3F1

Test 2 Notes

Module 3: Functions

Ann:
- retake exam in Nov.
  - only 1 exam, second score counts.
- results for exam 1 open sept 13th.
  - only can open once!
- post exam reflection in BB
  - Spanish,
- kC 2 opens Monday of week 5.
  - IDK adds to Hw.
  - Can lower Hw score
  - treat like IkC.

(P) review: 1) which are functions of x?

a) write $x = \frac{a}{b}$; $f(x) = 6$.

no, $\frac{a}{b} = \frac{2a}{2b}$ but $f(\frac{a}{b}) = 6 \neq f(\frac{2a}{2b}) = 26$

b) $x^2 + y^2 = 4^2$

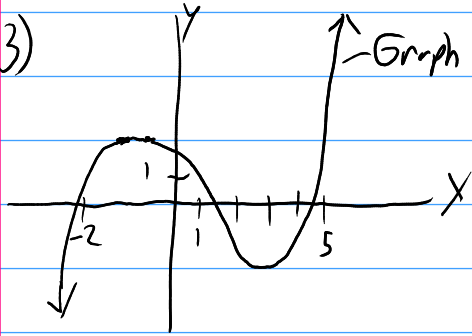
no, $(-2,0), (2,0) \in$ Graph $(x^2+y^2=4^2)$

fails vert line test.

3F2 2) $g(x) = -4x^2 + 5$

$f(z) = z - 2$

find $(g \circ f)(y)$

3) 

find $f^{-1}(1)$

Ans! $f^{-1}(1) = \{x \in \mathbb{R} : f(x) = 1\}$

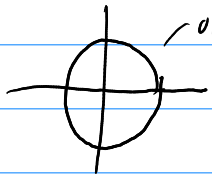
$f^{-1}(1) = \{-2, 1, 5\}$

Content: Functions

Def: A set of ordered pairs $(x,y) \in R \subseteq X \times Y$ is called a relation of X and Y. (Domain, Codomain)

Say xRy if $(x,y) \in R$.

ex) $\{(1,2), (3,2), (3,1)\} \subseteq \mathbb{Z} \times \mathbb{Z}$

ex)  ordered pairs on this graph

ex)

x	y
1	2
1	3
2	2

3F3

Def: A function is a relation F such that each input has only one output.
— "vertical line test"

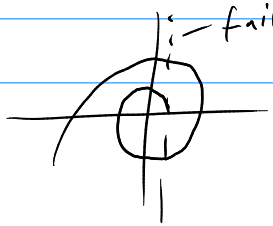
Notation:

$f: A \to B$ or $f(a) = \text{expression}(a)$

- f: function name
- A: domain
- B: codomain
- $a \mapsto \text{expression}(a)$: rule
- domain, codomain implicit or stated

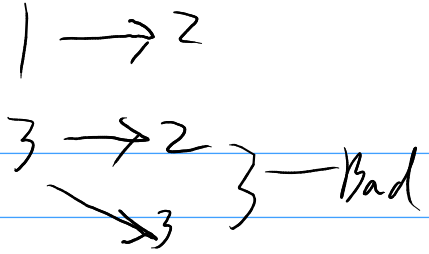
ex) $f: \mathbb{R} \to \mathbb{R}$ or $f(r) = 3r+1$
$r \mapsto 3r+1$

non-ex) $f\left(\frac{a}{b}\right) = a$; note $\frac{a}{b} = \frac{3a}{3b}$ yet $f\left(\frac{3a}{3b}\right) = 3a$.

non-ex) 

non-ex) $\{(1,2), (3,2), (1,2), (3,3)\}$

3F4



Def: The range of $f: A \to B$ is

$$f(A) = \{f(a) : a \in A\}$$
$$= \{b \in B : \exists a \in A \text{ w/ } f(a) = b\}$$

(exists)

"achieved elements in codomain".

BB — MB states class do bot of p1, bot of p2

Telling if equation is function:

$expr_1(x,y) = expr_2(x,y)$ is a function of x if you can isolate y uniquely.

non-ex) $y^2 - x = 4$

$y^2 = 4 + x$

$y = \pm\sqrt{4+x}$, not unique ans per x.

3F5

non-ex) $|y|+x=3$

in	out
0	3
	-3

test: set $x=0$

$|y|+0=3$

$y=\pm 3$

not func.

caution!

ex) $y^2+3x=2(x-4)+x+8$

$y^2+3x=2x-8+x+8$

$y^2+3x=3x$

$y^2=0$

$y=0$

Aug:
- test 1 graded
  - Avg 85ish
- no test ret. (yet)
- Today wk: 3
  - due 6 on Thursday
- KC 2 next week
- test make up day Nov. 18th

BB M3 notes p3 top.

4M1

(P)review: 1) is $(y-4)^2-4=x$ a func of x? (no) of y? (yes)

2) $h(x)=3x^2+5$

what is $(h \circ h)(x)$?

