CORTO #14 - DAVID CORZO

1) $I_1 = \int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \underbrace{(x^2+y^2)}_{r^2} \sin\underbrace{(x^2+y^2)}_{r^2} \underbrace{dy\,dx}_{r\,dr\,d\theta}$

$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$y^2 + x^2 = \underbrace{3^2}_{r}$

$r = 3$

$y = 0$

$0 \le y \le \sqrt{9-x^2}$

$\boxed{0 \le r \le 3}$

$0 \le \theta \le \pi$

$$\int_0^{\pi}\int_0^{3} r^2 \sin(r^2)\, r\, dr\, d\theta = \int_0^{\pi}\int_0^{3} r^3 \sin(r)\, dr\, d\theta$$

$\boxed{1}$ $\int_0^3 r^3 \sin(r^2)\, dr$

$u = r^2$ $\quad$ $dv = r\sin(r^2)$

$du = 2r\, dr$ $\quad$ $v = -\frac{1}{2}\cos(r^2)$

$= -\frac{r^2}{2}\cos(r^2) + \int_0^3 r\cos(r^2)\, dr$ $\quad$ $u = r^2$, $\frac{du}{2} = r\,dr$

$= -\frac{r^2}{2}\cos(r^2) + \frac{1}{2}\sin(r^2)\Big]_0^3$

$= \frac{1}{2}\left\{-9\cos(9) + \sin(9)\right\}$

$\boxed{2}$ $\frac{1}{2}\left(-9\cos(9) + \sin(9)\right)\int_0^{\pi} 1\, d\theta$

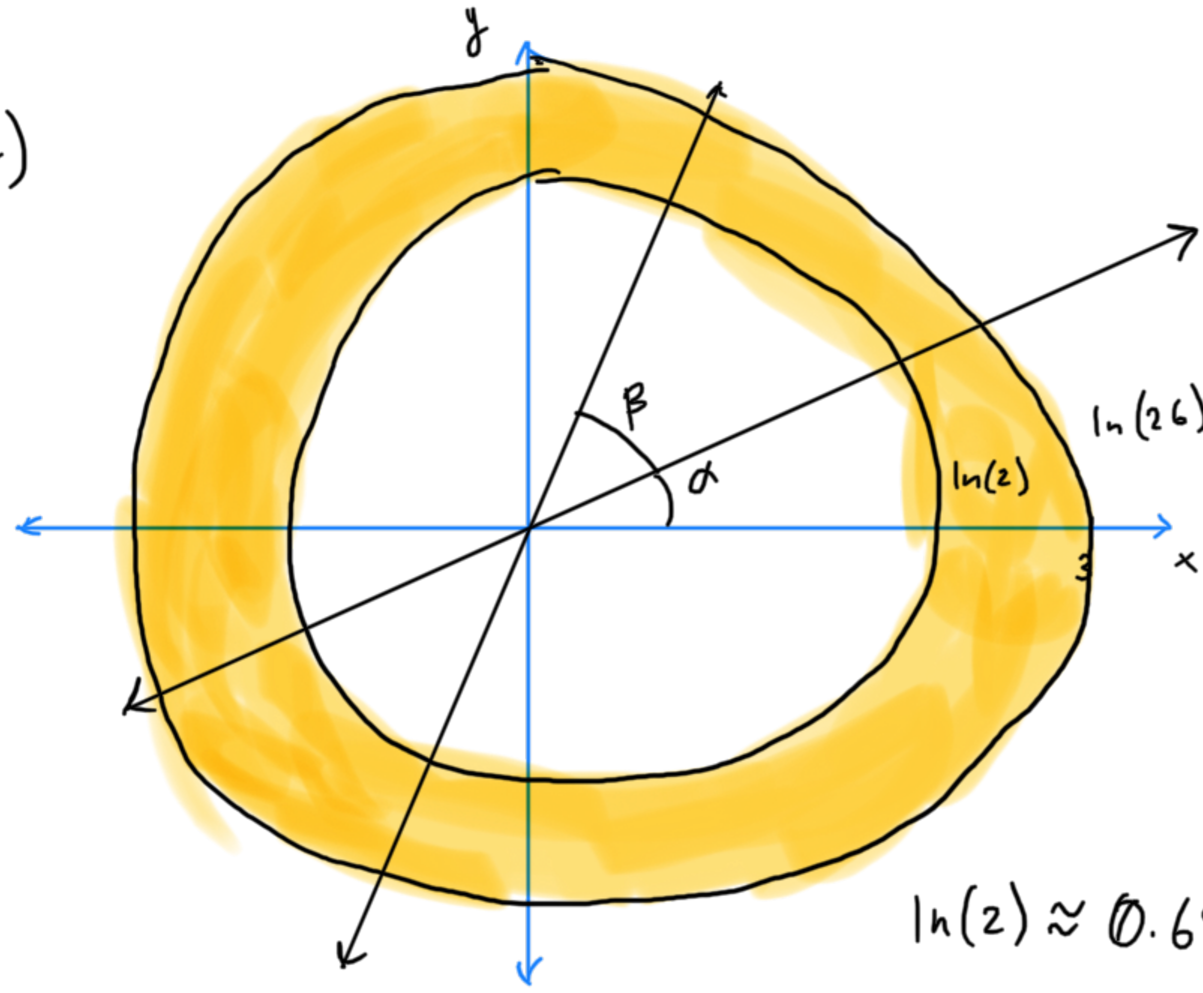
$= \frac{1}{2}\left(-9\cos(9) + \sin(9)\right)\, \theta\Big]_0^{\pi}$

