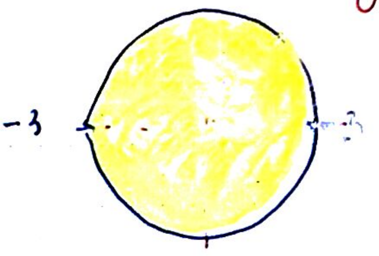
3. $\boxed{-3 \le x \le 3, -\sqrt{9-x^2} \le y \le \sqrt{9-x^2}}$

$3-\sqrt{9-x^2-y^2} \le z \le 3+\sqrt{9-x^2-y^2}$

$0 \le \theta \le 2\pi$
$0 \le r \le 3.$

$$\iiint_E (x^2+y^2+z^2)^{3/2}\,dV.$$



¿ $\le \varphi \le$, $\le \rho \le$?

$z = 3 \pm \sqrt{9-x^2-y^2}$ (x

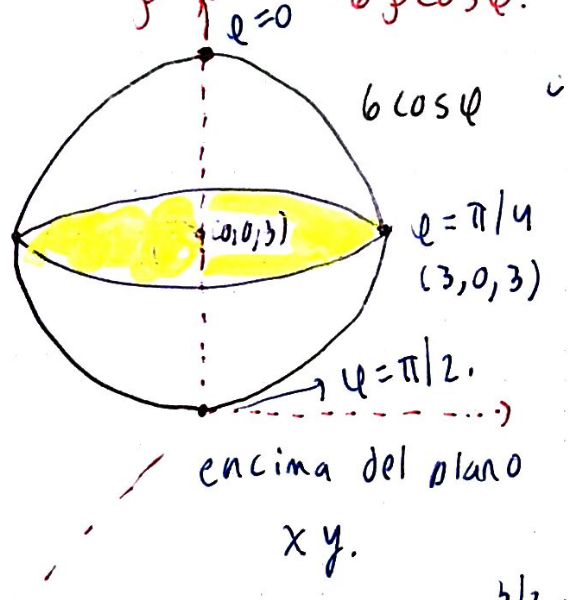
$(z-3)^2 = 9-x^2-y^2. \Rightarrow x^2+y^2+(z-3)^2 = 9.$

Esfera radio $\rho = 3$ centrada en $(0,0,3)$.

Escriba en términos de ρ, θ, φ.

$$\underbrace{x^2+y^2+z^2}_{\rho^2}\ \underbrace{-6z}_{-6\rho\cos\varphi}\ +\ \underline{\underline{9}} = \underline{\underline{9}} \Rightarrow \rho^2-6\rho\cos\varphi = 0.$$

$\rho = 6\cos\varphi.$



$$\boxed{\begin{aligned} &0 \le \theta \le 2\pi \\ &0 \le \rho \le 6\cos\varphi. \\ &0 \le \varphi \le \pi/2. \end{aligned}}$$

$z = \rho\cos\varphi.$

$3 = \sqrt{18}\cos\varphi. \quad \cos\varphi = \frac{3}{3\sqrt{2}}$

$\cos\varphi = \frac{1}{\sqrt{2}} \Rightarrow \varphi = \pi/4.$

J. $$\iiint_E (x^2+y^2+z^2)^{3/2}\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{6\cos\varphi.} \rho^3\ \overbrace{\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta}^{dV}.$$

$$\underline{\int_{-2}^{0}}\underline{\int_{0}^{\sqrt{4-y^2}}}\int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} x\,dz\,dx\,dy.$$

$0 \leq \rho \leq 4$

$0 \leq \varphi \leq \pi/6.$

proxección de $\sqrt{16-x^2-y^2}$ sobre xy.

$\rho = 4.$ (Semiesfera superior)

cono $z = \sqrt{3}\sqrt{x^2+y^2}$

$\varphi = \pi/6.$ (cono)

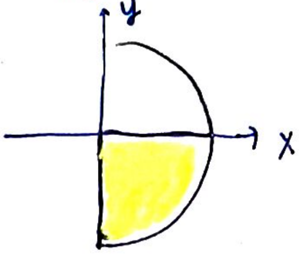
$$\rho\cos\varphi = \sqrt{3}\,r = \sqrt{3}\,\rho\sin\varphi. \Rightarrow \frac{1}{\sqrt{3}} = \tan\varphi.$$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2.$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta.$	1	$\sqrt{3}/2$	$\sqrt{2}/2.$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

$z = \rho\cos\varphi$

$r = \rho\sin\varphi.$

$$\int_{-2}^{0}\int_{0}^{\sqrt{4-y^2}}$$



$\frac{3\pi}{2} \leq \theta \leq 2\pi.$

$\cot\varphi = \sqrt{3}$

$$I_1 = \iiint_E x\,dV$$

$x = \rho\sin\varphi\cos\theta.$

$dV = \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta.$

$$I_1 = \int_{3\pi/2}^{2\pi}\int_{0}^{\pi/6.}\int_{0}^{4}\rho^3\sin^2\varphi\cos\theta.\,d\rho\,d\varphi\,d\theta.$$

