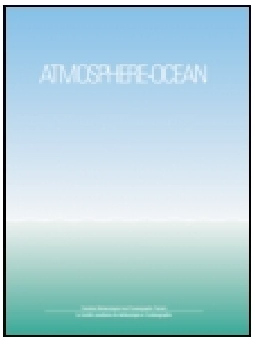

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Non-iterative surface flux parametrization for the unstable surface layer

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Non-iterative Surface Flux Parametrization for the Unstable Surface Layer

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ABSTRACT *This study presents a semi-analytic non-iterative solution for the Monin-Obukhov similarity equations under unstable surface conditions. The solution is represented in terms of the non-dimensional Monin-Obukhov stability parameter z/L. This parameter is given as a function of the bulk Richardson number and other surface parameters including the heat and momentum roughness lengths which are generally assumed to be different in this formulation. The proposed formulations give results that are both quantitatively and qualitatively consistent with the fully iterated numerical solution for a wide range of surface parameters.*

RESUMÉ *[Traduit par la rédaction] Cette étude présente une solution semi-analytique non itérative aux équations de similitude de Monin-Obukhov dans des conditions de surface instables. La solution est exprimée en fonction de la stabilité adimensionnelle z/L de Monin-Obukhov. Ce paramètre est donné comme une fonction du nombre de Richardson apparent et d'autres paramètres de surface, y compris les longueurs de rugosité de la chaleur et de la quantité de mouvement, qui sont généralement supposées différentes dans cette formulation. Les formulations proposées donnent des résultats qui s'accordent tant quantitativement que qualitativement avec la solution numérique pleinement itérée pour une vaste gamme de paramètres de surface.*

1 Introduction

The accurate determination of the surface fluxes of momentum, heat and moisture is of critical importance in predicting the atmospheric structure and its evolution in time since these fluxes represent the forcing and the lower boundary conditions in atmospheric boundary layer modelling. These fluxes are commonly described by the Monin-Obukhov (1954) similarity theory for a wide range of surface conditions.

The Monin-Obukhov similarity theory consists of several highly non-linear coupled sets of equations represented in terms of the non-dimensional Obukhov stability parameter $\zeta = \frac{z}{L}$ where z is a reference height above the surface, and L is the Obukhov stability length scale defined as

$$L = -\frac{u_*^3\overline{\theta_0}}{kg\overline{w'\theta'}}, \tag{1}$$

where u_* is the friction velocity, $\bar{\theta}_0$ is a reference mean temperature above the surface, k is the von Karman constant, g is acceleration due to gravity and $\overline{w'\theta'}$ is the heat flux.

Solving these equations numerically using iterative procedures is highly undesirable in numerical simulations with large atmospheric models since this can be computationally expensive and time consuming in terms of achieving numerical convergence. Therefore, accurate non-iterative solutions are preferable in order to minimize these computational costs and avoid difficulties related to numerical convergence. In light of this, several non-iterative solutions have been proposed during the last few decades.

Assuming the roughness length for momentum (z_o) and the roughness length for heat (z_h) to be identical, Louis (1979) proposed one of the earliest non-iterative approximations of the Monin-Obukhov similarity equations in terms of an expression for the stability parameter ζ as an explicit function of the bulk Richardson number Ri_b. With the same assumption of equal momentum and heat roughness length, Byun (1990) derived an analytical solution for ζ as a function of the gradient Richardson number. However, these approximations were inadequate since both experimental and theoretical evidence demonstrated that the momentum and heat roughness lengths could differ by several orders of magnitude (see Garratt (1978)) and neglecting the difference could result in inaccurate surface flux predictions (Braud et al., 1993).

More recently, alternative formulations have been developed without the restrictive assumption of equal momentum and heat roughness lengths. Mascart et al. (1995) used a higher order polynomial fit to obtain the explicit approximation in

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terms of Ri_b, while Launianen (1995) adopted a series expansion to arrive at an alternative expression. Their expressions gave an improved estimate of the transfer coefficients for momentum and heat. However, they were typically found to be applicable only to a limited range of the roughness ratio, $\frac{z_o}{z_h}$ (see Van Den Hurk and Holtslag (1997) for a review). Guilloteau (1998) obtained an expression for ζ as an analytic solution of a cubic equation. However, this scheme tends to underestimate both the momentum and the heat transfer coefficients.

Abdella and McFarlane (1996) presented another surface flux parametrization derived using a fitting function based on the asymptotic behaviour of ζ at large and small Ri_b. This formulation has been used by the Canadian Centre for Climate Modelling and Analysis (CCCma) (McFarlane et al., 1992). However, the chosen form of the fitting function leads to inaccurate results for some surface conditions. In particular, for rough surfaces when the ratio of the roughness length of momentum to that of heat is above unity, their formulation leads to an overestimation of the fluxes by up to 25%. In contrast, for smooth surfaces with roughness length ratios which are less than unity, their formulation leads to an underestimation of the fluxes. Moreover, for moderate to high Ri_b values and for some ranges of the roughness length ratio this formulation is inadequate. A more accurate solution, based on a semi-analytic approach, was recently proposed by De Bruin et al. (2000). However, the model parameters needed in their scheme are not analytic functions of height. Instead, the parameters are assumed to be constants which are determined by matching the exact and the approximate formulation at one selected value of ζ. As pointed out in De Bruin et al. (2000), the constant parameter assumption is inconvenient and limits the generality and reduces the accuracy of their scheme. Van den Hurk and Holtslag (1997) compared most of the approximate analytic methods. They reported that, due to the inaccuracies of the existing analytic solutions for a wide range of unstable conditions, the full iterative solution is still used in many operational large-scale models (see also Beljaars and Viterbo (1998) and De Bruin et al. (2000)).

In this paper, we present a semi-analytic non-iterative solution under unstable surface conditions in terms of ζ given as a function of Ri_b, z_o, z_h. The derivation of this formulation leads to a bi-quadratic polynomial equation whose analytic solution is readily available using Ferari's method. In contrast to the work of De Bruin et al. (2000) the model parameters which are approximated using their asymptotic behaviour are analytic functions of height and surface layer variables. For various, carefully selected, typical surface conditions the new scheme yields results that are both qualitatively and quantitatively consistent with the fully iterated solution.

