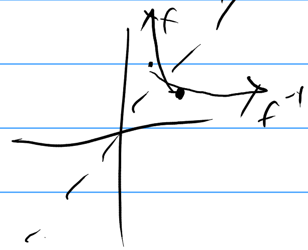
6W1

Mod 6 today

Test 3 notes

<u>Ann:</u> - Take test 3 today by 4:30 for full 75 min!

<u>Preview</u>: 1) $f(X) = 3(x-4)^2 + 2$ on $(-\infty, 4]$
find $f^{-1}(X)$.



$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[loop right, "id_B"] \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

$$\underbrace{f(f^{-1}(x))}_{= x} = 3(\overbrace{f^{-1}}^{\text{isolate}}(x) - 4)^2 + 2$$

$$\frac{x-2}{3} = \cancel{3}(f^{-1}(x)-4)^2$$

from domain rest. or pic.

$$-\sqrt{\frac{x-2}{3}} = f^{-1}(x) - 4$$

$$4 - \sqrt{\frac{x-2}{3}} = f^{-1}(x)$$

2) You're costs at Rob's Hotdogs per day is 400$ for cart and supplies and 50¢ per hotdog. Model this and interpret

6W2

its invers function.

$$D(h) = 400 + .5h$$

$$\begin{tikzcd}
\text{Hotdogs sold} \arrow[r, bend left, "D"] & \text{Daily cost} \arrow[l, bend left, "D^{-1}"]
\end{tikzcd}$$

$D^{-1}(C)$:

$$C = 400 + .5\underline{h}$$

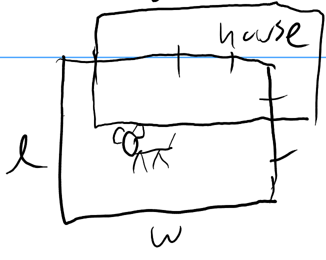
$$C - 400 = .5h$$

$$2C - 800 = h$$

$$D^{-1}(C) = 2C - 800$$

$D^{-1}(C)$ gives for a given daily cost, the number of hot dogs sold to achieve this cost.

3) (mod 7) Your making a rectangular yard. Half of and $\frac{3}{4}$ of 2 sides is encompased by your house. If you have 100 m of fence, what dimensions maximise doggo romp land?



6W3

$P = 100 = 1.5l + \frac{5}{4}w$

1) simplify

$4P = 400 = 6l + 5w$

$A = l \cdot w - \left(\frac{3}{4}l \cdot \frac{1}{2}w\right)$

$= l \cdot w - \frac{3}{8}lw$

$= \frac{5}{8}lw$

2) isolate $l = \frac{400 - 5w}{6}$

3) plug into other eq'ns

$A = \frac{5}{8}\left(\frac{400 - 5w}{6}\right)w$

4) maximum at vertex:
halfway between roots

$0 = \frac{5}{8}\left(\frac{5 \cdot 80 - 5w}{6}\right)w$

$0 = \frac{5 \cdot 5}{8 \cdot 6}(80 - w)w$

$0 = (80 - w)w$

$r_1 = 80, r_2 = 0$

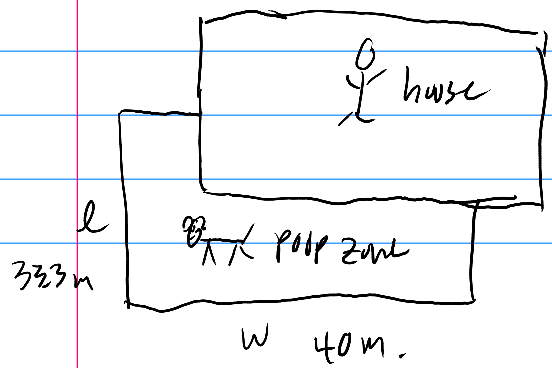
mid $= \frac{80 + 0}{2} = 40$

$W_{max} = 40$

l_{max} = plug in W_{max} into l eqn.

$= \frac{400 - 5 \cdot 40}{6}$

$= \frac{200}{6} = \frac{100}{3} \approx 33.3$



Now! BB notes M6 on PN except [illegible] one

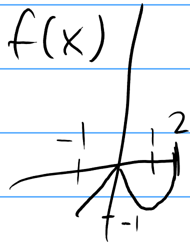
6F1

6.2 inverses

(P) review: 1) $f(x) = 3x^2$

find the line $\perp$ to the secant line on f through $x=2,4$ through $(3, f(3))$.

