Parcial 1: Viernes 11:00 AM

Resolución Corto 5

1. $x y' - y = x^2 \sin x$ ED lineal.

$y' - \frac{1}{x} y = x \sin x$ ED lineal estándar.

$e^{\int p\,dx} = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}.$

Multiplique la ED por el FI.

$\frac{1}{x} y' - \frac{1}{x^2} y = \sin x$

$\left(\frac{y}{x}\right)' = \sin x \rightarrow \frac{y}{x} = c - \cos x$

$\boxed{y = cx - x\cos x}$

2. $\underbrace{\left(x^2 + \frac{2y}{x}\right)}_{M} dx + \underbrace{\left(2\ln x - \frac{1}{\sqrt{y}}\right)}_{N.} dy = 0.$

$M_y = \frac{2}{x}$ $N_x = \frac{2}{x}$ ED exacta.

$\frac{\partial F}{\partial x} = x^2 + \frac{2y}{x}$ $\frac{\partial F}{\partial y} = 2\ln x - \frac{1}{\sqrt{y}}$

$F = \frac{1}{3} x^3 + 2y\ln x + A(y)$

$F_y = 2\ln x + A'(y) = 2\ln x - y^{-1/2}.$

$A'(y) = -y^{-1/2} \Rightarrow A(y) = -2y^{1/2}.$

$$\boxed{\frac{1}{3}x^3 + 2y\ln x - 2\sqrt{y} = C.}$$

3 $(\underbrace{-2xy\sin x + 2y\cos x}_{M})dx + \underbrace{2x\cos x}_{N}dy = 0.$

$M_y = -2x\sin x + 2\cos x$
$N_x = 2\cos x - 2x\sin x$ } iguales. ED Exacta.

(1) $\frac{\partial F}{\partial x} = -2xy\sin x + 2y\cos x$ (2) $\frac{\partial F}{\partial y} = 2x\cos x.$

IPP

Integre (2) $F = 2yx\cos x + A(x)$

$F_x = 2y\cos x - 2yx\sin x + A'(x) = -2xy\sin x + 2y\cos x$

$A'(x) = 0 \Rightarrow A(x) = 0.$

$$\boxed{2yx\cos x = C.} \Rightarrow y = \frac{C}{2x\cos x}$$

Integre (1) $F = -2y\int x\sin x\,dx + 2y\sin x + A(y)$

x	+	$\sin x$
1	−	$-\cos x$
0		$-\sin x$

$F = 2yx\cos x - 2y\sin x + 2y\sin x + A(y)$

$F = 2yx\cos x + A(y)$

4. $(y^2 + yx)\,dx + x^2\,dy = 0.$

EO homogénea de grado 2.

$y = vx \qquad dy = v\,dx + x\,dv.$

$(v^2x^2 + vx^2)\,dx + vx^2dx + x^3dv = 0$

$(v^2x^2 + 2vx^2)\,dx = -x^3\,dv.$

$x^2(v^2+2v)\,dx = -x^3\,dv \qquad w = 1 + \frac{2}{v}$

$v^2\left(1+\frac{2}{v}\right) \qquad dw = -\frac{2}{v^2}$

$-\frac{dx}{x} = \frac{dv}{v^2+2v+1-1}$

$\frac{A}{v} + \frac{B}{v+2} = \frac{1}{v(v+2)}$ fracciones Parciales

$A(v+2) + Bv = 1$

$v=0: \quad 2A = 1 \qquad v=-2: \quad -2B = 1$

Integre la EO Separable.

$-\int\frac{dx}{x} = \frac{1}{2}\int\frac{dv}{v} - \frac{1}{2}\int\frac{1}{v+2}.$

$-\ln x = \frac{1}{2}\ln v - \frac{1}{2}\ln(v+2) + C \qquad v = y/x$

$$\boxed{-2\ln x = \ln\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x}+2\right) + C}$$

$\ln\left(\frac{y/x}{y/x+2}\right)$

5. $\frac{dV}{dt} = rV + A(t) \qquad V(0) = V_0$

ED lineal, no separable. $\qquad B = 1000, \ r = 0.2$

$V_0 = 20{,}000$

$A(t) = Bt$

$V' - 0.2V = 1000t \quad , \qquad V(0) = 20{,}000$

F.I. $\quad e^{-0.2\int dt} = e^{-0.2t}.$

$e^{-0.2t} V' - 0.2 e^{-0.2t} V = 1000 t e^{-0.2t}.$

$(e^{-0.2t} V)' = 1000 t e^{-0.2t}.$

$e^{-0.2t} V = 1000 \int t e^{-0.2t} dt$

$$\begin{array}{ll} t & e^{-0.2t} \\ 1 & -5e^{-0.2t} \\ 0 & 25e^{-0.2t}. \end{array}$$

$e^{-0.2t} V = 1000(-5te^{-0.2t} - 25e^{-0.2t}) + C.$

Multiplique por $e^{0.2t}.$

$V = -5000t - 25{,}000 + Ce^{0.2t}.$

Use $V(0) = 20{,}000$ Para encontrar C.

