

2019-10-06

Webassign: Integrales impropias

①

$$\int_6^{\infty} \frac{1}{(x-5)^{3/2}} dx = \int_6^{\infty} \frac{1}{u^{3/2}} du = \int_6^{\infty} u^{-3/2} du = 2u^{-1/2}$$

$$u = (x-5)$$

$$du = dx$$

$$= 2 \cdot (x-5)^{-1/2} \Big|_6^{\infty} =$$

$$= \left[\left(\lim_{t \rightarrow \infty} \left(\frac{-2}{\sqrt{t-5}} \right) \right) - \left(\frac{-2}{\sqrt{1}} \right) \right] = \frac{2}{\infty} \rightarrow 0$$

②

$$\int_0^{\infty} \frac{1}{\sqrt[3]{1+x}} dx = \int_0^{\infty} (u)^{-1/3} du = \frac{-1/3 + 2/3}{-1/3} u^{-1/3 + 2/3} = \frac{2}{7} u^{7/8} \Big|_0^{\infty} =$$

$$u = 1+x$$

$$du = dx$$

$$= \left[\lim_{t \rightarrow \infty} \left(\frac{2}{7} (1+\infty)^{7/8} \right) - \left(\frac{2}{7} (1+0)^{7/8} \right) \right] = \infty$$

$$\textcircled{3} \int_2^{\infty} e^{-7p} dp = -\frac{1}{7} \int_2^{\infty} e^u du = -\frac{1}{7} (e^u) \Big|_2^{\infty} = -\frac{1}{7} (e^{-7p}) \Big|_2^{\infty}$$

$$\left. \begin{array}{l} u = -7p \\ du = -7 dp \\ -\frac{du}{7} = dp \end{array} \right\} = \left[\lim_{t \rightarrow \infty} \underbrace{\left(-\frac{1}{7} e^{7t} \right)}_0 - \left(-\frac{1}{7} e^{7(2)} \right) \right] = \frac{1}{7} e^{14}$$

$$\textcircled{4} \int_1^{\infty} \frac{e^{-\frac{1}{x}}}{x^2} dx = \int_1^{\infty} e^{-\frac{1}{x}} \cdot x^{-2} dx = \int_1^{\infty} e^u du = e^u \Big|_1^{\infty} = e^{-\frac{1}{x}} \Big|_1^{\infty} =$$

$$\left. \begin{array}{l} u = -\frac{1}{x} = -x^{-1} \\ du = x^{-2} dx \end{array} \right\} = \left[\lim_{t \rightarrow \infty} \left(\frac{1}{e^{\frac{1}{t}}} \right) - \left(e^{-\frac{1}{1}} \right) \right] =$$

$$\textcircled{5} \int_{-\infty}^0 \frac{z}{z^4 + 4} dz =$$

$$\left. \begin{array}{l} u = z^2 \\ \frac{1}{2} du = z dz \end{array} \right\} = \frac{1}{2} \int \frac{du}{u^2 + 4} =$$

$$\left. \begin{array}{l} u = 2v \\ du = 2dv \end{array} \right\} = \frac{1}{2} \int \frac{2dv}{4v^2 + 4} = \frac{1}{2} \int \frac{z dv}{4(v^2 + 1)} =$$

$$= \frac{1}{2} \cdot \frac{1}{2} \int \frac{1}{v^2 + 1} dv = \frac{1}{4} \arctan(v)$$

$$= \frac{1}{4} \left[\arctan\left(\frac{0^2}{2}\right) - \arctan\left(\frac{(-\infty)^2}{2}\right) \right] =$$

$$= \frac{1}{4} \left[\arctan(0) - \arctan(\infty) \right] = \frac{1}{4} \arctan\left(\frac{z^2}{2}\right) \Big|_{-\infty}^0 = \frac{1}{4} \left[\arctan(0) - \arctan(\infty) \right]$$

$$= \frac{1}{4} \left[0 - \frac{\pi}{2} \right] = -\frac{\pi}{8}$$

⑥

$$S = \{(x, y) \mid x \geq 1, 0 \leq y \leq e^{-x}\}$$

$$S = \int_1^{\infty} e^{-x} dx = - \int_1^{\infty} e^u du = -e^{-x} \Big|_1^{\infty} =$$

$$\begin{array}{l} u = -x \\ -du = dx \end{array} \left| = \left[\lim_{t \rightarrow \infty} \left(-\frac{1}{e^{\infty}} \right) - \left(-e^{-1} \right) \right] = \frac{e^{-1}}{1}$$

⑦

$$\int_0^{\infty} \frac{4}{\sqrt{x}(1+x)} dx = \int_0^1 \frac{4}{\sqrt{x}(1+x)} dx + \int_1^{\infty} \frac{4}{\sqrt{x}(1+x)} dx = 2\pi + 2\pi = 4\pi$$

$$u = \sqrt{x}$$

$$\sqrt{x} u^2 = x$$

$$2u du = dx$$

$$= 8 \left[\left(\arctan(\sqrt{x}) \right) - \left(\arctan(\sqrt{0}) \right) \right]$$

$$\frac{\pi}{4} = \frac{2\pi}{4}$$

$$2 \cdot 4 \int_0^{\infty} \frac{u du}{u(1+u^2)} = 8 \int_0^{\infty} \frac{u du}{1+u^2} = 8 \arctan(u)$$

$$8 \left[\left(\arctan(\infty) \right) - \left(\arctan(1) \right) \right] = 8 \left(\frac{2\pi}{4} - \frac{\pi}{4} \right) = 8 \left(\frac{\pi}{4} \right) = 2\pi$$

8

$$23 \int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx = -23 \cdot 2 \int_1^{\infty} e^u du = -23 \left(e^{-\sqrt{x}} \right) \Big|_1^{\infty} =$$

$$u = -\sqrt{x}$$

$$du = -\frac{1}{2\sqrt{x}} dx$$

$$2du = -\frac{1}{\sqrt{x}} dx$$

$$\left. \begin{aligned} &+ 23 \cdot 2 \left[\underbrace{\left(\lim_{x \rightarrow \infty} \left(\frac{1}{e^{\sqrt{x}}} \right) \right)}_0 - \left(e^{-1} \right) \right] = +23 \cdot 2 \left[-e^{-1} \right] = \\ &= 46 e^{-1} \end{aligned} \right\}$$