2019-11-01

10.2 Cálculo de ecuaciones paramétricas

1) Eq. of tangent curve:

$x = t^9 + 1 \quad ; \quad y = t^{10} + t \quad$ in $\quad t = -1$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10t^9 + 1}{9t^8} = \frac{10(-1)^9 + 1}{9(-1)^8} = \frac{-10+1}{9} = -1$$

$x(-1) = (-1)^9 + 1 = 0$

$y(-1) = (-1)^{10} - 1 = 0$

$\boxed{y = m(x - x_0) - y_0}$

$y = -x$

2) $x(t) = 8\sec t \quad ; \quad y(t) = 8\tan(t) \quad ; \quad t = \frac{\pi}{4}$

$x'(t) = 8\sec(t)\cdot\tan(t) \qquad y'(t) = 8\sec^2(t)$

$$\frac{dy}{dx} = \frac{\cancel{8\sec(t)}\cdot\sec(t)}{\cancel{8\sec(t)}\cdot\tan(t)} = \frac{\sec(t)}{\tan(t)} = \frac{\sec(\frac{\pi}{4})}{\tan(\frac{\pi}{4})} = \left[\frac{\frac{2}{\sqrt{2}}}{\frac{1}{1}}\right] = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$x(\frac{\pi}{4}) = \frac{16}{\sqrt{2}} = 2^4 \cdot 2^{-\frac{1}{2}} = 2^{\frac{7}{2}}$

$\frac{8}{2} - \frac{1}{2} = \frac{7}{2}$

$y(\frac{\pi}{4}) = 8 \cdot 1 = 8$

$y = \sqrt{2}(x - 2^{\frac{7}{2}}) + 8$

$y = \sqrt{2}x - \sqrt{2}\cdot 2^{\frac{7}{2}} + 8$

$y = \sqrt{2}x - 8$

3) $x = t^2 + 9$; $y = t^2 + 9t$

$$\frac{dy}{dx} = \frac{2t + 9}{2t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{2t+9}{2t}\right)}{(2t)^2} = \frac{\frac{(2t)(2) - (2t+9)(2)}{(2t)^2}}{\frac{(2t)}{1}}$$

$$= \frac{\cancel{4t} - \cancel{4t}^{0} - 18}{\frac{2^2 t^2}{(2t)}} = -\frac{18}{(2t)^3}$$

$$-\frac{18}{(2t)^3} = 0$$

$t = 0$

$t = 1$ $\quad$ $t = 2$

$-$ $\quad$ $+$

$t = -1$

4) $x = t\sin(t)$; $y = t^2 + 5t$

$$\frac{dy}{dx} = \frac{2t + 5}{\sin(t) + t\cos(t)}$$

5) $x = 3\cos(t)$ $\quad y = 2\sin(t)\cos(t)$

$$\frac{dy}{dx} = \frac{2\cdot\cos(2t)}{-3\sin(t)}\Bigg|_{\frac{\pi}{2},\, \frac{3\pi}{2}} \Bigg\} = 0$$

$$= \frac{2\cdot\cos(t)\cos(t) - 2\sin(t)\sin(t)}{-3\sin(t)}$$

$$= +\frac{2}{3}, \ -\frac{2}{3}$$

$$y = \frac{2}{3}(x - 0) + 0 = \frac{2}{3}x$$

$$y = -\frac{2}{3}(x - 0) + 0 = -\frac{2}{3}x$$

6) $x(t) = 7 + \ln(t)$ $\quad y(t) = t^2 + 3t$ $\quad (7, 4)$

$$\frac{dy}{dx} = \frac{2t + 3}{\frac{1}{t}} = t(2t + 3) = 2t^2 + 3t$$

7) $x = a\cos\theta$; $y = b\sin\theta$; $0 \leq \theta \leq 2\pi$

$$A = \int_{t_1}^{t_2} y\,dx$$

$$= \int_0^{2\pi} b\sin\theta \cdot a(-\sin\theta)\,d\theta = -ab\int_0^{2\pi} \sin^2\theta\,d\theta$$

$$= -ab\int_0^{2\pi}\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right)d\theta$$

$$= -ab\left[\frac{\theta}{2} - \frac{1}{4}\sin(2\theta)\right]_0^{2\pi} =$$

$$\underline{ab\pi}$$

8) $x = t^2 - 2t$ $\quad y = \sqrt{t}$

$$A = \int t^{\frac{1}{2}}(2t-2)\,dt = \int_2 2t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\,dt$$

$$= 2\left[\frac{2}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}}\right]_0$$

$0 = t^2 - 2t$
$0 = t(t-2)$
$t = 0$
$t = 2$

$$= 2\left[\left(\frac{2}{5}(2)^{\frac{5}{2}} - \frac{2}{3}(2)^{\frac{3}{2}}\right) - (0)\right]$$

$$= 2\left[\frac{2}{5}\sqrt{32} - \frac{2\sqrt{8}}{3}\right] = \frac{2^2}{5}\sqrt{32} - \frac{2\sqrt{8}}{3}$$

$$= \frac{2^4\sqrt{2}}{5} - \frac{2^2\sqrt{2}}{3}$$

$$= \sqrt{2}\left(\frac{2^4\cdot 3 - 2^2\cdot 5}{5\cdot 3}\right) = \underline{\frac{8\sqrt{2}}{15}}$$

