TAREA #6 - DAVID CORZO - 20190432 - 2020-02-17

1) Evalúe las integrales:

a) $$\int_0^1 \left(\underbrace{\frac{4}{1+t^2}}_{f(t)} \hat{j} + \underbrace{\frac{2t}{1+t^2}}_{g(t)} \hat{k} \right) dt$$

$$\int_0^1 f(t)\,dt = \int_0^1 \left(\frac{4}{1+t^2} \right) dt$$

1

$$= 4 \int_0^1 \left(\frac{1}{1+t^2} \right) dt$$

$$= 4 \arctan(t) \Big|_0^1$$

$$= 4 \left\{ \arctan(1) - \arctan(0) \right\}$$

$$= 4 \left\{ \frac{\pi}{4} - 0 \right\} = \boxed{\pi}$$

$$\int_0^1 g(t)\,dt = \int_0^1 \left(\frac{2t}{1+t^2} \right) dt$$

$$= \int_0^1 \left(\frac{2t}{1+t^2} \right) dt$$

$u = 1 + t^2$

$du = 2t\,dt$

$$= \int_0^1 \left(\frac{du}{u} \right)$$

$$= \ln|1+t^2| \Big|_0^1$$

$$= \left\{ \ln(2) - \cancelto{0}{\ln(1)} \right\} = \boxed{\ln(2)}$$

$$\therefore \left\langle 0, \pi, \ln(2) \right\rangle$$

b) $$\int^{\frac{\pi}{2}} \left(3\sin^2(t)\cos(t)\hat{\imath} + 3\sin(t)\cos^2(t)\hat{j} + 2\sin(t)\cos(t)\hat{k} \right) dt$$

$$\int_0 \Big(\underbrace{\qquad\qquad}_{f(t)} \; \underbrace{\qquad\qquad}_{g(t)} \; \underbrace{\qquad\qquad}_{h(t)} \Big)$$

$$\int_0^{\frac{\pi}{2}} f(t)\,dt = 3\int_0^{\frac{\pi}{2}} \sin^2(t)\cos(t)\,dt$$

$u = \sin(t)$

$du = \cos(t)\,dt$

$$= 3\int_0^{\frac{\pi}{2}} u^2\,du$$

$$= \frac{3}{3}u^3 = \sin^3(t)\Big|_0^{\frac{\pi}{2}}$$

$$= \left\{\sin^3\left(\tfrac{\pi}{2}\right) - \sin^3(0)\right\}$$

$$= \boxed{1}$$

$$\int_0^{\frac{\pi}{2}} g(t)\,dt = 3\int_0^{\frac{\pi}{2}} \Big(\sin(t)\cos^2(t)\Big)\,dt$$

$u = \cos(t)$

$-du = \sin(t)\,dt$

$$= -3\int_{u(0)}^{u(\frac{\pi}{2})} u^2\,du$$

$$= -\frac{3}{3}u^3 \Big|_0^{\frac{\pi}{2}}$$

$$= -\left\{\cos\left(\tfrac{\pi}{2}\right) - \cos(0)\right\}$$

$$= -\{0 - 1\} = \boxed{1}$$

$$\int_0^{\frac{\pi}{2}} h(t)\,dt = 2\int_0^{\frac{\pi}{2}} \Big(\sin(t)\cos(t)\Big)\,dt$$

$u = \sin(t)$

$du = \cos(t)\,dt$

$$= 2\int_{u(0)}^{u(\frac{\pi}{2})} u\,du$$

$$= u^2 \Big|_{u(0)}^{u(\frac{\pi}{2})=1} = \{1^2 - 0\} = \boxed{1}$$

$|u(0)| = 0 \quad (\quad)$

$$\therefore \langle 1, 1, 1 \rangle$$

3) $$\int \left(\underbrace{te^t \hat{\imath}}_{f(t)} + \underbrace{t^2 \ln(t)\, \hat{\jmath}}_{g(t)} + \underbrace{\frac{e^t}{\sqrt{1 - e^{2t}}}\, \hat{k}}_{h(t)} \right) dt$$

