TAREA #11 - DAVID CORZO

1) $\int_0^2 \int_1^2 (x - 3y^2)\, dy\, dx$

- $\int_1^2 (x - 3y^2)\, dy = xy - y^3 \Big]_{y=1}^{y=2}$

$= \{[x(2) - (2)^3] - [x(1) - (1)^3]\}$

$= 2x - 8 - x + 1 = x - 7$

- $\int_0^2 (x - 7)\, dx = \frac{x^2}{2} - 7x \Big]_{x=0}^{x=2} =$

$= \left\{\left[\frac{(2)^2}{2} - 7(2)\right] - [0]\right\} = \frac{4}{2} - 14 = 2 - 14 = \boxed{-12}$

2) $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$

- $\int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy = x \int_1^2 \frac{1}{y}\, dy + \frac{1}{x}\int_1^2 y\, dy = x \ln|y| + \frac{y^2}{2x}$

$= x\{\ln(2) - \ln(1)\} + \frac{1}{2x}\{4 - 1\} =$

$= x\ln(2) + \frac{3}{2x}$

- $\ln(2)\int_1^4 x\, dx + \frac{3}{2}\int_1^4 \frac{1}{x} dx = \frac{\ln(2)}{2} x^2 \Big]_{x=1}^{x=4} + \frac{3}{2}\ln|x| \Big]_{x=1}^{x=4}$

$= \frac{\ln(2)}{2}\{16 - 1\} + \frac{3}{2}\{\ln(4) - \cancel{\ln(1)}^{0}\} =$

$= \boxed{\ln(2)\frac{15}{2} + \frac{3}{2}\ln(4)}$

3) $\int_{-3}^3 \int_0^{\frac{\pi}{2}} (y + y^2\cos(x))\, dx\, dy =$

- $\int^{\frac{\pi}{2}} ($ [illegible]

$$\int_0 (y + y\cos(x))\,dx = yx + y^2\sin(x)\Big]_{x=0}^{x=\frac{\pi}{2}} =$$

$$= y\Big[x + y\sin(x)\Big]_{x=0}^{x=\frac{\pi}{2}} = y\left\{\left[\left(\frac{\pi}{2}\right) + y\sin\left(\frac{\pi}{2}\right)\right] - [0]\right\} =$$

$$= y\left\{\frac{\pi}{2} + y(1)\right\} = \frac{\pi}{2}y + y^2$$

$$\int_{-3}^{3}\left(\frac{\pi}{2}y + y^2\right)dy = \frac{\pi}{2}\int_{-3}^{3} y\,dy + \int_{-3}^{3} y^2\,dy =$$

$$= \frac{\pi}{2}\left(\frac{y^2}{2}\right)\Big]_{y=-3}^{y=3} + \left(\frac{y^3}{3}\right)\Big]_{y=-3}^{y=3}$$

$$= \frac{\pi}{4}\cancelto{0}{\left\{9 - 9\right\}} + \frac{1}{3}\left\{(3)^3 - (-3)^3\right\} =$$

$$= \frac{1}{3}\left\{27 + 27\right\} = \frac{54}{3} = \boxed{18}$$

4) $\iint_R x\sin(x+y)\,dA$ $\qquad R = [\underbrace{0}_{a}, \underbrace{\tfrac{\pi}{6}}_{b}] \times [\underbrace{0}_{c}, \underbrace{\tfrac{\pi}{3}}_{d}]$

$$\to \int_0^{\frac{\pi}{3}}\int_0^{\frac{\pi}{6}} (x\sin(x+y))\,dx\,dy \qquad 0 \le x \le \frac{\pi}{6},\quad 0 \le y \le \frac{\pi}{3}$$

$$\int_0^{\frac{\pi}{6}} x\sin(x+y)\,dx$$

$u = x$ $\qquad dv = \sin(x+y)$

$\qquad\qquad v = -\cos(x+y)$

$du = dx$

$$= -x\cos(x+y) - \int_0^{\frac{\pi}{6}} -\cos(x+y)\,dx$$

$$= -x\cos(x+y) + \sin(x+y)\Big]_{x=0}^{x=\frac{\pi}{6}}$$

$$= \left\{\left[-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right)\right] - \left[\sin(y)\right]\right\}$$

$= -\frac{\pi}{6}\cos(\frac{\pi}{}$ ...