$=h(h(x))=3(3x^2+5)^2+5$

$=3((3x^2)^2+2\cdot 3\cdot 5x^2+5^2)+5=27x^4+90x^2+80$

4/12

3) $g(x,y) = 7x + 2y$

what is $g(\frac{1}{7}, 4)$?
(x-slot, y-slot)

$= 7 \cdot \frac{1}{7} + 2 \cdot 4 = 1 + 8 = 9$

4) (previous) Domain of $\frac{\sqrt{x^2-4}}{\sqrt{9-x^2}}$ ($= f$, $= g$)

$\text{Dom}(\frac{f}{g}) = \text{Dom}(f) \cap \text{Dom}(g) \setminus g^{-1}(0)$ (domain)

$\text{Dom}(\sqrt{x^2-4})$: $\text{Dom}(\sqrt{x}) = [0, \infty)$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$(-\infty, 2] \cup [2, \infty)$

$\text{Dom}(\sqrt{9-x^2})$:

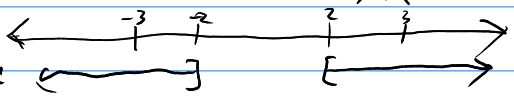
$9 - x^2 \geq 0$

$9 \geq x^2$

$[-3, 3]$

$g^{-1}(0)$: $0 = \sqrt{9-x^2}$

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$



Dom (Num):

Dom (Denom):

Denom$^{-1}(0)$:

Num $\cap$ Dom $\setminus$ Denom$^{-1}(0)$:

4M3 Ans: $(-3,-2]\cup[2,3)$

Now: wiki 3

Module 4

Functions (1) Dom trick 2) AROC

4W1 Ann: - kC 2 opens Monday
- do as soon as can, treat like initial.
- keeps all other activity from being done.

(P) review: 1) What is the average rate of change of $f(x)=x^2+x-1$ from 2 to 4?

Fact: $AROC(g,a,b)=\frac{g(b)-g(a)}{b-a}$

so $AROC(g,2,4)=\frac{(4^2+4-1)-(2^2+2-1)}{4-2}$

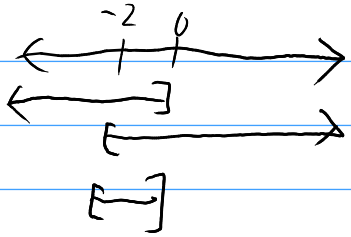
2) what is the domain of $f(x)=\sqrt{-x}-\sqrt{x+2}$

Fact: $Dom(g+h)=Dom(g)\cap Dom(h)$

$\sqrt{-x}$: $-x\geq 0 \rightarrow x\leq 0$ ie $(-\infty,0]$

4w2 $\sqrt{x+2}$: $x+2 \geq 0$ $[-2, \infty)$

$x \geq -2$



Domain $(f) = [-2, 0]$

3) Domain and range from graph of piece wise

Aleks

Content: Intervals

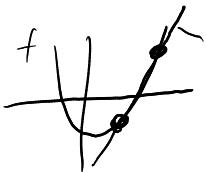
use BB M4 - Funcs P2

almost end on page 4, creating more examples if extra time.

- sec 12 end pg 3 to sec 14

last 3 minutes: AROC

$$AROC(f, a, b) = \frac{f(a) - f(b)}{a - b}$$



Slope of line through points on function.

4w3

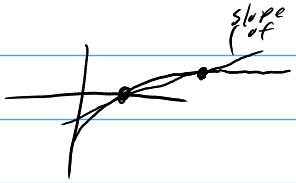
ex)

x	f(x)
1	8
2	12
3	34
7	89

AROC(f, 2, 7)?

$\frac{f(7)-f(2)}{7-2} = \frac{89-12}{7-2}$

ex) $\pi(y) = \sqrt{y} - 4$



AROC(π, 16, 4)?

$\pi(16) = \sqrt{16} - 4 = 0$

$\pi(4) = \sqrt{4} - 2 = -2$

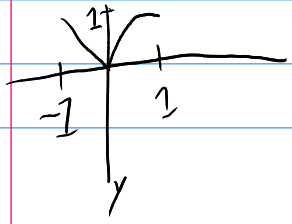
$\frac{0-(-2)}{16-4} = \frac{2}{8} = \frac{1}{4}$

Now wiki 4 (start module 5)

5M1

Start: GO over wiki 4 Rubric

(P)review: 1) f(x) graph



a) key points

X	f(x)
-1	1
0	0
1	1

$(in^{-1}(x), out(y))$ →

X'	g(X')
-1-1=-2	3(-1)-4=-1
0-1=-1	3(0)-4=-4
1-1=0	3(1)-4=-1

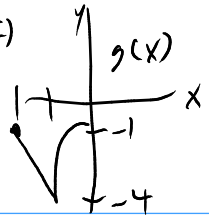
$g(x) = 3f(x-1) - 4$

b) $in(x) = x-1$ Proof

$in^{-1}(x) = x+1$

$out(x) = 3x - 4$

c)



