DERIVATION OF THE RELATIONSHIP BETWEEN THE OBUKHOV STABILITY PARAMETER AND THE BULK RICHARDSON NUMBER FOR FLUX-PROFILE STUDIES

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Abstract. Using the relationship between the bulk Richardson number R_z and the Obukhov stability parameter z/L (L is the Obukhov length), formally obtained from the flux-profile relationships, methods to estimate z/L are discussed. Generally, z/L can not be uniquely solved analytically from flux-profile relationships, and it may be defined using routine observations only by iteration. In this paper, relationships of z/L in terms of R_z obtained semianalytically were corrected for variable aerodynamic roughness z_0 and for aerodynamic-to-temperature roughness ratios z_0/z_T, using the flux-profile iteration procedure. Assuming the so-called log-linear profiles to be valid for the near-neutral and moderately stable region ($z/L < 1$), a simple relationship is obtained. For the extension to strong stability, a simple series expansion, based on utilisation of specified universal functions, is derived.

For the unstable region, a simple form based on utilisation of the Businger-Dyer type universal functions, is derived. The formulae yield good estimates for surfaces having an aerodynamic roughness of 10^{-5} to 10^{-1} m, and an aerodynamic-to-temperature roughness ratio of $z_0/z_T = 0.5$ to 7.3. When applied to the universal functions, the formulae yield transfer coefficients and fluxes which are almost identical with those from the iteration procedure.

1. Introduction

During the last few decades, the Monin–Obukhov similarity theory (Obukhov, 1946; Monin and Obukhov, 1954, see e.g. Lumley and Panofsky, 1964) has proved to be an essential basis for numerous surface boundary-layer studies. Accordingly, it has been used in estimating the boundary-layer stratification and the turbulent fluxes of momentum, heat, water vapour and other gases between the atmosphere and a water or land surface, and it has formed a relevant basis for modelling studies. This has been possible after finding tested and recognized parameterized forms for the universal stratification functions, see e.g. Högström (1988). These are e.g. the so-called Businger-Dyer formulation (Businger *et al.*, 1971; Dyer, 1974; Businger, 1988) for unstable stratification, and the forms by Webb (1970) and Holtslag and De Bruin (1988) for the stable region. For the ocean, additionally, the overall values found for the neutral drag coefficients. in particular those of Smith (1980, 1988, 1989) or Large and Pond (1981), have made the universal utilisation of the similarity hypothesis really possible. By introducing a simple stability dependency, one should also expect to obtain better results in large-scale wind stress studies and Sverdrup transport estimates (Trenberth *et al.*, 1990; Böning *et al.*, 1990), as well as in heat balance studies and coupled air-ice-ocean models (Häkkinen and

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Cavalieri, 1989; Darby and Wilmott. 1993). This should also be the case from the point of view of gas exchange, effects due to stratification having been neglected in calculations so far (Boutin and Etcheto, 1991: Etcheto *et al.*, 1991).

Difficulties may arise when using the similarity theory for simple practical routine purposes. The most reliable and well-tested universal function forms are given in terms of the Obukhov length (L), presupposing the buoyancy and momentum fluxes (the heat flux and friction velocity) to be known i.e. containing in fact those quantities to be derived using the similarity hypothesis. This leads to an iterative solution of the Obukhov stability parameter z/L (z being height) and the fluxes (Kondo,1975; Hicks, 1975; Beljaars and Holtslag, 1990; Launiainen and Vihma, 1990). Although an iterative solution can nearly always be found, and the procedure given in the last-mentioned reference is rather a flexible one, the iteration may however prove difficult or too cumbersome for some applications.

This report outlines a method for avoiding iteration, using the functional relationship (Deardorff, 1968: Launiainen, 1979; Hsu, 1989) between the stability parameter z/L and the bulk Richardson number R_z. This allows the transfer coefficients and fluxes for diabatic cases to be calculated accurately. The necessary background formulations are given in the Appendix. In spite of the fact that z/L can not be generally uniquely expressed in terms of R_z (Garratt, 1992) and can be analytically solved only for certain discrete cases, vital and reliable practical formulae can be found. This is the case especially for surfaces of small and moderate roughness, such as the sea, snow and ice surfaces.

