Test 2 notes

Module 3: Functions

Ann:-Retake exam in Nov.

    └ only 1 exam, second score counts.

- results for exam 1 open sept 13th.

    └ only can open once!

- post exam reflection in BB

    Soonish,

- KC 2 opens Monday of week 5.

    ├ IDK adds to HW.

    ├ can lower HW score

    └ treat like IKC.

(P)review: 1.) which are functions of x?

    a) write $x = \frac{a}{b}$; $f(x) = b$.

    no, $\frac{a}{b} = \frac{2a}{2b}$ but $f(\frac{a}{b}) = b \neq f(\frac{2a}{2b}) = 2b$

    b) $x^2 + y^2 = 2^2$
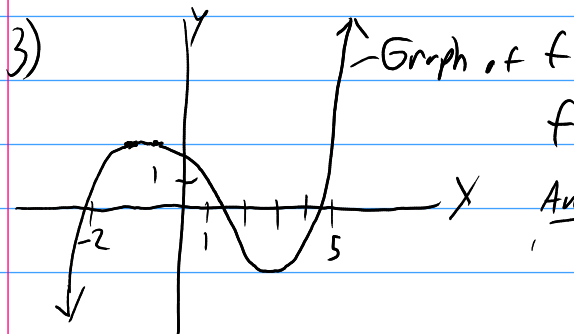
    no, $(-2,0), (2,0)$ ∈ Graph $(x^2 + y^2 = 4^2)$

    fails vert line test.

3F2

2) $g(x) = -4x^2 + 5$

$f(z) = z - 2$

find $(g \circ f)(y)$

3)

-Graph of f

find $f^{-1}(1)$

Ans! $f^{-1}(1) = \{x \in \mathbb{R} : f(x) = 1\}$

$f^{-1}(1) = \{-2, 1, 53\}$

Content: Functions

Def: A set of ordered pairs $(x,y) \in R \subseteq X \times Y$ is called a $\underline{relation}$ of X and Y. [Domain / Codomain]

say $x R y$ if $(x,y) \in R$.

ex) $\{(1,2), (3,2), (3,1)\} \subseteq \mathbb{Z} \times \mathbb{Z}$

ex)

ordered pairs on this graph

ex)

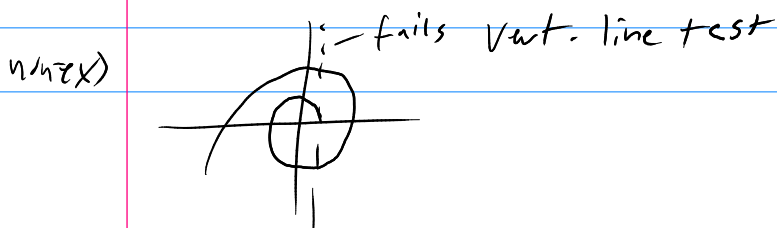| x	y
1	2
1	3
2	2

Def: A function is a relation
F such that each input has only
one output.
— "vertical line test"

Notation:

$$f : \underset{\text{domain}}{A} \to \underset{\text{codomain}}{B} \quad \text{or} \quad f(a) = \text{exprn}(a)$$

function name

$$a \longmapsto \underset{\text{rule}}{\text{exprn}(a)}$$

and domain, codomain implicit or stated.

ex) $f : \mathbb{R} \to \mathbb{R}$ or $f(r) = 3r+1$

$$r \longmapsto 3r+1$$

non-ex) $f\left(\dfrac{a}{b}\right) = a$; note $\dfrac{a}{b} = \dfrac{3a}{3b}$ yet

$$f\left(\dfrac{3a}{3b}\right) = 3a.$$

non-ex)

— fails vert. line test

non-ex) $\{(1,2), (3,2), (1,2), (3,3)\}$

$$1 \longrightarrow 2$$

$$3 \longrightarrow 2 \atop \searrow 3 \Big\} - Bad$$

Def: The range of $f: A \to B$ is

$$f(A) = \{f(x) : a \in A\}$$
$$= \{b \in B : \exists a \in A \text{ w/ } f(a) = b\}$$
exists

"achieved elements in codomain".

