1. $E: -3 \leq y \leq 0, \quad -\sqrt{9-y^2} \leq x \leq 0$

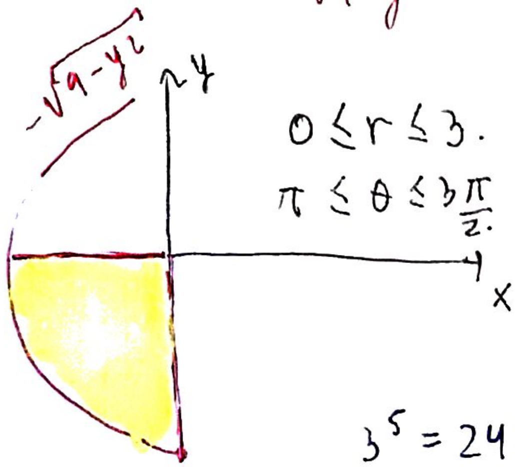
$$3x^2+3y^2 \leq z \leq 54-(x^2+y^2)^{3/2}.$$

$$\int_a^b dz = b-a.$$

a. Volumen del sólido.

$$V = \iiint_E dV = \int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{0}\int_{3x^2+3y^2}^{54-(x^2+y^2)^{3/2}} dz\, dx\, dy.$$

$$V = \int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{0}\left[54-(\underbrace{x^2+y^2}_{r^2})^{3/2}-3(\underbrace{x^2+y^2}_{r^2})\right]dx\,dy.$$



$0 \leq r \leq 3.$

$\pi \leq \theta \leq 3\frac{\pi}{2}.$

$$V = \int_{\pi}^{3\pi/2}\int_0^3 (54-r^3-3r^2)\, r\,dr\,d\theta.$$

$$V = \int_{\pi}^{3\pi/2} d\theta \int_0^3 54r - r^4 - 3r^3\, dr$$

$3^5 = 243$

$$V = \frac{\pi}{2}\left(27r^2-\frac{r^5}{5}-\frac{3}{4}r^4\Big]_0^3\right) = \frac{\pi}{2}\left(243-\frac{243}{5}-\frac{243}{4}\right)$$

$$V = 243\frac{\pi}{2}\left(1-\frac{1}{5}-\frac{1}{4}\right). = 243\frac{\pi}{2}\,\frac{11}{20}.$$

b. Masa del sólido $\rho(x,y) = 30xy$.

$$m = \iiint_E \rho \, dV = \underbrace{\int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0}}_{D} \int_{3x^2+3y^2}^{54-(x^2+y^2)^{3/2}} 30\,xy \, dz \, dx \, dy.$$

Cambie a coordenadas cilíndricas.

$$m = \iiint_E 30\,xy \, dV.$$

$dV = r\,dz\,dr\,d\theta.$

$x = r\cos\theta. \qquad x^2+y^2 = r^2.$

$y = r\sin\theta.$

$0 \le r \le 3, \quad \pi \le \theta \le \frac{3\pi}{2}. \qquad 3(x^2+y^2) \le z \le 54-(x^2+y^2)^{3/2}$

$3r^2 \le z \le 54 - r^3.$

$$m = \int_{\pi}^{3\pi/2} \int_0^3 \int_{3r^2}^{54-r^3} 30r^2 \cos\theta \sin\theta. \, r \, dz \, dr \, d\theta$$

$$m = \left(\int_{\pi}^{3\pi/2} \sin\theta\cos\theta \, d\theta \right) \int_0^3 \int_{3r^2}^{54-r^3} 30r^3 \, dz \, dr.$$

$$m = \frac{1}{2}\sin^2\theta \Big]_{\pi}^{3\pi/2} \int_0^3 30r^3(54-r^3-3r^2)\,dr.$$

$$m = 15\int_0^3 (54r^3 - r^6 - 3r^5)\,dr.$$

2. Adentro de la esfera $x^2+y^2+z^2=4$.
Encima del cono $z^2 = 3x^2+3y^2$.
Enfrente del plano $x=0$.

a. Coordenadas Esféricas r radio polar.

Esfera $\rho = 2$.