2. Relationship between z/L and R_z

2.1. Formal Relationship

In a stratified boundary layer, stratification effects may be given in terms of the Obukhov length (Obukhov, 1946; Monin and Obukhov, 1954) as the parameter

$$
\begin{aligned}
z/L = \zeta = -\frac{zgkH}{u_*^3 T_0 \rho c_p} & \times (1 + 0.61 T_0 c_p E/H) \\
= & \times \left(1 + 0.61 T_0 \frac{C_E(q_z - q_s)}{C_H(\Theta_z - \Theta_s)}\right),
\end{aligned}
\tag{1}
$$

given in the form in which the term in brackets represents the buoyancy correction for moisture suggested by Lumley and Panofsky (1964) and Deardorff (1968). In the above, z is the measurement height, H is the heat flux and E the moisture flux and u_* the friction velocity. T_0 is the reference temperature (mean absolute virtual temperature of the layer), g is the acceleration due to gravity, k is the von Karman constant and ρ and c_p denote the density and specific heat capacity of air. C_E and C_H denote the moisture and heat bulk transfer coefficients and, $q_z - q_s$ and

$\Theta_z - \Theta_s$ are the differences in specific humidity and potential temperature between the atmosphere and the surface, respectively. Except over the tropical oceans, the moisture correction effect is usually insignificant from the point of view of the bulk parameterization of turbulent fluxes, especially when stability lies far outside the near-neutral region ($|\zeta| > 0.3$; Launiainen, 1979).

Using the flux-profile relationships, ζ may be given in the form (Launiainen, 1979; Hsu, 1989)

$$\zeta = \alpha_{HN} \frac{(\ln z/z_0 - \psi_M(\zeta))^2}{(\ln z/z_T - \psi_H(\zeta))} \times R_z = kC_H C_D^{-3/2} R_z, \tag{2}$$

where

$$R_z = \frac{zg(\Theta_z - \Theta_s)}{T_0 V_z^2} \times \left[1 + 0.61 T_0 \frac{C_E(q_z - q_s)}{C_H(\Theta_z - \Theta_s)}\right],$$

in which R_z is the bulk Richardson number and z_0 and z_T are the roughness lengths for velocity and temperature. $\psi_M(\zeta)$ and $\psi_H(\zeta)$ are the integrated universal functions for velocity and temperature, C_D denotes the drag coefficient, V_z is the wind speed and α_{HN} is the ratio of the eddy diffusivities of heat and momentum in the neutral case, with $\alpha_{HN} \sim 1$ adopted in this paper (Högström, 1988). In ζ, the correction for moisture is that of R_z and Equation (2) is valid in cases whether the moisture correction is taken into account or not.

Because the parameter R_z can be calculated from routine observations, the relationship (2) can be used for derivation of the Obukhov parameter, provided the roughness parameters or neutral bulk transfer coefficients are known. This can be done e.g. by first constructing the relationship between ζ and R_z using arbitrary values of ζ, and then determining ζ from inverse mapping using R_z calculated from observations (Launiainen, 1983). An example of the ζ versus R_z relationship for conditions above water, based on the iteration procedure using arbitrary values for meteorological conditions, is to be seen in Figure 1. Hsu (1989) tabulated a dependence of ζ on R_z for various wind speeds and air-sea temperature differences, using the discrete literature values of C_D and C_H. The problematics are discussed more generally below and the practical $\zeta = f(R_z)$ relationship for aerodynamic roughness conditions of 10^{-4} to 10^{-1} m and aerodynamic-to-temperature roughness ratios of z_0/z_T of 0.5 to 7.3 (Garratt, 1978, 1992) is considered.