In the next section a brief summary of surface-flux theory is given. In Section 3 the derivation of the new solution is presented followed by some results and discussion in Section 4. Finally, concluding remarks are presented in Section 5.

2 Surface flux parametrizations

The surface fluxes of momentum τ, sensible heat H and moisture Q_E can be expressed in terms of the friction velocity u_*, the temperature scale θ_* and the humidity scale q_* respectively as

$$\tau = \rho u_*^2,\ H = \rho C_p \overline{w'\theta'} = -\rho C_p u_* \theta_*,\ Q_E = -\lambda \rho u_* q_* \quad (2)$$

where ρ is the density of air, C_p is the specific heat at constant pressure and λ is the latent heat of vaporization. Assuming horizontally homogeneous and stationary conditions, the Monin-Obukhov similarity theory states that the scaling parameters and the gradient of the mean field variables can be written as follows, in terms of the non-dimensional Monin-Obukhov parameter ζ which is negative for unstable conditions, positive for stable conditions and zero for neutral conditions:

$$\frac{\partial u}{\partial z}\frac{kz}{u_*} = \phi_m(\zeta),\ \frac{\partial \theta}{\partial z}\frac{kz}{\theta *} = \phi_h(\zeta),\ \text{and}\ \frac{\partial q}{\partial z}\frac{kz}{q_*} = \phi_q(\zeta) \quad (3)$$

where u is the magnitude of the mean wind, θ is the potential temperature, q is the specific humidity, and ϕ is the corresponding stability function which is typically determined empirically through observation. The commonly accepted forms of these functions in the unstable case (i.e., for $\frac{z}{L} < 0$) are:

$$\phi_m(\zeta) = (1-\gamma_m\zeta)^{-\frac{1}{4}},\ \text{and}\ \phi_h(\zeta) = \phi_q = pr_t(1-\gamma_h\zeta)^{-\frac{1}{2}} \quad (4)$$

where $pr_t = \left(\frac{\phi_h}{\phi_m}\right)_{\zeta=0}$ turbulent Prandtl number and γ_m and γ_h are constants.

Integrating Eq. (3) from the corresponding roughness height to z and using Eq. (4) for the ϕ functions we obtain the following well known flux profile relationship:

$$u_* = \frac{ku(z)}{\ln\left(\frac{z}{z_o}\right) - \left(\psi_m\left(\frac{z}{L}\right) - \psi_m\left(\frac{z_o}{L}\right)\right)} \quad (5)$$

$$\theta_* = \frac{k(\theta(z) - \theta_s)}{pr_t\left(\ln\left(\frac{z}{z_h}\right) - \left(\psi_h\left(\frac{z}{L}\right) - \psi_h\left(\frac{z_h}{L}\right)\right)\right)} \quad (6)$$

where $\theta_s = \theta(z_h)$ and the integrated stability functions are defined as

$$\psi_m\left(\frac{z}{L}\right) = \int \frac{1-\phi_m\left(\frac{z}{L}\right)}{z}dz,\quad \psi_h\left(\frac{z}{L}\right) = \int \frac{1-\phi_h\left(\frac{z}{L}\right)}{z}dz \quad (7)$$

and their integrated expressions are given by:

$$\psi_m\left(\frac{z}{L}\right) = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan(x) + \frac{\pi}{2} \quad (8)$$

$$\psi_h\left(\frac{z}{L}\right) = 2\ln\left(\frac{1+y}{2}\right) \quad (9)$$

where $x = \left(1 - \gamma_m \frac{z}{L}\right)^{\frac{1}{4}}$ and $y = \left(1 - \gamma_h \frac{z}{L}\right)^{\frac{1}{2}}$. Similar expressions can be written for q_*.

For a given set of mean and surface parameters, Eqs (1), (5) and (6) define a non-linear set of coupled equations for the variables L, u_* and θ_*. Due to the non-linearity of these equations, full analytic solutions are available only in special cases (Byun, 1990). Moreover, because numerical methods based on iterative procedures can be computationally costly, accurate non-iterative solutions need to be developed. In the next section, a new non-iterative solution is derived for the unstable surface layer.

3 New parametrizations

A new non-iterative parametrization for ζ in terms of Ri_b and other surface layer parameters will be derived in this section. The newly parametrized ζ can then be substituted into Eqs (5) and (6) to obtain the scaling parameters u_* and θ_* which yields the surface fluxes according to Eq. (2).

In order to derive the new parametrization for ζ, we first note that, Eqs (1), (5) and (6) can be combined to give the equation:

$$L = AL_0 \frac{f_h}{f_m^2} \quad (10)$$

where

$$L_0 = \frac{\overline{\theta_0}(u(z))^2}{g\Delta\theta}, A = \frac{pr_t \ln\left(\frac{z}{z_h}\right)}{\left(\ln\left(\frac{z}{z_o}\right)\right)^2}, \Delta\theta = \theta(z) - \theta_s \quad (11)$$

$$f_h = 1 - \frac{\psi_h\left(\frac{z}{L}\right) - \psi_h\left(\frac{z_h}{L}\right)}{\ln(z) - \ln(z_h)} \quad (12)$$

and

$$f_m = 1 - \frac{\psi_m\left(\frac{z}{L}\right) - \psi_m\left(\frac{z_o}{L}\right)}{\ln(z) - \ln(z_o)} \quad (13)$$

The expressions for f_h and f_m can be simplified by applying Cauchy's theorem for mean value, which states that if two functions $g_1(x)$ and $g_2(x)$ are differentiable in the interval $[a,b]$ then there is at least one value of x, say x' with $a < x' < b$, for which

$$\frac{g_1(b) - g_1(a)}{g_2(b) - g_2(a)} = \frac{g_1'(x')}{g_2'(x')} \quad (14)$$

where $g_1'(x') = \frac{\partial g_1}{\partial x}$ at $x = x'$ and $g_2'(x') = \frac{\partial g_2}{\partial x}$ at $x = x'$ (Franklin, 1960).

