MACHINE
BIT_DEFINITION

CONSTANTS
BIT,
bit_not,
bit_and,
bit_or,
bit_xor,
bool_to_bit

PROPERTIES
$BIT = 0 \mathinner{..} 1 \wedge$
$bit\_not \in BIT \rightarrow BIT \wedge$
$\forall\ (bb).\ (bb \in BIT \Rightarrow bit\_not(bb) = 1\text{-}bb) \wedge$
$bit\_and \in BIT \times BIT \rightarrow BIT \wedge$
$\forall\ (b1,\ b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow ((bit\_and(b1,\ b2) = 1) \Leftrightarrow (b1 = 1) \wedge (b2 = 1))) \wedge$
$bit\_or \in BIT \times BIT \rightarrow BIT \wedge$
$\forall\ (b1,\ b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow ((bit\_or(b1,\ b2) = 1) \Leftrightarrow (b1 = 1) \vee (b2 = 1))) \wedge$
$bit\_xor \in BIT \times BIT \rightarrow BIT \wedge$
$\forall\ (b1,\ b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow ((bit\_xor(b1,\ b2) = 1) \Leftrightarrow (b1 \neq b2))) \wedge$
$bool\_to\_bit \in \mathbf{BOOL} \rightarrow BIT \wedge$
$bool\_to\_bit = \{\ \mathbf{TRUE} \mapsto 1,\ \mathbf{FALSE} \mapsto 0\ \}$

ASSERTIONS
$bit\_not(0) = 1;$
$bit\_not(1) = 0;$
$\forall\ (bb).(bb \in BIT \Rightarrow bit\_not(bit\_not(bb)) = bb);$

$bit\_and(0,0) = 0;$
$bit\_and(0,1) = 0;$
$bit\_and(1,0) = 0;$
$bit\_and(1,1) = 1;$
$\forall\ (b1,b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow (bit\_and(b1,\ b2) = bit\_and(b2,b1)));$
$\forall\ (b1,b2,b3).(b1 \in BIT \wedge b2 \in BIT \wedge b3 \in BIT \Rightarrow (bit\_and(b1,\ bit\_and(b2,b3)) = bit\_and(bit\_and(b1,b2),b3)));$
$\forall\ (b1).(b1 \in BIT \Rightarrow (bit\_and(b1,\ 1) = b1));$
$\forall\ (b1).(b1 \in BIT \Rightarrow (bit\_and(b1,\ 0) = 0));$

$bit\_or(0,0) = 0;$
$bit\_or(0,1) = 0;$
$bit\_or(1,0) = 0;$
$bit\_or(1,1) = 1;$
$\forall\ (b1,\ b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow (bit\_or(b1,\ b2) = bit\_or(b2,b1)));$
$\forall\ (b1,b2,b3).(b1 \in BIT \wedge b2 \in BIT \wedge b3 \in BIT \Rightarrow (bit\_or(b1,\ bit\_or(b2,b3)) = bit\_or(bit\_or(b1,b2),b3)));$
$\forall\ (b1).(b1 \in BIT \Rightarrow (bit\_or(b1,\ 1) = 1));$

$\forall\ (b1).(b1 \in BIT \Rightarrow (bit\_or(b1,\ 0) = b1))$;

$bit\_xor(0,0) = 0$;
$bit\_xor(0,1) = 1$;
$bit\_xor(1,0) = 1$;
$bit\_xor(1,1) = 0$;
$\forall\ (b1,\ b2).(b1 \in BIT \wedge b2 \in BIT \Rightarrow (bit\_xor(b1,\ b2) = bit\_xor(b2,b1)))$;
$\forall\ (b1,b2,b3).(b1 \in BIT \wedge b2 \in BIT \wedge b3 \in BIT \Rightarrow (bit\_xor(b1,\ bit\_xor(b2,b3)) = bit\_xor(bit\_xor(b1,b2),b3)))$;
$\forall\ (bb).(bb \in BIT \Rightarrow bit\_xor(bb,bb) = 0)$;

$bool\_to\_bit(\mathbf{TRUE}) = 1$;
$bool\_to\_bit(\mathbf{FALSE}) = 0$

END