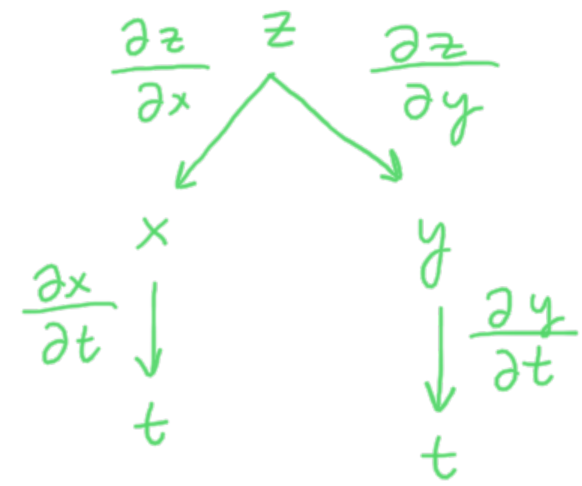
Regla de la cadena & derivación implícita

1) $z = xy^9 - x^2y$, $x = t^2+1$, $y = t^2 - 1$

find $\frac{\partial z}{\partial t}$



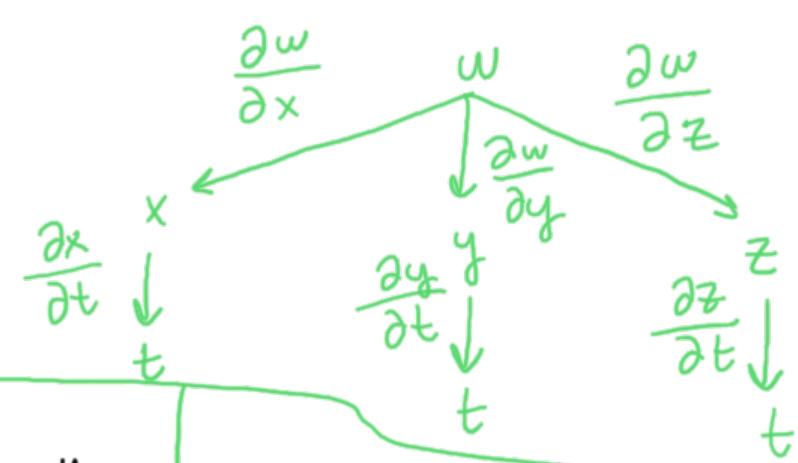
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = \left(y^9 - 2xy\right)(2t) + \left(9y^8x - x^2\right)(2t)$$

$$= 2ty^9 - 4txy + 18ty^8x - 2tx^2$$

2) $w = xe^{y/z}$ $\quad x = t^5$, $y = 4-t$, $z = 2+3t$

find $\frac{\partial w}{\partial t}$



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t}$$

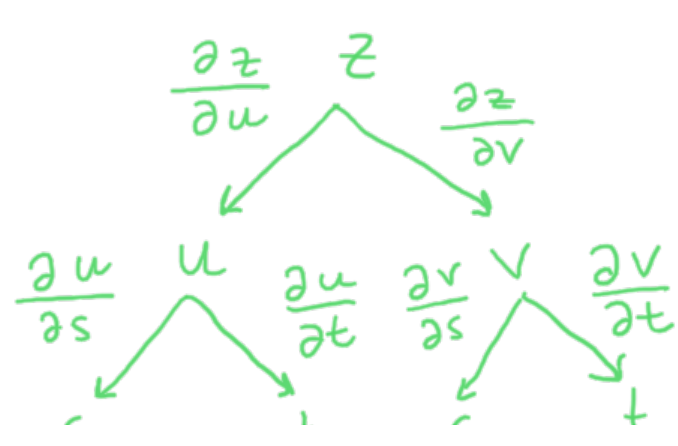
$\frac{\partial w}{\partial x} = e^{\frac{y}{z}}$	$\frac{\partial x}{\partial t} = 5t^4$	$\frac{\partial w}{\partial y} = \frac{xe^{\frac{y}{z}}}{z}$	$\frac{\partial y}{\partial t} = -1$;
$w = xe^{yz^{-1}} \Rightarrow \frac{\partial w}{\partial z} = -xe^{yz^{-1}}yz^{-2}$	$\frac{\partial z}{\partial t} = 3$		

