

1. For every $n, n \in \mathbb{Z}^+$, there is a special $n \times n$ matrix called the identity matrix, I_n . This special matrix is a matrix contains all zeros except on the principle diagonal, which only contains ones. For example, if $n = 3$, I_3 is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

An $n \times n$ matrix, M , is orthogonal if and only if $M(M^T) = I_n$. Write pseudocode to check if a matrix M is orthogonal. You may assume that you have access to a *mat_mul* function and *transpose* function.

2. Recall that a symmetric matrix is an $n \times n$ matrix M where the entries are such that $M_{ij} = M_{ji}$. Write pseudocode that accepts a matrix and returns *True* if is symmetric and *False* otherwise.
3. Explain why it useful to have special methods for representing sparse matrices.
4. Explain why an $n \times n$ upper triangular matrix has $\frac{1}{2}n(n+1)$ non-zero entries.
5. Consider the following sparse matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 9 \\ 2 & 3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 89 \end{bmatrix}$$

Illustrate the following sparse representations of this matrix:

- (a) LIL
- (b) COO
- (c) CSR