PARCIAL #2 - DAVID CORZO

1) Superficie S.

1, 3, 5, 7, 11

$$z^2 + zx + y^2 = 9 \rightarrow z^2 + zx + y^2 - 9 = 0$$

Encontrar $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$F_x = z$

$F_z = 2z + x$

$F_y = 2y$

$$\frac{\partial z}{\partial x} = -\frac{z}{2z + x}$$

$$\frac{\partial z}{\partial y} = -\frac{2y}{2z + x}$$

- Encuentre la ecuación del plano tangente en $P(4,2,1)$ $z^2 + zx + y^2 = 9$

$f(4,2) = 1$

$$\left.\frac{\partial z}{\partial x}\right|_{x_1,y_1,z_1} = -\frac{(1)}{2(1) + (4)} = -\frac{1}{2+4} = -\frac{1}{6}$$

$$\left.\frac{\partial z}{\partial y}\right|_{x_1,y_1,z_1} = -\frac{2(2)}{2(1) + (4)} = -\frac{4}{2+4} = -\frac{4}{6} = -\frac{2}{3}$$

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

$$z = 1 - \frac{1}{6}(x-4) - \frac{2}{3}(y-2)$$

3) Temperatura $P(x,y)$

$T(x,y) = 6\ln(x^3 + 2y^2 - 34)$

Punto : $(3, 2)$

dirección $\vec{u} = \langle 12, 5 \rangle$

La derivada direccional

$\ulcorner D_{\vec{u}} f = \nabla f \cdot \vec{u} \lrcorner$

$$T_x = \frac{6 \cdot 3x^2}{x^3 + 2y^2 - 34}$$

$$T_x\Big|_{P(x,y)} = \frac{18(3)^2}{(3)^3 + 2(2)^2 - 34} = \frac{162}{27 + 8 - 34} = 162$$

$$T_y = \frac{6 \cdot 4y}{x^3 + 2y^2 - 34}$$

$\nabla f = \langle 162, 24 \rangle$

$$T_y\Big|_{x,y} = \frac{12(2)}{(3)^3 + 2(2)^2 - 34} = 48$$

$$\vec{u} = \frac{1}{\sqrt{12^2 + 5^2}} \langle 12, 5 \rangle = \left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$$ vector unitario

$$D_{\vec{u}} f = \langle 162, 48 \rangle \cdot \left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$$

$$= (162)\left(\frac{12}{13}\right) + (48)\left(\frac{5}{13}\right)$$

$$= 168$$

5) $f(x,y) = (y^2 - 4)(e^x - 2)$

Sí :

$$f(x,y) = (y^2-4)e^x - 2(y^2-4)$$

$D(x,y) > 0$
$f_x > 0$ min

$D(x,y) > 0$
$f_x < 0$ máx

$D(x,y) = 0$ inconcluso

$D(x,y) < 0$ silla

$$f_x = (y^2-4)e^x = 0$$

$$e^x = 0 \quad \text{indef}$$

$$\downarrow$$

$$y^2 - 4 = 0$$

$$y^2 = 4 \rightarrow \boxed{y = \pm 2}$$

$$f(x,y) = y^2(e^x-2) - 4(e^x-2)$$

$$f_y = 2y(e^x-2) = 0 \longrightarrow e^x - 2 = 0$$

$$\boxed{y = 0} \qquad e^x = 2$$

$$\boxed{x = \ln(2)}$$

Puntos críticos $(\ln(2), \pm 2)$

el punto $y = 0$ no hace 0 las dos derivadas parciales.

$$f_{xx} = (y^2-4)e^x \Big|_{(\ln(2),0)} = (0-4)2 = -8$$

$$f_{xy} = 2ye^x \Big|_{(\ln(2),0)} = 2(0)2 = 0$$

$$f_{yy} = 2(e^x-2)\Big|_{(\ln(2),0)} = 2(2-2) = 0$$

$$f_{yx} = 2ye^x \Big|_{(\ln(2),0)} = 2(0)(2) = 0$$

$$D(\ln(2),0) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -8 & 0 \\ 0 & 0 \end{vmatrix} = (-8)(0) - (0)(0) = 0$$

