Notas de clase Cálculo integral
UFM
SECCIÓN B

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Capítulo 1

c. $2x + y = 2$
$-2x - y = -2.$

$R_1 + R_2$: $0 = 0.$ ✓ Ecs. repetidas.

infinitas soluciones, sistema consistente.

$x = t.$
$y = 2 - 2t$
$s = \begin{bmatrix} t \\ 2-2t \end{bmatrix}$

Sistemas Equivalentes.

Dos sistemas son equivalentes. si tienen la misma solución.

Considere los sigs. sistemas.

$$\begin{aligned} w + x - 2y - z &= -5 \\ 2w + x + y + z &= 7. \\ 2w + 2x + y + 2z &= 7. \\ 3w + x + y + z &= 8 \end{aligned}$$

$$\begin{aligned} w &= 1 \\ w + x &= 3 \\ w + x + y &= 8 \\ w + x + y + z &= 6 \end{aligned}$$

es más facil de resolver.

sustituyendo hacia adelante.

$w = 1$
$x = 3 - w = 2$
$y = 8 - w - x = 5$
$z = 6 - w - x - y = 5 - 7 = -2.$

$s = \begin{bmatrix} 1 \\ 2 \\ 5 \\ -2 \end{bmatrix}$

$w = 1$
$x = 2$
$y = 5$
$z = -2.$

los dos sistemas son equivalentes.

Realice operaciones algebraicas para simplificar cada sistema a una forma equivalente más sencilla.

Capítulo 2

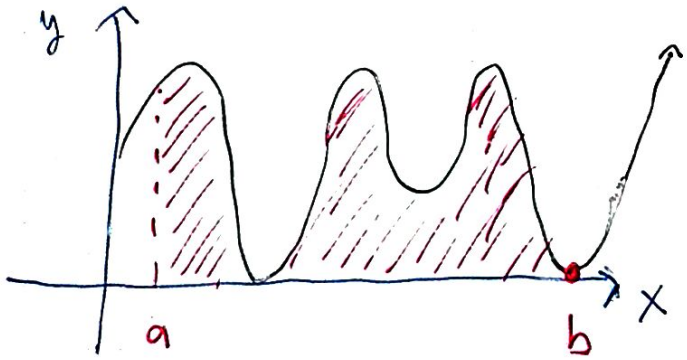
5.4 A'rea y Propiedades. Integral Definida.

El A'rea de una región $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

Integral definida de f en [a,b] f es continua $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

Interpretación Integral Definida.

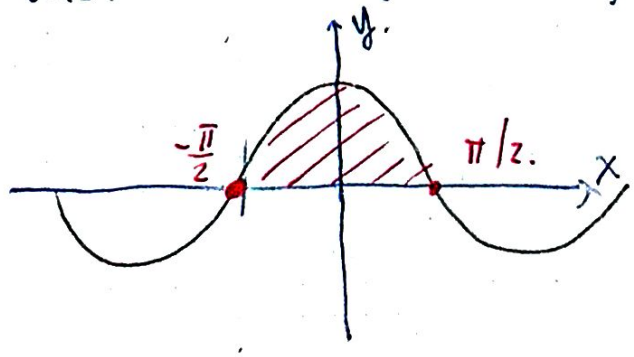
El área de la región bajo la curva $y = f(x)$, encima del eje-x, y entre las rectas verticales $x = a$ y $x = b$. es la integral definida de f en [a,b] $(f \geq 0)$



$$A = \int_a^b f(x)\,dx.$$

$$F(b) - F(a)$$

Considere la región debajo de $y = \cos x$ en $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



$$A = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2.$$

$$A = \sin x \Big]_{-\pi/2}^{\pi/2} = \cancel{\sin\frac{\pi}{2}}^{\,1} - (-1)$$
$$= 1 + 1 = 2.$$

$$A = -x^2\Big]_{-2}^{0} + x^2\Big]_{0}^{3} = -0-(-(-2)^2)+9-0$$
$$= 0+4+9 = 13.$$

Regla Integral Definida. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

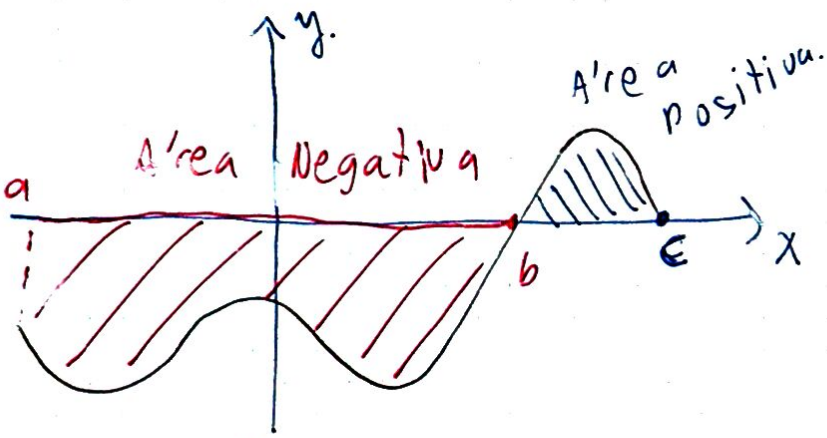
Invertir el orden

Ej: $\int_{-2}^{0} -2x\,dx = \int_0^{-2} 2x\,dx = x^2\Big]_0^{-2} = 4-0 = 4$

b) $\int_0^{\pi} \sin x\,dx = -\int_{\pi}^{0} \sin x\,dx = \cos x\Big]_{\pi}^{0} = 1-(-1) = 2.$

ó $-\cos x\Big]_0^{\pi} = -\underbrace{\cos(\pi)}_{-1} - (-\cos 0) = 1+1 = 2.$

¿Qué sucede cuando $f(x)$ es negativa?



Área de la región entre $f(x)$ & $y=0$

$$\int_{-1}^{0} x^3\,dx = \frac{x^4}{4}\Big]_{-1}^{0}$$
$$= 0 - \frac{1}{4} = -\frac{1}{4}.$$

$$A \neq \int_a^c f(x)\,dx$$

$$A = -\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$ ✓ valor absoluta

Definición Más compacta $A = \int_a^b |f(x)|\,dx$

Ejercicio 3: (Pág 16) Considere $f(x) = 4x^3 - 4$ en $-2 \leq x \leq 2$.

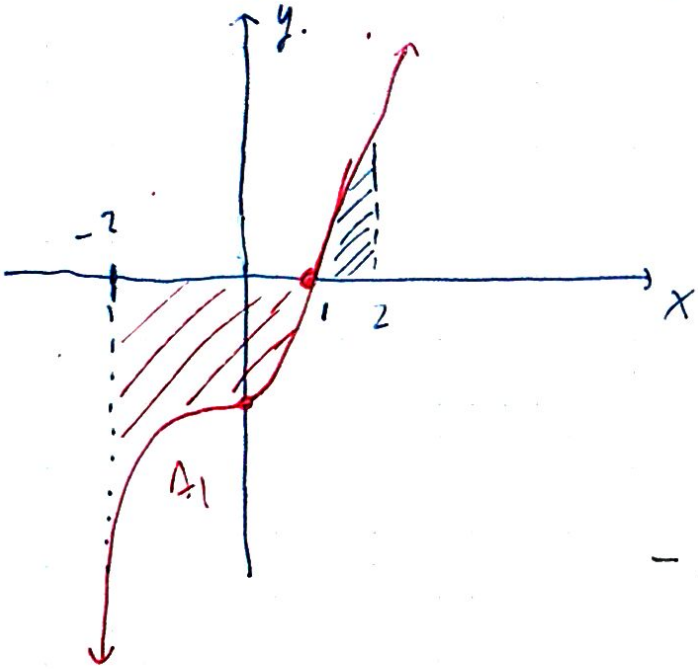
a. Evalúe $\int_{-2}^{2} (4x^3 - 4)\,dx = x^4 - 4x\Big]_{-2}^{2}$

$x^4\Big]_{-2}^{2} - 4x\Big]_{-2}^{2}$

$= (16-8) - (16+8)$

$= 8 - 24 = -16.$

b. Bosqueje la región y explique si la integral definida es igual al área de la región.



Intercepto-x: $4(x^3 - 1) = 0$

$x = 1.$

Intercepto-y: $0 - 4 = -4$

$$A \neq \int_{-2}^{2} f(x)\,dx$$

$-f(x) = -4x^3 + 4$

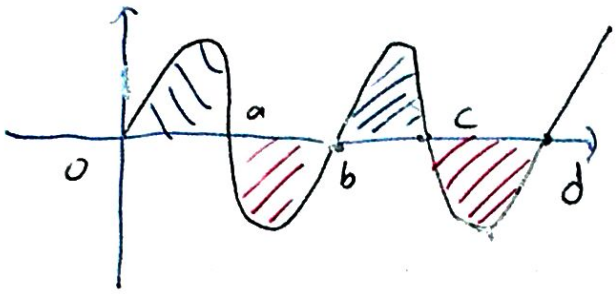
c. Encuentre el área de la región.

$$A = \int_{-2}^{1} \overbrace{(4 - 4x^3)}^{A_1}\,dx + \int_{1}^{2} (4x^3 - 4)\,dx.$$

$$A = 4x - x^4\Big]_{-2}^{1} + x^4 - 4x\Big]_{1}^{2} = \overbrace{(4-1) - (-8-16)}^{A_1}$$

$+(16-8) - (1-4)$

$A = 3 + 24 + 8 + 3. = 27 + 11 = 38.$



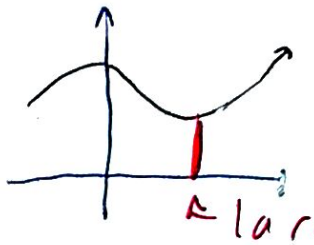
$$A = \int_0^a f\,dx - \int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx.$$

Propiedades Integrales Definidas.

1 y 2. $\int_a^b k_1 f(x) \pm k_2 g(x)\,dx = k_1 \int_a^b f(x)\,dx \pm k_2 \int_a^b g\,dx.$

$$\int_{\sqrt{2}}^{\sqrt{2}} e^{x^2 + \ln x + \sinh x}\,dx. = F(\sqrt{2}) - F(\sqrt{2}) = 0$$

3. $\int_a^a f(x)\,dx. = 0$



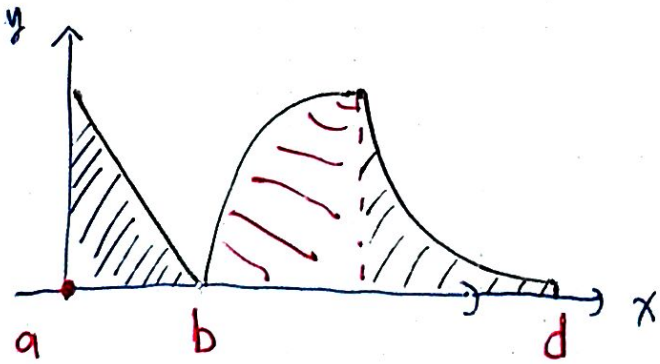
la región es sólo una línea.

4. $\int_a^b h\,dx = hx\Big]_a^b = h(b-a)$ rectángulo altura h largo b-a.

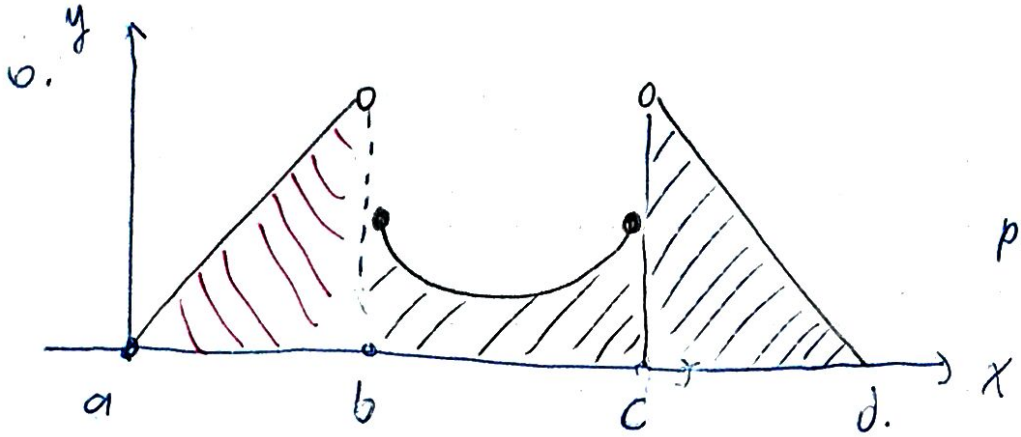
$$\int_e^{\sqrt{10}} \ln(10)\,dx = \ln(10)\left[\sqrt{10} - e\right]$$

$\int \ln x\,dx$

5.



$$\int_a^d f(x)\,dx = \int_a^b f\,dx. + \int_b^c f\,dx + \int_c^d f\,dx$$



continua por tramos

piecewise continuous.

$$\int_a^d f\,dx = \int_a^b f\,dx + \int_b^c f\,dx + \int_c^d f\,dx$$

$$2x\Big]_0^1 = 2-0$$

Ejercicio 5: Evalúe la sig. integral definida.

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2 & \text{si} \quad 0 \le x \le 1 \\ 4-2x & \text{si} \quad 1 \le x \le 2 \\ 6x-12 & \text{si} \quad 2 \le x \le 3 \end{cases}$$

$$\int_0^3 f\,dx = \int_0^1 2\,dx + \int_1^2 (4-2x)\,dx + \int_2^3 (6x-12)\,dx$$

$$= 2 + \left(4x - x^2\Big]_1^2\right) + \left(3x^2 - 12x\Big]_2^3\right)$$

$$= 2 + (4-3) + (-9-(-12))$$

$$= 2 + 1 + 3 = 6.$$

Derivación logarítmica, $y = a^x$.

$$y = \frac{a^x}{\ln a}, \quad y' = a^x \frac{\cancel{\ln a}}{\cancel{\ln a}}, \quad \ln y = x \ln a.$$

$$\frac{y'}{y} = \ln a. \Rightarrow y' = a^x \ln a.$$

Capítulo 3

5.4 Desplazamiento y Distancia

La integral de la derivada $f'(x)$ es la función original.

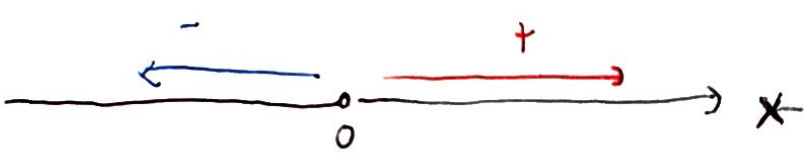
$$\int_a^b f'(x)\,dx = f(x)\Big]_a^b = \underbrace{f(b) - f(a)}_{\text{cambio neto.}}$$

Si se conoce la razón de cambio de una función, el cambio neto se obtiene integrando la razón de cambio.

Desplazamiento en 1-Dimensión

$$s = \int_a^b v(t)\,dt \quad ✓$$

$$\int_a^b s'(t)\,dt.$$



Costo Marginal: $C'(x)$ costo neto $= \int_a^b C'(x)\,dx.$

Población: población neta $= \int_a^b p'(t)\,dt.$

Ejemplo: Una partícula tiene una velocidad de $v(t) = \frac{2}{t^{4/3}}$ cm/s. Encuentre el desplazamiento entre $t=1$ y 8 s. $2 \cdot t^{-4/3}$

$$s = \int_1^8 v(t)\,dt = 2\int_1^8 t^{-4/3}\,dt. = -6\,t^{-1/3}\Big]_1^8 = 6t^{-1/3}\Big]_8^1$$

$$1 - \frac{1}{2} \qquad -\frac{1}{2} + 1$$

$$s = 6\left(\frac{1}{\sqrt[3]{1}} - \frac{1}{\sqrt[3]{8}}\right) = 6\left(\frac{1}{2}\right) = 3. \quad ó \quad 6\left(\frac{-1}{\sqrt[3]{8}} - \frac{(-1)}{\sqrt[3]{1}}\right)$$

Desplazamiento Neto es de 3 cm.

Ejercicio 1: Se lanza una pelota con una velocidad inicial de 64 pies/s, a nivel del suelo. Encuentre el desplazamiento de la pelota entre 1 y 3 s.

$v(t) = 64 - 32t.$ ✓ $\qquad g = -32$ pies/s²

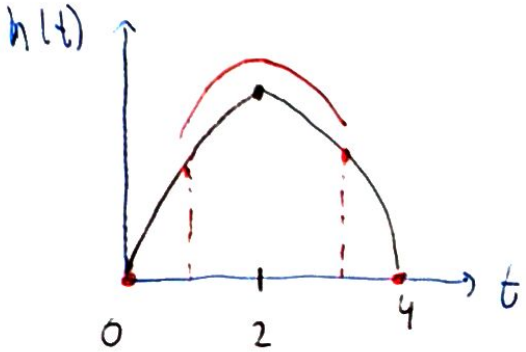
Respuesta:

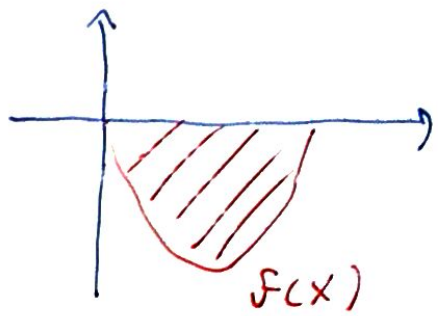
$$s = \int_1^3 v(t)\,dt. = \int_1^3 (64 - 32t)\,dt. = 64t\Big]_1^3 - 16t^2\Big]_1^3$$

$$s = 64(3-1) - 16(9-1) = 128 - 128 = 0$$

$$64 \cdot 2 \qquad 16 \cdot 4 \cdot 2 \qquad 48 - 48 = 0$$

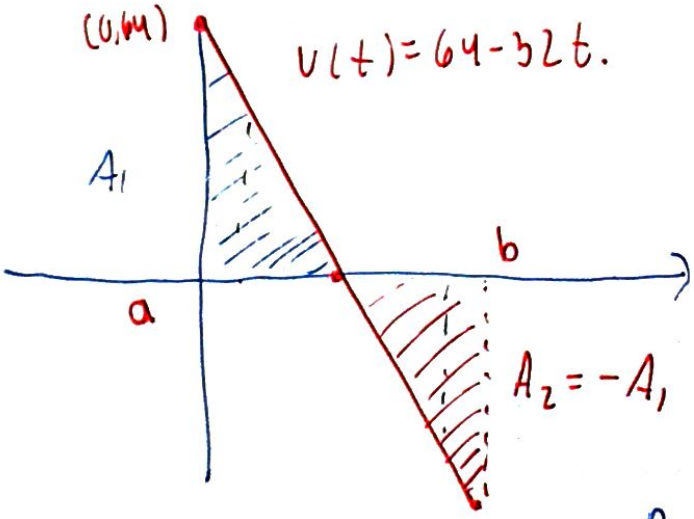
NO HAY CAMBIO NETO.





$$A = -\int_a^b f(x)\,dx$$

Distancia y Desplazamiento en una dimensión.



$-, 0, +$

Desplazamiento $s = \int_a^b v(t)\,dt.$

Para el ejercicio $s = 0.$

Distancia $d = \int_a^b |v(t)|\,dt.$

0 ó +.

Rapidez: $|v(t)|.$

Para este diagrama.

Desplazamiento $s = A_1 + A_2 \approx 0$ A_2 es negativo.

Distancia: $s = A_1 - A_2$.

Para el ejercicio 1j encuentre la distancia recorrida por la pelota entre $t = 1$ y 3 s.

$$d = \int_1^3 |v(t)|\,dt = \int_1^3 |\underbrace{64-32t}_{=0}|\,dt.$$

$v(t) = 0$ cuando $t = 2$.

$64-32t$ $+$ $\overset{2}{\vdots}$ $-$

$$d = \int_1^2 v(t)\,dt. - \int_2^3 v(t)\,dt$$

$64-32t = 0$

$64 = 32t$

$2. = t$

$$d = \int_1^2 64-32t\,dt + \int_2^3 32t-64\,dt.$$

$$d = 64t - 16t^2\Big]_1^2 + 16t^2 - 64t\Big]_2^3$$

$$d = 128 - 64 - (64-16) + 16(9-4) - 64(3-2)$$

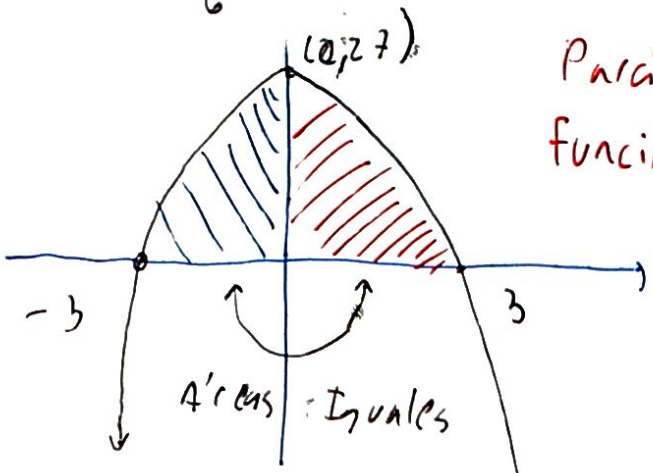
$$d = 64 - 48 + 80 - 64$$

$$d = 16 + 16 = 32 \text{ pies.}$$

Ejercicio 2: Un vehículo da vueltas en un circuito. una velocidad $v(t) = 27 - 3t^2$ millas/hora.

a. Plantee la integral para encontrar el desplazamiento del vehículo entre -6 y 6 horas.

$$s = \int_{-6}^{6} (27 - 3t^2)\,dt. = 2\int_{0}^{6} (27 - 3t^2)\,dt.$$



Parábola
función par

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

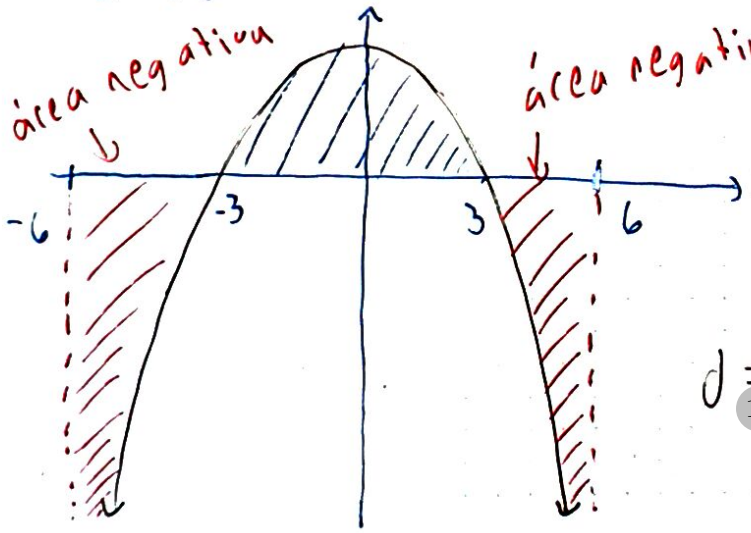
$$f(-x) = f(x)$$

$$s = 2\left(27t - t^3\Big]_0^6\right) = 2\left(\overset{27\cdot 3\cdot 2}{27\cdot 6} - 6^3\right) = 2(162 - 216)$$

$$2(-54) = -108.$$

millas.

b. Plantee la integral para encontrar la DISTANCIA. del vehículo entre -6 y 6 horas.



$$d = -A_1 + A_2 - A_3.$$

$$d = \int_{-6}^{6} |v(t)|\,dt.$$

$$d = -\int_{-6}^{-3} v(t)\,dt$$

$$d = -\int_{-6}^{-3}(27-3t^2)\,dt + \int_{-3}^{3}(27-3t^2)\,dt + \int_{3}^{6}\underbrace{(3t^2-27)}_{v(t)}\,dt$$

OLVIDARLO: $d = 2\int_0^3 v(t)\,dt - 2\int_3^6 v(t)\,dt.$

Integrales Indefinidas: desplazamiento, velocidad y aceleración.

$\int f(t)\,dt = F(t) + C$, función.

Dada la aceleración del objeto. $a(t) = v'(t)$.

Velocidad: $v(t) = \int a(t)\,dt + C_1$

Velocidad inicial: $v(0) = v_0$. reposo $v(0) = 0$.

Posición: $s(t) = \int v(t)\,dt + C_2$.

Posición inicial: $s(0) = s_0$. posición equilibrio $s(0) = 0$

Ejercicio 3: Un cohete despeja con una aceleración vertical de $a(t) = t^2\left(\frac{72}{t} - 36\right)$ ft/s².

La posición inicial es 0 pies snm y la velocidad inicial es de 400 ft/s.

sobre el nivel del mar.

a. Encuentre la posición vertical del cohete.

$$a(t) = 72t - 36t^2.$$

Velocidad: $v(t) = \int (72t - 36t^2)\,dt.$

$v(t) = 36t^2 - 12t^3 + C_1$

Use $v(0) = 400$: $v(0) = \boxed{C_1 = 400}$

Posición: $s(t) = \int v(t)\,dt = 12t^3 - 3t^4 + 400t + C_2.$

Use $s(0) = 0$: $s(0) = 0 + 0 + 0 + \boxed{C_2 = 0}$

Posición vertical es $\boxed{s(t) = 12t^3 - 3t^4 + 400t.}$

b. ¿Cuál es la rapidez y la velocidad a los $t = 10s$?

$v(10) = 36(100) - 12(1000) + 400 = 4{,}000 - 12{,}000$
$-8{,}000$ pies/s.

rapidez $|v(10)| = 8{,}000$ pies/s.

Ejercicio 4: Un resorte inicialmente en reposo y en su punto de equilibrio tiene una aceleración de

$a(t) = 4\cos t - 3\sin t.$ ✓

Encuentre la velocidad y posición del resorte.

$v(t) = \int (4\cos t - 3\sin t)\,dt = 4\sin t + 3\cos t + C_1$

$v(0) = 0 + 3 + C_1 = 0 \Rightarrow C_1 = -3.$

$v(t) = 4\sin t + 3\cos t - 3.$

: Desplazamiento

$$s(t) = \int (4\sin t + 3\cos t - 3)\,dt. \qquad s(0)=0$$

$$s(t) = -4\cos t + 3\sin t - 3t + C_2.$$

$$s(0) = -4 + 0 + 0 + C_2 = 0 \Rightarrow C_2 = 4.$$

$$s(t) = -4\cos t + 3\sin t - 3t + 4.$$

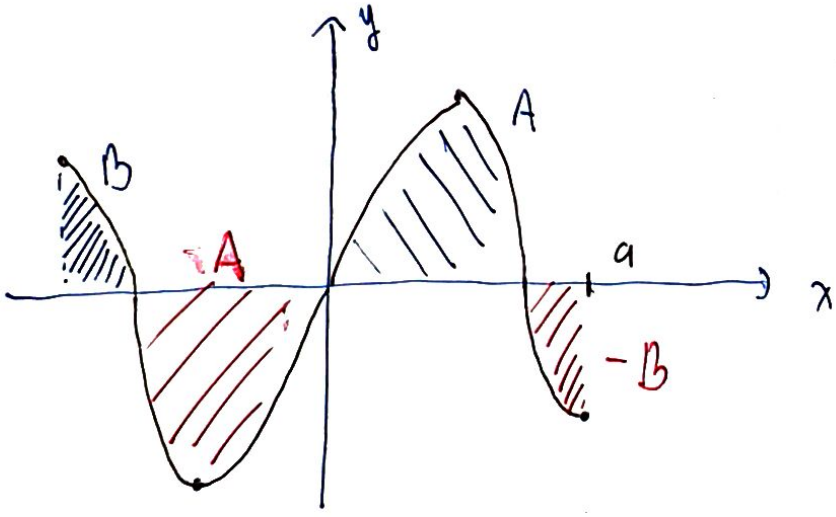
Función Pares e Impares

$$\int_{-100}^{100} (\sin x + x^3 + \tanh x)\,dx = 0.$$

Tres funciones son impares

$\sin(-x) = -\sin x$

$(-x)^3 = -x^3.$



Las áreas se cancelan entre sí.

$$\int_{-a}^{a} f_{impar}(x)\,dx = 0.$$

$$\int_0^1 e^x\,dx = e-1$$

$$\int_{-a}^{a} f_{par}(x)\,dx = 2\int_0^a f_{par}(x)\,dx$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-x^2}\,dx = 1$$

Capítulo 4

5.3 Teorema Fundamental del Cálculo.

si $f(x)$ es continua en $[a, x]$ entonces

PARTE 1: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ $\qquad \int_a^b$ $\qquad \int$

Integral y la derivada se cancelan entre sí.

Como la derivada de una antiderivada es la función original, entonces la $\int_a^x f(t)\,dt$ es la antiderivada de $f(x)$

variable temporal de integración.

se integra primero respecto y se deriva respecto a x.

$$f(x) = \int_{10}^{x} 4t^3\,dt = t^4\Big]_{t=10}^{t=x} = x^4 - 10^4.$$

$$y(t) = \int_0^t \sin(w)\,dw$$

$$f'(x) = 4x^3$$

$t \to x$

$$f'(x) = \frac{d}{dx}\int_{10}^{x} 4t^3\,dt = 4x^3$$

ATAJO.