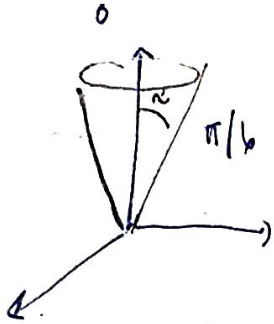
Cono $\rho^2\cos^2\varphi = 3r^2 = 3\rho^2\sin^2\varphi$

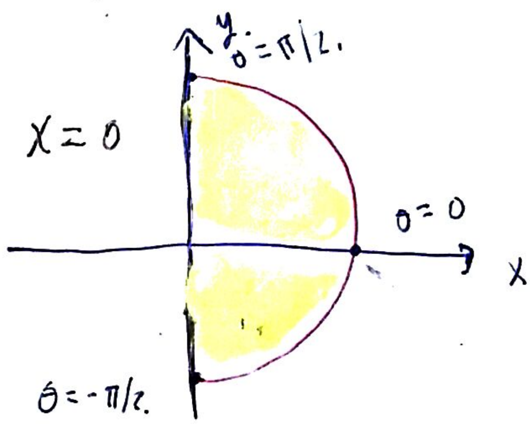
$$\cos^2\varphi = 3\sin^2\varphi.$$
$$\tan^2\varphi = \frac{1}{3}.$$
$$\varphi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}.$$

$x = \rho\sin\varphi\cos\theta$
$y = \rho\sin\varphi\sin\theta$
$z = \rho\cos\varphi.$

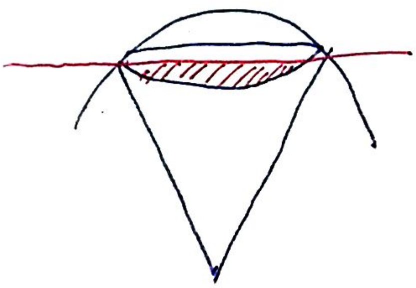
$\sin\pi/6 = 1/2$
$\cos\pi/6 = \sqrt{3}/2$

$\varphi = \alpha.$





encima y adentro.



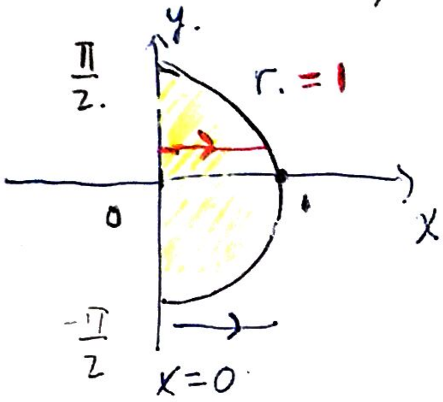
No hay cascarones. $0 \le \rho \le 2.$
Encima de un cono $0 \le \varphi \le \pi/6.$
Enfrente de $x=0$. $-\frac{\pi}{2} \le \theta \le \pi/2.$

$$V = \iiint_E dV = \int_{-\pi/2}^{\pi/2}\int_0^{\pi/6}\int_0^2 \rho^2\, d\rho.\ \sin\varphi\, d\varphi\, d\theta.$$

b. Cilíndricas. $z^2 = a(x^2+y^2)$

$$\rho^2\cos^2\varphi = a\rho^2\sin^2\varphi.$$

$$\varphi = \tan^{-1}\left(\frac{1}{\sqrt{a}}\right) \qquad \frac{1}{a} = \tan^2\varphi.$$



$$\sqrt{3}\,r \le z \le +\sqrt{4-r^2}$$

Esfera:

$$r^2+z^2=4.$$

$$z = \pm\sqrt{4-r^2}$$

Cono: $z^2 = 3(x^2+y^2) = 3r^2 \Rightarrow z = \sqrt{3}\,r.$

Intersección
$$\left.\begin{aligned} x^2+y^2+z^2 &= 4. \\ z^2 &= 3x^2+3y^2. \end{aligned}\right\}$$

$$x^2+y^2+3x^2+3y^2 = 4x^2+4y^2 = 4 \Rightarrow x^2+y^2=1$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \quad 0 \le r \le 1, \quad \sqrt{3}\,r \le z \le +\sqrt{4-r^2}$$

$$V = \iiint_E dV = \int_{-\pi/2}^{\pi/2}\int_0^1\int_{\sqrt{3}r}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

– Cartesianas. Esfera: $z^2 = 4-x^2-y^2$.
Cono $z^2 = 3(x^2+y^2)$

$$\sqrt{3}\sqrt{x^2+y^2} \le z \le +\sqrt{4-x^2-y^2}$$

$$-1 \le y \le 1, \quad 0 \le x \le \sqrt{1-y^2} \longrightarrow D$$

$$0 \le x \le 1, \quad -\sqrt{1-x^2} \le y \le \sqrt{1-y^2} \to D$$

$$V = \int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{4-x^2-y^2}} dz\, dx\, dy.$$ NO.

