module 9 Rational fncs. W9F1

Ann: T3 today - last day

Preview: $f(x) = \dfrac{2(x-1)^2(x-3)(x-4)}{(x-2)(x+1)(x-4)}$ - plot (include slant if app.)

1. Simp: $\tilde{f}(x) = \dfrac{2(x-1)^2(x-3)}{(x-2)(x+1)}$; $x \neq 4$
2. list holes poles roots: holes: $(4, \tilde{f}(4))$
   poles: 2 m1, -1 m1
   roots: 1 m2, 3 m1
3. deg: Σ root mult $-$ Σ pole mult.
   $= (2+1) - (1+1) = 1$
4. LC: 2 - positive
5. End behavior: $\sim LC \cdot x^{deg} \sim 2 \cdot x$
   when deg = 1, need slant asymptote
   ie, EB is $2x + \underline{d}$ need find

num of $f = 2(x^2-2x+1)(x-3)$
$= 2x^3 - 4x^2 + 2x$
$\quad -6x^2 + 12x - 6$
$= 2x^3 - 10x^2 + 14x - 6$

den of $f = x^2 - 2x + x - 2 = x^2 - x - 2$

Way 1: (hope, don't do)

$f(x) - 2x = \dfrac{2(x-1)^2(x-3)}{(x-2)(x+1)} - \dfrac{2x(x-2)(x+1)}{(x-2)(x+1)}$

$= \dfrac{2(x-1)^2(x-3) - 2x(x-2)(x+1)}{(x-2)(x+1)}$

$= \dfrac{2x^3 - 10x^2 + 14x - 6 - (2x^3 - 2x^2 - 2x)}{x^2 - x - 2}$

$= \dfrac{(2-2)x^3 + (-10+2)x^2 + (14+2)x - 6}{x^2 - x - 2}$

$= \dfrac{-8x^2 + 16x - 6}{x^2 - x - 2}$

$\lim_{x \to \infty} (f(x) - 2x) = \dfrac{LT}{LT} = \dfrac{-8x^2}{1x^2} = -8$

Slant $= 2x - 8$

Way 2:

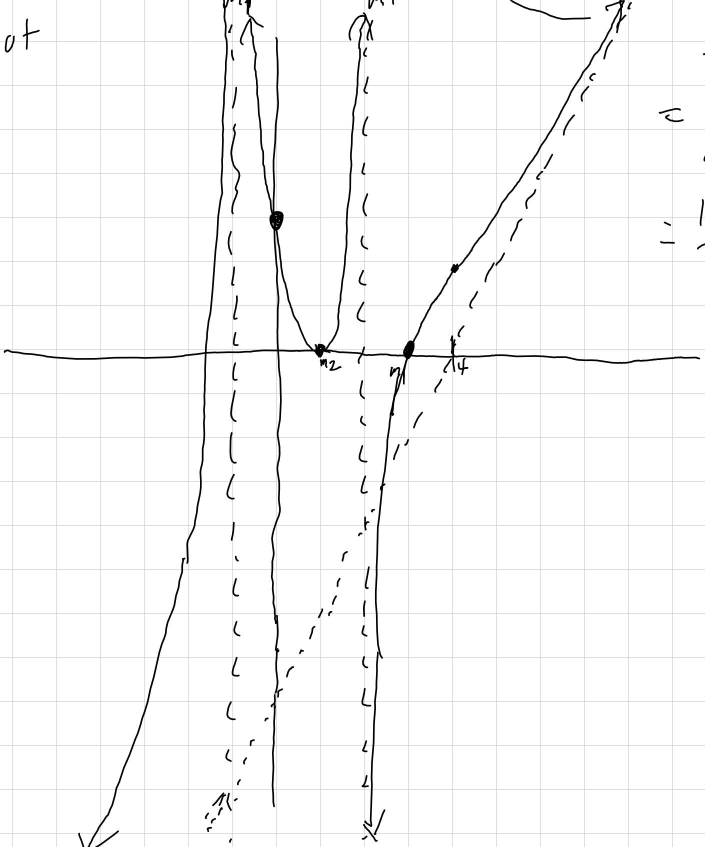
$$\begin{array}{r|l} & 2x - 8 \quad r\ 10x - 22 \\ x^2 - x - 2 & 2x^3 - 10x^2 + 14x - 6 \\ & -(2x^3 - 2x^2 - 4x) \\ & 0 - 8x^2 + 18x - 6 \\ & -(-8x^2 + 8x + 16) \\ & 0 + 10x - 22 \end{array}$$

