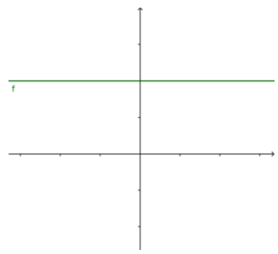
The Library of Functions

The Constant Function:

$f(x) = 1$

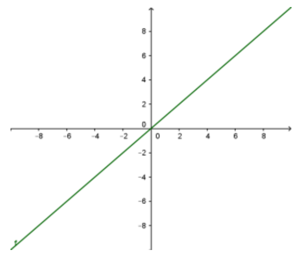
Domain: $(-\infty, \infty)$ Range: $\{1\}$



The Identity Function:

$f(x) = x$

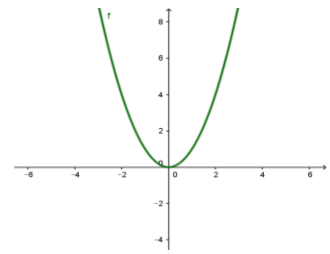
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$



The Quadratic Function:

$f(x) = x^2$

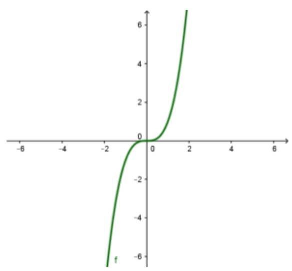
Domain: $(-\infty, \infty)$ Range: $[0, \infty)$



The Cubic Function:

$f(x) = x^3$

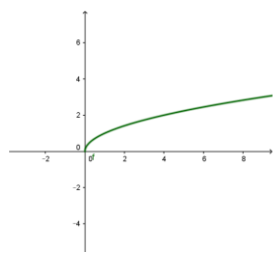
Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$



The Square Root Function:

$f(x) = \sqrt{x}$

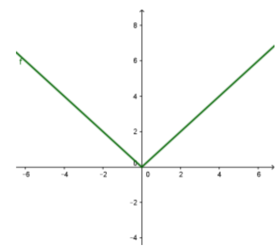
Domain: $[0, \infty)$ Range: $[0, \infty)$



The Absolute Value Function:

$f(x) = |x|$

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

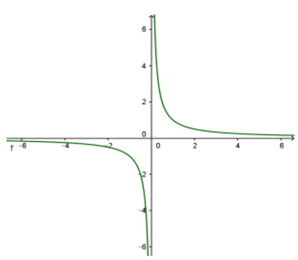


The Reciprocal Function:

$f(x) = \dfrac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

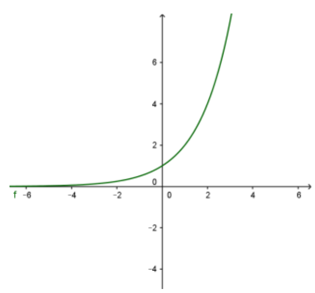
Range: $(-\infty, 0) \cup (0, \infty)$



The Exponential Function:

$f(x) = b^x \qquad (b > 0, b \neq 1)$

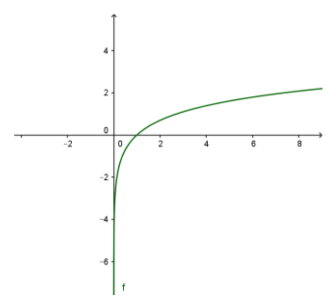
Domain: $(-\infty, \infty)$ Range: $(0, \infty)$



The Logarithmic Function:

$f(x) = \log_b x \qquad (b > 0, b \neq 1)$

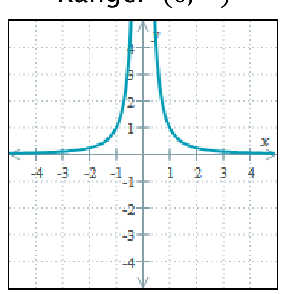
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$



$f(x) = \dfrac{1}{x^2}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(0, \infty)$



