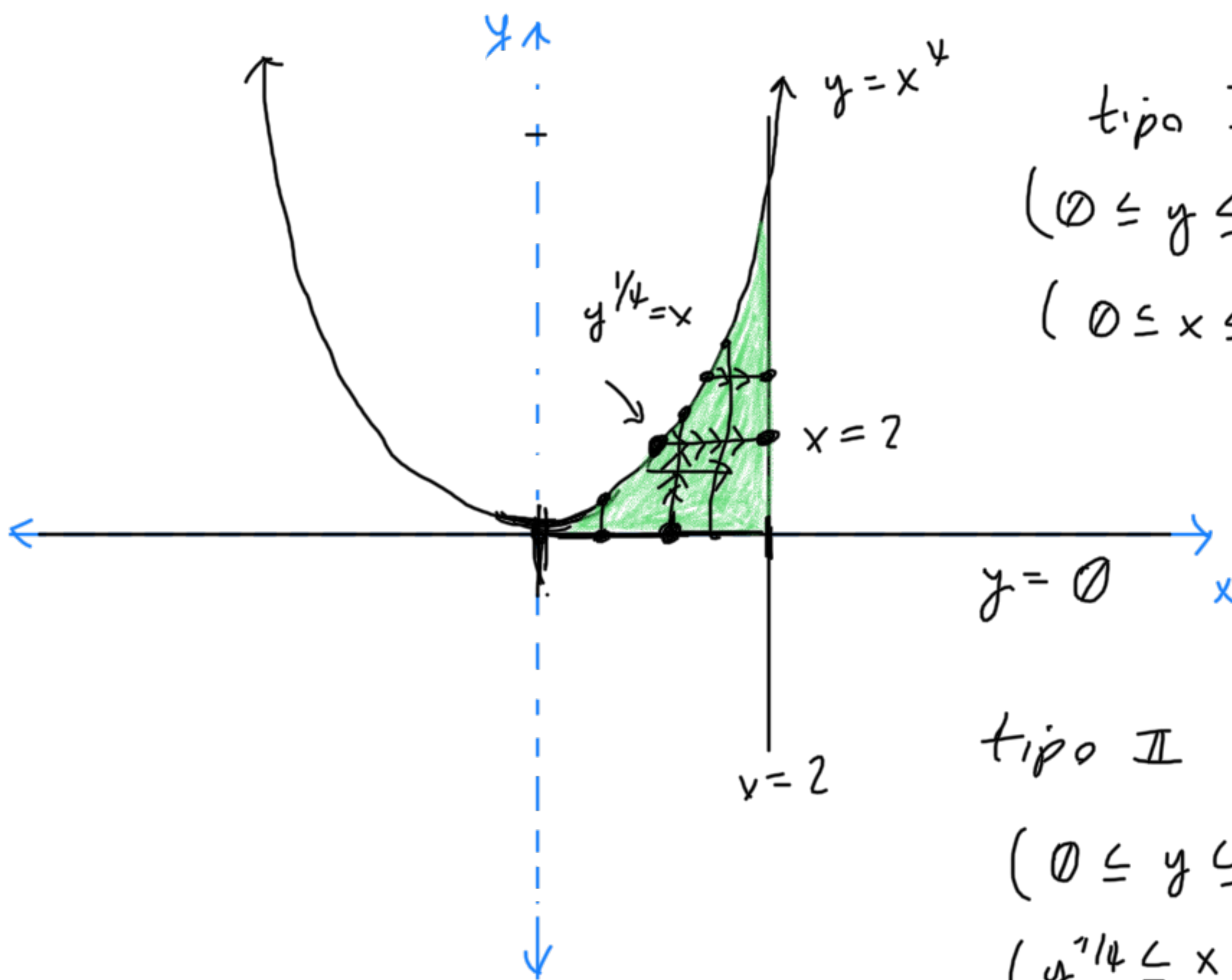
PARCIAL #3 - DAVID CORZO

tipo 1 abajo para arriba $dy\,dx$

tipo 2 izquierda a derecha $dx\,dy$

$x = y^{1/4}$ $\quad x = 2 \quad$ & $\quad y = 0$

a)



tipo I
$(0 \le y \le x^4)$
$(0 \le x \le 2)$

$x^4 = y$

$x = 2$

$y = 0$

tipo II
$(0 \le y \le 16)$
$(y^{1/4} \le x \le 2)$

b)

