Tarea #2
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Cálculo Multivariable

1) Vectores:

$a = \langle 5, -12, 0 \rangle$

$b = \langle 0, -3, -6 \rangle$

$c = \langle 1, 0, 2 \rangle$

$2b = \langle 0, -6, -12 \rangle$

a) $a + 2(b + c) - (a - 2b) =$

$= a + 2[(0+1), (-3+0), (-6+2)] - [(5-0), (-12+6), (0+12)]$

$= a + 2\langle 1, -3, -4 \rangle - \langle 5, -6, 12 \rangle$

$= a + \langle 2, -6, -8 \rangle - \langle 5, -6, 12 \rangle$

$= a + [(2-5), (-6+6), (-8-12)]$

$= a + \langle -3, 0, -20 \rangle$

$= \langle 5, -12, 0 \rangle + \langle -3, 0, -20 \rangle$

$= [(5-3), (-12+0), (0-20)]$

$= \langle 2, -12, -20 \rangle$

b) $2a \cdot (3b + 4c) - 2c \cdot (4a + 0b)$

$2a = \langle 16, -24, 0 \rangle$ $\quad$ $4a = \langle 20, -48, 0 \rangle$

$3b = \langle 0, -9, -24 \rangle$ $\quad$ $0b = \langle 0, 0, 0 \rangle$

$2c = \langle 2, 0, 4 \rangle$ $\quad$ # Sacar nuevos vectores

$4c = \langle 4, 0, 8 \rangle$

$= 2a \cdot \langle (0 + 4), (-9 + 0), (-24 + 8) \rangle - 2c \cdot \langle (20 + 0), (-48 + 0), (0 + 0) \rangle$

$= 2a \cdot \langle 4, -9, -16 \rangle - 2c \cdot \langle 20, -48, 0 \rangle$

$= \langle (16 \cdot 4), (-24 \cdot -9), (0 \cdot -16) \rangle - \langle (2 \cdot 20), (0 \cdot -48), (4 \cdot 0) \rangle$

$= \langle 40, 216, 0 \rangle - \langle 40, 0, 0 \rangle$

$= \langle (40 - 40), (216 - 0), (0, 0) \rangle$

$= \langle 0, 216, 0 \rangle$

c) $|a + c - (a + b)| =$

$a = \langle 5, -12, 0 \rangle \quad b = \langle 0, -3, -6 \rangle \quad c = \langle 1, 0, 2 \rangle$

$= |\langle (5+1), (-12+0), (0+2) \rangle - (\langle (5+0), (-12-3), (0-6) \rangle)|$

$= |\langle 6, -12, 2 \rangle - \langle 5, -15, -6 \rangle|$

$= |\langle (6-5), (-12+15), (2+6) \rangle|$

$= |\langle 1, 3, 8 \rangle| \Rightarrow \sqrt{(1)^2 + (3)^2 + (8)^2}$

$= \sqrt{1 + 9 + 64}$

$= \sqrt{74}$

d) $|a + c| - |a + b|$

$= |\langle (5+1), (-12+0), (0+2) \rangle| - |\langle (5+0), (-12-3), (0-6) \rangle|$

$= |\langle 6, -12, 2 \rangle| - |\langle 5, -15, -6 \rangle|$

$= \sqrt{6^2 + (-12)^2 + 2^2} - \sqrt{5^2 + (-15)^2 + (-6)^2}$

$= \sqrt{36 + 144 + 4} - \sqrt{25 + 225 + 36}$

$= \sqrt{184} - \sqrt{286}$

$\cos\theta = \dfrac{A \cdot B}{|A||B|}$

2) Misma dirección que el vector $\langle -3, 4, 6, -8 \rangle$

$$= \left| \langle -3, 4, 6, -8 \rangle \right|$$

Calcular magnitud

$$= \sqrt{(-3)^2 + (4)^2 + (6)^2 + (-8)^2}$$

$$= \sqrt{9 + 16 + 36 + 64}$$

$$= \sqrt{125}$$

Entonces...