BB~M3 notes Class do bot of p1, bot of p2

Telling if equation is function:
$expm_1(x,y) = expm_2(x,y)$ is a
function of x if you can isolate
y uniquely.

non-ex) $y^2 - x = 4$
$$y^2 = 4 + x$$
$$y = \pm\sqrt{4+x} \text{, not-unique ans per } x.$$

3F5

non-ex) $|y| + X = 3$ in   out

test: set $X=0$

$|y| + 0 = 3$

$y = \pm 3$

$0 \rightarrow 3$

$0 \rightarrow -3$

not func.

caution!

ex) $y^2 + 3X = 2(x-4) + X + 8$

$y^2 + 3X = 2x - 8 + X + 8$

$y^2 + 3X = 3x$

$y^2 = 0$

$y = 0$

Ann:
- test 1 graded
  ↳ Avg 85ish
- no test retl. (yet)
- Today wiki: 3
  ↳ done Guntermww.
- KC 2 next week
- test make up day Nov. 18th

BB M3notes p3 top.

4M1

(P)review: 1) is $(y-4)^2 - 4 = X$ a func of x? (no)

of y? (yes)

2) $h(X) = 3x^2 + 5$

what is $(h \circ h)(X)$?

$= h(h(x)) = 3(3x^2 + 5)^2 + 5$

$= 3((3x^2)^2 + 2 \cdot 3 \cdot 5 x^2 + 5^2) + 5 = 27x^4 + 90x^2 + 80$

3) $g(x,y) = 7x + 2y$

what is $g\left(\frac{1}{7}, 4\right)$?

      x-slit   y-slit

$$= 7 \cdot \frac{1}{7} + 2 \cdot 4 = 1 + 8 = 9$$

4) (preview) Domain of $\dfrac{\overset{=f}{\sqrt{x^2-4}}}{\underset{=g}{\sqrt{9-x^2}}}$

      domain

$\text{Dom}\left(\frac{f}{g}\right) = \text{Dom}(f) \cap \text{Dom}(g) \setminus g^{-1}(0)$

$\text{Dom}(\sqrt{x^2-4}):$ $\text{Dom}(\sqrt{x}) = [0, \infty)$

$$x^2 - 4 \geq 0$$
$$x^2 \geq 4$$
$$(-\infty, 2] \cup (2, \infty)$$

$\text{Dom}(\sqrt{9-x^2}):$

$$9 - x^2 \geq 0$$
$$9 \geq x^2$$
$$[-3, 3]$$

$g^{-1}(0):$ $0 = \sqrt{9-x^2}$
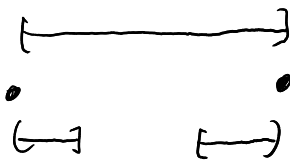
$$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$
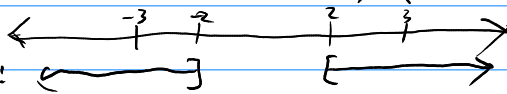
Dom (Num):

Dom (Denom):

Denom$^{-1}(0)$:

$N \cap D \setminus D^{-1}(0)$:

4M3   Ans: $(-3,-2] \cup [2,3)$

Now: wiki 3

Functions ①Pointwise
②AROC

4W1   Ann: -KC 2 opens Monday
       └ do   as   soon   as   can,
       ┌ treat like initial.
       └ keeps all other activity
         from being done.

(P)review: 1) What is the average rate
of change of $f(x) = x^2 + x - 1$
from 2 to 4?

Fact: $AROC(g, a, b) = \dfrac{g(b) - g(a)}{b - a}$

so $AROC(g, 2, 4) =$

$$\dfrac{(4^2 + 4 - 1) - (2^2 + 2 - 1)}{4 - 2}$$

2) what is the domain of
$f(x) = \sqrt{-x} - \sqrt{x+2}$

Fact: $Dom(g+h) = Dom(g) \cap Dom(h)$

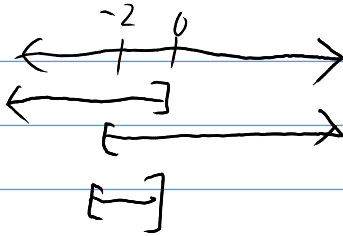$\sqrt{-x}: -x \geq 0 \rightarrow x \leq 0$ ie $(-\infty, 0]$

