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Corto #7 Cálculo Multivariable

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1. Encuentre el dominio de la función vectorial $\vec{r}(t) = \dfrac{t+4}{t-4}\hat{\imath} + \dfrac{2}{\sqrt{1-t}}\hat{\jmath} + \ln(t+2)\hat{k}$.

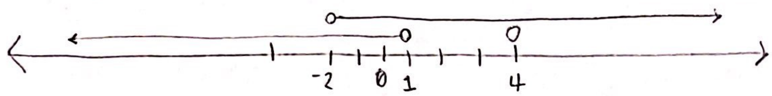
$\vec{r}(t) = \left\langle \dfrac{t+4}{t-4}, \dfrac{2}{\sqrt{1-t}}, \ln(t+2) \right\rangle$

$t - 4 \neq 0$ $\qquad$ $1 - t > 0$ $\qquad$ $t + 2 > 0$

$t \neq 4$ $\qquad$ $-t > -1$ $\qquad$ $t > -2$

$t < 1$

$\therefore \{ \vec{r}(t) \in \mathbb{R} \mid (t \neq 4) \land (t < 1) \land (t > -2) \}$



$ID: (-2, 1)$

2. Considere la función $g(x,y) = \dfrac{1}{\sqrt{9-x^2-y^2}}$.

(a) Encuentre y bosqueje el dominio de g.

(b) Encuentre y bosqueje las curvas de nivel para $k = \dfrac{1}{\sqrt{5}}, \dfrac{1}{\sqrt{8}}, \dfrac{1}{\sqrt{9}}$. en otra hoja

a) $g(x,y) = \dfrac{1}{\sqrt{9-x^2-y^2}}$

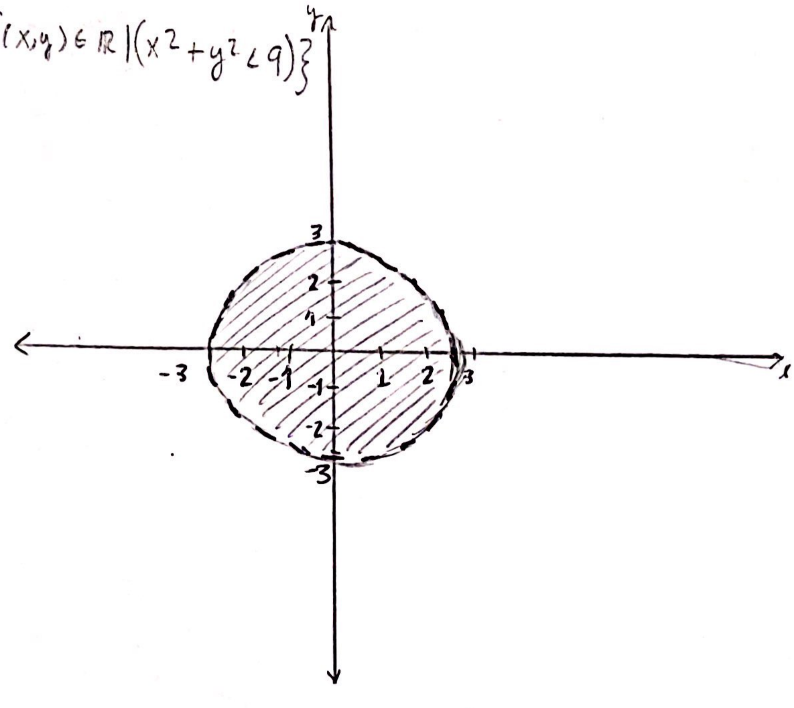
$9 - x^2 - y^2 > 0$

$-x^2 - y^2 > -9$

$x^2 + y^2 < 9$

a) $x^2 + y^2 < 3^2$

$ID: \{(x,y) \in \mathbb{R} \mid (x^2 + y^2 < 9)\}$



3. Considere la función de producción $P = 10e^{K+L-KL}$

(a) Encuentre las funciones de productividad marginal del trabajo y del capital.
L K

(b) Encuentre y simplifique $P_{LL} + P_{KK}$.