tipo 1 $\rightarrow$ $$\int_{x=0}^{x=2} \int_{y=0}^{y=x^4} f(x)\, dy\, dx$$

$$D: \{ (0 \le x \le 2) \wedge (0 \le y \le x^4) \}$$

tipo 2 $\rightarrow$ $$\int_{y=0}^{y=16} \int_{x=y^{1/4}}^{x=2} f(x)\, dx\, dy$$

$$D: \{ (y^{1/4} \le x \le 2) \wedge (0 \le y \le 16) \}$$

c)

$$I_1 = 30 \iint_D \sqrt{x^5 + 4} \, dA$$

[redacted] $$\int_{x=0}^{x=2} \int_{y=0}^{y=x^4} 30\sqrt{x^5+4} \, dy \, dx$$

$\boxed{1}$ $$\int_0^{x^4} 30\sqrt{x^5+4} \, dy$$

$$= 30\sqrt{x^5+4} \; y \Big]_0^{x^4} = 30\sqrt{x^5+4}\,(x^4)$$

$\boxed{2}$ $$\int_0^2 30\sqrt{x^5+4} \; x^4 \, dx = \int_{u(0)}^{u(2)} 6\sqrt{u} \, du$$

$$u = x^5 + 4$$
$$\frac{du}{5} = x^4 \, dx$$

$$= \frac{6 \cdot 2}{3} u^{\frac{3}{2}} \Big]_{u(0)}^{u(2)}$$

$$= 4 \, u^{\frac{3}{2}} \Big]_4^{36} = 4 \left\{ (36)^{\frac{3}{2}} - 8 \right\} =$$

$$= 4 \{ 216 - 8 \} = 4 \{ 208 \} = 832$$

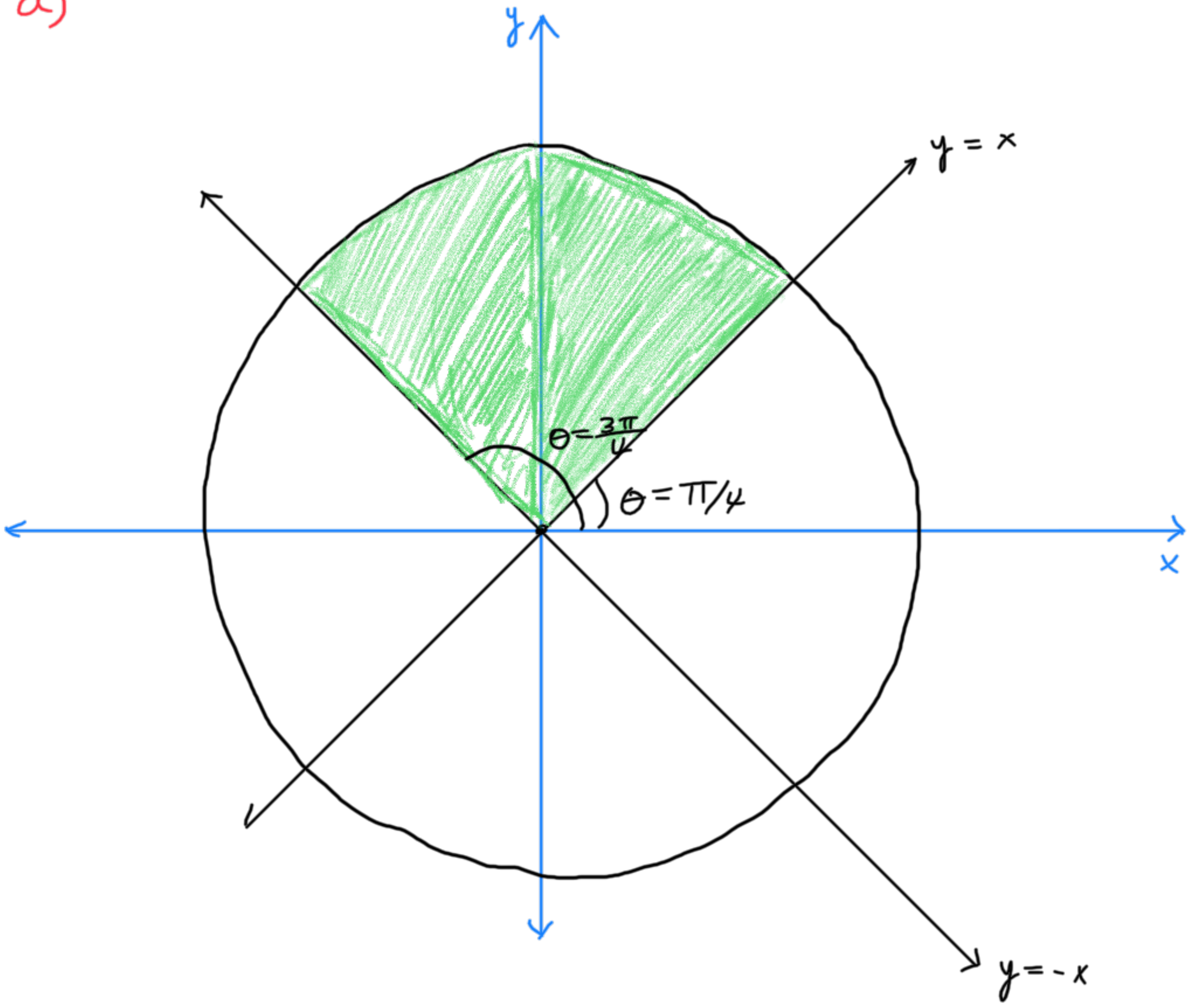
d) [redacted] $$\int_0^2 x^4 \, dx = \frac{x^5}{5} \Big]_0^2 = \frac{1}{5} \{ 2^5 - 0 \} = \frac{32}{5}$$

