Cluster Expansion of Thermal States using Tensor Networks

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Introduction

Variant A

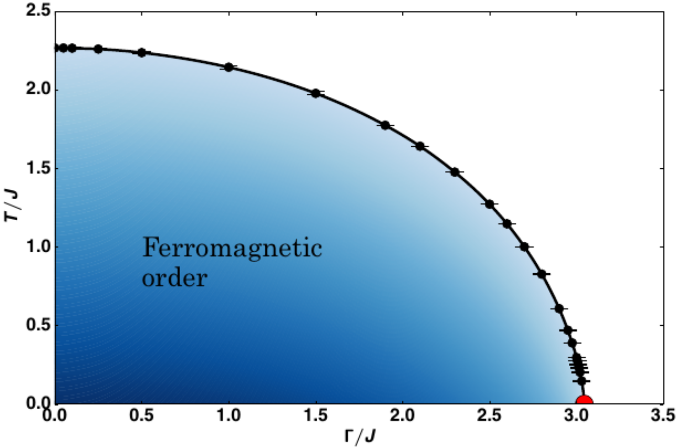


Figure: Figure taken from [1]

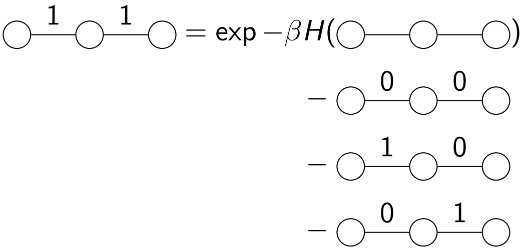
Cluster Expansion

General idea

$$\bigcirc = \exp\left(-\beta H(\bigcirc)\right) \tag{1}$$

$$\begin{aligned} \bigcirc\overset{1}{\text{——}}\bigcirc &= \exp -\beta H(\bigcirc\text{——}\bigcirc) \\ &\quad - \bigcirc\overset{0}{\text{——}}\bigcirc \end{aligned} \tag{2}$$

General idea

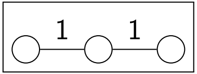


(3)

General idea

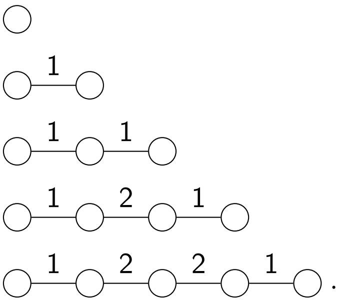
$$\bigcirc \overset{1}{\text{———}} \bigcirc \overset{1}{\text{———}} \bigcirc = \exp -\beta H(\bigcirc\text{———}\bigcirc\text{———}\bigcirc) - \bigcirc\text{———}\bigcirc\text{———}\bigcirc \tag{3}$$

General idea



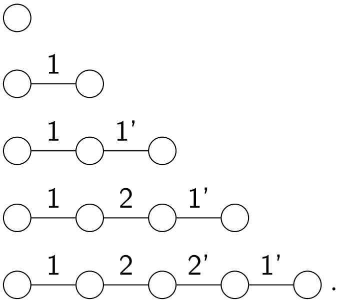
(3)

1D: Variant A



(4a)
(4b)
(4c)
(4d)
(4e)

1D: Variant E



(5a)
(5b)
(5c)
(5d)
(5e)

1D: Variant F

$$\bigcirc \tag{6a}$$

$$\bigcirc \overset{1'}{\text{———}} \bigcirc + \bigcirc \overset{1}{\text{———}} \bigcirc \tag{6b}$$

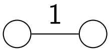
$$\bigcirc \overset{1}{\text{———}} \bigcirc \overset{1}{\text{———}} \bigcirc \tag{6c}$$

$$\bigcirc \overset{1}{\text{———}} \bigcirc \overset{2}{\text{———}} \bigcirc \overset{1}{\text{———}} \bigcirc +$$

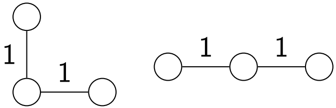
$$\bigcirc \overset{1}{\text{———}} \bigcirc \overset{2'}{\text{———}} \bigcirc \overset{1}{\text{———}} \bigcirc \tag{6d}$$

$$\bigcirc \overset{1}{\text{———}} \bigcirc \overset{2}{\text{———}} \bigcirc \overset{2}{\text{———}} \bigcirc \overset{1}{\text{———}} \bigcirc\,. \tag{6e}$$

