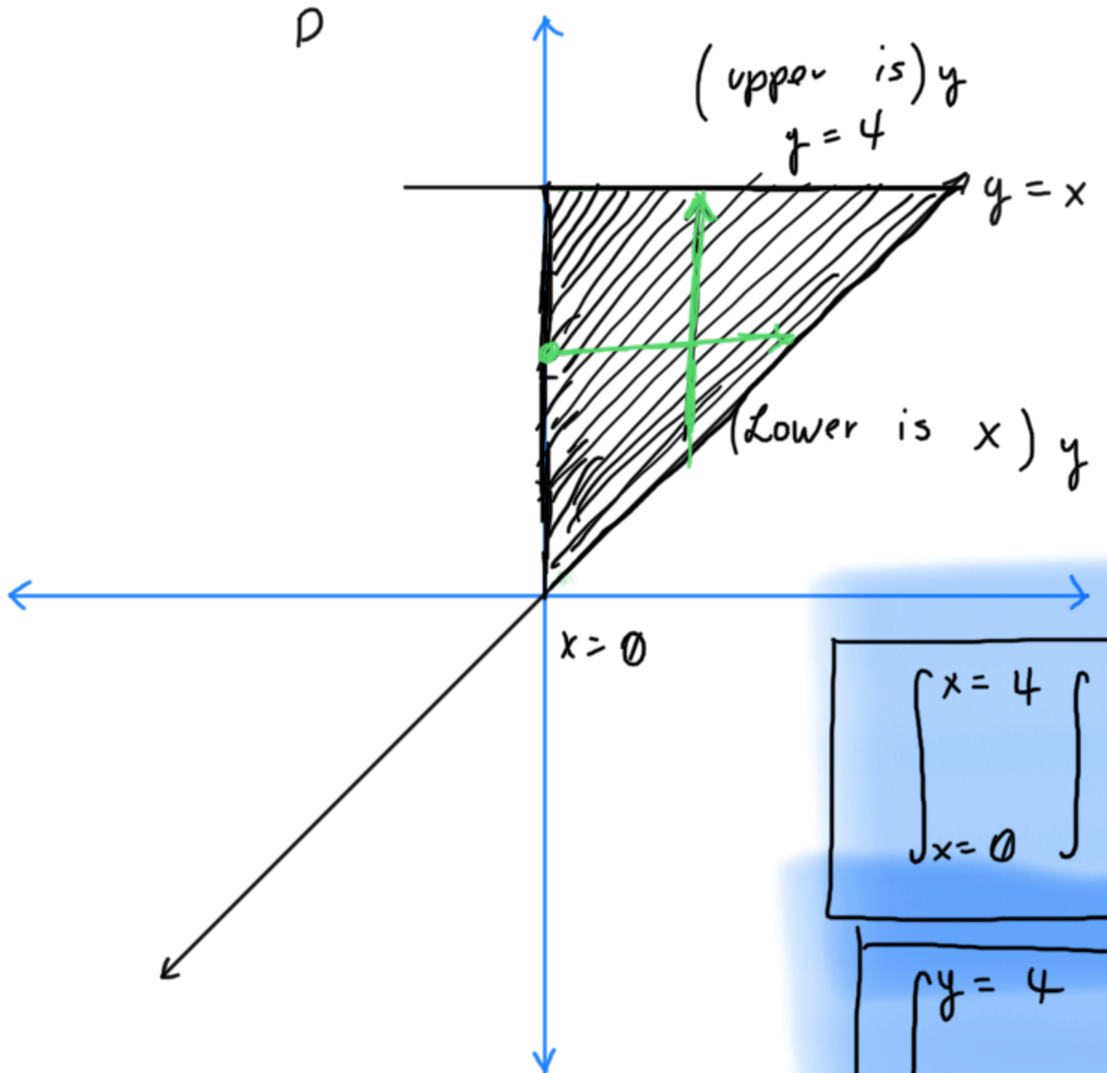
TAREA #13 - DAVID CORZO

1.a)

