College Algebra

Ann: BB gradesync is wrong, use Aleks for now

first: a few minutes of free talk, then move spots

2nd: Groups of 4

- designate talking stick
- in 2 minutes, give name, major, fears, and expectations for class
  - only talking stick person speaks
  - rotate till done

3rd: Quick walk trough on BB, sylabus, Aleks

4th: Get into rules/Axioms of Algebra

Notation: $\forall$ - for all, $\exists$ - there exists, $\in$ - in, s.t. - such that

$\exists!$ - exists a unique, $\mathbb{Z}$ - integers (zahlen)

$\mathbb{N}$ - natural numbers, iff - if and only if

Group: $(G, \times)$ is a group iff

Closure 1) for all x, y in G, $x \times y$ is in G.

$\forall x, y \in G$, $x \times y \in G$

Associativity 2) $\forall x, y, z \in G$, $(x \times y) \times z = x \times (y \times z)$

identity 3) $\exists! e \in G$ s.t. $\forall x \in G$, $x \times e = e \times x = x$.

invertability 4) for all x in G, there exists x^{-1} in G such that

$x \times x^{-1} = x^{-1} \times x = e$

example groups: $(\mathbb{Z},+)$, a rubicks cube w1d1p2

non-examples: $(\mathbb{N},+)$

Rings:

$(R,+,\bullet)$ is a ring if and only if

1) $(R,+)$ is a group

+ is communative 2) $a+b=b+a$ for all a,b in R

• is associative 3) $\forall a,b,c \in R$, $(a\bullet b)\bullet c = a\bullet(b\bullet c)$

Distributivity (most important) 4) for all a,b,c in R, $a\cdot(b+c)=a\cdot b+a\cdot c$
and $(b+c)\cdot a = b\cdot a + c\cdot a$

Unity 5) $\exists 1 \in R$ s.t. $\forall r \in R$ $r\cdot 1 = 1\cdot r = r$

Examples: $(\mathbb{Z},+,\bullet)$, working with remainders (like a clock)

$\mathbb{Q}$ - rationals, $\mathbb{R}$ - reals, $\mathbb{R}[X]$ - real polynomials
$\mathbb{R}(X)$ - real rational functions

Integral Domain (ID):

$(R,+,\cdot)$ is an Integral domain if and only if (iff)

1) $(R,+,\cdot)$ is a ring

• is communative 2) $\forall a,b \in R$, $a\cdot b = b\cdot a$

(cancelation property) 3) for all a,b,c in R with $a\neq 0$,
if $a\bullet b = a\cdot c$ then $b=c$.

Ex) $\mathbb{Z}$, all examples in rings

Field $(F,+,\cdot)$ is a field iff

1) $(F,+,\cdot)$ is an integral domain (ID)
2) for all $x \in F$ with $x \neq 0$, there exists x^{-1} such that $x \cdot x^{-1} = x^{-1} \cdot x = 1$

Side note on

Fractions From any integral domain R one may construct its field of fractions $\text{Frac}(R)$ by

$$\text{Frac}(R) = \{ \tfrac{a}{b} : a,b \in R,\ b \neq 0 \}$$

and $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

ex) $\text{Frac}(\mathbb{Z}) = \mathbb{Q}$

$\text{Frac}(\mathbb{R}[x]) = \mathbb{R}(x)$

Now toward a k-Algebra

(homomorphism)

Ring map: Let A, B be rings.

Then $f: A \to B$ is a ring map iff $\forall a, a' \in A$

1) $f(a+a') = f(a) + f(a')$

2) $f(a \cdot a') = f(a) f(a')$

3) $f(1_A) = 1_B$

K-Algebra Let K be a commutative ring. Then A is a K-Algebra iff A is a ring and there exists a ring map $\mu : K$

examples: $\mathbb{C} \sim \mathbb{R} \xhookrightarrow{\text{inclusion}} \mathbb{C}$

$\mathbb{Q} \sim \mathbb{Z} \hookrightarrow \mathbb{Q}$ — any ring is a $\mathbb{Z}$-algebra

$\mathbb{R}[x] \sim \mathbb{R} \hookrightarrow \mathbb{R}[x]$

Alternate view: its a ring with an ability to be scaled by a "smaller" ring.

w1d2p1

Day 2

Ann: - DO IKC
- Do HW1
- Quiz due Sunday
  - requires 80
- Alex and BB syncing weird
  - use Alex for grade
- introduce yourself activity is extra credit participation
- CEA talk to during Ohours or after class
- Ohours: SCEN 224
  MWF - 9:40-10:30
  - email / UA sucess to guarentee i'll be there
  - more to come

Today: exponents

Preview: 1) simplify $\dfrac{(7x)^{-2}}{7x^{-3}}$

w1d2p2

2) solve: $-3|v|+3=0$

3) translate: three times a number is its double plus 5.

skip — 4) multiply: $(3x)^2 y^3 x^{-2} y^4$

5) factor: $6w^2+48w-54$

$6(w^2+8w-9)$

$-1 \overset{\cdot}{\wedge} 9 \overset{+}{\rightarrow} 8$

$6(w-1)(w+9)$

6) solve: $|2y-14|=8$

Content: Exponents

Notation: $(\text{expr})^p$ means $\underbrace{\text{expr}\cdot\text{expr}\cdot\dots\cdot\text{expr}}_{p\text{-times}}$

where expr is some expression

ex) $5^3=5\cdot5\cdot5$

ex) $(7x+12)^{97}=\underbrace{(7x+12)(7x+12)\cdots(7x+12)}_{97\text{ times}}$

Properties:

1) $x^1=x$

2) $x^{-1}=1/x$

3) $x^0=1$

4) $x^a x^b = x^{a+b}$

v1d2p3

5) $(a \cdot b)^c = a^c \cdot b^c$

6) $(a^b)^c = a^{b \cdot c}$

"Proof" of 4+5: 4) $x^a \cdot x^b = \underbrace{x \cdots x}_{a \text{ times}} \cdot \overbrace{x \cdots x}^{b\text{-times}}$

$= \underbrace{x \cdots x}_{a+b\text{-times}}$

5) $(a \cdot b)^c = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) \cdots$

by associativity and commuunitivity,

$= \underbrace{a \cdot a \cdot a \cdots a}_{c\text{ times}} \cdot \underbrace{b \cdots b}_{c\text{-times}}$

Sec 12

$= a^c \cdot b^c$

ex) $\dfrac{(7x)^4}{7^3x^{-8}} = \dfrac{(7x)^4}{1} \cdot \dfrac{1}{7^3x^{-8}}$

$\overset{(2)}{=} (7x)^4 (7^3x^{-8})^{-1}$

$\overset{(5)}{=} 7^4x^4 \cdot (7^3)^{-1}(x^{-8})^{-1}$

$\overset{(6)}{=} 7^4x^4 \; 7^{-3} x^8$

$\overset{\text{Com}+(4)}{=} 7^{4+(-3)} x^{4+8}$

$= 7x^{12}$

BB Mod 1 - notes - Basic, me, you style

L1-9 as groups (3).

Sec 14

w1d3p1

Filling in yesterday: Simplifying exponents: a process

Steps	example
	$\dfrac{(7x^2)^3}{7y^{-4}x^2}$
1) distribute powers across multiplication and multiply powers	$= \dfrac{7^3 x^6}{7y^{-4}x^2 z}$
2) move denominator (bottom) to numerator (top), negating powers	$= 7^3 x^6 7^{-1} y^4 x^{-2} z^{-1}$
3) combine like terms	$= 7^2 x^4 y^4 z^{-1}$
4) move negative pieces to denominator negating power	$= \dfrac{7x^4 y^4}{z}$

Now 3 Q's from BB, me, them, them

w1d3p2

(P)review:

1) Solve for C: $1 = \frac{5A}{2C} + 3B$

ans→ $\Downarrow \cdot 2C$

$2C = 5A + 6BC$

$\Downarrow$ like on same side

$2C - 6BC = 5A$

$\Downarrow$ distributive prop.

$(2-6B)C = 5A$

$\Downarrow$

$C = \frac{5A}{2-6B}$

2) $f(x) = 4x^2 + 2x - 1$

evaluate when $x = \frac{1}{2}$

ans) $f(\frac{1}{2}) = 4(\frac{1}{2})^2 + 2(\frac{1}{2}) - 1$

$= 4(\frac{1}{4}) + 1 - 1$

$= 2 + 0$

$= 2$

3) factor: $x^2 + 2x - 8$ LC = 1 ✓

-8: $4, -2$; $+ \to 2$

$(x-2)(x+4)$

4) translate: the quotient of your score

W1d3p3

remove four and 72 is 84.

ans) $\frac{u-4}{72} = 84$

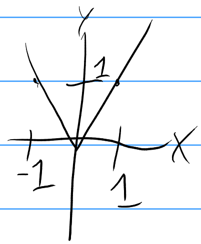
Content: Absolute value and factoring

π-Prog: (65%) (59%)

Absolute value:

Def: $|\cdot|$ is the absolute value function. It measures distance from zero.

graph: $y = |x|$



properties: a) $|x| \geq 0$

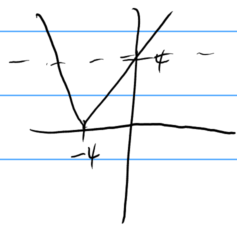
b) $|ab| = |a||b|$

c) $|a+b| \leq |a| + |b|$

ex) $3|x+4| - 2 = 10$

$3|x+4| = 12$

$|x+4| = 4$



$-x-4=4$ $\quad$ $x+4=4$

$x=8$ $\quad$ $x=0$

w1d3p4

procedure for abs. value:

rules) $|expr| = m$

a) isolate $|\cdot|$

b) split into 2 equations if not = neg.

$-expr = m$ $\quad$ $expr = m$

c) solve both

d) answer

ex) $4|x-1| - 2 = 6$

$= 4|x-1| = 8$

$= |x-1| = 2$

$x - 1 = 2$ $\quad$ $-x + 1 = 2$

$x = 3$ $\quad$ $-x = 1$

$x = 1$

$x = 3$ or 1

BB sheet you 4-5

sec 14

Factoring: taking an expesion and rewriting it as a product.

Process: for $ax^2 + bx + c$,

1) factor out a: $a(x^2 + \frac{b}{a}x + \frac{c}{a})$

$= a(x^2 + dx + e)$

2) find f, g s.t. $\begin{cases} f \cdot g = e \\ f + g = d \end{cases}$

3) $ax^2 + bx + c = a(x+f)(x+g)$

4) combine like terms if applicable.

w1d3p5

ex) $2x^2+4x+2$

$=2(x^2+2x+1)$

$\begin{array}{c} \wedge \\ 1\ \ 1 \end{array} \xrightarrow{+}_{\times} \begin{array}{c} 2 \\ 2 \end{array} \checkmark$

$=2(x+1)(x+1)$

$=2(x+1)^2$

sec12

BB Factoring quadratics odds till 10 min.