$V(0) = 20{,}000 = -25{,}000 + C.$

$C = 45{,}000$

Valor fondo de Amortización: $\boxed{V(t) = 45{,}000 e^{0.2t} - 5000t - 25{,}000.}$

$V(6) = 94{,}405.26.$

¿6. $\dfrac{dP}{dt} = P(10-P)$ $\qquad P(0) = 5$

ED Separable, No lineal.

$$\frac{dP}{P(10-P)} = dt. \qquad \frac{1}{P^2\left(\frac{10}{P}-1\right)} \qquad u = \frac{10}{P} - 1$$

$$\frac{A}{P} + \frac{B}{10-P} = \frac{1}{P(10-P)} = \frac{-1}{P(P-10)}$$

$$A(10-P) + BP = 1$$

$P = 0$: $10A = 1$ $\qquad P = 10$: $10B = 1.$

$$\frac{1}{10}\int \frac{dP}{P} + \frac{1}{10}\int \frac{dP}{10-P} = \int dt.$$

$$\frac{1}{10}\ln P - \frac{1}{10}\ln(10-P) = t + C.$$

$$\ln\left(\frac{P}{10-P}\right) = 10t + 10C. \qquad C_1 = e^{10C}$$

$$\frac{P}{10-P} = e^{10t+10C} = C_1 e^{10t}.$$

Use $P(0) = 5$ para encontrar C_1

$$\frac{5}{5} = C_1 e^0 \Rightarrow C_1 = 1.$$

Resuelva para y: $\dfrac{P}{10-P} = e^{10t}.$

$$P = 10e^{10t} - Pe^{10t}$$

$$P(1+e^{10t}) = 10e^{10t} \Rightarrow \boxed{P(t) = \frac{10e^{10t}}{1+e^{10t}}.}$$

Problema 4 Tarea 5.

$$\frac{dP}{dt} = P(a - bP) - h \qquad P(0) = 2.5$$

$$a = 5, \quad b = 1, \quad h = 6.$$

a. Resuelva la ED.

$$P(5-P) - 6 = 5P - P^2 - 6 = -(P^2 - 5P + 6)$$
$$-(P-3)(P-2).$$

$$\frac{dP}{(P-3)(P-2)} = -dt.$$

$$\frac{A}{P-3} + \frac{B}{P-2} = \frac{1}{(P-3)(P-2)}$$

$$A(P-2) + B(P-3) = 1$$

$P = 3$: $A = 1$

$P = 2$: $-B = 1$

$$\int \frac{1}{P-3}\, dP + \int \frac{1}{P-2}\, dP = \int -dt.$$

$$\ln(P-3) - \ln(P-2) = -t + C.$$

$$\ln\left(\frac{P-3}{P-2}\right) = -t + C \Rightarrow \frac{P-3}{P-2} = C_1 e^{-t}.$$

Use $P(0) = 2.5$ $\quad \frac{-0.5}{0.5} = C_1 e^0 \Rightarrow C_1 = -1$

$$\frac{P-3}{P-2} = -e^{-t} \Rightarrow P - 3 = -Pe^{-t} + 2e^{-t}.$$

$$P(1+e^{-t}) = 3 + 2e^{-t}$$

$$\boxed{P(t) = \frac{3+2e^{-t}}{1+e^{-t}}}$$

$$e^{+\infty} \to +\infty$$
$$e^{-\infty} \to 0.$$

b. Asíntotas Horizontales. $\lim_{t\to\infty} P(t)$, $\lim_{t\to-\infty} P(t)$

$$\lim_{t\to\infty} \frac{3+2e^{-t}}{1+e^{-t}} = \frac{3+0}{1+0} = 3.$$

$$\lim_{t\to-\infty} \frac{3+2e^{-t}}{1+e^{-t}} \underset{LH}{=} \lim_{t\to-\infty} \frac{-2\cancel{e^{-t}}}{-\cancel{e^{-t}}} = \frac{-2}{-1} = 2.$$

c. Gráfica de la solución.