

# Material de apoyo - Cálculo Multivariable 2

David Gabriel Corzo Mcmath

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Parte I

Ejercicios en clase

# Capítulo 1

Ejercicio en clase #08

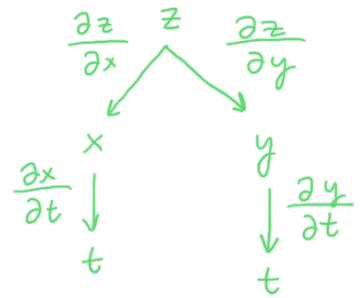
**Regla de la cadena & derivación implícita**

1)  $z = xy^9 - x^2y$ ,  $x = t^2 + 1$ ,  $y = t^2 - 1$   
find  $\frac{\partial z}{\partial t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = (y^9 - 2xy)(2t) + (9y^8x - x^2)(2t)$$

$$= 2ty^9 - 4txy + 18ty^8x - 2tx^2$$



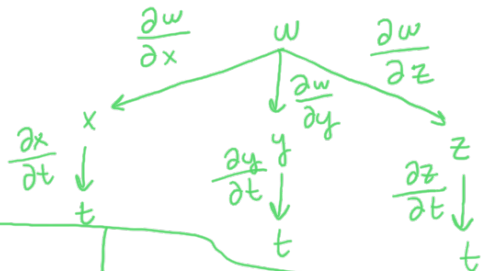
2)  $w = xe^{y/z}$ ,  $x = t^5$ ,  $y = 4 - t$ ,  $z = 2 + 3t$   
find  $\frac{\partial w}{\partial t}$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$\frac{\partial w}{\partial x} = e^{\frac{y}{z}}$	$\frac{\partial x}{\partial t} = 5t^4$	$\frac{\partial w}{\partial y} = \frac{xe^{\frac{y}{z}}}{z}$	$\frac{\partial y}{\partial t} = -1$ ;
$w = xe^{yz^{-1}} \Rightarrow \frac{\partial w}{\partial z} = -xe^{yz^{-1}} yz^{-2}$		$\frac{\partial z}{\partial t} = 3$	

$$\frac{\partial w}{\partial t} = \left(e^{y/z}\right)(5t^4) + \left(\frac{xe^{y/z}}{z}\right)(-1) + \left(\frac{-xe^{y/z}y}{z^2}\right)(3)$$

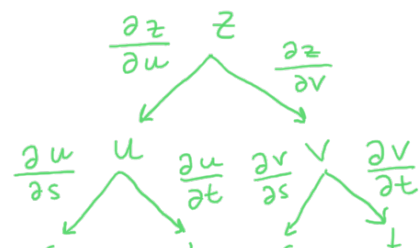
$$= 5t^4 e^{\frac{y}{z}} - \frac{xe^{\frac{y}{z}}}{z} - \frac{3xe^{\frac{y}{z}}y}{z^2}$$



3)  $z = \tan(u/v)$ ,  $u = 9s + 5t$ ,  $v = 5s - 9t$   
find  $\frac{\partial z}{\partial s}$ ,  $\frac{\partial z}{\partial t}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial t}$$



$$\frac{\partial z}{\partial u} = \sec^2(uv^{-1})v^{-1} \quad \frac{\partial z}{\partial v} = -\sec^2(uv^{-1})uv^{-2}$$

$$\frac{\partial u}{\partial s} = 9 \quad \frac{\partial u}{\partial t} = 5 \quad \frac{\partial v}{\partial s} = 5 \quad \frac{\partial v}{\partial t} = -9$$

$$\frac{\partial z}{\partial s} = 9 \sec^2(uv^{-1})v^{-1} - 5 \sec^2(uv^{-1})uv^{-2}$$

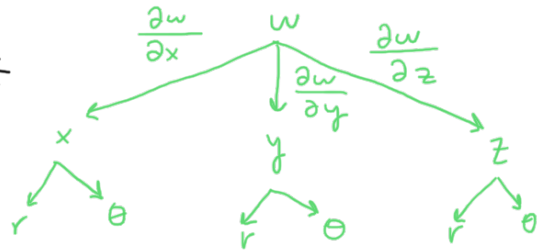
$$\frac{\partial z}{\partial t} = 5 \sec^2(uv^{-1})v^{-1} + 9 \sec^2(uv^{-1})uv^{-2}$$

4)  $w = xy + yz + zx$ ,  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$ ,  $z = r\theta$

$\frac{\partial w}{\partial r}$ ,  $\frac{\partial w}{\partial \theta}$  when  $r = 6$ ,  $\theta = \frac{\pi}{2}$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial \theta}$$



