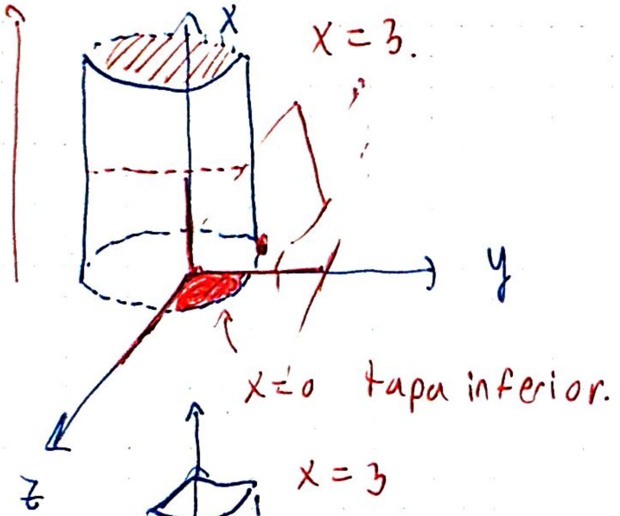
2. $y^2+z^2=9$ $\quad x=0,\ x=3,\ z=0$ en el 1er octante.

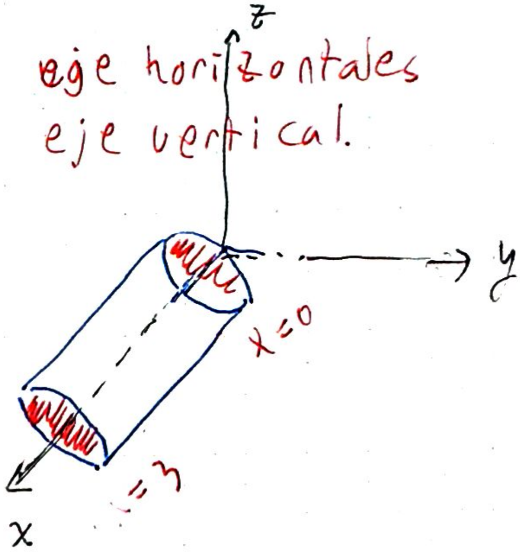
a. $\iiint_E z\, dV$ $\qquad$ b. $\rho(x,y,z)=x^2+y^2$

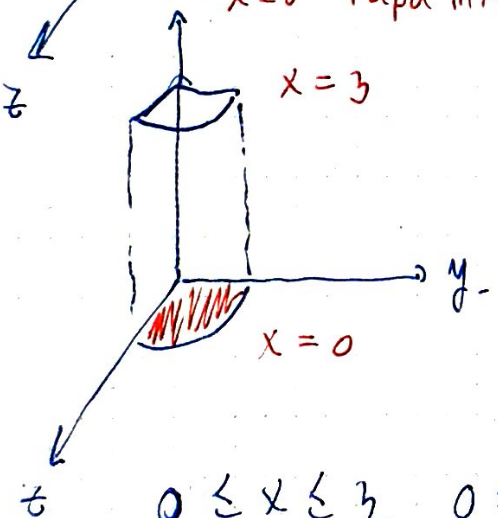
$x^2+y^2=r^2$

z eje horizontales

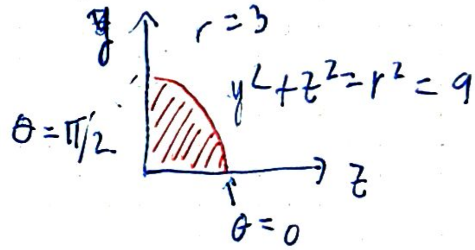
x eje vertical.







$0 \le x \le 3.$



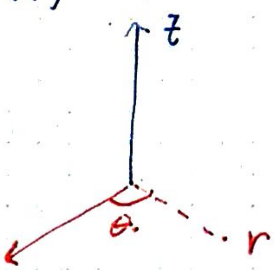
$y = r\sin\theta.$

$z = r\cos\theta.$

$0 \le x \le 3,\quad 0 \le r \le 3,\quad 0 \le \theta \le \pi/2.$

$$\iiint_E z\, dV = \int_0^{\pi/2}\int_0^3\int_0^3 r\cos\theta\; r\, dx\, dr\, d\theta.$$