$$\Rightarrow \int_0^2 \int_0^{x^4} 1 \, dy \, dx = \int_0^2 x^4 \, dx = \frac{32}{5}$$

2) $I_2 = \iint_D \frac{16}{x^2+y^2+1}\,dA$

$D: \{(y=-x) \wedge (y=x) \wedge (0 \leq x^2+y^2 \leq e-1)\}$

a)



$I_2 = \iint_D \frac{16}{\underbrace{x^2+y^2}_{r^2}+1}\,dA$

$\theta = \arctan\left(\frac{y}{x}\right)$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$x^2+y^2 = r^2$

$D: \{(y=-x) \wedge (y=x) \wedge (0 \leq \overbrace{x^2+y^2}^{r^2} \leq e-1)\}$

$\boxed{\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}}$

$0 \leq r^2 \leq e-1$

$\boxed{0 \leq r \leq \sqrt{e-1}}$

$dA = r\,dr\,d\theta$

b) $\int_{\theta=\frac{\pi}{4}}^{\theta=\frac{3\pi}{4}} \int_{r=0}^{r=\sqrt{e-1}} \frac{16}{r^2+1}\, r\,dr\,d\theta$

$\boxed{1}$ $\int_0^{\sqrt{e-1}} \frac{16r}{r^2+1}\,dr = 8\int_0^{\sqrt{e-1}} \frac{du}{u}$