Choosing $g_1 = \psi_h(z)$, $g_2 = \ln(z)$, $a = z_h$, and $b = z$, Eq. (12) gives

$$f_h = 1 - \frac{\psi_h\left(\frac{z}{L}\right) - \psi_h\left(\frac{z_h}{L}\right)}{\ln(z) - \ln(z_h)} = 1 - \frac{\psi_h'\left(\frac{z_t}{L}\right)}{\left(\frac{1}{z_t}\right)} \quad (15)$$

for some z_t such that $z_h < z_t < z$ and the derivative ψ_h' is with respect to z. However, using Eq. (7) this reduces to

$$f_h = 1 - \frac{\dfrac{1 - \phi_h\left(\frac{z_t}{L}\right)}{z_t}}{\left(\frac{1}{z_t}\right)} = \phi_h\left(\frac{z_t}{L}\right). \quad (16)$$

Similarly, choosing $g_1 = \psi_m(z)$, $g_2 = \ln(z)$, $a = z_o$, and $b = z$, Eq. (13) gives

$$f_m = 1 - \frac{\psi_m\left(\frac{z}{L}\right) - \psi_m\left(\frac{z_o}{L}\right)}{\ln(z) - \ln(z_o)} = 1 - \frac{\psi_m'\left(\frac{z_u}{L}\right)}{\left(\frac{1}{z_u}\right)} = \phi_m\left(\frac{z_u}{L}\right) \quad (17)$$

for some z_u such that $z_o < z_u < z$.

Using Eqs (4), (10), (16) and (17) we obtain

$$L = AL_0 \left(\frac{1 - \gamma_m \frac{z_u}{L}}{1 - \gamma_h \frac{z_t}{L}}\right)^{\frac{1}{2}} \quad (18)$$

which can be written as,

$$\gamma_m \alpha_m A^2 \zeta^3 - A^2 \zeta^2 - \gamma_h \alpha_h + Ri_b^2 \zeta + Ri_b^2 = 0 \quad (19)$$

where $\frac{z_h}{z} \leq \alpha_h = \frac{z_t}{z} \leq 1, \frac{z_o}{z} \leq \alpha_m = \frac{z_u}{z} \leq 1$, and $Ri_b = \frac{z}{L_0}$ is the bulk Richardson number. It should be noted that no approximating assumption has been made to obtain Eq. (19). However, in order to solve Eq. (19), the variables α_h and α_m must still be parametrized.

Using Eqs (4) and (16) we obtain

$$f_h\left(\frac{z}{L}\right) = \phi_h\left(\frac{z_t}{L}\right) = \left(1 - \gamma_h \frac{z_u}{L}\right)^{-\frac{1}{2}} = \left(1 - \gamma_h \alpha_h \zeta\right)^{-\frac{1}{2}} \quad (20)$$

which can be solved to obtain

$$\alpha_h = \frac{1}{\gamma_h \zeta}\left(1 - \frac{1}{f_h^2}\right). \quad (21)$$

Similarly using Eqs (4) and (17) it can be shown that

$$\alpha_m = \frac{1}{\gamma_m \zeta}\left(1 - \frac{1}{f_m^4}\right). \quad (22)$$

Using Eqs (12) and (21) we can obtain α_h as a function of ζ. In order to simplify Eq. (19) we consider the asymptotic behaviour of α_h. It can be shown that the asymptotic behaviour of α_h as $\zeta \to 0$ is given by

$$\alpha_{h0} = \frac{b_h - 1}{\ln(b_h)} \quad (23)$$

where $b_h = \frac{z_h}{z}$. Similarly the asymptotic behaviour of α_h for small ζ is given by

$$\alpha_{m0} = \frac{b_m - 1}{\ln(b_m)} \quad (24)$$

where $b_m = \frac{z_o}{z}$. In order to approximate α_h at large ζ (i.e. $\zeta \to -\infty$), we first approximate L using Eq. (10) as

$$L_\infty = AL_0 \quad (25)$$

so that

$$\alpha_{h\infty} = \frac{A}{\gamma_h Ri_b}\left(1 - \frac{1}{f_{h\infty}^2}\right) \quad (26)$$

where $f_{h\infty} = f_h\left(\frac{Ri_b}{A}\right)$. Similarly

$$\alpha_{m\infty} = \frac{A}{\gamma_m Ri_b}\left(1 - \frac{1}{f_{m\infty}^4}\right). \quad (27)$$

where $f_{m\infty} = f_m\left(\frac{Ri_b}{A}\right)$.

The variables α_h and α_m are then chosen to approach these asymptotic limits using the following expressions:

$$\alpha_h = \frac{s\alpha_{h0} - \alpha_{h\infty}\zeta}{s - \zeta} \quad (28)$$

and

$$\alpha_m = \frac{s\alpha_{m0} - \alpha_{m\infty}\zeta}{s - \zeta} \quad (29)$$

where s is a constant. While the approximation is not highly sensitive to the parameter s, $s = 0.25$ is chosen since this gives the best approximations for α over a wide range of surface parameters.

Note that both the α_h and α_m parameters are analytic functions of z, z_o, and z_h. This is in contrast to the work of De Bruin et al. (2000) where the corrsponding α parameters are constants that are determined by matching the exact and the approximate formulation at one selected value of L. As pointed out in De Bruin et al. (2000), the assumption that the α parameters are constants is inconvenient and limits the generality and reduces the accuracy of the outlined approach.

