Webassign Sustitución

① $\int \cos(6x)\,dx = \frac{1}{6}\int \cos(u)\,du = \frac{\sin(6x)}{6} + C$

$u = 6x$

$\frac{du}{6} = dx$

② $\int x^2\sqrt{x^3+13}\,dx = \int \sqrt{u}\,\frac{du}{3} = \frac{1}{3}\int u^{3/2} = \frac{1}{3}\,\frac{(x^3+13)^{3/2}}{3/2}$

$u = x^3 + 13$

$du = 3x^2 + 0\,dx$

$\frac{du}{3} = x^2\,dx$

$= \frac{1}{3}\cdot\left[\frac{\frac{(x^3+13)^{3/2}}{1}}{\frac{3}{2}}\right] =$

$= \frac{1}{3}\cdot\frac{2(x^3+13)^{3/2}}{3}$

$= \frac{2}{9}(x^3+13)^{3/2} + C$

③ $\int \sin^3\theta\,\cos\theta\,d\theta = u = \sin\theta$

$u = \sin\theta$

$du = \cos\theta$

$= \int u^3\,du = \frac{u^4}{4} + C$

$= \frac{\sin^4\theta}{4} + C$

④ $\int \frac{x^3}{x^4-2}\,dx = \int \frac{du}{u} = \ln|u| + C$

$= \frac{\ln|x^4-2|}{-4} + C$

$u = x^4 - 2$

$du = 4x^3 - 0\,dx$

$\frac{du}{4} = x^3\,dx$

(5) $\displaystyle\int (4-4x)^7\,dx = \int u^7 \cdot -\frac{du}{4} = -\frac{1}{4}\int u^7 = -\frac{1}{4}\cdot\frac{u^8}{8}$

$u = 4-4x$

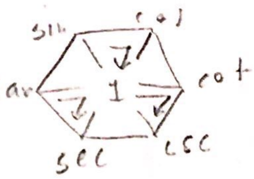
$du = -4\,dx$

$-\dfrac{du}{4} = dx$

$$= -\frac{(4-4x)^8}{8\cdot 4} + C$$

$$= -\frac{(4-4x)^8}{32} + C$$

(6) $\displaystyle\int \sin(t)\sqrt{1+\cos(t)}\,dt = -\int \sqrt{u}\,du$



$u = 1 + \cos(t)$

$du = (0 - \sin(t))\,dt$

$-du = \sin(t)\,dt$

$$= -\frac{u^{3/2}}{3/2} + C$$

$\sin^2\theta + \cos^2\theta = 1$

$$= -\frac{(1+\cos(t))^{3/2}}{3/2}$$

$$= -\frac{2(1+\cos(t))^{3/2}}{3} + C$$

(7) $\displaystyle\int \frac{e^u}{(1-e^u)^2}\,du = -\int \frac{1}{(u)^2}\,du = -\int u^{-2}\,du$

$$= -\left\{\frac{u^{-1}}{-1}\right\}$$

$v = 1 - e^u$

$dv = 0 - e^u\,du$

$$= -\left\{-\frac{1}{u}\right\}$$

$-dv = e^u\,du$

$$= \frac{1}{u} + C$$

$$= \frac{1}{(1-e^u)} + C$$

(8) $$\int \frac{a+bx^7}{\sqrt{8ax+bx^8}}\,dx = \frac{1}{8}\int \frac{du}{\sqrt{u}} = \frac{1}{8}\int u^{-1/2}\,du$$

$u = 8ax + bx^8$

$du = 8a + 8bx^7$

$du = 8(a+bx^7)$

$\frac{du}{8} = a + bx^7$

$$= \frac{1}{8}\left\{\frac{u^{1/2}}{1/2}\right\}$$

$$= \frac{1}{8}\left\{2u^{1/2}\right\}$$

$$= \frac{1}{\cancel{2}\cdot 4}\cdot \cancel{2}u^{1/2}$$

$$= \frac{u^{1/2}}{4} + C$$

$$= \frac{\sqrt{8ax+bx^8}}{4} + C$$

(9) $$\int \frac{\ln(x)^{16}}{x}\,dx = \int \ln(x)^{16}\cdot \underbrace{\left(\frac{1}{x}\,dx\right)}_{du}$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$$= \int u^{16}\,du = \frac{u^{17}}{17} + C = \frac{\ln^{17}(x)}{17} + C$$