$$u = r^2 + 1$$
$$du = 2r\,dr$$
$$8\,du = 16r\,dr$$

$$= 8 \ln|u| \Big]_{u(0)}^{u(\sqrt{e-1})}$$

$$= 8 \ln|r^2 + 1| \Big]_{0}^{\sqrt{e-1}}$$

$$= 8 \left\{ \ln|(\sqrt{e-1})^2 + 1| - \cancel{\ln|0+1|}^{\,0} \right\}$$

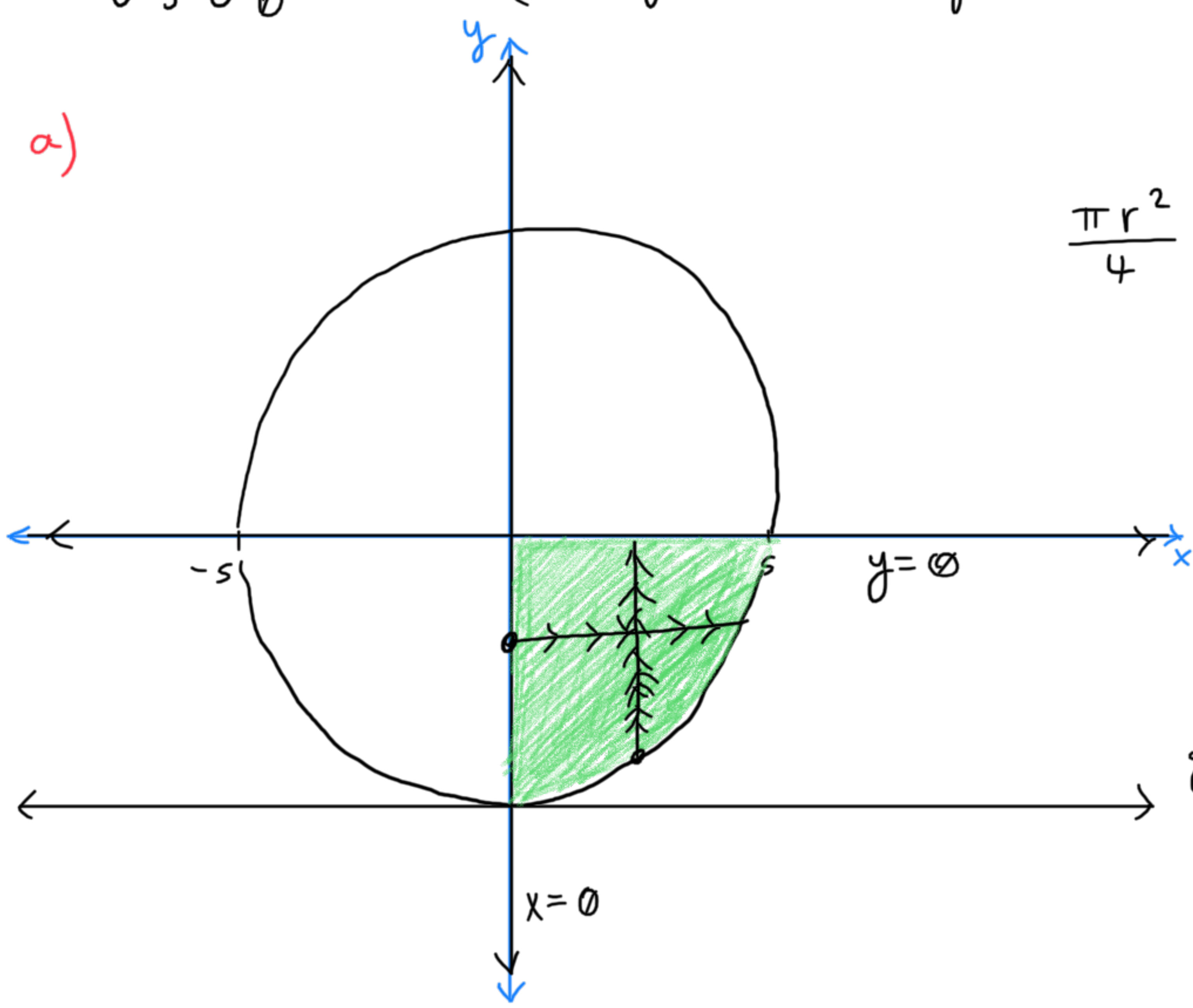
$$= 8 \left\{ \ln|e - \cancel{1} + \cancel{1}| \right\} = 8 \cancel{\ln|e|}^{\,1} = 8$$

$$\boxed{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 8\,d\theta = 8\theta \Big]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = 8\left\{\frac{3\pi}{4} - \frac{\pi}{4}\right\} = 8\left(\frac{\pi}{2}\right)$$

c) $= 4\pi$

3) $$\int_{-5}^{0} \int_{0}^{\sqrt{25-y^2}} 8\left(x^2+y^2\right)^{3/2}\,dx\,dy$$

a)



$$\frac{\pi r^2}{4} = \frac{\pi\, 25}{4}$$

$x = \sqrt{25 - y^2}$ $\quad x = 0 \quad y = -5 \quad y = 0$

$x^2 + y^2 = 5^2$

$r^2 = 5^2$ $\qquad (0 \le r \le 5)$ $\qquad \sqrt{x^2 - 5^2} = y$

$r = 5$

$$\int_{-5}^{0} \int_{0}^{\sqrt{25-y^2}} 8\underbrace{(x^2+y^2)}_{r^2}^{3/2} \, dx \, dy$$

b) tipo 1: abajo para arriba (dy dx)

$$\int_{x=0}^{x=5} \int_{y=\sqrt{x^2-25}}^{y=0} 8(x^2+y^2)^{3/2} \, dy \, dx$$

tipo 2: izquierda a derecha (dx dy)

$$\int_{y=-5}^{y=0} \int_{x=0}^{x=\sqrt{25-y^2}} 8(x^2+y^2)^{3/2} \, dx \, dy$$

$$\int_{-5}^{0} \int_{0}^{\sqrt{25-y^2}} 8(x^2+y^2)^{3/2} \, dx \, dy$$

en polares:

$$\int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} \int_{r=0}^{r=5} 8 \underbrace{(r^2)^{\frac{3}{2}}}_{\underbrace{r^{\cancel{2}\cdot\frac{3}{\cancel{2}}}}_{r^3 \cdot r = r^4}} r \, dr \, d\theta$$

$$\int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} \int_{r=0}^{r=5} 8r^4 \, dr \, d\theta$$

