Notas de clase Cálculo integral
UFM
SECCIÓN B

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Capítulo 1

La integral indefinida, notación de integral, Teorema de evalucación de integrales

5.4 La Integral Indefinida. (Pag 9)

Una antiderivada de f. es una función $F(x)$

$$F'(x) = f(x)$$

Por ejemplo, encuentre la antiderivada de $f(x) = 14x^6$.

$F(x) = 2 \cdot x^7$ $\qquad F'(x) = 14x^6 = f(x)$

$F(x) = 2 \cdot x^7 + \sqrt{10}$

$F(x) = 2 \cdot x^7 - 10^{20} + \ln(10)$

Antiderivada más general

$F(x) = 2x^7 + C$

La Integral Indefinida de $f(x)$ respecto a x, es la antiderivada más general de f.

$$\int f(x)\,dx = F(x) + C.$$

C es una constante de integración.

$\int (\ )\,dx$ integre la función es antiderivar.

signa $\int$ dx diferencial, integre respecto a x.

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

$u = g(x) \quad du = g'(x)\,dx$

$$\int 14x^6\,dx = 2x^7 + C.$$

Reglas de Integración Básicas.

$\frac{d}{dx}(\ln(-x)) = \frac{1}{x}$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

$n \neq -1$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

valor absoluto.

$$\int e^x \, dx = e^x + C.$$

$$\int a^x dx = \frac{a^x}{\ln a} + C.$$

0	$\sin x$	$\cos x$	
	$\cos x$	$-\sin x$	$\int dx$
$+C$	$\tan x$	$\sec^2 x$	
	$\cot x$	$-\csc^2 x$.	

$$\int \cos x \, dx = \sin x + C.$$

$$\int \sin x \, dx = -\cos x + C.$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C.$$

$$\int \sec x \tan x \, dx = \sec x + C.$$

$$\int \csc x \cot x \, dx = -\csc x + C.$$

Comprueba

Julio: $\int \tan x \, dx = \ln|\sec x| + C.$ $\quad \frac{\sec x \tan x}{\sec x} + 0.$

$$\int \cot x \, dx \qquad \int \sec x \, dx \qquad \int \csc x \, dx.$$

$$\int \frac{\cos x}{\sin x} dx$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C. \qquad \int \frac{1}{1+x^2} dx = \tan^{-1} x + C.$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C. \qquad \int \sinh x \, dx = \cosh x + C.$$

Suma/Diferencia $\int f(x) \pm g(x) \, dx = \int f(x) dx \pm \int g(x) dx$

Múltiplo Constante $\int a f(x) \, dx = a \int f(x) dx$

Reglas de Integración Básicas.

$\frac{d}{dx}(\ln(-x)) = \frac{1}{x}$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

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$$\int \csc^2 x \, dx = -\cot x + C.$$

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Comprueba

Julio: $\int \tan x \, dx = \ln|\sec x| + C.$ $\quad \frac{\sec x \tan x}{\sec x} + 0.$

$$\int \cot x \, dx \qquad \int \sec x \, dx \qquad \int \csc x \, dx.$$

$$\int \frac{\cos x}{\sin x} dx$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C. \qquad \int \frac{1}{1+x^2} dx = \tan^{-1} x + C.$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C. \qquad \int \sinh x \, dx = \cosh x + C.$$

Suma/Diferencia $\int f(x) \pm g(x) \, dx = \int f(x) dx \pm \int g(x) dx$

Múltiplo Constante $\int a f(x) \, dx = a \int f(x) dx$

Ejemplos pág 11.

a) $\int x^{50} + 2x^6 \, dx = \frac{x^{51}}{51} + \frac{2}{7} x^7 + C.$

b) $\int \frac{1}{1+x^2} + \frac{1}{x} + \frac{1}{x^2} \, dx = \tan^{-1} x + \ln|x| + x^{-1} + C.$

x^{-2}

c) $\int \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[5]{x^3}} \, dx$

$\int x^{1/2} + x^{-1/2} + x^{-3/5} \, dx = \frac{2}{3} x^{3/2} + 2x^{1/2} + \frac{5}{2} x^{2/5} + C.$

d) $\int x^{\ln 2} + x^{\sqrt{2}} + x^{\sin(2)} \, dx = c + \frac{x^{1+\ln 2}}{1+\ln 2} + \frac{x^{1+\sqrt{2}}}{1+\sqrt{2}} + \frac{x^{1+\sin(2)}}{1+\sin(2)}$

potencia

Ejercicio 1: Evalúe las sigs. integrales.

a) $\int x^e + e^x \, dx. = \frac{x^{e+1}}{e+1} + e^x + C.$

potencia exponencial

b) $\int \left(8 \cdot 10^x - \frac{2}{x}\right) dx = \frac{8 \cdot 10^x}{\ln(10)} - 2\ln|x| + C.$

Allgebra.

c) $\int (x-2)(x+2)(x^2+4) \, dx = \int (x^2-4)(x^2+4) \, dx$

$\int (x^4 - 16) \, dx = \frac{1}{5} x^5 - 16x + C.$

d) $\int e^{-4x}(e^{4x} + e^{5x}) \, dx = \int (1 + e^x) \, dx = x + e^x + C.$

Integrales Definidas

son integrales con límites de integración en x y $x = b$.

$$\int_a^b f(x)\,dx \qquad \int f(x)\,dx = F(x)$$

Teorema de Evaluación (TFC parte 2).

Si $f(x)$ es <u>continua</u> en $[a,b]$ entonces

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

utilizando la notación de corchete.

$$\int_a^b f(x)\,dx = F(x)\Big]_{x=a}^{x=b} \quad \text{luego evalúe.}$$

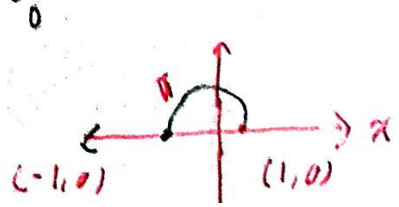
primero antiderive

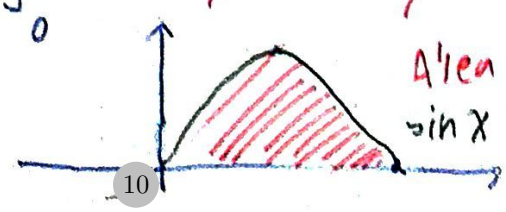
$$F(x) + C\Big]_{x=a}^{x=b} = F(b) + C - (F(a) + C) = F(b) - F(a)$$

Función es Integrable si $\int_a^b (x)\,dx$ existe.

Ejercicio 1: Evalúe las sig. integrales.

0. $\int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi} = -\cos\pi + \cos 0 = 1 + 1 = 2.$





Área bajo la curva.

5.

a. $\int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 = \frac{27}{3} - 0 = 9.$

b. $\int_9^{36} \underbrace{\sqrt{x}}_{x^{1/2}}\,dx = \frac{2}{3}x^{3/2}\Big]_9^{36.} = \frac{2}{3}\left((6^2)^{3/2} - \overbrace{9^{3/2}}^{3^3}\right)$

$= \frac{2}{3}(216-27) = 144 - 18$

$= 126.$

c. $\int_0^2 \frac{1}{1-x^2}\,dx$ no existe discontinua en $[0,2]$

se indefine en -1 y 1.

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C.$ $\quad \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$

d) $\int_1^4 \left(\underbrace{\frac{1}{\sqrt{x}}}_{x^{-1/2}} + 3\underbrace{\sqrt{x}}_{x^{1/2}}\right)dx = 2\cdot x^{1/2} + \frac{3\cdot 2}{3}x^{3/2}\Big]_1^4$

$= 2\sqrt{4} + 2(2^2)^{3/2} - (2\cdot 1^{1/2} + 2\cdot 1^{3/2})$

$= 4 + 16 - (2+2) = 16.$

3. $\frac{x^{3/2} * 2/3}{3/2 * 2/3} = \frac{3\cdot 2}{3}x^{3/2} = 2x^{3/2}.$ $\quad \int (\quad)\,dx.$

$\frac{d}{dx}(\quad)$

Capítulo 2

Áreas y propiedades de la integral definida

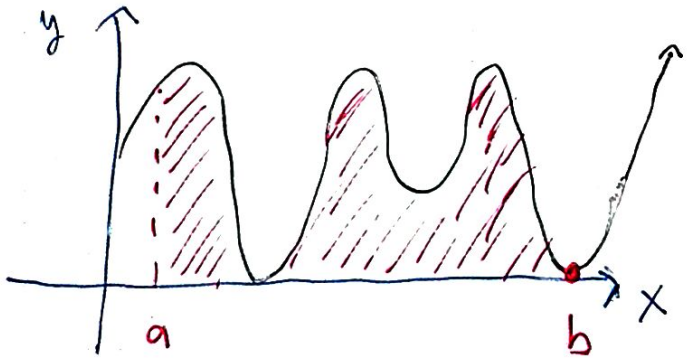
5.4 Área. y Propiedades. Integral Definida.

El Área de una región $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

Integral definida de f en [a,b] f es continua $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

Interpretación Integral Definida.

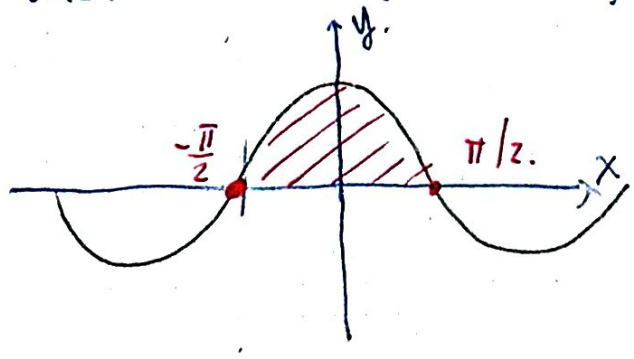
El área de la región bajo la curva $y = f(x)$, encima del eje-x, y entre las rectas verticales $x = a$ y $x = b$. es la integral definida de f en [a,b] $(f \geq 0)$



$$A = \int_a^b f(x)\,dx.$$

$$F(b) - F(a)$$

Considere la región debajo de $y = \cos x$ en $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



$$A = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2.$$

$$A = \sin x \Big]_{-\pi/2}^{\pi/2} = \cancel{\sin\frac{\pi}{2}}^{\,1} - (-1)$$

$$= 1 + 1 = 2.$$

$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_{0}^{3} = -0-(-(-2)^2)+9-0$$
$$= 0+4+9 = 13.$$

Regla Integral Definida. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

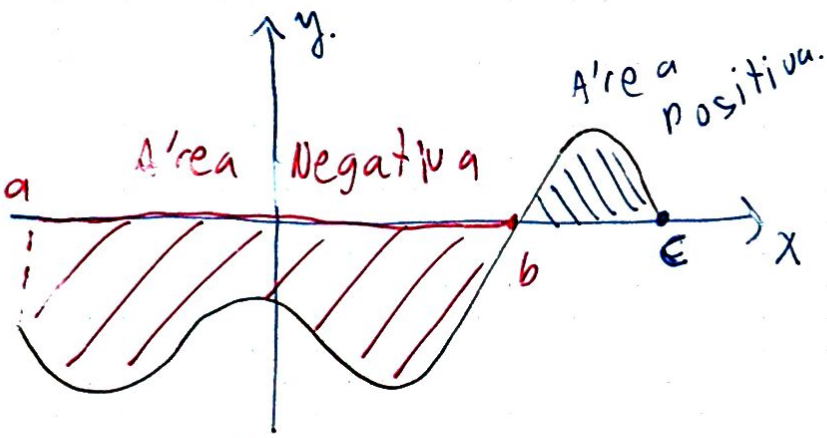
Invertir el orden

Ej: $\int_{-2}^{0} -2x\,dx = \int_0^{-2} 2x\,dx = x^2\Big]_0^{-2} = 4-0 = 4$

b) $\int_0^{\pi} \sin x\,dx = -\int_{\pi}^{0} \sin x\,dx = \cos x\Big]_{\pi}^{0} = 1-(-1) = 2.$

ó $-\cos x\Big]_0^{\pi} = -\underbrace{\cos(\pi)}_{-1} - (-\cos 0) = 1+1 = 2.$

¿Qué sucede cuando $f(x)$ es negativa?



Área de la región entre $f(x)$ & $y=0$

$$\int_{-1}^{0} x^3\,dx = \frac{x^4}{4}\Big]_{-1}^{0}$$
$$= 0 - \frac{1}{4} = -\frac{1}{4}.$$

$$A \neq \int_a^c f(x)\,dx$$

$$A = -\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$ ✓ valor absoluta

Definición Más compacta $A = \int_a^b |f(x)|\,dx$

Ejercicio 3: (Pág 16) Considere $f(x) = 4x^3 - 4$ en $-2 \leq x \leq 2$.

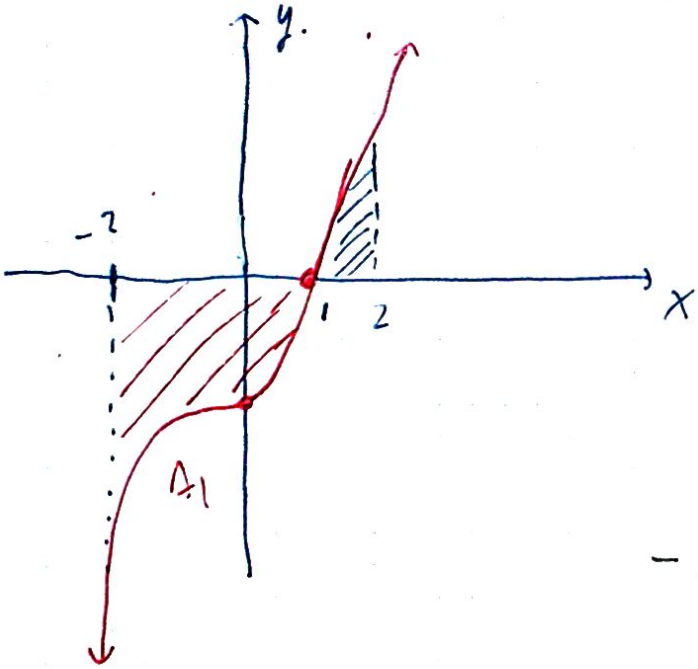
a. Evalúe $\int_{-2}^{2} (4x^3 - 4)\,dx = x^4 - 4x\Big]_{-2}^{2}$

$x^4\Big]_{-2}^{2} - 4x\Big]_{-2}^{2}$

$= (16-8) - (16+8)$

$= 8 - 24 = -16.$

b. Bosqueje la región y explique si la integral definida es igual al área de la región.



Intercepto-x: $4(x^3 - 1) = 0$

$x = 1.$

Intercepto-y: $0 - 4 = -4$

$$A \neq \int_{-2}^{2} f(x)\,dx$$

$-f(x) = -4x^3 + 4$

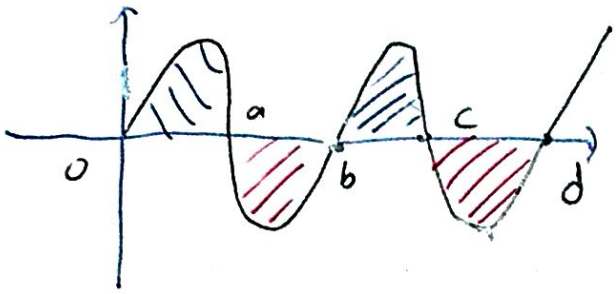
c. Encuentre el área de la región.

$$A = \int_{-2}^{1} \overbrace{(4 - 4x^3)}^{A_1}\,dx + \int_{1}^{2} (4x^3 - 4)\,dx.$$

$$A = 4x - x^4\Big]_{-2}^{1} + x^4 - 4x\Big]_{1}^{2} = \overbrace{(4-1) - (-8-16)}^{A_1}$$

$+(16-8) - (1-4)$

$A = 3 + 24 + 8 + 3. = 27 + 11 = 38.$



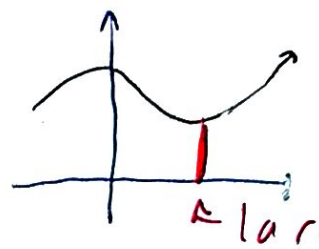
$$A = \int_0^a f\,dx - \int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx.$$

Propiedades Integrales Definidas.

1 y 2. $\int_a^b k_1 f(x) \pm k_2 g(x)\,dx = k_1 \int_a^b f(x)\,dx \pm k_2 \int_a^b g\,dx.$

$$\int_{\sqrt{2}}^{\sqrt{2}} e^{x^2 + \ln x + \sinh x}\,dx. = F(\sqrt{2}) - F(\sqrt{2}) = 0$$

3. $\int_a^a f(x)\,dx. = 0$



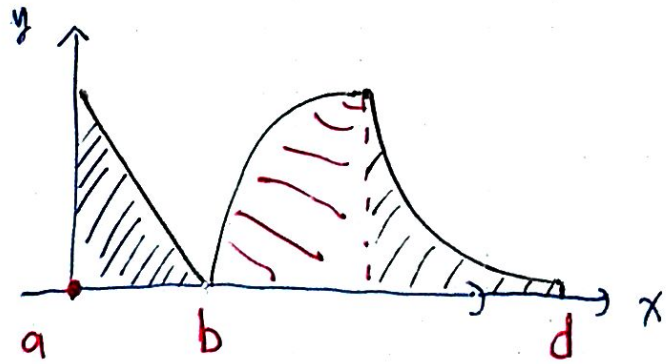
la región es sólo una línea.

4. $\int_a^b h\,dx = hx\Big]_a^b = h(b-a)$ rectángulo altura h largo b-a.

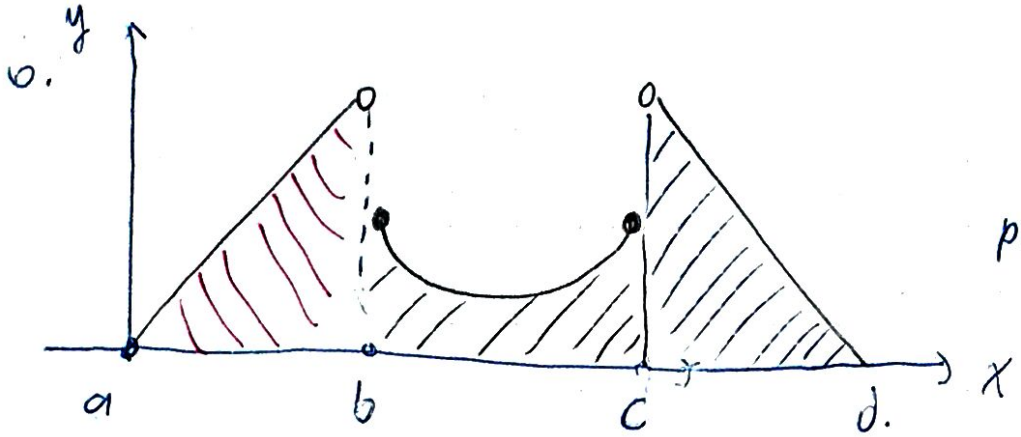
$$\int_e^{\sqrt{10}} \ln(10)\,dx = \ln(10)\left[\sqrt{10} - e\right]$$

$\int \ln x\,dx$

5.



$$\int_a^d f(x)\,dx = \int_a^b f\,dx. + \int_b^c f\,dx + \int_c^d f\,dx$$



continua por tramos

piecewise continuous.

$$\int_a^d f\,dx = \int_a^b f\,dx + \int_b^c f\,dx + \int_c^d f\,dx$$

$$2x\Big]_0^1 = 2-0$$

Ejercicio 5: Evalúe la sig. integral definida.

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2 & \text{si} \quad 0 \leq x \leq 1 \\ 4-2x & \text{si} \quad 1 \leq x \leq 2 \\ 6x-12 & \text{si} \quad 2 \leq x \leq 3 \end{cases}$$

$$\int_0^3 f\,dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx$$

$$= 2 + \left(4x - x^2\Big]_1^2\right) + \left(3x^2 - 12x\Big]_2^3\right)$$

$$= 2 + (4-3) + (-9-(-12))$$

$$= 2 + 1 + 3 = 6.$$

Derivación logarítmica, $y = a^x$.

$y = \dfrac{a^x}{\ln a}$, $y' = a^x \dfrac{\cancel{\ln a}}{\cancel{\ln a}}$, $\ln y = x \ln a$.

$\dfrac{y'}{y} = \ln a. \Rightarrow y' = a^x \ln a.$

Capítulo 3

5.4 Desplazamiento y distancias con integrales

5.4 Desplazamiento y Distancia

La integral de la derivada $f'(x)$ es la función original.

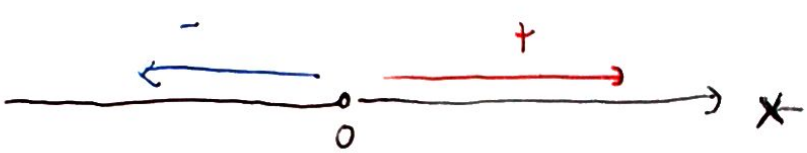
$$\int_a^b f'(x)\,dx = f(x)\Big]_a^b = \underbrace{f(b) - f(a)}_{\text{cambio neto.}}$$

Si se conoce la razón de cambio de una función, el cambio neto se obtiene integrando la razón de cambio.

Desplazamiento en 1-Dimensión

$$s = \int_a^b v(t)\,dt \quad ✓$$

$$\int_a^b s'(t)\,dt.$$



Costo Marginal: $C'(x)$ costo neto $= \int_a^b C'(x)\,dx.$

Población: población neta $= \int_a^b p'(t)\,dt.$

Ejemplo: Una partícula tiene una velocidad de $v(t) = \frac{2}{t^{4/3}}$ cm/s.

Encuentre el desplazamiento entre $t=1$ y 8 s. $2 \cdot t^{-4/3}$

$$s = \int_1^8 v(t)\,dt = 2\int_1^8 t^{-4/3}\,dt. = -6\,t^{-1/3}\Big]_1^8 = 6t^{-1/3}\Big]_8^1$$

$1 - \frac{1}{2}$ $\quad -\frac{1}{2} + 1$

$$s = 6\left(\frac{1}{\sqrt[3]{1}} - \frac{1}{\sqrt[3]{8}}\right) = 6\left(\frac{1}{2}\right) = 3. \quad ó \quad 6\left(\frac{-1}{\sqrt[3]{8}} - \frac{(-1)}{\sqrt[3]{1}}\right)$$

Desplazamiento Neto es de 3 cm.

Ejercicio 1: Se lanza una pelota con una velocidad inicial de 64 pies/s, a nivel del suelo. Encuentre el desplazamiento de la pelota entre 1 y 3 s.

$v(t) = 64 - 32t.$ ✓ $\qquad g = -32$ pies/s²

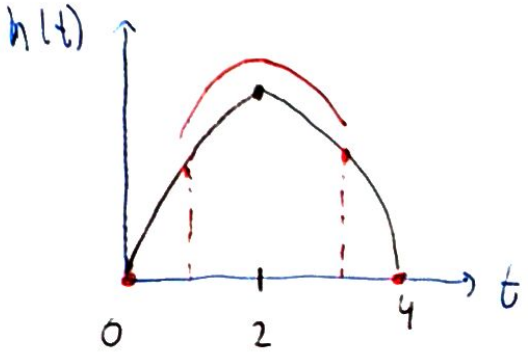
Respuesta:

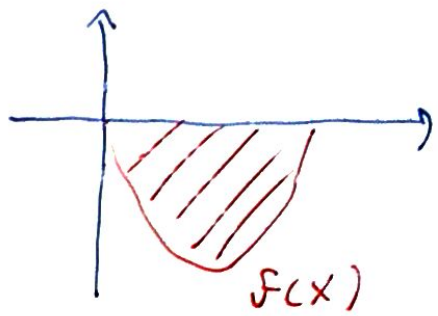
$$s = \int_1^3 v(t)\,dt. = \int_1^3 (64 - 32t)\,dt. = 64t\Big]_1^3 - 16t^2\Big]_1^3$$

$$s = 64(3-1) - 16(9-1) = 128 - 128 = 0$$

$$64 \cdot 2 \qquad 16 \cdot 4 \cdot 2 \qquad 48 - 48 = 0$$

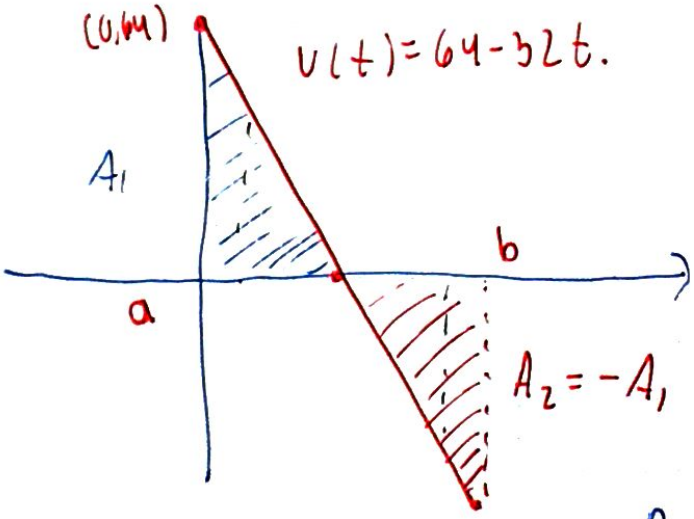
NO HAY CAMBIO NETO.





$$A = -\int_a^b f(x)\,dx$$

Distancia y Desplazamiento en una dimensión.



$-, 0, +$

Desplazamiento $s = \int_a^b v(t)\,dt.$

Para el ejercicio $s = 0.$

Distancia $d = \int_a^b |v(t)|\,dt.$

0 ó +.

Rapidez: $|v(t)|.$

Para este diagrama.

Desplazamiento $s = A_1 + A_2 \approx 0$ A_2 es negativo.

Distancia: $s = A_1 - A_2$.

Para el ejercicio 1j encuentre la distancia recorrida por la pelota entre $t = 1$ y 3 s.

$$d = \int_1^3 |v(t)|\,dt = \int_1^3 |\underbrace{64-32t}_{=0}|\,dt.$$

$v(t) = 0$ cuando $t = 2$.

$64-32t$ $+$ $\overset{2}{\vdots}$ $-$

$$d = \int_1^2 v(t)\,dt. - \int_2^3 v(t)\,dt$$

$64-32t = 0$

$64 = 32t$

$2. = t$

$$d = \int_1^2 64-32t\,dt + \int_2^3 32t-64\,dt.$$

$$d = 64t - 16t^2\Big]_1^2 + 16t^2 - 64t\Big]_2^3$$

$$d = 128 - 64 - (64-16) + 16(9-4) - 64(3-2)$$

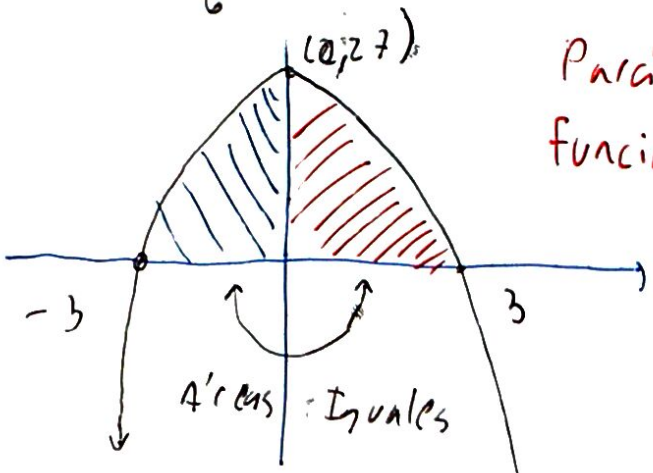
$$d = 64 - 48 + 80 - 64$$

$$d = 16 + 16 = 32 \text{ pies.}$$

Ejercicio 2: Un vehículo da vueltas en un circuito. una velocidad $v(t) = 27 - 3t^2$ millas/hora.

a. Plantee la integral para encontrar el desplazamiento del vehículo entre -6 y 6 horas.

$$s = \int_{-6}^{6} (27 - 3t^2)\,dt. = 2\int_{0}^{6} (27 - 3t^2)\,dt.$$



Parábola
función par

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

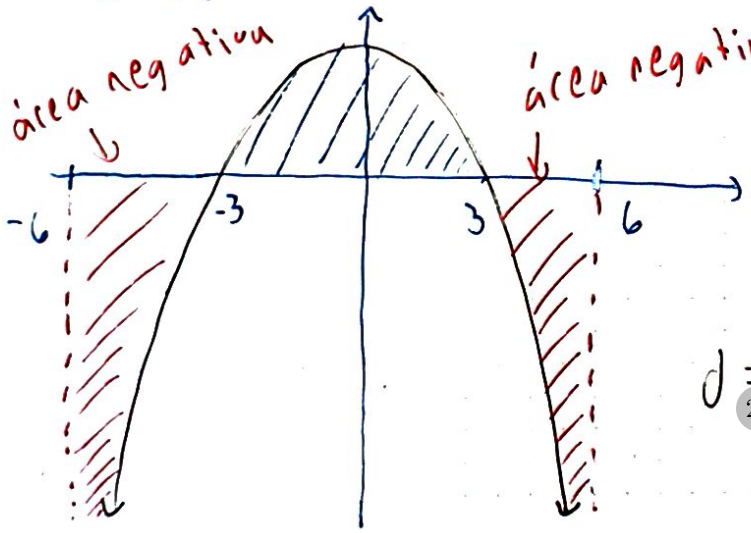
$$f(-x) = f(x)$$

$$s = 2\left(27t - t^3\Big]_0^6\right) = 2\left(\overset{27\cdot 3\cdot 2}{27\cdot 6} - 6^3\right) = 2(162 - 216)$$

$$2(-54) = -108.$$

millas.

b. Plantee la integral para encontrar la DISTANCIA. del vehículo entre -6 y 6 horas.



$$d = -A_1 + A_2 - A_3.$$

$$d = \int_{-6}^{6} |v(t)|\,dt.$$

$$d = -\int_{-6}^{-3} v(t)\,dt$$

$$d = -\int_{-6}^{-3}(27-3t^2)\,dt + \int_{-3}^{3}(27-3t^2)\,dt + \int_{3}^{6}\underbrace{(3t^2-27)}_{v(t)}\,dt$$

OLVIDARLO: $d = 2\int_0^3 v(t)\,dt - 2\int_3^6 v(t)\,dt.$

Integrales Indefinidas: desplazamiento, velocidad y aceleración.

$\int f(t)\,dt = F(t) + C$, función.

Dada la aceleración del objeto. $a(t) = v'(t)$.

Velocidad: $v(t) = \int a(t)\,dt + C_1$

Velocidad inicial: $v(0) = v_0$. reposo $v(0) = 0$.

Posición: $s(t) = \int v(t)\,dt + C_2$.

Posición inicial: $s(0) = s_0$. posición equilibrio $s(0) = 0$

Ejercicio 3: Un cohete despeja con una aceleración vertical de $a(t) = t^2\left(\frac{72}{t} - 36\right)$ ft/s².

La posición inicial es 0 pies snm y la velocidad inicial es de 400 ft/s.

sobre el nivel del mar.

a. Encuentre la posición vertical del cohete.

$$a(t) = 72t - 36t^2.$$

Velocidad: $v(t) = \int (72t - 36t^2)\,dt.$

$v(t) = 36t^2 - 12t^3 + C_1$

Use $v(0) = 400$: $v(0) = \boxed{C_1 = 400}$

Posición: $s(t) = \int v(t)\,dt = 12t^3 - 3t^4 + 400t + C_2.$

Use $s(0) = 0$: $s(0) = 0 + 0 + 0 + \boxed{C_2 = 0}$

Posición vertical es $\boxed{s(t) = 12t^3 - 3t^4 + 400t.}$

b. ¿Cuál es la rapidez y la velocidad a los $t = 10s$?

$v(10) = 36(100) - 12(1000) + 400 = 4{,}000 - 12{,}000$
$-8{,}000$ pies/s.

rapidez $|v(10)| = 8{,}000$ pies/s.

Ejercicio 4: Un resorte inicialmente en reposo y en su punto de equilibrio tiene una aceleración de

$a(t) = 4\cos t - 3\sin t.$ ✓

Encuentre la velocidad y posición del resorte.

$v(t) = \int (4\cos t - 3\sin t)\,dt = 4\sin t + 3\cos t + C_1$

$v(0) = 0 + 3 + C_1 = 0 \Rightarrow C_1 = -3.$

$v(t) = 4\sin t + 3\cos t - 3.$

: Desplazamiento

$$s(t) = \int (4\sin t + 3\cos t - 3)\,dt. \qquad s(0)=0$$

$$s(t) = -4\cos t + 3\sin t - 3t + C_2.$$

$$s(0) = -4 + 0 + 0 + C_2 = 0 \Rightarrow C_2 = 4.$$

$$s(t) = -4\cos t + 3\sin t - 3t + 4.$$

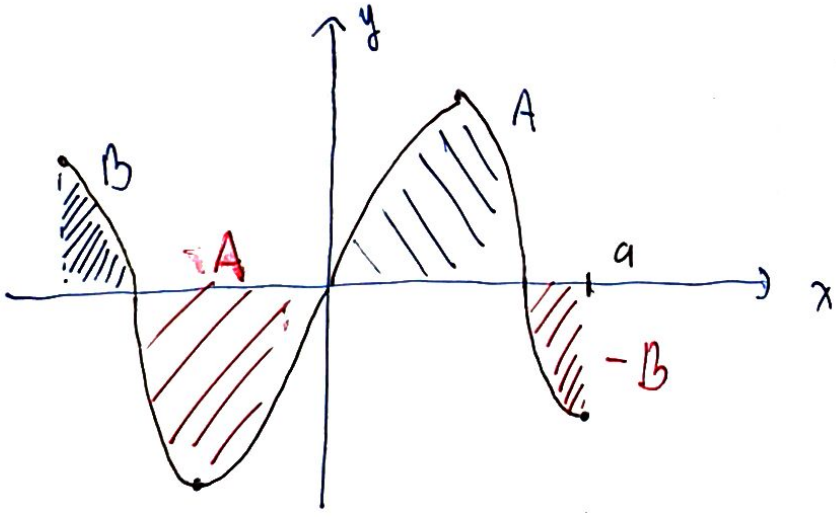
Función Pares e Impares

$$\int_{-100}^{100} (\sin x + x^3 + \tanh x)\,dx = 0.$$

Tres funciones son impares

$\sin(-x) = -\sin x$

$(-x)^3 = -x^3.$



Las áreas se cancelan entre sí.

$$\int_{-a}^{a} f_{impar}(x)\,dx = 0.$$

$$\int_0^1 e^x\,dx = e-1$$

$$\int_{-a}^{a} f_{par}(x)\,dx = 2\int_0^a f_{par}(x)\,dx$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-x^2}\,dx = 1$$

Capítulo 4

Teorema fundamental del cálculo, parte I & II, derivadas de funciones compuestas y definidas para integrales, generalizaciones de TFC

5.3 Teorema Fundamental del Cálculo.

si $f(x)$ es continua en $[a, x]$ entonces

PARTE 1: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ $\qquad \int_a^b$ $\qquad \int$

Integral y la derivada se cancelan entre sí.

Como la derivada de una antiderivada es la función original, entonces la $\int_a^x f(t)\,dt$ es la antiderivada de $f(x)$

variable temporal de integración.

se integra primero respecto y se deriva respecto a x.

$$f(x) = \int_{10}^{x} 4t^3\,dt = t^4\Big]_{t=10}^{t=x} = x^4 - 10^4.$$

$$y(t) = \int_0^t \sin(w)\,dw$$

$$f'(x) = 4x^3$$

$t \to x$

$$f'(x) = \frac{d}{dx}\int_{10}^{x} 4t^3\,dt = 4x^3$$

ATAJO.