TFC parte 2: $\int_a^b f(x)\,dx = F(x)\Big]_{x=a}^{x=b} = F(b) - F(a)$

$$\int_a^b f(w)\,dw = F(w)\Big]_{w=a}^{w=b} = F(b) - F(a)$$

Se pueden definir funciones por medio de integrales

Distribución normal $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$$P(x) = \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt.$$

no se puede integrar de manera explícita.

$$\int e^t\, dt = e^t + C \qquad \int e^{t^2} t\, dt. = \frac{1}{2} e^{t^2} + C.$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2}\, dt = 1$$

$$P'(x) = \frac{d}{dx} \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

Ejercicio 1: Derive las siguientes funciones. Pág 20.

a) $h(x) = \int_a^x 3\sqrt{t+1}\, dt.$ $\quad h'(x) = 3\sqrt{x+1}$

$t \to x$

b) $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$ $\quad S'(x) = \sin\left(\frac{\pi}{2} x^2\right)$

$t \to w$

c) $H(w) = \int_{-5}^{w} \frac{t+4}{t^4+t^2+2}\, dt.$ $\quad H'(w) = \frac{w+4}{w^4+w^2+2}$

TFC parte I y la regla de la cadena.

$$g(x) = \int_{100}^{x^5} e^t \, dt = e^t \Big]_{t=100}^{t=x^5} = e^{x^5} - e^{100}$$

$$g'(x) = e^{x^5} \underbrace{5x^4}_{\text{Regla de la cadena.}} - 0$$

$h(x) = \sin(x^5 + x^2)$ $\quad h'(x) = \cos(x^5+x^2)(5x^4+2x)$

$$f(x) = \int_a^{b(x)} g(t)\,dt. \qquad f'(x) = g(b(x))\, b'(x)$$

$a \leftarrow$ constante. $\quad t \to b(x)$

Ejercicio 2: Derive las siguientes funciones.

a. $g(x) = \int_5^{\ln x} \sqrt{t^2+1}\, dt.$ $\qquad h(x) = \int_5^{x} \sqrt{t^2+1}\, dt.$

$$g'(x) = \sqrt{(\ln x)^2 + 1} \cdot \underbrace{\frac{1}{x}}_{d(\ln x)}. \qquad h'(x) = \sqrt{x^2+1}$$

$t \to \ln x$

b. $h(x) = \int_{\sec x}^{8} \tan^{-1}(t)\, dt. = -\int_8^{\sec x} \tan^{-1}(t)\, dt.$

$$h'(x) = -\tan^{-1}(\sec x)\, \sec x \tan x$$

$t \to \sec x$

c. $\frac{d}{dx}\left(\int_{1000}^{x^5+x^3} \ln(t)\, dt.\right) = \ln(x^5+x^3)\,(5x^4+3x^2)$

$t \to x^5+x^3$ • derivada lím superior

Funciones con ambos límites dependiendo de x.

$$f(x) = \int_{\sinh x}^{\cosh x} \sec^2 t \, dt.$$

$$f(x) = \tan t \Big]_{\sinh x}^{\cosh x} = \tan(\cosh x) - \tan(\sinh x)$$

Derive: $f'(x) = \sec^2(\cosh x) \cdot \sinh x - \sec^2(\sinh x) \cosh x$

$$f'(x) = \frac{d}{dx} \int_{\tan x}^{\csc x} \sec^2 t \, dt = \sec^2(\csc x)(-\csc x \cot x) - \sec^2(\tan x)(\sec^2 x)$$

$t \to \csc x$

TFC parte 1 y la Regla de la Cadena

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(t) \, dt = f(b) b' - f(a) a'.$$

Ejercicio 3: Derive

$$\frac{d}{dx} \int_{\sin x}^{e^x} \sqrt{10 + 4t^4} \, dt = \sqrt{10 + 4e^{4x}} \, e^x - \sqrt{10 + 4\sin^4 x} \cos x$$

$$\frac{d}{dx} \int_{\ln x}^{\sin^{-1} x} \cosh \theta^3 \, d\theta = \cosh(\sin^{-1} x)^3 \cdot \frac{1}{\sqrt{1 - x^2}} - \cosh(\ln^3 x) \cdot \frac{1}{x}.$$

Ejercicio 4: Encuentre la ecuación de la recta tangente a $f(x) = \int_0^x \cosh^2 t \, dt$ en $t=0$

Ec. Recta Tangente $y = \underline{f(0)} + \underline{f'(0)}(x-0)$

$f(0) = \int_0^0 \cosh^2 t \, dt = 0$ $\qquad \int_a^a f(y)\,dy = 0.$

$f'(x) = \frac{d}{dx}\int_0^x \cosh^2 t \, dt = \cosh^2 x \cdot 1$

$f'(0) = (\cosh 0)^2 = 1^2 = 1$

Ec. Recta Tangente $y = 0 + 1(x-0)$

$y = x$

Capítulo 5

5.5 Regla de la sustitución

Objetivo: Integre $F(g(x))$ funciones compuestas.

a $\int 3\underbrace{(x+2)^2}_{x^2+4x+4}\,dx = \int (3x^2+12x+12)\,dx$

$= x^3+6x^2+12x+C.$

conjeturando $\int 3(x+2)^2\,dx = \underbrace{(x+2)^3+C.}_{\text{derivada } 3(x+2)^2\cdot 1+0}$

b. $\int 11(x-20)^{10}\,dx = (x-20)^{11}+C.$

$\longrightarrow$ derivando

Regla de la Potencia $\frac{d}{dx}[\underbrace{f(x)}]^{n+1} = (n+1)[f(x)]^n f'(x)$

$\longleftarrow$ integrando

Regla de la sustitución funciones Potencia

$$\int [\underbrace{f(x)}_{u}]^n \underbrace{f'(x)\,dx}_{du} = \frac{f(x)^{n+1}}{n+1}+C$$

$u=f(x) \quad du = f'(x)\,dx$

$$\int u^n\,du = \frac{u^{n+1}}{n+1}+C \quad \text{si } n\neq -1.$$

Ejercicio 1: Evalúe las sigs. integrales.

a. $\int (\overbrace{11x-20}^{u})^{10}\,\underbrace{11\,dx}_{du} = \int u^{10}\,du = \frac{u^{11}}{11}+C_1$

$u=11x-20 \quad du=11\,dx \qquad = \boxed{\frac{1}{11}(11x-20)^{11}+C_1}$

aa. $\int (\overbrace{x^2+x+3}^{u})^5\,\overbrace{(2x+1)\,dx}^{du} = \int u^5\,du. = \frac{1}{6}u^6+C_2.$

Pág. 30

$u=x^2+x+3, \quad du=(2x+1)\,dx \qquad = \frac{1}{6}(x^2+x+3)^6+C_2$

Multiplicar/Dividir por una constante.

b. $\int (30w^3-8)^{19} w^2 dw. = \int u^{19} \frac{du}{90}$

$u = 30w^3 - 8$, $du = 90 \underbrace{w^2 dw}$. $\Rightarrow w^2 dw = \frac{du}{90}$.

$dw = \frac{du}{90w^2}$

$\int u^{19} \frac{du}{90} = \frac{1}{90} \cdot \frac{1}{20} u^{20} + C_3 = \boxed{\frac{1}{1,800}(30w^3-8)^{20} + C_3}$.

c. $\int (30w^3-8)^{19} 90w^3 dw. = \int u^{19} w \, du.$ X

sustitución incompleta.

$\cancel{\int u^{19}\left(\frac{1}{30}(u+8)^{1/3}\right)du}$ sólo se puede integrar. por fuerza bruta.

d. $\int 8x^3\sqrt{8+x^4}\, dx = \int 2u^{1/2} du = 2 \cdot \frac{2}{3} u^{3/2} + C$

$u = 8 + x^4$ $du = 4x^3 dx$ $\Rightarrow$ $2du = 8x^3 dx$.

La integral es $\frac{4}{3}(8+x^4)^{3/2} + C$.

e. $\int (10x^2+6x)^2 dx = \int (100x^4 + 120x^3 + 36x^2) dx$

$= \boxed{20x^5 + 30x^4 + 12x^3 + C}$.

yo se usa la sustitución expanda y luego integre

Regla de la Cadena Derivadas. $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$

Regla de la Sustitución Cadena a la Inversa. $\int f'(g(x))\,g'(x)\,dx = \int f'(u)\,du = f(u)+C.$

$u = g(x), \quad du = g'(x)\,dx \qquad = f(g(x))+C.$

Ejercicio 2: Integre. Pág 32.

0. $\int \frac{(8+16x+48x^2)}{x+x^2+2x^3}\,dx = \int \frac{8\,du}{u} = 8\ln|u| + C$

$= \boxed{8\ln|x+x^2+2x^3| + C.}$

$u = x+x^2+2x^3 \qquad du = (1+2x+6x^2)\,dx$

$8\,du = (8+16x+48x^2)\,dx$

a. $\int e^{\overbrace{x^{10}+\sqrt{2}}^{u}}\,x^9\,dx = \int e^u \frac{du}{10} = \frac{1}{10}e^u + C$

$u = x^{10}+\sqrt{2} \qquad \frac{du}{10} = \frac{10x^9}{10}\,dx. \qquad = \frac{1}{10}e^{x^{10}+\sqrt{2}} + C.$

a2 $\int e^{x^{10}}\underbrace{x^8\,dx}_{\neq du} \qquad \int e^{x^{10}}\underbrace{dx}_{\neq du.}$ no se puede integrar.

b. $\int x^3(x^4+3)^2\sin(x^4+3)^3\,dx = \int u^2\sin u^3\,\frac{du}{4}.$

$u = (x^4+3) \qquad du = 4x^3\,dx \qquad \frac{du}{4} = x^3\,dx$

$u = (x^4+3)^3$ ✓

$\frac{1}{4}\int \sin(u^3)\,u^2\,du = \frac{1}{4}\int \sin t\,\frac{dt}{3} = -\frac{1}{4}\cdot\frac{1}{3}\cos t + C$

$t = u^3 \qquad dt = 3u^2\,du. \qquad = -\frac{1}{12}\cos u^3 + C$

$= -\frac{1}{12}\cos(x^4+3)^3 + C$

Una sóla sustitución.

$$\int \sin(x^4+3)^3 \overbrace{[(x^4+3)^2 x^3]\,dx}^{du/12} = \frac{1}{12}\int \sin u\,du.$$

$u = (x^4+3)^3$, $du = 3(x^4+3)^2\,4x^3\,dx$

$\frac{du}{12} = (x^4+3)^2 x^3\,dx$

c. $\int \cot x\,dx = \int \overbrace{\frac{\cos x}{\sin x}\,dx}^{du} = \int \frac{du}{u} = \ln|u| + C.$

$= \ln|\sin x| + C.$

$u = \sin x \quad du = \cos x\,dx$

d $\int \sec^2(\overbrace{e^x+x}^{u})\overbrace{(e^x+1)\,dx}^{du} = \int \sec^2 u\,du = \tan u + C.$

$u = e^x + x \quad du = (e^x+1)dx \qquad = \tan(e^x+x) + C.$

Sustitución Incompleta. $x = u - 4.$

e. $\int 28x(x+4)^{1/3}\,dx = \int 28x\,u^{1/3}\,du.$

$u = x+4 \quad du = 1\cdot dx \quad = \int 28(u-4)\,u^{1/3}\,du.$

$28\int u^{4/3} - 4u^{1/3}\,du = 28\left[\frac{3}{7}u^{7/3} - 4\cdot\frac{3}{4}u^{4/3}\right] + C$

$= 12(x+4)^{7/3} - 84(x+4)^{4/3} + C.$

Regla de la sustitución para Integrales Definidas

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$u = g(x)$ $\qquad u = g(a)$

$du = g'(x)dx$ $\qquad u = g(b)$

Cambie también los límites.

Ejercicio 3: Integre.

a. $\displaystyle\int_{-4}^{0} \frac{1}{3x-2}\,dx = \int_{-14}^{-2} \frac{1}{u}\,\frac{du}{3} = \frac{1}{3}\ln|u|\Big]_{-14}^{-2} = (\ln 2 - \ln 14)\frac{1}{3}$

$= -\dfrac{\ln 7}{3}$

f es continua en $x \neq \frac{2}{3}$

$u = 3x-2 \qquad du = 3\cdot dx$

$u(0) = 0-2 = -2, \qquad u(-4) = -12-2 = -14$

b. $\displaystyle\int_{0}^{1} \frac{8}{\pi}\,\frac{\sin^{-1} t}{\sqrt{1-t^2}}\,dt = \frac{8}{\pi}\int_{0}^{\pi/2} u\,du = \frac{8}{\pi}\,\frac{u^2}{2}\Big]_{0}^{\pi/2} = \frac{4}{\pi}\,\frac{\pi^2}{4}$

$= \pi$

¿u? $\quad u = \sin^{-1} t \qquad u(1) = \sin^{-1}(1) = \pi/2$

$du = \dfrac{dt}{\sqrt{1-t^2}} \qquad u(0) = \sin^{-1}(0) = 0$

Capítulo 6

7.1 Integración por partes

$\int \ln x \, dx \qquad \int \tan^{-1} x \, dx \qquad \int x^3 e^x \, dx \qquad \int e^x \cos x \, dx$

IPP: Integre productos de funciones "disimilares".

$$\int \underbrace{f(x)}\, \underbrace{g(x)\, dx} = ?$$

Regla del Producto para Derivadas.

$$(fg)' = f'g + fg'$$

$$(fg)' - f'g = fg'$$ Integre esta expresión.

IPP $$\boxed{\int fg' = fg - \int f'g}$$

f deriva.
g' integra.

$\int fg'$ $\qquad$ $\int f'g$ más simple que la integral original.

$$\int \underset{u}{f(x)}\, \underbrace{g(x)\, dx}_{dv} = uv - \int v\, du$$

$u = f(x) \qquad dv = g(x)\, dx$

$du = f'(x)\, dx \qquad v = G(x)$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ejercicio 1: Pág 39 Integre $\int x e^x \, dx$

Opción 1: $u = x \qquad dv = e^x dx$

$du = 1 \cdot dx \qquad v = e^x$

Opción 2: $\cancel{u = e^x \qquad dv = x\, dx}$

$\cancel{du = e^x dx \qquad v = \frac{1}{2} x^2}$

$$\int x e^x dx = \underbrace{xe^x}_{u \cdot v} - \underbrace{\int e^x dx}_{v\, du} = xe^x - e^x + C.$$

Derive y compruebe su respuesta.

Ejercicio 2: Integre $\int fg' = fg - \int f'g$.

; $\int u\,dv = uv - \int v\,du$

a. $\int 6x^2 \ln x \, dx = (\ln x)\, 2x^3 - \int \frac{1}{x} 2x^3 \, dx$ $\qquad \frac{x^3}{x} = x^2$

$u = \ln x \qquad dv = 6x^2$

$du = \frac{1}{x} \qquad v = 2x^3$

$2x^3 \ln x - \int 2x^2 \, dx$

$\boxed{2x^3 \ln x - \frac{2}{3}x^3 + C.}$

b. $\int \ln x \, dx = \underset{v}{x} \underset{u}{\ln x} - \int \underset{v}{x} \underset{du}{\frac{dx}{x}} = x\ln x - \int dx$

$u = \ln x \qquad dv = 1 \cdot dx$

$du = \frac{dx}{x} \qquad v = x$

$= \boxed{x\ln x - x + C.}$

c. $\int \tan^{-1} x \, dx = \underset{v}{x} \underset{u}{\tan^{-1} x} - \int \underset{v}{\frac{x}{1+x^2}} \underset{du}{dx}$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$\int \frac{1}{1+x^2} \underbrace{x\,dx}_{du/2.} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+x^2| + C.$

Sustitución. $u = 1+x^2 \qquad du = 2x\,dx$

$\int \tan^{-1} x \, dx = \boxed{x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| - C}$

J. $\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$

$u v$ $\qquad$ $v \, du$.

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x \, dx \qquad v = \sin x$

otra vez IPP.

$\int 2x \sin x \, dx = -2x \cos x + \int 2 \cos x \, dx = -2x \cos x + 2 \sin x + C$

$u = 2x \qquad dv = \sin x \, dx$

$du = 2 \, dx \qquad v = -\cos x$

$\int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C.$

ILATE.

Mnemotecnia.

Inversas trigonométricas	$\sin^{-1} x$, $\tan^{-1} x$
Logarítmicas	$\ln x$, $\log_a x$
Algebraicas	x^n, $\sqrt{x}$, $1/x^r$
Trigonométricas	$\sin x$, $\cos x$, $\tan x$
Exponenciales.	e^x, a^x

IPP:

Integrales Definidas:

no cambian los límites

$$\boxed{\int_a^b u \, dv = uv \Big]_a^b - \int_a^b v \, du}$$

Ejercicio 3: Evalúe. $\qquad$ a) Interesante.

b. $72 \int_1^2 \frac{\ln x}{x^4} \, dx. = 72 \ln x \left(\frac{-1}{3x^3} \right) \Big]_1^2 - \int_1^2 \frac{x^{-3}}{-3} x^{-1} \, dx$

$u = \ln x \qquad dv = x^{-4} \, dx$

$du = x^{-1} \, dx \qquad v = \frac{x^{-3}}{-3}$

$$-\frac{24}{1x^3}\ln x\Big]_1^2 - \int_2^1 \frac{x^{-4}}{3}\,dx \qquad -x^{-4} \to \frac{-x^{-3}}{-3}$$

$$-\frac{24}{8}\ln 2 + \cancel{\frac{24}{1}\ln 1}^{\,0} + \frac{1}{3x^3}\Big]_2^1$$

$$-3\ln 2 + \frac{1}{3} - \frac{1}{3\cdot 8}$$

Resuelto con detalles pág 42.

incógnita

$$\boxed{\int e^x \cos x\,dx} = e^x\cos x + \int e^x \sin x\,dx \qquad (1)$$

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x\,dx \qquad v = e^x$

$$\int e^x \sin x\,dx = e^x \sin x - \boxed{\int e^x \cos x\,dx} \qquad (2)$$

incógnita.

$u = \sin x \qquad dv = e^x\,dx$

$du = \cos x\,dx \qquad v = e^x$

$$\int e^x \cos x\,dx = e^x\cos x + e^x \sin x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x\cos x + e^x \sin x$$

$$\int e^x \cos x\,dx = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x \sin x + C.$$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x\cos x - \frac{1}{2}e^x \sin x + C.$$

Capítulo 7

7.2 Integrales Trigonométricas.

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x \quad \div \cos^2 x.$

$1 + \cot^2 x = \csc^2 x \quad \div \sin^2 x.$

I. Integrales de la forma $\int \sin^n x \cos^m x \, dx$

$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

$u = \sin x, \; du = \cos x \, dx \qquad u = \cos x, \; du = -\sin x \, dx$

Evalúe $\int \cos^5 x \, dx = \int \cos^4 x \, \overbrace{(\cos x \, dx)}^{du}$

Reescriba $\cos^4 x = (\cos^2 x)^2 = (1 - \sin^2 x)^2$

$\cos^2 x = 1 - \sin^2 x$

$$\int \cos^5 x \, dx = \int (1 - \sin^2 x)^2 \underbrace{(\cos x \, dx)}_{du = \cos x \, dx}$$

$u = \sin x$

$$= \int (1 - u^2)^2 \, du$$

$$= \int (1 - 2u^2 + u^4) \, du$$

$$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C.$$

$\sin^3 x \neq \sin x^3$

$(\sin x)^3$

$$= \boxed{\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C.}$$

Apante algún término sinx ó cosx

a. Potencias impares de seno o coseno.

Ejercicio 1: Evalúe.

a. $\int \cos^3 x \sin^6 x \, dx = \int \cos^2 x \sin^6 x (\cos x \, dx)$

$\int \underbrace{\cos^2 x}_{1-\sin^2 x} \sin^6 x \cos x \, dx$ ó $\int \cos^3 x \sin^{\overset{\text{impar}}{5}} x \sin x \, dx$

$1-\cos^2 x$ X

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$

$du = \cos x \, dx$

$(a+b+c)d$

$ad + bd + cd.$

$= \int (1-\sin^2 x) \sin^6 x \, (\cos x \, dx)$

$= \int (1-u^2) u^6 \, du$

$= \int (u^6 - u^8) \, du$

$= \frac{1}{7} u^7 - \frac{1}{9} u^9 + C.$

$= \boxed{\frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C.}$

b. $\int \cos^5 x \sin^3 x \, dx = \int \cos^5 x (\sin^2 x) \sin x \, dx$

$\int \underbrace{\cos^4 x}_{(1-\sin^2 x)^2} \sin^3 x \cos x \, dx$ ó $\int \cos^5 x \underbrace{\sin^2 x}_{1-\cos^2 x} \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= \int \cos^5 x (1 - \cos^2 x) \sin x \, dx$

$= -\int u^5 (1-u^2) \, du = \int (-u^5 + u^7) \, du$

$= -\frac{1}{6} u^6 + \frac{1}{8} u^8 + C.$

$= \boxed{-\frac{1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + C.}$

b) Potencias pares de seno y coseno

$$\int \cos^2 x \, dx = \int \frac{1}{2} + \frac{1}{2}\cos 2x \, dx = \boxed{\frac{x}{2} + \frac{1}{4}\sin 2x + C.}$$

$$1 = \cos^2 x + \sin^2 x. \quad (1)$$

$$\cos(x+x) = \cos^2 x - \sin^2 x \quad (2)$$

$$1 + \cos(2x) = 2\cos^2 x \quad \Rightarrow \quad \boxed{\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x}$$

(1) + (2)

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Ejercicio 2: Evalúe.

a $$\int_{-\pi}^{\pi} \sin^2 x \, dx = 2\int_0^{\pi} \sin^2 x \, dx = \frac{2}{2}\int_0^{\pi}(1 - \cos 2x)\, dx$$

PAR.

$$\int_{-\pi}^{\pi} \sin x \, dx = 0 \quad \text{impar.}$$

360°

$$\int_0^{\pi}(1 - \underbrace{\cos 2x}_{u = 2x \quad du = 2dx})\, dx = x - \frac{1}{2}\sin 2x \Big]_0^{\pi} = \pi - \frac{1}{2}\cancelto{0}{\sin 2\pi} - 0 + \frac{\sin 0}{2}$$

$$\int_{-\pi}^{\pi} \sin^2 x \, dx = \pi.$$

$$\cos^2 \square = \frac{1}{2}(1 - \cos 2\square)$$

b. $$\int \sin^2 x \cos^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x)\frac{1}{2}(1 + \cos 2x)\, dx$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

doble

$$= \frac{1}{4}\int (1 - \cos^2 \underline{2x})\, dx$$

$$\cos^2(2x) = \frac{1}{2}(1 + \cos 4x) = \frac{1}{4}\int \left(1 - \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\int\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \int \frac{1}{8} + \frac{1}{8}\cos 4x \; dx.$$

$\int a f \, dx$

$= aF + C$

$$= \boxed{\frac{x}{8} + \frac{1}{8\cdot 4}\sin 4x + C.}$$

II Forma $\int \tan^m x \sec^n x \, dx$

$(\tan x)' = \sec^2 x$ (aparte)	$(\sec x)' = \sec x \tan x$ (aparte.)
$u = \tan x$	$u = \sec x$
$\sec^2 x = \tan^2 x + 1$	$\tan^2 x = \sec^2 x - 1$

Ejercicio 3: Evalúe Pág. 48.

a. $\int \tan^5 x \sec^4 x \, dx$

$\int \tan^5 x \underbrace{\sec^2 x}_{\tan^2 x + 1} (\sec^2 x \, dx)$ ✓

$u = \tan x$

ó $\int \tan^4 x \sec^3 x (\tan x \sec x \, dx)$ ✓

$u = \sec x$

$(\tan^2 x)^2 = (\sec^2 x - 1)^2$

$\int \tan^5 x \sec^2 x (\sec^2 x \, dx)$

$\int \tan^5 x (\tan^2 x + 1)(\sec^2 x \, dx)$

$u = \tan^2 x$ ✗

$du = 2\tan x \sec^2 x \, dx$ ✗

$u = \tan x \qquad du = \sec^2 x \, dx$

$$\int u^5 (u^2 + 1) \, du = \int (u^7 + u^5) \, du = \frac{u^8}{8} + \frac{u^6}{6} + C.$$

$$= \boxed{\frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C.}$$

Sharon: Resumen Identidades y luego sustitución.

b. Alejandro $\int \tan^5 x \sec^5 x \, dx$

$\int \tan^4 x \sec^4 x (\sec x \tan x \, dx)$ ✓

$\int \tan^5 x \sec^3 x (\sec^2 x \, dx)$ ✗

$\int (\tan^2 x)^2 \sec^4 x (\sec x \tan x \, dx)$ $\tan^2 x = \sec^2 x - 1$

$\int (\sec^2 x - 1)^2 \sec^4 x (\sec x \tan x \, dx)$

$u = \sec x$ $du = \sec x \tan x \, dx.$

$\int (u^2 - 1)^2 u^4 \, du = \int (u^4 - 2u^2 + 1) u^4 \, du.$

$\int (u^8 - 2u^6 + u^4) \, du = \frac{1}{9} u^9 - \frac{2}{7} u^7 + \frac{1}{5} u^5 + C.$

$= \frac{1}{9} \sec^9 x - \frac{2}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C.$

c. $\int \tan^4 x \sec^4 x \, dx. = \int \tan^4 x \sec^2 x \sec^2 x \, dx$

sólo tan's.

$\sec^2 x = \tan^2 x + 1$ $= \int \tan^4 x (\tan^2 x + 1)(\sec^2 x \, dx)$

$u = \tan x$ $du = \sec^2 x \, dx$ $= \int u^4 (u^2 + 1) \, du.$

$= \int u^6 + u^4 \, du = \frac{1}{7} u^7 + \frac{1}{5} u^5 + C$

$= \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C.$

Casos especiales $\int \tan^m x\,dx$ $\int \sec^n x\,dx$

$$\int \tan x\,dx = \int \frac{\overbrace{\sin x\,dx}^{-du}}{\underbrace{\cos x}} = -\int \frac{du}{u} = -\ln|u| + C.$$

$u = \cos x$ $\quad du = -\sin x\,dx$

$$= -\ln|\cos x| + C.$$

$$\int \sec x\,dx = \int \sec x \frac{(\sec x + \tan x)}{\tan x + \sec x}\,dx$$ Brillante.

$$= \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x}\,dx$$

$u = \tan x + \sec x$ $\quad du = (\sec^2 x + \sec x \tan x)\,dx$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\tan x + \sec x| + C.$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C.$$

$$\int \sec^2 x\,dx = \tan x + C.$$

$\tan^2 x = \sec^2 x - 1$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C.$$

$$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$$

$u = \tan x$
$du = \sec^2 x\,dx$

$$= \int (\sec^2 x - 1)\tan x\,dx$$

$$= \int (\sec^2 x \tan x - \tan x)\,dx$$

$$= \int \underbrace{\tan x \sec^2 x\,dx}_{u \cdot du.} - \int \tan x\,dx$$

$$= \frac{1}{2}\tan^2 x - \ln|\cos x| + C.$$

7.

Más difícil $\int \sec^3 x \, dx$.

$$\int \sec x \sec^2 x \, dx = \sec x \tan x - \int \tan^2 x \sec x \, dx$$

IPP $u = \sec x$ $\qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx$ $\qquad v = \tan x$

$$\int \tan^2 x \sec x \, dx = \int (\sec^2 x - 1) \sec x \, dx$$

$$= \int (\sec^3 x - \sec x) \, dx$$

$$\underline{\int \sec^3 x \, dx} = \sec x \tan x - \underline{\int \sec^3 x \, dx} + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C.$$

$$\int \sec^3 x \, dx = \frac{1}{2} \text{derivada}(\sec) + \frac{1}{2} \text{integral}(\sec)$$

$$\frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C.$$

Capítulo 8

Lunes 26 de agosto Simulacro Parcial.
3 de septiembre Parcial 1
capítulos 5 y 7 Págs 11-70.

Integrales de la forma $\int \cot^n x \csc^m x \, dx$

$(\csc x)' = -\csc x \cot x$ $\qquad$ $(\cot x)' = -\csc^2 x$

$\cot^2 x = \csc^2 x - 1$ $\qquad$ $\csc^2 x = \cot^2 x + 1$

$u = \csc x$ $\qquad$ $u = \cot x$

Ejercicio 4: Integre (Pág 50).

a $\int \cot^2 x \csc^4 x \, dx$ $\qquad$ $\cot^2 \csc^2 x \csc^2 x$ ✓

$\times \cot x \csc^3 x (\csc x \cot x)$

$$\int \cot^2 x \csc^2 x (\csc^2 x \, dx) = \int \cot^2 x (\cot^2 x + 1) \csc^2 x \, dx$$

$\csc^2 x = \cot^2 x + 1$, $\quad u = \cot x$ $\qquad$ $du = -\csc^2 x \, dx$

$$= -\int u^2 (u^2 + 1) \, du$$

$$= -\int (u^4 + u^2) \, du = -\frac{u^5}{5} - \frac{u^3}{3} + C.$$

$$= -\frac{\cot^5 x}{5} - \frac{\cot^3 x}{3} + C.$$

b. $\int \cot^3 x \csc^3 x \, dx = \int \cot^2 x \csc^2 x (\cot x \csc x \, dx)$

$\cot^2 x = \csc^2 x - 1$ $\qquad$ $= \int (\csc^2 x - 1) \csc^2 x (\cot x \csc x \, dx)$

$u = \csc x$ $\quad$ $du = -\csc x \cot x \, dx$ $\qquad$ $= -\int (u^2 - 1)(u^2) \, du.$

$$-\int (u^4 - u^2)\,du = -\frac{u^5}{5} + \frac{u^3}{3} + C.$$

$$= -\frac{\csc^5 x}{5} + \frac{\csc^3 x}{3} + C.$$

casos especiales $\int \csc x\,dx$ $\int \csc^3 x\,dx$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C.$$

$$\int \csc x \frac{(\csc x + \cot x)}{\cot x + \csc x}\,dx = \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x}\,dx$$

*"1" especial.

