CORRESPONDENCE

Comments on "An Extremum Solution of the Monin–Obukhov Similarity Equations"

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1. Introduction

Recently, Wang and Bras (2010, hereafter WB10) postulated an extremum hypothesis (EH) of turbulent transport in the atmospheric surface layer (ASL). Based on this hypothesis, they derived a unique solution of the well-known Monin–Obukhov similarity theory (MOST) equations. Throughout the paper, WB10 highlighted the significance of EH and associated results. We summarize a few of their claims below (mostly in the authors' own words):

- EH implies that the atmosphere takes an optimal path toward a potential thermal equilibrium by maximizing the downward sensible heat flux in the stable surface layer (SSL).
- EH opens a possibility of simplifying the MOST formalism by replacing the two empirical stability functions by some empirical constants.
- EH has a solid foundation built on the modern nonequilibrium thermodynamics.
- The extremum solution is the only mathematically consistent and physically realistic solution of the MOST equations.
- The nonunique solutions of MOST equations signal potential loopholes in the classical treatment of the MOST, including violation of conservation laws and inconsistency in asymptotic properties.
- EH has overcome some technical difficulties (e.g., nonuniqueness and nonconvergence) in applying the MOST in modeling of turbulent transport in the ASL.
- EH retrieves a specific parameter (called the stationary Richardson number by WB10) that Monin and Obukhov did not predict.

After perusing WB10 meticulously, we felt obligated to write this comment, specifically for the following reasons:

- This paper sends out the wrong message that the SSL normally opts for a preferred state of thermal equilibrium (corresponding to maximum downward sensible heat flux). Over the past several decades, a wide range of observed atmospheric stabilities (from near-neutral

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to strongly stratified regimes) has been reported in the stable boundary layer literature. To the best of our knowledge, no observational evidence of a preferred stability state in SSL exists in the literature.

- Even though there is no consensus in the boundary layer meteorology field in terms of the exact form of the empirical stability correction functions (especially under the strongly stratified conditions), it is generally accepted that these functions are nondecreasing in nature. No observationalist has ever found these functions to be completely independent of stability. In other words, WB10's proposition to represent stability correction functions by two empirical constants cannot be substantiated from an observational standpoint.
- The claim that EH has a solid foundation built on the modern nonequilibrium thermodynamics is not supported by WB10. In the authors' own words:

> It remains uncertain at the moment whether the proposed extremum hypothesis can be ''proved'' using the MEP theory. Nonetheless, the extremum hypothesis appears to be consistent with the MEP theory, implying that the extremum hypothesis may result from some fundamental laws of nonequilibrium thermodynamics, an ongoing research subject.

- The dual nature (called nonuniqueness by WB10) of sensible heat flux under stably stratified condition can be explained in a physically consistent manner. In the very stable regime, due to the suppression of turbulence, the sensible heat flux should vanish. On the other hand, the sensible heat flux should also approach zero in the near-neutral limit, since temperature fluctuations become quite small. Between these two extreme regimes, the magnitude of downward sensible heat flux should maximize. Thus, the interpretation of WB10 regarding the nonuniqueness of sensible heat flux is fundamentally flawed.
- One does not need to invoke EH to apply MOST in ASL modeling. The nonconvergence issue of the MOST equations alluded by WB10 has quite different ramifications and will be discussed later in this comment.
- Finally, WB10 seem to imply that some of the major results of their paper (e.g., nonunique solutions of MOST equations, negative sensible heat flux maxima, stationary Richardson number) are their original contributions, but they missed the majority of relevant references published in the past 40 years. Taylor (1971) recognized that the MOST relationships lead to nonunique solutions. He used concepts from hydrodynamic instability to explain some of the results (e.g., multiple roots of the friction velocity equation). De Bruin (1994) confirmed the findings of Taylor (1971) and related them to the so-called temperature fluctuation method. He recognized that a physical explanation for the duality character of sensible heat flux under stably stratified condition exists. Malhi (1995) provided an analytical expression for surface sensible heat flux duality. He supplied evidence of sensible heat flux duality in nocturnal boundary layer data from Niger, West Africa. Derbyshire (1999) discussed various interrelated issues such as downward sensible heat flux maxima, decoupling of the boundary layer, positive feedback between temperature gradient and sensible heat flux in the very stable regime, and flow instabilities. Van de Wiel et al. (2007) proved that hydrodynamic instability sets in at the extremum point in the case of Couette flow. Basu et al. (2008) delineated the fundamental shortcomings of using sensible heat flux–based surface boundary conditions. Using an analytical approach, they were able to show that for reliable modeling accurate surface temperature prescription or prediction is needed.

