

TAREA #10 - DAVID CORZO

1.a) $f(x, y) = 2x^2 + 2xy + y^2 - 2x - 3$

$$f_x = 4x + 2y - 2$$

$$4x + 2y - 2 = 0$$

$$4x + 2y = 2$$

$$y = \frac{1}{2}(2 - 4x)$$

$$y = 1 - 2x$$

$$y = 1 - 2(1)$$

$$y = -1$$

$$f_y = 2x + 2y$$

$$2x + 2y = 0$$

$$2x + 2(1 - 2x) = 0$$

$$2x + 2 - 4x = 0$$

$$-2x = -2$$

$$x = 1$$

Ponto crítico em $y = -1$ $x = 1$

$$f_{xx} = 4$$

$$f_{yy} = 2$$

$$f_{xy} = 2$$

$$f_{yx} = 2$$

$$f(x, y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} = (4)(2) - (2)(2) = 4$$

$$f(1, -1) = 2(1)^2 + 2(1)(-1) + (-1)^2 - 2(1) - 3$$

$$= \cancel{2} - 2 + 1 - 2 - 3$$

$$= -4$$

Mínimo em $f(1, -1) = -4$

1.b) $g(x, y) = \sqrt{x^2 + y^2}$

$$g_x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$g_y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\frac{x}{\sqrt{x^2 + y^2}} = 0$$

$$x = 0$$

$$\frac{y}{\sqrt{x^2 + y^2}} = 0$$

$$y = 0$$

$$g_{xx} = (x^2 + y^2)^{-\frac{1}{2}} - \left[\frac{1}{2}(x^2 + y^2)^{-\frac{3}{2}} \cdot 2x \right] x$$

$$= \frac{1}{\sqrt{x^2 + y^2}} - \frac{x^2}{\sqrt{x^2 + y^2}} \Bigg|_{(0,0)} = \text{indef.}$$

$$g_{yy} = \frac{1}{\sqrt{x^2 + y^2}} - \frac{y^2}{\sqrt{x^2 + y^2}} \Bigg|_{(0,0)} = \text{indef.}$$

$$g_{xy} = -\frac{1}{2}(x^2 + y^2)^{-\frac{3}{2}} 2xy$$

$$= -\frac{xy}{(x^2 + y^2)^{\frac{3}{2}}}$$

$$-\frac{xy}{(x^2 + y^2)^{\frac{3}{2}}} = 0$$

= indef.

$$x = 0$$

$$y = 0$$

$$g_{yx} = -\frac{1}{2}(x^2 + y^2)^{-\frac{3}{2}} 2xy$$

$$= -\frac{xy}{(x^2 + y^2)^{\frac{3}{2}}} = \text{indef.}$$

Hay punto de silla por que se indefinen todos los componentes de $D(x, y)$.

\therefore No hay mínimo ni máximo

2. a.)

$$f(x, y) = (x^2 + y^2)^{\frac{1}{3}} + 2$$

$$f_x = \frac{1}{3}(x^2 + y^2)^{-\frac{2}{3}} \cdot 2x \quad f_y = \frac{1}{3}(x^2 + y^2)^{-\frac{2}{3}} \cdot 2y$$

$$= \frac{2x}{3(x^2 + y^2)^{\frac{2}{3}}}$$

$$= \frac{2y}{3(x^2 + y^2)^{\frac{2}{3}}}$$

$$\frac{2x}{3(x^2 + y^2)^{\frac{2}{3}}} = 0$$

$$2x = 0$$

$$x = 0$$

$$\frac{2y}{3(x^2 + y^2)^{\frac{2}{3}}} = 0$$

$$2y = 0$$

$$y = 0$$

\therefore El punto $(0, 0)$ es un punto de silla.

