W6M1

Module 6

Ann: Test 2 today
└ Hope to grade by wednesday

Preview: 1) Let $f(x) = 3x^3+2$; $g(x) = \sqrt{7-x}$

a) find $h = f-g$; Domain?

b) $h = \frac{g}{f}$; Domain?

c) Domains of $g \circ f$ and $f \circ g$?

2) Graph $g(x) = \begin{cases} 1 & -1 \le x < 0 \\ 3 & 0 < x \le 2 \\ -1 & 3 < x < 4 \end{cases}$

3) Find f^{-1} where $f(x) = 2 - \sqrt{x}$

$Dom(f) = [0,\infty)$
$Range(f) = (-\infty, 2]$

Note
$x = f(f^{-1}(x)) = 2 - \sqrt{f^{-1}(x)}$.
So, $x - 2 = -\sqrt{f^{-1}(x)}$
$-x+2 = \sqrt{f^{-1}(x)}$
$f^{-1}(x) = (2-x)^2$ is the rule for f^{-1}.

$$f^{-1} : (-\infty, 2] \longrightarrow [0,\infty) \quad \text{is } f^{-1}.$$
$$x \longmapsto (2-x)^2$$

Content: Inverses of functions.

Notation: $f \circ g$ is the function $f(g(x))$; $id(x) = x$.

Def: $g: B \to A$ is the inverse for a function $f: A \to B$ if $g \circ f = id_A$ and $f \circ g = id_B$.
Denote g by f^{-1}.

$$A \underset{f^{-1}}{\overset{f}{\rightleftarrows}} B$$

Fact: Inverses are unique.

Def: A function is invertable if it has an inverse.

Fact: A function f is invertable iff it is bijective, that is surjective (onto) and injective (1-1)

Recall: Surjective means every element of the codomain is acheived and f is injective if whenever $f(a) = f(b)$, we have $a = b$.

Horiz. line Test: A function $f: U \to V$, $U, V \subseteq \mathbb{R}$, $im(f) = V$ is invertable iff for every $c \in V$, the line $y = c$ intersects the graph of f only once. (this is a way to check injectivity visually)

Composing functions

Fact: for $f: A \to B$, $g: B \to C$, the domain of $g \circ f$ is A.

In this class, we give functions implicitly by their rules (grumble grumble), so here $Dom(g \circ f)$ is $g^{-1}(Dom(f))$.
Also, by $g \circ f$, we really mean $(\underbrace{g|_{im(f)}}_{\text{restrict } g\text{'s domain}} \circ f)$.

BB: 6.1, etc.

W6 W1

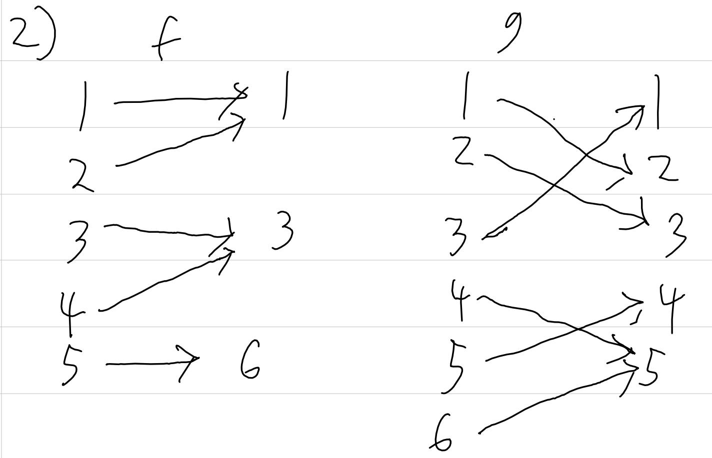
Preview: 1) $f(x) = \sqrt{16-x^2}$; $g(x) = x+4$

a) Domain of f/g ?

b) Domain of $g \circ f$?

c) Domain of $f \circ g$?

2)



Find $f \circ g$ and $g \circ f$.

