

Introduction

Construction 1D

Construction 2D

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2D Transversal
Ising Model

Conclusion and
outlook

PEPO cluster expansion of tensor exponential

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Problem Statement
Graphical notation
Cluster expansion

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Introduction

Statistical Quantum mechanics

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$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{Z} \quad (1)$$

$$\begin{aligned} Z &= \text{Tr}(e^{-\beta \hat{H}}) \\ \langle X \rangle &= \text{Tr}(\rho \hat{X}) \end{aligned} \quad (2)$$

Graphical notation

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$$\begin{array}{c} i \\ 0 \quad | \quad 0 \\ \hline \bigcirc \\ j \end{array} = \bigcirc \quad (3)$$

$$\begin{array}{c} i_1 \quad i_2 \\ 0 \quad | \quad 1 \quad | \quad 0 \\ \hline \bigcirc \quad \bigcirc \\ j_1 \quad j_2 \end{array} = \bigcirc \text{---} 1 \text{---} \bigcirc \quad (4)$$

$$\begin{array}{c} i_1 \quad i_2 \quad i_3 \\ 0 \quad | \quad 1 \quad | \quad 1 \quad | \quad 0 \\ \hline \bigcirc \quad \bigcirc \quad \bigcirc \\ j_1 \quad j_2 \quad j_3 \end{array} = \bigcirc \text{---} \bigcirc \text{---} \bigcirc \quad (5)$$

Graphical notation

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$$\hat{H} = \left(\sum_{\langle ij \rangle} H_2^i H_2^j + \sum_i H_1^i \right) \quad (6)$$

$$\begin{aligned} H \left(\bigcirc - \bigcirc - \bigcirc \right) = & H_1 \otimes 1 \otimes 1 \\ & + 1 \otimes H_1 \otimes 1 \\ & + 1 \otimes 1 \otimes H_1 \\ & + H_2 \otimes H_2 \otimes 1 \\ & + 1 \otimes H_2 \otimes H_2 \end{aligned} \quad (7)$$

General idea

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$$\bigcirc = \exp(-\beta H(\bigcirc)) \quad (8)$$

$$\overset{1}{\bigcirc - \bigcirc} = \exp -\beta H(\overset{1}{\bigcirc - \bigcirc} - \overset{0}{\bigcirc - \bigcirc}) \quad (9)$$

General idea

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$$\text{---} \overset{1}{\bigcirc} \text{---} \overset{1}{\bigcirc} \text{---} \bigcirc = \exp -\beta H(\text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc)$$

$$\text{---} \overset{0}{\bigcirc} \text{---} \overset{0}{\bigcirc} \text{---} \bigcirc$$

$$\text{---} \overset{1}{\bigcirc} \text{---} \overset{0}{\bigcirc} \text{---} \bigcirc$$

$$\text{---} \overset{0}{\bigcirc} \text{---} \overset{1}{\bigcirc} \text{---} \bigcirc$$

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Advantages

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- size extensive
- symmetry
- fast

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Variant A

Variant C

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Construction 1D

Variant A

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Construction 1D

Variant A

Variant C

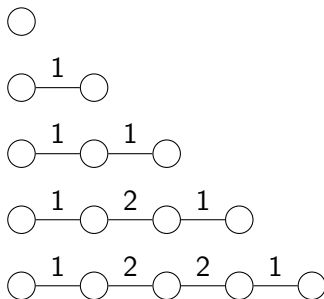
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Variant C

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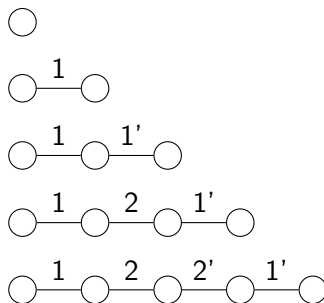
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Error measure

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$$\epsilon(\text{map}) = \frac{\|\exp - \beta H(\text{map}) - \text{MPO}(\text{map})\|}{\|\exp - \beta H(\text{map})\|} \quad (13)$$

