Intoduction
Construction 1D
Construction 2D
Solvers
2D Transversal Ising Model
Conclusion and outlook

PEPO cluster expansion of tensor exponential

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Intoduction
Problem Statement
Graphical notation
Cluster expansion
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Intoduction

Statistical Quantum mechanics

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$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{Z} \tag{1}$$

$$\begin{aligned} Z &= \mathrm{Tr}\left(e^{-\beta \hat{H}}\right) \\ \langle X \rangle &= \mathrm{Tr}\left(\rho \hat{X}\right) \end{aligned} \tag{2}$$

Graphical notation

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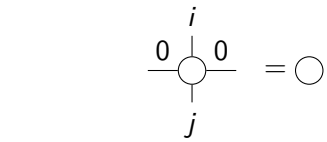

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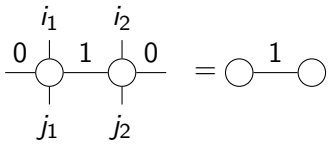

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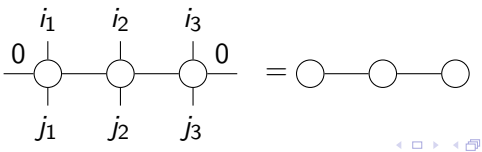

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Graphical notation

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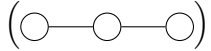
$$\hat{H} = \left(\sum_{<ij>} H_2^i H_2^j + \sum_i H_1^i \right) \tag{6}$$

H 

$$\begin{aligned} &= H_1 \otimes 1 \otimes 1 \\ &+ 1 \otimes H_1 \otimes 1 \\ &+ 1 \otimes 1 \otimes H_1 \\ &+ H_2 \otimes H_2 \otimes 1 \\ &+ 1 \otimes H_2 \otimes H_2 \end{aligned} \tag{7}$$

General idea

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$$\bigcirc = \exp\left(-\beta H(\bigcirc)\right) \tag{8}$$

$$\overset{1}{\bigcirc\!\!-\!\!\bigcirc} = \exp -\beta H(\bigcirc\!\!-\!\!\bigcirc) - \overset{0}{\bigcirc\!\!-\!\!\bigcirc} \tag{9}$$

General idea

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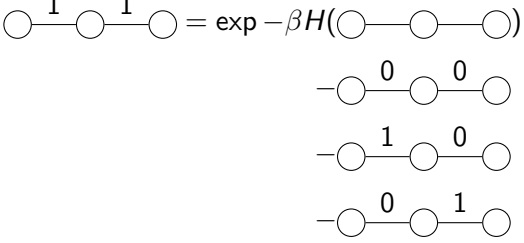

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Advantages

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- size extensive
- symmetry
- fast

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Construction 1D

Variant A

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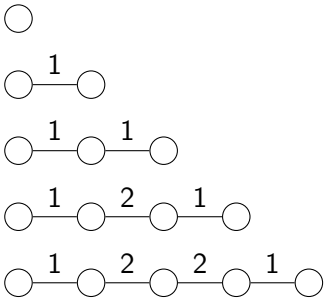

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Variant C

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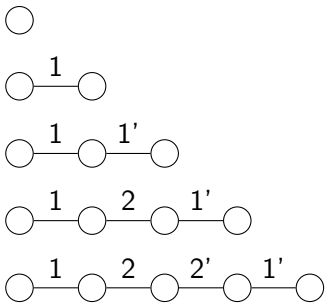

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Error measure

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$$\epsilon(\text{map}) = \frac{||\exp -\beta H(\text{map}) - \text{MPO}(\text{map})||}{||\exp -\beta H(\text{map})||} \tag{13}$$

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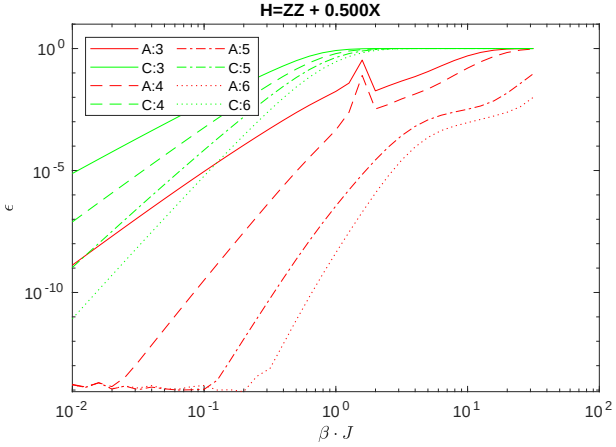

