Coeficientes indeterminados Caso II

Ejercicio 3: Determine la forma de y_p.
No encuentre los coeficientes.

a. $y'' + y' - 6y = 3xe^{2x} + 6e^{2x}$

Se propone que $y_p \neq Axe^{2x} + \underline{Be^{2x}}$.

Solución homogéneo. $m^2 + m - 6 = 0$.
$(m+3)(m-2) = 0. \Rightarrow m = 2, -3.$

$y_c = \underline{c_1 e^{2x}} + c_2 e^{-3x}$

Multiplique por x para evitar e^{2x} en y_p.

$$\boxed{y_p = Ax^2e^{2x} + Bxe^{2x}.}$$

$y = c_1 e^{2x} + c_2 e^{-3x} + Ax^2e^{2x} + Bxe^{2x}.$

b. $y'' + 2y' + y = 10e^{-x}$ Ae^{-x}

Soln. complementaria: $m^2 + 2m + 1 = 0.$
$(m+1)^2 = 0 \Rightarrow m = -1, -1$

$y_c = c_1 e^{-x} + c_2 x e^{-x}$

$y_p \neq Ae^{-x}$ $y_p \neq Axe^{-x}$ $\boxed{y_p = Ax^2e^{-x}}$

$y = c_1 e^{-x} + c_2 x e^{-x} + Ax^2 e^{-x}$

c. $y'' + 9y = 4\sin x + 2\cos 3x$

soln. complementaria: $m^2 + 9 = 0.$

$m = \pm 3i$

$y_c = c_1 \cos(3x) + c_2 \sin(3x).$

Inicial mente $A\sin x + B\cos x + \overbrace{\underbrace{C\sin 3x + D\cos 3x}_{\text{son parte de } y_c}}^{*\, x.}$

$$\boxed{y_p = A\sin x + B\cos x + Cx \sin 3x + Dx\cos 3x.}$$

d. $y^{(4)} + 8y'' + 16y = x + 4\sin 2x + x\sin 2x$

Soln complementaria: $m^4 + 8m^2 + 16 = (m^2+4)^2 = 0$

$m = \sqrt{-4} = \pm 2i, \pm 2i$

Raices complejas repetidas: $y_c = c_1 \sin 2x + c_2 \cos 2x + c_3 x \sin 2x + c_4 x\cos 2x$

Proponga $Ax + B + \underbrace{C\sin 2x + D\cos 2x + Ex\sin 2x + Fx\cos 2x}_{\text{son parte de } y_c.}$ $*\, x^2$

$$\boxed{\begin{aligned} y_p = {} & Ax + B. + Cx^2 \sin 2x + Dx^2 \cos 2x \\ & + Ex^3 \sin 2x + Fx^3 \cos 2x \end{aligned}}$$

e. $y^{(5)} + 4y^{(4)} = t^2 + e^{-4t}.$

$$m^5 + 4m^4 = m^4(m+4) = 0 \Rightarrow m_1 = -4$$
$$m_2 \ldots = 0, 0, 0, 0$$

$$y_c = c_1 + c_2 t + c_3 t^2 + c_4 t^3 + c_5 e^{-4t}.$$

Proponga $\underbrace{At^2 + Bt + C.}_{\text{es parte de } y_c} + \underbrace{De^{-4t}.}_{\text{es parte de } y_c.}$

$* t^4$ $\quad$ $* t.$

$$\boxed{y_p = At^6 + Bt^5 + Ct^4 + Dte^{-4t}.}$$

$e^x \cos x \rightarrow m = 1 \pm i, 1 \pm i$ $\quad$ $xe^x \cos x$

$$(m-1-i)(m-1+i) = m^2 - m + im$$
$$-m \quad +1 - i$$
$$-im \quad +i - i^2$$

$$(m-1-i)(m-1+i) = m^2 - 2m + 1 - (-1) = m^2 - 2m + 2.$$

$i = \sqrt{-1}$ $\quad$ $i^2 = -1$

$$m = \frac{2}{2} \pm \frac{1}{2}\sqrt{4-8} = 1 \pm i.$$

Raíces Repetidas

$$(m^2 - 2m + 2)^2 = 0.$$
$$m^4 + am^3 + bm^2 + cm + 4 = 0$$
$$\updownarrow$$
$$y^{(4)} + ay^{(3)} + by'' + cy' + 4y = 0.$$