TAREA #5 - DAVID CORZO - 20190432 - 2020-02-10

1.a) $r = \left\langle 3e^{-t}, \dfrac{\sin^2(\pi t)}{t}, \tan(2\pi t) \right\rangle$

¿Continua en $t = 0$?

$$\lim_{a\to 0}(r) = \left\langle \underbrace{\lim_{a\to 0}\left(3e^{-t}\right)}_{f(t)}, \underbrace{\lim_{a\to 0}\left(\frac{\sin^2(\pi t)}{t}\right)}_{g(t)}, \underbrace{\lim_{a\to 0}\left(\tan(2\pi t)\right)}_{h(t)} \right\rangle$$

$$\lim_{a\to 0}\left(f(t)\right) = \lim_{a\to 0}\left(3e^{-t}\right)$$

$$= 3e^{-0}$$

$$= \boxed{3}$$

$$\lim_{a\to 0}\left(g(t)\right) = \lim_{a\to 0}\left(\frac{\sin^2(\pi t)}{t}\right) \quad \frac{0}{0}$$

$$\overset{LH}{=} \lim_{a\to 0}\left(\frac{2\sin(\pi t)\cdot\cos(\pi t)\cdot\pi}{1}\right)$$

$$= \lim_{a\to 0}\left(2\sin(\pi t)\cos(\pi t)\cdot\pi\right)$$

$$= 2\sin(\pi\cdot 0)\cos(\pi\cdot 0)\cdot\pi = \boxed{0}$$

$$\lim_{a\to 0}\left(h(t)\right) = \lim_{a\to 0}\left(\tan(2\pi t)\right)$$

$$= \tan(2\pi\cdot 0) = \boxed{0}$$

$$\lim_{a \to 0} (r) = \langle 3, \emptyset, \emptyset \rangle \quad \text{es continua}$$

1.b) $$\lim_{a \to 1} (r) = \lim_{a \to 1} \left(\left\langle \underbrace{3e^{-t}}_{f(t)}, \underbrace{\frac{\sin^2(\pi t)}{t}}_{g(t)}, \underbrace{\tan(2\pi t)}_{h(t)} \right\rangle \right)$$

$$\lim_{a \to 1} (f(t)) = \lim_{a \to 1} (3e^{-t})$$

$$= 3e^{-1} = \frac{3}{e}$$

$$\lim_{a \to 1} (g(t)) = \lim_{a \to 1} \left(\frac{\sin^2(\pi t)}{t} \right)$$

$$= \frac{\sin^2(\pi)}{1} = \sin^2(\pi) = \emptyset$$

$$\lim_{a \to 1} (h(t)) = \lim_{a \to 1} (\tan(2\pi t))$$

$$= \tan(2\pi) = \emptyset$$

$$\lim_{a \to 1} (r) = \left\langle \frac{3}{e}, \emptyset, \emptyset \right\rangle \quad \text{es continua}$$

2) Determine el límite de las sigs funciones.

2a) $$\lim_{t \to 0} \left(\left\langle \underbrace{e^{-3t}}_{f(t)}, \underbrace{\frac{t^2}{\sin^2(t)}}_{g(t)}, \underbrace{\cos(2t)}_{h(t)} \right\rangle \right)$$

$$\lim_{a \to 0} \left(f(t) \right) = \lim_{a \to 0} \left(e^{-3t} \right)$$

$$= e^{-3 \cdot 0} = e^{0} = \boxed{1}$$

$$\lim_{a \to 0} \left(g(t) \right) = \lim_{a \to 0} \left(\frac{t^2}{\sin^2(t)} \right) \leftarrow \frac{0}{0} \text{ indef.}$$

$$\overset{LH}{=} \lim_{a \to 0} \left(\frac{2t}{\sin(t)\cos(t)} \right) \leftarrow \frac{0}{0} \text{ indef}$$

$f'g + fg'$

$$\overset{LH}{=} \lim_{a \to 0} \left(\frac{2}{\cos^2(t) - \sin^2(t)} \right)$$

$$= \frac{2}{1^2 - 0} = \boxed{2}$$

$$\lim_{a \to 0} \left(h(t) \right) = \lim_{a \to 0} \left(\cos(2t) \right)$$

$$= \boxed{1}$$

$$\boxed{\lim_{t \to 0} \left(\left\langle e^{-3t}, \frac{t^2}{\sin^2(t)}, \cos(2t) \right\rangle \right) = \left\langle 1, 2, 1 \right\rangle}$$

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2 b) $$\lim_{t \to \infty} \left(\left\langle \underbrace{\frac{1+t^2}{1-t^2}}_{f(t)}, \underbrace{\arctan(t)}_{g(t)}, \underbrace{\frac{1-e^{-2t}}{t}}_{h(t)} \right\rangle \right)$$

$$\lim_{a \to \infty} \left(f(t) \right) = \lim_{a \to \infty} \left(\frac{1+t^2}{1-t^2} \right) \leftarrow \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{a \to \infty} \left(\frac{0 + 2t}{0 - 2t} \right)$$

$$= \lim_{a\to\infty}\left(\frac{2t}{-2t}\right) = \boxed{-1}$$

$$\lim_{a\to\infty}(g(t)) = \lim_{a\to\infty}(\arctan(t))$$

$$= \boxed{\frac{\pi}{2}}$$

$$\lim_{a\to\infty}(h(t)) = \lim_{a\to\infty}\left(\frac{1-e^{-2t}}{t}\right) \leftarrow \frac{1^{\infty}}{\infty} \text{ algo así}$$

$$\overset{LH}{=} \lim_{a\to\infty}\left(\frac{-e^{-2t}\cdot -2}{1}\right)$$

$$= \lim_{a\to\infty}\left(e^{-2t}\right) = \lim_{a\to\infty}\left(\frac{1}{e^{2t}}\right) \leftarrow \frac{1}{\infty} \to \emptyset$$

$$= \boxed{\emptyset}$$

$$\lim_{t\to\infty}\left(\left\langle \frac{1+t^2}{1-t^2}, \arctan(t), \frac{1-e^{-2t}}{t}\right\rangle\right) = \left\langle -1, \frac{\pi}{4}, \emptyset\right\rangle$$