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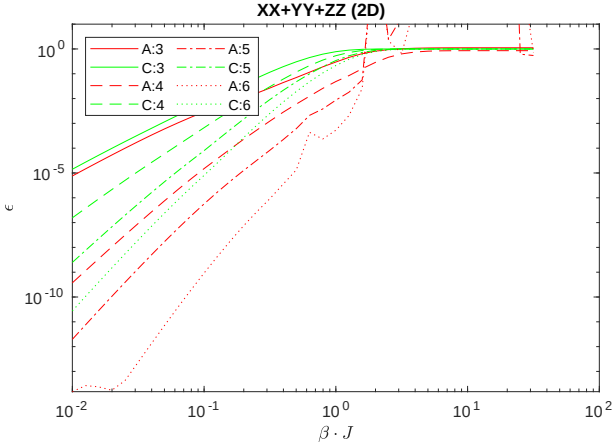

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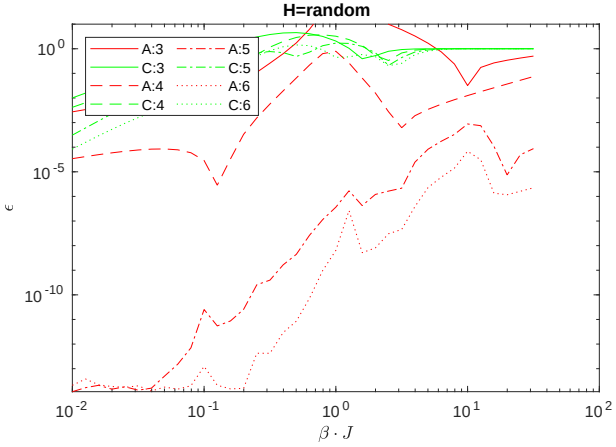

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Construction 2D

Construction 2D: Linear blocks

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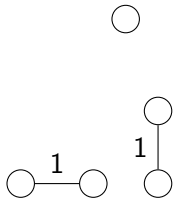

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Construction 2D: Linear blocks

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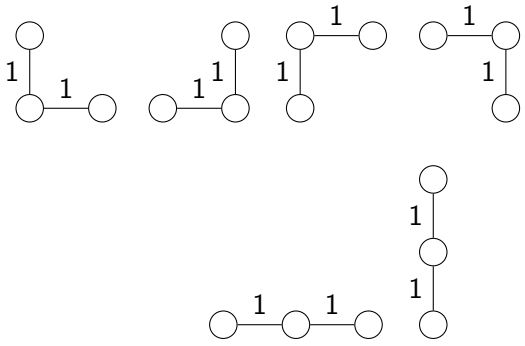

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Construction 2D: Linear blocks

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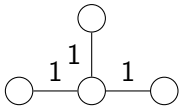

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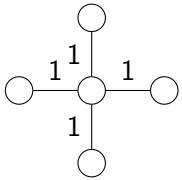

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Construction 2D: Linear blocks

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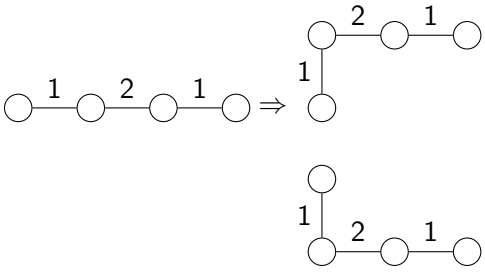

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And many more "linear" blocks

Loops

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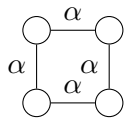

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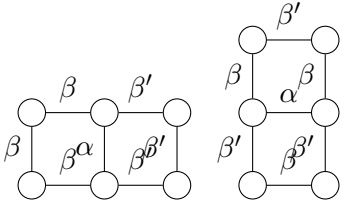

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- bond dim
- solver: see later

Unsolved

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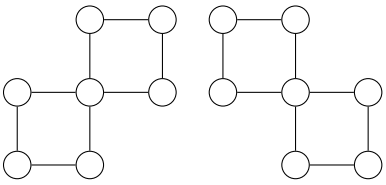

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Easy to solve on finite lattice, difficult in thermodynamic limit...

