College Algebra

w1d1p1

Ann: BB gradesync is wrong, use Aleks for now

first: a few minutes of free talk, then move spots

snd: Groups of 4

- designate talking stick
  - in 2 minutes, give name, major, fears, and expectations for class
  - only talking stick person speaks
  - rotate till done

3rd: Quick walk through on BB, Syllabus, Aleks

4th: Get into rules/Axioms of Algebra

Notation: $\forall$ - for all, $\exists$ - there exists, $\in$ - in, s.t. - such that

$\exists!$ - exists a unique, $\mathbb{Z}$ - integers (Zahlen)

$\mathbb{N}$ - natural numbers, iff - if and only if

Group: $(G, *)$ is a group iff

Closure 1) for all x, y in G, $x*y$ is in G.

$\forall x, y \in G$, $x*y \in G$

Associativity 2) $\forall x, y, z \in G$, $(x*y)*z = x*(y*z)$

identity 3) $\exists! e \in G$ s.t. $\forall x \in G$, $x*e = e*x = x$.

invertibility 4) for all x in G, there exists x^{-1} in G such that

$x*x^{-1} = x^{-1}*x = e$

Example groups: $(\mathbb{Z}, +)$, a rubicks cube w1d1p2

Non-examples: $(\mathbb{N}, +)$

Rings: $(R, +, \cdot)$ is a ring if and only if

1) $(R, +)$ is a group
2) $a+b=b+a$ for all a, b in R ($+$ is commutative)
3) $\forall a,b,c \in R$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ ($\cdot$ is associative)
4) (Distributivity, most important) for all a, b, c in R, $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(b+c) \cdot a = b \cdot a + c \cdot a$
5) (unity) $\exists 1 \in R$ s.t. $\forall r \in R$ $r \cdot 1 = 1 \cdot r = r$

Examples: $(\mathbb{Z}, +, \cdot)$, working with remainders (like a clock)

$\mathbb{Q}$ - rationals, $\mathbb{R}$ - reals, $\mathbb{R}[X]$ - real polynomials, $\mathbb{R}(X)$ - real rational functions

Integral Domain (ID): $(R, +, \cdot)$ is an Integral domain if and only if (iff)

1) $(R, +, \cdot)$ is a ring
2) ($\cdot$ is commutative) $\forall a, b \in R$, $a \cdot b = b \cdot a$
3) (Cancelation property) for all a, b, c in R with $a \neq 0$, if $a \cdot b = a \cdot c$ then $b = c$.

Ex) $\mathbb{Z}$, all examples in rings

Field $(F, +, \cdot)$ is a field iff

1) $(F, +, \cdot)$ is an integral domain (ID)
2) for all $x \in F$ with $x \neq 0$, there exists x^{-1} such that $x \cdot x^{-1} = x^{-1} \cdot x = 1$

side note on

Fractions From any integral domain R one may construct its **field of fractions** $Frac(R)$ by

$$Frac(R) = \left\{ \frac{a}{b} : a, b \in R,\ b \neq 0 \right\}$$

and $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

ex) $Frac(\mathbb{Z}) = \mathbb{Q}$

$Frac(\mathbb{R}[x]) = \mathbb{R}(x)$

Now toward a k-Algebra

Ring map (homomorphism): Let A, B be rings. Then $f: A \to B$ is a ring map iff $\forall a, a' \in A$

1) $f(a + a') = f(a) + f(a')$

2) $f(a \cdot a') = f(a) f(a')$

3) $f(1_A) = 1_B$

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K-Algebra Let K be a (Communative) ring. Then A is a K-Algebra iff A is a ring and there exists a ring map $\mu : K$

examples: $\mathbb{C} \rightsquigarrow \mathbb{R} \hookrightarrow \mathbb{C}$ (inclusion)

$\mathbb{Q} \rightsquigarrow \mathbb{Z} \hookrightarrow \mathbb{Q}$ — any ring is a $\mathbb{Z}$-algebra

$\mathbb{R}[x] \rightsquigarrow \mathbb{R} \hookrightarrow \mathbb{R}[x]$

Alternate view: its a ring with an ability to be scaled by a "smaller" ring.

w1d2p1

Day 2

Ann:
- DO IKC
- Do HW1
- Quiz due Sunday
  - requires 80
- Alex and BB syncing weird
  - use Alex for grade
- introduce yourself activity is extra credit participation
- CEA talk to during Ohours or after class
- Ohours: SCEN 224
  MWF - 9:40 - 10:30
  - email / UA success to guarantee i'll be there
  - more to come

Today: exponents

(Preview: 1) simplify $\frac{(7x)^{-2}}{7x^{-3}}$

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2) solve: $-3|v|+3=0$

3) translate: three times a number is its double plus 5.

skip — 4) multiply: $(3x)^2 y^3 x^{-2} y^4$

5) factor: $6w^2+48w-54$

$6(w^2+8w-9)$

$-1 \cdot 9 \to 8$

$6(w-1)(w+9)$

6) solve: $|2y-14|=8$

Content: Exponents

Notation: $(expr)^p$ means $expr \cdot expr \cdot \cdots \cdot expr$ (p times)

where expr is some expression

ex) $5^3 = 5 \cdot 5 \cdot 5$

ex) $(7x+12)^{97} = (7x+12)(7x+12)\cdots(7x+12)$ (97 times)