$$\left| \left\langle -\frac{3}{\sqrt{125}}, \frac{4}{\sqrt{125}}, \frac{6}{\sqrt{125}}, -\frac{8}{\sqrt{125}} \right\rangle \right| = 1$$

Comprobar...

$$= \sqrt{\frac{9}{125} + \frac{16}{125} + \frac{36}{125} + \frac{64}{125}} = \sqrt{\frac{125}{125}} = 1$$

3) Encuentre el ángulo de los vectores

a) $a = \langle 3, 0 \rangle$, $b = \langle 5, 5 \rangle$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$$

$$= \cos^{-1}\left(\frac{\langle (3 \cdot 5), (0 \cdot 5) \rangle}{|\langle 3, 0 \rangle||\langle 5, 5 \rangle|}\right)$$

$$= \cos^{-1}\left(\frac{15 + 0}{3 \cdot \sqrt{50}}\right) = \cos^{-1}\left(\frac{15}{3 \cdot \sqrt{2 \cdot 25}}\right)$$

$$= \cos^{-1}\left(\frac{15}{3 \cdot 5 \cdot \sqrt{2}}\right) = \cos^{-1}\left(\frac{15}{15} \cdot (2)^{-\frac{1}{2}}\right)$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

b)

$a = \langle 2, -4, 5 \rangle$

$b = \langle -2, 4, -5 \rangle$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|a||b|}\right)$$

$$= \cos^{-1}\left(\frac{-45}{(45)^{1/2}(45)^{1/2}}\right)$$

$$= \cos^{-1}\left(\frac{-45}{45}\right)$$

$$= \cos^{-1}(-1)$$

$$= \pi$$

$\vec{a} \cdot \vec{b}$

$\hookrightarrow$

2 -2	-4 4	5 -5
-4	-16	-25

$$= (-4) + (-16) + (-25)$$

$$= -20 - 25$$

$$= -45$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

$|a|$

$$= \sqrt{2^2 + (-4)^2 + (5)^2}$$

$$= \sqrt{4 + 16 + 25} = \sqrt{45}$$

$|b|$

$$= \sqrt{(-2)^2 + (4)^2 + (-5)^2}$$

$$= \sqrt{4 + 16 + 25} = \sqrt{45}$$

4) Determine si los vectores son ortogonales, paralelos o ninguno.

a) $a = \langle -5, 3, 7 \rangle$ $b = \langle 6, -8, 2 \rangle$

Producto punto de a & b

$-5 \cdot 6$	$3 \cdot -8$	$7 \cdot 2$
-30	-24	$+14$

$(-30) + (-24) + (14) = -40$ Ninguno

b) $a = \langle 4, 6 \rangle$

$b = \langle -3, 2 \rangle$

4	6
-3	2
-12	12

$\underbrace{(-12) + 12}_{0}$

Son ortogonales

$$\sum_{i=1}^{n} a_i b_i + \dots a_n b_n$$

c) $a = -i + 2j + 5k$

$b = 3i + 4j - k$

$a = \langle -1, 2, 5 \rangle$

$b = \langle 3, 4, -1 \rangle$

$$\begin{array}{|c|c|c|}\hline -1 & 2 & 5 \\ 3 & 4 & -1 \\ \hline -3 & 8 & -5 \\ \hline \end{array}$$

$$\underbrace{(-3) + 8 + (-5)}_{\downarrow}$$

$$\underbrace{-8 + 8}_{0}$$

Son ortogonales

d) $a = 2i + 6j - 4k$

$b = -3i - 9j + 6k$

$a = \langle 2, 6, -4 \rangle$

$b = \langle -3, -9, 6 \rangle$

$$\begin{array}{|c|c|c|}\hline 2 & 6 & -4 \\ -3 & -9 & 6 \\ \hline -6 & 54 & -24 \\ \hline \end{array}$$

$(-6) + 54 + (-24)$

$-30 + 54$

24

Son paralelos por $\vec{a} \cdot \vec{b}$ son multiplos entre sí.