$= \frac{1}{2}\left(-9\cos(9) + \sin(9)\right)(\pi - 0)$

$$= \frac{\pi}{2}\left(-9\cos(9) + \sin(9)\right)$$

2) $\iint_D e^{x^2+y^2}\, dA$ $\qquad D: \left\{ \left(y = \frac{x}{\sqrt{3}}\right) \wedge \left(y = \sqrt{3}x\right) \wedge \left(\ln(2) \le x^2+y^2 \le \ln(26)\right) \right\}$

a)



$y = \frac{\sqrt{3}}{\sqrt{3}} = 1$

$y = \sqrt{3}\,\frac{1}{\sqrt{3}} = 1$

$\ln(2) \approx 0.69$

$\ln(26) \approx 3.25$

$\sqrt{3}\,y = x$

$y = \sqrt{3}x$

$\ln(2) \le \underbrace{x^2 + y^2}_{r^2} \le \ln(26)$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \frac{\pi}{3}$

$\sqrt{\ln(2)} \le r \le \sqrt{\ln(26)}$

$\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = \frac{\pi}{6}$

b)

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \int_{\sqrt{\ln(2)}}^{\sqrt{\ln(26)}} e^{r^2}\, r\, dr\, d\theta$$

$\boxed{1}$ $\int_{\sqrt{\ln(2)}}^{\sqrt{\ln(26)}} e^{r^2}\, r\, dr \quad = \frac{1}{2}\int e^u\, du = \frac{e^u}{2} = \frac{e^{r^2}}{2}\Big]_{\sqrt{\ln(2)}}^{\sqrt{\ln(26)}} =$

$u = r^2$

$\frac{du}{2} = r\, dr$

c)

$= \frac{1}{2}\left\{ e^{(\ln(26))^2} - e^{(\ln(2))^2} \right\} = \frac{1}{2}\left\{ 26 - 2 \right\}$

$= 12$

$\boxed{2}$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 12 \, d\theta = 12\,\theta \Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{12\pi}{3} - \frac{12\pi}{6} = 2\pi$

3) $I_3 = \int_0^{e-1} \left(\int_{\ln(y+1)}^{1} 3\sqrt{e^x - x}\, dx \right) dy$

