TAREA #8 - David Corzo

1) Encontrar $\dfrac{dy}{dx}$

a) $y\tan^{-1}(x) = x\sin^{-1}(y) + x^2y^2$

$$\emptyset = x\sin^{-1}(y) + x^2y^2 - y\tan^{-1}(x)$$

$$\frac{\partial}{\partial y}: \frac{x}{\sqrt{1-y^2}} + x^2 \cdot 2y - \tan^{-1}(x)$$

$$\frac{\partial}{\partial x}: \sin^{-1}(y) + 2xy^2 - \frac{y}{x^2+1}$$

$$\frac{\partial y}{\partial x} = \frac{\dfrac{x}{\sqrt{1-y^2}} + x^2 2y - \tan^{-1}(x)}{\sin^{-1}(y) + 2xy^2 - \dfrac{y}{x^2+1}}$$

b) $yx + x^3\ln(y) = (x^2+y^2)^2$

$$\emptyset = (x^2+y^2)^2 - yx - x^3\ln(y)$$

$$\frac{\partial}{\partial y} = 2(x^2+y^2)\cdot 2y - x - \frac{x^3}{y}$$

$$\frac{\partial}{\partial x} = 2(x^2+y^2)\cdot 2x - y - 3x^2\ln(y)$$

$$\frac{\partial y}{\partial x} = \frac{4y(x^2+y^2) - x - \dfrac{x^3}{y}}{4x(x^2+y^2) - y - 3x^2\ln(y)}$$

2) Encontrar derivadas parciales de z.

a) $\sin(xy) + \cos(yz) = \cot(zx)$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\emptyset = \sin(xy) + \cos(yz) - \cot(zx)$$

$$F_x = \cos(xy)y + \csc^2(zx)\cdot z$$

$$F_z = -\sin(yz)\cdot y + \csc^2(zx)\cdot x$$

$$\frac{\partial z}{\partial x} = -\left(\frac{\cos(xy)y + \csc^2(zx)z}{-\sin(yz)y + \csc^2(zx)x}\right)$$

$$F_y = \cos(xy)x - \sin(yz)z$$

$$\frac{\partial z}{\partial y} = -\left(\frac{\cos(xy)x - \sin(yz)z}{-\sin(yz)y + \csc^2(zx)x}\right)$$

b) $\sqrt{x^2y^2 + y^2z^2} = \dfrac{1}{x - 2y - 3z}$

$$0 = \frac{1}{x - 2y - 3z} - \sqrt{x^2y^2 + y^2z^2}$$

$$= (x - 2y - 3z)^{-1} - (x^2y^2 + y^2z^2)^{\frac{1}{2}}$$

$$F_x = -1(x - 2y - 3z)^{-2} - \frac{1}{2}(x^2y^2 + y^2z^2)^{-\frac{1}{2}} \cdot (2xy^2)$$

$$F_y = -1(x - 2y - 3z)^{-2} \cdot (-2) - \frac{1}{2}(x^2y^2 + y^2z^2)^{-\frac{1}{2}} \cdot (2yx^2 + 2yz^2)$$

$$F_z = -1(x - 2y - 3z)^{-2} \cdot (-3) - \frac{1}{2}(x^2y^2 + y^2z^2)^{-\frac{1}{2}} \cdot (2zy^2)$$

$$\frac{\partial z}{\partial x} = -\left(\frac{\dfrac{-1}{(x-2y-3z)^2} - \dfrac{xy^2}{\sqrt{x^2y^2 + y^2z^2}}}{\dfrac{3}{(x-2y-3z)^2} - \dfrac{zy^2}{\sqrt{x^2y^2 + y^2z^2}}}\right)$$

$$\frac{\partial z}{\partial y} = -\left(\frac{\dfrac{2}{(x-2y-3z)^2} - \dfrac{yx^2 + yz^2}{\sqrt{x^2y^2 + y^2z^2}}}{\dfrac{3}{(x-2y-3z)^2} - \dfrac{zy^2}{\sqrt{x^2y^2 + y^2z^2}}}\right)$$

3) Encontrar ec. plano tangente.