TFC parte 2: $\int_a^b f(x)\,dx = F(x)\Big]_{x=a}^{x=b} = F(b) - F(a)$

$$\int_a^b f(w)\,dw = F(w)\Big]_{w=a}^{w=b} = F(b) - F(a)$$

Se pueden definir funciones por medio de integrales

Distribución normal $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$$P(x) = \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt.$$

no se puede integrar de manera explícita.

$$\int e^t\, dt = e^t + C \qquad \int e^{t^2} t\, dt. = \frac{1}{2} e^{t^2} + C.$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2}\, dt = 1$$

$$P'(x) = \frac{d}{dx} \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

Ejercicio 1: Derive las siguientes funciones. Pág 20.

a) $h(x) = \int_a^x 3\sqrt{t+1}\, dt.$ $\quad h'(x) = 3\sqrt{x+1}$

$t \to x$

b) $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$ $\quad S'(x) = \sin\left(\frac{\pi}{2} x^2\right)$

$t \to w$

c) $H(w) = \int_{-5}^{w} \frac{t+4}{t^4+t^2+2}\, dt.$ $\quad H'(w) = \frac{w+4}{w^4+w^2+2}$

TFC parte I y la regla de la cadena.

$$g(x) = \int_{100}^{x^5} e^t \, dt = e^t \Big]_{t=100}^{t=x^5} = e^{x^5} - e^{100}$$

$$g'(x) = e^{x^5} \underbrace{5x^4}_{\text{Regla de la cadena.}} - 0$$

$h(x) = \sin(x^5 + x^2)$ $\quad h'(x) = \cos(x^5+x^2)(5x^4+2x)$

$$f(x) = \int_a^{b(x)} g(t)\,dt. \qquad f'(x) = g(b(x))\, b'(x)$$

a ← constante. $\quad t \to b(x)$

Ejercicio 2: Derive las siguientes funciones.

a. $g(x) = \int_5^{\ln x} \sqrt{t^2+1}\, dt.$ $\qquad h(x) = \int_5^{x} \sqrt{t^2+1}\, dt.$

$$g'(x) = \sqrt{(\ln x)^2 + 1} \cdot \underbrace{\frac{1}{x}}_{d(\ln x)}. \qquad h'(x) = \sqrt{x^2+1}$$

$t \to \ln x$

b. $h(x) = \int_{\sec x}^{8} \tan^{-1}(t)\, dt. = -\int_8^{\sec x} \tan^{-1}(t)\, dt.$

$$h'(x) = -\tan^{-1}(\sec x)\, \sec x \tan x$$

$t \to \sec x$

c. $\frac{d}{dx}\left(\int_{1000}^{x^5+x^3} \ln(t)\, dt.\right) = \ln(x^5+x^3)\,(5x^4+3x^2)$

$t \to x^5+x^3$ • derivada lím superior

Funciones con ambos límites dependiendo de x.

$$f(x) = \int_{\sinh x}^{\cosh x} \sec^2 t \, dt.$$

$$f(x) = \tan t \Big]_{\sinh x}^{\cosh x} = \tan(\cosh x) - \tan(\sinh x)$$

Derive: $f'(x) = \sec^2(\cosh x) \cdot \sinh x - \sec^2(\sinh x) \cosh x$

$$f'(x) = \frac{d}{dx} \int_{\tan x}^{\csc x} \sec^2 t \, dt = \sec^2(\csc x)(-\csc x \cot x) - \sec^2(\tan x)(\sec^2 x)$$

$t \to \csc x$

TFC parte 1 y la Regla de la Cadena

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(t) \, dt = f(b) b' - f(a) a'.$$

Ejercicio 3: Derive

$$\frac{d}{dx} \int_{\sin x}^{e^x} \sqrt{10 + 4t^4} \, dt = \sqrt{10 + 4e^{4x}} \, e^x - \sqrt{10 + 4\sin^4 x} \cos x$$

$$\frac{d}{dx} \int_{\ln x}^{\sin^{-1} x} \cosh \theta^3 \, d\theta = \cosh(\sin^{-1} x)^3 \cdot \frac{1}{\sqrt{1 - x^2}} - \cosh(\ln^3 x) \cdot \frac{1}{x}.$$

Ejercicio 4: Encuentre la ecuación de la recta tangente a $f(x) = \int_0^x \cosh^2 t \, dt$ en $t=0$

Ec. Recta Tangente $y = \underline{f(0)} + \underline{f'(0)}(x-0)$

$f(0) = \int_0^0 \cosh^2 t \, dt = 0$ $\qquad \int_a^a f(y)\,dy = 0.$

$f'(x) = \frac{d}{dx}\int_0^x \cosh^2 t \, dt = \cosh^2 x \cdot 1$

$f'(0) = (\cosh 0)^2 = 1^2 = 1$

Ec. Recta Tangente $y = 0 + 1(x-0)$

$y = x$

Capítulo 5

Técnica de integración por sustitución, para integrales definidas e indefinidas

5.5 Regla de la sustitución

Objetivo: Integre $F(g(x))$ funciones compuestas.

a $\int 3\underbrace{(x+2)^2}_{x^2+4x+4}\,dx = \int (3x^2+12x+12)\,dx$

$= x^3+6x^2+12x+C.$

conjeturando $\int 3(x+2)^2\,dx = \underbrace{(x+2)^3+C.}_{\text{derivada } 3(x+2)^2\cdot 1+0}$

b. $\int 11(x-20)^{10}\,dx = (x-20)^{11}+C.$

$\longrightarrow$ derivando

Regla de la Potencia $\frac{d}{dx}[\underbrace{f(x)}]^{n+1} = (n+1)[f(x)]^n f'(x)$

$\longleftarrow$ integrando

Regla de la sustitución funciones Potencia

$$\int [\underbrace{f(x)}_{u}]^n \underbrace{f'(x)\,dx}_{du} = \frac{f(x)^{n+1}}{n+1}+C$$

$u=f(x) \quad du = f'(x)\,dx$

$$\int u^n\,du = \frac{u^{n+1}}{n+1}+C \quad \text{si } n\neq -1.$$

Ejercicio 1: Evalúe las sigs. integrales.

a. $\int (\overbrace{11x-20}^{u})^{10}\,\underbrace{11\,dx}_{du} = \int u^{10}\,du = \frac{u^{11}}{11}+C_1$

$u=11x-20 \quad du=11\,dx \qquad = \boxed{\frac{1}{11}(11x-20)^{11}+C_1}$

aa. $\int (\overbrace{x^2+x+3}^{u})^5\,\overbrace{(2x+1)\,dx}^{du} = \int u^5\,du. = \frac{1}{6}u^6+C_2.$

Pág. 30

$u=x^2+x+3, \quad du=(2x+1)\,dx \qquad = \frac{1}{6}(x^2+x+3)^6+C_2$

Multiplicar/Dividir por una constante.

b. $\int (30w^3-8)^{19} w^2 dw. = \int u^{19} \frac{du}{90}$

$u = 30w^3 - 8$, $du = 90 \underbrace{w^2 dw}$. $\Rightarrow w^2 dw = \frac{du}{90}$.

$dw = \frac{du}{90w^2}$

$\int u^{19} \frac{du}{90} = \frac{1}{90} \cdot \frac{1}{20} u^{20} + C_3 = \boxed{\frac{1}{1,800} (30w^3-8)^{20} + C_3}$.

c. $\int (30w^3-8)^{19} 90w^3 dw. = \int u^{19} w \, du.$ X

sustitución incompleta.

$\cancel{\int u^{19} \left(\frac{1}{30}(u+8)^{1/3}\right) du}$ sólo se puede integrar. por fuerza bruta.

d. $\int 8x^3 \sqrt{8+x^4} \, dx = \int 2u^{1/2} du = 2 \cdot \frac{2}{3} u^{3/2} + C$

$u = 8 + x^4$ $du = 4x^3 dx$ $\Rightarrow$ $2du = 8x^3 dx$.

La integral es $\frac{4}{3}(8+x^4)^{3/2} + C$.

e. $\int (10x^2+6x)^2 dx = \int (100x^4 + 120x^3 + 36x^2) dx$

yo se usa la sustitución expanda y luego integre

$= \boxed{20x^5 + 30x^4 + 12x^3 + C}$.

Regla de la Cadena Derivadas. $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$

Regla de la Sustitución Cadena a la Inversa. $\int f'(g(x))\,g'(x)\,dx = \int f'(u)\,du = f(u)+C.$

$u = g(x), \quad du = g'(x)\,dx \qquad = f(g(x))+C.$

Ejercicio 2: Integre. Pág 32.

0. $\int \frac{(8+16x+48x^2)}{x+x^2+2x^3}\,dx = \int \frac{8\,du}{u} = 8\ln|u| + C$

$= \boxed{8\ln|x+x^2+2x^3| + C.}$

$u = x+x^2+2x^3 \qquad du = (1+2x+6x^2)\,dx$

$8\,du = (8+16x+48x^2)\,dx$

a. $\int e^{\overbrace{x^{10}+\sqrt{2}}^{u}}\,x^9\,dx = \int e^u \frac{du}{10} = \frac{1}{10}e^u + C$

$u = x^{10}+\sqrt{2} \qquad \frac{du}{10} = \frac{10x^9\,dx}{10}. \qquad = \frac{1}{10}e^{x^{10}+\sqrt{2}} + C.$

a2 $\int e^{x^{10}}\underbrace{x^8\,dx}_{\neq du} \qquad \int e^{x^{10}}\underbrace{dx}_{\neq du.}$ no se puede integrar.

b. $\int x^3(x^4+3)^2\sin(x^4+3)^3\,dx = \int u^2\sin u^3\,\frac{du}{4}.$

$u = (x^4+3) \qquad du = 4x^3\,dx \qquad \frac{du}{4} = x^3\,dx$

$u = (x^4+3)^3$ ✓

$\frac{1}{4}\int \sin(u^3)\,u^2\,du = \frac{1}{4}\int \sin t\,\frac{dt}{3} = -\frac{1}{4}\cdot\frac{1}{3}\cos t + C$

$t = u^3 \qquad dt = 3u^2\,du. \qquad = -\frac{1}{12}\cos u^3 + C$

$= -\frac{1}{12}\cos(x^4+3)^3 + C$

Una sóla sustitución.

$$\int \sin(x^4+3)^3 \overbrace{[(x^4+3)^2 x^3]\,dx}^{du/12} = \frac{1}{12}\int \sin u\,du.$$

$u = (x^4+3)^3$, $du = 3(x^4+3)^2\,4x^3\,dx$

$\frac{du}{12} = (x^4+3)^2 x^3\,dx$

c. $\int \cot x\,dx = \int \overbrace{\frac{\cos x}{\sin x}\,dx}^{du} = \int \frac{du}{u} = \ln|u| + C.$

$= \ln|\sin x| + C.$

$u = \sin x \quad du = \cos x\,dx$

d. $\int \sec^2(\overbrace{e^x+x}^{u})\overbrace{(e^x+1)\,dx}^{du} = \int \sec^2 u\,du = \tan u + C.$

$u = e^x + x \quad du = (e^x+1)dx \qquad = \tan(e^x+x) + C.$

Sustitución Incompleta. $x = u - 4.$

e. $\int 28x(x+4)^{1/3}\,dx = \int 28x\,u^{1/3}\,du.$

$u = x+4 \quad du = 1\cdot dx \quad = \int 28(u-4)\,u^{1/3}\,du.$

$28\int u^{4/3} - 4u^{1/3}\,du = 28\left[\frac{3}{7}u^{7/3} - 4\cdot\frac{3}{4}u^{4/3}\right] + C$

$= 12(x+4)^{7/3} - 84(x+4)^{4/3} + C.$

Regla de la sustitución para Integrales Definidas

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$u = g(x)$ $\qquad u = g(a)$

$du = g'(x)dx \qquad u = g(b)$

Cambie también los límites.

Ejercicio 3: Integre.

a. $\displaystyle\int_{-4}^{0} \frac{1}{3x-2}\,dx = \int_{-14}^{-2} \frac{1}{u}\,\frac{du}{3} = \frac{1}{3}\ln|u|\Big]_{-14}^{-2} = (\ln 2 - \ln 14)\frac{1}{3}$

$= -\dfrac{\ln 7}{3}$

f es continua en $x \neq \frac{2}{3}$

$u = 3x-2 \qquad du = 3\cdot dx$

$u(0) = 0-2 = -2, \qquad u(-4) = -12-2 = -14$

b. $\displaystyle\int_{0}^{1} \frac{8}{\pi}\,\frac{\sin^{-1} t}{\sqrt{1-t^2}}\,dt = \frac{8}{\pi}\int_{0}^{\pi/2} u\,du = \frac{8}{\pi}\,\frac{u^2}{2}\Big]_{0}^{\pi/2} = \frac{4}{\pi}\,\frac{\pi^2}{4}$

$= \pi$

¿u? $\quad u = \sin^{-1} t \qquad u(1) = \sin^{-1}(1) = \pi/2$

$du = \dfrac{dt}{\sqrt{1-t^2}} \qquad u(0) = \sin^{-1}(0) = 0$

Capítulo 6

Técnica de integación por partes, para integrales definidas e indefinidas

7.1 Integración por partes

$\int \ln x \, dx \qquad \int \tan^{-1} x \, dx \qquad \int x^3 e^x \, dx \qquad \int e^x \cos x \, dx$

IPP: Integre productos de funciones "disimilares".

$$\int \underbrace{f(x)}\, \underbrace{g(x)\, dx} = ?$$

Regla del Producto para Derivadas.

$$(fg)' = f'g + fg'$$

$$(fg)' - f'g = fg'$$ Integre esta expresión.

IPP $$\boxed{\int fg' = fg - \int f'g}$$

f deriva.
g' integra.

$\int fg'$ $\qquad$ $\int f'g$ más simple que la integral original.

$$\int \underset{u}{f(x)}\, \underbrace{g(x)\, dx}_{dv} = uv - \int v\, du$$

$u = f(x) \qquad dv = g(x)\, dx$

$du = f'(x)\, dx \qquad v = G(x)$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ejercicio 1: Pág 39 Integre $\int x e^x \, dx$

Opción 1: $u = x \qquad dv = e^x dx$

$du = 1 \cdot dx \qquad v = e^x$

Opción 2: $\cancel{u = e^x \qquad dv = x\, dx}$

$\cancel{du = e^x dx \qquad v = \frac{1}{2} x^2}$

$$\int x e^x dx = \underbrace{xe^x}_{u \cdot v} - \underbrace{\int e^x dx}_{v\, du} = xe^x - e^x + C.$$

Derive y compruebe su respuesta.

Ejercicio 2: Integre $\int fg' = fg - \int f'g$.

; $\int u\,dv = uv - \int v\,du$

a. $\int 6x^2 \ln x \, dx = (\ln x)\, 2x^3 - \int \frac{1}{x} 2x^3 \, dx$ $\qquad \frac{x^3}{x} = x^2$

$u = \ln x \qquad dv = 6x^2 \qquad 2x^3 \ln x - \int 2x^2 \, dx$

$du = \frac{1}{x} \qquad v = 2x^3 \qquad \boxed{2x^3 \ln x - \frac{2}{3}x^3 + C.}$

b. $\int \ln x \, dx = \underset{v}{x} \underset{u}{\ln x} - \int \underset{v}{x} \underset{du}{\frac{dx}{x}} = x\ln x - \int dx$

$u = \ln x \qquad dv = 1 \cdot dx \qquad = \boxed{x\ln x - x + C.}$

$du = \frac{dx}{x} \qquad v = x$

c. $\int \tan^{-1} x \, dx = \underset{v}{x} \underset{u}{\tan^{-1} x} - \int \underset{du}{\frac{x}{1+x^2} dx}$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$\int \frac{1}{1+x^2} \underbrace{x\,dx}_{du/2.} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+x^2| + C.$

Sustitución. $u = 1+x^2 \qquad du = 2x\,dx$

$\int \tan^{-1} x \, dx = \boxed{x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| - C}$

J. $\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$

uv $\quad$ $v\,du.$

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x\,dx \qquad v = \sin x$

otra vez IPP.

$\int 2x \sin x \, dx = -2x \cos x + \int 2 \cos x \, dx = -2x\cos x + 2\sin x + C$

$u = 2x \qquad dv = \sin x \, dx$

$du = 2\,dx \qquad v = -\cos x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2\sin x + C.$

ILATE. $\quad$ Inversas trigonométricas $\sin^{-1}x$, $\tan^{-1}x$

Logarítmicas $\quad \ln x$, $\log_a x$

Mnemotecnia. $\quad$ Algebraicas $\quad x^n$, $\sqrt{x}$, $1/x^r$

Trigonométricas $\quad \sin x$, $\cos x$, $\tan x$

Exponenciales. $\quad e^x$, a^x

IPP:

Integrales Definidas:

no cambian los límites

$$\boxed{\int_a^b u \, dv = uv\Big]_a^b - \int_a^b v \, du}$$

Ejercicio 3: Evalúe. $\quad$ a) Interesante.

b. $72 \int_1^2 \frac{\ln x}{x^4} \, dx. = 72 \ln x \left(\frac{-1}{3x^3}\right)\Big]_1^2 - \int_1^2 \frac{x^{-3}}{-3} x^{-1} \, dx$

$u = \ln x \qquad dv = x^{-4} \, dx$

$du = x^{-1} \, dx \qquad v = \frac{x^{-3}}{-3}$

$$-\frac{24}{1x^3}\ln x\Big]_1^2 - \int_2^1 \frac{x^{-4}}{3}\,dx \qquad -x^{-4} \to \frac{-x^{-3}}{-3}$$

$$-\frac{24}{8}\ln 2 + \cancel{\frac{24}{1}\ln 1}^{\,0} + \frac{1}{3x^3}\Big]_2^1$$

$$-3\ln 2 + \frac{1}{3} - \frac{1}{3\cdot 8}$$

Resuelto con detalles pág 42.

incógnita

$$\boxed{\int e^x \cos x\,dx} = e^x\cos x + \int e^x \sin x\,dx \qquad (1)$$

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x\,dx \qquad v = e^x$

$$\int e^x \sin x\,dx = e^x \sin x - \boxed{\int e^x \cos x\,dx} \qquad (2)$$

incógnita.

$u = \sin x \qquad dv = e^x\,dx$

$du = \cos x\,dx \qquad v = e^x$

$$\int e^x \cos x\,dx = e^x\cos x + e^x \sin x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x\cos x + e^x \sin x$$

$$\int e^x \cos x\,dx = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x \sin x + C.$$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x\cos x - \frac{1}{2}e^x \sin x + C.$$

Capítulo 7

Integrales trigonométrias, manipulación con identidades trigonométricas de la forma

1) $\int \sin^n(x)\cos(x)^m dx$

2) $\int \tan^n(x)\sec(x)^m dx$

3) $\int \cot^n(x)\sec^m(x) dx$

7.2 Integrales Trigonométricas.

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x \quad \div \cos^2 x.$

$1 + \cot^2 x = \csc^2 x \quad \div \sin^2 x.$

I. Integrales de la forma $\int \sin^n x \cos^m x \, dx$

$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

$u = \sin x, \; du = \cos x \, dx \qquad u = \cos x, \; du = -\sin x \, dx$

Evalúe $\int \cos^5 x \, dx = \int \cos^4 x \, \overbrace{(\cos x \, dx)}^{du}$

Reescriba $\cos^4 x = (\cos^2 x)^2 = (1 - \sin^2 x)^2$

$\cos^2 x = 1 - \sin^2 x$

$$\int \cos^5 x \, dx = \int (1 - \sin^2 x)^2 \underbrace{(\cos x \, dx)}_{du = \cos x \, dx}$$

$u = \sin x$

$$= \int (1 - u^2)^2 \, du$$

$$= \int (1 - 2u^2 + u^4) \, du$$

$$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C.$$

$\sin^3 x \neq \sin x^3$

$(\sin x)^3$

$$= \boxed{\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C.}$$

Apante algún término sinx ó cosx

a. Potencias impares de seno o coseno.

Ejercicio 1: Evalúe.

a. $\int \cos^3 x \sin^6 x \, dx = \int \cos^2 x \sin^6 x (\cos x \, dx)$

$\int \underbrace{\cos^2 x}_{1-\sin^2 x} \sin^6 x \cos x \, dx$ ó $\int \cos^3 x \sin^{\overset{\text{impar}}{5}} x \sin x \, dx$

$1-\cos^2 x$ X

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$

$du = \cos x \, dx$

$(a+b+c)d$

$ad + bd + cd.$

$$= \int (1-\sin^2 x) \sin^6 x \, (\cos x \, dx)$$

$$= \int (1-u^2) u^6 \, du$$

$$= \int (u^6 - u^8) \, du$$

$$= \frac{1}{7} u^7 - \frac{1}{9} u^9 + C.$$

$$= \boxed{\frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C.}$$

b. $\int \cos^5 x \sin^3 x \, dx = \int \cos^5 x (\sin^2 x) \sin x \, dx$

$\int \underbrace{\cos^4 x}_{(1-\sin^2 x)^2} \sin^3 x \cos x \, dx$ ó $\int \cos^5 x \underbrace{\sin^2 x}_{1-\cos^2 x} \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int \cos^5 x (1-\cos^2 x) \sin x \, dx$$

$$= -\int u^5 (1-u^2) \, du = \int (-u^5 + u^7) \, du$$

$$= -\frac{1}{6} u^6 + \frac{1}{8} u^8 + C.$$

$$= \boxed{-\frac{1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + C.}$$

b) Potencias pares de seno y coseno

$$\int \cos^2 x \, dx = \int \frac{1}{2} + \frac{1}{2}\cos 2x \, dx = \boxed{\frac{x}{2} + \frac{1}{4}\sin 2x + C.}$$

$$1 = \cos^2 x + \sin^2 x. \quad (1)$$

$$\cos(x+x) = \cos^2 x - \sin^2 x \quad (2)$$

$$1 + \cos(2x) = 2\cos^2 x \quad \Rightarrow \quad \boxed{\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x}$$

(1) + (2)

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Ejercicio 2: Evalúe.

a $$\int_{-\pi}^{\pi} \sin^2 x \, dx = 2\int_0^{\pi} \sin^2 x \, dx = \frac{2}{2}\int_0^{\pi}(1 - \cos 2x)\, dx$$

PAR.

$$\int_{-\pi}^{\pi} \sin x \, dx = 0 \quad \text{impar.}$$

360°

$$\int_0^{\pi}(1 - \underbrace{\cos 2x}_{u = 2x \quad du = 2dx})\, dx = x - \frac{1}{2}\sin 2x \Big]_0^{\pi} = \pi - \frac{1}{2}\cancel{\sin 2\pi}^{\,0} - 0 + \frac{\sin 0}{2}$$

$$\int_{-\pi}^{\pi} \sin^2 x \, dx = \pi.$$

$$\cos^2 0 = \frac{1}{2}(1 - \cos 2\square)$$

b. $$\int \sin^2 x \cos^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x)\frac{1}{2}(1 + \cos 2x)\, dx$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

doble

$$= \frac{1}{4}\int (1 - \cos^2 \underline{2x})\, dx$$

$$\cos^2(2x) = \frac{1}{2}(1 + \cos 4x) = \frac{1}{4}\int \left(1 - \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\int\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \int \frac{1}{8} + \frac{1}{8}\cos 4x \; dx.$$

$\int a f \, dx$

$= aF + C$

$$= \boxed{\frac{x}{8} + \frac{1}{8\cdot 4}\sin 4x + C.}$$

II Forma $\int \tan^m x \sec^n x \, dx$

$(\tan x)' = \sec^2 x$ (aparte)	$(\sec x)' = \sec x \tan x$ (aparte.)
$u = \tan x$	$u = \sec x$
$\sec^2 x = \tan^2 x + 1$	$\tan^2 x = \sec^2 x - 1$

Ejercicio 3: Evalúe Pág. 48.

a. $\int \tan^5 x \sec^4 x \, dx$

$\int \tan^5 x \underbrace{\sec^2 x}_{\tan^2 x + 1} (\sec^2 x \, dx)$ ✓

$u = \tan x$

ó $\int \tan^4 x \sec^3 x (\tan x \sec x \, dx)$ ✓

$u = \sec x$

$(\tan^2 x)^2 = (\sec^2 x - 1)^2$

$\int \tan^5 x \sec^2 x (\sec^2 x \, dx)$

$\int \tan^5 x (\tan^2 x + 1)(\sec^2 x \, dx)$

$u = \tan^2 x$ ✗

$du = 2\tan x \sec^2 x \, dx$ ✗

$u = \tan x \qquad du = \sec^2 x \, dx$

$$\int u^5 (u^2 + 1) \, du = \int (u^7 + u^5) \, du = \frac{u^8}{8} + \frac{u^6}{6} + C.$$

$$= \boxed{\frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C.}$$

Sharon: Resumen Identidades y luego sustitución.

b. Alejandro $\int \tan^5 x \sec^5 x \, dx$

$\int \tan^4 x \sec^4 x (\sec x \tan x \, dx)$ ✓

$\int \tan^5 x \sec^3 x (\sec^2 x \, dx)$ ✗

$\int (\tan^2 x)^2 \sec^4 x (\sec x \tan x \, dx)$ $\tan^2 x = \sec^2 x - 1$

$\int (\sec^2 x - 1)^2 \sec^4 x (\sec x \tan x \, dx)$

$u = \sec x$ $du = \sec x \tan x \, dx.$

$\int (u^2 - 1)^2 u^4 \, du = \int (u^4 - 2u^2 + 1) u^4 \, du.$

$\int (u^8 - 2u^6 + u^4) \, du = \frac{1}{9} u^9 - \frac{2}{7} u^7 + \frac{1}{5} u^5 + C.$

$= \frac{1}{9} \sec^9 x - \frac{2}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C.$

c. $\int \tan^4 x \sec^4 x \, dx. = \int \tan^4 x \sec^2 x \sec^2 x \, dx$

sólo tan's.

$\sec^2 x = \tan^2 x + 1$ $= \int \tan^4 x (\tan^2 x + 1)(\sec^2 x \, dx)$

$u = \tan x$ $du = \sec^2 x \, dx$ $= \int u^4 (u^2 + 1) \, du.$

$= \int u^6 + u^4 \, du = \frac{1}{7} u^7 + \frac{1}{5} u^5 + C$

$= \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C.$

Casos especiales $\int \tan^m x\,dx$ $\int \sec^n x\,dx$

$$\int \tan x\,dx = \int \frac{\overbrace{\sin x\,dx}^{-du}}{\underbrace{\cos x}} = -\int \frac{du}{u} = -\ln|u| + C.$$

$u = \cos x$ $\quad du = -\sin x\,dx$

$$= -\ln|\cos x| + C.$$

$$\int \sec x\,dx = \int \sec x \frac{(\sec x + \tan x)}{\tan x + \sec x}\,dx$$ Brillante.

$$= \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x}\,dx$$

$u = \tan x + \sec x$ $\quad du = (\sec^2 x + \sec x \tan x)\,dx$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\tan x + \sec x| + C.$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C.$$

$$\int \sec^2 x\,dx = \tan x + C.$$

$\tan^2 x = \sec^2 x - 1$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C.$$

$$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$$

$u = \tan x$
$du = \sec^2 x\,dx$

$$= \int (\sec^2 x - 1)\tan x\,dx$$

$$= \int (\sec^2 x \tan x - \tan x)\,dx$$

$$= \int \underbrace{\tan x \sec^2 x\,dx}_{u \cdot du.} - \int \tan x\,dx$$

$$= \frac{1}{2}\tan^2 x - \ln|\cos x| + C.$$

7.

Más difícil $\int \sec^3 x \, dx$.

$$\int \sec x \sec^2 x \, dx = \sec x \tan x - \int \tan^2 x \sec x \, dx$$

IPP $u = \sec x$ $\qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx$ $\qquad v = \tan x$

$$\int \tan^2 x \sec x \, dx = \int (\sec^2 x - 1) \sec x \, dx$$

$$= \int (\sec^3 x - \sec x) \, dx$$

$$\underline{\int \sec^3 x \, dx} = \sec x \tan x - \underline{\int \sec^3 x \, dx} + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C.$$

$$\int \sec^3 x \, dx = \frac{1}{2} \text{derivada}(\sec) + \frac{1}{2} \text{integral}(\sec)$$

$$\frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C.$$

Capítulo 8

Integrales trigonométricas (continuación), forma
1) $\int \sin^n(x)\cos(x)^m dx$
2) $\int sec^n(x)tan^m(x)dx$
3) $\int \csc^n(x)\cot^m(x)dx$
introducción a integración por sustitución trigonométrica

Lunes 26 de agosto Simulacro Parcial.
3 de septiembre Parcial 1
capítulos 5 y 7 Págs 11-70.

Integrales de la forma $\int \cot^n x \csc^m x \, dx$

$(\csc x)' = -\csc x \cot x$ $\qquad$ $(\cot x)' = -\csc^2 x$

$\cot^2 x = \csc^2 x - 1$ $\qquad$ $\csc^2 x = \cot^2 x + 1$

$u = \csc x$ $\qquad$ $u = \cot x$

Ejercicio 4: Integre (Pág 50).

a $\int \cot^2 x \csc^4 x \, dx$ $\qquad$ $\cot^2 \csc^2 x \csc^2 x$ ✓

✗ $\cot x \csc^3 x (\csc x \cot x)$

$$\int \cot^2 x \csc^2 x (\csc^2 x \, dx) = \int \cot^2 x (\cot^2 x + 1) \csc^2 x \, dx$$

$\csc^2 x = \cot^2 x + 1$, $\quad u = \cot x$ $\qquad$ $du = -\csc^2 x \, dx$

$$= -\int u^2 (u^2 + 1) \, du$$

$$= -\int (u^4 + u^2) \, du = -\frac{u^5}{5} - \frac{u^3}{3} + C.$$

$$= -\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C.$$

b. $\int \cot^3 x \csc^3 x \, dx = \int \cot^2 x \csc^2 x (\cot x \csc x \, dx)$

$\cot^2 x = \csc^2 x - 1$ $\qquad$ $= \int (\csc^2 x - 1) \csc^2 x (\cot x \csc x \, dx)$

$u = \csc x$ $\quad$ $du = -\csc x \cot x \, dx$ $\qquad$ $= -\int (u^2 - 1)(u^2) \, du.$

$$-\int (u^4 - u^2)\,du = -\frac{u^5}{5} + \frac{u^3}{3} + C.$$

$$= -\frac{\csc^5 x}{5} + \frac{\csc^3 x}{3} + C.$$

casos especiales $\int \csc x\,dx$ $\int \csc^3 x\,dx$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C.$$

$$\int \csc x \frac{(\csc x + \cot x)}{\cot x + \csc x}\,dx = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x}\,dx$$

*"1" especial.

$u = \cot x + \csc x.$

$-du = (\csc^2 x + \csc x \cot x)\,dx$

$$= -\int \frac{du}{u} = -\ln|u| + C. = -\ln|\cot x + \csc x| + C.$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C.$$

$$\int \sec^3 x\,dx = \frac{1}{2}(\sec x)' + \frac{1}{2}\int \sec x\,dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C.$$

$$\int \csc^3 x\,dx = \frac{1}{2}(\csc x)' + \frac{1}{2}\int \csc x\,dx$$

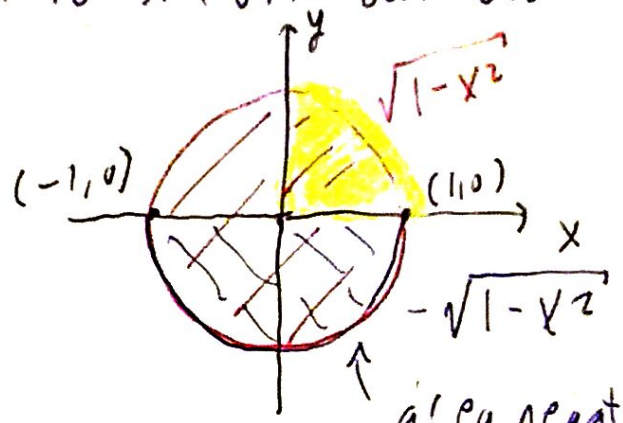
$$= -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + C.$$

Área de un círculo unitario sin utilizar Geometría

Ec: $x^2 + y^2 = 1$

Función: $y^2 = 1 - x^2$.

2 Funciones $y = \pm\sqrt{1-x^2}$



área negativa.

$$A = \int_{-1}^{1} \sqrt{1-x^2}\, dx + \int_{-1}^{1} \sqrt{1-x^2}\, dx \qquad \int_{1}^{-1} \sqrt{1-x^2}\, dx$$

$$A = 2\int_{-1}^{1} \sqrt{1-x^2}\, dx = 4\int_{0}^{1} \sqrt{1-x^2}\, dx$$

función par

$u = 1 - x^2$

$du = -2x\,dx$

Ni sustitución ni integración por partes.

$1 - \sin^2\theta = \cos^2\theta.$

$$A = 4\int_0^{\pi/2} \underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta.} \cos\theta\, d\theta.$$

$x = \sin\theta.$

$dx = \cos\theta\, d\theta.$

$$A = 4\int_0^{\pi/2} \cos^2\theta\, d\theta.$$

$x = 1 = \sin\theta \Rightarrow \theta = \pi/2$

$x = 0 = \sin\theta \Rightarrow \theta = 0$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$A = \frac{4}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta. = 2\left(\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}\Bigg)$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\sin \pi}^{\,0} - 0 - \frac{1}{2}\cancel{\sin 0}^{\,0}\right) = \frac{2\pi}{2} = \pi.$$

Área de un círculo de radio (1) $\pi(1)^2$.

7.3 Sustitución Trigonométrica (Pág 54).

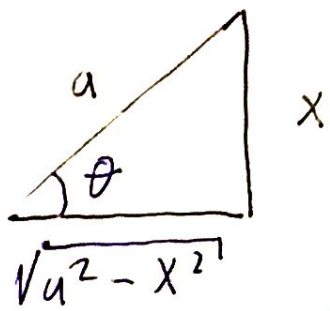
$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}\, g'(\theta)\,d\theta.$$

simplifique si es posible.

$x = g(\theta) \qquad dx = g'(\theta)\,d\theta$

$\sqrt{1-x^2}$	$\sqrt{1+x^2}$	$\sqrt{x^2-1}$	$\sqrt{\sec^2\theta - 1}$
$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta.$	$\sqrt{\tan^2\theta}$
$1-\sin^2\theta = \cos^2\theta.$	$1+\tan^2\theta = \sec^2\theta.$		$\sqrt{x^2-1} = \tan\theta.$
$\sqrt{1-x^2} = \cos\theta.$	$\sqrt{1+x^2} = \sec\theta.$		

forma $\sqrt{a^2 - x^2}$



$\sin\theta = \dfrac{C.O.}{H} = \dfrac{x}{a}.$

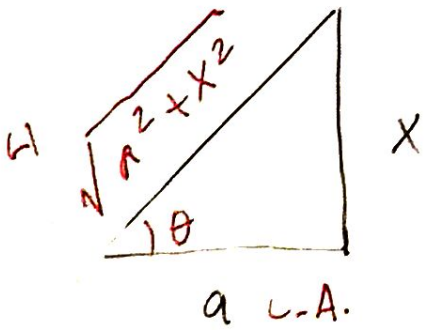
$\cos\theta = \dfrac{C.A.}{H} = \dfrac{\sqrt{a^2-x^2}}{a}.$

$x = a\cdot\sin\theta$

$dx = a\cdot\cos\theta\,d\theta.$

$\sqrt{a^2-x^2} = a\cdot\cos\theta.$

Forma $\sqrt{a^2+x^2}$



$\sin\theta = \dfrac{x}{\sqrt{a^2+x^2}} \qquad \tan\theta = \dfrac{x}{a}.$

$\dfrac{H}{C.A.} = \sec\theta = \dfrac{\sqrt{a^2+x^2}}{a}. \qquad \dfrac{C.A.}{H}$

$x = a\cdot\tan\theta.$

$dx = a\cdot\sec^2\theta\,d\theta. \qquad \sqrt{a^2+x^2} = a\cdot\sec\theta.$

Ejercicio 1: Evalúe.

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\cdot\frac{du}{2} = -\int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

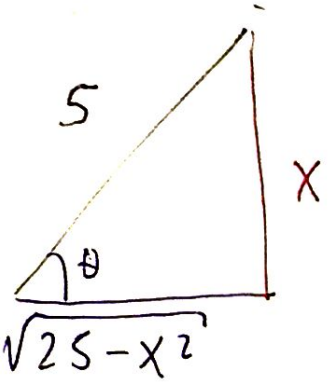
$u = 25 - x^2 \qquad du = -2x\,dx \Rightarrow dx = \dfrac{du}{-2x} \qquad = -u^{1/2} + C.$

$$= -\sqrt{25-x^2} + C.$$

Sustitución Trigonométrica.

$H = 5.$

$C.O = x$



$x = 5\sin\theta.$

$dx = 5\cos\theta\,d\theta.$

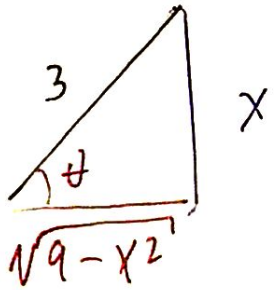
$\sqrt{25-x^2} = 5\cos\theta.$

$\dfrac{C.A.}{H.}$

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{5\cos\theta}\,5\cos\theta\,d\theta = \int 5\sin\theta\,d\theta.$$

$$= -5\cos\theta + C. = -\sqrt{25-x^2} + C.$$

a $$\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{27\sin^3\theta}{3\cancel{\cos\theta}}\,3\cancel{\cos\theta}\,d\theta = 27\int \sin^3\theta\,d\theta.$$



$\sin\theta = \dfrac{x}{3}$

$\cos\theta = \dfrac{1}{3}\sqrt{9-x^2}$

$x^3 = 27\sin^3\theta$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta.$

$$27\int \sin^2\theta\,\sin\theta\,d\theta = 27\int (1-\cos^2\theta)\sin\theta\,d\theta.$$

$u = \cos\theta \qquad du = -\sin\theta\,d\theta.$

$$= -27\int (1-u^2)\,du = -27\left(u - \frac{u^3}{3}\right) + C.$$

regrese a var. θ.

$$= -27u + 9u^3 + C.$$

$$= -27\cos\theta + 9\cos^3\theta + C$$

regrese a var x.

$$= -27\,\frac{1}{3}\sqrt{9-x^2} + 9\,\frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C.$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)^{3/2} + C.$$

Caso Integrales Trigonométricas.

$$\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$$

Ejercicio 5: Evalúe. (Pág 51)

a $\displaystyle\int_{-\pi}^{\pi} \sin \underset{m}{8}x \cos \underset{n}{4}x\,dx = 0.$

$$\frac{1}{2}\int_{-\pi}^{\pi}(\sin 4x + \sin 12x)\,dx = \frac{1}{2}\left(\left.\frac{-\cos 4x}{4}\right]_{-\pi}^{\pi} - \left.\frac{\cos 12x}{12}\right]_{-\pi}^{\pi}\right)$$

$$\frac{1}{2}\left(\underbrace{-\frac{\cos 4\pi}{4}}_{-\frac{1}{4}} + \underbrace{\frac{\cos(4\pi)}{4}}_{+\frac{1}{4}} - \frac{\cos 12\pi}{12} + \frac{\cos 12\pi}{12}\right) = \frac{1}{2}(0+0) = 0.$$

Capítulo 8. Integrales trigonométricas (continuación), forma
1) $\int \sin^n(x)\cos(x)^m dx$
2) $\int sec^n(x) tan^m(x) dx$
3) $\int \csc^n(x)\cot^m(x) dx$
introducción a integración por sustitución trigonométrica

Capítulo 9

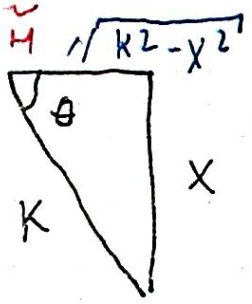
Sustitución trigonométrica por medio del triángulo pitagórico

7.3 Sustitución Trigonométrica

Forma $\sqrt{\underset{H}{K^2} - x^2}$

$H = K$

$C.O = x$

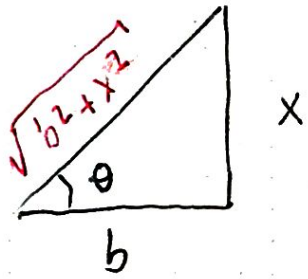


$\frac{C.O}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta.$

$dx = K\cos\theta\, d\theta.$

$\frac{\sqrt{K^2-x^2}}{K} = \cos\theta \Rightarrow \sqrt{K^2-x^2} = K\cdot\cos\theta.$

Forma $\sqrt{b^2 + x^2}$

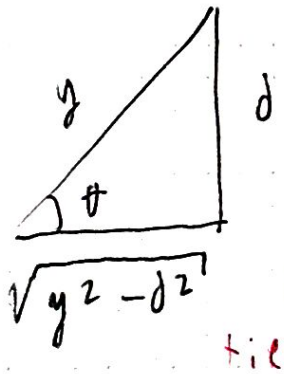


$\frac{x}{b} = \tan\theta. \Rightarrow x = b\cdot\tan\theta.$

$dx = b\cdot\sec^2\theta\, d\theta.$

$\frac{\sqrt{b^2+x^2}}{b} = \sec\theta \Rightarrow \sqrt{b^2+x^2} = b\sec\theta.$

Forma $\sqrt{y^2 - d^2}$



$\operatorname{sen}\theta = \frac{d}{y}$

$y = d\cdot\csc\theta.$

$dy = -d\cdot\csc\theta\cot\theta.$

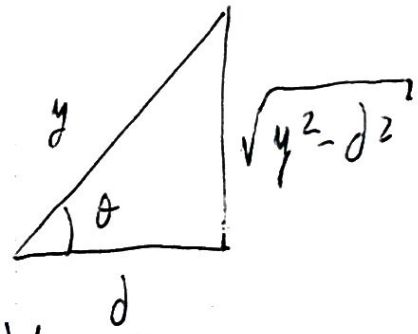
tiene signos negativos.