(10) $$\int \sec^2\theta \tan^3\theta\,d\theta = \int u^3\,du = \frac{u^4}{4} + C$$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$$= \frac{\tan^4\theta}{4} + C$$

(11) $$\int e^x\sqrt{7+e^x}\,dx = \int u\,du$$

$u = 7 + e^x$

$du = e^x\,dx$

$$= \frac{u^2}{2} + c$$

$$= \frac{(7+e^x)}{2} + C$$

(12) $$\int \frac{dx}{tx+g} = \frac{1}{t}\int \frac{1}{u}\,du = \frac{1}{t}\ln|tx+g| + C$$

$u = tx + g$

$du = (t + 0)\,dx$

$\frac{du}{t} = dx$

(18) $$\int \tan^8(\theta)\sec^2\theta\,d\theta = \int u^8\,du = \frac{u^9}{9} + C = \frac{\tan^9(\theta)}{9} + C$$

$u = \tan(\theta)$

$du = \sec^2\theta\,d\theta$

(17) $$\int \frac{x^5\,dx}{1+x^{12}} = \frac{1}{6}\int \frac{du}{1+u^2} = \frac{1}{6}\int \frac{1}{1+u^2}\,du$$

$u = x^6$

$du = 6x^5\,dx$

$\frac{du}{6} = x^5\,dx$

$$= \frac{1}{6}\tan^{-1}(u) + C$$

$$= \frac{1}{6}\tan^{-1}(x^6) + C$$

(16) $$\int \cot(18x)\,dx = \int \frac{\cos(18x)}{\sin(18x)}\,dx$$

$$= \frac{1}{18}\int \frac{1}{u}\,du = \frac{1}{18}\ln|u| + C$$

$$= \frac{1}{18}\ln|\sin(18x)| + C$$

$u = \sin(18x)$

$du = \cos(18x)\,18\,dx$

$\frac{du}{18} = \cos(18x)\,dx$

(15)

$$\int \frac{\sin(2x)}{28+\cos^2 x}\,dx = \int \frac{\sin(2x)}{28 + \left(\frac{1+\cos(2x)}{2}\right)}\,dx$$

$$= \int \frac{\sin(2x)}{\frac{56}{2} + \frac{1}{2} + \frac{\cos(2x)}{2}}\,dx$$

$$= \int \frac{\frac{\sin(2x)}{1}}{\frac{57+\cos(2x)}{2}}\,dx = \int \frac{2\sin(2x)}{57+\cos(2x)}\,dx = -\frac{2}{2}\int \frac{du}{57+u}$$

$u = \cos(2x)$

$du = -\sin(2x)\cdot 2\,dx$

$-\frac{du}{2} = \sin(2x)\,dx$

$$= -\int \frac{1}{57+u}\,du$$

$$= -\ln|57+\cos(2x)| + C$$

(14)

$$\int (\underbrace{\cot(x)}_{u})^{\frac{1}{30}} \underbrace{\csc^2(x)\,dx}_{du} = -\int u^{1/30}\,du = \frac{-(\cot x)^{31/30}}{31/30} + C$$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$\frac{1}{30} + \frac{30}{30} = \frac{31}{30}$

(13) $\int \frac{\cos\left(\frac{\pi}{x^{11}}\right)}{x^{12}} \, dx = \frac{1}{\pi}\int \cos(u)\, du = -\frac{1}{\pi} \sin(u) + C$

$= -\frac{1}{11\pi} \sin\left(\frac{\pi}{x^{12}}\right) + C$

$u = \frac{\pi}{x^{12}} = \pi \cdot x^{-12}$

$du = -11\pi\, x^{-12}$

$du = -\frac{\pi}{x^{12}}$

$-\frac{du}{11\pi} = \frac{1}{x^{12}}$

(11) $\int e^x \sqrt{7 + e^x} \, dx = \int \sqrt{u}\, du = \frac{u^{3/2}}{3/2} + C$

$u = 7 + e^x$

$du = e^x \, dx$

$= \frac{2(7+e^x)^{3/2}}{3} + C$