TAREA #12 - DAVID CORZO

1) Tema 1: Planos tangentes & aproximaciones lineales:

a) Encontrar la ec. plano tangente:

a.1) $z = x^2 + y^2$, $(1,1,3)$

$$\boxed{z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)}$$

$$f_x = 2x \Big|_{(1,1,3)} = 2(1) = 2$$

$$f_y = 2y \Big|_{(1,1,3)} = 2(1) = 2$$

$$f(1,1) = 1^2 + 1^2 = 2$$

$$z = 2 + 2(x-1) + 2(y-1)$$
$$= \cancel{2} + 2x \cancel{-2} + 2y - 2$$
$$= 2x + 2y - 2$$
$$z = 2x + 2y - 2$$

a.1) $z = x\sin(x+y)$, $(-1,1,0)$

$$f_x(x_0,y_0) = \sin(x+y) + x\cos(x+y)\Big|_{(-1,1,0)} =$$
$$= \sin(\cancel{-1+1})^{0} - 1\cancel{\cos(-1+1)}^{1} = -1$$

$$f_y(x_0,y_0) = x\cos(x+y)\Big|_{(-1,1,0)} = -1\cos(-1+1) = -1$$

$$f(-1,1) = \cancel{-1\sin(-1+1)}^{0} = 0$$

$$z = 0 - 1(x+1) - 1(y-1)$$
$$= -x \cancel{-1} - y \cancel{+1}$$
$$z = -x - y$$

b) Encontrar la aproximación lineal $L(x,y)$:

b.1) $z = xe^{xy}$ ...

$(1, 0)$

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

$$f_x(1,0) = e^{xy} + xye^{xy}\Big|_{(1,0)} = 1$$

$$f_y(1,0) = x^2e^{xy}\Big|_{(1,0)} = 1$$

$$f(1,0) = 1\cancel{e^{1\cdot 0}}^{1} = 1$$

$$z = 1 + 1(x-1) + 1(y-0)$$

$$= \cancel{1} + x - \cancel{1} + y$$

$$z = x + y$$

b.2) $z = \sqrt{x + e^{4y}}$ $(3, 0)$

$$f_x(3,0) = \frac{1}{\sqrt{x+e^{4y}}}\Big|_{(3,0)} = \frac{1}{\sqrt{3+1}} = \frac{1}{2}$$

$$f_y(3,0) = \frac{4e^{4y}}{\sqrt{x+e^{4y}}}\Big|_{(3,0)} = \frac{4}{\sqrt{3+1}} = 2$$

$$f(3,0) = \sqrt{3+1} = \sqrt{4} = 2$$

$$z = 2 + \frac{1}{2}(x-3) + 2(y-0)$$

$$= 2 + \frac{1}{2}x - \frac{3}{2} + 2y$$

$$= \frac{1}{2}x + 2y + \underbrace{\frac{4}{2} - \frac{3}{2}}_{\frac{1}{2}}$$

$$= \frac{1}{2}x + 2y + \frac{1}{2}$$

$$z = \frac{1}{2}x + 2y + \frac{1}{2}$$

2) Tema 2: Derivación Implícita

a) Encuentre $\frac{dy}{dx}$

dx

a.1) $y\cos(x) = x^2 + y^2$

$$0 = x^2 + y^2 - y\cos(x)$$

$$\frac{\partial}{\partial y} = 2y - \cos(x) \qquad \frac{\partial}{\partial x} = 2x + y\sin(x)$$

$$\boxed{\frac{\partial y}{\partial x} = \frac{2y - \cos(x)}{2x + y\sin(x)}}$$

a.2) $e^y \sin(x) = x + xy$

$$0 = x + xy - e^y \sin(x)$$

$$\frac{\partial}{\partial y} = x - e^y \sin(x) \qquad \frac{\partial}{\partial x} = 1 + y - e^y \cos(x)$$

$$\boxed{\frac{\partial y}{\partial x} = \frac{x - e^y \sin(x)}{1 + y - e^y \cos(x)}}$$

b) Encuentre la derivada parcial de z de:

$$e^z + e^{xy} = xyz$$

$$e^z + e^{xy} - xyz = 0$$

$$\boxed{\frac{\partial}{\partial z} = e^z - xy}$$

c) Encontrar la derivada parcial de z:

$$\cos(yx) + \sin(yz) = zx$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \cos(yx) + \sin(yz) - zx = 0$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{-\sin(yx)\,y - z}{\cos(yz)\,y - x}}$$

