TAREA #11 - DAVID CORZO

1) $\int_0^2 \int_1^2 (x - 3y^2)\, dy\, dx$

■ $\int_1^2 (x - 3y^2)\, dy = xy - y^3 \Big]_{y=1}^{y=2}$

$= \{[x(2) - (2)^3] - [x(1) - (1)^3]\}$

$= 2x - 8 - x + 1 = x - 7$

■ $\int_0^2 (x - 7)\, dx = \frac{x^2}{2} - 7x \Big]_{x=0}^{x=2} =$

$= \{[\frac{(2)^2}{2} - 7(2)] - [0]\} = \frac{4}{2} - 14 = 2 - 14 = -12$

2) $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$

■ $\int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy = \int_1^2 \left(xy^{-1} + yx^{-1}\right) dy =$

$= -xy^{-2} + x \Big]_{y=1}^{y=2} = -\frac{x}{y^2} + x \Big]_{y=1}^{y=2} =$

$= x\left(-\frac{1}{y^2} + 1\right)\Big]_{y=1}^{y=2} = x\{[-\frac{1}{4} + 1] - [-\frac{1}{1} + 1]\} =$

$= x\{[\frac{3}{4}] - [0]\} = \frac{3x}{4}$

■ $\frac{3}{4}\int_1^4 x\, dx = \frac{3}{4} \cdot \frac{x^2}{2} \Big]_{x=1}^{x=4} = \frac{3x^2}{8} \Big]_{x=1}^{x=4} = \frac{3}{8}\{(4)^2 - (1)^2\} =$

$= \frac{3}{8}\{16 - 1\} = \frac{3}{8}\{15\} = \frac{45}{8}$

3) $$\int_{-3}^{3}\int_{0}^{\frac{\pi}{2}}\left(y + y^2\cos(x)\right)\,dx\,dy =$$

■ $$\int_{0}^{\frac{\pi}{4}}\left(y + y^2\cos(x)\right)dx = yx + y^2\sin(x)\Big]_{x=0}^{x=\frac{\pi}{4}} =$$

$$= y\left[x + y\sin(x)\right]_{x=0}^{x=\frac{\pi}{4}} = y\left\{\left[\left(\frac{\pi}{4}\right) + y\sin\left(\frac{\pi}{4}\right)\right] - [0]\right\} =$$

$$= y\left\{\frac{\pi}{4} + y\frac{\sqrt{2}}{2}\right\} = \frac{\pi y}{4} + \frac{y^2\sqrt{2}}{2}$$

■ $$\int_{-3}^{3}\left(\frac{\pi}{4}y + \frac{\sqrt{2}}{2}y^2\right)dy = \frac{\pi}{4}\cdot\frac{y^2}{2} + \frac{\sqrt{2}}{2}\cdot\frac{y^3}{3}\Big]_{y=-3}^{y=3} =$$

$$= \left\{\left[\frac{\pi}{8}(3)^2 + \frac{\sqrt{2}}{6}(3)^3\right] - \left[\frac{\pi}{8}(-3)^2 + \frac{\sqrt{2}}{6}(-3)^3\right]\right\} =$$

$$= \left\{\left[\frac{9\pi}{8} + \frac{\sqrt{2}\,27}{6}\right] - \left[\frac{9\pi}{8} - \frac{\sqrt{2}\,27}{6}\right]\right\} =$$

$$= \left\{\cancel{\frac{9\pi}{8}} + \frac{\sqrt{2}\,27}{6} - \cancel{\frac{9\pi}{8}} + \frac{\sqrt{2}\,27}{6}\right\} = \frac{2\cdot\sqrt{2}\cdot 27}{6}$$

$$= 9\sqrt{2}$$

4) $$\iint_R x\sin(x+y)\,dA \qquad R = [\underbrace{0}_{a},\underbrace{\tfrac{\pi}{6}}_{b}] \times [\underbrace{0}_{c},\underbrace{\tfrac{\pi}{3}}_{d}]$$

$$\int_{0}^{\frac{\pi}{3}}\int_{0}^{\frac{\pi}{6}}\left(x\sin(x+y)\right)dx\,dy \qquad 0 \le x \le \frac{\pi}{6},\ 0 \le y \le \frac{\pi}{3}$$

