# Ejercicio algoritmia y complejidad - David Corzo

1) a) $a_n = 4_{n-1} - 4_{n-2}, \; n \geq 2, \; a_0 = 6 \quad a_1 = 8$

Polinomio Caract.: $r^2 - 4r + 4 = 0$

$$(r-2)^2 = 0$$

$$r = 2$$ Raices caracteristicas

Solución General: $a_n = c_1 2^n + c_2 n 2^n$

$a_0 = 6$

$6 = c_1 2^0 + c_2 \cdot 0 \cdot 2^0 \rightarrow 6 = c_1 2^0 \rightarrow \boxed{c_1 = 6}$

$a_1 = 8$

$8 = c_1 2^1 + c_2 (1)(2^1) \rightarrow 8 = c_1 2 + 2c_2$

$8 = 6 \cdot 2 + 2c_2$

$8 - 12 = 2c_2$

$\boxed{-2 = c_2}$

$a_n = 6 \cdot 2^n - 2n \cdot 2^n$

$$\boxed{a_n = 2^n (6 - 2n)}$$

b) $a(n) = \frac{1}{4} a(n-2), \; n \geq 2, \; a_0 = 1, \; a_1 = 0$

1 Polinom. Característico

$r^n = \frac{1}{4} r_{n-2}$

$r^2 - \frac{1}{4} = 0$

$(r - \frac{1}{2})(r + \frac{1}{2}) = 0$

$r = 1$

3 ecuacion general:

$a_n = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(-\frac{1}{2}\right)^n$

4 $a_0 = 1$

$1 = c_1 \cancel{\left(\frac{1}{2}\right)^0}^{1} + c_2 \cancel{\left(-\frac{1}{2}\right)^0}^{1}$

$1 = c_1 + c_2$

$r = -\frac{1}{2}$

2 Raíces características

6 Solución general:

$$a_n = \frac{1}{2}\left(\frac{1}{2}\right)^n + \frac{1}{2}\left(-\frac{1}{2}\right)^n$$

5 $a_1 = 0$

$$0 = c_1\left(\frac{1}{2}\right)^1 + c_2\left(-\frac{1}{2}\right)$$

$$0 = \frac{c_1}{2} - \frac{c_2}{2}$$

$$\frac{c_2}{2} = \frac{c_1}{2} \rightarrow c_2 = c_1$$

$$1 = c_1 + c_1$$

$$1 = 2c_1 \rightarrow c_1 = \frac{1}{2}$$

$$c_2 = \frac{1}{2}$$

c)

$$a(n) = a_{n-1} + n^2, \; n \geq 1, \; a_0 = 1$$

$$a(n) - a(n-1) - n^2 = 0$$

Polinomio característico:

$$r^2 - r = 0$$

$$r(r-1) = 0$$

$$r = 1$$

Solución característica: $a_c(n) = c_1 \cdot \overbrace{1^n}^{1} = c_1$

Propuesta general: $a_p(n) = (An^2 + Bn + C)n$

Sustituir $a_p(n)$ en $a(n) - a(n-1) - n^2$

$$An^3 + Bn^2 + Cn = A(n-1)^3 + B(n-1)^2 + C(n-1) + n^2$$

$$\cancel{An^3} + \cancel{Bn^2} + \cancel{Cn} = \cancel{An^3} - 3An^2 + 3An - A + \cancel{Bn^2} - 2Bn + B + \cancel{Cn} - C + n^2$$

$$0 = -3An^2 + 3An - A - 2B_n + B - C + n^2$$

Averiguar A, B, C, n.

$$-3An^2 + n^2 + 3An - 2Bn - A + B - C = 0$$

$$n^2(-3A+1) + n(3A-2B) + (-A+B-C) = 0$$

$-3A+1=0$ $\quad$ $\cancel{3}\cancel{A}^{1} - 2B = 0$ $\quad$ $-A+B-C=0$

$A = \frac{1}{3}$ $\quad$ $B = \frac{1}{2}$ $\quad$ $-\frac{1}{3} + \frac{1}{2} - C = 0$

$C = \frac{1}{6}$

$$\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = c_1 + \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$a_0 = 1 \to c_1 + \frac{1}{3}(0)^3 + \frac{1}{2}(0)^2 + \frac{1}{6}(0) \to c_1 = 1$$

