Ann: Test 2 today + Tuesday W6M1

6.1 piecewise

(P)review:

1) Rewrite w/ no neg. exponents

$$\frac{2^{-48}(z^3y^{-4})^2}{8^{17}z^0(xzy)^{-3}}$$

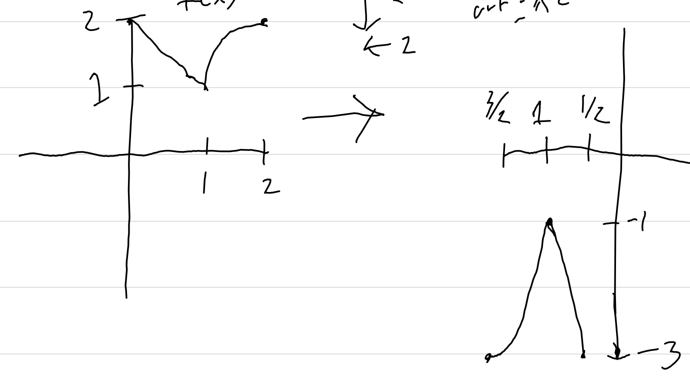
2) $x = 7y - 3$ find $\perp$ line through $(4,8)$

3) Is $\sqrt{x}+\sqrt{y} = 4z$ a function of (x,z)?

name x: $y^2 = x^2 - 1$

4 me) describe the transformation:

4 you)

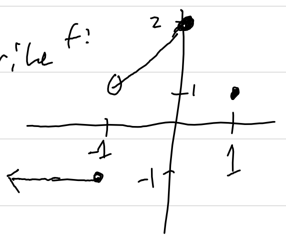


$g(x) = h(f(k(x)))$

$= 2f($

Preview:

5) describe f:



Ans: $f(x) = \begin{cases} -1 & x \leq 1 \\ x+2 & -1 < x \leq 0 \\ 1 & x = 1 \end{cases}$

6) $f(x) = \dfrac{\sqrt{x^3+3}}{4}$; find $h, g \neq id$ s.t. $h \circ g = f$.

Content: Piecewise defined functions

Def: If the rule for $f: A \to B$ on some $U \subset A$ and $A \setminus U$ is different, then f is piecewise defined.

ex)* $f: \mathbb{R} \to \mathbb{R}$, where $f(x) = x^2$ on $(0,1)$ and $f(x) = x+1$ elsewhere.

Notation: If $f: A \to B$ is piecewise defined on U its rule can be written as

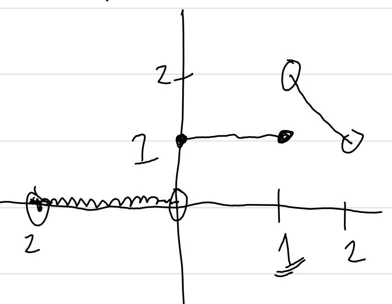
$$f(x) = \begin{cases} g(x) & x \in U \\ h(x) & x \notin U \ (\text{or } x \in A \setminus U) \end{cases}$$

ex)* $f(x) = \begin{cases} x^2 & x \in (0,1) \\ x+1 & x \in (-\infty,0] \cup [1,\infty) \end{cases}$

or

$f(x) = \begin{cases} x+1 & x \leq 0 \\ x^2 & 0 < x < 1 \\ x+1 & x \geq 1 \end{cases}$

you) write rule for



Now) 6.1

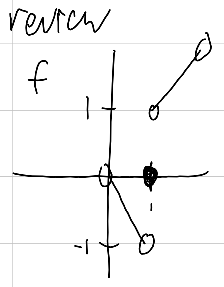
finish: Alex time.

w6w1

Composition

(P) review

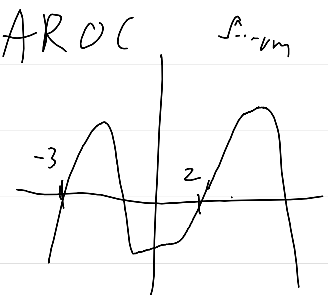
1) f



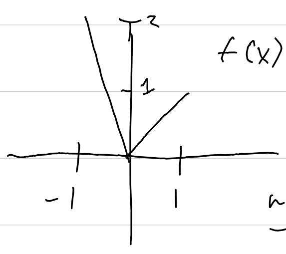
write the rule for f.

