

Difference-in-Differences

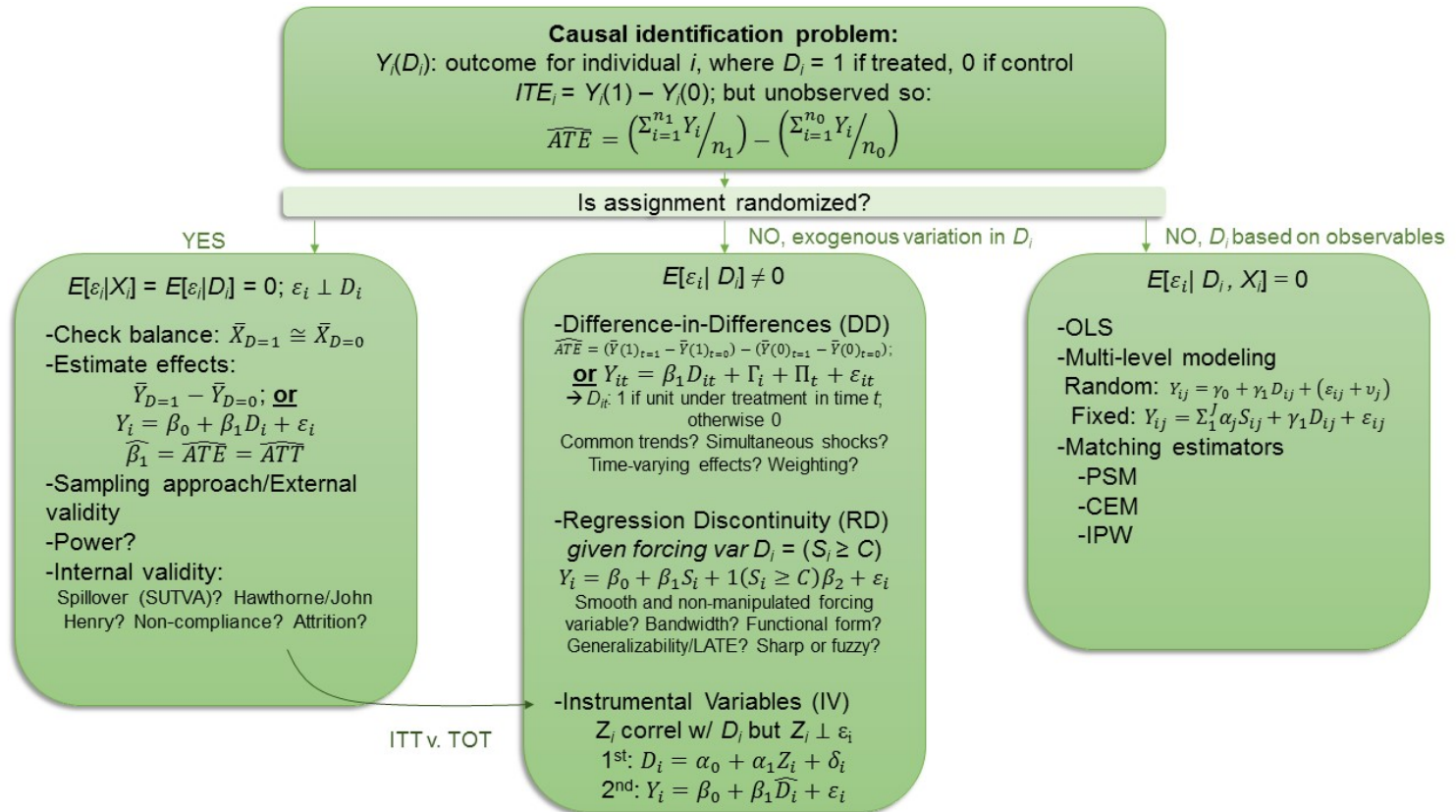
EDLD 650: Week 2

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Agenda

1. Roadmap and Goals
2. Estimating DD effects in data
3. Wrap-up
 - DARE #1

Roadmap



Goals

1. Describe threats to validity in difference-in-differences (DD) identification strategy and multiple approaches to address these threats
 - *from responding to Discussion Questions*
2. Using a cleaned dataset, estimate multiple DD specifications in R and interpret these results
 - *from this lecture and accompanying script*

Programming in EDLD 650

What you won't get 😞

- A heavy dose of data management and visualization strategies
- The most efficient code with extensive use of functions

What you will get 😊

- A review of the programming steps you should take as part of the **actual** research process
- *Some* model code for data management and visualization
- Programming strategies and packages that can be used to estimate the causal inference techniques we will study
- A community of skilled programmers who will expand our collective knowledge base!

Estimating a classic, two-period difference-in-differences (DD) model

Replicating Dynarski (2003)

Recall Dynarski's primary model (Eq. 2):

$$y_i = \alpha + \beta(\text{FATHERDEC}_i \times \text{BEFORE}_i) + \delta\text{FATHERDEC}_i + \theta\text{BEFORE}_i + v_i$$

Let's try to fit this in our data!

Reading in the data

I'm using the haven package to import a data file that is in the Stata .dta format. Lotsa options for importing file formats other than .csv (foreign and rio are two such ones)!

```
dynarski <- haven::read_dta(here("data/ch8_dynarski.dta"))  
  
head(dynarski)
```

```
#> # A tibble: 6 x 8  
#>       id   hhid   wt88   coll hgc23 yearsr fatherdec      offer  
#>   <dbl> <dbl>   <dbl> <dbl> <dbl>   <dbl> <dbl+lbl>   <dbl>  
#> 1     9     9 691916     1    13     81 0 [Father not deceased]     1  
#> 2    14    13 784204     1    16     81 0 [Father not deceased]     1  
#> 3    15    15 811032     1    16     82 0 [Father not deceased]     0  
#> 4    21    20 644853     1    16     79 0 [Father not deceased]     1  
#> 5    22    22 728189     1    16     80 0 [Father not deceased]     1  
#> 6    24    23 776590     0    12     79 0 [Father not deceased]     1
```


Viewing the data

Show entries

Search:

	id ▾	coll ▾	hgc23 ▾	yearsr ▾	fatherdec ▾	offer ▾
1	9	1	13	81	0	1
2	14	1	16	81	0	1
3	15	1	16	82	0	0
4	21	1	16	79	0	1
5	22	1	16	80	0	1
6	24	0	12	79	0	1
7	26	1	14	80	0	1

