Hashtables and their Applications

COMP2611: Data Structures

September 2019

Q: how would you build a (basic) search engine?

What's so Hard about Search Engines?

"The **Google** Search **index** contains hundreds of billions of webpages and is well over 100,000,000 gigabytes in **size**."

How Google Search Works | Crawling & Indexing
https://www.google.com › search › crawl...

About 40,600,000 results (0.93 seconds)

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Search Through Each Page?

- Assume Google indexes `200,000,000,000` pages
- If we could scan `1` page in `1` microsecond
  - one search would take `55` hours
- How do we improve search time
  - when we have to look through billions of documents?

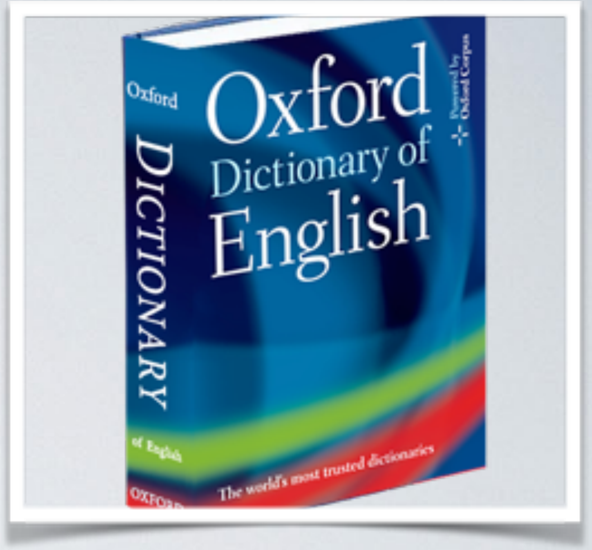
Outline

- Dictionary ADT
  - and its Naive Implementation
- Hashtables
  - Collisions and how to resolve them
  - Methods:
    - Chaining
    - Linear Probing
    - Quadrating Probing
    - Double Hashing
- Set ADT
  - Using Hashtables as sets

ADT - Abstract Data Type

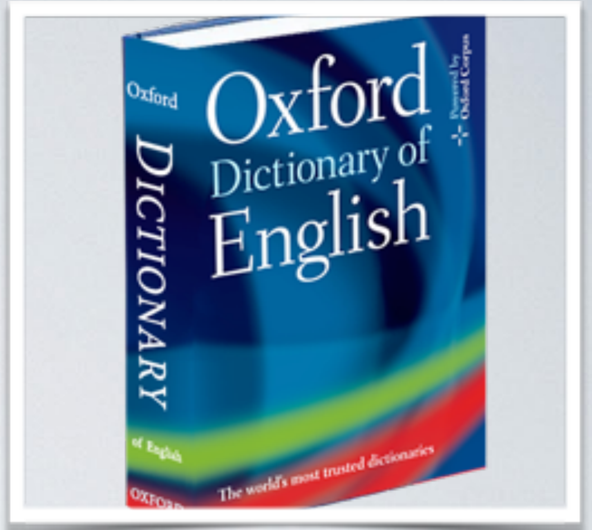
- Describes minimal operations that a data type must satisfy
- Think Interfaces from Java and C#
- Does not describe how to achieve functionality
  - Functionality achieved through data structures!
  - Different underlying data structures provide different performances

Dictionary ADT



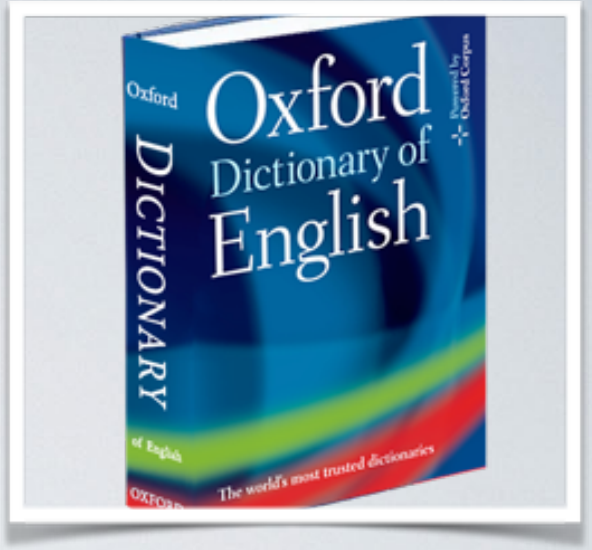
- Collection of key/value pairs
  - Each key is unique
  - Keys are **unordered**
- Primarily concerned with lookup and storage by key
- Sometimes also called a **map**
  - maps keys to values
  - ex: student id → student record; word → definition
  - one-to-one mapping but can emulate one-to-many using lists (index)

Dictionary ADT



- Few main operations
  - **insert**(key, value): inserts a key/value pair in the dictionary. If a value already exists for that key, it overwrites that value
  - **search**(key): searches the dictionary for a key and returns the associated value
  - **delete**(key): deletes this key and its associated value from the dictionary

Dictionary ADT



- Dictionaries are found everywhere!
  - Routing tables in Routers
  - Symbol Tables in Compilers and Interpreters
  - Indicies in Databases (unordered access)
  - DNA Sequence analysis (Assignment #1)
  - NLP techniques
  - etc…

Dynamic Array Based Dictionary

- **insert**(key, value): scans the array. If a value for this key exists, then replace it, else insert at the end
- **search**(key): scans array until it finds the key in the array
- **remove**(key): scans array, finds key in array and deletes element, moving previous entries up

Dynamic Array Based Dictionary

- **insert**(key, value): **scans the array**. If a value for this key exists, then replace it, else insert at the end
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Dynamic Array Based Dictionary

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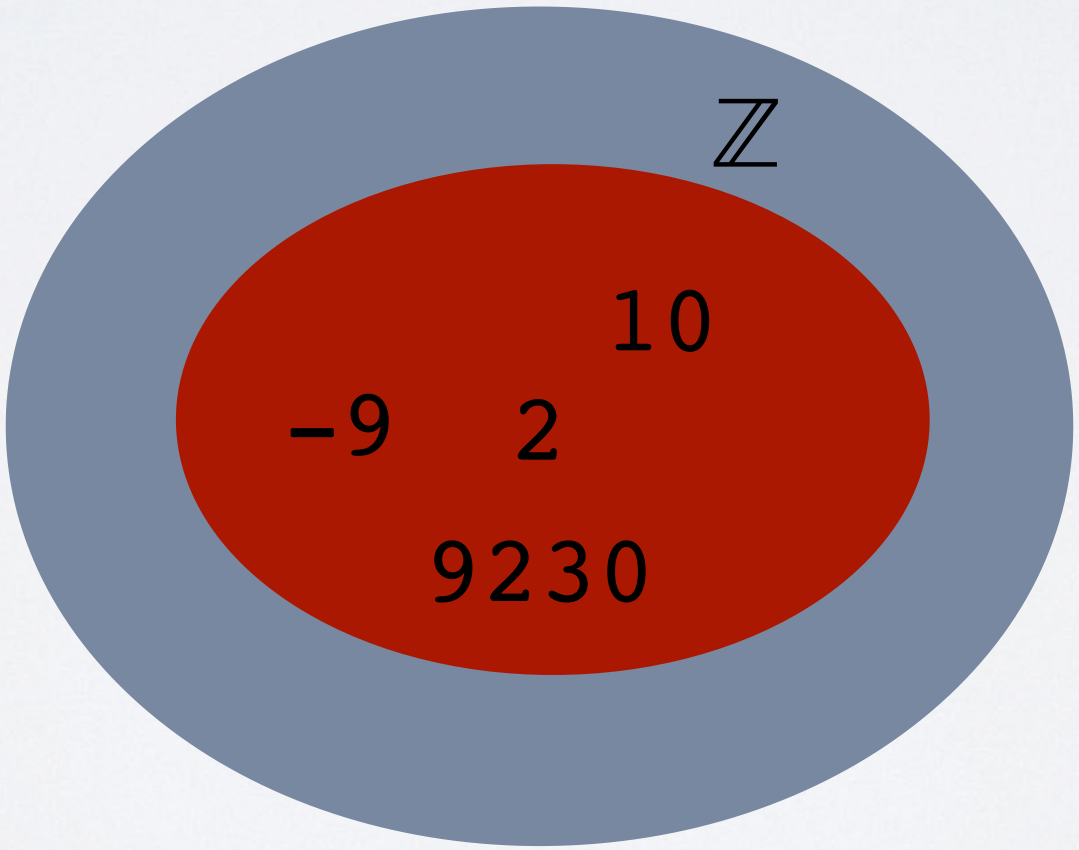
This is worst-case `O(n)`

Solution: Hashtables

- Dictionaries serve as the workhorse behind a lot of other algorithms
- They need to be efficient
- Array's are like dictionaries
  - keys are limited to `{0, 1, … (n - 1)}`
- Can we generalise arrays for natural keys outside of range
  - What about keys that are not numbers?

Solution: Hashtables

- Core insight: Even though we can have a large universe of keys, we will only deal with a finite subset of them in practice!



Solution: Hashtables

- Core insight: We can “shrink” a larger universe of keys to a smaller one
- Core insight: We can project keys from a larger universe of keys to the set of indices for an array!
- If we have an array index, then we can insert, search, and delete data using random access

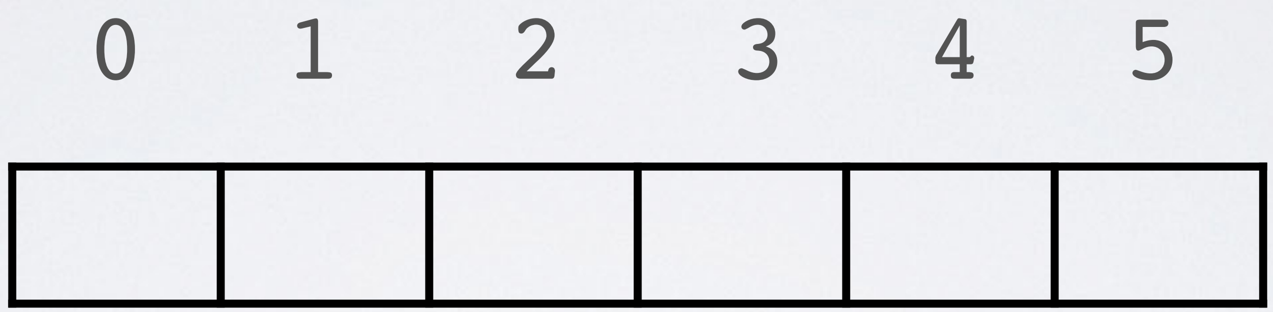
Random access on array indices is `O(1)`!

