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On the bulk Richardson number and flux–profile relations in an atmospheric surface layer under weak wind stable conditions

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Abstract

For describing the surface fluxes in the atmospheric dispersion models, the commonly used linear universal similarity functions ϕ_m and ϕ_h for nondimensional wind and temperature profiles (J. Atmos. Sci. 28 (1971) 181; Boundary-Layer Meteorol. 7 (1974) 363) are analyzed under weak wind stable conditions. The stability parameter z/L (z is the height above the ground and L is the Obukhov length) is not known a priori. On the other hand, the bulk Richardson number (Ri_B) is readily available from profile measurements. Thus, a systematic mathematical analysis is carried out to analyze the applicability of the linear functions in terms of Ri_B. These linear functions are found to be valid as long as Ri_B is smaller than $Pr_t\,\gamma/\beta^2$ where Pr_t is the turbulent Prandtl number and β and γ are constants appearing in the linear functions of ϕ_m and ϕ_h. The linear functional forms for ϕ_m and ϕ_h underestimate the Obukhov length resulting in the large value of the stability parameter when $Ri_B \geqslant Pr_t\,\gamma/\beta^2$ occurring in weak wind stable conditions.

Using the data from plume validation field experiment conducted by Electric Power Research Institute at Kincaid, it is found that in 70% of the weak wind cases, the Ri_B is larger than $Pr_t\,\gamma/\beta^2$. The similarity functions proposed by Beljaars and Holtslag (J. Appl. Meteorol. 30 (1991) 327) are shown to perform well in weak wind stable conditions for all the values of Ri_B. The computed surface fluxes are also compared with those based on turbulence measurements taken in Cooperative Atmospheric Surface Exchange Study.

Keywords: Flux–profile relationships; Weak and strong winds; Bulk Richardson number; Stability functions; Atmospheric surface layer; Weak wind dispersion

1. Introduction

A large number of theories and formulations describing the surface fluxes have been in use in atmospheric dispersion modelling. In all these formulations for nondimensional wind and temperature profiles in a surface layer, the most commonly used universal similarity functions are those of Businger et al. (1971) and Dyer (1974). However, these profiles are not valid for large Richardson numbers (Lee, 1997), which are found in a number of observational studies (e.g., Aggarwal et al., 1995; Yadav et al., 1996), specially in weak wind stable conditions. New formulations for the nondimensional wind and temperature profiles applicable at large Richardson numbers in stable conditions have been proposed (Carson and Richards, 1978; Holtslag and De Bruin, 1988; Beljaars and Holtslag, 1991; Lee, 1997). Byun (1990) has given an analytical solution for the nondimensional stability parameter z/L (z is the vertical coordinate and L is the Obukhov length) as a function of the bulk Richardson number (Ri_B) under stable conditions which is restricted to a value of $Ri_B < 0.2$. However, no proper justification is given for choosing this value. The surface fluxes become

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insignificant or zero as turbulence becomes negligible and the flow becomes strongly stable beyond this limit. However, the limit 0.2 of Ri_{B} could be higher (Stull, 1988) under weak wind conditions. Carson and Richards (1978) have found that the surface fluxes are finite but not zero beyond this limit and have reported a critical value of Ri_{B} as 1.33 up to which the fluxes at the surface are nonzero. Beljaars and Holtslag (1991) have improved the formulation of Holtslag and De Bruin (1988) by using a slightly different form of the similarity function for heat (ψ_{h}) under stable conditions, which can be used for large Ri_{B} values. However, they did not look into the implications with respect to Ri_{B}.

In many of the low wind dispersion studies (Aggarwal et al., 1995; Arya, 1995; Sharan and Yadav, 1998; Sharan et al., 2002) and modelling studies on nocturnal boundary layer (Estournel and Guedalia, 1985, 1987; Sharan et al., 1995, 1996a; Sharan and Gopalakrishnan, 1997; Gopalakrishnan et al., 1998; Sharan et al., 1999) in weak wind conditions, the surface layer parameters such as L, surface friction velocity (u_*), etc., are based on the linear universal similarity functions of Businger et al. (1971) and Dyer (1974). Many of the well-known mesoscale models (e.g., Pielke's model) are based on these profiles. Louis (1979) noted that Dyer's profiles produced unrealistic results within a weather forecast model, because the surface tended to become thermally disconnected from the atmosphere in stable conditions. This resulted in far more surface cooling than actually observed. This unrealistic model behavior was attributed to the limited applicability (Hicks, 1976; Carson and Richards, 1978; Holtslag, 1984) of linear functions of Businger et al. (1971) and Dyer (1974). Delage (1997) modified the Louis scheme for stable conditions and found the results are similar to those obtained using the similarity functions proposed by Beljaars and Holtslag (1991). Launiainen (1995) has derived analytically a relationship between z/L and Ri_{B}. Hogstrom (1996) has discussed various functional forms for the universal similarity functions in unstable and stable conditions. Howell and Sun (1999) have calculated the surface fluxes in stable conditions using Microfronts experimental tower data. Derbyshire (1999) and Mahrt (1999) have described the various aspects in a very stable nocturnal boundary layer. Zilitinkevich and Calanca (2000) have proposed an extended similarity theory for the stably stratified atmospheric surface layer. Most of these studies are restricted to strong or moderate wind conditions. However, the linear profiles of Businger et al. (1971) and Dyer (1974) work fairly well under moderate to strong wind conditions and may result in underestimation of surface fluxes under weak wind stable conditions and in turn influence the dispersion of air pollutants.

The commonly used similarity functions ϕ_{m} and ϕ_{h} are linear in z/L (Webb, 1970; Businger et al., 1971; Dyer, 1974). Webb (1970) has analyzed these linear functions and concluded that these are applicable for $z/L \leqslant 1$. This conclusion was based on the criterion when transition takes place from the log-linear profile to log profile. However, this was an ad hoc assumption and no systematic analysis was carried out to support this conclusion. In the analysis, a majority of the data points used are with $Ri_{\mathrm{B}} < 0.2$. Further, z/L is not known a priori. Instead, the stability parameter Ri_{B} can be easily computed from the meteorological measurements.

The use of Ri_{B} for determining the gradient Richardson number (Ri) and z/L was advocated by Golder (1972). However, since Ri_{B} is only a crude approximation of Ri, it requires nomograms (Golder, 1972; Panofsky and Dutton, 1984) relating it to Ri and $\ln(z/z_0)$. A good estimate of z_0 is essential. The nomograms proposed by Golder (1972) are based on the linear functions (Webb, 1970) having the limitations in weak wind stable conditions.

The objective of the present study is to examine the extent of applicability of the linear universal similarity functions under weak wind stable conditions, for which an upper limit for Ri_{B} is derived. The mathematical analysis is given in the next section. Section 3 describes briefly the data used from the plume validation experiment conducted by the Electric Power Research Institute (EPRI) at Kincaid, USA. Section 4 presents the results. The implications of this analysis are discussed in Section 5.