a) $P_L = 10\, e^{(K+L-KL)} \cdot (1-K)$

$P_K = 10\, e^{(K+L-KL)} \cdot (1-L)$

b) $10\, e^{K+L-KL} \cdot (2-2K+K^2-2L+L^2)$

b) $P_L = 10\, e^{(K+L-KL)} \cdot (1-K)$
$= 10 e^{(K+L-KL)} - 10K e^{(K+L-KL)}$

$P_{LL} = 10 e^{(K+L-KL)} \cdot (1-K) - 10 K e^{(K+L-KL)} \cdot (1-K)$

$P_K = 10 e^{(K+L-KL)} - 10 L e^{(K+L-KL)}$

$P_{KK} = 10 e^{(K+L-KL)} \cdot (1-L) - 10 L e^{(K+L-KL)} \cdot (1-L)$

continuación en hoja

4. Encuentre la longitud de arco de la curva descritas por las función vectorial $s(t) = \langle t\sin t + \cos t,\ t\cos t - \sin t\rangle$, $-2 \leqslant t \leqslant 0$

$s'(t) = \langle \cancel{\sin(t)} + t\cos(t) - \cancel{\sin(t)},\ \cancel{\cos(t)} - t\sin(t) - \cancel{\cos(t)} \rangle = \langle t\cos(t), -t\sin(t)\rangle$

$|s'(t)| = \sqrt{(t\cos(t))^2 + (-t\sin(t))^2}$
$= \sqrt{t^2\cos^2(t) + t^2\sin^2(t)}$
$= \sqrt{t^2} = |t|$

$\sqrt{t^2} = \begin{cases} t & t \geqslant 0 \\ -t & t < 0 \end{cases}$

$L = \int_{-2}^{0} t\, dt = -\frac{1}{2} t^2 \Big|_{-2}^{0} = -\frac{1}{2}\{0^2 - (-2)^2\} = -\frac{1}{2}(-2)^2 = 2$

$\boxed{L = 2}$

5. La curva C es descrita por la función vectorial $\mathbf{r}(t) = \left\langle \ln(t-2), \frac{1}{t-4}, 10\tan^{-1} t \right\rangle$.

(a) Encuentre la ec. vectorial de la recta tangente a C en $t = 3$.

(b) ¿Es perpendicular la recta tangente a la recta $2x - 1 = \frac{z}{-3}$, $y = 10$?

$$\vec{r}_T = \vec{r}(a) + t\vec{r}'(a)$$

$$\vec{r}'(t) = \left\langle \frac{1}{t-2}, -1(t-4)^{-2}, \frac{10}{t^2+1} \right\rangle$$

$$\vec{r}'(3) = \left\langle \frac{1}{3-2}, -1(3-4)^{-2}, \frac{10}{9+1} \right\rangle = \left\langle 1, -\frac{1}{(-1)^2}, \frac{10}{10} \right\rangle = \langle 1, -1, 1 \rangle$$

$$\vec{r}(3) = \langle 0, -1, 10\tan^{-1}(3) \rangle$$

a) $\vec{r}_T = \langle 0, -1, 10\tan^{-1}(3) \rangle + t\langle 1, -1, 1 \rangle$

b) No es perpendicular.

6. Una partícula dentro de un campo eléctrico experimenta la siguiente aceleración.

$$\mathbf{a}(t) = -6t^2\mathbf{i} + 8\sinh(2t)\mathbf{j} + e^{t/2}\mathbf{k}$$

(a) Encuentre la función de velocidad si $\mathbf{v}(0) = 10\mathbf{i} - 2\mathbf{k}$.

(b) Encuentre la función de posición si $\mathbf{r}(0) = 10\mathbf{j}$.

$$v(t) = \int a(t)\,dt = \int \langle \underbrace{-6t^2}_{f1}, \underbrace{8\sinh(2t)}_{2}, \underbrace{e^{\frac{t}{2}}}_{3} \rangle\, dt$$

a) $v(t) = \langle -2t^3 + 10,$
$4\cosh(2t) - 4,$
$2e^{\frac{t}{2}} - 4 \rangle$

$$\int f_1\,dt = -6\int t^2\,dt = -\frac{6}{3}t^3 + C_1$$

$$\int f_2\,dt = 8\int \sinh(2t)\,dt = \frac{8}{2}\int \sinh(u)\,du = 4\cosh(2t) + C_2$$

