

**TAREA #5 - DAVID CORZO - 20190432 - 2020-02-10**

$$1.a) \quad r = \left\langle 3e^{-t}, \frac{\sin^2(\pi t)}{t}, \tan(2\pi t) \right\rangle$$

ist continua an  $t = 0$ ?

$$\lim_{a \rightarrow 0} (r) = \left\langle \underbrace{\lim_{a \rightarrow 0} (3e^{-t})}_{f(t)}, \underbrace{\lim_{a \rightarrow 0} \left( \frac{\sin^2(\pi t)}{t} \right)}_{g(t)}, \underbrace{\lim_{a \rightarrow 0} (\tan(2\pi t))}_{h(t)} \right\rangle$$

$$\begin{aligned} \lim_{a \rightarrow 0} (f(t)) &= \lim_{a \rightarrow 0} (3e^{-t}) \\ &= 3e^{-0} \\ &= 3 \end{aligned}$$

$$\lim_{a \rightarrow 0} (g(t)) = \lim_{a \rightarrow 0} \left( \frac{\sin^2(\pi t)}{t} \right) \xrightarrow[0]{0}$$

$$\stackrel{LH}{=} \lim_{a \rightarrow 0} \left( \frac{2 \sin(\pi t) \cdot \cos(\pi t) \cdot \pi}{1} \right)$$

$$= \lim_{a \rightarrow 0} (2 \sin(\pi t) \cos(\pi t) \cdot \pi)$$

$$= 2 \sin(\pi \cdot 0) \cos(\pi \cdot 0) \cdot \pi = 0$$

$$\lim_{a \rightarrow 0} (h(t)) = \lim_{a \rightarrow 0} (\tan(2\pi t))$$

$$= \tan(2\pi \cdot 0) = 0$$

$$\lim_{a \rightarrow 0} (r) = \langle 3, 0, 0 \rangle$$

$$r(0) = \left\langle 3, \frac{0}{0}, 0 \right\rangle$$

Discontinuidad

∴ No es continua en  $t=0$  ya que

$$\lim_{a \rightarrow 0} (r) \neq r(0)$$

1.b)  $\lim_{a \rightarrow 1} (r) = \lim_{a \rightarrow 1} \left\langle \underbrace{3e^{-t}}_{f(t)}, \underbrace{\frac{\sin^2(\pi t)}{t}}_{g(t)}, \underbrace{\tan(2\pi t)}_{h(t)} \right\rangle$

$$\lim_{a \rightarrow 1} (f(t)) = \lim_{a \rightarrow 1} (3e^{-t})$$

$$= 3e^{-1} = \frac{3}{e}$$

$$\lim_{a \rightarrow 1} (g(t)) = \lim_{a \rightarrow 1} \left( \frac{\sin^2(\pi t)}{t} \right)$$

$$= \frac{\sin^2(\pi)}{1} = \sin^2(\pi) = 0$$

$$\lim_{a \rightarrow 1} (h(t)) = \lim_{a \rightarrow 1} (\tan(2\pi t))$$

$$= \tan(2\pi) = 0$$

$$\lim_{a \rightarrow 1} (r) = \left\langle \frac{3}{e}, 0, 0 \right\rangle$$

$$r(1) = \left\langle \frac{3}{e}, 0, 0 \right\rangle$$

Sí es continua en  $t=1$  ya que

$$\lim_{a \rightarrow 1} (r) = r(0)$$

2) Determine el límite de las sigs funciones.

2a)  $\lim_{t \rightarrow 0} \left( \left\langle e^{-3t}, \frac{t^2}{\sin^2(t)}, \cos(2t) \right\rangle \right)$

$$\lim_{a \rightarrow 0} (f(t)) = \lim_{a \rightarrow 0} (e^{-3t})$$

$$= e^{-3 \cdot 0} = e^0 = 1$$

$$\lim_{a \rightarrow 0} (g(t)) = \lim_{a \rightarrow 0} \left( \frac{t^2}{\sin^2(t)} \right) \leftarrow \frac{0}{0} \text{ indef.}$$

$$\stackrel{LH}{=} \lim_{a \rightarrow 0} \left( \frac{2t}{2 \sin(t) \cos(t)} \right) \leftarrow \frac{0}{0} \text{ indef.}$$

$$f'g + fg' = \lim_{a \rightarrow 0} \left( \frac{1}{\cos^2(t) - \sin^2(t)} \right)$$

$$= \frac{1}{1^2 - 0} = 1$$

$$\lim_{t \rightarrow 0} (h(t)) = \lim_{t \rightarrow 0} (\cos(2t)) \\ = 1$$

$$\lim_{t \rightarrow 0} \left( \left\langle e^{-3t}, \frac{t^2}{\sin^2(t)}, \cos(2t) \right\rangle \right) = \langle 1, 1, 1 \rangle$$