2) $f(x)$ draw $g(x) = 2f(-x)+3$



3) $f(x) = \sqrt{6x+30}$ on $[-5, \infty)$

plot $f^{-1}(x)$.

4) you: $f(x) = 2(x-1)^2$ on $[1, \infty)$

plot $f^{-1}(x)$.

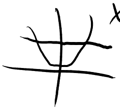
skip

5) you: decompose $H(x) = \dfrac{7x^2+2}{3x^2-1}$ into a composition of two funcs

Content: 6.2 Inverse Functions

Def: A function, f, is injective (or one-to-one) if when $f(a) = f(b)$, then $a = b$.

Also said to pass the "horizontal line test"

non-ex)  x^2

$(-2)^2 = (2)^2$

yet $-2 \neq 2$

6F2

Def: A function, $f:A\to B$, is <u>surjective</u> if the range of f is B. That is for every $b\in B$, there exists an $a\in A$ with $f(a)=b$.

non-ex) $f:\mathbb{Z}\to\mathbb{Z}$, $n\mapsto 2n$ — no input gives 3.

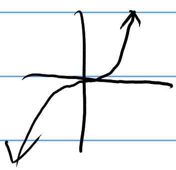
Def: A function, $f:A\to B$ and a function $g:B\to A$ are inverses, if $f\circ g=id_B$ and $g\circ f=id_A$ where $id_C(c)=c$.

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[loop right, "id_B"] \arrow[l, bend left, "g"] \end{tikzcd}$$

Alt: $f(g(x))=x$ and $g(f(y))=y$

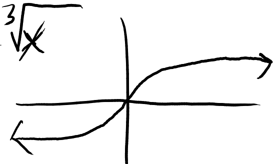
Fact: A function that is both injective and surjective is invertable.

ex) $f(x)=x^3$



is injective: pass H-line test ✓
surj: image is $\mathbb{R}$ ✓

$f^{-1}(x)=\sqrt[3]{x}$



7M1

(P)review:

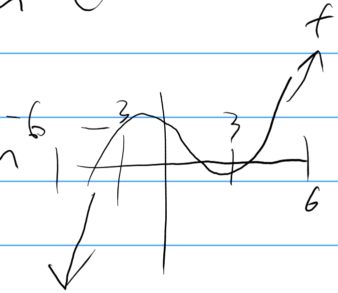
you 0) $f(x) = (x+3)^2$ on $(-\infty, -3]$

find f^{-1} and plot

6.3 peicewise

wiki 6

1) find x-ints on



2) find $f \circ g$ $\boxed{\text{Aleks}}$

Composition of two functions: Domain and range

3) you've got coupons for shoes.
Coupon f takes 20 usd off,
Coupon g take 20% off.
Which composition will the store use to maximize profit?

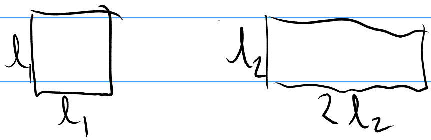
NOW) 6.3 peicewise graphing (7 min)

Then wiki 6 - unhide

7W1 Quadratics

Mod 7.1 warm up:

1) A wire is cut to make a square and a rectangle w/ its width twice its length. If the wire is 18cm, what are the dimensions that minimizes total area?



$$18 = 4l_1 + 6l_2 \rightarrow l_1 = \frac{18-6l_2}{4} = \frac{9-3l_2}{2}$$

$$A = l_1^2 + 2l_2^2$$

$$A = \left(\frac{9-3l_2}{2}\right)^2 + 2l_2^2$$

$$= \frac{1}{4}\left(9^2 - 2\cdot 9\cdot 3l_2 + 9l_2^2\right) + 2l_2^2$$

$$= \underbrace{\left(\frac{9}{4}+2\right)}_{a} l_2^2 - \underbrace{\frac{9\cdot 3}{2}}_{b} l_2 + \underbrace{\frac{9^2}{4}}_{c}$$

$$l_2^{min} = \frac{-b}{2a} \boxed{\pm\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}}$$

aside - quadratic formula

$$\frac{-b \pm \sqrt{b^2-4ac}}{\sqrt{2|a^3}}$$

$$= \frac{\frac{9\cdot 3}{2}}{2\left(\frac{9}{4}+2\right)} = \frac{\frac{9\cdot 3}{2}}{\frac{9}{2}+\frac{8}{2}} = \frac{9\cdot 3}{17} = \frac{9\cdot 3}{17} = \frac{27}{17}$$