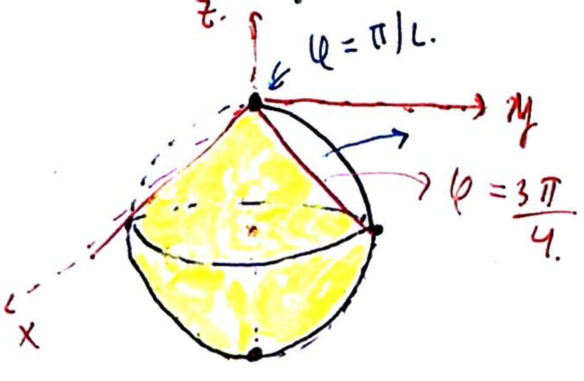
$$I_1 = \underbrace{\left(\int_{3\pi/2}^{2\pi}\cos\theta\,d\theta.\right)}_{\sin\theta\Big]_{3\pi/2.}^{2\pi}\ 1}\int_{0}^{\pi/6}\sin^2\varphi\,d\varphi\ \underbrace{\int_{0}^{4}\rho^3\,d\rho.}_{64.}$$

$$\frac{1}{2}(1+\cos 2\varphi)$$

$$x^2+y^2+z^2+2z+1=1$$

$x^2+y^2+(z+1)^2=1$ radio 1 centro $(0,0,-1)$

$(x-a)^2+(y-b)^2+(z-c)^2=r^2$ radio r centro (a,b,c).



E $\quad 0\le\rho\le-2\cos\varphi$.

$\frac{3\pi}{4}\le\varphi\le\pi$.

$0\le\theta\le2\pi$.

c. Volumen $\quad V=\iiint_E dV=\int_{3\pi/4}^{\pi}\int_0^{2\pi}\int_0^{-2\cos\varphi}\rho^2\sin\varphi\,d\rho\,d\theta.\,d\varphi.$

$$V=2\pi\int_{3\pi/4}^{\pi}\int_0^{-2\cos\varphi}\rho^2\,d\rho\,\sin\varphi\,d\varphi.$$

$$V=2\pi\int_{3\pi/4}^{\pi}-\frac{8}{3}\cos^3\varphi\,\sin\varphi\,d\varphi. \qquad -\int\cos^3\sin\varphi=\frac{1}{4}\cos^4\varphi$$

$$V=2\pi\;\frac{2}{3}\cos^4\varphi\Big]_{3\pi/4}^{\pi}=\frac{4\pi}{3}\left((-1)^4-\left(-\frac{\sqrt{2}}{2}\right)^4\right)$$

$$V=\frac{4\pi}{3}\left(1-\frac{4}{16}\right)=\frac{4\pi}{3}\left(1-\frac{1}{4}\right)=\frac{4\pi}{3}\,\frac{3}{4}=\pi.$$

d. masa $\quad m=\iiint_E\rho\,dV=\int_0^{2\pi}12\theta\,d\theta\int_{3\pi/4}^{\pi}\int_0^{-2\cos\varphi.}(\rho^3+1)^3\rho^2\,d\rho\,\sin\varphi\,d\varphi.$

4.

$$\rho^2 = x^2+y^2+z^2 = r^2+z^2 = \rho^2\sin^2\varphi + \rho^2\cos^2\varphi$$

$$(x^2+y^2+z^2)^{3/2} = (\rho^2)^{3/2} = \rho^3 = \rho^2.$$

$$I_3 = \int_0^{2\pi} d\theta . \int_0^{\pi/2}\int_0^{6\cos\varphi} \rho^5 \, d\rho . \sin\varphi \, d\varphi . \qquad \frac{\rho^6}{6}\Big]_0^{6\cos\varphi}$$

$$I_3 = 2\pi \int_0^{\pi/2} 6^5\cos^6\varphi \sin\varphi \, d\varphi.$$

$$I_3 = 2\pi \; \frac{-6^5}{7} \underbrace{\cos^7\varphi \Big]_0^{\pi/2}}_{-1} = 2\pi \frac{6^5}{7}.$$

5. $x^2+y^2+z^2+2z=0$ cono $z=-\sqrt{x^2+y^2}$

esfera. $(0,0,-1)$ radio 1.

a. Ec. esfera. c. Volumen

b. Los límites de E. d. masa $\rho = 12\theta(\rho^3+1)$?