So, $\tilde{f}(x) = 2x - 8 + \dfrac{10x - 22}{(x+1)(x-2)}$

6. y-int: $f(0) = -8 + \dfrac{-22}{-2}$
   $= -8 + 11 = 3$

holes: $(4, \tilde{f}(4))$
poles: 2 m1, -1 m1
roots: 1 m2, 3 m1

7. plot



$\tilde{f}(4) = \dfrac{40-22}{5 \cdot 2} = \dfrac{18}{10} = \dfrac{9}{5}$

Content BB mtg. *

W10M1

BB: 9.2 *

Exam Avg: 1 - 71, 2 - 68

Ann: wiki wednesday

Review: 1) you have a mentos and coke sprayer with arc modeled by
$h(x) = -(x-4)^2 + 8$

a) if you move the sprayer 2 feet upward 1 foot right, what is the new model?

b) where will the moved spray have its spray land on the ground?

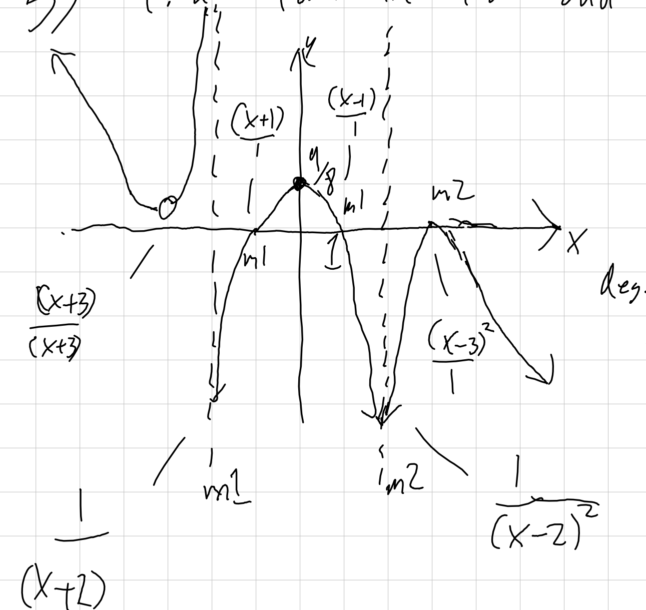
2) find f^{-1} for $f(x) = \sqrt{\frac{7x^3+3}{9}}$

3) find x when $2x^2 + 9x + 2 = 1 + x$

4) Suppose f models number of burgers to prep when given number catering, explain $f^{-1}(45)$ in this context.

Preview:

5) find formula for and EB.



$f(x) = a\,\frac{(x+1)(x-1)(x-3)^2(x+3)}{(x+2)(x-2)^2(x+3)}$

deg: 4 - 3 = 1

$f(0) = \frac{9}{8} = \frac{(1)(-1)(-3)^2}{(2)(-2)^2} \cdot a$

$= \frac{-9}{8} \cdot a$

$a = -1$

Slant: top: $-(x+1)(x-1)(x-3)^2$
$= -(x^2-1)(x^2-6x+9)$
$= -(x^4 - 6x^3 + 9x^2 - x^2 + 6x - 9)$
$= -(x^4 - 6x^3 + 8x^2 + 6x - 9)$

bot $= (x^2-4)(x-2)$
$= x^3 + 0x^2 - 4x - 2x^2 + 0 + 8$
$= x^3 - 2x^2 - 4x + 8$

$$\begin{array}{r} -x + 4 \\ -x^3+2x^2+4x-8 \overline{\big) x^4 - 6x^3 + 8x^2 + 6x - 9} \\ -(x^4 - 2x^3 - - - - -) \\ \hline -4x^3 \end{array}$$

need only

Lazy! need only top 2 terms of each!

Slant: $-x + 4$
Asymptote.