2. Methodology

2.1. Surface layer

Based on Monin–Obukhov similarity theory for the horizontally homogenous and stationary surface layer, the profiles of wind (U) and potential temperature (θ) are given by

$$U(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_0}\right) - \psi_{\mathrm{m}}\left(\frac{z}{L}\right)\right], \tag{1}$$

$$\theta(z) - \theta_0 = \frac{\theta_*}{k}\left[\ln\left(\frac{z}{z_0}\right) - \psi_{\mathrm{h}}\left(\frac{z}{L}\right)\right], \tag{2}$$

where θ_0 is the potential temperature at z_0, k is the von Karman constant, and θ_* is the friction temperature scale expressed as the ratio of the surface heat flux $-(\overline{w'\theta'})|_{\mathrm{s}}$ to u_*.

The functions ψ_{m} and ψ_{h} depending on the atmospheric stability are defined by

$$\psi_{\mathrm{m}}\left(\frac{z}{L}, \frac{z_0}{L}\right) = \int_{z_0/L}^{z/L} \left[1 - \phi_{\mathrm{m}}(\zeta')\right] \frac{\mathrm{d}\zeta'}{\zeta'}, \tag{3}$$

$$\psi_{\rm h}\left(\frac{z}{L},\frac{z_0}{L}\right)=\int_{z_0/L}^{z/L}\left[1-\phi_{\rm h}(\zeta')\right]\frac{{\rm d}\zeta'}{\zeta'}, \tag{4}$$

where $\zeta' = z'/L$. The stability parameter z/L is defined as (Stull, 1988)

$$\frac{z}{L}=\frac{kzg\theta_*}{\bar{\theta}\ u_*^2}, \tag{5}$$

where g is the acceleration due to gravity and $\bar{\theta}$ is the mean potential temperature.

2.2. The bulk Richardson number

The stability parameter z/L is difficult to measure and it is required for computing u_* and θ_* from Eqs. (1) and (2). In a layer close to the ground, another stability parameter for the atmospheric surface layer is the bulk Richardson number ($Ri_{\rm B}$):

$$Ri_{\rm B}=\frac{g}{\bar{\theta}}\frac{(\theta-\theta_0)(z-z_0)}{U^2}, \tag{6}$$

where the wind speed at z_0 is taken to be zero. The potential temperature at z_0 may be obtained by extrapolating θ from tower observations. Eq. (6) expresses $Ri_{\rm B}$ in terms of known variables U, θ, θ_0 and z.

The meteorological data collected by the EPRI at Kincaid during the plume validation experiment in 1980–81 are used in the present study (see details in Section 3). The variation of $Ri_{\rm B}$ computed from the observations using Eq. (6) with the wind speed is shown in Fig. 1. In 70% of the cases under weak wind conditions, $Ri_{\rm B}$ is found to be larger than 0.2. In the earlier studies, the weak wind conditions are characterized based on: (i) geostrophic wind ($U_{\rm g}$) (Estournel and Guedalia, 1987; Tjemkes and Duynkerke, 1989; Sharan et al., 1995) and (ii) wind speed ($U_{\rm s}$) measured at 10 m height (Segendorf and Dickson, 1974; Arya, 1995; Sharan et al., 1996b). In the present study, both the approaches were considered for selecting the weak wind conditions. The detailed classification of the weak wind conditions is given in Section 3.

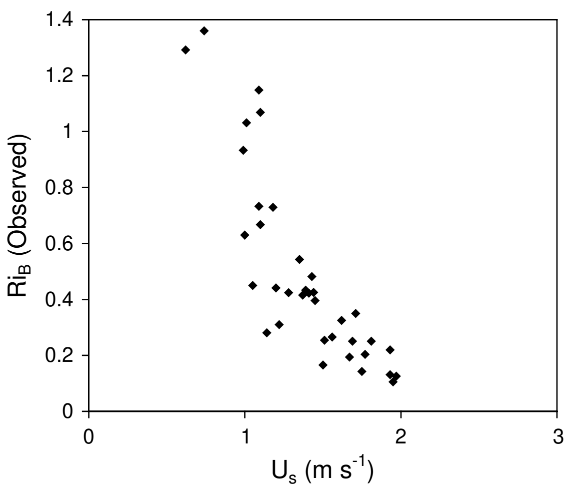

Fig. 1. Scattered diagram of bulk Richardson number ($Ri_{\rm B}$) obtained using the observed temperature and wind speed (which is designated as observed $Ri_{\rm B}$) as a function of wind speed ($U_{\rm s}$).

$Ri_{\rm B}$ in terms of stability parameter z/L is obtained from the Eqs. (1), (2) and (5) as

$$Ri_{\rm B}=\frac{z-z_0}{L}\left[\ln\frac{z}{z_0}-\psi_{\rm h}\left(\frac{z}{L}\right)\right]\left[\ln\frac{z}{z_0}-\psi_{\rm m}\left(\frac{z}{L}\right)\right]^{-2}. \tag{7}$$

This can be put in the functional form F as

$$Ri_{\rm B}=F(z/L,z_0/L). \tag{8}$$

The inverse of F in Eq. (8) can be used to determine z/L for given values of z/z_0 and $Ri_{\rm B}$.

2.3. Similarity functions ψ_m and ψ_h

For computing z/L from Eq. (7), one requires the expressions for $\psi_{\rm m}$ and $\psi_{\rm h}$. In principle, these can be calculated from Eqs. (3) and (4) for any functional form of $\phi_{\rm m}$ and $\phi_{\rm h}$. For stable conditions ($z/L>0$), Businger et al. (1971) have suggested the following forms for $\phi_{\rm m}$ and $\phi_{\rm h}$:

$$\phi_{\rm m}=1+\beta z/L, \tag{9}$$

$$\phi_{\rm h}=Pr_{\rm t}\,(1+\gamma z/L), \tag{10}$$

where β and γ are constants and $Pr_{\rm t}$ is the turbulent Prandtl number, which may be defined as the ratio $\phi_{\rm h}/\phi_{\rm m}$. The turbulent Prandtl number characterizes the difference in turbulent mixing of momentum and heat. The momentum transfer is more efficient for $Pr_{\rm t}>1$ (Howell and Sun, 1999). Webb (1970) has analyzed the linear functions using the observations from John Hopkins and Australian experiments and concluded that these are applicable for $z/L\leqslant 1$. Hicks (1976) has found that their applicability is restricted for $z/L\leqslant 0.5$. Hogstrom (1996) has reviewed several linear functional forms and found their applicability is limited to $z/L\leqslant 0.5$.

