - \cancel{4x^2} + \cancel{2x}$

$-7x^3 - 2x^3 - 3x^2 - 4x^2 + 8x + 8x + 2x + 3 - 2$

$-9x^3 - 7x^2 + 18x + 1$

㉕ $\dfrac{(x-6)\cancel{(x+6)}}{\cancel{(x+6)}(x+3)} = \dfrac{x-6}{x+3}$

㉔ $\dfrac{x^2 - 8x + 15}{3 - x} =$

$= \dfrac{(x-5)(x-3)}{(3-x)} \quad -x+3$

(23) $$\frac{3x+1}{5} + \frac{2-5x}{6} = \frac{6(3x+1)+5(2-5x)}{5\cdot 6} =$$

$$\frac{18x+6+10-25x}{30} = \frac{-7x+16}{30}$$

(22) $$\frac{2x}{x^2-4} + \frac{1}{x-2} + \frac{2}{x+2} = \frac{\cancel{2x}+\cancel{x}+2+\cancel{2x}-4}{(x+2)(x-2)}$$

$$\frac{4x+x-2}{(x+2)(x-2)} = \frac{5x-2}{(x+2)(x-2}$$

(21) $$\frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)} = \frac{(x+1)}{(x-1)} \div \frac{x^3-1}{x-1}$$

$$= \left[\frac{\frac{x+1}{x-1}}{\frac{x^3-1}{x-1}}\right] = \frac{(x-1)\cancel{(x-1)}}{\cancel{(x-1)}(x^3-1)}$$

$$\frac{x+1}{x^3-1}$$

x

(11) $16x^3 - 6x^2 + 12x - 72x^2 + 27x - 54$

$16x^3 - 78x^2 + 39x - 54$

(12) $(16x - y)(4x + y)$

$64x^2 - 4xy + 16xy - y^2$

$64x^2 + 12xy - y^2$

(13) $(y^3 - 3)^2$

$y^6 - 2y^3 \cdot 3 + 9$

$y^6 - 6y^3 + 9$

(14) $(4x^2 + 6y^4)^3$

$64x^6 + 3x$

(15) $\dfrac{27y^3z^3 - 15y^6z^2}{3y^2z}$ $\dfrac{\cancel{3y^2z}(9y^2z^2 - 5y^4z)}{\cancel{3y^2z}}$

(17) $-8y^4 + 12y^3 - 16y^2 + 4y$

$-4y(2y^2 - 3y^2 + 4y - 1)$

(18) $a^x(a^2 - 3a + 10)$

# Parte IX

# Honores del curso

# Capítulo 32

## Agradecimientos por programa en Python

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