1. Derivación Implícita. $(1, +1, 4)$ no es parte de S.

$16 - 2 - 2 - 12 = 0 \neq 0$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{+2}{2z} = +\frac{2}{8} = +\frac{1}{4}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{+2}{2z} = +\frac{2}{8} = +\frac{1}{4}$$

Ec. Plano Tangente. $z = 4 + \frac{1}{4}(x-1) + \frac{1}{4}(y-1).$

b. Derive explícitamente $z = \sqrt{12 + 2x + 2y}$

$z(1,-1) = \sqrt{12}$

$$z_x = \frac{1}{\sqrt{12 + 2x + 2y}} = \frac{1}{\sqrt{12}}$$

$$z_y = \frac{1}{\sqrt{12 + 2x + 2y}} = \frac{1}{\sqrt{12}}$$

Ec. Plano Tangente $z = \sqrt{12} + \frac{1}{\sqrt{12}}(x-1) + \frac{1}{\sqrt{12}}(y+1)$

2. $F(x,y) = \cos(yx) + 1 - \sec(zx) - \sin(yz) = 0$

$$\frac{\partial z}{\partial x} = \frac{-y\sin(yx) - z\sec(zx)\tan(zx) + 0}{x\sec(xz)\tan(xz) + y\cos(yz)} \qquad -\frac{F_x}{F_z}.$$

$$\frac{\partial z}{\partial y} = \frac{-x\sin(yx) - z\cos(yz)}{x\sec(xz)\tan(xz) + y\cos(yz)} \qquad -\frac{F_y}{F_z}.$$

$F(x,y) =$ constante $\qquad F(x,y) = G(x,y)$

Razón de cambio ó Instantánea $\frac{dW}{dt}$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}. \qquad \frac{\partial w}{\partial x} = -\frac{F_x}{F_w}.$$

Cuando $t=3$, $x = e^0 = 1$ $\quad y = \ln(1) + 3 - 2 = 1$

$$\frac{\partial w}{\partial x} = \frac{y}{1+(yx)^2}, \quad \left.\frac{\partial w}{\partial x}\right|_{(1,1)} = \frac{1}{1+1} = \frac{1}{2}.$$

$$\frac{\partial w}{\partial y} = \frac{x}{1+(yx)^2}, \quad \left.\frac{\partial w}{\partial y}\right|_{(1,1)} = \frac{1}{2}.$$

$$\frac{dx}{dt} = 2e^{2t-6}, \quad x'(3) = 2e^0 = 2.$$

$$\frac{dy}{dt} = \left(\frac{2}{2t-5}+1\right) \quad y'(3) = \frac{2}{1}+1 = 3.$$

$$\frac{dw}{dt} = \frac{1}{2}(2) + \frac{1}{2}(3) = \frac{5}{2}$$

4. Derivada Direccional $\quad D_u T = \nabla T \cdot \vec{u}. = \frac{\nabla T \cdot \vec{v}}{|\vec{v}|}$

$$\nabla T = \langle \sin(\pi y z), x\pi z\cos(\pi y z), \pi x y\cos(\pi y z)\rangle.$$

$$\nabla T(1,1,2) = \langle \sin(\cancel{2}\pi), 2\pi\cos(\cancel{2}\pi), \pi\cos(2\pi)\rangle.$$

$$\nabla T(1,1,2) = \langle 0, 2\pi, \pi\rangle.$$

$$\vec{u} = \frac{\langle 1,4,8\rangle}{\sqrt{1+16+64}} = \frac{1}{9}\langle 1,4,8\rangle.$$

vector unitario. $|u| = 9.$

$$D_u T = \frac{1}{9}\langle 0, 2\pi, \pi\rangle \cdot \langle 1,4,8\rangle. = \frac{8\pi + 8\pi}{9} = \frac{16\pi}{9}.$$

6. Presupuesto = 20,000 x periódicos y televisión

gasto gasto.

$x + y = 20{,}000$

Ventas $S = 80x^{1/4}y^{3/4}$ margen de utilidad 10%

$U = 0.10S - 20{,}000 = 8x^{1/4}y^{3/4} - 200{,}000$

$U = 80x^{1/4}y^{3/4} - 72x^{1/4}y^{3/4} - 20{,}000$

pedido es de 72.

