Ann: Final day of test   w9F1
9.1 rational

(P)review:

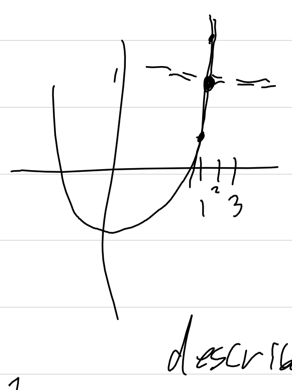
1) $f(x) = x^2 - 3$ write an approximation of the perpendicular line through $f(2)$.

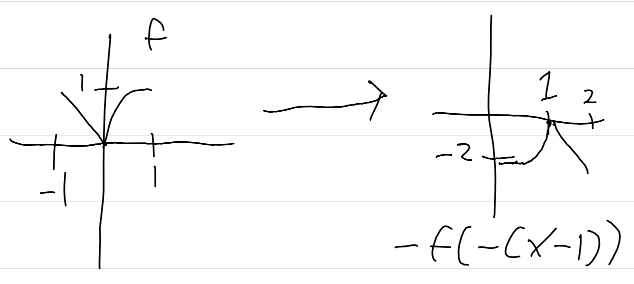
Ans: $AROC(f, 2-1, 2+1) = \frac{(3)^2-3-[(1)^2-3]}{3-1}$
$= \frac{6-(-2)}{2} = 4$

$m_\perp = \frac{-1}{4}$

$f(2) = 4-3 = 1$

$y - 1 = -\frac{1}{4}(x-2)$



2) 

describe transformation

$-f(-(x-1))$ ✓

Check: $-f(-(1-1)) = f(0)$

x-axis $: -f$
y-axis $: f(-x)$
→ 1 : $f(x-1)$

3) $f(x) = (x-2)^2 + 4$; $x \in (-\infty, 2]$
$g(x) = -\sqrt{x-4} + 2$
find $f \circ g$, $g \circ f$ dom + range

4) describe $f(x) = -2(x-7)^2(\frac{x}{3}-1)(2-x)$
LT:
LC:
EB:
roots w/ mult:
y-int:
sketch:

5) plot $f(x) = \frac{(2x-2)^2(x-1)(x-7)}{(x-1)(x+3)^2}$

$\tilde{f}(x) = \frac{4(x-1)^2(x-7)}{(x+3)^2}$; $x \neq 1$

holes: 1
roots: 1m2, 7m1
poles: -3m2

EB: $\sim \frac{4x^3}{x^2} \sim 4x$

num: $4x^3 + 4x^2(-2-7) + \text{lower}$

$$\begin{array}{r} 4x \\ x^2+6x \overline{\big) 4x^3 - 36x^2} \\ \vdots \end{array}$$

Content: Rational functions

Build up: Solve $0 = az - b$
$\frac{b}{a} = z$

so, $0 = (x+1)(x-3)z - (x+3)$
$\frac{(x+3)}{(x+1)(x-3)} = z$

Def: A rational function is a fraction $\frac{p(x)}{q(x)}$ where both p and q $(q \neq 0)$ are polynomials. The set of all rational functions with coefficients in $\mathbb{R}$ and indeterminant x is $\mathbb{R}(x)$.

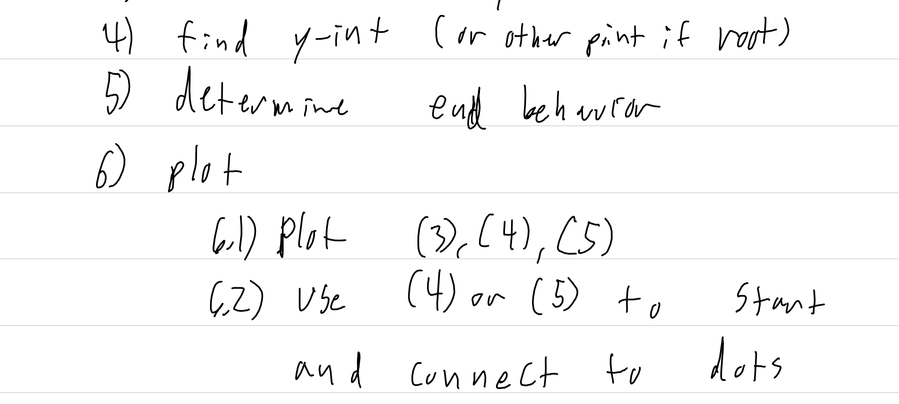
Def: For $f(x) = \frac{p(x)}{q(x)}$, holes/removable singularities are where $p(x) = q(x) = 0$
roots/x-ints/... are where $p(x) = 0$ and $q(x) \neq 0$
restricted values are where $p(x) = 0$
Vert. asymptotes/poles are where $q(x) = 0$ and $p(x) \neq 0$