$\frac{y}{d} = \sec\theta.$

$y = d\sec\theta.$

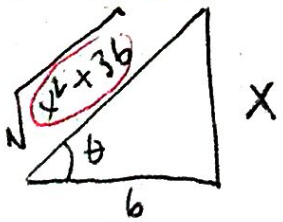
$dy = d\cdot\sec\theta\tan\theta.$

$\sqrt{y^2-d^2} = d\tan\theta.$



Ejercicios 2 y 3 Pág 58 y 59.

2o) $\int \frac{1}{x^2+36}\,dx = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$



$x = 6\cdot\tan\theta.$

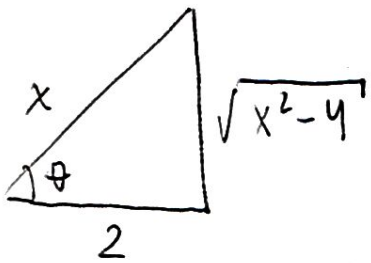
$dx = 6\cdot\sec^2\theta\, d\theta.$

$x^2 + 36 = 36\tan^2\theta + 36) = 36\sec^2\theta$

$$\frac{1}{6}\int d\theta = \frac{1}{6}\theta + C. = \boxed{\frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C.}$$

$$\tan^{-1}\left(\frac{x}{6}\right) = \theta.$$

3a. $$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \int \frac{2^3\tan^3\theta}{2^6\cdot\sec^6\theta}\; 2\cdot\sec\theta\tan\theta\,d\theta.$$



$$\frac{2}{x} = \cos\theta. \rightarrow x = \frac{2}{\cos\theta} = 2\sec\theta.$$

$$dx = 2\sec\theta\tan\theta\,d\theta.$$

$$\sqrt{x^2-4} = 2\tan\theta.$$

$$\left[(x^2-4)^{1/2}\right]^3 = 8\tan^3\theta.$$

$$\frac{1}{2^2}\int \frac{\tan^4\theta}{\sec^5\theta}\,d\theta. = \frac{1}{4}\int \frac{\sin^4\theta}{\cos^4\theta}\cos^5\theta\,d\theta.$$

$$= \frac{1}{4}\int \underbrace{\sin^4\theta}_{u^4}\,\underbrace{\cos\theta\,d\theta}_{du}. = \boxed{\frac{1}{4}\cdot\frac{1}{5}\sin^5\theta + C.}$$

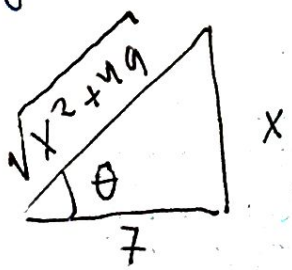
$u = \sin\theta \quad du = \cos\theta\,d\theta$

$\frac{1}{4}\int u^4\,du.$

Regrese a la variable x $\quad \sin\theta = \frac{\sqrt{x^2-4}}{x}, \quad \sin^5\theta = \frac{(x^2-4)^{5/2}}{x^5}$

$$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \boxed{\frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C.}$$

2a $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{49\;\cancel{7}\sec^2\theta}{49\tan^2\theta\cdot\cancel{7}\sec\theta}\,d\theta. = \int \frac{\sec\theta}{\underbrace{\tan^2\theta}_{\sec^2\theta-1}}\,d\theta.$$

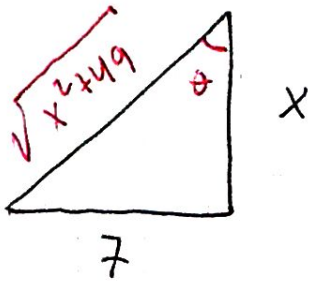


$$\frac{x}{7} = \tan\theta \qquad x = 7\tan\theta.$$

$$dx = 7\sec^2\theta\,d\theta.$$

$$\frac{H}{C.A.} = \sec\theta. \qquad \sqrt{x^2+49} = 7\sec\theta.$$

2a) $\displaystyle\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49\cdot 7\csc^2\theta}{49\cot^2\theta\; 7\csc\theta}\,d\theta. = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta.$



$\dfrac{x}{7} = \cot\theta \Rightarrow x = 7\cot\theta.$

$dx = -7\csc^2\theta\,d\theta.$

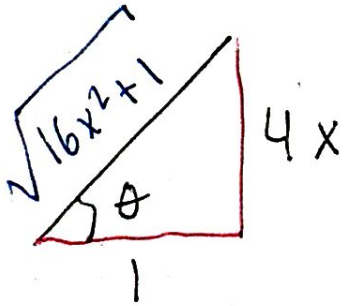
$\dfrac{\sqrt{x^2+49}}{7} = \csc\theta \Rightarrow \sqrt{x^2+49} = 7\csc\theta.$

$$-\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\,\frac{\sin^2\theta}{\cos^2\theta}\,d\theta. = -\int \frac{\sin\theta}{\cos\theta}\,\frac{1}{\cos\theta}\,d\theta.$$

$$= -\int \sec\theta\tan\theta\,d\theta.$$

$$= -\sec\theta + C. = -\frac{\sqrt{x^2+49}}{x} + C.$$

3 b) $\displaystyle\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta.}{(1/4)\tan\theta.\sec\theta.} = \int \frac{\sec\theta}{\tan\theta}\,d\theta.$



$\dfrac{4x}{1} = \tan\theta. \Rightarrow x = \frac{1}{4}\tan\theta.$

$dx = \frac{1}{4}\sec^2\theta\,d\theta.$

$\sqrt{16x^2+1} = \sec\theta.$

$$\int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{1}{\cos\theta}\,\frac{\cos\theta}{\sin\theta}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta.$$

$$= -\ln\left|\csc\theta + \cot\theta\right| + C.$$

$$= -\ln\left|\frac{\sqrt{16x^2+1}}{4x} + \frac{1}{4x}\right| + C.$$

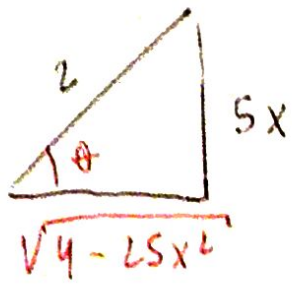
Capítulo 10

Más problemas de integración por sustitución trigonométrica

Parcial Lunes 2 septiembre 1:00 PM. D-503 B.
2:30 PM D-505 A

sub 5.

1. $\int 5^8 x^7 \sqrt{4-25x^2}\, dx = 5^8 \int \frac{2^7}{5^7} \sin^7\theta \, 2\cdot\cos\theta \cdot \frac{2}{5}\cos\theta \, d\theta.$



$\frac{5x}{2} = \sin\theta \qquad x = \frac{2}{5}\sin\theta.$

$dx = \frac{2}{5}\cos\theta \, d\theta$

$\sqrt{4-25x^2} = 2\cos\theta.$

$x^7 = \frac{2^7}{5^7}\sin^7\theta.$

$\frac{5^8}{5^8} 2^9 \int \sin^7\theta \cdot \cos^2\theta \, d\theta = 512 \int \sin^6\theta \cos^2\theta \, \underline{\sin\theta \, d\theta}.$

$\sin^6\theta = (\sin^2\theta)^3 = (1-\cos^2\theta)^3 \qquad = 512\int (1-\cos^2\theta)^3 \cos^2\theta \sin\theta \, d\theta.$

$u = \cos\theta \quad du = -\sin\theta \, d\theta. \qquad = 512 \int \underline{(1-u^2)^3} u^2 \, du.$

$(1-u^2)^3 = 1 - 3u^2 + 3u^4 - u^6$

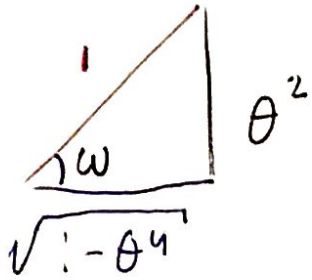
$(1-u^2)^3 u^2 = u^2 - 3u^4 + 3u^6 - u^8$

$\cos\theta.$

$\cos\theta = \frac{(4-25x^2)^{1/2}}{2}.$

2. $\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta. = \frac{4}{\pi}\int_0^1 \underbrace{\sqrt{1-\theta^4}}_{\cos\omega}\ \underbrace{\theta\,d\theta}_{\frac{\cos\omega}{2}d\omega}.$

Lab. 5.



$\sin\omega = \theta^2.$

$\cos\omega\,d\omega = 2\theta\,d\theta.$

$\sqrt{1-\theta^4} = \cos\omega$

$\sin\omega = 1^2$

$\omega = \pi/2.$

$\sin\omega = 0.$

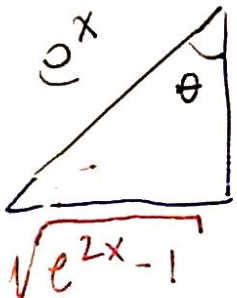
$\omega = 0.$

$$\frac{2}{\pi}\int_0^{\pi/2} \cos^2\omega\,d\omega = \frac{1}{\pi}\int_0^{\pi/2}(1+\cos 2\omega)\,d\omega. \qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$= \frac{1}{\pi}\left(\omega + \frac{1}{2}\sin 2\omega\Big]_0^{\pi/2}\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right) = \boxed{\frac{1}{2}.}$$

5. $\int_0^{\ln\sqrt{2}} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx. = \int \frac{e^{3x}}{\sqrt{e^{2x}-1}}\,e^x\,dx = \int \frac{\sec^3\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta.$



$e^x = \sec\theta$

$\sqrt{e^{2x}-1} = \tan\theta\,d\theta.$

$e^{3x} = \sec^3\theta.$

$e^x\,dx = \sec\theta\tan\theta\,d\theta.$

$\sec\theta = \cancel{e^{\ln\sqrt{2}}} = \sqrt{2}.$

$\sec\theta = e^0 = 1$

$\theta = 0.$

$\sec 0 = \frac{1}{\cos 0} = 1$

$$\int_0^{\pi/4} \sec^4\theta\,d\theta. = \int_0^{\pi/4} \sec^2\theta\,\sec^2\theta\,d\theta.$$

$$\int_0^{\pi/4}(1+\tan^2\theta)\sec^2\theta\,d\theta. = \int_0^1 (1+u^2)\,du.$$

$u = \tan\theta \qquad du = \sec^2\theta\,d\theta \qquad u(\pi/4) = \tan\pi/4 = 1$

$u(0) = \tan 0 = 0$

$$\int_0^1 (1+u^2)\,du = u + \frac{1}{3}u^3\Big]_0^1 = 1 + \frac{1}{3} = \frac{4}{3}.$$

5. $$\int_0^{\ln\sqrt{2}} \frac{e^{2x}}{\sqrt{e^{2x}-1}}\, e^{2x}\,dx. = \int_0^1 \frac{e^{2x}}{\sqrt{u}}\,\frac{du}{2}. \qquad e^{2x} = u+1$$

$$u = e^{2x} - 1 \qquad u(\ln\sqrt{2}) = e^{2\cdot\ln 2^{1/2}} - 1 = 2 - 1 = 1$$

$$\frac{du}{2} = e^{2x}\,dx \qquad u(0) = e^0 - 1 = 0$$

$$\frac{1}{2}\int_0^1 \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\int_0^1 u^{1/2} + u^{-1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\Big]_0^1\right)$$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{6}{3}\right) = \frac{4}{3}.$$

$$\int \frac{\overbrace{e^{2x}}^{u+1}}{\sqrt{e^{2x}-1}}\,\overbrace{e^{2x}\,dx}^{du/2.} \qquad \int \frac{u+1}{\sqrt{u}}\,\frac{du}{2}$$

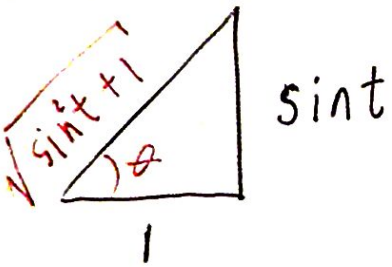
$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C.$$

$$= \frac{1}{3}(e^{2x}-1)^{3/2} + (e^{2x}-1)^{1/2}\Big]_0^{\ln\sqrt{2}}$$

Problema 2 b) Simulacro.

$$\int_0^{\pi/2} \frac{\cos t}{\sqrt{\sin^2 t + 1}}\, dt = \int \frac{\sec^2\theta\, d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta\, d\theta.$$



$\sin\theta$

$\tan\theta = \sin t.$

$\sec^2\theta\, d\theta = \cos t\, dt.$

$\sqrt{\sin^2 t + 1} = \sec\theta.$

$\tan\theta = \sin\frac{\pi}{2} = 1$

$\theta = \tan^{-1}(1) = \pi/4$

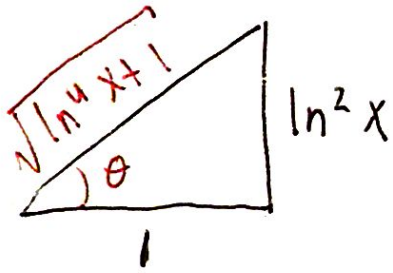
$\tan\theta = \sin 0 = 0$

$\theta = 0$

$$\int_0^{\pi/4} \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4} = \ln|\sec\pi/4 + \tan\pi/4) - \ln(\sec 0 + \tan 0)$$

$$= \ln(\sqrt{2}+1) - \cancel{\ln(1)}^{\,0} = \underline{\ln(\sqrt{2}+1)}$$

$$\int \frac{4}{\sqrt{\ln^4 x + 1}} \frac{2(\ln x)}{x}\, dx = 4\int \frac{\sec^2\theta\, d\theta}{\sec\theta} = 4\int \sec\theta\, d\theta.$$



$\tan\theta = [\ln x]^2$

$\sec^2\theta\, d\theta = 2\ln x \frac{1}{x}\, dx.$

$\sqrt{\ln^4 x + 1} = \sec\theta.$

$$4\int \sec\theta\, d\theta = 4\ln|\sec\theta + \tan\theta| + C.$$

$$4\ln\left|\sqrt{\ln^4 x + 1} + \ln^2 x\right| + C.$$

$$\int \frac{(x-2)^3}{\sqrt{x^2-4x+13}}\,dx.$$

$u = x^2-4x+13$

$du = 2x-4 = 2(x-2)\,dx$

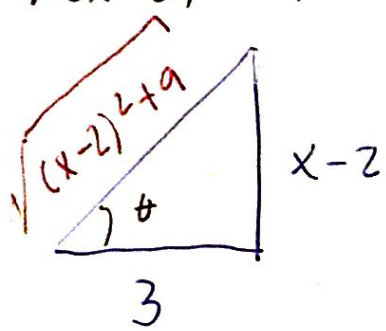
completed cuadrado. $(x^2-4x+4)+13-4$

$(x-2)^2+9.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = a\cdot \operatorname{sen}\theta.$

$1\cdot dx = a\cdot\cos\theta\, d\theta.$



$3\cdot\tan\theta = x-2$

$3\sec^2\theta\, d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$(x-2)^3 = 3^3\tan^3\theta.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\, d\theta.$$

$9\tan^2\theta + 9$

$9\sec^2\theta.$

$$= 3^3\int \tan^3\theta\sec\theta\, d\theta.$$

$$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\, d\theta)$$

$$= 27\int(\sec^2\theta-1)\,(\tan\theta\sec\theta\, d\theta).$$

$du = \sec\theta\tan\theta\, d\theta.$

$u = \sec\theta$

$$= 27\int(u^2-1)\,du = 9u^3 - 27u + C.$$

$\sec\theta = \dfrac{\sqrt{x^2-4x+13}}{3}.$

$9\sec^3\theta - 27\sec\theta + C.$

$$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C.$$

$$\int \frac{x^3}{\sqrt{x^2-4x+13}}\,dx$$

$u = x^2-4x+13$

$du = (2x-4)\,dx = 2(x-2)\,dx$

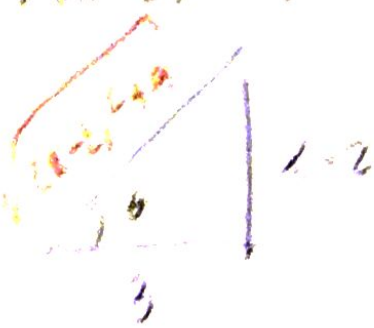
completa el cuadrado $x^2-4x+4 = 13-4$

$(x-2)^2 = 9$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = 3\sec\theta$

$dx = 3\cos\theta\,d\theta$



$3\tan\theta = x-2$

$3\sec^2\theta\,d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$(x-2)^3 = 3^3\tan^3\theta.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\,d\theta.$$

$1\tan^2\theta + 9$

$9\sec^2\theta$

$= 3^3\int \tan^3\theta\sec\theta\,d\theta.$

$= 27\int \tan^2\theta\,\tan\theta\sec\theta\,d\theta;$

$= 27\int(\sec^2\theta-1)(\tan\theta\sec\theta\,d\theta);$

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$= 27\int(u^2-1)\,du = 9u^3 - 27u + C.$

$\sec\theta = \frac{\sqrt{x^2-4x+13}}{3}$

$9\sec^3\theta - 27\sec\theta + C.$

$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C.$

Capítulo 11

Simulacro de parcial # 1

Simulacro Parcial

a) $\int x \tan^{-1} x^2 \, dx$ $\qquad \int x f(x) \, dx$ IPP.

ILATE

$y = x^2 \qquad dy = 2x\,dx$

$$\int \tan^{-1}(x^2)(x\,dx) = \frac{1}{2}\int \tan^{-1}(y)\,dy.$$

IPP: $u = \tan^{-1} y \qquad dv = \frac{dy}{2}$

$du = \frac{1}{1+y^2} \qquad v = \frac{y}{2}.$

$$uv - \int v\,du = \frac{1}{2} y \tan^{-1} y - \frac{1}{2}\int \frac{y}{1+y^2}\,dy.$$

$w = 1 + y^2$

$dw = 2y\,dy.$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\int \frac{dw}{w}$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4} \ln|w| + C.$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4} \ln|1+y^2| + C.$$

$$= \boxed{\frac{1}{2} x^2 \tan^{-1}(x^2) - \frac{1}{4} \ln|1+x^4| + C.}$$

b) $\int \frac{xe^x}{(x+1)^2}\,dx$

Tres funciones

	Der.			
	x Int.			
	xe^x	ó	$\frac{x}{(x+1)^2}$	x Int
Der. x				x der
Int ✓	$\frac{1}{(x+1)^2}$		e^x	✓

$u = xe^x \qquad dv = (x+1)^{-2}\,dx$

$du = (e^x + xe^x)\,dx, \quad v = \dfrac{(x+1)^{-1}}{-1} = \dfrac{-1}{(x+1)}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = \frac{-xe^x}{(x+1)} + \int \frac{(e^x + xe^x)\,dx}{(x+1)}$$

inspiración,

$$= \frac{-xe^x}{(x+1)} + \int e^x\,dx$$

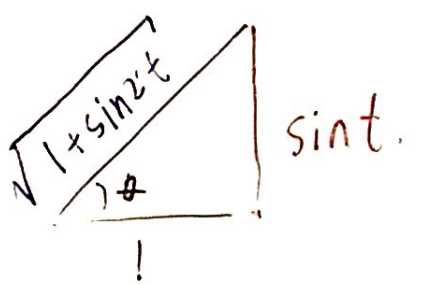
$$= \frac{-xe^x}{(x+1)} + e^x + C$$

Otras sugerencias: $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx \qquad u = x+1, \quad du = dx, \quad x = u-1.$

$\displaystyle\int \frac{(u-1)}{u^2}\,e^{u-1}\,dx$ NO AYUDA. "MUCHO"

2b) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt. = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta\,d\theta.$

$\sin t = \tan\theta. \qquad \cos t\,dt = \sec^2\theta\,d\theta.$



$\sqrt{1+\sin^2 t} = \sec\theta.$

cambie los límites.

$\tan\theta = \sin \pi/2 = 1 \Rightarrow \theta_b = \pi/4.$

$\tan\theta = \sin 0 = 0 \Rightarrow \theta_a = 0$

$$\int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}$$

$\sec 0 = 1 \quad \tan 0 = 0$

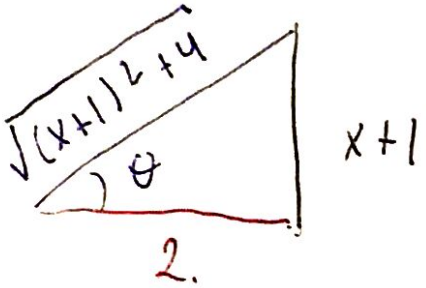
$\sec\frac{\pi}{4} = \sqrt{2} \quad \tan\frac{\pi}{4} = 1$

$$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \cancel{\ln(1)} = \ln(\sqrt{2} + 1).$$

Conto 4A. $\displaystyle\int \frac{1}{(x^2+2x+5)^2}\,dx = \int \frac{1}{[(x+1)^2+4]^{4/2}}\,dx$

$x^2+2x+1 \quad +5-1 = (x+1)^2+4$



$\tan\theta = \dfrac{x+1}{2}$ $\qquad x+1 = 2\tan\theta.$

$dx = 2\sec^2\theta\, d\theta.$

$\dfrac{((x+1)^2+4)^{1/2}}{2} = \sec\theta.$

$[(x+1)^2+4)]^2 = 16\sec^4\theta.$

$$\int \frac{1}{[(x+1)^2+4]^2}\,dx = \int \frac{2\sec^2\theta\, d\theta}{16\sec^4\theta} = \frac{1}{8}\int \frac{1}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{8}\int \cos^2\theta\, d\theta. = \frac{1}{16}\int 1+\cos(2\theta)\, d\theta.$$

$$= \frac{1}{16}\left(\theta + \frac{1}{2}\underline{\sin(2\theta)}\right) + C.$$

$\underline{2\sin\theta\cos\theta}$

$$= \frac{1}{16}(\theta + \sin\theta\cos\theta) + C.$$

$\tan\theta = \dfrac{x+1}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{x+1}{2}\right).$ $\quad \sin\theta = \dfrac{x+1}{\sqrt{x^2+2x+5}}$

$\cos\theta = \dfrac{2}{\sqrt{x^2+2x+5}}$

$$\int \frac{1}{(x^2+2x+5)^2}\,dx = \frac{\tan^{-1}\left(\frac{x+1}{2}\right)}{16} + \frac{1}{16}\,\frac{(x+1)}{\sqrt{x^2+2x+5}}\,\frac{2}{\sqrt{x^2+2x+5}} + C.$$

$$\frac{1}{16}\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{8}\,\frac{x+1}{x^2+2x+5} + C.$$

Inp $\int (x-1)\sin \pi x \, dx$ $\int e^{-\theta}\cos 2\theta \, d\theta.$

$u = x-1$ $dv = \sin \pi x$

$du = dx$ $v = \frac{-1}{\pi}\cos \pi x$

$$\int (x-1)\sin \pi x \, dx = \frac{-(x-1)}{\pi}\cos \pi x + \int \frac{1}{\pi}\cos \pi x \, dx$$

$$= \frac{(1-x)}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + C.$$

Ciclico

$$* \int e^{-\theta}\cos 2\theta \, d\theta. = -\frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta \, d\theta$$

$u = e^{-\theta}$ $dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta} d\theta.$ $v = \frac{1}{2}\sin 2\theta.$

$$\int e^{-\theta}\sin 2\theta \, d\theta. = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta \, d\theta. *$$

$u = e^{-\theta}$ $dv = \sin 2\theta \, d\theta.$

$du = -e^{-\theta} d\theta$ $v = -\frac{1}{2}\cos 2\theta$

$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta. - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}\int e^{-\theta}\cos 2\theta \, d\theta.$$

$$\frac{5}{4}\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta.$$

$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C.$$

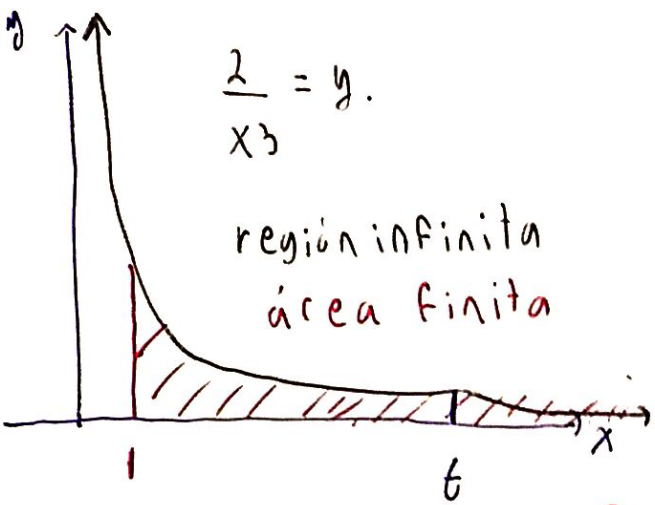
Capítulo 12

Integrales impropias

Fracciones Parciales Pág. 66 (Posteriormente).

7.8 Integrales Impropias.

Considere la región bajo la curva $y = \frac{2}{x^3}$ encima del eje-x y a la derecha de la recta $x = 1$.



$\frac{2}{x^3} = y.$

región infinita

área finita

$$A = \int_1^t 2x^{-3}\,dx$$

$$A = \frac{2}{-2}\,x^{-2}\Big]_1^t$$

$$A = -1 \cdot t^{-2} + \underbrace{1 \cdot 1^{-2}}_{1} = 1 - \frac{1}{t^2}.$$

$$\lim_{t\to\infty} A = \lim_{t\to\infty} 1 - \cancel{\frac{1}{t^2}}^{\,0} = 1$$

$$\boxed{\int_1^\infty \frac{2}{x^3}\,dx = 1}$$

Límites Básicos

a. $\lim_{x\to\infty} \frac{1}{x^r} = 0$ $\quad \frac{1}{\infty}.$ $\qquad \lim_{x\to\infty} x^r = +\infty.$

r positivo

b. $\lim_{x\to\infty} e^x = \infty$ $\quad e^\infty$ $\qquad \lim_{x\to-\infty} e^x = 0$ $\quad \frac{1}{e^\infty} \to 0.$

c. $\lim_{x\to 0^+} \ln x = -\infty$ $\qquad \lim_{x\to\infty} \ln x = +\infty.$

$$\log_{10} \underbrace{10^{-48}}_{0^+} = -48 \qquad \log_{10} 10^{-10{,}000} = -10{,}000$$

Integrales Impropias:

Tipo 1: Intervalos infinitos $\pm\infty$.

Tipo 2: Funciones discontinuas. (AVs en $x = \pm a$).

Integrales Impropias Tipo 1: (Pág. 74)

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

Integre 1ro
Evalúe límite.

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^a f(x)\,dx.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

CONVERGENTE: (Se acerca a un número) el límite existe.

DIVERGENTE: (La integral a $\pm\infty$) el límite no existe.

Ejercicio 1: Evalúe. (P. 74) $\sqrt{\infty} \Rightarrow \infty$

a. $$\int_1^{\infty} x^{-1/2}\,dx = 2x^{1/2}\Big]_1^{\infty} = \lim_{x\to\infty} 2\sqrt{x} - 2 = \infty.$$

no existe.

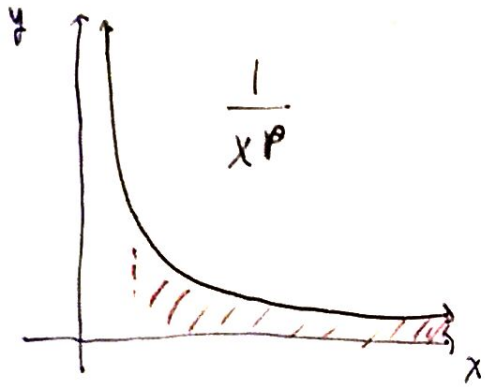
$p = \frac{1}{2}$ DIVERGENTE.

b. $$\int_1^{\infty} \frac{1}{x}\,dx = \ln x\Big]_1^{\infty} = \lim_{x\to\infty} \ln x - 0 = \infty.$$

no existe.

$p = 1$

DIVERGENTE



$\int_1^\infty \frac{1}{x^p}\,dx$ no necesariamente existe.

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} p \leq 1 & \text{DIVERGE.} \\ p > 1 & \text{converge.} \end{cases}$$

$\therefore p = 0.99$ $\quad \int_1^\infty x^{-0.99}\,dx = \left.\frac{x^{0.01}}{0.01}\right]_1^\infty = \lim_{x\to\infty} x^{0.01} - \frac{1}{0.01} = +\infty.$

DIVERGE.

$p = 1.001$ $\quad \int_1^\infty x^{-1.001}\,dx = \left.\frac{x^{-0.001}}{-0.001}\right]_1^\infty = \lim_{x\to\infty} \frac{1000}{x^{0.001}} + \frac{1}{0.001}$

$x^{-0.001}$

$\frac{1000}{\infty} \to 0$

$= \left.\frac{1000}{x^{0.001}}\right]_\infty^1 = 1000 - \lim_{x\to\infty} \frac{1000}{x^{0.001}}$ 0

$= 1000$ converge.

Ejercicio 3: Evalúe.

a. $\int_{-\infty}^0 e^{-x^2} \underbrace{x\,dx}_{du/-2} = \int_{-\infty}^0 e^u \frac{du}{-2} = -\frac{1}{2} e^u \Big]_{-\infty}^0 = -\frac{1}{2} e^0 + \cancel{\frac{1}{2} e^{-\infty}}$ 0

$u = -x^2$ $\qquad u(0) = -0^2$

$du = -2x\,dx$ $\qquad u(-\infty) = -(-\infty)^2 = -\infty$

$= -\frac{1}{2} + 0$

$= -\frac{1}{2}$ converge.

Notación Abreviada: $\quad e^{-\infty} = \lim_{x\to-\infty} e^x = 0$

$$f(\infty) = \lim_{x\to\infty} f(x)$$

b. $\frac{1}{2}\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1}(x)\Big]_{-\infty}^{\infty} = \frac{\pi}{4}+\frac{\pi}{4} = \frac{\pi}{2}.$

$= \frac{1}{2}\tan^{-1}(\infty) - \frac{1}{2}\tan^{-1}(-\infty)$ CONVERGE

$\tan x$ ID: $(-\frac{\pi}{2}, \frac{\pi}{2})$

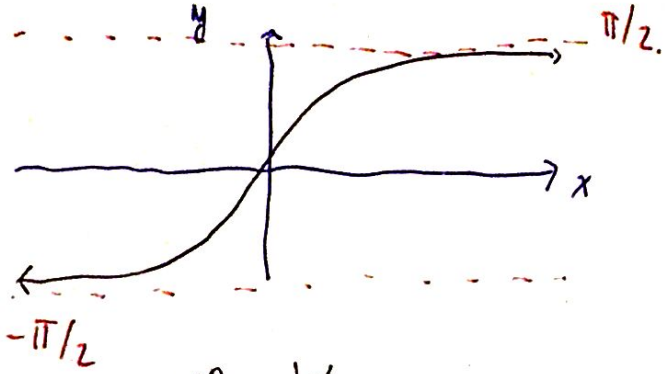
R: $(-\infty, \infty)$

A.V $x = -\pi/2, +\frac{\pi}{2}$

$\tan^{-1} x$ ID: $(-\infty, \infty)$

R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

A.H. $y = \pm \pi/2.$



$\tan^{-1}(\infty) = \pi/2$

$\tan^{-1}(-\infty) = -\pi/2.$

David Corzo.

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \underline{\underline{\pi}}. \approx \int_{-1000}^{1000}\frac{dx}{1+x^2} \approx \sum_{i=1}^{n}\frac{1}{1+x_i^2}\Delta x$$

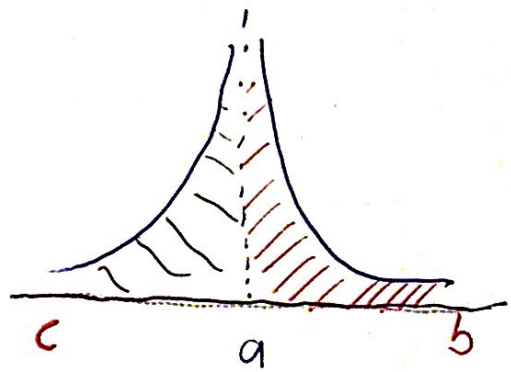
PYTHON.

Integrales Impropias Tipo 2.

Hay una asíntota vertical en $x = a$.

$$\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$$

$$\int_c^a f(x)dx = \lim_{t\to a^-}\int_c^t f(x)dx$$



AV $x=a$

$$\int_c^b f(x)dx = \int_c^a f(x)dx + \int_a^b f(x)dx$$

Ejercicio 4: Evalúe. Indique donde es discontinua.

a. $\displaystyle\int_{1}^{9} \frac{1}{\sqrt[3]{x-1}}\,dx = \int_{0}^{8} u^{-1/3}\,du = \frac{3}{2}u^{2/3}\Big]_{0^+}^{8} =$

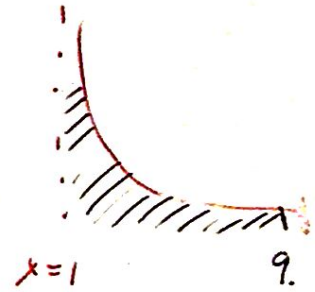
Discontinua: denominador igual a cero $1/0$.

en $x=1$ logaritmo de cero $\ln(0) \to -\infty$.

raíz cuadrada de un número negativo.

$u = x-1 \qquad u(9) = 8$

$du = dx \qquad u(1) = 0$



$$\frac{3}{2}u^{2/3}\Big]_{0^+}^{8} = \frac{3}{2}(8^2)^{1/3} - \frac{3}{2}\lim_{u\to 0^+} u^{2/3}.$$

$$\frac{3}{2}\sqrt[3]{64} - 0 = \frac{3}{2}\cdot 4 = 6 \qquad \text{CONVERGE}$$

b. $\displaystyle\int_{-2}^{3} \frac{3}{x^4}\,dx = \underbrace{\int_{-2}^{0} 3x^{-4}\,dx}_{(1)} + \underbrace{\int_{0}^{3} 3x^{-4}\,dx}_{(2)} = \infty$ diverge.

Discontinua en $x=0$

(2) $\displaystyle\int_{0}^{3} 3x^{-4}\,dx = -x^{-3}\Big]_{0^+}^{3} = -\frac{1}{3^3} + \overbrace{\lim_{x\to 0^+}\frac{1}{x^3}}^{+\infty} = +\infty.$

(1) $\displaystyle\int_{-2}^{0} 3x^{-4}\,dx = -x^{-3}\Big]_{-2}^{0^-} = \lim_{x\to 0^-} \underbrace{-\frac{1}{x^3}}_{+\infty} + \frac{1}{(-2)^3} = +\infty.$

6.

c. $\int_0^1 \ln x \, dx$

$\int \ln x \, dx = x\ln x - \int dx = x\ln x - x + C.$

$u = \ln x \qquad dv = dx$

$du = \frac{dx}{x} \qquad v = x$

$0 \cdot \infty$

$0 \cdot \ln 0$

discontinua en $x=0$

$\int_0^1 \ln x \, dx = x\ln x - x \Big]_{0^+}^{1} = 1\cdot\ln(1) - 1 - \lim_{x\to 0^+} x\ln x$

A.V.

$= -1 - \boxed{\lim_{x\to 0^+} x\ln x}$

Regla de L'Hospital $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$

$0/0$ ó ∞/∞.

También aplica para $0\cdot\infty$, 1^∞, ∞^0, 0^0.

$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1}} \overset{LH}{=} \lim_{x\to 0^+} \frac{1}{x(-x^{-2})}$

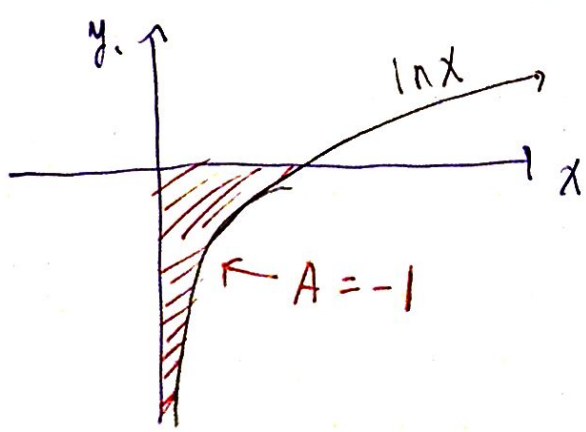
∞/∞

$(\ln x)^{-1} \to -(\ln x)^{-2}\frac{1}{x}.$

$\lim_{x\to 0^+} \frac{1}{-x^{-1}} = \lim_{x\to 0^+} -x$

$= 0$

$\int_0^1 \ln x \, dx = -1 - \lim_{x\to 0} x\ln x = -1 + 0$ CONVERGE.



$\frac{\ln x}{x} \to \frac{x^{-1}}{-x^{-2}} = -x$

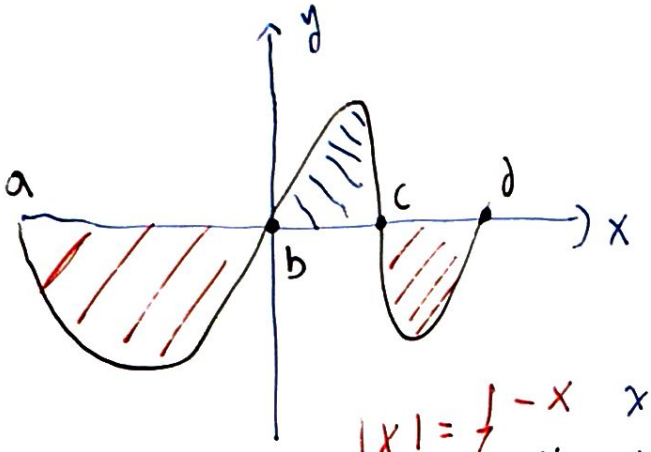
$\lim_{x\to 0^+} -x = 0.$

Capítulo 13

6.1 Área entre curvas

6.1 Áreas entre curvas (p 79)

Región entre la curva $y = f(x)$ y el eje x.