BB content 9.2 *

lifc: early voting

W10W1

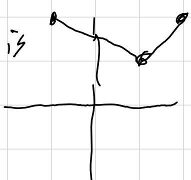
BB 9.2b Rules and down
9.3*

Review: 1) L_1 is through $(1,2)$ and $(-4,3)$
find $\perp$ line through $(-1,2)$

2) which are functions of x? y?

	x	y
a) $x=\frac{2}{y}$	✓	✓
b) $6\lvert x\rvert+y=4$	✓	X
c) $x^2+\lvert y\rvert=9$	X	X
d) $6x=y^2$	X	✓

3) Domain of
$f(x)=\frac{\sqrt{x+5}}{3x+8}$

4) $h(x)$ is 

x	y
-1	2
1	1
2	2

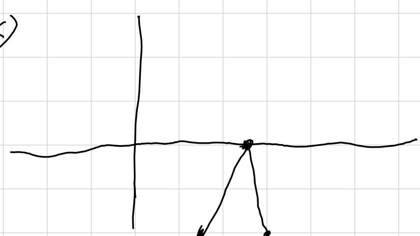
$\xrightarrow{(in^{-1}, out)}$

x	y
$\frac{1}{2}(-1+4)=\frac{3}{2}$	$-4+2=-2$
$\frac{1}{2}(1+4)=\frac{5}{2}$	0
$\frac{2+4}{2}=3$	-2

graph $g(x)=-2h(2x-4)+2$

$in(x)=\frac{x}{2}-4$ $\quad in^{-1}(x)=(x+4)\frac{1}{2}$
$out(x)=-2\cdot x+2$

g(x)



Now BB (see top) + wiki

wiki 9 1st 10 minutes

W10F1

Module 10: Exponential functions and logarithms.

Recall: the function b^n is

$\underbrace{b \cdot b \cdot b \cdots b}_{n\text{-many times}}$

extension: For $b^{p/q} = \sqrt[q]{b^p} = \left(\sqrt[q]{b}\right)^p$ for $p/q \in \mathbb{Q}$

For $r \in \mathbb{R}$, b^r is approximated by $b^{p/q}$ for $p/q \to r$.

Defn: The exponential function with base b with initial value (leading coefficient) C is $C \cdot b^x$. Its inverse function is $\log_b(x/C)$.

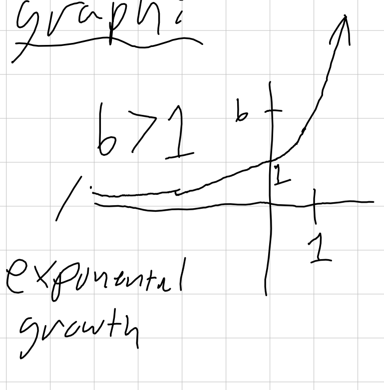
Properties: a) $b^x \cdot b^y = b^{x+y}$ — takes addition inside to product outside.

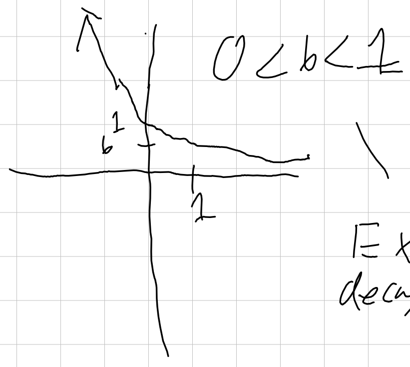
b) $b^0 = 1$

c) $Dom(b^x) = (-\infty, \infty)$

d) $Range(b^x) = (0, \infty)$

graph:





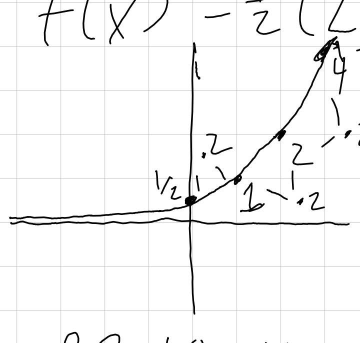
graphing: for $C \cdot b^x = f(x)$ note

$f(0) = C \cdot b^0 = C$

and $f(n) = C \cdot b^n$ while $f(n+1) = C \cdot b^{n+1}$

so $f(n+1) = b \cdot f(n)$. ie to go over one from current location multiply by b to get new y-coord.