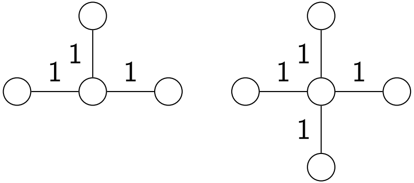
2D: Linear Blocks



(7a)

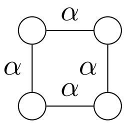


(7b)

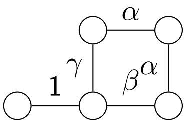


(7c)

2D: Nonlinear Blocks



(8)



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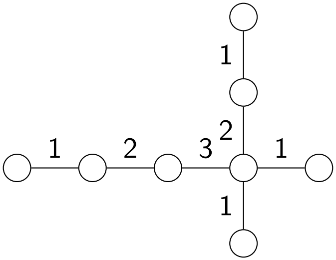
(9)

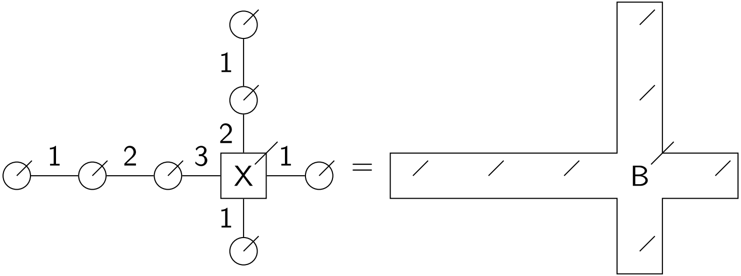
Solvers

Linear Solver: Standard Form



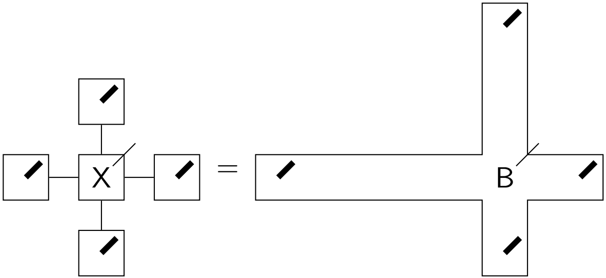
(10)

Linear Solver: Standard Form



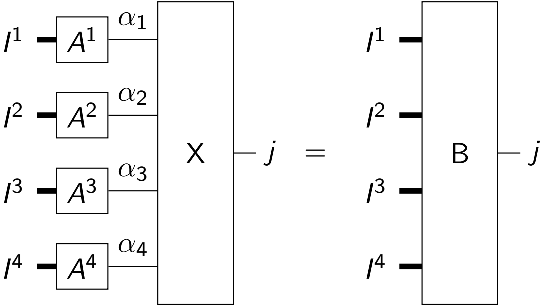
(10)

Linear Solver: Standard Form



(10)

Linear Solver: Standard Form



(10)

Linear Solver: Inversion Scheme

- Invert A^i separately
  - Fast
  - Numerically unstable

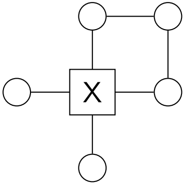
Linear Solver: Inversion Scheme

- Invert A^i separately
- Full inversion $A = A^1 \otimes A^2 \cdots \otimes A^m$
  - Slow
  - Stable for pseudoinverse

Linear Solver: Inversion Scheme

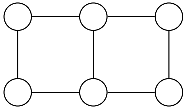
- Invert A^i separately
- Full inversion $A = A^1 \otimes A^2 \cdots \otimes A^m$
- Sparse full inversion
  - $A^i = U \Sigma V^\dagger$
  - $S = S^1 \otimes S^2 \cdots \otimes S^m$

Linear Solver: Applicability



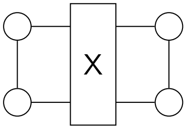
(11)

Linear Solver: Applicability



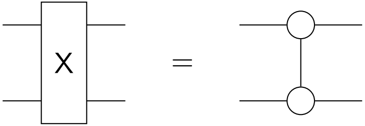
(12)

Linear Solver: Applicability



(12)

Linear Solver: Applicability



(12)

Nonlinear Solver

- Nonlinear least squares
- Jacobian
- Permutations

Sequential Linear Solver

- Based on linear solver
- Sweep over unknown tensors
- Permutations

Results

Conclusion

References I

S. Hesselmann, S. Wessel, Thermal Ising transitions in the vicinity of two-dimensional quantum critical points, PHYSICAL REVIEW B 93 (2016) 155157.
doi:10.1103/PhysRevB.93.155157.