Área de la región

$$A = \int_a^d |f(x)|\,dx$$

$$A = -\int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx$$

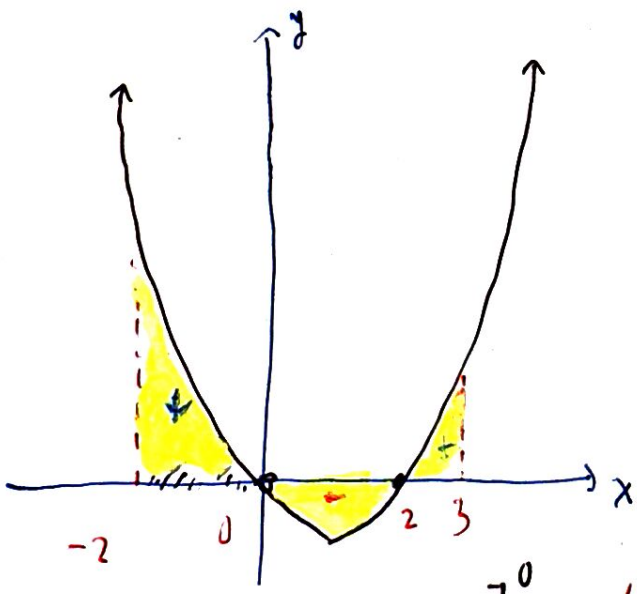
$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

Intersecciones y bosquejar la curva y la región,

Ejercicio 1: Bosqueje y encuentre el área de la región limitada por $y = 3x^2 - 6x$, $x = -2$, $x = 3$ & $y = 0$.

Interceptos-x

$$y = 3x^2 - 6x = 0$$
$$3x(x-2) = 0 \Rightarrow x = 0,\ x = 2.$$



Sume el área de 3 subregiones

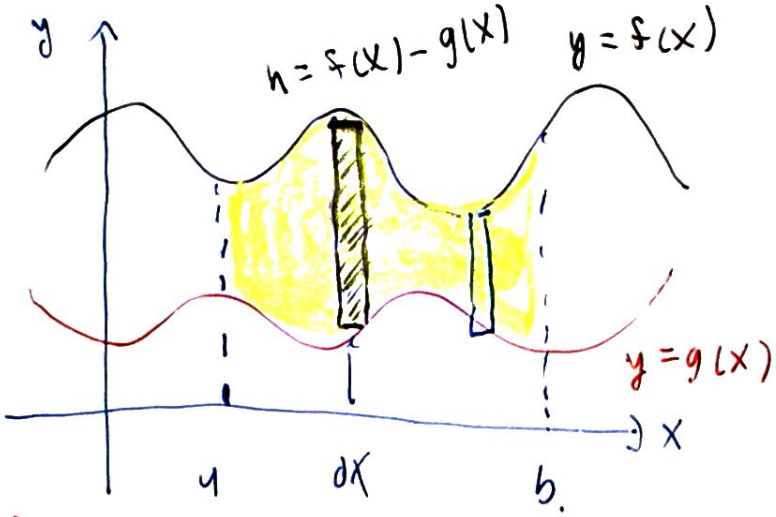
$$A = \int_{-2}^{0} 3x^2 - 6x\,dx + \int_0^2 -3x^2 + 6x\,dx + \int_2^3 3x^2 - 6x\,dx$$

$$A = x^3 - 3x^2\Big]_{-2}^{0} + \left(-x^3 + 3x^2\Big]_0^2\right) + x^3 - 3x^2\Big]_2^3$$

$$A = 0 - (-8 - 12) + (-8 + 12) + (\cancel{27 - 27}^{\,0} - 8 + 12) = 28$$

$A = 20 + 4 + 4 = 28$

¿Qué sucede cuando hay una curva inferior?



Región: $g(x) \leq y \leq f(x)$

$a \leq x \leq b.$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Diferencia de áreas.

$A = bh.$

$$\boxed{A = \int_a^b y_{arriba} - y_{abajo}\,dx}$$

Franja horizontal ó rectangulo infinitesimal. ancho dx

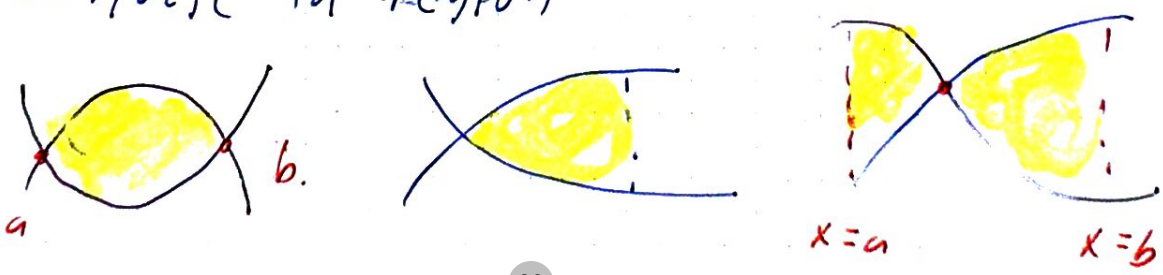
altura $f(x) - g(x)$

$$dA = (f(x) - g(x))\,dx$$

Integrando en $a \leq x \leq b$. $\int dA = A$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

1. Bosquejar las curvas f & g.
2. Intersectos entre las curvas.
3. Bosqueje la región

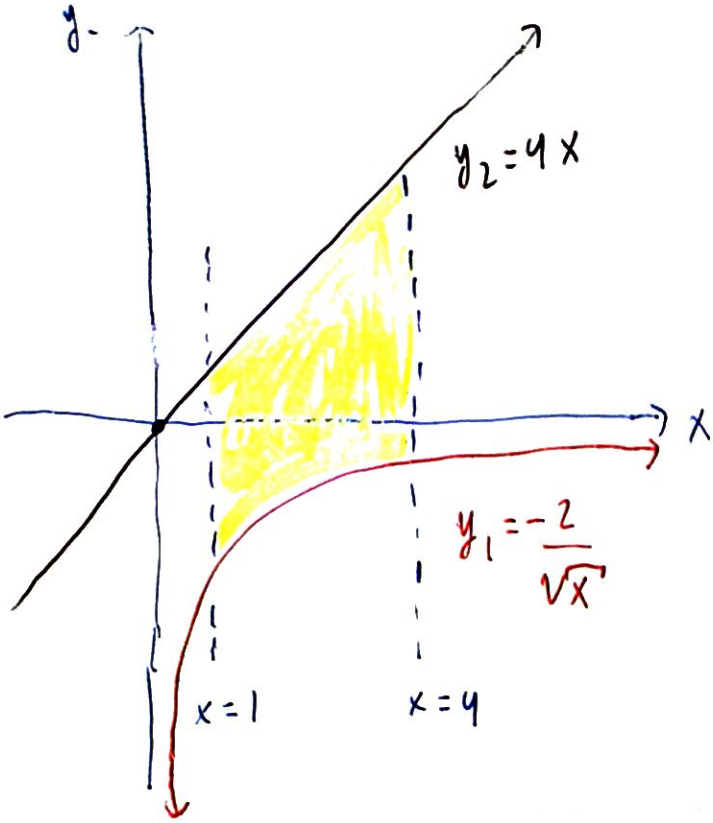


Ejemplo: Bosqueje y encuentre el área entre

$y_1 = \frac{-2}{\sqrt{x}}$ & $y_2 = 4x$ en $1 \leq x \leq 4$.

¿ $y_2 > y_1$? ó ¿ $y_1 > y_2$?

$\frac{1}{x^r}$



$$A = \int_1^4 y_2 - y_1 \, dx$$

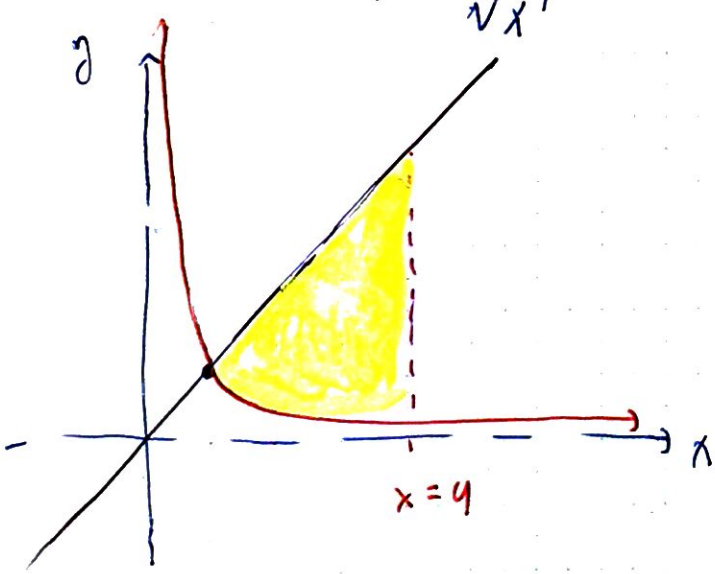
$$A = \int_1^4 4x + 2x^{-1/2} \, dx$$

$$A = 2x^2 + 4x^{1/2} \Big]_1^4$$

$$A = 2 \cdot 16 + 4 \cdot 2 - (2 + 4).$$

$$A = 32 + 8 - 6 = 34.$$

Variación B: $y_1 = \frac{+4}{\sqrt{x}}$ & $y_2 = 4x$ y la recta $x = 4$.



$$A = \int_1^4 y_2 - y_1 \, dx$$

intersección entre y_1 & y_2.

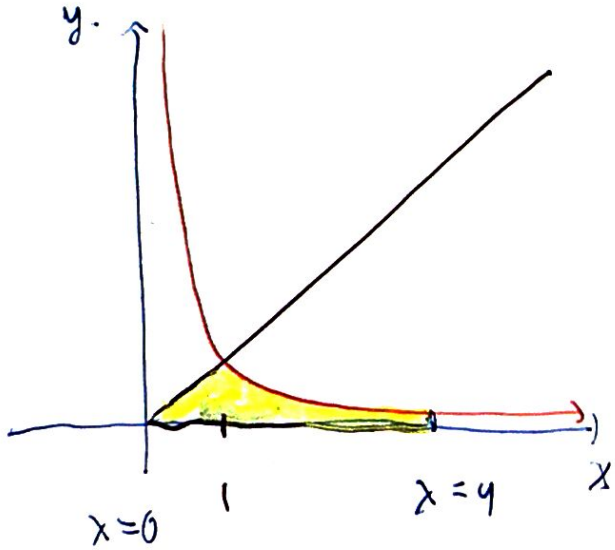
$$\frac{4}{x^{1/2}} = 4x$$

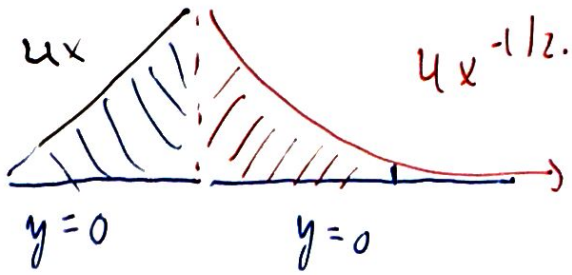
$$1 = x^{3/2} \Rightarrow x = 1$$

$$A = \int_1^4 4x - 4x^{-1/2}\,dx = 2x^2 - 8x^{1/2}\Big]_1^4$$

$$A = 32 - 16 - (2-8) = 16 + 6 = 22.$$

Variación C: Área de la región entre $y_1 = 4x^{-1/2}$, $y_2 = 4x$, $x = 4$ & $y = 0$.



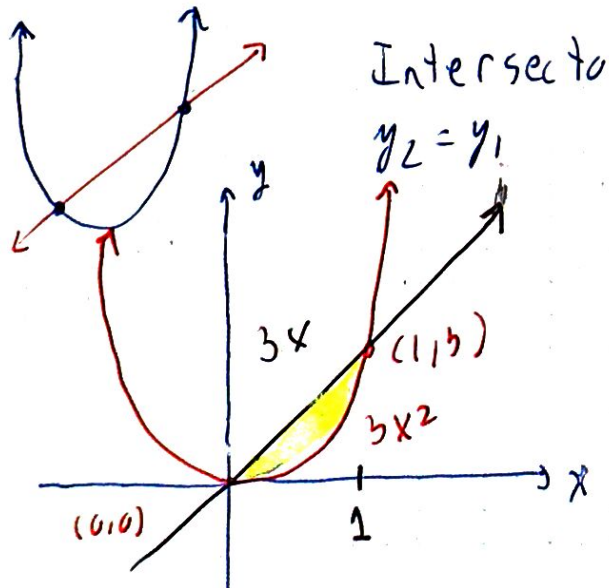


$$A = \int_0^1 4x\,dx + \int_1^4 4x^{-1/2}\,dx$$

$$A = 2x^2\Big]_0^1 + 8x^{1/2}\Big]_1^4$$

$$A = 2 + 16 - 8 = 10$$

Ejemplo: Encuentre el área de la región entre las curvas $y_1 = 3x$ & $y_2 = 3x^2$.



Intersectos $y_2 = y_1$

$3x^2 = 3x$ $\quad x=0$ & $x=1$

$3x^2 - 3x = 3x(x-1) = 0$

$$A = \int_0^1 (3x - 3x^2)\,dx$$

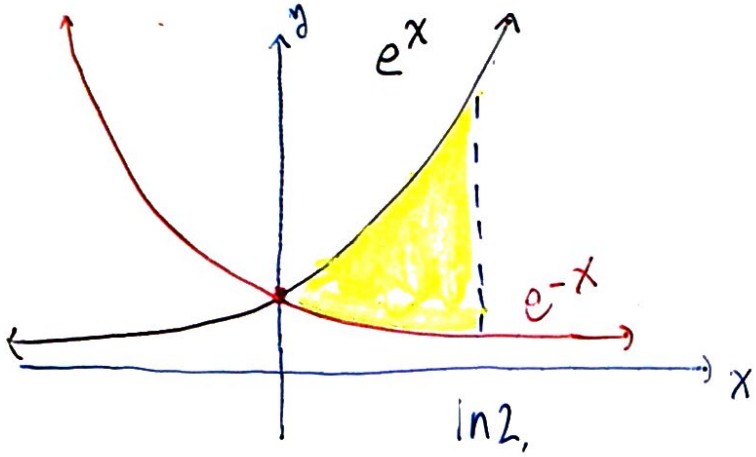
$$A = \frac{3}{2}x^2 - x^3\Big]_0^1 = \frac{3}{2} - 1 - 0$$

$$A = \frac{1}{2}.$$

Ejercicio 2: Bosqueje y encuentre el área de la región entre las curvas.

a) $y_1 = e^x$, $y_2 = e^{-x}$, $x = 0$, $x = \ln(2)$.

crece $\to +\infty$ decrece $\to 0$ $x = 0$ $y_1 = y_2 = 1$



$$A = \int_0^{\ln 2} (e^x - e^{-x})\,dx$$

$$A = e^x + e^{-x} \Big|_0^{\ln 2}$$

$$e^{\ln 2} = 2, \quad e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$$

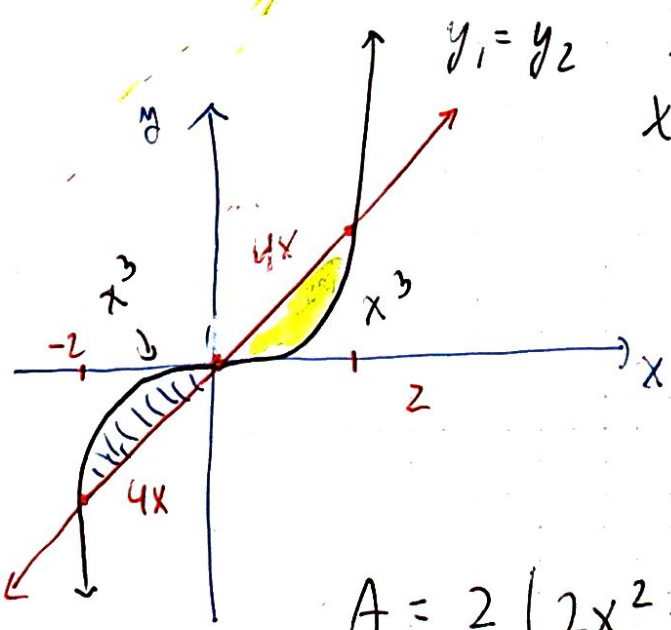
$$e^{\ln 2^{-1}}$$

$$A = e^{\ln 2} + e^{-\ln 2} - (e^0 + e^{-0})$$

$$A = 2 + \frac{1}{2} - 2 = \frac{1}{2}.$$

b) $y_1 = x^3$ & $y_2 = 4x$

2 regiones distintas y 3 intersectos.



$y_1 = y_2$ $\quad x^3 - 4x = 0$

$x(x^2 - 4) = 0 \Rightarrow x = 0, \pm 2.$

$y_1 = y_2(2) = 8.$

$$A = \int_{-2}^{0} (x^3 - 4x)\,dx + \int_0^2 (4x - x^3)\,dx$$

$$A = 2\int_0^2 (4x - x^3)\,dx$$

$$A = 2\left(2x^2 - \frac{x^4}{4}\Big]_0^2\right) = 2\left(8 - \frac{16}{4}\right) = 2 \cdot 4 = 8$$

c) $y_1 = x^2 - 4x + 4$, $y_2 = 10 - x^2$.

Dos parábolas.

$$A = \int_a^b y_2 - y_1 \, dx$$

Intersecciones $y_1 = y_2$. $x^2 - 4x + 4 = 10 - x^2$.

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 2(x-3)(x+1) =$$

intersectos en $x = -1$ y $x = 3$.

$$A = \int_{-1}^{3} 10 - x^2 - (x^2 - 4x + 4) \, dx$$

$$A = \int_{-1}^{3} (6 - 2x^2 + 4x) \, dx$$

$$A = 6x - \frac{2}{3}x^3 + 2x^2 \Big]_{-1}^{3} = 18 - 18 + 18 - \left(-6 + \frac{2}{3} + 2\right)$$

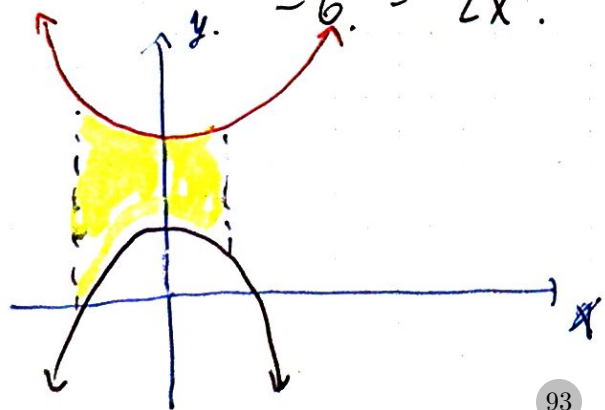
$$18 + 4 - 2/3 = 22 - \frac{2}{3}.$$

Alejandro Área entre $y_1 = 4 - x^2$ & $y_2 = 10 + x^2$, $x = a$, $x = b$

$y_1 = y_2$: $4 - x^2 = 10 + x^2$

$-6 = 2x^2$.

No hay intersectos.



$$A = \int_a^b y_2 - y_1 \, dx.$$

Capítulo 14

Área entre curvas, integración respecto al eje-y, respecto al eje-x, introducción a volúmenes de sólidos, volúmen de un cilíndro

Avisos: Participación Actividad Mate Compu.
Hasta 1.5 netos y 0.5 neto mínimo.
Lunes y martes.

Zona WA (9 pts.). → 18-17 pts Parcial.

Parcial 10 octubre, 3 octubre. Simulacro 2.
Bota Parcial 1.

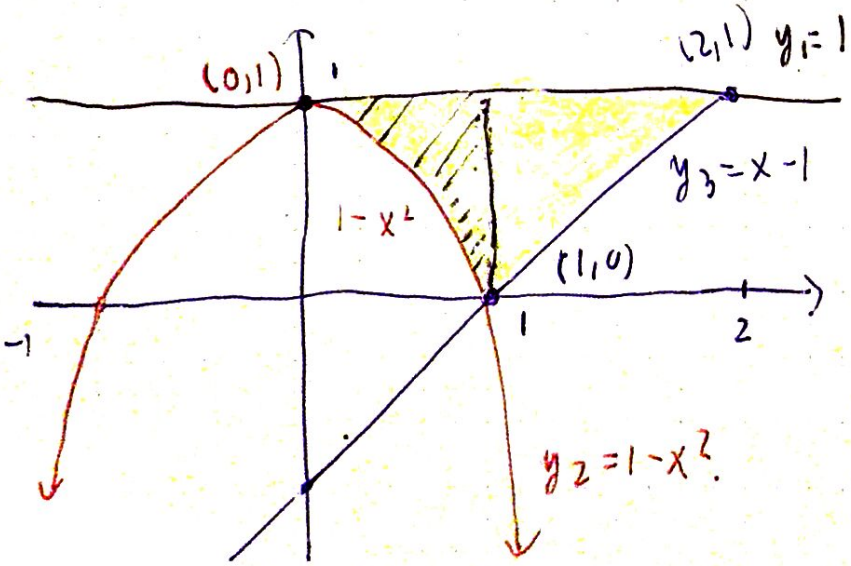
2:30 PM CES.

1-2:30 PM Almuerzo o Repaso con Samuel

Aplicaciones Integración:

- Planteamiento
- Gráfica de una Región. → área / volumen sólido en revolución

Ejercicio 3: Encuentre de la región entre $y_1 = 1$, $y_2 = 1 - x^2$ & $y_3 = x - 1$.



$y_1 = y_2$ D.I (0,1)

$y_1 = y_3$ $x - 1 = 1$
$x = 2$ (2,1)

$y_2 = y_3$ en (1,0)

$\int f - g \, dx$

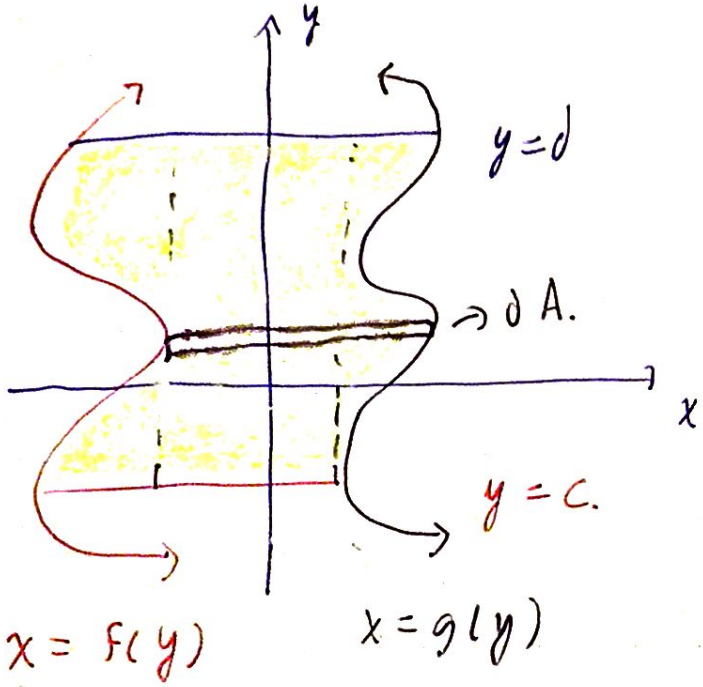
$$A = \int_0^1 (1-(1-x^2))\,dx + \int_1^2 \underbrace{1-(x-1)}_{1/2.}\,dx$$

$$A = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx$$

$$A = \frac{1}{3}x^3\Big]_0^1 + 2x - \frac{x^2}{2}\Big]_1^2$$

$$A = \frac{1}{3} + 4 - \frac{4}{2} - \left(2-\frac{1}{2}\right) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

Integración en el eje-y: Franjas Horizontales
Derecha - Izquierda.



Región S $\quad f(y) \le x \le g(y)$
$c \le y \le d.$

rectángulo infinitesimal.
altura dy.
ancho $g(y) - f(y)$
$dA = [g(y) - f(y)]\,dy.$

$$A = \int_c^d [g(y) - f(y)]\,dy.$$

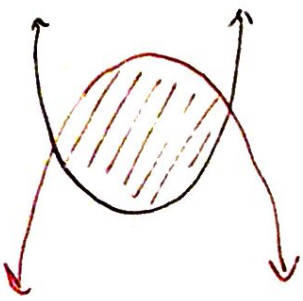
$$A = \int_a^b y_{arriba} - y_{abajo}\,dx$$

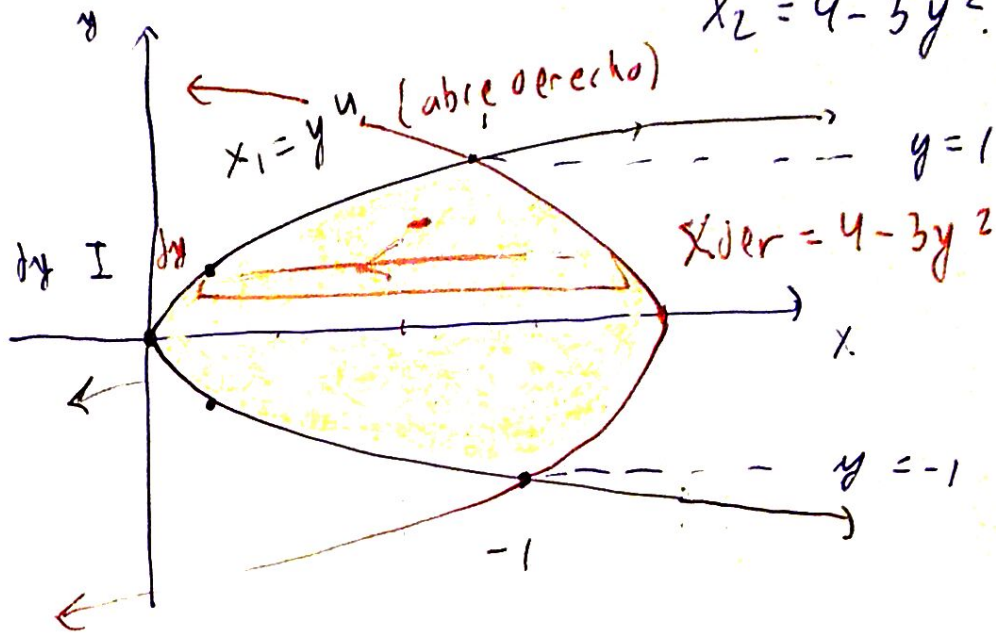
$y_{arriba} = f(x)$
$y_{abajo} = g(x)$

$$A = \int_c^d x_{der} - x_{izq}.\,dy.$$

$x_d = g(y) \quad x_{iz} = f(y).$

Ejercicio 4: Encuentre el área entre $x_1 = y^4$ & $x_2 = 4-3y^2$.





$x_{izq} = y^4$

(4 intersecc(s))

Pts. de intersección $x_1 = x_2$: $y^4 = 4-3y^2$

$y^4 + 3y^2 - 4 = 0$

$(y^2 \quad +4)(y^2-1) = 0$

$y^2 \neq -4$. imaginarios.

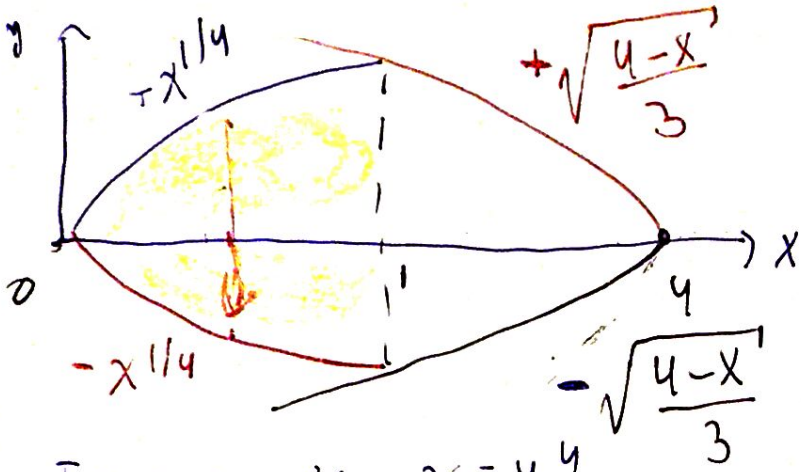
$y^2 = 1 \Rightarrow y = \pm 1$

$$A = \int_{-1}^{1} x_{der} - x_{izq}\, dy = \int_{-1}^{1} 4-3y^2 - y^4\, dy.$$ PAR

$$A = 2\int_0^1 4-3y^2-y^4\, dy = 2\left(4y - y^3 - \frac{y^5}{5}\Big]_0^1\right)$$

$$A = 2\left(3 - \frac{1}{5}\right) = 2\left(\frac{15}{5} - \frac{1}{5}\right) = \frac{28}{5}.$$

Integrando en el eje - x.



$\int f(x)-g(x)\,dx$

1. $x=y^4 \qquad x=4-3y^2$

Resuelva para x

$a^2=b \Rightarrow a=\pm\sqrt{b}$

2. $\int_0^1 f-g\,dx+\int_1^4 h-i\,dx$

Inversa de $x=y^4$

$y=\pm x^{1/4}$; $\pm\sqrt[4]{x}$

$x=4-3y^2 \Rightarrow x-4=-3y^2. \qquad y^2=\frac{4-x}{3}, \; y=\pm\sqrt{\frac{4-x}{3}}$

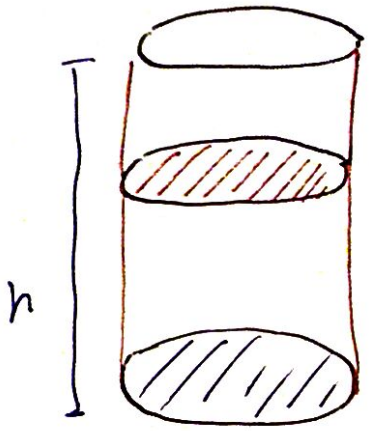
$$A=\int_0^1 x^{1/4}-(-x^{1/4})\,dx+\int_1^4\sqrt{\frac{4-x}{3}}-\left(-\sqrt{\frac{4-x}{3}}\right)dx$$

$$A=2\int_0^1 x^{1/4}dx+2\int_1^4\left(\frac{4-x}{3}\right)^{1/2}dx=\frac{28}{5}.$$

Menos 40 (2pts. P1) >50 (1 pt. P1.

6.2. Volúmenes (P 87)

Volumen de un cilindro circular



(rebanado)

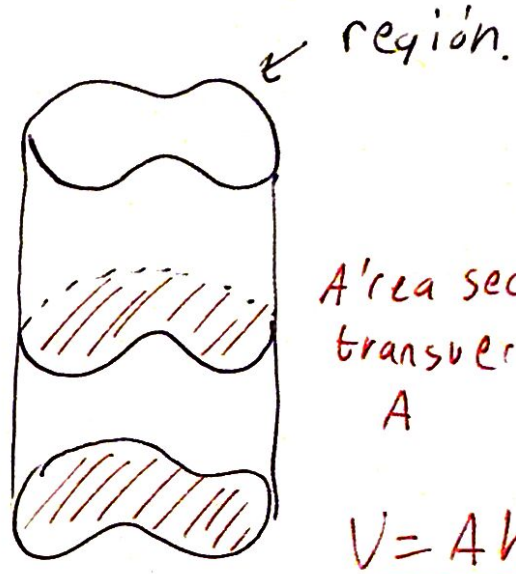
sección transversal

$A = \pi r^2$.

$V = Ah$.

$V = \pi r^2 h$.

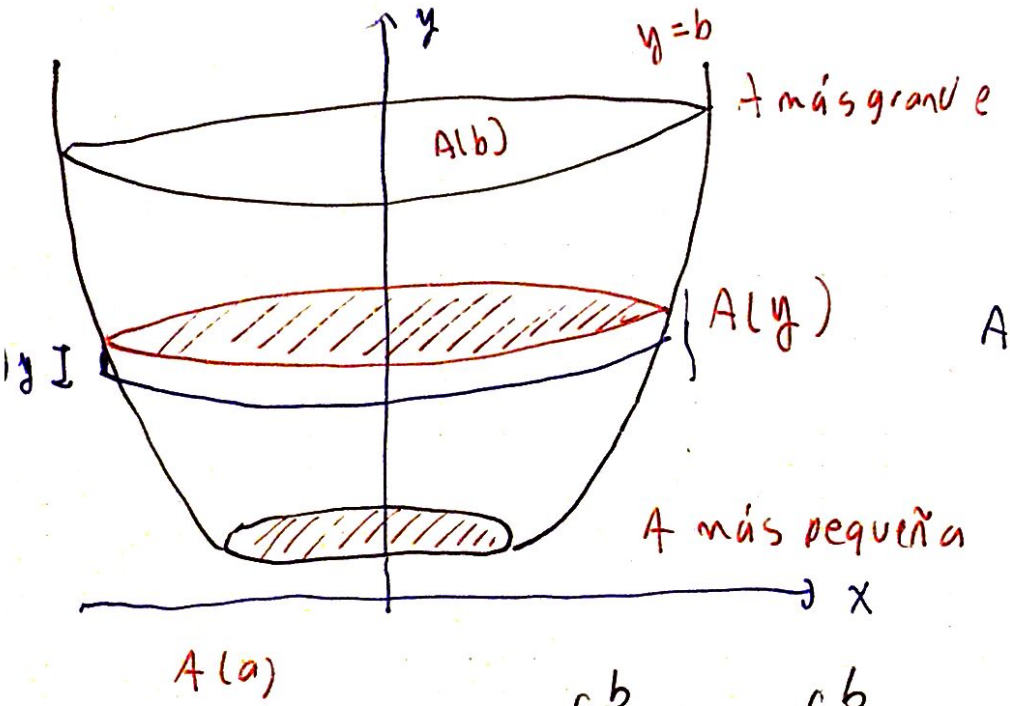
círculo de radio r



Área sección transversal A

$V = Ah$.

Volumen de un sólido S.



Parte infinitesimal de este sólido.

cilindro

Área transversal $A(y)$

Altura dy.

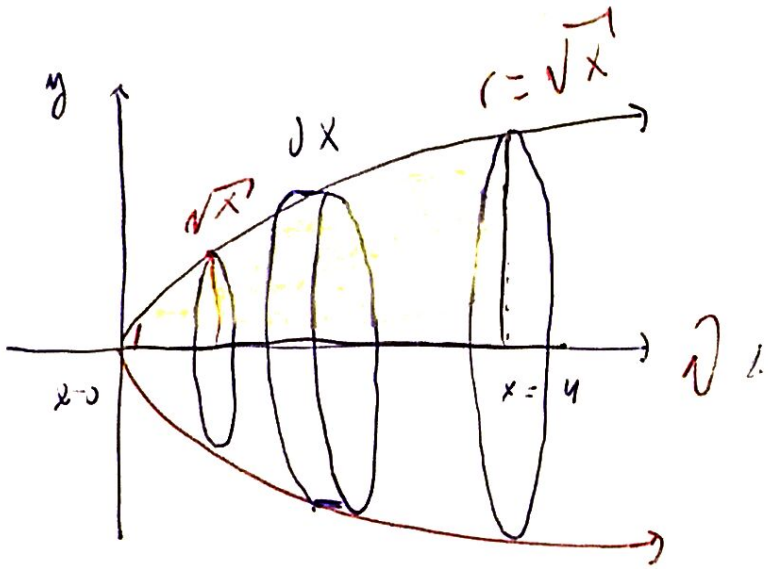
$dV = A(y)\,dy$.

Integrando $V = \int_a^b dV = \int_a^b A(y)\,dy$.

¿Cuál es el área transversal?

Ejemplo: Considere la región $0 \leq y \leq \sqrt{x}$

$0 \leq x \leq 4.$

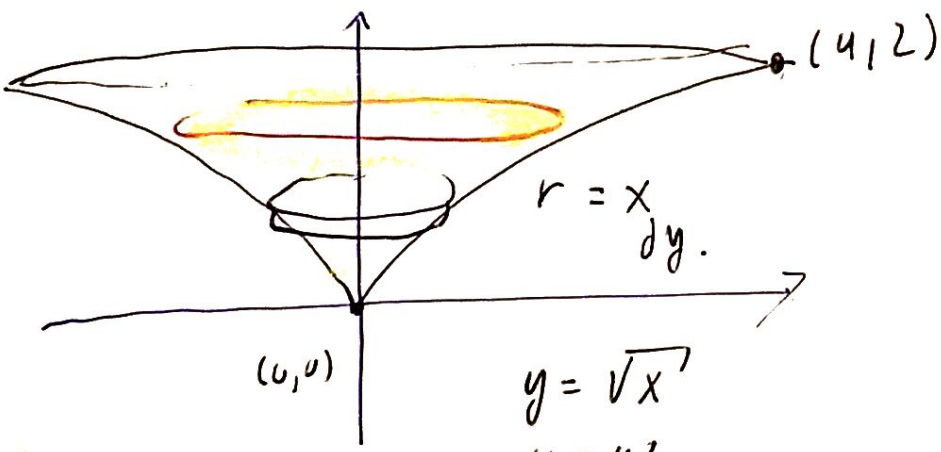


Rote alrededor del eje-x para obtener un sólido en revolución

Volumen del sólido es, sección transversal es un cilindro disco de radio $r = \sqrt{x}$

$$dV = \pi r^2 dx = \pi x dx.$$

$$V = \int_0^4 \pi r^2 dx = \pi \int_0^4 x dx = \frac{\pi x^2}{2}\Big]_0^4 = \frac{16\pi}{2} = 8\pi$$



$r = x$

$dy.$

$y = \sqrt{x}$

$x = y^2$

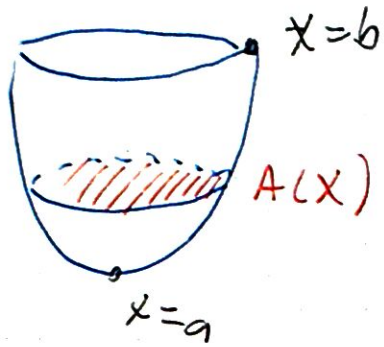
$$V = \pi \int \underline{\underline{x}}^2 dy.$$

$$V = \pi \int_0^2 y^4 dy.$$

Capítulo 15

Volúmenes sólidos en revolución, introducción a arandelas

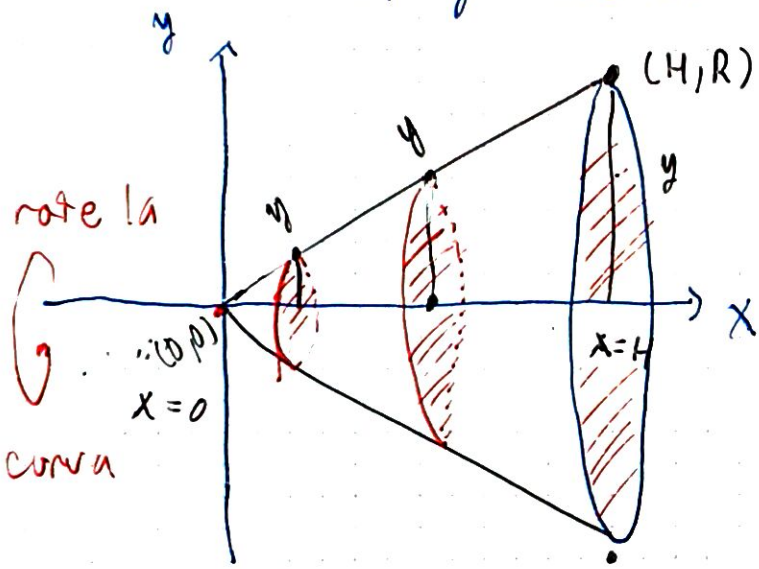
Volúmenes.



$$V = \int_a^b A(x)\,dx$$

área sección transversal.