$$\iint_D y^2 e^{xy} \, dA, \qquad D\ \{(y=x) \wedge (y=4) \wedge (x=0)\}$$

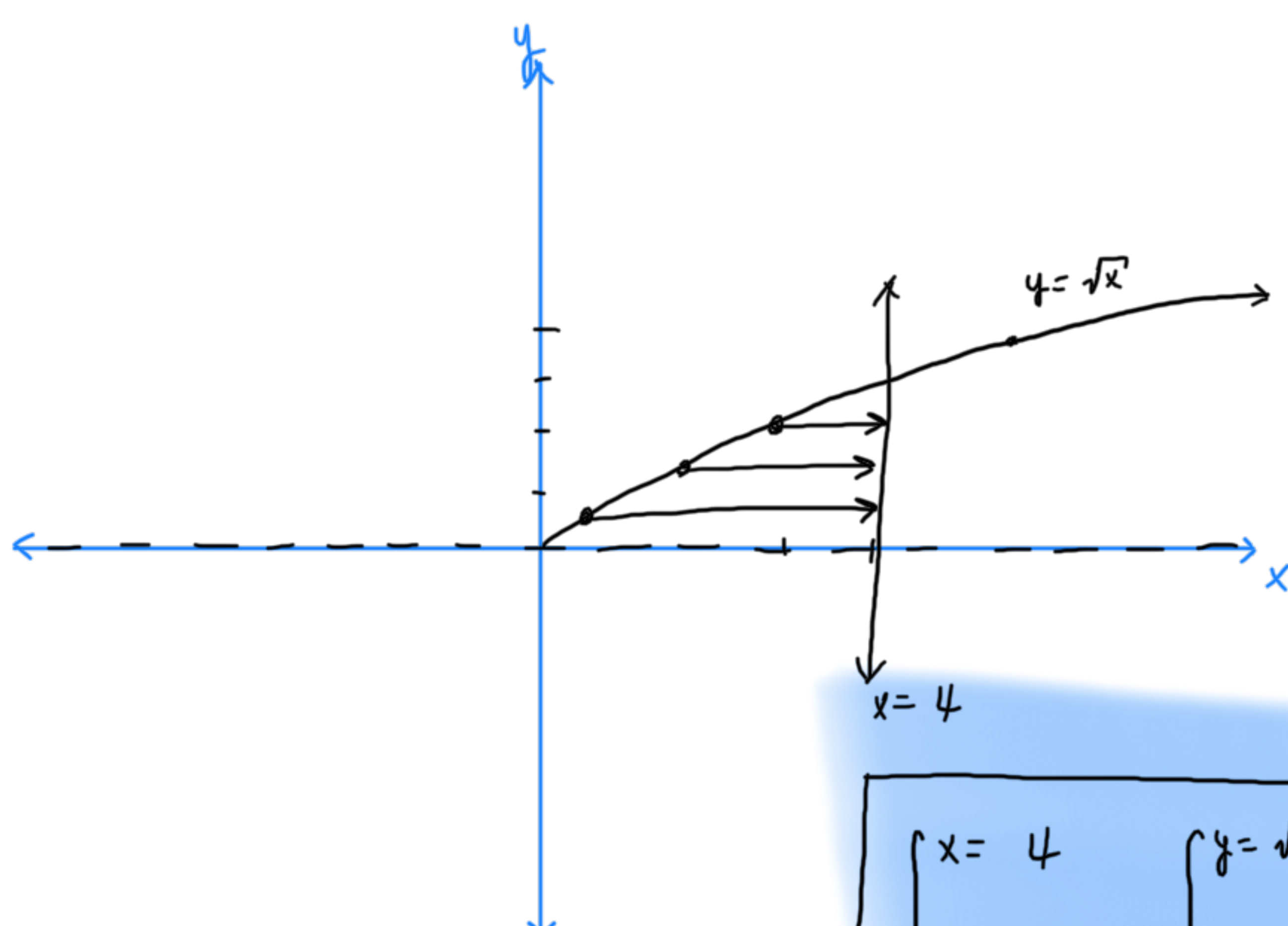


$$\boxed{\int_{x=0}^{x=4} \int_{y=x}^{y=4} y^2 e^{xy} \, dy \, dx}$$

$$\boxed{\int_{y=0}^{y=4} \int_{x=0}^{x=y} y^2 e^{xy} \, dy \, dx}$$

1.b)

$$\iint_D \frac{y}{1+x^2} \, dA \qquad D: \{(y=0) \wedge (y=\sqrt{x}) \wedge (x=4)\}$$