$$\frac{\partial w}{\partial t} = \left(e^{y/z}\right)\left(5t^4\right) + \left(\frac{xe^{y/z}}{z}\right)(-1) + \left(\frac{-xe^{\frac{y}{z}}y}{z^2}\right)(3)$$

$$= 5t^4e^{\frac{y}{z}} - \frac{xe^{\frac{y}{z}}}{z} - \frac{3xe^{\frac{y}{z}}y}{z^2}$$

3) $z = \tan(u/v)$, $u = 9s + 5t$, $v = 5s - 9t$

find $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial s} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial t}$$

$$\frac{\partial z}{\partial u} = \sec^2\left(uv^{-1}\right)v^{-1} \qquad \frac{\partial z}{\partial v} = -\sec^2\left(uv^{-1}\right)uv^{-2}$$

$$\frac{\partial u}{\partial s} = 9 \qquad \frac{\partial u}{\partial t} = 5 \qquad \frac{\partial v}{\partial s} = 5 \qquad \frac{\partial v}{\partial t} = -9$$

$$\frac{\partial z}{\partial s} = 9\sec^2\left(uv^{-1}\right)v^{-1} - 5\sec^2\left(uv^{-1}\right)uv^{-2}$$

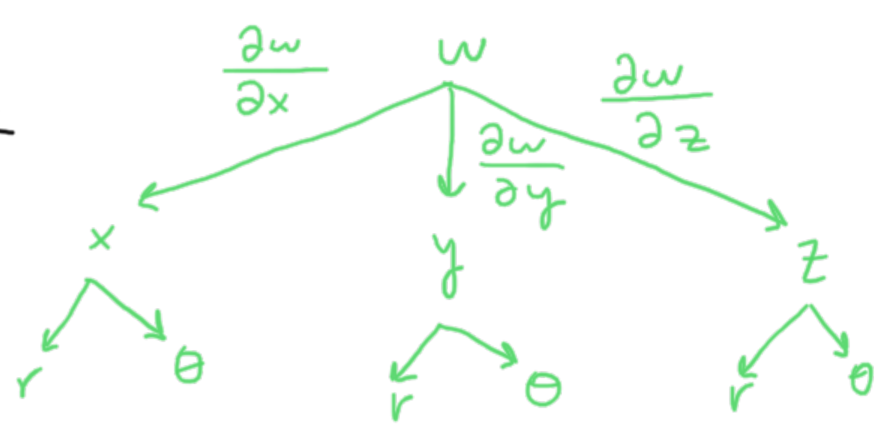
$$\frac{\partial z}{\partial t} = 5\sec^2\left(uv^{-1}\right)v^{-1} + 9\sec^2\left(uv^{-1}\right)uv^{-2}$$

4) $w = xy + yz + zx$, $\quad x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = r\theta$

$\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial \theta}$ when $r = 6$, $\theta = \frac{\pi}{2}$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial \theta}$$



$\frac{\partial w}{\partial x} = y + z$	$\frac{\partial w}{\partial y} = x + z$	$\frac{\partial w}{\partial z} = y + x$
$\frac{\partial x}{\partial r} = \cos(\theta)$	$\frac{\partial y}{\partial r} = \sin(\theta)$	$\frac{\partial z}{\partial r} = \theta$
$\frac{\partial x}{\partial \theta} = -r\sin(\theta)$	$\frac{\partial y}{\partial \theta} = r\cos(\theta)$	$\frac{\partial z}{\partial \theta} = r$

$$\frac{\partial w}{\partial r} = (y+z)\cos(\theta) + (x+z)\sin(\theta) + (y+x)\theta$$

