1. $E: -3 \le y \le 0, \quad -\sqrt{9-y^2} \le x \le 0$

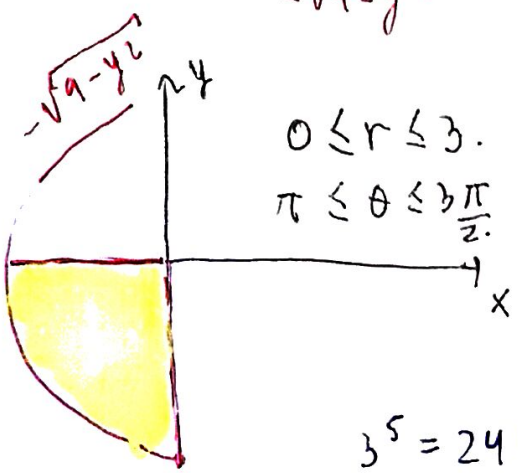
$$3x^2+3y^2 \le z \le 54-(x^2+y^2)^{3/2}.$$

$\int_a^b dz = b-a.$

a. Volumen del sólido.

$$V = \iiint_E dV = \int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0} \int_{3x^2+3y^2}^{54-(x^2+y^2)^{3/2}} dz\,dx\,dy.$$

$$V = \int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0} \left[54 - \underbrace{(x^2+y^2)^{3/2}}_{r^3} - 3\underbrace{(x^2+y^2)}_{r^2}\right] dx\,dy.$$



$0 \le r \le 3.$

$\pi \le \theta \le 3\frac{\pi}{2}.$

$$V = \int_{\pi}^{3\pi/2} \int_{0}^{3} (54 - r^3 - 3r^2)\, r\, dr\, d\theta.$$

$$V = \int_{\pi}^{3\pi/2} d\theta \int_{0}^{3} 54r - r^4 - 3r^3\, dr$$

$3^5 = 243$

$$V = \frac{\pi}{2}\left(27r^2 - \frac{r^5}{5} - \frac{3}{4}r^4\right]_0^3\right) = \frac{\pi}{2}\left(243 - \frac{243}{5} - \frac{243}{4}\right)$$

$$V = 243\frac{\pi}{2}\left(1 - \frac{1}{5} - \frac{1}{4}\right) = 243\frac{\pi}{2}\,\frac{11}{20}.$$

b. Masa del sólido $\rho(x,y) = 30xy$.

$$m = \iiint_E \rho \, dV = \underbrace{\int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0}}_{D} \int_{3x^2+3y^2}^{54-(x^2+y^2)^{3/2}} 30xy \, dz \, dx \, dy.$$

Cambie a coordenadas cilíndricas..

$$m = \iiint_E 30xy \, dV.$$

$dV = r \, dz \, dr \, d\theta$.

$x = r\cos\theta$. $\quad x^2+y^2 = r^2$.

$y = r\sin\theta$.

$0 \le r \le 3, \quad \pi \le \theta \le \frac{3\pi}{2}$.

$3(x^2+y^2) \le z \le 54-(x^2+y^2)^{3/2}$.

$3r^2 \le z \le 54 - r^3$.

$$m = \int_{\pi}^{3\pi/2} \int_{0}^{3} \int_{3r^2}^{54-r^3} 30r^2 \cos\theta \sin\theta \cdot r \, dz \, dr \, d\theta$$

$$m = \left(\int_{\pi}^{3\pi/2} \sin\theta\cos\theta \, d\theta \right) \int_{0}^{3} \int_{3r^2}^{54-r^3} 30r^3 \, dz \, dr.$$

$$m = \frac{1}{2}\sin^2\theta \Big]_{\pi}^{3\pi/2} \int_{0}^{3} 30r^3(54-r^3-3r^2) \, dr.$$

$$m = 15 \int_{0}^{3} (54r^3 - r^6 - 3r^5) \, dr.$$

2. Adentro de la esfera $x^2+y^2+z^2=4$.
Encima del cono $z^2=3x^2+3y^2$.
Enfrente del plano $x=0$.

4. Coordenadas Esféricas r radio polar.

$x=\rho\sin\varphi\cos\theta$
$y=\rho\sin\varphi\sin\theta$
$z=\rho\cos\varphi$.

Esfera $\rho=2$.

Cono $\rho^2\cos^2\varphi = 3r^2 = 3\rho^2\sin^2\varphi$

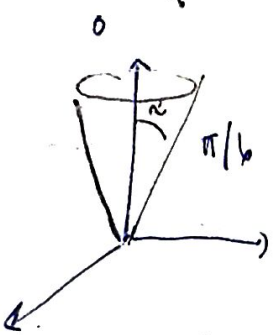
$\cos^2\varphi = 3\sin^2\varphi$.