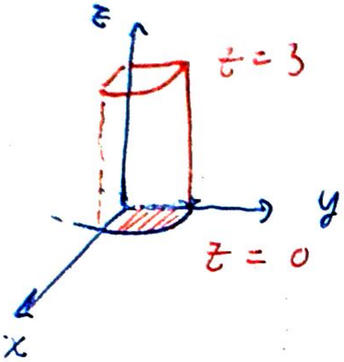
$f(z,y,x) \rightarrow f(r\cos\theta, r\sin\theta, x)$



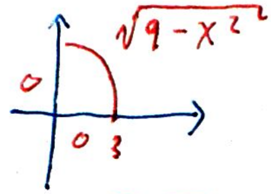
2. $x^2 + y^2 = 9$, entre $z=0$, $z=3$, $x=0$

en el 1er octante.

$z \leftrightarrow x$



$0 \le z \le 3$, $0 \le x \le 3$, $0 \le y \le \sqrt{9-x^2}$



$$\text{Masa} = \iiint_E x\, dV = \int_0^3 \int_0^3 \int_0^{\sqrt{9-x^2}} x \, dy\, dx\, dz. \qquad V = \iiint_E dV$$

volumen * altura.

$$\left(\int_0^3 dz\right) \int_0^3 \int_0^{\sqrt{9-x^2}} x \, dy \, dx$$

$$3 \cdot \int_0^3 x\sqrt{9-x^2}\, dx.$$

$0 \le z \le 3$, $0 \le r \le 3$

$0 \le \theta \le \pi/2$.

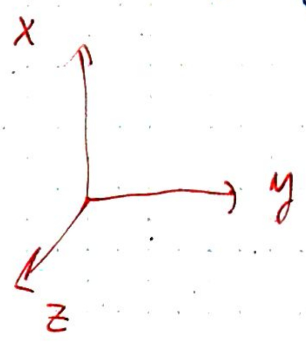
2 b) $$m = \iiint_E (z^2 + y^2)\, dV$$

$x = r\cos\theta$

$y = r\sin\theta$.

$$m = \int_0^3 \int_0^3 \int_0^{\pi/2} (z^2 + r^2 \sin^2\theta)\, r\, d\theta\, dr\, dz.$$

$$m = \int_0^3 \int_0^3 \int_0^{\sqrt{9-x^2}} (z^2 + y^2)\, dy\, dx\, dz.$$



$$m = \iiint_E (x^2 + y^2)\, dV.$$

$x = x$

$y = r\sin\theta$.

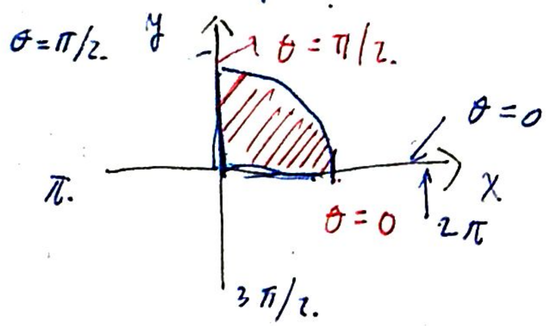
$z = r\cos\theta$.

3. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx.$

$0 \leq x \leq 2 \quad , \quad 0 \leq y \leq +\sqrt{4-x^2} \qquad 0 \leq z \leq \sqrt{16-x^2-y^2}$

Hemisferio superior de radio $\rho = 4$.

$x^2+y^2+z^2=16.$



$0 \leq \rho \leq 4$

$0 \leq \theta \leq \pi/2.$

$0 \leq \varphi \leq \pi/2.$ (Norte)

$x^2+y^2=r^2=\rho^2\sin^2\varphi.$

$$\iiint_E \sqrt{x^2+y^2}\, dV. = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^4 \rho\sin\varphi.\ \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta.$$

b, Esféricas.

$$= \int_0^{\pi/2} d\theta. \int_0^{\pi/2} \sin^2\varphi\, d\varphi \int_0^4 \rho^3\, d\rho.$$

---

a. Cilíndricas (r, θ, z) No hay ángulo φ.