$\theta = \frac{\pi}{2}$; $r = 6$; $x = 0$; $y = 6$; $z = 3\pi$

$$= \cancel{(6+3\pi)\cos\left(\tfrac{\pi}{2}\right)}^{0} + (0+3\pi)\,\cancel{\sin(\theta)}^{1} + (6+0)\frac{\pi}{2}$$

$$= 3\pi + \frac{6\pi}{2} = \boxed{6\pi}$$

$$\frac{\partial w}{\partial \theta} = -(y+z)\,r\sin(\theta) + (x+z)\,r\cos(\theta) + (y+x)\,r$$

$$= -(6+3\pi)\left(6\cdot\sin\left(\tfrac{\pi}{2}\right)\right) + \cancel{(0+3\pi)\left(6\cdot\cos\left(\tfrac{\pi}{2}\right)\right)}^{0} + (6)\,6$$

$$= -(6+3\pi)(6) + 36$$

$$= -\cancel{36} - 18\pi \cancel{+36}$$

$$= \boxed{-18\pi}$$

5) Use :

$$\frac{dy}{dx} = -\frac{\frac{dF}{dy}}{\frac{dF}{dx}} = -\frac{F_x}{F_y}$$

to find : $\frac{dy}{dx}$: $\left[\,4y\cos(x) = x^2 + y^2\,\right]$

$$4y\cos(x) - x^2 - y^2 = 0$$

$$\boxed{\frac{\partial y}{\partial x} = -\frac{-4y\sin(x) - 2x}{4\cos(x) - 2y}}$$

6) $4\tan^{-1}(x^2y) = x + xy^2$ $\qquad \frac{\partial y}{\partial x}$

$$4\tan^{-1}(x^2y) - x - xy^2$$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = \boxed{-\frac{\left(\frac{8xy}{(x^2y)^2+1}\right) - 1 - y^2}{\left(\frac{4x^2}{(x^2y)^2+1}\right) - 2xy}}$$

7) $x^2 + 8y^2 + 3z^2 = 1$

$\dfrac{\partial z}{\partial x}, \quad \dfrac{\partial z}{\partial y}$

$x^2 + 8y^2 + 3z^2 - 1 = 0$

$\dfrac{\partial z}{\partial x} = -\dfrac{2x}{6z} = \boxed{-\dfrac{x}{3z}}$

$\dfrac{\partial z}{\partial y} = -\dfrac{16y}{6z} = \boxed{-\dfrac{8y}{3z}}$

8) $x = \sqrt{2+t}, \quad y = 4 + \frac{1}{2}t$

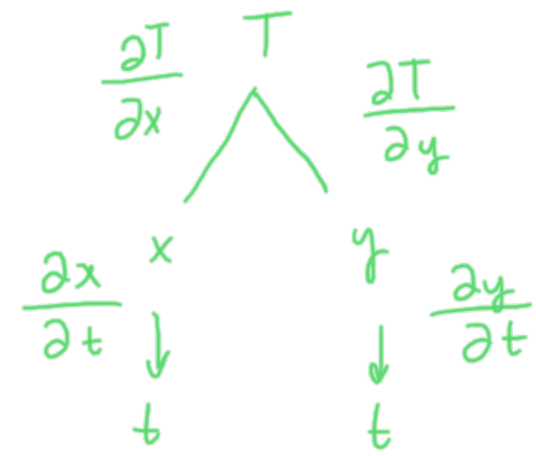
x, y measured in cm.