∂z

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\frac{\partial z}{\partial x} = -\frac{-\sin(yx)\,x + \cos(yz)\,z}{\cos(yz)\,y - x}$$

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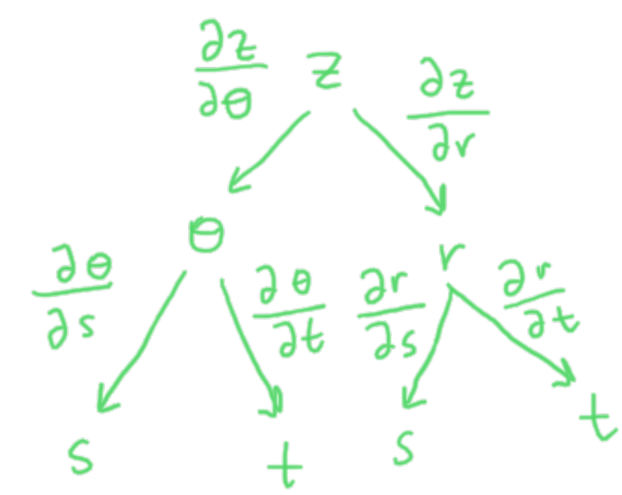
3) Tema 3: Regla de la cadena

a) Encuentre $\partial z/\partial s$ y $\partial z/\partial t$

a.1) $z = e^{r}\cos(\theta)$ $\quad \theta = \sqrt{s^2+t^2}$ $\quad r = st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta}\cdot\frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial r}\cdot\frac{\partial r}{\partial s}$$



$$\frac{\partial z}{\partial s} = \left(-e^{r}\sin(\theta)\right)\left(\frac{1}{2}(s^2+t^2)^{-\frac{1}{2}}2s\right) + \left(e^{r}\cos(\theta)\right)(t)$$

$$\frac{\partial z}{\partial s} = -\frac{e^{r}\sin(\theta)\,s}{\sqrt{s^2+t^2}} + e^{r}\cos(\theta)\,t$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial \theta}\cdot\frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial r}\cdot\frac{\partial r}{\partial t}$$

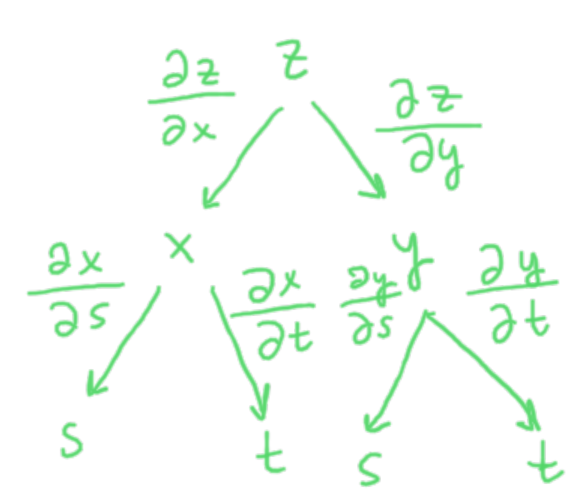
$$= \left(-e^{r}\sin(\theta)\right)\left(\frac{1}{\cancel{2}}(s^2+t^2)^{-\frac{1}{2}}\cancel{2}t\right) + \left(e^{r}\cos(\theta)\right)(s)$$

$$\frac{\partial z}{\partial r} = -\frac{e^{r}\sin(\theta)\,t}{\sqrt{s^2+t^2}} + e^{r}s\cos(\theta)$$

a.2) $z = \arcsin(x-y)$ $\quad x = s^2+t^2$ $\quad y = 1-2st$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$



$$\frac{\partial z}{\partial t} = \left(\frac{1}{\quad}\right)\left(\quad\right)\left(\quad -1 \quad\right)\left(\quad\right)$$

$$\frac{\partial z}{\partial t} \left(\frac{}{(x-y)^2+1}\right)(2t) + \left(\frac{}{(x-y)^2+1}\right)(-2s)$$

$$\boxed{\frac{\partial z}{\partial t} = \frac{2t}{(x+y)^2+1} + \frac{2s}{(x-y)^2+1}}$$

$$\frac{\partial z}{\partial s} = \left(\frac{1}{(x-y)^2+1}\right)(2s) + \left(\frac{-1}{(x-y)^2+1}\right)(-2s)$$

