```
> x := (r, theta, phi) → r·sin(theta)·cos(phi)
```

$$x := (r, \theta, \phi) \mapsto r \sin(\theta) \cos(\phi) \tag{1}$$

```
> y := (r, theta, phi) → r·sin(theta)·sin(phi)
```

$$y := (r, \theta, \phi) \mapsto r \sin(\theta) \sin(\phi) \tag{2}$$

```
> z := (r, theta, phi) → r·cos(theta)
```

$$z := (r, \theta, \phi) \mapsto r \cos(\theta) \tag{3}$$

```
> M :=
    [[diff(x(r, theta, phi), r), diff(x(r, theta, phi), phi), diff(x(r, theta, phi
    ), theta)],
    [diff(y(r, theta, phi), r), diff(y(r, theta, phi), phi), diff(y(r, theta, phi), theta)
    ],
    [diff(z(r, theta, phi), r), diff(z(r, theta, phi), phi), diff(z(r, theta, phi), theta)
    ]]
```

$$M := \begin{bmatrix} \sin(\theta) \cos(\phi) & -r \sin(\theta) \sin(\phi) & r \cos(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) & r \sin(\theta) \cos(\phi) & r \cos(\theta) \sin(\phi) \\ \cos(\theta) & 0 & -r \sin(\theta) \end{bmatrix} \tag{4}$$

```
> simplify(M^(-1))
```

$$\begin{bmatrix} \sin(\theta) \cos(\phi) & \sin(\theta) \sin(\phi) & \cos(\theta) \\ -\frac{\sin(\phi)}{r \sin(\theta)} & \frac{\cos(\phi)}{r \sin(\theta)} & 0 \\ \frac{\cos(\phi) \cos(\theta)}{r} & \frac{\sin(\phi) \cos(\theta)}{r} & -\frac{\sin(\theta)}{r} \end{bmatrix} \tag{5}$$

```
>
>
> restart;
>
> x := (r, theta, phi) → sqrt(r^2 + a^2)·sin(theta)·cos(phi)
```

$$x := (r, \theta, \phi) \mapsto \sqrt{a^2 + r^2} \sin(\theta) \cos(\phi) \tag{6}$$

```
> y := (r, theta, phi) → sqrt(r^2 + a^2)·sin(theta)·sin(phi)
```

$$y := (r, \theta, \phi) \mapsto \sqrt{a^2 + r^2} \sin(\theta) \sin(\phi) \tag{7}$$

```
> z := (r, theta, phi) → r·cos(theta)
```

$$z := (r, \theta, \phi) \mapsto r \cos(\theta) \tag{8}$$

```
> M :=
    [[diff(x(r, theta, phi), r), diff(x(r, theta, phi), phi), diff(x(r, theta, phi
    ), theta)],
    [diff(y(r, theta, phi), r), diff(y(r, theta, phi), phi), diff(y(r, theta, phi), theta)
    ],
    [diff(z(r, theta, phi), r), diff(z(r, theta, phi), phi), diff(z(r, theta, phi), theta)
```

]]

$$M := \begin{bmatrix} \frac{\sin(\theta)\cos(\phi)\, r}{\sqrt{a^2+r^2}} & -\sqrt{a^2+r^2}\sin(\theta)\sin(\phi) & \sqrt{a^2+r^2}\cos(\theta)\cos(\phi) \\ \frac{\sin(\theta)\sin(\phi)\, r}{\sqrt{a^2+r^2}} & \sqrt{a^2+r^2}\sin(\theta)\cos(\phi) & \sqrt{a^2+r^2}\cos(\theta)\sin(\phi) \\ \cos(\theta) & 0 & -r\sin(\theta) \end{bmatrix} \quad (9)$$

> $simplify(M^{-1})$

$$\begin{bmatrix} \frac{\sqrt{a^2+r^2}\, r\cos(\phi)\sin(\theta)}{\cos(\theta)^2 a^2 + r^2} & \frac{\sqrt{a^2+r^2}\, r\sin(\theta)\sin(\phi)}{\cos(\theta)^2 a^2 + r^2} & \frac{(a^2+r^2)\cos(\theta)}{\cos(\theta)^2 a^2 + r^2} \\ -\frac{\sin(\phi)}{\sqrt{a^2+r^2}\sin(\theta)} & \frac{\cos(\phi)}{\sqrt{a^2+r^2}\sin(\theta)} & 0 \\ \frac{\sqrt{a^2+r^2}\cos(\phi)\cos(\theta)}{\cos(\theta)^2 a^2 + r^2} & \frac{\sqrt{a^2+r^2}\sin(\phi)\cos(\theta)}{\cos(\theta)^2 a^2 + r^2} & -\frac{r\sin(\theta)}{\cos(\theta)^2 a^2 + r^2} \end{bmatrix} \quad (10)$$

>

>

>