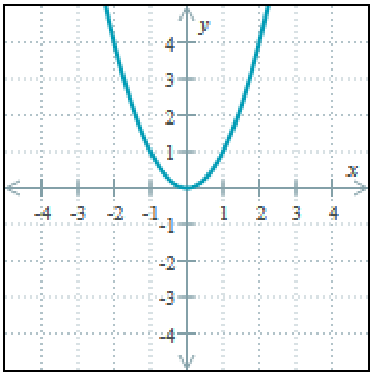
VERTICAL		
Graph	***Function Notation***	***Point***
"up"	$g(x) =$	
"down"	$g(x) =$	
"Stretching"	$g(x) =$	
"Shrinking"	$g(x) =$	
HORIZONTAL		
Graph	***Function Notation***	***Point***
"left"	$g(x) =$	
"Right"	$g(x) =$	
"Shrinking"	$g(x) =$	
"Stretching"	$g(x) =$	
REFLECTIONS		
Graph	***Function Notation***	***Point***
"Across the x-axis"	$g(x) =$	
"Across the y-axis"	$g(x) =$	

Examples:

Let $f(x) = x^2 \quad g(x) = \lvert x \rvert \quad h(x) = \sqrt{x}$		
Horizontal shift to the right 1 & vertical shift up 2.	Vertical stretch by a factor of 2.	Horizontal shrink by a factor of 2.

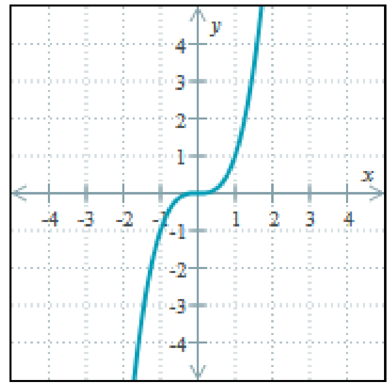
For the following problems, the graph of f is drawn. Use the graphing feature on your calculator to find out how the following transformations affect the graphs of these functions. Then come up with a general rule on the right that applies to any number c.

1. If $f(x) = x^2$
   a. Find $f(x+3) =$
   b. Sketch the graph of the transformed function:



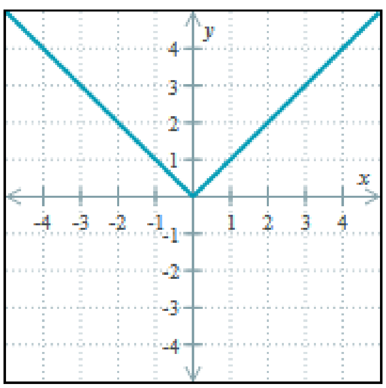
   c. **The rule for $f(x+c)$ is: If you *add* a number directly to x, you shift the graph to the ______________________ that many units.**

2. If $f(x) = x^3$
   a. Find $f(x-2) =$
   b. Sketch the graph of the transformed function:



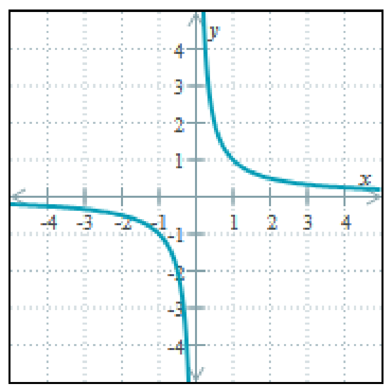
   c. **The rule for $f(x-c)$ is: If you *subtract* a number directly from x, you shift the graph to the ______________________ that many units.**

3. If $f(x) = |x|$,
   a. Find $f(x) + 1 =$
   b. Sketch the graph of the transformed function:



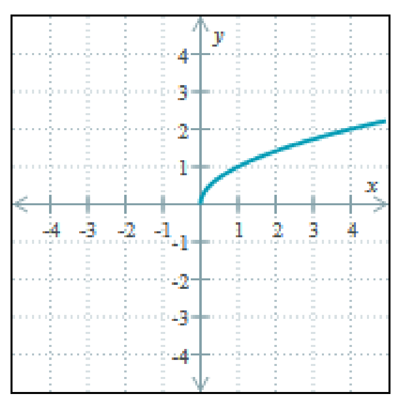
   c. **The rule for $f(x) + c$ is: If you *add* a number to the entire function it shifts the graph _______________ that many units.**

4. If $f(x) = \frac{1}{x}$
   a. Find $f(x) - 2 =$
   b. Sketch the graph of the transformed function:

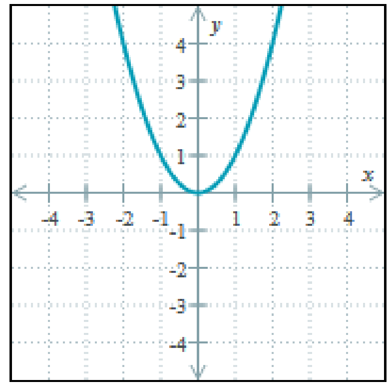


   c. **The rule for $f(x) - c$ is: If you subtract a number from the entire function, it shifts the graph ______________ that number of units.**

