Cálculo Multivariable

Parcial 1

5to Semestre 2020 **(2 horas)**

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Tema:	1	2	3	4	5	6	Total
Puntos:	18	16	16	16	18	16	100
Nota:	18	14	13	12	18	16	89

~~89~~ 91

1. Considere la función en dos variables $f(x, y) = \ln(10 - x^2 - y)$.
   (a) (06 **pts.**) Encuentre y bosqueje el dominio de la función.
   (b) (12 **pts.**) Encuentre y bosqueje las curvas de nivel de f para $k = 0,\ \ln(6),\ \ln(10)$.

B en hoja.

a) $f(x,y) = \ln(10 - x^2 - y)$

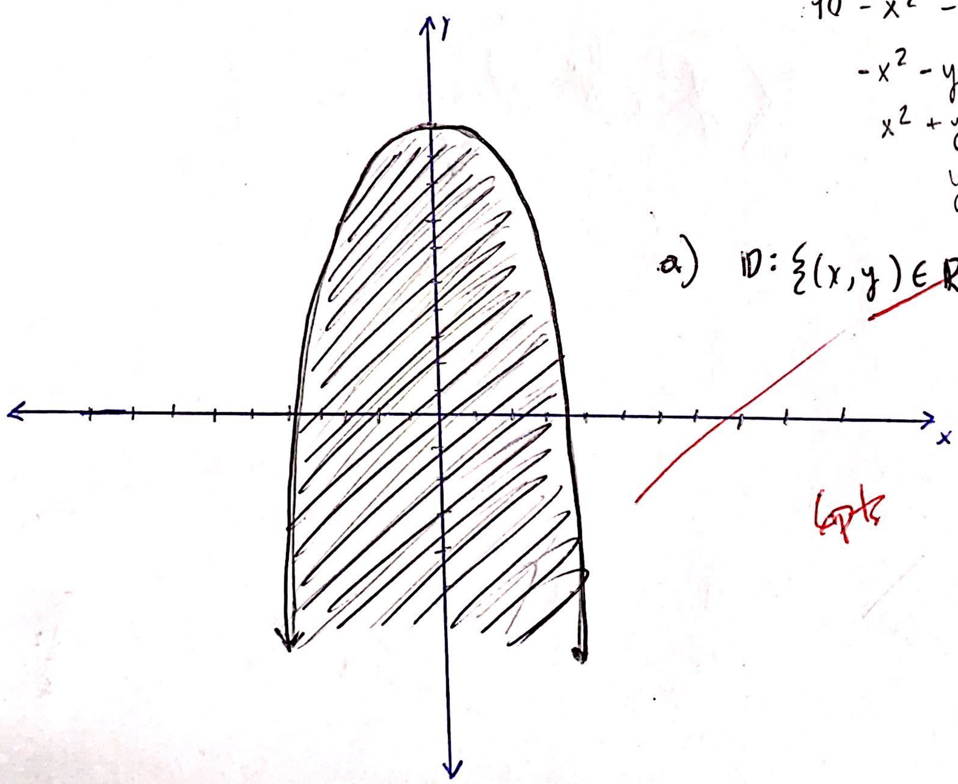
$\mathbb{D}$ $10 - x^2 - y > \emptyset$

$-x^2 - y > -10$

$x^2 + y < 10$

$y < 10 - x^2$

a) $\mathbb{D}: \{(x,y) \in \mathbb{R} \mid (y < 10 - x^2)\}$



6pts

1. b) $K = 0, \ln(6), \ln(10)$

$0 = \ln(10 - x^2 - y)$

$e^0 = 10 - x^2 - y$

$1 = 10 - x^2 - y$

$1 - 10 = -x^2 - y$

$-9 = -x^2 - y$

$9 = x^2 + y$

$\boxed{9 - x^2 = y}$

$\ln(6) = \ln(10 - x^2 - y)$

$6 = 10 - x^2 - y$

$6 - 10 = -x^2 - y$

$-4 = -x^2 - y$

$4 = x^2 + y$

$\boxed{4 - x^2 = y}$

$\ln(10) = \ln(10 - x^2 - y)$

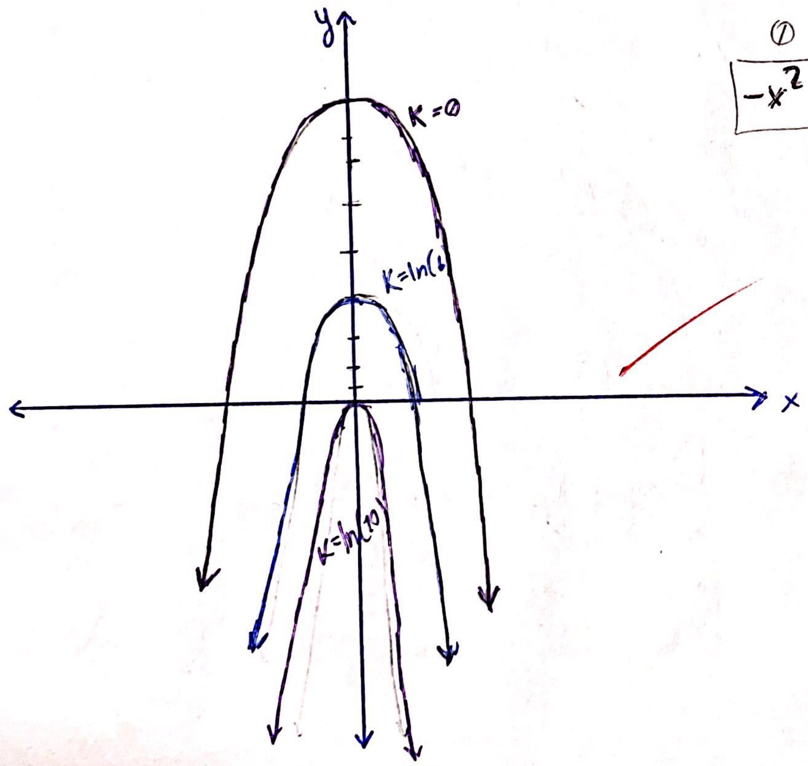
