W6M1

Module 6

Anni: Test 2 today
- Hope to grade by wednesday

Preview: 1) Let $f(x) = 3x^3 + 2$; $g(x) = \sqrt{7-x}$

a) find $h = f - g$; Domain?

b) $h = \frac{g}{f}$; Domain?

c) Domains of $g \circ f$ and $f \circ g$?

2) Graph $g(x) = \begin{cases} 1 & -1 \le x < 0 \\ 3 & 0 < x \le 2 \\ -1 & 3 < x < 4 \end{cases}$

3) Find f^{-1} where $f(x) = 2 - \sqrt{x}$

$\text{Dom}(f) = [0, \infty)$

$\text{Range}(f) = (-\infty, 2]$

Note

$x = f(f^{-1}(x)) = 2 - \sqrt{f^{-1}(x)}$.

So, $x - 2 = -\sqrt{f^{-1}(x)}$

$-x + 2 = \sqrt{f^{-1}(x)}$

$f^{-1}(x) = (2-x)^2$ is the rule for f^{-1}.

$f^{-1}: (-\infty, 2] \longrightarrow [0, \infty)$ is f^{-1}.

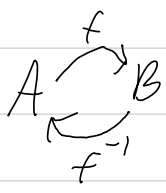
$x \longmapsto (2-x)^2$

Content: Inverses of functions.

Notation: $f \circ g$ is the function $f(g(x))$; $id(x) = x$.

Def: $g: B \to A$ is the inverse for a function $f: A \to B$ if $g \circ f = id_A$ and $f \circ g = id_B$.

Denote g by f^{-1}.



Fact: Inverses are unique.

Def: A function is invertable if it has an inverse.

Fact: A function f is invertable iff it is bijective, that is surjective (onto) and injective (1-1).

Recall: Surjective means every element of the codomain is acheived and f is injective if whenever $f(a) = f(b)$, we have $a = b$.

Horiz. line Test: A function $f: U \to V$, $U, V \subseteq \mathbb{R}$, $\text{im}(f) = V$ is invertable iff for every $c \in V$, the line $y = c$ intersects the graph of f only once. (this is a way to check injectivity visually)

Composing functions

Fact: for $f: A \to B$, $g: B \to C$, the domain of $g \circ f$ is A.

In this class, we give functions implicitly by their rules (grumble grumble), so here $\text{Dom}(g \circ f)$ is $g^{-1}(\text{Dom}(f))$.

Also by $g \circ f$, we really mean $(g|_{\text{im}(f)} \circ f)$.

restrict g's domain

BB: 6.1, etc.

W6.W1

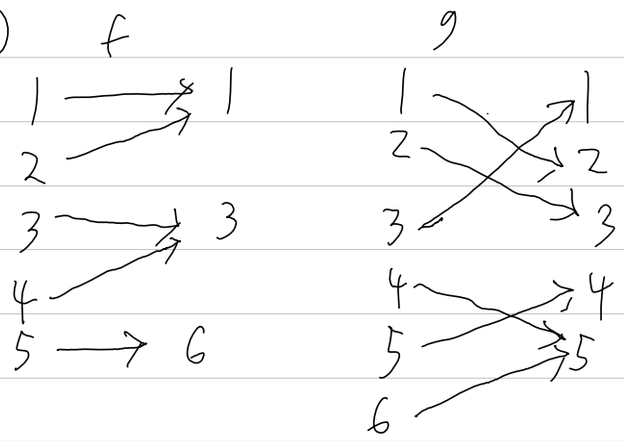
Preview: 1) $f(x) = \sqrt{16 - x^2}$; $g(x) = x + 4$

a) Domain of f/g?

b) Domain of $g \circ f$?

c) Domain of $f \circ g$?

2)



Find $f \circ g$ and $g \circ f$.

3) Let $f(x) = \frac{7}{\sqrt{3x^3 - 9}}$; Find f^{-1}.

4) Tires cost 45 USD per tire. To put n wheels on an n-wheeler, it costs 35 times square root n plus tire costs. You need to find how many wheels each transaction had on an order. Find such a formula.

5) A really cool spherical bubble is growing at a rate of $3 \frac{cm}{min}$. from a vial of liquid of volume $1 cm^3$. If the bubble needs a thickness of at least 1mm, how long till the bubble bursts? Surface area of a sphere is $4\pi r^2$. How large did the bubble get?

Content: BB 6.2, etc.

