TAREA #4 - DAVID CORZO - 20190432 - 2020-02-05

1) Planos $\underbrace{x + 3y + 2z = 3}_{\hat{n}_1}$ & $\underbrace{-2x + y + 3z}_{\hat{n}_2} = 8$

a) Encontrar el ángulo de intersección.

$$\hat{n}_1 = \langle 1, 3, 2 \rangle$$

$$\hat{n}_2 = \langle -2, 1, 3 \rangle$$

$$\cos\theta = \frac{\hat{n}_1 \cdot \hat{n}_2}{|\hat{n}_1||\hat{n}_2|}$$

$$\cos\theta = \frac{(1 \cdot -2) + (3 \cdot 1) + (2 \cdot 3)}{\left[\sqrt{(1)^2 + (3)^2 + (2)^2}\right]\left[\sqrt{(-2)^2 + (1)^2 + (3)^2}\right]}$$

$$= \frac{-2 + 3 + 6}{\left(\sqrt{1 + 9 + 4}\right)\left(\sqrt{4 + 1 + 9}\right)}$$

$$= \frac{7}{\sqrt{14} \cdot \sqrt{14}} = \frac{7}{\ }$$

$$\theta = \cos^{-1}\left(\frac{7}{14}\right) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$ ✓

B) Recta de intersección:

Resta de ecuaciones

$r = \vec{r_0} + t\underbrace{\vec{v}}_{\text{vector director}}$

$$2(x + 3y + 2z = 3)$$
$$-2x + y + 3z = 8$$

$$2x + 6y + 4z = 6$$
$$-2x + y + 3z = 8$$

$$\tfrac{1}{7}(0x + 7y + 7z = 14)$$

$$y + z = 2$$
$$y = 2 - z$$

Encuentro dos puntos en común para
encontrar el vector director

Cuando $\boxed{z = 0}$ $\boxed{y = 2}$ $\langle -3, 2, 0\rangle$

$$x = 3 - 3(2) - 2(0)$$
$$x = 3 - 6$$
$$\boxed{x = -3}$$

Cuando $\boxed{z = 1}$ $\boxed{y = 1}$ $\langle -2, 1, 1\rangle$

$$x = 3 - 3(1) - 2(1)$$
$$x = 3 - 3 - 2$$
$$\boxed{x = -2}$$

$$\vec{v} = \overrightarrow{PQ} = \langle(-2+3), (1-2), (1-0)\rangle$$
$$\vec{v} = \langle 1, -1, 1\rangle$$

$$\vec{r}_0 = \langle -3, 2, 0\rangle - t\langle 1, -1, 1\rangle$$ ✓

2) Considere: $P(-2,5,7)$ & $Q(1,3,4)$.

¿Es perpendicular $A(4,3,2)$ $B(3,-1,8)$

$$\vec{u} = \overrightarrow{PQ} = \langle (-2-1), (5-3), (7-4) \rangle$$

$$\vec{w} = \overrightarrow{AB} = \langle (4-3), (3+1), (2-8) \rangle$$

$$\vec{u} = \langle -3, 2, 3 \rangle$$

$$\vec{w} = \langle 1, 4, -6 \rangle$$

$$\begin{aligned}
\vec{u} \cdot \vec{w} &= \langle -3, 2, 3 \rangle \cdot \langle 1, 4, -6 \rangle \\
&= (-3 \cdot 1) + (2 \cdot 4) + (3 \cdot -6) \\
&= -3 + 8 - 18 \\
&= -21 + 8 \\
&= -13
\end{aligned}$$

No son perpendiculares

---

3) Encuentre la ecuación del plano: $A(0,1,1)$ & $B(1,0,1)$ & $C(1,1,0)$:

$$\begin{aligned}
\vec{u} = \overrightarrow{AB} &= \langle (1-0), (0-1), (1-1) \rangle \\
&= \langle 1, -1, 0 \rangle
\end{aligned}$$

$$\begin{aligned}
\vec{w} = \overrightarrow{AC} &= \langle (1-0), (1-1), (0-1) \rangle \\
&= \langle 1, 0, -1 \rangle
\end{aligned}$$

$$\hat{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \ldots$$

$$\ldots = \hat{\imath}\left[(-1\cdot -1) - (0\cdot 0)\right] - \hat{\jmath}\left[(1\cdot -1) - (1\cdot 0)\right] + \hat{k}\left[(1\cdot 0) - (1\cdot -1)\right]$$

$$= \hat{\imath}[1] - \hat{\jmath}[-1] + \hat{k}[1]$$

$$= \langle 1, 1, 1 \rangle$$

$$\hat{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

$$\hat{\imath}(x - x_0) + \hat{\jmath}(y - y_0) + \hat{k}(z - z_0) = 0$$

$$1(x-1) + 1(y-1) + 1(z-1) = 0$$

$$x - 1 + y - 1 + z - 1 = 0$$

$$\boxed{x + y + z - 3 = 0}$$

---

4) Encuentre la ec. del plano que pasa $Q(1, 4, -7)$ & contiene a $z = 2y = 3x$

#Empieza en el origen $(0,0,0)$ ¿es porque si

#$z = 0 \rightarrow y = 0 \rightarrow x = 0$

$$z = 2y$$
$$z = 3x$$

$$\vec{w} = \underbrace{\left\langle \frac{1}{3}, \frac{1}{2}, 1 \right\rangle}_{\text{El recíproco}}$$