Showing 1 to 7 of 3,986 entries

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Next

Understanding the data (1)

```
d <- select(dynarski, coll, hgc23, fatherdec, offer)

summary(d)
```

#>	coll	hgc23	fatherdec	offer
#>	Min. :0.0000	Min. :10.00	Min. :0.000000	Min. :0.000
#>	1st Qu.:0.0000	1st Qu.:12.00	1st Qu.:0.000000	1st Qu.:0.000
#>	Median :0.0000	Median :12.00	Median :0.000000	Median :1.000
#>	Mean :0.4579	Mean :13.14	Mean :0.04792	Mean :0.723
#>	3rd Qu.:1.0000	3rd Qu.:14.00	3rd Qu.:0.000000	3rd Qu.:1.000
#>	Max. :1.0000	Max. :19.00	Max. :1.000000	Max. :1.000

```
sum(is.na(coll))
```

```
#> [1] 0
```

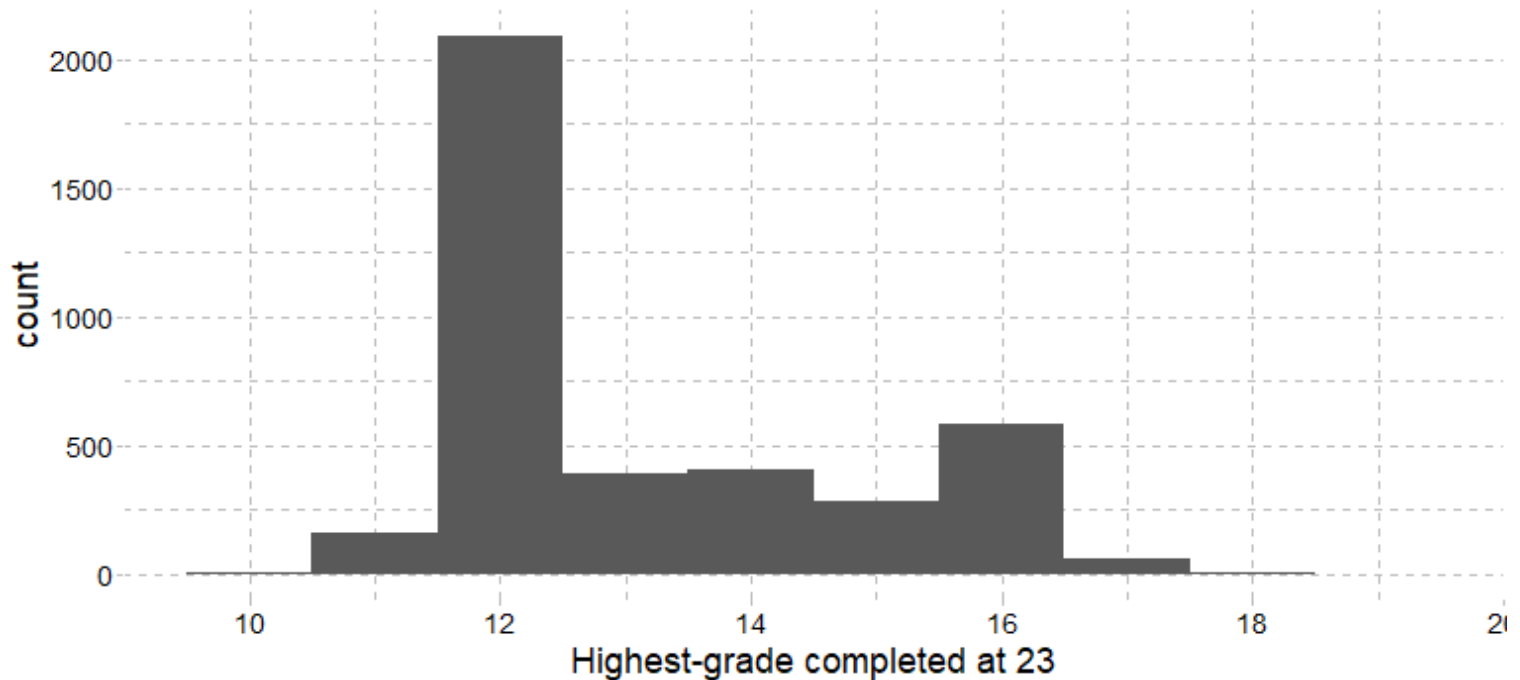
Understanding the data (2)

```
college <- table(dynarski$fac_fatherdec, dynarski$fac_coll)
college
```

```
#>
#>               No College College
#> Father not deceased      2059    1736
#> Father deceased         102     89
```

Plot outcome data

```
hg <- ggplot(dynarski, aes(hgc23)) + geom_histogram(binwidth=1)
hg + scale_x_continuous(name="Highest-grade completed at 23",
                        breaks=c(10, 12, 14, 16, 18, 20)) +
  theme_pander(base_size=18)
```