$\tan^2\varphi = \frac{1}{3}$.

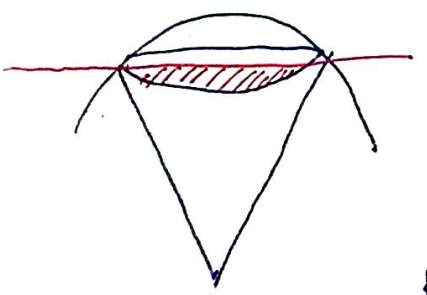
$\varphi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$.

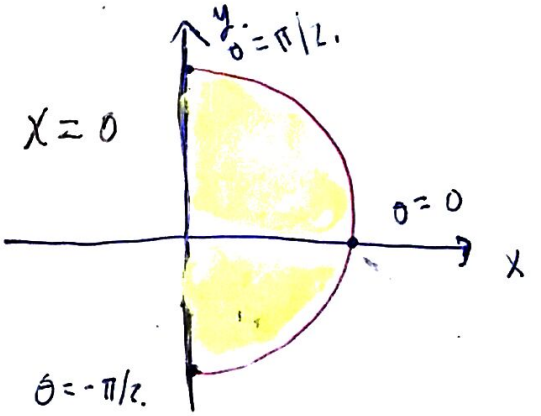
$\sin\pi/6 = 1/2$
$\cos\pi/6 = \sqrt{3}/2$

$\varphi=\alpha$.



encima y adentro.





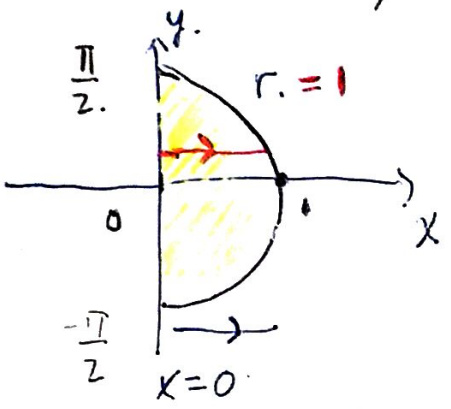
No hay cascarones. $0\le\rho\le 2$.
Encima de un cono $0\le\varphi\le\pi/6$.
Enfrente de $x=0$. $-\frac{\pi}{2}\le\theta\le\pi/2$.

$$V=\iiint_E dV = \int_{-\pi/2}^{\pi/2}\int_0^{\pi/6}\int_0^2 \rho^2\, d\rho\cdot\sin\varphi\, d\varphi\, d\theta.$$

b. Cilíndricas. $z^2 = a(x^2+y^2)$

$\rho^2\cos^2\varphi = a\rho^2\sin^2\varphi$.

$\varphi = \tan^{-1}\left(\frac{1}{\sqrt{a}}\right)$ $\quad \frac{1}{a} = \tan^2\varphi$.



$\sqrt{3}\,r \le z \le +\sqrt{4-r^2}$

Esfera:

$r^2 + z^2 = 4$.

$z = \pm\sqrt{4-r^2}$

Cono: $z^2 = 3(x^2+y^2) = 3r^2 \Rightarrow z = \sqrt{3}\,r$.

Intersección $\left.\begin{array}{l} x^2+y^2+z^2 = 4. \\ z^2 = 3x^2+3y^2. \end{array}\right\}$

$x^2+y^2+3x^2+3y^2 = 4x^2+4y^2 = 4 \Rightarrow x^2+y^2 = 1$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, $0 \le r \le 1$, $\sqrt{3}\,r \le z \le +\sqrt{4-r^2}$

$$V = \iiint_E dV = \int_{-\pi/2}^{\pi/2}\int_0^1\int_{\sqrt{3}\,r}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

- Cartesianas. Esfera: $z^2 = 4-x^2-y^2$

Cono $z^2 = 3(x^2+y^2)$

$\sqrt{3}\sqrt{x^2+y^2} \le z \le +\sqrt{4-x^2-y^2}$

$-1 \le y \le 1$, $0 \le x \le \sqrt{1-y^2}$ $\longrightarrow$ D

$0 \le x \le 1$, $-\sqrt{1-x^2} \le y \le \sqrt{1-y^2}$ $\rightarrow$ D

$$V = \int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{4-x^2-y^2}} dz\, dx\, dy. \quad \text{NO.}$$

3. $$V = \int_{0}^{\pi/6} \int_{-\pi/2}^{\pi/2} \int_{0}^{2} \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi.$$