$u = \cot x + \csc x.$

$-du = (\csc^2 x + \csc x \cot x)\,dx$

$$= -\int \frac{du}{u} = -\ln|u| + C. = -\ln|\cot x + \csc x| + C.$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C.$$

$$\int \sec^3 x\,dx = \frac{1}{2}(\sec x)' + \frac{1}{2}\int \sec x\,dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C.$$

$$\int \csc^3 x\,dx = \frac{1}{2}(\csc x)' + \frac{1}{2}\int \csc x\,dx$$

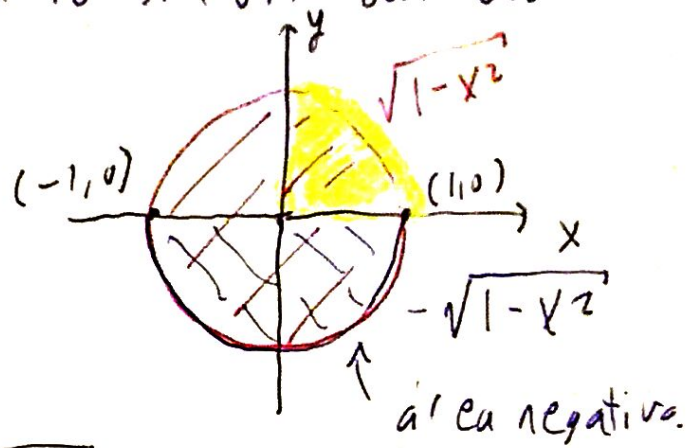
$$= -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + C.$$

Área de un círculo unitario sin utilizar Geometría

Ec: $x^2 + y^2 = 1$

Función: $y^2 = 1 - x^2$.

2 Funciones $y = \pm\sqrt{1-x^2}$



$$A = \int_{-1}^{1} \sqrt{1-x^2}\,dx + \int_{-1}^{1} \sqrt{1-x^2}\,dx \qquad \int_{1}^{-1} \sqrt{1-x^2}\,dx$$

$$A = 2\int_{-1}^{1} \sqrt{1-x^2}\,dx = 4\int_{0}^{1} \sqrt{1-x^2}\,dx$$

función par

$u = 1 - x^2$

$du = -2x\,dx$

Ni sustitución ni integración por partes,

$1 - \sin^2\theta = \cos^2\theta.$

$$A = 4\int_0^{\pi/2} \underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta.}\,\cos\theta\,d\theta.$$

$x = \sin\theta.$

$dx = \cos\theta\,d\theta.$

$$A = 4\int_0^{\pi/2} \cos^2\theta\,d\theta.$$

$x = 1 = \sin\theta \Rightarrow \theta = \pi/2$

$x = 0 = \sin\theta \Rightarrow \theta = 0$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$A = \frac{4}{2}\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta. = 2\left(\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}\Bigg)$$

$$A = 2\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\sin\pi}^{\,0} - 0 - \frac{1}{2}\cancel{\sin 0}^{\,0}\right) = \frac{2\pi}{2} = \pi.$$

Área de un círculo de radio (1) $\pi(1)^2$.

7.3 Sustitución Trigonométrica (Pág 54).

$$\int f(x)\,dx = \int \underbrace{f(g(\theta))}\, g'(\theta)\,d\theta.$$

simplifique si es posible.

$x = g(\theta) \qquad dx = g'(\theta)\,d\theta$

$\sqrt{1-x^2}$

$x = \sin\theta$

$1-\sin^2\theta = \cos^2\theta.$

$\sqrt{1-x^2} = \cos\theta.$

$\sqrt{1+x^2}$

$x = \tan\theta$

$1+\tan^2\theta = \sec^2\theta.$

$\sqrt{1+x^2} = \sec\theta.$

$\sqrt{x^2-1}$

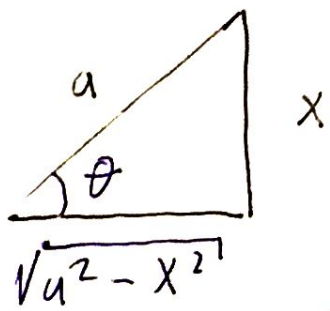
$x = \sec\theta.$

$\sqrt{\sec^2\theta - 1}$

$\sqrt{\tan^2\theta}$

$\sqrt{x^2-1} = \tan\theta.$

forma $\sqrt{a^2-x^2}$



$\sin\theta = \dfrac{C.O.}{H} = \dfrac{x}{a}.$

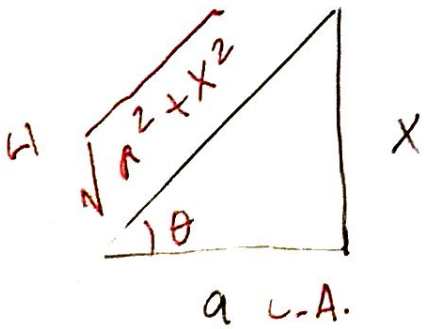
$\cos\theta = \dfrac{C.A.}{H} = \dfrac{\sqrt{a^2-x^2}}{a}.$

$x = a\cdot\sin\theta$

$dx = a\cdot\cos\theta\,d\theta.$

$\sqrt{a^2-x^2} = a\cdot\cos\theta.$

Forma $\sqrt{a^2+x^2}$



$\sin\theta = \dfrac{x}{\sqrt{a^2+x^2}} \qquad \tan\theta = \dfrac{x}{a}.$

$\dfrac{H}{C.A.} = \sec\theta = \dfrac{\sqrt{a^2+x^2}}{a}. \qquad \dfrac{C.A.}{H}$

$x = a\cdot\tan\theta.$

$dx = a\cdot\sec^2\theta\,d\theta. \qquad \sqrt{a^2+x^2} = a\cdot\sec\theta.$

Ejercicio 1: Evalúe.

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{-1}{\sqrt{u}}\cdot\frac{du}{2} = -\int \frac{u^{-1/2}}{2}\,du = -\frac{2u^{1/2}}{2} + C$$

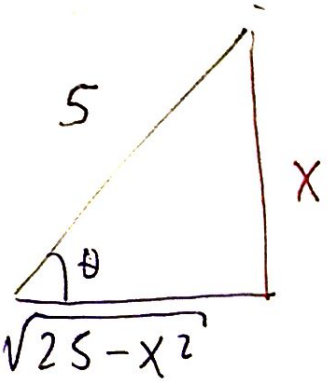
$u = 25 - x^2 \qquad du = -2x\,dx \Rightarrow dx = \dfrac{du}{-2x} \qquad = -u^{1/2} + C.$

$$= -\sqrt{25-x^2} + C.$$

Sustitución Trigonométrica.

$H = 5.$

$C.O = x$



$x = 5\sin\theta.$

$dx = 5\cos\theta\,d\theta.$

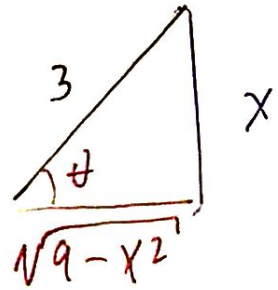
$\sqrt{25-x^2} = 5\cos\theta.$

$\dfrac{C.A.}{H.}$

$$\int \frac{x}{\sqrt{25-x^2}}\,dx = \int \frac{5\sin\theta}{5\cos\theta}\,5\cos\theta\,d\theta = \int 5\sin\theta\,d\theta.$$

$$= -5\cos\theta + C. = -\sqrt{25-x^2} + C.$$

a $$\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{27\sin^3\theta}{3\cancel{\cos\theta}}\,3\cancel{\cos\theta}\,d\theta = 27\int \sin^3\theta\,d\theta.$$



$\sin\theta = \dfrac{x}{3}$

$\cos\theta = \dfrac{1}{3}\sqrt{9-x^2}$

$x^3 = 27\sin^3\theta$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta.$

$$27\int \sin^2\theta\,\sin\theta\,d\theta = 27\int (1-\cos^2\theta)\sin\theta\,d\theta.$$

$u = \cos\theta \qquad du = -\sin\theta\,d\theta.$

$$= -27\int (1-u^2)\,du = -27\left(u - \frac{u^3}{3}\right) + C.$$

regrese a var. θ.

$$= -27u + 9u^3 + C.$$

$$= -27\cos\theta + 9\cos^3\theta + C$$

regrese a var x.

$$= -27\,\frac{1}{3}\sqrt{9-x^2} + 9\,\frac{1}{27}\left(\sqrt{9-x^2}\right)^3 + C.$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)^{3/2} + C.$$

Caso Integrales Trigonométricas.

$$\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin(m-n)x + \sin(m-n)x\right)$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos(m-n)x - \cos(m-n)x\right)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$$

Ejercicio 5: Evalúe. (Pág 51)

a $\displaystyle\int_{-\pi}^{\pi} \underset{m}{\sin 8x}\ \underset{n}{\cos 4x}\,dx = 0.$

$$\frac{1}{2}\int_{-\pi}^{\pi}(\sin 4x + \sin 12x)\,dx = \frac{1}{2}\left(\left.\frac{-\cos 4x}{4}\right]_{-\pi}^{\pi} - \left.\frac{\cos 12x}{12}\right]_{-\pi}^{\pi}\right)$$

$$\frac{1}{2}\left(\frac{-\overbrace{\cos 4\pi}^{0}}{4} + \frac{\cos(4\pi)}{4} - \frac{\cos 12\pi}{12} + \frac{\cos 12\pi}{12}\right)$$

$$-\frac{1}{4} \qquad +\frac{1}{4}. \qquad = \frac{1}{2}(0+0) = 0.$$

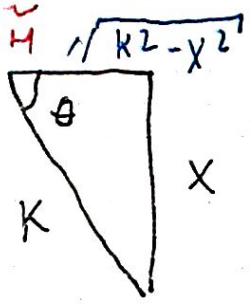
Capítulo 9

7.3 Sustitución Trigonométrica

Forma $\sqrt{\underset{H}{K^2} - x^2}$

$H = K$

$C.O = x$

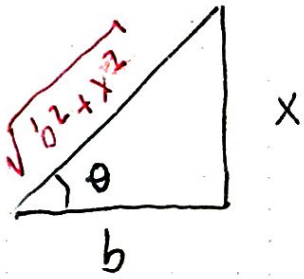


$\frac{C.O}{H} = \sin\theta = \frac{x}{K} \Rightarrow x = K\sin\theta.$

$dx = K\cos\theta\, d\theta.$

$\frac{\sqrt{K^2-x^2}}{K} = \cos\theta \Rightarrow \sqrt{K^2-x^2} = K\cdot\cos\theta.$

Forma $\sqrt{b^2 + x^2}$

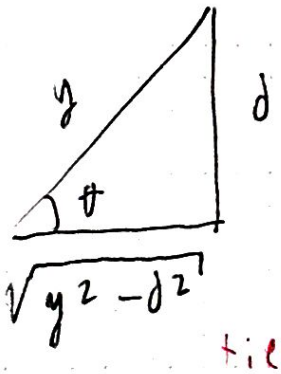


$\frac{x}{b} = \tan\theta. \Rightarrow x = b\cdot\tan\theta.$

$dx = b\cdot\sec^2\theta\, d\theta.$

$\frac{\sqrt{b^2+x^2}}{b} = \sec\theta \Rightarrow \sqrt{b^2+x^2} = b\sec\theta.$

Forma $\sqrt{y^2 - d^2}$



$\operatorname{sen}\theta = \frac{d}{y}$

$y = d\cdot\csc\theta.$

$dy = -d\cdot\csc\theta\cot\theta.$

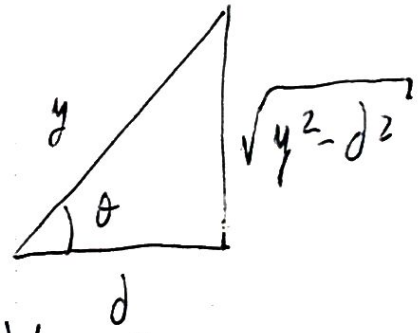
tiene signos negativos.

$\frac{y}{d} = \sec\theta.$

$y = d\sec\theta.$

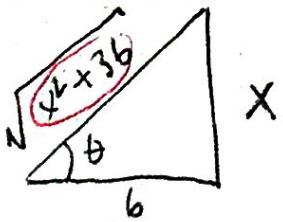
$dy = d\cdot\sec\theta\tan\theta.$

$\sqrt{y^2-d^2} = d\tan\theta.$



Ejercicios 2 y 3 Pág 58 y 59.

2o) $\int \frac{1}{x^2+36}\,dx = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{\theta}{6} + C$



$x = 6\cdot\tan\theta.$

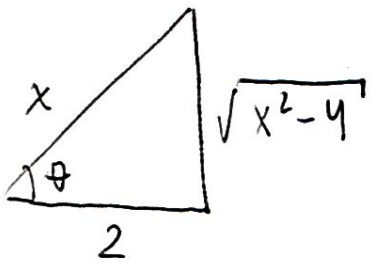
$dx = 6\sec^2\theta\, d\theta.$

$x^2 + 36 = 36\tan^2\theta + 36) = 36\sec^2\theta$

$$\frac{1}{6}\int d\theta = \frac{1}{6}\theta + C. = \boxed{\frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C.}$$

$$\tan^{-1}\left(\frac{x}{6}\right) = \theta.$$

3a. $$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \int \frac{2^3\tan^3\theta}{2^6\cdot\sec^6\theta}\; 2\cdot\sec\theta\tan\theta\,d\theta.$$



$$\frac{2}{x} = \cos\theta. \rightarrow x = \frac{2}{\cos\theta} = 2\sec\theta.$$

$$dx = 2\sec\theta\tan\theta\,d\theta.$$

$$\sqrt{x^2-4} = 2\tan\theta.$$

$$\left[(x^2-4)^{1/2}\right]^3 = 8\tan^3\theta.$$

$$\frac{1}{2^2}\int \frac{\tan^4\theta}{\sec^5\theta}\,d\theta. = \frac{1}{4}\int \frac{\sin^4\theta}{\cos^4\theta}\cos^5\theta\,d\theta.$$

$u = \sin\theta \quad du = \cos\theta\,d\theta$

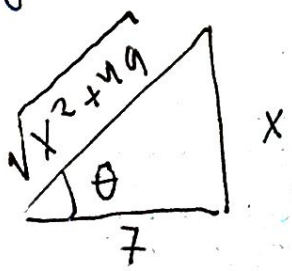
$\frac{1}{4}\int u^4\,du.$

$$= \frac{1}{4}\int \underbrace{\sin^4\theta}_{u^4}\,\underbrace{\cos\theta\,d\theta}_{du}. = \boxed{\frac{1}{4}\cdot\frac{1}{5}\sin^5\theta + C.}$$

Regrese a la variable x $\quad \sin\theta = \frac{\sqrt{x^2-4}}{x}, \quad \sin^5\theta = \frac{(x^2-4)^{5/2}}{x^5}$

$$\int \frac{(x^2-4)^{3/2}}{x^6}\,dx = \boxed{\frac{1}{20}\,\frac{(x^2-4)^{5/2}}{x^5} + C.}$$

2a $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{49\;\cancel{7}\sec^2\theta}{49\tan^2\theta\cdot\cancel{7}\sec\theta}\,d\theta. = \int \frac{\sec\theta}{\underbrace{\tan^2\theta}_{\sec^2\theta-1}}\,d\theta.$$

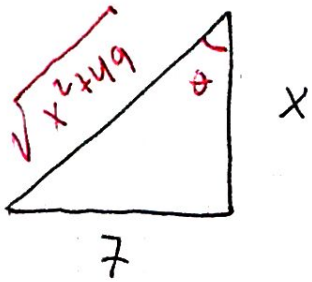


$$\frac{x}{7} = \tan\theta \qquad x = 7\tan\theta.$$

$$dx = 7\sec^2\theta\,d\theta.$$

$$\frac{H}{C.A.} = \sec\theta. \qquad \sqrt{x^2+49} = 7\sec\theta.$$

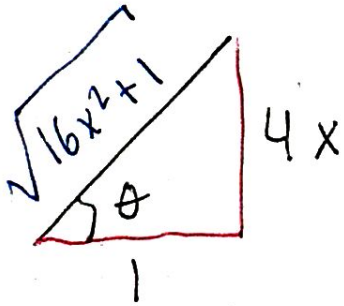
2a) $$\int \frac{49}{x^2\sqrt{x^2+49}}\,dx = \int \frac{-49\cdot 7\csc^2\theta}{49\cot^2\theta\; 7\csc\theta}\,d\theta. = -\int \frac{\csc\theta}{\cot^2\theta}\,d\theta.$$



$$\frac{x}{7} = \cot\theta \Rightarrow x = 7\cot\theta.$$
$$dx = -7\csc^2\theta\,d\theta.$$
$$\frac{\sqrt{x^2+49}}{7} = \csc\theta \Rightarrow \sqrt{x^2+49} = 7\csc\theta.$$

$$-\int \frac{\csc\theta}{\cot^2\theta}\,d\theta = -\int \frac{1}{\sin\theta}\frac{\sin^2\theta}{\cos^2\theta}\,d\theta. = -\int \frac{\sin\theta}{\cos\theta}\frac{1}{\cos\theta}\,d\theta.$$
$$= -\int \sec\theta\tan\theta\,d\theta.$$
$$= -\sec\theta + C. = -\frac{\sqrt{x^2+49}}{x} + C.$$

3 b) $$\int \frac{1}{x\sqrt{16x^2+1}}\,dx = \int \frac{(1/4)\sec^2\theta\,d\theta.}{(1/4)\tan\theta.\sec\theta.} = \int \frac{\sec\theta}{\tan\theta}\,d\theta.$$



$$\frac{4x}{1} = \tan\theta. \Rightarrow x = \frac{1}{4}\tan\theta.$$
$$dx = \frac{1}{4}\sec^2\theta\,d\theta.$$
$$\sqrt{16x^2+1} = \sec\theta.$$

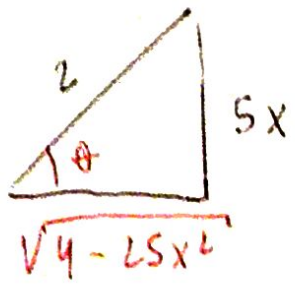
$$\int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta}\,d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \csc\theta\,d\theta.$$
$$= -\ln\left|\csc\theta + \cot\theta\right| + C.$$
$$= -\ln\left|\frac{\sqrt{16x^2+1}}{4x} + \frac{1}{4x}\right| + C.$$

Capítulo 10

Parcial Lunes 2 septiembre 1:00 PM. D-503 B.
2:30 PM D-505 A

sub 5.

1. $\int 5^8 x^7 \sqrt{4-25x^2}\, dx = 5^8 \int \frac{2^7}{5^7} \sin^7\theta \, 2\cdot\cos\theta \cdot \frac{2}{5}\cos\theta \, d\theta.$



$\frac{5x}{2} = \sin\theta \qquad x = \frac{2}{5}\sin\theta.$

$dx = \frac{2}{5}\cos\theta \, d\theta$

$\sqrt{4-25x^2} = 2\cos\theta.$

$x^7 = \frac{2^7}{5^7}\sin^7\theta.$

$\frac{5^8}{5^8} 2^9 \int \sin^7\theta \cdot \cos^2\theta \, d\theta = 512 \int \sin^6\theta \cos^2\theta \, \underline{\sin\theta \, d\theta}.$

$\sin^6\theta = (\sin^2\theta)^3 = (1-\cos^2\theta)^3 \qquad = 512\int (1-\cos^2\theta)^3 \cos^2\theta \sin\theta \, d\theta.$

$u = \cos\theta \quad du = -\sin\theta \, d\theta. \qquad = 512 \int \underline{(1-u^2)^3} u^2 \, du.$

$(1-u^2)^3 = 1 - 3u^2 + 3u^4 - u^6$

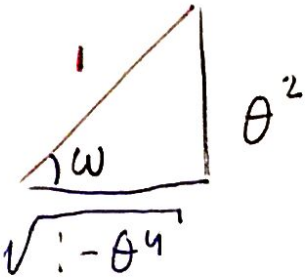
$(1-u^2)^3 u^2 = u^2 - 3u^4 + 3u^6 - u^8$

$\cos\theta.$

$\cos\theta = \frac{(4-25x^2)^{1/2}}{2}.$

2. $\frac{4}{\pi}\int_0^1 \theta\sqrt{1-\theta^4}\,d\theta. = \frac{4}{\pi}\int_0^1 \underbrace{\sqrt{1-\theta^4}}_{\cos\omega}\ \underbrace{\theta\,d\theta}_{\frac{\cos\omega}{2}d\omega}.$

Lab. 5.



$\sin\omega = \theta^2.$

$\cos\omega\,d\omega = 2\theta\,d\theta.$

$\sqrt{1-\theta^4} = \cos\omega$

$\sin\omega = 1^2$

$\omega = \pi/2.$

$\sin\omega = 0.$

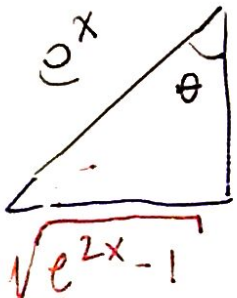
$\omega = 0.$

$$\frac{2}{\pi}\int_0^{\pi/2} \cos^2\omega\,d\omega = \frac{1}{\pi}\int_0^{\pi/2}(1+\cos 2\omega)\,d\omega. \qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$= \frac{1}{\pi}\left(\omega + \frac{1}{2}\sin 2\omega\Big]_0^{\pi/2}\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right) = \boxed{\frac{1}{2}.}$$

5. $\int_0^{\ln\sqrt{2}} \frac{e^{4x}}{\sqrt{e^{2x}-1}}\,dx. = \int \frac{e^{3x}}{\sqrt{e^{2x}-1}}\,e^x\,dx = \int \frac{\sec^3\theta}{\tan\theta}\sec\theta\tan\theta\,d\theta.$



$e^x = \sec\theta$

$\sqrt{e^{2x}-1} = \tan\theta\,d\theta.$

$e^{3x} = \sec^3\theta.$

$e^x\,dx = \sec\theta\tan\theta\,d\theta.$

$\sec\theta = \cancel{e^{\ln\sqrt{2}}} = \sqrt{2}.$

$\sec\theta = e^0 = 1$

$\theta = 0.$

$\sec 0 = \frac{1}{\cos 0} = 1$

$$\int_0^{\pi/4} \sec^4\theta\,d\theta. = \int_0^{\pi/4} \sec^2\theta\,\sec^2\theta\,d\theta.$$

$$\int_0^{\pi/4}(1+\tan^2\theta)\sec^2\theta\,d\theta. = \int_0^1 (1+u^2)\,du.$$

$u = \tan\theta \qquad du = \sec^2\theta\,d\theta \qquad u(\pi/4) = \tan\pi/4 = 1$

$u(0) = \tan 0 = 0$

$$\int_0^1 (1+u^2)\,du = u + \frac{1}{3}u^3\Big]_0^1 = 1 + \frac{1}{3} = \frac{4}{3}.$$

5. $$\int_0^{\ln\sqrt{2}} \frac{e^{2x}}{\sqrt{e^{2x}-1}}\, e^{2x}\,dx. = \int_0^1 \frac{e^{2x}}{\sqrt{u}}\,\frac{du}{2}. \qquad e^{2x} = u+1$$

$$u = e^{2x} - 1 \qquad u(\ln\sqrt{2}) = e^{2\cdot\ln 2^{1/2}} - 1 = 2 - 1 = 1$$

$$\frac{du}{2} = e^{2x}\,dx \qquad u(0) = e^0 - 1 = 0$$

$$\frac{1}{2}\int_0^1 \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\int_0^1 u^{1/2} + u^{-1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\Big]_0^1\right)$$

$$= \frac{1}{2}\left(\frac{2}{3} + \frac{6}{3}\right) = \frac{4}{3}.$$

$$\int \frac{\overbrace{e^{2x}}^{u+1}\ \overbrace{e^{2x}\,dx}^{du/2.}}{\sqrt{e^{2x}-1}} \qquad \int \frac{u+1}{\sqrt{u}}\ \frac{du}{2}$$

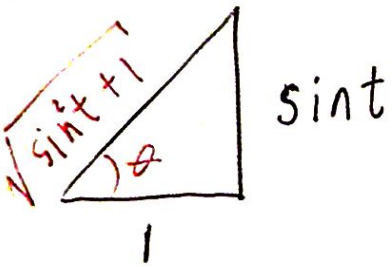
$$= \frac{1}{2}\int u^{1/2} + u^{-1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C.$$

$$= \frac{1}{3}(e^{2x}-1)^{3/2} + (e^{2x}-1)^{1/2}\Big]_0^{\ln\sqrt{2}}$$

Problema 2 b) Simulacro.

$$\int_0^{\pi/2} \frac{\cos t}{\sqrt{\sin^2 t+1}}\,dt = \int \frac{\sec^2\theta\,d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta\,d\theta.$$



$\sin\theta$

$\tan\theta = \sin t.$

$\sec^2\theta\,d\theta = \cos t\,dt.$

$\sqrt{\sin^2 t+1} = \sec\theta.$

$\tan\theta = \sin\frac{\pi}{2} = 1$

$\theta = \tan^{-1}(1) = \pi/4$

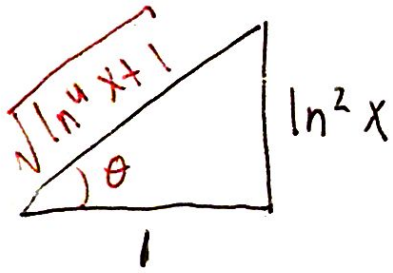
$\tan\theta = \sin 0 = 0$

$\theta = 0$

$$\int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|\Big]_0^{\pi/4} = \ln|\sec\pi/4 + \tan\pi/4) - \ln(\sec 0 + \tan 0)$$

$$= \ln(\sqrt{2}+1) - \cancel{\ln(1)}^{\,0} = \underline{\ln(\sqrt{2}+1)}$$

$$\int \frac{4}{\sqrt{\ln^4 x+1}}\,\frac{2(\ln x)}{x}\,dx = 4\int \frac{\sec^2\theta\,d\theta}{\sec\theta} = 4\int \sec\theta\,d\theta.$$



$\tan\theta = [\ln x]^2$

$\sec^2\theta\,d\theta = 2\ln x\,\frac{1}{x}\,dx.$

$\sqrt{\ln^4 x+1} = \sec\theta.$

$$4\int \sec\theta\,d\theta = 4\ln|\sec\theta+\tan\theta| + C.$$

$$4\ln\left|\sqrt{\ln^4 x+1} + \ln^2 x\right| + C.$$

$$\int \frac{(x-2)^3}{\sqrt{x^2-4x+13}}\,dx.$$

$u = x^2-4x+13$

$du = 2x-4 = 2(x-2)\,dx$

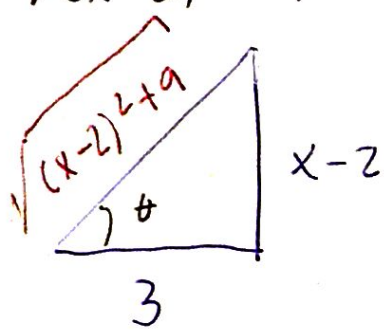
completed cuadrado. $(x^2-4x+4)+13-4$

$(x-2)^2+9.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = a\cdot \operatorname{sen}\theta.$

$1\cdot dx = a\cdot\cos\theta\, d\theta.$



$3\cdot\tan\theta = x-2$

$3\sec^2\theta\, d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$(x-2)^3 = 3^3\tan^3\theta.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\, d\theta.$$

$9\tan^2\theta + 9$

$9\sec^2\theta.$

$$= 3^3\int \tan^3\theta\sec\theta\, d\theta.$$

$$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\, d\theta)$$

$$= 27\int(\sec^2\theta-1)\,(\tan\theta\sec\theta\, d\theta).$$

$du = \sec\theta\tan\theta\, d\theta.$

$u = \sec\theta$

$$= 27\int(u^2-1)\,du = 9u^3-27u+C.$$

$\sec\theta = \dfrac{\sqrt{x^2-4x+13}}{3}.$

$9\sec^3\theta - 27\sec\theta + C.$

$$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C.$$

$$\int \frac{x^3}{\sqrt{x^2-4x+13}}\,dx$$

$x^2-4x+13$

$du = 2x-4 = 2(x-2)\,dx$

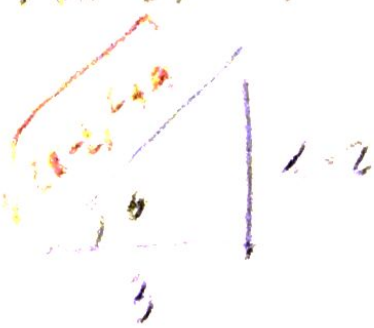
completa el cuadrado $x^2-4x+4 = 13-4$

$(x-2)^2+9$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx$$

$x = 3\sec\theta$

$dx = 3\cos\theta\,d\theta$



$3\tan\theta = x-2$

$3\sec^2\theta\,d\theta = dx$

$\sqrt{(x-2)^2+9} = 3\sec\theta.$

$(x-2)^3 = 3^3\tan^3\theta.$

$$\int \frac{(x-2)^3}{\sqrt{(x-2)^2+9}}\,dx = \int \frac{3^3\tan^3\theta}{3\sec\theta}\,3\sec^2\theta\,d\theta.$$

$1\tan^2\theta + 9$

$9\sec^2\theta$

$= 3^3\int \tan^3\theta\sec\theta\,d\theta.$

$= 27\int \tan^2\theta\,(\tan\theta\sec\theta\,d\theta)$

$= 27\int(\sec^2\theta-1)\,(\tan\theta\sec\theta\,d\theta)$

$du = \sec\theta\tan\theta\,d\theta$

$u = \sec\theta$

$= 27\int(u^2-1)\,du = 9u^3 - 27u + C.$

$\sec\theta = \frac{\sqrt{x^2-4x+13}}{3}$

$9\sec^3\theta - 27\sec\theta + C.$

$= \frac{9}{27}(x^2-4x+13)^{3/2} - \frac{27}{3}(x^2-4x+13)^{1/2} + C.$

Capítulo 11

Simulacro Parcial

a) $\int x \tan^{-1} x^2 \, dx$ $\qquad \int x f(x) \, dx$ IPP.