Hashtables

- Consider that we have a universe of keys **`X`**
- Hashtables:
  - An array (may or may not be static)
    - Each space in the array is called a bucket
  - A hash function, **`h: X⟶Y`**
    - Here **`Y = {0, 1, … n-1}`** where **`n`** is the length of the array
    - Applying a hash function is called hashing
    - The result of the application of a hash function is called the digest or hash value
    - In practice, hash functions used in **hash tables** also take the length of the array
    - Other uses of hashing!

Hashtables

Suppose that we need
to store the pair **`(k, v)`**



Hashtables

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0	1	2	3	4	5

$$\texttt{k} \longrightarrow \texttt{h(x)} \longrightarrow \texttt{2}$$

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to store the pair `(k, v)`

0	1	2	3	4	5
		`(k, v)`			

`k` ⟶ `h(x)` ⟶ `2`

```
function insert(table, h, k, v)
    index = h(k, length(table))
    arr[index] = (k, v)
```

O(1)

```
function search(table, h, k)
    index = h(k, length(table))
    // arr is initially filled with NIL
    return arr[index]
```

O(1)

```
function delete(table, h, k)
    index = h(k, length(table))
    // arr is initially filled with NIL
    arr[index] = NIL
```

O(1)

Assuming that `h` is `O(1)`

Hashtables: Trouble in Paradise

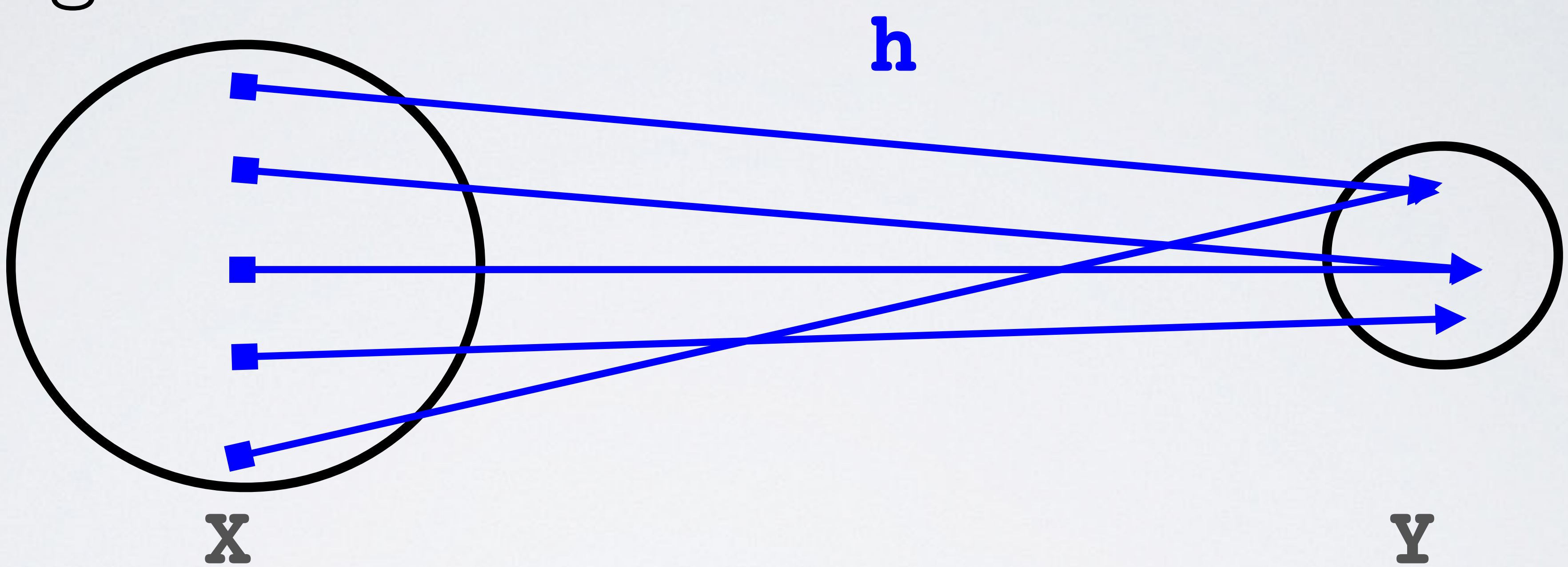
- Hashtables are wonderful, but there are challenges
- `|X| > |Y|`
  - When shrinking universe of keys, will have multiple keys hashing to the same hash value. (Pigeon-hole principle)
  - This is called a collision
- The concept of a hashtable is easy!
- Handling these collisions is the hard part!

Hashtables: Fixing the problems

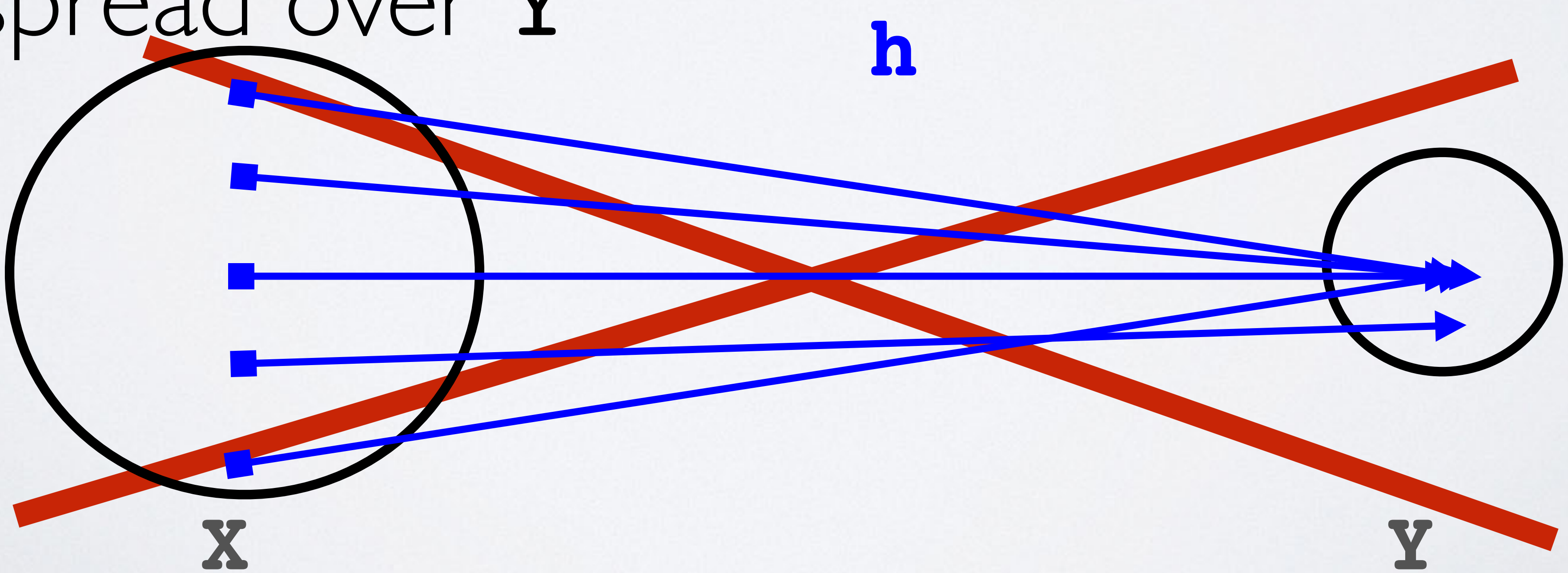
- We have control over `h`
  - We can design a good `h`
  - What are the properties of a good hash function?

Properties of a Good Hash Function

- Shrinking



- Well-spread over **`Y`**



Properties of a Good Hash Function

- A good hash function should consider all of the data available in the key
  - Consider the strings “abc” and “abd”.
  - One character different, but this would cause them to map to different digests
- A good hash function should depend solely on the the input alone.
  - External data might inadvertently increase collisions

Properties of a Good Hash Function

1. **Well-spread**: a good hash function is uniformly spread across the set of hash digests
2. **Deterministic**: considers only its input
3. **Comprehensive**: considers all of its input

Most hash functions we would use in this course will be of the form

`(af(x) + b) mod n`

where

`n` is the length of the array

`a = 0, b = 0` for the most part

`f` is a function that interprets `x` as an integer

Interpreting as an Integer

- We can also the case of non-integer keys
- To use the hash function on the last slide, we need to interpret or cast the input to an integer
- Suppose that we had the string “foobar”?
  - Operate on character byte representation to generate hash code, then use hash code to generate digest for the hashtable
  - In Python, we can do this using the (poorly named) *hash* function

Code to do this C/C++

```python
s = 'foobar'
i = hash(s)
print(i)
```

```c
int string_hash(char* s)
{
  long code = 0
  long i = 1
  while(*s)
  {
    code += (long)(s) * i
    i += 1
    s += 1
  }
  return code
}
```

```java
String s = "foobar";
Integer i = s.hashCode()
System.out.println(i)
```

Integer computed
using underlying bytes

Collision Resolution

- A good hash function would minimise the probability of collisions
  - But these theoretical guarantees are hard to come by and prove in practice
- Still need to handle collisions when they (inevitably) happen

Collision Resolution Strategies

- Chaining
- Linear Probing
- Quadrating Probing
- Double Hashing
- Hopscotch Hashing
- Cuckoo Hashing

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Open Addressing

Collision Resolution Strategies

- **Chaining**
- **Linear Probing**
- **Quadrating Probing**
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Open Addressing

Chaining

- Core idea: Allow each **bucket** to hold more than one item
- Uses additional data structure to store multiple key/value pairs. Notice each value still maps to **single** value
- Hashtable becomes array of this data structure
- Data Structures used:
  - Linked Lists
  - Binary Search Trees
  - Dynamic Arrays
  - Another Hashtable

Chaining

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  - **Dynamic Arrays**
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Will use Dynamic Arrays
But result generalise!