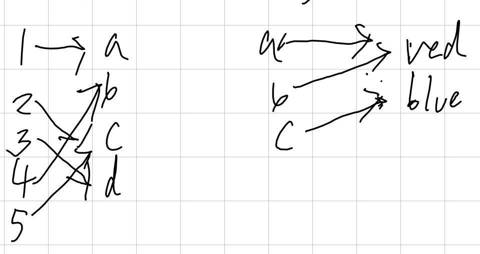
ex) $f(x) = \frac{1}{2}(2)^x$



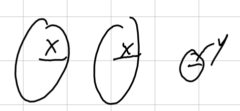
Now BB 10.*

Review

1) Domain of $g \circ f$? Range?



2) 3 circles are cut from 8π cm of wire. Two are equal sized. What are the circles radii that maximizes Area?



$8\pi = 2 \cdot 2\pi x + 2\pi y$

$4 - 2x = y$

$A = \pi(2x^2 + y^2)$

$= \pi(2x^2 + (4-2x)^2)$

$= 2\pi x^2 + 16\pi - 16\pi x + 4\pi x^2$

$= 6\pi x^2 - 16\pi x + 16\pi$

$V_x = \frac{16\pi}{12\pi}$

$= \frac{4}{3}$

V_y: $8\pi = 2\cdot 2\pi \cdot \frac{4}{3} + 2\pi y$

$24\pi = 16\pi + 6\pi y$

$8\pi = 6\pi y$

$y = \frac{4}{3}$

radii are $\frac{4}{3}$ and $\frac{4}{3}$ cm.

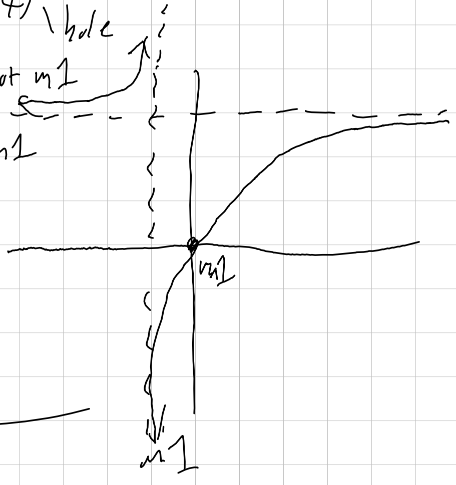
3) Graph $\frac{3x^2+12x}{x^2+5x+4} = f(x)$