$T_x(2,5) = 8, \quad T_y(2,5) = 5$

how fast is the temperature rising when:

$t = 2$

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\partial T}{\partial x}}_{8} \cdot \frac{\partial x}{\partial t} + \underbrace{\frac{\partial T}{\partial y}}_{5} \cdot \frac{\partial y}{\partial t}$$



$$\frac{\partial x}{\partial t} = \frac{1}{2}(2+t)^{-\frac{1}{2}} = \frac{1}{2\sqrt{2+t}}\Big|_{t=2} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

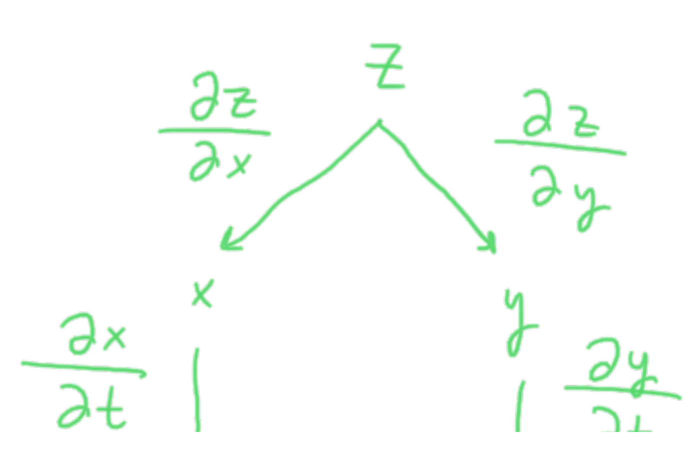
$$\frac{\partial y}{\partial t} = \frac{1}{2}\Big|_{t=3} = \frac{1}{2}$$

$$\frac{\partial T}{\partial t} = (8)\left(\frac{1}{4}\right) + (5)\left(\frac{1}{2}\right)$$

$$= 2 + \frac{5}{2} = \boxed{\frac{9}{2}}$$

9) $z = \cos(x + 7y), \quad x = 2t^3, \quad y = \dfrac{4}{t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$



$$\frac{\partial z}{\partial x} = -\sin(x + 7y) \cdot 1$$

$$\frac{\partial x}{\partial t} = 6t^2$$

$$\frac{\partial y}{\partial t} = -4t^{-2} = -\frac{4}{t^2}$$

$$\frac{\partial z}{\partial y} = -\sin(x+7y)\,7$$

$$\frac{\partial z}{\partial t} = \left(-\sin(x+7y)\right)\left(6t^2\right) + \left(-\sin(x+7y)\,7\right)\left(-\frac{4}{t^2}\right)$$