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Chaining

- Let's consider a practical example
  - Assume student IDs are integers (note that is not the best type for them!)
    - Why?
  - Assume a hashtable has 7 buckets
    - `h(x) = x mod 7`
    - Will guarantee that every student ID maps to a single bucket
    - Why?

Chaining

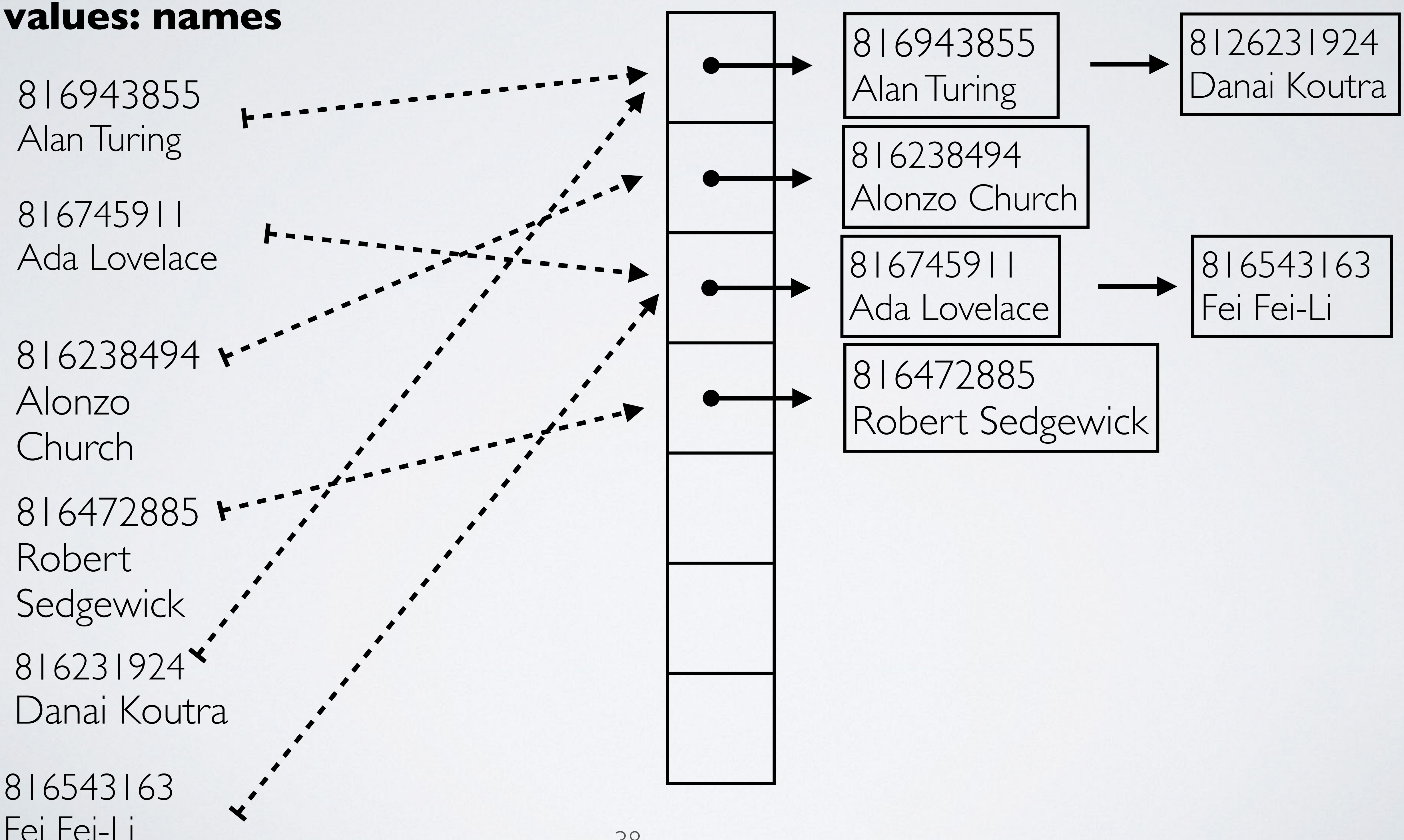
- Let's consider a practical example
  - Assume student IDs are integers (note that is not the best type for them!)
    - Student IDs can start with 0 (and that is semantically relevant)
    - We don't use arithmetic operations on student IDs!
    - We will consider how to handle strings later (focus on resolution for now)
  - Assume a hashtable has 7 buckets
    - `h(x) = x mod 7`
    - Will guarantee that every student ID maps to a single bucket
    - Taking modulo of an integer with 7 finds the remainder after dividing by 7.
    - This is bounded between 0 (inclusive) and 6 (inclusive)

Chaining - Insertion

Array of buckets w/ key/value pairs

keys: student IDs
values: names

h(key)=key%7



Chaining - Searching

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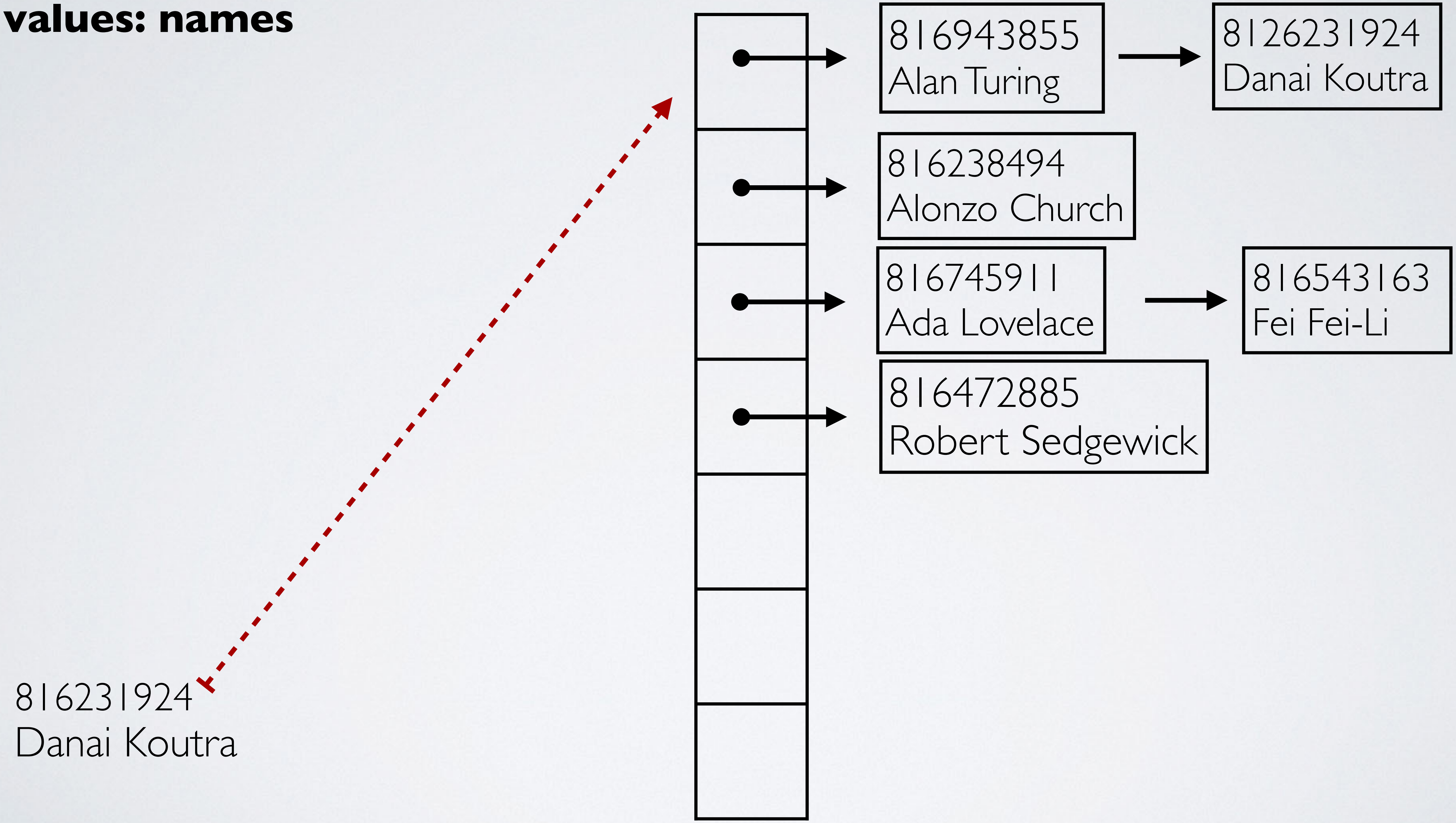

