

TAREA #12 - DAVID CORZO

1) Tarea 1: Planos tangentes & aproximaciones lineales:

a) Encontrar la ec. plano tangente:

$$a.1) z = x^2 + y^2, \quad (1, 1, 3)$$

$$\boxed{z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

$$f_x = 2x \Big|_{(1, 1, 3)} = 2(1) = 2$$

$$f_y = 2y \Big|_{(1, 1, 3)} = 2(1) = 2$$

$$f(1, 1) = 1^2 + 1^2 = 2$$

$$\begin{aligned} z &= 2 + 2(x - 1) + 2(y - 1) \\ &= \cancel{2} + 2x - \cancel{2} + 2y - 2 \\ &= 2x + 2y - 2 \end{aligned}$$

$$\boxed{z = 2x + 2y - 2}$$

$$a.1) z = x \sin(x+y), \quad (-1, 1, 0)$$

$$\begin{aligned} f_x(x_0, y_0) &= \sin(x+y) + x \cos(x+y) \Big|_{(-1, 1, 0)} = \\ &= \sin(-1+1) - 1 \cos(-1+1) = -1 \end{aligned}$$

$$f_y(x_0, y_0) = x \cos(x+y) \Big|_{(-1, 1, 0)} = -1 \cos(-1+1) = -1$$

$$f(-1, 1) = -\cancel{1} \sin(-1+1) \cancel{0} = \emptyset$$

$$\begin{aligned} z &= \emptyset - 1(x+1) - 1(y-1) \\ &= -x - \cancel{1} - y + \cancel{1} \end{aligned}$$

$$\boxed{z = -x - y}$$

b) Encontrar la aproximación lineal $L(x, y)$:

$$L(1) = \dots$$

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$\boxed{z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)}$$

$$f_x(1, 0) = e^{xy} + xy e^{xy} \Big|_{(1, 0)} = 1$$

$$f_y(1, 0) = x^2 e^{xy} \Big|_{(1, 0)} = 1$$

$$f(1, 0) = 1 e^{1 \cdot 0^2} = 1$$

$$\begin{aligned} z &= 1 + 1(x - 1) + 1(y - 0) \\ &= 1 + x - 1 + y \end{aligned}$$

$$z = x + y$$

$$b.2) z = \sqrt{x + e^{4y}} \quad (3, 0)$$

$$f_x(3, 0) = \frac{1}{\sqrt{x + e^{4y}}} \Big|_{(3, 0)} = \frac{1}{\sqrt{3 + 1}} = \frac{1}{2}$$

$$f_y(3, 0) = \frac{4e^{4y}}{\sqrt{x + e^{4y}}} \Big|_{(3, 0)} = \frac{4}{\sqrt{3 + 1}} = 2$$

$$f(3, 0) = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$z = 2 + \frac{1}{2}(x - 3) + 2(y - 0)$$

$$= 2 + \frac{1}{2}x - \frac{3}{2} + 2y$$

$$= \frac{1}{2}x + 2y + \underbrace{\frac{4}{2} - \frac{3}{2}}_{\frac{1}{2}}$$

$$= \frac{1}{2}x + 2y + \frac{1}{2}$$

$$z = \frac{1}{2}x + 2y + \frac{1}{2}$$

2) Tema 2: Derivación Implícita

a) Encuentre $\frac{dy}{dx}$

$$a.1) \quad y \cos(x) = x^2 + y^2$$

$$\emptyset = x^2 + y^2 - y \cos(x)$$

$$\frac{\partial}{\partial y} = 2y - \cos(x) \quad \frac{\partial}{\partial x} = 2x + y \sin(x)$$

$$\frac{\partial y}{\partial x} = \frac{2y - \cos(x)}{2x + y \sin(x)}$$

$$a.2) \quad e^y \sin(x) = x + xy$$

$$\emptyset = x + xy - e^y \sin(x)$$

$$\frac{\partial}{\partial y} = x - e^y \sin(x) \quad \frac{\partial}{\partial x} = 1 + y - e^y \cos(x)$$

