2.

5. Masa del sólido $p(x,y) = 30xy$.

$$m = \iiint_E \rho\, dV = \int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0} \int_{3x^2+3y^2}^{54-(x^2+y^2)^{3/2}} 30xy\, dz\, dx\, dy.$$

Cambie a coordenadas cilíndricas.

$$m = \iiint_E 30xy\, dV.$$

$dV = r\,dz\,dr\,d\theta.$

$x = r\cos\theta.$ $\quad x^2+y^2 = r^2.$

$y = r\sin\theta.$

$0 \le r \le 3, \quad \pi \le \theta \le \dfrac{3\pi}{2}.$

$3(x^2+y^2) \le z \le 54-(x^2+y^2)^{3/2}$

$\qquad 3r^2 \quad \le z \le 54-r^3$

$$m = \int_{\pi}^{3\pi/2} \int_{0}^{3} \int_{3r^2}^{54-r^3} 30r^2\cos\theta\sin\theta \cdot r\, dz\, dr\, d\theta$$

$$m = \left(\int_{\pi}^{3\pi/2} \sin\theta\cos\theta\, d\theta\right) \int_{0}^{3} \int_{3r^2}^{54-r^3} 30r^3\, dz\, dr.$$

$$m = \left.\frac{1}{2}\sin^2\theta\right]_{\pi}^{3\pi/2} \int_{0}^{3} 30r^3(54-r^3-3r^2)\, dr.$$

$$m = 15 \int_{0}^{3} (54r^3 - r^6 - 3r^5)\, dr.$$

$$m = 15\left(\frac{27}{2}r^4 - \frac{1}{7}r^7 - \frac{3}{6}r^6\right]_{0}^{3}\right)$$

$$m = 15\left(\frac{3^7}{2} - \frac{3^7}{7} - \frac{3^7}{6}\right) = 15\cdot 3^7\left(\frac{1}{2} - \frac{1}{7} - \frac{1}{6}\right)$$

$$m = \frac{15\cdot 3^7}{84}(42-12-14) = 15\cdot 3^7\frac{16}{84} = \frac{5\cdot 3^7\cdot 4}{7} = \frac{20\cdot 3^7}{7}.$$