10M1 Test 4 prep

Module 9: Rational functions

1) Sgt. Space square's planets are in syzygy. Since his universe is 2-d, gravity falls at a rate of $\frac{M}{|r|}$ where $|r|$ is distance from center. Below is his system

mass	.001	.05	.01	1
				
	Pludo	Urmope	mors	
dist from Sun	1500	X	500	100	0

If Sgt. space □ is X away from Sol 2.0, how much gravity does he feel?

Ans: $g(x) = \frac{.001}{|1500-x|} + \frac{.05}{|500-x|} + \frac{.01}{|100-x|} + \frac{1}{|x|}$

$$= \left| \begin{array}{l} .001(500-x)^2(100-x)^2x^2 + .05(1500-x)^2(100-x)^2x^2 \\ +.01(1500-x)^2(500-x)^2x^2 + (1500-x)^2(500-x)^2(100-x)^2 \end{array} \right|$$

10 M2

$|(1500-x)(500-x)(100-x)x|$

Def/Notation: Let k be a field, then $f(x) = \frac{a(x-r_1)^{m_1}\cdots(x-r_n)^{m_n}}{(x-p_1)^{n_1}\cdots(x-p_r)^{n_r}} \in k[x, \frac{1}{x-p_1}, \ldots, \frac{1}{x-p_r}]$

$\| \Lambda$

$k(x)$

is a rational function with poles/vertical asymptotes $p_1, \ldots, p_r$ with multiplicities $n_1, \ldots, n_r$ respectively.

$k(x)$ is called the field of fractions.

$k(x) = \{\frac{p(x)}{q(x)} : p(x), q(x) \in k[x], q(x) \neq 0\}$

ex) $f(x) = \frac{x^2+1}{x^2-1} = \frac{x^2+1}{(x-1)(x+1)} = \frac{p(x)}{q(x)}$ $p(x) = x^2+1$, $q(x) = x^2-1$

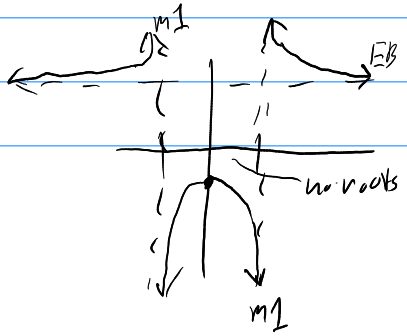
EB: $\sim \frac{LT}{LT} \sim \frac{x^2}{x^2} \sim 1$

y-int: $f(0) = \frac{1}{-1} = -1$

roots: $p^{-1}(0) = \emptyset$ — empty set

poles/V.A.: $q^{-1}(0) = \{1, -1\}$

mult: 1, 1



10M3 Behavior near poles: $f(1+\varepsilon) = \frac{(1+\varepsilon)^2+1}{(1+\varepsilon-1)(1+\varepsilon+1)} = \frac{2+2\varepsilon+\varepsilon^2}{\varepsilon\cdot(2+\varepsilon)}$