ILATE

$y = x^2 \qquad dy = 2x\,dx$

$$\int \tan^{-1}(x^2)(x\,dx) = \frac{1}{2}\int \tan^{-1}(y)\,dy.$$

IPP: $u = \tan^{-1} y \qquad dv = \frac{dy}{2}$

$du = \frac{1}{1+y^2} \qquad v = \frac{y}{2}.$

$$uv - \int v\,du = \frac{1}{2} y \tan^{-1} y - \frac{1}{2}\int \frac{y}{1+y^2}\,dy.$$

$w = 1 + y^2$

$dw = 2y\,dy.$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\int \frac{dw}{w}$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\ln|w| + C.$$

$$= \frac{1}{2} y \tan^{-1} y - \frac{1}{4}\ln|1+y^2| + C.$$

$$= \boxed{\frac{1}{2} x^2 \tan^{-1}(x^2) - \frac{1}{4}\ln|1+x^4| + C.}$$

b) $\int \frac{xe^x}{(x+1)^2}\,dx$

Tres funciones

	Der.			
	x Int.			
	xe^x	ó	$\frac{x}{(x+1)^2}$	x Int
Der. x				x der
Int ✓	$\frac{1}{(x+1)^2}$		e^x	✓

$u = xe^x \qquad dv = (x+1)^{-2}\,dx$

$du = (e^x + xe^x)\,dx, \quad v = \dfrac{(x+1)^{-1}}{-1} = \dfrac{-1}{(x+1)}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = \frac{-xe^x}{(x+1)} + \int \frac{(e^x + xe^x)\,dx}{(x+1)}$$

inspiración,

$$= \frac{-xe^x}{(x+1)} + \int e^x\,dx$$

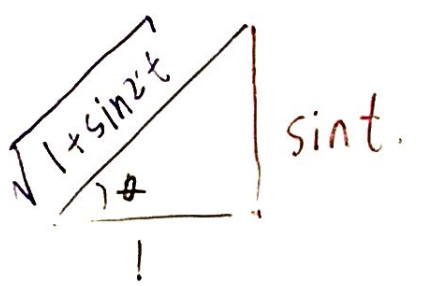
$$= \frac{-xe^x}{(x+1)} + e^x + C$$

Otras sugerencias: $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx \qquad u = x+1, \quad du = dx, \quad x = u-1.$

$\displaystyle\int \frac{(u-1)}{u^2}\,e^{u-1}\,dx$ NO AYUDA. "MUCHO"

2b) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt. = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec\theta} = \int_0^{\pi/4} \sec\theta\,d\theta.$

$\sin t = \tan\theta. \qquad \cos t\,dt = \sec^2\theta\,d\theta.$



$\sqrt{1+\sin^2 t} = \sec\theta.$

cambie los límites.

$\tan\theta = \sin \pi/2 = 1 \Rightarrow \theta_b = \pi/4.$

$\tan\theta = \sin 0 = 0 \Rightarrow \theta_a = 0$

$$\int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}$$

$\sec 0 = 1 \quad \tan 0 = 0$

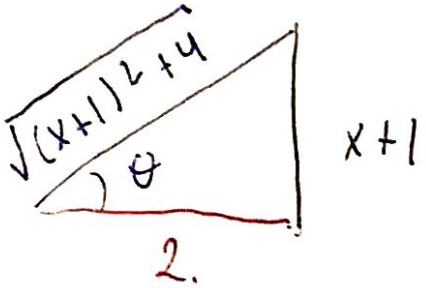
$\sec\frac{\pi}{4} = \sqrt{2} \quad \tan\frac{\pi}{4} = 1$

$$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \ln(1) = \ln(\sqrt{2} + 1).$$

Cont. 4A. $\displaystyle\int \frac{1}{(x^2+2x+5)^2}\,dx = \int \frac{1}{[(x+1)^2+4]^{4/2}}\,dx$

$x^2+2x+1 \quad +5-1 = (x+1)^2+4$



$\tan\theta = \dfrac{x+1}{2}$ $\qquad x+1 = 2\tan\theta.$

$dx = 2\sec^2\theta\, d\theta.$

$$\frac{((x+1)^2+4)^{1/2}}{2} = \sec\theta.$$

$$[(x+1)^2+4)]^2 = 16\sec^4\theta.$$

$$\int \frac{1}{[(x+1)^2+4]^2}\,dx = \int \frac{2\sec^2\theta\, d\theta}{16\sec^4\theta} = \frac{1}{8}\int \frac{1}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{8}\int \cos^2\theta\, d\theta. = \frac{1}{16}\int 1+\cos(2\theta)\, d\theta.$$

$$= \frac{1}{16}\left(\theta + \frac{1}{2}\underline{\sin(2\theta)}\right) + C.$$

$\underline{2\sin\theta\cos\theta}$

$$= \frac{1}{16}(\theta + \sin\theta\cos\theta) + C.$$

$\tan\theta = \dfrac{x+1}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{x+1}{2}\right).$ $\quad \sin\theta = \dfrac{x+1}{\sqrt{x^2+2x+5}}$

$$\cos\theta = \frac{2}{\sqrt{x^2+2x+5}}$$

$$\int \frac{1}{(x^2+2x+5)^2}\,dx = \frac{\tan^{-1}\left(\frac{x+1}{2}\right)}{16} + \frac{1}{16}\,\frac{(x+1)}{\sqrt{x^2+2x+5}}\,\frac{2}{\sqrt{x^2+2x+5}} + C.$$

$$\frac{1}{16}\tan^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{8}\,\frac{x+1}{x^2+2x+5} + C.$$

Inp $\int (x-1)\sin \pi x \, dx$ $\int e^{-\theta}\cos 2\theta \, d\theta.$

$u = x-1$ $dv = \sin \pi x$

$du = dx$ $v = \frac{-1}{\pi}\cos \pi x$

$$\int (x-1)\sin \pi x \, dx = \frac{-(x-1)}{\pi}\cos \pi x + \int \frac{1}{\pi}\cos \pi x \, dx$$

$$= \frac{(1-x)}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + C.$$

Ciclico

$$* \int e^{-\theta}\cos 2\theta \, d\theta. = -\frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta \, d\theta$$

$u = e^{-\theta}$ $dv = \cos 2\theta \, d\theta$

$du = -e^{-\theta} d\theta.$ $v = \frac{1}{2}\sin 2\theta.$

$$\int e^{-\theta}\sin 2\theta \, d\theta. = -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta \, d\theta. *$$

$u = e^{-\theta}$ $dv = \sin 2\theta \, d\theta.$

$du = -e^{-\theta} d\theta$ $v = -\frac{1}{2}\cos 2\theta$

$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta. - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}\int e^{-\theta}\cos 2\theta \, d\theta.$$

$$\frac{5}{4}\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta.$$

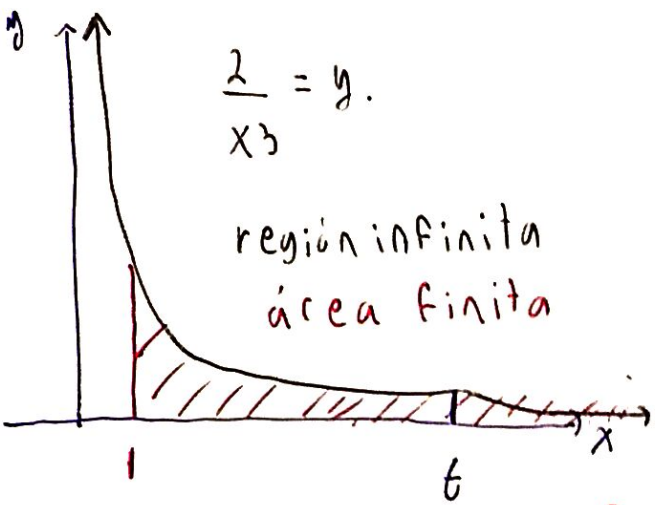
$$\int e^{-\theta}\cos 2\theta \, d\theta = -\frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C.$$

Capítulo 12

Fracciones Parciales Pág. 66 (Posteriormente).

7.8 Integrales Impropias.

Considere la región bajo la curva $y = \frac{2}{x^3}$ encima del eje-x y a la derecha de la recta $x = 1$.



$\frac{2}{x^3} = y.$

región infinita

área finita

$$A = \int_1^t 2x^{-3}\,dx$$

$$A = \frac{2}{-2}\,x^{-2}\Big]_1^t$$

$$A = -1 \cdot t^{-2} + \underbrace{1 \cdot 1^{-2}}_{1} = 1 - \frac{1}{t^2}.$$

$$\lim_{t\to\infty} A = \lim_{t\to\infty} 1 - \cancel{\frac{1}{t^2}}^{\,0} = 1$$

$$\boxed{\int_1^\infty \frac{2}{x^3}\,dx = 1}$$

Límites Básicos

a. $\lim_{x\to\infty} \frac{1}{x^r} = 0$ $\quad \frac{1}{\infty}.$ $\qquad \lim_{x\to\infty} x^r = +\infty.$

r positivo

b. $\lim_{x\to\infty} e^x = \infty$ $\quad e^\infty$ $\qquad \lim_{x\to-\infty} e^x = 0$ $\quad \frac{1}{e^\infty} \to 0.$

c. $\lim_{x\to 0^+} \ln x = -\infty$ $\qquad \lim_{x\to\infty} \ln x = +\infty.$

$$\log_{10} \underbrace{10^{-48}}_{0^+} = -48 \qquad \log_{10} 10^{-10{,}000} = -10{,}000$$

Integrales Impropias:

Tipo 1: Intervalos infinitos $\pm\infty$.

Tipo 2: Funciones discontinuas. (AVs en $x = \pm a$).

Integrales Impropias Tipo 1: (Pág. 74)

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

Integre 1ro
Evalúe límite.

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^a f(x)\,dx.$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

CONVERGENTE: (Se acerca a un número) el límite existe.

DIVERGENTE: (La integral a $\pm\infty$) el límite no existe.

Ejercicio 1: Evalúe. (P. 74) $\sqrt{\infty} \Rightarrow \infty$

a. $$\int_1^{\infty} x^{-1/2}\,dx = 2x^{1/2}\Big]_1^{\infty} = \lim_{x\to\infty} 2\sqrt{x} - 2 = \infty.$$

no existe.

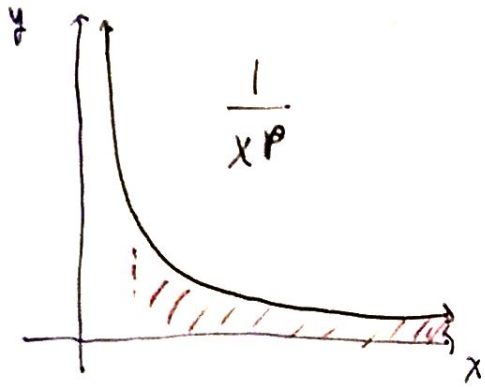
$p = \frac{1}{2}$ DIVERGENTE.

b. $$\int_1^{\infty} \frac{1}{x}\,dx = \ln x\Big]_1^{\infty} = \lim_{x\to\infty} \ln x - 0 = \infty.$$

no existe.

$p = 1$

DIVERGENTE



$\int_1^\infty \frac{1}{x^p}\,dx$ no necesariamente existe.

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} p \leq 1 & \text{DIVERGE.} \\ p > 1 & \text{converge.} \end{cases}$$

$\therefore p = 0.99$ $\quad \int_1^\infty x^{-0.99}\,dx = \left.\frac{x^{0.01}}{0.01}\right]_1^\infty = \lim_{x\to\infty} x^{0.01} - \frac{1}{0.01} = +\infty.$

DIVERGE.

$p = 1.001$ $\quad \int_1^\infty x^{-1.001}\,dx = \left.\frac{x^{-0.001}}{-0.001}\right]_1^\infty = \lim_{x\to\infty} \frac{1000}{x^{0.001}} + \frac{1}{0.001}$

$x^{-0.001}$

$\frac{1000}{\infty} \to 0$

$= \left.\frac{1000}{x^{0.001}}\right]_\infty^1 = 1000 - \lim_{x\to\infty} \frac{1000}{x^{0.001}}$ 0

$= 1000$ converge.

Ejercicio 3: Evalúe.

a. $\int_{-\infty}^0 e^{-x^2} \underbrace{x\,dx}_{du/-2} = \int_{-\infty}^0 e^u \frac{du}{-2} = -\frac{1}{2} e^u \Big]_{-\infty}^0 = -\frac{1}{2} e^0 + \cancel{\frac{1}{2} e^{-\infty}}$ 0

$u = -x^2$ $\qquad u(0) = -0^2$

$du = -2x\,dx$ $\qquad u(-\infty) = -(-\infty)^2 = -\infty$

$= -\frac{1}{2} + 0$

$= -\frac{1}{2}$ converge.

Notación Abreviada: $\quad e^{-\infty} = \lim_{x\to-\infty} e^x = 0$

$f(\infty) = \lim_{x\to\infty} f(x)$

b. $\frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1}(x)\Big]_{-\infty}^{\infty} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}.$

$= \frac{1}{2}\tan^{-1}(\infty) - \frac{1}{2}\tan^{-1}(-\infty)$ CONVERGE

$\tan x$ ID: $(-\frac{\pi}{2}, \frac{\pi}{2})$

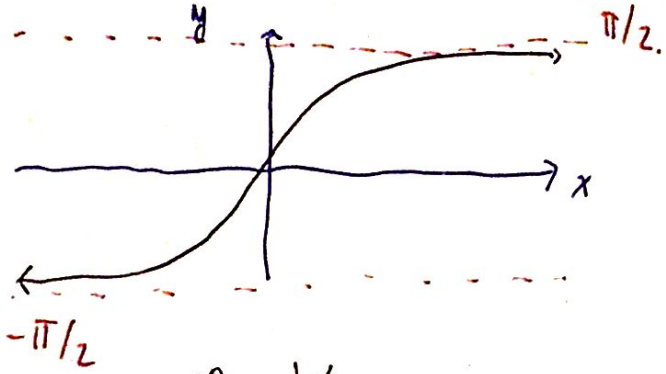
R: $(-\infty, \infty)$

A.V $x = -\pi/2, +\frac{\pi}{2}$

$\tan^{-1} x$ ID: $(-\infty, \infty)$

R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

A.H. $y = \pm \pi/2.$



$\tan^{-1}(\infty) = \pi/2$

$\tan^{-1}(-\infty) = -\pi/2.$

David Corzo.

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \underline{\underline{\pi}}. \approx \int_{-1000}^{1000} \frac{dx}{1+x^2} \approx \sum_{i=1}^{n} \frac{1}{1+x_i^2}\Delta x$$

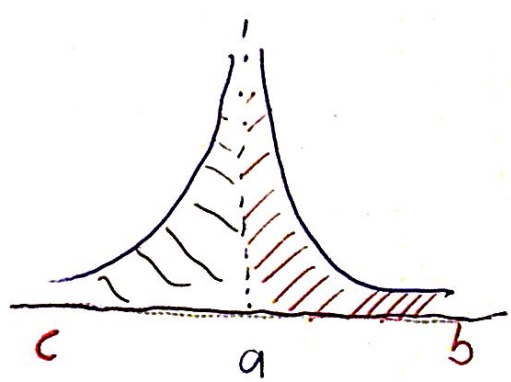
PYTHON.

Integrales Impropias Tipo 2.

Hay una asíntota vertical en $x = a$.

$$\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$$

$$\int_c^a f(x)dx = \lim_{t\to a^-}\int_c^t f(x)dx$$



AV $x=a$ $\int_c^b f(x)dx = \int_c^a f(x)dx + \int_a^b f(x)dx$

Ejercicio 4: Evalúe. Indique donde es discontinua.

a. $\displaystyle\int_{1}^{9} \frac{1}{\sqrt[3]{x-1}}\,dx = \int_{0}^{8} u^{-1/3}\,du = \frac{3}{2}u^{2/3}\Big]_{0^+}^{8} =$

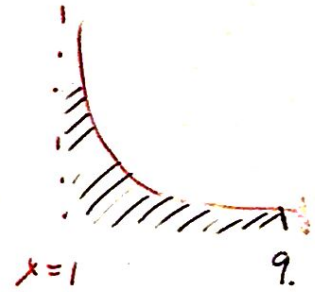
Discontinua: denominador igual a cero $1/0$.

en $x=1$ logaritmo de cero $\ln(0) \to -\infty$.

raíz cuadrada de un número negativo.

$u = x-1 \qquad u(9) = 8$

$du = dx \qquad u(1) = 0$



$$\frac{3}{2}u^{2/3}\Big]_{0^+}^{8} = \frac{3}{2}(8^2)^{1/3} - \frac{3}{2}\lim_{u\to 0^+} u^{2/3}.$$

$$\frac{3}{2}\sqrt[3]{64} - 0 = \frac{3}{2}\cdot 4 = 6 \qquad \text{CONVERGE}$$

b. $\displaystyle\int_{-2}^{3} \frac{3}{x^4}\,dx = \underbrace{\int_{-2}^{0} 3x^{-4}\,dx}_{(1)} + \underbrace{\int_{0}^{3} 3x^{-4}\,dx}_{(2)} = \infty$ diverge.

Discontinua en $x=0$

(2) $\displaystyle\int_{0}^{3} 3x^{-4}\,dx = -x^{-3}\Big]_{0^+}^{3} = -\frac{1}{3^3} + \overbrace{\lim_{x\to 0^+}\frac{1}{x^3}}^{+\infty} = +\infty.$

(1) $\displaystyle\int_{-2}^{0} 3x^{-4}\,dx = -x^{-3}\Big]_{-2}^{0^-} = \lim_{x\to 0^-} \underbrace{-\frac{1}{x^3}}_{+\infty} + \frac{1}{(-2)^3} = +\infty.$

c. $\int_0^1 \ln x \, dx$

$\int \ln x \, dx = x\ln x - \int dx = x\ln x - x + C.$

$u = \ln x \qquad dv = dx$

$du = \frac{dx}{x} \qquad v = x$

$0 \cdot \infty$

$0 \cdot \ln 0$

discontinua en $x=0$

$$\int_0^1 \ln x \, dx = x\ln x - x \Big]_{0^+}^{1} = 1\cdot\ln(1) - 1 - \lim_{x\to 0^+} x\ln x$$

A.V.

$$= -1 - \boxed{\left(\lim_{x\to 0^+} x\ln x\right)}$$

Regla de L'Hospital $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$

$0/0$ ó ∞/∞.

También aplica para $0\cdot\infty$, 1^∞, ∞^0, 0^0.

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1}} \overset{LH}{=} \lim_{x\to 0^+} \frac{1}{x(-x^{-2})}$$

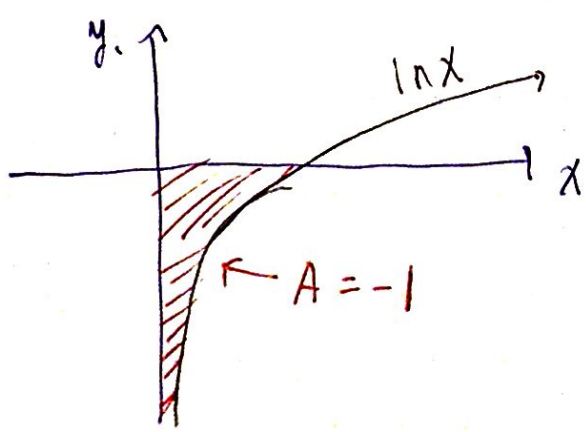
∞/∞

$$\lim_{x\to 0^+} \frac{1}{-x^{-1}} = \lim_{x\to 0^+} -x$$

$$= 0$$

$(\ln x)^{-1} \xrightarrow{\prime} -(\ln x)^{-2}\frac{1}{x}.$

$$\int_0^1 \ln x \, dx = -1 - \lim_{x\to 0} x\ln x = -1 + 0 \quad \text{CONVERGE.}$$



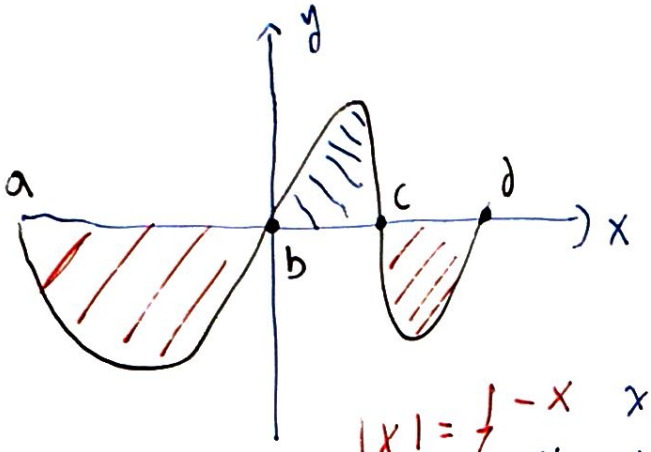
$$\frac{\ln x}{x} \to \frac{x^{-1}}{-x^{-2}} = -x$$

$$\lim_{x\to 0^+} -x = 0.$$

Capítulo 13

6.1 Áreas entre curvas (p 79)

Región entre la curva $y = f(x)$ y el eje x.



$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

Área de la región

$$A = \int_a^d |f(x)|\,dx$$

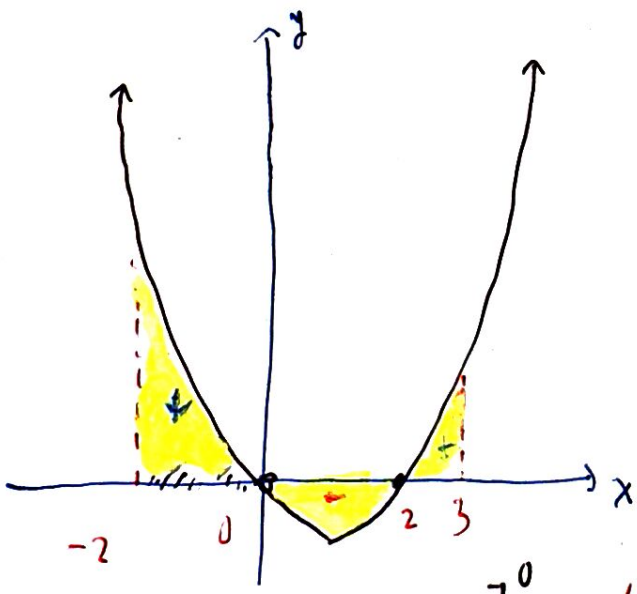
$$A = -\int_a^b f\,dx + \int_b^c f\,dx - \int_c^d f\,dx$$

Intersecciones y bosquejar la curva y la región,

Ejercicio 1: Bosqueje y encuentre el área de la región limitada por $y = 3x^2 - 6x$, $x = -2$, $x = 3$ & $y = 0$.

Interceptos-x

$$y = 3x^2 - 6x = 0$$
$$3x(x-2) = 0 \Rightarrow x = 0,\ x = 2.$$



Sume el área de 3 subregiones

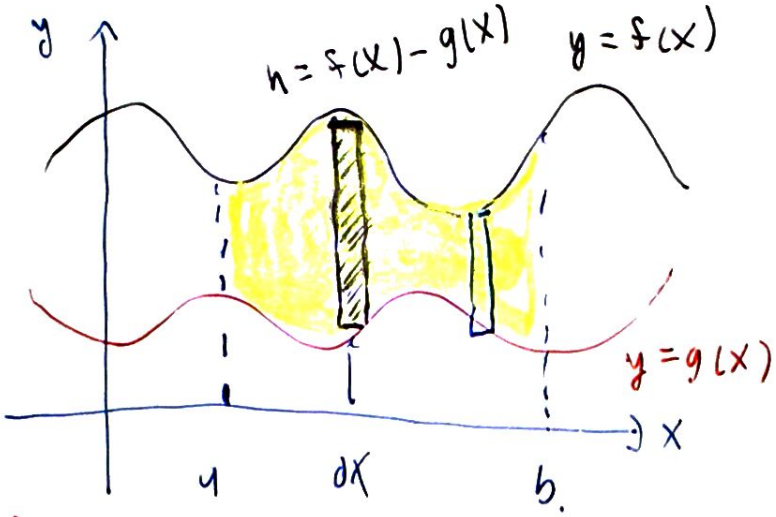
$$A = \int_{-2}^{0} 3x^2 - 6x\,dx + \int_0^2 -3x^2 + 6x\,dx + \int_2^3 3x^2 - 6x\,dx$$

$$A = x^3 - 3x^2\Big]_{-2}^{0} + \left(-x^3 + 3x^2\Big]_0^2\right) + x^3 - 3x^2\Big]_2^3$$

$$A = 0 - (-8 - 12) + (-8 + 12) + (\cancel{27 - 27}^{\,0} - 8 + 12) = 28$$

$$A = 20 + 4 + 4 = 28$$

¿Qué sucede cuando hay una curva inferior?



Región: $g(x) \leq y \leq f(x)$

$a \leq x \leq b.$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Diferencia de áreas.

$A = bh.$

$$\boxed{A = \int_a^b y_{arriba} - y_{abajo}\,dx}$$

Franja horizontal ó rectangulo infinitesimal. ancho dx

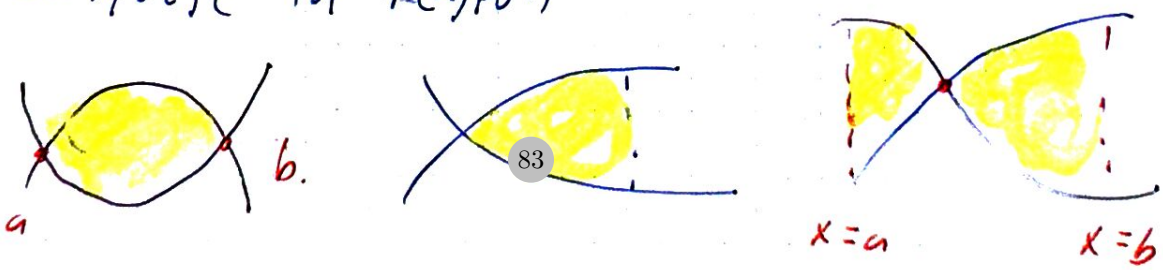
altura $f(x) - g(x)$

$$dA = (f(x) - g(x))\,dx$$

Integrando en $a \leq x \leq b$. $\int dA = A$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

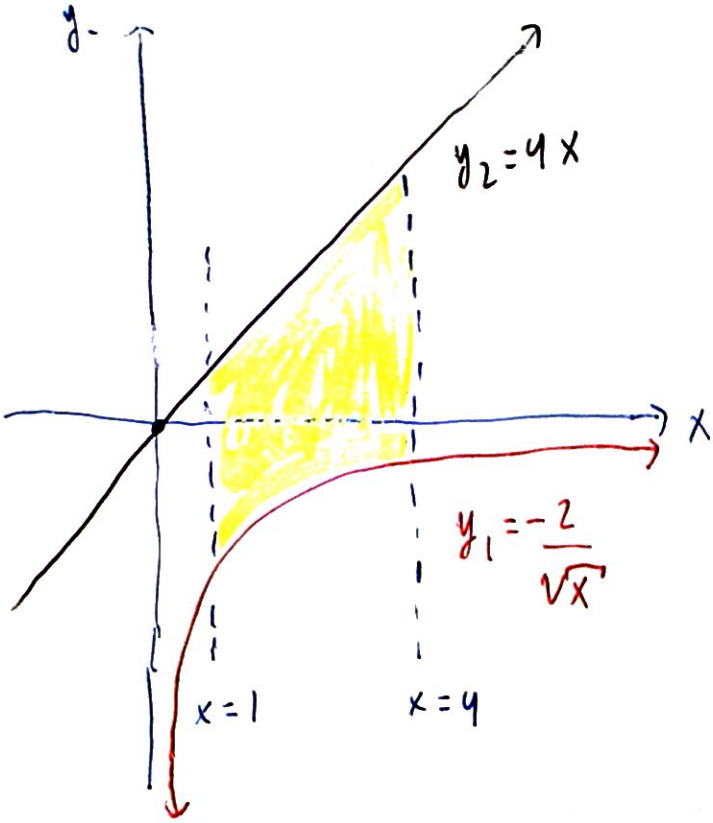
1. Bosquejar las curvas f & g.
2. Intersectos entre las curvas.
3. Bosqueje la región



Ejemplo: Bosqueje y encuentre el área entre

$y_1 = \frac{-2}{\sqrt{x}}$ & $y_2 = 4x$ en $1 \leq x \leq 4$.