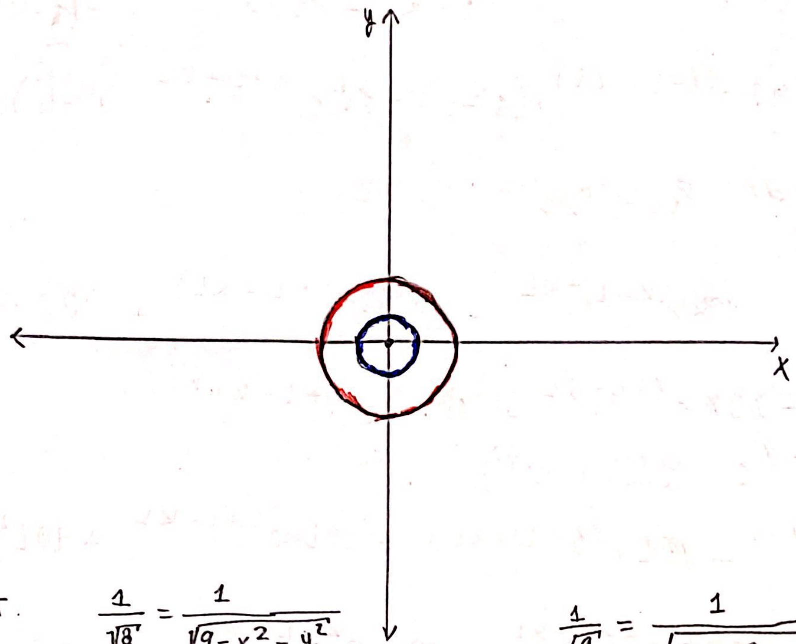
$u = 2t$
$du = 2dt$
$\frac{1}{2}du = dt$

b) $r(t) = \langle -\frac{1}{2}t^4 + 10t,$

$2\sinh(2t) - 4t + 10,$
$4e^{\frac{t}{2}} - 4t \rangle$

$$\int f_3\,dt = \int e^{\frac{t}{2}}dt = 2\int e^u\,du = 2e^{\frac{t}{2}} + C_3$$

$u = \frac{t}{2}$
$du = \frac{1}{2}dt \Rightarrow 2\,du = dt$



$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{9-x^2-y^2}}$$

$$\frac{\sqrt{9-x^2-y^2}}{\sqrt{5}} = 1$$

$$\sqrt{9-x^2-y^2} = \sqrt{5}$$

$$9-x^2-y^2 = 5$$

$$-x^2-y^2 = 5-9$$

$$-x^2-y^2 = -4$$

$$x^2+y^2 = 4$$

$$x^2+y^2 = 2^2$$

$$\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{9-x^2-y^2}}$$

$$\sqrt{9-x^2-y^2} = \sqrt{8}$$

$$9-x^2-y^2 = 8$$

$$-x^2-y^2 = 8-9$$

$$x^2+y^2 = 1$$

$$\frac{1}{\sqrt{9}} = \frac{1}{\sqrt{9-x^2-y^2}}$$

$$\sqrt{9-x^2-y^2} = \sqrt{9}$$

$$9-x^2-y^2 = 9$$

$$x^2+y^2 = 0$$

3) $P_{LL} + P_{KK} = 10e^{(K+L-KL)} \cdot (1-K) - 10Ke^{(K+L-KL)} \cdot (1-K) + ($ ↳

$10e^{(K+L-KL)} \cdot (1-L) - 10Le^{(K+L-KL} \cdot (1-L)$

Simplificación de $P_{LL} + P_{KK}$

$P_{LL} = 10e^{K+L-KL} - 10Ke^{K+L-KL} - 10Ke^{(K+L-KL)} + 10K^2e^{(K+L-KL)}$

$= 10e^{K+L-KL} - 20Ke^{K+L-KL} + 10K^2e^{(K+L-KL)}$

$= 10e^{K+L-KL}(1 - 2K + K^2)$

$P_{KK} = 10e^{(K+L-KL)} - 10Le^{(K+L-KL)} - 10Le^{K+L-KL} + 10L^2e^{K+L-KL}$

$= 10e^{K+L-KL} - 20Le^{K+L-KL} + 10Le^{K+L-KL}$

$= 10e^{K+L-KL}(1 - 2L + L^2)$

$P_{LL} + P_{KK} = 10e^{K+L-KL}(1-2K+K^2) + 10e^{K+L-KL}(1-2L+L^2)$

$= 10e^{K+L-KL}([1-2K+K^2] + [1-2L+L^2])$

$= 10e^{K+L-KL} \cdot (2-2K+K^2-2L+L^2)$

b)

$$v(t) = \langle -2t^3 + C_1,\ 4\cosh(2t) + C_2,\ 2e^{\frac{t}{2}} + C_3 \rangle$$

$$v(0) = \langle 10, 0, -2 \rangle$$

$\cancel{-2(0)^3}^{0} + C_1 = 10$ $\qquad$ $4\cancel{\cosh(2(0))}^{?\ 1} + C_2 = 0$ $\qquad$ $2\cancel{e^{\frac{0}{2}}}^{1} + C_3 = -2$