7W2 $\quad l_1^{min} \approx \frac{9 - 3 \cdot \frac{27}{17}}{2}$

Content: Quadratics

Forms: Vertex $\quad f(x) = (x - V_x)^2 + V_y$ $\quad$ Alex's $-h$, $+k$

Vertex @ (V_x, V_y)

Standard: $f(x) = ax^2 + bx + c$

Def: The vertex of a quadratic is its extrema, that is its min/max.

Formula: for $f(x) = ax^2 + bx + c$, the vertex of f is at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

Fact/Def: the axis of symmetry of f is $x = \frac{-b}{2a}$.

In particular, the roots of f lie symmetric about its vertex.

7W3

Def: let $f(x) = ax^2+bx+c$, then the roots / x-intercepts / where f vanishes / zeros of f is $f^{-1}(0)$.

Formula: if $f(x) = ax^2+bx+c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

V_x

Now BB notes

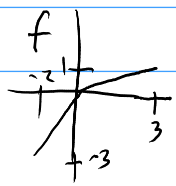
7F1

7.2 - vertex

Ann: - wiki 7 monday

- Fall break 13th + 14th
- test 3 Oct 20th + 21st
- KC 12th oct

Preview:

1) 

plot $g(x) = -f(x-3)$

2) $f(x) = \dfrac{\sqrt{x^2-5x+2}}{(x-1)(x+2)}$ what is the domain of f?

$\sqrt{\ }$: $x^2-5x+2 \ge 0$

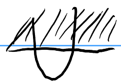
$\frac{1}{\ }$: $(x-1)(x+2) \ne 0$

$x \ne 1,\ x \ne -2$

$x^2-5x+2=0$

$$x = \frac{+5 \pm \sqrt{25-4\cdot 2\cdot 1}}{2\cdot 1}$$

$$= \frac{5 \pm \sqrt{17}}{2}$$



Note $\dfrac{5+\sqrt{17}}{2} < \dfrac{5+\sqrt{25}}{2} = 1$

$\dfrac{5-\sqrt{17}}{2} > 0 > -2$

Domain: $(-\infty,-2) \cup \left(-2, \frac{5-\sqrt{17}}{2}\right] \cup \left[\frac{5+\sqrt{17}}{2}, 1\right) \cup (1,\infty)$

Now BB M7 notes - pg 3-5

↳ groups of 3 wiki style

~ pg 5 around me lecture quickly

8M1 Graphing Quadratics

wiki: 7 + 7.3 graphing

Cool trick: for $y = x^2$, note differences

x	x^2		
-3	9		
		-5	
-2	4		2
		-3	
-1	1		2
		-1	
0	0		2
		1	
1	1		2
		3	
2	4		2
		5	
3	9		

ie, when you go one more from V_x in x, you increase how much you go up by 2.

ex) plot $-2(x-2)^2+4 = f(x)$

x	f
1	$4-2\cdot 1 = 2$
2	$4+0$
3	2 — axis of sym

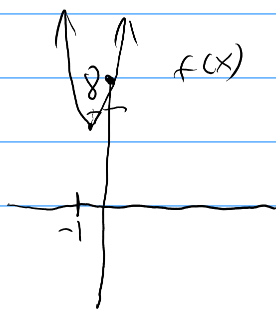
ex) $3x^2 + 4x + 8 = f(x)$

$LC = a = 3$ $\qquad$ $y\text{-int} = 8$

$V_x = \frac{-4}{2\cdot 3} = \frac{-2}{3}$

$V_y = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) + 8$

$= \frac{+4}{3} - \frac{8}{3} + 8 = 8 - \frac{4}{3}$

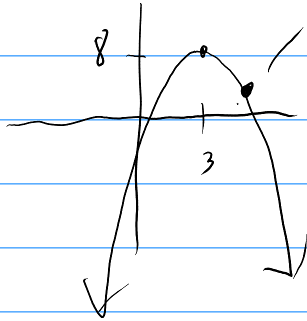


$f(x)$

$f(x) = 3(x - V_x)^2 + V_y$

8M2

ex)



