







10M1 Test 4 prep

Module 9: Rational functions

1) Sgt. Space squirrel's planets are in syzygy. Since his universe is 2-d, gravity falls at a rate of $\frac{M}{|r|}$ where $|r|$ is distance from center. Below is his system

mass	.001	.05	.01	1
				
	Pluto	Ur-nope	Mars	
dist from sun	1500	500	100	0



If Sgt. Space  is x away from Sol/2.0, how much gravity does he feel?

$$\text{Ans: } g(x) = \frac{.001}{|1500-x|} + \frac{.05}{|500-x|} + \frac{.01}{|100-x|} + \frac{1}{|x|}$$

$$= \left| \begin{aligned} &.001(500-x)^2(100-x)^2x^2 + .05(1500-x)^2(100-x)^2x^2 \\ &+ .01(1500-x)^2(500-x)^2x^2 + (1500-x)^2(500-x)^2(100-x)^2 \end{aligned} \right|$$

10 m2

$$(1500-x)(500-x)(100-x)x$$

Def/Notation: Let K be a field,

$$\text{then } f(x) = \frac{a(x-r_1)^{m_1} \dots (x-r_n)^{m_n}}{(x-p_1)^{n_1} \dots (x-p_r)^{n_r}} \in K\left[x, \frac{1}{x-r_1}, \dots, \frac{1}{x-r_n}\right] \cap K(x)$$

is a rational function with
poles/vertical asymptotes p_1, \dots, p_r with
multiplicities n_1, \dots, n_r respectively.

$K(x)$ is called the field of functions.

$$K(x) = \left\{ \frac{p(x)}{q(x)} : p(x), q(x) \in K[x], q(x) \neq 0 \right\}$$

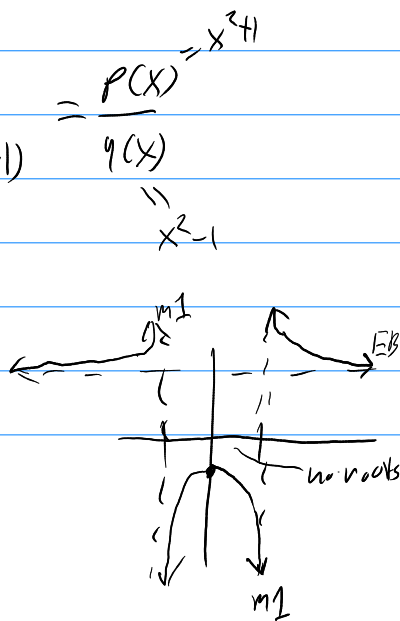
$$\text{ex) } f(x) = \frac{x^2+1}{x^2-1} = \frac{x^2+1}{(x-1)(x+1)} = \frac{p(x)}{q(x)} = \frac{x^2+1}{x^2-1}$$

$$EB: \sim \frac{L^+}{L^-} \sim \frac{x^2}{x^2} \sim 1$$

$$y\text{-int: } f(0) = \frac{1}{-1} = -1$$

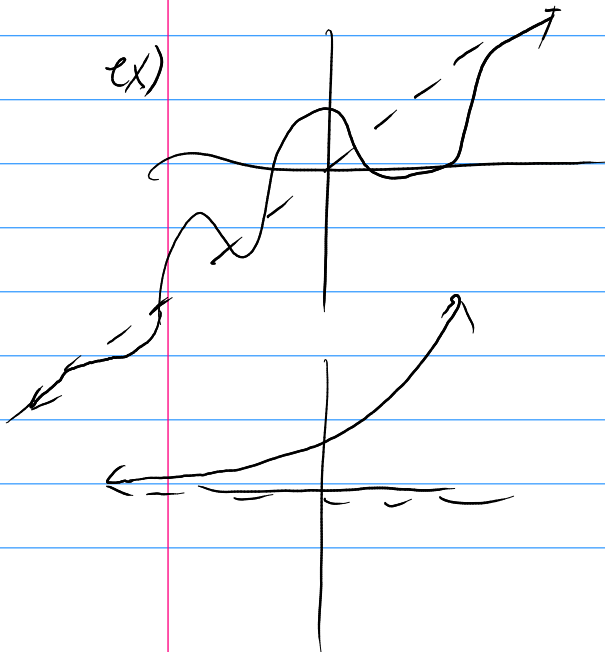
roots: $p^{-1}(0) = \emptyset$ - empty set

poles/V.A.: $q^{-1}(0) = \{1, -1\}$
mult: 1, 1



10M3 Behaviour near poles: $f(1+\varepsilon) = \frac{(1+\varepsilon)^3 + 1}{(1+\varepsilon-1)(1+\varepsilon+1)} = \frac{2+2\varepsilon+\varepsilon^2}{\varepsilon \cdot (2+\varepsilon)}$
 let $\varepsilon > 0$ approach 0,
 as $\varepsilon \rightarrow 0$, $f(1+\varepsilon) \rightarrow +\infty$

Def! Asymptote: A line $L: ax+by=c$ is a slant asymptote for $f(x)$ if for any $\varepsilon > 0$, there exists X such that $|L(x') - f(x')| < \varepsilon$ for all $x' > X$ or all $x' < -X$.
 $L: x=a$ is a vertical asymptote if for all N , there exists $\varepsilon > 0$ such that for all x with $|x-a| < \varepsilon$ $|f(x)| > N$.