$$\int f(t)\,dt = \int \underbrace{te^t}\,dt$$

$u = t \qquad dv = e^t\,dt$

$du = dt \qquad v = e^t$

$$= te^t - \int e^t\,dt$$

$$= \boxed{te^t - e^t + C_1}$$

$$\int g(t)\,dt = \int t^2 \ln(t)\,dt$$

$u = \ln(t) \qquad dv = t^2\,dt$

$du = \frac{1}{t}\,dt \qquad v = \frac{1}{3}t^3$

$$= \ln(t) \cdot \frac{1}{3}t^3 - \frac{1}{3}\int \frac{t^3}{t}\,dt$$

$$= \frac{1}{3}t^3 \ln(t) - \frac{1}{3} \cdot \frac{1}{3} t^3 + C_2$$

$$\boxed{= \frac{1}{3}t^3 \ln(t) - \frac{1}{9}t^3 + C_2}$$

$$\int h(t)\,dt = \int \frac{e^t}{\sqrt{1 - e^{2t}}}\,dt$$

$u = e^t$

$du = e^t\,dt$

$$\int \frac{du}{\quad}$$

$$= \int \frac{}{\sqrt{1-u^2}}$$

$$= \arcsin(u) + C_3$$

$$\boxed{= \arcsin(e^t) + C_3}$$

$$\therefore \left\langle te^t - e^t + C_1 \, , \, \frac{1}{3}t^3 \ln(t) - \frac{1}{9}t^3 + C_2 \, , \, \arcsin(e^t) + C_3 \right\rangle$$

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2) Dada la posición $r(t) = t\hat{\imath} + \sin(3t)\hat{\jmath} + \cos(3t)\hat{k}$

a) Encuentre la función de velocidad:

$$r'(t) = v(t) = \hat{\imath} + \cos(3t)\cdot 3\,\hat{\jmath} + \sin(3t)\cdot 3\,\hat{k}$$

b) Encuentre la función de aceleración:

$$r''(t) = a(t) = 0\hat{\imath} - 3\sin(3t)\cdot 3\,\hat{\jmath} + 3\cdot 3\cos(3t)\,\hat{k}$$

c) Encuentre la función de rapidez:

$$|v(t)| = \sqrt{(1)^2 + (3\cos(3t))^2 + (3\sin(3t))^2}$$

$$= \sqrt{1 + 9\cos^2(3t) + 9\sin^2(3t)}$$

$$= \sqrt{1+9} = \sqrt{10}$$

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3) Dada la función de aceleración:

$$a(t) = \left\langle e^t, \sin(t)\cos(t), \frac{1}{(t+1)^2} \right\rangle$$

$$v(0) = \langle 3, -1, 2 \rangle$$

$v(0) = \langle 3, -1, 2 \rangle$

$$p(0) = \langle 0, 2, 0 \rangle$$

a) Encontrar la función de aceleración:

$$\int a(t)dt = v(t)$$

$$a(t) = \langle \underbrace{e^t}_{f(t)}, \underbrace{\sin(t)\cos(t)}_{g(t)}, \underbrace{\frac{1}{(t+1)^2}}_{h(t)} \rangle$$

$$\int f(t)\,dt = \boxed{e^t + C_1}$$

$$\int g(t)\,dt = \int \sin(t)\cos(t)\,dt = \boxed{\frac{1}{2}\sin^2(t) + C_2}$$

$u = \sin(t)$

$du = \cos(t)dt$

$$\int h(t)\,dt = \int \frac{1\,dt}{(t+1)^2} = \int \frac{du}{u^2} = \frac{1}{-2+1}u^{-2+1} = -\frac{1}{u} = -\frac{1}{t+1} + C_3$$

$u = t+1$

$du = 1\,dt$

$$\boxed{= -\frac{1}{t+1} + C_3}$$

$$v(t) = \langle e^t + C_1, \frac{1}{2}\sin^2(t) + C_2, -\frac{1}{t+1} + C_3 \rangle$$

Encontrar constantes

$v(0) = \langle 3, -1, 2 \rangle$

$e^0 + C_1 = 3$

$1 + C_1 = 3$

$C_1 = 3 - 1$

$C_1 = 2$

$\frac{1}{2}\sin^2(0) + C_2 = -1$

$0 + C_2 = -1$

$C_2 = -1$

$$-\frac{1}{0+1} + C_3 = 2$$

$$-1 + C_3 = 2$$

$$C_3 = 2 + 1$$

$$C_3 = 3$$

función de velocidad:

$$v(t) = \left\langle e^t + 2,\ \frac{1}{2}\sin^2(t) - 1,\ -\frac{1}{t+1} + 3 \right\rangle$$