Using Eqs. (9) and (10) in Eqs. (3) and (4), we find

$$\psi_{\rm m}=\left(\frac{z}{L},\frac{z_0}{L}\right)=-\beta\frac{(z-z_0)}{L} \tag{11}$$

and

$$\psi_{\rm h}=\left(\frac{z}{L},\frac{z_0}{L}\right)=(1-Pr_t)\ln\left(\frac{z}{z_0}\right)-Pr_t\,\gamma\frac{(z-z_0)}{L}. \tag{12}$$

2.4. Analytical expression for stability parameter z/L

Substituting $\psi_{\rm m}$ and $\psi_{\rm h}$ from Eqs. (11) and (12), in Eq. (7), we get

$$Ri_B = Pr_t\,\zeta\left[\ln\frac{z}{z_0}+\gamma\zeta\right]\left[\ln\frac{z}{z_0}+\beta\zeta\right]^{-2} \tag{13}$$

in which $\zeta = (z - z_0)/L$. Eq. (13) becomes a quadratic in $x = \zeta/\ln(z/z_0)$:

$$x^2(Pr_t\,\gamma - \beta^2\,Ri_B) + x(Pr_t - 2\beta\,Ri_B) - Ri_B = 0. \tag{14}$$

In Eq. (14), when x is zero in neutral conditions, Ri_B is also zero. Further, as x becomes infinity, Ri_B approaches a limiting value, denoted as the upper value of Ri_B (Ri_{Bu}) given by

$$Ri_{Bu} = \frac{Pr_t\gamma}{\beta^2}. \tag{15}$$

Note that the product of the roots of the quadratic Eq. (14) is $Ri_B/(\beta^2\,Ri_B - Pr_t\,\gamma)$, which has the negative sign as $(\beta^2\,Ri_B - Pr_t\,\gamma)<0$, for $Ri_B<Ri_{Bu}$ in stable conditions. This implies that the roots of Eq. (14) have the opposite signs. Here, we are primarily interested in a positive root that is given by

$$x = \frac{-(Pr_t - 2\beta\,Ri_B) + \sqrt{(Pr_t - 2\beta\,Ri_B)^2 + 4Ri_B(Pr_t\gamma - \beta^2 Ri_B)}}{2(Pr_t\,\gamma - \beta^2\,Ri_B)} \tag{16}$$

Expressing Eq. (16) in terms of Ri_{Bu}:

$$x = \frac{-\left(1 - \frac{2\gamma}{\beta}\frac{Ri_B}{Ri_{Bu}}\right) + \sqrt{\left(1 - \frac{2\gamma}{\beta}\frac{Ri_B}{Ri_{Bu}}\right)^2 + 4\frac{Ri_B}{Ri_{Bu}}\frac{\gamma^2}{\beta^2}\left(1 - \frac{Ri_B}{Ri_{Bu}}\right)}}{2\gamma\left(1 - Ri_B/Ri_{Bu}\right)},$$
$$0 \leqslant Ri_B < Ri_{Bu}. \tag{17}$$

The values of β and γ are related as $\beta = \gamma Pr_t$ in Eq. (17). The stability parameter is given by

$$\frac{z}{L} = \left[\frac{z/(z - z_0)\ln(z/z_0)}{2\gamma\left(1 - Ri_B/Ri_{Bu}\right)}\right]\left[-\left(1 - \frac{2}{Pr_t}\frac{Ri_B}{Ri_{Bu}}\right) + \sqrt{\left(1 - \frac{2}{Pr_t}\frac{Ri_B}{Ri_{Bu}}\right)^2 + \frac{4}{Pr_t^2}\frac{Ri_B}{Ri_{Bu}}\left(1 - \frac{Ri_B}{Ri_{Bu}}\right)}\right],$$
$$0 \leqslant Ri_B < Ri_{Bu}. \tag{18}$$

In the experiments, the values of Ri_B are observed to be larger than Ri_{Bu} especially in weak wind conditions (Fig. 1) and the corresponding critical value of the Ri_B could be higher than that of Ri_{Bu} (Carson and Richards, 1978; Stull, 1988). Thus, we examine the case when $Ri_B \geqslant Ri_{Bu}$.

The condition $Ri_B > Ri_{Bu}$ implies that the term $(Pr_t\,\gamma - \beta^2\,Ri_B)<0$ and the product of the roots of Eq. (14) become positive. Thus, both the roots of Eq. (14) will have the same sign. Both are either positive or negative. The sum of the roots of Eq. (14) is $(Pr_t - 2\beta\,Ri_B)/(\beta^2\,Ri_B - Pr_t\,\gamma)$. Now $(\beta^2\,Ri_B - Pr_t\,\gamma) > 0$ and $(Pr_t - 2\beta\,Ri_B)<0$ for $Ri_B > Ri_{Bu}$ implying that both the roots of Eq. (14) are negative. Hence, there is no positive root of Eq. (14) for the values of $Ri_B > Ri_{Bu}$. This implies that a positive value of z/L corresponding to stable conditions does not exist for the Businger et al. (1971) and Dyer (1974) similarity functions for $Ri_B > Ri_{Bu}$. In other words, these linear functions cannot be used in the surface layer Monin–Obukhov similarity theory for $Ri_B \geqslant Ri_{Bu}$, normally found in weak wind stable conditions.

Eq. (15) provides the upper limit of Ri_B for which the linear similarity functions ϕ_m and ϕ_h (Businger et al., 1971; Dyer, 1974) can be used for describing the fluxes in the surface layer in weak wind stable conditions. However, this upper limit need not necessarily be the critical value of the bulk Ri_B (Stull, 1988).

We conclude that the applicability of linear forms of ϕ_m and ϕ_h in terms of z/L (Eqs. (9) and (10)) in the surface layer under weak wind stable conditions is limited to $Ri_B < Pr_t\,\gamma/\beta^2$. The range of applicability increases with increase in the value of $Pr_t\,\gamma/\beta^2$. For Dyer (1974) relations $\beta = \gamma$ and $Pr_t = 1$, Eq. (17) reduces to

$$x = \frac{Ri_B}{(1 - Ri_B/Ri_{Bu})}, \quad 0 < Ri_{Bu} < \frac{1}{\beta}. \tag{19}$$

This expression is the same as that given by Arya (1988). For the Businger's relationships

$$\beta = 4.7, \quad \gamma = 6.35, \quad Pr_t \approx 0.74 \tag{20}$$

are taken. Ri_{Bu} becomes $1/\beta$.

The values of Ri_{Bu} in Table 1 are based on the values of the constants used by various investigators in the linear functions for ϕ_m and ϕ_h. The value of Ri_{Bu} in each case indicates the applicability of the corresponding linear functions ϕ_m and ϕ_h for calculating the surface fluxes in weak wind stable conditions. The values of the constants β and γ are based on different k values with Pr_t equals to 1 in the original relations except those for Businger et al. (1971) profiles. Yaglom (1977) recalculated all these formulae for ϕ_m and ϕ_h using a common value of k as 0.4 and found large differences between these formulations. Hogstrom (1988, 1996) modified the values of the coefficients corresponding to $k = 0.4$ and showed that $Pr_t = 0.95$ is ideal in stable conditions. It may be seen that the Ri_{Bu} obtained from the modified functions for ϕ_m and ϕ_h (Hogstrom, 1988, 1996) are not significantly different from the Ri_{Bu} obtained from the corresponding original relations.

