1. 14.5 Regla de la cadena

- Explicación:

$$
\begin{aligned}
& y = f(g(t)) \qquad y = f(x) \quad x = g(t) \\
& y \implies x \implies t \qquad \frac{\partial y}{\partial t} = \frac{\partial y}{\partial x} \cdot \frac{\partial x}{\partial t} \\
\text{Caso 1: } & z = f(x, y) \quad x = g(t) \quad y = h(t)
\end{aligned}
$$

- Caso 1: **¿**Cómo se encuentra $\frac{\partial z}{\partial t}$ **?**:

$$z = f(x(t), y(t))$$

  - Variable independiente z
  - Variable intermedia x, y
  - Variable independiente t

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

- Caso 2: $z = f(x, y)$, $x = g(s, t)$, $y = h(s, t)$:

$$
\begin{aligned}
\frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \\
\frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}
\end{aligned}
$$

1.1. Ejercicios

1. Suponga que el costo de producir x unidades de A y de y unidades de B es:

$$C(x, y) = \left(3x^2 + y^3 + 4\right)^{\frac{1}{3}}$$

Las funciones de producción para cada producto es:

$$x = 10KL \qquad\qquad y = 5k^2 + 4L$$

Encuentre la razón de cambio de C respecto al capital y al trabajo.

$$
\begin{aligned}
&\frac{\partial C}{\partial K} = \frac{\partial C}{\partial x}\frac{\partial x}{\partial K} + \frac{\partial C}{\partial y}\frac{\partial y}{\partial K} \\
&\frac{\partial C}{\partial L}\frac{\partial C}{\partial x}\frac{\partial X}{\partial L} + \frac{\partial C}{\partial y}\frac{\partial y}{\partial L} \\
&\frac{\partial C}{\partial K} = \frac{1}{3}6x\left(3x^2 + y^3 + 4\right)^{-\frac{2}{3}} \cdot 10 + \frac{1}{3}\frac{3y^2}{\left(3x^2 + y^3 + 4\right)^{\frac{2}{3}}} \\
&\frac{\partial C}{\partial K} = \frac{2x}{\left(3x^2 + y^3 + 4\right)^{\frac{2}{4}}} \cdot 10K + \frac{y^2}{\left(3x^2 + y^2 + 4\right)}(4)
\end{aligned}
$$

2. Suponga que $z = f(u, v, w)$ y que u, v, w son funciones de t. Encuentre $\frac{\partial z}{\partial t}$:

1.2. Ejercicios varios

1. Encuentre las derivadas parciales indicadas:

$$\frac{\partial w}{\partial p} \quad = \quad \frac{\partial w}{\partial x} \quad \frac{\partial x}{\partial p} \quad + \quad \frac{\partial w}{\partial y} \quad \frac{\partial y}{\partial p}$$

$$\frac{\partial w}{\partial p} \quad = \mathrm{x}\frac{}{(x^2+y^2)^{\frac{1}{2}} \cdot 2p + \frac{y}{(x^2+y^2)^{\frac{1}{2}}} \frac{1}{p}}$$

$$\mathrm{x}(1{,}0{,}3) = 1^2 - 0^3 + 3 - 1 = 3$$

$$\mathrm{y}\ (1{,}0{,}3) = \ln(1) + e^0 + e^{\ln(3)} = 0 + 1 + 3 = 4$$

$$\frac{\partial w}{\partial p} \quad \Big|_{(1,0,3)} \quad = 3\frac{}{\sqrt{9+16} \cdot 2 + \frac{4}{5} \cdot \frac{1}{1} = \frac{6}{5} + \frac{4}{5} = 2}$$

2. $h = 4 - t^2$, $t = 2a + 3b + 4c$, $\frac{\partial h}{\partial b}\Big|_{(4,2,3)}$:

$$\frac{\partial h}{\partial b} = -2\,(2a + 3b + 4c) \cdot 3$$

3. $w = \ln(x, y, z)$, $x = r^2 - s^2$, $y = rs$, $z = r^2 + s^2$:

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

$$w_x = \frac{yz}{xyz} = \frac{1}{x}$$

$$\frac{\partial w}{\partial r} = \frac{2r}{x} + \frac{\partial s}{\partial y} + \frac{\partial 2r}{\partial z}$$

2. Derivación implícita, planos y rectas tangentes

IEncuentre las ecs. paramétricas de las rectas tangentes a $z = \sin(x)\tan(x)$ en la dirección de x & y en el punto $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$:

1.

$$\text{En la dirección de x: } m_x = z_x\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$$

$$\text{de y: } m_y = z_y\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$$

$$z_x = \cos(x)\tan(y) z_x\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

$$z_x = \cos()$$