2) $\sqrt{y} = \sqrt{x} - 1$ is it a function of x? domain?

3) AROC from $x = -3$ to $x = 2$ on



4 you)



$f(x)$; draw $-f(\frac{1}{2}x) + 2$

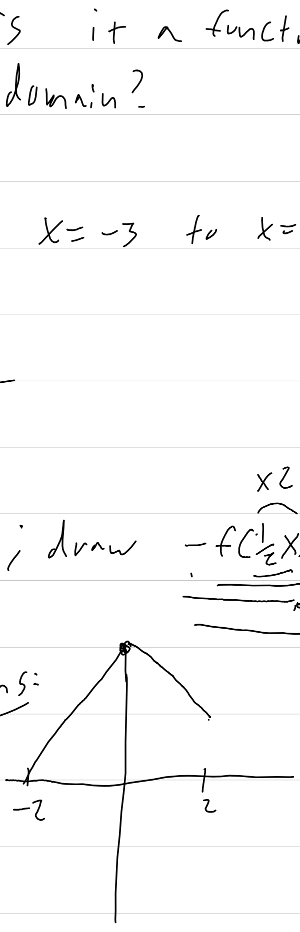
x2 on x

in

out

$\uparrow 2$

ans:



5) $H(x) = (6x-2)^2$

find f, g ≠ id s.t. $f \circ g = H$.

$$(6\underline{x} - 2)^2 = \big(\overbrace{(x \mapsto x^2)}^{f} \circ \overbrace{(x \mapsto x-2) \circ (x \mapsto 6x)}^{g}\big)(x)$$

read right to left.

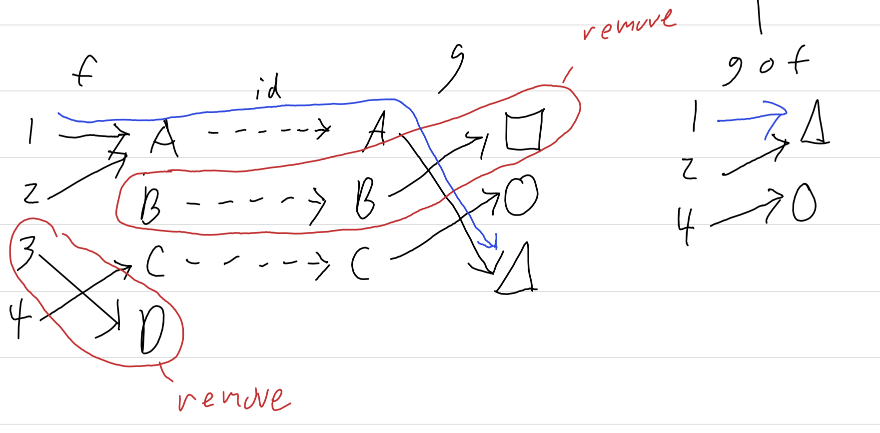
Content: Composition.

Def: Suppose $f: A \to B$ and $g: B \to C$, then $g \circ f : A \to C$ is the function $(g \circ f)(x) = g(f(x))$ "g (post) composed with f" "f pre composed w/ g"

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & B \arrow[r, "g"] & C \end{tikzcd}$$

Here: if $f: A \to B$, $g: C \to D$ where $C \subset B$, then "$g \circ f$" $= g \circ (f|_{f^{-1}(C)})$

ex)



ex) $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2 - 1$ $g: \mathbb{R}_{\geq 0} \to \mathbb{R}$, $x \mapsto \sqrt{x}$