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b) función posición:

$$\int v(t)\,dt = r(t)$$

$$v(t) = \left\langle \underbrace{e^t + 2}_{f(t)},\ \underbrace{\frac{1}{2}\sin^2(t) - 1}_{g(t)},\ \underbrace{-\frac{1}{t+1} + 3}_{h(t)} \right\rangle$$

$$\int f(t)\,dt = \int (e^t + 2)\,dt$$

$$\boxed{= e^t + 2t + C_1}$$

$$\int g(t)\,dt = \frac{1}{2}\int (\sin^2(t) - 1)\,dt = -\frac{1}{2}\int \cos^2(t)\,dt = -\frac{1}{2}\cdot\frac{1}{2}\int (1 - \cos(2t))\,dt$$

$$\boxed{= -\frac{1}{4}\left(t - \frac{1}{2}\sin(2t)\right) + C_2}$$

$$\int h(t)\,dt = -\int\left(\frac{1}{t+1} + 3\right)dt = -\left(\ln|t+1| + 3t\right) + C_3$$

$$\boxed{= -\ln|t+1| + 3t + C_3}$$

Encontrar constantes

$$r(0) = \langle 0, 2, 0 \rangle$$

$$e^0 + 2(0) + C_1 = 0$$

$$1 + 0 + C_1 = 0$$

$$-\frac{1}{4}\left(0 - \frac{1}{2}\sin(2\cdot 0)\right) + C_2 = 2$$

$$-\frac{1}{4}\cdot\cancel{0}^{0} + \frac{1}{\ }\cancel{\sin(0)}^{0} + C_2 = 2$$

$\frac{}{8}\sin(\ldots) + C_2$

$$C_1 = -1$$

$$C_2 = 2$$

$$-\ln|0+1| + 3(0) + C_3 = 0$$

$$0 + 0 + C_3 = 0$$

$$C_3 = 0$$

funcione posición :

$$r(t) = \left\langle e^t + 2t - 1,\ -\frac{1}{4}\left(t - \frac{1}{2}\sin(2t)\right) + 2,\ -\ln|t+1| + 3t + 0 \right\rangle$$

4) Calcule la longitud de arco de la helice circular de la ecuación vectorial:

$$r(t) = \cos(t)\,\hat{i} + \sin(t)\,\hat{j} + t\,\hat{k}$$

desde el punto $(1,0,0)$ hasta el punto $(1,0,2\pi)$.

$$L = \int_a^b |r'(t)|\,dt$$

$$r'(t) = -\sin(t)\,\hat{i} + \cos(t)\,\hat{j} + \hat{k}$$

$$|r'(t)| = \sqrt{(-\sin(t))^2 + (\cos(t))^2 + (1)^2}$$

$$= \sqrt{\sin^2(t) + \cos^2(t) + 1}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$L = \int_a^b \sqrt{2}\,dt = \sqrt{2}\,t \Big|_{a=0}^{b=2\pi}$$