Table 1
Values of upper limit of bulk Richardson number (Ri_{Bu}) for various linear functions $\phi_{\mathrm{m}} = 1 + \beta z/L$ and $\phi_{\mathrm{h}} = Pr_{\mathrm{t}} + (1 + \gamma z/L)$

Reference	k	β	γ	Pr_{t}	Ri_{Bu}
Businger et al. (1971)					
Original	0.35	4.7	6.35	0.74	0.21
Modified (Hogstrom, 1996)	0.40	6.0	8.42	0.95	0.22
Dyer (1974)					
Original	0.41	5.0	5.0	1.00	0.20
Modified (Hogstrom, 1996)	0.40	4.8	4.74	0.95	0.20
Zilitinkevich and Chailikov (1968)					
Original	0.43	9.9	9.9	1.00	0.10
Modified (Hogstrom, 1996)	0.40	9.4	9.4	0.95	0.10
Webb (1970)					
Original	0.41	5.2	5.2	1.00	0.19
Modified (Hogstrom, 1996)	0.40	4.2	7.4	0.95	0.20
Hicks (1976)	0.41	5.0	5.0	1.00	0.20

2.5. *Estimation of z/L for all Ri_B*

The validity of the formulation of z/L (Eq. 18) based on similarity functions proposed by Businger et al. (1971) and Dyer (1974) in stable conditions is limited to $Ri_{\mathrm{B}} < Ri_{\mathrm{Bu}} = 1/\beta$. As the large values of Ri_{B} are observed in weak wind stable conditions, we require a procedure for computing z/L when $Ri_{\mathrm{B}} \geqslant Ri_{\mathrm{Bu}}$. Various functional forms available in the literature for ϕ_{m} and ϕ_{h} are examined here for this purpose. Carson and Richards (1978) have summarized the limits of applicability of linear and nonlinear functions proposed by Businger et al. (1971), Webb (1970), Clarke (1970) and Hicks (1976) in the range of z/L from 0 to 10. Some of these are the combination of functions valid in different ranges of z/L. However, due to nonavailability of z/L a priori, it is difficult to decide about the range of z/L and an appropriate choice of functional forms for ϕ_{m} and ϕ_{h}. On the other hand, Beljaars and Holtslag (1991) have proposed a single expression for each ϕ_{m} and ϕ_{h} applicable in the range $0 < z/L < 10$. We adopt these here and accordingly, the stability functions ψ_{m} and ψ_{h} are given by

$$\psi_{\mathrm{m}}\left(\frac{z}{L}, \frac{z_0}{L}\right) = -a\left(\frac{z}{L} - \frac{z_0}{L}\right) - y + y_0, \tag{21}$$

$$\psi_{\mathrm{h}}\left(\frac{z}{L}, \frac{z_0}{L}\right) = -\left(1 + \frac{2}{3}\frac{az}{L}\right)^{3/2} + \left(1 + \frac{2}{3}\frac{az_0}{L}\right)^{3/2} - y + y_0, \tag{22}$$

where

$$y = b\left(\frac{z}{L} - \frac{c}{d}\right)\exp\left(-d\frac{z}{L}\right), \tag{23}$$

$$y_0 = b\left(\frac{z_0}{L} - \frac{c}{d}\right)\exp\left(-d\frac{z_0}{L}\right) \tag{24}$$

in which $a = 1$, $b = 0.667$, $c = 5$ and $d = 0.35$.

Substitution of ψ_{m} and ψ_{h} from Eqs. (21) and (22) in Eq. (7) leads to an equation involving the exponential function of z/L. From the resulting equation, z/L cannot be expressed explicitly in terms of Ri_{B}. In fact, this can be solved numerically (Press et al., 1986) to obtain z/L for a given Ri_{B}. Note that z/L is, also, obtained by solving Eqs. (1) and (2) using ψ_{m} and ψ_{h} from Eqs. (21) and (22) after substituting for u_* and θ_* in Eq. (5). The values of z/L obtained from these alternative ways are found to be the same. However, the computation of z/L from observed Ri_{B} using the former approach has numerical advantages for calculating the surface fluxes in atmospheric models.

3. Data

The EPRI conducted a plume validation field experiment in 1980 and 1981 at Kincaid (39°35′N; 89°25′W), Illinois. During the experiment, extensive meteorological observations were taken along with the stack characteristics and tracer data (Bowne et al., 1983). A total of 104 h during 20 nights are taken for the analysis out of the total data of 250 days based on the selection criteria described in Sharan et al. (2003). The screened data are further classified into weak and strong winds based on the surface wind measured at 10 m level (U_{s}) from the tower and the geostrophic wind (U_{g}) observed from tethered balloon (TSONDE). The wind at 850 hPa level from TSONDE is considered as the geostrophic wind. The wind is considered as weak wind when (i) the wind speed is $\leqslant 4\,\mathrm{m\,s^{-1}}$ within 100 m from the tower, (ii) $U_{\mathrm{s}} \leqslant 2\,\mathrm{m\,s^{-1}}$ and (iii) $U_{\mathrm{g}} \leqslant 4\,\mathrm{m\,s^{-1}}$, otherwise it is considered as strong wind. The weak winds at the surface are strongly correlated with winds aloft (Sharan et al., 2003).

The nondimensional stability parameters Ri_{B} and z/L are calculated using the tower observations. The stability parameter z/L is computed in two alternative ways: (i) from Eqs. (1)–(5) based on similarity theory for given functional forms of ϕ_{m} and ϕ_{h} using an iterative procedure and (ii) from Eq. (7) for an observed Ri_{B}. The hourly surface temperature was obtained by extrapolating the temperature profile from the tower. The hourly temperatures at the surface and at 10 m level and the hourly wind speed measured at 10 m from the tower are used in the calculations of Ri_{B} and z/L with a roughness length of $z_0 = 0.1$ m (Sharan and Gopalakrishnan, 1997).

4. Results and discussion

Ri_B is obtained from Eq. (6) using the observed temperature and wind speed and it is denoted as the observed Ri_B. z/L is computed from Eqs. (1), (2) and (5) with ϕ_m and ϕ_h from Eqs. (9) and (10) of Businger et al. (1971) and Dyer (1974) using an iterative procedure. The computed value of z/L is used in Eq. (13) to calculate Ri_B. This is designated as the computed Ri_B. Further, z/L is also computed from Eq. (18) using the observed values of Ri_B.

The value of Ri_{Bu} obtained from Eq. (15) is 0.21 for Businger et al. (1971) and 0.2 for Dyer (1974) profiles. In 70% of the selected cases, Ri_B is found to be larger than its upper limit (Fig. 1). In all these cases, the value of z/L obtained from Eqs. (1)–(5) is very large (Fig. 2a). Thus, the surface layer theory with Businger's profiles underestimates the value of L resulting in a large value of z/L implying negligible magnitude of surface fluxes for $Ri_B \geqslant Ri_{Bu}$. Fig. 2b reveals that the calculated value of Ri_B is equal to Ri_{Bu} for all observed values of $Ri_B \geqslant Ri_{Bu}$. This confirms the result obtained analytically in Section 2 that the similarity functions of Businger et al. (1971) and Dyer (1974) are valid for $Ri_B < Ri_{Bu}$ and are not applicable for $Ri_B \geqslant Ri_{Bu}$. We wish to point out that a positive root of z/L cannot be obtained (Eq. (18)) for linear functions when $Ri_B \geqslant Ri_{Bu}$. On the other hand, the iterative procedure using Eqs. (1)–(5) gives rise to a positive large value of z/L for $Ri_B \geqslant Ri_{Bu}$ because this procedure does not involve the information on Ri_B. The computed large value of z/L in fact corresponds to an asymptotic value of Ri_B (≈ 0.21) as shown in Fig. 2b rather than the observed Ri_B.