a) $z = \dfrac{2x + 3}{4y + 1}$ $\quad (0, 0, 0)$

$$z - f(x_0,y_0) = f_x(x_0,y_0)(x-x_0) - f_y(x_0,y_0)(y-y_0)$$

$$f(a,b) = \frac{2(0)+3}{4(0)+1} = 3$$

$$f_x = \frac{2}{4y+1}\Bigg|_{(0,0)} = \frac{2}{1} = 2$$

$$f_y = (2x+3)\left[-1(4y+1)^{-2}\cdot 4\right] = (2x+3)\left(\frac{-4}{(4y+1)^2}\right)\Bigg|_{(0,0)}$$

$$= \frac{-4(3)}{1} = -12$$

$$z = 2x - 12y + 3$$

b) $z = \sec(xy^2)$ $\left(\frac{\pi}{3}, 1, 2\right)$

$$f\left(\frac{\pi}{3}, 1\right) = \sec\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{\cos\left(\frac{\pi}{3}\right)} = 2$$

$$f_x = \sec(xy^2)\tan(xy^2)\,y^2\Bigg|_{\left(\frac{\pi}{3},1\right)} = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

$$f_y = \sec(xy^2)\tan(xy^2)\,2xy\Bigg|_{\left(\frac{\pi}{3},1\right)} = \frac{4\sqrt{3}\pi}{3}$$

$$z = 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + \frac{4\pi\sqrt{3}}{3}(y-1)$$

4) Encuentre la aproximación lineal.

a) $z = \dfrac{x}{x+y}$ $(4,-2)$

$$L = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$f(a,b) = 4$ ...

$$\frac{\quad}{4-2} = \frac{4}{2} = 2$$

$$f_x(a,b) = \left.\frac{(x+y)+x}{(x+y)^2}\right|_{(a,b)} = \frac{2(-2)-2}{(4-2)^2} = \frac{-4-2}{4} = -\frac{1}{2}$$

$$f_y(a,b) = -x(x+y)^{-2}\Big|_{(a,b)} = -4(4-2)^{-2} = \frac{-4}{4} = -1$$

$$z = 2 - \frac{1}{2}(x-4) - 1(y+2)$$

b) $z = e^{-xy}\sin(y) \qquad \left(\frac{\pi}{2}, 0\right)$

$$f(a,b) = e^{-(\frac{\pi}{2})(0)}\sin(0) = 0$$

$$f_x(a,b) = -y\,e^{-xy}\sin(y)\Big|_{(a,b)} = 0$$

$$f_y(a,b) = -x\,e^{-xy}\sin(y) + e^{-xy}\cos(y)\Big|_{(a,b)} =$$

$$= \cancel{-\left(\frac{\pi}{2}\right)e^{-(\frac{\pi}{2})(0)}\sin(0)} + 1 = 1$$

$$z = y + 1$$

5) Encuentre las ec. paramétricas de la recta tangente

$L_1 \rightarrow$ tangente en la dirección de x.

$L_2 \rightarrow$ tangente en la dirección de y.

a) $z = \sqrt{x^2 + y^2} \qquad (3, 4)$

Dirección de x no hay cambio en y.

$$x = t$$

$$y = 4$$

$$z = f(t,4) = \sqrt{t^2 + 4^2}$$

$$f_x = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x \Big|_{(3,4)} = \frac{3}{\sqrt{9+16}} = \frac{3}{5}$$

$$f_y = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y \Big|_{(3,4)} = \frac{4}{5}$$

$$\vec{r}(t) = \langle t, 4, \sqrt{t^2+4} \rangle$$
$$\vec{r}\,'(t) = \langle 1, 0, \frac{t}{\sqrt{t^2+4}} \rangle$$

$x = t$
$y = 4$
$z = \sqrt{t^2+4}$

$$\vec{r}(3) = \langle 3, 4, \sqrt{13} \rangle$$
$$\vec{r}\,'(3) = \langle 1, 0, \frac{3}{\sqrt{13}} \rangle$$

$$L_1 = \vec{r}(x) + t\,\vec{r}\,'(x)$$
$$= \langle 3, 4, \sqrt{13} \rangle + t\langle 1, 0, \frac{3}{\sqrt{13}} \rangle$$

direción x

$x = 3 + t$
$y = 4$
$z = \sqrt{13} + t\frac{3}{\sqrt{13}}$

Dirección de y no hay cambio en x

$$L_2 = \vec{r}(t) + t\,\vec{r}\,'(t)$$
$$\vec{r}(t) = \langle 3, t, \sqrt{9+t^2} \rangle$$
$$\vec{r}\,'(t) = \langle 0, 1, \frac{2t}{\sqrt{9+t^2}} \rangle$$