$$= \sqrt{2}\cdot\{2\pi - 0\} = \sqrt{2}\cdot 2\pi$$

$P(1,0,0)$ & $Q(1,0,2\pi)$

$$r(t) = \langle \cos(t),\ \sin(t),\ t \rangle$$

$x = \cos(t)$ $\qquad \cos(t) = 1 \rightarrow t = 0$

$y = \sin(t)$ $\qquad \sin(t) = 0 \rightarrow t = 0$

$z = t$ $\qquad t = 0 \longrightarrow t = 0$

$\cos(t) = 1 \rightarrow t = 2\pi$

$\sin(t) = 0 \rightarrow t = 2\pi$

$t = 2\pi \rightarrow t = 2\pi$

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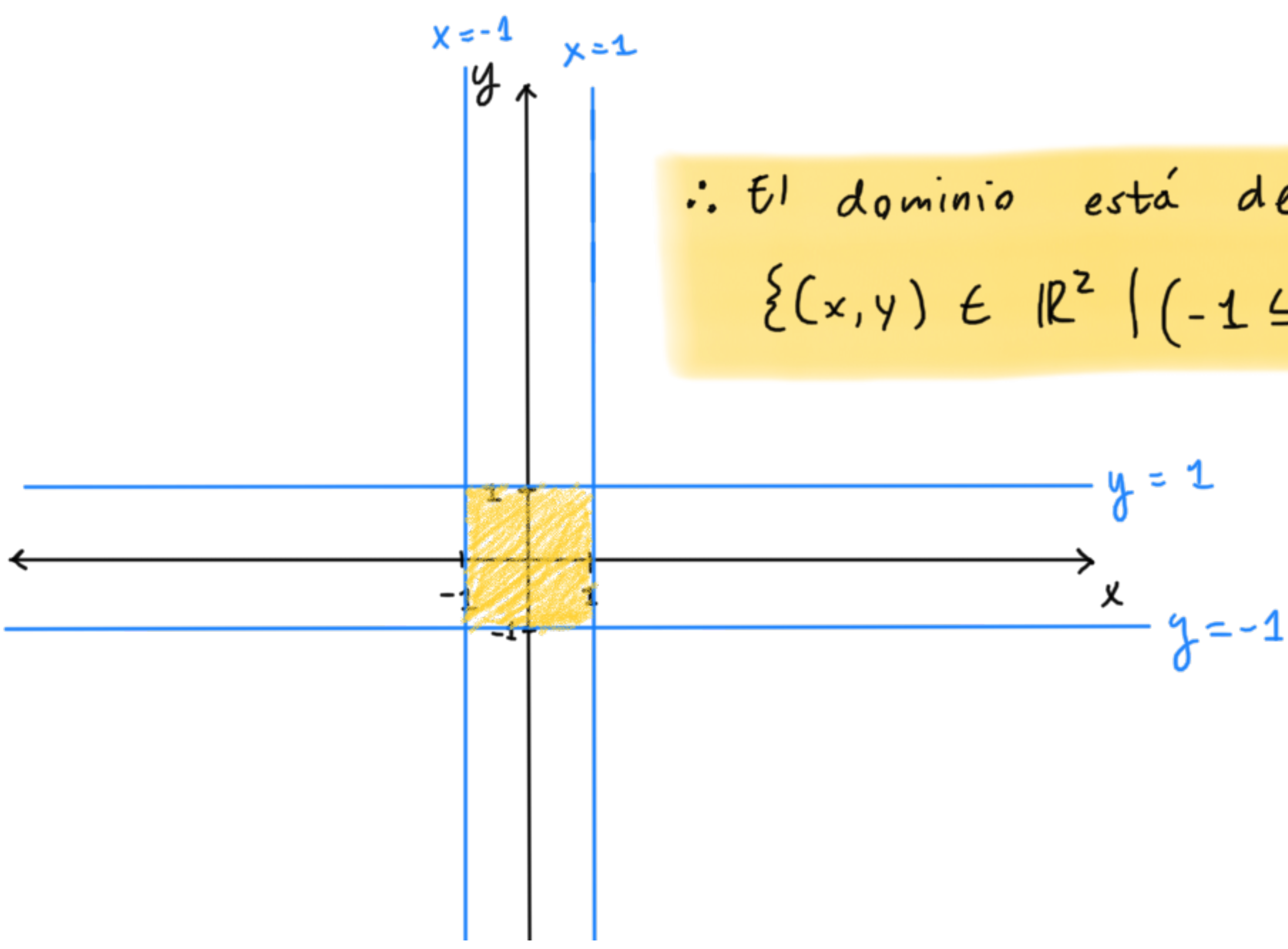
5) a) $f(x,y) = \sqrt{\underbrace{1 - x^2}} - \sqrt{1 - y^2}$

$$1 - x^2 \geq 0$$
$$-x^2 \geq -1$$
$$x^2 \leq 1$$
$$x \leq \pm 1$$
$$-1 \leq x \leq 1$$

$$1 - y^2 \geq 0$$
$$-y^2 \geq -1$$
$$y^2 \leq 1$$
$$y \leq \pm 1$$
$$-1 \leq y \leq 1$$