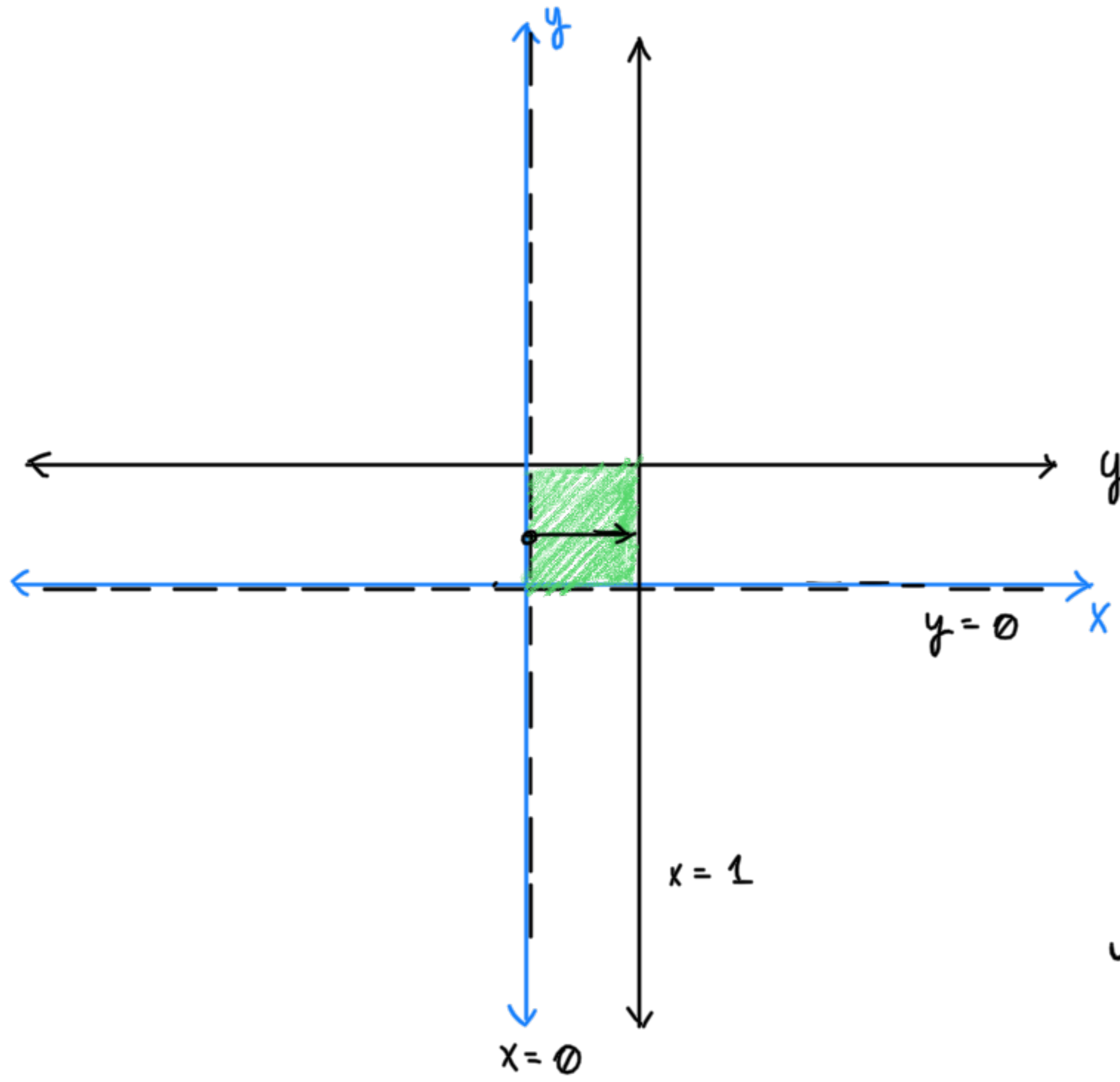
$\sqrt{x} = 4$

$x = 16$

$$\boxed{\int_{x=0}^{x=4} \int_{y=0}^{y=\sqrt{x}} \frac{y}{1+x^2} \, dy \, dx}$$

$$\boxed{\int_{y=0}^{y=2} \int_{x=y^2}^{x=4} \frac{y}{1+x^2} \, dx \, dy}$$

2. a) $$\int_0^1 \left(\int_0^{\frac{\pi}{2}} y\cos(xy)\, dy \right) dx$$



$$\int_{y=0}^{y=\frac{\pi}{2}} \int_{x=0}^{x=1} y\cos(xy)\, dx\, dy$$

$\boxed{1}$ $$\int_0^1 y\cos(xy)\, dx = \sin(xy)\Big]_0^1 = \left\{ \sin(y) - \cancel{\sin(0)}^{\,0} \right\}$$

$$= \sin(y)$$

$\boxed{2}$ $$\int_0^{\frac{\pi}{2}} \sin(y)\, dy = -\cos(y)\Big]_0^{\frac{\pi}{2}} = -\left\{ \cancel{\cos\left(\tfrac{\pi}{2}\right)}^{\,0} - \cos(0) \right\}$$

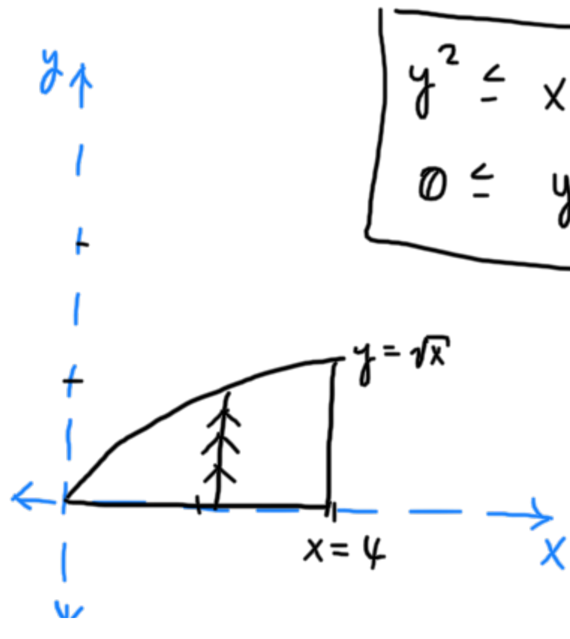
$$= -\{-1\} = 1$$

2. b) $$\int_0^2 \int_{y^2}^4 y^2 \sqrt{x}\, \sin(x)\, dx\, dy$$

$$\boxed{\begin{array}{l} y^2 \leq x \leq 4 \\ 0 \leq y \leq 2 \end{array}}$$

D: $0 \leq y \leq \sqrt{x}$

$0 \leq x \leq 4$



$$= \int_0^4 \int_0^{\sqrt{x}} y^2 \sqrt{x}\, \sin(x)\, dy\, dx$$

$\boxed{1}$ $$\int_0^{\sqrt{x}} y^2 \sqrt{x} \sin(x)\, dy = \frac{\sqrt{x}\sin(x)}{3} y^3 \Big]_0^{\sqrt{x}} =$$

$$= \frac{\sqrt{x}\sin(x)}{3} \left\{ (\sqrt{x})^3 - (0)^3 \right\} = \frac{x^2 \sin(x)}{3}$$