$x = \ln(y+1)$ $\boxed{x = 1}$

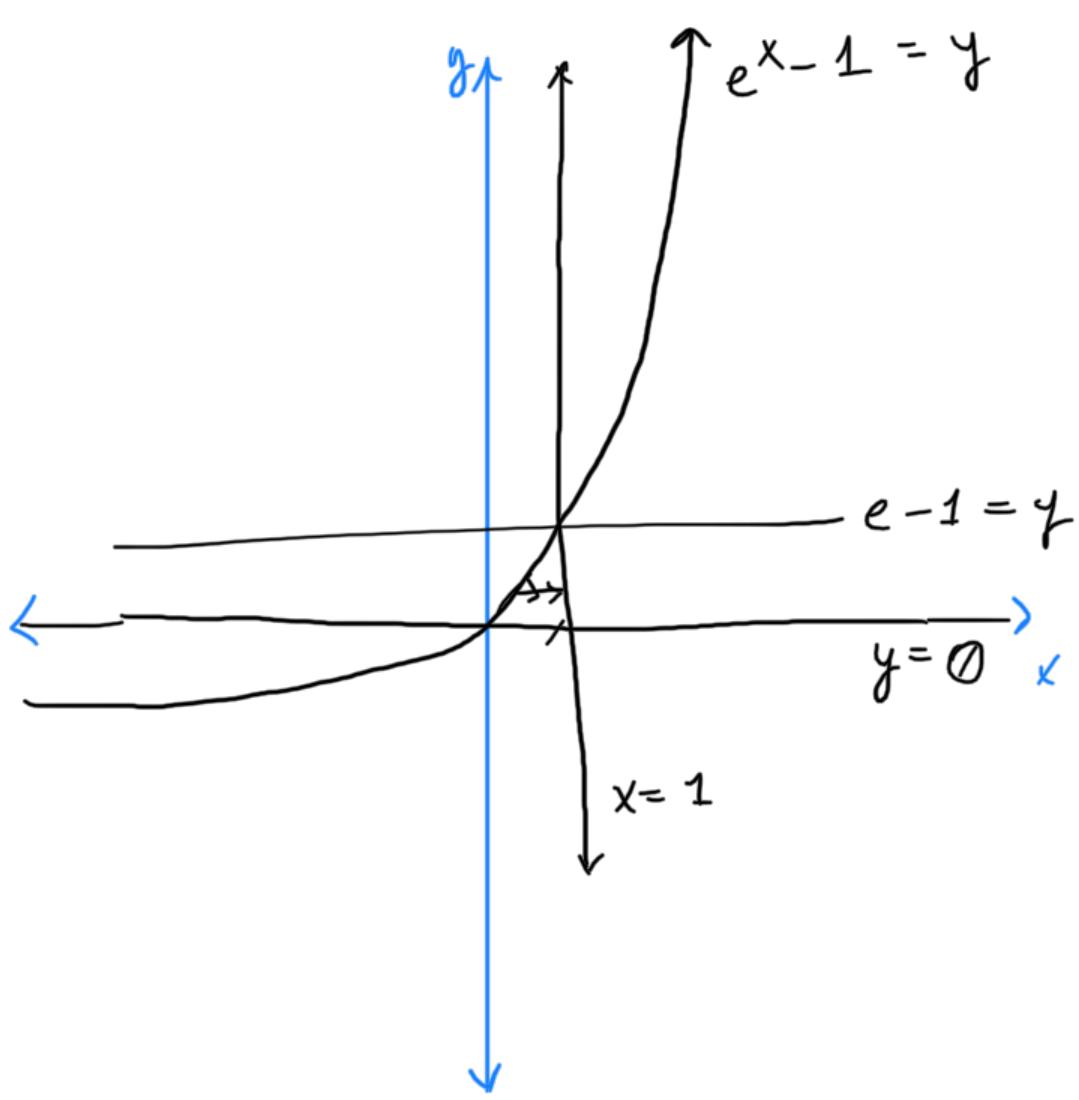
$e^x = y + 1$

$\boxed{e^x - 1 = y}$

$y = e - 1$

$y = 0$

a)



b) tipo 1: respecto y:

$\int^{x=1} \int^{e^x - 1 = y}$

$$\int_{x=0}^{} \int_{0=y}^{} 3\sqrt{e^x - x} \; dy \, dx$$

tipo 2: respecto x:

$$\int_{y=0}^{y=e-1} \int_{x=\ln(y+1)}^{x=1} 3\sqrt{e^x - x} \, dx \, dy$$

c) $\boxed{1}$ $\displaystyle\int_0^{e^x-1} 3\sqrt{e^x - x} \, dy = \left. 3\sqrt{e^x - x} \; y \right]_0^{e^x - 1}$

$$= 3\sqrt{e^x - x}\,\{e^x - 1\}$$

$\boxed{2}$ $\displaystyle 3\int \sqrt{e^x - x} \; e^x - 1 \; dx = 3\int u^{\frac{1}{2}} du = \frac{3}{\left(\frac{2}{3}\right)} u^{\frac{3}{2}}.$

$u = e^x - x$

$du = e^x - 1 \; dx$

$$= \frac{9}{2} u^{\frac{3}{2}} = \left. \frac{9}{2}(e^x - x)^{\frac{3}{2}} \right]_0^1$$

$$= \frac{9}{2}\{(e^1 - 1) - (e^0 - 0)\} = \frac{9}{2}\{e - 1 - 1\}$$

d) $= \dfrac{9}{2}\{e - 2\}$ área