The end behavior (EB) $f(x)$ is $\lim_{|x| \to \infty} f(x)$, ie how $f(x)$ behaves as $|x|$ grows.

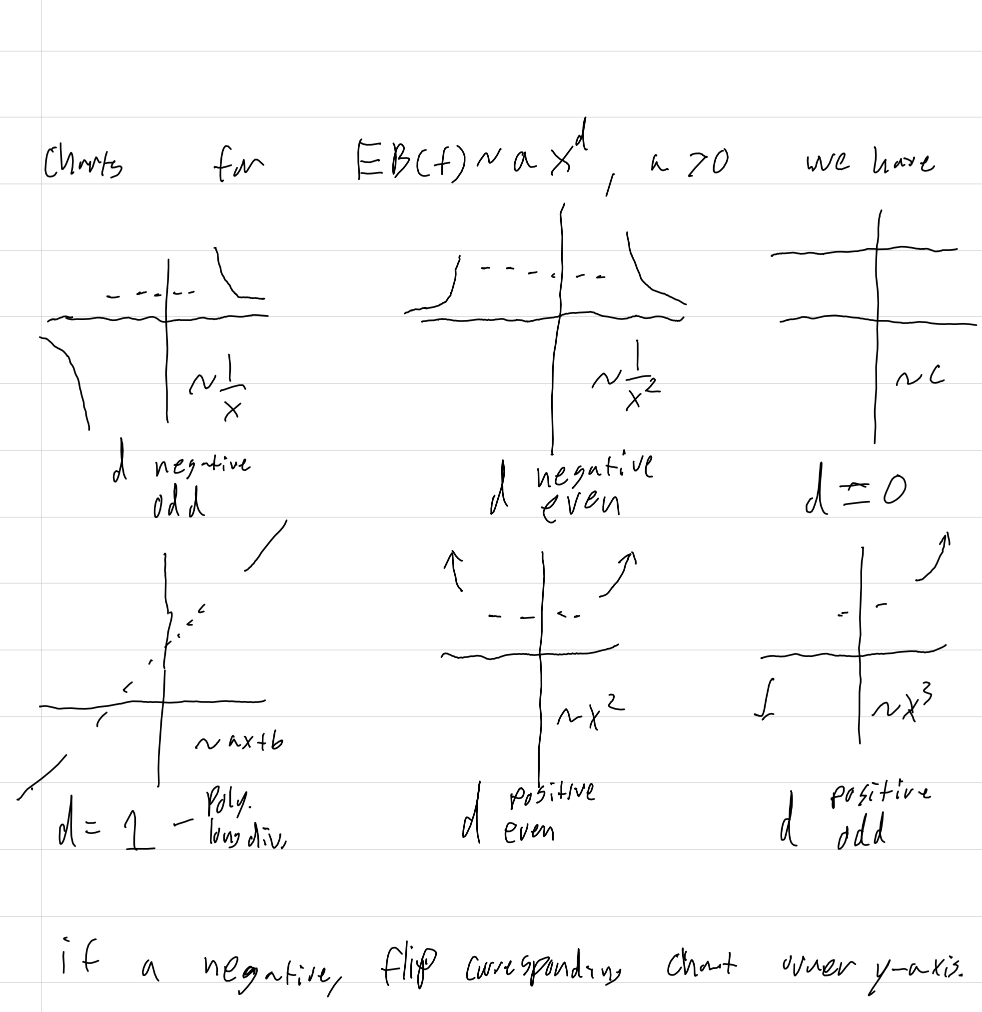
$EB(f) \approx \frac{LT(p)}{LT(q)} = \frac{LC(p)}{LC(q)} x^{\deg(p)-\deg(q)}$
if $\deg(p) - \deg(q) = 1$, need slant.