By substituting Eqs (28) and (29) into Eq. (19) we obtain the following bi-quadratic polynomial equation for ζ:

$$\zeta^4 + a_0\zeta^3 + b_0\zeta^2 + c_0\zeta + d_0 = 0 \quad (30)$$

where

$$a_0 = -\left(\frac{1 + \gamma_m s\alpha_{h0}}{\gamma_m \alpha_{h\infty}}\right), \; b_0 = -\frac{\left(\gamma_h \alpha_{m\infty} Ri_b^2 - sA^2\right)}{\gamma_m \alpha_{h\infty} A^2} \quad (31)$$

$$c_0 = \left(\frac{Ri_b}{A}\right)^2 \left(\frac{1 + \gamma_h s\alpha_{m0}}{\gamma_m \alpha_{h\infty}}\right) \; d_0 = -\left(\frac{Ri_b}{A}\right)^2 \left(\frac{s}{\gamma_m \alpha_{h\infty}}\right). \quad (32)$$

Since Eq. (30) is valid only for unstable conditions, negative real roots are the only acceptable solutions. It can be shown using the Intermediate Value Theorem and Descartes' Rule of Signs that, for a given set of surface parameters, there exists a unique negative real root of Eq. (30). This unique solution can be obtained using Cardan's formulae and Ferrari's method (Uspensky, 1948) and is given by

$$\zeta = \frac{\left(\operatorname{sgn}(b)\sqrt{a} - \frac{a_0}{2}\right) - \sqrt{\left(\frac{a_0}{2} - \operatorname{sgn}(b)\sqrt{a}\right)^2 - 4\left(\frac{q_0}{2} - \sqrt{c}\right)}}{2} \quad (33)$$

where sgn represents the sign function and the other parameters are given by:

$$a = \frac{a_0^2}{4} - b_0 + q_0, \quad b = \frac{1}{2} q_0 a_0 - c_0, \quad c = \frac{1}{4} q_0^2 - d_0 \tag{34}$$

$$q_0 = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right)^{\frac{1}{3}} + \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} - \frac{p^3}{27}} \right)^{\frac{1}{3}} + \frac{b_0}{3}, \tag{35}$$

$$p = \left(a_0 c_0 - 4d_0\right) - \frac{b_0^2}{3}, q = \left(4b_0 d_0 - a_0^2 d_0 - c_0^2\right) + \frac{b_0}{3}\left(a_0 c_0 - 4d_0\right) - \frac{2}{27} b_0^3. \tag{36}$$

The fully detailed derivation as well as the proof of uniqueness of Eq. (33) may be found in Assafa (2004).

Equation (33) is the new approximation for ζ which can be substituted into Eqs (5) and (6) to obtain the friction velocity u_* and the temperature scale θ_* respectively. The sensible heat and the momentum fluxes are then computed using Eq. (2).

Note that in Eq. (33), ζ is a function of the roughness parameters z_o and z_h which are related non-linearly to the turbulent fluxes of heat and momentum in the surface layer. For example, over the ocean surface z_o is proportional to u_*^2 according to the well-known Charnock's formula (Charnock, 1955). Due to the complex non-linearity, these relationships cannot be solved analytically. Therefore, most surface flux schemes, including the scheme presented in this paper, neglect these non-linear effects and assume the roughness parameters to be known quantities in their derivation.

In the next section we will test the accuracy of this new formulation by comparing it with the fully iterated solution of the non-linear system.

4 Results and discussions

The new non-iterative solution given by Eq. (33) is compared with the fully iterated solution of the coupled non-linear system of Eqs (1), (5), (6), (8) and (9). Although the new formulation is valid for any choice of k, γ_m, γ_h and pr_t, the values suggested by Dyer (1974) are used in this paper; $k = 0.4$, $pr_t = 1$, $\gamma_m = 16$ and $\gamma_h = 16$. Following Abdella and McFarlane (1996) and others, the comparison is carried out by computing the normalized momentum and heat transfer coefficients F_m and F_h respectively which are given by

$$F_m = \frac{u_*^2}{C_{mn} u(z)^2}, F_h = \frac{pr_t \overline{w'\theta'}}{C_{hn} u(z) \Delta\theta} \tag{37}$$

where C_{mn} and C_{hn} are the neutral transfer coefficients for momentum and heat respectively.

A wide range of values is considered for both $\Delta\theta$ and the mean wind $u(z)$ at the top of the surface layer. The effect of the roughness parameters b_m, b_h and their ratio

$$r = \frac{b_m}{b_h} = \frac{z_o}{z_h} \tag{38}$$

are also considered in order to test the performance of the new formulation under the full range of surface conditions. Over rough land surfaces $r \geq 1$, and in some cases $r >> 1$, reflecting the more efficient transfer of momentum to the surface compared to the heat and mass transfer as reported by Beljaars and Holtslag (1991) for the Cabauw data and as reported by Beljaars (1995) for the Boundary Layer Experiment-1983 (BLX83) in Oklahoma. In contrast, over the sea surface and over smooth land surfaces $r \leq 1$ (Brutsaert, 1982; Guilloteau, 1998). Various typical surface conditions were considered in order to illustrate the validity of the new non-iterative formulation. Figure 1a and Fig. 1b respectively depict the momentum and heat transfer coefficients with $r = 1$ over a smooth surface of roughness length $z_o = 10^{-5}$. This is a condition that is typical of a sea surface. As the figures clearly show, the non-iterative solution is in excellent agreement with the fully iterated numerical solution. Figure 2 also depicts a smooth surface but with $r < 1$ representing a condition in which the roughness length of heat exceeds the roughness length for momentum. Again the new formulation yields very accurate results. For the sake of comparison with other schemes, Fig. 3 also includes the solution of the Abdella and McFarlane (1996) scheme as well as that of De Bruin et al. (2000) for the $r < 1$ case. Both the Abdella and McFarlane (1996) and De Bruin et al. (2000) schemes lead to an underestimation of both the momentum and heat transfer coefficients. The case of rough surfaces is depicted in Fig. 4 with $z_o = 0.1$ and $r = 1.0$ where the non-iterative solution and the numerical solution are in clear agreement. Similar accuracy is obtained for rough surfaces with $r >> 1$ as depicted in Fig. 5. However, as shown in Fig. 6 with $r = 100$, while the De Bruin et al. (2000) scheme slightly overestimates, the Abdella and MacFarlane (1996) scheme significantly overestimates both the heat and the momentum transfer coefficients for these rough surface cases. Further investigation of the new formulation using a wide range of momentum roughness lengths and the ratio r resulted in a similarly accurate approximation of the fluxes. Moreover, as the selected figures demonstrate, the new formulation works well for large and small Ri_b characterizing the success of the new formulation for a wide range of unstable surface conditions.