c) $\int^{2\pi} \int^{5} \ldots r^4 \, dr \, d\theta$

$$\int_{\frac{3\pi}{2}} \int_0 8r \; dr\,d\theta$$

$\boxed{1}$ $$\int_0^5 8r^4 \, dr = \frac{8r^5}{5}\Big]_0^5 =$$

$$= \frac{8}{5}\{5^5 - 0\} = \frac{8}{5}\{3125\} = 5{,}000$$

$\boxed{2}$ $$\int_{\frac{3\pi}{2}}^{2\pi} 5{,}000 \, d\theta = 5000\,\theta\Big]_{\frac{3\pi}{2}}^{2\pi} = 5000\left\{2\pi - \frac{3\pi}{2}\right\}$$

$$= \frac{5000}{2}\pi = 2500\pi$$

d) $$\int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} \int_{r=0}^{r=5} 1 \; r\,dr\,d\theta$$

$\boxed{1}$ $$\int_0^5 r\,dr = \frac{r^2}{2}\Big]^5 = \frac{25}{2}$$

$\boxed{2}$ $$\frac{25}{2}\int_{\frac{3\pi}{2}}^{2\pi} 1 \, d\theta = \frac{25}{2}\,\theta\Big]_{\frac{3\pi}{2}}^{2\pi} = \frac{25}{2}\left\{2\pi - \frac{3\pi}{2}\right\}$$

$$= \frac{25}{2}\left\{\frac{\pi}{2}\right\} = \frac{25\pi}{4}$$

fórmula círculo πr^2

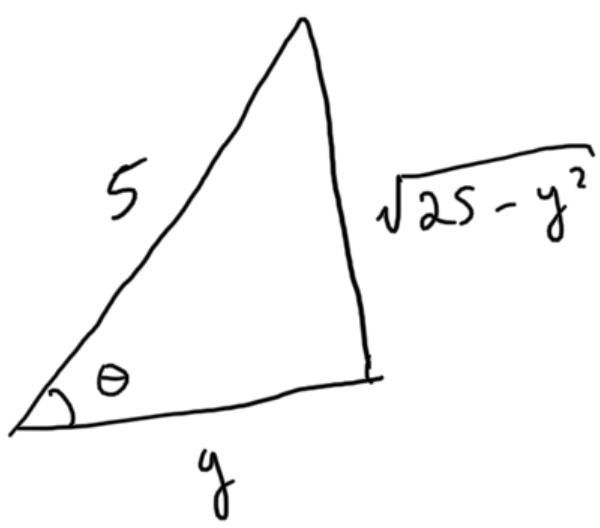
Para este $\frac{\pi r^2}{4} \rightarrow \frac{\pi}{4}(5)^2 = \frac{25\pi}{4}$

integración trigonométrica intento:

$$\rightarrow \int_{y=-5}^{y=0} \int_{x=0}^{x=\sqrt{25-y^2}} 1 \quad dx\,dy$$

$\boxed{1}$ $\int_0^{\sqrt{25-y^2}} dx = \sqrt{25-y^2}$

$\boxed{2}$ $\int_{-5}^{0} \sqrt{25-y^2}\, dy$



$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

$C\frac{H}{O}$ $S\frac{H}{A}$ $C\frac{A}{O}$

$\theta = \cos^{-1}\left(\frac{y}{5}\right)$

$\cos(\theta) = \frac{y}{5}$

$-\sin(\theta) = \frac{1}{5}\frac{dy}{d\theta}$

$-5\sin(\theta)\, d\theta = dy$

$5\sin(\theta) = \sqrt{25-y^2}$

$\int -25\sin(\theta)\sin(\theta)\, dy = -25\int \sin^2(\theta)$

$= -25\int\left(\frac{1-\cos(2\theta)}{2}\right)d\theta$

$= -25\int \frac{1}{2} - \frac{\cos(2\theta)}{2}\, d\theta$

$= -25\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)$

$= -25\left(\cos^{-1}\left(\frac{y}{5}\right) - \frac{\sin\left(2\cos^{-1}\left(\frac{y}{5}\right)\right)}{4}\right]_{-5}^{0}$

$= -25\left\{\cos^{-1}(0) - \frac{\sin\left(2\cos^{-1}(0)\right)}{4} - \right.$

$\cos^{-1}($

$$\cos^{-1}(-1) - \frac{\sin\left(2\cos^{-1}(-1)\right)}{4} \Bigg\}$$

$$= -25\left\{\frac{\pi}{2} - 0 - \pi - 0\right\} = \boxed{\frac{25\pi}{4}}$$