To analyze further, we have used ψ_m and ψ_h from Beljaars and Holtslag (1991) in Eqs. (1), (2) and (7) for computing Ri_B and z/L. A linear relationship is found (Fig. 3a) between the observed and calculated values of Ri_B. This shows that the relationships for ψ_m and ψ_h proposed by Beljaars and Holtslag (1991) perform well for all values of Ri_B. It is not possible at this juncture to prove it mathematically as Eq. (13) is not reducible into a functional form in z/L, which can be treated analytically. The relationship between Ri_B and z/L computed using profiles of ψ_m and ψ_h from Beljaars and Holtslag (1991) is almost the same to the one obtained from Businger et al. (1971) and Dyer (1974) similarity functions for $z/L < 1$. Thus, Businger et al. (1971) and Dyer (1974) functions can be used in the surface similarity theory when z/L is less than 1 (Webb, 1970; Hogstrom, 1996; Howell and Sun, 1999). Fig. 3b reveals that the Obukhov length is underpredicted with Businger et al. (1971) and Dyer (1974) similarity functions for $z/L \geqslant 1$ resulting in large z/L. On the other hand, z/L computed using ψ_m and ψ_h from Beljaars and Holtslag (1991) appears to be reasonable implying a higher value of critical Ri_B than the Ri_{Bu}.

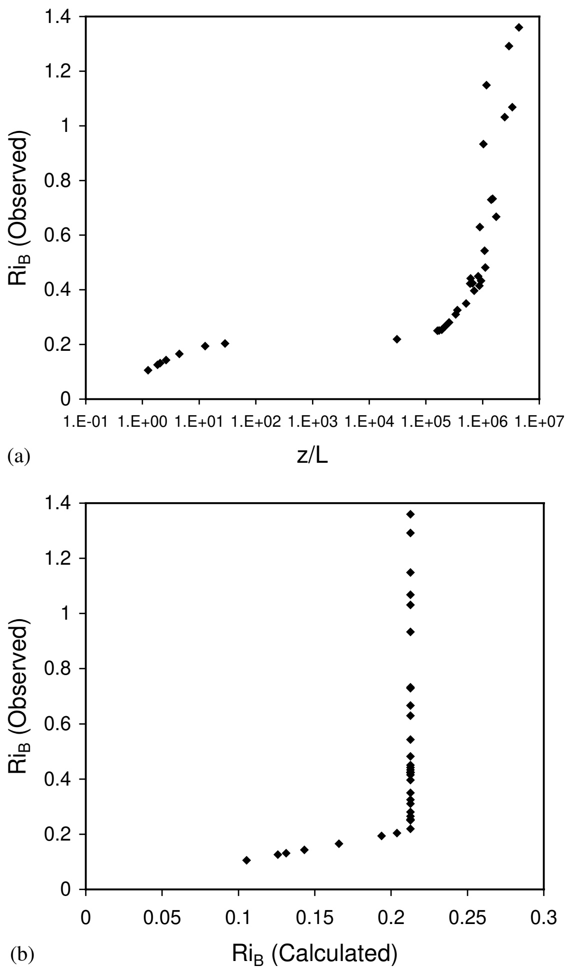

Fig. 2. (a) Scattered diagram of observed Ri_B as a function of the stability parameter z/L obtained from similarity theory using Businger et al. (1971) profiles. (b) Observed Ri_B and the corresponding calculated Ri_B using Businger et al. (1971) profiles. The calculated Ri_B is obtained from Eq. (7) for the computed value of z/L from Eqs. (1)–(5).

Unfortunately turbulence data were not available in the EPRI data set for calculating the surface fluxes. However, we have taken a different data set (Cooperative Atmospheric Surface Exchange Study in October 1999 (CASES99) conducted at Leon (37°38′N; 96°14′W), Kansas, USA) consisting of turbulence data for validation of fluxes. The turbulence data were measured at 1.5/2.5, 10, 20, 30, 40, 50 and 55 m levels on a 55 m micro-meteorological tower. The turbulence data consisting of wind components (u, v, w) and temperature were measured with hot-film anemometers at 1.5/2.5 m level and from sonic anemometer at other levels (Mahrt et al., 2001). The data set considered here is obtained after preliminary analysis of turbulence data enabling the stable conditions during nighttime (Sharan

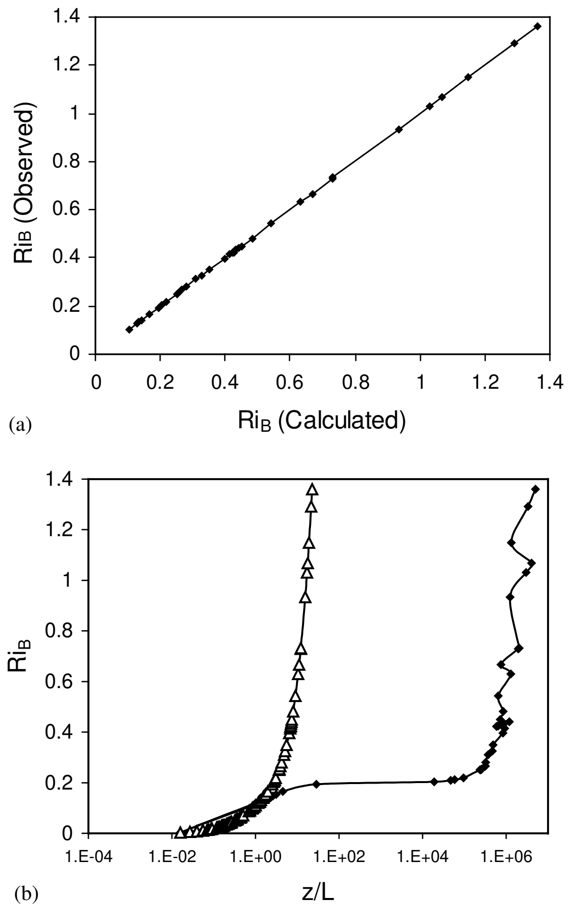

Fig. 3. (a) Observed Ri_B and the corresponding calculated Ri_B using Beljaars and Holtslag (1991) functions. The calculated Ri_B is obtained from Eq. (7) for the computed value of z/L from Eqs. (1)–(5). The continuous curve is obtained by joining the points. (b) Observed Ri_B vs. z/L. The continuous curve is obtained by joining the points at which data are available. The curve (–△–△–△–△–) corresponds to the z/L obtained from Beljaars and Holtslag (1991) where as the curve (–◆–◆–◆–◆–) denotes z/L obtained from Businger et al. (1971).

et al., 2003). About 14 h of data in weak wind stable conditions during the period 4–30 October are obtained. The turbulence fluxes are obtained at 10 m level using the eddy correlation method (Arya, 1999). The surface layer parameters such as u_*, θ_* and L, and thus, fluxes are computed with the hourly averaged temperatures at the surface and 10 m level and wind speed at 10 m level using the similarity theory with Businger et al. (1971) and Beljaars and Holtslag (1991) profiles. The hourly averaged surface temperatures are obtained by extrapolating the temperature profile from the tower.