$$\boxed{\frac{\partial z}{\partial s} = \frac{2s}{(x-y)^2+1} + \frac{2s}{(x-y)^2+1}}$$

b) función de Producción:

$$P(L,K) = 10\,L^{\frac{1}{2}}\,K^{\frac{1}{2}}$$

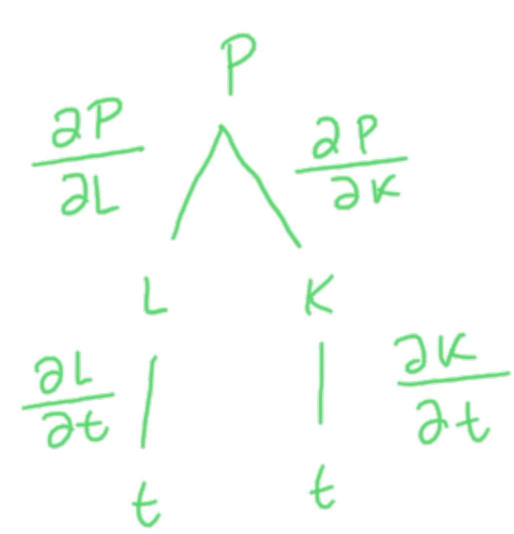
L : h en miles ; K : h de capital

$L = 25$; $K = 16$ $P(L,K)$ esta en toneladas

$\Delta L = -2$ $\Delta K = 3$

Encontrar tasa de cambio de la producción:

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial L}\frac{\partial L}{\partial t} + \frac{\partial P}{\partial K}\frac{\partial K}{\partial t}$$



$$\frac{\partial P}{\partial L} = \frac{5\sqrt{K}}{\sqrt{L}}\Bigg|_{L=25,\,K=16} = \frac{5\sqrt{16}}{\sqrt{25}} = \frac{20}{5} = 4$$

$$\frac{\partial P}{\partial K} = \frac{5\sqrt{L}}{\sqrt{K}}\Bigg|_{L=25,\,K=16} = \frac{5\sqrt{25}}{\sqrt{16}} = \frac{25}{4} = 5$$

$$\frac{\partial P}{\partial t} = (4)(-2) + (5)(3)$$

$$= -8 + 15$$

$= 7$

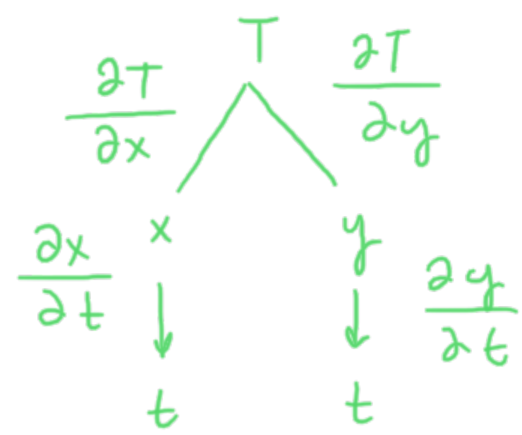
c) La temperatura (x,y) medida en Celsius,

$x = \sqrt{1+t}$ $\quad y = 2 + \frac{1}{3}t$ $\quad$ # x, y en cm.

$T_x(2,3) = 4 \quad T_y(2,3) = 3$

¿Cuán rápido está aumentando la temperatura en $t = 3$?

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\partial T}{\partial x}}_{4} \cdot \frac{\partial x}{\partial t} + \underbrace{\frac{\partial T}{\partial y}}_{3} \cdot \frac{\partial y}{\partial t}$$



$$\frac{\partial x}{\partial t} = \frac{1}{2}(1+t)^{-\frac{1}{2}} \cdot 1 = \frac{1}{2\sqrt{1+t}} \Bigg|_{t=3} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\frac{\partial y}{\partial t} = \frac{1}{3} \Big|_{t=3} = \frac{1}{3}$$

$$\frac{\partial T}{\partial t} = (4)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{3}\right) = 1 + 1 = 2$$

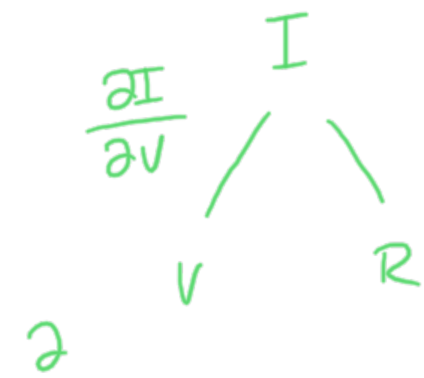
d) Voltage ↓ ; Resistencia ↑ ; ley Ohm: $V = IR$