$P(0,0,0)$
$Q(1,4,-7)$

$$\vec{u} = \overrightarrow{PQ} = \langle (1-0), (4-0), (-7-0) \rangle$$

$$\vec{u} = \langle 1, 4, -7 \rangle$$

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{3} & \frac{1}{2} & 1 \\ 1 & 4 & -7 \end{vmatrix} = \quad \ldots$$

$$= \hat{i}\left[\left(\tfrac{1}{2} \cdot -7\right) - (1 \cdot 4)\right] - \hat{j}\left[\left(\tfrac{1}{3} \cdot -7\right) - (1 \cdot 1)\right] + \hat{k}\left[\left(\tfrac{1}{3} \cdot 4\right) - \left(\tfrac{1}{2} \cdot 1\right)\right] =$$

$$= -\frac{15}{2}\hat{i} + \frac{10}{3}\hat{j} + \frac{5}{6}\hat{k}$$

$$\vec{u} \times \vec{w} = \left\langle -\frac{15}{2}, \frac{10}{3}, \frac{5}{6} \right\rangle$$

$$= -\frac{15}{2}\left(x - \overbrace{x_0}^{q.x}\right) + \frac{10}{3}\left(y - \overbrace{y_0}^{q.y}\right) + \frac{5}{6}\left(z - \overbrace{z_0}^{q.z}\right)$$

$$= -\frac{15}{2}(x-1) + \frac{10}{3}(y-4) + \frac{5}{6}(z+7)$$

5) Considere los planos:

$P_1: \; 3x + 6y - 3z = 3$ $\qquad$ $P_2: \; 2y = x - z - 2$

$P_3: \; 4x - 12y + 5z = 8$ $\qquad$ $P_4: \; 9y = 3x + 6z - 6$

a) ¿Paralelas?

b) ¿idénticas?

$(P_1 \& P_3) \vee (P_3 \& P_1)$:

$$\begin{aligned} 3x + 6y - 3z &= 3 \\ 4x - 12y + 5z &= 8 \end{aligned}$$

$$x\left(\begin{matrix}3\\4\end{matrix}\right. \quad \left(\begin{matrix}6\\-12\end{matrix}\right. \quad \left(\begin{matrix}-3\\5\end{matrix}\right. \quad \left(\begin{matrix}3\\8\end{matrix}\right.$$

no son paralelas

$(P_1 \& P_2) \vee (P_2 \& P_1)$:

$$\begin{aligned} 3x + 6y - 3z &= 3 \\ x - 2y - z &= 2 \end{aligned}$$

$$\left(\begin{matrix}3\\1\end{matrix}\right. \quad \left(\begin{matrix}6\\-2\end{matrix}\right. \quad \left(\begin{matrix}-3\\-1\end{matrix}\right. \quad \left(\begin{matrix}3\\2\end{matrix}\right.$$

no son paralelas

$(P_1 \& P_4) \vee (P_4 \& P_1)$:

$$\begin{aligned} 3x + 6y - 3z &= 3 \\ 3x - 9y + 6z &= 6 \end{aligned}$$

$$\left(\begin{matrix}3\\3\end{matrix}\right. \quad \left(\begin{matrix}6\\-9\end{matrix}\right. \quad \begin{matrix}-3\\6\end{matrix} \quad \begin{matrix}3\\6\end{matrix}$$

No son paralelas

$(P_2 \& P_3) \vee (P_3 \& P_2)$:

$$\begin{aligned} x - 2y - 7 &= 2 \\ 4x - 12y + 5z &= 8 \end{aligned}$$

$$\left(\begin{matrix}1\\4\end{matrix}\right. \quad \left(\begin{matrix}-2\\-12\end{matrix}\right. \quad \left(\begin{matrix}-7\\5\end{matrix}\right. \quad \left(\begin{matrix}2\\8\end{matrix}\right.$$

No son paralelas

$(P_2 \& P_4) \vee (P_4 \& P_2)$:

$$\begin{aligned} x - 2y - z &= 2 \\ 3x - 9y + 6z &= 6 \end{aligned}$$

$$\left(\begin{matrix}1\\3\end{matrix}\right. \quad \left(\begin{matrix}-2\\-9\end{matrix}\right. \quad \left(\begin{matrix}-1\\6\end{matrix}\right. \quad \left(\begin{matrix}2\\6\end{matrix}\right.$$