$$\frac{\partial y}{\partial x} = \frac{x - e^y \sin(x)}{1 + y - e^y \cos(x)}$$

b) Encuentre la derivada parcial de z de:

$$e^z + e^{xy} = xyz$$

$$e^z + e^{xy} - xyz = \emptyset$$

$$\frac{\partial}{\partial z} = e^z - xy$$

c) Encontrar la derivada parcial de z :

$$\cos(yx) + \sin(yz) = zx$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \cos(yx) + \sin(yz) - zx = \emptyset$$

$$\frac{\partial z}{\partial x} = -\frac{-\sin(yx)y - z}{\cos(yz)y - x}$$

$$\frac{\partial z}{\partial y} = - \frac{F_y}{F_z}$$

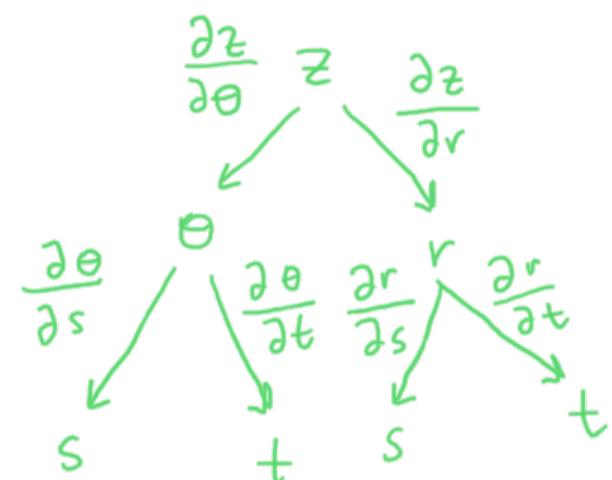
$$\frac{\partial z}{\partial x} = - \frac{-\sin(yx)x + \cos(yz)z}{\cos(yz)y - x}$$

3) Tema 3: Regla de la cadena

a) Encuentre $\frac{\partial z}{\partial s}$ y $\frac{\partial z}{\partial t}$

a.1) $z = e^r \cos(\theta)$ $\theta = \sqrt{s^2 + t^2}$ $r = st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s}$$



$$\frac{\partial z}{\partial s} = (-e^r \sin(\theta)) \left(\frac{1}{2}(s^2 + t^2)^{-\frac{1}{2}} 2s \right) + (e^r \cos(\theta))(t)$$

$$\frac{\partial z}{\partial s} = -\frac{e^r \sin(\theta)s}{\sqrt{s^2 + t^2}} + e^r \cos(\theta)t$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t}$$

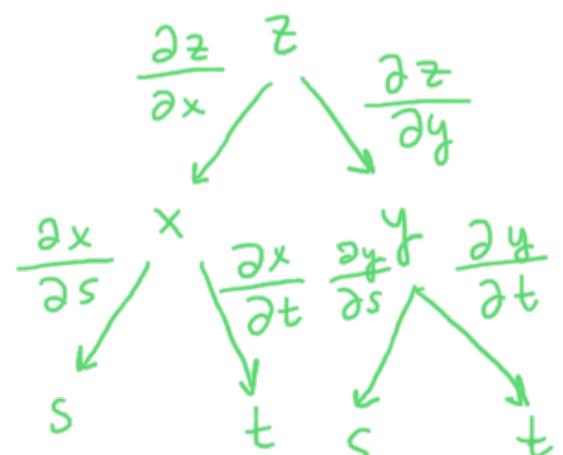
$$= (-e^r \sin(\theta)) \left(\frac{1}{2}(s^2 + t^2)^{-\frac{1}{2}} 2t \right) + (e^r \cos(\theta))(s)$$

$$\frac{\partial z}{\partial r} = -\frac{e^r \sin(\theta)t}{\sqrt{s^2 + t^2}} + e^r s \cos(\theta)$$

a.2) $z = \arcsin(x-y)$ $x = s^2 + t^2$ $y = 1 - 2st$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$



$$\frac{\partial z}{\partial t} = \int \frac{1}{\sqrt{1-(x-y)^2}} \cdot 2t \cdot 1 - 2s(-2st) \cdot 1$$