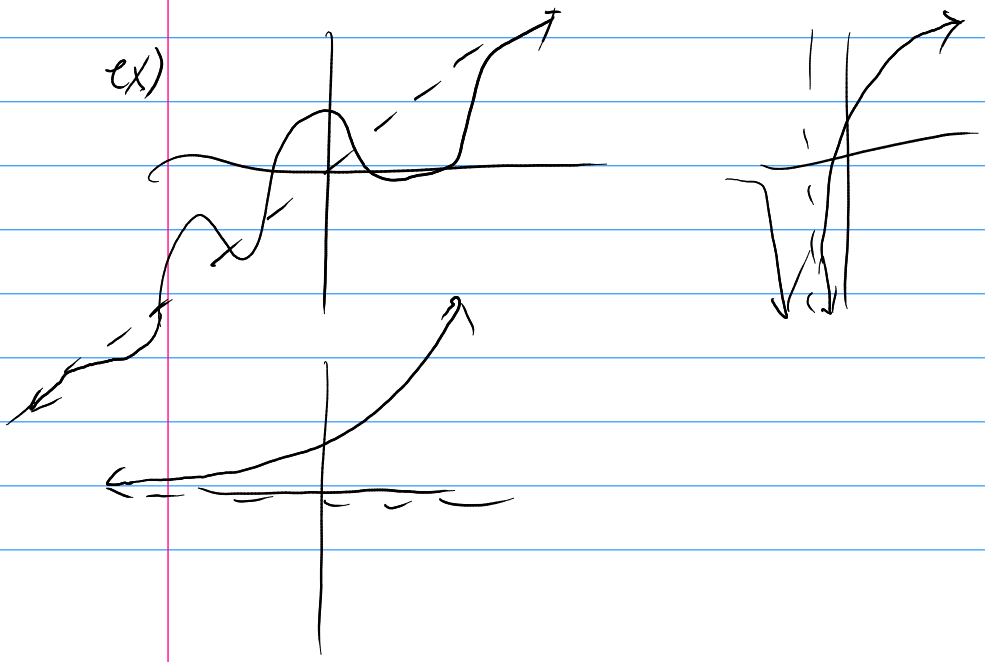
let $\varepsilon > 0$ represent 0,

as $\varepsilon \to 0$, $f(1+\varepsilon) \to +\infty$

Def: Asymptote: A line $L: ax+by=0$ is a slant asymptote for $f(x)$ if for any $\varepsilon > 0$, there exists X such that $|L(x') - f(x')| < \varepsilon$ for all $x' > X$ or all $x' < X$.

$L: x=a$ is a vertical asymptote if for all N, there exists $\varepsilon > 0$ such that for all x with $|x-a| < \varepsilon$ $|f(x)| > N$.

ex)



Now: BB M9 notes me, you Style.

10W1

Ann: -Support
- success.uwec.edu (Coord)
  - help w/ study habits / etc
- MRTC
  - math tutoring
- Health.uark.edu (Pat walker)
  - mental health
- myself — UA success link on BB
  - Others: 9:40-10:30 MWF Scen224
    2:30-3:30 MWF (appointments)
    10:30-11 Thursday

-Test 3 plan to be graded by Monday.
-test make up day Nov. 18th

today 9.2
VA/hdes

(P) review!

1) Find the line perpendicular to $3x+2y=4$ through the left-most x-intercept of $25x^2+16y^2=36$

2) Is $(y-4)^2-x=5$ a function of x? y?

10w2

3) give a sketch of

$$f(x) = \frac{\sqrt{4-2x}}{10-2x}$$

4) graph $\frac{3}{x-4} + 3$ via transformations.

5) $f(x) = x^2 + 1$ on $(-\infty, 0]$.
find f^{-1}.

Now mix of poly trational functions game.
↳ teams of 6 to start.

10FL

today: Wiki & Graphing

Aug: test 3 Alexis A's graded
Aug +3?

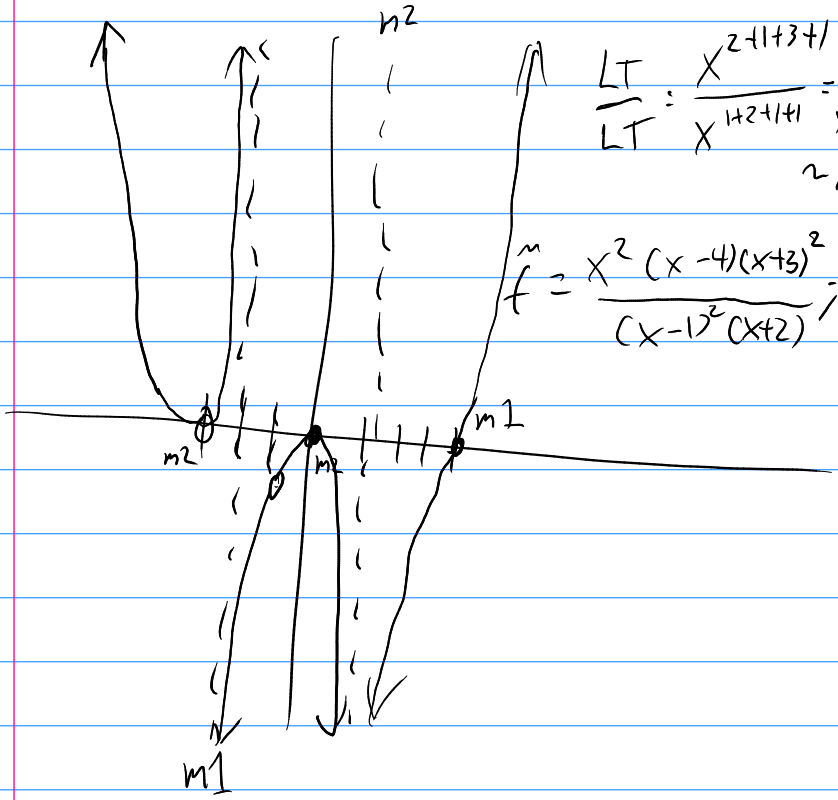
review) 11) $f(x) = \frac{7}{\sqrt{3x+2}}$