WGF 1

Ann i-Test 4 Aug 84%

– wiki 6 today

Contents: C.2, C.3, wiki

Review: 1) Suppose $Ax - By = C$ with $A > B > 0 > C$ graph this line.

$$-By = C - Ax$$

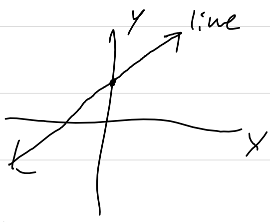
$$y = \frac{C - Ax}{-B} = \frac{C}{-B} + \frac{-Ax}{-B} = -\frac{C}{B} + \frac{Ax}{B}$$

$A, B > 0$, so $A/B > 0$

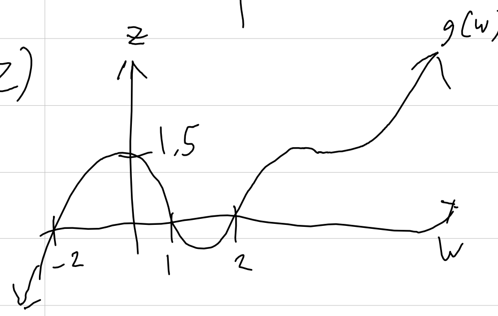
$C < 0$ and $B > 0$,

so $-\frac{C}{B}$ has signs $- \frac{-}{+} = +$

So



2)

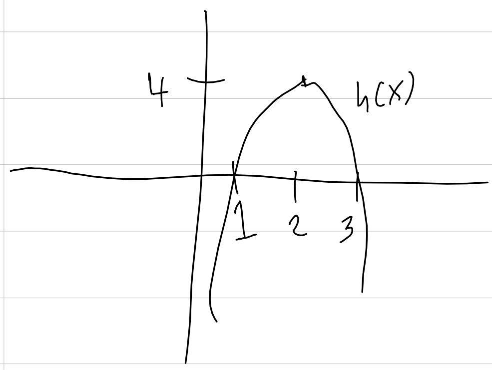


what are g(w)'s w-ints and z-ints?

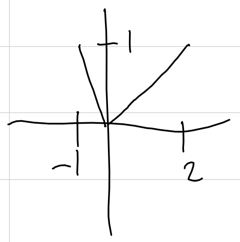
w-int: (−2,0), (1,0), (2,0)

z-int: (0,1.5)

3) Find AROC for h between $x = 2$ and $x = 3$.

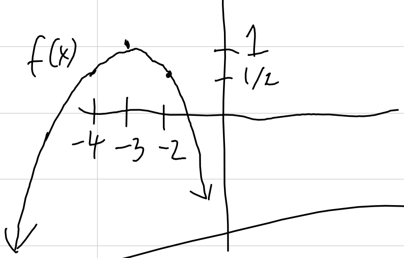


4) h(x) is ; graph $3h(\frac{x}{2} - 1) + 1$.



5) f(x) is a transformation of x^2,

write the function of f.



Module 7 W7M1

Preview ① An isosceles right triangle and a square are cut out of m meters of metal. What side lengths minimize area?

m is given
b is what we want and ℓ

$A_\Delta(b) = \frac{1}{2}b^2$, $A_\square(\ell) = \ell^2$

also, $4\ell + 2b + \sqrt{2}b = m$

so $\ell = \frac{m-(2+\sqrt{2})b}{4}$

$$A_{tot}(b) = \frac{1}{2}b^2 + \left(\frac{m-(2+\sqrt{2})b}{4}\right)^2$$
$$= \frac{1}{2}b^2 + \left(\frac{m}{4}\right)^2 - 2m\cdot\frac{(2+\sqrt{2})}{4}b + \left(\frac{(2+\sqrt{2})}{4}\right)^2 b^2$$

Fact: min/max of poly is @ vertex!
Formula: For $f(x) = ax^2+bx+c$, vertex is at $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Here we have poly in b

$$\left(\frac{1}{2}+\left(\frac{(2+\sqrt{2})}{4}\right)^2\right)b^2 - 2m\left(\frac{2+\sqrt{2}}{4}\right)b + \left(\frac{m}{4}\right)^2$$

so vertex is at $\dfrac{-2m\left(\frac{2+\sqrt{2}}{4}\right)}{-2\left(\frac{1}{2}+\left(\frac{(2+\sqrt{2})}{4}\right)^2\right)} = b_{min}$

Hence the base and length that minimize area are respectively

$$\frac{m\left(\frac{2+\sqrt{2}}{4}\right)}{\frac{1}{2}+\left(\frac{2+\sqrt{2}}{4}\right)^2} \quad \text{and} \quad \frac{m-(2+\sqrt{2})\cdot}{4}$$

Content: Def: A function f is a quadratic if f is of the form $f(x) = ax^2+bx+c$.

ex) 1) $4x^2+3x+2$

2) $xy^4 + zy^2 + 1$; here note
$= x(y^2)^2 + z(y^2)+1$

Def: A root/zero of f is a $x \in \mathbb{R}$ such that $f(x)=0$

Formula: for $f(x) = ax^2+bx+c$, the roots of $f(x)$ are

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\left(\frac{-b}{2a}\right)^2 - \frac{c}{a}}$$