Ejemplo: Encuentre el volumen de un cono de altura H y base circular de radio R.



las secciones transversales son círculos de radio $y(x)$.

$A = \pi y^2$

$$V = \int_0^H \pi y^2\,dx$$

$y(0) = 0 + b = 0$

$b = 0$

$y = mx + b.$ $\quad m = \dfrac{R-0}{H-0} = \dfrac{R}{H}$ $\quad y = \dfrac{R}{H}x + \cancel{b}.$

$(0,0)$ y (H,R)

$$V = \int_0^H \pi \frac{R^2 x^2}{H^2}\,dx = \frac{\pi R^2}{H^2}\int_0^H x^2\,dx$$

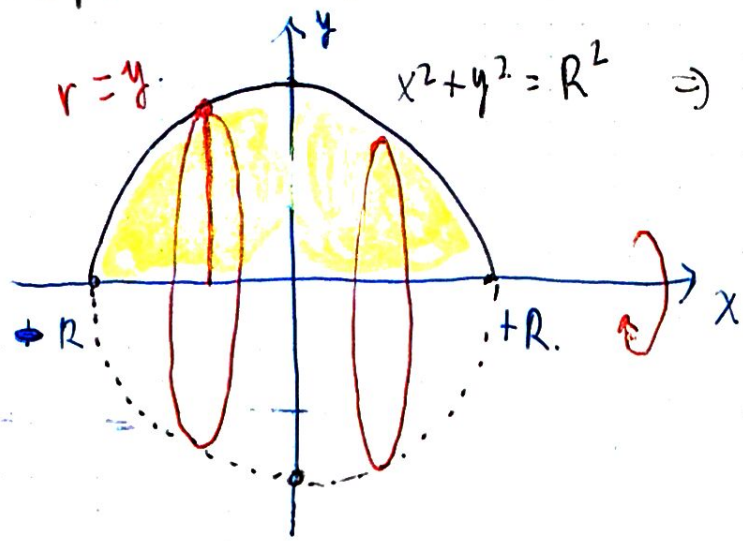
$$= \frac{\pi R^2}{H^2}\left.\frac{x^3}{3}\right]_0^H = \frac{\pi R^2}{H^2}\,\frac{H^3}{3}$$

P. 88.

$$V = \frac{\pi R^2 H}{3}$$

Ejercicio 1: p. 89 Volumen de una Esfera de radio R.

La esfera se obtiene al girar el círculo $x^2+y^2 \leq R^2$ respecto al eje-x.



$\Rightarrow$ $y^2 = R^2 - x^2$ ID $-R \leq x \leq R$

$y = \sqrt{R^2 - x^2}$

$x^2 = R^2$ $x = \pm R$

Sección transversal círculo de radio $y = r$

$A(x) = \pi y^2$

$$V = \int_{-R}^{R} A(x)\,dx = \pi \int_{-R}^{R} (R^2 - x^2)\,dx = 2\pi \int_{0}^{R} (R^2 - x^2)\,dx$$

R es constante.

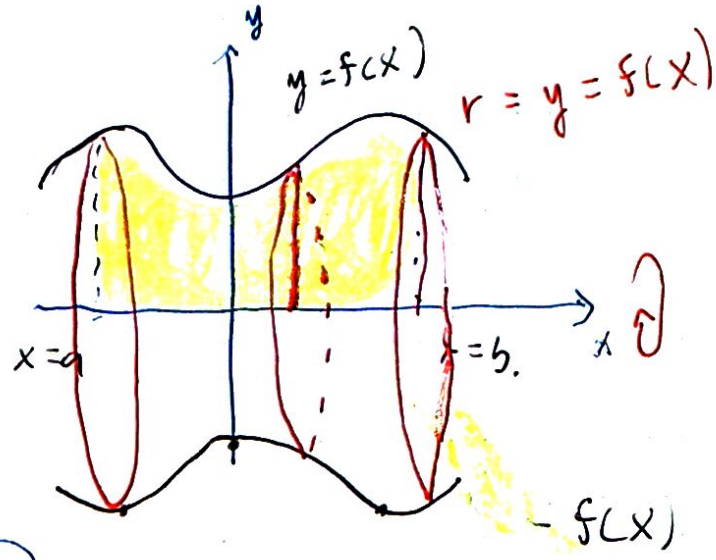
$$V = 2\pi \left(R^2 x - \frac{x^3}{3} \Big]_0^R \right) = 2\pi \left(R^3 - \frac{R^3}{3} \right) = 2\pi R^3 \left(1 - \frac{1}{3} \right)$$

$$\boxed{V = \frac{4\pi}{3} R^3}$$

Sólidos de Revolución.

$R: a \leq x \leq b.$
$0 \leq y \leq f(x)$

gire respecto al eje-x para obtener el sólido de revolución

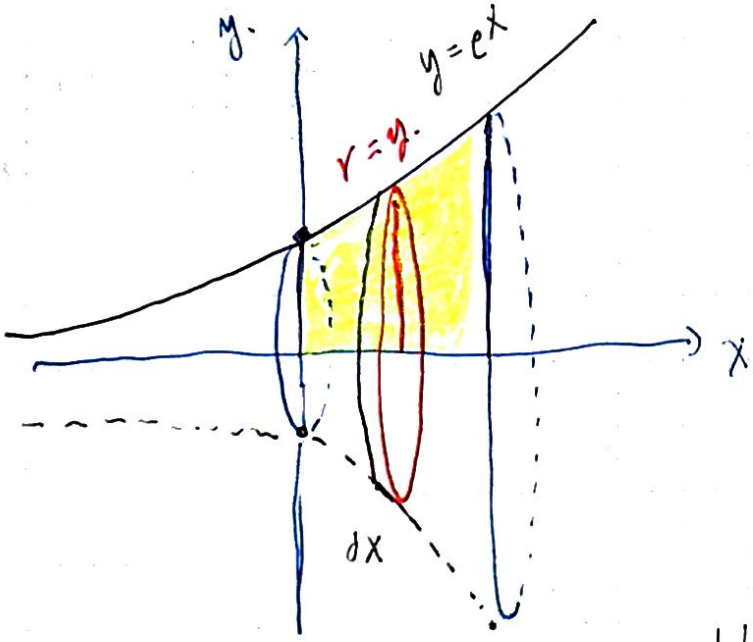


Área transversal : $\boxed{A = \pi y^2} = \pi f^2(x)$

$$V = \int_a^b \pi y^2 \, dx = \int_a^b \pi f^2(x) \, dx$$

Ejercicio 4: Encuentre el Volumen del sólido que se obtiene al girar la región $R: 0 \leq x \leq \ln 3 \quad 0 \leq y \leq e^x$ respecto al eje-x. (Pag. 91)

$y = e^x$ curva.



$e^{\ln[\,]} = [\,]$

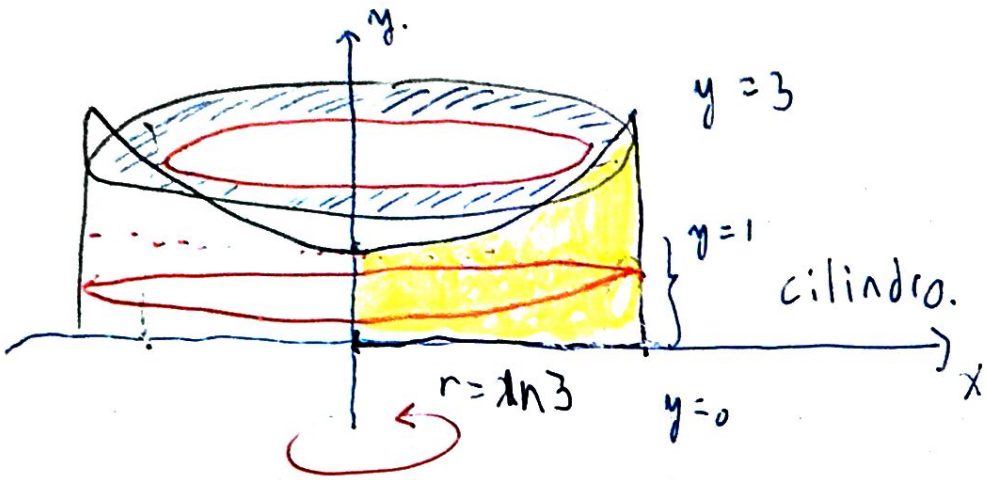
$A = \pi y^2 = \pi e^{2x}$

$$V = \int_0^{\ln 3} \pi e^{2x} \, dx$$

$$V = \frac{\pi}{2} e^{2x} \Big]_0^{\ln 3} = 4\pi.$$

$$V = \frac{\pi}{2}\left(e^{\ln 3^2} - e^0\right) = \frac{\pi}{2}(9-1)$$

Girando la misma región respecto al eje - y.



$f(0) = 1$

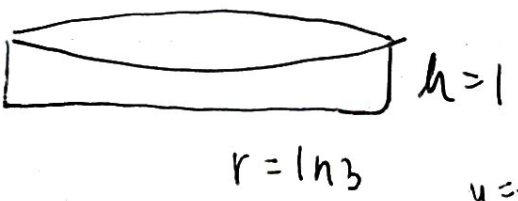
$f(\ln 3) = 3$

Difícil de visualizar.

Volumen = $V_1 + V_2$.

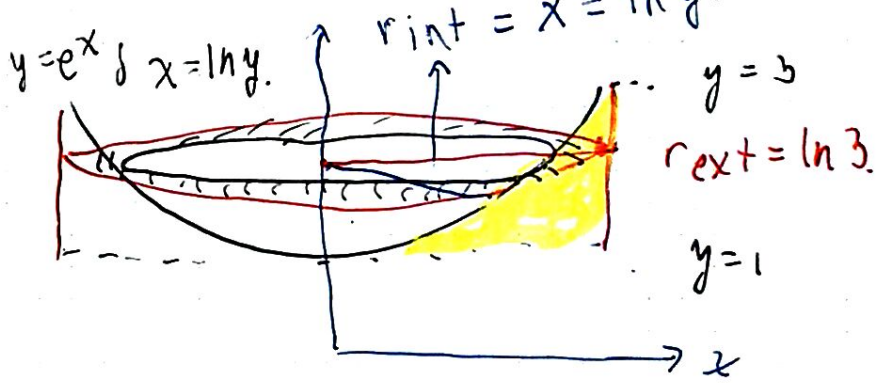
V_1 = cilindro

$V_1 = \pi r^2 h = \pi \ln^2 3.$

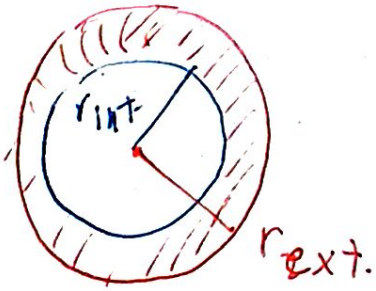


$y = e^x$

V_2 sólido hueco:



Área transversal



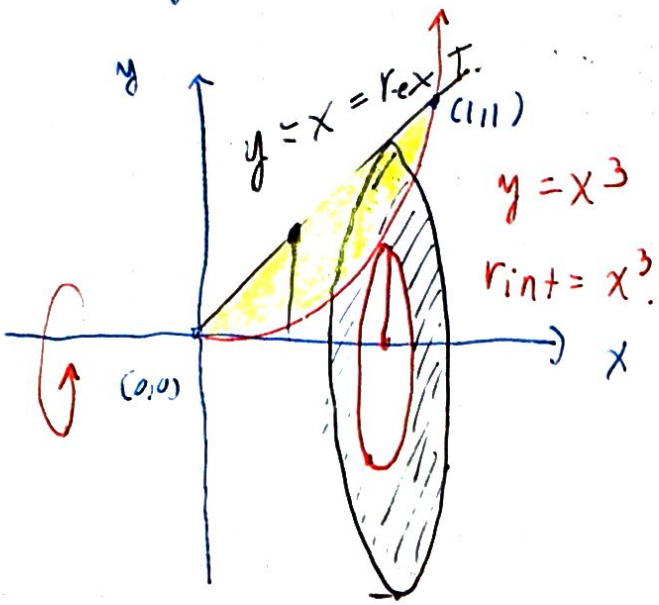
anillo

$A = \pi r_{ext}^2 - \pi r_{int}^2$

$A = \pi (\ln 3)^2 - \pi (\ln y)^2$

$$V_2 = \int_1^3 \pi (\ln 3)^2 - \pi (\ln y)^2 \, dy.$$

Ejercicio 5: Encuentre el volumen del sólido obtenido al girar la región entre las curvas $y = x$ & $y = x^3$ en el 1er cuadrante respecto al eje-x.



Área Anillo.

círculo grande − Cír pequeño.

$r_{ext} = x \qquad r_{int} = x^3$

$A = \pi r_{ext}^2 - \pi r_{int}^2$

$A = \pi x^2 - \pi x^6$

Volumen $V = \int_0^1 A\,dx = \int_0^1 (\pi x^2 - \pi x^6)\,dx$

$V = \pi\left(\frac{x^3}{3} - \frac{x^7}{7}\Big]_0^1\right) = \pi\left(\frac{1}{3} - \frac{1}{7}\right)$

Transformadas Laplace.

$$\int_0^\infty f(x)\, e^{-st}\, dt$$

$$f(t) = e^t \qquad = \int_0^\infty e^t e^{-st}\, dt. = \frac{-1}{1-s} = \frac{1}{s-1}$$

$$= \int_0^\infty e^{(1-s)t}\, dt.$$

$$\left.\frac{e^{(1-s)t}}{1-s}\right]_0^\infty = \cancel{\lim_{t\to\infty} \frac{e^{(1-s)t}}{1-s}}^{\,0} - \frac{e^0}{1-s}$$

$$\lim_{t\to\infty} e^{\overbrace{(1-s)}t} = 0$$

se necesita que $1-s<0.$

$1<s$

$s>1.$

$e^{-\infty} \to 0.$

$$\int_0^\infty t e^{-st} = -\frac{t}{s} e^{-st}\Big]_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\, dt. \qquad \text{IPP.}$$

$s>0.$

$$= \underbrace{\frac{-1}{s}\lim_{t\to\infty} t e^{-st}}_{\text{cuanto } s\ 0.\ 0} + \frac{0e^0}{s} + \left.\frac{e^{-st}}{s(-s)}\right]_0^\infty$$

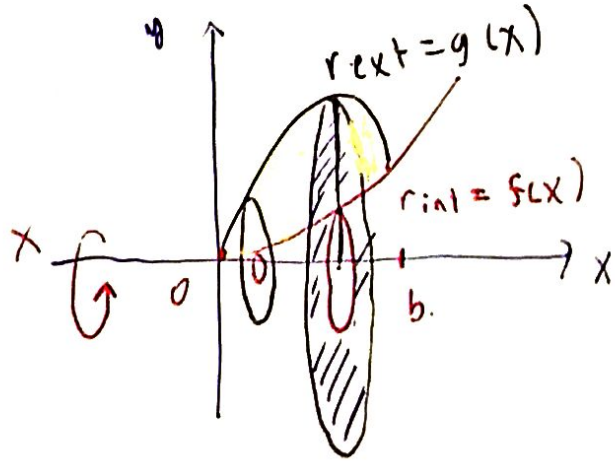
$$= 0 - \frac{1}{s^2}\cancel{\lim_{t\to\infty} e^{-st}}^{\,0} - \frac{e^0}{-s^2} = \frac{-1}{-s^2} = \frac{1}{s^2}.$$

Capítulo 16

Volúmenes por arandelas y cascarones cilindricos del cuadrado infinitesimal

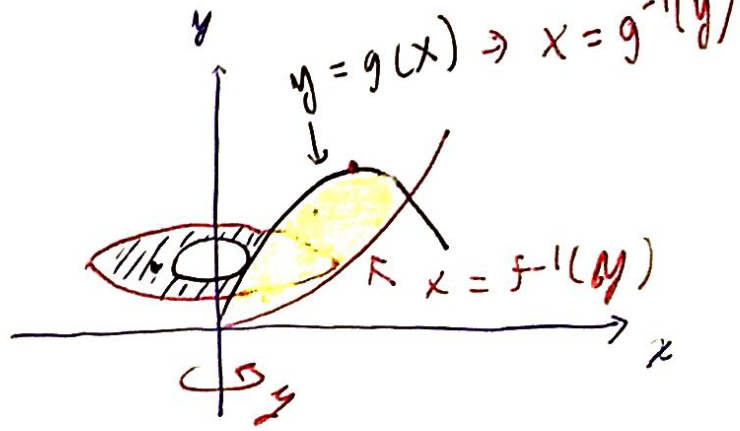
Volúmenes:

- Región.
- Identificar la recta de rotación.
- Identificar las funciones de radio para cada anillo
- Escoger la variable de integración.



$$A = \pi r_{ext}^2 - \pi r_{int}^2$$

$$V = \pi \int_0^b r_{ext}^2 - r_{int}^2 \, dx$$



Refleje respecto al eje-y.

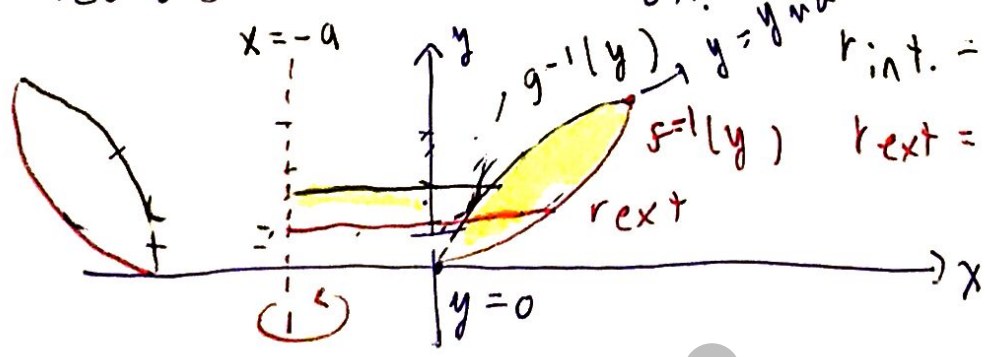
$0 \leq y \leq y_{max}$

$r_{int} = x = g^{-1}(y)$

$r_{ext} = x = f^{-1}(y)$

Volumen $V = \pi \int_0^{y_{max}} r_{ext}^2 - r_{int}^2 \, dy = \pi \int_0^{y_{max}} (f^{-1})^2 - (g^{-1})^2 \, dy.$

Rectas de Rotación.



$r_{int.} = a + g^{-1}(y)$

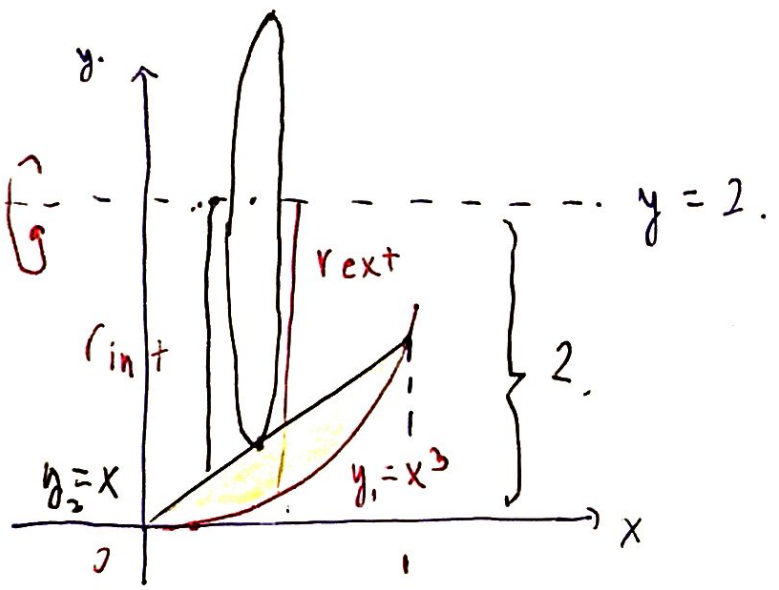
$r_{ext} = a + f^{-1}(y)$

$0 \leq y \leq y_{max}.$

$$V = \pi \int_0^{y_{max}} r_{ext}^2 - r_{int}^2 \, dy = \pi \int_0^{y_{max}} (a+f^{-1})^2 - (a+g^{-1})^2 \, dy.$$

Ejercicio 6: Considerar la región entre $f(x) = x$ & $g(x) = x^3$ en el 1er cuadrante. (P 96)

a. Plantee la integral del volumen del sólido al girar la región respecto a $y = 2$.



$r_{int} = 2 - y_2 = 2 - x.$

$r_{ext} = 2 - y_1 = 2 - x^3$

$A = \pi (r_{ext}^2 - r_{int}^2)$

Integre en x.

$x^3 - x = 0$

$x(x^2 - 1) = 0 \quad x = 0, 1$

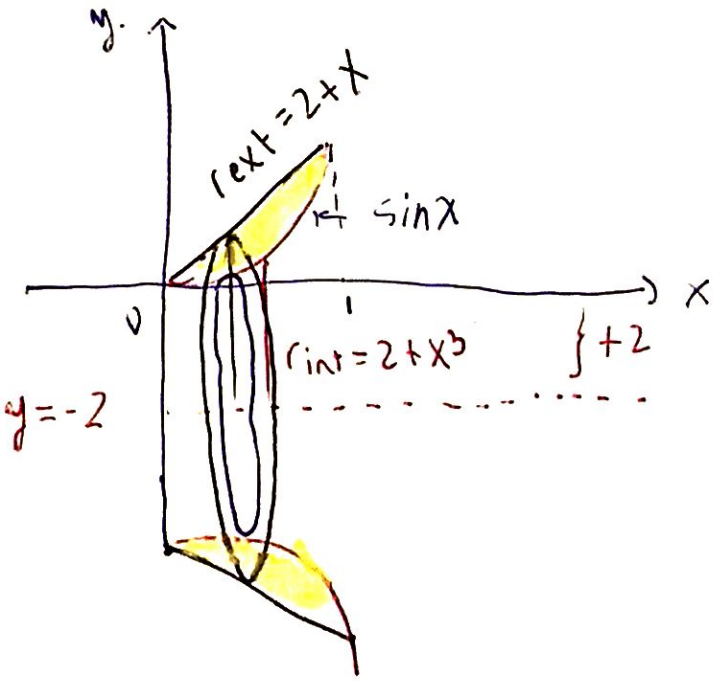
$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dx = \pi \int_0^1 (2-x^3)^2 - (2-x)^2 \, dx$$

$$\pi \int_0^1 x^6 - 4x^3 - x^2 + 4x \, dx$$

$$= 17\pi / 21.$$

b. Plantee la integral del volumen del sólido que se obtiene al girar la región respecto a $y=-2$.

dado.



$r_{int} = 2 + x^3$

$r_{ext} = 2 + x$

integre en $0 \leq x \leq 1$

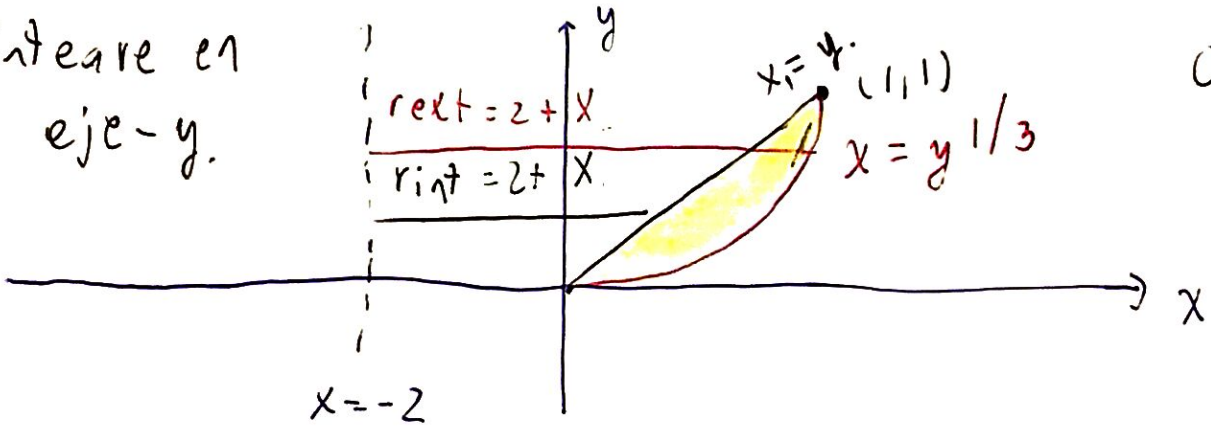
$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dx$$

$$V = \pi \int_0^1 (2+x)^2 - (2+x^3)^2 \, dx$$

$$V = 32\pi/21$$

c. Rote la región respecto a $x=-2$.

Integre en el eje-y.



$0 \leq y \leq 1$

$r_{ext} = 2 + y^{1/3}$

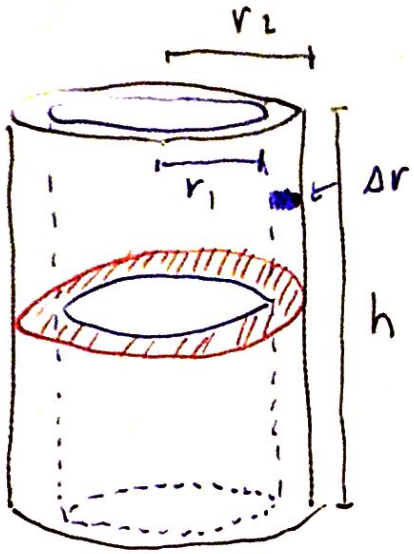
$r_{int} = 2 + y$

$0 \leq y \leq 1$

$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dy$$

$$V = \pi \int_0^1 (2+y^{1/3})^2 - (2+y)^2 \, dy.$$

6.3 Volumen con Cascarones Cilíndricos (Latas)



r_1 interno $\quad r_2$ externo. -te.

Área anillo $\pi r_2^2 - \pi r_1^2$

Volumen $V = \pi h (r_2^2 - r_1^2)$

grosor $\Delta r = r_2 - r_1 = dr$

radio promedio: $r = \frac{1}{2}(r_2 + r_1)$

$$V = \pi h (r_2 + r_1)(r_2 - r_1) = \pi h (2r)\, dr.$$

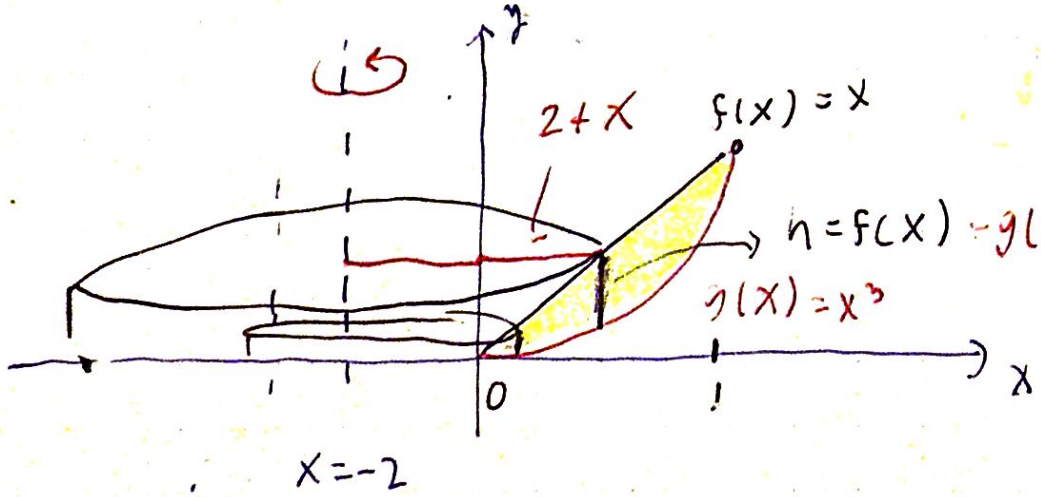
Volumen Cascarón $\quad V = \pi h r^2$

Derive respecto a r $\quad dV = 2\pi h r\, dr$

Integre en $a \le x \le b$ $\quad \boxed{V = 2\pi \int_a^b h r\, dr}$

Cascarones cilíndricos.

Otra vez inciso c) $y = x^3$ & $y = x$ gire a $x = -2$.



$0 \le x \le 1$

$h = x - x^3 \qquad r = 2 + x \qquad 0 \le x \le 1$

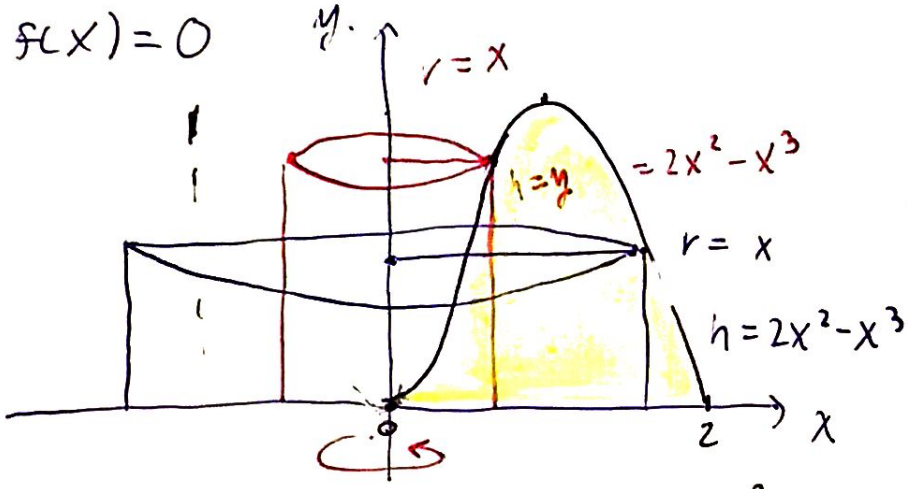
$$V = 2\pi \int_0^1 h\, r\, dx = 2\pi \int_0^1 (x - x^3)(2 + x)\, dx$$

ó $\qquad V = \pi \int_0^1 (2 + y^{1/3})^2 - (2 + y)^2 \, dy.$

Ejemplo: Encuentre el volumen del sólido que se obtiene al girar la región entre el eje-x y la curva $f(x) = 2x^2 - x^3$ en el 1er cuadrante respecto al eje-y.

Intersectos-x $\quad 2x^2 - x^3 = x^2(2 - x) = 0 \qquad x = 0, \; x = 2.$

$f(x) = 0$



radio $\quad r = x$

altura $\quad h = 2x^2 - x^3.$

limites $\quad 0 \le x \le 2.$

$$V = 2\pi \int_0^2 h\, r\, dx = 2\pi \int_0^2 (2x^2 - x^3)\, x \, dx$$

$$V = 2\pi \int_0^2 2x^3 - x^4 \, dx = 2\pi \left(\frac{2x^4}{4} - \frac{x^5}{5} \Big]_0^2 \right)$$

$$V = 2\pi \left(8 - \frac{32}{5} \right)$$

Rotando con un eje horizontal

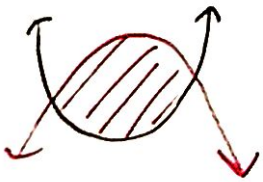
Anillos $\quad V = \int_a^b \pi (r_2^2 - r_1^2)\, dx$

eje-x
$y = 0$
$y = cte$

Rotando con un eje vertical $\quad x = 0$ ó $x = cte.$

Cilindros $\quad V = 2\pi \int_a^b h\, r\, dx$

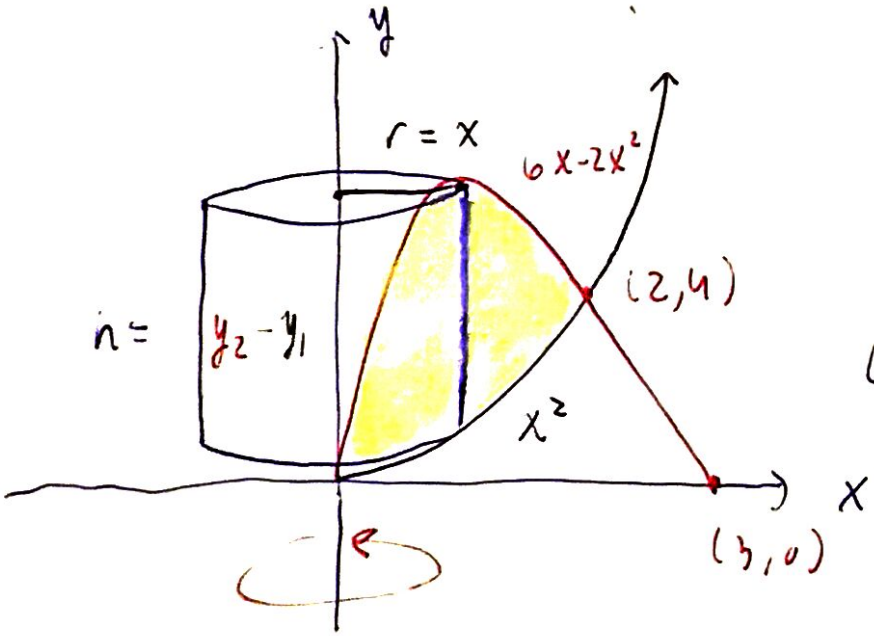
Ejercicio 2: Encuentre el volumen del sólido obtenido al girar la región entre $y_1 = x^2$ & $y_2 = 6x - 2x^2$ alrededor del eje-y.



$y_2 = 0 \quad 2x(3-x) = 0 \Rightarrow x = 0, 3.$

$y_1 = y_2 \quad x^2 = 6x - 2x^2.$

$3x^2 - 6x = 3x(x-2) = 0 \Rightarrow x = 0, 2.$



Altura. $h = y_2 - y_1$

$h = 6x - 3x^2.$

Radio $\quad r = x$

Límites. $0 \leq x \leq 2.$

$V = 2\pi \int_0^2 h\, r\, dx.$

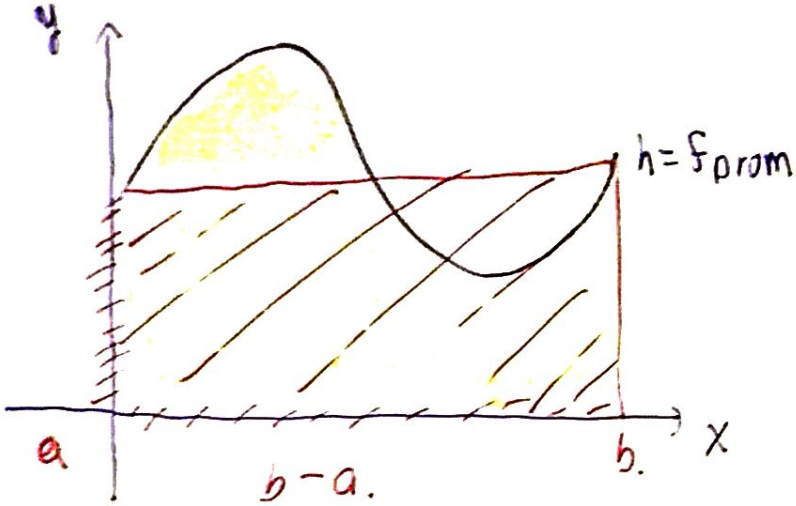
$$V = 2\pi \int_0^2 6x^2 - 3x^3 \, dx = 2\pi \left(2x^3 - \frac{3}{4} x^4 \Big]_0^2 \right)$$

$$V = 2\pi \left(16 - 3 \cdot 4 \right) = 2\pi (4) = 8\pi$$

Capítulo 17

6.5 Valor promedio de una función

6.5 Valor Promedio de una función



¿Promedio altura y. ? $f_{prom.}$

Iguale el área del rectángulo con el área de la región amarilla

misma área

Ancho $b-a$

altura f_{prom}

Área $f_{prom}(b-a) = \int_a^b f(x)\,dx.$

Área Región Amarilla

$$\boxed{f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$f(x)$ sea continua en $[a,b]$.