¿ $y_2 > y_1$? ó ¿ $y_1 > y_2$? $\frac{1}{x^r}$



$$A = \int_1^4 y_2 - y_1 \, dx$$

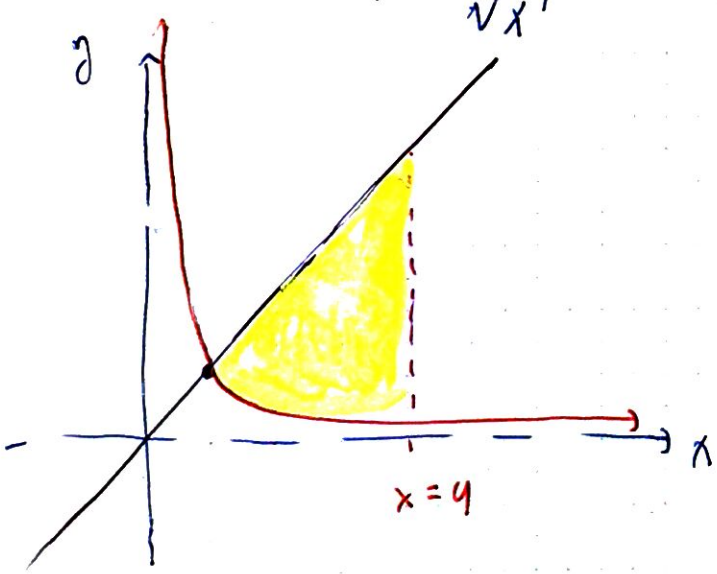
$$A = \int_1^4 4x + 2x^{-1/2} \, dx$$

$$A = 2x^2 + 4x^{1/2} \Big]_1^4$$

$$A = 2 \cdot 16 + 4 \cdot 2 - (2 + 4).$$

$$A = 32 + 8 - 6 = 34.$$

Variación B: $y_1 = \frac{+4}{\sqrt{x}}$ & $y_2 = 4x$ y la recta $x = 4$.



$$A = \int_1^4 y_2 - y_1 \, dx$$

intersección entre y_1 & y_2.

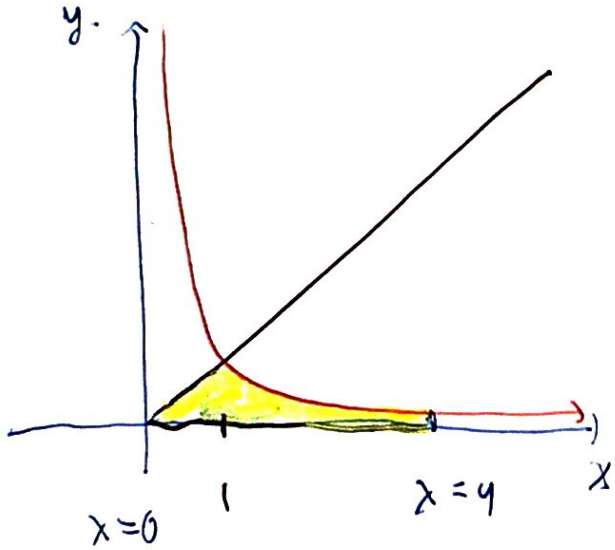
$$\frac{4}{x^{1/2}} = 4x$$

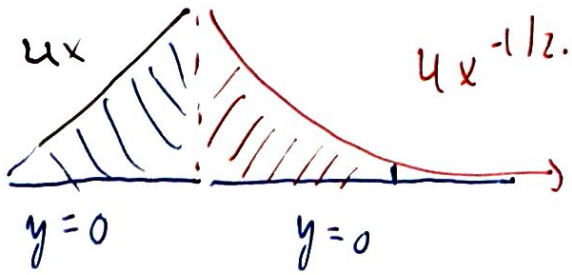
$$1 = x^{3/2} \Rightarrow x = 1$$

$$A = \int_1^4 4x - 4x^{-1/2}\,dx = 2x^2 - 8x^{1/2}\Big]_1^4$$

$$A = 32 - 16 - (2-8) = 16 + 6 = 22.$$

Variación C: Área de la región entre $y_1 = 4x^{-1/2}$, $y_2 = 4x$, $x = 4$ & $y = 0$.



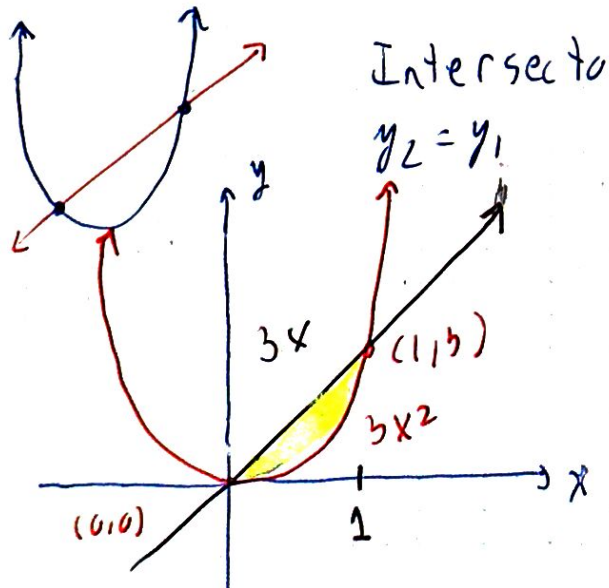


$$A = \int_0^1 4x\,dx + \int_1^4 4x^{-1/2}\,dx$$

$$A = 2x^2\Big]_0^1 + 8x^{1/2}\Big]_1^4$$

$$A = 2 + 16 - 8 = 10$$

Ejemplo: Encuentre el área de la región entre las curvas $y_1 = 3x$ & $y_2 = 3x^2$.



Intersectos $y_2 = y_1$

$3x^2 = 3x$ $\quad x=0$ & $x=1$

$3x^2 - 3x = 3x(x-1) = 0$

$$A = \int_0^1 (3x - 3x^2)\,dx$$

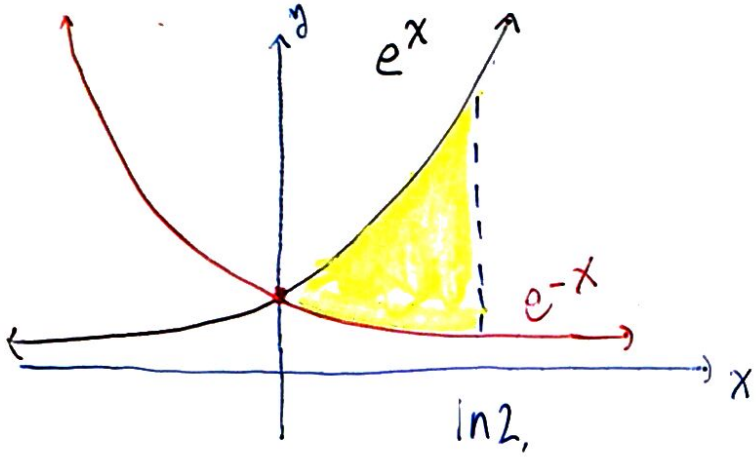
$$A = \frac{3}{2}x^2 - x^3\Big]_0^1 = \frac{3}{2} - 1 - 0$$

$$A = \frac{1}{2}.$$

Ejercicio 2: Bosqueje y encuentre el área de la región entre las curvas.

a) $y_1 = e^x$, $y_2 = e^{-x}$, $x = 0$, $x = \ln(2)$.

crece $\to +\infty$ decrece $\to 0$ $x = 0$ $y_1 = y_2 = 1$



$$A = \int_0^{\ln 2} (e^x - e^{-x})\,dx$$

$$A = e^x + e^{-x} \Big|_0^{\ln 2}$$

$$e^{\ln 2} = 2, \quad e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$$

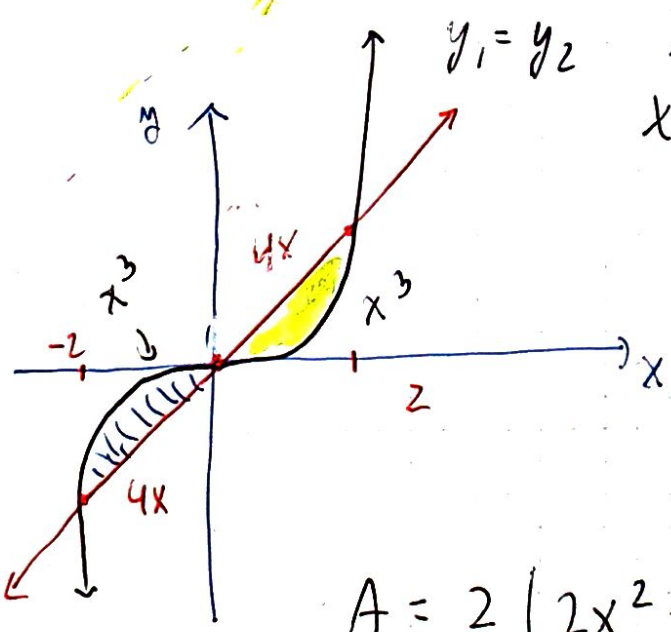
$$e^{\ln 2^{-1}}$$

$$A = e^{\ln 2} + e^{-\ln 2} - (e^0 + e^{-0})$$

$$A = 2 + \frac{1}{2} - 2 = \frac{1}{2}.$$

b) $y_1 = x^3$ & $y_2 = 4x$

2 regiones distintas y 3 intersectos.



$y_1 = y_2$ $\quad x^3 - 4x = 0$

$x(x^2 - 4) = 0 \Rightarrow x = 0, \pm 2.$

$y_1 = y_2(2) = 8.$

$$A = \int_{-2}^{0} (x^3 - 4x)\,dx + \int_0^2 (4x - x^3)\,dx$$

$$A = 2\int_0^2 (4x - x^3)\,dx$$

$$A = 2\left(2x^2 - \frac{x^4}{4}\Big]_0^2\right) = 2\left(8 - \frac{16}{4}\right) = 2 \cdot 4 = 8$$

c) $y_1 = x^2 - 4x + 4$, $y_2 = 10 - x^2$.

Dos parábolas.

$$A = \int_a^b y_2 - y_1 \, dx$$

Intersecciones $y_1 = y_2$. $x^2 - 4x + 4 = 10 - x^2$.

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 2(x-3)(x+1) =$$

intersectos en $x = -1$ y $x = 3$.

$$A = \int_{-1}^{3} 10 - x^2 - (x^2 - 4x + 4) \, dx$$

$$A = \int_{-1}^{3} (6 - 2x^2 + 4x) \, dx$$

$$A = 6x - \frac{2}{3}x^3 + 2x^2 \Big]_{-1}^{3} = 18 - 18 + 18 - \left(-6 + \frac{2}{3} + 2\right)$$

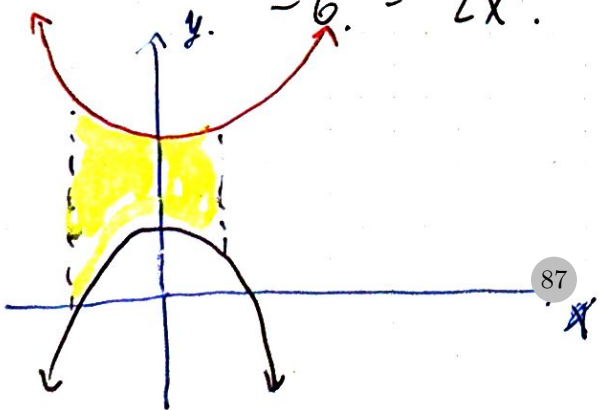
$$18 + 4 - 2/3 = 22 - \frac{2}{3}.$$

Alejandro Área entre $y_1 = 4 - x^2$ & $y_2 = 10 + x^2$, $x = a$, $x = b$

$y_1 = y_2$: $4 - x^2 = 10 + x^2$

$-6 = 2x^2$.

No hay intersectos.



$$A = \int_a^b y_2 - y_1 \, dx.$$

Capítulo 14

Avisos: Participación Actividad Mate Compu.
Hasta 1.5 netos y 0.5 neto mínimo.
Lunes y martes.

Zona WA (9 pts.). → 18-17 pts Parcial.

Parcial 10 octubre, 3 octubre. Simulacro 2.
Bota Parcial 1.

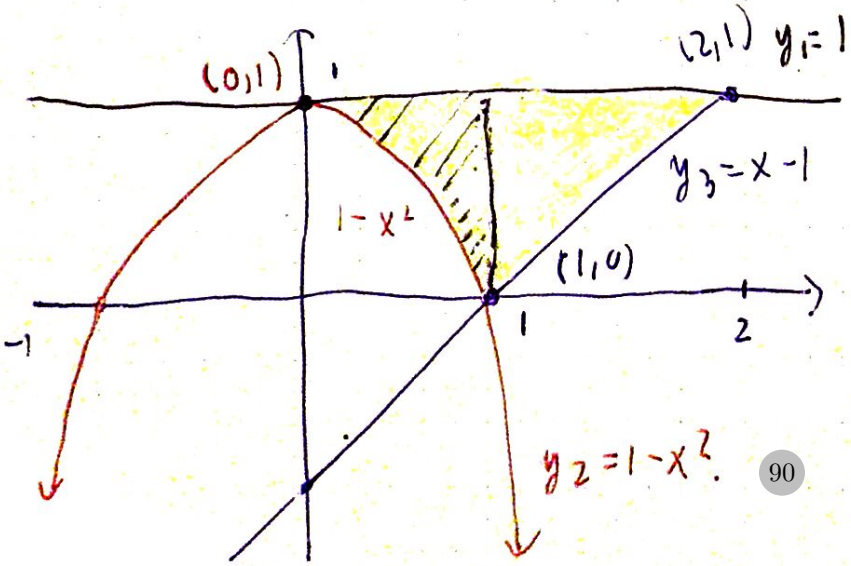
2:30 PM CES.

1-2:30 PM Almuerzo o Repaso con Samuel

Aplicaciones Integración:

- Planteamiento
- Gráfica de una Región. → área / volumen sólido en revolución

Ejercicio 3: Encuentre de la región entre $y_1 = 1$, $y_2 = 1 - x^2$ & $y_3 = x - 1$.



$y_1 = y_2$ D.I (0,1)

$y_1 = y_3$ $x - 1 = 1$
$x = 2$ (2,1)

$y_2 = y_3$ en (1,0)

$\int f - g \, dx$

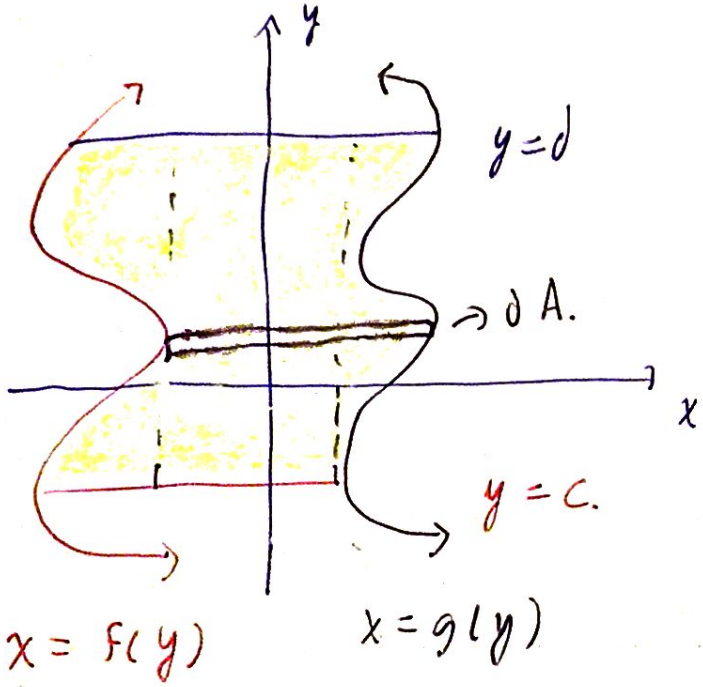
$$A = \int_0^1 (1-(1-x^2))\,dx + \int_1^2 \underbrace{1-(x-1)}_{1/2.}\,dx$$

$$A = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx$$

$$A = \frac{1}{3}x^3\Big]_0^1 + 2x - \frac{x^2}{2}\Big]_1^2$$

$$A = \frac{1}{3} + 4 - \frac{4}{2} - \left(2-\frac{1}{2}\right) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

Integración en el eje-y: Franjas Horizontales
Derecha - Izquierda.



Región S $\quad f(y) \le x \le g(y)$
$c \le y \le d.$

rectángulo infinitesimal.
altura dy.
ancho $g(y) - f(y)$
$dA = [g(y) - f(y)]\,dy.$

$$A = \int_c^d [g(y) - f(y)]\,dy.$$

$$A = \int_a^b y_{arriba} - y_{abajo}\,dx$$

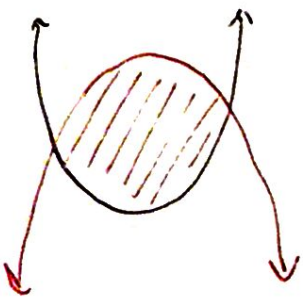
$y_{arriba} = f(x)$
$y_{abajo} = g(x)$

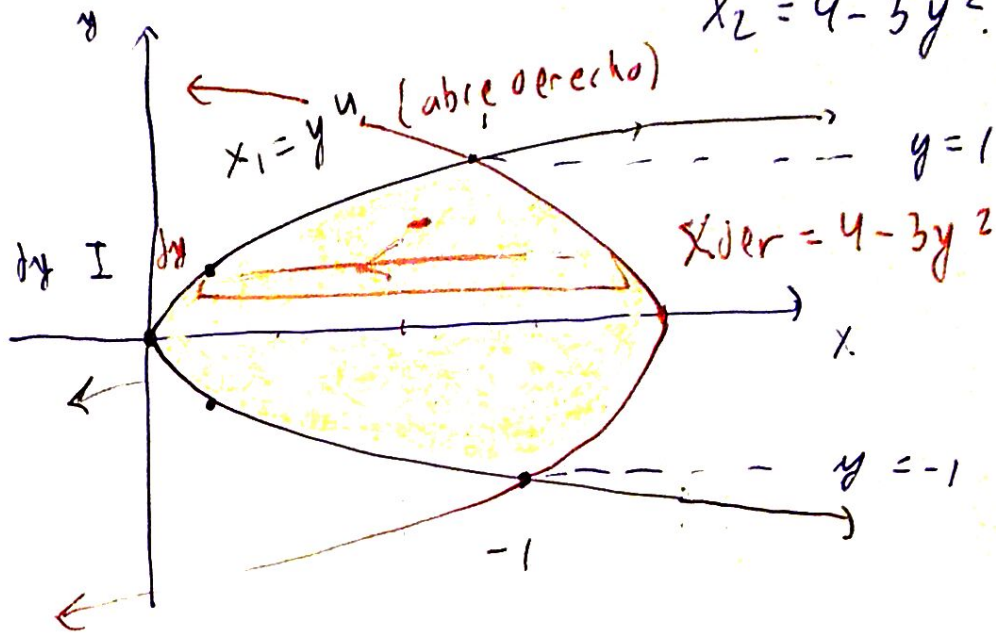
$$A = \int_c^d x_{der} - x_{izq}.\,dy.$$

$x_d = g(y) \quad x_{iz} = f(y).$

Ejercicio 4: Encuentre el área entre $x_1 = y^4$ & $x_2 = 4-3y^2$.

$x_2 = 4-3y^2$.





$x_{izq} = y^4$

(4 intersecc(s))

Pts. de intersección $x_1 = x_2$: $y^4 = 4-3y^2$.

$y^4 + 3y^2 - 4 = 0$

$y^2 \neq -4$. imaginarios.

$(y^2 \quad +4)(y^2-1) = 0$

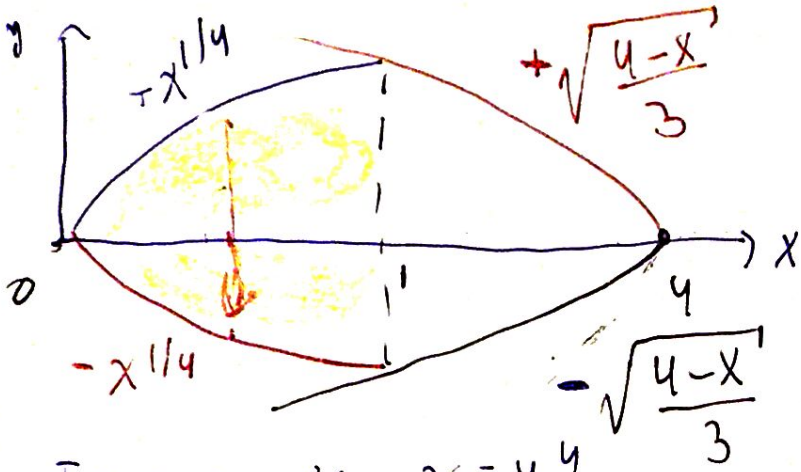
$y^2 = 1 \Rightarrow y = \pm 1$

$$A = \int_{-1}^{1} x_{der} - x_{izq}\, dy = \int_{-1}^{1} 4-3y^2 - y^4\, dy.$$ PAR

$$A = 2\int_0^1 4-3y^2-y^4\, dy = 2\left(4y - y^3 - \frac{y^5}{5}\Big]_0^1\right)$$

$$A = 2\left(3 - \frac{1}{5}\right) = 2\left(\frac{15}{5} - \frac{1}{5}\right) = \frac{28}{5}.$$

Integrando en el eje - x.



$\int f(x)-g(x)\,dx$

1. $x=y^4 \qquad x=4-3y^2$

Resuelva para x

$a^2=b \Rightarrow a=\pm\sqrt{b}$

2. $\int_0^1 f-g\,dx+\int_1^4 h-i\,dx$

Inversa de $x=y^4$

$y=\pm x^{1/4}$; $\pm\sqrt[4]{x}$

$x=4-3y^2 \Rightarrow x-4=-3y^2. \qquad y^2=\frac{4-x}{3}, \; y=\pm\sqrt{\frac{4-x}{3}}$

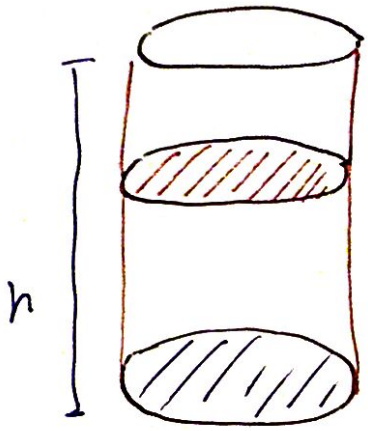
$$A=\int_0^1 x^{1/4}-(-x^{1/4})\,dx+\int_1^4\sqrt{\frac{4-x}{3}}-\left(-\sqrt{\frac{4-x}{3}}\right)dx$$

$$A=2\int_0^1 x^{1/4}dx+2\int_1^4\left(\frac{4-x}{3}\right)^{1/2}dx=\frac{28}{5}.$$

Menos 40 (2pts. P1) >50 (1 pt. P1.

6.2. Volúmenes (P 87)

Volumen de un cilindro circular



(rebanado)

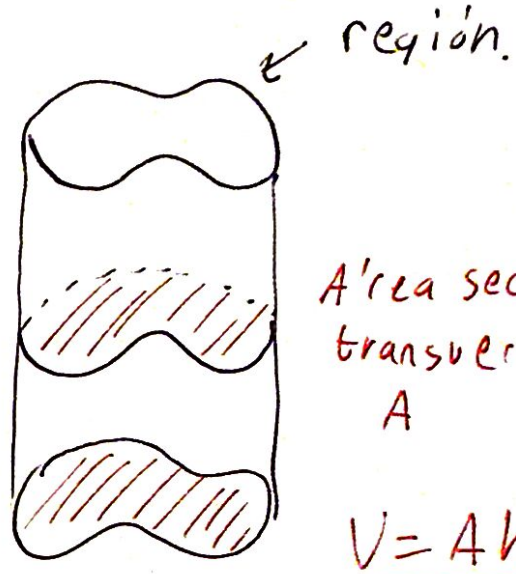
sección transversal

$A = \pi r^2$.

$V = Ah$.

$V = \pi r^2 h$.

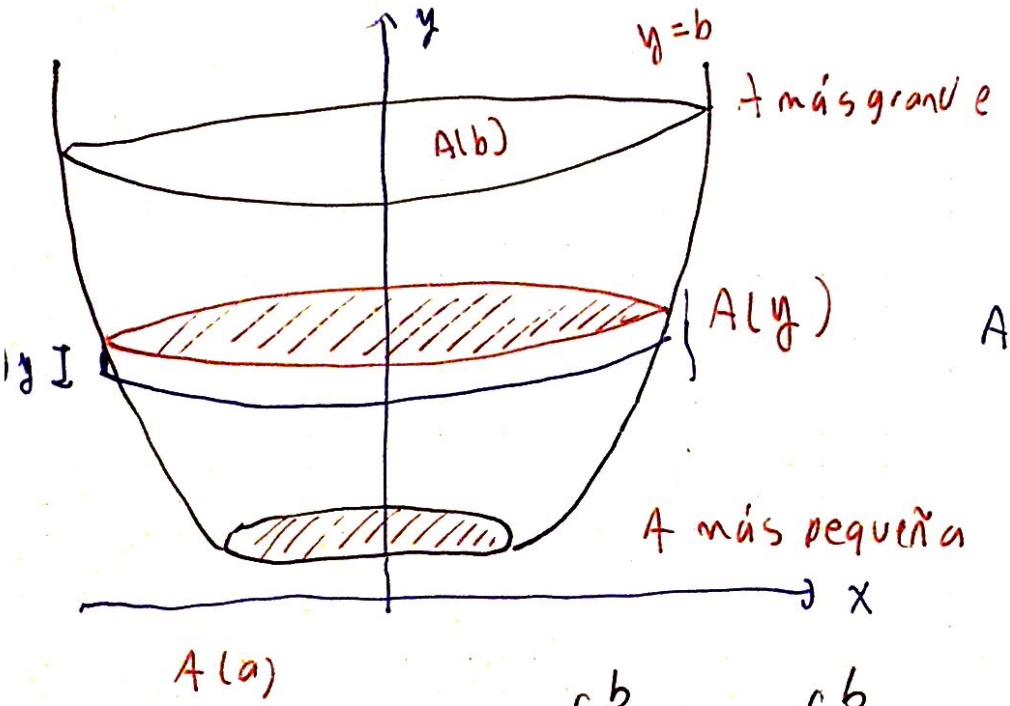
círculo de radio r



Área sección transversal A

$V = Ah$.

Volumen de un sólido S.



Parte infinitesimal de este sólido.

cilindro

Área transversal $A(y)$

Altura dy.

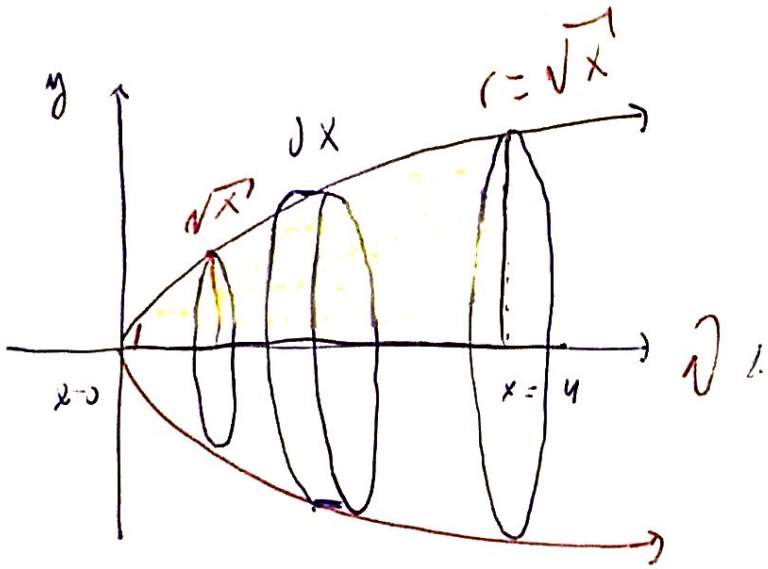
$dV = A(y)\,dy$.