$\boxed{C_1 = 10}$ $\qquad$ $4 + C_2 = 0$ $\qquad$ $2 + C_3 = -2$

$\boxed{C_2 = -4}$ $\qquad$ $C_3 = -2 - 2$

$$v(t) = \langle -2t^3 + 10,\ 4\cosh(2t) - 4,\ 2e^{\frac{t}{2}} - 4 \rangle$$

$\boxed{C_3 = -4}$

$$\int v(t)\,dt = r(t) = \int \langle \underbrace{-2t^3 + 10}_{f_1},\ \underbrace{4\cosh(2t) - 4}_{f_2},\ \underbrace{2e^{\frac{t}{2}} - 4}_{f_3} \rangle\,dt$$

$$\int f_1\,dt = \int (-2t^3 + 10)\,dt = -\frac{2}{4}t^4 + 10t + C_1$$

$$\int f_2\,dt = \int (4\cosh(2t) - 4)\,dt = 2\sinh(2t) - 4t + C_2$$

$$\int f_3\,dt = \int (2e^{\frac{t}{2}} - 4)\,dt = 4e^{\frac{t}{2}} - 4t + C_3$$

$$r(t) = \langle -\frac{1}{2}t^4 + 10t + C_1,\ 2\sinh(2t) - 4t + C_2,\ 4e^{\frac{t}{2}} - 4t + C_3 \rangle$$

$$r(0) = \langle 0, 10, 0 \rangle$$

$\cancel{-\frac{1}{2}(0)^2}^{0} + \cancel{10(0)}^{0} + C_1 = 0$ $\qquad$ $\cancel{2\sinh(2(0))}^{0} - \cancel{4(0)}^{0} + C_2 = 10$ $\qquad$ $\cancel{-4(0)}^{0} + C_3 = 0$

$\boxed{C_1 = 0}$ $\qquad$ $\boxed{C_2 = 10}$ $\qquad$ $\boxed{C_3 = 0}$

$$r(t) = \langle -\frac{1}{2}t^4 + 10t,\ 2\sinh(2t) - 4t + 10,\ 4e^{\frac{t}{2}} - 4t \rangle$$

b) $2x - 1 = \frac{z}{-3}, \; y = 10$

$t = 2x - 1 \rightarrow \frac{t+1}{2} = x$

$t = \frac{z}{-3} \longrightarrow -3t = z$ $\qquad \vec{u} = \langle \frac{t+1}{2}, 10, -3t \rangle$

$y = 10 \qquad y = 10 + 0(t) \qquad \vec{w} = \langle t, -1-t, 10\tan^{-1}(3) + t \rangle$

$\langle \frac{1}{2}, 0, -3 \rangle$

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{t+1}{2} & 10 & -3t \\ t & -1-t & 10\tan^{-1}(3)+t \end{vmatrix} = \hat{\imath}\left[(10)(10\tan^{-1}(3)+t) - (-3t)(-1-t)\right]$$

$$- \hat{\jmath}\left[\left(\frac{t+1}{2}\right)(10\tan^{-1}(3)+t) - (-3t)(t)\right] +$$

$$\hat{k}\left[\left(\frac{t+1}{2}\right)(-1-t) - (10)(t)\right]$$

$$= \hat{\imath}\left[100\tan^{-1}(3) + 10t - (3t + 3t^2)\right] -$$

$\hat{\jmath}\,[$

$$\langle 1, -1, 1 \rangle \cdot \langle \tfrac{1}{2}, 0, -3 \rangle = (1)(\tfrac{1}{2}) + (-1)(0) + (1)(-3)$$

$$= \frac{1}{2} + 0 - 3$$

$$= \frac{1}{2} - 3 = \frac{1}{2} - \frac{3 \cdot 2}{2} = \frac{1-6}{2} = -\frac{5}{2}$$

no es perpendicular