$\boxed{2}$ $$\int_0^4 \frac{x^2 \sin(x)}{3} dx = -x^2\cos(x) + \underbrace{2\int \cos(x)\, x dx}_{IPP}$$

$$u = x^2 \qquad dv = \sin(x)$$
$$du = 2x\, dx \qquad v = -\cos(x)$$

$$2\int \cos(x)\, x = 2\left(x\sin(x) - \int \sin(x)\, dx\right) = 2\left(x\sin(x) + \cos(x)\right)$$

$$u = x \qquad dv = \cos(x)$$
$$du = dx \qquad v = \sin(x)$$

$$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x)\Big]_0^4 =$$

$$= -(4)^2\cos(4) + 2(4)\sin(4) + 2\cos(4) - (0 + 0 + 2)$$

$$= -16\cos(4) + 8\sin(4) + 2\cos(4) - 2$$

$$= -14\cos(4) + 8\sin(4) - 2$$

3. a) $$\int_0^2 \int_0^{\sqrt{2x - x^2}} \underbrace{\sqrt{x^2 + y^2}}_{r}\, dy\, dx$$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$dA = dy\,dx$

$dy\,dx = r\,dr\,d\theta$

$y = \sqrt{2x - x^2}$

$y^2 + x^2 = 2x$

$y^2 + (x^2 - 2x + 1) = 1$

$y^2 + (x - 1)^2 = 1$

$y^2 + (x-1)^2 = 1$ ... $r \, dr \, d\theta$

$$r^2\sin^2(\theta) + (r\cos(\theta) - 1)^2 = 1$$

$$r^2\cancel{\sin^2(\theta)} + r^2\cancel{\cos^2(\theta)} - 2r\cos(\theta) + \cancel{1} = \cancel{1}$$

$$r^2 - 2\cos(\theta) = 0$$

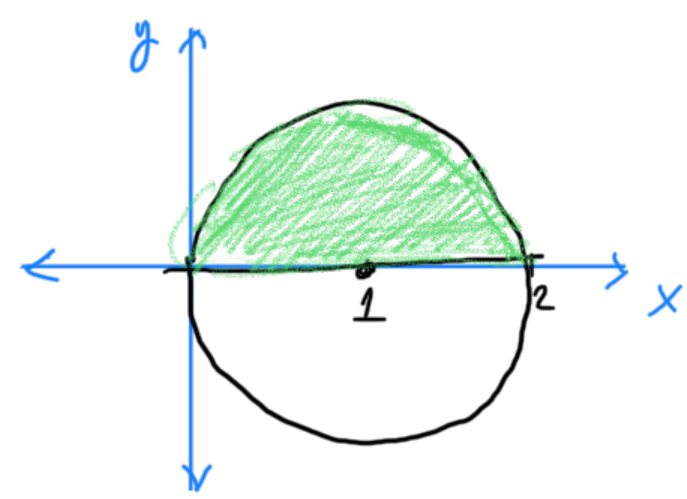
$$r = \sqrt{2\cos(\theta)}$$

$$r = 0$$

$$0 \le r \le \sqrt{2\cos(\theta)}$$

$$0 \le \theta \le \frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2\cos(\theta)}} r^2 \, dr \, d\theta$$