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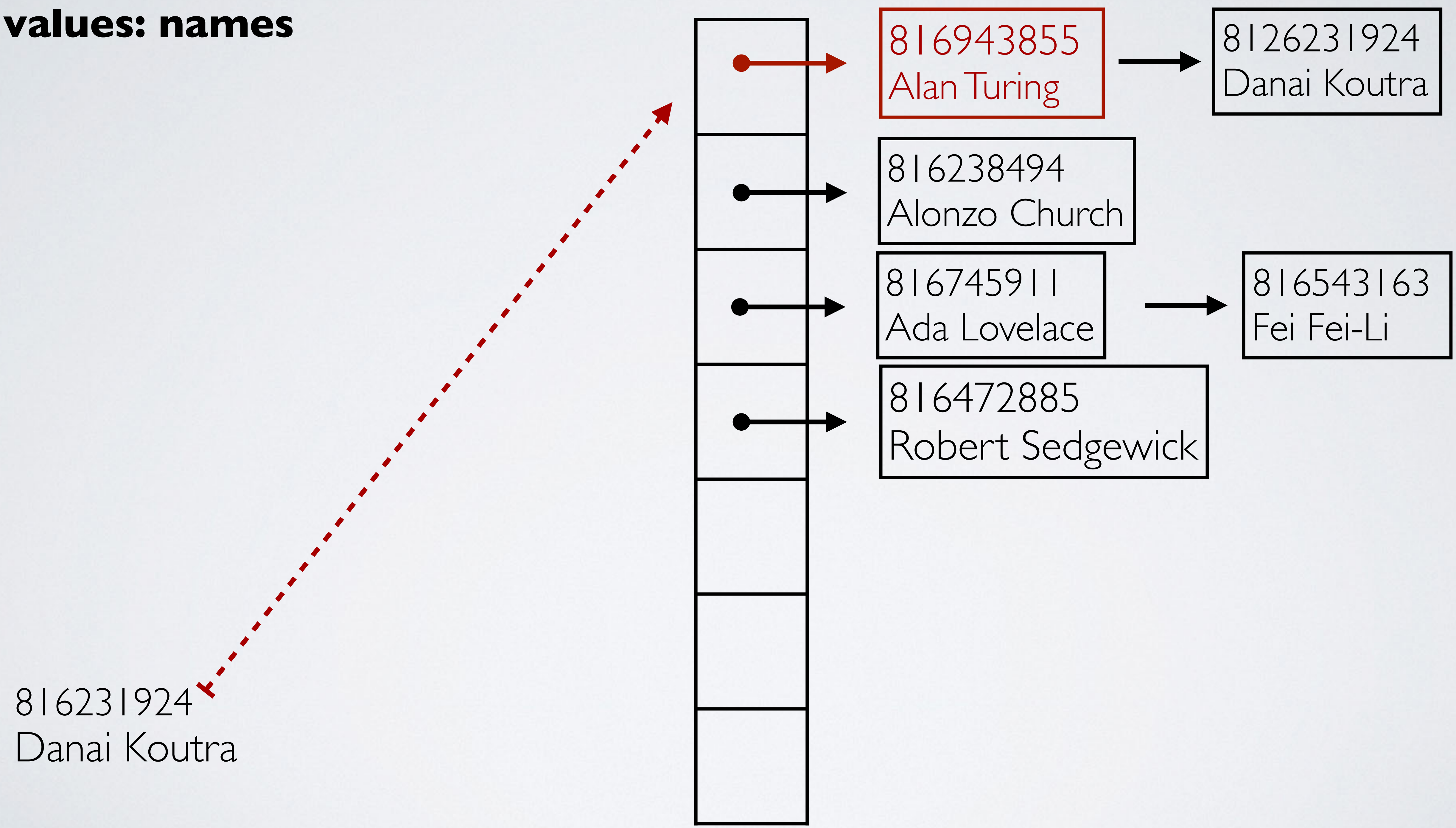

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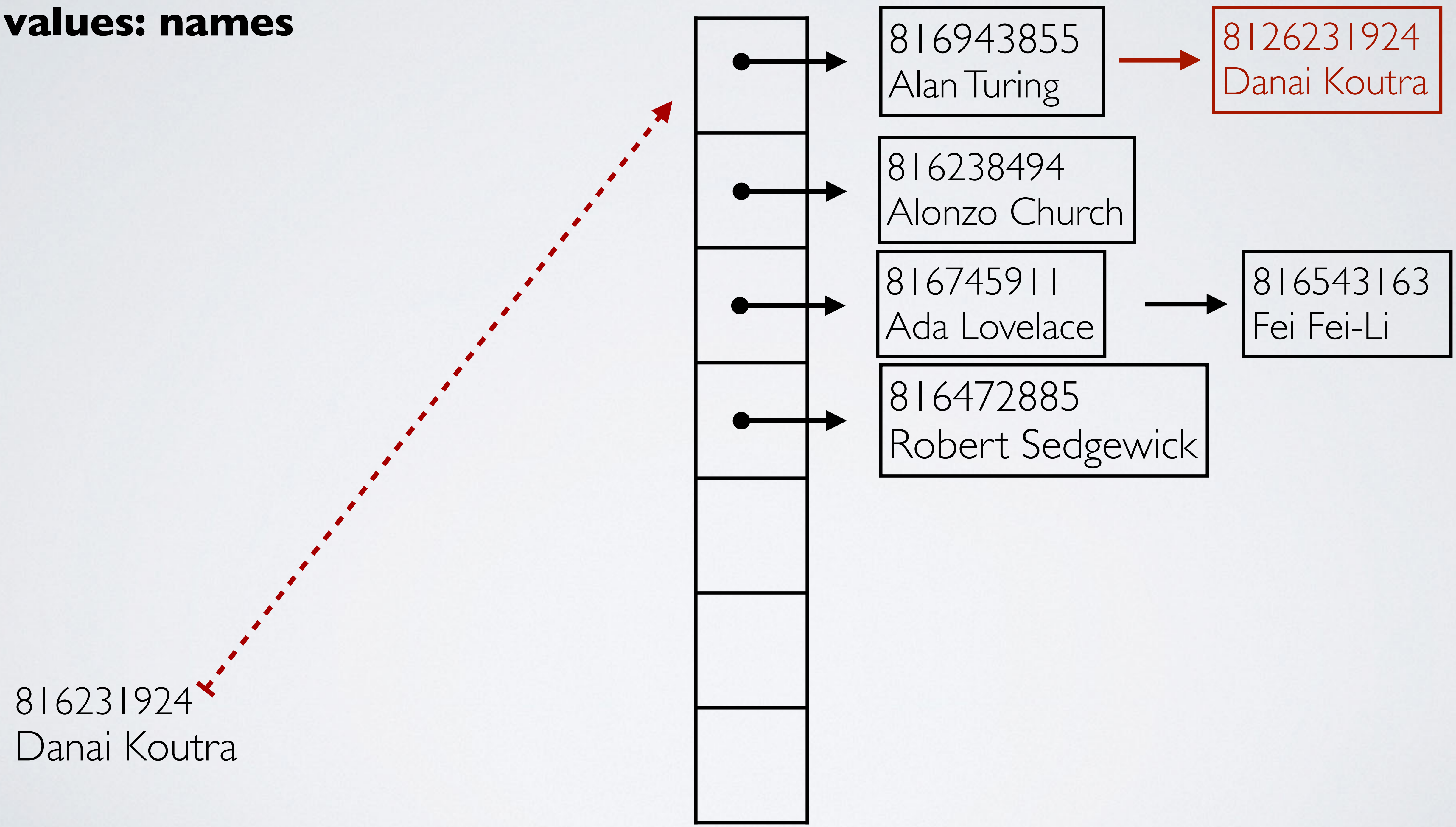


Chaining - Deletion

- Deletion just involves locating the key/value pair in the appropriate location and removing it

Chaining - Pseudocode

Pseudocode has redundancy (Try to eliminate it)

```
function insert(table, h, k, v)
   index = h(k, length(table))
   // if the bucket is empty
   if length(arr[index]) == 0:
     arr[index].append((k, v))
   else:
      inserted = False
     for j, (k_prime, v_prime) in arr[index]:
       // update value if key exists
       if k == k_prime:
          arr[index][j] = (k, v)
          inserted = True
     // insert if it doesn't
     if inserted == False:
        arr[index].append((k, v))
```

Chaining - Pseudocode

```
function search(table, h, k)
   index = h(k, length(table))
   // if the bucket is empty
   if arr[index] is NIL:
     return NIL
   else:
      // search bucket for (key, value) pair
      for (k_prime, v_prime) in arr[index]:
        if k_prime == k:
          return v_prime
      // if didn't return, key not found
      return NIL
```

Chaining - Pseudocode

```
function delete(table, h, k)
   index = h(k, length(table))
   if arr[index] is not NIL:
     for i, (kp, vp) in enumerate(arr[index]):
       if k == kp:
         delete(arr[index], i)
```

```python
class ChainingHashTable:
  def __init__(self, n, h):
    # create blank list with n elements
    self.table = [[]] * n
    self.n = n

  def insert(self, key, value):
    i = hash(key)
    i = i % self.n
    if self.table[i] is None:
      self.table[i] = [(key, value)]
    else:
      inserted = False
      for j, (k_prime, v_prime) in enumerate(self.table[i]):
        if key == k_prime:
          self.table[i][j] = value
          inserted = True
      if inserted == False:
        self.table[i].append((key, value))

    def search(self, key, value):
      i = hash(key)
      i = i % self.n
      if self.table[i] is None:
        return None
      for (k_prime, v_prime) in self.table[i]:
        if key == k_prime:
          return value
      return None

    def delete(self, key):
      i = hash(key)
      i = i % self.n
      if self.table[i] is not None:
        for j, (k_prime, v_prime) in enumerate(self.table[i]):
          if key == k_prime:
            # Python remove element from index j
            # moves all other elements up accordingly
            self.table[i].pop(j)
```

Chaining

- Advantages:
  - Very Easy to implement
  - Can have more entries than buckets without expanding the number of buckets
  - Deletion is trivial in Chaining. Not so much in open addressing
- Disadvantages
  - Large number of collisions lead to large buckets!
  - Has worst case `O(n)`
  - Amoritized and average case is better
  - But we want **stronger** guarantees

Open Addressing

- In Open Addressing collision resolution schemes, each bucket contains only a single element
- In Open Addressing, we access key/value pairs directly by an index
- Underlying generalisation: if a bucket is filled, we compute new bucket indices in a **deterministic** way until we find a free space

Linear Probing

- Uses number of misses in computing the new space.
  - misses = # of buckets that we've seen already filled during our search for a free space
- If a bucket is filled, we move in increments of `1` until we find a free bucket
  - Consider `(k, v)`. Let `i = h(k)`. If we find a place after `m` misses, we put the data into index `(i + m) mod n` where `n` is the number buckets in the hashtable
  - We use `mod n` to ensure that we stay in the bounds of the table

Double Hashing

- Double Hashing is a generalisation of linear probing where the fixed increment is a function of the input
  - Primary hash function computes initial index, $h_1(x)$
  - Use secondary hash function $h_2(x)$ to compute increment
  - Consider `(k, v)`. Let $i = h_1(x)$, $j = h_2(x)$. If we find a place after `m` misses, we put the data into index `(i + m * j) mod n` where `n` is the number buckets in the hashtable
- Think of linear probing as double hashing where $h_2(x)$ always returns a `1`