■ $$\int_{0}^{\frac{\pi}{6}}\left(x\sin(x+y)\right)dx =$$

$u = x$ $\quad$ $dv = \sin(x+y)$

$du = dx$

$w = x+y$

$dw = 1$

$v = -\cos(x+y)$

$$= -x\cos(x+y) + \int\cos(x+y)\,dx$$

$$= -x\cos(x+y) + \sin(x+y)\Big]_{x=0}^{x=\frac{\pi}{6}} =$$

$$= \left\{ \left[-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) \right] - \left[\sin(y) \right] \right\}$$

$$= -\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) - \sin(y)$$

$$\int_0^{\frac{\pi}{3}} \left(-\frac{\pi}{6}\cos\left(\frac{\pi}{6}+y\right) + \sin\left(\frac{\pi}{6}+y\right) - \sin(y) \right) dy$$

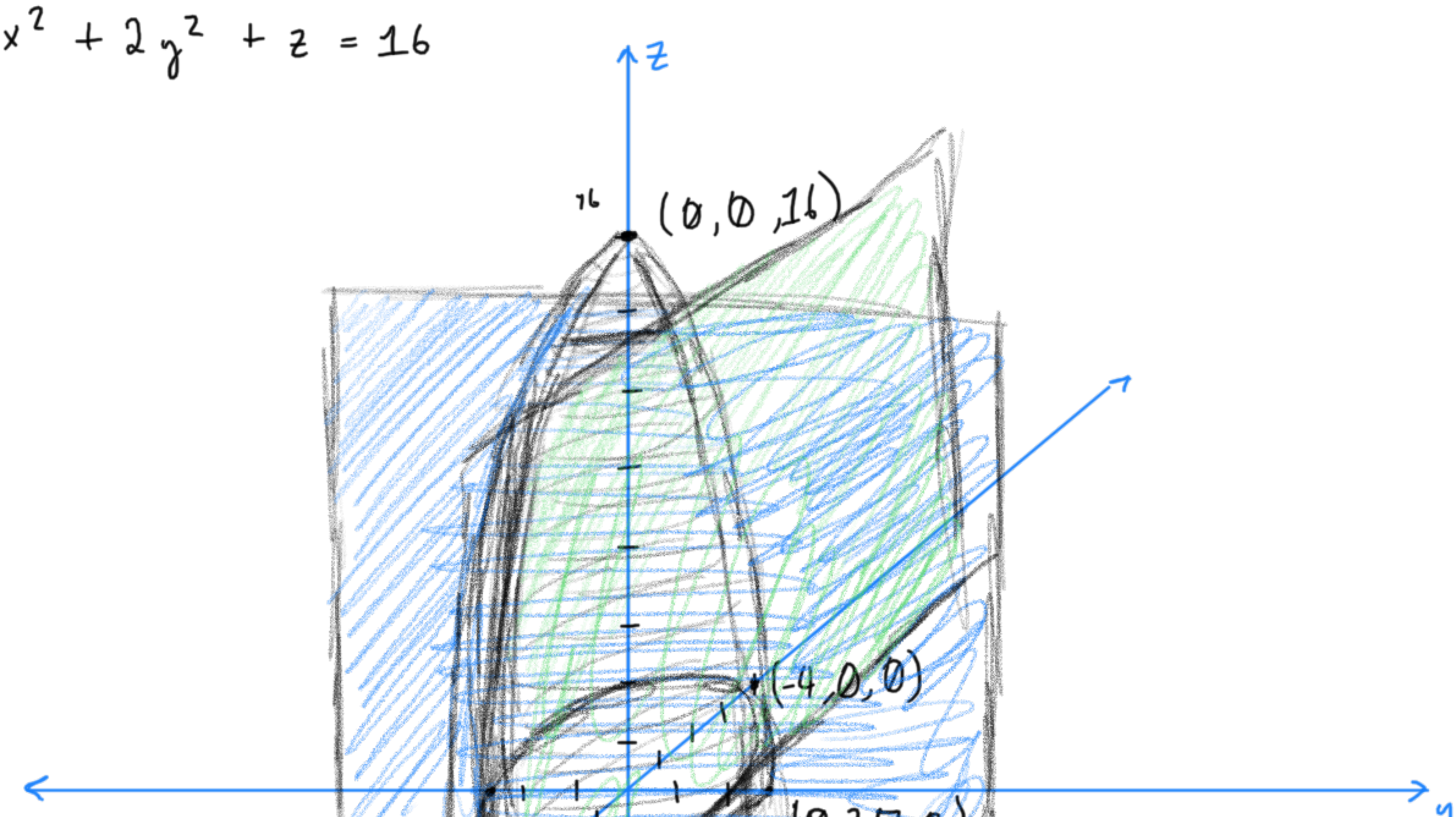
$$= -\frac{\pi}{6}\sin\left(\frac{\pi}{6}+y\right) - \cos\left(\frac{\pi}{6}+y\right) + \cos(y) \Bigg]_{y=0}^{y=\frac{\pi}{3}}$$

