Financial Applications

1. Let's say you put **$100** into a savings account that earns **3% annual interest** compounded **annually**.

a) How much interest would you earn in the first year?

b) Now add that amount to your principal. How much interest would you earn on this new amount in the second year?

c) Use the table to repeat the process five times. How much is in the account after five years

d) How much (A) would be in the account after t years?

$A =$

Year	Use exponents	End of year
1	$100(1+.03)^1$	$\$103$
2	$100(1+.03)^2$	
3		
4		
5		

2. The formula you have derived works if you compound the interest once per year (annually). Now what would you have to do with this formula if the same interest rate was:

a) Compounded monthly?

b) What about semi-annually?

c) What about $\boldsymbol{n}$ times per year?

Compound Interest Formula:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

	What the variable represents
A	
P	
r	
t	
n	

3. Using this formula, how much money would you have using the same scenario above if you compounded interest every second?

Start here

Compounding Continuously

Eulers Constant

The ~~**Natural Number**~~ is a constant like π that is so special it has its own letter, $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \cong 2.71$. This is a very common base in exponential functions. It's just a number! Compare this to your equation above. This is what happens when we compound *infinitely many times* per year. We call that ***compounding <u>continuously</u>***. If we do some fancy algebra using the definition of $\boldsymbol{e}$ we get:

$$A = Pe^{rt}$$

takes Calculus

L'Hopitals rule + $e^{\ln}$ trick

Lab 10.3 – Exponential Applications

groups of 3, each do one,

Effective Rate or Annual Percentage Yield (APY)

me

Monotonically increasing and bounded

4. Find the accumulated value of **$1** after **1 year** at an annual interest rate of **7%** if the money is compounded at the following frequencies. Give answers to at least four decimal places.			
Semiannually	**Quarterly**	**Monthly**	**Continuously**
$\left(1+\frac{.07}{2}\right)^2$	$\left(1+\frac{.07}{4}\right)^4$	$\left(1+\frac{.07}{12}\right)^{12}$	$\lim_{n\to\infty}\left(1+\frac{.07}{n}\right)^n$
Now **subtract the $1** you invested in each case. How much money did you actually earn? (leave the answer to five decimal places)			
$	$	$	$
What **percentage** did you actually earn in each scenario? (leave the answer to three decimal places)			
%	%	%	%
This is called the "**effective rate**". How could this be useful?		Is it better for the saver to have interest compounded more or less often?	

Other Applications of Continuous Growth and Decay

Recall the formula for continuously compounding interest. It turns out that it can be used to model many things that grow or decay exponentially. Since e is a positive number greater than 1, we have to rely on the exponent to tell whether the population is growing or decaying.

Recall $a^{bc}=(a^b)^c$ so $A=A_0e^{rt}=A_0(e^r)^t$; i.e. base $=e^r$

If $r>0$ then $A=A_0e^{rt}$ represents (growth / decay) If $r<0$ then $A=A_0e^{rt}$ represents (growth / decay)

relative rate

1. The number of bacteria in a certain population is predicted to increase according to a *continuous exponential growth model*, at a relative rate of 18% per hour. Suppose that a sample culture has an initial population of 88 bacteria. Find the population predicted after two hours, according to the model.

$A(t)=88e^{.18t}$; $A(2)=88\cdot e^{.18(2)}=88*e\wedge(.18*2)$ in calc

near π

2. The mass of a radioactive substance follows a *continuous exponential decay model*. A sample of this radioactive substance has an initial mass of 342 kg and decreases continuously at a relative rate of 19% per day. Find the mass of the sample after four days.

$M(4)=342e^{-.19(4)}$