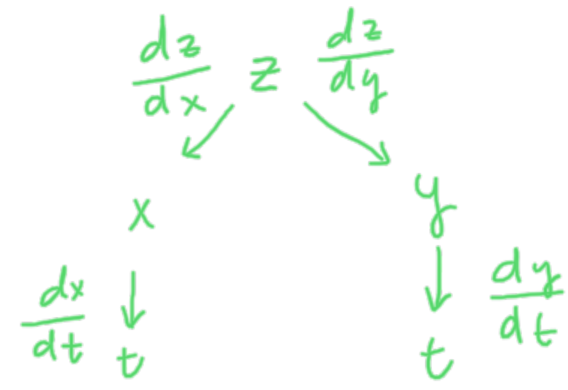
TAREA #9 - DAVID CORZO

1) Encuentre $\frac{\partial z}{\partial t}$

a) $z = x^2 + y^2 + xy$, $x = \sin(t)$ $y = e^t$

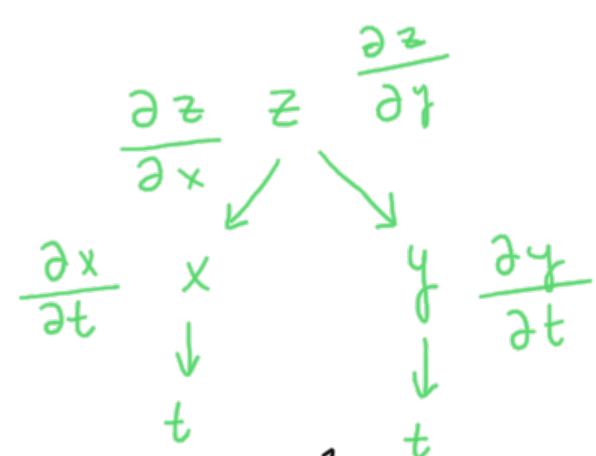
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$



$$\frac{\partial z}{\partial t} = (2x + y)\cos(t) + (2y + x)e^t$$

b) $z = \sqrt{1 + x^2 + y^2}$, $x = \ln(t)$ $y = \cos(t)$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$



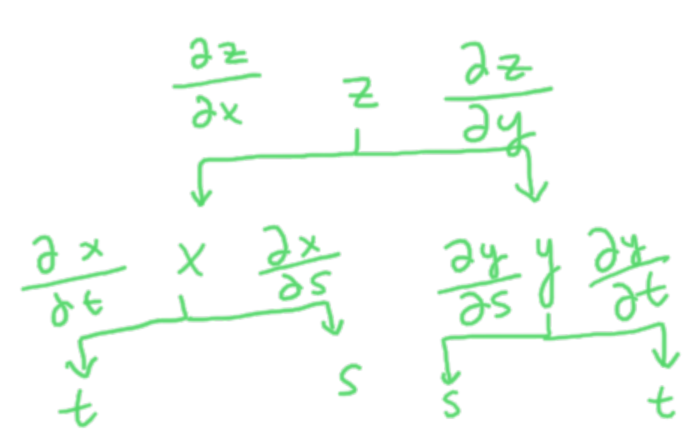
$$\frac{\partial z}{\partial t} = \frac{1}{\cancel{2}}(1+x^2+y^2)^{-\frac{1}{2}}\cdot\cancel{2}x \cdot \frac{1}{t} + \frac{1}{\cancel{2}}(1+x^2+y^2)^{-\frac{1}{2}}\cdot\cancel{2}y \cdot(-\sin(t))$$

$$= \frac{x}{t\sqrt{1+x^2+y^2}} - \frac{y\sin(t)}{\sqrt{1+x^2+y^2}}$$

2) Encuentre $\frac{\partial z}{\partial s}$ y $\frac{\partial z}{\partial t}$

a) $z = x^2y^3$, $x = s\cos(t)$ $y = s\sin(t)$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$



$$\frac{\partial z}{\partial s} = (2xy^3)(\cos(t)) + (3y^2x^2)(\sin(t))$$

$$\frac{\partial z}{\partial s} = 2\cos(t)\,xy^3 + 3\sin(t)\,y^2x^2$$

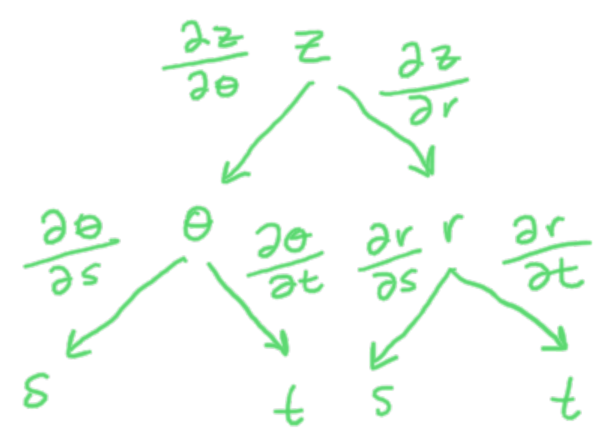
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = (2xy^3)(-s\sin(t)) + (3y^2x^2)(s\cos(t))$$