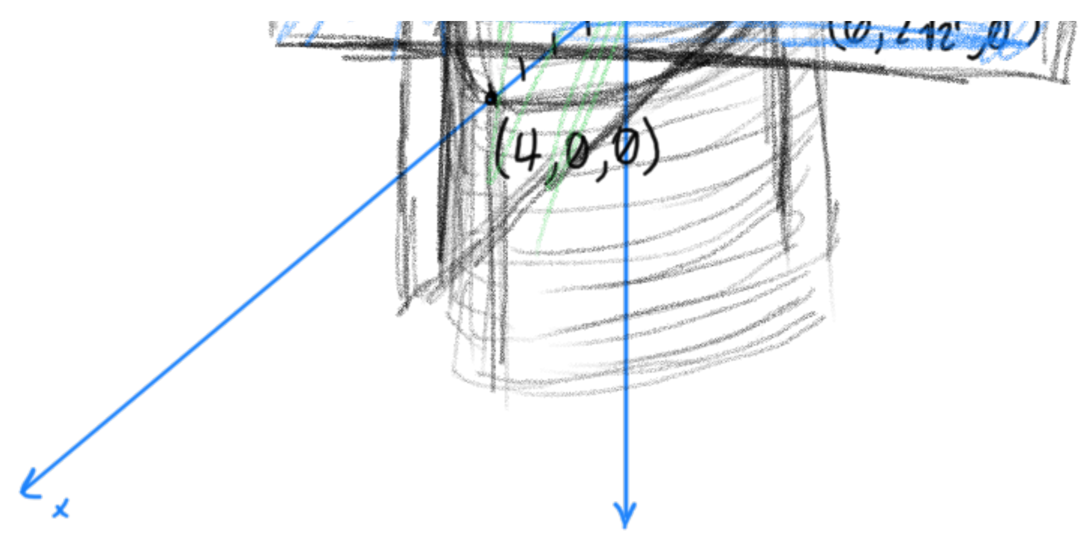
$$= \left\{ \left[-\frac{\pi}{6}\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) \right] - \left[-\frac{\pi}{6}\sin\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right) + \cos(0) \right] \right\}$$

$$= \left\{ \left[-\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1 \right] - \left[-\frac{\pi}{12} - \frac{\sqrt{3}}{2} + 1 \right] \right\}$$

$$= \boxed{\frac{\pi}{4}}$$

5) $x^2 + 2y^2 + z = 16$





$x = 0, \quad y = 0$

$(0)^2 + 2(0)^2 + z = 16$

$z = 16$

$z = 0, \quad y = 0$

$x = \pm 4$

$z = 0, \quad x = 0$

$y = \pm\sqrt{8} = \pm 2\sqrt{2}$

$0 \leq x \ 2$

$0 \leq y \leq 2$

$$= \underbrace{\int_0^2 \int_0^2 (16 - x^2 - 2y^2)\, dy\, dx}_{\int_c^d \int_a^b f(x,y)\, dy\, dx}$$

$$\int_0^2 (16 - x^2 - 2y^2)\, dy = 16y - yx^2 - \frac{2}{4}y^4 \Big]_{y=0}^{y=2} =$$

$$= \left\{\left[16(2) - (2)x^2 - \frac{1}{2}(2)^4\right] - [0]\right\} = 32 - 2x^2 - 8 = -2x^2 + 24$$

$$\int_0^2 (-2x^2 + 24)\, dx = -\frac{2}{3}x^3 + 24x \Big]_{x=0}^{x=2} =$$

$$= \left\{\left[-\frac{2}{3}(2)^3 + 24(2)\right] - [0]\right\} = \frac{128}{3} \approx \boxed{42.\overline{66}}$$