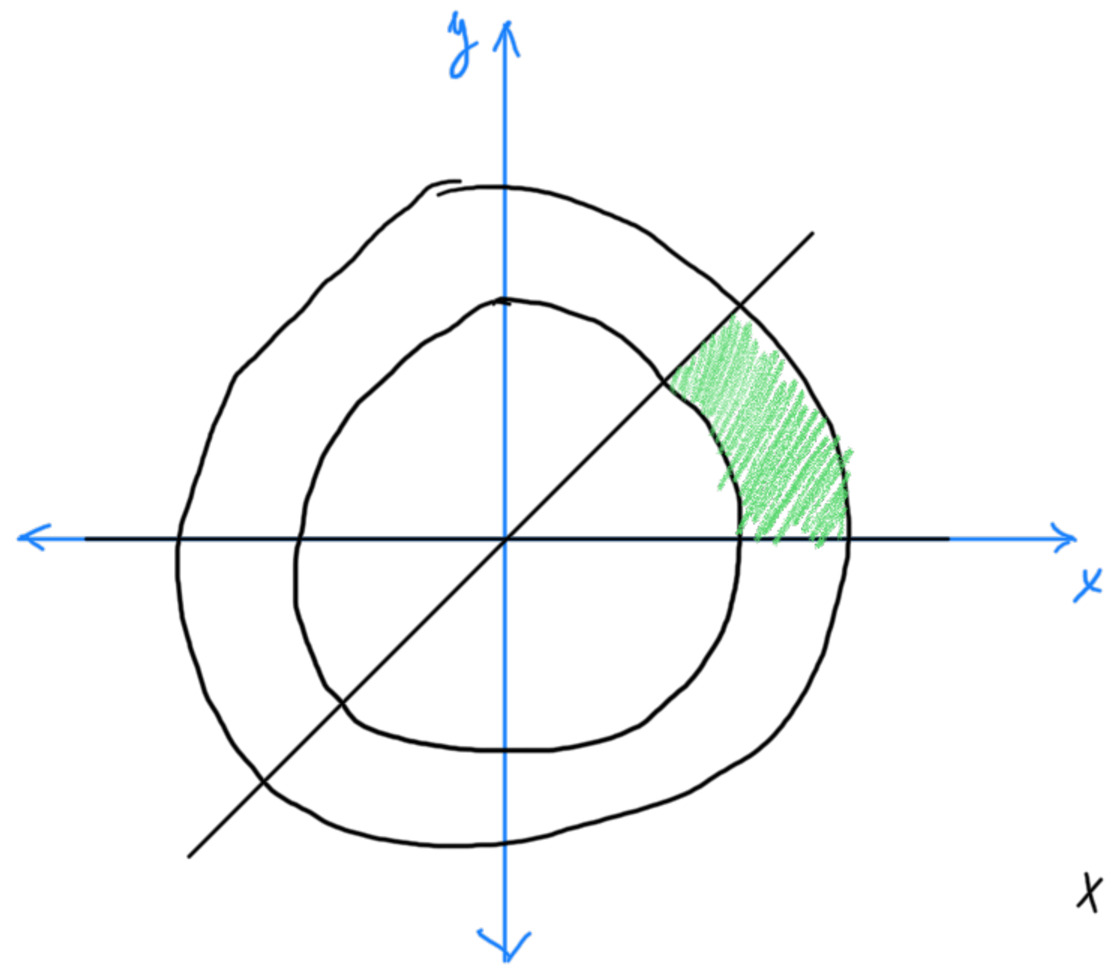
$\boxed{1}$ $$\int_0^{\sqrt{2\cos(\theta)}} r^2 \, dr$$

$$= \frac{r^3}{3}\Bigg]_0^{\sqrt{2\cos(\theta)}} = \frac{1}{3}\left\{(2\cos(\theta))^{\frac{3}{2}} - 0\right\}$$

$$= \frac{1}{3}\left(2^{\frac{3}{2}}\cos^{\frac{3}{2}}(\theta)\right) = \frac{2^{\frac{3}{2}}}{3}\cos^{2/3}(\theta)$$

$\boxed{2}$ $$\frac{2^{\frac{3}{2}}}{3}\int_0^{\pi/2} (\cos(\theta))^{\frac{3}{2}} \, d\theta$$

3.b) $\displaystyle\iint_D \arctan\left(\frac{y}{x}\right) dA$ $\quad D: \{(x,y) \mid 1 \le x^2+y^2 \le 4,\ 0 \le y \le x\}$



$1 \le r \le 2$
$0 \le \theta \le \frac{\pi}{4}$

$y = 0$
$y = x$

$x^2 + y^2 = 2^2$
$x^2 + y^2 = 1^2$

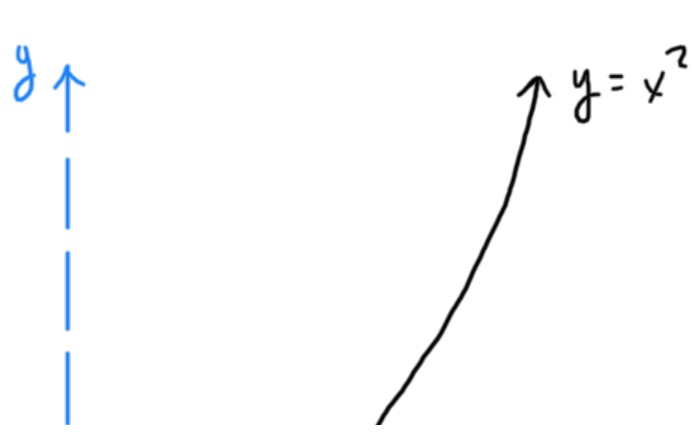
$$\int_1^2 \int_0^{\frac{\pi}{4}} \theta r \, d\theta \, dr$$

$\boxed{1}$ $\displaystyle\int_0^{\frac{\pi}{2}} \theta r \, d\theta = r \left.\frac{\theta^2}{2}\right]_0^{\frac{\pi}{2}} = \frac{r}{2}\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8} r$