No son paralelas

$(P_3 \& P_4) \vee (P_4 \& P_3)$:

$$\begin{aligned} 4x - 12y + 5z &= 8 \\ 3x - 9y + 6z &= 6 \end{aligned}$$

$$\left(\begin{matrix}4\\3\end{matrix}\right. \quad \left(\begin{matrix}-12\\-9\end{matrix}\right. \quad \left(\begin{matrix}5\\6\end{matrix}\right. \quad \left(\begin{matrix}8\\6\end{matrix}\right.$$

No son paralelas

b) No hay identicas

b)

$\mathcal{L}_1: \quad x = 1 + 6t$
$\quad\quad y = 1 - 3t$
$\quad\quad z = 12t + 5$

$\mathcal{L}_2: \quad 2x - 2 = 4 - 4y = z + 1$

$\mathcal{L}_3: \quad x = 1 + 2t$
$\quad\quad y = t$
$\quad\quad z = 1 + 4t$

$\mathcal{L}_4: \quad r = \langle 3, 1, 5 \rangle + t \langle 4, 2, 8 \rangle$

a) ¿Paralelas?
b) ¿idénticas?

if (paralelas) {
verificar si son identicas,
todos tienen que ser
iguales;
} else {
¬paralelas & ¬idénticas;
}

$\mathcal{L}_1$ & $\mathcal{L}_3$:

$\mathcal{L}_1: \quad x = 1 + 6t$
$\quad\quad y = 1 - 3t$
$\quad\quad z = 12t + 5$

$\mathcal{L}_3: \quad x = 1 + 2t$
$\quad\quad y = t$
$\quad\quad z = 1 + 4t$

$\mathcal{L}_1$ & $\mathcal{L}_3$ No son paralelas

L_1 & L_2 :

L_1 :
$$x = 1 + 6t$$
$$y = 1 - 3t$$
$$z = 12t + 5$$

L_2 :
$$t = 2x - 2 \Rightarrow x = \frac{t + 2}{2}$$
$$t = 4 - 4y \Rightarrow y = \frac{4 - t}{4}$$
$$t = z + 1 \Rightarrow z = t - 1$$

L_1 & L_2 son paralelas

Agarro los coheficientes (1, 1, 5)

$$\left.\begin{aligned} x &= \frac{1 + 2}{2} = \frac{3}{2} \\ y &= \frac{4 - 1}{4} = \frac{3}{4} \\ z &= 5 - 1 = 4 \end{aligned}\right\} \text{No iguales}$$

L_1 & L_2 : Son paralelas pero no iguales

L_1 & L_4 :

L_1 : $x = 1 + 6t$

$y = 1 - 3t$

L_4 : $r = \langle 3, 1, 5 \rangle + t\langle 4, 2, 8 \rangle$

$x = 3 + 4t$

$$z = 12t + 5 \qquad\qquad x = 3 + 4t$$
$$y = 1 + 2t$$
$$z = 5 + 8t$$

L_1 & L_4 no son paralelas.

L_3 & L_4:

L_3: $x = 1 + 2t$
$y = t$
$z = 1 + 4t$

L_4: $r = \langle 3, 1, 5\rangle + t\langle 4, 2, 8\rangle$
$x = 3 + 4t$
$y = 1 + 2t$
$z = 5 + 8t$

de L_3 extraigo pt. $(1, 0, 1)$

$x = 3 + 4 = 7$

$y = 1 + 0 = 1$

$z = 5 + 8 = 13$

L_3 & L_4 son paralelas pero no iguales

L_3 & L_2:

L_2: $x = \dfrac{t + 2}{2}$

$y = \dfrac{4 - t}{4}$

$z = t - 1$

L_4: $x = 3 + 4t$
$y = 1 + 2t$
$z = 5 + 8t$

...alelas.

$\mathcal{L}_3$ & $\mathcal{L}_2$ no son para...

7) a)

$\mathcal{L}_1$: $x = 3 + 2t$
$y = 4 - t$
$z = 12t + 5$

$\mathcal{L}_2$: $x = 1 + 4s$
$y = 3 - 2s$
$z = 4 + 5s$

$\mathcal{L}_1$: $(3,4,1) + t(2,-1,3)$ $\quad \vec{v} = \langle 2,-1,3 \rangle$

$\mathcal{L}_2$: $(1,3,4) + s(4,-2,5)$ $\quad \vec{u} = \langle 4,-2,5 \rangle$

a.1) ¿Paralelas?

$$\underbrace{\vec{u} = k\vec{v}}_{\text{Para ser paralelas}}$$

$$(4,-2,5) = k(2,-1,3)$$

$4 = 2k \Rightarrow k = 2$

$-2 = -1k \Rightarrow k = 2$

$5 = 3k \Rightarrow k = 5/3$

$\therefore$ No son vectores paralelos

a.2) ¿Pts en común?