Quadratic Probing

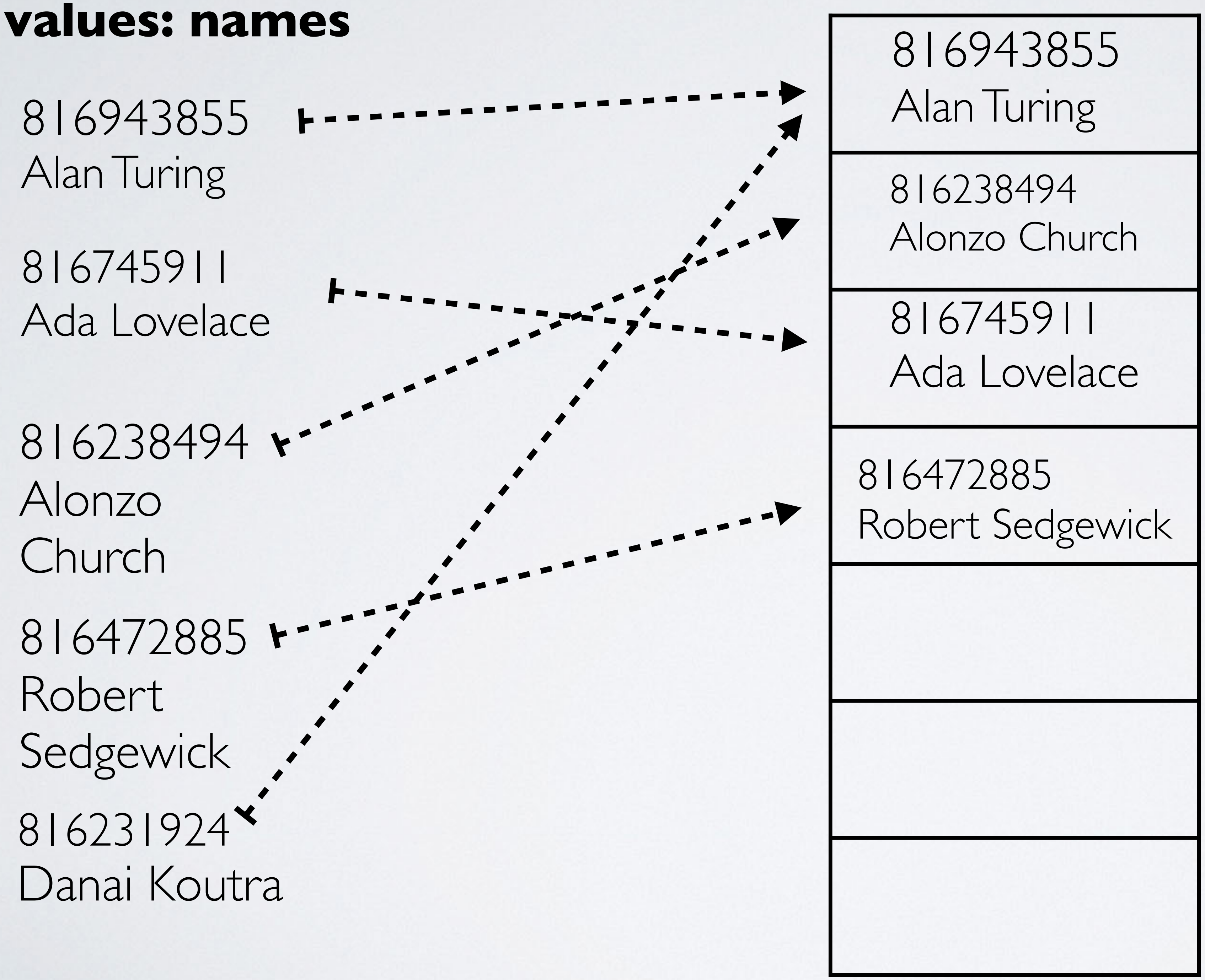
- Uses number of misses in computing the new space.
- increment is the square of the number of misses!
- Consider `(k, v)`. Let `i = h(k)`. If we find a place after `m` misses, we put the data into index `(i + m²) mod n` where `n` is the number buckets in the hashtable
- Will will cover Linear Probing pseudocode in lecture
- Quadratic Probing and Double Hashing in labs!
  - Make sure that you know the pseudocode for all, can implement all of them, and can trace through all collision resolution methods
- Unifying principle: move in increments

Linear Probing- Insertion

Array of buckets w/ key/value pairs

keys: student IDs $h(key)=key\%7$

values: names



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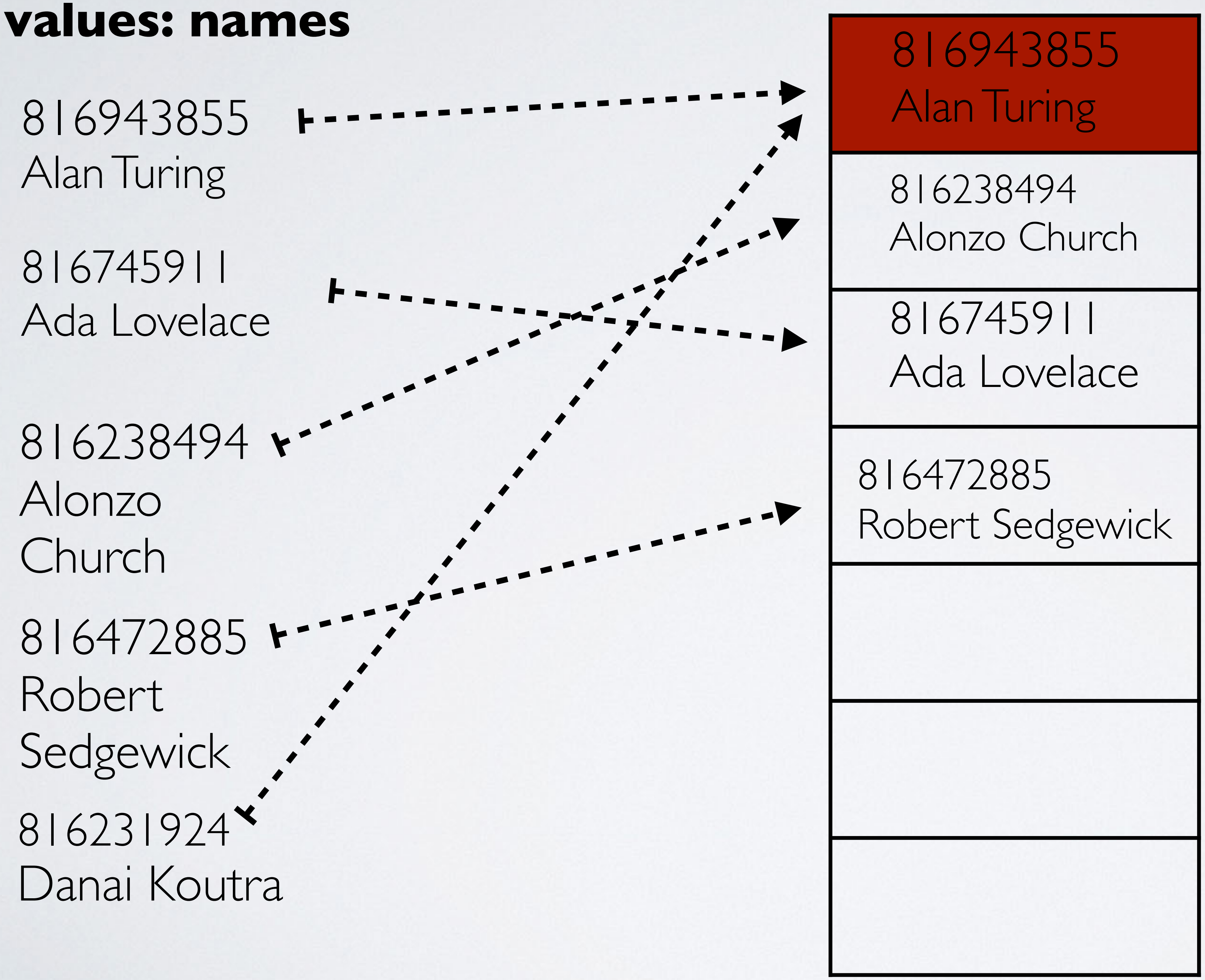

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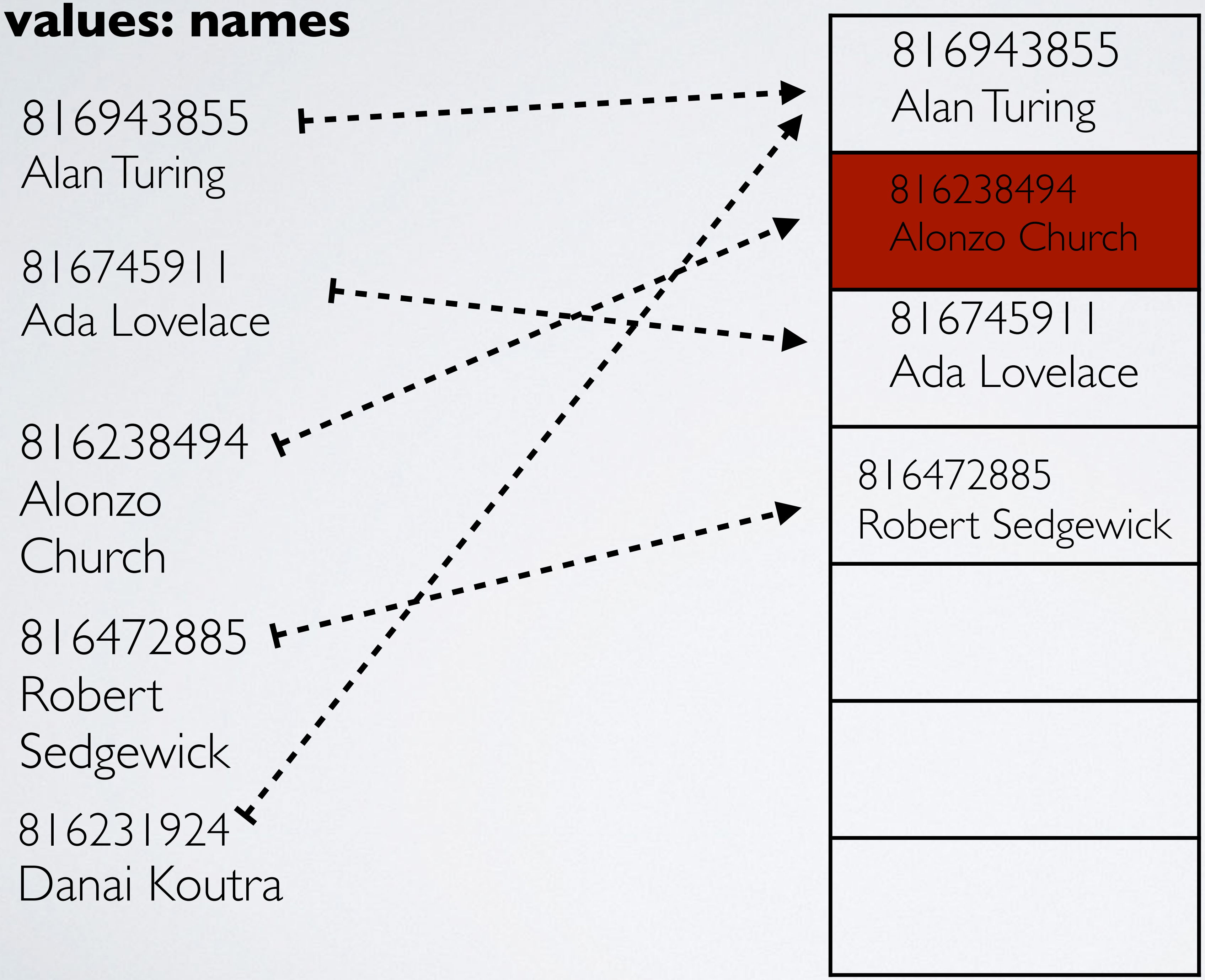