$10 = 10 - x^2 - y$

$10 - 10 = -x^2 - y$

$0 = -x^2 - y$

$0 = x^2 + y$

$\boxed{-x^2 = y}$



1pt

3)

$$|r(t)| = \sqrt{\underbrace{\frac{16t^2}{9}(1+t^2) + \frac{16t^2}{9}(1-t^2) + \frac{1}{9}t^2}}$$

$$\frac{1}{9}t^2\left[16(1+t^2) + 16(1-t^2)\right]$$

$$\frac{1}{9}t^2\left[\underbrace{16 + \cancel{16t^2} + 16 - \cancel{16t^2}}_{\underbrace{16+16}_{32}}\right]$$

$$\frac{32}{9}t^2$$

$2 \cdot 2 = 4 \cdot 2 = 8 \cdot 2 = 16 \cdot 2 = 32$

2^5

$$|r'(t)| = \sqrt{\frac{32}{9}t^2} = \frac{\sqrt{32}}{\sqrt{9}}\sqrt{t^2} = \frac{2^{\frac{5}{2}}}{3}t$$

$$L = \int_2^8 \frac{2^{\frac{5}{2}}}{3}|t|\,dt$$

$$\cancel{= \frac{2^{\frac{5}{2}}}{3}\int_5^8 |t|\,dt = \frac{2^{\frac{5}{2}}}{6}|t|^2 + C}$$

$$\cancel{= \frac{2^{\frac{5}{2}}}{6}\{8^2 - 5^2\}}$$

$$\cancel{= \frac{2^{\frac{5}{2}}}{6}\{64 - 25\}}$$

$$\cancel{= \frac{2^{\frac{5}{2}}}{6}\{39\} = \frac{\sqrt{32}\cdot 39}{6}}$$

2. **(16 pts.)** Encuentre las ecuaciones simétricas de la recta tangente a $\mathbf{r(t)} = \langle\, 2\sqrt{t+2},\ \sin(\pi t),\ t\ln(t-1) - 2t \,\rangle$ en el punto donde $t = 2$.