Solución:

$$\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + 1$$

d) $a_n = 2a_{n-1} + n + 5$, $n \geq 1$, $a_0 = 4$

$r = 2$

$1 - \frac{2}{r} = 0$ $\quad$ $a_c(n) = c_1 \cdot 2^n$ $\quad$ $a_p(n) = (c \cdot n + d)$

$$cn + d = 2(c(n-1)+d) + n + 5$$

$$cn + d = 2(cn - c + d) + n + 5$$

$$cn + d = \cancel{2cn}^{cn} - 2c + \cancel{2d}^{d} + n + 5$$

$$0 = cn - 2c + d + n + 5$$

$$0 = (cn + n) + (-2c + d + 5)$$

$n(c+1) = 0$ $\quad$ $-2c + d + 5 = 0$

$c = -1$ $\quad$ $7 + d = 0 \to d = -7$

$a_p(n) = -n - 7$ $\quad$ $c_1 \cdot 2^n - n - 7$

$$a_0 = 4 \to c_1 \cdot \cancel{2^0}^{1} - (0) - 7 = 4$$

$$c_1 = 11$$

Sol:

$$11 \cdot 2^n - 7 - n$$

e) $a_n = 2a_{n\ldots}\ \ldots\ ^n$

$$\ldots_{n-1} + 2^n \quad , n \geq 1, \ a_0 = 2$$

$$a_c(n) = c_1 \cdot 2^n \qquad a_p(n) = 2^n + d$$

$$2^n + d = 2(2^{n-1} + d) + 2^n$$

$$2^n + d = 2^n + 2d + 2^n \longrightarrow c_1 \cdot 2^n + n2^n$$

$$a_0 = 2 \rightarrow c_1 \cdot \cancel{2^0}^{1} = 2 \rightarrow c_1 = 2$$

Sol: $2(2^n) + n2^n$

2) a) $\underbrace{\ln(n)}_{\text{este año}} = \dfrac{\overbrace{\ln(n-1)}^{\text{año ant.}} + \overbrace{\ln(n-2)}^{\text{hace dos años}}}{\underbrace{2}_{\text{años}}} \qquad n > 2$

b) Polinomio característico:

$$r^2 - \frac{1}{2}r + \frac{1}{2} = 0 \qquad a(1) = 100{,}000$$

Raices características: $\qquad a(2) = 300{,}000$

$$r_1 = 1 \qquad r_2 = -\frac{1}{2} \longrightarrow c_1 (1)^n + c_2\left(-\frac{1}{2}\right)^n$$

$$a(1) = c_1(1)^1 + c_2\left(-\frac{1}{2}\right)^1 = 100{,}000$$

$$c_1 - \frac{c_2}{2} = 100{,}000$$

$$a(2) = c_1(1)^2 + c_2\left(-\frac{1}{2}\right)^2 = 300{,}000$$

$$c_1 + \frac{c_2}{4} = 300{,}000$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 100{,}000 \\ 1 & \frac{1}{4} & 300{,}000 \end{array}\right]_{F_2 - F_1} = \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 100{,}000 \\ 0 & \frac{3}{4} & 300{,}000 \end{array}\right] F_2 \frac{4}{3}$$

$$= \left[\begin{array}{cc|c} 1 & -1/2 & 100{,}000 \\ 0 & 1 & 800000/3 \end{array}\right] F_1 + \tfrac{1}{2}F_2 = \left[\begin{array}{cc|c} 1 & 0 & 700{,}000/3 \\ 0 & 1 & 800\,000/3 \end{array}\right]$$

$$c_1 = 700{,}000 \ldots$$

$$c_1 = \frac{700,000}{3} \qquad c_2 = \frac{800,000}{3}$$

$$\text{Sol.:} \quad \frac{700,000}{3}\left(1\right)^n + \frac{800,000}{3}\left(-\frac{1}{2}\right)^n$$

3) a)

$$P(n) = \underbrace{0.2\,(n-1)}_{\text{depositado al año}} + \underbrace{0.45\,(n-2)}_{\text{año anterior}} \qquad n > 2$$

b)