$$\partial_t \left((x-y)^2 + 1 \right) (2t) + \left(\frac{1}{(x-y)^2 + 1} \right) (-2s)$$

$$\frac{\partial z}{\partial t} = \frac{2t}{(x+y)^2 + 1} + \frac{2s}{(x-y)^2 + 1}$$

$$\frac{\partial z}{\partial s} = \left(\frac{1}{(x-y)^2 + 1} \right) (2s) + \left(\frac{-1}{(x-y)^2 + 1} \right) (-2s)$$

$$\frac{\partial z}{\partial s} = \frac{2s}{(x-y)^2 + 1} + \frac{2s}{(x-y)^2 + 1}$$

b) función de Producción:

$$P(L, K) = 10 L^{\frac{1}{2}} K^{\frac{1}{2}}$$

L: h en miles ; K: h de capital

L = 25 ; K = 16 P(L, K) esta en toneladas

$$\Delta L = -2 \quad \Delta K = 3$$

Encontrar tasa de cambio de la producción:

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial L} \frac{\partial L}{\partial t} + \frac{\partial P}{\partial K} \frac{\partial K}{\partial t}$$

$$\begin{array}{ccc} P & & \\ \swarrow \frac{\partial P}{\partial L} & & \searrow \frac{\partial P}{\partial K} \\ L & & K \\ \frac{\partial L}{\partial t} & | & | \frac{\partial K}{\partial t} \\ t & & t \end{array}$$

$$\frac{\partial P}{\partial L} = \frac{5\sqrt{K}}{\sqrt{L}} \Big|_{L=25, K=16} = \frac{5\sqrt{16}}{\sqrt{25}} = \frac{20}{5} = 4$$

$$\frac{\partial P}{\partial K} = \frac{5\sqrt{L}}{\sqrt{K}} \Big|_{L=25, K=16} = \frac{5\sqrt{25}}{\sqrt{16}} = \frac{25}{4} = 5$$

$$\frac{\partial P}{\partial t} = (4)(-2) + (5)(3)$$

$$= -8 + 15$$

= 7

c) La temperatura (x, y) medida en Celsius,

$$x = \sqrt{1+t} \quad y = 2 + \frac{1}{3}t \quad \# x, y \text{ en cm.}$$

$$T_x(2,3) = 4 \quad T_y(2,3) = 3$$

¿Cuán rápido estará aumentando la temperatura en $t = 3$?

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial t}}_4 + \underbrace{\frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial t}}_3$$

$$\begin{array}{c}
 T \\
 / \quad \backslash \\
 \frac{\partial T}{\partial x} \quad x \quad \frac{\partial T}{\partial y} \\
 \downarrow \quad \downarrow \\
 \frac{\partial x}{\partial t} \quad t \quad \frac{\partial y}{\partial t} \quad t
 \end{array}$$

$$\frac{\partial x}{\partial t} = \frac{1}{2}(1+t)^{-\frac{1}{2}} \cdot 1 = \frac{1}{2\sqrt{1+t}} \quad \Big|_{t=3} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\frac{\partial y}{\partial t} = \frac{1}{3} \Big|_{t=3} = \frac{1}{3}$$

$$\frac{\partial T}{\partial t} = (4) \left(\frac{1}{4}\right) + (3) \left(\frac{1}{3}\right) = 1 + 1 = 2$$

d) Voltage \downarrow ; Resistencia \uparrow ; ley Ohm: $V = IR$

$$R = 400 \Omega \quad I = 0.08 A$$

I: Corriente
R: Resistencia

$$\# V = IR \quad \frac{dV}{dt} = -0.01 \text{ V/s} \quad \frac{dR}{dt} = 0.03 \Omega/\text{s} \quad V: \text{Voltage}$$

$$\# \frac{\partial I}{\partial t}$$

$$\begin{array}{c}
 I \\
 / \quad \backslash \\
 \frac{\partial I}{\partial V} \quad V \quad R \\
 2
 \end{array}$$