$f(x) = a(x-3)^2 + 8$

$3.2 = a(5.1-3)^2 + 8$

$\frac{3.2-8}{2.1^2} = a$

Now wk: 7, when done Aleks time

8W1 Ann: - Fall break 13^{th} + 14^{th}

– KC opens 12^{th}

module 8

- test 3 20^{th}–21^{st}

L M6 – 8

(Preview: 1) plot $f(x) = \frac{(x+7)(x-1)^3}{(x-1)(x+2)}$

module 9

End behavior: tot deg = 4 – 2 = 2 ~ like x^2

$\frac{x^4}{x^2}$

LC: $\frac{LC}{LC} = 1$

"↑ ↑"

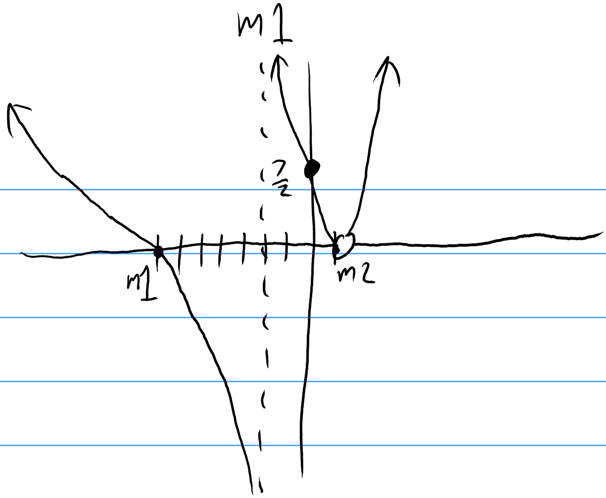
Simplified: $\tilde{f}(x) = \frac{(x+7)(x-1)^2}{x+2}$; $x \neq 1$

holes: $x = 1$

roots: –7 m1, 1 m2

poles: –2 m1

other point: $f(0) = \frac{7(-1)^2}{2} = \frac{7}{2}$



Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

Range: $(-\infty,\infty)$

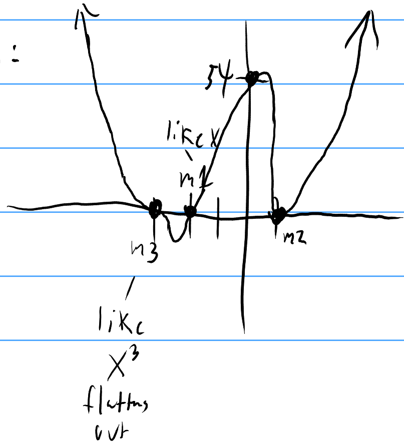
2) $f(x)=(x+2)(x-1)^2(x+3)^3$

End behavior? $LT(f\cdot g)=LT(f)\cdot LT(g)$, ie $x\cdot x^2\cdot x^3 = x^6$ — even positive

roots w/ multiplicities: $-2m1$, $1m2$, $-3m3$

y-int? $f(0)=2\cdot(-1)^2(3)^3=54$

graph:



8W2 Content: polynomials

Def: Let R be a ring. The set of polynomials in one variable over R is denoted $R[X]$.

$$R[X] = \{\sum_{i=0}^{n} a_i x^i : a_i \in R, n \in \mathbb{N}\}$$

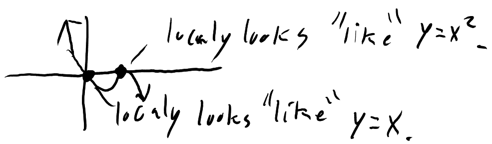
ie things of the form

$$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n.$$

non-ex) $f(x) = \frac{1}{x} + x^{2.3}$
Need natural powers on our variable.

Def: Let $f(x) = (x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_k)^{m_k}$ $\in R[X]$. The multiplicity of a root r_i is m_i. It determines what the graph looks like localy at r_i upto scaling.

ex) $f(x) = -(x-1)^2 x$



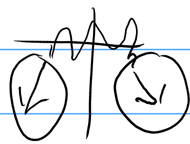
8W3

Def: The <u>end behaviour</u> of $f(x)$ is $\lim_{x \to \pm\infty} f(x)$, ie what does f approach as x grows. It is determined by the parity of the <u>leading term</u>, the largest non zero term in f in standard form.