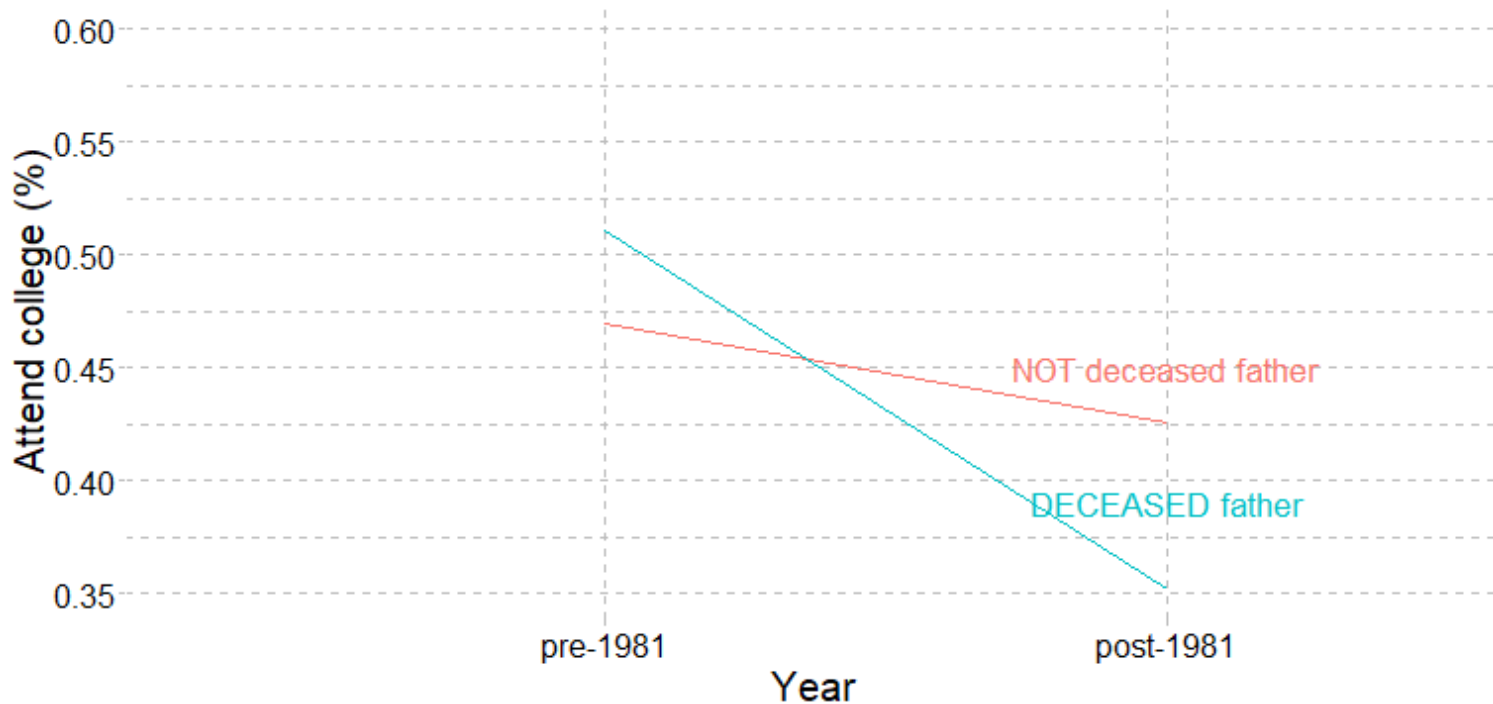
Summary statistics table

Table 1. Descriptive Statistics

Statistic	N	Mean	St. Dev.
Attend college at 23	3,986	0.46	0.50
Years schooling at 23	3,986	13.14	1.63
Father deceased	3,986	0.05	0.21
Offer	3,986	0.72	0.45

Notes: This table presents unweighted means and standard deviations from the NLSY poverty and random samples used in the Dynarski (2003) paper.

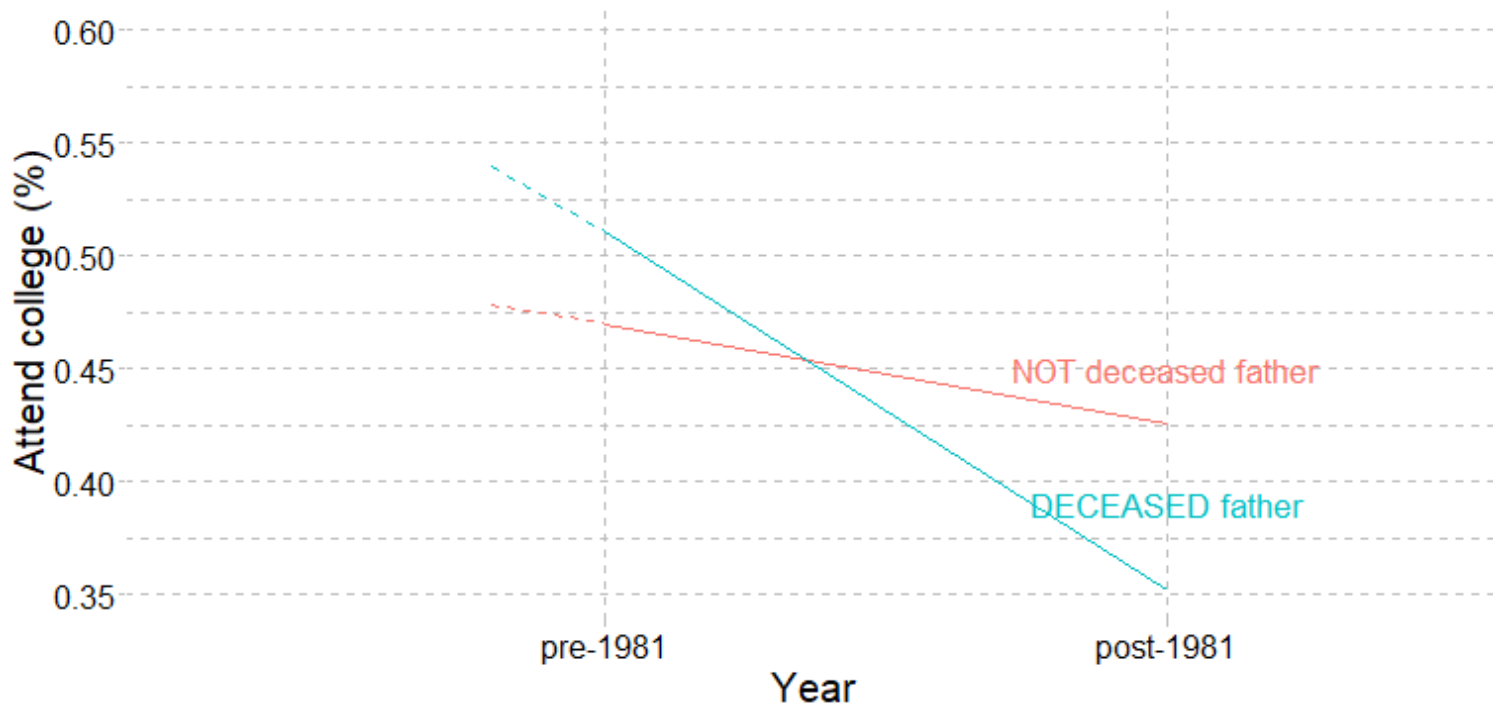
Graphical DD



What is treatment effect?

What is the core **identifying assumption** assumption underlying the DD framework? How do we know whether we've satisfied it?

Graphical DD



What would you think if you "knew" this was the pattern?