3) $r = \langle \sin(2t), t^2, \cos(2t)\rangle$

a) $r'(t)$

$$r'(t) = \langle 2\cos(2t), 2t, -\sin(2t)\cdot 2\rangle$$

b) $r''(t)$

$$r''(t) = \langle -4\sin(2t), 2, -\cos(2t)\cdot 4\rangle$$

c) $r''(t) \cdot r'''(t)$

$$r'''(t) = \langle -8\cos(2t),\ 0,\ \sin(2t)\cdot 8 \rangle$$

$$= \langle -4\sin(2t),\ 2,\ -4\cos(2t) \rangle \cdot \langle -8\cos(2t),\ 0,\ 8\sin(2t) \rangle$$

$$= [-4\sin(2t)\cdot -8\cos(2t)] + [2\cdot 0] + [-4\cos(2t)\cdot 8\sin(2t)]$$

$$= 32\sin(2t)\cos(2t) - 32\sin(2t)\cos(2t)$$

$$= \boxed{0}$$

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d) $r''(t) \times r'(t)$

$$r''(t) = \langle -4\sin(2t),\ 2,\ -\cos(2t)\cdot 4 \rangle$$

$$r'(t) = \langle 2\cos(2t),\ 2t,\ -\sin(2t)\cdot 2 \rangle$$

$$r''(t) \times r'(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4\sin(2t) & 2 & -4\cos(2t) \\ 2\cos(2t) & 2t & -2\sin(2t) \end{vmatrix} = \ldots$$

$$\ldots = \hat{i}\left[(2\cdot -2\sin(2t)) - (2t\cdot -4\cos(2t))\right] -$$

$$\hat{j}\left[(-4\sin(2t)\cdot -2\sin(2t)) - (2\cos(2t)\cdot -4\cos(2t))\right] +$$

$$\hat{k}\left[(-4\sin(2t)\cdot 2t) - (2\cos(2t)\cdot 2)\right] = \ldots$$

$$\ldots = \hat{\imath}\left[-4\sin(2t) - 8t\cos(2t)\right] -$$
$$\hat{\jmath}\left[8\sin^2(2t) + 8\cos^2(2t)\right] +$$
$$\hat{k}\left[-8t\sin(2t) - 4\cos(2t)\right] = \ldots$$

Expansión de $\hat{\imath}$

$$\boxed{-\hat{\imath}\, 4\sin(2t) - \hat{\imath}\, 8t\cos(2t)}$$

Expansión de $\hat{\jmath}$

$$8(\sin^2(t) + \cos^2(t)) = (1)8 \qquad \text{id. Pitagórica}$$
$$8\sin^2(t) + 8\cos^2(t) = 8$$

$$\boxed{8\hat{\jmath}}$$

Expansión de $\hat{k}$

$$-\hat{k}\, 8t\sin(2t) - \hat{k}\, 4\cos(2t)$$

Pendiente

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4)
$$x = t \qquad P(0,1,0)$$
$$y = e^{-t}$$
$$z = 2t - t^2$$

Encontramos t

$x:\quad t = 0 \Rightarrow t = 0$

$y:\quad e^{-t} = 1 \Rightarrow t = 0$

$z:\quad 2t - t^2 = 0 \Rightarrow t = 0$

Armamos $\vec{r}$

$$\vec{r}(t) = \langle t, e^{-t}, 2t - t^2\rangle$$
$$\vec{r}(0) = \langle 0, 1, 0\rangle$$

Derivamos:

$$\vec{r}\,'(t) = \langle 1, -e^{-t}, 2 - 2t\rangle$$

$r'(0) = \langle 1, -1, 2 \rangle$

Armamos la fórmula de recta tangente a vector:

$$\vec{r}_T = \vec{r}(a) + t\,\vec{r}\,'(a)$$

$a = 0$

$$\vec{r}_T = \langle 0, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$$

Ecs. Paramétricas:

$$x = 0 + 1t$$
$$y = 1 - 1t$$
$$z = 0 + 2t$$

5) $r_1 = \langle \sin(t), t^2, t^4 \rangle$

$r_2 = \langle \sin(t), \sin(2t), t^3 \rangle$

$$\vec{r}_1\,'(t) = \langle \cos(t), 2t, 4t^3 \rangle$$
$$\vec{r}_1\,'(0) = \langle 1, 0, 0 \rangle$$

$$\vec{r}_2\,'(t) = \langle \cos(t), 2\cos(2t), 3t^2 \rangle$$
$$\vec{r}_2\,'(0) = \langle 1, 2, 0 \rangle$$

Evaluar el ángulo:

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\cos\theta = \langle 1, 0, 0 \rangle \cdot \langle 1, 2, 0 \rangle$$

$$\overline{\sqrt{1^2+0^2+0^2}\cdot\sqrt{1^2+2^2+0^2}}$$

$$\cos\theta = \frac{1+0+0}{1\cdot\sqrt{5}}$$

$$\boxed{\cos\theta = \frac{1}{\sqrt{5}}}$$