Material de apoyo - Cálculo Multivariable 2

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Índice general

Parte I

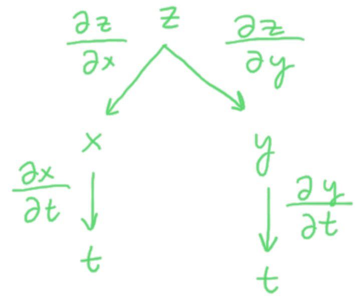
Ejercicios en clase

Capítulo 1

Ejercicio en clase #08

1) $z = xy^9 - x^2 y$, $x = t^2 + 1$, $y = t^2 - 1$

find $\frac{\partial z}{\partial t}$



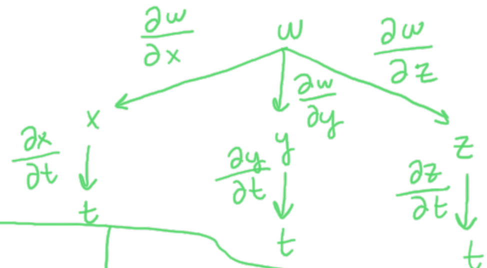
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = \left(y^9 - 2xy\right)(2t) + \left(9y^8 x - x^2\right)(2t)$$

$$= 2ty^9 - 4txy + 18ty^8 x - 2tx^2$$

2) $w = xe^{y/z}$ $x = t^5$, $y = 4 - t$, $z = 2 + 3t$

find $\frac{\partial w}{\partial t}$



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t}$$

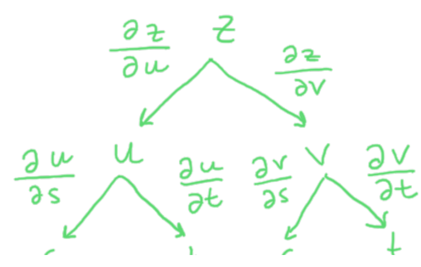
$\frac{\partial w}{\partial x} = e^{\frac{y}{z}}$	$\frac{\partial x}{\partial t} = 5t^4$	$\frac{\partial w}{\partial y} = \frac{xe^{\frac{y}{z}}}{z}$	$\frac{\partial y}{\partial t} = -1$;
$w = xe^{yz^{-1}} \Rightarrow \frac{\partial w}{\partial z} = -xe^{yz^{-1}} yz^{-2}$	$\frac{\partial z}{\partial t} = 3$		

$$\frac{\partial w}{\partial t} = \left(e^{y/z}\right)\left(5t^4\right) + \left(\frac{xe^{y/z}}{z}\right)(-1) + \left(\frac{-xe^{\frac{y}{z}} y}{z^2}\right)(3)$$

$$= 5t^4 e^{\frac{y}{z}} - \frac{xe^{\frac{y}{z}}}{z} - \frac{3xe^{\frac{y}{z}} y}{z^2}$$

3) $z = \tan(u/v)$, $u = 9s + 5t$, $v = 5s - 9t$

find $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial t}$$

$\partial u \quad \partial t \quad , \quad \partial v \quad \partial t$

$$\frac{\partial z}{\partial u} = \sec^2(uv^{-1})v^{-1} \qquad \frac{\partial z}{\partial v} = -\sec^2(uv^{-1})\,uv^{-2}$$

$$\frac{\partial u}{\partial s} = 9 \qquad \frac{\partial u}{\partial t} = 5 \qquad \frac{\partial v}{\partial s} = 5 \qquad \frac{\partial v}{\partial t} = -9$$

$$\frac{\partial z}{\partial s} = 9\sec^2(uv^{-1})v^{-1} - 5\sec^2(uv^{-1})\,uv^{-2}$$

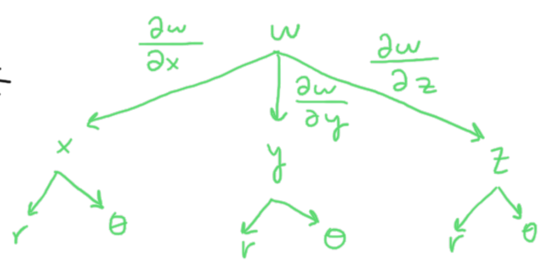
$$\frac{\partial z}{\partial t} = 5\sec^2(uv^{-1})v^{-1} + 9\sec^2(uv^{-1})\,uv^{-2}$$

4) $w = xy + yz + zx, \quad x = r\cos(\theta), \quad y = r\sin(\theta), \quad z = r\theta$

$\frac{\partial w}{\partial r}, \frac{\partial w}{\partial \theta}$ when $r = 6, \ \theta = \frac{\pi}{2}$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial \theta}$$