$-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) - \sin(y)$

$$\int_0^{\frac{\pi}{3}} \left(-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) - \sin(y)\right) dy$$

$$= \underbrace{-\frac{\pi}{6}\int_0^{\frac{\pi}{3}} \cos\left(\frac{\pi}{6}+y\right) dy}_{1} + \underbrace{\int_0^{\frac{\pi}{3}} \sin\left(\frac{\pi}{6}+y\right) dy}_{2} - \underbrace{\int_0^{\frac{\pi}{3}} \sin(y)\, dy}_{3}$$

1 $\quad -\frac{\pi}{6}\left(\sin\left(\frac{\pi}{6}+y\right)\Big]_0^{\frac{\pi}{3}}\right) =$

$$= -\frac{\pi}{6}\left(\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}\left\{1 - \frac{1}{2}\right\} = -\frac{\pi}{6}\cdot\frac{1}{2} = -\frac{\pi}{12}$$

2 $\quad -\cos\left(\frac{\pi}{6}+y\right)\Big]_0^{\frac{\pi}{3}} =$

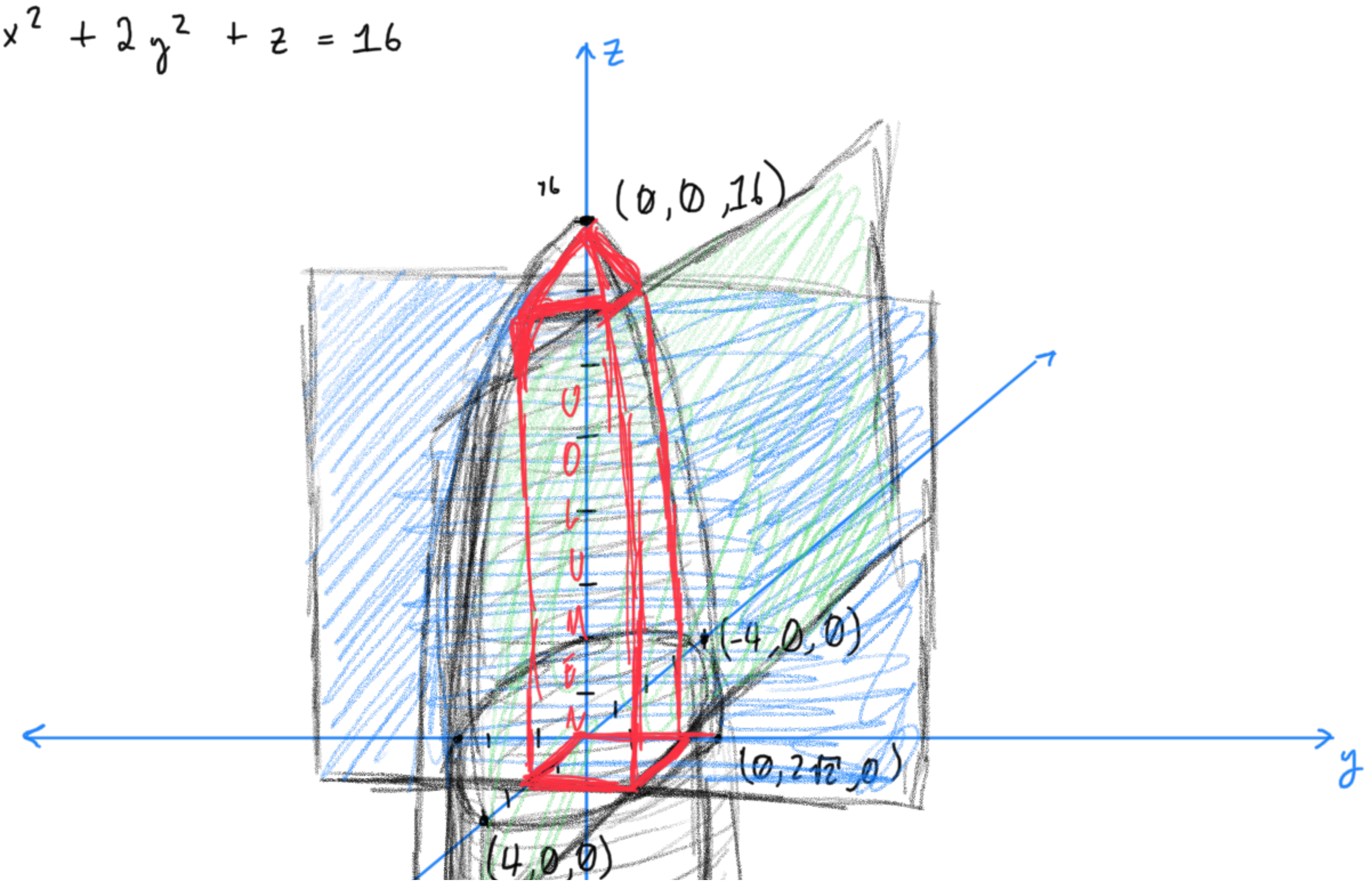
$$= -\left\{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}\right)\right\} = -\left\{0 - \frac{\sqrt{3}}{2}\right\} = \frac{\sqrt{3}}{2}$$