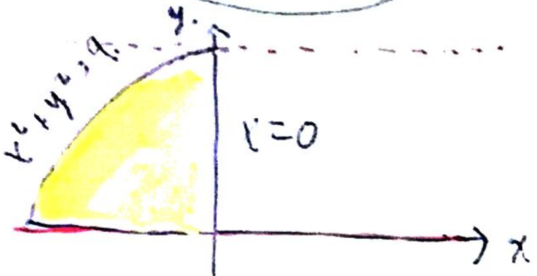
J. $$V = \int_{0}^{\pi/6} \int_{-\pi/2}^{\pi/2} \int_{0}^{2} \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi.$$

$$V = \int_{0}^{\pi/6} \sin\varphi \, d\varphi \int_{-\pi/2}^{\pi/2} d\theta . \int_{0}^{2} \rho^2 \, d\rho.$$

$$V = -\cos\varphi \Big]_{0}^{\pi/6} \left(\frac{\pi}{2} + \frac{\pi}{2}\right) \frac{1}{3} \rho^3 \Big]_{0}^{2}$$

$$V = \left(1 - \frac{\sqrt{3}}{2}\right) \pi \, \frac{8}{3}.$$

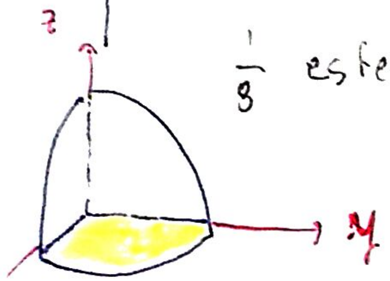
5. $$\int_0^3 \int_{-\sqrt{9-y^2}}^{0} \int_0^{\sqrt{9-x^2-y^2}} 6z(x^2+y^2+z^2)\, dz\, dx\, dy.$$ → esfera.



Esféricas	Cilíndricas
$\frac{\pi}{2} \leq \theta \leq \pi$ ✓	$\frac{\pi}{2} \leq \theta \leq \pi$. ✓
	$0 \leq r \leq 3$

$z^2 = 9 - x^2 - y^2$.

$\rho^2 = 9 \Rightarrow \rho = 3$.



$\frac{1}{8}$ esfera.

$0 \leq \rho \leq 3$. $0 \leq \varphi \leq \pi/2$.

$\frac{\pi}{2} \leq \theta \leq \pi$.

Toda la esfera $$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{+\sqrt{9-x^2-y^2}} f\, dz\, dx\, dy$$

$$I = \iiint_E 6\underbrace{z}_{\rho\cos\varphi}\underbrace{(x^2+y^2+z^2)}_{\rho^2}\, dV.$$

$$I = \int_{\pi/2}^{\pi} \int_0^{\pi/2} \int_0^3 6\rho^3 \cos\varphi\, \rho^2\, d\rho.\, \sin\varphi\, d\varphi\, d\theta.$$

$z = \sqrt{K^2 - x^2 - y^2}$ esferas. $\rho = K$.

b. Cilíndricas.

$$I = \iint_{D} \int_0^{\sqrt{9-\underbrace{x^2-y^2}_{r}}} 6z(x^2+y^2+z^2)\,dz\,dA.$$

$D:\ 0 \le r \le 3$

$\frac{\pi}{2} \le \theta \le \pi.$

$dA = r\,dr\,d\theta.$

$z = z$

$x = r\cos\theta,\ y = r\sin\theta.$

$$I = \int_{\pi/2}^{\pi} \int_0^{3} \int_0^{\sqrt{9-r^2}} 6z(r^2+z^2)\,dz.\,r\,dr\,d\theta$$

c. Evalúe la integral, prefiera esféricas.

$$I = \int_{\pi/2}^{\pi} d\theta.\int_0^{\pi/2} \sin\varphi\cos\varphi\,d\varphi.\int_0^{3} 6\rho^5\,d\rho.$$

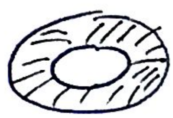
$$I = \frac{\pi}{2}\quad \frac{1}{2}\sin^2\varphi\Big]_0^{\pi/2}\quad \rho^6\Big]_0^{3}$$

$$I = \frac{\pi}{4}\,729$$

$\rho\cos\varphi$

$\rho\sin\varphi.$

$z = \pm\sqrt{x^2+y^2}$ conos



$\theta = 0$

$z = \pm\sqrt{K^2-x^2-y^2}$ esferas.

dos esferas.

esfera y cono

$0 \le \rho \le K.$

$0 \le \varphi \le \alpha.$

$0 \le \varphi \le \pi.$

$K_1 \le \rho \le K_2.$