Fig. 4 represents a comparison of u_* and heat flux computed using similarity functions of Businger et al. (1971) and Beljaars and Holtslag (1991) with those observed on the nights of 10–11 October 1999 ($Ri_B < 0.2$) and 10 and 19 October 1999 ($Ri_B \geqslant 0.2$). As the data in weak wind stable conditions is sparse and not continuous, the hourly variation is shown for two different nights (10 and 19 October). For $Ri_B < 0.2$, the calculated u_* from both profiles are close to each other as well as to those based on observations (Fig. 4a). However for $Ri_B \geqslant 0.2$, the values of u_* based on observations lies in the range 0.03–0.14 m s^{-1} whereas the computed values from Beljaars and Holtslag (1991) profiles are in the range 0.01–0.04 m s^{-1} (Fig. 4b). The computed values of u_* from the profiles of Businger et al. (1971) are found to be in the range 5.0×10^{-08}–7.3×10^{-06} and thus, not included in Fig. 4b. A similar trend is observed in the hourly variation of surface heat flux for $Ri_B < 0.2$ (Fig. 4c) and for $Ri_B \geqslant 0.2$ (Fig. 4d). For $Ri_B \geqslant 0.2$, the magnitude of the heat flux based on the observations lies in the range 1.2–8.7 W m^{-2} whereas the corresponding computed fluxes using Beljaars and Holtslag (1991) profiles are in the range 0.5–2.8 W m^{-2}. We wish to point out the magnitude of the heat fluxes obtained using Businger et al. (1971) profiles are found to be in the range 6.55×10^{-12}–9.11×10^{-08}. However, the fluxes are finite (Figs. 4b and d) but not negligible for $Ri_B \geqslant 0.2$ which correspond to the weak wind stable conditions.

The whole data set considered here is divided into two groups corresponding to $Ri_B < 0.2$ and $Ri_B \geqslant 0.2$. The points with $Ri_B \geqslant 0.2$ correspond to weak wind stable conditions. Figs. 5a–d give the overall picture of the u_* and the heat flux as a function of Ri_B. For $Ri_B < 0.2$, there is a good agreement between the observed and computed fluxes (Figs. 5a and c) obtained using the profiles of Beljaars and Holtslag (1991). The observed (computed) values of u_* lies in the range 0.03–0.6 m s^{-1} (0.05–0.65 m s^{-1}) and the magnitude of heat flux lies in the range 1.5–63 W m^{-2} (3.9–54 W m^{-2}). The magnitude of u_* ranges from 0.037 to 0.14 m s^{-1} in the observations and from 0.015 to 0.055 m s^{-1} in the computations for $Ri_B \geqslant 0.2$. Similarly, the magnitude of the heat flux varies from 1.2 to 8.7 W m^{-2} in observations and from 0.35 to 3.13 W m^{-2} in computations for $Ri_B \geqslant 0.2$. Thus, the fluxes are finite, but not negligible in the weak wind stable conditions. This is in conformity with the findings of Carson and Richards (1978). A large scatter in both observed and calculated heat fluxes (Figs. 5c and d) may be attributed to the hourly variations in the differences in temperatures at the surface and at 10 m level from tower and the averaging period used to compute surface fluxes from turbulence data (Howell and Sun, 1999).

In general, the effective momentum (z_{0m}) and heat (z_{0h}) roughness lengths are assumed to be the same and equal to z_0. This is normally accepted in the atmospheric modelling for a homogenous surface layer. However, under extreme conditions the ratio of z_{0m} and z_{0h} may be different from unity (Beljaars and Holtslag, 1991). A similar analysis may be carried out by taking $z_{0m} \neq z_{0h}$ (Blumel, 2000). It is unlikely that the surface layer

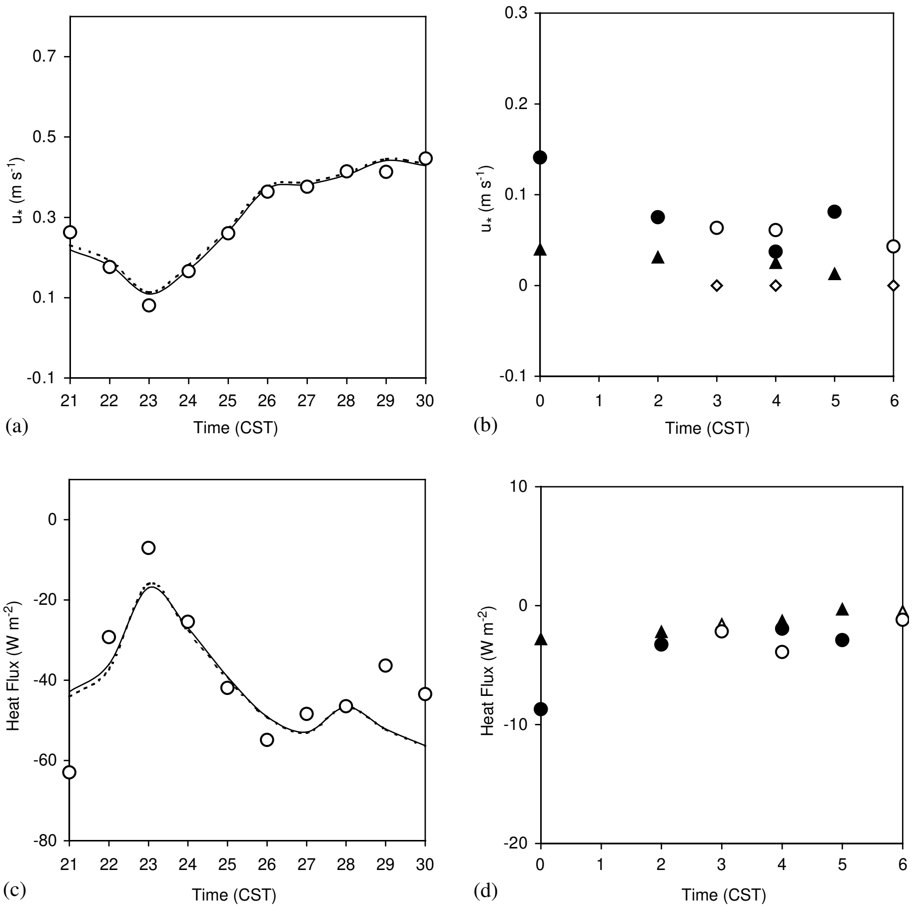

Fig. 4. Hourly variation of surface friction velocity (a and b) and heat flux (c and d) in CASES99. (a) and (c) are for 10–11 October 1999 ($Ri_B < 0.2$), Beljaars and Holtslag (1991) (———); Businger et al. (1971) (– – – –); observed (○ ○ ○ ○) whereas (b) and (d) are for 10 and 19 October 1999 ($Ri_B > 0.2$), Beljaars and Holtslag (1991) (△△△△); (▲▲▲▲); observed (○ ○ ○ ○); (●●●●); hollow symbols correspond to 0300, 0400, 0600 CST on 10 October 1999 and dark symbols correspond to 0000, 0200, 0400, 0500 CST on 19 October 1999.

similarity theory breaks down for very stable conditions and upside-down boundary layers (Mahrt, 1999), which normally occur during the light wind conditions at night. Surface layer and local similarity theory may lead to self-correlations where the predicted variables and the predictors are functions of the same input quantities. The weak wind nocturnal boundary layer structure can be very complex and the assumption that it can be described in terms of the flux–profile similarity relations determined in a well-behaved slightly stable surface layer requires a further investigation.