Esfera: $\rho^2 + 2\rho\cos\varphi = 0 \Rightarrow \rho = -2\cos\varphi.$

Cono: $\rho\cos\varphi = -\rho\sin\varphi. \Rightarrow \tan\varphi = -1 \Rightarrow \varphi = -\pi/4$
$\varphi = 3\pi/4.$

$$\sqrt{x^2+y^2} = \sqrt{\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta}$$

$$\sqrt{\rho^2\sin^2\varphi(\cos^2\theta+\sin^2\theta)} = \rho\sin\varphi.$$

1. $x^2+y^2+z^2=9$ & $x^2+y^2+z^2=36.$

Fuera $z=\sqrt{3}\sqrt{x^2+y^2}$ $z=-\frac{1}{\sqrt{3}}\sqrt{x^2+y^2}$

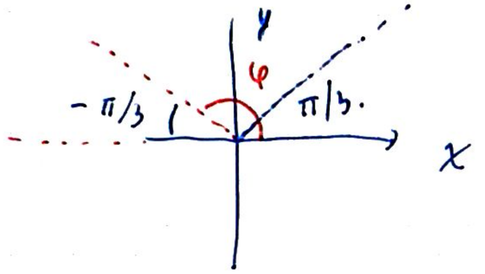
Tipo I: $a\le x\le b$ $f(x)\le y\le g(x)$ $u_1\le z\le u_2.$

Tipo II: $a\le y\le b$ $f(y)\le x\le g(y)$ $u_1\le z\le u_2.$

$3\le\rho\le 6.$ Afuera de los conos

$$\frac{1}{\sqrt{3}}=\frac{\sqrt{x^2+y^2}}{z}=\frac{\rho\sin\varphi}{\rho\cos\varphi}\Rightarrow\varphi=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}.$$

$$\rho\cos\varphi=-\frac{1}{\sqrt{3}}\rho\sin\varphi.\Rightarrow-\sqrt{3}=\tan\varphi\quad\varphi=\frac{-\pi}{3}+\pi=\frac{1\pi}{3}.$$



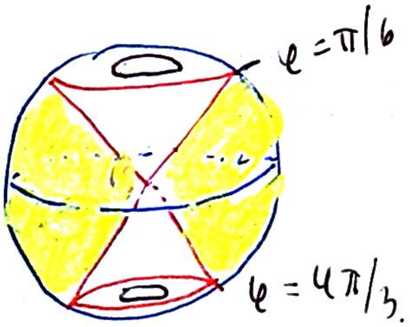
$\tan^{-1}\sqrt{3}$

$\tan^{-1}(-\sqrt{3})=-\frac{\pi}{3}$

$0\le\theta\le 2\pi.$

Describa el sólido E $3\le\rho\le6,$ $0\le\theta\le2\pi.$

$$\frac{\pi}{6}\le\varphi\le\frac{4\pi}{3}.$$



$$V=\int_0^{2\pi}d\theta\int_{\pi/6}^{4\pi/3}\sin\varphi\,d\varphi\int_3^6\rho^2\,d\rho.$$

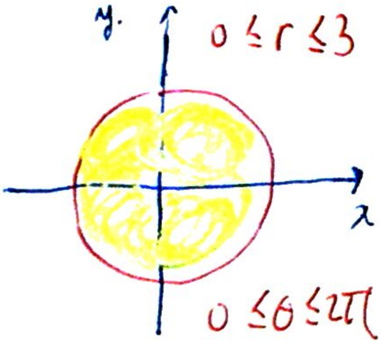
Adentro de los conos y esferas
2 integrales

$$\int_0^{2\pi}\int_0^{\pi/6}\int_3^6 dV+\int_0^{2\pi}\int_{4\pi/3}^{\pi}\int_3^6 dV.$$

3. Cilindricas. , $y = \pm\sqrt{9-x^2}$

$$E = \{-3 \le x \le 3, \quad -\sqrt{9-x^2} \le y \le \sqrt{9-x^2}\}$$

$$3 - \sqrt{9-x^2-y^2} \le z \le 3 + \sqrt{9-x^2-y^2}$$



$$V = \iiint_E r\,dz\,dr\,d\theta. \qquad y^2 = 9 - x^2 \Rightarrow x^2+y^2=9.$$

$x^2+y^2=r^2$. $\quad z = 3 \pm \sqrt{9-x^2-y^2}$ se reescribe como

$$z = 3 \pm \sqrt{9-r^2}$$

$$E = \{0 \le \theta \le 2\pi, \; 0 \le r \le 3, \; 3-\sqrt{9-r^2} \le z \le 3+\sqrt{9-r^2}\}.$$

$$V = \int_0^{2\pi} d\theta . \int_0^3 \int_{3-\sqrt{9-r^2}}^{3+\sqrt{9-r^2}} dz\, r\,dr.$$

$$V = 2\pi \int_0^3 z \Big]_{3-\sqrt{9-r^2}}^{3+\sqrt{9-r^2}} r\,dr$$

$$V = 2\pi \int_0^3 3+\sqrt{9-r^2}-3+\sqrt{9-r^2}\; r\,dr.$$

$$V = 2\pi \int_0^3 (\overset{u}{9}-r^2)^{1/2}.\, \overbrace{2r\,dr}^{-du}.$$

$$V = -2\pi\, \frac{2}{3}(9-r^2)^{3/2}.\Big]_0^3 = -\frac{4\pi}{3}\left(0^{3/2}-(3^2)^{3/2}\right)$$

$$= \frac{4\pi}{3}\, 3^3 = 36\pi.$$

en esféricas

$$V = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{6\cos\varphi} \rho^2\,d\rho\,\sin\varphi\,d\varphi\,d\theta.$$