Estimate classic two-period DD

Dynarski's original model:

$$y_i = \alpha + \beta(\text{FATHERDEC}_i \times \text{BEFORE}_i) + \delta\text{FATHERDEC}_i + \theta\text{BEFORE}_i + v_i$$

Murnane and Willet have renamed the variable to make clear that a value of 1 means individuals are eligible for aid, so we'll do the same:

$$y_i = \alpha + \beta(\text{FATHERDEC}_i \times \text{OFFER}_i) + \delta\text{FATHERDEC}_i + \theta\text{OFFER}_i + v_i$$

Estimate classic two-period DD

$$y_i = \alpha + \beta(\text{FATHERDEC}_i \times \text{OFFER}_i) + \delta\text{FATHERDEC}_i + \theta\text{OFFER}_i + v_i$$

```
lm(coll ~ fatherdec*offer, data=dynarski)
```

```
#>
```

```
#> Call:
```

```
#> lm(formula = coll ~ fatherdec * offer, data = dynarski)
```

```
#>
```

```
#> Coefficients:
```

#> (Intercept)	fatherdec	offer	fatherdec:offer
#> 0.42571	-0.07386	0.04387	0.11523

This doesn't quite match, let's add the weights in...

Estimate classic two-period DD

$$y_i = \alpha + \beta(\text{FATHERDEC}_i \times \text{OFFER}_i) + \delta\text{FATHERDEC}_i + \theta\text{OFFER}_i + v_i$$

```
lm(coll ~ fatherdec*offer, data=dynarski,  
    weights=dynarski$wt88)
```

```
#>
```

```
#> Call:
```

```
#> lm(formula = coll ~ fatherdec * offer, data = dynarski, weights = dynarski
```

```
#>
```

```
#> Coefficients:
```

#>	(Intercept)	fatherdec	offer	fatherdec:offer
#>	0.47569	-0.12348	0.02601	0.18223

Pretty underwhelming output?

Under the hood

```
est_dynarski <- lm(coll ~ fatherdec*offer,  
                   data=dynarski, weights=dynarski$wt88)  
est_dynarski %>% names()
```

```
#> [1] "coefficients" "residuals"      "fitted.values" "effects"  
#> [5] "weights"      "rank"          "assign"        "qr"  
#> [9] "df.residual"  "xlevels"       "call"          "terms"  
#> [13] "model"
```

```
est_dynarski %>% tidy()
```

```
#> # A tibble: 4 x 5  
#>   term          estimate std.error statistic    p.value  
#>   <chr>          <dbl>     <dbl>     <dbl>    <dbl>  
#> 1 (Intercept)    0.476     0.0150     31.8 7.12e-198  
#> 2 fatherdec    -0.123     0.0752     -1.64 1.01e- 1  
#> 3 offer         0.0260     0.0178      1.46 1.43e- 1  
#> 4 fatherdec:offer 0.182     0.0893      2.04 4.14e- 2
```

Further under the hood

```
summary(est_dynarski)
```

```
...
```

```
#>      Min      1Q  Median      3Q      Max  
#> -490.9 -230.3 -138.6  247.7  554.0
```

```
#>
```

```
#> Coefficients:
```

#>		Estimate	Std. Error	t value	Pr(> t)
#>	(Intercept)	0.47569	0.01496	31.793	<2e-16 ***
#>	fatherdec	-0.12348	0.07520	-1.642	0.1007
#>	offer	0.02601	0.01777	1.463	0.1435
#>	fatherdec:offer	0.18223	0.08931	2.041	0.0414 *

```
#> ---
```

```
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
#>
```

```
#> Residual standard error: 285.7 on 3982 degrees of freedom
```

```
#> Multiple R-squared:  0.001961,    Adjusted R-squared:  0.001209
```

```
#> F-statistic: 2.607 on 3 and 3982 DF,  p-value: 0.04998
```

```
...
```

Making a no-fuss table

```
stargazer(est_dynarski, type='html', single.row = T)
```

	<i>Dependent variable:</i>
	coll
fatherdec	-0.123 (0.075)
offer	0.026 (0.018)
fatherdec:offer	0.182** (0.089)
Constant	0.476*** (0.015)
Observations	3,986
R ²	0.002
Adjusted R ²	0.001
Residual Std. Error	285.711 (df = 3982)
F Statistic	2.607** (df = 3; 3982)

Central DD assumptions

In order to fully trust that the estimates produced by a DD analysis are unbiased by endogeneity, we need to make (and defend) the following two assumptions:

1. Not-treated (or not-yet-treated) units are **valid counterfactuals**
 - Parallel trends?
 - Selection into treatment? (non-exogeneity)
2. There are no **simultaneous shocks** or unobserved **secular trends**
 - Other observed and unobserved events or patterns?

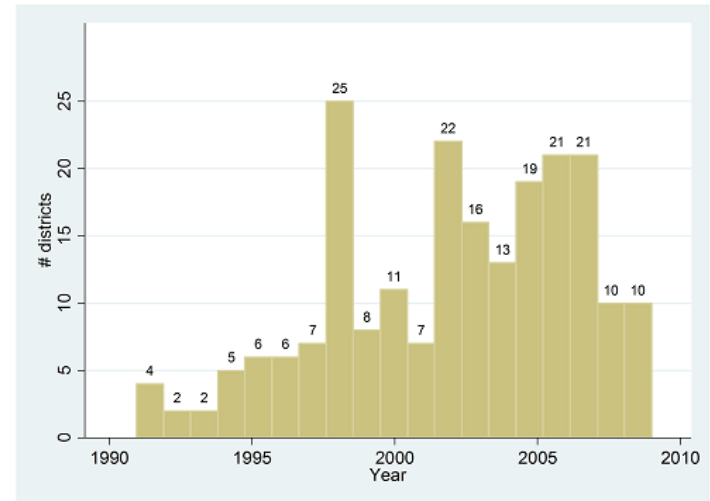
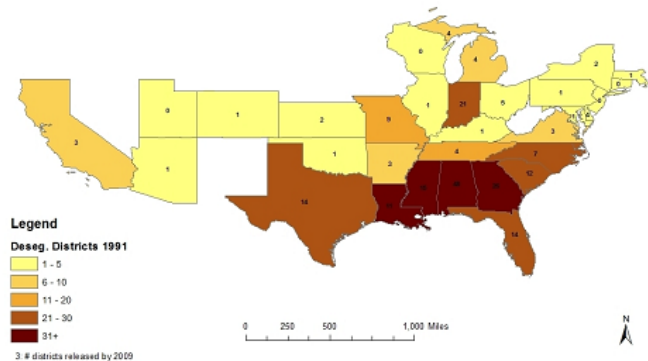
We'll look at how to address some of these in the next section of the lecture, and you'll read more about how to do so in the readings and DARE for next week!