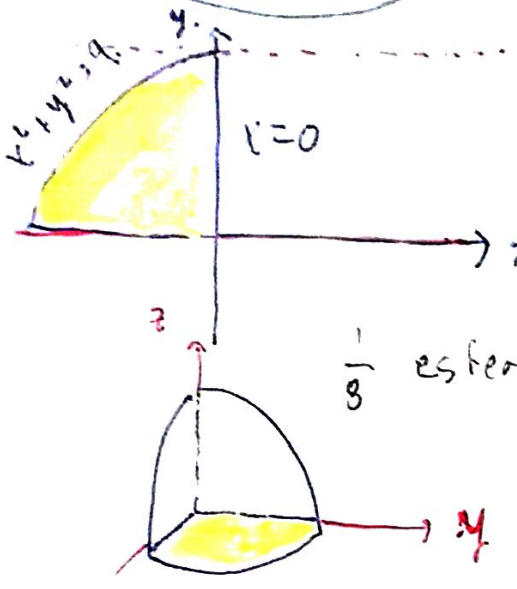
$$V = \int_{0}^{\pi/6} \sin\varphi \, d\varphi \int_{-\pi/2}^{\pi/2} d\theta \int_{0}^{2} \rho^2 \, d\rho.$$

$$V = -\cos\varphi \Big]_{0}^{\pi/6} \left(\frac{\pi}{2} + \frac{\pi}{2}\right) \frac{1}{3} \rho^3 \Big]_{0}^{2}.$$

$$V = \left(1 - \frac{\sqrt{3}}{2}\right) \pi \, \frac{8}{3}.$$

5. $\int_0^3 \int_0^{-\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}}$ → esfera.

$$6z(x^2+y^2+z^2)\,dz\,dx\,dy.$$



Esféricas

$\frac{\pi}{2} \le \theta \le \pi$ ✓

Cilíndricas

$\frac{\pi}{2} \le \theta \le \pi$. ✓

$0 \le r \le 3$

$z^2 = 9 - x^2 - y^2$.

$\rho^2 = 9 \Rightarrow \rho = 3$.

$0 \le \rho \le 3$. $\quad 0 \le \varphi \le \pi/2$.

$\frac{\pi}{2} \le \theta \le \pi$.

toda la esfera

$$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{+\sqrt{9-x^2-y^2}} 6\,dz\,dx\,dy$$

$$I = \iiint_E 6z\underbrace{(x^2+y^2+z^2)}_{\rho^2}\,dV.$$

$\rho\cos\varphi$

$$I = \int_{\pi/2}^{\pi} \int_0^{\pi/2} \int_0^3 6\rho^3 \cos\varphi \cdot \rho^2 \, d\rho \cdot \sin\varphi \, d\varphi \, d\theta.$$

$= \sqrt{K^2-x^2-y^2}$ esferas. $\quad \rho = K.$

b. Cilíndricas.

$$I = \iint_D \int_0^{\sqrt{9-x^2-y^2}} 6z(x^2+y^2+z^2)\,dz\,dA.$$

$\sqrt{x^2+y^2} = r$

$D: \quad 0 \le r \le 3$

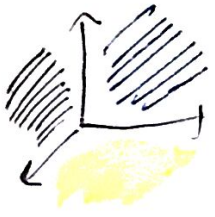
$\frac{\pi}{2} \le \theta \le \pi.$

$dA = r\,dr\,d\theta.$

$z = z$

$x = r\cos\theta, \quad y = r\sin\theta.$

$$I = \int_{\pi/2}^{\pi} \int_0^3 \int_0^{\sqrt{9-r^2}} 6z(r^2+z^2)\,dz.\,r\,dr\,d\theta \leftarrow$$



∴ Evalúe la integral, prefiera esféricas.

$$I = \int_{\pi/2}^{\pi} d\theta \cdot \int_0^{\pi/2} \sin\varphi\cos\varphi\,d\varphi \cdot \int_0^3 6\rho^5\,d\rho.$$

$$I = \frac{\pi}{2} \cdot \frac{1}{2}\sin^2\varphi\Big]_0^{\pi/2} \cdot \rho^6\Big]_0^3.$$

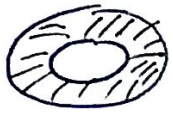
$\rho\cos\varphi$

$\rho\sin\varphi.$

$$I = \frac{\pi}{4} \cdot 729.$$

$z = \pm\sqrt{x^2+y^2}$ conos

$z = \pm\sqrt{k^2-x^2-y^2}$ esferas.



$\theta = 0$

dos esferas.

esfera y cono

$0 \le \rho \le k.$

$0 \le \varphi \le \alpha.$

$0 \le \varphi \le \pi.$

$k_1 \le \rho \le k_2.$