3) Let $f(x) = \frac{7}{\sqrt{3x^3-9}}$; Find f^{-1}.

~~4) tires cost 45 USD per tire. To put n wheels on an n-wheeler, it costs 35 times square root n plus tire costs. You need to find how many weels each transaction had on an order. Find such a formula.~~

5) A realy cool spherical bubble is growing at a rate of $3 \frac{cm}{min}$ from a vial of liquid of volume $1 cm^3$. If the bubble needs a thickness of at least 1mm, how long till the bubble bursts? Surface area of a sphere is $4\pi r^2$. How large did the bubble get?

Content: BB 6.2, etc.

WGF 1

Anni-Test 4 Aug 84%

— wiki 6 today

Content: C,2 ✓ C,3, wiki

Review: 1) Suppose $Ax - By = C$ with $A > B > 0 > C$ graph this line.

$-By = C - Ax$

$y = \frac{C - Ax}{-B} = \frac{C}{-B} + \frac{-Ax}{-B}$

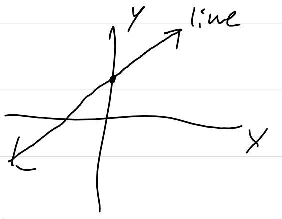
$= -\frac{C}{B} + \frac{Ax}{B}$

$A, B > 0$, so $A/B > 0$

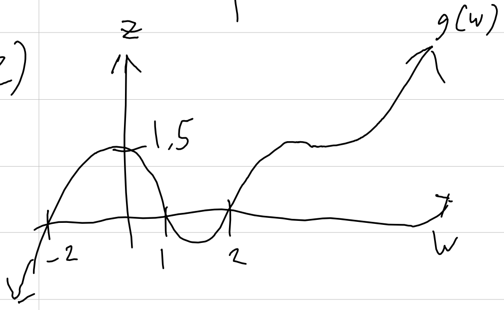
$C < 0$ and $B > 0$,

so $-\frac{C}{B}$ has signs $- \frac{-}{+} = +$

So



2)

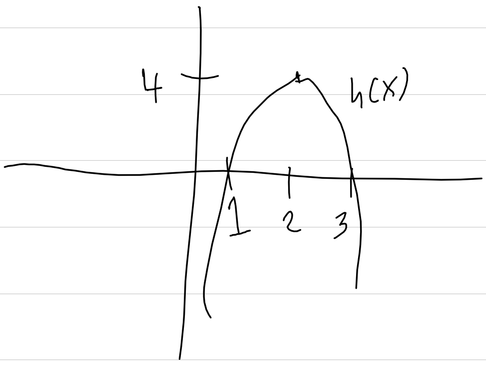


what are g(w)'s w-ints and z-ints?

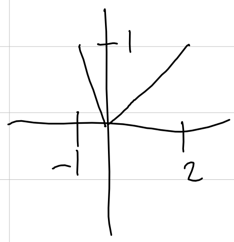
w-int: (−2,0),(1,0),(2,0)

z-int: (0,1.5)

3) Find AROC for h between $x = 2$ and $x = 3$.

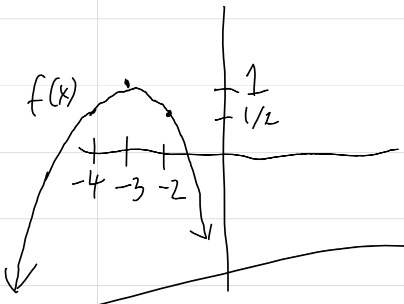


4) h(x) is ; graph $3h(\frac{x}{2} - 1) + 1$.



5) f(x) is a trasformation of x^2,

write the function of f.