DD in panel data

- A. The two-way fixed effect (TWFE) estimator for staggered implementation
- B. Appropriate statistical inference
- C. Assessing the parallel trends assumption (PTA)
- D. The modern event-study approach

End of desegregation

- In 1991, 480 school districts were under court desegregation order
- In following two decades, nearly half (215) were released and returned to neighborhood assignment patterns
- Timing of release was arguably **exogenous** and **quasi-random**
- This provides strong support to the claim that the districts which were not (or *not yet*) released from court orders were on **parallel trends** in their outcomes with districts that were released and, thus, serve as **valid counterfactuals**¹



[1] Liebowitz (2018)

End of desegregation data

Show entries

Search:

	leadid	year	STATE	unitary	sd_dropout_prop_b	yrdis
1	3904375	2010	39	1	0.0929224863648415	1991
2	0102160	1990	01	0	0.133906632661819	1991
3	4828500	2000	48	1	0.120370373129845	1991
4	4815270	1990	48	0	0.0985714271664619	1992
5	4702940	2010	47	1	0.0839421972632408	1992
6	4702940	1990	47	0	0.150237992405891	1992
7	2200540	2000	22	0	0.144326865673065	2003
8	0101410	2000	01	0	0.150395780801773	2003
9	1300001	1990	13	0	0.0979623794555664	2003

Showing 1 to 9 of 9 entries

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Estimate DD in panel data (1)

$$\text{DROPOUT_BLACK}_{jt} = \beta_1 \text{UNITARY}_{jt} + \Gamma_j + \Pi_t + \epsilon_j$$

Take a minute to write down what this model does in words. Use the terms **mean effect**, **time series**, **fixed effects** and **causal parameter of interest**.

The model takes advantage of **time series (or panel or repeated measure)** data in which the Black dropout rate in each district is observed at three points in time. The model regresses the Black dropout rate in a **fixed effect** model in which observations are clustered in two dimensions: within district (Γ_j) and also within time (Π_t). Note: Γ_j represents a vector of dummy indicators that take the value of one if observation j is equal to district j and zero otherwise. Π_t represents a vector of dummy indicators that take the value of one if observation j is in time t (1990, 2000 or 2010). β_1 estimates the **average treatment effect** of being observed after being declared unitary and is the **causal parameter of interest** reflecting the effect of being released from a desegregation order UNITARY_{jt} on the black dropout rate.

In this case, the estimates rely on **repeated cross-sectional** panel data. We could also implement the same framework in **longitudinal** panel data.

Estimate DD in panel data (2)

We are going to shift to using the `fixest` [package](#); an incredibly versatile and robust tool for regression analysis in R from Laurent Berge.

```
ols_unitary1 <- feols(sd_dropout_prop_b ~ unitary | leaid + year,  
                      data=desegregation,  
                      vcov = "iid", weights=desegregation$sd_t_1619_b)  
summary(ols_unitary1)
```

```
#> OLS estimation, Dep. Var.: sd_dropout_prop_b  
#> Observations: 1,403  
#> Weights: desegregation$sd_t_1619_b  
#> Fixed-effects: leaid: 476, year: 3  
#> Standard-errors: IID  
#>           Estimate Std. Error t value  Pr(>|t|)  
#> unitary 0.018185    0.003121 5.82642 7.8155e-09 ***  
#> ---  
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
#> RMSE: 1.17345      Adj. R2: 0.547167  
#>           Within R2: 0.035437
```

Can you interpret this output? (ignore un-highlighted line for now)

Addressing serial correlation

The worry: within-unit correlation of outcomes (e.g., within-state, across state-years) results in correlated (and therefore too small) standard errors. As a result out **statistical inference** will be incorrect.

The solution: **cluster-robust standard errors**¹. Clustering standard errors by the k^{th} regressor inflates iid OLS standard errors by:

$$\tau_k \simeq 1 + \rho_{x_k} \rho_\mu (\bar{N}_g - 1)$$

where ρ_{x_k} is the within-cluster correlation of regressor x_{igk} , ρ_μ is the within-cluster error correlation and \bar{N}_g is the average cluster size. τ_k is **asymptotically** correct as number of clusters increase. Current consensus: this estimate of τ_k is accurate with **~45 clusters**. Fewer than 40, and this approach can dramatically under-estimate SEs (consider bootstrapping).

Best practice: cluster at the unit of treatment (or consider two-way clustering).²

[1] Read all about cluster-robust standard errors in [Cameron & Miller's \(2015\)](#) accessible practitioner's guide to standard errors.

[2] [Bertrand, Mullainathan & Duflo \(2004\)](#) and [Abadie et al. \(2017\)](#).

Clustered standard errors (1)

```
ols_unitary2 <- feols(sd_dropout_prop_b ~ unitary | leaid + year,  
                      data=desegregation,  
                      weights=desegregation$sd_t_1619_b)  
  
summary(ols_unitary2)
```

```
#> OLS estimation, Dep. Var.: sd_dropout_prop_b  
#> Observations: 1,403  
#> Weights: desegregation$sd_t_1619_b  
#> Fixed-effects: leaid: 476, year: 3  
#> Standard-errors: Clustered (leaid)  
#>      Estimate Std. Error t value Pr(>|t|)  
#> unitary 0.018185 0.004851 3.74879 0.00019958 ***  
#> ---  
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
#> RMSE: 1.17345 Adj. R2: 0.547167  
#> Within R2: 0.035437
```

Default behavior in `fixest` is to cluster standard errors on the first fixed effect.

Clustered standard errors (2)

```
ols_unitary3 <- feols(sd_dropout_prop_b ~ unitary | leaid + year,  
                      data=desegregation,  
                      vcov = ~ leaid^year,  
                      weights=desegregation$sd_t_1619_b)  
  
summary(ols_unitary3)
```

```
#> OLS estimation, Dep. Var.: sd_dropout_prop_b  
#> Observations: 1,403  
#> Weights: desegregation$sd_t_1619_b  
#> Fixed-effects: leaid: 476, year: 3  
#> Standard-errors: Clustered (leaid^year)  
#>      Estimate Std. Error t value  Pr(>|t|)  
#> unitary 0.018185    0.004816 3.77557 0.00016631 ***  
#> ---  
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
#> RMSE: 1.17345      Adj. R2: 0.547167  
#>      Within R2: 0.035437
```

We are going to cluster our standard errors **at the level of assignment to treatment**: the district-year.