$\vec{r}_t = \vec{r}(a) + t\,\vec{r}\,'(a)$

$$r'(t) = \left\langle 2\cdot\frac{1}{2}(t+2)^{-\frac{1}{2}},\ \cos(\pi t),\ \ln(t-1) + t\cdot\frac{1}{t-1} - 2 \right\rangle$$

$$= \left\langle 1(t+2)^{-\frac{1}{2}},\ \cos(\pi t),\ \ln(t-1) + \frac{t}{t-1} - 2 \right\rangle$$

$$r'(2) = \left\langle (2+2)^{-\frac{1}{2}},\ \cos(2\pi),\ \ln(2-1) + \frac{2}{2-1} - 2 \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{4}},\ 1,\ \ln(1) \cancel{+ 2 - 2} \right\rangle$$

$$= \left\langle \frac{1}{2},\ 1,\ \emptyset \right\rangle$$

$$r(2) = \left\langle 2\sqrt{2+2},\ \sin(2\pi),\ 2\ln(2-1) - 2(2) \right\rangle$$

$$= \langle 2\cdot 2,\ 0,\ 2\ln(1) - 4 \rangle$$

$$= \langle 4,\ \emptyset,\ -4 \rangle$$

$$\vec{r}_T = \langle 4, \emptyset, -4 \rangle + t\left\langle \frac{1}{2}, 1, \emptyset \right\rangle$$

$x = 4 + t\frac{1}{2} \longrightarrow 2x - 8 = t$

$y = \emptyset + t \longrightarrow y = t$

$z = -4 + \emptyset t \longrightarrow z - 4 = \emptyset t$

$$\boxed{2x - 8 = y\ ,\ z = 4}$$

13 pts

3. (16 pts.) Encuentre la longitud (simplifique a un entero) de la curva descrita por la ecuación vectorial: $\mathbf{r(t)} = \frac{4}{9}(1+t^2)^{3/2}\mathbf{i} + \frac{4}{9}(1-t^2)^{3/2}\mathbf{j} + \frac{1}{6}t^2\mathbf{k}$ en $2 \leqslant t \leqslant 8$.

$$r(t) = \underbrace{\frac{4}{9}(1+t^2)^{\frac{3}{2}}\hat{\imath}}_{f(t)} + \underbrace{\frac{4}{9}(1-t^2)^{\frac{3}{2}}\hat{\jmath}}_{g(t)} + \underbrace{\frac{1}{6}t^2\hat{k}}_{h(t)}$$

$$f'(t) = \frac{4}{9}\cdot\frac{3}{2}(1+t^2)^{\frac{3}{2}-\frac{2}{2}}\cdot 2t$$

$$= \frac{12}{18}(1+t^2)^{\frac{1}{2}}\cdot 2t = \frac{2}{3}\cdot 2t\,(1+t^2)^{\frac{1}{2}}$$

$$= \frac{4t}{3}(1+t^2)^{\frac{1}{2}} \xrightarrow{2} \frac{16t^2}{9}(1+t^2)$$

$$g'(t) = \frac{4}{9}\cdot\frac{3}{2}(1-t^2)^{\frac{1}{2}}\cdot -2t$$

$$= \frac{12}{18}\cdot -2t\,(1-t^2)^{\frac{1}{2}} = -\frac{24t}{18}(1-t^2)^{\frac{1}{2}}$$

$$= -\frac{4\cdot\cancel{6}t}{3\cdot\cancel{6}}(1-t^2)^{\frac{1}{2}} = -\frac{4t}{3}(1-t^2)^{\frac{1}{2}} \xrightarrow{2} \frac{16t^2}{9}(1-t^2)$$

$$h'(t) = \frac{1}{6}\cdot 2t = \frac{\cancel{2}}{\cancel{2}\cdot 3}t = \frac{1}{3}t \xrightarrow{2} \frac{1}{9}t^2$$