2.2. Near-Neutral and Stable Stratification

In cases when $C_D \simeq C_H$ (i.e. $\ln z/z_0 \simeq \ln z/z_T$ as a Reynolds analogy) as is generally true over water in light to moderate winds, snow or smooth ground surfaces, and for the universal functions $\psi_M \sim \psi_H$, as regularly reported for near-neutral and stable stratification, (2) reduces to the following simple form

$$\zeta = (\ln z/z_0 - \psi_M(\zeta))R_z = kC_D^{-1/2} R_z = (kC_{DN}^{-1/2} - \psi_M(\zeta))R_z. \tag{3}$$

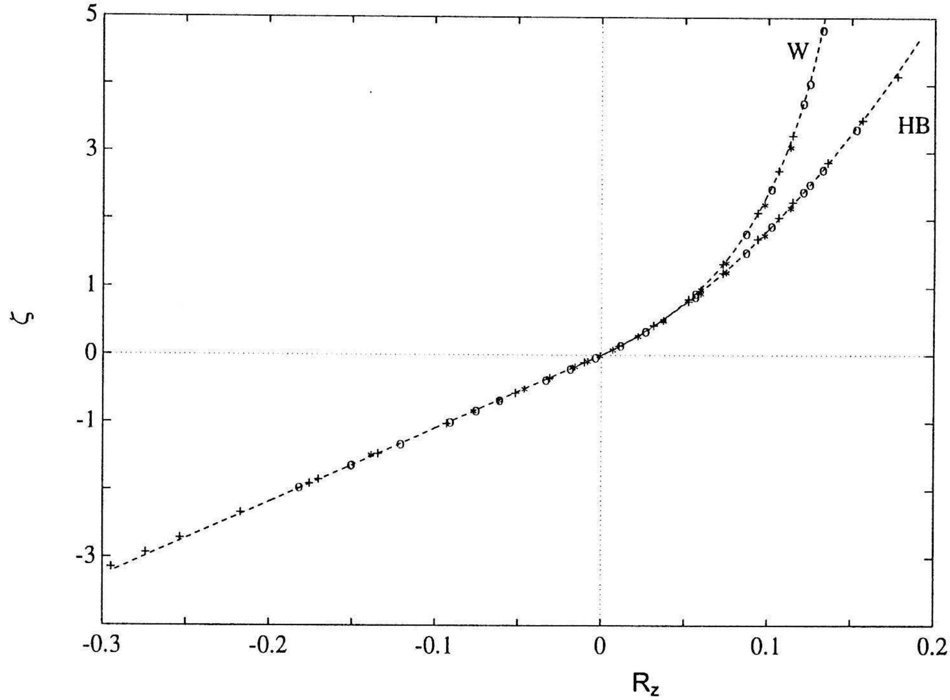

Fig. 1. Obukhov stability parameter ζ versus bulk Richardson number R_z, given by an iteration procedure and by the parameterized $\zeta = f(R_z)$ equations ($z = 10$ m). Samples from the iteration procedure (Launiainen and Vihma, 1990) denoted by +, o and * correspond to cases of wind speed of 3, 5 and 7 m s^{-1}, respectively. The dashed line W gives the ζ estimate according to Equation (4), corresponding to the linear universal function (Webb, 1970). HB denotes the estimate of Equation (9) based on the stable region nonlinear universal functions of Holtslag and De Bruin (1988). For the unstable region, the dashed line given by Equation (11) corresponds to the Businger *et al.* (1971) and Dyer (1974) type universal-function formulation. The roughness lengths used, $z_0 = 5 \times 10^{-5}$ m and $z_0/z_T = 0.5$, correspond to those characteristic of water in light winds.

If a linear form (Webb, 1970; Monin and Yaglom, 1971; see Appendix) of $\psi_M = -b\zeta$ is adopted then

$$\zeta = \frac{(\ln z/z_0)R_z}{1 - bR_z} = kC_{DN}^{-1/2}\frac{R_z}{1 - bR_z}, \qquad |R_z| \leq 1/b. \tag{4}$$

The above yields for the stability-dependent bulk transfer coefficients the simple parabolic formula (Launiainen, 1979)

$$\frac{C_D}{C_{DN}} \simeq \frac{C_H}{C_{HN}} \simeq \frac{C_E}{C_{EN}} = (1 - bR_z)^2. \tag{5}$$

Relationship (4) is shown in Figure 1, using the ocean drag coefficient (or z_0) from the formula of Smith (1980, 1988) and the universal function coefficient $b = 5.2$

(Webb, 1970). As can be seen, Equation (4) follows the iterated results nicely for conditions $z_0 \simeq z_T$.