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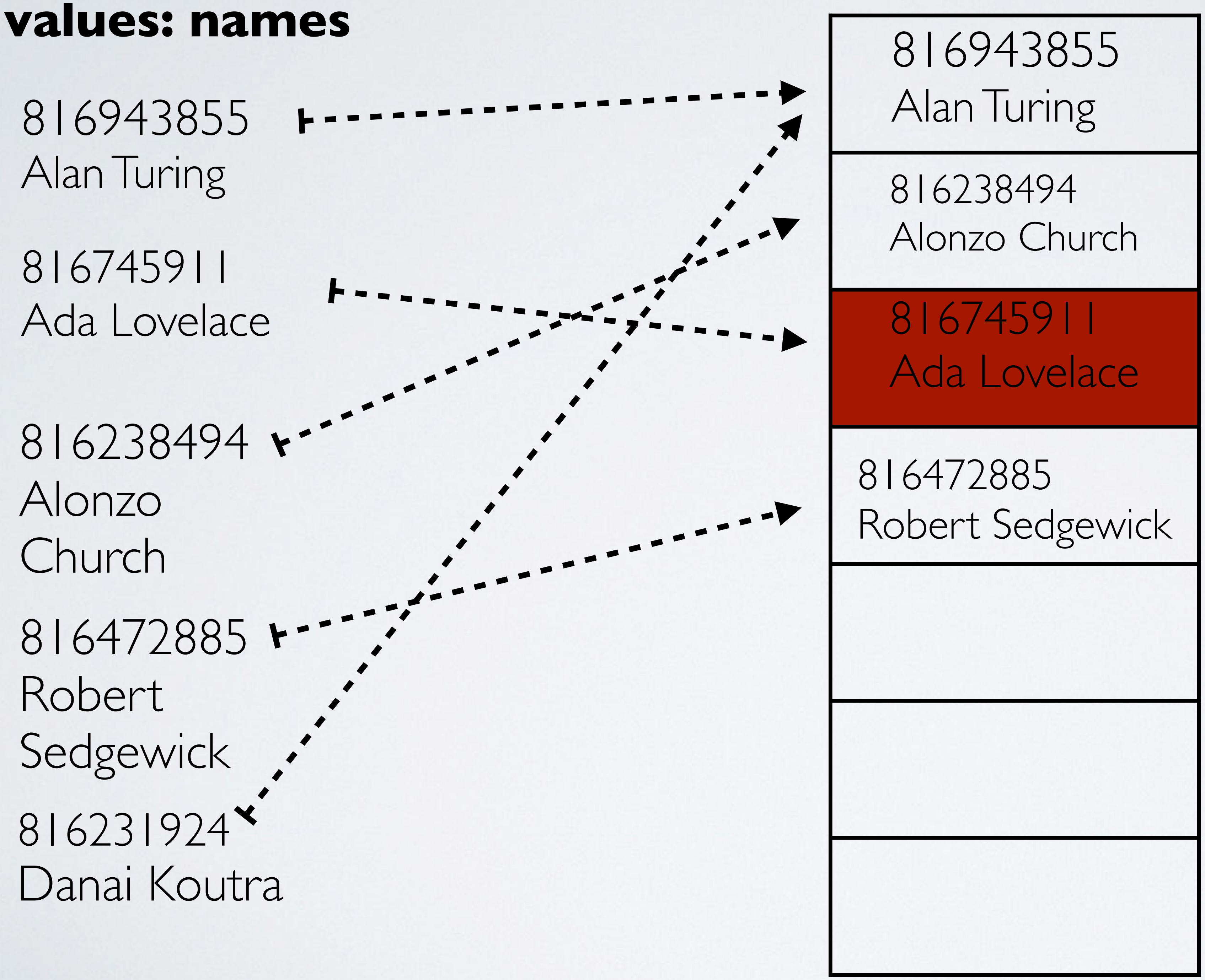

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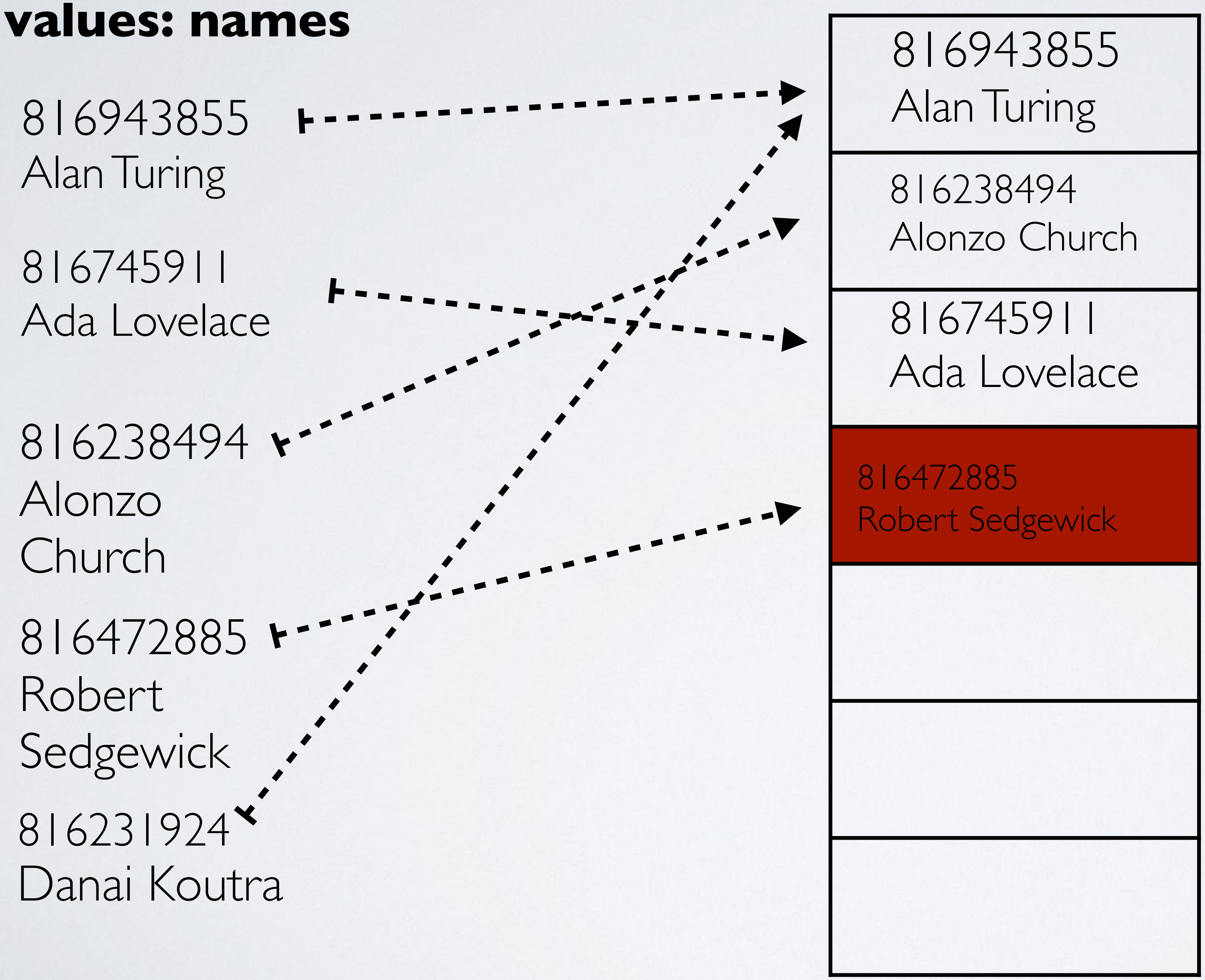