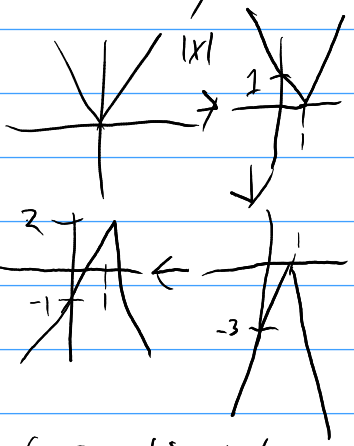
2) Plot $-3|x-1|+2$

1) plant 0 → 1

2) ↕ -axis, scale by 3

3) ↑ 2

"inside acts inversely"



Content: Transformations of functions

5W1

5.2

GO to mid 5 transformation notes on pinenote.

(Preview: 1) Give a linear model for $f(x) = 3x^2 - 4$ from 3 to 6.

what's it's AROC here?

$$\text{AROC} = m_{\parallel} = \frac{f(6)-f(3)}{6-3} = \frac{(3\cdot 36-4)-(3\cdot 9-4)}{6-3} = \frac{3(36-4-(9-4))}{3}$$

$$= 36-9 = 27$$

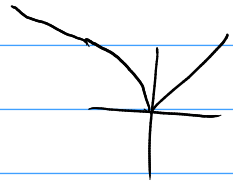
$$y-(3\cdot 9-4) = 27(x-3)$$

5W2

2) IS $(y+3)^2 - x = 25$ a func of y? x?

yes no

3) $f(x) = \begin{cases} x & x \geq 0 \\ \sqrt{-x} & x < 0 \end{cases}$



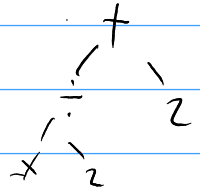
plot

$g(x) = \frac{1}{2} f\left(-\frac{x}{2} + 2\right) + 1$

a)

parent: f

$in(x) = \frac{-x}{2} + 2$



$x = \frac{-in^{-1}(x)}{2} + 2$

$x - 2 = \frac{-in^{-1}(x)}{2}$

$2(x-2) = -in^{-1}(x)$

$-2x + 2 = in^{-1}(x)$

$out(y) = \frac{1}{2}y + 1$

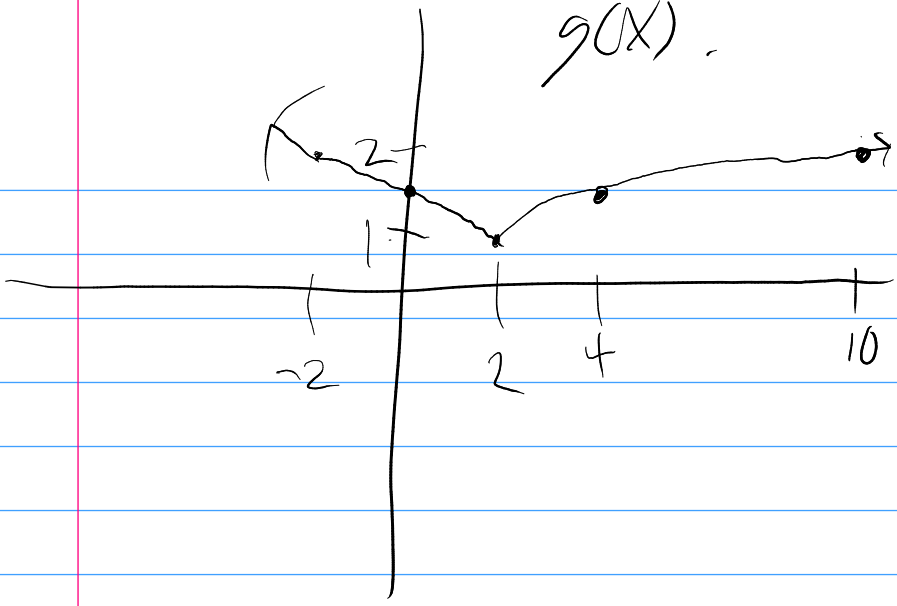
b)

x	$f(x)$
-4	2
-1	1
0	0
1	1
2	2

(in, out) →

x'	$g(x')$
$-2(-4)+2 = 10$	$\frac{2}{2}+1 = 2$
$-2(-1)+2 = 4$	$\frac{1}{2}+1 = 1.5$
$0+2 = 2$	$0+1 = 1$
$-2+2 = 0$	1.5
$-2(2)+2 = -2$	2

5W3



for 15min, do in-class Alekes practise transformation.

rest of class: practise graphing as game.