3).

$$r(t) = \left\langle \underbrace{\frac{4}{9}(1+t^2)^{\frac{3}{2}}}_{f(t)}, \underbrace{\frac{4}{9}(1-t^2)^{\frac{3}{2}}}_{g(t)}, \underbrace{\frac{1}{6}t^2}_{h(t)} \right\rangle \qquad 2 \le t \le 8$$

$$f'(t) = \frac{4}{9} \cdot \frac{3}{2}(1+t^2)^{\frac{1}{2}} \cdot 2t$$

$$= \frac{12}{18}(1+t^2)^{\frac{1}{2}} \cdot 2t = \frac{24t}{18}(1+t^2)^{\frac{1}{2}} = \frac{\cancel{6} \cdot 4t}{\cancel{6} \cdot 3}(1+t^2)^{\frac{1}{2}}$$

$$= \frac{4t}{3}(1+t^2)^{\frac{1}{2}} = \frac{4t}{3}\sqrt{1+t^2}$$

$$g'(t) = \frac{4}{9} \cdot \frac{3}{2}(1-t^2)^{\frac{1}{2}} \cdot -2t = \frac{12}{18}\sqrt{1-t^2} \cdot -2t = -\frac{24t}{18}\sqrt{1-t^2}$$

$$= -\frac{4}{3}t\sqrt{1-t^2}$$

$$h'(t) = \frac{1}{6} \cdot 2t = \frac{1}{3}t$$

$$|r'(t)| = \sqrt{\left(\frac{4}{3}t\sqrt{1+t^2}\right)^2 + \left(-\frac{4}{3}t\sqrt{1-t^2}\right)^2 + \left(\frac{1}{3}t\right)^2}$$

$$= \sqrt{\frac{16}{9}t^2(1+t^2) + \left(\frac{16}{9}t^2(1-t^2)\right) + \left(\frac{1}{9}t^2\right)}$$

$$= \sqrt{\frac{1}{9}t^2\left(16(1+t^2) + 16(1-t^2)\right)}$$

$$= \sqrt{\frac{1}{9}t^2\left(16 + \cancel{16t^2} + 16 - \cancel{16t^2}\right)}$$

$$= \sqrt{\frac{1}{9}t^2(32)} = \frac{1}{3} \cdot t \cdot \sqrt{32} = \frac{\sqrt{32}}{3}t$$

$$L = \int |v'(t)|\,dt = \frac{\sqrt{32}}{3}\int t\,dt = \frac{\sqrt{32}}{6}t^2 + C \Big]_2^8$$

14 pts

$$\frac{\sqrt{32}}{6}\{64 - 4\} = \frac{\sqrt{32}}{6} \cdot 60 = \boxed{10 \cdot \sqrt{32}}$$

4. Considere la función vectorial: $\mathbf{r}(t) = \left\langle \overbrace{\ln(t^2-1)}^{f(t)}, \overbrace{\frac{5t-15}{t^2-9}}^{g(t)}, \overbrace{\frac{\sinh(2t-4)}{t-2}}^{h(t)} \right\rangle$.

(a) (10 pts.) Encuentre el dominio de **r**. Utilice notación de intervalo.

(b) (06 pts.) Evalúe $\lim_{t\to 2} \mathbf{r}(t)$.