Plotting:
1) find holes
2) "simplify"
3) find roots and poles w/ multiplicities
4) find y-int (or other point if root)
5) determine end behavior
6) plot
 6.1) plot (3), (4), (5)
 6.2) use (4) or (5) to start and connect to dots

Locally: up to linear transformations, locally, poles look like



EB charts for $EB(f) \sim ax^d$, $a > 0$ we have



if a negative, flip corresponding chart over y-axis.

NOW: BB notes 9.1

Ann: - Test 3 Avg ~63ish   w10M1

└ no pered yet + no written resp graded) 9.1-9.2 ...

- Spring break next week

Plotting:

1) find holes
2) "simplify"
3) find roots and poles w/ multiplicities
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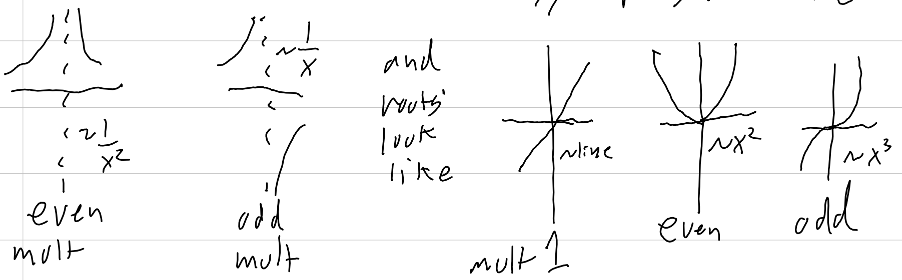

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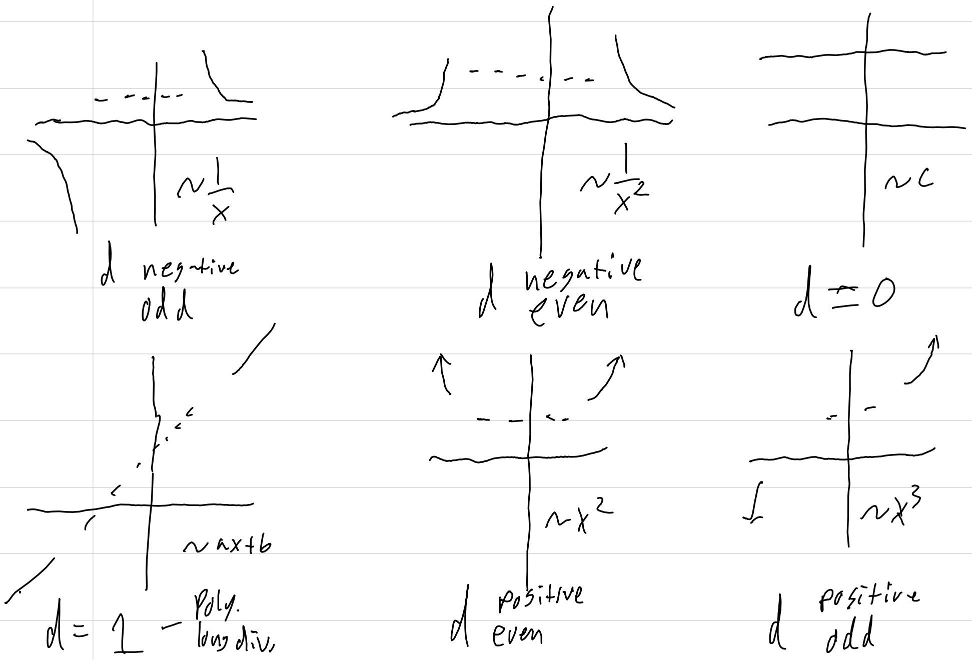