$0 \leq z \leq \sqrt{16-x^2-y^2}$

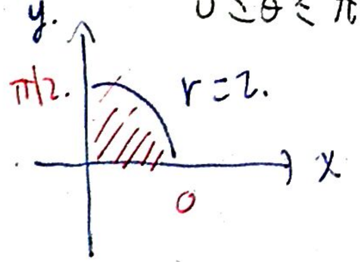
$z = \sqrt{16-x^2-y^2}$

$z = \sqrt{16-r^2\cos^2\theta - r^2\sin^2\theta}$

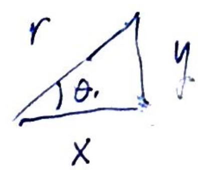
$0 \leq z \leq \sqrt{16-r^2}$

$z = \sqrt{16-r^2}$

$0 \leq \theta \leq \pi/2, \quad 0 \leq r \leq 2.$



$$\iiint_E \underline{\sqrt{x^2+y^2}}\, dV.$$



$$\int_0^{\pi/2}\int_0^2\int_0^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta = \frac{\pi}{2}\int_0^2 \sqrt{16-r^2}\,r\,dr.$$

Chivo Esfera

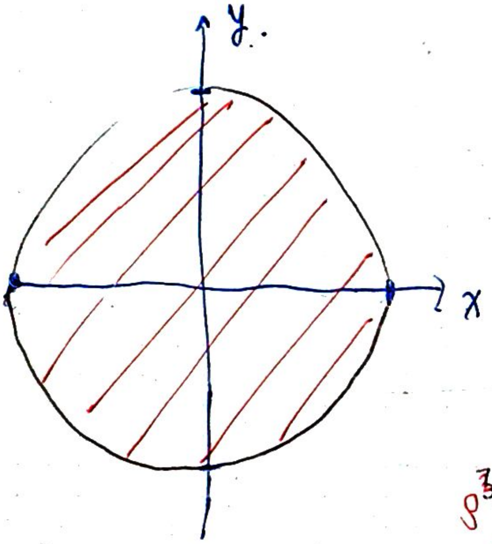
Cartesianas $x^2+y^2+z^2=K^2$ ó $z=\pm\sqrt{K^2-x^2-y^2}$

cilíndricas: $r^2+z^2=K^2$ ó $z=\pm\sqrt{K^2-r^2}$

Esféricas $\rho = K.$

Antes 3 $-3\le x\le 3$, $-\sqrt{9-x^2}\le y\le\sqrt{9-x^2}$

$-\sqrt{9-x^2-y^2}\le z\le\sqrt{9-x^2-y^2}$



$0\le\theta\le 2\pi.$

los dos hemisferios $\rho=3.$

$0\le\rho\le 3.$ $0\le\varphi\le\pi.$

norte sur.

$$\iiint_E (\overbrace{x^2+y^2+z^2}^{\rho^3})^{3/2}\,dV.$$

$(\rho^2)^{3/2}=\rho^3$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^3 \rho^3\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta.$$

$$2\pi\int_0^{\pi}\sin\varphi\int_0^3 \rho^5\,d\rho$$

3. $3-\sqrt{9-x^2-y^2} \leq z \leq 3+\sqrt{9-x^2-y^2}$ $(0,0,3)$

$0 \leq \theta \leq 2\pi.$

$\rho^2 = 9$

esfera de radio 3 centrada en $(0,0,0)$.