"$g \circ f$" $= g \circ \tilde{f}$ where $\tilde{f}$ is f with restricted domain so that $\tilde{f}$'s range is $\mathbb{R}_{\geq 0}$

ie, $x^2 - 1 \geq 0 \Rightarrow |x| \geq 1 \Rightarrow x \in (-\infty, -1] \cup [1, \infty)$

"$g \circ f$" $: (-\infty, -1] \cup [1, \infty) \to \mathbb{R}$

$x \mapsto \sqrt{x^2 - 1}$

you) $f(x) = (x+1)(x-2)$; $g(x) = \frac{1}{x}$

find $g \circ f$'s rule and domain.

$\boxed{\text{Now BB 6.2}}$

Ann: - test Monday W6F1

Content: #5 + Content of yesterday (remove BB)

Today:

Content: Inverses of functions.

notation: $f \circ g$ is the function $f(g(x))$; $id(x) = x$.

Def: $g: B \to A$ is the inverse for a function $f: A \to B$ if $g \circ f = id_A$ and $f \circ g = id_B$.

Denote g by f^{-1}.

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[l, bend left, "f^{-1}"] \arrow[loop right, "id_B"] \end{tikzcd}$$

Fact: Inverses are unique.

Def: A function is invertable if it has an inverse.

Fact: A function f is invertable iff it is bijective, that is surjective (onto) and injective (1-1).

Def: Surjective means every element of the codomain is acheived and f is injective if whenever $f(a) = f(b)$, we have $a = b$.

Horiz. line Test: A function $f: U \to V$, $U, V \subseteq \mathbb{R}$, $im(f) = V$ is invertable iff for every $c \in V$, the line $y = c$ intersects the graph of f only once. (this is a way to check injectivity visually)

BB: 6.2, 6.3

W7M1

Ann: - Today is test 2
- wiki 6 today

Wiki prep: a) optin 1 costs $45m + 10$ per a mile, while optin 2 costs $45m + 75$.
which is better? prove it! (not an example)

b) $f(x) = 7x + 2$; $g(x) = 3 - x$
$f \circ g$? $g \circ f$?

c) $h: (-\infty, -3] \to \mathbb{R}$
$x \longmapsto (x+3)^2 - 2$
find $h^{-1}(x)$

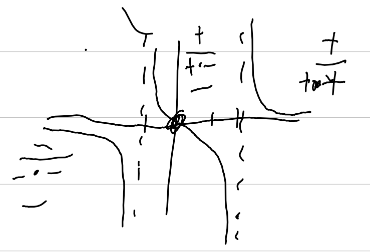
d) π: avocadoes $\to$ toast
what could $\pi(12) = 17$ mean in words?
what would $\pi^{-1}(3)$ mean?

review + preview long prob.

1) $f(x) = \frac{x}{(x+1)(x-2)}$; $g(x) = \frac{\sqrt{7x}}{x-1}$

find $g \circ f$

Ans: want $7\left(\frac{x}{(x+1)(x-2)}\right) \geq 0$
root:
$(-1,0) \cup (2,\infty)$



den of g root:

$\frac{x}{(x+1)(x-2)} \neq 1$

$x \neq (x+1)(x-2)$
$x \neq x^2 - x - 2$
$0 \neq x^2 - 2x - 2$

$x \neq \frac{-(-2) \pm \sqrt{4 - 4(1)(-2)}}{2(1)}$
$\neq \frac{4 \pm 2\sqrt{1+2}}{2}$
$\neq \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$2 - \sqrt{3} \in (0,2)$ already out
$2 + \sqrt{3} \in (2, \infty)$

restrictions from f:
$x \neq -1, 2$ - already have from g

$g \circ f: (-1, 0) \cup (2, 2+\sqrt{3}) \cup (2+\sqrt{3}, \infty)$

$$(g \circ f)(x) = \frac{\sqrt{7\frac{x}{(x+1)(x-2)}}}{\frac{x}{(x+1)(x-2)} - 1}$$

Now, Aleks time.

