2019-10-29

- Lab 13
- Lunes 4 noviembre Lab 13
- Corto II Jueves 31 10.2 Longitud de arco y Área
  Jueves 6 10.4 Áreas coordenadas polares.
- Simulacro 3: 11 de noviembre
- Parcial 3: Martes 12 de noviembre.

Curvas Polares

Función polar $r = f(\theta)$ (θ variable independiente)

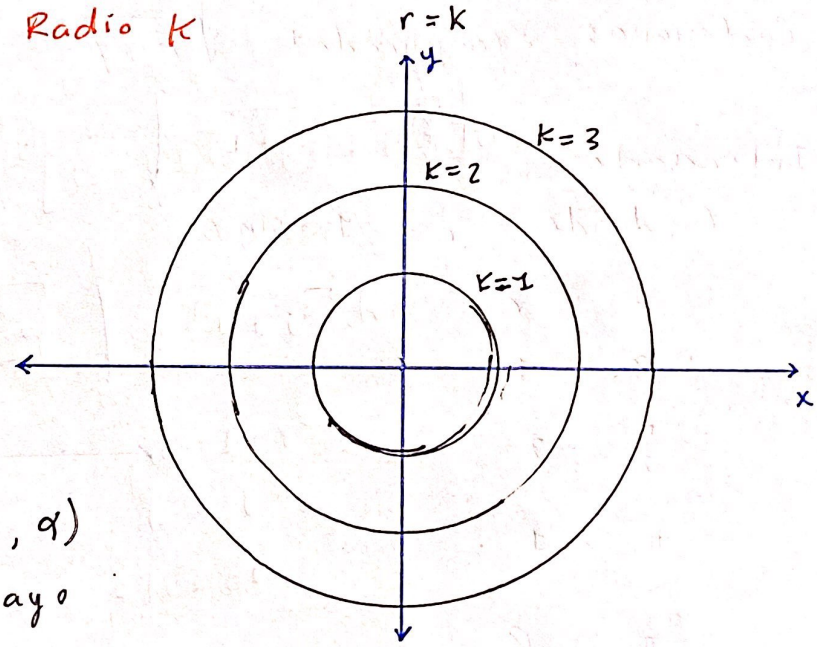
r es dependiente

a) Circunferencia de Radio K

$$x^2 + y^2 = K^2$$

$$r^2 = K^2$$

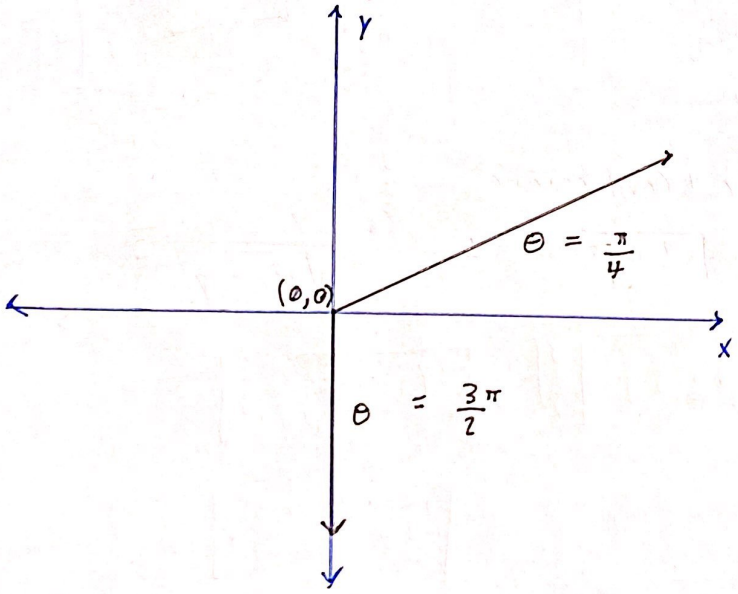
$$r = K$$



b) Línea o rayo $\theta = \alpha$

$(0, \alpha)$ $(1, \alpha)$ $(2, \alpha)$

son parte del rayo



Recordar:

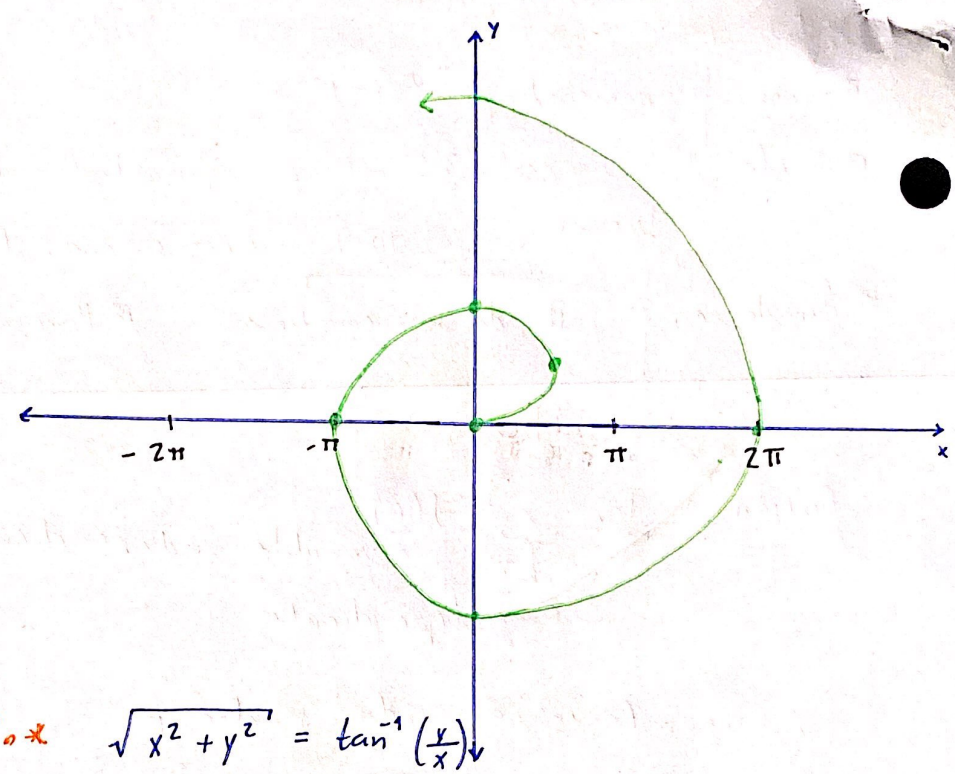
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \alpha$$

$$y = x(\tan \alpha)$$

cartesianas

Espiral $r = \theta$

θ	r
0	0
$\frac{\pi}{2}$	$\frac{\pi}{2}$
π	π
$\frac{3\pi}{2}$	$\frac{3}{2}\pi$
2π	2π

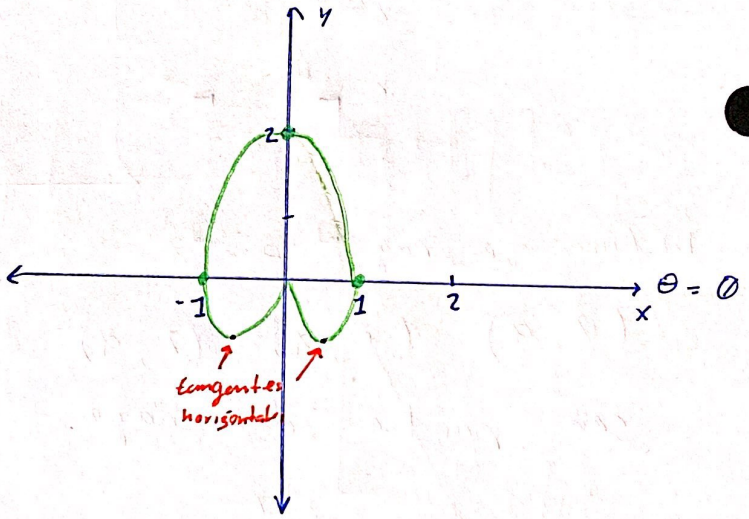


Cartesianas *complicado* $\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$

Interesante:

- Cardioide $r = 1 + \sin\theta$

θ	r
0	1
$\frac{\pi}{2}$	2
π	1
$\frac{3}{2}\pi$	0
2π	1
$\frac{7\pi}{6}$	$\frac{1}{2}$
$\frac{11}{6}\pi$	$\frac{1}{2}$



Ec. Cartesiana

$$\sqrt{x^2+y^2} = 1 + \frac{x}{\sqrt{x^2+y^2}}$$

Rosa o pétalos de 4 hojas:

$r = \cos(2\theta) = 0$

$\cos(u) = 0$ cuando $2\theta = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi$