$3x(x+4)$

$(x+1)(x+4)$ hole

$\tilde{f}(x) = \frac{3x}{x+1}$ root m1, pole m1

EB: $\frac{LT}{LT} = \frac{3x}{x} = 3$



BB 10.d2 Growth + Decay

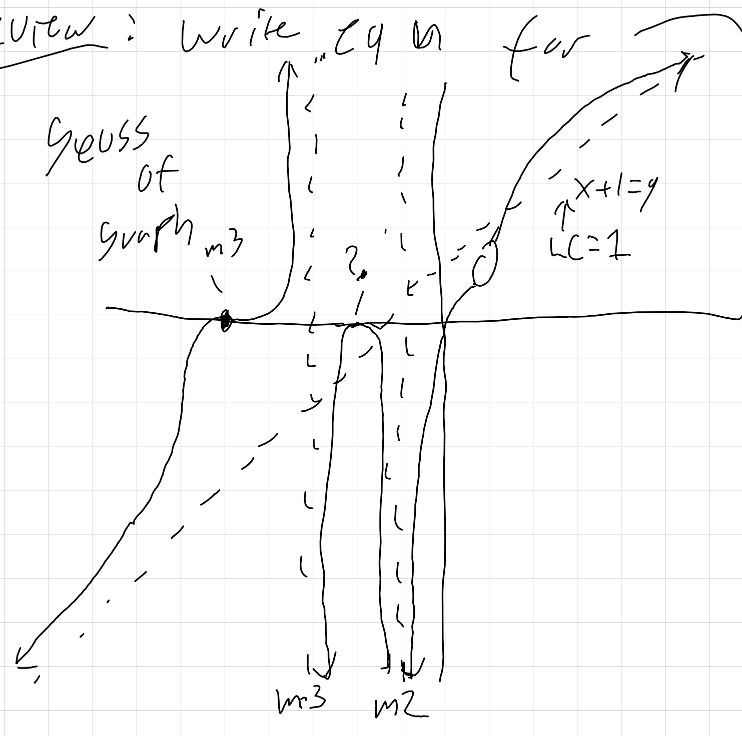
10.d3 Applications

W11 M1

Any-Test 4 Nov. 7-8.
- wiki monday
  due that day
- KC opens

rational funcs
through logs
Modules 9-11

review: write eqn for



f where f has
a hole at $x=1$
roots w/ mult. at
-5 m3, origin m1,
and some other point
w/ mult. 2.
poles at $x=-3$ and
$x=-1$.
and slant asymptote
$y=x+1$.

Then graph it!

holes: $1 = x$
roots: -5 m3, $-r$ m2, 0 m1
poles: -3 m3, -1 m2
Slant: $y=x+1$

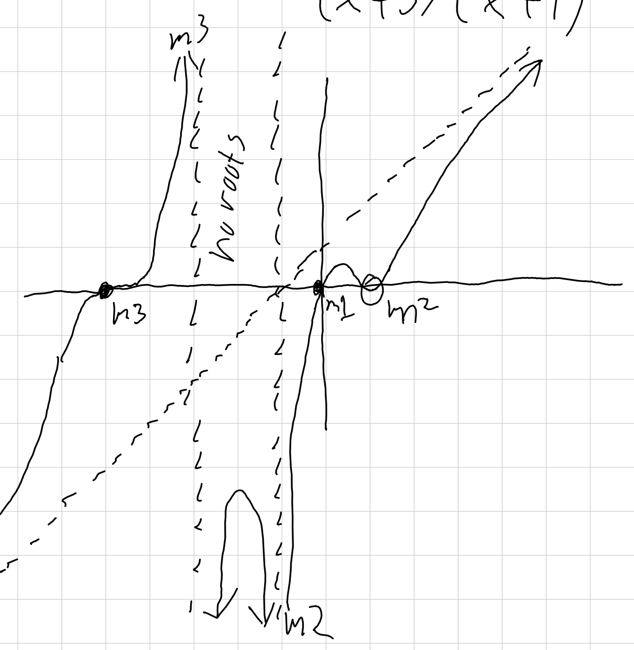
guess: $f(x) = \dfrac{(x+5)^3(x+r)^2(x-1)x}{(x+3)^3(x+1)^2(x-1)}$

long div: num: $x^{3+2+1} + (3\cdot5+2r-1)x^5$ + lower terms
Den: $x^{3+2} + (3\cdot3+2)x^4$ + lower terms

$$\begin{array}{r} x + 1 \\ x^5+11x^4 \overline{\big) x^6 + (14+2r)x^5} \\ -\underline{(x^6+11x^5 \quad)} \\ (3+2r)x^5 \end{array}$$

$3+2r=1$
$2r=-2$
$r=-1$

So, $f(x) = \dfrac{(x+5)^3(x-1)^2(x-1)x}{(x+3)^3(x+1)^2(x-1)}$



Now BB Content 10.d3++
10.d4 do below first

logs:

$$\begin{tikzcd} \mathbb{R} \arrow[loop left, "id_{\mathbb{R}}"] \arrow[r, bend left, "b^{-}"] & (0,\infty) \arrow[l, bend left, "\log_b(-)"] \arrow[loop right, "id_{(0,\infty)}"] \end{tikzcd}$$

Defn: $\log_b(x)$ is the inverse function to b^x ie

So, $\log_b(b^x)=x$

Hence $b^x=y$ iff $\log_b(y)=x$

Properties: 1) $b^x=y$ iff $\log_b(y)=x$

2) $\log_b(a\cdot c) = \log_b(a)+\log_b(c)$

3) $\log_b(a^r) = r\log_b(a)$

4) $\dfrac{\log_b(a)}{\log_b(c)} = \log_c(a)$

Proof of 4) Notice:

$a = c^{\log_c(a)}$
$= c^{\log_c(a)}$
$= \left(b^{\log_b(c)}\right)^{\log_c(a)}$
$= b^{\log_b(c)\cdot\log_c(a)}$