$$= -6t^2 \sin(x+7y) + \frac{4\cdot 7}{t^2}\sin(x+7y)$$

$$= -\sin(x+7y)\left[6t^2 + \frac{4\cdot 7}{t^2}\right]$$

$$= -\sin(x+7y)\left[6t^2 + \frac{28}{t^2}\right]$$

10) If $z = f(x,y)$

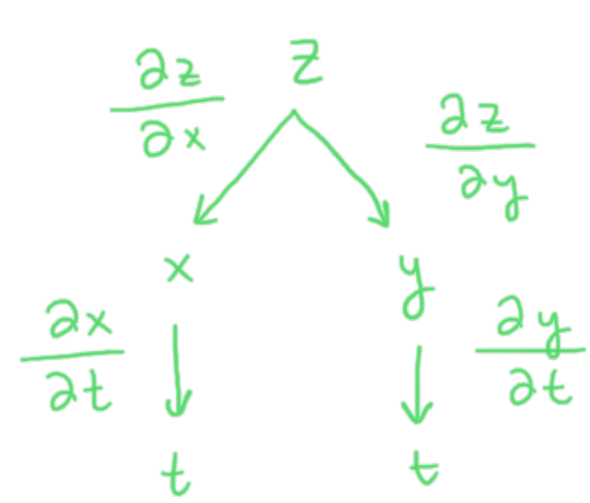
$x = g(t)$ $\qquad$ $y = h(t)$

$g(5) = -7$ $\qquad$ $h(5) = 8$

$g'(5) = 4$ $\qquad$ $h'(5) = -5$

$f_x(-7,8) = 2$ $\qquad$ $f_y(-7,8) = -6$

find $\frac{\partial z}{\partial t}$ when $t = 5$



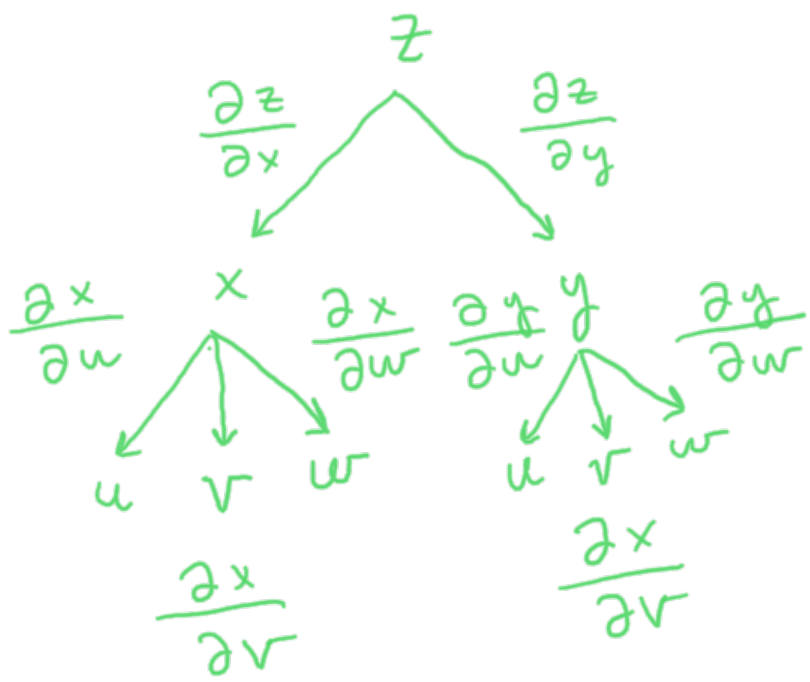
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= (2)(4) + (-6)(-5)$$

$$= 8 + 30 = 38$$

11) $z = x^3 + xy^4, \quad x = uv^4 + w^3, \quad y = u + ve^w$

when $u=1$, $v=1$, $w=0$ $\qquad \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{w}$



$$\frac{\partial z}{\partial x} = 3x^2 + y^4 \Big|_{\substack{u=1\\v=1\\w=0}} = 3(1\cdot 1+0)^2 + (1+1\cdot 1)^4$$
$$= 3 + 16 = 19$$

$$\frac{\partial z}{\partial y} = 4xy^3 \Big|_{\substack{u=1\\v=1\\w=0}} = 4(uv^4+w)(u+ve^w)^3$$
$$= 4(1+0)(1+1)^3 = 4(1+0)(2)^3$$
$$= 4\cdot 8 = 32$$

$$\frac{\partial x}{\partial u} = v^4 \Big|_{v=1} = 1 \qquad \frac{\partial y}{\partial u} = 1$$

$$\frac{\partial x}{\partial v} = 4uv^3 \Big|_{\substack{u=1\\v=1}} = 4 \qquad \frac{\partial y}{\partial v} = ve^w \Big|_{\substack{v=1\\w=0}} = 1e^0 = 1$$

$$\frac{\partial x}{\partial w} = 3w^3 = 0 \qquad \frac{\partial y}{\partial w} = ve^w \Big|_{\substack{v=1\\w=0}} = 1e^0 = 1$$

- $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$

  $= (19)(1) + (32)(1) = 19 + 32 = 51$

- $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$

  $= (19)(4) + (32)(1) = 19\cdot 4 + 32 = 108$

[redacted] $\dfrac{\partial z}{\partial w} = \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial x}{\partial w} + \dfrac{\partial z}{\partial y}\cdot\dfrac{\partial y}{\partial w}$

$$= \overset{0}{\cancel{(19)(0)}} + (32)(1) = \boxed{32}$$

13) $yz = 4\ln(x+z) \rightarrow 0 = 4\ln(x+z) - yz$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{\dfrac{4}{x+z}}{\dfrac{4}{x+z} - y}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{z}{\dfrac{4}{x+z} - y}$$