Ejemplo: Encuentre el valor promedio de $f(x) = \csc^2 x$ en $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

$b-a = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$ $\quad \frac{1}{b-a} = \frac{4}{\pi}$

$$f_{prom} = \frac{4}{\pi}\int_{\pi/4}^{\pi/2} \csc^2 x\,dx = -\frac{4}{\pi}\cot x\Big]_{\pi/4}^{\pi/2}$$

$$f_{prom} = \cancel{-\frac{4}{\pi}\cot \pi/2}^{\,0} + \frac{4}{\pi}\cot \pi/4. = \frac{4}{\pi}.$$

$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1}$ $\qquad \tan\frac{\pi}{4} = 1$

Ejercicio 1: Encuentre el valor promedio de $f(x)$.

a. $f(t) = \cos^4 t \sin t$ en $[0, \pi]$.

$$f_{prom} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{\pi - 0}\int_0^{\pi} \underbrace{\cos^4 t}_{u^4}\, \overbrace{\sin t\, dt}^{-du}.$$

$u = \cos t \qquad du = -\sin t\, dt. \qquad u(\pi) = \cos\pi = -1$

$u(0) = \cos 0 = 1$

$$f_{prom} = \frac{-1}{\pi}\int_1^{-1} u^4\, du = \frac{1}{\pi}\int_{-1}^{1} u^4\, du = \frac{2}{\pi}\int_0^1 u^4\, du.$$

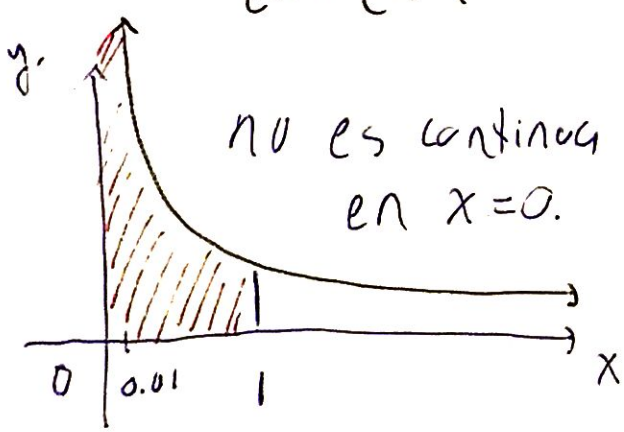
par.

$$f_{prom} = \frac{2}{\pi}\left.\frac{u^5}{5}\right]_0^1 = \frac{2}{5}\cdot\frac{1}{\pi}.$$

b. $g(x) = \dfrac{1}{x}$ en $[e^4, e^{10}]$.

$$y_{prom} = \frac{1}{e^{10}-e^4}\int_{e^4}^{e^{10}} \frac{1}{x}\, dx = \frac{1}{e^{10}-e^4}\left.\ln|x|\right]_{e^4}^{e^{10}}$$

$$y_{prom} = \frac{1}{e^{10}-e^4}\left(\ln e^{10} - \ln e^4\right) = \frac{6}{e^{10}-e^4}$$



no es continua en $x=0$.

Valor promedio de f en $0 \le x \le 1$

$$y_{prom} = \frac{1}{1}\int_0^1 \frac{1}{x}\, dx$$

$$= \left.\ln x\right]_0^1 = -\lim_{x\to 0^+} \ln x$$

$y_{prom} = -\lim_{x \to 0^+} \ln x = +\infty$ no existe.

La función no tiene valor promedio

$4^{-1/2}$
$2\,4^{1/2}$

c. $h(x) = \dfrac{3}{(4+x)^{1/2}}$ en $[-4, 5]$.

no es continua en $x = -4$.

$$h_{prom} = \frac{1}{5-(-4)} \int_{-4}^{5} 3(4+x)^{-1/2}\, dx$$

$(x)' = 1.$

$$h_{prom} = \frac{3}{9}\, 2(4+x)^{1/2} \Big]_{-4}^{5}$$

$(4-4)^{1/2} = 0^{1/2}$

$$h_{prom} = \frac{2}{3}\left(9^{1/2} - \lim_{x \to -4^+} \cancel{(4+x)^{1/2}}^{\,0} \right) = \frac{2}{3} \cdot 3 = 2.$$

Ejercicio 2: Densidad Lineal $\rho = 12(x+1)^{-1/2}$.

la varilla tiene 8 m. de longitud.

a. Encuentre la densidad promedio de la varilla.

$$\rho_{prom} = \frac{1}{8} \int_0^8 12(x+1)^{-1/2}\, dx$$

$$\rho_{prom} = \frac{24}{8} (x+1)^{1/2} \Big]_0^8 = 3\left(9^{1/2} - 1^{1/2} \right)$$

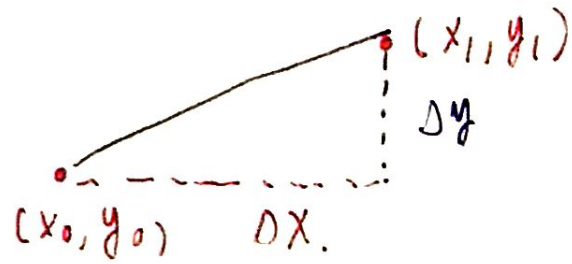
$3(3-1) = 6$ Kg/m.

Capítulo 18

8.1 Longitud de arco

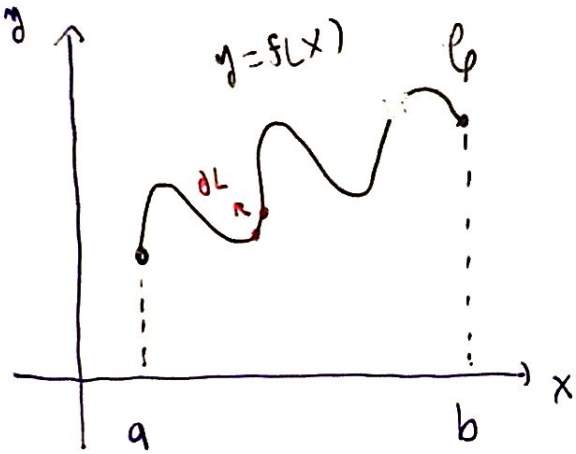
8.1 Longitud de Arco.

Derivación Fórmula.



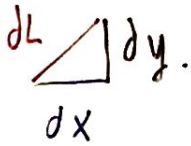
$$L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



largo de la curva $\mathcal{C}$.

$a \leq x \leq b, \quad y = f(x)$



Longitud infinitesimal del segmento.

$$dL = \sqrt{(dy)^2 + (dx)^2}$$

$$dL = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\; dx.$$

Integrando

Longitud de arco de $\mathcal{C}$: $\boxed{L = \int_a^b \sqrt{1 + [y']^2}\, dx}$

derivada.

Valor Promedio: $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx$

No es necesario graficar ninguna curva.

Ejemplo. Encuentre la longitud de la curva

$$y(x) = 1 + 2x^{3/2} \quad \text{en} \quad 0 \leq x \leq 8/9$$

Simplifique $1 + (y')^2$ antes de integrar.

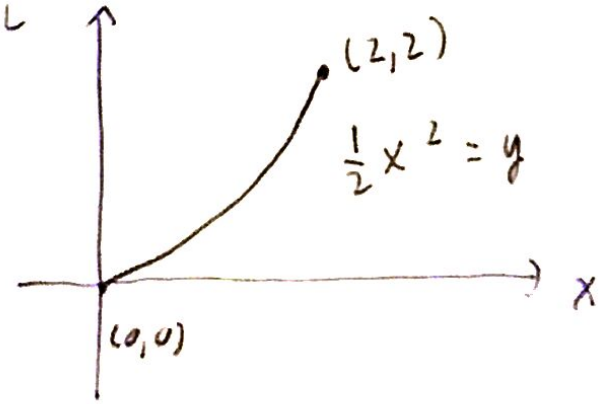
$$y' = 3x^{1/2} \qquad (y')^2 = 9x \qquad 1 + (y')^2 = 1 + 9x$$

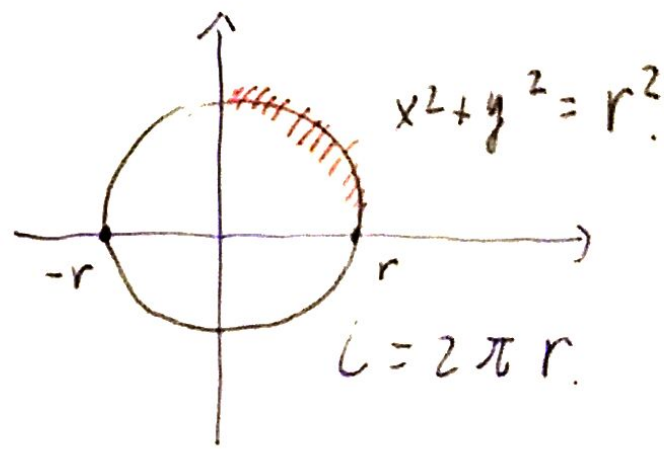
Longitud Arco: $L = \int_0^{8/9} (1+9x)^{1/2}\, dx$ $\qquad u = 1+9x$, $du = 9 \cdot dx$

$$L = (1+9x)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{9} \Big]_0^{8/9}$$

$$L = \frac{2}{27}\Big(\underbrace{9^{3/2}}_{(3^2)^{3/2}} - \underbrace{1^{3/2}}_{1} \Big) = \frac{2}{27}\left(3^3 - 1\right) = \frac{2}{27} \cdot 26.$$

Ejercicio (a): Parábola (Planteado)





$$y' = x$$

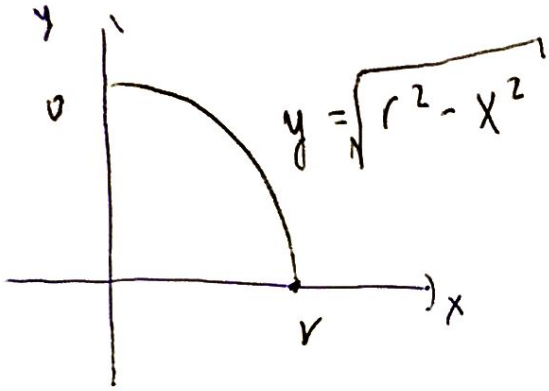
$$1 + (y')^2 = 1 + x^2$$

$$L = \int_0^2 \sqrt{1+x^2}\, dx \qquad x = \tan\theta, \quad dx = \sec^2\theta \cdot d\theta.$$

Ejercicio 1b: Longitud de una circunferencia de radio r. $r = cte.$

Ec: $x^2 + y^2 = r^2$

$$L = 4\int_0^r \sqrt{1+(y')^2}\, dx.$$



$$y' = \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x)$$

$$y' = \frac{-x}{\sqrt{r^2 - x^2}}$$

$$1 + (y')^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2 - x^2 + x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}.$$

$$L = 4\int_0^r \sqrt{\frac{r^2}{r^2 - x^2}}\, dx = 4r\int_0^r \frac{dx}{\sqrt{r^2 - x^2}}$$

$\sqrt{r^2} = r$

$$= 4r \sin^{-1}\left(\frac{x}{r}\right)\Big]_0^r$$

$$= 4r\left[\sin^{-1}\left(\frac{r}{r}\right) - \sin^{-1}(0)\right]$$

$\sin\left(\frac{\pi}{2}\right) = 1$

$$= 4r\left[\overset{\pi/2}{\cancel{\sin^{-1} 1}} - \overset{0}{\cancel{\sin^{-1}(0)}}\right]$$

$$= 4r\frac{\pi}{2} = 2\pi r.$$

Longitud de circunferencia $2\pi r. = L$

Área de un círculo $\pi r^2 = A$

Volumen $4\pi r^3/3 = V$

1.

Longitud: Un cable telefónico cuelga entre dos postes con posiciones horizontales en $x = \pm 25$. El cable tiene una curva con ec. $y = -5 + 25\cosh\left(\frac{x}{25}\right)$

Encuentre la longitud del cable.

$$L = \int_{-25}^{25} \sqrt{1+(y')^2}\, dx$$

$\frac{d}{dx}\cosh x = \sinh x$

$$y' = 25\sinh\left(\frac{x}{25}\right)\cdot\frac{1}{25} = \sinh\left(\frac{x}{25}\right)$$

se cancelan los 25's.

$$1+(y')^2 = 1+\sinh^2\left(\frac{x}{25}\right) = \cosh^2\left(\frac{x}{25}\right)$$

identidad hiperbólica

$$L = \int_{-25}^{25} \sqrt{\cosh^2(x/25)}\, dx = \int_{-25}^{25} \cosh\left(\frac{x}{25}\right) dx$$

$$L = 2\int_{0}^{25} \cosh\left(\frac{x}{25}\right) dx = 2\cdot 25\sinh\left(\frac{x}{25}\right)\Big]_0^{25}$$

$$L = 50\left[\sinh(1) - \cancel{\sinh(0)}^{\,0}\right] \approx 58.7600$$

función tiene diferente variable independiente.

Ejercicio 3: Pág 112. Encuentre la longitud para las siguientes curvas.

a. $\mathcal{C}_1 : x = \frac{y^3}{6} + \frac{1}{2y} \qquad 1 \leq y \leq 2.$

Utilice el eje-y para integrar

$$L = \int_a^b \sqrt{1+(y')^2}\, dx \quad ; \quad \int_a^b \sqrt{1+(x')^2}\, dy.$$

Objetivo: SIMPLIFIQUE $1+(x')^2$.

$$x' = \frac{3y^2}{6} - \frac{1}{2}y^{-2} = \frac{1}{2}\left(y^2 - y^{-2}\right) \qquad y^2 y^{-2} = y^0$$

$$(x')^2 = \frac{1}{4}\left(y^2 - y^{-2}\right)^2 = \frac{1}{4}\left(y^4 - 2\cdot 1 + y^{-4}\right)$$

$$1+(x')^2 = 1 + \frac{1}{4}\left(y^4 - 2 + y^{-4}\right)$$ Simplifique y factorice.

$$= \frac{1}{4}\left(4 + y^4 - 2 + y^{-4}\right) \qquad a^2 + 2a + 1 \quad (a+1)^2$$

$$= \frac{1}{4}\left(y^4 + 2 + y^{-4}\right) \qquad 1 + \frac{1}{4}a = \frac{4+a}{4}$$

$y^2 \cdot y^{-2} = 1$

$$= \frac{1}{4}\left(y^2 + y^{-2}\right)^2 = 1 + (x')^2.$$

$$L = \int_1^2 \sqrt{1+(x')^2}\, dy = \int_1^2 \sqrt{\frac{1}{4}\left(y^2+y^{-2}\right)^2}\, dy.$$

$$= \frac{1}{2}\int_1^2 (y^2 + y^{-2})\,dy = \frac{1}{2}\left(\frac{y^3}{3} - \frac{1}{y}\Big]_1^2\right)$$

$$L = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right)$$

b. $\mathcal{C}_2$: $y = \ln(\sec\underline{\underline{\theta}})$ $\quad 0 \leq \underline{\underline{\theta}} \leq \pi/4.$

$$y'(\theta) = \frac{\sec\theta\tan\theta}{\sec\theta} = \tan\theta.$$

$$1 + (y')^2 = 1 + \tan^2\theta = \sec^2\theta.$$

$$L = \int_0^{\pi/4} \sqrt{1 + (y')^2}\,d\theta = \int_0^{\pi/4} \sqrt{\sec^2\theta}\,d\theta.$$

$$L = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}.$$

$$L = \ln(\sec\pi/4 + \tan\pi/4) - \ln(\overset{1}{\cancel{\sec 0}} + \overset{0}{\cancel{\tan 0}})$$

$$= \ln(\sqrt{2} + 1) - \ln 1 = \ln(\sqrt{2} + 1)$$

Longitud de Arco: curva está en $a \leq t \leq x$.

límite superior indefinido.

$$S(x) = \int_a^x \sqrt{1 + (y')^2}\,dt \Big\} \text{ Función de Longitud de Arco.}$$

Ejercicio 4: Encuentre la función de longitud de arco para la curva $y = \ln(\sin t)$ en $-\frac{\pi}{2} \leq t \leq \underline{\underline{x}}$.

$$y' = \frac{\cos t}{\sin t} = \cot(t) \qquad \int \csc^2 t \, dt = \cot x$$

$$1 + (y')^2 = 1 + \cot^2(t) = \csc^2(t).$$

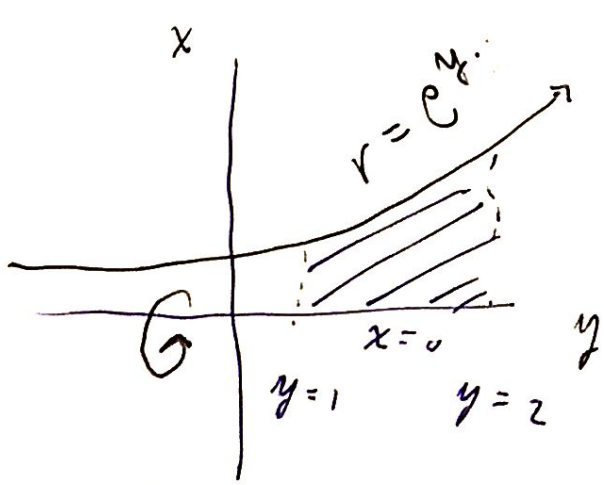
$$L(x) = \int_{-\pi/2}^{x} \sqrt{1 + (y')^2} \, dt = \int_{-\pi/2}^{x} \sqrt{\csc^2 t} \, dt.$$

$$L(x) = \int_{-\pi/2}^{x} \csc t \, dt. = -\ln|\csc t + \cot t| \Big]_{-\pi/2}^{x}$$

$$= -\ln|\csc x + \cot x| + \ln|\csc(-\pi/2) + \cot(-\pi/2)|$$

$$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = 0 \qquad \csc(-\pi/2) = \frac{1}{\sin(-\pi/2)} = -1$$

$$= -\ln|\csc x + \cot x| + \cancel{\ln|-1|}^{\,0.}$$

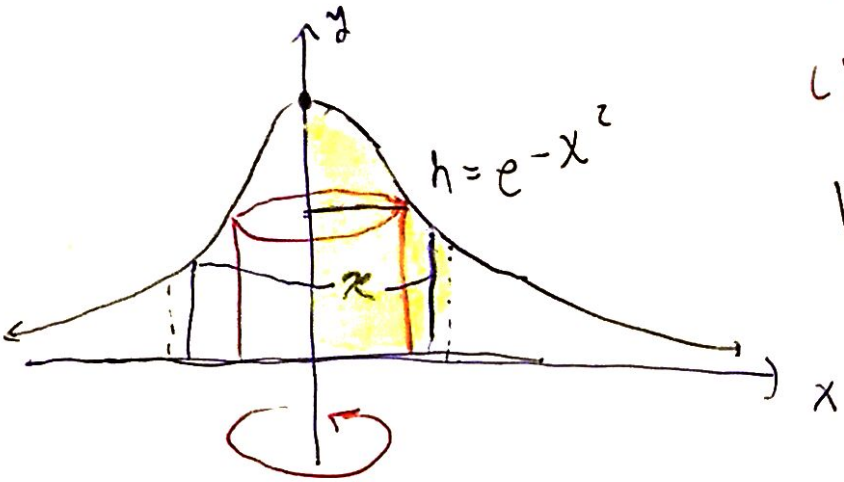


Ej: 3 Lab 8.

Ej: 4 $\quad y = e^{-x^2} \quad y = 0, \quad x = -1 \quad x = 1$

Lab 8.

a) alrededor del eje-y.



Cilindros.

$$V = 2\pi \int_0^1 h r \, dx$$

Discos

$$V = \pi \int_0^1 r^2(y)\, dy.$$

$$V = \pi \int_0^1 e^{-x^2} \overbrace{2x\,dx}^{-du} = \pi \int_0^{-1} -e^u \, du = -\pi e^u \Big]_0^{-1}$$

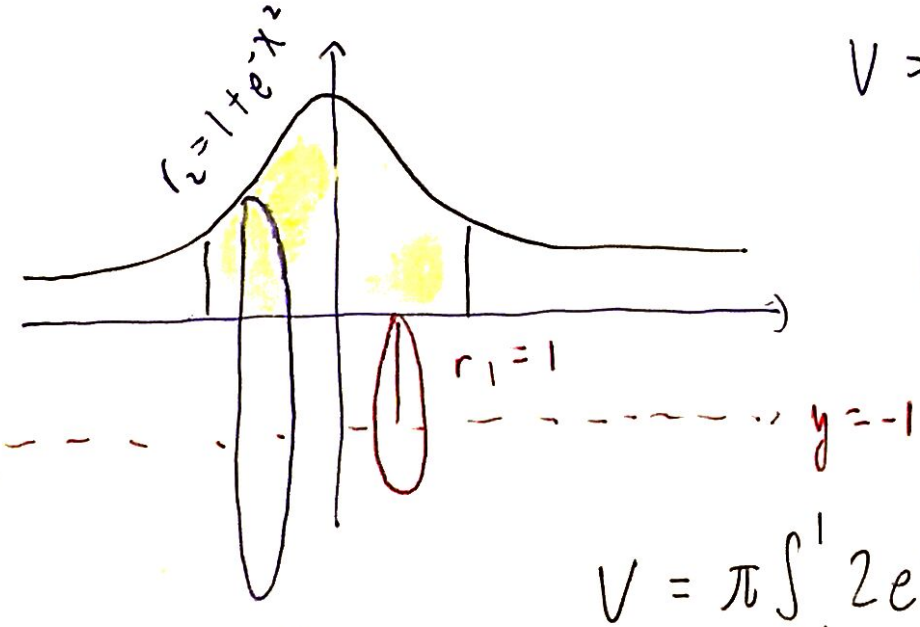
$u = -x^2 \qquad du = -2x\,dx$

$u(0) = 0 \qquad u(1) = -1$

$$-\pi(e^{-1} - e^0)$$

$$\pi(e^0 - e^{-1}) = \pi\left(1 - \frac{1}{e}\right)$$

b) alrededor de $y = -1$.



$$V = \pi \int_{-1}^1 r_2^2 - r_1^2 \, dx.$$

$$(1 + e^{-x^2})^2 - 1$$

$$1 + 2e^{-x^2} + e^{-x^4} - 1$$

$$2e^{-x^2} + e^{-x^4}$$

$$V = \pi \int_{-1}^1 2e^{-x^2} + e^{-x^4} \, dx$$

Capítulo 19

8.5 Probabilidad

a) Distribución uniforme

b) Distribución exponencial

c) Distribución normal

Simulacro Lunes 7 de octubre. 2:30 PM CES.

$0 \leq$ Notas ≤ 60 Bola Parcial. Res

7.8 Integrales Improp 8.5 Probabilidades.
76-128. sin 8.2 Área Superficial.

Lunes 14 de octubre Parcial 2: 2:30 PM CES.
Integr. Funciones Parciales.

Probabilidad (p. 123).

Un evento puede ser discreto o puede ser continuo.

Discreto: hay un número finito o contable de eventos.

Goles en un partido, lanzamiento de dados y monedas.

$$\text{Probabilidad} = \frac{\text{\# veces de que ocurra un evento}}{\text{\# total de eventos.}}$$

Dado
$E = \{1, 2, 3, 4, 5, 6\}$.

Probabilidad $P(x \geq 5) = \frac{2}{6} = \frac{1}{3}$.

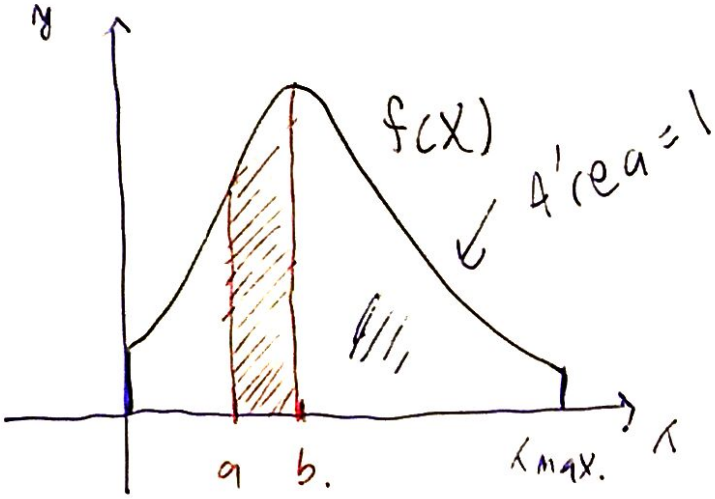
Probabilidad de que ocurra cualquier evento está entre 0 y 1 $0 \leq P(x) \leq 1$.

$0.0\% \leq P(x) \leq 100\%$

Probabilidad de que ocurran todos los eventos $\sum_{i=1}^{n} P(x_i)$

Enfoque: Statistical I y Mate Discreta

Continuo: el número de eventos no es contable.
El dominio de los eventos son los números reales.



Mediciones de cantidades como pesos, alturas, tiempos, volúmenes, áreas.

La probabilidad de que ocurra un evento entre a y b es el área bajo la curva.

$$p(a \leq x \leq b) = \int_a^b \underline{\underline{f(x)}}\, dx$$

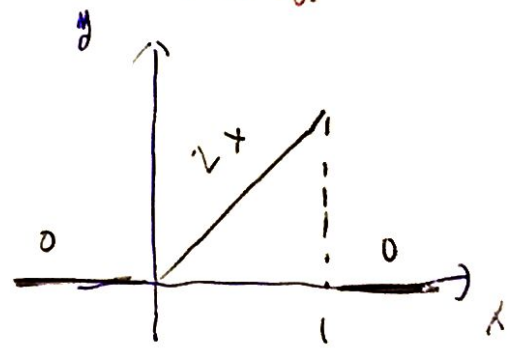
Función de densidad de probabilidad. $f(x)$.

Condiciones.

i. $f(x) \geq 0$ en $-\infty \leq x \leq \infty$.

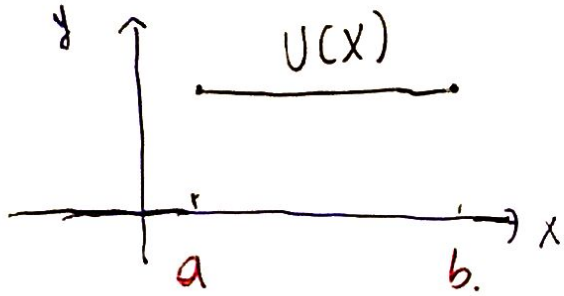
ii $\int_{-\infty}^{\infty} f(x)\, dx = 1$ Probabilidad tiene que ser del 100%.

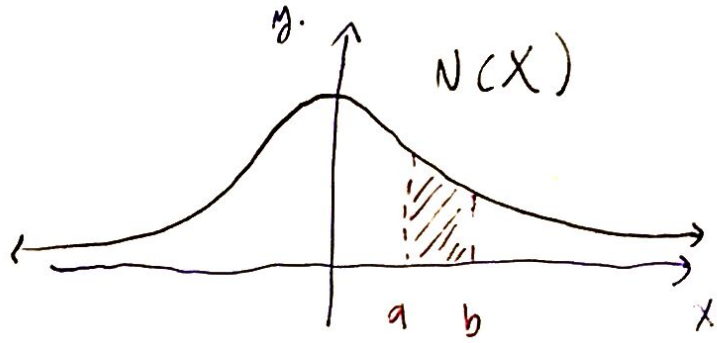
Ejemplo: $f(x) = \begin{cases} 0 & x < 0. \\ 2x & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$



$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx = x^2\Big]_0^1 = 1 \quad \checkmark$$

3 Distribuciones de Probabilidad Comunes.





$$U(X) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a \leq x \leq b \\ 0 & x > b. \end{cases}$$

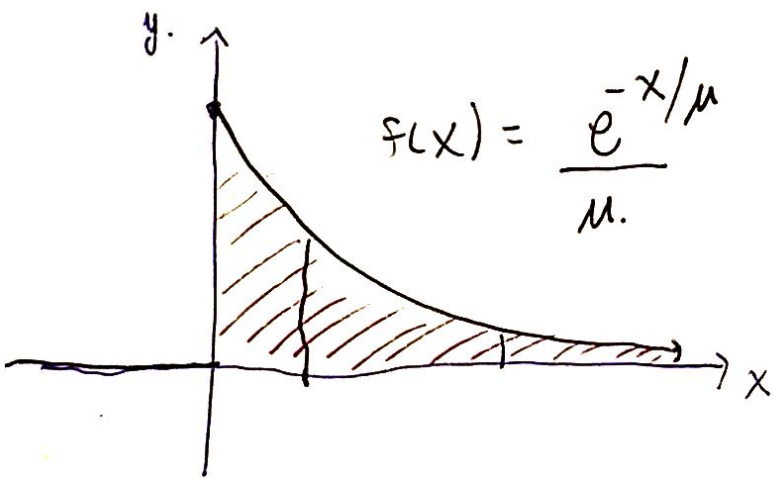
uniforme.

$$N(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Normal media 0
desv. estándar 1.

$$\int_{-\infty}^{\infty} N(x)\,dx = 1.$$

Exponencial.



$$f(x) = \frac{e^{-x/\mu}}{\mu}.$$

letra griega.

μ = media, mio
todos los eventos

$$\int_0^{\infty} f(x)\,dx = 1.$$

mediciones, tiempos, pesos

Ejercicio: Compruebe de que la distribución uniforme y la distribución exponencial son funciones de densidad

$$U(x) = \frac{1}{b-a} \quad a \leq x \leq b \qquad \underline{f}(x) = \frac{1}{\mu} e^{-x/\mu}, \; x \geq 0.$$

$$P(-\infty \leq x \leq \infty) = \int_{-\infty}^{\infty} U(x)\,dx = \int_a^b \frac{1}{b-a}\,dx$$

a, b son constantes

↑ probabilidad

$$= \frac{x}{b-a}\Big]_a^b = \frac{b-a}{b-a} = 1$$

$U(x) \geq 0$ en $\mathbb{R}$.

$U(x)$ es una función de densidad de probabilidad.

Exponencial:

$$P(0 \leq x \leq \infty) = \int_0^{\infty} e^{-x/\mu} \frac{dx}{\mu}.$$

μ tiempo promedio constante.

$$w = \frac{-x}{\mu} \qquad dw = \frac{-dx}{\mu}$$

$u(\infty) = -\infty.$
$u(0) = 0.$

$$P(0 \leq x \leq \infty) = -\int_0^{-\infty} e^{w}\,dw = -e^{w}\Big]_0^{-\infty}$$

$e^{-\infty} \to 0$

$$= -\lim_{w \to -\infty} e^{w} - (-e^{0}) = 1$$

Capítulo 19. 8.5 Probabilidad
a) Distribución uniforme
b) Distribución exponencial
c) Distribución normal

Capítulo 20

8.5 Probabilidad, media, varianza, desviación estándar, mediana

3.5 Probabilidad

Variable continua $-\infty \leq x \leq \infty$.

Probabilidad P, de que x ocurra entre a y b es:

$$\boxed{P(a \leq x \leq b) = \int_a^b f(x)\,dx}$$

Distribución Uniforme: $f(x) = \frac{1}{b-a}$ $\quad a \leq x \leq b.$

μ = media $\qquad$ Exponencial: $f(x) = \frac{1}{\mu} e^{-x/\mu}$ $\quad x \geq 0.$

Probabilidad es del 100%. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

P. 125.

Ejercicio 1: Un contenedor tiene mercancía cuyo peso tiene una distribución uniforme entre 2 y 4 toneladas.

a. Calcule la probabilidad de que el contenedor pese entre 2.5 y 3.5 toneladas. $\quad a = 2 \quad b = 4.$

$f(x) = \frac{1}{b-a} = \frac{1}{2}$ $\qquad 2 \leq x \leq 4.$ $\qquad f(x) = 0$ si $x > 4.$

$$P(2.5 \leq x \leq 3.5) = \int_{2.5}^{3.5} \frac{1}{2}\,dx = \frac{x}{2}\Big]_{2.5}^{3.5} = \frac{3.5-2.5}{2.} = \frac{1}{2.}$$

La probabilidad de que pese entre 2.5 y 3.5 es del 50%

b. calcule la probabilidad de que el contenedor pese más de 2.5 toneladas.

$$P(X \geq 2.5) = \cancel{\int_{2.5}^{\infty} f(x)\,dx} = \int_{2.5}^{4} \frac{1}{2}\,dx = \frac{x}{2}\Big]_{2.5}^{4} = \frac{4-2.5}{2} = \frac{1.5}{2}.$$

$$P(X \geq 2.5) = \frac{3}{4} \text{ ó } 75\%.$$

$$f(x) = \begin{cases} 0 & x < 2 \\ 1/2 & 2 \leq x \leq 4 \\ 0 & x > 4. \end{cases}$$

Ejercicio 2: El tiempo de espera promedio para recibir una orden a domicilio es de 0.5 hora y tiene una distribución exponencial.

a. Encuentre la probabilidad de que la comida se entregue después de media hora.

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad \mu \text{ -media} \qquad \mu = 0.5.$$
$$x \geq 0.$$

$$f(x) = \frac{1}{0.5} e^{-x/0.5} = 2e^{-2x} \qquad x \geq 0.$$

$$P(x \geq 0.5) = \int_{0.5}^{\infty} 2e^{-2x}\,dx \qquad \frac{2e^{-2x}}{-2}$$

$$P(X \geq 0.5) = -e^{-2x}\Big]_{0.5}^{\infty} \qquad e^{-\infty} \to 0$$

$$= -\lim_{x \to \infty} e^{-2x} + e^{-2(0.5)} = e^{-1} = \frac{1}{e}.$$

Existe una probabilidad del 36.78% de que la entrega se realice 0.5 hora después.

b. ¿Cuál es la probabilidad de que la entrega se realice en menos de 15 min? ; 1/4 de hora

$$P(X \leq 1/4) = \int_0^{1/4} 2e^{-2x}\,dx = \int_{1/4}^{0} e^{\overbrace{-2x}^{u}}\,\overbrace{(-2\,dx)}^{du}$$

$u = -2x \quad du = -2\,dx \qquad u(0) = 0 \qquad u(1/4) = -1/2$

$$P(X \leq 1/4) = \int_{-1/2}^{0} e^{u}\,du = e^{u}\Big]_{-1/2}^{0} = 1 - e^{-1/2} \approx 0.3934$$

Estadísticos Importantes: Media, Mediana y Varianza

Variables	x_1	2	3	4	5	6.
Discretas.	1/6	1/6	1/6	1/6	1/6	1/6.
	$p(x_1)$					

Media $= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + \dots 6\left(\frac{1}{6}\right)$

Dados: $= \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5$

Media $\mu = \sum_{i=1}^{n} x_i f(x_i)$ $f(x_i)$ peso, frecuencia probabilidad.

Discretas

Variable continua: se integra $x f(x)$.

Media $\mu = \int_{-\infty}^{\infty} x f(x)\, dx$ el dominio de $f(x)$ puede restringir el intervalo de integración.

(mu)

realice IPP $u = x$
$dv = f(x)dx$.

Mediana: el número m que tiene una probabilidad acumulada del 50%.

$$\int_{-\infty}^{m} f(x)\, dx = 0.5$$

Resuelva para m.

$$F(m) = 0.5 \Rightarrow m = F^{-1}(0.5)$$

Varianza: la desviación cuadrado respecto a la media $(x-\mu)^2$.

(sigma)

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

Realice IPP dos veces $u = (x-\mu)^2$.

por partes.

Ejercicio 3: Considere la distribución exponencial.

$$f(x) = \frac{1}{100} e^{-x/100}. \qquad x \geq 0.$$

a. Encuentre la media de $f(x)$

$$\mu = \int_0^\infty x \frac{e^{-x/100}}{100} dx = -xe^{-x/100}\Big]_0^\infty + \int_0^\infty e^{-x/100} dx$$

$u = x \qquad dv = \frac{1}{100} e^{-x/100} dx$

$du = dx \qquad v = -e^{-x/100}$

$$\mu = -xe^{-x/100}\Big]_0^\infty - 100e^{-x/100}\Big]_0^\infty$$

$$\mu = -\lim_{x\to\infty} xe^{-x/100} + 0 \cdot e^{-0} - 100 \lim_{x\to\infty} e^{-x/100} + 100 e^{-0}$$

$e^{-\infty} \to 0.$

$$\mu = 100 - \overbrace{\lim_{x\to\infty} \underbrace{xe^{-x/100}}_{L.H.}}^{0.}$$

$\infty \cdot e^{-\infty} = \infty \cdot 0$

$$\lim_{x\to\infty} \frac{x}{e^{x/100}} \underset{LH}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{100} e^{x/100}}$$

$\frac{0}{0}$ ó $\frac{\infty}{\infty}$ LH.

$\frac{1}{e^\infty} \to 0$

$$\mu = 100.$$

b. Encuentre la mediana de $f(x)$.

$$\int_0^m f(x)\,dx = 0.5. \qquad f(x) = \frac{1}{100} e^{-x/100}.$$

$$\frac{1}{100}\int_0^m e^{-x/100}\,dx = -\frac{100}{100} e^{-x/100}\Big]_0^m = -e^{-m/100} + e^0.$$

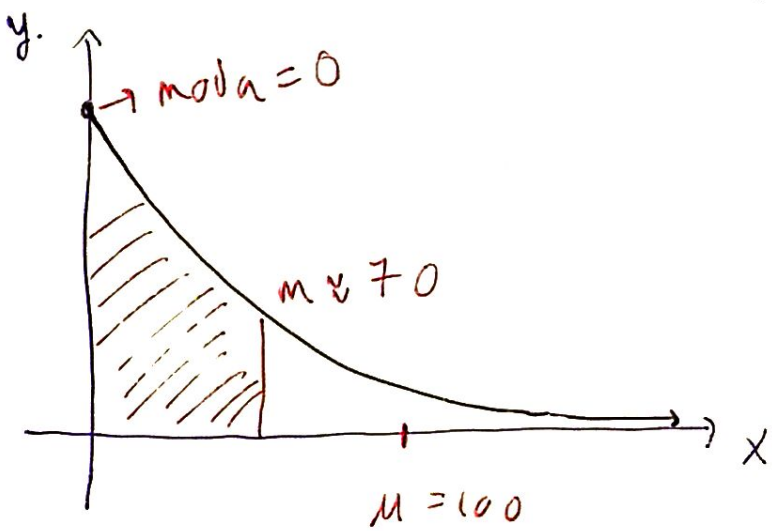
$$1 - e^{-m/100} = 0.5. \qquad \text{Resuelva para } m.$$

$$-e^{-m/100} = -0.5$$

$$e^{-m/100} = 0.5.$$

Aplique ln's: $\frac{-m}{100} = \ln(0.5) \rightarrow m = -100\,\underbrace{\ln(0.5)}_{-0.69}$

$$m \approx 69.31$$



Varianza $\sigma^2 = \int_0^\infty (x-100)^2 \frac{e^{-x/100}}{100}\,dx$

$$u = (x-100)^2 \qquad dv = \frac{e^{-x/100}}{100}\,dx$$

$$du = 2(x-100) \qquad v = -e^{-x/100}$$

$$\sigma^2 = -(x-100)^2 e^{-x/100}\Big]_0^{\infty} + \int_0^{\infty} 2(x-100)\, e^{-x/100}\, dx$$

$$\sigma^2 = -\lim_{x\to\infty} \frac{(x-100)^2}{e^{x/100}} + 100^2 e^{-0}$$

L'H IPP

$$\sigma^2 = 100^2 \qquad \sigma = 100$$

Ejercicio 4: Encuentre la media, mediana, varianza y desviación estándar de la distribución uniforme.

$$f(x) = \frac{1}{b-a} \qquad a \le x \le b. \qquad \text{para } b=7,\ a=1$$

Media: $\mu = \int_a^b x f(x)\, dx = \int_1^7 \frac{x}{6}\, dx = \frac{x^2}{12}\Big]_1^7$

$$\mu = \frac{49-1}{12} = \frac{48}{12} = 4.$$

Mediana: $\int_a^m f(x)\, dx = 0.5 \qquad \int_1^m \frac{1}{6}\, dx = \frac{x}{6}\Big]_1^m$

$$\frac{m-1}{6} = 0.5. \Rightarrow m = 0.5(6)+1 = 4.$$

No hay moda.

