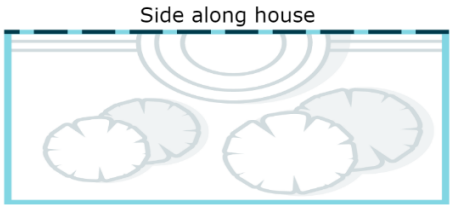
# The Vertex

1. Carlos has 360 meters of fencing. He will use it to form three sides of a rectangular garden. The fourth side will be along a house and will not need fencing. One of the sides is length $x$ (in meters).
2. .

Step 1: Write a function $A(x)$ that represents the enclosed area in terms of $x$ (in meters).




$A(x) = x(360-2x) = -2x^2+360x$

$\ell + 2x = 360$

$\ell = 360 - 2x$

vertex = x coord

roots $= 0, 180$, by symmetry, $V_x = \frac{0+180}{2} = 90$

Step 2: Find the vertex using the vertex formula: $x = -\frac{b}{2a}$

$x = \frac{-360}{2(-2)} = \frac{360}{4} = 90$

Step 3: Find the dimensions that maximize the enclosed area. What is the maximum area?

$x = 90$

$\ell = 360 - 90(2) = 180$

max area: $90 \cdot 180 \text{ m}^2$

$= 2\cdot(100-10)^2$

$= 2\cdot 10^2(10-1)^2$

$= 200(10^2 - 20 + 1)$

$20000 - 4000 + 200 = 16{,}200 \text{ m}^2$.

## General (or Standard) Form

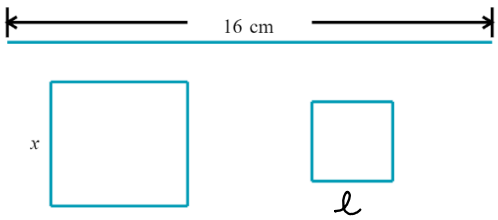
Quadratic functions in **general form** look like this: $f(x) = ax^2 + bx + c$

a) The vertex can be found using the **vertex formula**: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

b) This is just saying find $x = -\frac{b}{2a}$, then plug it back into the function to find $y$.

**You try…**

A wire that is 16 centimeters is cut into two pieces, and each piece is bent to form the shape of a square. One square has side length $x$ cm.




a. Find a function that gives the total area, $A(x)$ enclosed by both squares in terms of $x$.

$A(x) = x^2 + (4-x)^2$

$4x + 4\ell = 16 \Rightarrow \ell = \frac{16-4x}{4} = 4-x$

$= x^2 + (x-4)^2 = 2x^2 + (-2)(4)x + 16$

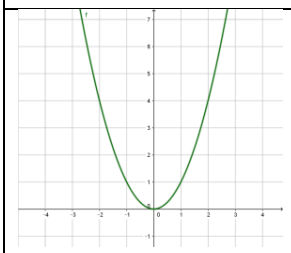
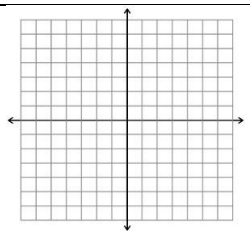
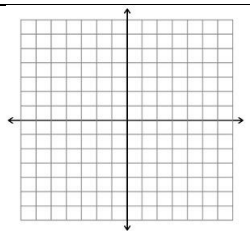
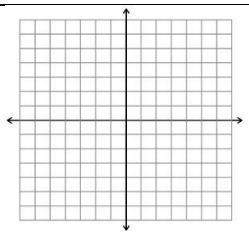
b. Find the side length that minimizes the total area of the two squares.

$\text{argmin}(A) = -\frac{-8}{2(2)} = \frac{8}{4} = 2$

c. What is the minimum area enclosed by the two squares?

$\min(A) = A(\text{argmin}(A)) = A(2) = 2^2 + (4-2)^2 = 2\cdot 2^2 = 8$

## Vertex Form

| $f(x) = x^2$ | Function: | $g(x) = -(x-3)^2 + 2$ | $h(x) = (x-3)^2 - 1$ | $j(x) = -(x+1)^2 + 3$ |
|---|---|---|---|---|
|  | Use transform-ations to graph the other functions. |  |  |  |
| $(0,0)$ | Vertex | $(3,2)$ | $(3,1)$ | $(-1,3)$ |
| $x = 0$ | Axis of symmetry | | | |
| Minimum | Maximum or minimum? | | | |
| Minimum is 0 | What is the max/min? (y-value) | | | |
| occurs at $x = 0$ | Where does the max/min occur? (x-value) | | | |
| $(-\infty, \infty)$ | Domain | | | |
| $[0, \infty)$ | Range | | | |

3. Each of the above functions are in **vertex form:** $\boldsymbol{f(x) = a(x-h)^2 + k}$. How can you use this form to find the vertex?

   The vertex is: ( $h$ , $k$ ) "$(V_x, V_y)$"

4. Look at the equations of the functions that have a maximum. What do they have in common? What about the ones that have a minimum? What rule can you come up with based on this observation?

   max: $a < 0$ min: $a > 0$

5. How can the vertex help you find the maximum/minimum?

   extrema @ $(V_x, V_y)$

6. How can the vertex help you find the axis of symmetry?

   $x = V_x$

7. What does the vertex have to do with the range of the function?

   $\text{Range}(f) = \begin{cases} [k, \infty) & a > 0 \\ \{k\} & a = 0 \\ (-\infty, k] & a < 0 \end{cases}$

**Vertex Form**

All of the functions above are in **Vertex form**:

$f(x) = a(x-h)^2 + k$ where $(h, k)$ is the vertex.

Furthermore the vertex is always the **minimum** (if $a$ is positive) OR the **maximum** (if $a$ is negative)