$x = 3$
$y = t$
$z = \sqrt{9+t^2}$

$t = 4$

$$\vec{r}(4) = \langle 3, 4, 5 \rangle$$
$$\vec{r}\,'(4) = \langle 0, 1, \frac{8}{5} \rangle$$

dirección y:

$x = 3$
$y = 4 + t$
$z = 5 + \frac{8}{5}t$

b) $z = 2\sin(3x - 2y) + \underbrace{4\cos^2(x+y)}_{4(\cos(x+y))^2}$ $\quad \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$$f_x = 2\cos(3x-2y)\cdot 3 + 8\cos(x+y)\sin(x+y)$$

$$f_y = 2\cos(3x-2y)\cdot 2 + 8\cos(x+y)\sin(x+y)$$

dir. x $\quad t = \frac{\pi}{4}$

$x = t$

$y = \frac{\pi}{4}$

$z = 2\sin(3t - \frac{\pi}{2}) + 4\cos^2(t + \frac{\pi}{4})$

$\vec{r}(t) = \langle t, \frac{\pi}{4}, 2\sin(3t - \frac{\pi}{2}) + 4\cos^2(t + \frac{\pi}{4})$

$\vec{r}'(t) = \langle 1, 0, 2\cos(3t - \frac{\pi}{2}) \cdot 3 + 8\cos(t + \frac{\pi}{4})\sin(t + \frac{\pi}{4}) \rangle$

$\vec{r}(\frac{\pi}{4}) = \langle \frac{\pi}{4}, \frac{\pi}{4}, 2\sin(\frac{3\pi}{4} - \frac{\pi}{2}) + 4\cos^2(\frac{\pi}{4}) \rangle$

$= \langle \frac{\pi}{4}, \frac{\pi}{4}, \sqrt{2} + \frac{3}{2} \rangle$

$\vec{r}'(\frac{\pi}{4}) = \langle 1, 0, 2\cos(\frac{3\pi}{4} - \frac{\pi}{2}) \cdot 3 + 8\cos(\frac{\pi}{2})\sin(\frac{\pi}{2}) \rangle$

$= \langle 1, 0, 3\sqrt{2} \rangle$

$l_1 = \langle \frac{\pi}{4}, \frac{\pi}{4}, \sqrt{2} + \frac{3}{2} \rangle + t \langle 1, 0, 3\sqrt{2} \rangle$

$x = \frac{\pi}{4} + t$ dir x

$y = \frac{\pi}{4}$

$z = \sqrt{2} + \frac{3}{2} + t\, 3\sqrt{2}$

dir y:

$t = \frac{\pi}{4}$ $x = \frac{\pi}{4}$

$y = t$

$z = 2\sin(\frac{3\pi}{4} - 2t) + 4\cos^2(\frac{\pi}{4} + t)$

$\vec{r}(t) = \langle \frac{\pi}{4}, \frac{\pi}{4}, 2\sin(\frac{3\pi}{4} - 2(\pi/4)) + 4\cos^2(\frac{\pi}{4} + \frac{\pi}{4}) \rangle$

$= \langle \frac{\pi}{4}, \frac{\pi}{4}, \sqrt{2} \rangle$

$\vec{r}(t) = \langle 0, 1, 2\cos(\frac{3\pi}{4} - 2t) \cdot (-2) + 8\cos(\frac{\pi}{4} + t)\sin(\frac{\pi}{4} + t) \rangle$

$= \langle 0, 1, -2\sqrt{2} \rangle$

$$L_2 = \left\langle \frac{\pi}{4}, \frac{\pi}{4}, \sqrt{2} \right\rangle + t\left\langle 0, 1, -2\sqrt{2} \right\rangle$$

$$x = \frac{\pi}{4}$$

$$y = \frac{\pi}{4} + t$$

$$z = \sqrt{2} - \sqrt{2}\,2t$$