Properties: 1) $x^1 = x$ 3) $x^0 = 1$

2) $x^{-1} = \frac{1}{x}$ 4) $x^a x^b = x^{a+b}$

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5) $(a \cdot b)^c = a^c \cdot b^c$

6) $(a^b)^c = a^{b \cdot c}$

"Proof" of 4+5: 4) $x^a \cdot x^b = \underbrace{x \cdots x}_{a \text{ times}} \cdot \overbrace{x \cdots x}^{b \text{ times}}$

$= \underbrace{x \cdots x}_{a+b \text{ times}}$

5) $(a \cdot b)^c = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) \cdots$

by associativity and commutivity,

$= \underbrace{a \cdot a \cdot a \cdots a}_{c \text{ times}} \cdot \underbrace{b \cdots b}_{c \text{ times}}$

$= a^c \cdot b^c$

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ex) $\frac{(7x)^4}{7^3 x^{-8}} = \frac{(7x)^4}{1} \cdot \frac{1}{7^3 x^{-8}}$

$\overset{(2)}{=} (7x)^4 (7^3 x^{-8})^{-1}$

$\overset{(5)}{=} 7^4 x^4 \cdot (7^3)^{-1} (x^{-8})^{-1}$

$\overset{(6)}{=} 7^4 x^4 \, 7^{-3} x^8$

$\overset{\text{Comt}(4)}{=} 7^{4+(-3)} x^{4+8}$

$= 7 x^{12}$

BB Mod 1 - notes - Basic, me, you style

L 1-9 as groups (3).

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W1d3p1

Filling in yesterday: Simplifying exponents: a process

Steps	example $\frac{(7x^2)^3}{7y^{-4}x^2}$
1) distribute power across multiplication and multiply powers	$= \frac{7^3 x^6}{7y^{-4}x^2 z}$
2) move denominator (bottom) to numerator (top), negating powers	$= 7^3 x^6 7^{-1} y^4 x^{-2} z^{-1}$
3) combine like terms	$= 7^2 x^4 y^4 z^{-1}$
4) move negative pieces to denominator negating power	$= \frac{7x^4y^4}{z}$

Now 3 Q's from BB, me, them, them

w1d3p2

(P)review:

1) Solve for C: $1 = \frac{5A}{2C} + 3B$

ans?

$\Downarrow \cdot 2C$

$2C = 5A + 6BC$

$\Downarrow$ like on same side

$2C - 6BC = 5A$

$\Downarrow$ distributive prop.

$(2 - 6B)C = 5A$

$\Downarrow$

$C = \frac{5A}{2 - 6B}$

2) $f(x) = 4x^2 + 2x - 1$

evaluate when $x = \frac{1}{2}$

ans) $f(\frac{1}{2}) = 4(\frac{1}{2})^2 + 2(\frac{1}{2}) - 1$

$= 4(\frac{1}{4}) + 1 - 1$

$= 2 + 0$

$= 2$

3) factor: $x^2 + 2x - 8$ LC = 1 ✓

$4 \cdot -2 \rightarrow -2 + 4 \rightarrow 2$

$(x - 2)(x + 4)$

4) translate: the quotient of your score

W1d3p3

remove four and 72 is 84.

ans) $\frac{u-4}{72} = 84$

Content: Absolute value and factoring

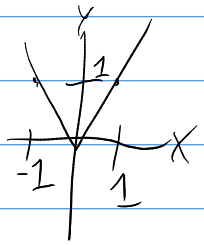
π-prog: (657.) (597.)

Absolute value:

Def: $|-|$ is the absolute value function.

It measures distance from zero.

graph: $y = |x|$



properties: a) $|x| \geq 0$

b) $|ab| = |a||b|$

c) $|a+b| \leq |a|+|b|$

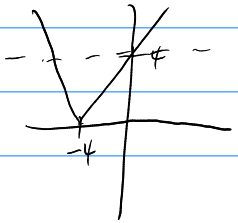
ex) $3|x+4|-2=10$

$3|x+4| = 12$

$|x+4| = 4$

$-x-4=4$ $\quad$ $x+4=4$

$x=8$ $\quad$ $x=0$



w1d3p4

procedure for absvalue:

rules) |expr| = n

a) isolate |·|

b) split into 2 equations if not neg.

−expr = n expr = n

c) solve both

d) answer

ex) $4|x-1|-2=6$

$= 4|x-1| = 8$

$= |x-1| = 2$

$x-1=2$ $-x+1=2$

$x=3$ $-x=1$, $x=1$

$x=3$ or 1

BB Sheet yw 4-5

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Factoring: taking an expression and rewriting it as a product.

process: for ax^2+bx+c,

1) factor out a: $a(x^2+\frac{b}{a}x+\frac{c}{a})$

$= a(x^2+dx+e)$

2) find f, g s.t. $\begin{cases} f\cdot g = e \\ f+g=d \end{cases}$

3) $ax^2+bx+c = a(x+f)(x+g)$

4) combine like terms if applicable.

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ex) $2x^2 + 4x + 2$

$= 2(x^2 + 2x + 1)$

$1 \cdot 1 \to +2 \quad 2 \cdot 2 \checkmark$

$= 2(x+1)(x+1)$

$= 2(x+1)^2$

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BB Factoring quadratics odds fill 10 min.