$R = 400\,\Omega \quad I = 0.08\,A$

I: Corriente
R: Resistencia
V: Voltage

$V = IR$ $\quad \frac{dV}{dt} = -0.01\ V/s \quad \frac{dR}{dt} = 0.03\ \Omega/s$

$\frac{\partial I}{\partial t}$



4) Tema 4: Derivada Direccional

a) Hallar la derivada direccional del vector $\vec{v}$

a.1) $f(x,y) = x^3 - y^3 \qquad P(4,3) \qquad \vec{v} = \frac{\sqrt{2}}{2}(\hat{\imath} + \hat{\jmath})$

$f_x = 3x^2 \qquad f_y = -3y \qquad \vec{v} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

$f_x(4,3) = 48 \qquad f_y(4,3) = -9$

$\nabla f(4,3) = \langle 48, -9 \rangle$

$$D_u f = \langle 48, -9 \rangle \cdot \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

$$= 24\sqrt{2} - \frac{9}{2}\sqrt{2} = \sqrt{2}\left(24 - \frac{9}{2}\right) = \boxed{\sqrt{2}\,\frac{39}{2}}$$

a.2) $g(x,y,z) = x^2 + y^2 + z^2 \qquad P(1,1,1)$

$\vec{v} = \frac{\sqrt{3}}{3}(\hat{\imath} - \hat{\jmath} + \hat{k})$

$f_x(1,1,1) = 2x\big|_{(1,1,1)} = 2$

$f_y(1,1,1) = 2y\big|_{(1,1,1)} = 2 \qquad \vec{v} = \left\langle \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle$

$f_z(1,1,1) = 2z\big|_{(1,1,1)} = 2 \qquad \nabla f = \langle 2, 2, 2 \rangle$

$$D_u f = \langle 2,2,2 \rangle \cdot \left\langle \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle$$

$$= \cancel{\frac{2}{3}\sqrt{3} - \frac{2}{3}\sqrt{3}}^{\,0} + \frac{2}{3}\sqrt{3} = \boxed{\frac{2}{3}\sqrt{3}}$$

b) Halle la razón de cambio instantánea de la función en P, la dirección $\vec{v}$:

b.1) $f(x,y) = e^x \sin(y) \qquad P\left(1, \frac{\pi}{2}\right) \qquad \vec{v} = -\hat{\imath}$

$D_u f = \nabla f \cdot \vec{v}$

$\nabla f = \langle e^x \sin(y), e^x \cos(y) \rangle$

$\nabla f\left(1, \frac{\pi}{2}\right) = \left\langle e \sin\left(\frac{\pi}{2}\right), e \cos\left(\frac{\pi}{2}\right) \right\rangle$

$= \langle e, 0 \rangle$

$$\vec{v} = \langle -1, 0 \rangle$$

$$D_u f = \langle e, 0 \rangle \cdot \langle -1, 0 \rangle$$
$$= -e + 0 = \boxed{-e}$$

b.2) $g(x,y,z) = x^2 + y^2 + z^2 \quad P(1,1,1) \quad \vec{v} = \frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} + \hat{k})$

$$\nabla f(1,1,1) = \langle 2x, 2y, 2z \rangle \Big|_{(1,1,1)} = \langle 2,2,2 \rangle$$

$$\vec{v} = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$D_u f = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle \cdot \langle 2,2,2 \rangle$$
$$= \cancel{\frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}}} + \frac{2}{\sqrt{3}} = \boxed{\frac{2}{\sqrt{3}}}$$

c) Hallar la derivada direccional del vector unitario $\vec{u} = \cos(\theta)\,\hat{\imath} + \sin(\theta)\,\hat{\jmath}$

c.1) $f(x,y) = \sin(2x+y) \qquad \theta = \frac{\pi}{3}$

$$D_u f = \nabla f \cdot \vec{v}$$

$$\nabla f = \langle 2\cos(2x+y), \cos(2x+y) \rangle$$

$$\vec{v} = \langle \cos(\theta), \sin(\theta) \rangle \Big|_{\theta = \frac{\pi}{3}} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