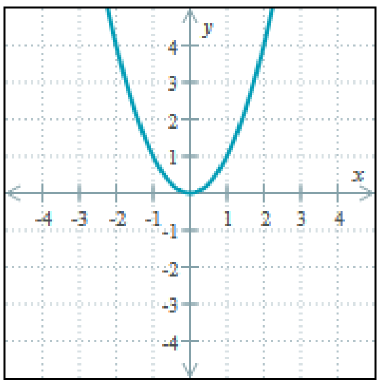
5. If $f(x) = \sqrt{x}$
   a. Find $f(-x) =$
   b. Sketch the graph of the transformed function:
   c. **The rule for $f(-x)$ is: If you make the x negative, it _______________ the graph over the ___-axis.**



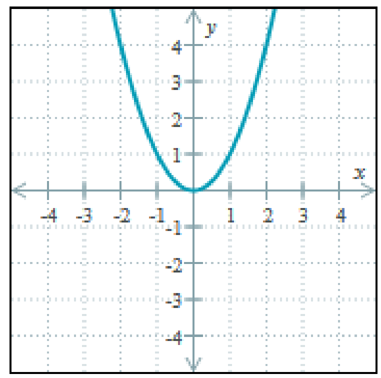
6. If $f(x) = x^2$
   a. Find $-f(x) =$
   b. Sketch the graph of the transformed function:
   c. **The rule for $-f(x)$ is: If you make the entire function negative, it ________________ the graph over the ____-axis.**



7. If $f(x) = x^2$
   a. Find $f(2x) =$
   b. Sketch the graph of the transformed function:

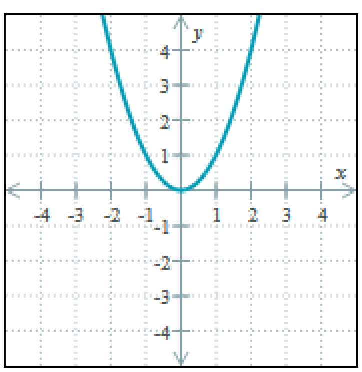


8. If $f(x) = x^2$
   a. Find $f\left(\frac{1}{2}x\right) =$
   b. Sketch the graph of the transformed function:

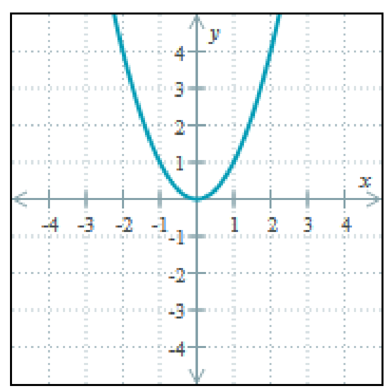


c. **The rule for for $f(c \cdot x)$ is: If you multiply x by $c > 1$ the graph is __________________ horizontally.**
If you multiply x by $0 < c < 1$ the graph is __________________ horizontally

9. If $f(x) = x^2$
   a. Find $3f(x) =$
   b. Sketch the graph of the transformed function:



10. If $f(x) = x^2$
    a. Find $\frac{1}{3}f(x) =$
    b. Sketch the graph of the transformed function:



c. **The rule for $c \cdot f(x)$ is: If you multiply $f(x)$ by $c > 1$ the graph is __________________ vertically.**
If you multiply $f(x)$ by $0 < c < 1$ the graph is __________________ vertically.

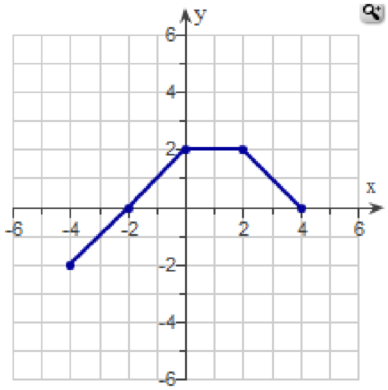
Explain how you can tell the difference in a horizontal transformation and a vertical transformation if given the equation of the function.