$\log_b(a) = \log_c(a)\log_b(c)$
$b^{\log_b(a)} = b^{\log_c(a)\log_b(c)}$
$a = \left(b^{\log_b(c)}\right)^{\log_c(a)}$
$a = c^{\log_c(a)} = a$

as $a = b^{\log_b(a)}$ we have
$b^{\log_b(a)} = b^{\log_b(c)\cdot\log_c(a)}$
so by applying $\log_b(-)$ to both sides,
$\log_b(a) = \log_b(c)\cdot\log_c(a)$
So $\dfrac{\log_b(a)}{\log_c(a)} = \log_b(c)$. ∎

Now to BB M10.d4

Ann - Test 4 Nov. 7-8.

- wiki monday
  due that da

rational funcs through logs

modules 9-11

Review:

module 6) $f(x) = \dfrac{(x-2)(x+3)}{x^2-4}$

$g(x) = \sqrt{4x^2-4}$

Find domain of $\frac{g}{f}$. and roles

$Dom(f) = \mathbb{R} \setminus \{-2, 2\}$

$Dom(g)$: $4x^2 - 4 \geq 0$

$4x^2 \geq 4$

$x^2 \geq 1$

$(-\infty, -1) \cup (1, \infty)$

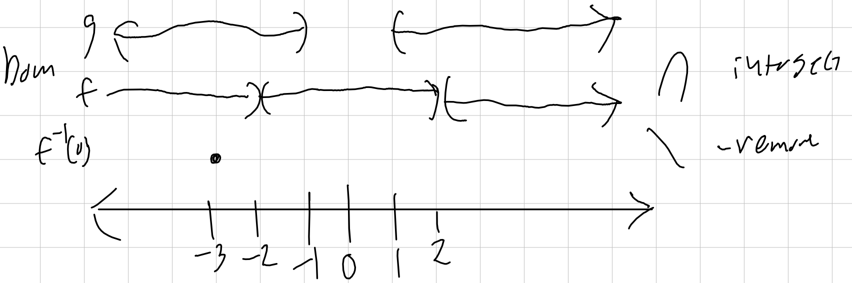
$Dom(g/f) = Dom(g) \cap Dom(f) \setminus f^{-1}(0)$

$f^{-1}(0)$: $\dfrac{\cancel{(x-2)}(x+3)}{x^2-4} = 0$

$(3+x)\underbrace{(x-2)}_{\text{hole}} = 0$

only $x = -3$

$Dom(g/f)$:



Ans:

$Dom(g/f) = (-\infty, -3) \cup (-3, -2) \cup (-2, -1) \cup (1, 2)$
$\cup (2, \infty)$

Now BB 10d4 finish

11*

Ann T4 Thurs + Friday
└ M9-11

W12 M1

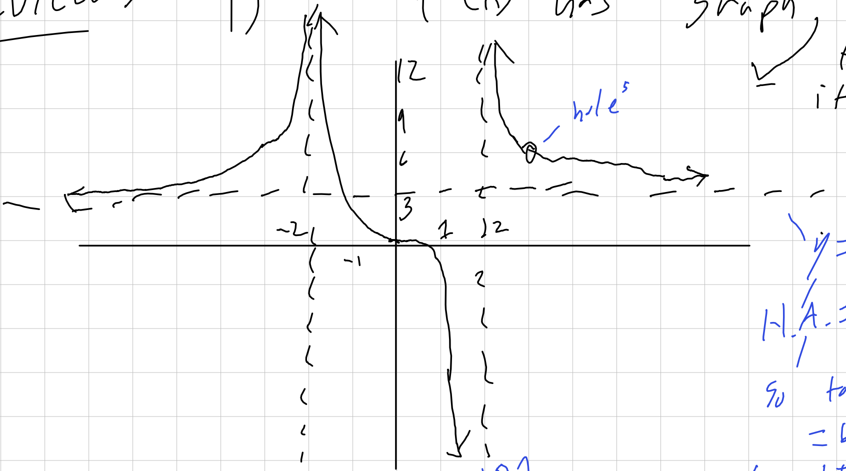
W12 W1

Content: - BB 11.2 *
- wiki 11

- Same remove BB 11.2

- finish wiki 10 (not 11)

Review: 1) m1 9 $f(x)$ has graph, find it's eqn.