5 Conclusion

The surface flux equations for the unstable surface layer consist of several coupled non-linear equations obtained through the Monin-Obukhov similarity theory. Due to high computational costs associated with the fully iterated numerical solutions,

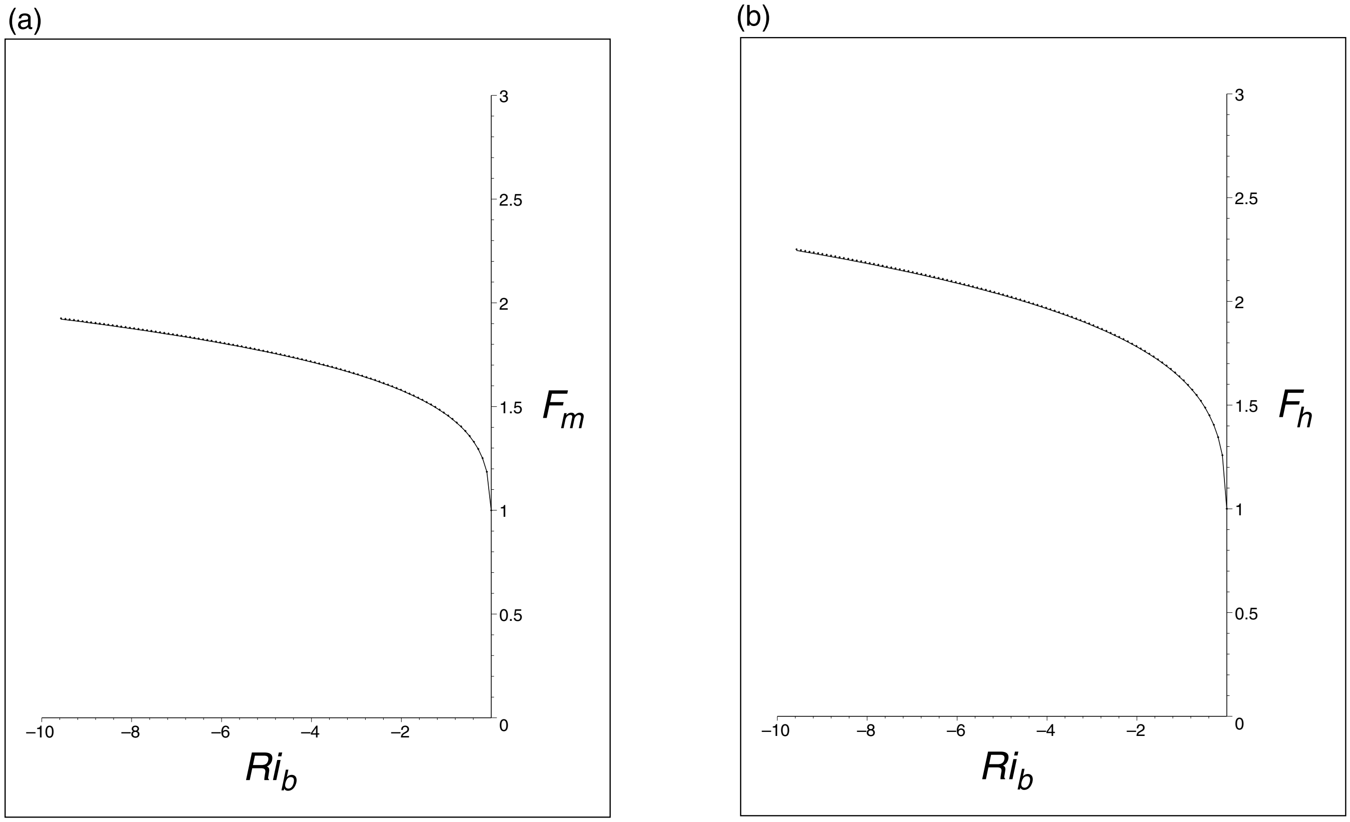

Fig. 1 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 10^{-5}$, and $r = 1$ computed using the fully iterated numerical solution (solid line) and the new non-iterative formulation (dotted line).