Module 7 W7M1

Preview ① An isocelese right triangle and a square are cut out of m meters of metal. What side lengths minimize area?

m is given
b is what we want
and l

$A_\Delta(b) = \frac{1}{2}b^2$, $A_\square(l) = l^2$

also, $4l + 2b + \sqrt{2}b = m$

so $l = \frac{m-(2+\sqrt{2})b}{4}$

$$A_{tot}(b) = \frac{1}{2}b^2 + \left(\frac{m-(2+\sqrt{2})b}{4}\right)^2$$
$$= \frac{1}{2}b^2 + \left(\frac{m}{4}\right)^2 - 2m\cdot\frac{2+\sqrt{2}}{4}b + \left(\frac{2+\sqrt{2}}{4}\right)^2 b^2$$

Fact: min/max of poly is @ vertex!

Formula: For $f(x)=ax^2+bx+c$, vertex is at $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Here we have poly in b

$$\left(\frac{1}{2}+\left(\frac{2+\sqrt{2}}{4}\right)^2\right)b^2 - 2m\left(\frac{2+\sqrt{2}}{4}\right)b + \left(\frac{m}{4}\right)^2$$

so vertex is at $\dfrac{-2m\left(\frac{2+\sqrt{2}}{4}\right)}{-2\left(\frac{1}{2}+\left(\frac{2+\sqrt{2}}{4}\right)^2\right)} = b_{min}$

Hence the base and length that minimize area are respectfully

$$\frac{m\left(\frac{2+\sqrt{2}}{4}\right)}{\frac{1}{2}+\left(\frac{2+\sqrt{2}}{4}\right)^2} \text{ and } \frac{m-(2+\sqrt{2})\cdot}{4}$$

Content: Def: A function f is a quadratic if f is of the form $f(x) = ax^2+bx+c$.

ex) 1) $4x^2+3x+2$

2) $xy^4 + zy^2 + 1$; here note
$= x(y^2)^2 + z(y^2)+1$

Def: A root/zero of f is a $x \in \mathbb{R}$ such that $f(x)=0$

Formula: for $f(x) = ax^2+bx+c$, the roots of $f(x)$ are

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\left(\frac{-b}{2a}\right)^2 - \frac{c}{a}}$$

Proof: $0 = ax^2+bx+c$

$-c = ax^2+bx$

$\frac{-c}{a} = x^2 + \frac{b}{a}x$

$\frac{-c}{a} = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$

$\frac{-c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$

$\frac{-c}{a} + \left(\frac{b}{2a}\right)^2 = \left(x+\frac{b}{2a}\right)^2$

$\pm\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} = x + b/2a$

$$x = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Note: Vertex on graph of parabola is symmetric about roots hence Vertex's x-coord is $\frac{-b}{2a}$

Now BB 7.1, etc.

W7W2

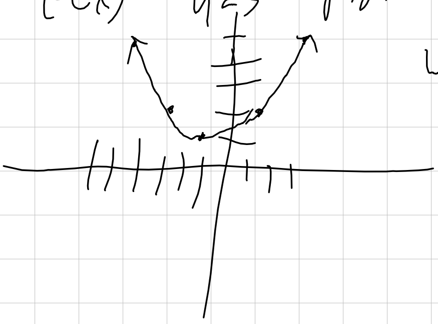
Content: BB d2 + d3

Thu: Aleks walk around and help.

Ann: wiki is graded

Preview: 1) Dale needs to add a privacy fence to his rectangular resort. A cliff covers one side of his property. If xe bought 1km of fence, how much area can xe cover?

2) f(x) has plot



What is formula for f?

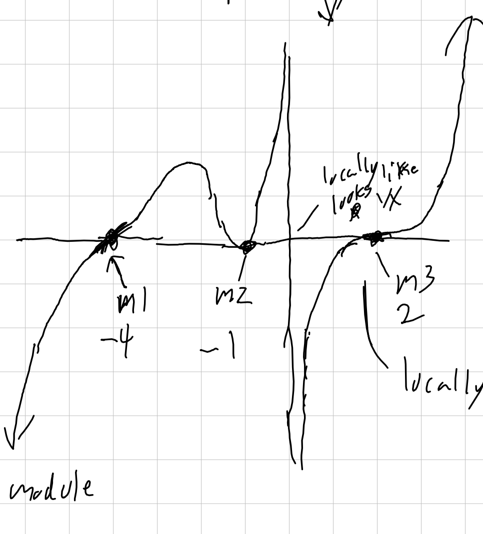
3) $f(x) = x^2 + x$; $g(x) = \sqrt{x-2}$
find fog domain:

dom(fog) ⊆ dom(g)
dom(f) = ℝ, so no restrictions from here.