$x^2+y^2+z^2=\rho^2.$

$z-3 = \pm\sqrt{9-x^2-y^2}$

$(z-3)^2 = 9-x^2-y^2.$

$z^2-6z+\cancel{9} = \cancel{9}-x^2-y^2.$

$z^2+x^2+y^2 = 6z.$

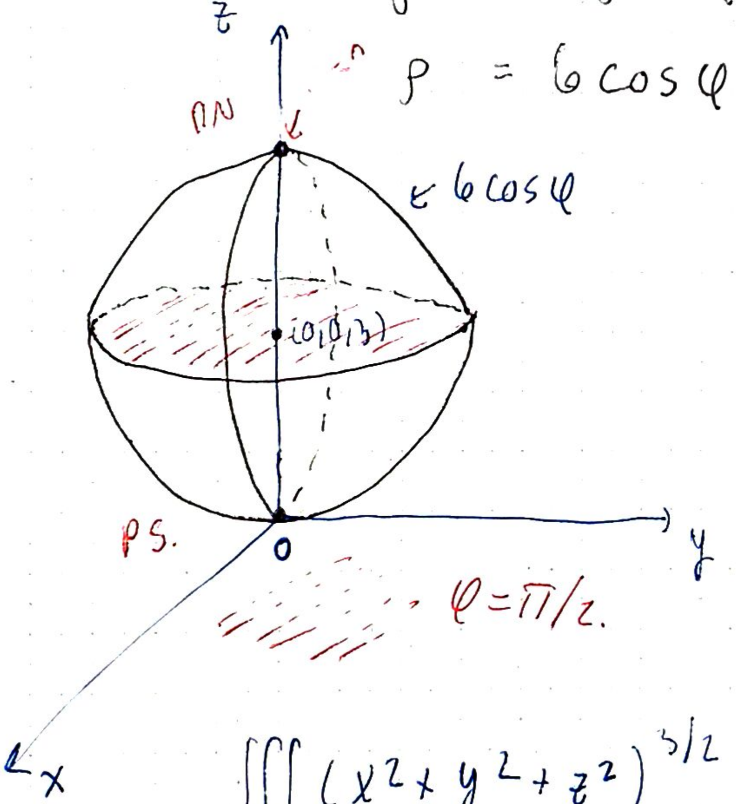
$\rho^2 = 6\rho\cos\varphi$

$\rho = 6\cos\varphi$

$x = \rho\sin\varphi\cos\theta.$

$y = \rho\sin\varphi\sin\theta.$

$z = \rho\cos\varphi.$



$0 \leq \rho \leq \underline{6\cos\varphi}.$

$0 \leq \theta \leq 2\pi.$

$0 \leq \varphi \leq \underline{\pi/2}$

encima del plano xy.
sobre el hemisferio norte.

$$\iiint_E (x^2+y^2+z^2)^{3/2}\, dV.$$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{6\cos\varphi} \rho^3 \cdot \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta.$$

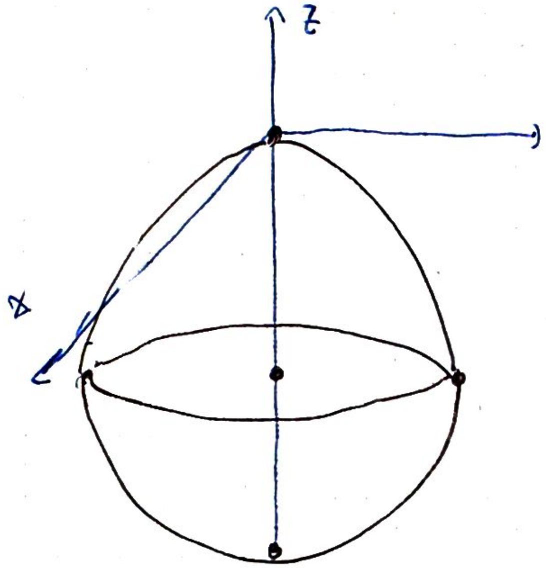
$$2\pi \int_0^{\pi/2}\int_0^{6\cos\varphi} \rho^5 \sin\varphi \, d\rho\, d\varphi.$$

$$\frac{2\pi}{6}\int_0^{\pi/2} \rho^6\Big]_0^{6\cos\varphi} \sin\varphi \, d\varphi.$$

$$\frac{\pi}{3}\int_0^{\pi/2} 6^6 \cos^6\varphi \sin\varphi \, d\varphi.$$

Variación: $-3-\sqrt{9-x^2-y^2} \leq z \leq -3+\sqrt{9-x^2-y^2}$

$z = \pm 6\cos\varphi$ centrada en $(0,0,-3)$



completamente debajo del plano xy (Sólo en el hemisferio sur).

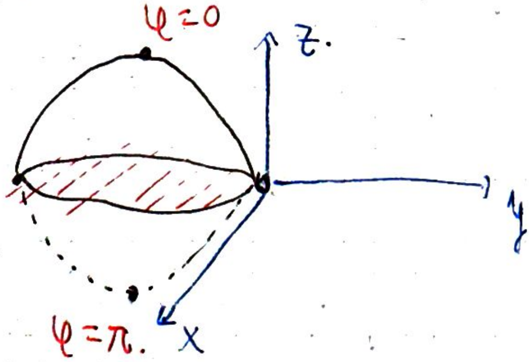
$\frac{\pi}{2} \leq \varphi \leq \pi.$

$$-\sqrt{9-(x+3)^2-y^2} \leq z \leq \sqrt{9-(x+3)^2 - y^2}$$

$z = \pm 6\cos\theta.$ $0 \leq \rho \leq 6\cos\theta.$

Centrada en $(-3,0,0)$.

Plano xy.



$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}.$

$0 \leq \varphi \leq \pi$