$$\frac{\partial z}{\partial t} = -2s\,xy^3\sin(t) + 3s\,y^2x^2\cos(t)$$

b) $z = e^r\cos(\theta)\sin(\phi)$, $r = st$, $\theta = \sqrt{s^2+t^2}$, $\phi = \ln[\tan(s) + \sinh(t)]$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s}$$



$$= \left(-\sin(\theta)\,e^r\sin(\phi)\right)\left(\frac{s}{\sqrt{s^2+t^2}}\right) +$$

$$\left(e^r\cos(\theta)\sin(\phi)\right)(t)$$

$$= -\frac{\sin(\theta)\,e^r\sin(\phi)\,s}{\sqrt{s^2+t^2}} + t\,e^r\cos(\theta)\sin(\phi)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t}$$

$$= \left(-e^r\sin(\theta)\sin(\phi)\right)\left(\frac{t}{\sqrt{s^2+t^2}}\right) + \left(e^r\cos(\theta)\sin(\phi)\right)(s)$$

$$= -\frac{e^r\sin(\theta)\sin(\phi)\,t}{\sqrt{s^2+t^2}} + e^r\cos(\theta)\sin(\phi)\,s$$

3) Determine la derivada direccional de $f(x,y) = x^3y^4 + x^4y^3$ en el punto $(1,1)$ en la dirección del vector unitario $\vec{u} = \langle \cos(\theta), \sin(\theta) \rangle$ $\theta = \frac{\pi}{6}$

$$D_u = \nabla f \cdot \vec{u}$$

$$f_x = 3x^2y^4 + 4x^3y^3 \Big|_{(1,1)} = 3 + 4 = 7$$

$$f_y = 4y^3x^3 + 3y^2x^4 \Big|_{(1,1)} = 4 + 3 = 7$$

$$D_u = \langle 7, 7 \rangle \cdot \langle \cos(\theta), \sin(\theta) \rangle$$

$$\nabla f = \langle 7, 7 \rangle$$

$= 7\cos(\theta) + 7\sin(\theta)$

$\vec{u} = \sqrt{\cos^2\theta + \sin^2(\theta)} = 1$

$\vec{u} = \langle \cos(\theta), \sin(\theta) \rangle$

4) Encuentre la razon de cambio de $f(x,y,z) = e^{x-1}\sin(y) + (x+1)^2 \ln(z+1)$ en el punto $(1, \frac{\pi}{3}, 0)$ en la dirección del vector $\vec{v} = \langle -1, 4, -8 \rangle$

$$D_u f(1, \tfrac{\pi}{3}, 0) = \nabla f(1, \tfrac{\pi}{3}, 0) \cdot \vec{v}$$

$$\nabla f(x,y,z) = \left\langle e^{x-1}\sin(y) + 2(x+1)\ln(z+1),\ e^{x-1}\cos(y),\ \frac{(x+1)^2}{z+1} \right\rangle$$

$$\nabla f(1, \pi/3, 0) = \left\langle e^0 \sin(\tfrac{\pi}{3}) + \cancel{4\ln(1)}^{0},\ e^0 \cos(\tfrac{\pi}{3}),\ \frac{4}{1} \right\rangle$$

$$= \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, 4 \right\rangle$$

$$\vec{v} = \frac{1}{9}\langle -1, 4, -8 \rangle$$

$$D_u f(1, \tfrac{\pi}{3}, 0) = \left\langle -\frac{1}{9}, \frac{4}{9}, -\frac{8}{9} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, 4 \right\rangle$$

$$= \left(-\frac{1}{9}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{4}{9}\right)\left(\frac{1}{2}\right) + \left(-\frac{8}{9}\right)\left(\frac{4}{1}\right)$$

$$= -\frac{\sqrt{3}}{18} + \frac{2}{9} - \frac{32}{9} = -\frac{\sqrt{3}}{18} - \frac{10}{3}$$

5) Dada la función:

$$f(x,y) = \sin(2x + 3y)$$

a) Determine el gradiente de f.

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$f_x = 2\cos(2x + 3y)$$

$f_y = 3\cos(2x+3y)$

$$\boxed{\nabla f = \langle 2\cos(2x+3y),\ 3\cos(2x+3y) \rangle}$$

b) Evalúe el gradiente en el pt. $P(-6\pi, 4\pi)$

$$\nabla f(-6\pi, 4\pi) = \langle 2\cos(-12\pi + 12\pi),\ 3\cos(-12\pi + 12\pi) \rangle$$

$$= \langle 2\,\cancel{\cos(0)}^{1},\ 3\,\cancel{\cos(0)}^{1} \rangle = \boxed{\langle 2, 3 \rangle}$$

c) Encuentre la razón de cambio de f en P en la dirección del vector $u = \frac{1}{2}\left(\sqrt{3}\,\hat{\imath} - \hat{\jmath}\right)$.

$$\vec{u} = \underbrace{\frac{1}{2}\langle \sqrt{3}, -1 \rangle}_{1?}$$

$$\frac{1}{2}\sqrt{3+1} = 1 \checkmark$$

$$D_u f = \nabla f \cdot \vec{u}$$

$$= \langle 2, 3 \rangle \cdot \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle = (2)\left(\frac{\sqrt{3}}{2}\right) + (3)\left(-\frac{1}{2}\right)$$

$$\boxed{= \sqrt{3} - \frac{3}{2}}$$