6W1

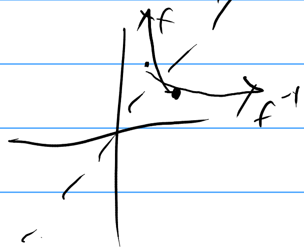
mod 6 today

Test 3 notes

Ann: - Take test 3 today by 4:30 for full 75 min!

Preview: 1) $f(x) = 3(x-4)^2 + 2$ on $(-\infty, 4]$

find $f^{-1}(x)$.



$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[rr, bend left, "f"] & & B \arrow[loop right, "id_B"] \arrow[ll, bend left, "f^{-1}"] \end{tikzcd}$$

$$f(f^{-1}(x)) = 3(\overbrace{f^{-1}}^{\text{isolate}}(x) - 4)^2 + 2$$

$$\| \quad x$$

$$\frac{x-2}{3} = (f^{-1}(x) - 4)^2$$

from domain rest. on pic

$$-\sqrt{\frac{x-2}{3}} = f^{-1}(x) - 4$$

$$4 - \sqrt{\frac{x-2}{3}} = f^{-1}(x)$$

2) You're costs at Rob's Hotdogs per day is 400\$ for cart and supplies and 50¢ per hotdog. Model this and interpret

6W2

its inverse function.

$$D(h) = 400 + .5h$$

$$\begin{tikzcd} \text{Hotdogs sold} \arrow[r, bend left, "D"] & \text{Daily cost} \arrow[l, bend left, "D^{-1}"] \end{tikzcd}$$

$D^{-1}(C)$:

$$C = 400 + .5h$$

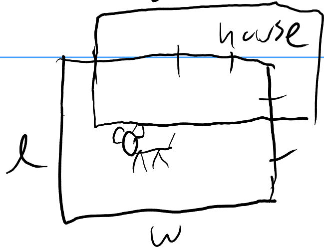
$$C - 400 = .5h$$

$$2C - 800 = h$$

$$D^{-1}(C) = 2C - 800$$

$D^{-1}(C)$ gives for a given daily cost, the number of hot dogs sold to achieve this cost.

3) (mod 1) Your making a rectangular yard. Half of and $\frac{3}{4}$ of 2 sides is encompassed by your house. If you have 100 m of fence, what dimensions maximise doggo romp land?



6W3

$$P = 100 = 1.5l + \frac{5}{4}w$$

1) simplify

$4P = 400 = 6l + 5w$

$$A = l \cdot w - \left(\frac{3}{4}l \cdot \frac{1}{2}w\right)$$
$$= l \cdot w - \frac{3}{8}lw$$
$$= \frac{5}{8}lw$$

2) isolate $l = \frac{400-5w}{6}$

3) plug into other eq'ns

$$A = \frac{5}{8}\left(\frac{400-5w}{6}\right)w$$

4) maximum at vertex:
 halfway between roots

$$0 = \frac{5}{8}\left(\frac{5 \cdot 80 - 5w}{6}\right)w$$

$$0 = \frac{5 \cdot 5}{8 \cdot 6}(80 - w)w$$

$$0 = (80 - w)w$$

$V_1 = 80, V_2 = 0$

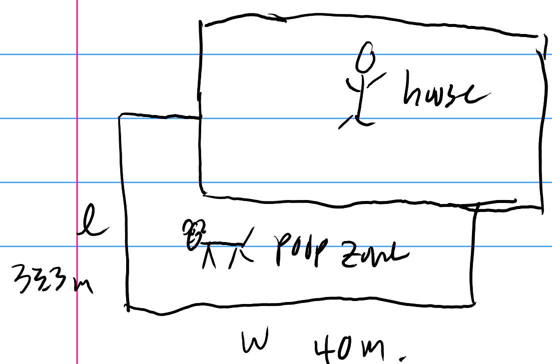
mid $= \frac{80+0}{2} = 40$

$W_{max} = 40$

l_{max} = plug in W_{max} into l eqn.

$$= \frac{400 - 5 \cdot 40}{6}$$

$$= \frac{200}{6} = \frac{100}{3} \approx 33.3$$



Now: BB notes M6 on PN except ~~20~~ ~~10~~ one

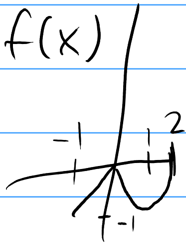
6F1

6.2 inverses

(P) review: 1) $f(x) = 3x^2$

find the line $\perp$ to the secant line on f through $x=2,4$ through $(3, f(3))$.

2) $f(x)$ draw $g(x) = 2f(-x)+3$



3) $f(x) = \sqrt{6x+30}$ on $[-5, \infty)$

plot $f^{-1}(x)$.

4) you: $f(x) = 2(x-1)^2$ on $[1, \infty)$

plot $f^{-1}(x)$.