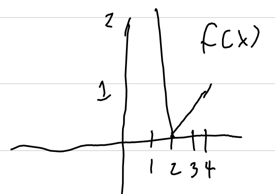
Ann: - KC 3 opens monday
- hope to grade wiki + tests today

w7w1
7.1 Quadratics

(P) review: 1) write the line AROC calculates the slope of from $x=2$ to $x=4$ on $f(x)=\sqrt{x-1}$

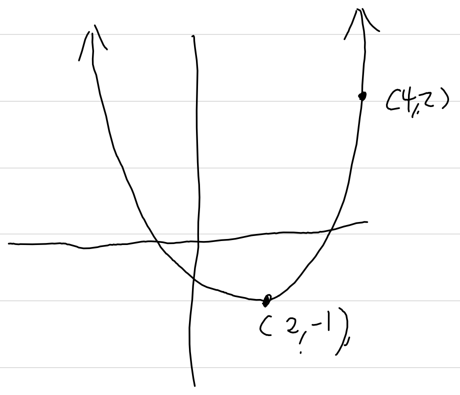
2) is $x=\frac{7}{y}$ a func of x?

3)



draw $\frac{1}{2}f(x+2)-1$

4)



is quadratic. find its formula.

Ans: vertex: $(2,-1)$

$f(x)=a(x-2)^2-1$

$2=a(4-2)^2-1$

$3=a(4)$

$a=3/4$

Content: Def: A function f is a quadratic if f is of the form $f(x)=ax^2+bx+c$.

ex) 1) $4x^2+3x+2$

2) xy^4+2y^2+1; here note
$=x(y^2)^2+2(y^2)+1$

Def: A root/zero of f is a $x\in\mathbb{R}$ such that $f(x)=0$

Formula: for $f(x)=ax^2+bx+c$,
the roots of $f(x)$ are
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-b}{2a}\pm\sqrt{\left(\frac{-b}{2a}\right)^2-\frac{c}{a}}$$

Proof: $0=ax^2+bx+c$ (Hint: $(x+d)^2=x^2+2dx+d^2$)

$-c=ax^2+bx$

$\frac{-c}{a}=x^2+\frac{b}{a}x$

$\frac{-c}{a}=x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2$

$\frac{-c}{a}+\left(\frac{b}{2a}\right)^2=x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2$

$-\frac{c}{a}+\left(\frac{b}{2a}\right)^2=\left(x+\frac{b}{2a}\right)^2$

$\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}=x+b/2a$

$$x=\frac{-b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}$$

Note: vertex on graph of parabola is symmetric about roots hence vertex's x-coord is $\frac{-b}{2a}$

Now BB 7.1, etc.

W7F

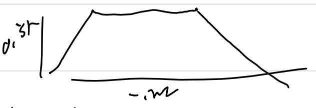
7.2 the Vertex

Ann: – no fast masquera for T2
└ could still change

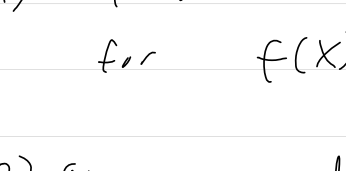
– T3 March 13–14th

Review: 1) find the x and ~~y~~ intercepts for $f(x) = 3\sqrt{x+5} - 4$

2) Quick wA for describing like graphs, each piece is at const speed.



3) A 48 cm wire is cut two form two boxes. What side lengths give maximal area?



$48 = 4x + 4y \rightarrow 12 = x + y$

$A = x^2 + y^2$

$A = x^2 + (12-x)^2$

$= x^2 + 144 - 24x + x^2$

$= 2x^2 - 24x + 144$

max @ vertex
/min

$V_x = \frac{-b}{2a} = \frac{24}{4} = 6 \rightarrow y = 12 - 6 = 6$

$x = 6$

$V_A = 2(6^2 - 12 \cdot 6 + 72)$

4) find the range of $g(x) = 2x^2 + 8x + 5$

LC: + ∪

$V_x: \frac{-8}{4} = -2$ Ans: $[-2, \infty)$

Content: Graphing Quadratics / Vertex

def: The vertex of a quadratic is its unique extrema (max/min)

for $ax^2 + bx + c = f(x)$
vertex is at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

def: The leading coefficient (LC) of a polynomial is the coefficient on the leading term, that is the term with highest degree (to).

property: If LC is positive, vertex is a min
" " negative, vertex is a max.