The organization of this comment is as follows. In the following section, we analytically derive a sensible heat flux relationship (corresponding to stably stratified condition), which portrays a nontrivial dual character. We show that this duality is both mathematically and physically sound. In section 3, we explain why the extremum solution cannot be the ''only'' physically realistic case. We conclude this comment in section 4 by discussing the implications of sensible heat flux duality for ABL modeling.

2. Surface sensible heat flux duality

The MOST equations for mean velocity (U) and potential temperature (Θ) gradients are traditionally written as

$$\left(\frac{\kappa z}{u_*}\right)\left(\frac{\partial U}{\partial z}\right) = \phi_m\left(\frac{z}{L}\right), \tag{1a}$$

$$\left(\frac{\kappa z}{\theta_*}\right)\left(\frac{\partial \Theta}{\partial z}\right) = \phi_h\left(\frac{z}{L}\right), \tag{1b}$$

where u_*, θ_*, and L denote friction velocity, surface temperature scale, and Obukhov length, respectively; also, ϕ_m and ϕ_h are empirical gradient functions, κ is the von Kármán constant (equal to 0.4), and z is the height from the ground surface.

For stable surface layers, WB10 utilized the widely used gradient functions proposed by Businger et al. (1971):

$$\phi_m = \phi_h = 1 + \beta\frac{z}{L}, \quad \text{for} \quad 0 \le \frac{z}{L} < 1. \tag{2}$$

It is straightforward to rewrite these equations in terms of gradient Richardson number Ri as follows (Arya 1999):

$$\phi_m = \phi_h = 1 + \frac{\beta \mathrm{Ri}}{1 - \beta \mathrm{Ri}}, \quad \text{for} \quad 0 \le \mathrm{Ri} < \frac{1}{\beta}, \tag{3}$$

where $\mathrm{Ri} = \dfrac{\dfrac{g}{\Theta_o}\dfrac{\partial \Theta}{\partial z}}{\left(\dfrac{\partial U}{\partial z}\right)^2}$.

By definition, sensible heat flux is

$$\frac{H}{\rho C_p} = -u_* \theta_*. \tag{4}$$

By algebraic manipulations of Eqs. (1a), (1b), (3), and (4), one can arrive at

$$\frac{H}{\rho C_p} = -\left(\frac{\Theta_o}{g}\right)(\kappa z)^2 \mathrm{Ri}(1 - \beta \mathrm{Ri})^2 \left(\frac{\partial U}{\partial z}\right)^3. \tag{5}$$

This equation portrays the nonlinear (cubic) relationship of sensible heat flux with gradient Richardson number. Earlier, we mentioned that H should go to zero at the neutral and very stable regimes. Mathematically, these conditions can be written as: $H \rightarrow 0$ when $\mathrm{Ri} \rightarrow 0$ (neutral) as well as when $\mathrm{Ri} \rightarrow 1/\beta$ (very stable).

For fixed shear condition, the magnitude of H is maximized (i.e., $d|H|/d\mathrm{Ri} = 0$ and $d^2|H|/d\mathrm{Ri}^2 < 0$) when $\mathrm{Ri} = 1/3\beta$. In this case, the maximum downward sensible heat flux becomes

$$\frac{H_{\max}}{\rho C_p} = -\frac{4}{27}\left(\frac{\Theta_o}{g}\right)(\kappa z)^2 \left(\frac{\partial U}{\partial z}\right)^3. \tag{6}$$

Therefore, in nondimensional form, Eq. (5) reads (see also Fig. 1)

$$\frac{H}{H_{\max}} = \frac{27}{4}\beta \mathrm{Ri}(1 - \beta \mathrm{Ri})^2. \tag{7}$$

Almost identical equations have been reported by Malhi (1995), van de Wiel et al. (2007), and Basu et al. (2008). The analytical derivations of Malhi (1995) and Basu et al. (2008) were in terms of z/L.