Integrando $V = \int_a^b dV = \int_a^b A(y)\,dy$.

¿Cuál es el área transversal?

Ejemplo: Considere la región $0 \leq y \leq \sqrt{x}$

$0 \leq x \leq 4.$

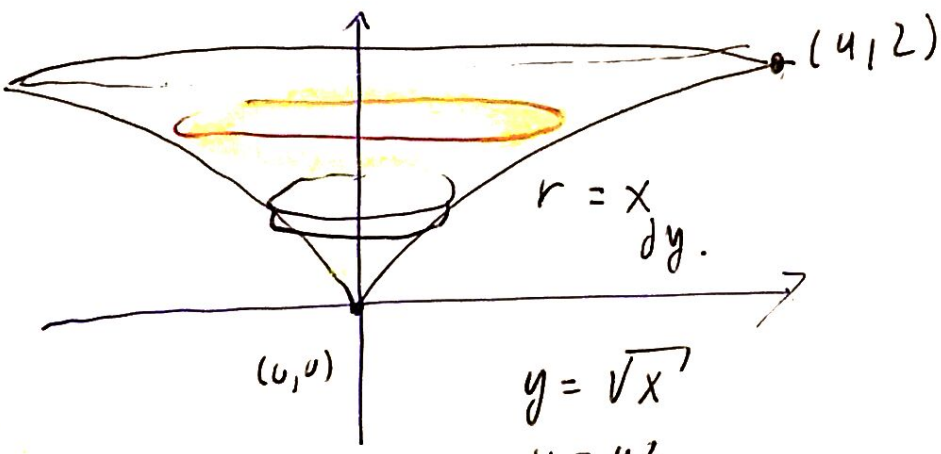


Rote alrededor del eje-x para obtener un sólido en revolución

Volumen del sólido es, sección transversal es un cilindro disco de radio $r = \sqrt{x}$

$$dV = \pi r^2 dx = \pi x dx.$$

$$V = \int_0^4 \pi r^2 dx = \pi \int_0^4 x dx = \frac{\pi x^2}{2}\Big]_0^4 = \frac{16\pi}{2} = 8\pi$$



$r = x$

$dy.$

$y = \sqrt{x}$

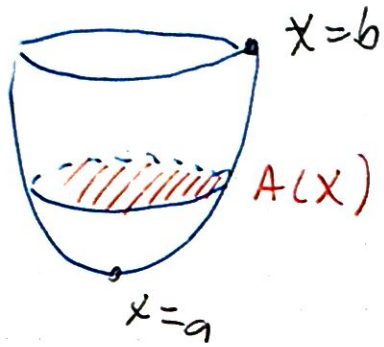
$x = y^2$

$$V = \pi \int \underline{\underline{x}}^2 dy.$$

$$V = \pi \int_0^2 y^4 dy.$$

Capítulo 15

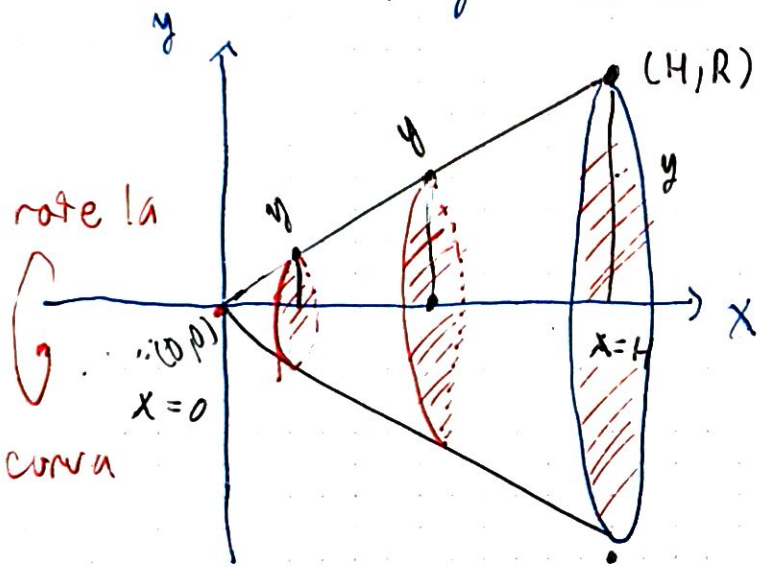
Volúmenes.



$$V = \int_a^b A(x)\,dx$$

área sección transversal.

Ejemplo: Encuentre el volumen de un cono de altura H y base circular de radio R.



las secciones transversales son círculos de radio $y(x)$.

$A = \pi y^2$

$$V = \int_0^H \pi y^2\,dx$$

$y(0) = 0 + b = 0$

$b = 0$

$y = mx + b.$ $\quad m = \frac{R-0}{H-0} = \frac{R}{H}$ $\quad y = \frac{R}{H}x + \cancel{b}.$

$(0,0)$ y (H,R)

$$V = \int_0^H \pi \frac{R^2 x^2}{H^2}\,dx = \frac{\pi R^2}{H^2}\int_0^H x^2\,dx$$

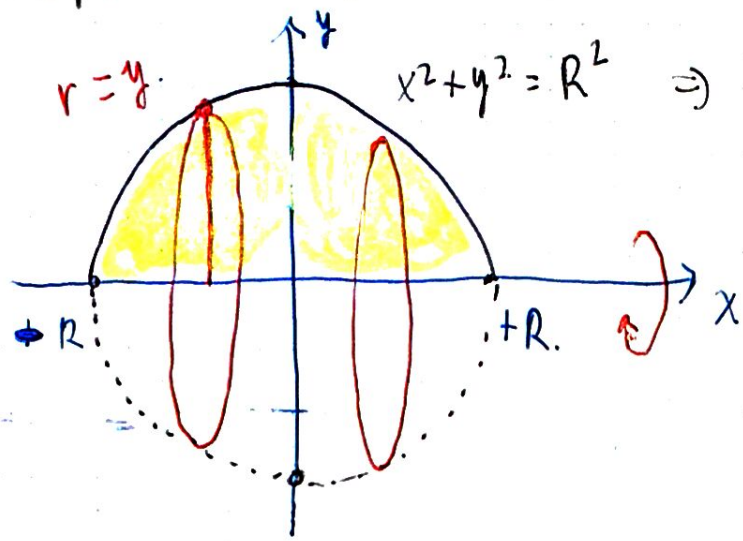
$$= \frac{\pi R^2}{H^2}\left.\frac{x^3}{3}\right]_0^H = \frac{\pi R^2}{H^2}\,\frac{H^3}{3}$$

P. 88.

$$V = \frac{\pi R^2 H}{3}$$

Ejercicio 1: p. 89 Volumen de una Esfera de radio R.

La esfera se obtiene al girar el círculo $x^2+y^2 \leq R^2$ respecto al eje-x.



$\Rightarrow$ $y^2 = R^2 - x^2$ ID $-R \leq x \leq R$

$y = \sqrt{R^2 - x^2}$

$x^2 = R^2$ $x = \pm R$

Sección transversal círculo de radio $y = r$

$A(x) = \pi y^2$

$$V = \int_{-R}^{R} A(x)\,dx = \pi \int_{-R}^{R} (R^2 - x^2)\,dx = 2\pi \int_{0}^{R} (R^2 - x^2)\,dx$$

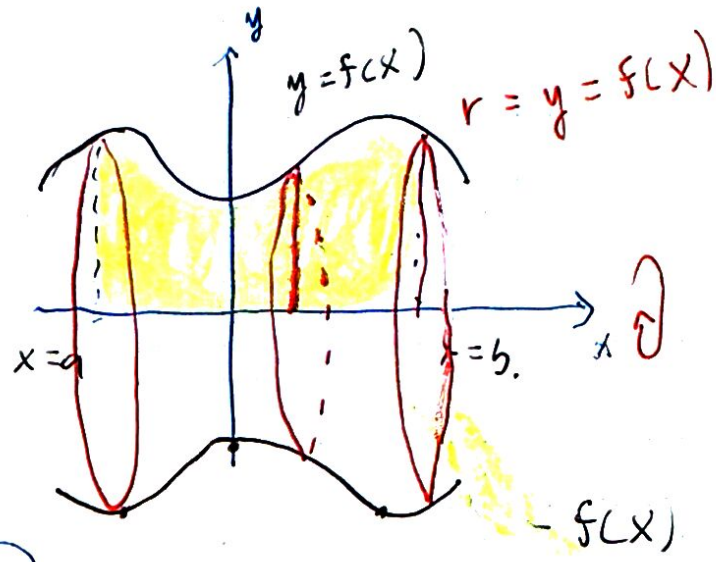
R es constante.

$$V = 2\pi \left(R^2 x - \frac{x^3}{3} \Big]_0^R \right) = 2\pi \left(R^3 - \frac{R^3}{3} \right) = 2\pi R^3 \left(1 - \frac{1}{3} \right)$$

$$\boxed{V = \frac{4\pi}{3} R^3}$$

Sólidos de Revolución.

$R: \; a \le x \le b.$
$\quad 0 \le y \le f(x)$



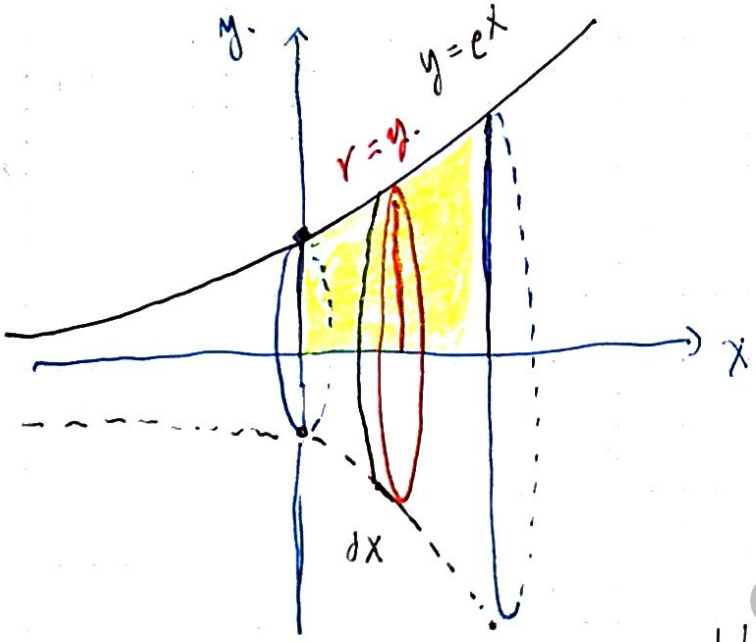
gire respecto al eje-x para obtener el sólido de revolución

Área transversal : $\boxed{A = \pi y^2} = \pi f^2(x)$

$$V = \int_a^b \pi y^2 \, dx = \int_a^b \pi f^2(x) \, dx$$

Ejercicio 4: Encuentre el Volumen del sólido que se obtiene al girar la región $R: 0 \le x \le \ln 3 \;\; 0 \le y \le e^x$ respecto al eje-x. (Pag. 91)

$y = e^x$ curva.



$e^{\ln[\,]} = [\,]$

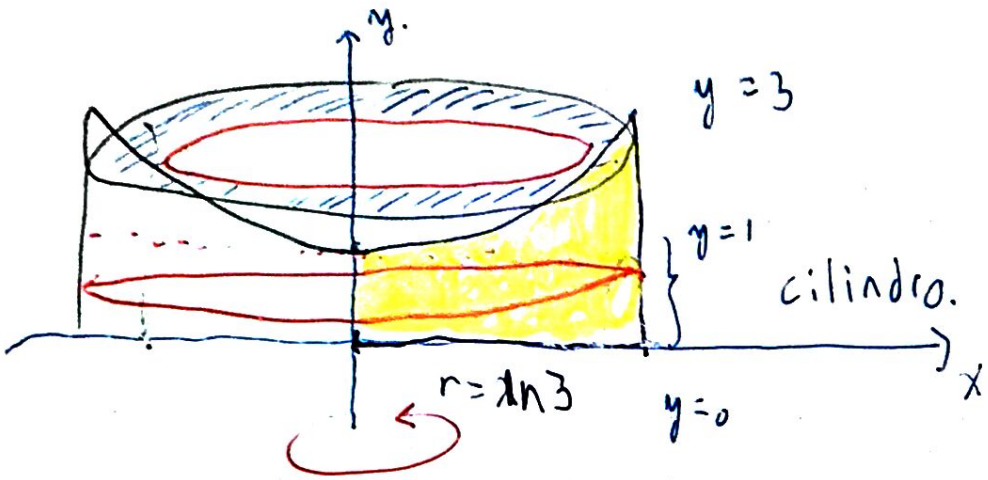
$A = \pi y^2 = \pi e^{2x}$

$$V = \int_0^{\ln 3} \pi e^{2x} \, dx$$

$$V = \frac{\pi}{2} e^{2x} \Big]_0^{\ln 3} = 4\pi.$$

$$V = \frac{\pi}{2}\left(e^{\ln 3^2} - e^0\right) = \frac{\pi}{2}(9-1)$$

Girando la misma región respecto al eje - y.



$f(0) = 1$

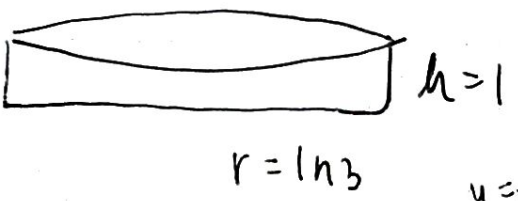
$f(\ln 3) = 3$

Difícil de visualizar.

Volumen = $V_1 + V_2$.

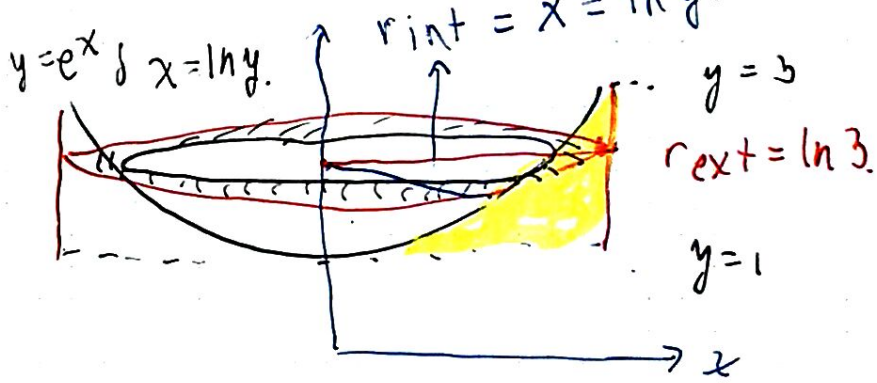
V_1 = cilindro

$V_1 = \pi r^2 h = \pi \ln^2 3.$

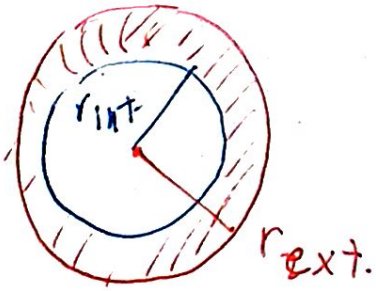


$y = e^x$

V_2 sólido hueco:



Área transversal



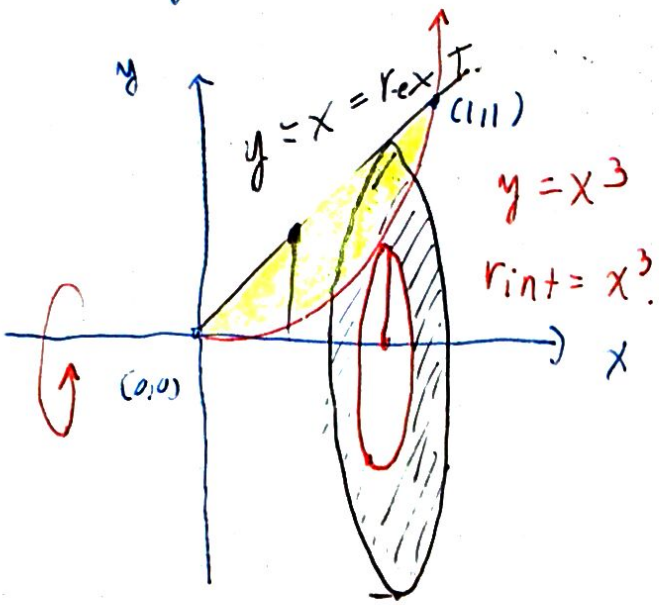
anillo

$$A = \pi r_{ext}^2 - \pi r_{int}^2$$

$$A = \pi (\ln 3)^2 - \pi (\ln y)^2$$

$$V_2 = \int_1^3 \pi (\ln 3)^2 - \pi (\ln y)^2 \, dy.$$

Ejercicio 5: Encuentre el volumen del sólido obtenido al girar la región entre las curvas $y = x$ & $y = x^3$ en el 1er cuadrante respecto al eje-x.



Área Anillo.

círculo grande − Cír pequeño.

$r_{ext} = x \qquad r_{int} = x^3$

$A = \pi r_{ext}^2 - \pi r_{int}^2$

$A = \pi x^2 - \pi x^6$

Volumen $V = \int_0^1 A\,dx = \int_0^1 (\pi x^2 - \pi x^6)\,dx$

$V = \pi\left(\frac{x^3}{3} - \frac{x^7}{7}\Big]_0^1\right) = \pi\left(\frac{1}{3} - \frac{1}{7}\right)$

Transformadas Laplace.

$$\int_0^\infty f(x)\, e^{-st}\, dt$$

$$f(t) = e^t \qquad = \int_0^\infty e^t e^{-st}\, dt. = \frac{-1}{1-s} = \frac{1}{s-1}$$

$$= \int_0^\infty e^{(1-s)t}\, dt.$$

$$\left.\frac{e^{(1-s)t}}{1-s}\right]_0^\infty = \cancel{\lim_{t\to\infty} \frac{e^{(1-s)t}}{1-s}}^{\,0} - \frac{e^0}{1-s}$$

$$\lim_{t\to\infty} e^{\overbrace{(1-s)}t} = 0$$ se necesita que $1-s<0.$

$e^{-\infty} \to 0.$

$$\begin{array}{r} 1<s \\ \hline s>1. \end{array}$$

$$\int_0^\infty t e^{-st} = -\frac{t}{s} e^{-st}\Big]_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\, dt. \qquad \text{IPP.}$$

$s>0.$

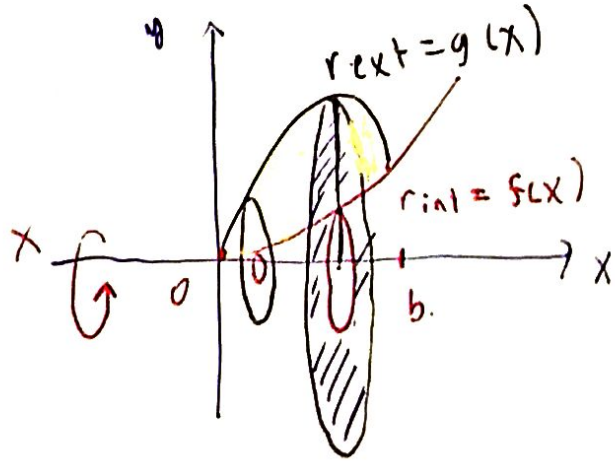
$$= \underbrace{\frac{-1}{s}\lim_{t\to\infty} t e^{-st}}_{\text{corto } s\ 0.\ 0} + \frac{0e^0}{s} + \left.\frac{e^{-st}}{s(-s)}\right]_0^\infty$$

$$= 0 - \frac{1}{s^2}\cancel{\lim_{t\to\infty} e^{-st}}^{\,0} - \frac{e^0}{-s^2} = \frac{-1}{-s^2} = \frac{1}{s^2}.$$

Capítulo 16

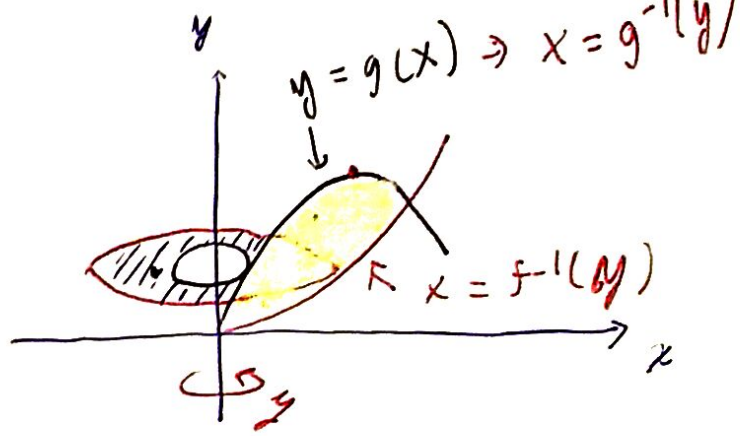
Volúmenes:

- Región.
- Identificar la recta de rotación.
- Identificar las funciones de radio para cada anillo
- Escoger la variable de integración.



$$A = \pi r_{ext}^2 - \pi r_{int}^2$$

$$V = \pi \int_0^b r_{ext}^2 - r_{int}^2 \, dx$$



Refleje respecto al eje-y.

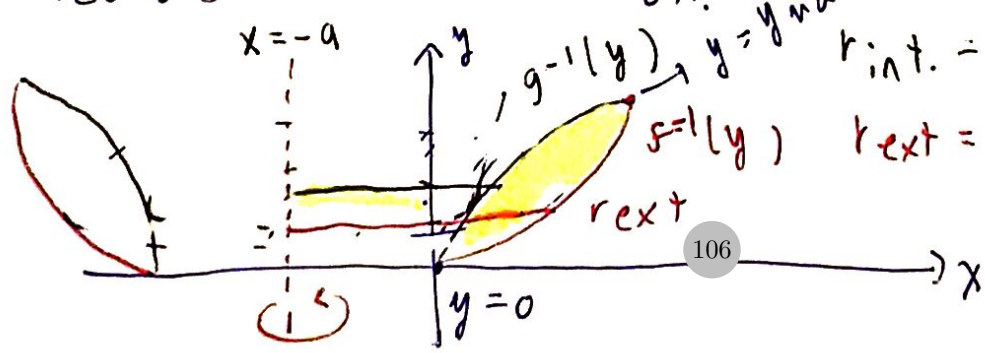
$0 \leq y \leq y_{max}$

$r_{int} = x = g^{-1}(y)$

$r_{ext} = x = f^{-1}(y)$

Volumen $V = \pi \int_0^{y_{max}} r_{ext}^2 - r_{int}^2 \, dy = \pi \int_0^{y_{max}} (f^{-1})^2 - (g^{-1})^2 \, dy.$

Rectas de Rotación.



$r_{int.} = a + g^{-1}(y)$

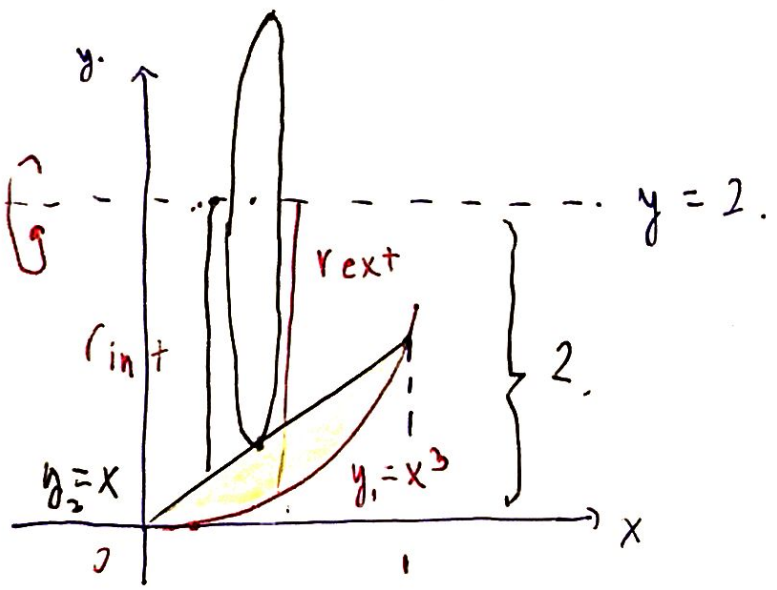
$r_{ext} = a + f^{-1}(y)$

$0 \leq y \leq y_{max}.$

$$V = \pi \int_0^{y_{max}} r_{ext}^2 - r_{int}^2 \, dy = \pi \int_0^{y_{max}} (a+f^{-1})^2 - (a+g^{-1})^2 \, dy.$$

Ejercicio 6: Considerar la región entre $f(x) = x$ & $g(x) = x^3$ en el 1er cuadrante. (p 96)

a. Plantee la integral del volumen del sólido al girar la región respecto a $y = 2$.



$r_{int} = 2 - y_2 = 2 - x.$

$r_{ext} = 2 - y_1 = 2 - x^3$

$A = \pi (r_{ext}^2 - r_{int}^2)$

Integre en x.

$x^3 - x = 0$

$x(x^2 - 1) = 0 \quad x = 0, 1$

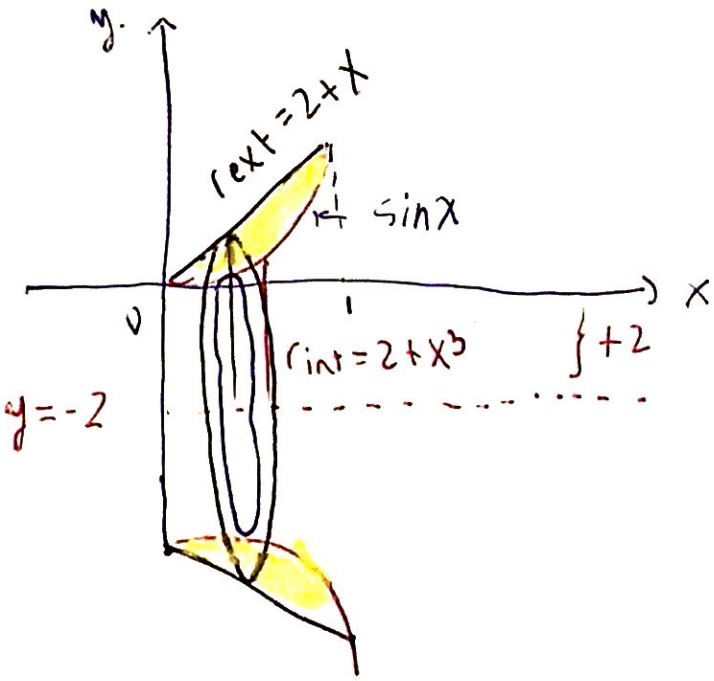
$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dx = \pi \int_0^1 (2-x^3)^2 - (2-x)^2 \, dx$$

$$\pi \int_0^1 x^6 - 4x^3 - x^2 + 4x \, dx$$

$$= 17\pi / 21.$$

b. Plantee la integral del volumen del sólido que se obtiene al girar la región respecto a $y=-2$.

dado.



$r_{int} = 2 + x^3$

$r_{ext} = 2 + x$

integre en $0 \leq x \leq 1$

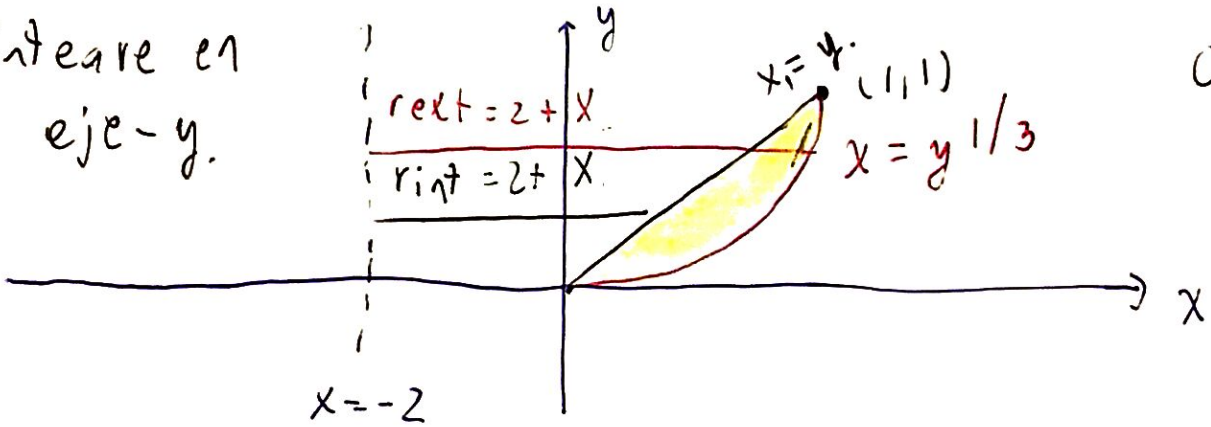
$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dx$$

$$V = \pi \int_0^1 (2+x)^2 - (2+x^3)^2 \, dx$$

$$V = 32\pi/21$$

c. Rote la región respecto a $x=-2$.

Integre en el eje-y.