$y = 3$

H.A. = EB

so top deg = bot deg

since $\frac{LT}{LT} = 3$

m2 m3 m1

$$f(x) = \frac{3_4 \, (x)_2^3 \, (x-3)_5}{(x+2)_1^2 \, (x-2)_3 \, (x-3)_5}$$

- Subscripts are just notes to help correspond

2) Graph $f(x) = \dfrac{(7x-14)(x-2)(x+1)^2}{(x+4)^2(x-2)}$

$7(x-2)$

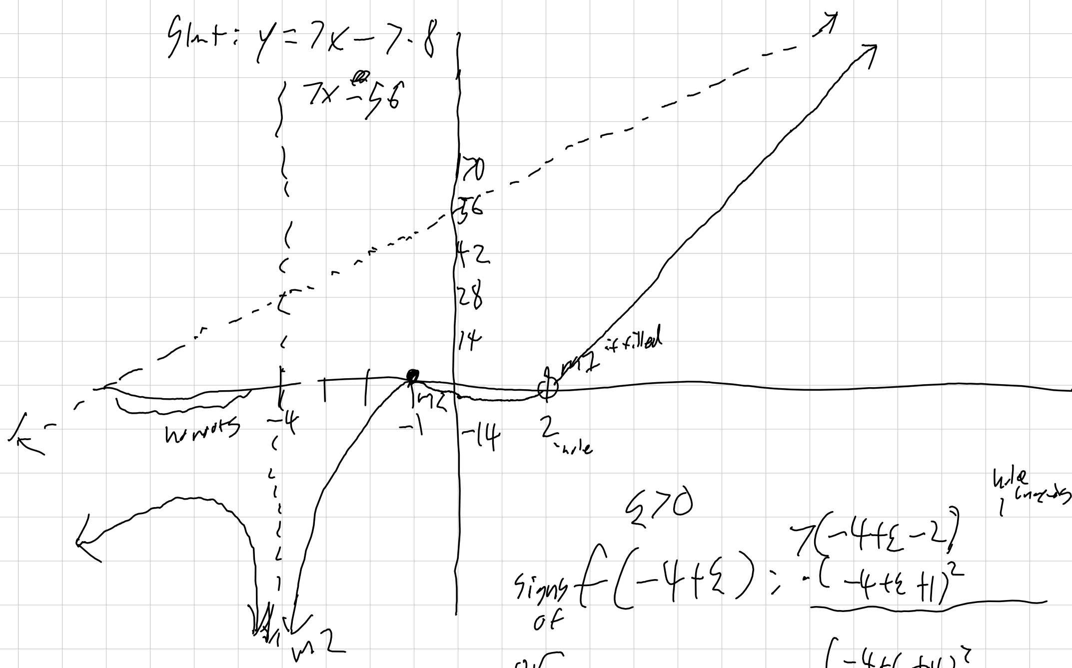
EB: $\dfrac{7x^{1+1+2}}{x^{2+1}} = 7x + b$

$(7x-14)(x+1)(x+1)$

$7x^3 + (-14x^2 + 7x^2 + 7x^2)$ + lower terms

$$\begin{array}{r} 7x - 7\cdot 8 \\ x^2+8x+\text{lower} \overline{) \, 7x^3 + 0x^2 + \text{lower}} \\ -\,(7x^3 + 7\cdot 8x^2 + \text{lower}) \\ \hline 0 - 7\cdot 8x^2 + \text{lower} \end{array}$$

Slant: $y = 7x - 7\cdot 8$
$7x - 56$



≥ 70

signs of $f(-4+\varepsilon) = \dfrac{7(-4+\varepsilon-2)(-4+\varepsilon+1)^2}{(-4+\varepsilon+4)^2}$

hole Corrects

$= \dfrac{7 \cdot \text{neg} \cdot \text{pos}}{\text{pos}} = \text{neg}$

or

$f(0) = \dfrac{7(-2)(1)}{4^2} = \dfrac{-14}{16}$

NOW Wiki / Test 4 review time.