Addressing serial correlation

A taxonomy of models estimating the end of school desegregation on the black dropout rate, by std. error clustering approach

	Unclustered	Clustered (Unit)	Clustered (Unit*Period)
unitary	0.018***	0.018***	0.018***
	(0.003)	(0.005)	(0.005)
Num.Obs.	1403	1403	1403
R2	0.702	0.702	0.702
Std.Errors	IID	by: leaid	by: leaid^year
FE: leaid	X	X	X
FE: year	X	X	X

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

Notes: The table displays coefficients from Equation X with standard errors in parentheses.

Doesn't make too much of a difference here... *Note:* Using `modelsummary` package, but `fixest` comes with the powerful `etable` function.

Addressing parallel trends

A parametric approach

$$\text{DROPOUT_BLACK}_{jt} = \beta_1 \text{UNITARY}_{jt} + \beta_2 (\text{UNITARY} \times \text{REL_YEAR})_{jt} + \beta_3 \text{REL_YEAR}_{jt} + \Gamma_j + \Pi_t + \epsilon_j$$

What is this REL_YEAR_{jt} and how do we code it?

```
desegregation <- desegregation %>%  
  mutate(rel_yr = case_when(  
    !is.na(yrdiss) ~ (year - yrdiss),  
    is.na(yrdiss) ~ -1 ## <-- this is funky, let's talk about it  
  ))  
summary(desegregation$rel_yr)
```

```
#>      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
#> -19.00  -1.00   -1.00   -1.51  -1.00   19.00
```


Peek at REL_YEAR

Show entries

Search:

	leadid	year	STATE	unitary	sd_dropout_prop_b	yrdiss	rel_yr
1	3904375	2010	39	1	0.0929224863648415	1991	19
2	0102160	1990	01	0	0.133906632661819	1991	-1
3	4828500	2000	48	1	0.120370373129845	1991	9
4	4815270	1990	48	0	0.0985714271664619	1992	-2
5	4702940	2010	47	1	0.0839421972632408	1992	18
6	4702940	1990	47	0	0.150237992405891	1992	-2
7	2200540	2000	22	0	0.144326865673065	2003	-3
8	0101410	2000	01	0	0.150395780801773	2003	-3
9	1300001	1990	13	0	0.0979623794555664	2003	-13

Showing 1 to 9 of 9 entries

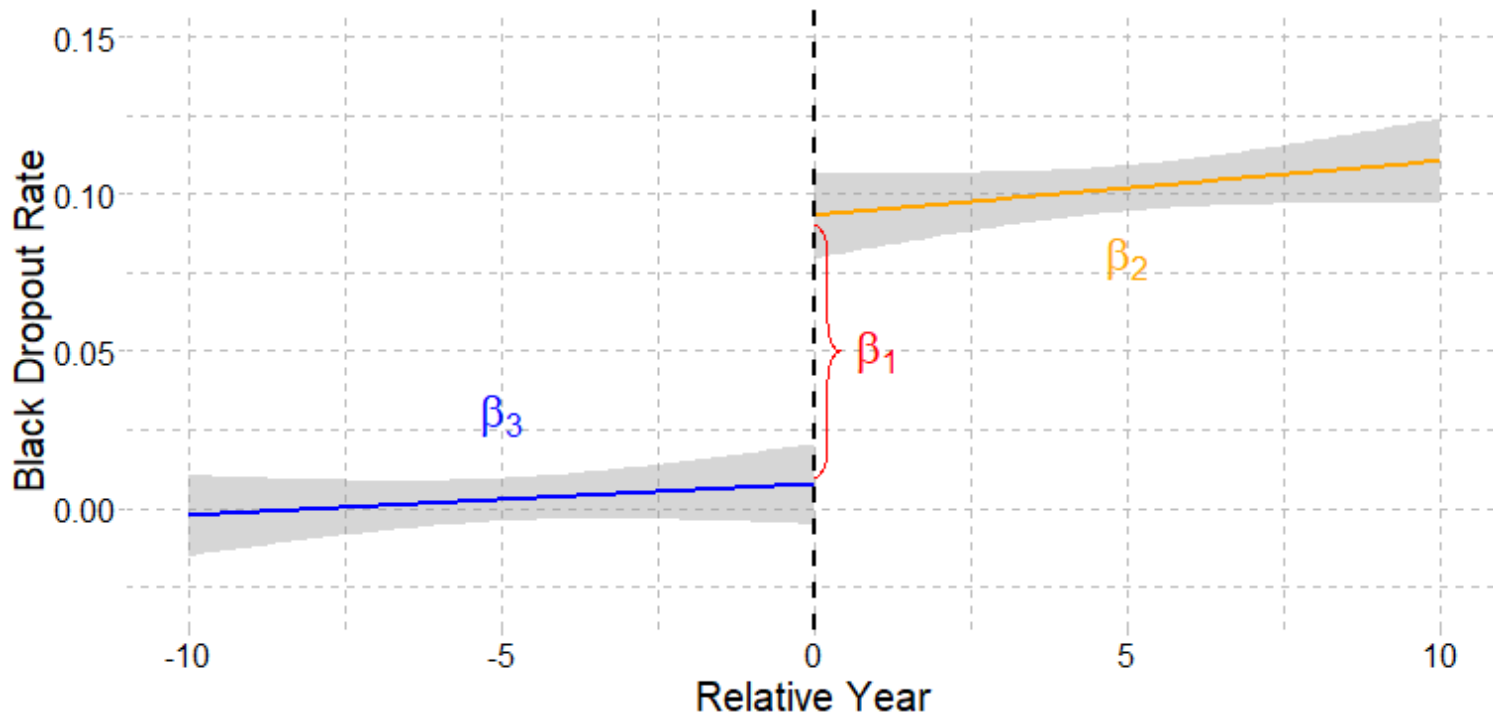
Previous

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Map coefficients to graph

$$\text{DROPOUT_BLACK}_{jt} = \beta_1 \text{UNITARY}_{jt} + \beta_2 (\text{UNITARY} \times \text{REL_YEAR})_{jt} + \beta_3 \text{REL_YEAR}_{jt} + \Gamma_j + \Pi_t + \epsilon_j$$



Remember: given the structure of our model, these parameters are estimated *relative to untreated and not-yet-treated districts*.

