TAREA #10 - DAVID CORZO

1.a) $f(x,y) = 2x^2 + 2xy + y^2 - 2x - 3$

$f_x = 4x + 2y - 2$

$4x + 2y - 2 = 0$

$4x + 2y = 2$

$y = \frac{1}{2}(2 - 4x)$

$y = 1 - 2x$

$y = 1 - 2(1)$

$y = -1$

$f_y = 2x + 2y$

$2x + 2y = 0$

$2x + 2(1 - 2x) = 0$

$2x + 2 - 4x = 2$

$\cancel{-}2x = \cancel{-}2$

$x = 1$

Punto crítico en $y = -1$ $x = 1$

$f_{xx} = 4$ $f_{yy} = 2$

$f_{xy} = 2$ $f_{yx} = 2$

$$f(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} = (4)(2) - (2)(2) = 4$$

$f(1,-1) = 2(1)^2 + 2(1)(-1) + (-1)^2 - 2(1) - 3$

$= \cancel{2 - 2} + 1 - 2 - 3$

$= -4$

Mínimo en $f(1,-1) = -4$

1.b) $g(x,y) = \sqrt{x^2 + y^2}$

$g_x = \frac{x}{\sqrt{x^2 + y^2}}$ $g_y = \frac{y}{\sqrt{x^2 + y^2}}$

$$\frac{x}{\sqrt{x^2+y^2}} = 0 \qquad\qquad \frac{y}{\sqrt{x^2+y^2}} = 0$$

$$x = 0 \qquad\qquad y = 0$$

$$g_{xx} = (x^2+y^2)^{-\frac{1}{2}} - \left[\tfrac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot 2x\right] x$$

$$= \frac{1}{\sqrt{x^2+y^2}} - \frac{x^2}{\sqrt{x^2+y^2}}\Bigg|_{(0,0)} = \text{indef.}$$

$$g_{yy} = \frac{1}{\sqrt{x^2+y^2}} - \frac{y^2}{\sqrt{x^2+y^2}}\Bigg|_{(0,0)} = \text{indef.}$$

$$g_{xy} = -\tfrac{1}{\cancel{2}}(x^2+y^2)^{-\frac{3}{2}} \cancel{2}y\,x$$

$$= -\frac{y\,x}{(x^2+y^2)^{\frac{3}{2}}}$$

$$g_{yx} = -\tfrac{1}{\cancel{2}}(x^2+y^2)^{-\frac{3}{2}} \cancel{2}x\,y$$

$$= -\frac{x\,y}{(x^2+y^2)^{\frac{3}{2}}} = \text{indef}$$

$$-\frac{y\,x}{(x^2+y^2)^{\frac{3}{2}}} = 0$$

$$= \text{indef.}$$

$$x = 0$$

$$y = 0$$

Hay punto de silla por que se indefinen todos los componentes de $D(x,y)$.

$\therefore$ No hay mínimo ni máximo

2. a.) $f(x,y) = (x^2+y^2)^{\frac{1}{3}} + 2$

$$f_x = \tfrac{1}{3}(x^2+y^2)^{-\frac{4}{3}} \cdot 2x \qquad f_y = \tfrac{1}{3}(x^2+y^2)^{-\frac{4}{3}} \cdot 2y$$

$$= \frac{2x}{3(x^2+y^2)^{4/3}} \qquad\qquad = \frac{2y}{3(x^2+y^2)^{4/3}}$$

$$\frac{2x}{3(x^2+y^2)^{4/3}} = 0 \qquad\qquad \frac{2y}{3(x^2+y^2)^{4/3}} = 0$$

$$2x = 0 \qquad\qquad 2y = 0$$

$$x = 0 \qquad\qquad y = 0$$

$\therefore$ El punto $(0,0)$ es un punto de silla.