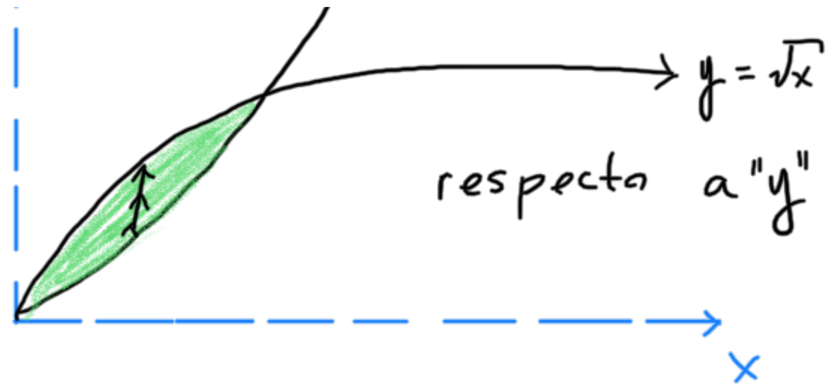
$\boxed{2}$ $\displaystyle\int_1^2 \frac{\pi^2}{8} r \, dr = \frac{\pi^2}{16}\left\{2^2 - 1^2\right\} = \frac{\pi^2}{16}\left\{4 - 1\right\} = \frac{\pi^2}{16}(3)$

$= \frac{3}{16}\pi^2$

4.a) $z = 7x + 2y$ $\quad D: \{(y = x^2) \wedge (x = y^2)\}$



$0 \le x \le 1$
$x^2 \le y \le \sqrt{x}$



$$\int_0^1 \int_{x^2}^{\sqrt{x}} 7x + 2y \, dy \, dx$$

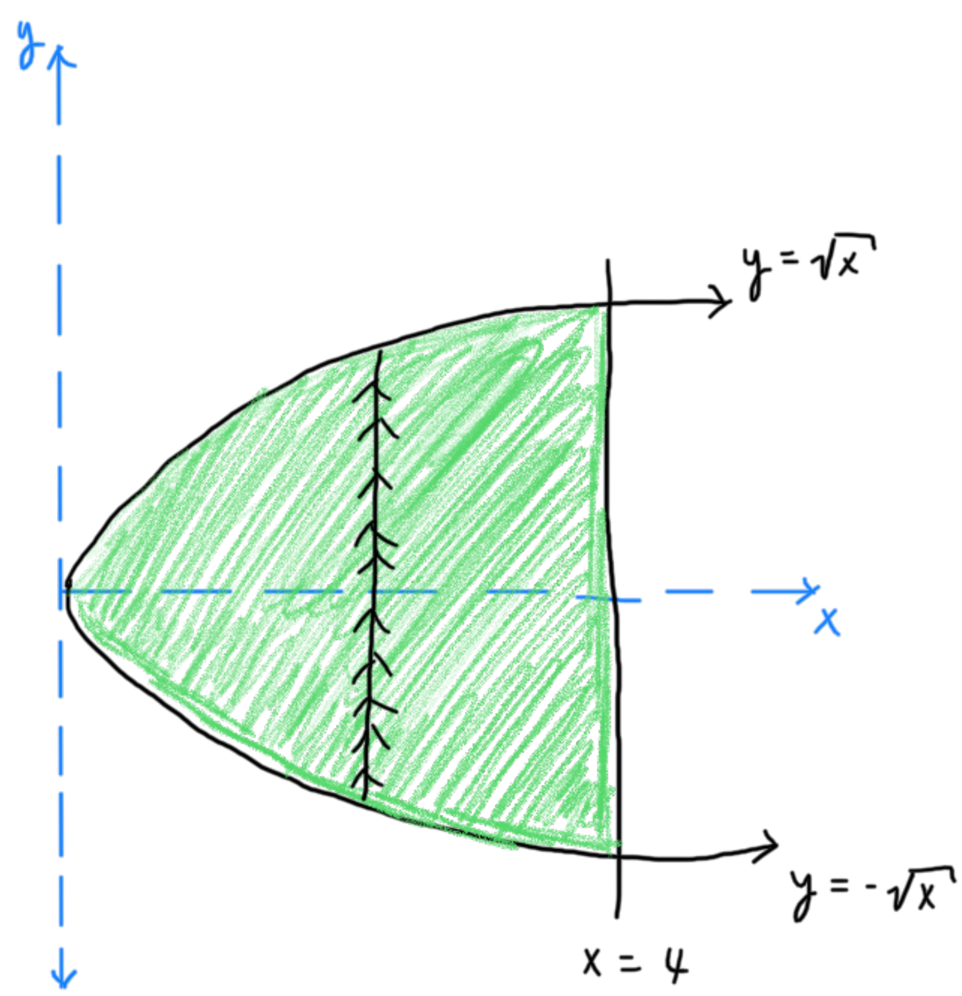
$\boxed{1}$
$$\int_{x^2}^{\sqrt{x}} 7x + 2y \, dy = 7xy + y^2 \Big]_{x^2}^{\sqrt{x}}$$

$$= 7x\sqrt{x} + x - 7x^3 - x^4 = 7x^{\frac{3}{2}} + x - 7x^3 - x^4$$

$\boxed{2}$
$$\int_0^1 7x^{\frac{3}{2}} + x - 7x^3 - x^4 \, dx$$

$$= \frac{35}{2} x^{\frac{5}{2}} + \frac{x^2}{2} - \frac{7}{4} x^4 - \frac{x^5}{5} \Big]_0^1 = \frac{27}{20}$$