Varianza: $\int_a^b (x-\mu)^2 f(x)\, dx = \int_1^7 (x-4)^2 \underbrace{\frac{1}{6}}_{f(x)}\, dx$

$$u = x-4 \qquad du = dx \qquad \begin{array}{l} u(7) = 3 \\ u(1) = -3 \end{array}$$

$$\frac{1}{6}\int_1^7 (x-4)^2\,dx = \frac{1}{6}\int_{-3}^{3} u^2\,du = \frac{2}{6}\int_0^3 u^2\,du$$

$$\sigma^2 = \frac{1}{3}\,\frac{1}{3}\,u^3\Big]_0^3 = \frac{27}{9} = 3$$

Desviación estándar: $\sigma = \sqrt{\text{varianza}} = \sqrt{3}$.

4. Longitud de arco $\int \sqrt{1+(y')^2}\,dx$ cab 9.

$$\int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right)\Big]$$

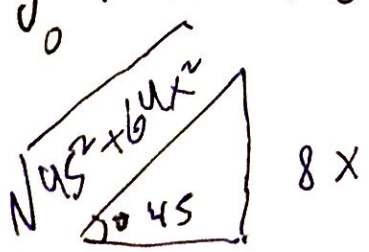
$x = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$.

5. $y = 180 - \left(\frac{2x}{45}\right)^2 = 180 - \frac{4x^2}{45^2}$.

$y' = \frac{-8x}{45^2}$ $\qquad 1+(y')^2 = 1 + \frac{64x^2}{45^2}$.

$$= \frac{1}{45^2}\left(45^2 + 64x^2\right).$$

$$L = \int_0^{90} \sqrt{45^2 + 64x^2}\,dx$$



$\tan\theta = \frac{8x}{45}$ $\qquad \sec\theta = \frac{\sqrt{(\ )}}{45}$.

$$x = \frac{45}{8}\tan\theta \qquad dx = \frac{45}{8}\sec^2\theta\, d\theta.$$

$$L = \int_0^1 \sqrt{45^2 + 64x^2}\, dx = \int_0^1 45\sec\theta \cdot \frac{45}{8}\sec^2\theta\, d\theta.$$

$$L = \frac{45^2}{8}\int \sec^3\theta\, d\theta. = \frac{45^2}{16}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)$$

Capítulo 21

7.4 Fracciones parciales

Caso 1: Factores lineales distintos

Caso 2: Factores lineales repetidos

Lab Lunes 1-2:30 PM.
sim2 2:30 PM Centro Estudiantil.

7.4 Fracciones Parciales ✓

se utiliza para integrar funciones racionales.

$$\frac{P(x)}{Q(x)} = f(x)$$

P polinomio de grado n
Q polinomio de grado m.

condición: Denominador > numerador. } ✓

En caso de que el grado del numerador sea mayor que o igual. del denominador, realice la división larga.

$$\frac{6}{x^2-9}, \quad \frac{x+3}{x^3-9x}, \quad \frac{1}{x^2+4}, \ldots,$$

denominador es el más grande

$$\left.\frac{x^2+3}{x^2-9}, \quad \frac{x^4+x^2+1}{x^3-4x}\right\}$$ división larga

simplifique la función en dos o más fracciones parciales.

$$\frac{6}{x^2-9} = \frac{6}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

no puede integrar

A, B dos coeficientes "desconocidos"

$$\frac{6}{x^2-9} = \frac{A(x+3) + B(x-3)}{x^2-9}.$$

Igualando los numeradores cero en $x=-3, 3$.

$$6 = A(x+3) + B(x-3)$$

$x=3$: $6 = 6A + 0 \Rightarrow A = 1$

$x=-3$: $6 = 0 - 6B \Rightarrow B = -1$

$$\frac{6}{x^2-9} = \frac{1}{x-3} - \frac{1}{x+3}$$

$$\int \frac{6}{x^2-9}\,dx = \int \frac{dx}{x-3} - \int \frac{dx}{x+3} = \ln|x-3| - \ln|x+3| + C.$$

Finalmente integre.

Caso 1: Factores lineales distintos.

$$\frac{p(x)}{(x^2-9)(x^2-4)} = \frac{p(x)}{(x-3)(x+3)(x-2)(x+2)}$$

$$= \frac{A}{x-3} + \frac{B}{x+3} + \frac{C}{x-2} + \frac{D}{x+2}$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + c.$$

Encuentre A, B, C y D.

Caso 2: Factores Lineales Repetidos.

$$f(x) = \frac{x^2+x+1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$x+1$ Encuentre A, B y C.

$$\int f(x) = A\ln|x+1| - B(x+1)^{-1} - \frac{1}{2}C(x+1)^{-2} + K.$$

$$\int (x+a)^{-t}\,dx = \frac{(x+a)^{-t+1}}{1-t} + K.$$

Ejercicio 1: $\displaystyle\int \frac{18z}{2z^2+7z-4}\,dz$ P(64)

1. Factorice dem: $2z^2+7z-4 = (2z-1)(z+4)$.

$2z \quad -1$

$z \quad 4$

$2(z^2+3.5z+2)$

2. Ceros Den. $2z-1=0 \Rightarrow z=0.5$. } lineales distintos.

$z+4=0 \Rightarrow z=-4$

$$\frac{18z}{2z^2+7z-4} = \frac{A}{2z-1} + \frac{B}{z+4}$$

¿A, B?

Multiplique por $(2z-1)(z+4)$

$$18z = A(z+4) + B(2z-1)$$

$z=0.5$: $9 = 4.5A + 0 \Rightarrow A = 9/4.5 = 2.$

$z=-4$: $18(-4) = 0 - 9B \Rightarrow B = \frac{-18}{-9}(4) = 8$

$$\int \frac{18z}{2z^2+7z-4} = \int \frac{2}{2z-1}\,dz + 8\int \frac{dz}{z+4}$$

$$= \frac{2}{2}\ln|2z-1| + 8\ln|z+4| + C.$$

Ejercicio 2: Integre las sigs. funciones.

a) $\displaystyle\int \frac{4x+2}{x^2+2x+1}\,dx$ b) $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

a) $x^2+2x+1 = (x+1)^2$. factor lineal repetido.

$$\frac{4x+2}{x^2+2x+1} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \quad \therefore \quad \text{¿ } A, B \text{ ?}$$

Multiplique por $(x+1)^2$ denominador se hace cero en $x=-1$

$$4x+2 = A(x+1) + B.$$

$x=-1$: $-2 = 0 + B \Rightarrow B=-2.$

$x=0$: $2 = A + B. \Rightarrow A = 2 - B = 2+2 = 4.$

$$\int \frac{4x+2}{x^2+2x+1}\,dx = \int \frac{4}{x+1}\,dx + \int -2(x+1)^{-2}\,dx$$

$$= 4\ln|x+1| + 2(x+1)^{-1} + C.$$

b) $\int \frac{x^2+2x-1}{x^3-x}\,dx$ $\qquad x^3-x = x(x^2-1)$
$= x(x+1)(x-1)$

ceros en $0, -1, 1$ (factores lineales distintos)

$$\frac{x^2+2x-1}{x^3-x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$x^2+2x-1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

$x=0$: $-1 = -A + 0 + 0 \Rightarrow A = 1$

$x=-1$: $-2 = 0 + 2B + 0 \Rightarrow B = -1$

$x=1$: $2 = 0 + 0 + 2C \Rightarrow C = 1$

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{dx}{x} - \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$$

$$= \ln|x| - \ln|x+1| + \ln|x-1| + C.$$

Capítulo 22

7.4 Fracciones parciales

Caso 3: Factores cuadráticos irreducibles

Caso 4: Factores cuadráticos repetidos

sesión Resolución Dudas Lun 1-2:30 PM
D-503.

Parcial 2 Lun 2:30 CES.

Conto 9 | Fracciones Parciales.

Lab 11 Hoy Entrega Jueves 17 octubre.

Fracciones Parciales.

Caso 1 y 2: Factores Lineales.

$$\frac{P(x)}{x(x+1)^2(x+2)^3} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+2)} + \frac{E}{(x+2)^2} + \frac{F}{(x+2)^3}$$

$$\int \frac{A\,dx}{x+a} = A\ln|x+a| + C \qquad \int B(x+a)^{-n}\,dx \quad -\frac{B}{-n+1}\frac{1}{(x+a)^{n-1}} + C.$$

Ejemplo 1: $\int \frac{x^2+x}{(x+2)^2(x-1)}\,dx$ $\qquad (x^2+4x+4)(x-1)$

$$\frac{x^2+x}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)} \quad * (x+2)^2(x-1)$$

$$x^2+x = A(x+2)(x-1) + B(x-1) + C(x+2)^2.$$

cero en $x=-2$ y en $x=1$

$x=-2$: $2 = 0 - 3B + 0$ $\qquad B = -2/3 = -6/9$

$x=1$: $1^2+1 = 0 + 0 + 9C$ $\qquad C = 2/9$

$x=0$: $0 = -2A - B + 4C.$ $\qquad 2A = -B - 4C = -2/9$

$A = -1/9, \quad B = -2/3 \quad C = 2/9. \quad \int (x+2)^{-2}\,dx$

$$\int \frac{x^2+x}{(x+2)^2(x-1)}\,dx = -\frac{1}{9}\int \frac{dx}{x+2} - \frac{2}{3}\int \frac{dx}{(x+2)^2} + \frac{2}{9}\int \frac{dx}{x-1}$$

$$= C \quad -\frac{1}{9}\ln|x+2| + \frac{2}{3}\,\frac{1}{x+2} + \frac{2}{9}\ln|x-1|$$

Caso 3: Factores Cuadráticos Irreducibles.

no tienen raíces reales. $b^2-4ac<0$ imaginario

x^2+36 $\qquad$ x^2+x+1 $\qquad$ no se pueden factorizar.

$x^2=-36.$ $\qquad$ $x^2+x+1=0$

cuadrática: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-4}}{2}$ no tiene soln reales

$$\frac{P(x)}{(x^2+36)(x^2+x+1)} = \frac{Ax+B}{x^2+36} + \frac{Cx+D}{x^2+x+1}$$

$$\int \frac{x}{\underbrace{x^2+k^2}_{u}}\,dx \qquad \int \frac{du}{2u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+k^2| + C.$$

$du/2$

$$\int \frac{1}{x^2+k^2}\,dx = \int \frac{k\sec^2\theta\,d\theta}{k^2\sec^2\theta} = \frac{1}{k}\int d\theta = \frac{1}{k}\theta + C.$$

$x = k\tan\theta.$
$dx = k\sec^2\theta\,d\theta$

$$\boxed{\frac{1}{k}\tan^{-1}\left(\frac{x}{k}\right) + C.}$$

Ejercicio 6: Integre P. 69.

b. $\displaystyle\int \frac{2x^2-x-4}{x^3+4x}\,dx$ $\qquad x(x^2+4) = x^3+4x$

$$\frac{2x^2-x-4}{x^3+4x} = \frac{A}{x} + \frac{Bx}{x^2+4} + \frac{C}{x^2+4} \quad * \; x^3+4x.$$

$$2x^2-x-4 = Ax^2+4A + Bx^2 + Cx$$

$$2x^2-x-4 = (A+B)x^2 + Cx + 4A.$$

Igualando los coeficientes.

Grado 2: $A+B = 2. \Rightarrow B = 2-A. = 3.$

Grado 1: $C = -1 \Rightarrow C = -1$

Grado 0: $4A = -4 \Rightarrow A = -1$

$A = -1,\ B = 3,\ C = -1$

$$\int \frac{2x^2-x-4}{x^3+4x}\,dx = -\int \frac{dx}{x} + 3\int \frac{x\,dx}{x^2+4} - \int \frac{dx}{x^2+4}$$

$$= -\ln|x| + \frac{3}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C.$$

b. $\int \frac{1}{x^4-1}\, dx$ 1.

$$x^4-1 = (x^2+1)(x^2-1) = (x^2+1)(x-1)(x+1)$$

$$\frac{1}{x^4-1} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+1}$$

$$1 = (Ax+B)(x^2-1) + C(x^2+1)(x+1) + D(x^2+1)(x-1)$$

$$0x^3+0x^2+1 = Ax^3+Bx^2-Ax-B+Cx^3+Cx^2+Cx+C.$$
$$+0x \qquad +Dx^3-Dx^2+Dx-D$$

Igualando coeficientes.

(1) $A+C+D=0$ (1)+(3) $2C+2D=0$ (5)

(2) $B+C-D=0$ (2)+(4) $2C-2D=1$ (6)

(3) $-A+C+D=0$ $4C=1$

(4) $-B+C-D=1$

$C=1/4$ $D=-C=-1/4.$

$A=-C-D=-1/4+1/4=0$ $B=D-C=-\frac{1}{4}-\frac{1}{4}=-\frac{1}{2}.$

$$\int \frac{dx}{x^4-1} = \int \frac{-1/2}{x^2+1} + \frac{1/4}{x-1} - \frac{1/4}{x+1}\, dx$$

$$= -\frac{1}{2}\tan^{-1}x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + C.$$

Factores Cuadráticos repetidos (Caso 4) P. 70.

$$\frac{P(x)}{(x^2+25)^2(x^2+36)^3} = \frac{Ax+B}{(x^2+25)} + \frac{Cx+D}{(x^2+25)^2} + \frac{Ex+F}{(x^2+36)}$$

$$+ \frac{Gx+H}{(x^2+36)^2} + \frac{Ix+J}{(x^2+36)^3}$$

Ejercicio 7: $\int \frac{x^2+2}{x^5+8x^3+16x}\,dx$

Cuadrado Perfecto

$$\int \frac{x^2+2}{x^5+8x^3+16x}\,dx \qquad x(x^4+8x^2+16)$$

$$x(x^2+4)^2.$$

$$\frac{x^2+2}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}.$$

Multiplique por $x(x^2+4)^2$.

$$x^2+2 = A(x^4+8x^2+16) + (Bx+C)(x^3+4x) + Dx^2 + Ex$$

$$x^2+2 = Ax^4 + 8Ax^2 + 16A + Bx^4 + Cx^3 + 4Bx^2 + 4Cx$$
$$Dx^2 \qquad + Ex.$$

Grado Cuatro: $A + B = 0 \Rightarrow B = -A = -1/8.$

Grado tres: $C = 0 \Rightarrow C = 0$

Grado Dos: $8A + 4B + D = 1 \qquad D = 1 - 1 + 4/8 = 1/2.$

Grado Uno: $4C + E = 0 \Rightarrow E = -4C = 0$

Grado cero. $16A = 2 \Rightarrow A = 1/8$

6.

$A = 1/8, \quad B = -1/8, \quad C = 0, \quad D = 1/2, \quad E = 0.$

$$\int \frac{x^2+2}{x(x^2+4)^2}\,dx = \frac{1}{8}\int \frac{dx}{x} - \frac{1}{8}\int \frac{x}{x^2+4}\,dx + \frac{1}{2}\int \frac{x}{(x^2+4)^2}\,dx$$

$$= \frac{1}{8}\ln|x| - \frac{1}{16}\ln|x^2+4| - \frac{1}{2}\,\frac{1}{2}\,\frac{1}{(x^2+4)} + C$$

$$8A + 4B + D = 1$$

$$1 - \frac{4}{8} + D = 1 \quad \Rightarrow \quad D = 1 - 1 + \frac{4}{8} = \frac{4}{8}$$

$$\frac{P(x)}{(x+1)(x+2)^2(x^2+4)(x^2+9)^2}$$

Capítulo 23

Lab. 10 Resueltos 3 y 4., Sección A.

5. Distribución exponencial $\mu = 4$. $f(x) = \frac{1}{4} e^{-x/4}$

¿Cuál es la probabilidad de que se atienda a la persona en menos de 3 minutos?

$$P(X < 3) = \int_0^3 \frac{1}{4} e^{-x/4}\, dx = -e^{-x/4}\Big]_0^3 = e^{-x/4}\Big]_3^0$$

$$= e^0 - e^{-3/4}$$

$$= 1 - e^{-3/4} \approx 52.76\%.$$

1. $f(x) = \dfrac{c}{1+x^2}$ $\quad -\infty \leq x \leq \infty$.

a. ¿Cuál es el valor de c para que f(x) sea función de probabilidad?

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \qquad c\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx = 1$$

$$2c\int_0^{\infty} \underbrace{\frac{1}{1+x^2}}_{\tan^{-1} x}\, dx = 2c\tan^{-1} x\Big]_0^{\infty}$$

$\tan(0) = 0$

$$= 2c \lim_{x\to\infty} \tan^{-1} x - \cancel{\tan^{-1}(0)}^{\,0} = 1$$

tan AV. en $x = \pi/2$.

$\tan^{-1}$ AH en $y = \pi/2$.

$$= 2\frac{\pi}{2} c = \pi c = 1 \Rightarrow c = \frac{1}{\pi}$$

b. ¿Cuál es la probabilidad de x esté entre -1 y 1?

$$P(-1 \leq x \leq 1) = \int_{-1}^{1} \frac{1}{\pi} \frac{1}{1+x^2}\, dx = \frac{2}{\pi}\int_{0}^{1} \frac{1}{1+x^2}\, dx$$

$$P(-1 \leq x \leq 1) = \frac{2}{\pi} \tan^{-1} x \Big]_0^1 = \frac{2}{\pi}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$\tan \frac{\pi}{4} = 1$

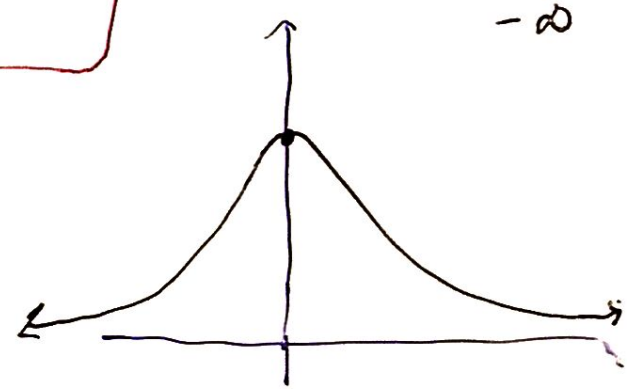
$$= \frac{2}{\pi} \cdot \frac{\pi}{4} = \frac{1}{2}.$$

$\tan 0 = 0$

c. ¿Cuál es la media de $f(x)$? $\infty - \infty$.

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \boxed{\int_{-\infty}^{\infty} \frac{x}{1+x^2}\, dx} = \frac{1}{2} \ln|1+x^2| \Big]_{-\infty}^{\infty}$$

$\frac{\text{Impar}}{\text{par.}} = \text{Impar}$ $\quad \mu = 0$

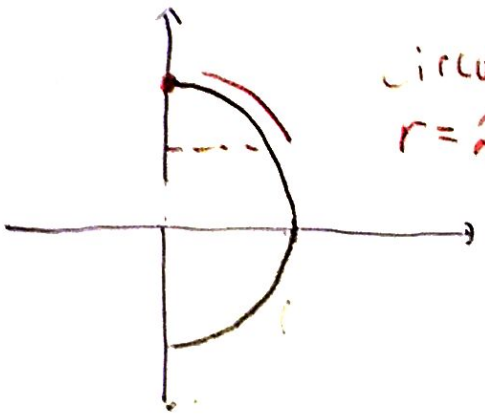


media = mediana = moda $= 0$.

Lab. 9. Prob 4.

$x = \sqrt{4-y^2}$ $\quad 1 \leq y \leq 2. \quad y = \sqrt{4-x^2}$

Encuentre la longitud de arco de x.



circunferencia
$r = 2.$

$$L = \int_1^2 \sqrt{1+(x')^2}\, dy.$$

$$L = \int_a^b \sqrt{1+(y')^2}\, dx$$

$$x' = \frac{1}{2}(4-y^2)^{-1/2}(-2y) = \frac{-y}{\sqrt{4-y^2}}$$

$$1+(x')^2 = 1 + \frac{y^2}{4-y^2} = \frac{4-y^2+y^2}{4-y^2} = \frac{4}{4-y^2}$$

$$\sqrt{1+(x')^2} = \frac{\sqrt{4}}{\sqrt{4-y^2}} = \frac{2}{\sqrt{4-y^2}}$$

$$L = \int_1^2 \frac{2}{\sqrt{4-y^2}}\, dy. = 2\sin^{-1}\left(\frac{y}{2}\right)\Big]_1^2 = 2\sin^{-1}(1) - 2\sin^{-1}(0.5)$$

$y = 2\sin\theta.$

$dy = 2\cos\theta\, d\theta.$

$\sqrt{4-y^2} = 2\cos\theta.$

$$\int \frac{2}{2\cos\theta}\, 2\cos\theta\, d\theta = \int 2\, d\theta.$$

$$= 2\theta + C.$$

$$= 2\sin^{-1}\left(\frac{y}{2}\right) + C.$$

$L = 2\sin^{-1}(1) - 2\sin^{-1}(0.5)$ $\qquad \sin \pi/2 = 1$

$\sin \pi/6 = 0.5$

$L = 2 \cdot \frac{\pi}{2} - 2\frac{\pi}{6} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$

Lab 6 Prob (b)

b) $\int_{-2}^{3} \frac{1}{\sqrt[4]{x+2}}\, dx$ $\qquad$ ¿Dónde es impropia?

en $x=-2$ discontinuidad.

$\int_{-2}^{3} (x+2)^{-1/4}\, dx = \frac{4(x+2)^{3/4}}{3}\Big]_{-2}^{3}$ $\qquad 0^{3/4} = 0$

$= \frac{4}{3}\left(5^{3/4} - \lim_{x\to -2^+} (x+2)^{3/4}\right)$

$x^2(2+x)$ $\qquad = \frac{4}{3} 5^{3/4}$ $\qquad$ CONVERGENTE

b) Simulacro $\int_{0}^{1} \frac{12x^2+4x}{2x^3+x^2}\, dx$ $\qquad$ indefinida en $x=0$.

$\frac{12x^2+4x}{x^2(2+x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$] MÁS LARGA.

ATAJO. $\qquad u = 2x^3 + x^2 \qquad du = (6x^2+2x)\,dx$

$2\int_{0}^{1} \frac{(6x^2+2x)}{2x^3+2x}\, dx = 2\int_{0}^{3} \frac{du}{u} = 2\ln u\Big]_{0}^{3}$ DIVERGENTE

porque $\lim_{u\to 0^+} \ln u = -\infty$.

Integre en fracciones parciales

$$\int \frac{12x^2+4x}{x^2(2+x)} = \int \frac{A}{x}\,dx + \int \frac{B\,dx}{x^2} + \int \frac{C}{x+2}\,dx$$

* $12x^2+4x.$

$$12x^2+4x = Ax(x+2) + B(x+2) + Cx^2.$$

$$12x^2+4x+0 = Ax^2+2Ax+Bx+2B+Cx^2.$$

$$A+C=12 \qquad C=12-A=10$$

$$2A+B=4 \qquad 2A=4 \quad A=2$$

$$2B=0 \Rightarrow B=0$$

$$\int \frac{12x^2+4x}{x^2(2+x)} = \int \frac{2}{x}\,dx + 0 + \int \frac{10}{x+2}\,dx$$

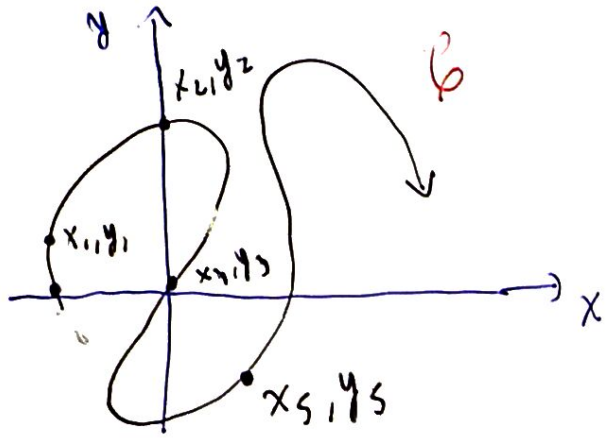
$$= 2\ln|x| + 10\ln|x+2|$$

Capítulo 24

10.1 Ecuaciones paramétricas

10.1 Ecuaciones Paramétricas

Describen el comportamiento de una partícula que se mueve a lo largo de una curva $\mathcal{C}$ en 2-D.



curva $\mathcal{C}$ no se puede representar por medio de $x = f(y)$ ó $y = g(x)$

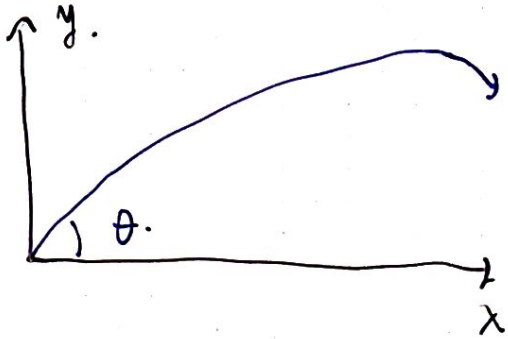
Los puntos sobre la curva paramétrica $\mathcal{C}$ se pueden representar por medio de las ecs. paramétricas

$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\} \text{Ecs. Paramétricas}$$

variable independiente t, conocida como un paramétro.

Ejercicio 1: $x = 4t - t^2$, $y = t + 2$.

Movimiento Parabólico en 2-D.



$$x = V_0(\cos\theta)t$$
$$y = V_0(\sin\theta)t - \frac{1}{2}gt^2.$$

$V_0 = \sqrt{2}\,10 \quad \theta = \pi/4 \quad y = 10$

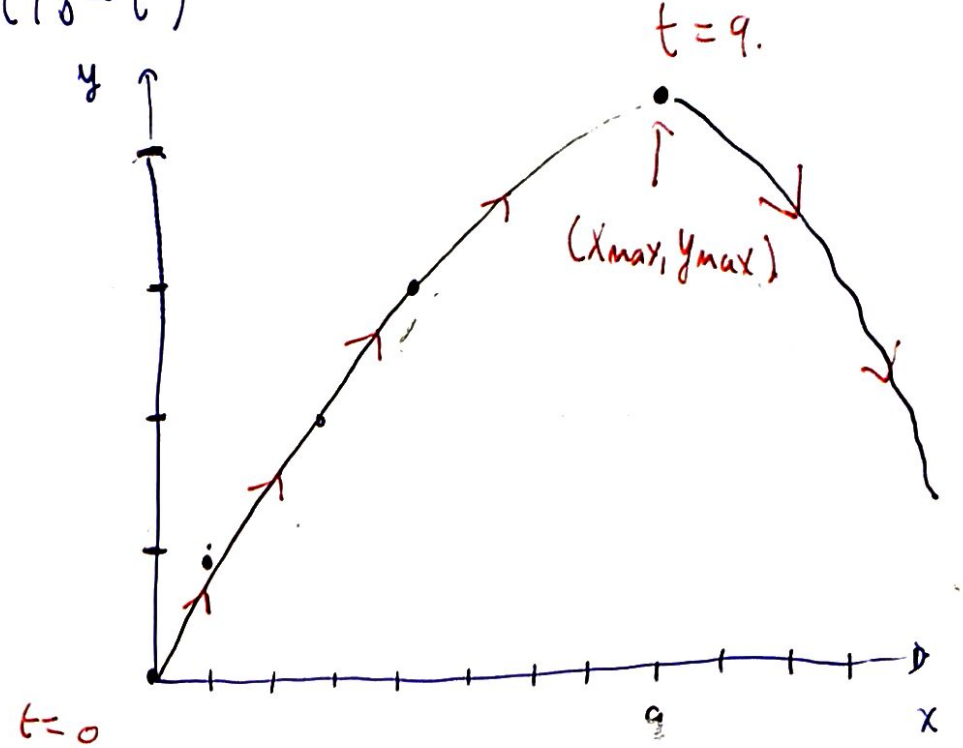
$$x = \sqrt{2}\,\frac{90}{\sqrt{2}}\,t = 90t. \qquad y = \frac{90\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}\,t - 5t^2.$$

a. Use una tabla de valores para bosquejar la curva paramétrica

$x = 90t$

$y = 90t - 5t^2 = 5t(18-t)$

t	x	y.
0	0	0
1	90	85
2	180	160
3	270	225
4	360	320
9	810	405.
13		325
18.	1620	0



traza una parábola

b. Elimine el parámetro para encontrar la ec. de la curva paramétrica.

$x = 90t \Rightarrow t = \frac{x}{90}$ sustituya en y:

$y = 90t - 5t^2.$

$$y = 90\left(\frac{x}{90}\right) - 5\left(\frac{x}{90}\right)^2 = x - \frac{5x^2}{90^2}.$$

Altura máxima cuando $y' = 1 - \frac{10x}{90 \cdot 90} = 0$

$10x = 90 \cdot 90$

$x = 810.$

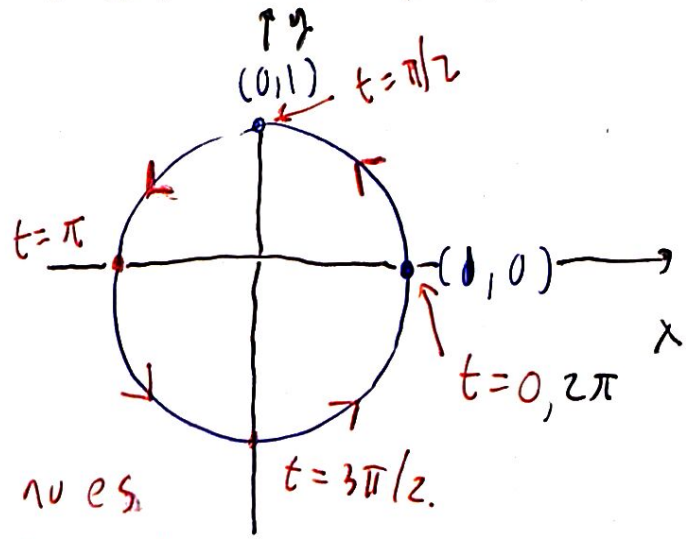
$x = \frac{90}{10} 90 = 9 \cdot 90 = 810$

Ejercicio 2: ¿Qué curva representan las sigs. ecuaciones paramétricas? P. 130.

a. $x = \cos t$, $y = \sin t$., $0 \leq t \leq 2\pi$

Elimine el parámetro t. Como $\cos^2 t + \sin^2 t = 1$

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$



Ec. Circunferencia de radio 1 $x^2 + y^2 = 1$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	1	0	-1	0	1
y	0	1	0	-1	0

no es función

Curva Paramétrica, tiene un sentido antihorario, se le da una vuelta al círculo.

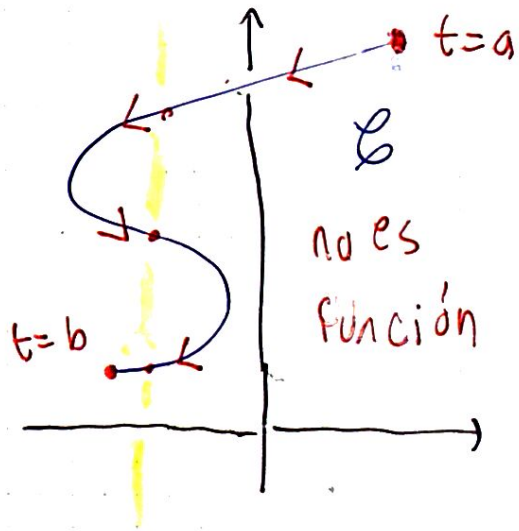
Una curva paramétrica $\mathcal{C}: x = f(t)$, $y = g(t)$ $a \leq t \leq b$.

tiene.

i un punto inicial en $t = a$

ii. un punto terminal en $t = b$.

iii orientación o sentido (el cual se indica con flechas)



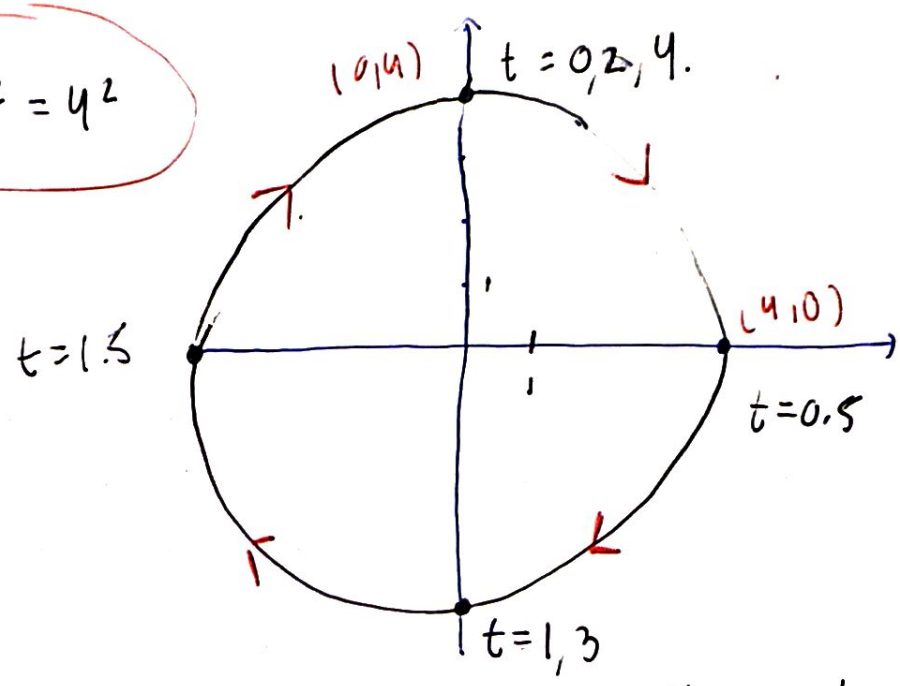
b. $x = 4\sin \pi t$, $y = 4\cos \pi t$, $0 \le t \le 4$.

$$x^2 + y^2 = 16\sin^2 \pi t + 16\cos^2 \pi t = 16.$$

Circunferencia
Radio 4

$$x^2 + y^2 = 4^2$$

t	x	y	
0	0	4	
0.5	4	0	
1	0	-4	
1.5	-4	0	
2	4	0	1 vuelta
3	0	-4	
4	4	0	2 vueltas.



$$\frac{2\pi}{\pi} = 2.$$ 1 vuelta cada 2 s.

sentido horario, 2 vueltas a la circunferencia

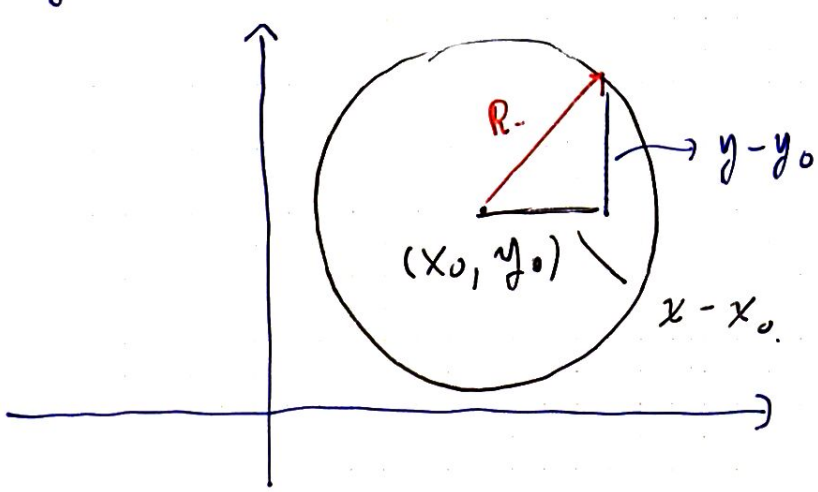
Ejercicio 3: Encuentre unas ecuaciones paramétricas que representen a una circunferencia con centro (x_0, y_0) y radio R.

Ec. Cartesiana

$$\underbrace{(x - x_0)}_{R\cos\theta}{}^2 + \underbrace{(y - y_0)}_{R\sin\theta}{}^2 = R^2.$$

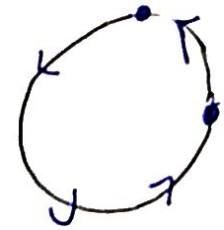
$$\sin^2 + \cos^2\theta = 1$$

$$R^2\cos^2\theta + R^2\sin^2\theta = R^2.$$



$x - x_0 = R\cos\theta.$ $\Rightarrow$ $x = x_0 + R\cos\theta$

$y - y_0 = R\sin\theta.$ $\quad$ $y = y_0 + R\sin\theta$



Anti horario $\quad 0 \leq \theta \leq 2\pi.$

$(x - x_0)^2 + (y - y_0)^2 = R^2$

$-R\sin\theta \qquad R\cos\theta.$

Otra parametrización

$x = x_0 - R\sin\theta$

$y = x_0 + R\cos\theta.$

$0 \leq \theta \leq 2\pi.$

$(x - x_0)^2 + (y - y_0)^2 = R^2$

$R\cos(8\theta) \qquad R\sin(8\theta)$

Otra más.

$x = x_0 + R\cos(8\theta)$

$y = y_0 + R\sin(8\theta)$

$0 \leq \theta \leq 2\pi.$

Para que sea única, se necesitan más condiciones (horario/antihorario, vueltas, punto inicial y terminal).