Max $U = 8x^{1/4}y^{3/4} - 20{,}000$, $x + y = 20{,}000$

$20000\lambda - \lambda x - \lambda y.$

Lagrange $F(x,y,\lambda) = 8x^{1/4}y^{3/4} - 20000 + \lambda(20000 - x - y)$

$f_x = 2x^{-3/4}y^{3/4} - \lambda = 0 \Rightarrow \lambda = 2y^{3/4}x^{-3/4}$

$F_y = 6x^{1/4}y^{-1/4} - \lambda = 0 \qquad \lambda = 6x^{1/4}y^{-1/4}$

$F_\lambda = 20000 - x - y = 0.$

Lagrange $f(x,y,z) + \lambda[c - g(x,y,z)]$

Igualando λ's: $\dfrac{2y^{3/4}}{x^{3/4}} = \dfrac{6x^{1/4}}{y^{1/4}} \Rightarrow 2y = 6x$

Sustituya $y = 3x$ en f_λ. Minimizando la pérdida.

$20{,}000 - x - 3x = 0 \qquad 4x = 20{,}000 \Rightarrow x = 5{,}000.$

$y = 15{,}000$

No realice la prueba de la 2da Derivada. $\lambda \approx 4.56.$

Utilidad Máxima. $U(5000, 15000) = 8 \cdot 5000^{1/4}\, 15{,}000^{3/4} - 20{,}000.$

$9{,}118.02 - 20{,}000.$

$U = 8x^{1/4}y^{3/4} - 20000 \qquad x+y = 20{,}000$

Sustituya $y = 20{,}000 - x$ en $U(x)$.

$U(x) = 8x^{1/4}(20{,}000-x)^{3/4} - 20$ mil

$U'(x) = 2x^{-3/4}(20{,}000-x)^{3/4} - 6x^{1/4}(20000-x)^{-1/4} = 0$

$$\frac{2}{x^{3/4}}(20{,}000-x)^{3/4} = \frac{6x^{1/4}}{(20{,}000-x)^{1/4}}$$

$(20000-x) = 3x$

$20000 = 4x \Rightarrow x = 5{,}000$

$y = 15{,}000.$

Método 3: Microeconomía. $\quad P = x^{\alpha}y^{\beta} \qquad x+y = 20$ mil

$\alpha + \beta = 1.$

Producción óptima $\quad x = \alpha \cdot 20$ mil $\quad \alpha = 1/4$

$y = \beta \cdot 20$ mil $\quad \beta = 3/4.$

$\frac{P_A}{P_B} = \frac{\alpha y}{\beta x}$ | Precaución.

Costo: $c = 2x + 4y$. Costo Promedio $\frac{c}{x} = 2$.

Ingresos: $I = p_A x + p_B y$.

Utilidad $U = p_A x + p_B y - 2x - 4y$.

$$U(p_A, p_B) = p_A(16 - p_A + p_B) + p_B(24 - 2p_A - 4p_B) - 2(16 - p_A + p_B) - 4(24 - 2p_A - 4p_B)$$

$$U(p_A, p_B) = 26p_A - p_A^2 - p_A p_B - 4p_B^2 + 38p_B - 128.$$

$$U_{p_A} = 26 - 2p_A - p_B = 0 \qquad 2p_A + p_B = 26$$

$$U_{p_B} = -p_A - 8p_B + 38 = 0 \qquad p_A + 8p_B = 38.$$

$p_B = 26 - 2p_A \Rightarrow p_A + 26 \cdot 8 - 16p_A = 38.$

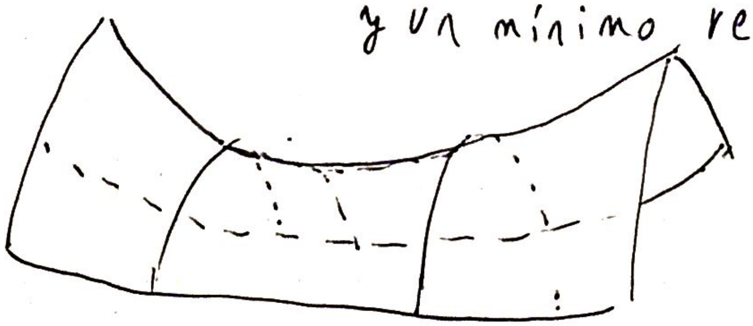
$p_A = 11.\overline{33}$ $\qquad 26 \cdot 8 - 38 = 15p_A.$ $\quad p_A = \frac{170}{15}$