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NOW: BB notes 9.1

Polynomial long division

use 9.2 BB on white board

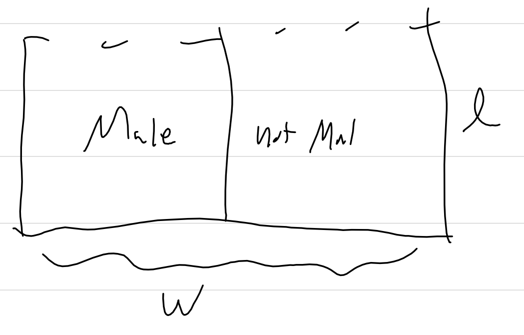
Aim:-wiki Friday

W10.W1

BB 9.3

(Preview: 1) Dan and Joé are making an onsen and need to fence 3 sides plus a divider. What dimensions maximize area if they have F many ft of fence?

just speak



$A = l \cdot w$

$F = 3l + w \Rightarrow w = F - 3l$

$A = l(F-3l)$

$= lF - 3l^2$

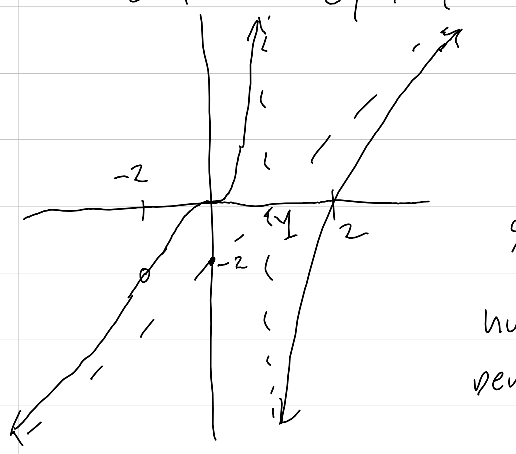
$l_{max} = \frac{-F}{-6} = \frac{F}{6}$

$w_{max} = F - 3\frac{F}{6}$

$= F/2$

$A_{max} = \frac{F^2}{12}$

2) write the eqn for



$\frac{a\ x^3(x-2)(x+2)}{(x+1)^3(x+2)}$

slant: $ax - 2$

num: $ax^4 - 2x^3$

den: $x^3 + 3x^2$

$$\begin{array}{r} ax - 2 \\ x^3+3x^2 \overline{\big)\, ax^4 - 2ax^3} \\ -(ax^4 + 3ax^3) \\ (-5a)x^3 \end{array}$$

$-5a = -2$

$a = \frac{2}{5}$

3) graph $\left(\frac{1}{3}\right)^{x-1}$

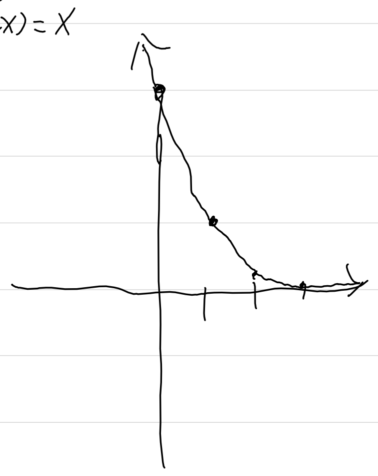
$in_f(x) = x-1$

$out_f(x) = x$

x	$(1/3)^{x}$
-2	9
-1	3
0	1
1	1/3
2	1/9

$(x+1, y)$

x	$(1/3)^{x-1}$
-1	9
0	3
1	1
2	1/3
3	1/9



Dom: $\mathbb{R}$

Image: $(0,\infty)$

Content: BB 9.2* - 9.3*

9.3 game (Subdivide)

$a\,g(x)$

$1 = a\,g(4)$

$\frac{1}{n} = a \quad n$

(will spring break)

W12M7

(10.1)

Ann: - T4 - 4/14

- Makeup day - 4/24

(Preview!!) wiki: a) Dom $\left(\frac{x+2}{(x-2)}\frac{(x-4)}{(x-4)}\right) = (-\infty,2)\cup(2,4)\cup(4,\infty)$

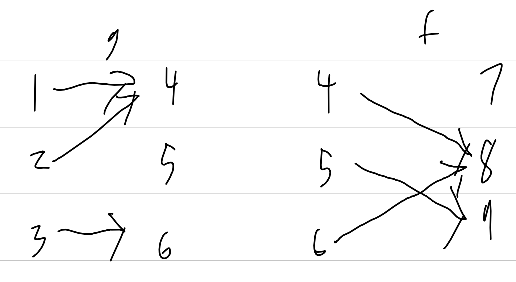
b) Eqn of graph w/ roots $x=4,7$

poles at $x=-1,3$ and a hole @ $x=2$

all mult. = 1.