Ejercicio 4: Trace la curva con ecs. paramétricas

$$x = \cos\theta \qquad y = \cos^4\theta$$

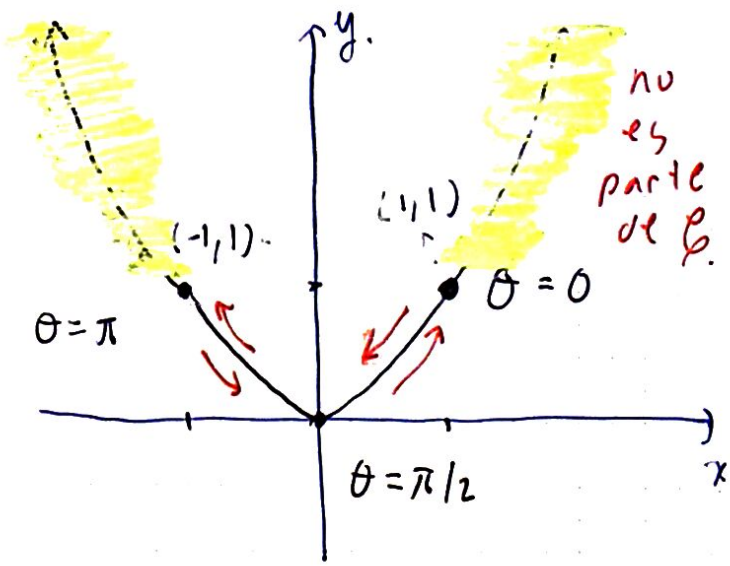
Indique con flechas la dirección de la curva.

Elimine el parámetro θ.

$\cos\theta = x, \quad y = (\cos\theta)^4$

$y = x^4$

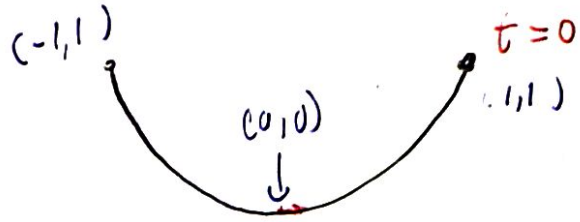
θ	0	$\pi/2$	π	$3\pi/2$	2π
x	1	0	-1	0	1
y	1	0	1	0	1



curva paramétrica $-1 \leq \cos\theta \leq 1$

$y = x^4, \quad -1 \leq x \leq 1$

$x = \cos\theta \qquad y = \cos^4\theta.$



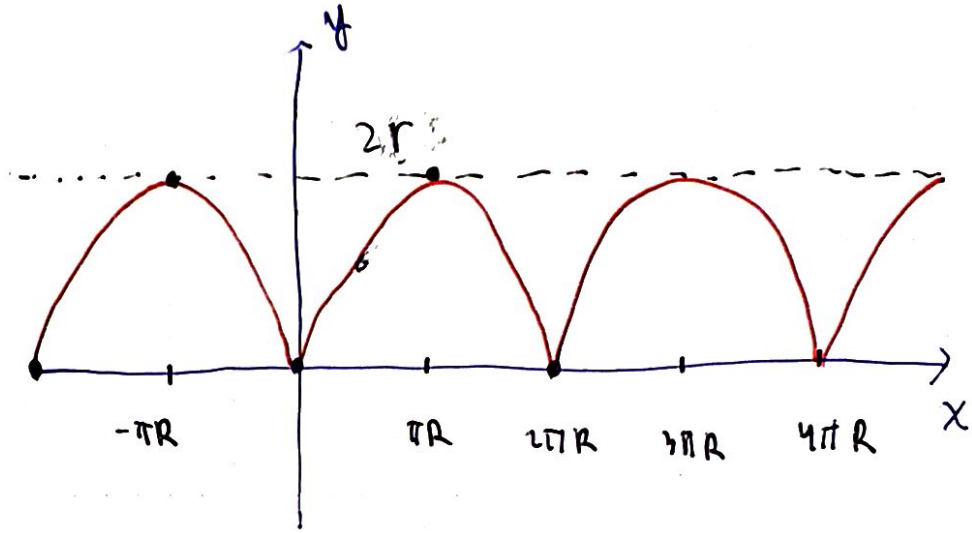
sube y baja la rampa

La Cicloide: es difícil $y = f(x)$

$x = r(\theta - \sin\theta)$

$y = r(1 - \cos\theta)$

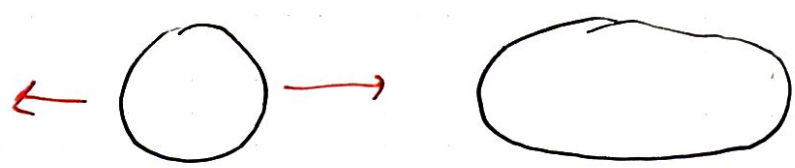
θ	x	y.
-2π	$-2\pi r$	0
$-\pi$	$-\pi r$	$2r$
0	0	0
$\pi/2$.	$r(\frac{\pi}{2} - 1)$	r
π	πr	$2r$
2π	$2\pi r$	0



$\sin(-2\pi) = -\sin(2\pi) \qquad \cos(n\pi) = \pm 1$

$\sin(n\pi) = 0.$

Elipse.



Ec. Cartesiana. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad a \neq b.$

$x = a \cdot \cos\theta. \qquad y = b \cdot \sin\theta.$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2\cos^2\theta}{a^2} + \frac{b^2}{b^2}\sin^2\theta = \cos^2\theta + \sin^2\theta = 1.$$

$x = a \cdot \cos\theta$

$y = b \cdot \sin\theta.$

$0 \leq \theta \leq 2\pi$

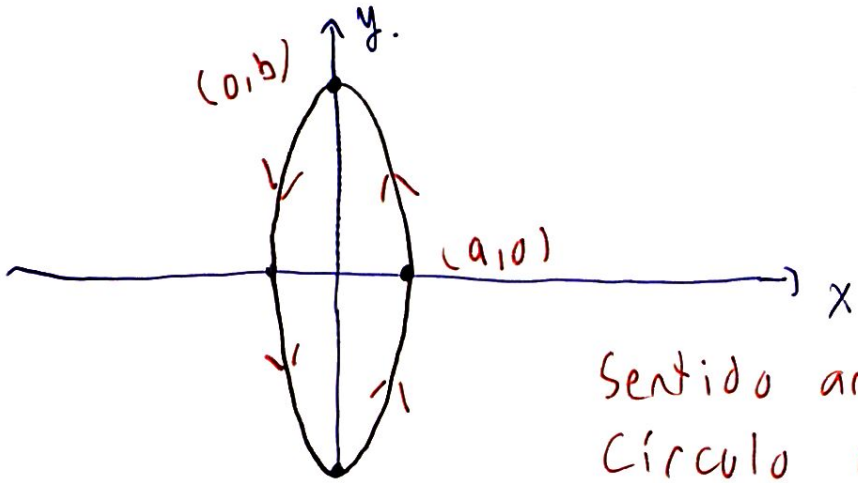
$$x^2 + \frac{y^2}{9} = 1$$

$a = 1$

$b = 3$

$x = 0: \quad y^2 = 9 \Rightarrow y = \pm 3. \quad (0, \pm 3)$

$y = 0: \quad x^2 = 1 \Rightarrow x = \pm 1 \quad (\pm 1, 0)$



Sentido antihorario

Círculo Achatado.

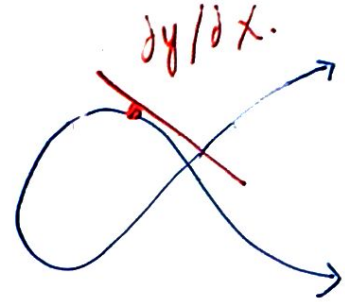
$$\underbrace{\frac{dy}{dx} = \frac{y'(t)}{x'(t)}}_{\text{Jueves 10.2 derivadas}} \qquad \underbrace{L = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt.}_{\text{Longitud de Arco.}}$$

Capítulo 25

10.2 Calculo con ecuaciones paramétricas

10.2 Cálculo con Ecuaciones Paramétricas

1. Derivadas y Rectas Tangentes.



curva $\mathcal{C}$: $x = f(t)$ ✓ $a \leq t \leq b$.
$y = g(t)$ ✓

A veces no se puede eliminar el paramétro t. para encontrar y en función de x.

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} \checkmark$$

$y \to x \to t$. en función de t.

Pendiente recta tangente en $t = t_0$. $m = \left.\frac{dy}{dx}\right|_{t=t_0}$.

Ec. Recta Tangente $y = \underline{\underline{y_1}} + m(x - \underline{\underline{x_1}})$

$$y = y(t_0) + m(x - x(t_0))$$

Ejercicio 1: p.135 Encuentre la ec. de la recta tangente a la curva $\mathcal{C}$: $x = f(t)$, $y = g(t)$ en el valor $t = a$

a. $x = 1 + 2t$, $y = 2 - 2t^3$. en $t = 1$.

Solución 1: $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-6t^2}{2} = -3t^2$. ✓

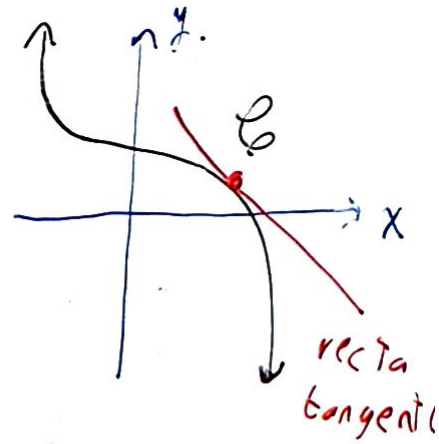
Pendiente : $m = \left.\dfrac{dy}{dx}\right|_{t=1} = -3(1)^2 = -3.$ ✓

evalúe en $t=1$

Solución 2: Elimine el parámetro t.

$2t = x-1 \Rightarrow t = \frac{1}{2}(x-1)$ sustituya en y:

$t^3 = \frac{1}{8}(x-1)^3 \Rightarrow y = 2 - \frac{2}{8}(x-1)^3.$



$\dfrac{dy}{dx} = -\dfrac{3}{4}(x-1)^2.$ $\quad x(1) = 1+2 = 3$

$m = \left.\dfrac{dy}{dx}\right|_{x=3} = -\dfrac{3}{4}\,2^2 = -3$ misma respuesta.

Recta Tangente: $x(1) = 1+2 = 3$
$y(1) = 2-2 = 0$

Ec. Recta Tangente: $y = 0 - 3(x-3) = \underline{-3x+9}$

a. $x = 1 + 2t^{1/2}$ $\quad y = e^{t^2}$ en $t = 1$.

Derivada: $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)} = \dfrac{2te^{t^2}}{\div^{-1/2}} = 2t\,t^{1/2}e^{t^2}$

$$\frac{dy}{dx} = 2t^{3/2}e^{t^2}.$$

Pendiente: $\left.\dfrac{dy}{dx}\right|_{t=1} = 2\cdot 1^{3/2}e^{1^2} = \underline{2e}$ *

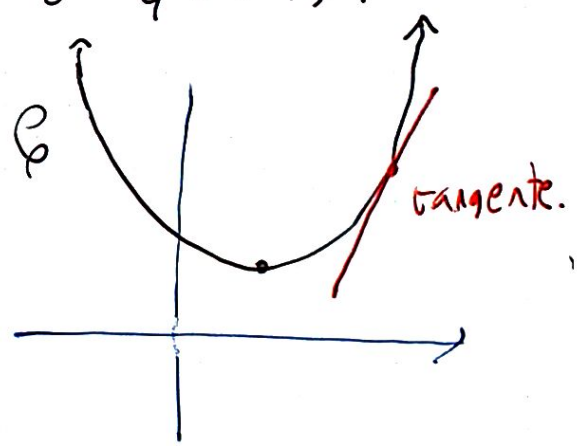
Coordenadas: $x(1) = 1 + 2\sqrt{1} = 3$ $\quad y(1) = e^1 = e.$

Ec. Recta Tangente: $y = e + 2e(x-3) = \boxed{-5e + 2ex}$

* Elimine el parámetro t.

$x - 1 = 2t^{1/2}.$ $\quad (x-1)^2 = 4t \Rightarrow t = \frac{1}{4}(x-1)^2.$

Sustituya en y: $y = e^{\frac{1}{16}(x-1)^4}$



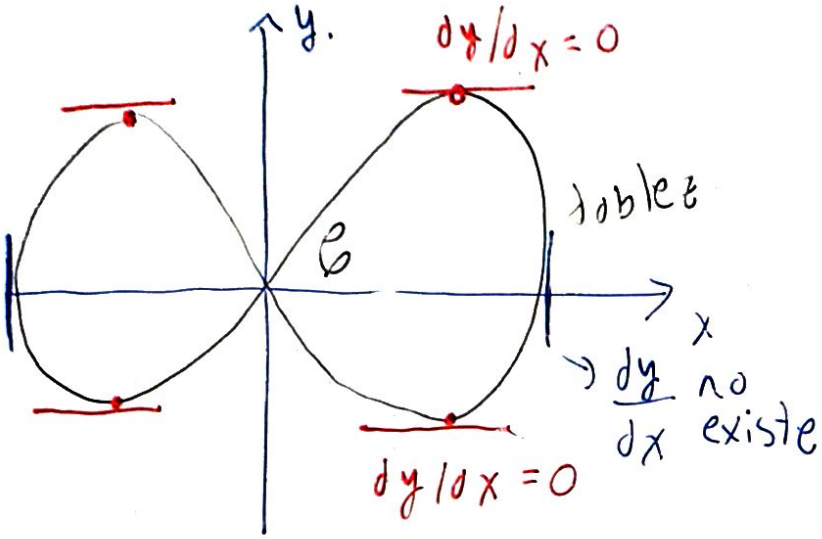
$$\frac{dy}{dx} = e^{\frac{1}{16}(x-1)^4}\cdot\frac{1}{4}(x-1)^3\cdot 1$$

$$\left.\frac{dy}{dx}\right|_{x=3} = e^{\frac{1}{16}2^4}\cdot\frac{1}{4}2^3$$

$$= e^{16/16}\cdot\frac{8}{4} = \underline{2e}\ *$$

misma respuesta

Tangentes horizontales y verticales



Tangente horizontal cuando $\frac{dy}{dx} = 0$

Tangente vertical cuando $\frac{dy}{dx}$ no existe.

$C: x = f(t), \; y = g(t)$

$\frac{a}{b} = 0 \Rightarrow a = 0$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = 0 \Rightarrow$ tangente horizontal cuando $y'(t) = 0$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$ se indefine $\Rightarrow$ tangente vertical cuando $x'(t) = 0$.

$\frac{1}{0}$ indefinido.

En Resumen, dada una curva C hay.

Tangentes Horizontales $y'(t) = 0$ & $x'(t) \neq 0$.

Tangentes Verticales. $x'(t) = 0$ & $y'(t) \neq 0$.

Indeterminado: cuando $x'(t) = y'(t) = 0$.

$\lim_{t \to t_0} \frac{x'(t)}{y'(t)} \quad \frac{0}{0}$ use la Regla de L'Hospital para encontrar la derivada.

Ejercicio 2: La curva $\mathcal{C}$ es definida por

$$x = t^3 - 3t \qquad y = t^3 - 3t^2.$$

a. Encuentre dy/dx. $\dfrac{dy}{dx} = \dfrac{3t^2 - 6t}{3t^2 - 3}$. $\qquad \dfrac{y'(t)}{x'(t)}$.

b. ¿En cuáles puntos (x, y) la tangente es horizontal a la curva $\mathcal{C}$?

Hay Tangentes horizontales cuando $y'(t) = 0$.

$$3t^2 - 6t = 3t(t-2) = 0 \Rightarrow t = 0, 2. \checkmark$$

$x'(0) = 0 - 3 \neq 0 \qquad x'(2) = 12 - 3 = 9 \neq 0.$

Puntos: $x(0) = 0 - 0 = 0 \qquad y(0) = 0 - 0 = 0 \qquad (0,0)$

$x(2) = 8 - 6 = 2 \qquad y(2) = 8 - 12 = -4 \qquad (2,-4)$

Tangentes horizontales en $(0,0)$ y $(2,-4)$.

c. ¿Dónde hay tangentes verticales?

$\dfrac{dy}{dx} = \dfrac{3t^2 - 6t}{3(t^2-1)}$ se indefine en $t = \pm 1$

Puntos: $x(1) = 1 - 3 = -2 \qquad y(1) = 1 - 3 = -2.$

$x(-1) = -1 + 3 = 2 \qquad y(-1) = -1 - 3 = -4$

Tangentes verticales en $(-2,-2)$ y $(-2,-4)$

J. Bosqueje la curva utilizando sólo las tangentes horizontales y verticales.

Tangentes Horizontales: $(0,0)$ y $(2,-4)$

Verticales: $(-2,-2)$ y $(2,-4)$

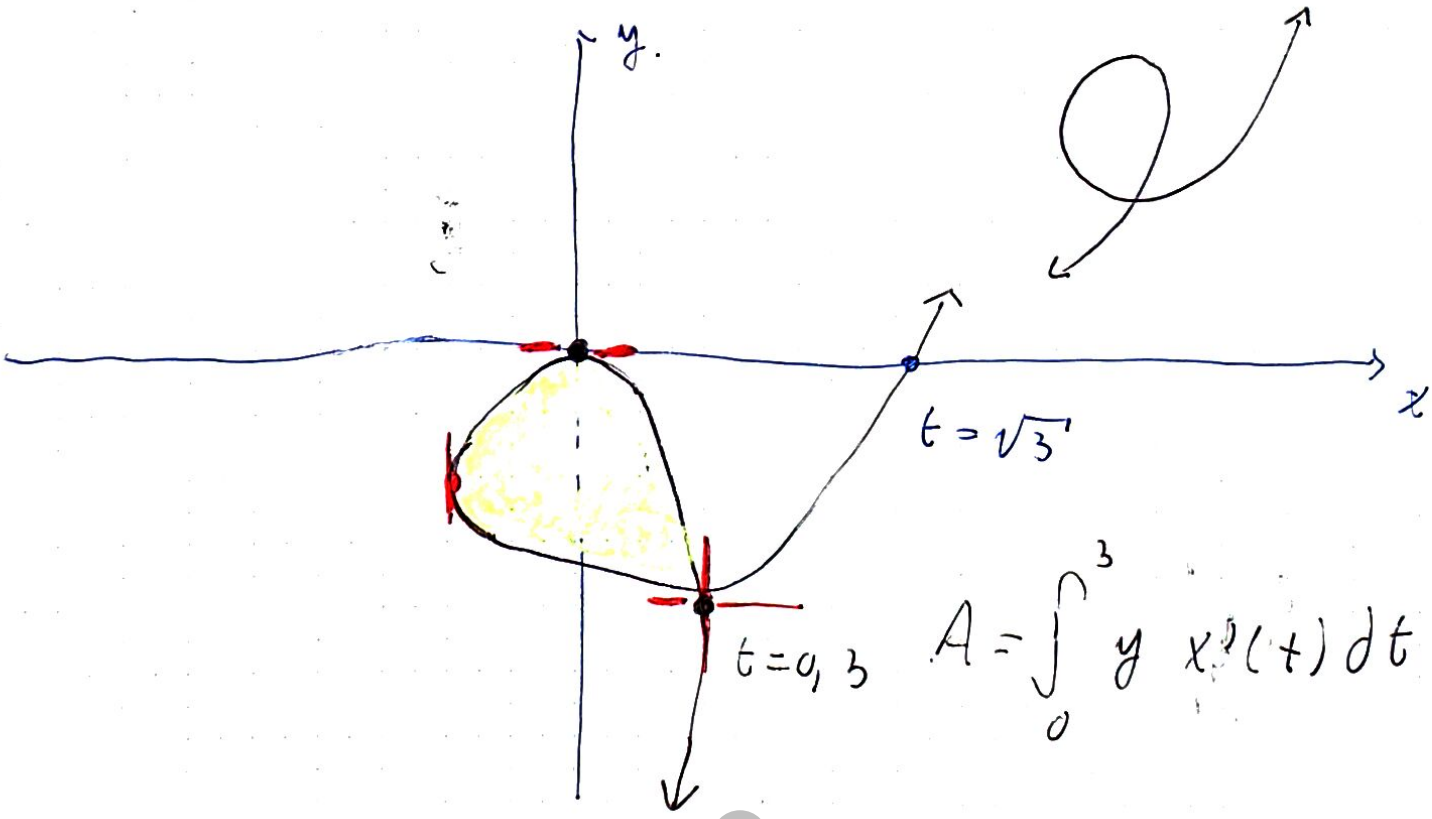
Luis $x=0$ cuando $t(t^2-3)=0$ $\quad t=0$, $t=\pm\sqrt{3}$

corte el eje-y en tres puntos

$y=0 \quad t^3-3t^2=t^2(t-3)=0 \quad t=0$, $t=3$.

$y(3)=27-27=0$

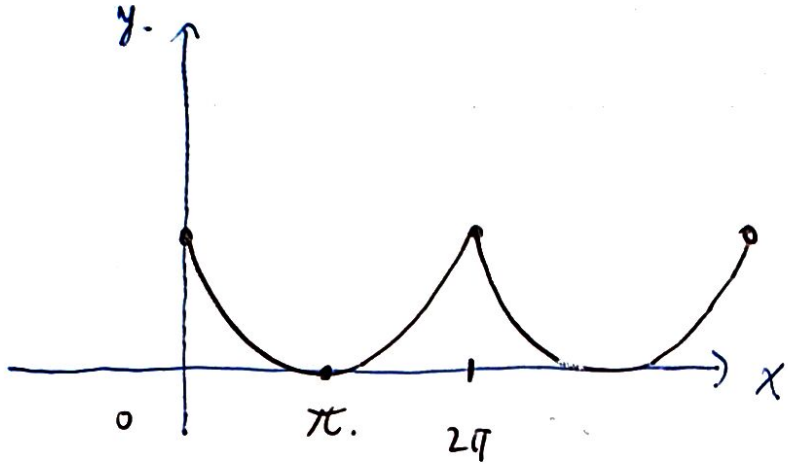
Se Pasa por el origen dos veces.



$$A=\int_0^3 y\, x'(t)\,dt$$

Ejercicio 3: Considere un arco del cicloide "invertido"

$$x = r(\theta + \cos\theta) \qquad 0 \leq \theta \leq 2\pi.$$
$$y = r(1 + \sin\theta)$$



a. Encuentre la derivada., r es constante.

$$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{r(\cos\theta)}{r(1-\sin\theta)} = \frac{\cos\theta}{1-\sin\theta}.$$

b. Encuentre la ec. de la recta tangente en $\theta = \pi/6$

Pendiente: $\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \frac{\cos\pi/6}{1-\sin\pi/6} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$

Coordenadas: $x(\pi/6) = r\left(\frac{\pi}{6} + \cos\frac{\pi}{6}\right) = r\left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)$

$$y(\pi/6) = r\left(1 + \sin\frac{\pi}{6}\right) = 3r/2.$$

Ec. Recta tangente: $y = \frac{3r}{2} + \sqrt{3}\left(x - \frac{\pi}{6}r - \frac{\sqrt{3}}{2}r\right)$

c. Encuentre dónde hay tangentes horizontales y verticales.

$$\frac{dy}{dx} = \frac{\cos\theta}{1-\sin\theta}$$

Horizontales: $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}.$

es cero.

Verticales: $1-\sin\theta = 0 \Rightarrow \sin\theta = 1 \quad \theta = \frac{\pi}{2}.$

no existe

Como $x'(3\pi/2) \neq 0$, hay tangente horizontal cuando $\theta = 3\pi/2$.

En $x'(\pi/2) = y'(\pi/2) = 0$, indeterminada.

$$\lim_{\theta \to \pi/2} \frac{\cos\theta}{1-\sin\theta} = \lim_{\theta \to \pi/2} \frac{-\sin\theta}{-\cos\theta} = \lim_{\theta \to \pi/2} \frac{\sin\theta}{\cos\theta} = \infty.$$

$0/0.$ $\qquad$ $-1/0.$ no existe

Hay una tangente vertical en $\theta = \pi/2$.

Coordenadas $x = r(\theta + \cos\theta)$, $y = (1+\sin\theta)r$

TH $\quad x(3\pi/2) = \frac{3\pi}{2} r \qquad y(3\pi/2) = 1 - 1 = 0.$

$(3\pi r/2, 0)$

TV $\quad x(\pi/2) = r\frac{\pi}{2}. \qquad y(\pi/2) = r(1+1) = 2r.$

TV en $(\frac{\pi}{2} r, 2r)$

Capítulo 26

Cálculo con ecuaciones paramétricas

Cálculo con ecuaciones paramétricas.

a. Primera Derivada $\quad \dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)}$

b. Tangentes Horizontales: $y'(t) = 0 \qquad x'(t) \neq 0.$

Tangentes Verticales: $x'(t) = 0 \qquad y'(t) \neq 0.$

c. Segunda Derivada. P. 139.

Dada la curva $\mathcal{C}$: $x = f(t) \quad y = g(t)$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\frac{d}{dt}(y)}{x'(t)} \qquad \frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dx)}{x'(t).} \quad , \quad \frac{d^3y}{dx^3} = \frac{\frac{d}{dt}\left(d^2y/dx^2\right)}{x'(t)}$$

$x = f(t) \quad y = dy/dx$

Ejercicio 4: Encuentre la primera, segunda y tercera derivada de las sigs. curvas paramétricas.

a. $x = 3t^2 \qquad y = t^3 + 3t^6.$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3t^2 + 18t^5}{6t.} = 0.5t + 3t^4$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dx)}{x'(t)} = \frac{0.5 + 12t^3}{6t.} = \frac{1}{12t} + 2t^2$$

$$\frac{\partial^3 y}{\partial x^3} = \frac{\frac{\partial}{\partial t}\left(\partial^2 y/\partial x^2\right)}{x'(t)} = \frac{1}{6t}\left(\frac{-1}{12t^2} + 4t\right)$$

b. $x = e^t \qquad y = \underline{te^t}$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{1e^t + te^t}{e^t} = 1 + t.$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\frac{\partial}{\partial t}\left(\partial y/\partial x\right)}{x'(t)} = \frac{1}{e^t} = e^{-t} = \frac{1}{x}$$

$$\frac{\partial^3 y}{\partial x^3} = \frac{\frac{\partial}{\partial t}\left(\partial^2 y/\partial x^2\right)}{x'(t)} = \frac{-e^{-t}}{e^t} = \frac{-1}{e^{2t}}$$

c. $x = \cos\theta \qquad y = \underline{\cos 2\theta}. \qquad -1 \le x \le 1$

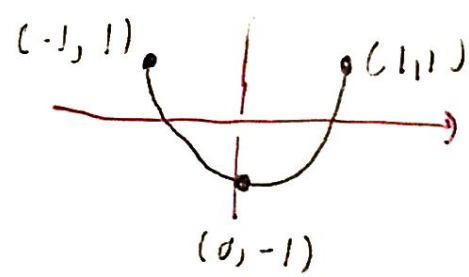
$$\underline{\cos^2\theta} = \frac{1}{2}\left(1 + \cos 2\theta\right)$$

$$2x^2 = 1 + \cos 2\theta. \qquad \cos 2\theta = 2x^2 - 1$$

$$y = 2x^2 - 1$$

$$\frac{dy}{dx} = 4x$$

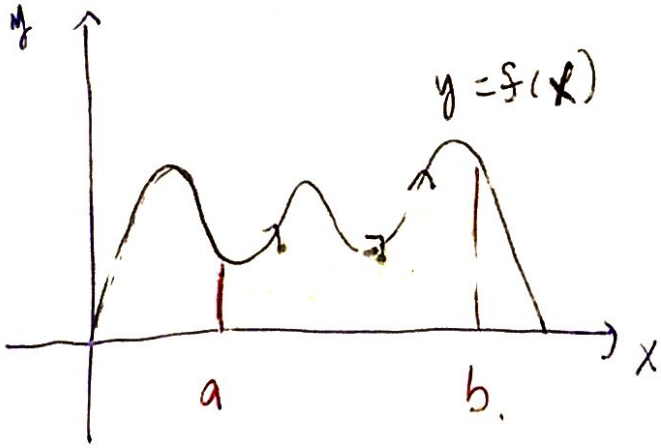
$$\frac{d^2y}{dx^2} = 4 \qquad \frac{\partial^3 y}{\partial x^3} = 0$$



Paramétricas $\quad \frac{dy}{dx} = \frac{-2\sin 2\theta}{-\sin\theta} = \frac{4\sin\theta\cos\theta}{\sin\theta} = 4\cos\theta$

$$\frac{\partial^2 y}{\partial x^2} = \frac{-4\sin\theta}{-\sin\theta} = 4.$$

Área de una Región encerrada por una curva.



$$A = \int_a^b y\,dx.$$

ó $A = \int_c^d x\,dy$) no se usa mucho.

$\mathcal{C}$: $x = f(t)$ $\quad dx = f'(t)\,dt.$

$y = g(t).$

$t_1 \le t \le t_2.$

Reescriba.

$$A = \int_a^b y\,dx = \boxed{\int_{t_1}^{t_2} g(t)\,f'(t)\,dt}$$

Ejercicio 6: P.141 Encuentre el área de la región dada.

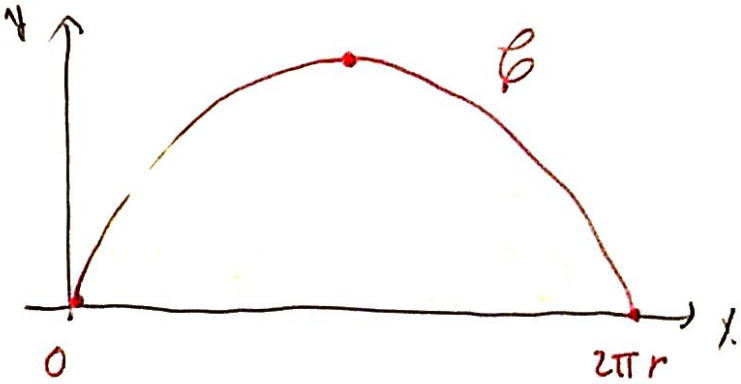
a. La región debajo de un arco de la cicloide

$x = r(\theta - \sin\theta)$

$y = r(1 - \cos\theta)$

$0 \le \theta \le 2\pi.$

r es constante



$$A = \int_0^{2\pi} y\,dx = \int_0^{2\pi} r(1-\cos\theta)\,r(1-\cos\theta)\,d\theta.$$

$dx = r(1-\cos\theta)\,d\theta.$

$$A = r^2 \int_0^{2\pi} (1-\cos\theta)^2\,d\theta.$$

$$A = r^2 \int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\, d\theta.$$

Identidad Doble Ángulo: $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta.)$

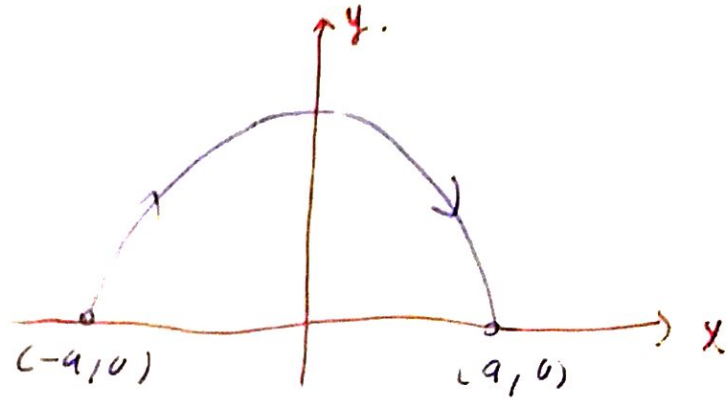
$$A = r^2 \int_0^{2\pi} \left(1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta.$$

$$A = r^2 \left(\frac{3\theta}{2} - 2\sin\theta + \frac{1}{2\cdot 2}\sin 2\theta \Big]_0^{2\pi} \right)$$ Sino $\pm\theta$

$$A = r^2 \left(\frac{3\cdot 2\pi}{2} - 2\cancelto{0}{\sin 2\pi} + \frac{1}{4}\cancelto{0}{\sin 4\pi} - 0 \right)$$

$$\boxed{A = 3\pi r^2.}$$ 3 veces el área del círculo. πr^2.

b. Media elipse.



$x = -a\cos\theta$
$y = b\sin\theta.$
$0 \leq \theta \leq \pi.$

$$A = \int_0^{\pi} y\, dx = \int_0^{\pi} b\cdot\sin\theta\; a\cdot\sin\theta\, d\theta. = ab\int_0^{\pi} \sin^2\theta\, d\theta.$$

$dx = a\cdot\sin\theta\, d\theta$

$$A = \frac{ab}{2}\int_0^{\pi} 1 - \cos 2\theta\; d\theta. = \frac{ab}{2}\left(\theta - \frac{1}{2}\sin 2\theta\Big]_0^{\pi}\right)$$

$$A = \frac{ab}{2}\left(\pi - \frac{1}{2}\cancelto{0}{\sin 2\pi} - 0\right) = \frac{\pi ab}{2}$$ 1/2 Elipse.

Área toda la elipse. $A = \pi ab$.

de un círculo: $A = \pi a^2$. $a = b$.

d. 6: $x = t^3$, $y = 64 - t^2$ y el eje-x.

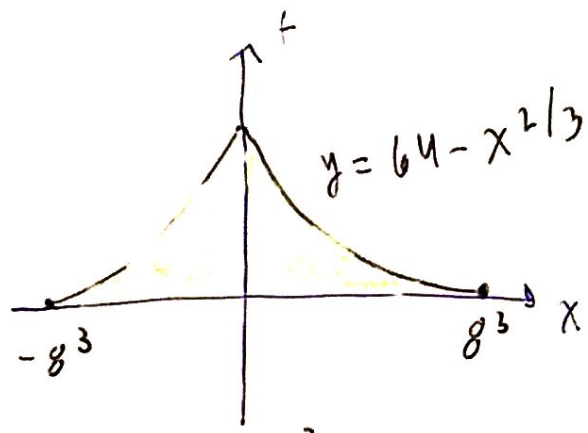
Elimine el parámetro ó $\int y\,dx$ $dx = x'(t)\,dt$.

$t = x^{1/3} \Rightarrow y = 8^2 - x^{2/3}$.

$(\cdot^{2/3})^{3/2}$.

Intersecciones-x: $y = 0$: $x^{2/3} = 8^2$

$x = (8^2)^{3/2} = \pm 8^3$



$$A = \int_{-8^3}^{8^3} y\,dx$$

$$A = \int_{-8^3}^{8^3} (8^2 - x^{2/3})\,dx$$

$$A = 2\int_0^{8^3} (8^2 - x^{2/3})\,dx = 2\left(8^2 x - \frac{3}{5}x^{5/3}\Big]_0^{8^3}\right)$$

$$A = 2\left(8^5 - \frac{3}{5}(8^3)^{5/3}\right) = 2\left(8^5 - \frac{3}{5}8^5\right) = 2\cdot 8^5\left(1 - \frac{3}{5}\right)$$

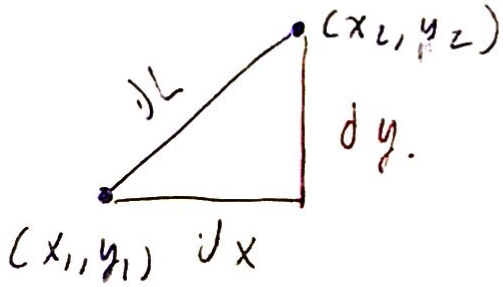
Soln 2: $y = 64 - t^2 = 0 \Rightarrow t^2 = 64 \Rightarrow t = \pm 8$

$$A = \int_{-8}^{8} y\,x'(t)\,dt. = \int_{-8}^{8} (64 - t^2)\,3t^2\,dt.$$

$$A = 2\int_0^8 64\cdot 3t^2 - 3t^4\,dt = 2\left(8^2\cdot t^3 - \frac{3}{5}t^5\Big]_0^8\right)$$

$$A = 2\left(8^5 - \frac{3}{5}8^5\right)$$

e. Longitud de Arco.



$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$\mathcal{C} : \quad x = f(t) \qquad y = g(t)$$

$$dL = \sqrt{(dx)^2 + (dy)^2}\,\frac{dt}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

Longitud de Arco de una curva $\mathcal{C}: x = f(t),\ y = g(t)$ en $a \le t \le b$.

$$L = \int_a^b \sqrt{(x')^2 + (y')^2}\,dt.$$

Cartesiana: $\quad L = \int_a^b \sqrt{1 + (y')^2}\,dx$

Ejercicio 7: Encuentre la longitud exacta de la curva dada.

a. Longitud de una circunferencia de radio 4.

$$x = 4\cos\theta, \quad y = -4\sin\theta, \quad 0 \le \theta \le 2\pi.$$

$$x'(\theta) = -4\sin\theta \qquad y'(\theta) = -4\cos\theta. \qquad y' = \sqrt{16 - x^2}$$

$$(x')^2 + (y')^2 = 16\sin^2\theta + 16\cos^2\theta = 16.$$

$$L = \int_0^{2\pi} \sqrt{(x')^2 + (y')^2}\,d\theta = 4\int_0^{2\pi} d\theta = 4\theta\Big]_0^{2\pi} = 8\pi.$$

c. $x = e^t \cos t \qquad y = e^t \sin t \qquad 0 \leq t \leq \ln 2.$

Use la Regla del Producto.

$x'(t) = e^t \cos t - e^t \sin t. \qquad e^t e^t = e^{2t}$

$y'(t) = e^t \sin t + e^t \cos t. \qquad \sin t \sin t = \sin^2 t.$

$$(x')^2 = e^{2t}\cos^2 t - 2e^{2t}\cos t \sin t + e^{2t}\sin^2 t$$
$$(y')^2 = e^{2t}\sin^2 t + 2e^{2t}\cos t \sin t + e^{2t}\cos^2 t.$$

$$(x')^2 + (y')^2 = e^{2t} + 0 + e^{2t} = 2e^{2t}.$$

$$L = \int_0^{\ln 2} \sqrt{(x')^2 + (y')^2}\, dt. = \int_0^{\ln 2} \left(2e^{2t}\right)^{1/2} dt.$$

$$L = 2^{1/2} \int_0^{\ln 2} e^t\, dt. = 2^{1/2} e^t \Big]_0^{\ln 2}$$

$$L = 2^{1/2}\left(e^{\ln 2} - e^0\right) = \sqrt{2}\,(2-1) = \sqrt{2}$$