4w2 $\sqrt{x+2}$: $x+2 \geq 0$ $[-2, \infty)$

          $x \geq -2$

Domain $(f) = [-2, 0]$

3) Domain and range from graph of
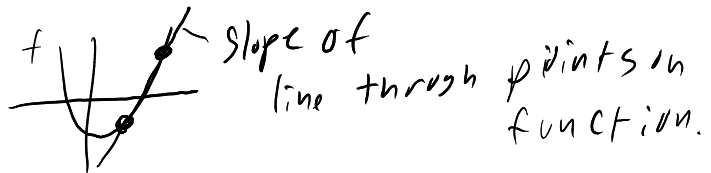   peice wise $\boxed{\text{Aleks}}$

Content: Intervals

Use Bb M4 -Funcs P2

  almost end   -sec 12 end pg 3 +sec14
          on page 4, creating more examples
       if extra time.

last 3 minutes: AROC
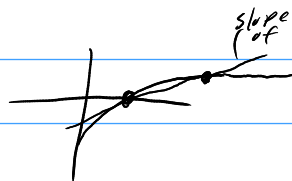
$$AROC(f, a, b) = \frac{f(a) - f(b)}{a - b}$$

 slope of
              line through points on
                    function.

4w3

ex)

x	f(x)
1	8
2	12
3	34
7	89

AROC(f, 2, 7)?

$$\frac{f(7)-f(2)}{7-2} = \frac{89-12}{7-2}$$

ex) $\Pi(y) = \sqrt{y} - 4$

slope of f

AROC(Π, 16, 4)?

$\Pi(16) = \sqrt{16} - 4 = 0$

$\Pi(4) = \sqrt{4} - 2 = -2$

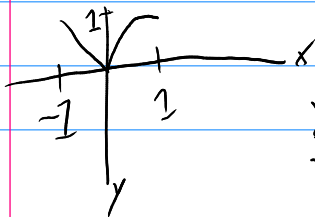$$\frac{0 - (-2)}{16 - 4} = \frac{2}{8} = \frac{1}{4}$$

Now wiki 4   (Start module 5)

5M1

Start: GO over wiki 4 Rubrics
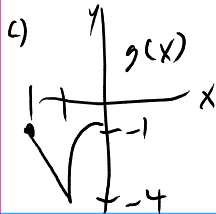
(Preview) 1) f(x)   graph $g(x) = 3f(x-1) - 4$

a) key points

b) in (x) = x-1   print = f
in⁻¹(x) = x+1
out(x) = 3x - 4

x	f(x)
-1	1
0	0
1	1

(in⁻¹, out)

(in⁻¹(x), out(y))

x'	g(x')
-1-1 = -2	3(-1)-4 = -1
0-1 = -1	3(0)-4 = -4
1-1 = 0	3(1)-4 = -1

c)

2) Plot $-3|x-1|+2$

1.) parent

$0 \to 1$

2.) ↻ -x-axis, scale by 3

3) ↑ 2

"inside acts inversly"
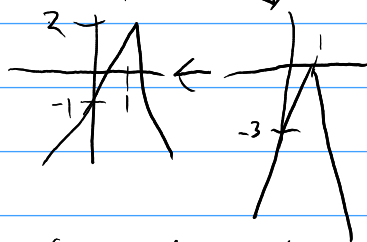
Content: Transformations
of functions

5w1

5.2

GO to mod 5 transformation notes
on pine note.

(Preview: 1) Give a linear model for
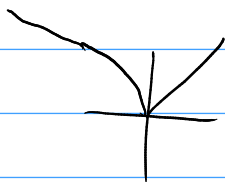$f(x) = 3x^2 - 4$ from 3 to 6.
whats it's AROC here?

$$AROC = m_0 = \frac{f(6)-f(3)}{6-3} = \frac{(3\cdot36-4)-(3\cdot9-4)}{6-3} = \frac{3(36-4-(9-4))}{3}$$

$$= 36 - 9 = 27$$

$$y - (3\cdot9-4) = 27(x-3)$$

5W2

2) IS $(y+3)^2 - X = 25$ ~ func of y? X?.
  Yes   no.