$\frac{\partial w}{\partial x} = y + z$	$\frac{\partial w}{\partial y} = x + z$	$\frac{\partial w}{\partial z} = y + x$
$\frac{\partial x}{\partial r} = \cos(\theta)$	$\frac{\partial y}{\partial r} = \sin(\theta)$	$\frac{\partial z}{\partial r} = \theta$
$\frac{\partial x}{\partial \theta} = -r\sin(\theta)$	$\frac{\partial y}{\partial \theta} = r\cos(\theta)$	$\frac{\partial z}{\partial \theta} = r$

$$\frac{\partial w}{\partial r} = (y+z)\cos(\theta) + (x+z)\sin(\theta) + (y+x)\,\theta$$

$$\theta = \frac{\pi}{2}; \ r = 6; \ x = 0; \ y = 6; \ z = 3\pi$$

$$= \cancel{(6+3\pi)\cos\left(\tfrac{\pi}{2}\right)^0} + (\emptyset + 3\pi)\cancel{\sin(\theta)}^1 + (6+0)\frac{\pi}{2}$$

$$= 3\pi + \frac{6\pi}{2} = \boxed{6\pi}$$

$$\frac{\partial w}{\partial \theta} = -(y+z)r\sin(\theta) + (x+z)r\cos(\theta) + (y+x)r$$

$$= -(6+3\pi)\left(6\cdot\sin\left(\tfrac{\pi}{2}\right)\right) + \cancel{(\emptyset + 3\pi)\left(6\cdot\cos\left(\tfrac{\pi}{2}\right)\right)}^{\emptyset} + (6)6$$

$$= -(6+3\pi)(6) + 36$$

$$= -\cancel{36} - 18\pi \cancel{+36}$$

$$\boxed{= -18\pi}$$

5) Use :

$$\frac{dy}{dx} = -\frac{\frac{dF}{dy}}{\frac{dF}{dx}} = -\frac{F_x}{F_y}$$

to find: $\frac{dy}{dx}$: $\ulcorner 4y\cos(x) = x^2 + y^2 \lrcorner$

$$4y\cos(x) - x^2 - y^2 = \emptyset$$

$$\boxed{\frac{\partial y}{\partial x} = -\frac{-4y\sin(x) - 2x}{4\cos(x) - 2y}}$$

6) $4\tan^{-1}(x^2y) = x + xy^2$ $\qquad \frac{\partial y}{\partial x}$

$$4\tan^{-1}(x^2y) - x - xy^2$$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = \boxed{-\frac{\left(\frac{8xy}{(x^2y)^2+1}\right) - 1 - y^2}{\left(\frac{4x^2}{(x^2y)^2+1}\right) - 2xy}}$$

7) $x^2 + 8y^2 + 3z^2 = 1$

$x^2 + 8y^2 + 3z^2 - 1 = \emptyset$

$\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x} = -\frac{2x}{6z} = \boxed{-\frac{x}{3z}}$$

$$\frac{\partial z}{\partial y} = -\frac{16y}{6z} = \boxed{-\frac{8y}{3z}}$$

8) $x = \sqrt{2+t}$, $y = 4 + \frac{1}{2}t$

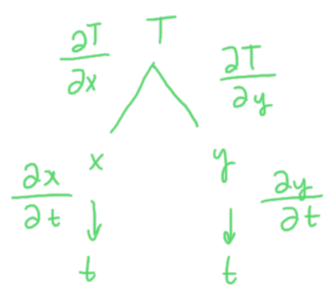
x, y measured in cm.

$T_x(2,5) = 8$, $T_y(2,5) = 5$

how fast is the temperature rising when:

$t = 2$

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\partial T}{\partial x}}_{8} \cdot \frac{\partial x}{\partial t} + \underbrace{\frac{\partial T}{\partial y}}_{5} \cdot \frac{\partial y}{\partial t}$$



$$\frac{\partial x}{\partial t} = \frac{1}{2}(2+t)^{-\frac{1}{2}} = \frac{1}{2\sqrt{2+t}}\Big|_{t=2} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