WB10 called $\mathrm{Ri} = 1/3\beta$ the stationary Richardson number and claimed that they found this critical value that Monin and Obukhov were unable to find in 1954. Again, this result has been reported by others [e.g., Eq. (11b) of Basu et al. (2008)].

We would also like to emphasize that the dual nature of sensible heat flux is not a numerical artifact: it has been reported in several observational studies. We believe its existence was first documented by Malhi (1995).

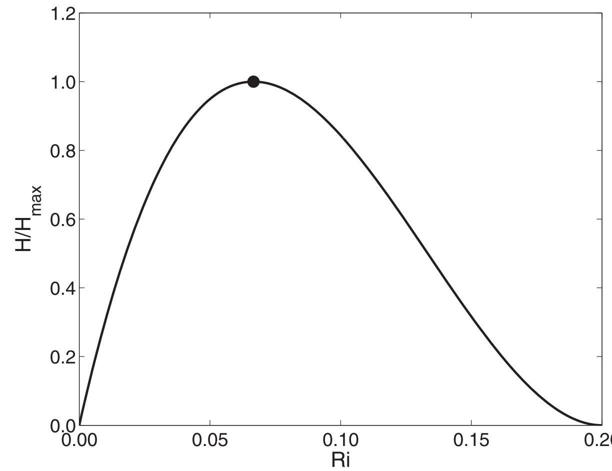

FIG. 1. The duality of surface sensible heat flux based on Eq. (7), illustrated with $\beta = 0.2$. The circle denotes the extremum point ($\mathrm{Ri} = 1/3\beta, H/H_{\max} = 1$).

Mahrt (1998) found its evidence in the Microfronts Experiment data. Basu et al. (2006) performed extensive analyses of turbulence data from several field campaigns and wind-tunnel experiments. They also provided convincing evidence of the duality of sensible heat flux. That is to say, WB10's assertion that "the nonunique solutions of MOST equations signal potential loopholes in the classical treatment of the MOST including violation of conservation laws and inconsistency in asymptotic properties" is deplorable.

3. Is the extremum solution the "only" physically realistic state?

The point ($\mathrm{Ri} = 1/3\beta, H/H_{\max} = 1$), denoted by a circle in Fig. 1, is the extremum solution.[1] WB10 hypothesized that the extremum solution is the "only mathematically consistent and physically realistic solution." Following this hypothesis, all the points corresponding to $\mathrm{Ri} \neq 1/3\beta$ in the Ri versus $H/H_{\max}$ graph (Fig. 1) should be physically unrealistic. Based on the past literature (e.g., Beljaars and Holtslag 1991; Högström 1996; Howell and Sun 1999; Cheng and Brutsaert 2005; Grachev et al. 2007), we know that it is definitely not the case. A large range of Ri (or z/L) values are observed in nature. Therefore, WB10's hypothesis that nature would tend to a state represented by the stationary Ri (equal to $1/3\beta$) cannot hold, not even in an approximate sense. It is clear that the extremum case can only be regarded as one out of many possible states, nothing more.

[1] In terms of the stability parameter z/L, the extremum point can be written as ($z/L = 1/2\beta, H/H_{\max} = 1$).

4. Implications of surface sensible heat flux duality for numerical modeling

WB10 wanted to find "a rationale leading to a unique solution to avoid the iterative procedure in the theoretical and modeling applications of the MOST." In this context, they stated that "in practice, we sometimes face the problem of a nonconverging iteration." This numerical problem was discussed at length in Basu et al. (2008). It arises from the fact that a fixed surface sensible heat flux as a boundary condition is not physically realistic for atmospheric boundary layer simulations. Surface sensible heat flux is an internal variable of the atmospheric system and is expected to respond to varying turbulence intensity near the surface. This problem can be circumvented in a physical manner by prediction of time-dependent surface temperature (Holtslag et al. 2007; Basu et al. 2008); there is no need to invoke the extremum hypothesis.

Surface sensible heat flux duality has further theoretical implications (e.g., instability and regime transitions) that are beyond the scope of this comment. Please refer to Derbyshire (1999) and van de Wiel et al. (2007) for details.

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