Note $g(x) = \frac{7}{x}$ has $(g \circ h) = f$
$h(x) = \sqrt{3x+2}$

1 OF 2

2) find f^{-1} for $f(x) = \frac{2x^3+4}{3} - 8$

3) plot $f(x) = \frac{x^2(x-4)(x+3)^3(x+1)}{(x+3)(x-1)^2(x+2)(x+1)}$



$\frac{LT}{LT} = \frac{x^{2+1+3+1}}{x^{1+2+1+1}} = \frac{x^7}{x^5} = x^2$

$\tilde{f} = \frac{x^2(x-4)(x+3)^2}{(x-1)^2(x+2)}$; $x \neq -3, -1$

Now wiki: 9, then Alekstime.

11/M1 10.1 Exponential functions

(Bruno: 1) You plot

$$f(x) = \frac{(x+1)(x+2)^2}{(x+1)(x-1)}$$

2 indv. 3 points, rest me

Ans: EB: $x-1 \overline{)\, x^2+4x+4}$ $\quad x+5$ rem 9

$-(x^2 - x)$

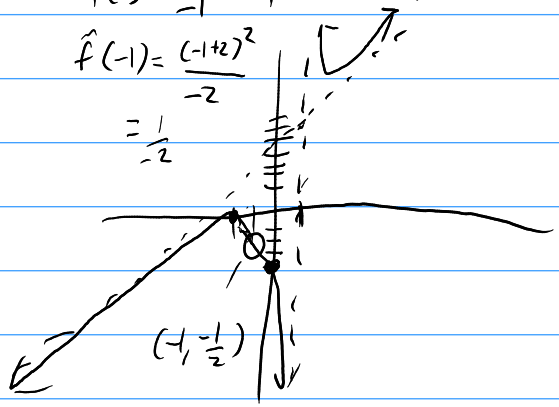
$5x+4$

$-(5x-5)$

9

$f(0) = \frac{2^2}{-1} = -4$

$f(-1) = \frac{(-1+2)^2}{-2} = \frac{1}{-2}$



(100 yr avg)

2) Interest is about 3.3% per year.

A placard in a museum says "in 1942, these boots cost 12 usd"

How pricey was that in today dollars?

11M2

Ans:

$12 \cdot 1.033^{2025-1942}$

$= 12 \cdot 1.033^{83}$

$= 12 \cdot 7.73$

≈ 92.79 USD.

Content: Expmantial functions

Defn: $f(x) = ab^x$ is called an expmantial function.

(Recall: expmentation has higher precedence than multiplication)

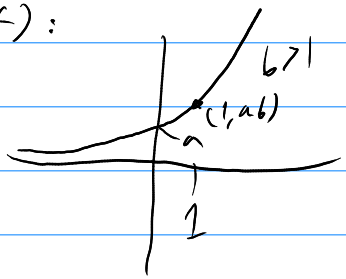
a: initial value; b: Base

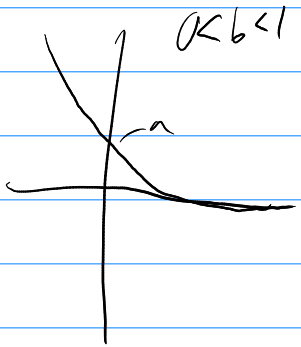
Say $a > 0$, then

Domain $(f) = (-\infty, \infty)$

Range $(f) = (0, f)$

Plot (f):





Key points: $f(0) = a$, $f(1) = ab$) note $\frac{f(1)}{f(0)} = b$

11M3

this holds true allways:

$$\frac{f(n+1)}{f(n)} = b$$ if f is an exponential function.

This says we multiplicatively grow by a factor of b each time the input increases by 1.

Bit of theory: Note $2^3 = 2 \cdot 2 \cdot 2$

$2^{1/2} = x$ such that $x^2 = 2$.

What does 2^π mean?

its the limit of sucessive approximations

$2^3, 2^{\frac{31}{10}}, 2^{\frac{314}{100}}, \ldots$

This how we get a smooth curve from exponential functions.

Now BB notes

11W1

10.2 Growth Decay (Preview: 1) A few years ago I invested enough for my kid in 26 yrs to go to a cheap college (36,000 USD). If stocks return about 7% after inflation, How much did I invest?