$f\circ g = \sqrt{x-2}^2 + \sqrt{x-2}$
$= x-2 + \sqrt{x-2}$
dom(fog) = dom(g) = $[2, \infty)$

Preview for end goal of class
4) f(x) has plot

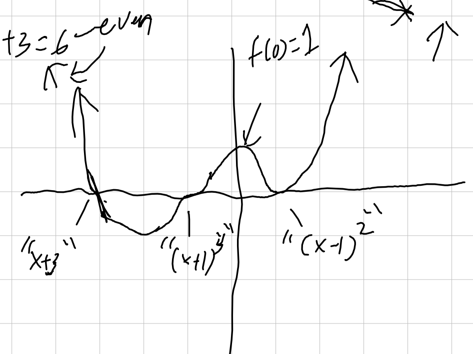
What is it's function upto LC?
LC pos or neg?
as $x \to \infty$ $f(x) \to \infty$
so LC pos.



$$f(x) = \frac{a(x+4)(x+1)^2(x-2)^3}{x}$$

Preview for next module
5) plot $f(x) = \frac{1}{3}(x-1)^2(x+3)(x+1)^3$

LC: pos
deg: 2+1+3 = 6 even



Wiki

W8M1

review) 1) M2 – 63%

$3x+2y=3$; Find line $\perp$ through $(7,2)$.

2) M3 ~ 50%

is $(y+3)^2 = x - y^2 + 5y$

a function of x?

3) M4 – 50%

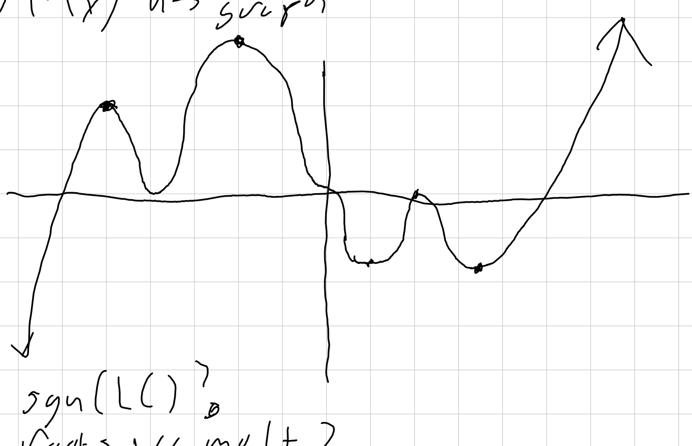
Domain of $\frac{\sqrt{x+7}}{\sqrt{3x-10}}$

M6 – 90%

4) $f(x) = \frac{x+2}{3x-1}$; $g(x) = 2x-1$; Find $f \circ g$.

Preview

5) $f(x)$ has graph



sgn(LC)?

roots w/ possible mult?

inc/dec?

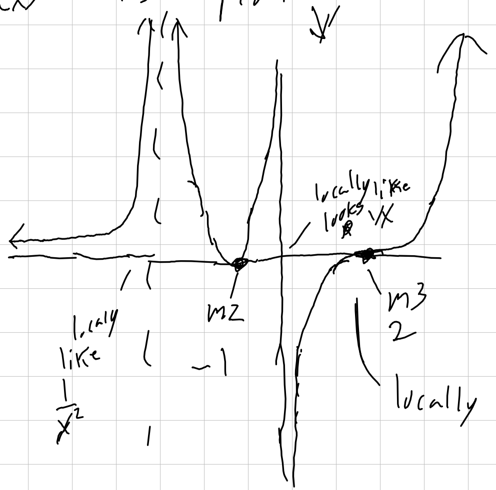
EB?