5. Conclusions

In general, the linear functions of ϕ_m and ϕ_h are used to prescribe the surface fluxes of momentum and heat in the atmospheric dispersion modelling in strong as well as weak wind conditions. The stability parameter z/L is calculated, in general, from turbulence flux measurements. In the absence of such measurements, z/L is not known a priori. On the other hand, the bulk Richardson number (Ri_B) can be readily calculated using routinely available meteorological (tower) observations. Ri_B can be utilized to compute z/L and in turn the fluxes. Thus, a systematic mathematical analysis has been presented to analyze the extent of applicability of the linear functions in terms of Ri_B.

Large values of Ri_B are often encountered in observational studies especially in weak wind stable conditions. We have analyzed the applicability of the formulae for nondimensional wind and temperature profiles based on similarity functions of Businger et al. (1971) and Dyer (1974) in the surface layer in terms of

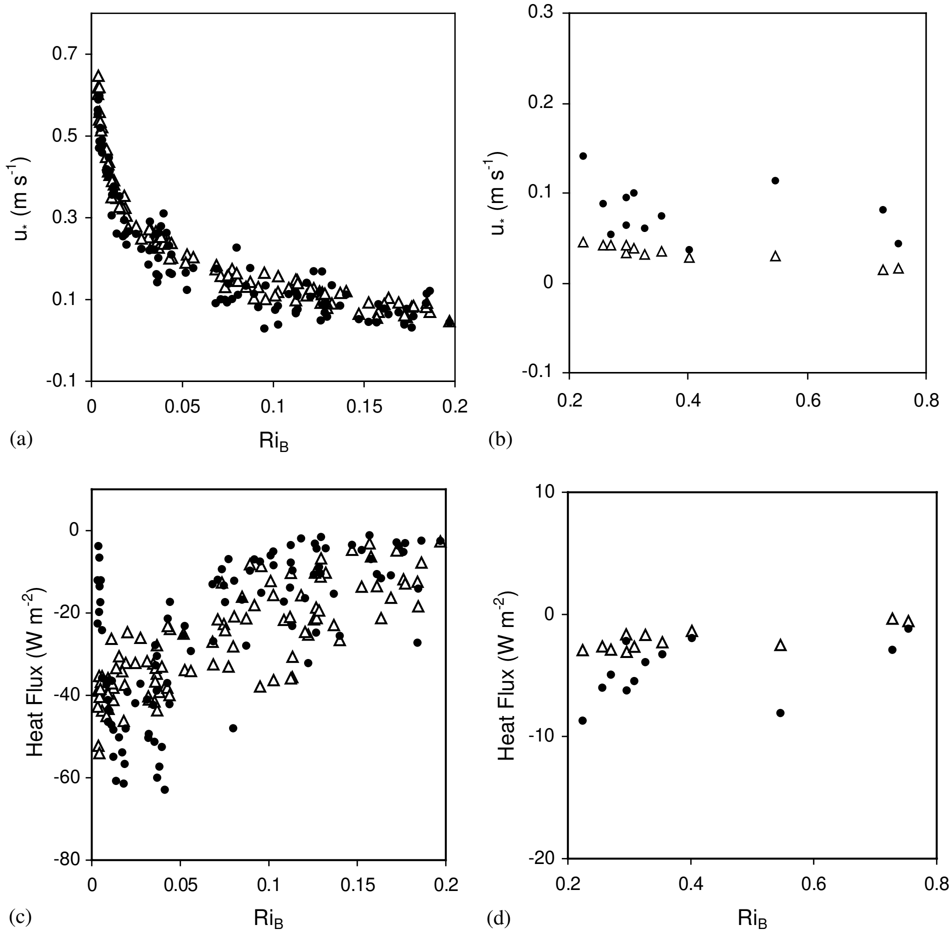

Fig. 5. Scatter diagram of surface friction velocity and heat flux in CASES99. Observed (●●●●); calculated (△△△△). (a) and (c) for $0<Ri_B<0.2$ and (b) and (d) for $0.2\leqslant Ri_B<0.8$.

Ri_B. These linear functions are found to be valid as long as the value of Ri_B is less than $Pr_t\,\gamma/\beta^2$. The linear functional forms for ϕ_m and ϕ_h lead to underestimation of the Obukhov length resulting in excessively large values of z/L when $Ri_B\geqslant Pr_t\,\gamma/\beta^2$ occurring in weak wind stable conditions.

The EPRI data have been used for the purpose of testing the above in this study. It is found that in 70% of the weak wind cases, Ri_B is larger than $Pr_t\,\gamma/\beta^2$. The formulation proposed by Beljaars and Holtslag (1991) is found to perform well in stable conditions for all Ri_B. The observed values of Ri_B are in close agreement with those calculated using Beljaars and Holtslag (1991) formulations. The computation of z/L from observed Ri_B from Eq. (7) using Eqs. (21) and (22) has advantages for calculating the surface fluxes.

The surface layer parameters such as u_*, θ_* and L, and thus, fluxes are computed from the turbulence data collected during the CASES99. The observed fluxes are compared with those computed using the Beljaars and Holtslag (1991) profiles for the similarity functions. In general, very limited data are available in the stable conditions, especially in the weak wind conditions. The performance of the ψ_m and ψ_h relations proposed by Beljaars and Holtslag (1991) need to be further verified as and when extensive observations in weak wind stable conditions become available.

In most of the atmospheric dispersion modelling studies, the fluxes in the surface layer in stable conditions are computed from the linear functional forms of ϕ_m and ϕ_h in terms of z/L (Businger et al., 1971; Dyer, 1974). The stability parameter z/L is not known initially and instead the stability parameter Ri_B can be calculated from the profile measurements. Thus, the analysis presented here will help in describing the surface fluxes accurately for modelling atmospheric dispersion and boundary layer processes, especially in weak wind stable conditions.

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References

Aggarwal, P., Yadav, A.K., Gulati, A., Raman, S., Rao, S., Singh, M.P., Nigam, S., Reddy, N., 1995. Surface layer turbulence processes in low wind speeds over land. Atmospheric Environment 29, 2089–2098.

Arya, S.P.S., 1988. Introduction to Micrometeorology. Academic Press, San Diego, CA, 307pp.

Arya, S.P.S., 1995. Modelling and parameterization of near-source diffusion in weak winds. Journal of Applied Meteorology 34, 1112–1122.

Arya, S.P.S., 1999. Air Pollution Meteorology and Dispersion. Oxford University Press, Oxford, 310pp.

Beljaars, A.C.M., Holtslag, A.A.M., 1991. Flux parameterization over land surfaces for atmospheric models. Journal of Applied Meteorology 30, 327–341.

Blumel, K., 2000. An approximate analytical solution of flux–profile relationships for the atmospheric surface layer with different momentum and heat roughness lengths. Boundary-Layer Meteorology 97, 251–271.