4.b) $z = 1 + x^2 y^2$ $\quad x = y^2$ & $x = 4$



$$\iint_D 1 + x^2 y^2 \, dA$$

$D: -\sqrt{x} \leq y \leq \sqrt{x}$

$2 \left(0 \leq y \leq \sqrt{x} \right)$

$0 \leq x \leq 4$

$$2 \int_0^4 \int_0^{\sqrt{x}} 1 + x^2 y^2 \, dy \, dx$$

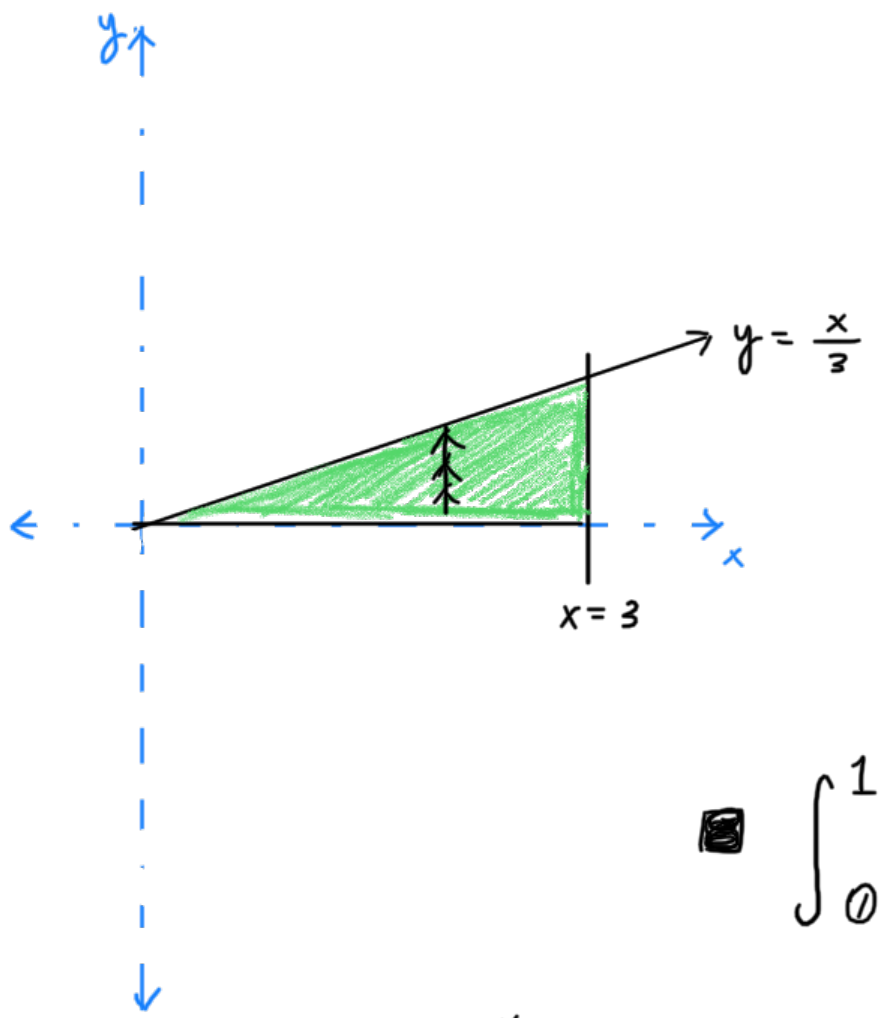
$\boxed{1}$
$$\int_0^{\sqrt{x}} 1 + x^2 y^2 \, dy = y + \frac{x^2}{3} y^3 \Big]_0^{\sqrt{x}} =$$

$$= \left\{ \sqrt{x} + \frac{x^2}{3} (\sqrt{x})^3 - 0 \right\} = \left(\sqrt{x} + \frac{x^{\frac{7}{2}}}{3} \right) 2$$

$\boxed{2}$ $2\int_0^4 \sqrt{x} + \frac{x^{\frac{7}{2}}}{3}\, dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{27}x^{\frac{9}{2}}\Big]_0^4$

$= 2\left(\frac{2}{3}(4)^{\frac{3}{2}} + \frac{2}{27}(4)^{\frac{9}{2}} - 0\right) = \frac{2336}{27}$

5. a) $\int_0^1 \int_{3y}^3 5e^{x^2}\, dx\, dy$



$3y \le x \le 3$

$0 \le y \le 1$

$0 \le x \le 1$

$0 \le y \le \frac{x}{3}$

$\int_0^1 \int_0^{\frac{x}{3}} 5e^{x^2}\, dy\, dx$

$\boxed{1}$ $\int_0^{\frac{x}{3}} 5e^{x^2}\, dy = 5e^{x^2}\left\{\frac{x}{3} - 0\right\} = \frac{5}{3}e^{x^2}x$

