

Corto #3 Cálculo Multivariable (20 min)

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90/100

Resuelva los siguientes problemas:

1. (50 pts.) Determine el área del triángulo entre los puntos $P = (0, -2, 0)$, $Q = (4, 1, -2)$ y $R = (5, 3, 1)$.

$$T_{\Delta} = \frac{1}{2} | \vec{u} \times \vec{w} |$$

$$\vec{u} = \overrightarrow{PQ} = \langle (4-0), (1-(-2)), (-2-0) \rangle = \langle 4, 3, -2 \rangle$$

$$\vec{w} = \overrightarrow{PR} = \langle (5-0), (3-(-2)), (1-0) \rangle = \langle 5, 5, 1 \rangle$$

$$| \vec{u} \times \vec{w} | = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & -2 \\ 5 & 5 & 1 \end{vmatrix} = \hat{i} [(3 \cdot 1) - (-2 \cdot 5)] - \hat{j} [(4 \cdot 1) - (-2 \cdot 5)] + \hat{k} [(4 \cdot 5) - (3 \cdot 5)]$$

$$= \hat{i} [3 + 10] - \hat{j} [4 + 10] + \hat{k} [20 - 15]$$

$$= 13\hat{i} - 14\hat{j} + 5\hat{k}$$

$$= \langle 13, -14, 5 \rangle$$

$$\begin{array}{r} 11 \\ 769 \\ + 496 \\ \hline 665 \\ 25 \\ \hline 690 \end{array}$$

$$\begin{array}{r} 1.4 \\ 1.4 \\ \cdot 14 \\ \hline 56 \\ 44 \\ \hline 496 \end{array}$$

$$= \sqrt{(13)^2 + (-14)^2 + (5)^2}$$

$$= \sqrt{169 + 196 + 25}$$

$$= \sqrt{690}$$

$$\therefore \frac{1}{2} \sqrt{690}$$

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2. (50 pts.) Encuentre el volumen del paralelepípedo determinado por los vectores $a = \langle 1, 5, -2 \rangle$, $b = \langle 3, -1, 0 \rangle$, y $c = \langle 5, 9, -4 \rangle$.

$$P_{\square} = | c \cdot (a \times b) |$$

$$(a \times b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 5 & -2 \\ 3 & -1 & 0 \end{vmatrix} = \hat{i} [(5 \cdot 0) - (-2 \cdot -1)] - \hat{j} [(1 \cdot 0) - (-2 \cdot 3)] + \hat{k} [(1 \cdot -1) - (5 \cdot 3)]$$

$$= \hat{i} [0 - 2] - \hat{j} [0 + 6] + \hat{k} [-1 - 15]$$

$$= -2\hat{i} - 6\hat{j} - 16\hat{k}$$

$$= \langle -2, -6, -16 \rangle$$

$$c \cdot (a \times b) = \langle 5, 9, -4 \rangle \cdot \langle -2, -6, -16 \rangle$$

$$= (5 \cdot -2) + (9 \cdot -6) + (-4 \cdot -16)$$

$$= (-10) + (-54) + 64$$

$$= -64 + 64 = 0$$

Hay un plano no un paralelepípedo

$$| c \cdot (a \times b) | = \sqrt{(-10)^2 + (-54)^2 + (64)^2} = \sqrt{100 + 2916 + 4096} = \sqrt{7102}$$

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$$\begin{array}{r} 2 \\ 54 \\ \cdot 54 \\ \hline 216 \\ 270 \\ \hline 2916 \end{array} \quad \begin{array}{r} 2 \\ 64 \\ \cdot 64 \\ \hline 1246 \\ 384 \\ \hline 4086 \end{array}$$

$$\begin{array}{r} 111 \\ 2916 \\ + 4086 \\ \hline 7002 \\ + 100 \\ \hline 7102 \end{array}$$