corto #12 David Corzo

1) Ec. Plano tangente

$$z^2 - 2x - 2y - 12 = 0 \qquad P(1,-1,4)$$

Plano tangente:

$$z - \underbrace{f(x_0, y_0)}_{4} = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$f_x(x_0,y_0) = -2 \Big|_{(1,-1)} = -2$$

$$f_y(x_0,y_0) = -2 \Big|_{(1,-1)} = -2$$

$$z - 4 = -2(x-1) - 2(y+1)$$

2) Primeras derivadas par. de z:

$$\cos(xy) + 1 = \sec(zx) + \sin(yz)$$

$$\cos(xy) + 1 - \sec(zx) - \sin(yz) = 0$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{-\sin(xy)\cdot y - \sec(zx)\tan(zx)\cdot z}{-\sec(zx)\tan(zx)\cdot x + \cos(yz)\cdot y}$$

$$= \frac{\sin(xy)\cdot y + \sec(zx)\tan(zx)\cdot z}{-\sec(zx)\tan(zx)\cdot x + \cos(yz)\cdot y}$$

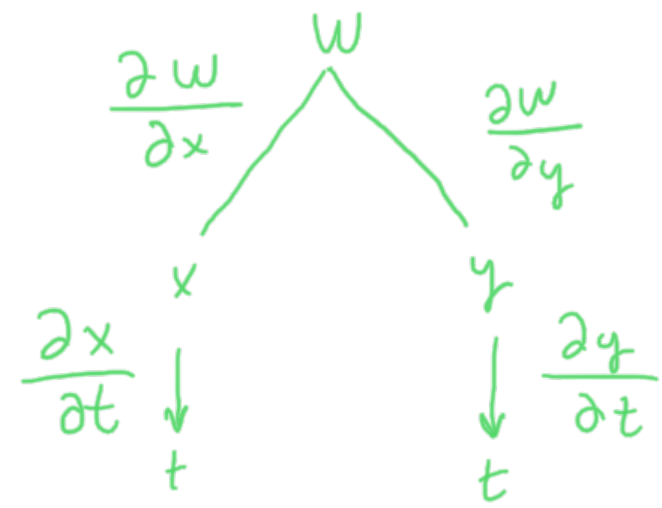
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-\sin(xy)\cdot x + \cos(yz)\cdot z}{-\sec(zx)\tan(zx)\cdot x + \cos(yz)\cdot y}$$

3)

$$w(x,y) = \tan^{-1}(yx)$$

$$x = e^{2t-6}$$

$$y = \ln(2t-5) + t - 2$$



$$\boxed{\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t}}$$

$$\frac{\partial w}{\partial x} = \frac{y}{(xy)^2+1} \qquad \frac{\partial x}{\partial t} = e^{2t-6}\cdot 2$$

$$\frac{\partial w}{\partial y} = \frac{x}{(xy)^2+1} \qquad \frac{\partial y}{\partial t} = \frac{2}{2t-5} + 1$$

$$\frac{\partial w}{\partial t} = \left(\frac{y}{(xy)^2+1}\right)\left(e^{2t-6}\cdot 2\right) + \left(\frac{x}{(xy)^2+1}\right)\left(\frac{2}{2t-5} + 1\right)$$

$$\Big|_{\substack{t=3\\ x=1\\ y=1}} = \frac{1}{1+1}\cdot \cancel{e^{2(3)-6}}^{\,1}\cdot 2 + \frac{1}{\cancel{1+1}}\cdot\left(\frac{\cancel{2}}{1} + \cancel{1}\right)$$

$$= 2 + 1 = 3$$

4) Temperatura en un lago punto $P(x,y,z)$ es:

$$T(x,y,z) = x\sin(\pi y z)$$

Encontrar razón de cambio en $P(1,1,2)$ en $\vec{u} = \langle 1, 4, 8\rangle$

$$|\vec{u}| = \sqrt{1+16+64}$$
$$= \sqrt{81}$$
$$= 9$$

$$D_{\vec{u}} f(\vec{x}_0) = \nabla f \cdot \vec{u}$$

$$\vec{u} = \frac{1}{9}\langle 1, 4, 8\rangle$$

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right\rangle$$

$$\frac{\partial f}{\partial x} = \sin(\pi y z) \qquad \frac{\partial f}{\partial y} = x\cos(\pi y z)\cdot z\pi$$

$$\frac{\partial f}{\partial z} = x\cos(\pi y z)\cdot \pi y$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,1,2)} = \sin(\pi 2) = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,1,2)} = \cos(\pi 2)\cdot 2\pi = 2\pi$$

$$\nabla f(\vec{x}) = \langle 0, 2\pi, \pi\rangle$$

$$\left.\frac{\partial f}{\partial z}\right|_{(1,1,2)} = \cos(\pi 2)\cdot \pi = \pi$$

$$D_{\vec{u}} f(\vec{x}) = \langle 0, 2\pi, \pi\rangle \cdot \left\langle \frac{1}{9}, \frac{4}{9}, \frac{8}{9}\right\rangle$$

$$= (0)\left(\frac{1}{9}\right) + (2\pi)\left(\frac{4}{9}\right) + (\pi)\left(\frac{8}{9}\right)$$

$$= \frac{8\pi}{9} + \frac{8\pi}{9}$$

$$\boxed{= \frac{16\pi}{9}}$$

5) Demanda:

$$x = 16 - P_A + P_B \qquad y = 24 - 2P_A - 4P_B$$

Costo

$$A = 2 \quad ; \quad B = 4$$

$$u = (x P_A + y P_B) - (2x + 4y)$$

$$= \left[(16 - P_A + P_B)P_A + (24 - 2P_A - 4P_B)P_B\right] - \left[2(16 - P_A + P_B) + 4(24 - 2P_A - 4P_B)\right]$$

$$= 16P_A - P_A^2 + P_BP_A + 24P_B - 2P_AP_B - 4P_B^2 - (32 - 2P_A + 2P_B + 96 - 8P_A - 16P_B$$

$$= \cancel{16P_A} - \cancel{P_A^2} + \cancel{P_BP_A} + \cancel{24P_B} - \cancel{2P_AP_B} - 4P_B^2 - \cancel{32} + \cancel{2P_A} - \cancel{2P_B} - \cancel{96} + \cancel{8P_A} - \cancel{16P_B}$$

$$= 16P_A + 2P_A + 8P$$

$$\ldots - 16P_B - 2P_B + 24P_B - 32 - 96 + P_BP_A - 2P_AP_B$$

$$= 26P_A - 6P_B - 128 + P_AP_B - P_A^2 - 4P_B^2$$

$$\frac{\partial u}{\partial P_A} = -2P_A + P_B + 26 = 0$$

$$P_B = -26 + 2P_A$$

$$\frac{\partial u}{\partial P_B} = -6 + P_A - 8P_B = 0$$

$$-6 + P_A - 8(-26 + 2P_A) = 0$$

$$-6 + P_A + 208 - 16P_A = 0$$

$$-15P_A + 202 = 0$$

$$P_A = \frac{202}{15} \longrightarrow P_B = -26 + 2\left(\frac{202}{15}\right) = \frac{14}{15}$$

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & -8 \end{vmatrix} = (-2)(-8) - 1 = 16 - 1$$

$$= 15 > 0$$

$f_{xx} = -2 < 0$ máx relativo

6) Publicidad Q 20,000 $\rightarrow y$

Periódico x

Utilidad = Ingresos − Costos