Ann: Test 2 Monday

5 from yesterday)

$H(x) = ((6x-2))^2$; $f, g \neq id$ s.t. $f \circ g = H$

P
E
M
D
A
S

$$\overbrace{[(x \mapsto x^2) \circ (x \mapsto x-2)}^{f} \circ \overbrace{(x \mapsto 6x)}^{g} \leftarrow](x)$$

$$[\underbrace{(x \mapsto x^2)}_{f} \circ \underbrace{(x \mapsto x-2) \circ (x \mapsto 6x)}_{g}](x)$$

$g(x) = 6x - 2$

$f(x) = x^2$

Composition:

Defintion: Suppose f:A->B and g:B->C, then gof:A->C is the function
(gof)(x)=g(f(x)) "g (post) composed with f"
"f pre-composed with g"

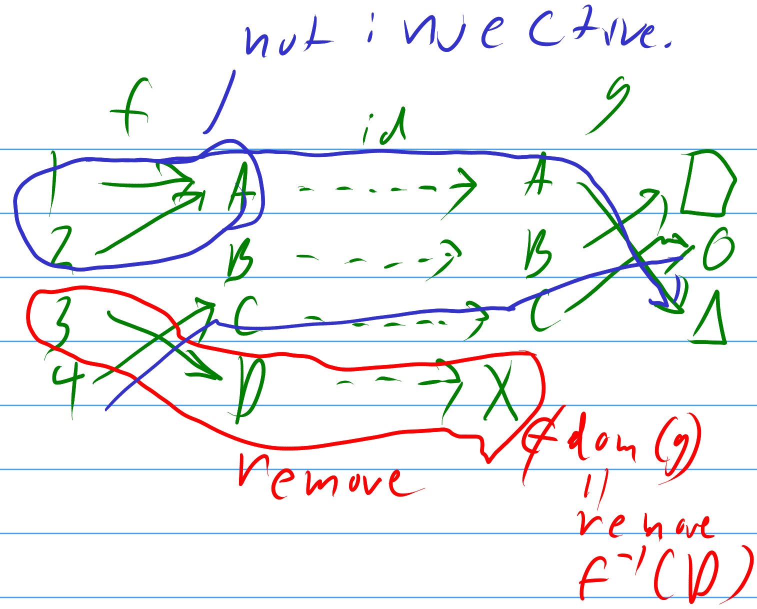
$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & B \arrow[r, "g"] & C \end{tikzcd}$$

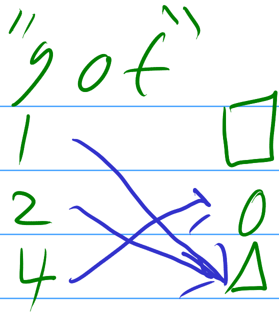
$f: f^{-1}(C) \to C$

<u>Here</u> if $f: A \to B$ and $g: C \to D$ w/ $C \subset B$, then

$$\text{"}g \circ f\text{"} = g \circ (f|_{f^{-1}(C)})$$

ex)





ex) $f: \mathbb{R} \to \mathbb{R}$ $\quad$ $g: [0, \infty) \to \mathbb{R}$

$x \mapsto x^2 - 1$ $\quad$ $x \mapsto \sqrt{x}$

landing

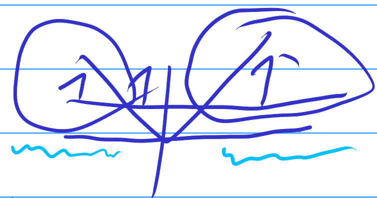
$\Downarrow$

"g∘f"?

$x^2 - 1 \geq 0$

$\sqrt{x^2} \geq \sqrt{1}$

$|x| \geq 1$



domain: $(-\infty, -1] \cup [1, \infty)$

rule: $(g \circ f)(x) = g(f(x)) = \sqrt{x^2 - 1}$

ex) $f(x) = (x+1)(x-2)$; $g(x) = \frac{1}{x}$

$f: \mathbb{R} \to \mathbb{R}$ $\qquad$ $g: \mathbb{R} \setminus \{0\} \to \mathbb{R}$

need $(x+1)(x-2) \neq 0$

$x \neq -1, 2 \Rightarrow -1 < 2$

g∘f dom: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

Today: Inverses of functions

Definition: g:B->A is the inverse of f:A->B if gof=id_A and fog=id_B. Denote g by f^{-1}

f^{-1}

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[loop right, "id_B"] \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

$id(x)=x$

Fact: Inverses are unique

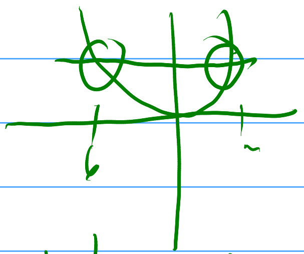
Definition: A function is invertable if it has an inverse

A function is invertable if and only if it is bijective (both injective and surjective)

Definition: A function is surjective if its codomain equals its range/image.
Also called onto.

Definition: A function is injective if whenever f(a)=f(b), then a=b. also called 1-1.

non-ex)



$f(a)=f(b)$
yet $a \neq b$

Fails Horizontal line test.

ex)

only 1 - intersection.