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Solvers

Linear solver

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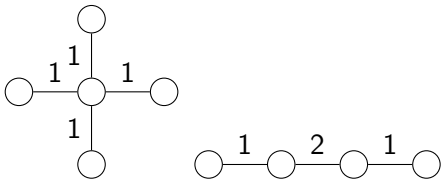
- pseudoinverse
- optimisation for tree graphs
- implemented for any shape



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```
[map, ~] = create_map([0, 0, 5, 0;
                       0, 0, 6, 0;
                       1, 2, 3, 4;
                       0, 0, 7, 0], obj.numopts);
pattern = {[2, 2, 1, 1]};
[obj, ~, ~, ln_prefact, ~] = solve_lin_and_assign(obj, map, pattern, ln_prefact);
```

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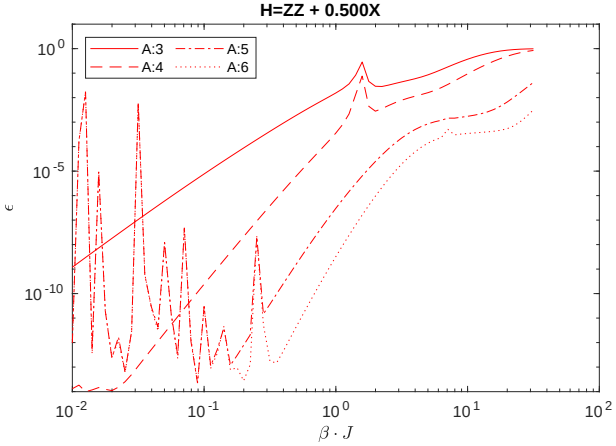

sequential linear

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- initialize randomly
- use linear sovler for 1 tensor
- fast

true non-linear solver

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- Matlab fsolve
- exact jocobian
- multiple patterns
- multiple maps

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2D Transversal Ising Model

Overview

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$$\hat{H} = -J \left(\sum_{<ij>} \sigma_i^z \sigma_j^z + \Gamma \sum_i \sigma_i^x \right) \tag{24}$$

Overview

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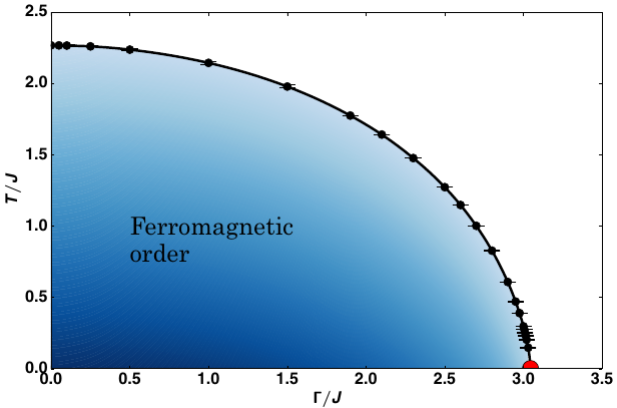

Figure: figure taken from [1]

First results

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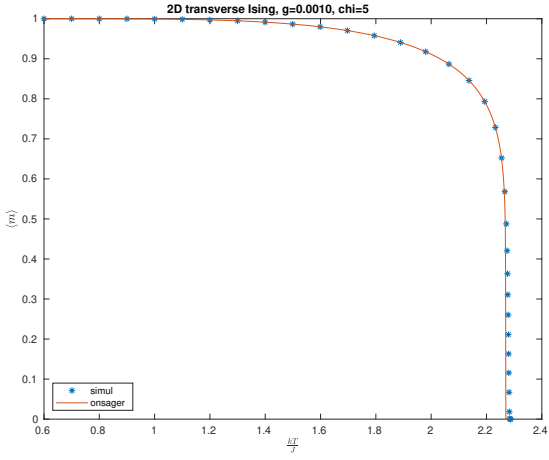

First results

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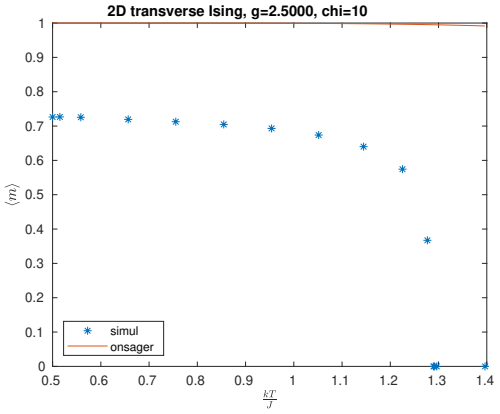

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Conclusion and outlook

Conclusion

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- Working code for 1D and 2D
- General solvers
- Promising first results in 2D

Outlook: short term

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- Accurate estimate transversal Ising quantum critical point
- Improve blocks for loops
- continuous improvements framework

Outlook: long term

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- Incorporate symmetries of Hamiltonians
- Look at other (types of) Hamiltonians
- Generalize for other lattice geometries
- Generalize to 3D

References I

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S. Hesselmann, S. Wessel, Thermal ising transitions in the vicinity of two-dimensional quantum critical points, Phys. Rev. B 93 (2016) 155157.
doi:10.1103/PhysRevB.93.155157.
URL https://link.aps.org/doi/10.1103/PhysRevB.93.155157