ID $f(t)$:

$t^2 - 1 > 0$

$t^2 > 1$

$t > \pm 1$

$-1 > t > \cancel{1}$ 1

ID $g(t)$:

$t^2 - 9 \neq 0$

$t^2 \neq 9$

$t \neq \pm 3$

ID $h(t)$:

$t \neq 2$

~~ID: $(-\infty, -1) \cup (1, 2) \cup (2, 3) \cup (3, \infty)$~~

-3 -1 0 1 2 3

$\boxed{D: (-\infty, -3) \cup (1, 2) \cup (2, 3) \cup (3, \infty)}$

6 6pts

b) $\lim_{t\to 2} \left\langle \ln(t^2-1), \frac{5t-15}{t^2-9}, \frac{\sinh(2t-4)}{t-2} \right\rangle$

$\lim_{t\to 2} (f(t)) \overset{LI}{=} \ln(4-1) = \ln(3)$

$\boxed{\lim_{t\to 2} (r) = \langle \ln(3), 1, 2 \rangle}$

6pts

$\lim_{t\to 2} (g(t)) = \frac{5(2)-15}{(2)^2-9} = \frac{10-15}{4-9} = \frac{-5}{-5} = 1$

$\lim_{t\to 2} (h(t)) = \frac{\sinh(2(2)-4)}{2-2}$ $\frac{0}{0}$ indef.

$\overset{LH}{=} \lim_{t\to 2} \left(\frac{\cosh(2t-4)\cdot 2}{1} \right) = \frac{1\cdot 2}{1} = 2$

5. Encuentre las operaciones indicadas para las funciones dadas.

(a) (08 **pts.**) $T(x,y,z) = \dfrac{-200}{x^2+y^2+z^2}$, Simplifique $\dfrac{x}{2}\dfrac{\partial T}{\partial x} + \dfrac{y}{2}\dfrac{\partial T}{\partial y} + \dfrac{z}{2}\dfrac{\partial T}{\partial z}$.

(b) (10 **pts.**) $u(w,x,y,z) = \sinh(w^2+x^3)\ln(\sec(y)+\tan(z))$, u_{ww}, u_{zx} .

a) $\dfrac{\partial T}{\partial x} = T_x = -200 \cdot -1\,(x^2+y^2+z^2)^{-2} \cdot 2x$

$= 400x\,(x^2+y^2+z^2)^{-2}$

$= \dfrac{400x}{(x^2+y^2+z^2)^2}$

$\dfrac{x}{2}\dfrac{\partial T}{\partial x} = \dfrac{x}{2} \cdot \dfrac{400x}{(x^2+y^2+z^2)^2} = \dfrac{400x^2}{2(x^2+y^2+z^2)^2}$

$\dfrac{\partial T}{\partial y} = T_y = -200 \cdot -1\,(x^2+y^2+z^2)^{-2} \cdot 2y$

$= \dfrac{400y}{(x^2+y^2+z^2)^2}$

$\dfrac{y}{2}\dfrac{\partial T}{\partial y} = \dfrac{400y^2}{2(x^2+y^2+z^2)^2}$

$\dfrac{\partial T}{\partial z} = -200 \cdot -1\,(x^2+y^2+z^2)^{-2} \cdot 2z$

$= \dfrac{400z}{(x^2+y^2+z^2)^2}$

$\dfrac{z}{2}\dfrac{\partial T}{\partial z} = \dfrac{400z^2}{2(x^2+y^2+z^2)^2}$

Respuesta en hoja

5) a)

$$\frac{x}{2}\frac{\partial T}{\partial x} + \frac{y}{2}\frac{\partial T}{\partial y} + \frac{z}{2}\frac{\partial T}{\partial z} = \frac{200x^2}{(x^2+y^2+z^2)^2} + \frac{200y^2}{(x^2+y^2+z^2)^2} + \frac{200z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{200}{(x^2+y^2+z^2)^2}\left(x^2+y^2+z^2\right)$$

