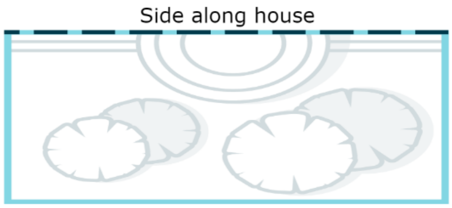
The Vertex

1. Carlos has 360 meters of fencing. He will use it to form three sides of a rectangular garden. The fourth side will be along a house and will not need fencing. One of the sides is length x (in meters).
2. .



X

l

$l + 2X = 360$
$l = 360 - 2X$

Step 1: Write a function $A(x)$ that represents the enclosed area in terms of x (in meters).

$A(x) = x(360 - 2x) = -2x^2 + 360x$

vertex x coord

roots $\nearrow = 0, 180$, by symmetry, $V_x = \frac{180+0}{2} = 90$

Step 2: Find the vertex using the vertex formula: $x = -\frac{b}{2a}$

$X = \frac{-360}{2(-2)} = \frac{360}{4} = 90$

Step 3: Find the dimensions that maximize the enclosed area. What is the maximum area?

$X = 90$
$l = 360 - 90(2) = 180$

max area: $90 \cdot 180 \, m^2$
$= 2 \cdot (100 - 10)^2$
$= 2 \cdot 10^2 (10 - 1)^2$
$= 200(10^2 - 20 + 1)$

$$\begin{array}{r} 20000 \\ -4000 \\ +200 \\ \hline 16{,}200 \end{array}$$
m^2.

General (or Standard) Form

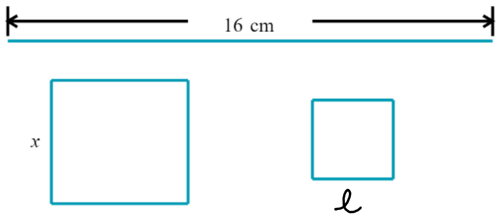
Quadratic functions in **general form** look like this: $f(x) = ax^2 + bx + c$

a) The vertex can be found using the **vertex formula**: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

b) This is just saying find $x = -\frac{b}{2a}$, then plug it back into the function to find y.

You try…

A wire that is 16 centimeters is cut into two pieces, and each piece is bent to form the shape of a square. One square has side length x cm.



l

a. Find a function that gives the total area, $A(x)$ enclosed by both squares in terms of x.

$A(x) = x^2 + (4 - x)^2$
$= x^2 + (x-4)^2 = 2x^2 + (-2)(4)x + 16$

$4x + 4l = 16 \Rightarrow \frac{16 - 4x}{4} = l$
$= 4 - x$

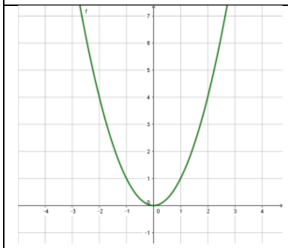
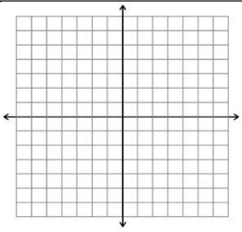
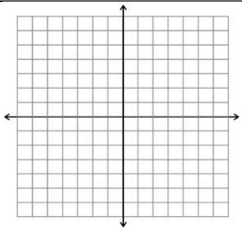
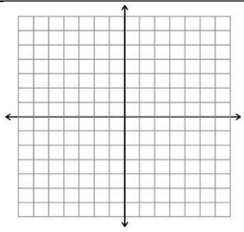
b. Find the side length that minimizes the total area of the two squares.

$\arg\min(A) = -\frac{-8}{2(2)} = \frac{8}{4} = 2$

c. What is the minimum area enclosed by the two squares?

$\min(A) = A(\arg\min(A)) = A(2) = 2^2 + (4-2)^2 = 2 \cdot 2^2 = 8$

Vertex Form

$f(x) = x^2$	Function:	$g(x) = -(x-3)^2 + 2$	$h(x) = (x-3)^2 - 1$	$j(x) = -(x+1)^2 + 3$
	Use transformations to graph the other functions.			
(0,0)	Vertex	(3,2)	(3,1)	(-1, 3)
$x = 0$	Axis of symmetry			
Minimum	Maximum or minimum?			
Minimum is 0	What is the max/min? (y-value)			
occurs at $x = 0$	Where does the max/min occur? (x-value)			
$(-\infty, \infty)$	Domain			
$[0, \infty)$	Range			

3. Each of the above functions are in **vertex form:** $\boldsymbol{f(x) = a(x - h)^2 + k}$. How can you use this form to find the vertex?

V_x V_y

The vertex is: (h , k) "(V_x, V_y)"

4. Look at the equations of the functions that have a maximum. What do they have in common? What about the ones that have a minimum? What rule can you come up with based on this observation?

max: $a < 0$ min: $a > 0$

5. How can the vertex help you find the maximum/minimum?

extrema @ (V_x, V_y)

6. How can the vertex help you find the axis of symmetry?

$x = V_x$

7. What does the vertex have to do with the range of the function?

$$\text{Range}(f) = \begin{cases} [k, \infty) & a > 0 \\ \{k\} & a = 0 \\ (-\infty, k] & a < 0 \end{cases}$$

Vertex Form

All of the functions above are in **Vertex form**:

$f(x) = a(x-h)^2 + k$ where (h, k) is the vertex.

Furthermore the vertex is always the **minimum** (if a is positive) OR the **maximum** (if a is negative)