2.b) $g(x,y) = x^2 - 3xy - y^2$

$g_x = 2x - 3y$ $\qquad$ $g_y = -3x - 2y$

$2x - 3y = 0$ $\qquad$ $-3x - 2y = 0$

$$D(x,y) = \begin{vmatrix} 2 & -3 \\ -3 & -2 \end{vmatrix} = 5$$

Punto mínimo en 0

3) $P(L,K) = 2LK - 3K^2 - 2L^2 - 2L + 21K$

$P_L = 2K - 4L - 2$ $\qquad$ $P_K = 2L - 6K + 21$

$2K - 4L - 2 = 0$ $\qquad$ $2L - 6K + 21 = 0$

$L = 1{,}500$ $\qquad$ $K = 4{,}000$

$$D(x,y) = \begin{vmatrix} -4 & 2 \\ 2 & -6 \end{vmatrix} = 20$$

máximo en

$L = 1{,}500$ & $K = 4{,}000$

4)

$q_A = 100 - 5x - 2y$ $\qquad$ $q_B = 250 - 3x - 5y$

$u = x q_A + y q_B - 20 q_A - 10 q_B$

$u = x(100 - 5x - 2y) + y(250 - 3x - 5y) - 20(100 - 5x - 2y) - 10(250 - 3x - 5y)$

$u = -5x^2 - 5y^2 - 5xy + 230x + 340y - 4{,}500$

$u_x = -10x - 5y + 230$ $\qquad$ $u_y = -10y - 5x + 340$

$-10x - 5y + 230 = 0$ $\qquad$ $-10y - 5x + 340 = 0$

$x = 8$ $\qquad$ $y = 30$

$$u(x,y) = \begin{vmatrix} -10 & -5 \\ -5 & -10 \end{vmatrix} = 75$$

Precios de venta maximos $x = 8$ $y = 30$

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6)
- función de producción

$$P(L,K) = 118L + 20K + 3LK - L^2 - 2K^2$$

- L : mano de obra
- K : Capital utilizado
- P : nivel de producción

- Costos unitarios:
  $L = 80$
  $K = 160$
- Presupuesto:
  $5640

$$\nabla f = \lambda \nabla g$$

$$P_L = \lambda g_L \qquad P_L = 118 + 3K - 2L$$
$$P_K = \lambda g_K \qquad g_L = 80$$

$$P_K = 20 + 3L - 4K$$
$$g_K = 160$$

$$118 + 3K - 2L = 80\lambda \qquad 20 + 3L - 4K = 160\lambda$$

$$\frac{1}{80}(118 + 3K - 2L) = \lambda \qquad \frac{1}{160}(20 + 3L - 4K) = \lambda$$

$$\frac{1}{80}(118 + 3K - 2L) = (20 + 3L - 4K)\frac{1}{160}$$

$$160(118 + 3K - 2L) = (20 + 3L - 4K)80$$

$$18{,}880 + 480K - 320L = 1600 + 240L - 320K$$

$$18{,}880 - 1{,}600 = 240L + 320L - 320K + 480K$$

$$17{,}280 = 560L - 160K$$

$$\frac{17{,}280 + 160K}{560} = L$$

- $80L + 160K = 5{,}640$

$$80\left(\frac{17{,}280 + 160K}{560}\right) + 160K = 5{,}640$$

$$\frac{80 \cdot 17{,}280}{560} + \frac{80 \cdot 160K}{560} + 160K = 5640$$

$$\frac{160}{7}K + 160K = 5640 - \frac{80 \cdot 17{,}280}{560}$$

$$\frac{1280}{7}K = \frac{22{,}200}{7}$$

$$K = \frac{\cancel{7} \cdot 22{,}200}{\cancel{7} \cdot 1280}$$

$$K = \frac{555}{32}$$

$$80L + 160\left(\frac{555}{32}\right) = 5{,}640$$

$$L = \frac{5{,}640 - 160\left(\frac{555}{32}\right)}{80}$$

$$L = \frac{573}{16}$$

maximiza en $K = \frac{555}{32}$ & $L = \frac{573}{16}$