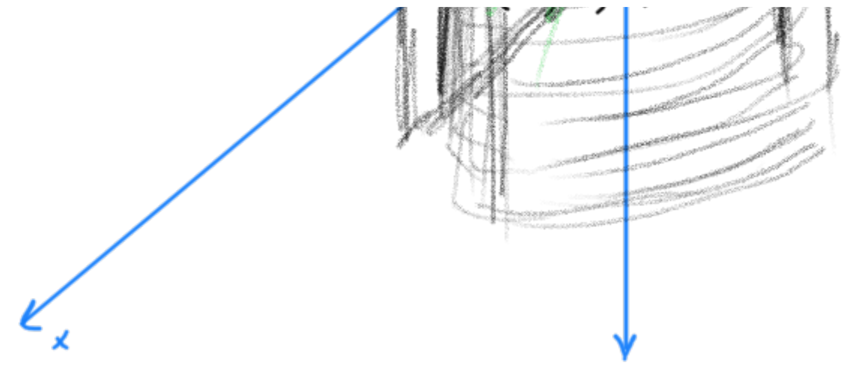
3 $\quad \cos(y)\Big]_0^{\frac{\pi}{3}} =$

$$= \left\{\cos\left(\frac{\pi}{3}\right) - \cos(0)\right\} = \left\{\frac{1}{2} - 1\right\} = -\frac{1}{2}$$

$$\therefore\ -\frac{\pi}{12} + \frac{\sqrt{3}}{2} - \frac{1}{2}$$

5) $x^2 + 2y^2 + z = 16$





$x = 0, \quad y = 0$

$(0)^2 + 2(0)^2 + z = 16$

$z = 16$

$z = 0, \quad y = 0$

$x = \pm 4$

$z = 0, \quad x = 0$

$y = \pm\sqrt{8} = \pm 2\sqrt{2}$

$0 \leq x \; 2$

$0 \leq y \leq 2$

$$= \underbrace{\int_0^2 \int_0^2 (16 - x^2 - 2y^2)\, dy\, dx}_{\int_c^d \int_a^b f(x,y)\, dy\, dx}$$

$$\int_0^2 (16 - x^2 - 2y^2)\, dy = 16y - yx^2 - \frac{2}{3}y^3 \Big]_{y=0}^{y=2} =$$

$$= \left\{ \left[16(2) - (2)x^2 - \frac{2}{3}(2)^3 \right] - [0] \right\} = 32 - 2x^2 - \frac{16}{3}$$

$$\int_0^2 \left(32 - 2x^2 - \frac{16}{3}\right) dx$$

$$= 32x - \frac{2}{3}x^3 - \frac{16}{3}x \Big]_{x=0}^{x=2}$$

$$= \left\{ \left[32(2) - \frac{2}{3}(2)^3 - \frac{16}{3}(2) \right] - [0] \right\} = 64 - \frac{16}{3} - \frac{32}{3}$$

$$= 64 - \frac{48}{3} = 64 - 16 = \boxed{48}$$