For conditions of $z_0 \neq z_T$, or rather $\ln(z/z_0) \neq \ln(z/z_T)$, expression (3) is less satisfactory and ζ and R_z cannot then be simply related in a strict way. Then, the applicability of (3) must be tested against iteration results or an error analysis. For practical purposes, however, the form (4) can be corrected. An analysis with the aid of an iteration procedure using various roughness lengths and meteorological conditions resulted in the following linear correction expression

$$\zeta = \left(\frac{(\ln z/z_0)}{1 - bR_z} - 1.3 \ln z_0/z_T\right) R_z, \tag{6}$$

for $0.2 < \zeta < 2$ or $0.02 < R_z < 0.08$. Tests in conditions of z_0 from 10^{-5} to 10^{-1} m and z_0/z_T from 0.5 to 7.3 showed (6) to generally yield estimates within a few percent of those found from the iteration. This finally provides good estimates for the bulk transfer coefficients.

In addition to the above simplified solutions, ζ can be determined (Choudhury *et al.*, 1986) from a somewhat longer complete analytic solution of (2)

$$\zeta = \frac{-t + 2mbR_z + (t^2 - 4tmbR_z + 4bm^2R_z)^{1/2}}{2b(1 - bR)}, \tag{7}$$

where $m = \ln(z/z_0)$ and $t = \ln(z/z_T)$, and which reduces to (4) for $m = t$. It is to be noted that an analytic solution is obtainable only for linear universal function forms (A8) of ψ_M and ψ_H.

2.3. Extension to Strong Stability

The linear universal function forms (A8) yielding the log-linear profile relationship have been found to be fairly consistent with most data for $0 < \zeta < 0.5$ (Högström, 1988; Beljaars and Holtslag, 1991) whereas they tend to yield too pronounced stratification effects for stronger stability (Hicks, 1976; Louis, 1979; Holtslag, 1984) and, e.g. cause the transfer coefficients to vanish for $R_z = 1/b$ (when $\zeta \to \infty$). Holtslag and De Bruin (1988) proposed for the stable region a form (A9) which for small values of ζ behaves like (A8), cf. Figure 1, but which should give realistic values even up to $\zeta \sim 7$ or so. However, we may note that the derived gradient form of (A9) follows from the high-quality experimental data set collected by Högström (1988: his Figure 6) for $\zeta > 0.5$ better than the earlier well-known forms listed in his compilation of universal functions. Unfortunately, an analytic solution for the above cannot be obtained. First constructing the numerical relationship $R_z = f(\zeta)$ for arbitrary values of ζ in Equation (3), for various roughness lengths, the parameterized ζ may then be expressed as a series expansion