a) Hallar la derivada direccional del vector \vec{v}

a.1) $f(x,y) = x^3 - y^3$ $P(4,3)$ $\vec{v} = \frac{\sqrt{2}}{2}(\hat{i} + \hat{j})$

$$\begin{aligned} f_x &= 3x^2 & f_y &= -3y^2 & \vec{v} &= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \\ f_x(4,3) &= 48 & f_y(4,3) &= -9 \end{aligned}$$

$$\nabla f(4,3) = \langle 48, -9 \rangle$$

$$\begin{aligned} D_u f &= \langle 48, -9 \rangle \cdot \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \\ &= 24\sqrt{2} - \frac{9}{2}\sqrt{2} = \sqrt{2} \left(24 - \frac{9}{2} \right) = \sqrt{2} \frac{39}{2} \end{aligned}$$

a.2) $g(x,y,z) = x^2 + y^2 + z^2$ $P(1,1,1)$

$$f_x(1,1,1) = 2x \Big|_{(1,1,1)} = 2$$

$$f_y(1,1,1) = 2y \Big|_{(1,1,1)} = 2 \quad \vec{v} = \left\langle \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle$$

$$f_z(1,1,1) = 2z \Big|_{(1,1,1)} = 2 \quad \nabla f = \langle 2, 2, 2 \rangle$$

$$D_u f = \langle 2, 2, 2 \rangle \cdot \left\langle \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle$$

$$= \cancel{\frac{2}{3}\sqrt{3}} - \cancel{\frac{2}{3}\sqrt{3}}^0 + \frac{2}{3}\sqrt{3} = \frac{2}{3}\sqrt{3}$$

b) Halle la razón de cambio instantánea de la función en P , la dirección \vec{v} :

b.1) $f(x,y) = e^x \sin(y)$ $P(1, \frac{\pi}{2})$ $\vec{v} = -\hat{i}$

$$D_u f = \nabla f \cdot \vec{v}$$

$$\nabla f = \langle e^x \sin(y), e^x \cos(y) \rangle$$

$$\begin{aligned} \nabla f(1, \frac{\pi}{2}) &= \langle e \sin(\frac{\pi}{2}), e \cos(\frac{\pi}{2}) \rangle \\ &= \langle e, 0 \rangle \end{aligned}$$

$$\vec{v} = \langle -1, 0 \rangle$$

$$D_u f = \langle e, 0 \rangle \cdot \langle -1, 0 \rangle \\ = -e + 0 = -e$$

b.2) $g(x, y, z) = x^2 + y^2 + z^2 \quad P(1, 1, 1) \quad \vec{v} = \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

$$\nabla f(1, 1, 1) = \left\langle 2x, 2y, 2z \right\rangle \Big|_{(1, 1, 1)} = \langle 2, 2, 2 \rangle$$

$$\vec{v} = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$D_u f = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle \cdot \langle 2, 2, 2 \rangle \\ = \cancel{\frac{2}{\sqrt{3}}} - \cancel{\frac{2}{\sqrt{3}}} + \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

c) Hallar la derivada direccional del vector unitario
 $\vec{u} = \cos(\theta)\hat{i} + \sin(\theta)\hat{j}$

c.1) $f(x, y) = \sin(2x + y) \quad \theta = \frac{\pi}{3}$

$$D_u f = \nabla f \cdot \vec{v}$$

$$\nabla f = \langle 2\cos(2x + y), \cos(2x + y) \rangle$$

$$\vec{v} = \langle \cos(\theta), \sin(\theta) \rangle \Big|_{\theta=\frac{\pi}{3}} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

$$D_u f = \sqrt{2}\cos(2x + y) + \frac{\sqrt{2}}{2}\cos(2x + y)$$

c.2) $f(x, y) = \frac{y}{x + u} \quad \theta = -\frac{\pi}{6}$

$$f_x = -y(x-y)^{-2} \cdot 1$$

$$f_y = \frac{1(x+y) - y}{(x+y)} = \frac{x}{x+y}$$

$$\nabla f = \left\langle -\frac{y}{(x-y)^2}, \frac{x}{x+y} \right\rangle$$

$$\vec{v} = \left\langle \cos\left(-\frac{\pi}{6}\right), \sin\left(-\frac{\pi}{6}\right) \right\rangle = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$$

$$\begin{aligned} D_u f &= \left\langle -\frac{y}{(x-y)^2}, \frac{x}{x+y} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle \\ &= -\frac{\sqrt{3}}{2(x-y)^2} - \frac{x}{2(x+y)} \end{aligned}$$