Now 7.2 BB,

– — — — —

Graphing Quadratics:

Steps: 1 find Vertex
2 find LC
3 plot $LC(x - V_x)^2 + V_y$
└ think LCx^2 with (V_x, V_y) as origin.

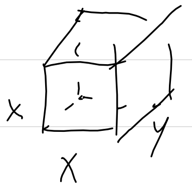
Now 7.3 B

Ann: kcc 3 today $\qquad$ W8M1

- wiki 7 today (30 min) $\quad$ BB: 7.3

(P) review: 1) you have 12 m of wire to make the frame for a rectangular frame with a square base for your fish. what side lengths maximize surface area?

Ans:



$12 = 8x + 4y$

$3 = 2x + y \rightarrow y = 3 - 2x$

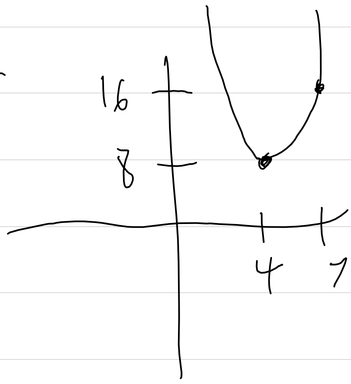
$$SA(x,y) = 2x^2 + 4xy$$
$$= 2x^2 + 4x(3-2x)$$
$$= 2(x^2 + 6x - 4x^2)$$
$$= 2(-3x^2 + 6x)$$
$$= (-6x)(x-2)$$

mid point: $1 = x_{max}$ $\qquad$ $3 = 2 \cdot 1 + y$

$y_{max} = 1$

$SA_{max} = 2 + 4 = 6$

2) eqn for



$f(x) = a(x-4)^2 + 8$

$f(7) = 16 = a(7-4)^2 + 8$

$8 = a \cdot 9$

$a = \frac{8}{9}$

3) $-x^2 - 12x + 11 < -11$

$x^2 + 12x - 22 > 0$

$$x = \frac{-12}{2} \pm \sqrt{(6)^2 - \frac{-22}{1}}$$
$$= -6 \pm \sqrt{36 + 22}$$
$$= -6 \pm \sqrt{48}$$
$$= -6 \pm 4\sqrt{3}$$

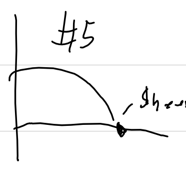
48
2 24
2 12
4 3

Now 7.3 BB. till wiki time (35 min @ end)

Ann: - KC 3 do Soon

W8W1
BB 8.1

wiki:



sharp - need take positive root

will take off for on test

- need quadratic formula or work shown.