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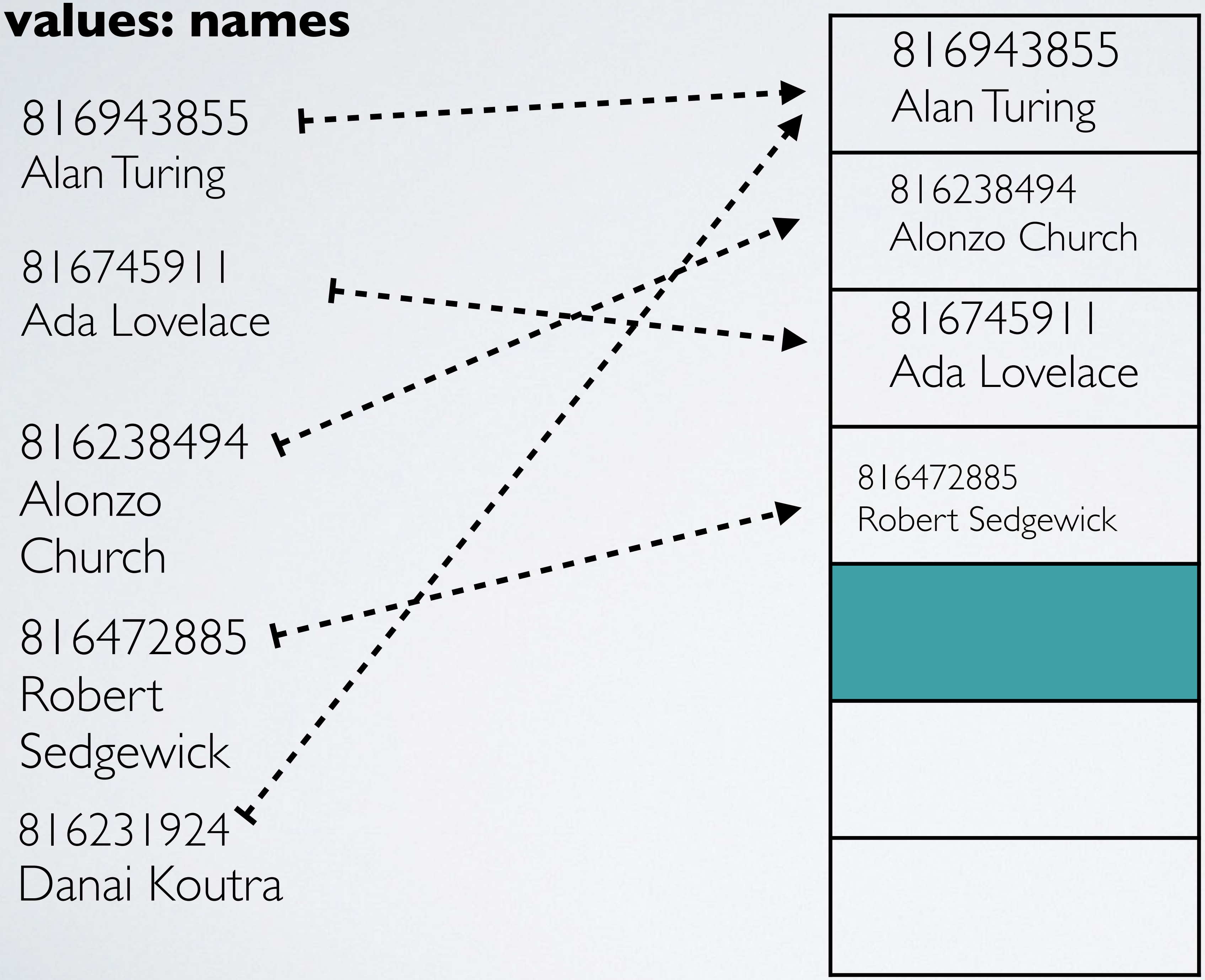

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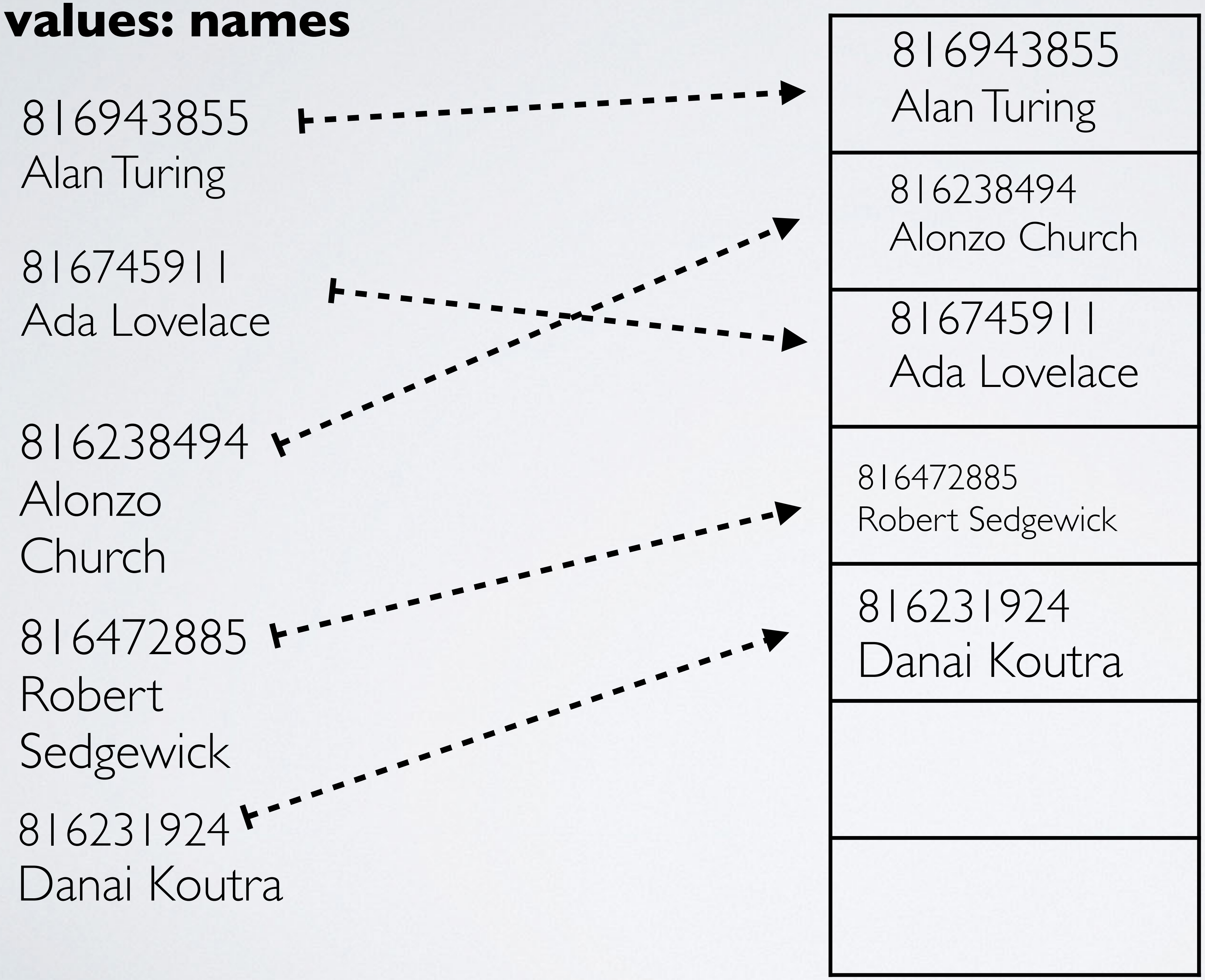

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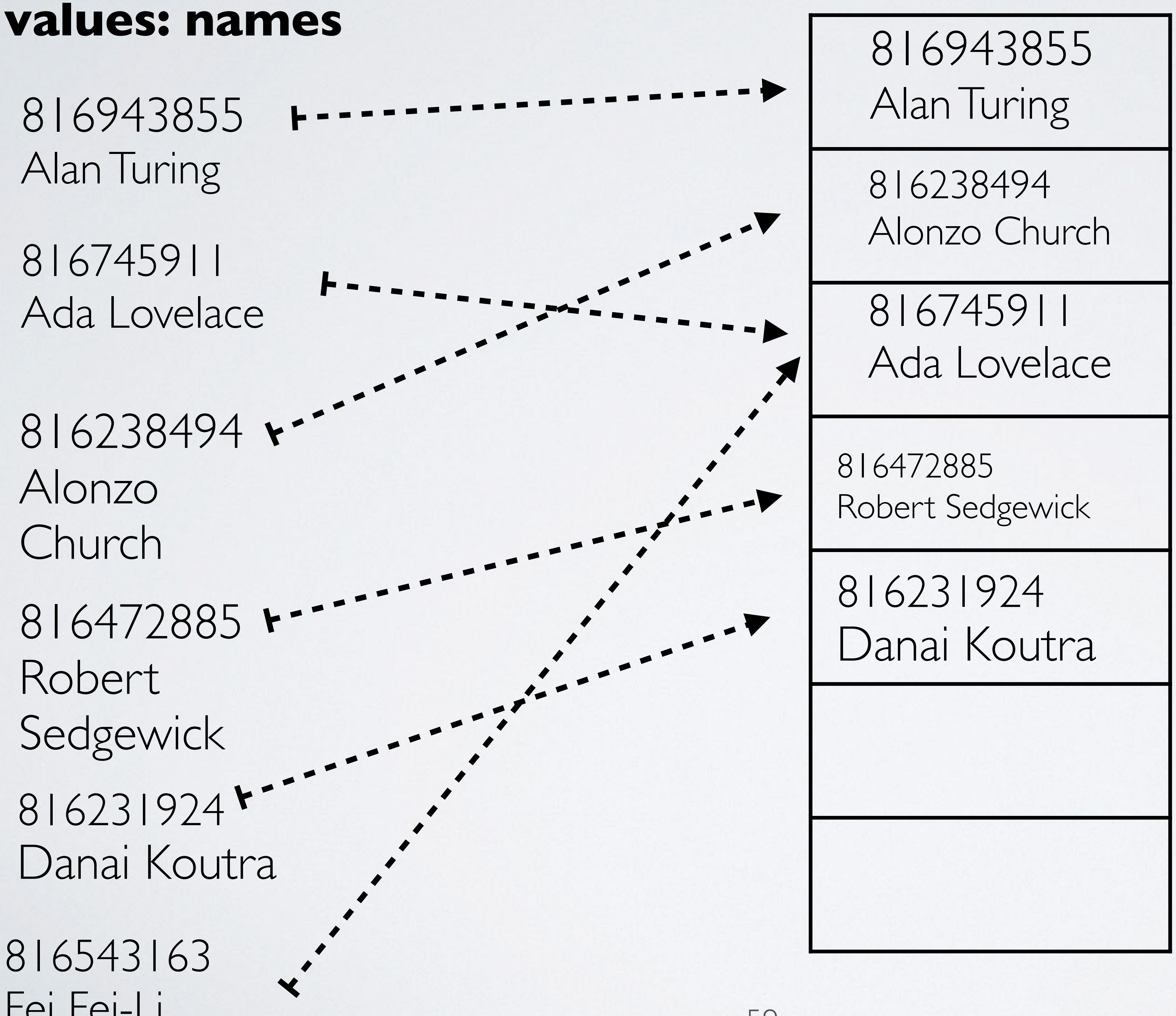