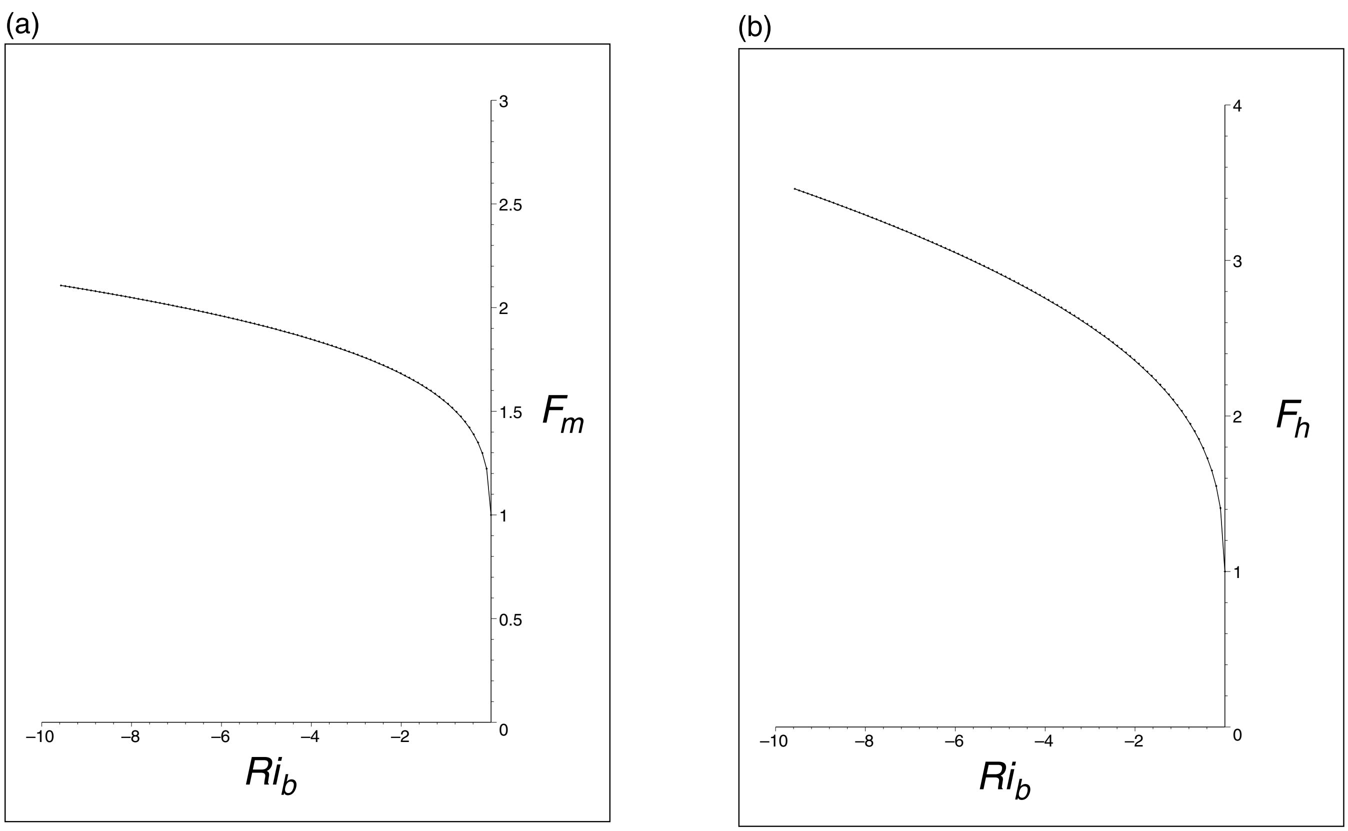

Fig. 2 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 10^{-5}$, and $r = 0.01$ computed using the fully iterated numerical solution (solid line) and the new non-iterative formulation (dotted line).

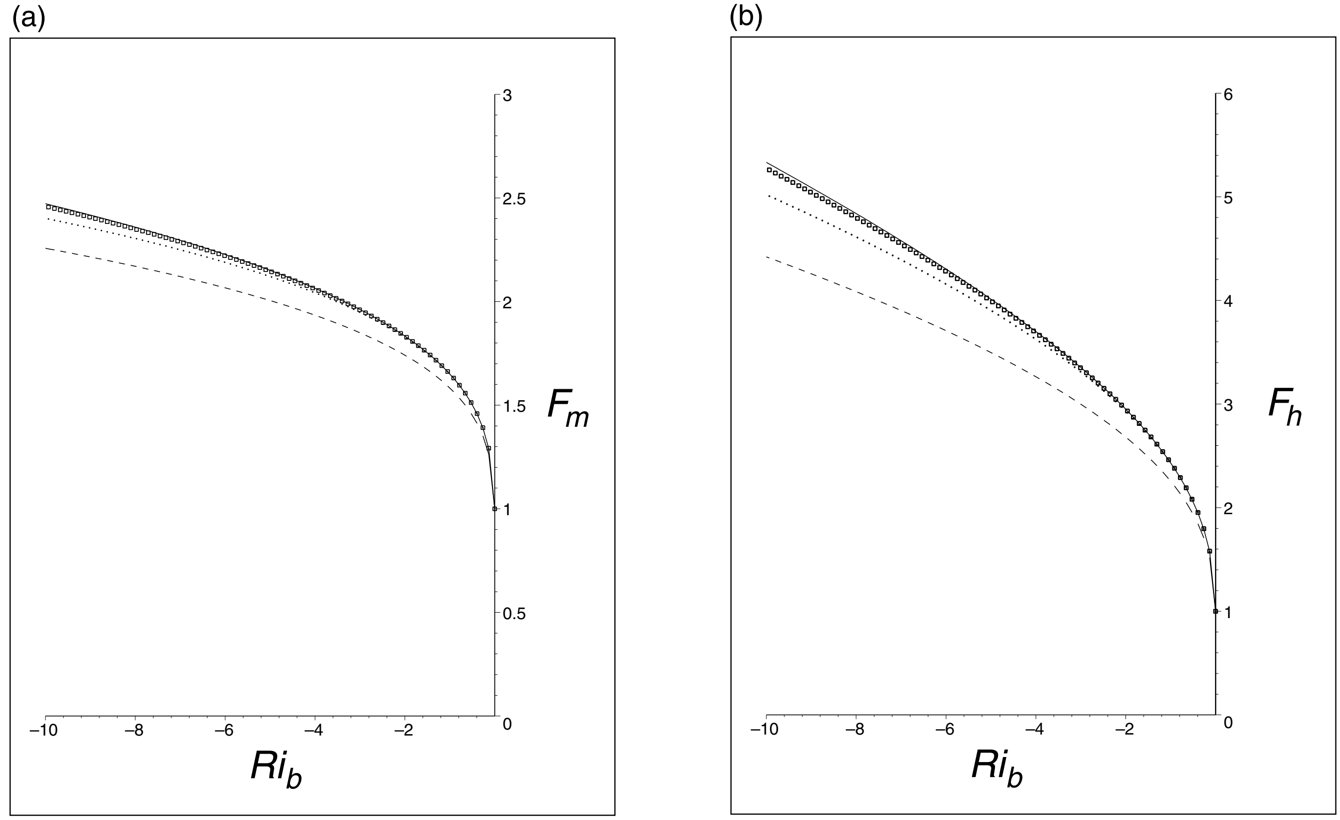

Fig. 3 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 10^{-5}$, and $r = 0.005$ computed using the fully iterated numerical solution (solid line), the new non-iterative formulation (boxed symbol), the Abdella and McFarlane (1996) formulation (dashed line) and the de Bruin et al. (2000) formulation (dotted line).