(P)review: 1) plot $f(x) = \frac{2(x-3)^2(x+2)}{(x-3)(x-1)^2}$

holes: $x=3$

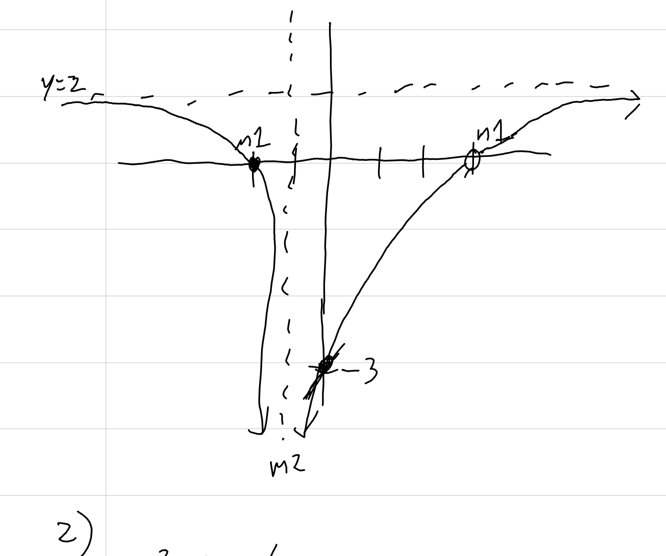
roots: $x = +3_{m1}$ - 2_{m1} kinda

$\tilde{f}(x) = \frac{1}{2}\frac{(x-3)(x+2)}{(x-1)^2}$

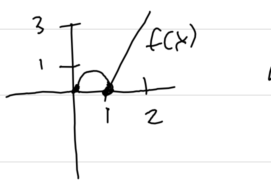
poles: $x = -1_{m2}$

y-int: $\frac{\frac{1}{2}(-3)(2)}{(-1)^2} = -3$

EB: $\tilde{f} \sim \frac{2x^2}{x^2} = 2$

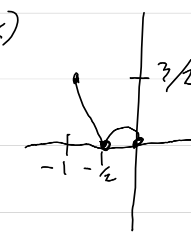


2)

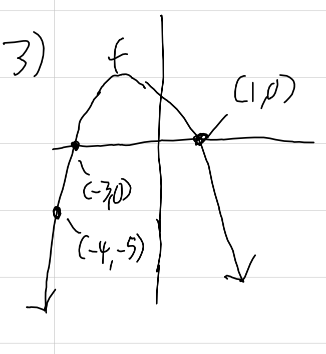


draw $\frac{1}{2}f(2x)$

Ans:



3)



find formula:

$f(x) = a(x+3)(x-1)$

$-5 = a(-1)(-5)$

$a = -1$

Content: Polynomials

Def: A polynomial is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ where x is a variable/indeterminant and $a_i \in R$ for some ring R. The collection of all such expressions is called $R[x]$. "R adjoin x".

example: $5x^4 + 3x^2 + 1 \in \mathbb{Z}[x]$

Def: The degree of a polynomial is the largest non-zero n such that $a_n \neq 0$.
The leading term (LT) is $a_n x^n$ and the leading coefficient (LC) is a_n.

ex) $\pi x^7 + \underbrace{6x^8}_{LT} - 12x + 2$

8 is degree
6 is LC.

ex) $(x+2)^n$ gives # of d-dimensional cells in n-cube as coeffs.

Recall: a root of a polynomial is where it evaluates to zero.

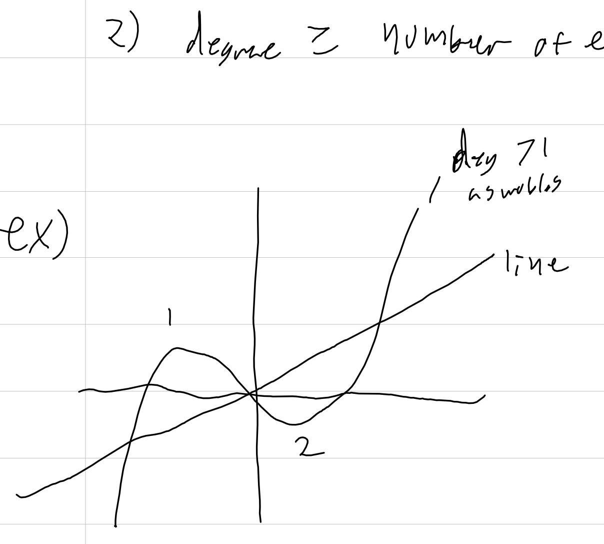
Rmk: A polynomial has at most its degree many roots.

Finding degree graphically:

1) degree = maximal # of intersections with lesser degree polynomial.