8pts

b) $u(w,x,y,z) = \sinh(w^2+x^3)\ln(\sec(y)+\tan(z))$ $\quad u_{ww}$, u_{zx}

$$u_w = \ln(\sec(y)+\tan(z))\cosh(w^2+x^3)\cdot 2w$$

$$u_{ww} = \ln(\sec(y)+\tan(z))\left(\sinh(w^2+x^3)\cdot 4w^2 + \cosh((w^2+x^3)\cdot 2)\right)$$

$$u_z = \sinh(w^2+x^3)\frac{\sec^2(z)}{\sec(y)+\tan(z)}$$

$$u_{zx} = \frac{\sec^2(z)}{\sec(y)+\tan(z)}\cdot\cosh(w^2+x^3)\cdot 3x^2$$

10

6. Un proyectil se lanza con una velocidad inicial v_o y con un ángulo θ respecto a la horizontal. Si se toma en cuenta su velocidad terminal mg/k, su función de velocidad es:

$$v(t) = \left\langle\ v_o \cos\theta,\ \frac{mg}{k} + \left(v_o \sin\theta - \frac{mg}{k}\right) e^{-kt/m}\ \right\rangle$$

Los términos v_o, m, g, θ y k son constantes.

(a) **(05 pts.)** Encuentre la función de aceleración del objeto.

(b) **(11 pts.)** Encuentre la función de posición del objeto si $\mathbf{r}(0) = 10\mathbf{i}$.

$v'(t) = a(t)$

$$v(t) = \langle \underbrace{V_o \cos(\theta)}_{f(t)},\ \underbrace{\frac{mg}{K} + \left(V_o \sin\theta - \frac{mg}{K}\right) e^{\frac{-Kt}{m}}}_{g(t)} \rangle$$

$f'(t) = \emptyset$ $\quad g'(t) = \emptyset + \left(V_o \sin(\theta) - \frac{mg}{K}\right) e^{\frac{-Kt}{m}} \cdot -\frac{K}{m}$

a) $a(t) = \langle \emptyset,\ \left(V_o \sin(\theta) - \frac{mg}{K}\right) e^{\frac{-Kt}{m}} \cdot -\frac{K}{m} \rangle$

$\int v(t)dt = r(t)$

$r(t) = \langle V_o \cos(\theta) t + C_1,$

Resp. en hoja

$\int g(t)\, dt = \frac{mg}{K} t + \left(V_o \sin(\theta) - \frac{mg}{K}\right) \int e^{\frac{-Kt}{m}} dt$

$u = -\frac{K}{m} t$

$du = -\frac{K}{m} dt$

$-\frac{m}{K} du = dt$

6) $\int g(t)\,dt = \frac{mg}{k}t + \left(V_0 \sin(\theta) - \frac{mg}{k}\right)\left(-\frac{m}{k}\int e^{u}\,du\right)$

$= \frac{mg}{k}t + \left(V_0 \sin(\theta) - \frac{mg}{k}\right)\left(-\frac{m}{k}e^{-\frac{k}{m}t}\right) + C$

$r(0) = \langle 10, 0 \rangle$

$\cancel{V_0 \cos(\theta)(0)}^{0} + C_1 = 10$

$C_1 = 10$

$\cancel{\frac{mg}{k}(0)}^{0} + \left(V_0 \sin(\theta) - \frac{mg}{k}\right)\left(-\frac{m}{k}e^{-\frac{k}{m}t(0)^{0}}\right) + C_2 = 0$

$\left(V_0 \sin(\theta) - \frac{mg}{k}\right)\left(-\frac{m}{k}\right) + C_2 = 0$

$-\frac{V_0 m \sin(\theta)}{k} + \frac{m^2 g}{k^2} + C_2 = 0$

b) $C_2 = \frac{V_0 m \sin(\theta)}{k} - \frac{m^2 g}{k^2} = \frac{m}{k}\left(V_0 \sin(\theta) - \frac{m}{k}g\right)$

$r(t) = \Big\langle V_0 \cos(\theta)\,t + 10,$

$\frac{mg}{k}t + \left(V_0 \sin(\theta) - \frac{mg}{k}\right)\left(-\frac{m}{k}e^{-\frac{k}{m}t}\right) + \frac{m}{k}\left(V_0 \sin(\theta) - \frac{m}{k}g\right)\Big\rangle$

11 pts