Ans: $f(x) = \frac{(x-4)(x-7)}{(x+1)(x-3)}\boxed{\frac{(x-2)}{(x-2)}}$

2) $f\circ g$?



3) $f(x)=\sqrt{x+5}$; $g(x)=\log_5(x-7)$

Dom of $\frac{f\circ g}{f}$?

$f\circ g$: g: $x\in(7,\infty)$

f: $g(x)\in(-5,\infty)$

ie $x\in(7+5^{-5},\infty)$

$-5=\log_5(x-7)$

$5^{-5}=x-7$

$7+5^{-5}=x$

$\frac{\sim}{f}$: $x\neq-5$

Dom: $(7+5^{-5},\infty)$

4) graph $f(x)=-2\log_3(x+1)$

Content:

Module 10: Exponential functions and logarithms.

Recall: the function b^n is

$\underbrace{b\cdot b\cdot b\cdots b}_{n\text{-many times}}$

extension: For $b^{p/q}=\sqrt[q]{b^p}=\left(\sqrt[q]{b}\right)^p$ for $p/q\in\mathbb{Q}$

For $r\in\mathbb{R}$, b^r is approximated by $b^{p/q}$ for $p/q\to r$.

Defn: The exponential function with base b with initial value (leading coefficient) C is $C\cdot b^x$. Its inverse function is $\log_b(x/C)$.

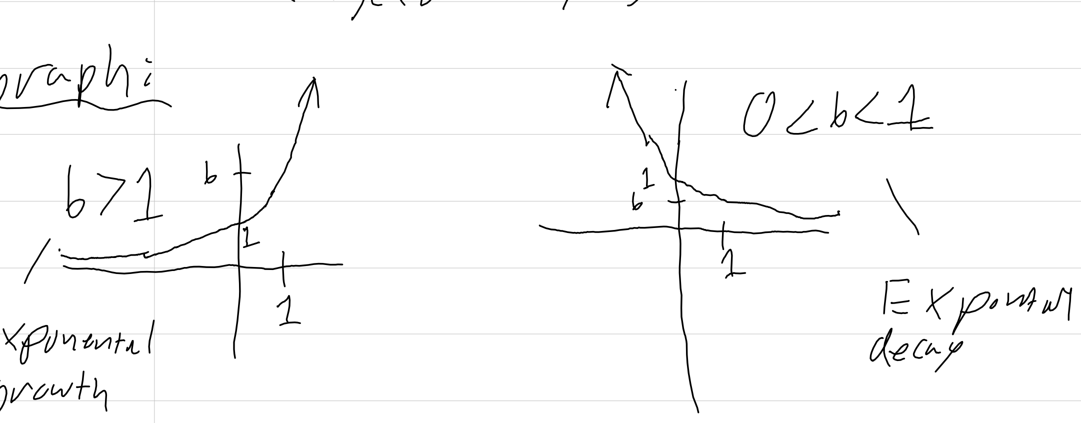
Properties: a) $b^x\cdot b^y=b^{x+y}$ — takes addition inside to product outside.

b) $b^0=1$

c) $\text{Dom}(b^x)=(-\infty,\infty)$

d) $\text{Range}(b^x)=(0,\infty)$

graph:



exponental growth

Exponential decay

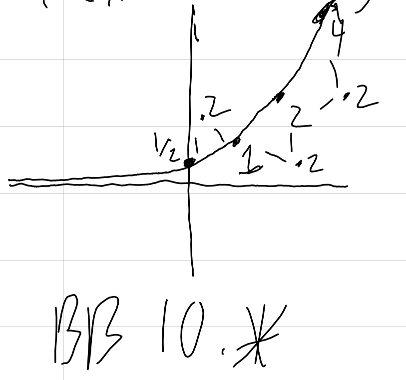
graphing: for $C\cdot b^x=f(x)$ note

$f(0)=C\cdot b^0=C$

and $f(n)=C\cdot b^n$ while $f(n+1)=C\cdot b^{n+1}$

so $f(n+1)=b\cdot f(n)$. ie to go over one from current location multiply by b to get new y-coord.