2) degree $\geq$ number of extrema +1

ex)



degree ≥ 3

2 extrema
deg ≥ 3

Fact: if $f(x) = g(x)h(x)$, then $\deg(f) = \deg(g) + \deg(h)$

ex) $f(x) = (x^2 - 7)(x^{14} - 2x^{11} + 3x)$

$\deg(f) = 2 + 14 = 16.$

-- no need to expand!

Now BB 8.1 — not for dan's class

w8f1

(1) review

8.2
- zeros + mult.
- end behavior

1) $f:[-10,\infty)\to\mathbb{R}$; $f(x) = 5x+4$

$g(x) = \dfrac{\sqrt{x+2}}{x-2}$

what is domain + rule for $g\circ f$?

rule: $\dfrac{\sqrt{5x+6}}{5x+2}$

domain: - must be subset $[-10,\infty)$

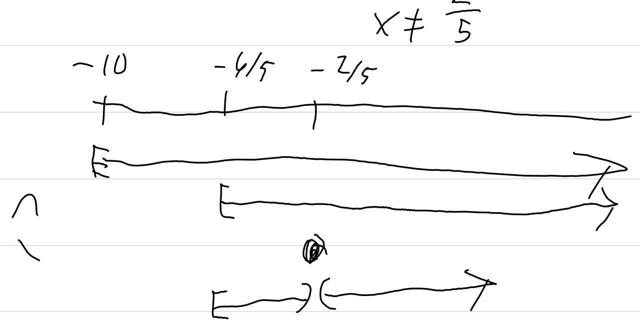
- need $\sqrt{5x+6} \geq 0$

$5x+6 \geq 0$

$x \geq \frac{-6}{5}$

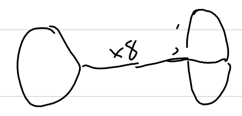
- need $5x+2 \neq 0$

$x \neq \frac{-2}{5}$



Dom: $[-6/5, -2/5) \cup (-2/5, \infty)$

2) you are making a cylindrical cage with 2 circles and 8 connecting beams. If you have 4 m of metal, what is the maximal surface you can make?



SA : $2\pi r \cdot l$

Frame: $4\pi r + 8l = 4$

$\pi r + 2l = 1$

$l = \frac{1-\pi r}{2}$

$SA(r) = 2\pi r \cdot \frac{(1-\pi r)}{2}$

$= \pi r - \pi^2 r^2$

$r_{max} = \frac{-\pi}{-2\pi^2} = \frac{1}{2\pi}$

$l_{max} = \frac{1-\pi(\frac{1}{2\pi})}{2} = \frac{1-\frac{1}{2}}{2} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$SA_{max} = 2\pi\left(\frac{1}{4}\right)\left(\frac{1}{2\pi}\right) = \frac{1}{4}\ m^2$

3) plot $f(x) = \dfrac{(x-2)^3(x+7)}{(x-3)^2(x+2)}$

holes: none

roots: +2 m3, -7 m1

poles: 3 m2, -2 m1

EB: $\sim \frac{x^4}{x^3} \sim x$ — need slant asymptote

Other point:

$f(1) = \dfrac{(-1)^3(8)}{(2)^2(3)} = \frac{8}{4\cdot 3} = \frac{2}{3} \Rightarrow (1, \frac{2}{3})$

close to largest multiplicity

Slant: num: $(x-2)(x-2)(x-2)(x+7)$

$= x^4 + (-2-2-2+7)x^3 + \text{lower}$

den: $x^3 + (-3(2)+2)x^2 + \text{lower}$

$$\begin{array}{r} x+5 \\ x^3-4x^2 \overline{\smash{)}\ x^4+x^3} \\ -(x^4-4x^3) \\ 5x^3 \end{array}$$

slant asymp: $x+5$

Now: run through BB 8.2 - *

me, me, you style same.

8.3 as competition.

Ann: - wiki; 8 - (30 min)

W9M1

- Test 3 Thurs + Friday

BB: 8.2 b + 8.3a

teams of 4 ish

- Aleks wlw done