Content: M8. *

Preview for end goal of class

1) $f(x)$ h-s plot. What is it's function up to LC?

LC pos or neg?



as $x \to \infty$ $f(x) \to \infty$, so LC pos.

$$f(x) = \frac{(x+1)^2(x-2)^3}{(x-3)^2 x}$$

2) You need to make a square prism whose frame is from 20m of steel. If you needn't paint one square, what dimensions maximize painting?

$$W = 8h + 4l = 20$$

$$4l = 20 - 8h$$

$$l = \frac{20-8h}{4} = 5 - 2h$$

$$SA = h^2 + 4hl$$
$$= h^2 + 4h(5-2h)$$
$$= h^2 + 20h - 8h^2$$
$$= -7h^2 + 20h$$

$$h_{max} = \frac{-20}{-2\cdot 7} = \frac{20}{14} = \frac{10}{7} \qquad l_{max} = 5 - 2\cdot\frac{10}{7}$$

3) Find a few polynomials of degree 5 with roots $-3, 2, 4$.

a) $7(x+3)^2(x-2)(x-4)(x+\pi)$

b) $-84(x+3)^3(x-2)(x-4)$

c) $\sqrt{74}\,(x+3)(x-2)^2(x-4)^2$

Content: - Do M8.* slides
- then if time game of plotting these-

VW8F1

Ann: - KC opens
L Do way before quiz

- Aleks test review
L Do, Aleks adaptive, you may have skipped some prob types.

- T3 - 17^{th} + 18^{th}

Mod 9 preview - not on test

graph $f(x) = \dfrac{(x^2-4)(x-1)^3(x+1)}{(x^2+3x+2)(x+3)^2}$

$= \dfrac{(x-2)(x+2)(x-1)^3(x+1)}{(x+2)(x+1)(x+3)^2}$

$f|_{x\neq -2,-1} = \tilde{f}$ $\quad \tilde{f}(x) = \dfrac{(x-2)(x-1)^3}{(x+3)^2}$; $x \neq -2, -1$

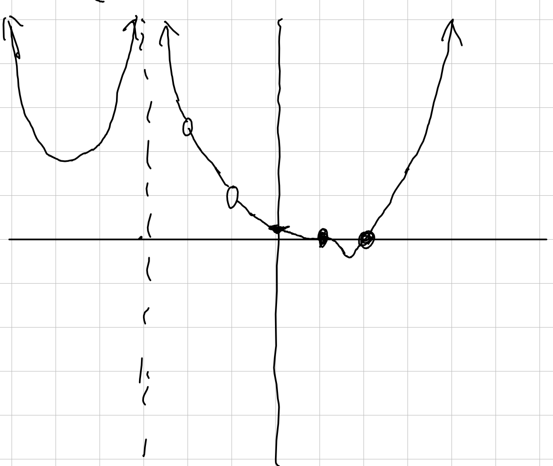
roots: 2m1, 1m3
poles: -3m2
holes$_x$: -2, -1
EB: $\sim + x^{4-2} = x^2$

y-int: $\tilde{f}(0) = \dfrac{-2(-1)^3}{3^2} = \dfrac{2}{9}$
also can see hold hold

$\tilde{f}(-1) = \dfrac{-3\cdot(-2)^3}{2^2} = 6$

$\tilde{f}(-2) = \dfrac{-4\cdot(-3)^3}{1^2} = 4\cdot 27 = 108$

holes: (-1, 6), (-2, 108)



Content: Mod 3.*, wiki

W9 W1

Test review day

preview 1) f(x) has graph

write it's eqn.

$f(x) = a(x+1)(x-1)^3(x-2)^2$

$-2 = a(1)(-1)^3(-2)^2$

$a = \frac{-2}{(-1)(4)} = \frac{1}{2}$

$f(x) = \frac{1}{2}(x+1)(x-1)^3(x-2)$