Parallel trends?

```
ols_unitary_run <- feols(sd_dropout_prop_b ~ unitary*rel_yr |  
  leaid + year, data=desegregation,  
  vcov = ~leaid^year, weights=desegregation$sd_t_1619_b)  
summary(ols_unitary_run)
```

```
#> OLS estimation, Dep. Var.: sd_dropout_prop_b
```

```
#> Observations: 1,403
```

```
#> Weights: desegregation$sd_t_1619_b
```

```
#> Fixed-effects: leaid: 476, year: 3
```

```
#> Standard-errors: Clustered (leaid^year)
```

```
#>           Estimate Std. Error  t value Pr(>|t|)
```

```
#> unitary      0.014584   0.005860  2.48893 0.012928 *
```

```
#> rel_yr       0.001027   0.000579  1.77312 0.076426 .
```

```
#> unitary:rel_yr -0.001367  0.000689 -1.98458 0.047386 *
```

```
#> ---
```

```
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
#> RMSE: 1.16896      Adj. R2: 0.54965
```

```
#>           Within R2: 0.042803
```

How would this graph look different than the one on previous slide?

A complete table!

Table 2. Effects of end of school desegregation on black dropout rate

	1	2	3
Unitary status	0.018***	0.018***	0.015*
	(0.005)	(0.005)	(0.006)
Pre-trend			0.001+
			(0.001)
Unitary x Relative-Year			-0.001*
			(0.001)
Covariates?		X	X
Num.Obs.	1403	1403	1403
R2	0.702	0.702	0.704

Notes: +p<0.1, *p<0.05, **p<0.01, ***p<0.001. Table displays coefficients and district-by-year clustered standard errors in parentheses. All models include fixed effects for year and district. Models 2 and 3 adjust for proportion of 16–19 year-olds residing in district in 1990 who were Black, interacted with year.

A flexible approach

What if, instead of assigning a particular functional form to our treatment effects over time (either mean, linear or higher-order polynomial), we specified an entirely flexible model?

$$\begin{aligned}\text{DROPOUT_BLACK}_{jt} = & \beta_1 \text{pre}_{jt}^{-n} + \beta_2 \text{pre}8 + \beta_3 \text{pre}7_{jt} + \dots \\ & + \beta_m \text{post}0_{jt} + \dots + \beta_n \text{post}_{jt}^n + \Gamma_j + \Pi_t + \epsilon_j\end{aligned}$$

Could also write as:

$$\text{DROPOUT_BLACK}_{jt} = \sum_{t=-10}^n 1(t = t_j^*) \beta_t + \Gamma_j + \Pi_t + \epsilon_j$$

Think for a moment what this model does?

The model adjusts its estimates of the mean rate of Black dropout in district j by the mean rate of Black dropout in year t across all districts. Then, it estimates what effect does being t years pre- or post-unitary have. The comparison in each of these β s is to being never or not yet *UNITARY*.