3) $f(x) = \begin{cases} X & X \geq 0 \\ \sqrt{-X} & X < 0 \end{cases}$

plot
$g(x) = \frac{1}{2} f(-\frac{x}{2} + 2) + 1$
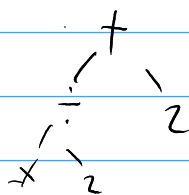
a) process: f
$in(x) = -\frac{X}{2} + 2$

$X = -\frac{in^{-1}(X)}{2} + 2$
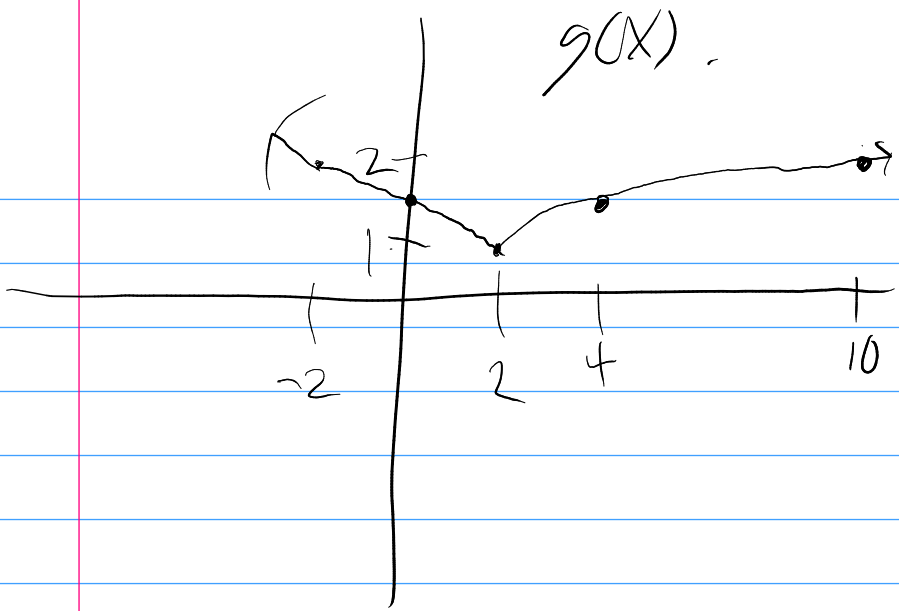
$X - 2 = -\frac{in^{-1}(X)}{2}$

$2(x-2) = -in^{-1}(X)$

$-2X + 2 = in^{-1}(X)$

$out(y) = \frac{1}{2}y + 1$

6)

X	f(x)		X'	g(x')
-4	2		$-2(-4)+2 = 10$	$\frac{2}{2} + 1 = 2$
-1	1	(in⁻¹, out) $-2(-1)+2 = 4$	$\frac{1}{2}+1 = 1.5$
0	0	$0+2=2$	$0+1 = 1$
1	1	$-2+2=0$	1.5
2	2	$-2(2)+2 = -2$	2

5W3

g(X).

for 15min, do in-class Aleks practise
transformation.

rest of class: practise graphing
        as game.

wik:5
+ test 2
review

5F1
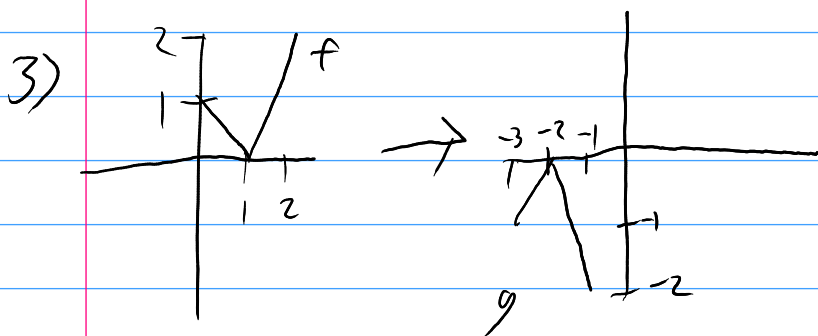
<u>Ann</u>:- KCZ due Monday
        └ do it now
    - test 2 tuesday + wednesday next week

5F2

1) $3x + 2y = 5$; find line $\perp$ through $(1,3)$

2) $f(x) = \dfrac{x}{-3x^2 + 2}$; find $f(5-x)$

3)

describe $g(x)$ in terms of $f(x)$.

4) (preview) $H(x) = (4 - 5x)^4$
find f, g st. $H = f \circ g$.

Now: wiki 5
when done: test 2 review / Aleks.

Ann :-Test 2 tues twednesday

     —it below 18/25 on wiki§, check own and

shw desmos graph duiring test 2 review tme /Aleks.

Now: test 2 review/Aleks.