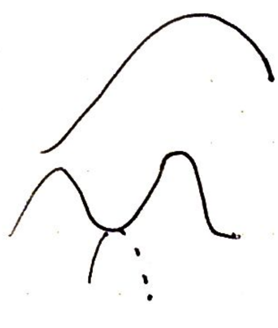
$p_B = 26 - 2(11.\overline{33}) = \frac{10}{3} = 3.\overline{33}$

Prueba 2da Derivada: $D(x, y) = \begin{vmatrix} -2 & -1 \\ -1 & -8 \end{vmatrix} = 16 - 1 = 15 > 0.$

$U_{p_A p_A} < 0$ hay un máximo relativo.

Punto de silla transición entre un máximo relativo y un mínimo relativo.





$U = 8x^{1/4}y^{3/4} - 20000$ $\quad x + y = 20,000$

sustituya $y = 20,000 - x$ en $U(x)$.

$U(x) = 8x^{1/4}(20,000 - x)^{3/4}. - 20$ mil

$U'(x) = 2x^{-3/4}(20,000 - x)^{3/4} - 6x^{1/4}(20000 - x)^{-1/4} = 0$

$$\frac{2}{x^{3/4}}(20,000 - x)^{3/4} = \frac{6x^{1/4}}{(20,000 - x)^{1/4}}$$

$(20000 - x) = 3x$

$20000 = 4x \Rightarrow x = 5,000$

$y = 15,000.$

Método 3: Microeconomía. $\quad P = x^{\alpha} y^{\beta} \quad x + y = 20$ mil

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$\frac{P_A}{P_B} = \frac{\alpha y}{\beta x}$ | Precaución.

3 d) $V = RI$. $\quad R = f(t) \quad I = g(t)$.

$$V \to R \to t, \quad V \to I \to t.$$

$$\frac{dV}{dt} = \underbrace{\frac{\partial V}{\partial R}}_{I} \frac{dR}{dt} + \underbrace{\frac{\partial V}{\partial I}}_{R.} \frac{dI}{dt}$$

$R = 400$, $I = 0.08$ $\quad \frac{dV}{dt} = -0.01 \quad \frac{dR}{dt} = -0.03$

$$-0.01 = 0.08(-0.03) + 400 \frac{dI}{dt}.$$

$$400 \frac{dI}{dt} = -0.01 + 0.0024 = -7.6 \times 10^{-3}$$

$$\frac{dI}{dt} = \frac{-7.6 \times 10^{-3}}{4 \times 10^{2}} = -1.9 \times 10^{-5} \quad A/s.$$

$-19 \ \mu A/s$

3 c) $T(x, y)$ $\quad x = \sqrt{1+t}$, $\quad y = 2 + \frac{t}{3}$.

$T_x(2,3) = 4$, $\quad T_y(2,3) = 3.$, $\quad t = 3$

$$\frac{dT}{dt} = \underbrace{\frac{\partial T}{\partial x}}_{4} \frac{dx}{dt} + \underbrace{\frac{\partial T}{\partial y}}_{3.} \frac{dy}{dt}.$$

$x(3) = 2$
$y(3) = 2 + 1 = 3.$

$x'(t) = \frac{1}{2}(1+t)^{-1/2} \quad x'(3) = \frac{1}{2}\frac{1}{\sqrt{4}} = \frac{1}{4}$

$y'(t) = \frac{1}{3} \quad y'(3) = \frac{1}{3}.$

$$\frac{dT}{dt} = 4\left(\frac{1}{4}\right) + 3\left(\frac{1}{3}\right) = 1 + 1 = 2.$$