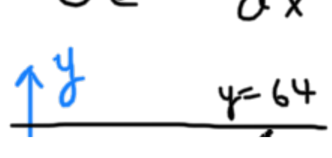
$\boxed{2}$ $\frac{5}{3}\int_0^1 e^{x^2}x\, dx = \frac{5}{6}\int_0^1 e^u\, du = \frac{5}{6}e^u\Big]_{u(0)}^{u(1)} =$

$u = x^2$

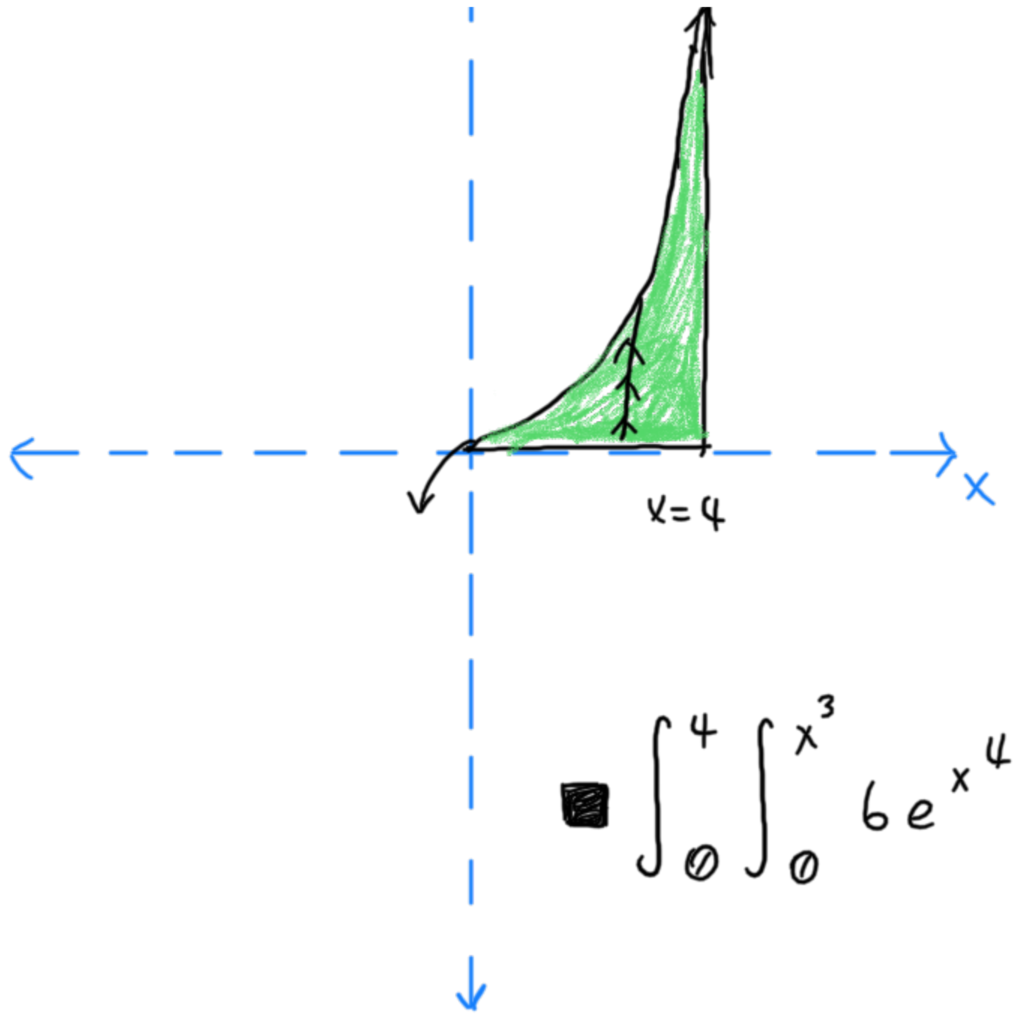
$\frac{du}{2} = x\,dx$

$= \frac{5}{6}\left[e^1 - e^0\right] = \frac{5}{6}\left[e - 1\right]$

5. b) $\int_0^{64} \int_{\sqrt[3]{y}}^4 6e^{x^4}\, dx\, dy$



$\sqrt[3]{y} \le x \le 4$



$0 \le y \le 4$

$0 \le y \le x^3$
$0 \le x \le 4$

$$\blacksquare \int_0^4 \int_0^{x^3} 6e^{x^4} \, dy \, dx$$

$\boxed{1}$ $$\int_0^{x^3} 6e^{x^4} \, dy = 6e^{x^4} \{ x^3 - 0 \}$$

$$= 6e^{x^4} x^3$$

$\boxed{2}$ $$\int_0^4 6e^{x^4} x^3 \, dx = \frac{6}{4} \int_0^4 e^u \, du = \frac{6}{4} e^u \Big]_{u(0)}^{u(4)}$$

$u = x^4$
$\frac{du}{4} = x^3 dx$

$$= \frac{6}{4} \{ e^{256} - 1 \}$$

$$= \frac{6}{4} \left(e^{256} - 1 \right)$$