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Variant A

Variant C

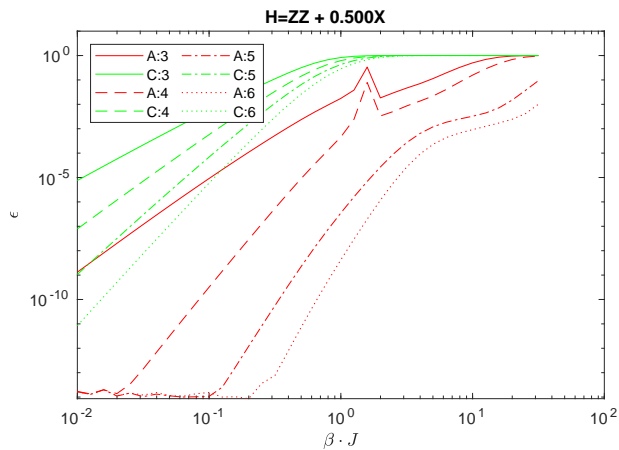
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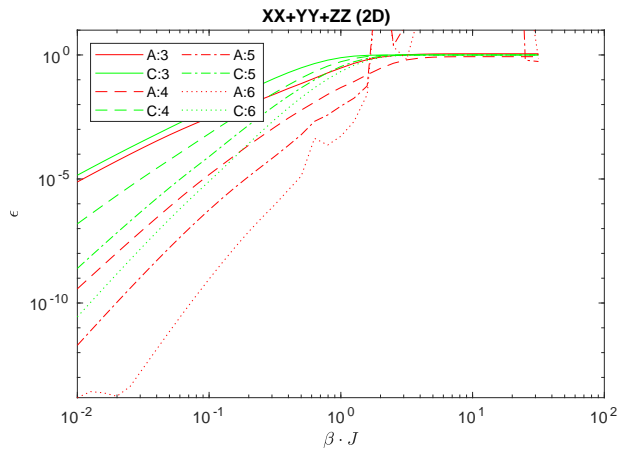
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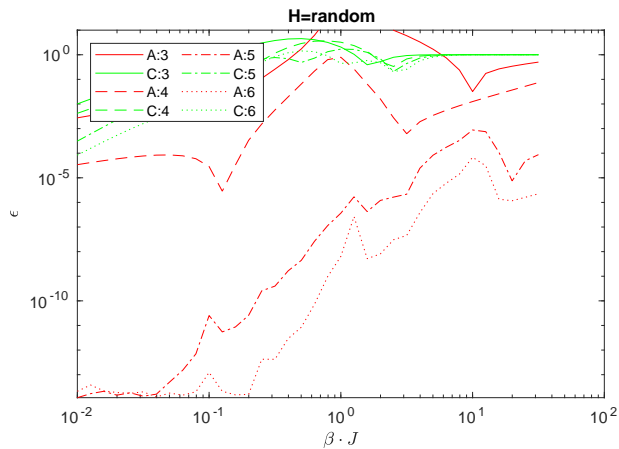
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Linear blocks

Loops

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Construction 2D

Construction 2D: Linear blocks

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Construction 2D

Linear blocks

Loops

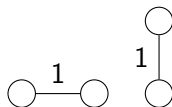
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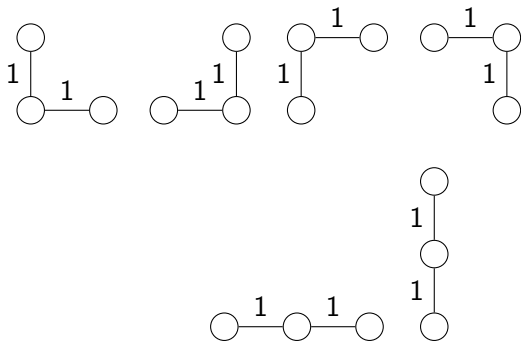


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Construction 2D: Linear blocks



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Construction 2D: Linear blocks

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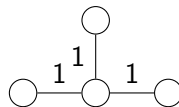
Linear blocks

Loops

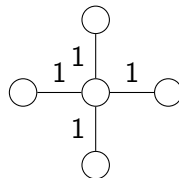
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Construction 2D: Linear blocks

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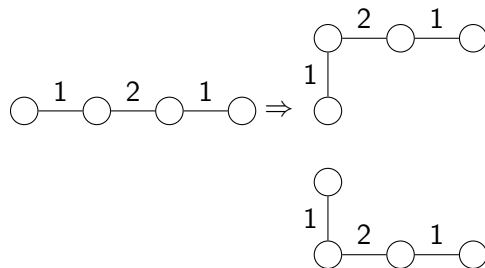
Linear blocks

Loops

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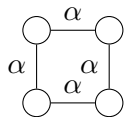
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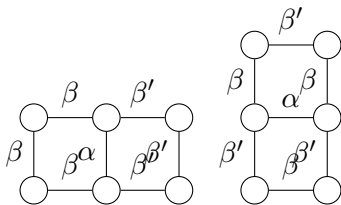
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And many more "linear" blocks

Loops



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- bond dim
- solver: see later

Unsolved

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Construction 1D

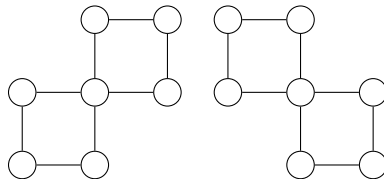
Construction 2D

Loops

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Easy to solve on finite lattice, difficult in thermodynamic limit...