5) Considerar vectores:

$a = \langle 3, 6, -2 \rangle$ esc: $\text{Proy}_a b = \dfrac{a \cdot b}{|a|}$

$b = \langle 1, 2, 3 \rangle$ vec: $\text{Proy}_a b = \dfrac{a \cdot b}{|a|} \dfrac{a}{|a|}$

a) Proyección de b sobre a:

Escalar:

$$\text{proy}_a b = \frac{3 + 12 - 6}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{9}{\sqrt{9 + 36 + 4}} =$$

$$= \frac{9}{\sqrt{49}} = \frac{9}{7}$$

Vectorial:

$$\text{proy}_a b = \frac{9}{7} \cdot \frac{1}{\sqrt{49}} \langle 3, 6, -2 \rangle$$

$$= \frac{9}{(49)^{\frac{1}{2}} \cdot (49)^{\frac{1}{2}}} \langle 3, 6, -2 \rangle$$

$$= \frac{9}{49} \langle 3, 6, -2 \rangle$$

$$= \left\langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49} \right\rangle$$

b) a sobre b:

$a = \langle 3, 6, -2 \rangle$

$b = \langle 1, 2, 3 \rangle$

$$\text{proy}_b\, a = \frac{a \cdot b}{|b|} \cdot \frac{b}{|b|}$$

$$\text{proy}_b\, a = \frac{a \cdot b}{|b|}$$

escalar: $\text{proy}_b\, a = \dfrac{9}{\sqrt{1^2+2^2+3^2}} = \dfrac{9}{\sqrt{14}}$

vectorial: $\text{proy}_b\, a = \dfrac{9}{\sqrt{14}} \cdot \dfrac{1}{\sqrt{14}} \left[\langle 1, 2, 3 \rangle \right]$

$$= \frac{9}{(14)^{\frac{1}{2}} \cdot (14)^{\frac{1}{2}}} \langle 1, 2, 3 \rangle$$

$$= \frac{9}{14} \langle 1, 2, 3 \rangle$$

$$= \left\langle \frac{9}{14}, \frac{18}{14}, \frac{27}{14} \right\rangle$$

$$= \left\langle \frac{9}{14}, \frac{9}{7}, \frac{27}{14} \right\rangle$$

c) proyección de b sobre a no es igual a proyección de a sobre b; si estos cumplen la condición de $\vec{a} \cdot \vec{b} = 0$ si resultan ser la misma proyección.

BONO: Encontrar tal valor de x que $\langle 2, 1, -1 \rangle$ & $\langle 1, x, 0 \rangle$ es de $45°$.

θ tiene que ser igual a $45°$ ó $\frac{\pi}{4}$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|a||b|}\right)$$

$$= \cos^{-1}\left(\frac{[(2 \cdot 1) + (1 \cdot x) + \cancel{(-1 \cdot 0)}^{\,0}]}{\sqrt{2^2 + 1^2 + (-1)^2} \cdot \sqrt{1^2 + x^2 + 0^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 + x}{\sqrt{4 + 1 + 1}\sqrt{1 + x^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 + x}{\sqrt{6}\sqrt{1 + x^2}}\right)$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{2 + x}{\sqrt{6}\sqrt{1 + x^2}}$$

$$\cos\left(\frac{\pi}{4}\right) \cdot \sqrt{6} \cdot \sqrt{1 + x^2} = 2 + x$$

$$\left(\frac{\sqrt{2}}{2} \cdot \sqrt{6}\right)\sqrt{1 + x^2}$$

$$\frac{\sqrt{12}}{2}\sqrt{1 + x^2} = 2 + x$$

$$\left(\frac{\sqrt{12}}{2}\sqrt{1 + x^2}\right)^2 = (2 + x)^2$$

$$3(1 + x^2) = x^2 + 4x + 4$$

$$3 + 3x^2 = x^2 + 4x + 4$$

$$3 + 3x^2 = x^2 + 4x + 4$$

$$0 = x^2 - 3x + 4x + 4 - 3$$

$$0 = -2x^2 + 4x + 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{16 - 4(-2)(1)}}{2(-2)}$$

$$= \frac{-4 \pm \sqrt{24}}{-4}$$

$$= \frac{-4 \pm \sqrt{24}}{-4} \approx -0.22$$

$$\approx 2.22$$