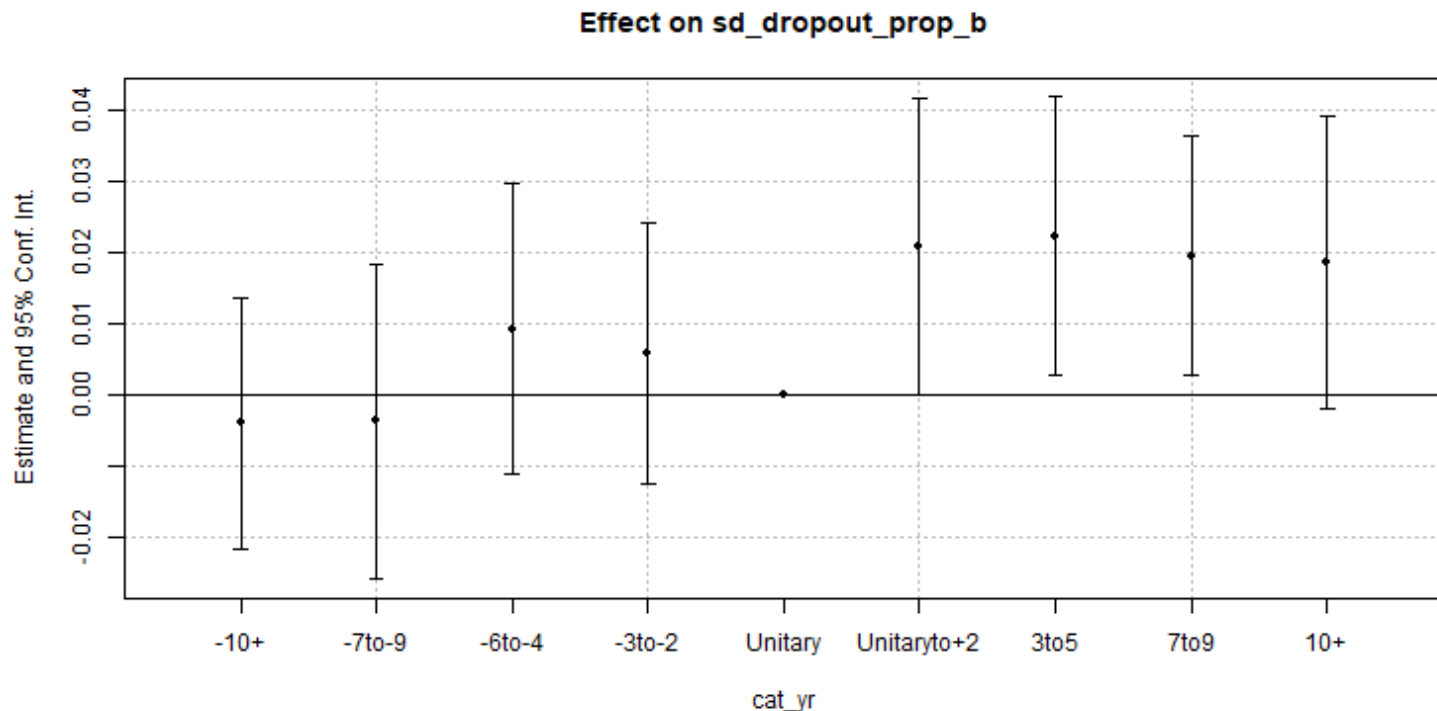
Event study

This would permit a **fully flexible specification**, permitting us to both evaluate **violations of the PTA** and assess potential **dynamic effects** of the treatment:

```
...  
#> Observations: 1,403  
#> Weights: desegregation$sd_t_1619_b  
#> Fixed-effects: year: 3, lead: 476  
#> Standard-errors: Clustered (lead^year)  
#>               Estimate Std. Error   t value Pr(>|t|)  
#> cat_yr::-10+      -0.004020    0.009034 -0.444977 0.656405  
#> cat_yr::-7to-9     -0.003755    0.011329 -0.331417 0.740379  
#> cat_yr::-6to-4      0.009199    0.010449  0.880378 0.378806  
#> cat_yr::-3to-2      0.005798    0.009318  0.622229 0.533892  
#> cat_yr::Unitaryto+2 0.020860    0.010611  1.965823 0.049516 *  
#> cat_yr::3to5        0.022258    0.010005  2.224797 0.026254 *  
#> cat_yr::7to9        0.019450    0.008586  2.265370 0.023642 *  
#> cat_yr::10+        0.018580    0.010454  1.777409 0.075718 .  
#> ---  
...
```

What has happened to our standard errors? (think about **bias v. variance tradeoff**)

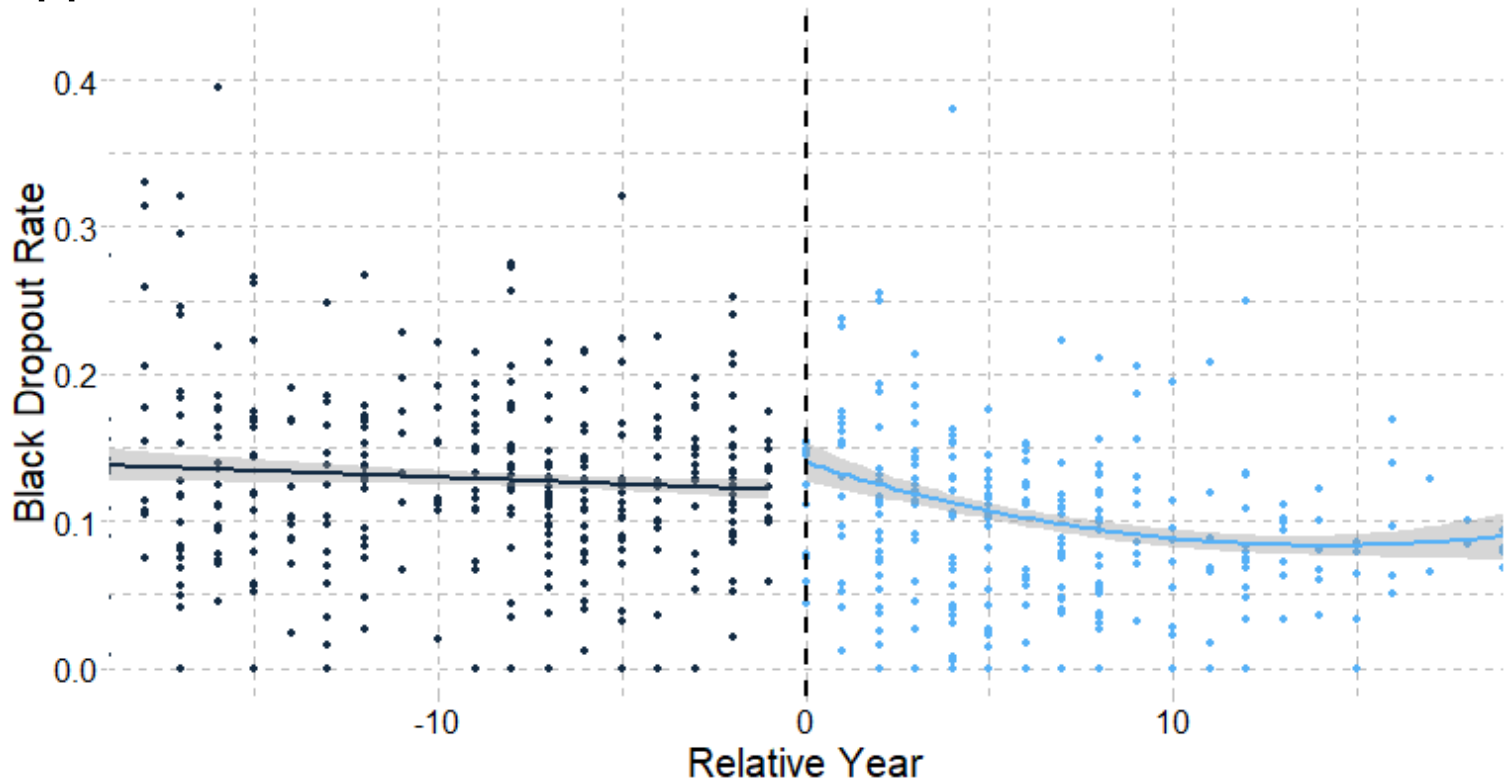
Event study visualized



The end of desegregation efforts had a causal effect on the Black dropout rate, resulting in a discontinuous and persistent increase of between 1 and 2 percentage points (*caveats, caveats*).

C-ITS

An aside on the related Comparative-Interrupted Time Series approach:



C-ITS considered

Strengths

- Takes advantage of full range of data
- Compared to mean-effect-only DD, allows differentiation of discontinuous jump vs. post-trend
- Permits modeling of fully flexible functional form (can include quadratic, cubic, quartic relationships, interactions and more!)
- Data-responsive approach

Weaknesses

- Encourages over-fitting
- Functional-form dependent
- Risks generating unstable models

Note that a fully-saturated C-ITS model (i.e., a model that estimates a coefficient on an indicator for each time period) is identical to an event study.

Wrap-up

Goals

1. Describe threats to validity in difference-in-differences (DD) identification strategy and multiple approaches to address these threats.
2. Using a cleaned dataset, estimate multiple DD specifications in R and interpret these results

To-Dos

Reading: Liebowitz, Porter & Bragg (2022)

- Critical to read the paper and answer a small set of questions as preparation for DARE
- *Further*: MHE: Ch. 5, 'Metrics: Ch. 5, Mixtape:

DARE #1

- Let's look at assignment
- Submit code and memo in response to questions
- Indicate partners (or not)
- I am available for support!

Research Project Proposal due 11:59pm, 1/28

- Talk to me!