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Linear solver

Non-linear solvers

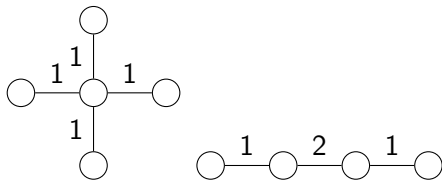
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Solvers

Linear solver

- pseudoinverse
- optimisation for tree graphs
- implemented for any shape



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```
[map, ~] = create_map([0, 0, 5, 0;  
                     0, 0, 6, 0;  
                     1, 2, 3, 4;  
                     0, 0, 7, 0], obj.numopts);  
pattern = {[2, 2, 1, 1]};  
[obj, ~, ~, ln_prefact, ~] = solve_lin_and_assign(obj, map, pattern, ln_prefact);  
o.
```


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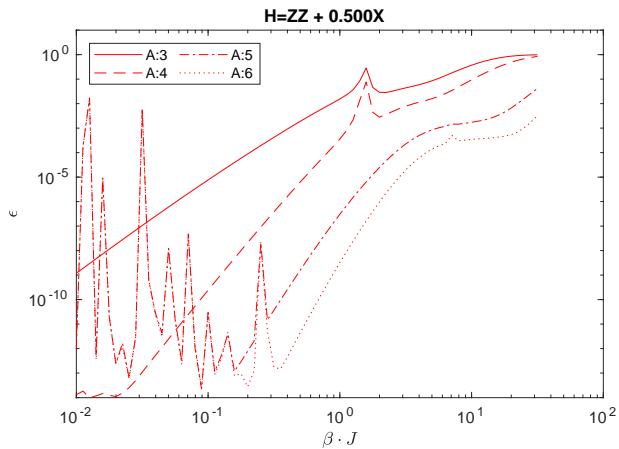
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sequential linear

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- initialize randomly
- use linear solver for 1 tensor
- fast

true non-linear solver

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- Matlab fsolve
- exact jacobian
- multiple patterns
- multiple maps

2D Transversal Ising Model

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First results

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$$\hat{H} = -J \left(\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + \Gamma \sum_i \sigma_i^x \right) \quad (24)$$

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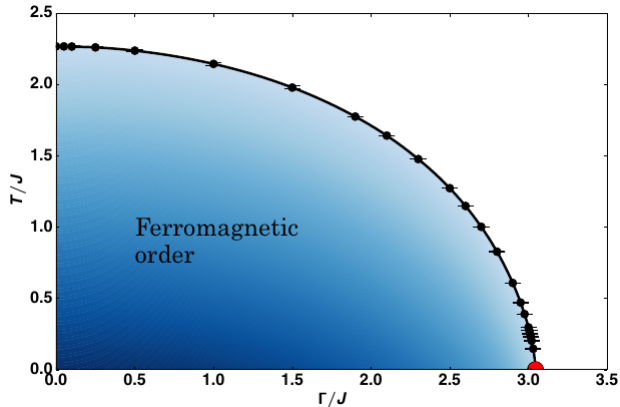
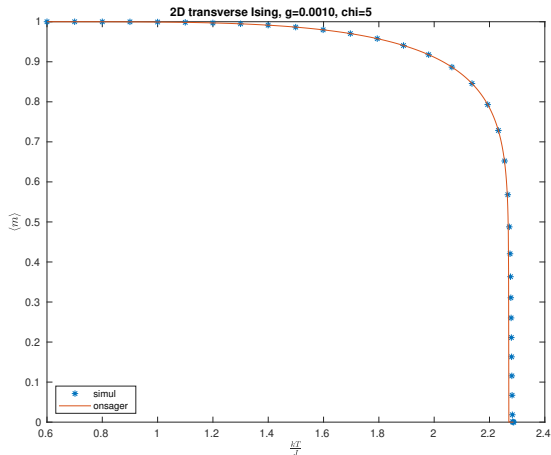
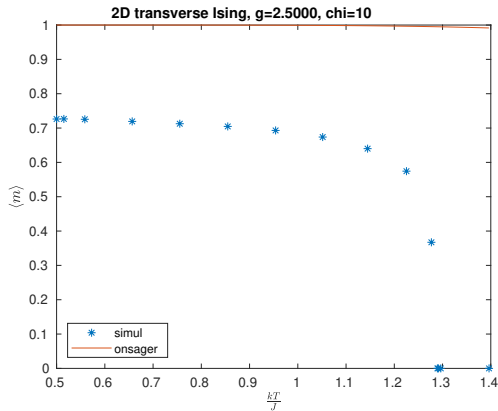


Figure: figure taken from [1]

First results



First results



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Conclusion and outlook

Conclusion

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- Working code for 1D and 2D
- General solvers
- Promising first results in 2D

Outlook: short term

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- Accurate estimate transversal Ising quantum critical point
- Improve blocks for loops
- continuous improvements framework

Outlook: long term

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Conclusion and
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- Incorporate symmetries of Hamiltonians
- Look at other (types of) Hamiltonians
- Generalize for other lattice geometries
- Generalize to 3D

References I

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S. Hesselmann, S. Wessel, Thermal ising transitions in the vicinity of two-dimensional quantum critical points, Phys. Rev. B 93 (2016) 155157.

doi:10.1103/PhysRevB.93.155157.

URL <https://link.aps.org/doi/10.1103/PhysRevB.93.155157>