Examen Corto - Algoritmia - David Corzo

1) Muestre que

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

$$\sum_{i=1}^{n} \left(\frac{1}{(3n-2)(3n+1)} \right) = \frac{n}{3n+1}$$

Prueba por inducción

Paso base: $n = 1$

$$\frac{1}{(3(1)-2)(3(1)+1)} = \frac{1}{3(1)+1}$$

$$\frac{1}{(3-2)(3+1)} = \frac{1}{3+1}$$

$$\frac{1}{(1)(4)} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4} \checkmark$$

Paso inductivo:

$$P(n) \rightarrow P(n+1) \qquad n = k$$

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

$$\underbrace{\frac{1}{1.4} + \frac{1}{4.7} + \dots + \frac{1}{(3n-2)(3n+1)}} + \frac{1}{(3(n+1)-2)(3(n+1)+1)} =$$

$$= \frac{n}{(3n+1)} + \frac{1}{(3(n+1)-2)(3(n+1)+1)}$$

$$= \frac{n}{(3n+1)} + \frac{1}{(3n+3-2)(3n+3+1)}$$

$$= \frac{n}{(3n+1)} + \frac{1}{(3n+1)(3n+4)}$$

$$= \frac{1}{(3n+1)} \left(n + \frac{1}{3n+4} \right)$$

$$= \frac{1}{(3n+1)} \left(\frac{n(3n+4)}{(3n+4)} + \frac{1}{3n+4} \right)$$

$$= \frac{1}{(3n+1)} \left(\frac{3n^2+4n+1}{(3n+4)} \right)$$

$$= \frac{3n^2+4n+1}{(3n+1)(3n+4)} = \frac{3n^2+4n+1}{9n^2+12n+3n+4}$$

$$= \frac{3n^2+4n+1}{9n^2+15n+4} = \frac{\cancel{(3n+1)}(n+1)}{\cancel{(3n+1)}(3n+4)}$$

$$= \frac{n+1}{3n+3+1} = \frac{n+1}{3(n+1)+1} \quad \square$$

2) Forma recursiva $P_m(n)$. Producto entero de m n.

$n \times m$ es n sumado a sí mismo m veces

5×3 5 sumado a si mismo 3 veces

$$\overbrace{5+5+5}^{3}$$

<u>Caso base:</u> cuando el que usemos como contador se vuelva ≤ 0 devolvemos 0

```
def mult(n, m):
    if (m <= 0):
        retur 0

    return n + mult(n, m-1)
```

mult(5,3) 5

5 + mut(5,2) 5 10

5 + mult(5,1) 5

5 + mut(5,0) 0