$0 \leq y \leq 1$

$r_{ext} = 2 + y^{1/3}$

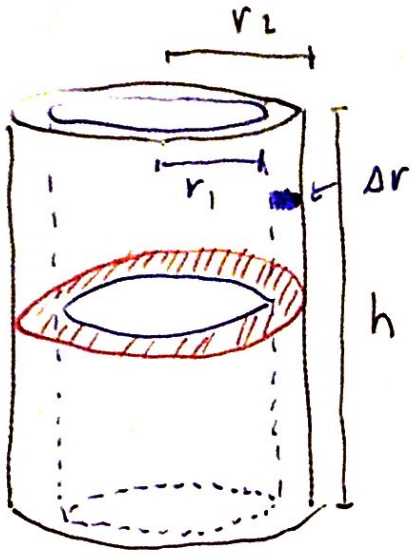
$r_{int} = 2 + y$

$0 \leq y \leq 1$

$$V = \pi \int_0^1 r_{ext}^2 - r_{int}^2 \, dy$$

$$V = \pi \int_0^1 (2+y^{1/3})^2 - (2+y)^2 \, dy.$$

6.3 Volumen con Cascarones Cilíndricos (Latas)



r_1 interno $\quad r_2$ externo. -te.

Área anillo $\pi r_2^2 - \pi r_1^2$

Volumen $V = \pi h (r_2^2 - r_1^2)$

grosor $\Delta r = r_2 - r_1 = dr$

radio promedio: $r = \frac{1}{2}(r_2 + r_1)$

$$V = \pi h (r_2 + r_1)(r_2 - r_1) = \pi h (2r)\, dr.$$

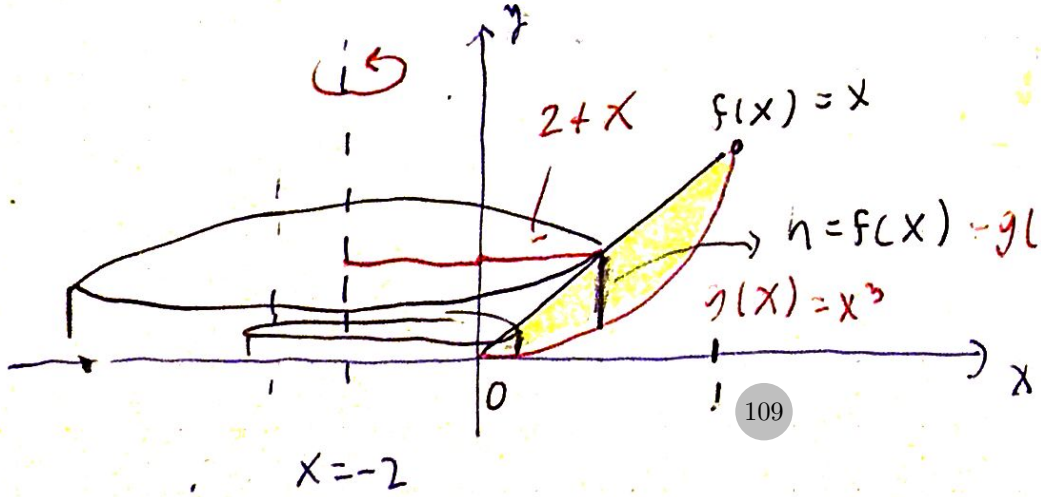
Volumen Cascarón $\quad V = \pi h r^2$

Derive respecto a r $\quad dV = 2\pi h r\, dr$

Integre en $a \le x \le b$ $\quad \boxed{V = 2\pi \int_a^b h r\, dr}$

Cascarones cilíndricos.

Otra vez inciso c) $y = x^3$ & $y = x$ gire a $x = -2$.



$0 \le x \le 1$

$h = x - x^3 \qquad r = 2 + x \qquad 0 \le x \le 1$

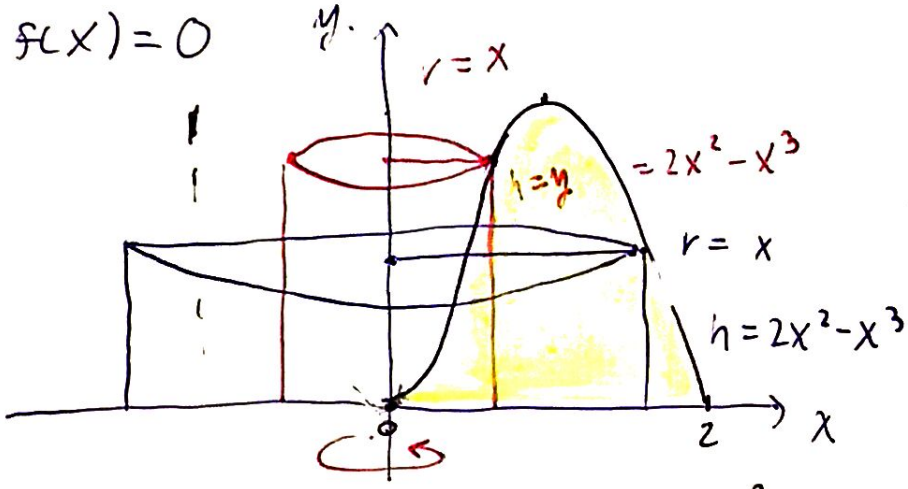
$$V = 2\pi \int_0^1 h\, r\, dx = 2\pi \int_0^1 (x - x^3)(2 + x)\, dx$$

ó $\qquad V = \pi \int_0^1 (2 + y^{1/3})^2 - (2 + y)^2 \; dy.$

Ejemplo: Encuentre el volumen del sólido que se obtiene al girar la región entre el eje-x y la curva $f(x) = 2x^2 - x^3$ en el 1er cuadrante respecto al eje-y.

Intersectos-x $\quad 2x^2 - x^3 = x^2(2 - x) = 0 \qquad x = 0, \; x = 2.$

$f(x) = 0$



radio $\quad r = x$

altura $\quad h = 2x^2 - x^3.$

limites $\quad 0 \le x \le 2.$

$$V = 2\pi \int_0^2 h\, r\, dx = 2\pi \int_0^2 (2x^2 - x^3)\, x\, dx$$

$$V = 2\pi \int_0^2 2x^3 - x^4\, dx = 2\pi \left(\frac{2x^4}{4} - \frac{x^5}{5} \Big]_0^2 \right)$$

$$V = 2\pi \left(8 - \frac{32}{5} \right)$$

Rotando con un eje horizontal

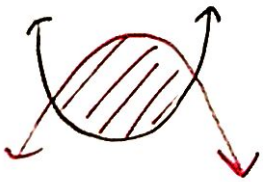
Anillos $\quad V = \int_a^b \pi (r_2^2 - r_1^2)\, dx$

eje-x
$y = 0$
$y = cte$

Rotando con un eje vertical $\quad x = 0$ ó $x = cte.$

Cilindros $\quad V = 2\pi \int_a^b h\, r\, dx$

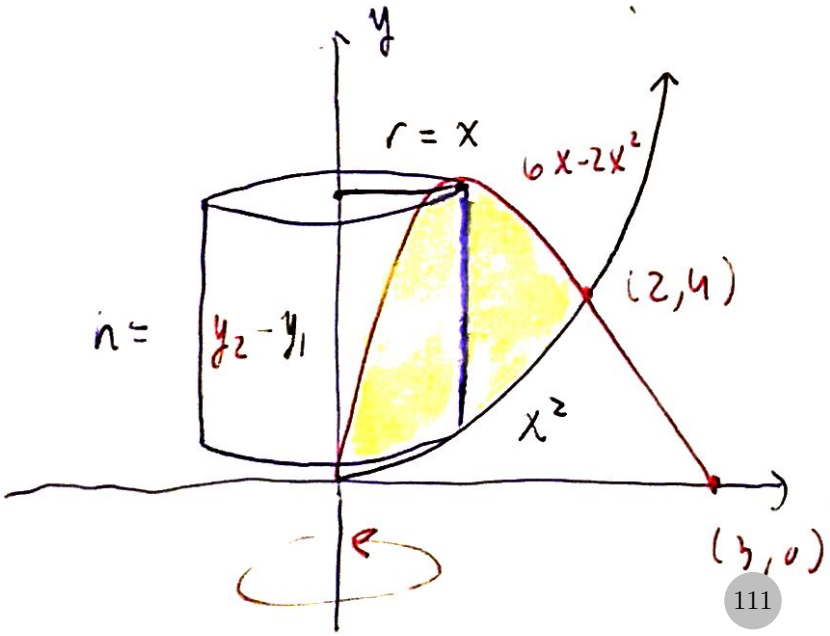
Ejercicio 2: Encuentre el volumen del sólido obtenido al girar la región entre $y_1 = x^2$ & $y_2 = 6x - 2x^2$ alrededor del eje-y.



$y_2 = 0 \quad 2x(3-x) = 0 \Rightarrow x = 0, 3.$

$y_1 = y_2 \quad x^2 = 6x - 2x^2.$

$3x^2 - 6x = 3x(x-2) = 0 \Rightarrow x = 0, 2.$



Altura. $h = y_2 - y_1$

$h = 6x - 3x^2.$

Radio $\quad r = x$

Límites. $0 \leq x \leq 2.$

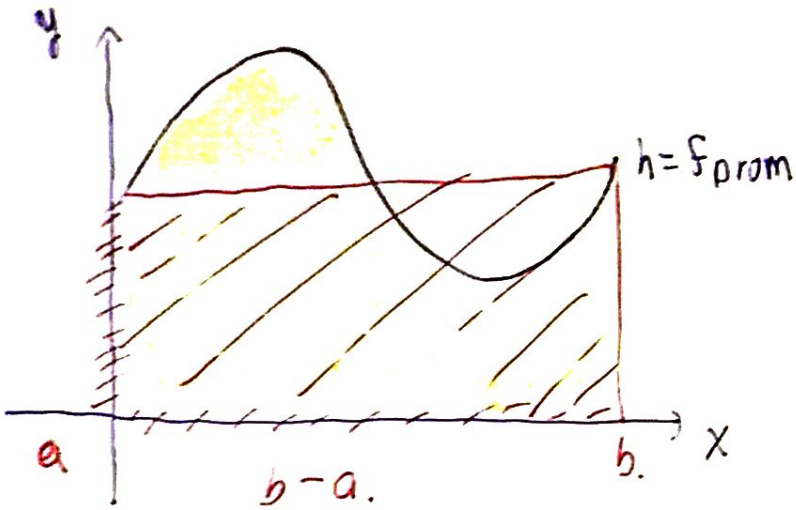
$V = 2\pi \int_0^2 h\, r\, dx.$

$$V = 2\pi \int_0^2 6x^2 - 3x^3 \, dx = 2\pi \left(2x^3 - \frac{3}{4} x^4 \Big]_0^2 \right)$$

$$V = 2\pi \left(16 - 3 \cdot 4 \right) = 2\pi (4) = 8\pi$$

Capítulo 17

6.5 Valor Promedio de una Función



¿Promedio altura y. ? $f_{prom.}$

Iguale el área del rectángulo con el área de la región amarilla

misma área

Ancho $b-a$

altura f_{prom}

Área $f_{prom}(b-a) = \int_a^b f(x)\,dx.$

Área Región Amarilla

$$\boxed{f_{prom} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$f(x)$ sea continua en $[a,b]$.

Ejemplo: Encuentre el valor promedio de $f(x) = \csc^2 x$ en $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

$b-a = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$ $\quad \frac{1}{b-a} = \frac{4}{\pi}$

$$f_{prom} = \frac{4}{\pi}\int_{\pi/4}^{\pi/2} \csc^2 x\,dx = -\frac{4}{\pi}\cot x\Big]_{\pi/4}^{\pi/2}$$

$$f_{prom} = \cancel{-\frac{4}{\pi}\cot \pi/2}^{\,0} + \frac{4}{\pi}\cot \pi/4. = \frac{4}{\pi}.$$

$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1}$ $\qquad \tan\frac{\pi}{4} = 1$

Ejercicio 1: Encuentre el valor promedio de $f(x)$.

a. $f(t) = \cos^4 t \sin t$ en $[0, \pi]$.

$$f_{prom} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{\pi - 0}\int_0^{\pi} \underbrace{\cos^4 t}_{u^4}\, \overbrace{\sin t\, dt}^{-du}.$$

$u = \cos t \qquad du = -\sin t\, dt. \qquad u(\pi) = \cos\pi = -1$

$u(0) = \cos 0 = 1$

$$f_{prom} = \frac{-1}{\pi}\int_1^{-1} u^4\, du = \frac{1}{\pi}\int_{-1}^{1} u^4\, du = \frac{2}{\pi}\int_0^1 u^4\, du.$$

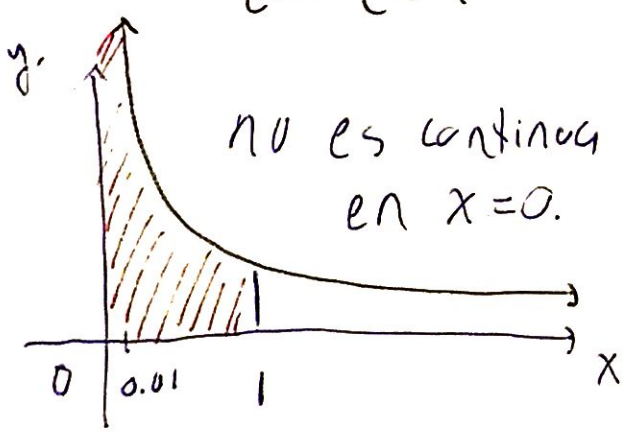
par.

$$f_{prom} = \frac{2}{\pi}\left.\frac{u^5}{5}\right]_0^1 = \frac{2}{5}\cdot\frac{1}{\pi}.$$

b. $g(x) = \dfrac{1}{x}$ en $[e^4, e^{10}]$.

$$y_{prom} = \frac{1}{e^{10}-e^4}\int_{e^4}^{e^{10}} \frac{1}{x}\, dx = \frac{1}{e^{10}-e^4}\left.\ln|x|\right]_{e^4}^{e^{10}}$$

$$y_{prom} = \frac{1}{e^{10}-e^4}\left(\ln e^{10} - \ln e^4\right) = \frac{6}{e^{10}-e^4}$$



no es continua en $x=0$.

Valor promedio de f en $0 \le x \le 1$

$$y_{prom} = \frac{1}{1}\int_0^1 \frac{1}{x}\, dx$$

$$= \left.\ln x\right]_0^1 = -\lim_{x\to 0^+} \ln x$$

$y_{prom} = -\lim_{x \to 0^+} \ln x = +\infty$ no existe.

La función no tiene valor promedio

$4^{-1/2}$
$2\,4^{1/2}$

c. $h(x) = \dfrac{3}{(4+x)^{1/2}}$ en $[-4, 5]$.

no es continua en $x = -4$.

$$h_{prom} = \frac{1}{5-(-4)} \int_{-4}^{5} 3(4+x)^{-1/2}\, dx$$

$(x)' = 1.$

$$h_{prom} = \frac{3}{9}\, 2(4+x)^{1/2} \Big]_{-4}^{5}$$

$(4-4)^{1/2} = 0^{1/2}$

$$h_{prom} = \frac{2}{3}\left(9^{1/2} - \lim_{x \to -4^+} \cancel{(4+x)^{1/2}}^{\,0} \right) = \frac{2}{3} \cdot 3 = 2.$$

Ejercicio 2: Densidad Lineal $\rho = 12(x+1)^{-1/2}$.

la varilla tiene 8 m. de longitud.

a. Encuentre la densidad promedio de la varilla.

$$\rho_{prom} = \frac{1}{8} \int_0^8 12(x+1)^{-1/2}\, dx$$

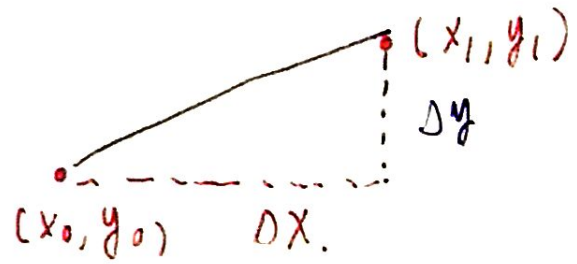
$$\rho_{prom} = \frac{24}{8} (x+1)^{1/2} \Big]_0^8 = 3\left(9^{1/2} - 1^{1/2} \right)$$

$3(3-1) = 6$ Kg/m.

Capítulo 18

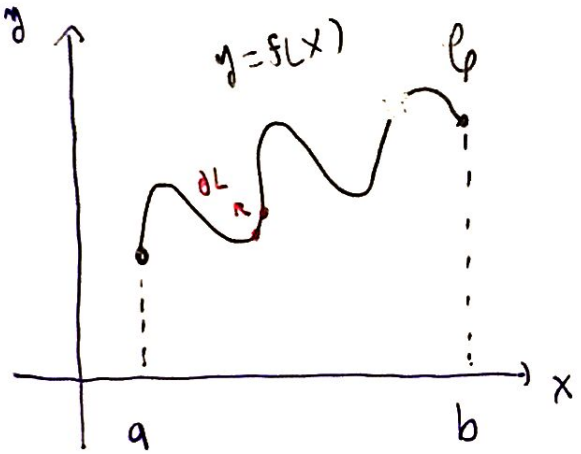
8.1 Longitud de Arco.

Derivación Fórmula.



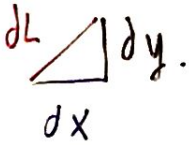
$$L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



largo de la curva $\mathcal{C}$.

$a \leq x \leq b, \quad y = f(x)$



Longitud infinitesimal del segmento.

$$dL = \sqrt{(dy)^2 + (dx)^2}$$

$$dL = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\; dx.$$

Integrando

Longitud de arco de $\mathcal{C}$: $\boxed{L = \int_a^b \sqrt{1 + [y']^2}\, dx}$

derivada.

Valor Promedio: $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx$

No es necesario graficar ninguna curva.

Ejemplo. Encuentre la longitud de la curva

$$y(x) = 1 + 2x^{3/2} \quad \text{en} \quad 0 \leq x \leq 8/9$$

Simplifique $1 + (y')^2$ antes de integrar.

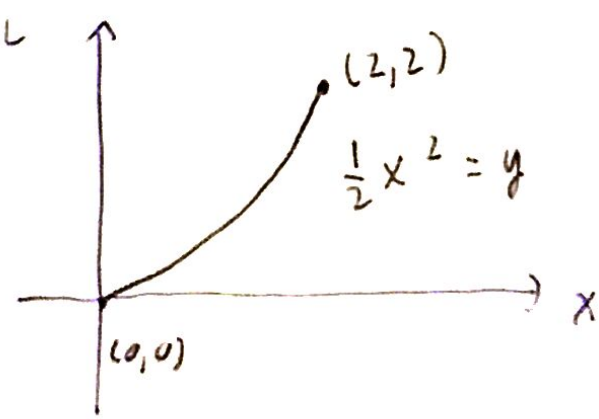
$$y' = 3x^{1/2} \qquad (y')^2 = 9x \qquad 1 + (y')^2 = 1 + 9x$$

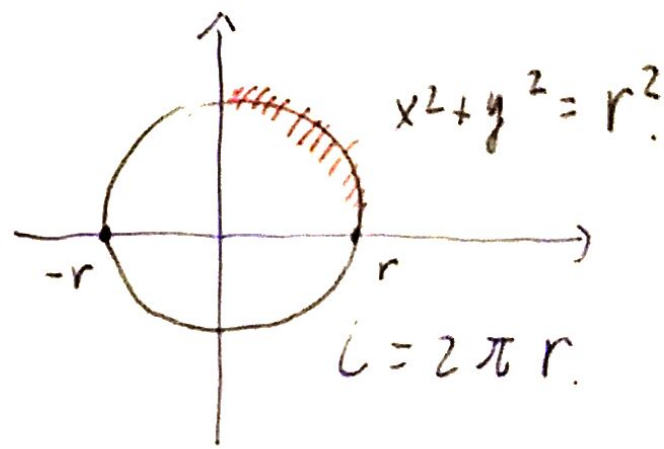
Longitud Arco: $L = \int_0^{8/9} (1+9x)^{1/2}\, dx$ $\qquad u = 1+9x$, $du = 9 \cdot dx$

$$L = (1+9x)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{9} \Big]_0^{8/9}$$

$$L = \frac{2}{27}\Big(\underbrace{9^{3/2}}_{(3^2)^{3/2}} - \underbrace{1^{3/2}}_{1} \Big) = \frac{2}{27}\left(3^3 - 1\right) = \frac{2}{27} \cdot 26.$$

Ejercicio (a): Parábola (Planteado)





$$y' = x$$

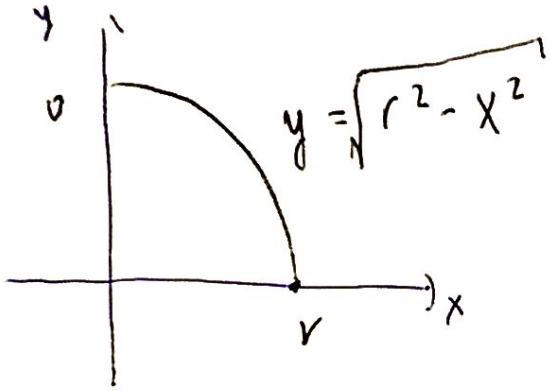
$$1 + (y')^2 = 1 + x^2$$

$$L = \int_0^2 \sqrt{1+x^2}\, dx \qquad x = \tan\theta, \quad dx = \sec^2\theta \cdot d\theta.$$

Ejercicio 1b: Longitud de una circunferencia de radio r. $r = cte.$

Ec: $x^2 + y^2 = r^2$

$$L = 4\int_0^r \sqrt{1+(y')^2}\, dx.$$



$$y' = \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x)$$

$$y' = \frac{-x}{\sqrt{r^2 - x^2}}$$

$$1 + (y')^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2 - x^2 + x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}.$$

$$L = 4\int_0^r \sqrt{\frac{r^2}{r^2 - x^2}}\, dx = 4r\int_0^r \frac{dx}{\sqrt{r^2 - x^2}}$$

$\sqrt{r^2} = r$

$$= 4r \sin^{-1}\left(\frac{x}{r}\right)\Big]_0^r$$

$$= 4r\left[\sin^{-1}\left(\frac{r}{r}\right) - \sin^{-1}(0)\right]$$

$\sin\left(\frac{\pi}{2}\right) = 1$

$$= 4r\left[\overset{\pi/2}{\cancel{\sin^{-1} 1}} - \overset{0}{\cancel{\sin^{-1}(0)}}\right]$$

$$= 4r\frac{\pi}{2} = 2\pi r.$$

Longitud de circunferencia $2\pi r. = L$

Área de un círculo $\pi r^2 = A$

Volumen $4\pi r^3/3 = V$

1.

Longitud: Un cable telefónico cuelga entre dos postes con posiciones horizontales en $x = \pm 25$. El cable tiene una curva con ec. $y = -5 + 25\cosh\left(\frac{x}{25}\right)$

Encuentre la longitud del cable.

$$L = \int_{-25}^{25} \sqrt{1+(y')^2}\, dx \qquad \frac{d}{dx}\cosh x = \sinh x$$

$$y' = 25\sinh\left(\frac{x}{25}\right)\cdot\frac{1}{25} = \sinh\left(\frac{x}{25}\right)$$

se cancelan los 25's.

$$1+(y')^2 = 1+\sinh^2\left(\frac{x}{25}\right) = \cosh^2\left(\frac{x}{25}\right)$$

identidad hiperbólica

$$L = \int_{-25}^{25} \sqrt{\cosh^2(x/25)}\, dx = \int_{-25}^{25} \cosh\left(\frac{x}{25}\right) dx$$

$$L = 2\int_{0}^{25} \cosh\left(\frac{x}{25}\right) dx = 2\cdot 25\sinh\left(\frac{x}{25}\right)\Big]_0^{25}$$

$$L = 50\left[\sinh(1) - \cancel{\sinh(0)}^{\,0}\right] \approx 58.7600$$

función tiene diferente variable independiente.

Ejercicio 3: Pág 112. Encuentre la longitud para las siguientes curvas.

a. $\mathcal{C}_1 : x = \frac{y^3}{6} + \frac{1}{2y} \qquad 1 \leq y \leq 2.$

Utilice el eje-y para integrar

$$L = \int_a^b \sqrt{1+(y')^2}\, dx \quad ; \quad \int_a^b \sqrt{1+(x')^2}\, dy.$$

Objetivo: SIMPLIFIQUE $1+(x')^2$.

$$x' = \frac{3y^2}{6} - \frac{1}{2}y^{-2} = \frac{1}{2}\left(y^2 - y^{-2}\right) \qquad y^2 y^{-2} = y^0$$

$$(x')^2 = \frac{1}{4}\left(y^2 - y^{-2}\right)^2 = \frac{1}{4}\left(y^4 - 2\cdot 1 + y^{-4}\right)$$

$$1+(x')^2 = 1 + \frac{1}{4}\left(y^4 - 2 + y^{-4}\right)$$ Simplifique y factorice.

$$= \frac{1}{4}\left(4 + y^4 - 2 + y^{-4}\right) \qquad a^2 + 2a + 1 \quad (a+1)^2$$

$$= \frac{1}{4}\left(y^4 + 2 + y^{-4}\right) \qquad 1 + \frac{1}{4}a = \frac{4+a}{4}$$

$y^2 \cdot y^{-2} = 1$

$$= \frac{1}{4}\left(y^2 + y^{-2}\right)^2 = 1 + (x')^2.$$

$$L = \int_1^2 \sqrt{1+(x')^2}\, dy = \int_1^2 \sqrt{\frac{1}{4}\left(y^2+y^{-2}\right)^2}\, dy.$$

$$= \frac{1}{2}\int_1^2 (y^2 + y^{-2})\,dy = \frac{1}{2}\left(\frac{y^3}{3} - \frac{1}{y}\Big]_1^2\right)$$

$$L = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right)$$

b. $\mathcal{C}_2$: $y = \ln(\sec \underline{\underline{\theta}})$ $\quad 0 \leq \underline{\underline{\theta}} \leq \pi/4.$

$$y'(\theta) = \frac{\sec\theta \tan\theta}{\sec\theta} = \tan\theta.$$

$$1 + (y')^2 = 1 + \tan^2\theta = \sec^2\theta.$$

$$L = \int_0^{\pi/4} \sqrt{1 + (y')^2}\,d\theta = \int_0^{\pi/4} \sqrt{\sec^2\theta}\,d\theta.$$

$$L = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big]_0^{\pi/4}.$$

$$L = \ln(\sec \pi/4 + \tan \pi/4) - \ln(\overset{1}{\cancel{\sec 0}} + \overset{0}{\cancel{\tan 0}})$$

$$= \ln(\sqrt{2} + 1) - \ln 1 = \ln(\sqrt{2} + 1)$$

Longitud de Arco: curva está en $a \leq t \leq x$.

límite superior indefinido.

$$S(x) = \int_a^x \sqrt{1 + (y')^2}\,dt \quad \Big\} \text{ Función de Longitud de Arco.}$$

Ejercicio 4: Encuentre la función de longitud de arco para la curva $y = \ln(\sin t)$ en $-\frac{\pi}{2} \leq t \leq \underline{\underline{x}}$.

$$y' = \frac{\cos t}{\sin t} = \cot(t) \qquad \int \csc^2 t \, dt = \cot x$$

$$1 + (y')^2 = 1 + \cot^2(t) = \csc^2(t).$$

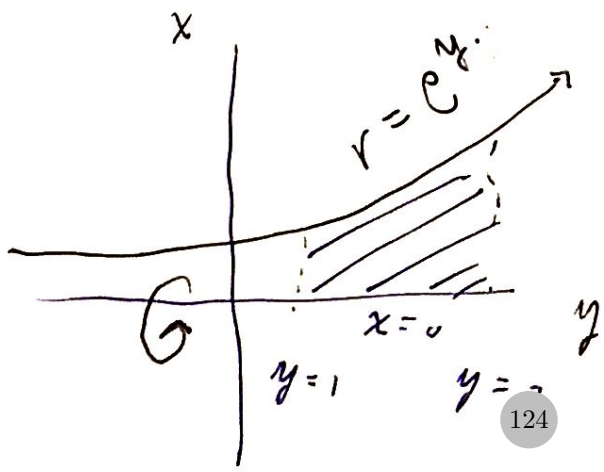
$$L(x) = \int_{-\pi/2}^{x} \sqrt{1 + (y')^2} \, dt = \int_{-\pi/2}^{x} \sqrt{\csc^2 t} \, dt.$$

$$L(x) = \int_{-\pi/2}^{x} \csc t \, dt. = -\ln|\csc t + \cot t| \Big]_{-\pi/2}^{x}$$

$$= -\ln|\csc x + \cot x| + \ln|\csc(-\pi/2) + \cot(-\pi/2)|$$

$$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = 0 \qquad \csc(-\pi/2) = \frac{1}{\sin(-\pi/2)} = -1$$

$$= -\ln|\csc x + \cot x| + \cancelto{0}{\ln|-1|}$$

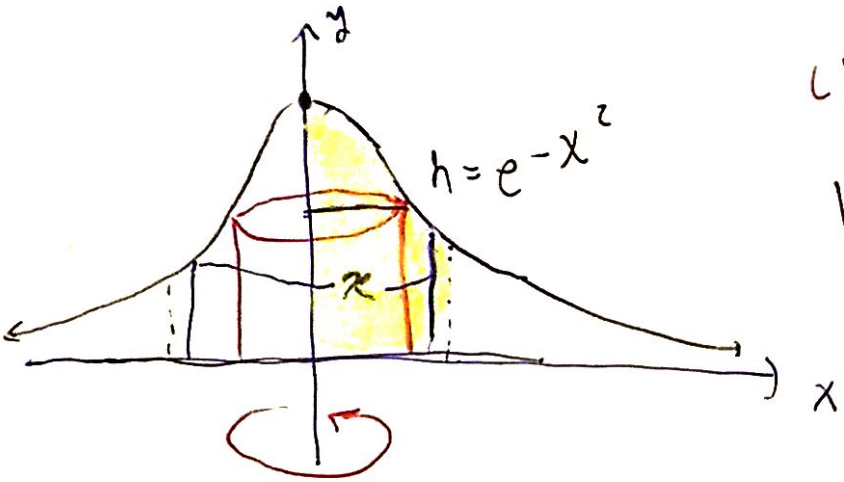


Ej: 3 Lab 8.