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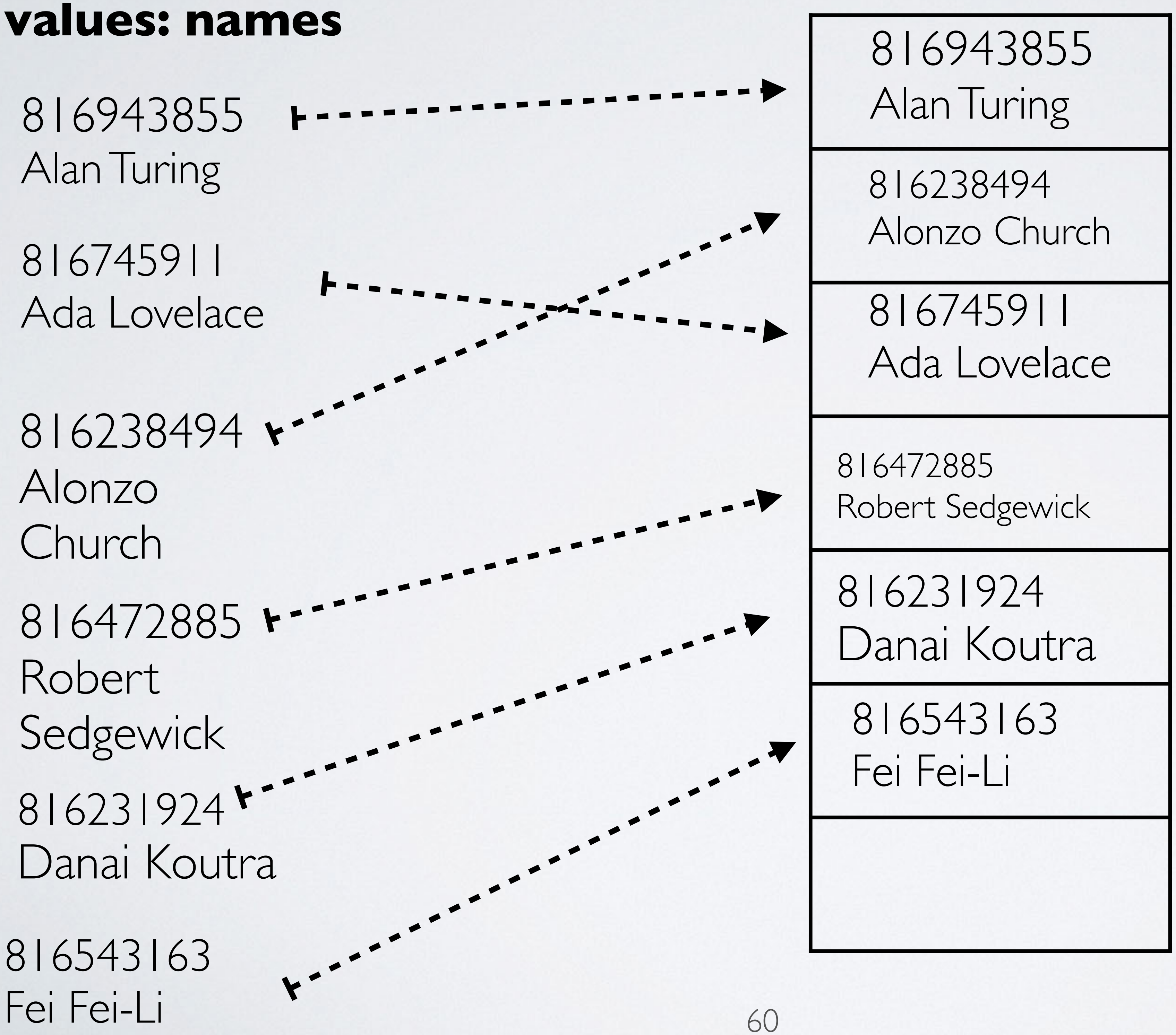


Linear Probing - Pseudocode

```
function insert(table, h, k, v)
   index = h(k, length(table))
   // if the bucket is empty
   if arr[index] is NIL:
     arr[index] = (k, v)
   else:
      n = length(table)
      misses = 1
      new_index = (index + misses) mod n
      while arr[new_index] is not NIL:
        misses += 1
        new_index = (index + misses) mod n
      arr[new_index] = (k, v)
```

Open Addressing Searching

- Recall that we insert key/value pair into the first free bucket
- Hence if are searching for a key….
- And come across an empty bucket….
- This means that ….

Open Addressing Searching

- Recall that we insert key/value pair into the first free bucket
- Hence if are searching for a key….
- And come across an empty bucket….
- This means that ….
- Key is not in the hashtable

Linear Probing - Pseudocode

```
function search(table, h, k)
    index = h(k, length(table))
    misses = 0
    n = length(table)
    new_index = (index + misses) mod n
    while (arr[index] is not NIL):
        key_prime, value_prime = arr[index]
        if key_prime == k:
            return value_prime
        misses += 1
       new_index = (index + misses) mod n
    return NIL
```

Open Addressing Deletion

- Deletion is more complicated when using open addressing
- If searching assumes that seeing an empty space means the key is not present
- Simply setting the bucket to NIL will not work!
- We need to move content of other buckets backwards

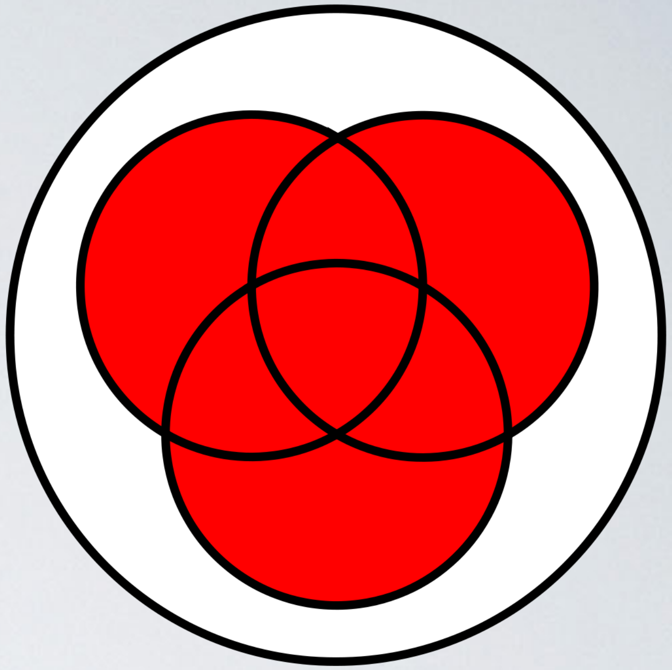
Linear Probing - Pseudocode

```
function delete(table, h, k)
    index = h(k, length(table))
    misses = 0
    n = length(table)
    new_index = (index + misses) mod n
    while (arr[index] is not NIL):
       key_prime, value_prime = arr[index]
       if key_prime == k:
          arr[new_index] = NIL
          prev_index = new_index
          curr = misses + 1
          new_index = (index + curr) mod n
          while (arr[new_index] is not NIL):
            arr[prev_index] = arr[new_index]
            prev_index = new_index
            curr += 1
            new_index = (index + curr) mod n
```

Liner Probing Problems

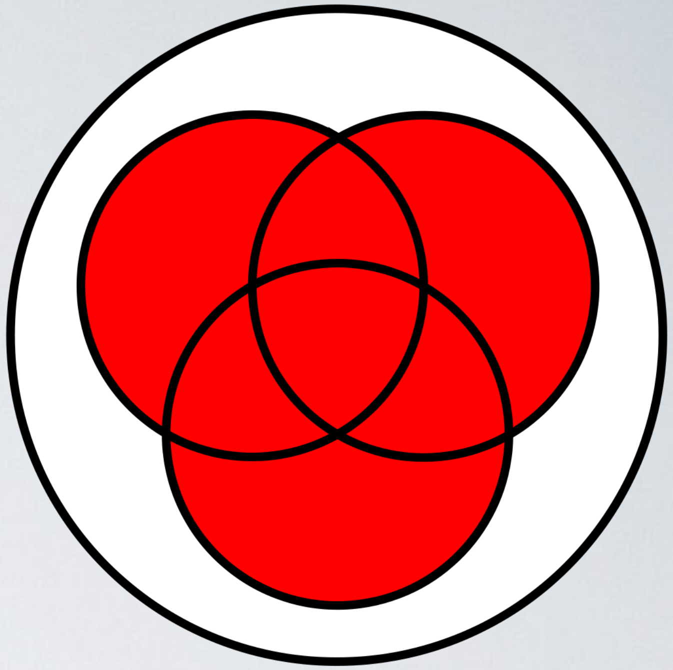
- Finding a free space by incrementing by `1` is not very efficient
- Using quadrating probing or double hashing are better as you hit fewer filled buckets on insertion
- Deletion is slightly more cumbersome though

Sets



- Collection of elements that are
  - distinct
  - unordered (unlike lists or arrays)

Set ADT



- **add**(object):
  - adds object to set if not there
- **remove**(object):
  - removes object from set if there
- boolean **contains**(object):
  - checks if object is in set

Set Use Cases

- Any application that needs to check if something is unique or is in a particular collection of items
  - Blacklists or whitelists on proxy servers
  - Privileged users in a system
  - Nodes already visited in a graph or network
  - Anti-viruses (Assignment #1)
  - etc ….

Sets as special cases of Dictionaries

- If you have a type implementing the Dictionary ADT, you can use that to implement Set ADT
- **add**(object) = **insert**(object, object)
- **remove**(object) = **delete**(object)
- **contains**(object) = **search**(object) `!= NIL`

References

- Some examples taken from Brown's CS61
- Template adapted from CS61
- Wikipedia
- Sedgewick
- Kalicharan