2.b)

$$g(x, y) = x^2 - 3xy - y^2$$

$$g_x = 2x - 3y$$

$$2x - 3y = 0$$

$$g_y = -3x - 2y$$

$$-3x - 2y = 0$$

$$D(x, y) = \begin{vmatrix} 2 & -3 \\ -3 & -2 \end{vmatrix} = 5$$

Punto mínimo en
0

3)

$$P(L, K) = 2LK - 3K^2 - 2L^2 - 2L + 21K$$

$$P_L = 2K - 4L - 2$$

$$2K - 4L - 2 = 0$$

$$L = 1,500$$

$$P_K = 2L - 6K + 21$$

$$2L - 6K + 21 = 0$$

$$K = 4,000$$

$$D(x, y) = \begin{vmatrix} -4 & 2 \\ 2 & -6 \end{vmatrix} = 20$$

máximo en

$$L = 1,500 \text{ \& } K = 4,000$$

4)

$$q_A = 100 - 5x - 2y$$

$$q_B = 250 - 3x - 5y$$

$$U = x q_A + y q_B - 20 q_A - 10 q_B$$

$$U = x(100 - 5x - 2y) + y(250 - 3x - 5y) - 20(100 - 5x - 2y) - 10(250 - 3x - 5y)$$

$$U = -5x^2 - 5y^2 - 5xy + 230x + 340y - 4,500$$

$$U_x = -10x - 5y + 230$$

$$-10x - 5y + 230 = 0$$

$$x = 8$$

$$U_y = -10y - 5x + 340$$

$$-10y - 5x + 340 = 0$$

$$y = 30$$

$$U(x, y) = \begin{vmatrix} -10 & -5 \\ -5 & -10 \end{vmatrix} = 75$$

Precios de venta máximos $x = 8$ $y = 30$

6) • función de producción

$$P(L, K) = 118L + 20K + 3LK - L^2 - 2K^2$$

• L : mano de obra

• K : Capital utilizado

• P : nivel de producción

• Costos unitarios:

$$L = 80$$

$$K = 160$$

• Presupuesto:

$$\$5640$$

$$\nabla f = \lambda \nabla g$$

$$P_L = \lambda g_L$$

$$P_L = 118 + 3K - 2L$$

$$P_K = \lambda g_K$$

$$g_L = 80$$

$$P_K = 20 + 3L - 4K$$

$$g_K = 160$$

$$118 + 3K - 2L = 80\lambda$$

$$\frac{1}{80}(118 + 3K - 2L) = \lambda$$

$$20 + 3L - 4K = 160\lambda$$

$$\frac{1}{160}(20 + 3L - 4K) = \lambda$$

$$\frac{1}{80}(118 + 3K - 2L) = (20 + 3L - 4K) \frac{1}{160}$$

$$160(118 + 3K - 2L) = (20 + 3L - 4K)80$$

$$18,880 + 480K - 320L = 1600 + 240L - 320K$$

$$18,880 - 1,600 = 240L + 320L - 320K + 480K$$

$$17,280 = 560L - 160K$$

$$\frac{17,280 + 160K}{560} = L$$

$$\blacksquare \quad 80L + 160K = 5,640$$

$$80 \left(\frac{17,280 + 160K}{560} \right) + 160K = 5,640$$

$$\frac{80 \cdot 17,280}{560} + \frac{80 \cdot 160K}{560} + 160K = 5,640$$

$$\frac{160}{7}K + 160K = 5,640 - \frac{80 \cdot 17,280}{560}$$

$$\frac{1280}{7}K = \frac{22,200}{7}$$

$$K = \frac{7 \cdot 22,200}{7 \cdot 1280}$$

$$K = \frac{555}{32}$$

$$80L + 160 \left(\frac{555}{32} \right) = 5,640$$

$$L = \frac{5,640 - 160 \left(\frac{555}{32} \right)}{80}$$

$$L = \frac{573}{16}$$

$$\text{maximiza en } K = \frac{555}{32}$$

$$\& \quad L = \frac{573}{16}$$