

1. Derivación Implícita. $(1, +1, 4)$ no es parte de S.

$$16 \frac{z}{2} = 2 - 12 = -10 \neq 0$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{+2}{2z} = -\frac{2}{8} = -\frac{1}{4}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{+2}{2z} = -\frac{2}{8} = -\frac{1}{4}$$

Ec. Plano Tangente. $z = 4 + \frac{1}{4}(x-1) + \frac{1}{4}(y-1)$.

b. Derive explícitamente $z = \sqrt{12 + 2x + 2y}$

$$z(1, -1) = \sqrt{12}$$

$$z_x = \frac{1}{\sqrt{12 + 2x + 2y}} = \frac{1}{\sqrt{12}}$$

$$z_y = \frac{1}{\sqrt{12 + 2x + 2y}} = \frac{1}{\sqrt{12}}$$

Ec. Plano Tangente $z = \sqrt{12} + \frac{1}{\sqrt{12}}(x-1) + \frac{1}{\sqrt{12}}(y+1)$

2. $F(x, y) = \cos(yx) + 1 - \sec(zx) - \sin(yz) = 0$

$$\frac{\partial z}{\partial x} = \frac{-y \sin(yx) - z \sec(zx) \tan(zx) + 0}{x \sec(xz) \tan(xz) + y \cos(yz)} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = \frac{-x \sin(yx) - z \cos(yz)}{x \sec(xz) \tan(xz) + y \cos(yz)} = -\frac{F_y}{F_z}$$

$$F(x, y) = \text{constante}$$

$$F(x, y) = G(x, y)$$

Razón de cambio instantánea

$$\frac{dT}{dt}$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt}$$

$$\frac{\partial w}{\partial x} = -\frac{F_x}{F_w}$$

Cuando $t=3$, $x=e^0=1$ $y=\ln(1)+3-2=1$

$$\frac{\partial w}{\partial x} = \frac{1}{1+(yx)^2}, \quad \left. \frac{\partial w}{\partial x} \right|_{(1,1)} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{\partial w}{\partial y} = \frac{x}{1+(yx)^2}, \quad \left. \frac{\partial w}{\partial y} \right|_{(1,1)} = \frac{1}{2}$$

$$\frac{dx}{dt} = 2e^{2t-6}, \quad x'(3) = 2e^0 = 2$$

$$\frac{dy}{dt} = \left(\frac{2}{2t-5} + 1 \right) \quad y'(3) = \frac{2}{1} + 1 = 3$$

$$\frac{dw}{dt} = \frac{1}{2}(2) + \frac{1}{2}(3) = \frac{5}{2}$$

4. Derivada Direccional $D_u T = \nabla T \cdot \vec{u} = \frac{\nabla T \cdot \vec{v}}{|\vec{v}|}$

$$\nabla T = \langle \sin(\pi y z), x \pi z \cos(\pi y z), \pi x y \cos(\pi y z) \rangle$$

$$\nabla T(1,1,2) = \langle \sin(2\pi), 2\pi \cos(2\pi), \pi \cos(2\pi) \rangle$$

$$\nabla T(1,1,2) = \langle 0, 2\pi, \pi \rangle$$

$$\vec{u} = \frac{\langle 1, 4, 8 \rangle}{\sqrt{1+16+64}} = \frac{1}{9} \langle 1, 4, 8 \rangle$$

Vector unitario.

$$|u| = 9$$

$$D_u T = \frac{1}{9} \langle 0, 2\pi, \pi \rangle \cdot \langle 1, 4, 8 \rangle = \frac{8\pi + 8\pi}{9} = \frac{16\pi}{9}$$

6. Presupuesto = 20,000 x periódicos y televisión
gasto gasto.

$$x + y = 20,000$$

Ventas $S = 80x^{1/4}y^{3/4}$ margen de utilidad 10%

$$U = 0.10S - 20,000 = 8x^{1/4}y^{3/4} - 20,000$$

$$J = 80x^{1/4}y^{3/4} - 72x^{1/4}y^{3/4} - 20,000$$

pérdida es de 72.