Inconcluso en $(\ln(2), 0)$

Pt. $(\ln(2), -2)$

$f_{xx} = (4-4)^{\ln(2)}$

$f_{xx} = (4-4)e^{...} = \emptyset$

$f_{yy} = 2(2-2) = \emptyset$

$f_{yx} = 2(-2)\,e^{\ln(2)} = -4 \cdot 2 = -8$

$f_{xy} = 2(-2)\,e^{\ln(2)} = -4(2) = -8$

$$D(\ln(2),-2) = \begin{vmatrix} \emptyset & -8 \\ -8 & \emptyset \end{vmatrix} = (0)(0) - (-8)(-8) = -64 < \emptyset$$

Punto de silla en $(\ln(2),-2)$

Pt. $(\ln(2), 2)$

$f_{xx} = (y^2 - 4)e^x \Big|_{\ln(2),2} = (4-4)e^{\ln(2)} = \emptyset$

$f_{xy} = 2ye^x \Big|_{\ln(2),2} = 2(2)(2) = 8$

$f_{yy} = 2(e^x - 2)\Big|_{(\ln(2),2)} = 2(2-2) = \emptyset$

$f_{yx} = 2ye^x \Big|_{(\ln(2),2)} = 2(2)(2) = 8$

$$D(\ln(2),2) = \begin{vmatrix} \emptyset & 8 \\ 8 & \emptyset \end{vmatrix} = (\emptyset)(0) - (8)(8) = -64 < \emptyset$$

Punto de silla en $(\ln(2),2)$

7) función producción:

$$Q = LK$$

Presupuesto = 640 mil

L = 10 mil K = 8 mil

restricción:

$$10L + 8K = 640$$

Lagrange

$$F(x,y,\lambda) = \underbrace{f(x,y)}_{\text{objetivo}} + \lambda(\overset{640}{c} - \overset{10L+8k}{g(x,y)})$$

$$= LK + \lambda(640 - 10L - 8K)$$
$$= LK + \lambda 640 - \lambda L 10 - 8\lambda K$$

$F_L = K - \lambda 10 = 0$

$K = \lambda 10$

$\frac{K}{10} = \lambda$

$F_K = L - \lambda 8 = 0$

$L = \lambda 8$

$\frac{L}{8} = \lambda$

$\frac{K}{10} = \frac{L}{8}$

$8K = 10L$

$\frac{8}{10}K = L$

$L = \frac{8}{10}(40) = 32$

$F\lambda = 640 - L10 - 8K = 0$

$640 - \frac{8 \cdot \cancel{10}}{\cancel{10}}K - 8K = 0$

$640 - 8K - 8K = 0$

$640 = 16K$

$40 = K$

$\lambda = 4$

$K = 40$

$L = 32$

40 mil máquinas y 32 mil trabajadores.

11)

x: iphones y: air books

$$U(x,y) = 400(x+y)^{\frac{3}{2}} - 2x^2 - 3y^2$$

$$x(t) = 10e^{(t-1)/10} + 2\ln(t)\Big|_1 = 10 = x(1)$$

$$y(t) = 14\sqrt{t} + t^2\Big|_1 = 14 + 1 = 15 = y(1)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\begin{tikzcd}
 & u \arrow[dl, "\frac{\partial u}{\partial x}"'] \arrow[dr, "\frac{\partial u}{\partial y}"] & \\
x \arrow[d, "\frac{\partial x}{\partial t}"'] & & y \arrow[d, "\frac{\partial y}{\partial t}"] \\
t & & t
\end{tikzcd}$$

$$\frac{\partial u}{\partial x} = 400\cdot\frac{3}{2}(x+y)^{\frac{1}{2}} - 4x$$

$$\boxed{\frac{\partial u}{\partial x} = 600\sqrt{x+y} - 4x}\ \Bigg|_{t=1} = 600\sqrt{10+15} - 4(10)$$

$$= 2960$$

$$\boxed{\frac{\partial u}{\partial y} = 600\sqrt{x+y} - 6y}\ \Bigg|_{t=1} = 600\sqrt{10+15} - 6(15)$$

$$= 2910$$

$$\frac{\partial x}{\partial t} = \cancel{10}e^{\frac{t-1}{10}}\cdot\frac{1}{\cancel{10}} + \frac{2}{t} \xrightarrow{\frac{\partial x}{\partial t} =} \boxed{e^{\frac{t-1}{10}} + \frac{2}{t}}$$

$$\Big|_{t=1} = 1 + 2 = 3$$

$$\boxed{\frac{\partial y}{\partial t} = 7(t)^{-\frac{1}{2}} + 2t}$$

$$\Big|_1 = 7 + 2 = 9$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial t} = (2960)(3) + (2910)(9) = 35{,}070$$