$\frac{\partial w}{\partial x} = y + z$	$\frac{\partial w}{\partial y} = x + z$	$\frac{\partial w}{\partial z} = y + x$
$\frac{\partial x}{\partial r} = \cos(\theta)$	$\frac{\partial y}{\partial r} = \sin(\theta)$	$\frac{\partial z}{\partial r} = \theta$
$\frac{\partial x}{\partial \theta} = -r \sin(\theta)$	$\frac{\partial y}{\partial \theta} = r \cos(\theta)$	$\frac{\partial z}{\partial \theta} = r$

$$\frac{\partial w}{\partial r} = (y+z) \cos(\theta) + (x+z) \sin(\theta) + (y+x) \theta$$

$$\theta = \frac{\pi}{2}; r = 6; x = 0; y = 6; z = 3\pi$$

$$= \cancel{(6 + 3\pi) \cos(\frac{\pi}{2})} + (0 + 3\pi) \cancel{\sin(\theta)} + (6 + 0) \frac{\pi}{2}$$

$$= 3\pi + \frac{6\pi}{2} = 6\pi$$

$$\frac{\partial w}{\partial \theta} = -(y+z)r \sin(\theta) + (x+z)r \cos(\theta) + (y+x)r$$

$$= -(6 + 3\pi)(6 \cdot \sin(\frac{\pi}{2})) + \cancel{(0 + 3\pi)(6 \cdot \cos(\frac{\pi}{2}))} + (6)6$$

$$= -(6 + 3\pi)(6) + 36$$

$$= \cancel{-36} - 18\pi + \cancel{36}$$

$$= -18\pi$$

5) Use :

$$\frac{dy}{dx} = - \frac{\frac{dF}{dy}}{\frac{dF}{dx}} = - \frac{F_x}{F_y}$$

to find:  $\frac{dy}{dx}$  :  $\Gamma 4y \cos(x) = x^2 + y^2 \rfloor$

$$4y \cos(x) - x^2 - y^2 = 0$$

$$\frac{\partial y}{\partial x} = - \frac{-4y \sin(x) - 2x}{4 \cos(x) - 2y}$$

6)  $4 \tan^{-1}(x^2 y) = x + x y^2$   $\frac{\partial y}{\partial x}$

$$4 \tan^{-1}(x^2 y) - x - x y^2$$

$$\frac{\partial y}{\partial x} = - \frac{F_x}{F_y} = - \frac{\left( \frac{8xy}{(x^2 y)^2 + 1} \right) - 1 - y^2}{\left( \frac{4x^2}{(x^2 y)^2 + 1} \right) - 2xy}$$



$$7) \quad x^2 + 8y^2 + 3z^2 = 1$$

$$x^2 + 8y^2 + 3z^2 - 1 = 0$$

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = - \frac{2x}{6z} = - \frac{x}{3z}$$

$$\frac{\partial z}{\partial y} = - \frac{16y}{6z} = - \frac{8y}{3z}$$

$$8) \quad x = \sqrt{2+t}, \quad y = 4 + \frac{1}{2}t$$

$x, y$  measured in cm.

$$T_x(2,5) = 8, \quad T_y(2,5) = 5$$

how fast is the temperature rising when:

$$t = 2$$

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\partial T}{\partial x}}_8 \cdot \frac{\partial x}{\partial t} + \underbrace{\frac{\partial T}{\partial y}}_5 \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial x}{\partial t} = \frac{1}{2} (2+t)^{-\frac{1}{2}} = \frac{1}{2\sqrt{2+t}} \Big|_{t=2} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$$\frac{\partial y}{\partial t} = \frac{1}{2} \Big|_{t=2} = \frac{1}{2}$$

$$\begin{array}{c} \frac{\partial T}{\partial x} \quad T \quad \frac{\partial T}{\partial y} \\ \swarrow \quad \searrow \\ \frac{\partial x}{\partial t} \quad x \quad y \quad \frac{\partial y}{\partial t} \\ \downarrow \quad \downarrow \\ t \quad t \end{array}$$

$$\frac{\partial T}{\partial t} = (8) \left( \frac{1}{4} \right) + (5) \left( \frac{1}{2} \right)$$

$$= 2 + \frac{5}{2} = \frac{9}{2}$$

$$9) \quad z = \cos(x + 7y), \quad x = 2t^3, \quad y = \frac{4}{t}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial x} = -\sin(x + 7y) \cdot 1$$

$$\frac{\partial x}{\partial t} = 6t^2$$

$$\begin{array}{c} z \\ \swarrow \quad \searrow \\ \frac{\partial z}{\partial x} \quad x \quad y \quad \frac{\partial z}{\partial y} \\ \downarrow \quad \downarrow \\ \frac{\partial x}{\partial t} \quad \frac{\partial y}{\partial t} \end{array}$$