2 b)

$$\lim_{t \rightarrow \infty} \left( \left\langle \underbrace{\frac{1+t^2}{1-t^2}}_{f(t)}, \underbrace{\arctan(t)}_{g(t)}, \underbrace{\frac{1-e^{-2t}}{t}}_{h(t)} \right\rangle \right)$$

$$\lim_{t \rightarrow \infty} (f(t)) = \lim_{t \rightarrow \infty} \left( \frac{1+t^2}{1-t^2} \right) \leftarrow \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{t \rightarrow \infty} \left( \frac{0+2t}{0-2t} \right)$$

$$= \lim_{t \rightarrow \infty} \left( \frac{2t}{-2t} \right) = -1$$

$$\lim_{t \rightarrow \infty} (g(t)) = \lim_{t \rightarrow \infty} (\arctan(t))$$

$$= \frac{\pi}{2}$$

$$\lim_{t \rightarrow \infty} (h(t)) = \lim_{t \rightarrow \infty} \left( \frac{1-e^{-2t}}{t} \right) \leftarrow \frac{1^\infty}{\infty} \text{ algo asi.}$$

$$\stackrel{LH}{=} \lim_{t \rightarrow \infty} \left( \frac{-e^{-2t} \cdot -2}{1} \right)$$

$$= \lim_{a \rightarrow \infty} \left( e^{-2t} \right) = \lim_{a \rightarrow \infty} \left( \frac{1}{e^{2t}} \right) \leftarrow \frac{1}{\infty} \rightarrow 0$$

$$= \emptyset$$

$$\lim_{t \rightarrow \infty} \left( \left\langle \frac{1+t^2}{1-t^2}, \arctan(t), \frac{1-e^{-2t}}{t} \right\rangle \right) = \left\langle -1, \frac{\pi}{2}, \emptyset \right\rangle$$

3)  $r = \langle \sin(2t), t^2, \cos(2t) \rangle$

a)  $r'(t)$

$$r'(t) = \langle 2\cos(2t), 2t, -\sin(2t) \cdot 2 \rangle$$

b)  $r''(t)$

$$r''(t) = \langle -4\sin(2t), 2, -\cos(2t) \cdot 4 \rangle$$

c)  $r''(t) \cdot r'''(t)$

$$r'''(t) = \langle -8\cos(2t), 0, \sin(2t) \cdot 8 \rangle$$

$$= \langle -4\sin(2t), 2, -4\cos(2t) \rangle \cdot \langle -8\cos(2t), 0, 8\sin(2t) \rangle$$

$$= [-4\sin(2t) \cdot -8\cos(2t)] + [2 \cdot 0] + [-4\cos(2t) \cdot 8\sin(2t)]$$

$$= 32\sin(2t)\cos(2t) - 32\sin(2t)\cos(2t)$$

$$= \emptyset$$

$$d) \quad r''(t) \times r'(t)$$

$$r''(t) = \langle -4\sin(2t), 2, -\cos(2t) \cdot 4 \rangle$$

$$r'(t) = \langle 2\cos(2t), 2t, -\sin(2t) \cdot 2 \rangle$$

$$r''(t) \times r'(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4\sin(2t) & 2 & -4\cos(2t) \\ 2\cos(2t) & 2t & -2\sin(2t) \end{vmatrix} = \dots$$

$$\dots = \hat{i} \left[ (2)(-2\sin(2t)) - (2t)(-4\cos(2t)) \right] -$$

$$\hat{j} \left[ (-4\sin(2t))(-2\sin(2t)) - (2\cos(2t))(-4\cos(2t)) \right] +$$

$$\hat{k} \left[ (-4\sin(2t))(2t) - (2\cos(2t))(2) \right] =$$

$$= \hat{i} \left[ -4\sin(2t) + 8t\cos(2t) \right] -$$

$$\hat{j} \left[ 8\sin^2(2t) + 8\cos^2(2t) \right] +$$

$$\hat{k} \left[ -8t\sin(2t) - 4\cos(2t) \right]$$

$$= \langle -4\sin(2t) + 8t\cos(2t), -8, -8t\sin(2t) - 4\cos(2t) \rangle$$

4)

$$\begin{aligned} x &= t \\ y &= e^{-t} \end{aligned}$$

$$P(0, 1, 0)$$

$$z = 2t - t$$

# Encuentramos  $t$

$$x: \quad t = 0 \Rightarrow t = 0$$

$$y: \quad e^{-t} = 1 \Rightarrow t = 0$$

$$z: \quad 2t - t^2 = 0 \Rightarrow t = 0$$

# Armamos  $\vec{r}$

$$\vec{r}(t) = \langle t, e^{-t}, 2t - t^2 \rangle$$

$$\vec{r}(0) = \langle 0, 1, 0 \rangle$$

# Devolvemos:

$$\vec{r}'(t) = \langle 1, -e^{-t}, 2 - 2t \rangle$$

$$\vec{r}'(0) = \langle 1, -1, 2 \rangle$$

# Armamos la fórmula de recta tangente a vector:

$$\vec{r}_T = \vec{r}(a) + t \vec{r}'(a)$$

$$a = 0$$

$$\vec{r}_T = \langle 0, 1, 0 \rangle + t \langle 1, -1, 2 \rangle$$

# Ecs. Paramétricas:

$$x = 0 + 1t$$

$$y = 1 - 1t$$

$$z = 0 + 2t$$

5)

$$\vec{r}_1 = \langle \sin(t), t^2, t^4 \rangle$$

$$\vec{r}_2 = \langle \sin(t), \sin(2t), t^3 \rangle$$

$$\vec{r}_1'(t) = \langle \cos(t), 2t, 4t^3 \rangle$$

$$\vec{r}_1'(0) = \langle 1, 0, 0 \rangle$$

$$\vec{r}_2'(t) = \langle \cos(t), 2\cos(2t), 3t^2 \rangle$$

$$\vec{r}_2'(0) = \langle 1, 2, 0 \rangle$$

# Evaluar el ángulo:

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$\cos \theta = \frac{\langle 1, 0, 0 \rangle \cdot \langle 1, 2, 0 \rangle}{\sqrt{1^2 + 0^2 + 0^2} \cdot \sqrt{1^2 + 2^2 + 0^2}}$$

$$\cos \theta = \frac{1 + 0 + 0}{1 \cdot \sqrt{5}}$$

$$\cos \theta = \frac{1}{\sqrt{5}}$$