Now: BB M9 notes me, you style.

10w1

Ann: -Support

- Success-univ.edu (Coord)
 - ↳ help w/ study habits / etc.
- MRTC
 - ↳ math tutoring
- Health.univ.edu (Pat Walker)
 - ↳ mental health
- myself — UA success link on BB
 - ↳ Oltwrs: 9:40 - 10:30 MWF Sec 224
 - 2:30 - 3:30 MWF (appointments)
 - 10:30 - 11 Thursday

- Test 3 plan to be graded by Monday.
- Test make up day Nov. 18th

today 9.2
VA/hws

(P) review!

1) Find the line perpendicular to
 $3x + 2y = 4$ through the ^{leftmost} x -intercept of
 $25x^2 + 16y^2 = 36$

2) Is $(y-4)^2 - x = 3$ a function
of x ? y ?

10w2

3) give a sketch of

$$f(x) = \frac{\sqrt{4-2x}}{10-2x}$$

4) graph $\frac{3}{x-4} + 3$ via transformations.

5) $f(x) = x^2 + 1$ on $(-\infty, 0]$.
find f^{-1} .

Now mix of poly rational functions
game.

↳ terms of 6 to start.

10F1

today: Wiki & Graphing

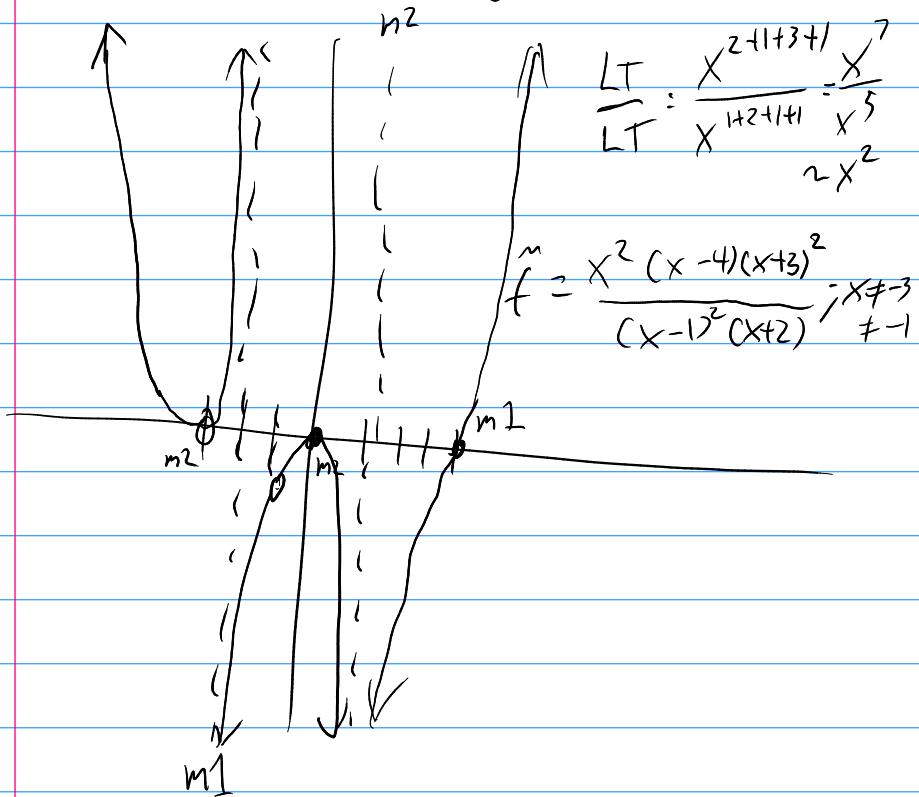
Ann: test + 3 A-levels & 3's graded
Avg +37.

review) 1) $f(x) = \frac{7}{\sqrt{3x+2}}$

Note $g(x) = \frac{7}{x}$ h.c., $(g \circ h) = f$
 $h(x) = \sqrt{3x+2}$

2) find f^{-1} for $f(x) = \frac{2x^3 + 4}{3} - 8$

3) plot $f(x) = \frac{x^2(x-4)(x+3)^3(x+1)}{(x+3)(x-1)^2(x+2)(x+1)}$



Now with: 9, then Aleks + 5, 6,