Ej: 4 $\quad y = e^{-x^2} \quad y = 0, \quad x = -1 \quad x = 1$

Lab 8.

a) alrededor del eje-y.



Cilindros.

$$V = 2\pi \int_0^1 h r \, dx$$

Discos

$$V = \pi \int_0^1 r^2(y)\, dy.$$

$$V = \pi \int_0^1 e^{-x^2} \overbrace{2x\,dx}^{-du} = \pi \int_0^{-1} -e^u \, du = -\pi e^u \Big]_0^{-1}$$

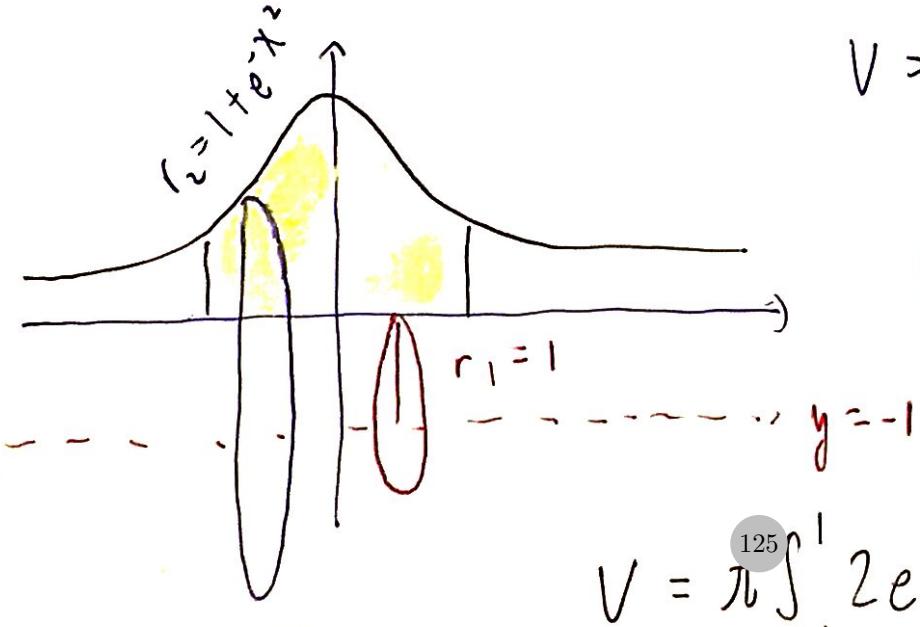
$u = -x^2 \qquad du = -2x\,dx$

$u(0) = 0 \qquad u(1) = -1$

$$-\pi(e^{-1} - e^0)$$

$$\pi(e^0 - e^{-1}) = \pi\left(1 - \frac{1}{e}\right)$$

b) alrededor de $y = -1$.



$$V = \pi \int_{-1}^1 r_2^2 - r_1^2 \, dx.$$

$$(1 + e^{-x^2})^2 - 1$$

$$1 + 2e^{-x^2} + e^{-x^4} - 1$$

$$2e^{-x^2} + e^{-x^4}$$

$$V = \pi \int_{-1}^1 2e^{-x^2} + e^{-x^4} \, dx$$

Capítulo 19

Simulacro Lunes 7 de octubre. 2:30 PM CES.

$0 \leq$ Notas ≤ 60 Bola Parcial. Res

7.8 Integrales Improp 8.5 Probabilidades.
76-128. sin 8.2 Área Superficial.

Lunes 14 de octubre Parcial 2: 2:30 PM CES.
Integr. Funciones Parciales.

Probabilidad (p. 123).

Un evento puede ser discreto o puede ser continuo.

Discreto: hay un número finito o contable de eventos.

Goles en un partido, lanzamiento de dados y monedas.

$$\text{Probabilidad} = \frac{\#\text{ veces de que ocurra un evento}}{\#\text{ total de eventos.}}$$

Dado
$E = \{1, 2, 3, 4, 5, 6\}$.

Probabilidad $P(x \geq 5) = \frac{2}{6} = \frac{1}{3}$.

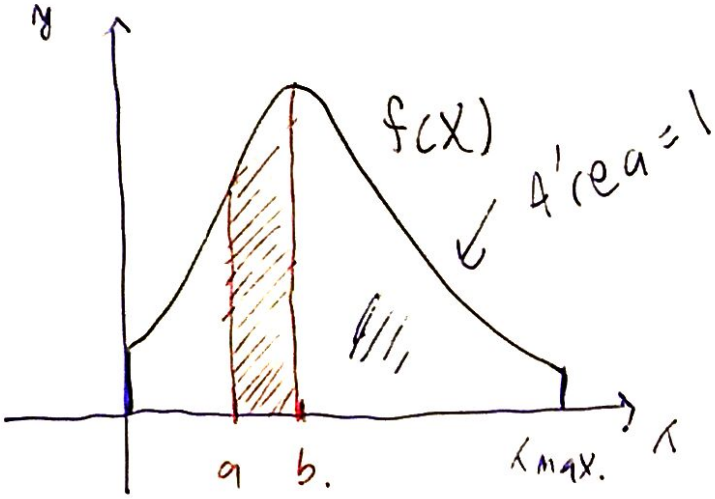
Probabilidad de que ocurra cualquier evento está entre 0 y 1 $0 \leq P(x) \leq 1$.

$0.0\% \leq P(x) \leq 100\%$

Probabilidad de que ocurran todos los eventos $\sum_{i=1}^{n} P(x_i)$

Enfoque: Statistical I y Mate Discreta

Continuo: el número de eventos no es contable.
El dominio de los eventos son los números reales.



Mediciones de cantidades como pesos, alturas, tiempos, volúmenes, áreas.

La probabilidad de que ocurra un evento entre a y b es el área bajo la curva.

$$p(a \leq x \leq b) = \int_a^b \underline{\underline{f(x)}}\, dx$$

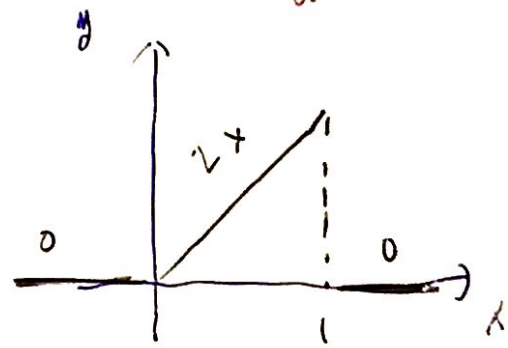
Función de densidad de probabilidad. $f(x)$.

Condiciones.

i. $f(x) \geq 0$ en $-\infty \leq x \leq \infty$.

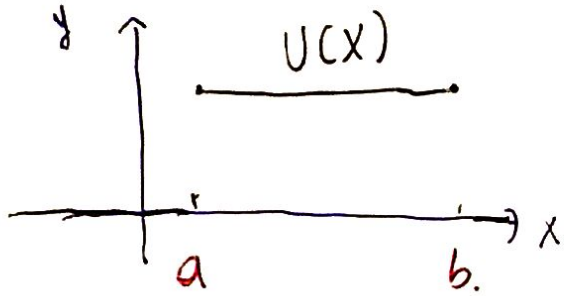
ii $\int_{-\infty}^{\infty} f(x)\, dx = 1$ Probabilidad tiene que ser del 100%.

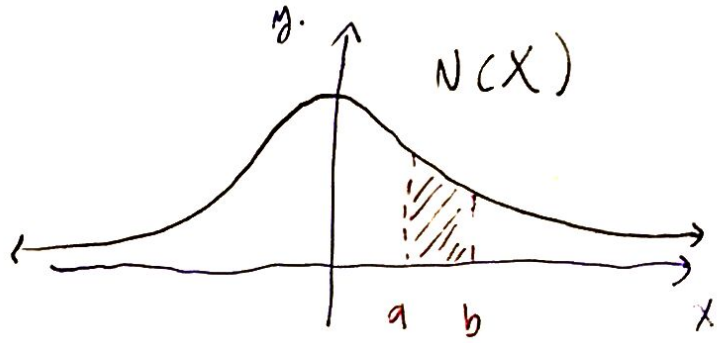
Ejemplo: $f(x) = \begin{cases} 0 & x < 0. \\ 2x & 0 \leq x \leq 1 \\ 0 & > 1 \end{cases}$



$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx = x^2\Big]_0^1 = 1 \quad \checkmark$$

3 Distribuciones de Probabilidad Comunes.





$$U(X) = \begin{cases} 0 & x < a \\ \frac{1}{b-a} & a \leq x \leq b \\ 0 & x > b. \end{cases}$$

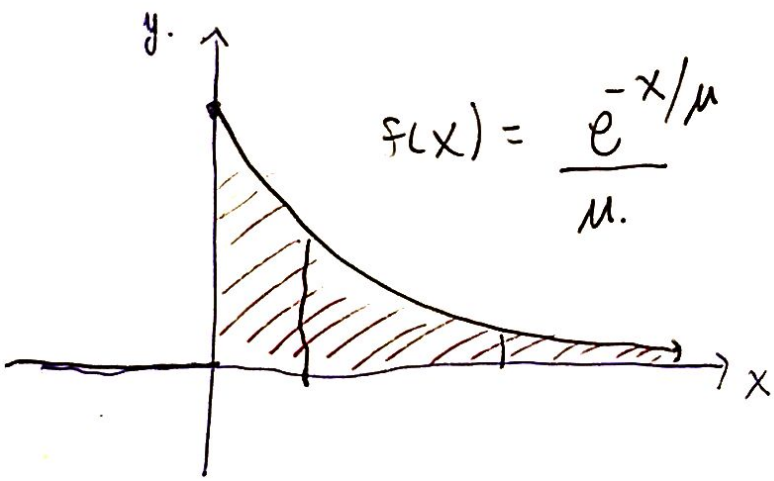
uniforme.

$$N(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Normal media 0
desv. estándar 1.

$$\int_{-\infty}^{\infty} N(x)\,dx = 1.$$

Exponencial.



$$f(x) = \frac{e^{-x/\mu}}{\mu}.$$

letra griega.

μ = media, mio

todos los eventos

$$\int_0^{\infty} f(x)\,dx = 1.$$

mediciones, tiempos, pesos

Ejercicio: Compruebe de que la distribución uniforme y la distribución exponencial son funciones de densidad

$$U(x) = \frac{1}{b-a} \quad a \leq x \leq b \qquad \underline{f}(x) = \frac{1}{\mu} e^{-x/\mu}, \; x \geq 0.$$

$$P(-\infty \leq x \leq \infty) = \int_{-\infty}^{\infty} U(x)\,dx = \int_a^b \frac{1}{b-a}\,dx$$

a, b son constantes

↑ probabilidad

$$= \frac{x}{b-a}\Big]_a^b = \frac{b-a}{b-a} = 1$$

$U(x) \geq 0$ en $\mathbb{R}$.

$U(x)$ es una función de densidad de probabilidad.

Exponencial:

$$P(0 \leq x \leq \infty) = \int_0^{\infty} e^{-x/\mu} \frac{dx}{\mu}.$$

μ tiempo promedio constante.

$$w = \frac{-x}{\mu} \qquad dw = \frac{-dx}{\mu}$$

$u(\infty) = -\infty.$
$u(0) = 0.$

$$P(0 \leq x \leq \infty) = -\int_0^{-\infty} e^{w}\,dw = -e^{w}\Big]_0^{-\infty}$$

$e^{-\infty} \to 0$

$$= -\lim_{w \to -\infty} e^{w} - (-e^{0}) = 1$$

Capítulo 20

3.5 Probabilidad

Variable continua $-\infty \leq x \leq \infty$.

Probabilidad P, de que x ocurra entre a y b es:

$$\boxed{P(a \leq x \leq b) = \int_a^b f(x)\,dx}$$

Distribución Uniforme: $f(x) = \frac{1}{b-a}$ $a \leq x \leq b$.

μ = media Exponencial: $f(x) = \frac{1}{\mu} e^{-x/\mu}$ $x \geq 0$.

Probabilidad es del 100%. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

P. 125.

Ejercicio 1: Un contenedor tiene mercancía cuyo peso tiene una distribución uniforme entre 2 y 4 toneladas.

a. Calcule la probabilidad de que el contenedor pese entre 2.5 y 3.5 toneladas. $a = 2$ $b = 4$.

$f(x) = \frac{1}{b-a} = \frac{1}{2}$ $2 \leq x \leq 4$. $f(x) = 0$ si $x > 4$.

$$P(2.5 \leq x \leq 3.5) = \int_{2.5}^{3.5} \frac{1}{2}\,dx = \frac{x}{2}\Big]_{2.5}^{3.5} = \frac{3.5-2.5}{2} = \frac{1}{2}.$$

La probabilidad de que pese entre 2.5 y 3.5 es del 50%

b. calcule la probabilidad de que el contenedor pese más de 2.5 toneladas.

$$P(X \geq 2.5) = \cancel{\int_{2.5}^{\infty} f(x)\,dx} = \int_{2.5}^{4} \frac{1}{2}\,dx = \frac{x}{2}\Big]_{2.5}^{4} = \frac{4-2.5}{2} = \frac{1.5}{2}.$$

$$P(X \geq 2.5) = \frac{3}{4} \text{ ó } 75\%.$$

$$f(x) = \begin{cases} 0 & x < 2 \\ 1/2 & 2 \leq x \leq 4 \\ 0 & x > 4. \end{cases}$$

Ejercicio 2: El tiempo de espera promedio para recibir una orden a domicilio es de 0.5 hora y tiene una distribución exponencial.

a. Encuentre la probabilidad de que la comida se entregue después de media hora.

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad \mu \text{ -media} \qquad \mu = 0.5.$$
$$x \geq 0.$$

$$f(x) = \frac{1}{0.5} e^{-x/0.5} = 2e^{-2x} \qquad x \geq 0.$$

$$P(x \geq 0.5) = \int_{0.5}^{\infty} 2e^{-2x}\,dx \qquad \frac{2e^{-2x}}{-2}$$

$$P(X \geq 0.5) = -e^{-2x}\Big]_{0.5}^{\infty} \qquad e^{-\infty} \to 0$$

$$= -\lim_{x \to \infty} e^{-2x} + e^{-2(0.5)} = e^{-1} = \frac{1}{e}.$$

Existe una probabilidad del 36.78% de que la entrega se realice 0.5 hora después.

b. ¿Cuál es la probabilidad de que la entrega se realice en menos de 15 min? ; 1/4 de hora

$$P(X \leq 1/4) = \int_0^{1/4} 2e^{-2x}\,dx = \int_{1/4}^{0} e^{\overbrace{-2x}^{u}}\,\overbrace{(-2\,dx)}^{du}$$

$u = -2x \quad du = -2\,dx \quad u(0) = 0 \quad u(1/4) = -1/2$

$$P(X \leq 1/4) = \int_{-1/2}^{0} e^{u}\,du = e^{u}\Big]_{-1/2}^{0} = 1 - e^{-1/2} \approx 0.3934$$

Estadísticos Importantes: Media, Mediana y Varianza

	x_i					
Variables Discretas.	1	2	3	4	5	6.
	1/6	1/6	1/6	1/6	1/6	1/6.
	$p(x_i)$					

Media $= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + \dots 6\left(\frac{1}{6}\right)$

Dados: $= \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5$

Media $\mu = \sum_{i=1}^{n} x_i f(x_i)$ $f(x_i)$ peso, frecuencia probabilidad.

Discretas

Variable continua: se integra $x f(x)$.

Media $\mu = \int_{-\infty}^{\infty} x f(x)\, dx$ el dominio de $f(x)$ puede restringir el intervalo de integración.

(mu)

realice IPP $u = x$
$dv = f(x)dx$.

Mediana: el número m que tiene una probabilidad acumulada del 50%.

$$\int_{-\infty}^{m} f(x)\, dx = 0.5$$

Resuelva para m.

$$F(m) = 0.5 \Rightarrow m = F^{-1}(0.5)$$

Varianza: la desviación cuadrado respecto a la media $(x-\mu)^2$.

(sigma)

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

Realice IPP dos veces $u = (x-\mu)^2$.

por partes.

Ejercicio 3: Considere la distribución exponencial.

$$f(x) = \frac{1}{100} e^{-x/100} \qquad x \geq 0.$$

a. Encuentre la media de $f(x)$

$$\mu = \int_0^\infty x \frac{e^{-x/100}}{100} dx = -xe^{-x/100}\Big]_0^\infty + \int_0^\infty e^{-x/100} dx$$

$u = x \qquad dv = \frac{1}{100} e^{-x/100} dx$

$du = dx \qquad v = -e^{-x/100}$

$$\mu = -xe^{-x/100}\Big]_0^\infty - 100e^{-x/100}\Big]_0^\infty$$

$$\mu = -\lim_{x\to\infty} xe^{-x/100} + 0 \cdot e^{-0} - 100 \lim_{x\to\infty} e^{-x/100} + 100 e^{-0}$$

$\infty \cdot e^{-\infty} = \infty \cdot 0$

$e^{-\infty} \to 0.$

$$\mu = 100 - \overbrace{\lim_{x\to\infty} \underbrace{xe^{-x/100}}_{L.H.}}^{0.}$$

$$\lim_{x\to\infty} \frac{x}{e^{x/100}} \underset{LH}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{100} e^{x/100}} \qquad \frac{\infty}{0} \text{ ó } \frac{\infty}{\infty} \text{ LH.}$$

$\frac{1}{e^{\infty}} \to 0$

$$\mu = 100.$$

b. Encuentre la mediana de $f(x)$.

$$\int_0^m f(x)\,dx = 0.5. \qquad f(x) = \frac{1}{100} e^{-x/100}.$$

$$\frac{1}{100}\int_0^m e^{-x/100}\,dx = -\frac{100}{100} e^{-x/100}\Big]_0^m = -e^{-m/100} + e^0.$$

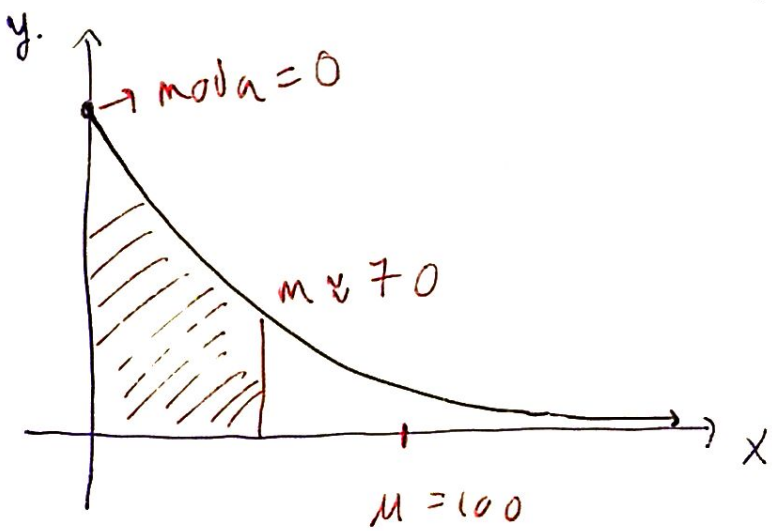
$$1 - e^{-m/100} = 0.5. \qquad \text{Resuelva para } m.$$

$$-e^{-m/100} = -0.5$$

$$e^{-m/100} = 0.5.$$

Aplique ln's: $\frac{-m}{100} = \ln(0.5) \rightarrow m = -100\,\underbrace{\ln(0.5)}_{-0.69}$

$$m \approx 69.31$$



Varianza $\sigma^2 = \int_0^\infty (x-100)^2 \frac{e^{-x/100}}{100}\,dx$

$u = (x-100)^2 \qquad dv = \frac{e^{-x/100}}{100}\,dx$

$du = 2(x-100) \qquad v = -e^{-x/100}$

$$\sigma^2 = -(x-100)^2 e^{-x/100}\Big]_0^{\infty} + \int_0^{\infty} 2(x-100)\, e^{-x/100}\, dx$$

$$\sigma^2 = -\lim_{x\to\infty} \frac{(x-100)^2}{e^{x/100}} + 100^2 e^{-0}$$

L'H IPP

$$\sigma^2 = 100^2 \qquad \sigma = 100$$

Ejercicio 4: Encuentre la media, mediana, varianza y desviación estándar de la distribución uniforme.

$$f(x) = \frac{1}{b-a} \qquad a \le x \le b. \qquad \text{para } b=7,\ a=1$$

Media: $\mu = \int_a^b x f(x)\, dx = \int_1^7 \frac{x}{6}\, dx = \frac{x^2}{12}\Big]_1^7$

$$\mu = \frac{49-1}{12} = \frac{48}{12} = 4.$$

Mediana: $\int_a^m f(x)\, dx = 0.5 \qquad \int_1^m \frac{1}{6}\, dx = \frac{x}{6}\Big]_1^m$

$$\frac{m-1}{6} = 0.5. \Rightarrow m = 0.5(6)+1 = 4.$$

No hay moda.

Varianza: $\int_a^b (x-\mu)^2 f(x)\, dx = \int_1^7 (x-4)^2 \underbrace{\frac{1}{6}}_{f(x)}\, dx$

$$u = x-4 \qquad du = dx \qquad \begin{array}{l} u(7) = 3 \\ u(1) = -3 \end{array}$$

$$\frac{1}{6}\int_1^7 (x-4)^2\,dx = \frac{1}{6}\int_{-3}^{3} u^2\,du = \frac{2}{6}\int_0^3 u^2\,du$$

$$\sigma^2 = \frac{1}{3}\,\frac{1}{3}\,u^3\Big]_0^3 = \frac{27}{9} = 3$$

Desviación estándar: $\sigma = \sqrt{\text{varianza}} = \sqrt{3}$.

4. Longitud de arco $\int \sqrt{1+(y')^2}\,dx$ cab 9.

$$\int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right)\Big]$$

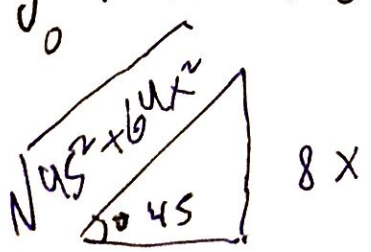
$x = 2\sin\theta$
$dx = 2\cos\theta\,d\theta$.

5. $y = 180 - \left(\frac{2x}{45}\right)^2 = 180 - \frac{4x^2}{45^2}$.

$y' = \frac{-8x}{45^2}$ $\qquad 1+(y')^2 = 1 + \frac{64x^2}{45^2}$.

$$= \frac{1}{45^2}\left(45^2 + 64x^2\right).$$

$$L = \int_0^{90} \sqrt{45^2 + 64x^2}\,dx$$



$\tan\theta = \frac{8x}{45}$ $\qquad \sec\theta = \frac{\sqrt{(\ )}}{45}$.

$$x = \frac{45}{8}\tan\theta \qquad dx = \frac{45}{8}\sec^2\theta\, d\theta.$$

$$L = \int_0 \sqrt{45^2 + 64x^2}\, dx = \int_0^{?} 45\sec\theta \cdot \frac{45}{8}\sec^2\theta\, d\theta.$$

$$L = \frac{45^2}{8}\int \sec^3\theta\, d\theta. = \frac{45^2}{16}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)$$

Capítulo 21

Lab Lunes 1-2:30 PM.
sim2 2:30 PM Centro Estudiantil.

7.4 Fracciones Parciales ✓

se utiliza para integrar funciones racionales.

$$\frac{P(x)}{Q(x)} = f(x)$$

P polinomio de grado n
Q polinomio de grado m.

condición: Denominador > numerador. } ✓

En caso de que el grado del numerador sea mayor que o igual. del denominador, realice la división larga.

$$\frac{6}{x^2-9}, \quad \frac{x+3}{x^3-9x}, \quad \frac{1}{x^2+4}, \ldots,$$

denominador es el más grande

$$\left.\frac{x^2+3}{x^2-9}, \quad \frac{x^4+x^2+1}{x^3-4x}\right\}$$

División larga

simplifique la función en dos o más fracciones parciales.

$$\frac{6}{x^2-9} = \frac{6}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

no puede integrar

A, B dos coeficientes "desconocidos"

$$\frac{6}{x^2-9} = \frac{A(x+3) + B(x-3)}{x^2-9}.$$

Igualando los numeradores cero en $x=-3, 3$.

$$6 = A(x+3) + B(x-3)$$

$x=3$: $\quad 6 = 6A + 0 \Rightarrow A = 1$

$x=-3$: $\quad 6 = 0 - 6B \Rightarrow B = -1$

$$\frac{6}{x^2-9} = \frac{1}{x-3} - \frac{1}{x+3}$$

$$\int \frac{6}{x^2-9}\,dx = \int \frac{dx}{x-3} - \int \frac{dx}{x+3} = \ln|x-3| - \ln|x+3| + C.$$

Finalmente integre.

Caso 1: Factores lineales distintos.

$$\frac{p(x)}{(x^2-9)(x^2-4)} = \frac{p(x)}{(x-3)(x+3)(x-2)(x+2)}$$

$$= \frac{A}{x-3} + \frac{B}{x+3} + \frac{C}{x-2} + \frac{D}{x+2}$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + c.$$

Encuentre A, B, C y D.

Caso 2: Factores Lineales Repetidos.

$$f(x) = \frac{x^2+x+1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$x+1$ Encuentre A, B y C.

$$\int f(x) = A\ln|x+1| - B(x+1)^{-1} - \frac{1}{2}C(x+1)^{-2} + K.$$

$$\int (x+a)^{-t}\,dx = \frac{(x+a)^{-t+1}}{1-t} + K.$$

Ejercicio 1: $\displaystyle\int \frac{18z}{2z^2+7z-4}\,dz$ P(64)

1. Factorice dem: $2z^2+7z-4 = (2z-1)(z+4)$.

$2z \quad -1$

$z \quad 4$

$2(z^2+3.5z+2)$

2. Ceros Den. $2z-1=0 \Rightarrow z=0.5$. $z+4=0 \Rightarrow z=-4$ } lineales distintos.

$$\frac{18z}{2z^2+7z-4} = \frac{A}{2z-1} + \frac{B}{z+4}$$

¿A, B?

Multiplique por $(2z-1)(z+4)$

$$18z = A(z+4) + B(2z-1)$$

$z=0.5$: $9 = 4.5A + 0 \Rightarrow A = 9/4.5 = 2.$

$z=-4$: $18(-4) = 0 - 9B \Rightarrow B = \frac{-18}{-9}(4) = 8$

$$\int \frac{18z}{2z^2+7z-4} = \int \frac{2}{2z-1}\,dz + 8\int \frac{dz}{z+4}$$

$$= \frac{2}{2}\ln|2z-1| + 8\ln|z+4| + C.$$

Ejercicio 2: Integre las sigs. funciones.

a) $\displaystyle\int \frac{4x+2}{x^2+2x+1}\,dx$ b) $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

a) $x^2+2x+1 = (x+1)^2$. factor lineal repetido.

$$\frac{4x+2}{x^2+2x+1} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \quad \therefore \quad \text{¿} A, B?$$

Multiplique por $(x+1)^2$ denominador se hace cero en $x=-1$

$$4x+2 = A(x+1) + B.$$

$x=-1$: $-2 = 0 + B \Rightarrow B=-2.$

$x=0$: $2 = A + B. \Rightarrow A = 2 - B = 2+2 = 4.$

$$\int \frac{4x+2}{x^2+2x+1}\,dx = \int \frac{4}{x+1}\,dx + \int -2(x+1)^{-2}\,dx$$

$$= 4\ln|x+1| + 2(x+1)^{-1} + C.$$

b) $\int \frac{x^2+2x-1}{x^3-x}\,dx$ $\qquad x^3-x = x(x^2-1)$
$\qquad\qquad\qquad\qquad\qquad\qquad = x(x+1)(x-1)$

ceros en $0, -1, 1$ (factores lineales distintos)

$$\frac{x^2+2x-1}{x^3-x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$x^2+2x-1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

$x=0$: $-1 = -A + 0 + 0 \Rightarrow A = 1$

$x=-1$: $-2 = 0 + 2B + 0 \Rightarrow B = -1$

$x=1$: $2 = 0 + 0 + 2C \Rightarrow C = 1$

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{dx}{x} - \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$$

$$= \ln|x| - \ln|x+1| + \ln|x-1| + C.$$