Transformation Examples:

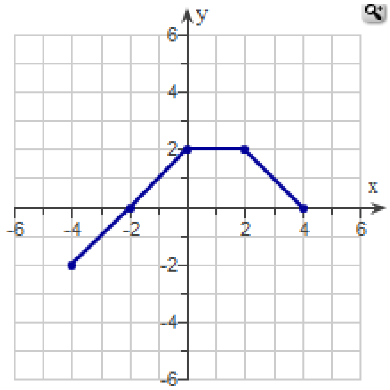
1. The graph of function f is given below. Sketch the graphs of the other functions, each of which is a transformation of the graph of function f.

Reflections

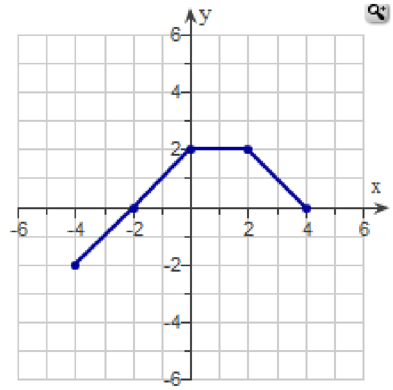
$Let\ y = f(x)$



a) $g(x) = f(-x)$

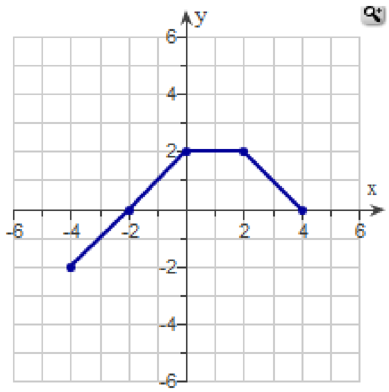


b) $h(x) = -f(x)$

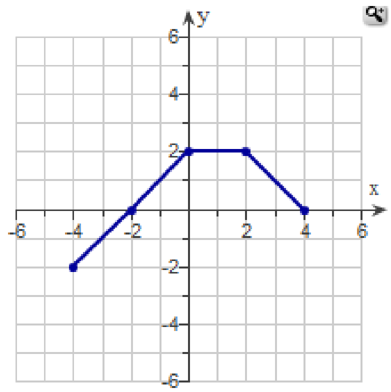


Translations

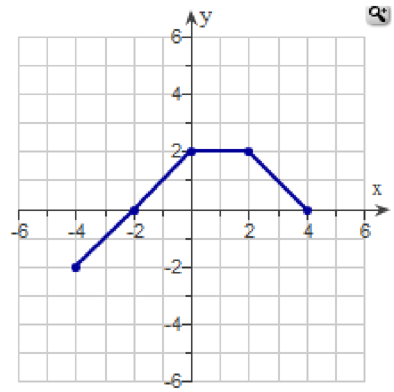
c) $j(x) = f(x) + 2$



d) $k(x) = f(x - 1)$



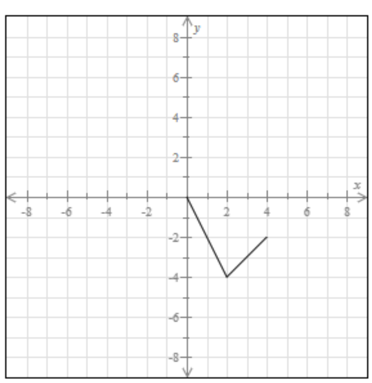
e) $s(x) = f(x + 2) + 3$



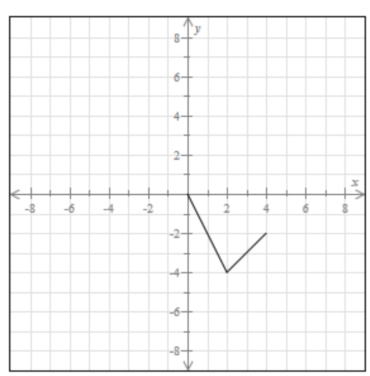
Horizontal Stretching and Shrinking

2. The graph of function f is given below. Sketch the graphs of the other functions, each of which is a transformation of the graph of function f.