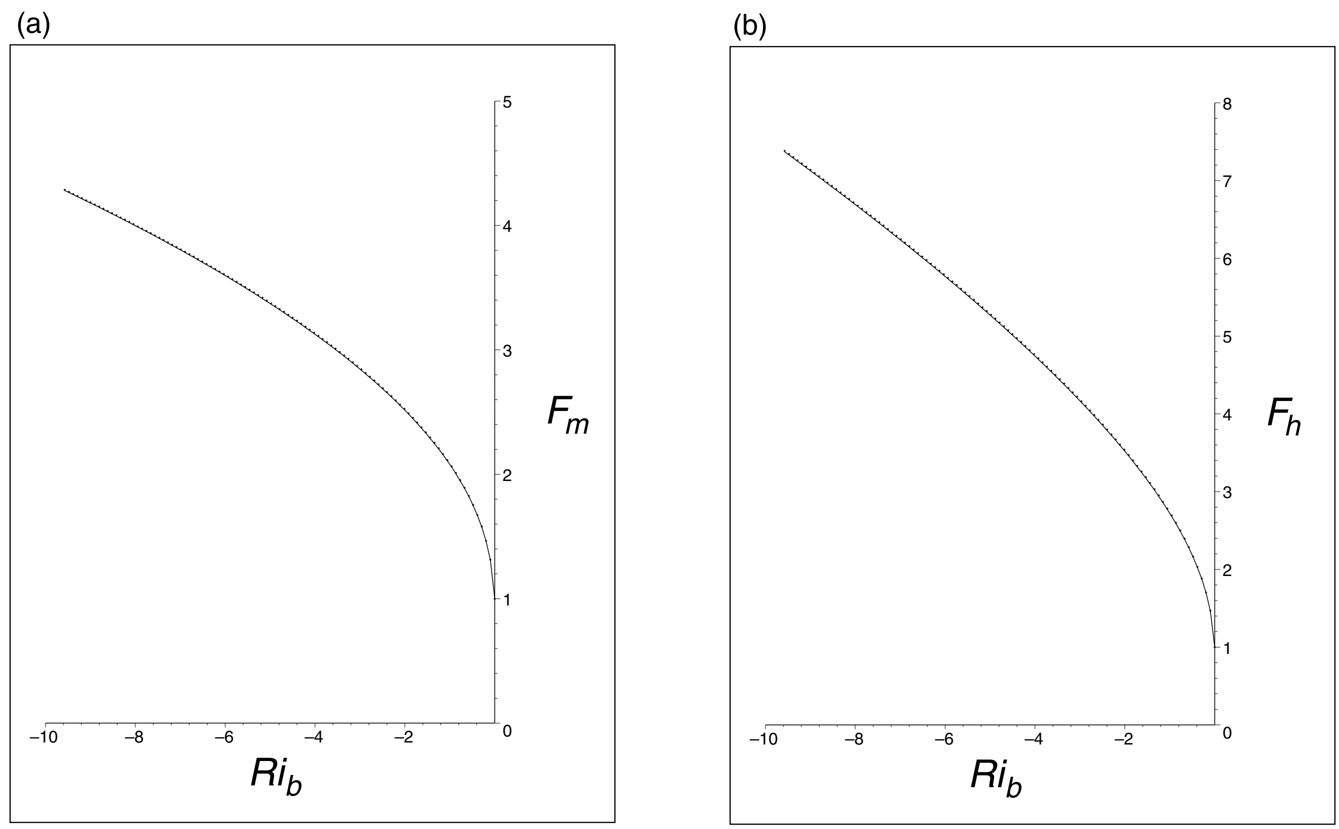

Fig. 4 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 0.1$, and $r = 1$ computed using the fully iterated numerical solution (solid line) and the new non-iterative formulation (dotted line).