$r=0$ en $\left\{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

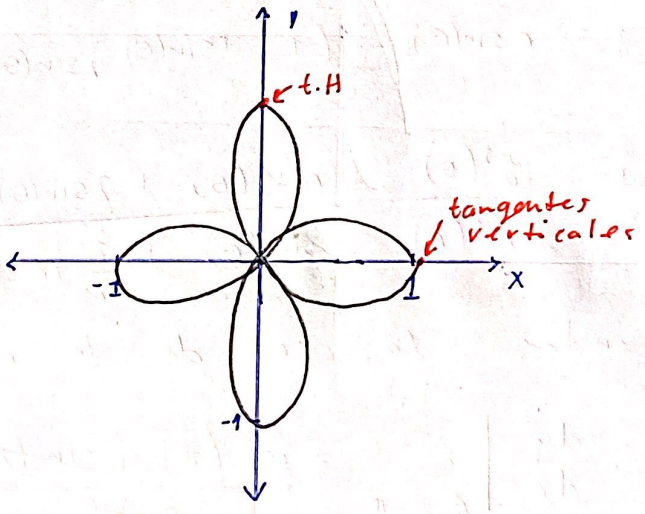
$r = \cos(2\theta) = 1$ $\quad 2\theta = 0, 2\pi, 4\pi$

$\theta = 0, \pi, 2\pi$

$r = \cos(2\theta) = -1$ $\quad 2\theta = \pi, 3\pi$

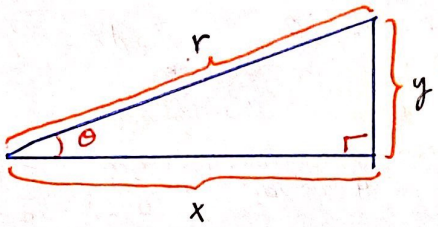
cuando $\theta = \frac{\pi}{2}, \frac{3}{2}\pi$

θ	r
0	1
45	0
10	-1
135	0
180	1
225	0
270	-1
315	0
360	1



Derivada $\frac{dy}{dx}$ de una curva polar $r = f(\theta)$

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$



Ecs. Paramétricas

$$x = f(\theta)\cos(\theta)$$
$$y = f(\theta)\sin(\theta)$$

θ es el parámetro

$r = f(a)$ $\quad (a, f(a))$

Ejercicio 5: Considere el cardioide $r = 1 + \sin(\theta)$

a) Encuentre $\frac{dy}{dx}$

$$x = r\cos(\theta) = (1 + \sin(\theta))\cos\theta = \cos\theta + \underbrace{\sin\theta\cos\theta}_{\frac{1}{2}\sin(2\theta)}$$

$$y = r\sin(\theta) = (1 + \sin(\theta))\sin(\theta) = \sin(\theta) + \sin^2(\theta)$$

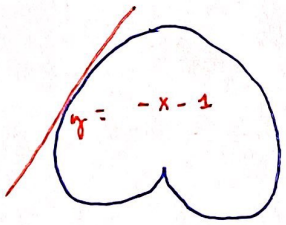
$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{\cos(\theta) + 2\sin(\theta)\cos(\theta)}{-\sin(\theta) + \frac{2}{2}\cos(2\theta)} =$$

Encuentra la ec. de la recta tangente en $\theta = \pi$

$$m = \left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\cos(\pi) + \cancel{\sin(2\pi)}^{0}}{\cancel{-\sin(\pi)}^{0} + \cos(2\pi)} = \frac{-1}{1} = -1$$

$$x(\pi) = \cancel{\cos(\pi)}^{-1} + \frac{1}{2}\cancel{\sin(2\pi)}^{0} = -1$$

$$y(\pi) = (1 + \sin(\pi))\sin(\pi) = 1 \cdot 0 = 0$$



Ec. Recta tangente:

$$y = y(\pi) + m(x - x(\pi)) = 0 + -1(x + 1)$$
$$= -x - 1$$

Ej 4: Circunferencias $r = A\cos(\theta) \pm B\sin(\theta)$ con centro fuera del origen.

Sea $r = 2\sin\theta$

a) Encuentre una ecuación cartesiana para la curva

$r = \sqrt{x^2 + y^2}$ Elimine θ & r

$y = r\sin(\theta) \Rightarrow \sin(\theta) = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$

$\underbrace{r = 2\sin(\theta)}_{\text{dado}} \Rightarrow \sqrt{x^2 + y^2} = \frac{2y}{\sqrt{x^2 + y^2}}$

$x^2 + y^2 = 2y$ Ec. circunferencia $(x-a)^2 + (y-b)^2 = R^2$

$x^2 + y^2 - 2y \underbrace{+1 = +1}_{\text{completación al cuadrado}}$

$\underbrace{x^2 + (y-1)^2 = 1}_{\text{Circunferencia de Radio 1 centrada en } (0,1).}$

b) Grafique la función polar $r = 2\sin(\theta)$