Ex) $f(x) = 3x^7 + x^8 - 2x^{12}$

Leading term $= -2x^{12}$

$-2x^{12} \underset{EB}{\sim} -2x^2$ as even.

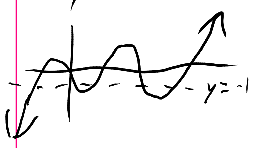


Fact: $LT(f \cdot g) = LT(f) \cdot LT(g)$

So for instance: $LT((3x-7)^4(-2x+2))$

$= (3x)^4 \cdot (-2x)$

Def: The degree of a polynomial is the degree of its leading term. Its also max # of extrema plus 1. Also # times a polynomial of lesser degree intersects it.

ex) 

extrema: 4

times poly y=-1 intersects: 5

roots: 5

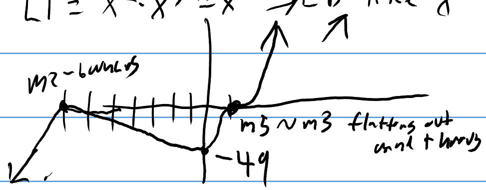
EB: ↙ ↗

so degree is odd.

8W 4

graphing ex) $f(x) = (x+7)^2(x-1)^5$

$LT = x^2 \cdot x^5 = x^7 \Rightarrow$ EB like x



$f(0) = 7^2 \cdot (-1)^5$
$= -49$

YOU) graph $f(x) = -x(x-1)^2(x+2)^4$

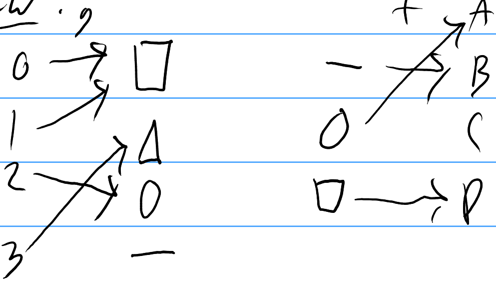
Now BB Mod 8 Notes.

8F1

8.2 roots + multiplicities

(P)review:

1) you

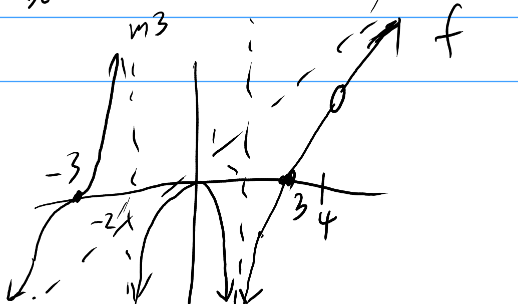


find a) $f \circ g$

b) Domain

c) Range

2)



8F2

$\text{tot deg} = 1 = \underset{6}{(3+2+1)} - \underset{3\ \ 4}{(1+2)}$ — need a pole of higher degree

holes: $x=4$

roots: -3 m 3, 0 m 2, 3 m 1

poles: -2 m 3, 1 m 2

$$f(x) = \frac{a(x+3)^3 x^2 (x-3)\ (x-4)}{(x+2)^3 (x-1)^2 (x-4)}$$

Slant: need $mx+b$ from poly long division.

Num: $a(x^3 \cdot x^2 x) + a\left(3\binom{3}{1}x^2 x^2 x + x^3 x^2 (-3)\right) + \text{lower}$

Den: $x^5 + \left(x^2 \binom{3}{1} \cdot 2x^2 + x^3 \binom{2}{1}(-1)x\right) + \text{lower}$

need only first 2 terms since after $mx+b$

Num: $ax^6 + 6ax^5 + \text{lower}$

Den: $x^5 + 4x^4 + \text{lower}$

$$\begin{array}{r|l} & ax + 2a \\ \hline x^5 + 4x^4 & ax^6 + 6ax^5 \\ & -ax^6 - 4ax^5 \\ \hline & 0 \quad 2ax^5 \end{array}$$

y-int of slant $=1$

so $2a = 1$

$a = \frac{1}{2}$

$$f(x) = \frac{1}{2} \cdot \frac{(x+3)^3 x^2 (x-3)(x-4)}{(x+2)^3 (x-1)^2 (x-4)}$$

8F3

Now: finish BB M8 notes. Then Aleks time.