9) $x = 4t + 2$; $y = 3t - 4$ $0 \le t \le 4$

$$L = \int \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$= \int_0^4 \sqrt{(4)^2 + (3)^2}\, dt$$

$$= \int_0^4 \sqrt{25}\, dt = 5t\Big]_0^4 = 5[(4) - (0)] = \underline{20}$$

10) $x = 8 + 12t^2$ $y = 8 + 8t^3$ $0 \le t \le 5$

$$L = \int \sqrt{(24t)^2 + (24t^2)^2}\, dt$$

$$= \int \sqrt{24^2 t^2 + 24^2 t^4}\, dt = \int \sqrt{24^2 t^2 (1 + t^2)}\, dt$$

$$= \int 24t \sqrt{1 + t^2}\, dt = 12 \int \sqrt{u}\, du$$

$u = 1 + t^2$
$du = 2t$
$12du = 24t$

$$= 12 \cdot \frac{2}{3} u^{\frac{3}{2}}\Big]_{u(0)}^{u(5)} = \cancel{3} \cdot 4 \cdot \frac{2}{\cancel{3}}$$

$$= 8 u^{\frac{3}{2}}\Big]_{u(0)}$$

$u(5) = 26$
$u(0) = 1$

$$= 8\left[(26)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right] = 8\left[26^{\frac{3}{2}} - 1\right]$$

11) $x = e^t - t \; ; \quad y = 4e^{\frac{t}{2}} \; ; \; 0 \leq t \leq 4$

$$L = \int \sqrt{(e^t - 1)^2 + \left(4e^{\frac{t}{2}} \cdot \frac{1}{2}\right)^2} \, dt$$

$$= \int \sqrt{e^{2t} - 2e^t + 1 + \left(\frac{16}{4} e^t\right)} \, dt$$

$$= \int \sqrt{e^{2t} \underbrace{- 2e^t + 4e^t}_{2e^t} + 1}$$

$$= \int \sqrt{e^{2t} + 2e^t + 1} \, dt$$

$$= \int \sqrt{(e^t + 1)(e^t + 1)} \, dt$$

$$= \int \sqrt{(e^t + 1)^2} \, dt = \int e^t + 1 \, dt = e^t + t \Big]_0^4$$

$$= \Big[(e^4 + 4) - (1 + 0)\Big] = \underline{e^4 + 3}$$

12) $x = 5\cos(t) - \cos(5t)$; $y = 5\sin(t) - \sin(5t)$ $0 \le t \le \pi$

$$L = \int \sqrt{\underbrace{(-5\sin(t) + \sin(5t)\cdot 5)^2}_{1} + \underbrace{(5\cos(t) - \cos(5t)\cdot 5)^2}_{2}}\, dt$$

1 $(-5\sin(t) + \sin(5t)\cdot 5)^2 =$
$= (5(\sin(5t) - \sin(t)))^2 =$
$= 25\,(\sin(5t) - \sin(t))^2 =$
$= 25\left[\cancel{\sin^2(5t)} - 2\sin(t)\sin(5t) + \cancel{\sin^2(t)}\right]$

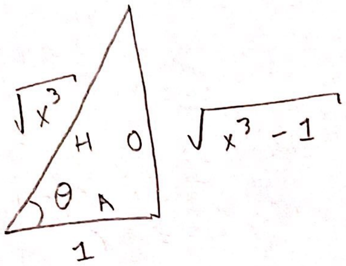
2 $(5(\cos(t) - \cos(5t)))^2 =$
$= 25(\cos(t) - \cos(5t))^2$
$= 25\left[\cancel{\cos^2(t)} - 2\cos(t)\cos(5t) + \cancel{\cos^2(5t)}\right]$

$$\underbrace{25\sin^2(5t) + 25\cos^2(5t)}_{25} + \underbrace{25\cos^2(t) + 25\sin(t)}_{25}$$

$25 + 25 \quad -2\cdot 25\sin(t)\sin(5t) \quad -2\cdot 25\sin(t)\sin(5t)$

$50 \quad -100\sin(t)\sin(5t)$

$$y = \int_{1}^{4} \sqrt{x^3 - 1}\, dx$$



S $\frac{O}{H}$ C $\frac{A}{H}$ T $\frac{O}{A}$

C $\frac{H}{O}$ S $\frac{H}{A}$ C $\frac{A}{O}$

$\tan\theta = \sqrt{x^3 - 1}$

$\sec\theta = x^{\frac{3}{2}}$

$(\sec\theta)^{\frac{2}{3}} = x$

$\frac{2}{3}(\sec\theta)^{\frac{1}{2}} \cdot \sec\theta\tan\theta\, d\theta = dx$

$$\frac{2}{3}\int \tan^2\theta \left(\sec^{\frac{3}{2}}\theta\right)$$

Ver 8

$$P(x) = \int e^{-\frac{x}{\mu}} \frac{1}{\mu} \, dx$$

$0 \le x \le 1$ $\qquad \mu = 1.6$

$$P(0 \le x \le 2) = \int_0^1 e^{-\frac{x}{1.6}} \cdot \frac{1}{1.6} \, dx = -\int_{u(0)}^{u(1)} e^{u} \, du = -e^{u} \Big| = -e^{-\frac{x}{1.6}} \Big]_0^1$$

$u = \frac{-x}{1.6}$

$du = -\frac{1}{1.6} dx = -$

$-du = \frac{1}{1.6} dx$

$$= -\left[\left(e^{-\frac{1}{1.6}}\right) - \left(e^{0}\right)\right] =$$

$$= -e^{-\frac{1}{1.6}} + 1 \approx \underline{0.465}$$

$$P(3 \le x \le \infty) = \left(-e^{-\frac{x}{1.6}}\right)\Big]_3^a =$$

$$= -\left[\underbrace{\left(\lim_{a\to\infty}\left(e^{-\frac{a}{1.16}}\right)\right)}_{\frac{1}{e^{\infty}} \to 0} - \left(e^{-\frac{3}{1.6}}\right)\right] = -\left(-e^{-\frac{3}{1.6}}\right) = e^{-\frac{3}{1.6}} \approx 0.153$$