ex) $f(x)=\frac{1}{2}(2)^x$



Now BB 10.*

w12w1

10.2 growth/decay

(P) Review: 1) $f(x) = 5x^2 - 4$

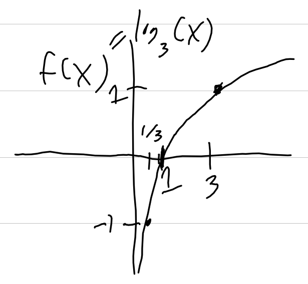
$g(x) = \frac{1}{3x^2+5}$

Domain of $\frac{f}{g}$? $f-g$?

BB 10.1 Graphs

me you me

2) $f(x) = \log_3(x)$



draw $2f(1-\frac{1}{2}x)+4 = g(x)$

$in(x) = 1 - \frac{1}{2}x$

$in^{-1}(x)$: $x = 1 - \frac{1}{2}y$

$1 - x = \frac{1}{2}y$

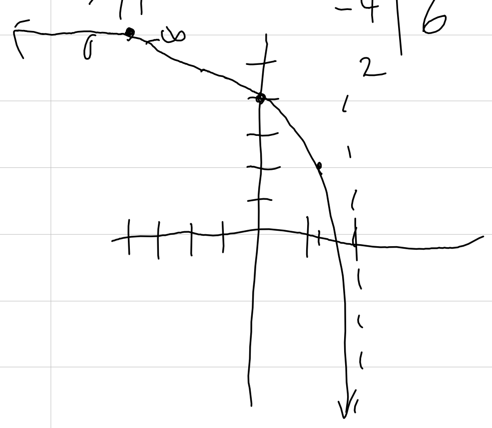
$2 - 2x = y$

$out(y) = 2y + 4$

x	y
$1/3$	-1
1	0
3	1

$\xrightarrow{(2-2x,\ 2y+4)}$

x	$g(x)$
$2-\frac{2}{3}$	2
0	4
-4	6



3) Find x: $\log_6(18) + \log_6(2) = x$

$6^{\log_6(18)} \cdot 6^{\log_6(2)} = 6^x$

$18 \cdot 2 = 6^x$

$6 \cdot 3 \cdot 2 = 6^x$

$6 \cdot 6 = 6^x \Rightarrow x = 2$

$\log_6(a) + \log_6(c) = \log_6(a \cdot c)$

BB: Do 10.1 graphs me, you, you

BB: 10.2, 10.3

Content:

Logs:

Defn: $\log_b(x)$ is the inverse function to b^x ie

$$\begin{tikzcd} \mathbb{R} \arrow[loop left, "id_{\mathbb{R}}"] \arrow[r, bend left, "b^{-}"] & (0,\infty) \arrow[l, bend left, "\log_b(-)"] \arrow[loop right, "id_{(0,\infty)}"] \end{tikzcd}$$

So, $\log_b(b^x) = x$

Hence $b^x = y$ iff $\log_b(y) = x$

Properties: 1) $b^x = y$ iff $\log_b(y) = x$

2) $\log_b(a \cdot c) = \log_b(a) + \log_b(c)$

3) $\log_b(a^r) = r\log_b(a)$

4) $\frac{\log_b(a)}{\log_b(c)} = \log_c(a)$

Proof of 4) Notice:

$a = c^{\log_c(a)}$

$= c^{\log_c(a)}$

$= (b^{\log_b(c)})^{\log_c(a)}$

$= b^{\log_b(c) \cdot \log_c(a)}$

$\log_b(a) = \log_c(a)\log_b(c)$

$b^{\log_b(a)} = b^{\log_c(a)\log_b(c)}$

$a = (b^{\log_b(c)})^{\log_c(a)}$

$a = c^{\log_c(a)} = a$

as $a = b^{\log_b(a)}$ we have

$b^{\log_b(a)} = b^{\log_b(c) \cdot \log_c(a)}$

so by applying $\log_b(-)$ to both sides,

$\log_b(a) = \log_b(c) \cdot \log_c(a)$

So $\frac{\log_b(a)}{\log_c(a)} = \log_b(c)$. ∎

Now to BB M10.d4