$$\boxed{D_u f = \sqrt{2}\cos(2x+y) + \frac{\sqrt{2}}{2}\cos(2x+y)}$$

c.2) $f(x,y) = \frac{y}{x+y} \qquad \theta = -\frac{\pi}{6}$

$x + y$

$y(x+y)^{-1}$

$\frac{}{B} = \frac{A'B - AB'}{(B)^2}$

$$f_x = -y(x-y)^{-2} \cdot 1$$

$$f_y = \frac{1(\cancel{x+y}) - \cancel{y}}{(x+y)} = \frac{x}{x+y}$$

$$\nabla f = \left\langle -\frac{y}{(x-y)^2}, \frac{x}{x+y} \right\rangle$$

$$\vec{v} = \left\langle \cos\left(-\frac{\pi}{6}\right), \sin\left(-\frac{\pi}{6}\right) \right\rangle = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$$

$$D_u f = \left\langle -\frac{y}{(x-y)^2}, \frac{x}{x+y} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$$

$$= -\frac{\sqrt{3}}{2(x-y)^2} - \frac{x}{2(x+y)}$$

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5) Tema 5: Optimización

Dulces A, B

$CP_A = 60 \qquad CP_B = 70$

Demanda A, B:

$q_A = 5(P_B - P_A)$

$q_B = 500 + 5(P_A - 2P_B)$

- maximizar precio

$$C = 60 q_A + 70 q_B$$

$$U(q_A, q_B) = P_A q_A + P_B q_B - (60 q_A + 70 q_B)$$

$$= P_A(5P_B - 5P_A) + P_B(500 + 5P_A - 10P_B) - 60 q_A - 70 q_B$$

$$= 5P_B P_A - 5P_A^2 + 500 P_B + 5P_A P_B - 10 P_B^2 - 60 q_A - 70 q_B$$

$$= -5P_A^2 + 10 P_B P_A + 500 P_B - 10 P_B^2 - 60(5P_B - 5P_A) - 70(500 + 5P_A - 10P_B)$$

$$= \cancel{-5P_A^2} + \cancel{10 P_B P_A} + \cancel{500 P_B} - \cancel{10P_B^2} - \cancel{300 P_B} + \cancel{300 P_A} - 35{,}000 - \cancel{350 P_A} + \cancel{700 P_B}$$

$$= -5P_A^2 + 10 P_B P_A + 500 P_B - 300 P_B + 700 P_B - 10 P_B^2 + 300 P_A - 350 P_A - 35{,}000$$

$$= -5P_A^2 + 10 P_B P_A + 900 P_B - 10 P_B^2 - 50 P_A - 35000$$

$$\frac{\partial U}{\partial P_A} = -5 \cdot 2\, P_A + 10 P_B - 50 = 0$$

$$= -10 P_A + 10 P_B - 50 = 0$$

$$10 P_B - 50 = 10 P_A$$

$$\boxed{P_B - 5 = P_A}$$

$$\frac{\partial U}{\partial P_B} = 10 P_A + 900 - 20 P_B = 0$$

$$P_A = \frac{1}{10}\left(-900 + 20 P_B\right)$$

$$\boxed{P_A = -90 + 2 P_B}$$

$$P_B - 5 = -90 + 2 P_B$$

$$-5 + 90 = 2 P_B - P_B$$

$$85 = P_B$$

$$80 = P_A$$

Precios que maximizan

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6) Tema 6: Multiplicadores La'Grange.

$$f(q_1, q_2) = 2q_1^2 + q_1 q_2 + q_2^2 + 200$$

$$q_1 + q_2 = 200$$

$$F(q_1, q_2, \lambda) = 2q_1^2 + q_1 q_2 + q_2^2 + 200 + \lambda(200 - q_2 - q_1)$$

$$= 2q_1^2 + q_1 q_2 + q_2^2 + 200 + 200\lambda - \lambda q_2 - \lambda q_1$$

$$\frac{\partial F}{\partial q_1} = 4q_1 + q_2 - \lambda = 0$$

$$4q_1 + q_2 = \lambda$$

$$\frac{\partial F}{\partial q_2} = q_1 + 2q_2 - \lambda = 0$$

$$4q_1 + q_2 = q_1 + 2q_2$$

$$\boxed{3q_1 = q_2}$$

$q_1 + 2q_2 - \lambda$

$$\frac{\partial F}{\partial \lambda} = 200 - q_2 - q_1 = 0$$

$$200 - 3q_1 - q_1 = 0$$

$$4q_1 = 200$$

$$q_1 = \frac{200}{4}$$

$$q_1 = 50$$

distribución de producción

$$200 - 50 = q_2$$

$$150 = q_2$$