$$\text{Max } U = 8x^{1/4}y^{3/4} - 20,000, \quad x + y = 20,000$$

$$\text{Lagrange } F(x, y, \lambda) = 8x^{1/4}y^{3/4} - 20,000 + \lambda(20,000 - x - y)$$

$$f_x = 2x^{-3/4}y^{3/4} - \lambda = 0 \Rightarrow \lambda = 2y^{3/4}x^{-3/4}$$

$$f_y = 6x^{1/4}y^{-1/4} - \lambda = 0 \Rightarrow \lambda = 6x^{1/4}y^{-1/4}$$

$$F_\lambda = 20,000 - x - y = 0.$$

$$\text{Lagrange } f(x, y, z) + \lambda[C - g(x, y, z)]$$

$$\text{Igualando } \lambda's: \frac{2y^{3/4}}{x^{3/4}} = \frac{6x^{1/4}}{y^{1/4}} \Rightarrow 2y = 6x$$

Sustituya $y = 3x$ en f_λ . Minimizando la pérdida.

$$20,000 - x - 3x = 0 \quad 4x = 20,000 \Rightarrow x = 5,000.$$

$$y = 15,000$$

No realice la prueba de la 2ª Derivada. $\lambda \approx 4.56$.

$$\text{Utilidad Máxima. } U(5,000, 15,000) = 8 \cdot 5,000^{1/4} 15,000^{3/4} - 20,000 = 9,118.02 - 20,000.$$

$$U = 8x^{1/4}y^{3/4} - 20000 \quad x+y = 20,000$$

Substituya $y = 20,000 - x$ en $U(x)$.

$$U(x) = 8x^{1/4}(20,000-x)^{3/4} - 20 \text{ mil}$$

$$U'(x) = 2x^{-3/4}(20,000-x)^{3/4} - 6x^{1/4}(20,000-x)^{-1/4} = 0$$

$$\frac{2}{x^{3/4}}(20,000-x)^{3/4} = \frac{6x^{1/4}}{(20,000-x)^{1/4}}$$

$$(20,000-x) = 3x$$

$$20,000 = 4x \Rightarrow x = 5,000$$

$$y = 15,000$$

Método 3: Microeconomía.

$$P = x^\alpha y^\beta \quad x+y = 20 \text{ mil}$$

$$\alpha + \beta = 1$$

$$\text{Producción óptima} \quad x = \alpha \cdot 20 \text{ mil} \quad \alpha = 1/4$$

$$y = \beta \cdot 20 \text{ mil} \quad \beta = 3/4$$

$$\frac{P_A}{P_B} = \frac{\alpha y}{\beta x} \quad | \text{Precaución.}$$

Costo: $C = 2x + 4y$.

Ingresos: $I = p_A x + p_B y$.

Utilidad $U = p_A x + p_B y - 2x - 4y$.

Costo Promedio $\frac{C}{x} = 2$.

$$U(p_A, p_B) = p_A(16 - p_A + p_B) + p_B(24 - 2p_A - 4p_B) - 2(16 - p_A + p_B) - 4(24 - 2p_A - 4p_B)$$

$$U(p_A, p_B) = 26p_A - p_A^2 - p_A p_B - 4p_B^2 + 38p_B - 128.$$

$$U_{p_A} = 26 - 2p_A - p_B = 0$$

$$2p_A + p_B = 26$$

$$U_{p_B} = -p_A - 8p_B + 38 = 0$$

$$p_A + 8p_B = 38.$$

$$p_B = 26 - 2p_A \Rightarrow p_A + 26 \cdot 8 - 16p_A = 38.$$

$$p_A = 11.\overline{33}$$

$$26 \cdot 8 - 38 = 15p_A. \quad p_A = \frac{170}{15}$$

$$p_B = 26 - 2(11.\overline{33}) = \frac{10}{3} = 3.\overline{33}$$

Prueba 2da

Derivada:

$$D(x, y) = \begin{vmatrix} -2 & -1 \\ -1 & -8 \end{vmatrix} = 16 - 1 = 15 > 0.$$

$U_{p_A p_A} < 0$ hay un máximo relativo.

Punto de Silla transición entre un máximo relativo y un mínimo relativo.

