

1. EXAMPLE SECTION

Theorem 1.1. *There are infinitely many primes*

Proof. Suppose that $p_1 < p_2 < \dots < p_n$ are all of the primes. Let $P = 1 + \prod_{i=1}^n p_i$ and let p be a prime dividing P .

Then p can not be any of p_i , for otherwise p would divide the difference $P - (\prod_{i=1}^n p_i) = 1$, which is impossible. So this prime p is still another prime, and p_1, p_2, \dots, p_n cannot be all of the primes. \square

Exercise 1.2. *Give an alternative proof that there are an infinite number of prime numbers.*

To solve this exercise, we first introduce the following lemma.

Lemma 1.3. *The Fermat numbers $F_n = 2^{2^n} + 1$ are pairwise relatively prime.*

Proof. It is easy to show by induction that

$$F_m - 2 = F_0 F_1 \dots F_{m-1}.$$

This means that if d divides both F_n and F_m (with $n < m$), then d also divides $F_m - 2$. Hence, d divides 2. But every Fermat number is odd, so d is necessarily one. This proves the lemma. \square

We can now provide a solution to the exercise.

Theorem 1.4. *There are infinitely many prime numbers (Goldbach's proof)*

Proof. Choose a prime divisor p_n of each Fermat number F_n . By the lemma we know these primes are all distinct, showing there are infinitely many primes. \square

2. L^AT_EX EXAMPLES

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- (1) Example list item
- (2) Another list item