$$\frac{\partial y}{\partial t} = -4t^{-2} = -\frac{4}{t^2}$$

$$\frac{\partial z}{\partial y} = -\sin(x + 7y) \cdot 7$$

$$\begin{aligned}\frac{\partial z}{\partial t} &= \left(-\sin(x + 7y)\right) \left(6t^2\right) + \left(-\sin(x + 7y) \cdot 7\right) \left(-\frac{4}{t^2}\right) \\ &= -6t^2 \sin(x + 7y) + \frac{4 \cdot 7}{t^2} \sin(x + 7y) \\ &= -\sin(x + 7y) \left[6t^2 + \frac{4 \cdot 7}{t^2}\right] \\ &= -\sin(x + 7y) \left[6t^2 + \frac{28}{t^2}\right]\end{aligned}$$

10) If  $z = f(x, y)$

$$x = g(t)$$

$$g(s) = -7$$

$$g'(s) = 4$$

$$f_x(-7, 8) = 2$$

$$y = h(t)$$

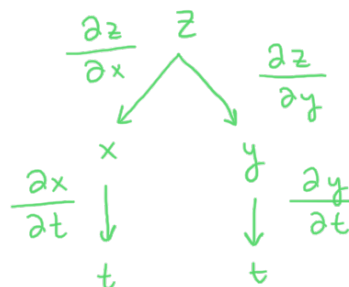
$$h(s) = 8$$

$$h'(s) = -5$$

$$f_y(-7, 8) = -6$$

find  $\frac{\partial z}{\partial t}$  when  $t = 5$

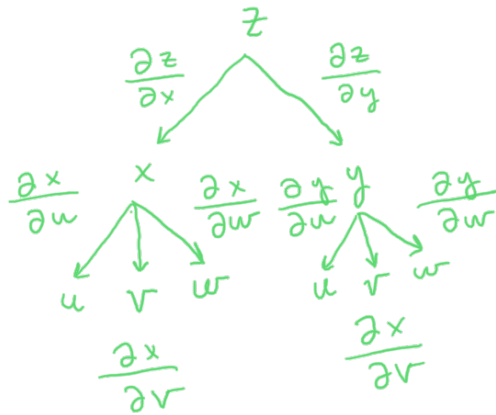
$$\begin{aligned}\frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} \\ &= (2)(4) + (-6)(-5) \\ &= 8 + 30 = 38\end{aligned}$$



11)  $z = x^3 + xy^4$ ,  $x = ur^4 + w^3$ ,  $y = u + ve^w$

when  $u=1, v=1, w=0$

$$\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$$



$$\frac{\partial z}{\partial x} = 3x^2 + y^4 \Big|_{\substack{u=1 \\ v=1 \\ w=0}} = 3(1 \cdot 1 + 0)^2 + (1 + 1 \cdot 1)^4 = 3 + 16 = 19$$

$$\frac{\partial z}{\partial y} = 4xy^3 \Big|_{\substack{u=1 \\ v=1 \\ w=0}} = 4(uv^4 + w)(u + ve^w)^3 = 4(1 + 0)(1 + 1)^3 = 4(1+0)(2)^3 = 4 \cdot 8 = 32$$

$$\frac{\partial x}{\partial u} = v^4 \Big|_{v=1} = 1 \quad \frac{\partial y}{\partial u} = 1$$

$$\frac{\partial x}{\partial v} = 4uv^3 \Big|_{\substack{u=1 \\ v=1}} = 4 \quad \frac{\partial y}{\partial v} = ve^w \Big|_{\substack{v=1 \\ w=0}} = 1e^0 = 1$$

$$\frac{\partial x}{\partial w} = 3w^3 = 0 \quad \frac{\partial y}{\partial w} = ve^w \Big|_{\substack{v=1 \\ w=0}} = 1e^0 = 1$$

$$\blacksquare \quad \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$= (19)(1) + (32)(1) = 19 + 32 = 51$$

$$\blacksquare \quad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$= (19)(4) + (32)(1) = 19 \cdot 4 + 32 = 108$$

$$\blacksquare \frac{\partial z}{\partial w} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial w} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial w}$$

$$= \cancel{(19)}(0) + (32)(1) = 32$$

13)

$$yz = 4 \ln(x+z) \rightarrow 0 = 4 \ln(x+z) - yz$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} = - \frac{\frac{4}{x+z}}{\frac{4}{x+z} - y}$$

$$\frac{\partial z}{\partial y} = - \frac{F_y}{F_z} = \frac{z}{\frac{4}{x+z} - y}$$