Ans:

$$36{,}000 = P \cdot (1.07)^{26}$$

$$P = \frac{36000}{(1.07)^{25}} = 6{,}199.$$

you) Calculate how much you would need to invest for your kid(s).

2) Does your CEO make too much?

your company has a tour per 10 under thing in leadership, each with a pay increase of 40%. You are at the base making 20,000 $/yr. How much should

11W2

the CEO make if the company has 100,000 employees?

Ans:

#terms: $\log_{10}(100{,}000) = 5$

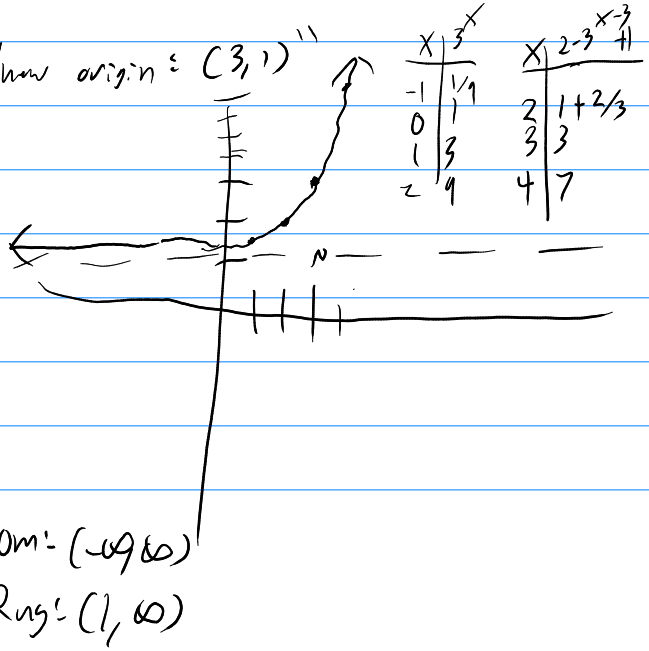
$P(t) = 20000 \cdot 1.4^t$

Pay @ t'th term

$P(5) = 20000 \cdot 1.4^5$

$= 107568$ USD

Me: graph $2 \cdot 3^{x-3} + 1 = f(x)$



Dom: $(-\infty, \infty)$

Rng: $(1, \infty)$

11w3

You: graph $\frac{1}{2} \cdot 2^{x-1} = f(x)$

Finance: Formulas: If an account has a rate of r, compounded n times per year. Then

$$A(t) = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

where t = time (yrs), A = Amount at end, P = Principal (initial investment).

As $n \to \infty$, we get Compounding Continously:

$$A = Pe^{rt}$$ where e = eulers Constant $\approx 2.71828\ldots$

ex) Wealthfront offers daily Compounding at an interest rate of 3.4403%. I've got 3,500 in it. How much interest will I gain in a month? you

What if it was Continous? you

11W4

Defn: APY or **Effective rate**

"put a dollar in for year and it's the interest"

$$Eff(r,n) = \left(\left(1+\frac{r}{n}\right)^n - 1\right)\cdot 100$$

(r: as decimal; $\cdot 100$: convert to %)

ex) $Eff(.034403, 365) = 3.5\%$

My bank has an APY of 3.5%.

Now BB notes p 5 – end. — didn't get to

11F1 Ann: – wiki today

– KC 4 opens Sunday

– TH Nov. 10 + 11th

↳ get $A = P\left(1+\frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ formulas

– Nov 18th – make up day.

11F2

Defn: Percent Inc/Decrease:

For $f(x) = a \cdot b^x$

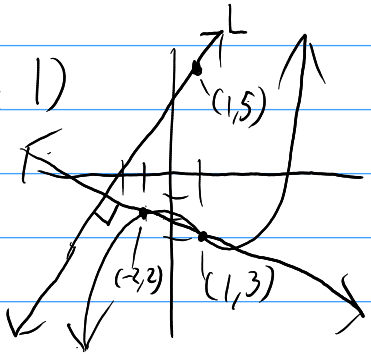
the percent inc/dec. per unit is

$100 \cdot (b-1)$. If negative, its decrease.
If positive, its increase.

ex) $f(x) = 10 \cdot (.7)^x$

% dec = 30%

(Preview: 1)



what is the equation for L?