Bowne, N.E., Londergan, R.J., Murray, D.R., Borenstein, H.S., 1983. Overview, results and conclusions for the EPRI plume model validation and development project: plains site, EA-3074, Research Project 1616-1. Palo Alto, CA, 234pp.

Businger, J.A., Wyngaard, J.C., Izumi, Y., Bradley, E.F., 1971. Flux–profile relationships in the atmospheric surface layer. Journal of Atmospheric Science 28, 181–189.

Byun, D., 1990. On the analytical solutions of flux–profile relationships for the atmospheric surface layer. Journal of Applied Meteorology 29, 652–657.

Carson, D.J., Richards, P.J.R., 1978. Modelling surface turbulent fluxes in stable conditions. Boundary-Layer Meteorology 14, 67–81.

Clarke, R.H., 1970. Observational studies in the atmospheric boundary layer. Quaternary Journal of Royal Meteorological Society 96, 91–114.

Delage, Y., 1997. Parameterizing sub-grid scale vertical transport in atmospheric models under statically stable conditions. Boundary-Layer Meteorology 82, 23–48.

Derbyshire, S.H., 1999. Stable boundary-layer modelling: established approaches and beyond. Boundary-Layer Meteorology 90, 423–446.

Dyer, A.J., 1974. A review of flux–profile relationships. Boundary-Layer Meteorology 7, 363–372.

Estournel, C., Guedalia, D., 1985. Influence of geostrophic wind on atmospheric nocturnal cooling. Journal of Atmospheric Science 42, 2695–2698.

Estournel, C., Guedalia, D., 1987. A new parameterization of eddy diffusivities for nocturnal boundary-layer modelling. Boundary-Layer Meteorology 39, 191–203.

Golder, D., 1972. Relations among stability parameters in the surface layer. Boundary-Layer Meteorology 3, 47–58.

Gopalakrishnan, S.G., Sharan, M., McNider, R.T., Singh, M.P., 1998. Study of radiative and turbulent processes in the stable boundary layer under weak wind conditions. Journal of Atmospheric Science 55, 954–960.

Hicks, B.B., 1976. Wind profiles relationships from the "Wangara" experiments. Quaternary Journal of Royal Meteorological Society 102, 535–551.

Hogstrom, U., 1988. Non-dimensional wind and temperature profiles in the atmospheric surface layer. Boundary-Layer Meteorology 42, 263–270.

Hogstrom, U., 1996. Review of some basic characteristics of the atmospheric surface layer. Boundary-Layer Meteorology 78, 215–246.

Holtslag, A.A.M., 1984. Estimates of dibatic wind speed profiles from near-surface weather observations. Boundary-Layer Meteorology 29, 225–250.

Holtslag, A.A.M., de Bruin, H.A., 1988. Applied modelling of the nighttime surface energy balance over land. Journal of Applied Meteorology 27, 689–704.

Howell, J.F., Sun, J., 1999. Surface-layer fluxes in stable conditions. Boundary-Layer Meteorology 90, 495–520.

Launiainen, J., 1995. Derivation of the relationship between the Obukhov stability parameters and bulk Richardson number for flux–profile studies. Boundary-Layer Meteorology 76, 165–179.

Lee, H.N., 1997. Improvement of surface flux calculations in the atmospheric surface layer. Journal of Applied Meteorology 36, 1416–1423.

Louis, J.F., 1979. A parametric model of vertical eddy fluxes in the atmosphere. Boundary-Layer Meteorology 17, 187–202.

Mahrt, L., 1999. Stratified atmospheric boundary layers. Boundary-Layer Meteorology 90, 375–396.

Mahrt, L., Vickers, D., Nakamura, R., Soler, M.R., Sun, J., Burns, S., Lenschow, D.H., 2001. Shallow drainage flows. Boundary-Layer Meteorology 101, 243–260.

Panofsky, H.A., Dutton, J.A., 1984. Atmospheric Turbulence. Wiley, New York, 397pp.

Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., 1986. Numerical Recipes: The Art of Scientific Computing. Cambridge University Press, Cambridge, 818pp.

Segendorf, J.F., Dickson, C.R., 1974. Diffusion under low wind speed inversion conditions, NOAA Technical Memo-ERL-ARL-52, Air Resources Laboratories, Sliver Spring.

Sharan, M., Gopalakrishnan, S.G., 1997. Comparative evaluation of eddy exchange coefficients for strong and weak wind stable boundary layer modelling. Journal of Applied Meteorology 36, 545–559.

Sharan, M., Yadav, A.K., 1998. Simulation of diffusion experiments under light wind, stable conditions by a variable *k*-theory model. Atmospheric Environment 32, 3481–3492.

Sharan, M., McNider, R.T., Gopalakrishnan, S.G., Singh, M.P., 1995. Bhopal gas leak: a numerical simulation of episodic dispersion. Atmospheric Environment 29, 2061–2070.

Sharan, M., Gopalakrishnan, S.G., McNider, R.T., Singh, M.P., 1996a. Bhopal gas leak: a numerical investigation of prevailing meteorological conditions. Journal of Applied Meteorology 3, 1637–1656.

Sharan, M., Singh, M.P., Yadav, A.K., 1996b. A mathematical model for atmospheric dispersion in low winds with eddy diffusivities as linear functions of downwind distance. Atmospheric Environment 30, 1137–1145.

Sharan, M., Gopalakrishnan, S.G., McNider, R.T., 1999. A local parameterization scheme for sig-w under stable conditions. Journal of Applied Meteorology 38, 617–622.

Sharan, M., Yadav, A.K., Modani, M., 2002. Simulation of short range diffusion experiment in low wind convective conditions. Atmospheric Environment 36, 1901–1906.

Sharan, M., Rama Krishna, T.V.B.P.S., Aditi, 2003. Surface layer characteristics in strong and weak wind stable boundary layers. Boundary Layer Meteorology 108, 257–288.

Stull, R.B., 1988. An Introduction to Boundary Layer Meteorology. Kluwer Academic Publishers, Dordrecht, The Netherlands, 666pp.

Tjemkes, S.A., Duynkerke, P.G., 1989. The nocturnal boundary layer: model calculations compared with observations. Journal of Applied Meteorology 28, 161–175.

Webb, E.K., 1970. Profile relationships: the log-linear range, and extension to strong stability. Quaternary Journal of Royal Meteorological Society 96, 67–90.

Yadav, A.K., Raman, S., Sharan, M., 1996. Surface layer turbulence and spectra and eddy dissipation during low winds in tropics. Boundary-Layer Meteorology 79, 205–223.

Yaglom, A.M., 1977. Comments on wind and temperature flux–profile relationships. Boundary-Layer Meteorology 11, 89–102.

Zilitinkevich, S.S., Calanca, P., 2000. An extended similarity theory for the strongly stratified atmospheric surface layer. Quaternary Journal of Royal Meteorological Society 126, 1913–1923.

Zilitinkevich, S.S., Chailikov, D.V., 1968. Determining the universal wind velocity and temperature profiles in the atmospheric boundary layer. Izvestiya, Atmospheric and Oceanic Physics 4, 165–170 (English translation).