$\mathbb{D}: \mathbb{R}^2 - \{1 - x^2 \leq 0\}$ & $\{1 - y^2 \leq 0\}$



$\therefore$ El dominio está definido tal que:

$$\{(x,y) \in \mathbb{R}^2 \mid (-1 \leq x \leq 1)\ \&\ (-1 \leq y \leq 1)\}$$

b) $g(x, y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$

$y - x^2 \geq 0$

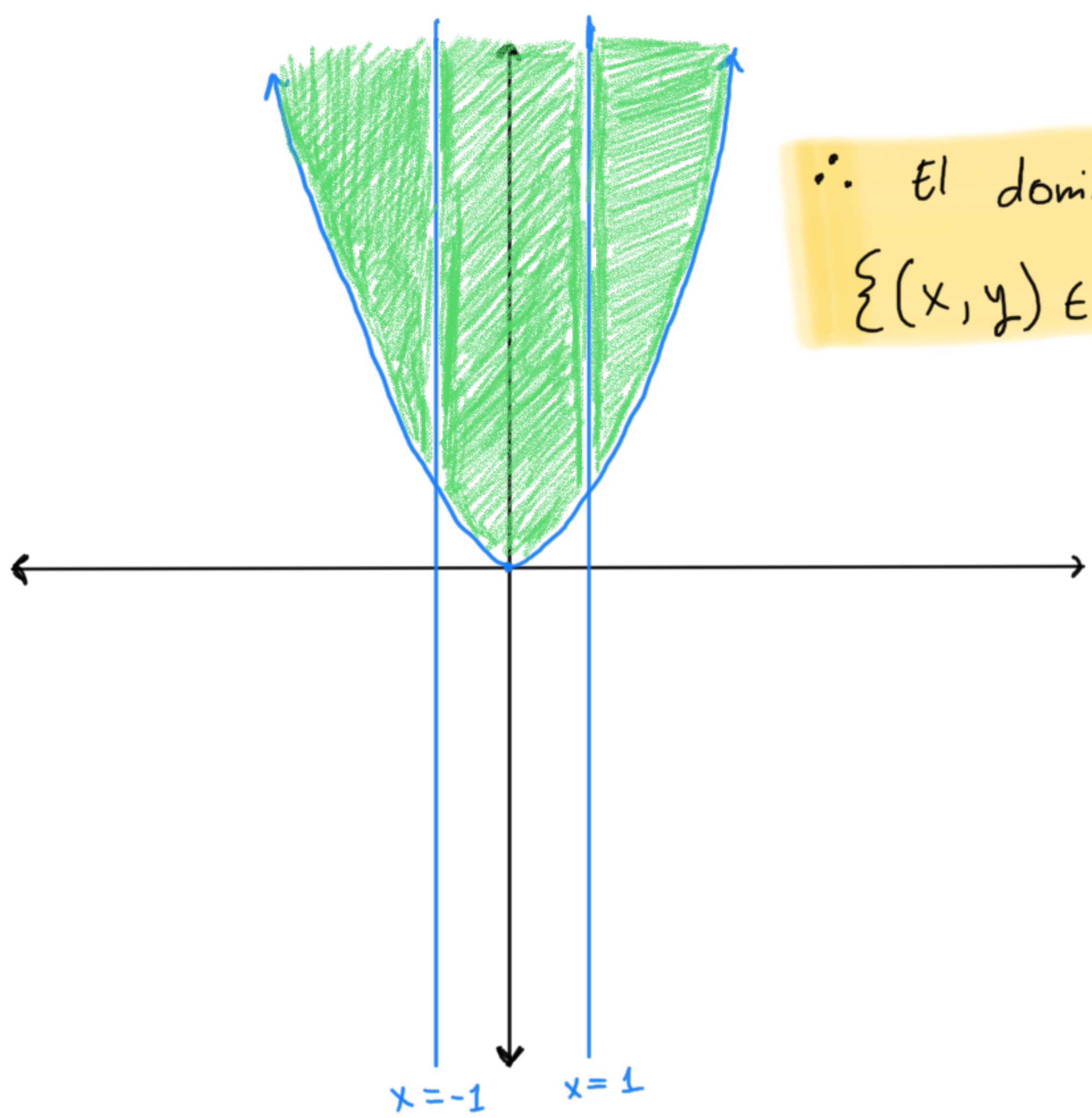
$y \geq x^2$

$1 - x^2 \neq 0$

$-x^2 \neq -1$

$x^2 \neq 1$

$x \neq \pm 1$



∴ El dominio está definido tal que:

$\{(x, y) \in \mathbb{R}^2 \mid (y \geq x^2) \ \& \ (x \neq \pm 1)\}$

c) $h(x, y) = \dfrac{9}{9 - x - y}$

$9 - x - y \neq 0$

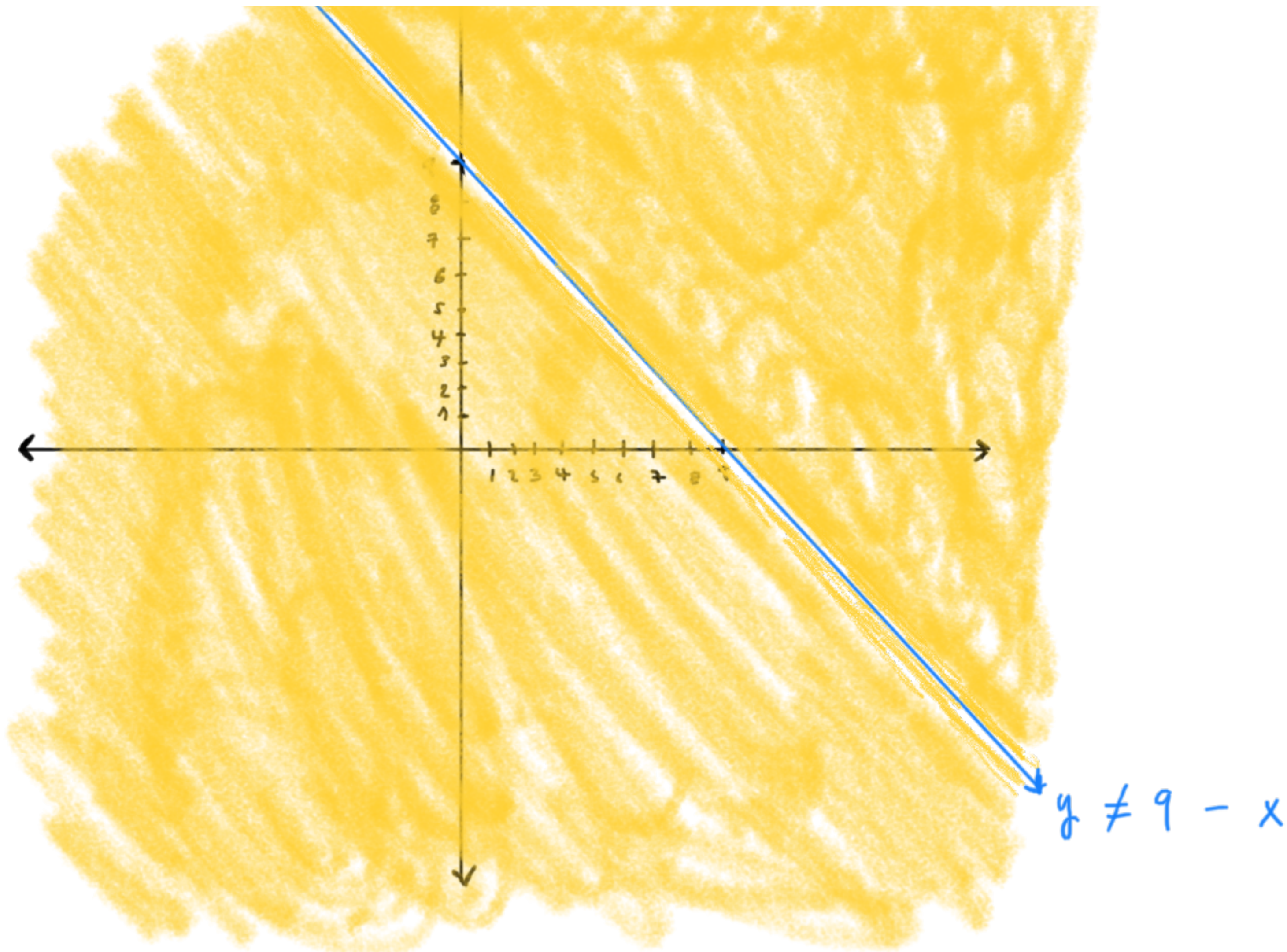
$-x - y \neq -9$

$x + y \neq 9$

$\neg (y = 9 - x)$ #excluir

∴ El dominio está definido tal que:

$\{(x, y) \in \mathbb{R}^2 \mid (x + y \neq 9)\}$



1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9
y ≠ 9 − x