2) what is the domain of

$$f(x) = \log\left(\frac{\sqrt{x}-1}{(x-\frac{1}{2})}\right) ?$$

Ans: $x \neq 1/2$

$x \geq 0$

$\log(\sim)$: $\sim > 0$

$$\frac{\sqrt{x}-1}{(x-\frac{1}{2})} > 0$$

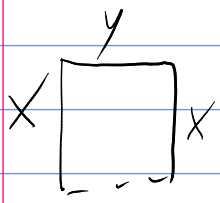
signs:

	0	1/4	1/2	3/4	1	2
Num	−	−	−	−	0	+
Den	−	−	0	+	+	+
Tot	+	+	NA	−	0	+

11F3

Domain: $[0, \frac{1}{2}) \cup (1, \infty)$

3) Sally and Sue's fun time sun park needs a privacy fence. They have 400m of fence. One side is touching the sun and takes a one way glass fence. How can they maximize these sun zone's area (assume a rectangle).



$400 = 2x + y$

$A = x \cdot y$

$y = -2x + 400$

$A = x(400 - 2x)$

$= 400x - 2x^2$

$x_{max} = \frac{-b}{2a} = \frac{-400}{-4} = 100$

$y_{max} = -2 \cdot 100 + 400 = 200.$

They maximize the sun zone by making 100 x 200 meters.

Now: wiki time

12M1

Logs

10,3/+11

Ass: T 4 Nov. 10+11th
-remake Nov. 18th

(P)review: 1) graph $f(x) = \log_3(x+1) - 1$

2) $\log_5(x+7) = 1 + \log_5(x+3)$

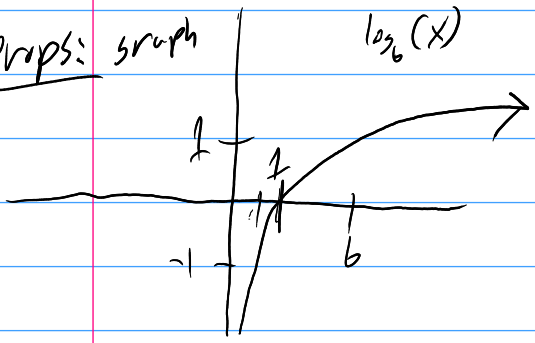
3) If interest is 12%. How long till your account doubles?

4) Bob, Sue, and Xeph Halifax want to send there kiddo to school. They have 12000 to invest and need 54000 in 16 years. What is the minimal rate they need for
a) Semi anual Compounding.
b) Continous Compounding.

12M2 Content: Logarithms

Def: The logarithm is the inverse function of exponentiation

Props: graph $\log_b(x)$



good pts:

x	$\log_b(x)$
$1/b$	-1
1	0
b	1
b^2	2
$\vdots$	

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Equational properties:

a) $\log_b(a \cdot c) = \log_b(a) + \log_b(c)$

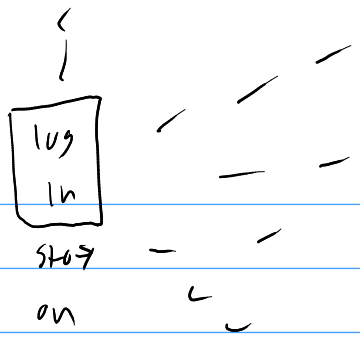
b) $\log_b(a^c) = c \log_b(a)$

c) $\log_b(a) = \dfrac{\log_c(a)}{\log_c(b)}$ "Change of base formula"

or $\log_c(a) = \log_c(b) \cdot \log_b(a)$

12M3

On calculator:



and 2nd → catalog 0 → log Base

or alpha → window f2 → log Base

or math → A: log Base

Now BB mod 10 part 2.

12W1 Ann: - T4 10th + 11th

- Test make up Nov. 18th
- thanks giving break Nov 26-28th

- first get with wiki 0 groups and go over wiki.
  - if pair work together to understand. I'll go around and update grades
  - if good, then Aleks time.

12W2

_I do one of if % < 70% known

Game day: - 4 rounds of
Aleks needed Mod 10

12F1

- 4 rounds of mod 9

- repeat till Class time.

Ann: - Test 4 Monday 4 tuesday
- retake nov. 18th
  - last score counts
- Final Sunday 11-7
  Wednesday 9-7
  of finals week

Sec 14: Continuous growth/ continuous compounding
use same formula.

Now Aleks test review
- first 20 min rush through
  skipping know how to do, skip don't know.
- 15 min of together
- 15 min of me go over.