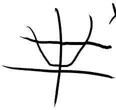
skip

5) you: decompose $H(x) = \frac{7x^2+2}{3x^2-1}$ into a composition of two funcs

Content: 6.2 Inverse functions

Def: A function, f, is injective (or one-to-one) if when $f(a) = f(b)$, then $a = b$.

Also said to pass the "horizontal line test"

non-ex)  x^2

$(-2)^2 = (2)^2$

yet $-2 \neq 2$

6F2

Def: A function, $f: A \to B$, is <u>surjective</u> if the range of f is B. That is for every $b \in B$, there exists an $a \in A$ with $f(a) = b$.

non-ex) $f: \mathbb{Z} \to \mathbb{Z}$, $n \mapsto 2n$ no input gives 3.

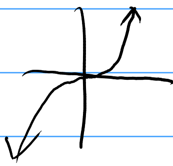
Def: A function, $f: A \to B$ and a function $g: B \to A$ are inverses, if $f \circ g = id_B$ and $g \circ f = id_A$ where $id_C(c) = c$.

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[loop right, "id_B"] \arrow[l, bend left, "g"] \end{tikzcd}$$

Alt: $f(g(x)) = x$ and $g(f(y)) = y$

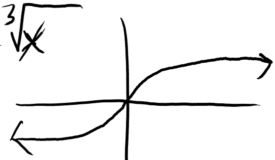
Fact: A function that is both injective and surjective is invertable.

ex) $f(x) = x^3$



is injective: pass H-line test ✓
surj: image is $\mathbb{R}$ ✓

$f^{-1}(x) = \sqrt[3]{x}$



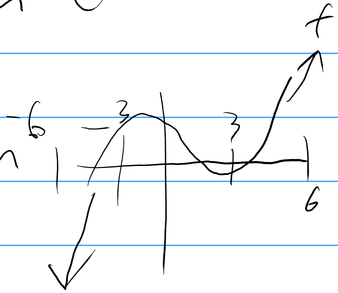
7M1

(P)review:

you 0) $f(x) = (x+3)^2$ on $(-\infty, -3]$
find f^{-1} and plot

6.3 peicewise 1) find x-ints on

wiki 6



2) find $f \circ g$ $\boxed{\text{Aleks}}$
Composition of two functions: Domain and range

3) you've got coupons for shoes.
Coupon f takes 20 USD off,
Coupon g take 20% off.
Which composition will the store use to maximize profit?

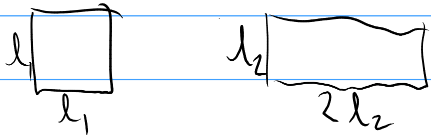
NOW) 6.3 peicewise graphing (7 min)

Then wiki 6 - unhide

7.1 Quadratics

Mod 7.1 warm up:

1) A wire is cut to make a square and a rectangle w/ its width twice its length. If the wire is 18 cm, what are the dimensions that minimizes total area?



$$18 = 4l_1 + 6l_2 \rightarrow l_1 = \frac{18-6l_2}{4} = \frac{9-3l_2}{2}$$

$$A = l_1^2 + 2l_2^2$$

$$A = \left(\frac{9-3l_2}{2}\right)^2 + 2l_2^2$$

$$= \frac{1}{4}\left(9^2 - 2\cdot 9\cdot 3 l_2 + 9l_2^2\right) + 2l_2^2$$

$$= \underbrace{\left(\frac{9}{4}+2\right)}_{a} l_2^2 - \underbrace{\frac{9\cdot 3}{2}}_{b} l_2 + \underbrace{\frac{9^2}{4}}_{c}$$

$$l_2^{min} = \frac{-b}{2a} \boxed{\pm\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}}$$

inside quadratic formula

$$\frac{-b \pm \sqrt{b^2-4ac}}{\sqrt{2|a^3}}$$

$$= \frac{\frac{9\cdot 3}{2}}{2\left(\frac{9}{4}+2\right)} = \frac{\frac{9\cdot 3}{2}}{\frac{9}{2}+\frac{8}{2}} = \frac{9\cdot 3}{17} = \frac{9\cdot 3}{17} = \frac{27}{17}$$

7W2 $\quad l_1^{min} \approx \frac{9-3\cdot\frac{27}{17}}{2}$

Content: Quadratics

Forms: Vertex $\quad f(x) = (x-V_x)^2 + V_y$ $\quad$ Aleks: $-h$, $+k$

Vertex @ (V_x, V_y)

Standard: $f(x) = ax^2+bx+c$

Def: The vertex of a quadratic is its extrema, that is its min/max.

Formula: for $f(x) = ax^2+bx+c$, the vertex of f is at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

Fact/Def: the axis of symmetry of f is $x = \frac{-b}{2a}$.

In particular, the roots of f lie symmetric about its vertex.

7W3

Def: let $f(x) = ax^2+bx+c$, then the roots / x-intercepts, where f vanishes / zeros of f is $f^{-1}(0)$.

Formula: if $f(x) = ax^2+bx+c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

v_x

Now BB notes