$$\zeta = (1.89 \ln z/z_0 + 44.2)R_z^2 + (1.18 \ln z/z_0 - 1.37)R_z, \tag{8}$$

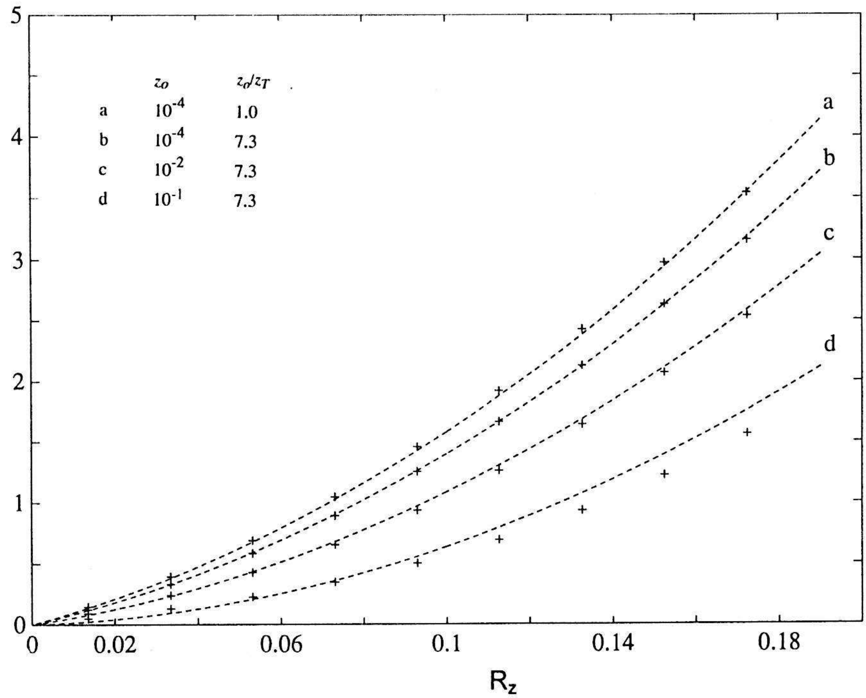

Fig. 2. ζ from the iteration procedure (crosses) and that calculated (dashed lines) from Equation (9) for the stable region for the various z_0 and z_0/z_T ratios listed.

which gives ζ to good accuracy for z_0 from 10^{-5} to 10^{-2} m and somewhat less accurately up to 10^{-1} m. An example is given in Figure 1 and shows (8) to precisely follow the iterated results. Again, because (8) was derived from (3) it holds strictly for conditons $z_0/z_T = 1$. For more general use, a correction for the above was studied with respect to the resistance ratio $\ln(z_0/z_T)$ ($= kB^{-1}$ concept; Owen and Thompson, 1963; Garratt, 1992). For practical purposes a linear correction and the formula

$$\begin{aligned} \zeta &= \zeta_{\text{Eq.}(8)} - 1.5\ln(z_0/z_T)R_z \\ &= (1.89\ln z/z_0 + 44.2)R_z^2 + (1.18\ln z/z_0 - 1.5\ln z_0/z_T - 1.37)R_z \qquad (9) \end{aligned}$$

was obtained for z_0 from 10^{-5} to 10^{-1} m and z_0/z_T in the range 0.5 to 7.3 i.e. $\ln(z_0/z_T)$ from -0.6 to 2. Estimates for various roughnesses and resistance ratios are given in Figure 2, from which it can be seen that (9) generally yields good estimates, but is somewhat less accurate for conditions of large roughness ($> 10^{-2}$ m) and large R_z. Finally, the above results produce very accurate bulk transfer coefficient results, as can be seen below.

2.4. The Unstable Region

For unstable conditions i.e. $\zeta < 0$, the current universal functions are non-linear and of a different form ($\psi_M \neq \psi_H$), and an analytic solution for Equation (2) cannot be found; we can, however, derive a practical form simply as follows.

For the first approach, we may roughly assume (cf. Launiainen, 1983; Figure 5) the universal functions to follow the following forms

$$\psi_H = r_1(-\zeta)^{r_3} = x \quad \text{and,} \quad \psi_M = r_2 x$$

the coefficients r_1, r_2, r_3 being of the order of 1.7, 0.5 and 4/7, respectively, if the Businger–Dyer type universal functions (A10) and (A11) are adopted. By replacing the above in Equation (2),

$$\zeta = \frac{(\ln z/z_0 - r_2 x)^2}{(\ln z/z_T - x)} R_z.$$

Expressed as a Taylor series and neglecting third and higher order terms we get

$$\zeta = \frac{(\ln z/z_0)^2}{(\ln z/z_T)}(1 - c)R_z \tag{10}$$

in which $c = -(x/t) + 2r_2(x/m) + 2r_2(x^2/mt) - \cdots$ and $m = \ln z/z_0$ and $t = \ln z/z_T$. c is a rather small coefficient of the order of 0.02 to 0.15 but is rather laborious to determine strictly for all cases. Equation (10) explains the "almost" linear relationship between ζ and R_z earlier found in numerical experiments (Kondo, 1975; Launiainen, 1979; Large and Pond, 1982; Donelan, 1982; Andreas and Murphy, 1986). With the aid of an iteration procedure, a study of various roughness and z_0/z_T conditions gave for practical calculations a correction

$$c = 0.55/\frac{(\ln z/z_0)^2}{(\ln z/z_T)},$$

whence

$$\zeta = \left(\frac{(\ln z/z_0)^2}{(\ln z/z_T)} - 0.55\right) R_z. \tag{11}$$

Relationship (11) is shown in Figure 1 and in Figure 3 in comparison with the iterated results, and can be seen to produce good estimates for z_0 from 10^{-5} to 10^{-1} m and z_0/z_T in the range 0.5–7.3. As is the case for the results and formulae presented for the stable region, (11) is practically independent of z and valid for any practical z level to be considered. For the oceanic conditions, the above gives an approximation $\zeta \sim 11 R_z$.