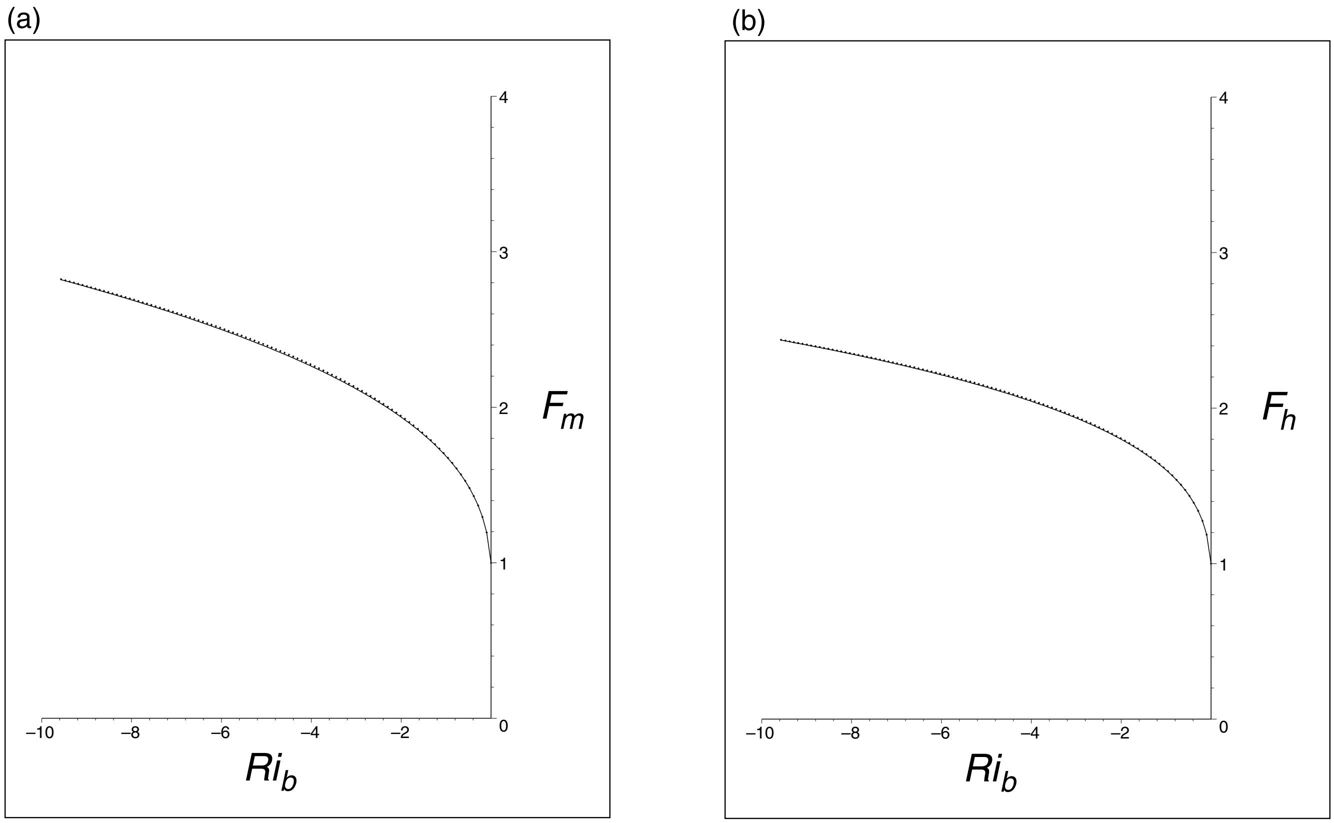

Fig. 5 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 0.1$, and $r = 1000$ computed using the fully iterated numerical solution (solid line) and the new non-iterative formulation (dotted line).

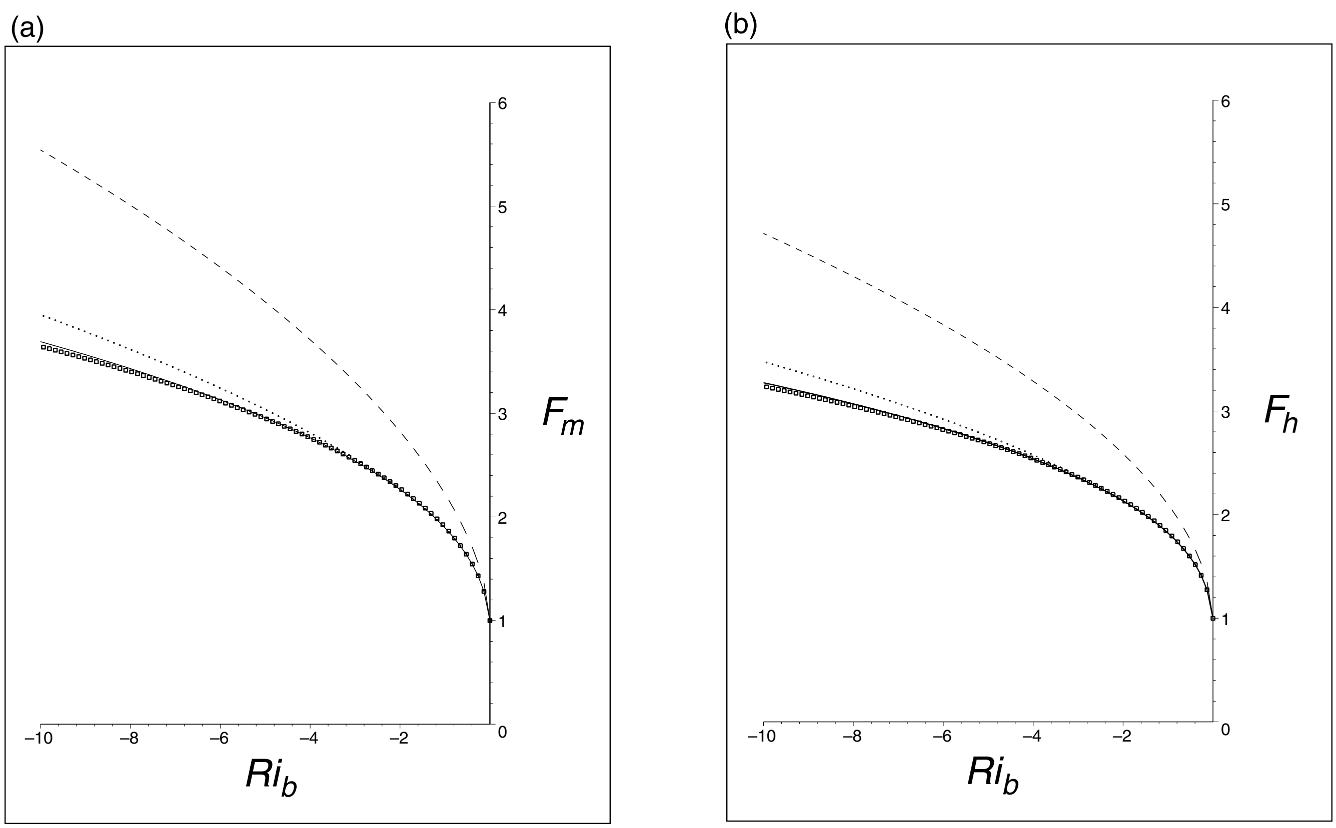

Fig. 6 a) Momentum and b) heat transfer coefficients versus Ri_b for $z_o = 0.1$, and $r = 100$ computed using the fully iterated numerical solution (solid line), the new non-iterative formulation (boxed symbol), the Abdella and McFarlane (1996) formulation (dashed line) and the De Bruin et al. (2000) formulation (dotted line).

non-iterative solutions are highly desirable in large-scale atmospheric models.

This paper proposes a new non-iterative formulation for approximating the Obukhov stabilty parameter ζ needed for computing the surface fluxes of momentum and heat. In this formulation, ζ is represented as a function of the bulk Richardson number Ri_b and makes no assumptions regarding the ratio of the roughness length of momentum to that of heat.

The non-iterative solution is derived using the generalized mean-value theorem which leads to a bi-quadratic equation for ζ. The analytic solution of the bi-quadratic equation is obtained using Ferrari's method and the existence and uniqueness of the unstable ζ in the new formulation is also established.

Investigations carried out to test the new formulation demonstrate its validity. It is shown that the new formulation yields approximations that are in excellent agreement with the fully iterated numerical solution for a wide range of surface fluxes.

The possibility of implementing similar approaches for the stable surface layer will be investigated in the future. Further studies will also involve the inclusion of free-convection effects for low wind surface conditions. For free-convective conditions, preliminary investigations demonstrate that the approach used in this paper leads to equations that are too complex to simplify. Therefore, the present approach would need to be modified in order to include free-convection effects.

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