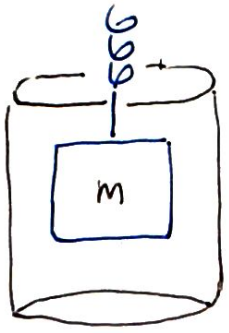
Movimiento Libre Amortiguado.



Fuerza de fricción $\beta y'$ β Kg/s.

ED Movimiento

$$m y'' + \beta y' + k y = 0$$

$$y'' + 2\left(\frac{\beta}{2m}\right) y' + \frac{k}{m} y = 0$$

λ $\qquad$ ω^2

$$\boxed{y'' + 2\lambda y + \omega^2 y = 0.}$$

$y = e^{rt}$

Ec. Auxiliar $\quad r^2 + 2\lambda r + \omega^2 = 0.$

$$r = -\lambda \pm \frac{\sqrt{4\lambda^2 - 4\omega^2}}{2} = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$$

$\lambda = \frac{\beta}{2m}$ amortiguamiento $\qquad$ $\omega^2 = \frac{k}{m}$ frec. circular,

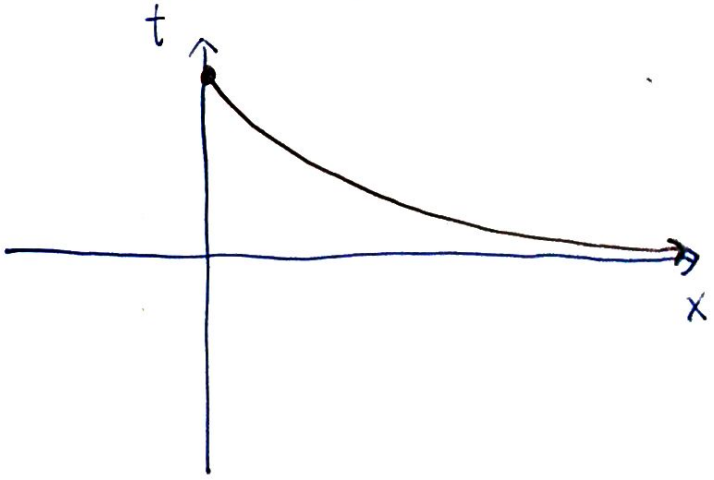
Casos de Movimiento:

Raíces distintas $(\lambda > \omega)$ Sobreamortiguado.

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$r_1, r_2 < 0$

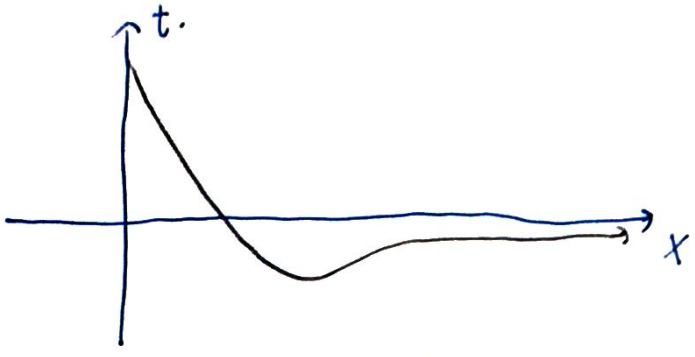
$y(t) \to 0 \quad t \to \infty.$



Raíces Repetidas: $\lambda = \omega$ Críticamente Amortiguado

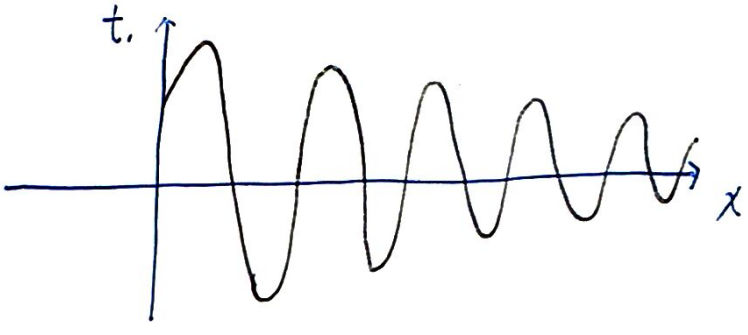
$r_1 = -\lambda$

$y = c_1 e^{-\lambda t} + c_2 t e^{-\lambda t}$



Raíces complejas: $\lambda < \omega$ Sub amortiguado.