5) Tema 5: Optimización

Dulces A, B

$$CP_A = 60 \quad CP_B = 70$$

maximizar precio

$$C = 60q_A + 70q_B$$

Demanda A, B:

$$q_A = S(P_B - P_A)$$

$$q_B = 500 + S(P_A - 2P_B)$$

$$U(q_A, q_B) = P_A q_A + P_B q_B - (60q_A + 70q_B)$$

$$= P_A (SP_B - SP_A) + P_B (500 + SP_A - 10P_B) - 60q_A - 70q_B$$

$$= SP_B P_A - SP_A^2 + 500 P_B + SP_A P_B - 10P_B^2 - 60q_A - 70q_B$$

$$= -SP_A^2 + 10P_B P_A + 500 P_B - 10P_B^2 - 60(SP_B - SP_A) - 70(500 + SP_A - 10P_B)$$

$$= -\cancel{SP_A^2} + \cancel{10P_B P_A} + \cancel{500 P_B} - \cancel{10P_B^2} - \cancel{300 P_B} + \cancel{300 P_A} - 35,000 - 350P_A + \cancel{700P_B}$$

$$= -SP_A^2 + 10P_B P_A + 900 P_B - 10P_B^2 - 50P_A - 350P_A - 35,000$$

$$\begin{aligned}\frac{\partial U}{\partial P_A} &= -5 \cdot 2 P_A + 10 P_B - 50 = 0 \\ &= -10 P_A + 10 P_B - 50 = 0 \\ 10 P_B - 50 &= 10 P_A \\ P_B - 5 &= P_A\end{aligned}$$

$$\begin{aligned}\frac{\partial U}{\partial P_B} &= 10 P_A + 900 - 20 P_B = 0 \\ P_A &= \frac{1}{10} (-900 + 20 P_B) \\ P_A &= -90 + 2 P_B\end{aligned}$$

$$P_B - 5 = -90 + 2 P_B$$

$$-5 + 90 = 2 P_B - P_B$$

$$85 = P_B$$

$$80 = P_A$$

Precios que maximizan

6) Tema 6: Multiplicadores La'Grange.

$$f(q_1, q_2) = 2q_1^2 + q_1 q_2 + q_2^2 + 200$$

$$q_1 + q_2 = 200$$

$$\begin{aligned}F(q_1, q_2, \lambda) &= 2q_1^2 + q_1 q_2 + q_2^2 + 200 + \lambda(200 - q_2 - q_1) \\ &= 2q_1^2 + q_1 q_2 + q_2^2 + 200 + 200\lambda - \lambda q_2 - \lambda q_1\end{aligned}$$

$$\frac{\partial F}{\partial q_1} = 4q_1 + q_2 - \lambda = 0$$

$$4q_1 + q_2 = \lambda$$

$$\begin{aligned}\frac{\partial F}{\partial q_2} &= q_1 + 2q_2 - \lambda = 0 \\ &\quad \sim \quad \sim \quad - \quad \sim\end{aligned}$$

$$\begin{aligned}4q_1 + q_2 &= q_1 + 2q_2 \\ 3q_1 &= q_2\end{aligned}$$

$$q_1 + \lambda q_2 = 1$$

$$\boxed{1 - \lambda}$$

$$\frac{\partial F}{\partial \lambda} = 200 - q_2 - q_1 = 0$$

$$200 - 3q_1 - q_1 = 0$$

$$4q_1 = 200$$

$$q_1 = \frac{200}{4}$$

$$q_1 = 50$$

distribución
de producción

$$200 - 50 = q_2$$

$$150 = q_2$$