Proof: $0 = ax^2+bx+c$

$-c = ax^2+bx$

$\frac{-c}{a} = x^2 + \frac{b}{a}x$

$\frac{-c}{a} = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$

$\frac{-c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$

$\frac{-c}{a} + \left(\frac{b}{2a}\right)^2 = \left(x+\frac{b}{2a}\right)^2$

$\pm\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} = x + b/2a$

$$x = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Note: Vertex on graph of parabola is symmetric about roots hence Vertex's x-coord is $\frac{-b}{2a}$

Now BB 7.1, etc.

W7W2

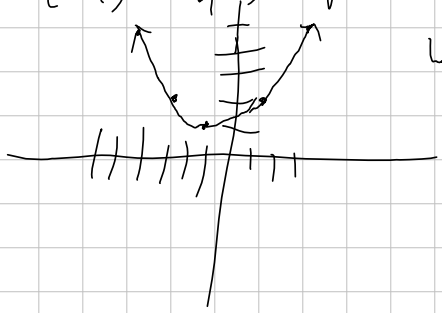
Content: BB d^2+d3

Thm: Aleks walk around and help.

Ann: wiki is graded

Preview: 1) Dale needs to add a privacy fence to his rectangular resort. A cliff covers one side of his property. If xe bought 1km of fence, how much area can xe cover?

2) f(x) has plot



What is formula for f?

3) $f(x)=x^2+x$; $g(x)=\sqrt{x-2}$

find fog domain:

$dom(f\circ g) \subseteq dom(g)$

$dom(f)=\mathbb{R}$, so no restrictions from here.

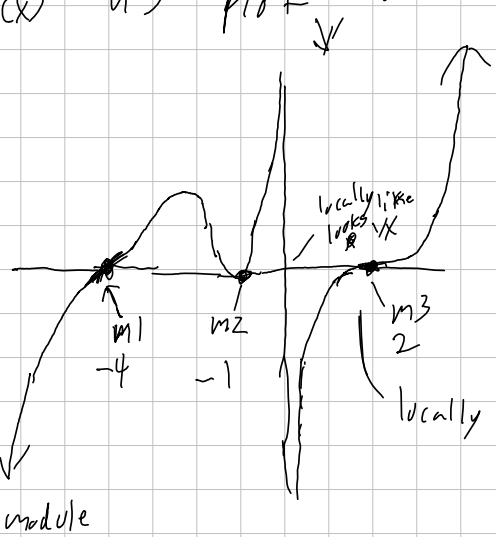
$f\circ g=\sqrt{x-2}^2+\sqrt{x-2}$

$=x-2+\sqrt{x-2}$

$dom(f\circ g)=dom(g)=[2,\infty)$

Preview for end goal of class

4) f(x) has plot



What is it's function up to LC?

LC pos or neg?

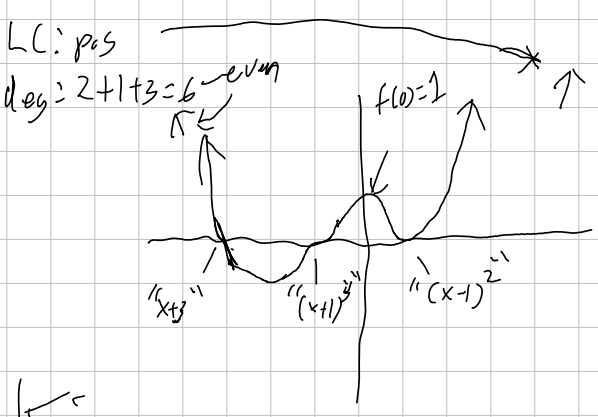
as $x\to\infty$ $f(x)\to\infty$

so LC pos.

$f(x)=\frac{a(x+4)(x+1)^2(x-2)^3}{x}$

Preview for next module

5) plot $f(x)=\frac{1}{3}(x-1)^2(x+3)(x+1)^3$



Wiki

W 8 M1

Review) 1) M2 – 63%

$3x+2y=3$; Find line $\perp$ through $(7,2)$.

2) M3 – 50%

is $(y+3)^2 = x - y^2 + 5y$

a fcn of x?

3) M4 – 50%

Domain of $\frac{\sqrt{x+7}}{3x-10}$

M6 – 90%

4) $f(x) = \frac{x+2}{3x-1}$; $g(x) = 2x-1$; Find $f \circ g$.

Preview

5) $F(x)$ has graph

sgn(LC)?

roots w/ mult?

possible

inc/dec?

EB?

Content: M8. *