$y = c_1 e^{-\lambda t} \cos(\beta t) + c_2 e^{-\lambda t} \sin(\beta t)$



Ejercicio 2: Encuentre la ec. de movimiento y clasifique el movimiento.

a. $y'' + 5y' + 4y = 0 \qquad y(0) = 1 \quad y'(0) = 2.$

$m = 1, \quad \kappa = 4, \quad \beta = 5.$

$m^2 + 5m + 4 = (m+4)(m+1) = 0 \Rightarrow m = -1, -4$

$y = c_1 e^{-t} + c_2 e^{-4t}$ Mov. Sobre Amortiguado.

$y(0) = c_1 + c_2 = 1$

$y'(0) = -c_1 - 4c_2 = 2.$

$y'(t) = -c_1 e^{-t} - 4c_2 e^{-4t}$

$-3c_2 = 3 \Rightarrow c_2 = -1$ y $c_1 = 1 - c_2 = 2.$

Ec. Movimiento: $\boxed{y = -e^{-t} + 2e^{-4t}}$ $\frac{N}{m^2}$ presión.

b. $0.25y'' + 4y' + 16y = 0. \quad y(0) = 5 \quad y'(0) = -5$

$$0.25m^2 + 4m + 16. = 0$$
$$m^2 + 16m + 64 = 0$$
$$(m+8)(m+8) = 0 \Rightarrow m = -8, -8.$$

Raíz Repetida

Críticamente Amortiguado:

$$y = c_1 e^{-8t} + c_2 t e^{-8t}$$

$$y'(t) = -8c_1 e^{-8t} + c_2 e^{-8t} - 8c_2 t e^{-8t}.$$

$$y(0) = c_1 + 0 = 5 \Rightarrow c_1 = 5$$
$$y'(0) = -8c_1 + c_2 = -5 \Rightarrow c_2 = -5 + 8c_1 = 35$$

Ec. Movimiento: $y(t) = 5e^{-8t} + 35te^{-8t}$

c. $y'' + 2y' + 10y = 0 \quad y(0) = 2 \quad y'(0) = 1$

$$m^2 + 2m + 10 = 0 \qquad m = \frac{-2}{2} + \frac{1}{2}\sqrt{4 - 40}$$
$$m = -1 + \frac{1}{2}\sqrt{-36} = -1 \pm \frac{1}{2}6i = -1 \pm 3i$$

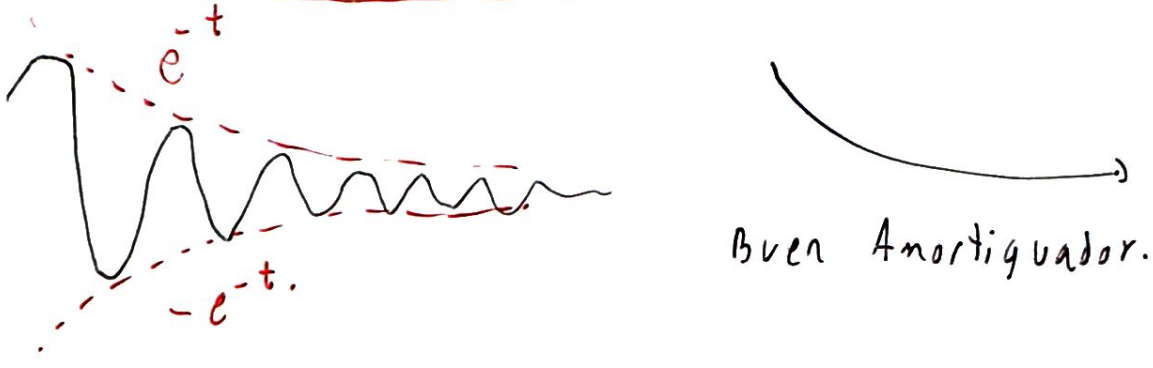
$$y(t) = c_1 e^{-t}\cos 3t + c_2 e^{-t}\sin 3t$$ SubAmortiguado.

$$y'(t) = -c_1 e^{-t}\cos 3t - 3c_1 e^{-t}\sin 3t.$$
$$-c_2 e^{-t}\sin 3t + 3c_2 e^{-t}\cos 3t.$$

$y(0) = c_1 + 0 = 2 \Rightarrow c_1 = 2$

$y'(0) = -c_1 + 3c_2 = 1 \qquad 3c_2 = 1 + c_1 = 3 \Rightarrow c_2 = 1.$

Ec. Movimiento: $\boxed{y(t) = 2e^{-t}\cos 3t + e^{-t}\sin 3t.}$



Corto 9 Lunes 90 mins.

Miércoles Resolución

Viernes 11 AM.

1. EDs Lineales
2. Coeficientes Ind.
3. Variación de Parámetros $f(x) = \tan x, \ln x$
4. ED Cauchy - Euler