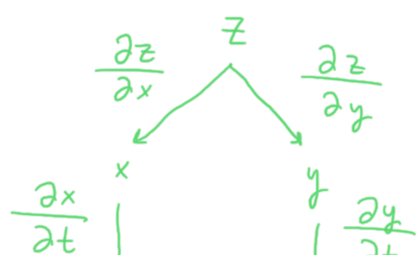
$$\frac{\partial y}{\partial t} = \frac{1}{2}\Big|_{t=3} = \frac{1}{2}$$

$$\frac{\partial T}{\partial t} = (8)\left(\frac{1}{4}\right) + (5)\left(\frac{1}{2}\right)$$

$$= 2 + \frac{5}{2} = \boxed{\frac{9}{2}}$$

9) $z = \cos(x + 7y)$, $x = 2t^3$, $y = \frac{4}{t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$



$$\frac{\partial z}{\partial x} = -\sin(x + 7y) \cdot 1$$

$$\frac{\partial x}{\partial t} = 6t^2$$

$$\frac{\partial y}{\partial t} = -4t^{-2} = -\frac{4}{t^2}$$

$$\frac{\partial z}{\partial y} = -\sin(x+7y)\,7$$

$$\frac{\partial z}{\partial t} = \Big(-\sin(x+7y)\Big)\Big(6t^2\Big) + \Big(-\sin(x+7y)\,7\Big)\Big(-\frac{4}{t^2}\Big)$$

$$= -6t^2\sin(x+7y) + \frac{4\cdot 7}{t^2}\sin(x+7y)$$

$$= -\sin(x+7y)\left[6t^2 + \frac{4\cdot 7}{t^2}\right]$$

$$= -\sin(x+7y)\left[6t^2 + \frac{28}{t^2}\right]$$

10) If $z = f(x,y)$

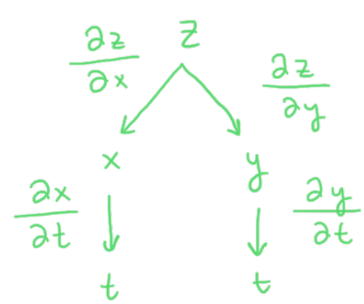
$x = g(t)$ $\qquad$ $y = h(t)$

$g(5) = -7$ $\qquad$ $h(5) = 8$

$g'(5) = 4$ $\qquad$ $h'(5) = -5$

$f_x(-7,8) = 2$ $\qquad$ $f_y(-7,8) = -6$

find $\frac{\partial z}{\partial t}$ when $t = 5$



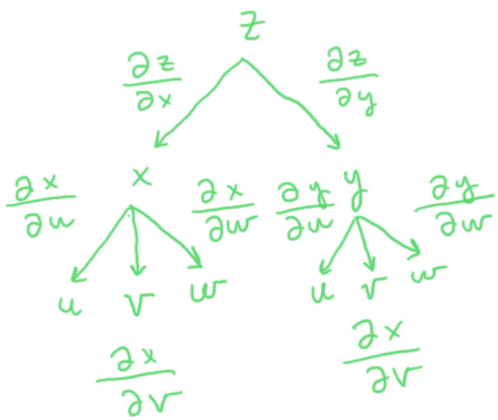
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= (2)(4) + (-6)(-5)$$

$$= 8 + 30 = 38$$

11) $z = x^3 + xy^4, \quad x = uv^4 + w^3, \quad y = u + ve^w$

when $u = 1$, $v = 1$, $w = 0$ $\quad \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{w}$



$$\frac{\partial z}{\partial x} = 3x^2 + y^4 \Big|_{\substack{u=1\\v=1\\w=0}} = 3(1\cdot 1 + 0)^2 + (1 + 1\cdot 1)^4$$

$$= 3 + 16 = 19$$

$$\frac{\partial z}{\partial y} = 4xy^3 \Big|_{\substack{u=1\\v=1\\w=0}} = 4(uv^4 + w)(u + ve^w)^3$$

$$= 4(1+0)(1+1)^3 = 4(1+0)(2)^3$$

$$= 4\cdot 8 = 32$$

$$\frac{\partial x}{\partial u} = v^4 \Big|_{v=1} = 1 \qquad \frac{\partial y}{\partial u} = 1$$

$$\frac{\partial x}{\partial v} = 4uv^3 \Big|_{\substack{u=1\\v=1}} = 4 \qquad \frac{\partial y}{\partial v} = ve^w \Big|_{\substack{v=1\\w=0}} = 1e^0 = 1$$

$$\frac{\partial x}{\partial w} = 3w^3 = 0 \qquad \frac{\partial y}{\partial w} = ve^w \Big|_{\substack{v=1\\w=0}} = 1e^0 = 1$$

- $$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

  $$= (19)(1) + (32)(1) = 19 + 32 = 51$$

- $$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$$

  $$= (19)(4) + (32)(1) = 19\cdot 4 + 32 = 108$$

[redacted] $$\frac{\partial z}{\partial w} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial w} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial w}$$

$$= \cancel{(19)(0)}^{\,0} + (32)(1) = \boxed{32}$$

13) $$yz = 4\ln(x+z) \rightarrow 0 = 4\ln(x+z) - yz$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \boxed{-\frac{\frac{4}{x+z}}{\frac{4}{x+z} - y}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \boxed{\frac{z}{\frac{4}{x+z} - y}}$$