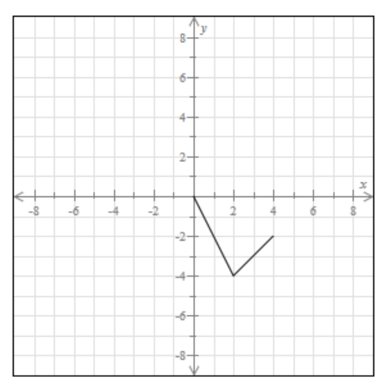
$Let\ y = f(x)$



a) $g(x) = f(2x)$

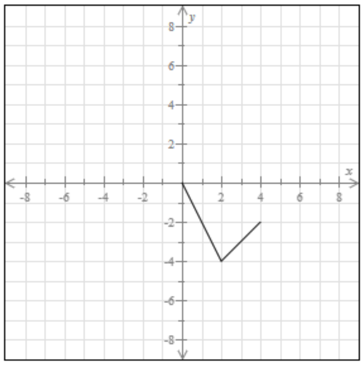


b) $h(x) = f\left(\frac{1}{2}x\right)$

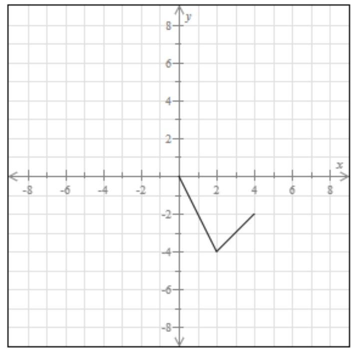


Vertical Shrinking and Stretching:

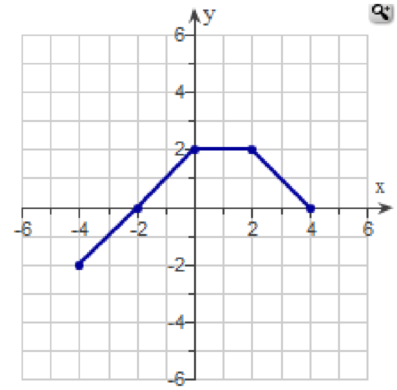
$j(x) = 2f(x)$



a) $g(x) = \frac{1}{2}f(x)$



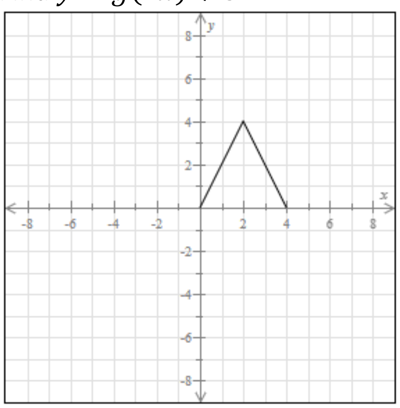
b) $h(x) = 2f(x)$



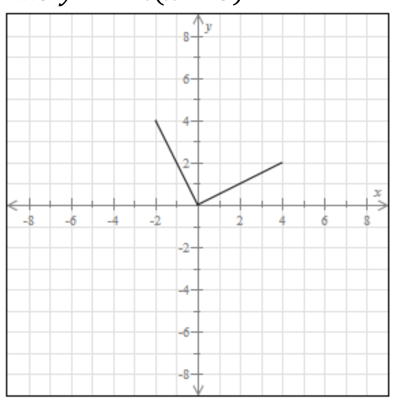
Multiple Transformations:

Order Matters!

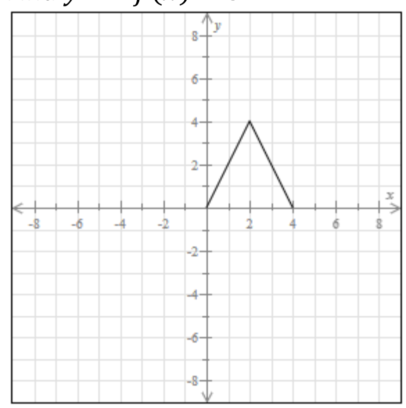
$y = g(x)$ is shown.
Find $y = g(2x) + 3$



$y = h(x)$ is shown.
Find $y = -h(x - 3)$



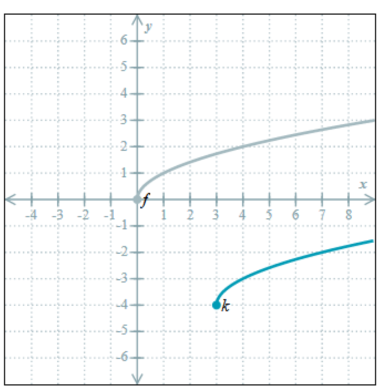
$y = f(x)$ is shown.
Find $y = 2f(x) - 3$



Write the Equation:

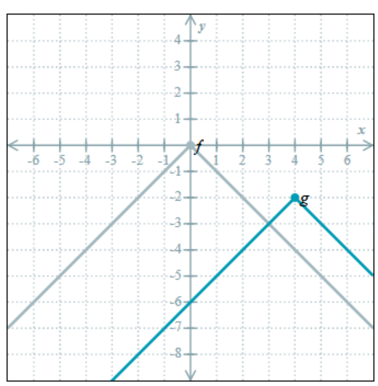
If $f(x) = \sqrt{x}$,

then $k(x) =$



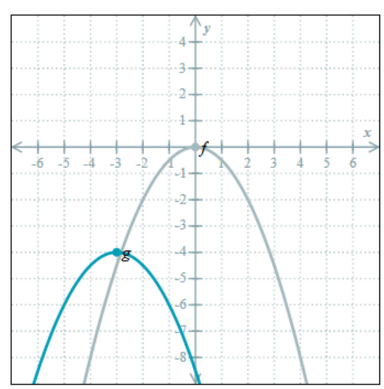
If $f(x) = -|x|$,

then $g(x) =$



If $f(x) = -\frac{1}{2}x^2$,

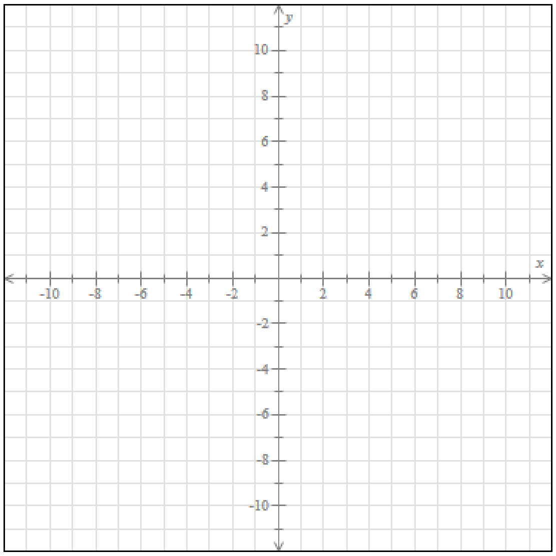
then $g(x) =$

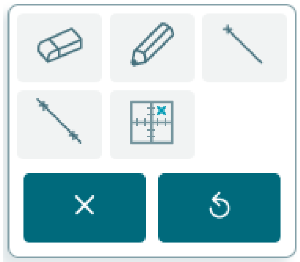


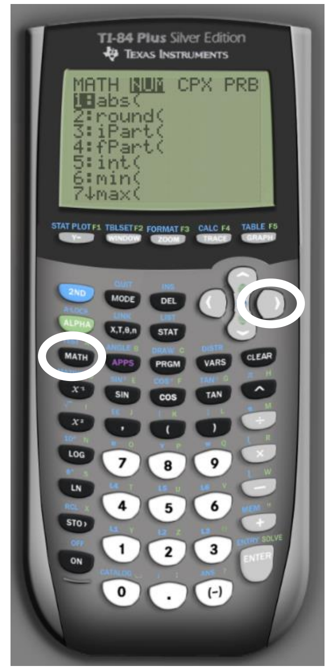
Graphing Absolute Value Equations:

Graph the equation:

$$y = -4|x + 2| + 3$$



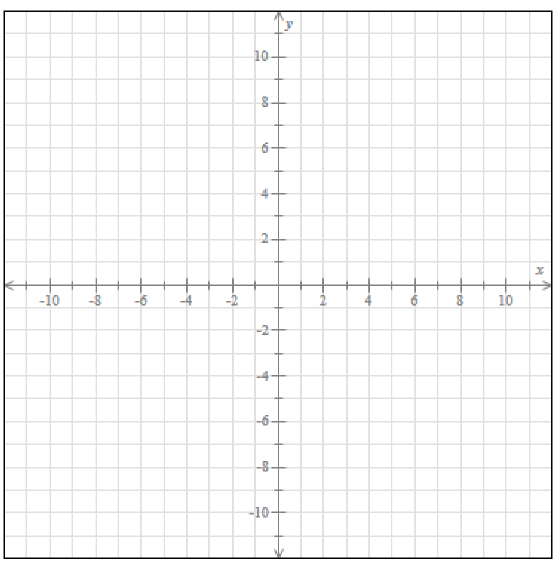




Graphing Quadratic Equations:

Graph the equation:

$$y = (x + 4)^2 - 3$$



Graphing Square Root Equations:

Graph the equation:

$$y = 3\sqrt{x - 2}$$