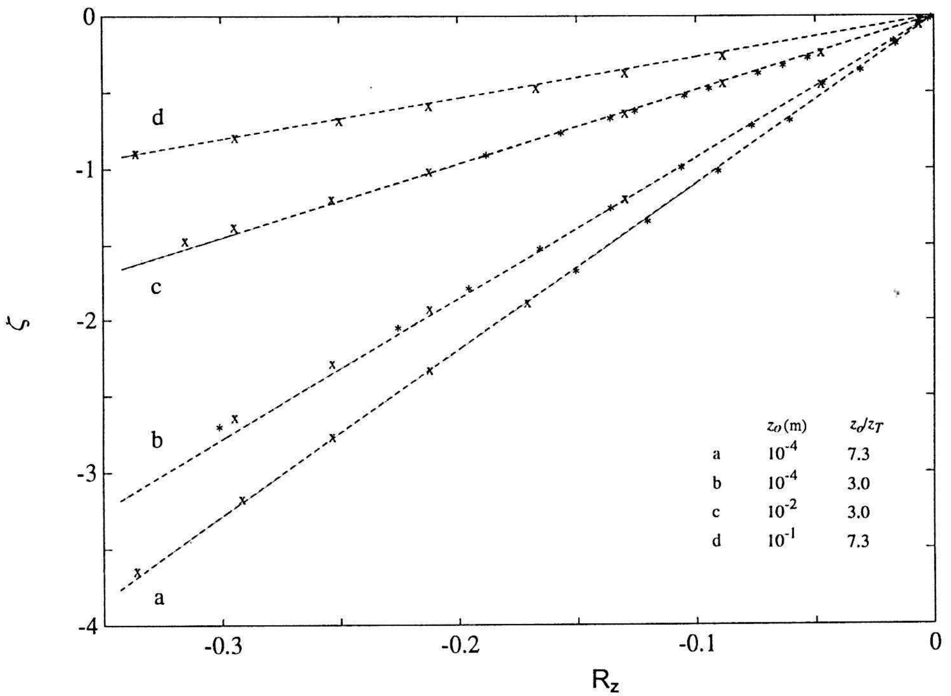

Fig. 3. ζ from the iteration procedure (+ and * corresponding to 3 m s^{-1} and 6 m s^{-1}) and that calculated from Equation (11) as dashed lines for the unstable region for various z_0 and z_0/z_T ratios.

As a minor point we may note that in the above the correction c was determined using the original values $\gamma_1 \simeq \gamma_3 \simeq 16$ in (A10) and (A11). If the somewhat different values of $\gamma_1 = 19.3$ and $\gamma_2 = 12$ suggested by Högström (1988) are used, the correction coefficient of 0.55 in (11) is to be replaced by 0.9. Then, the estimated ζ values are quantitatively comparable with those obtained from the iteration procedure.

3. Application to Transfer Coefficients

3.1. Bulk Transfer Coefficients and Flux Calculations

When the ζ results above are applied to the universal functions, the bulk transfer coefficients for diabatic conditions may be calculated from Equations (A5)–(A7) and the fluxes from Equations (A1) to (A3). In terms of bulk transfer coefficients an example is given in Figures 4 and 5 for two different surface roughnesses and heat transfer resistance ratios i.e. for $z_0 = 10^{-4}$ m and $z_0/z_T = 1$, corresponding to oceanic conditions in moderate winds, and for a rougher surface of 10^{-2} m and $z_0/z_T = 7.3$ respectively. For the stable region ζ was calculated from (9) and

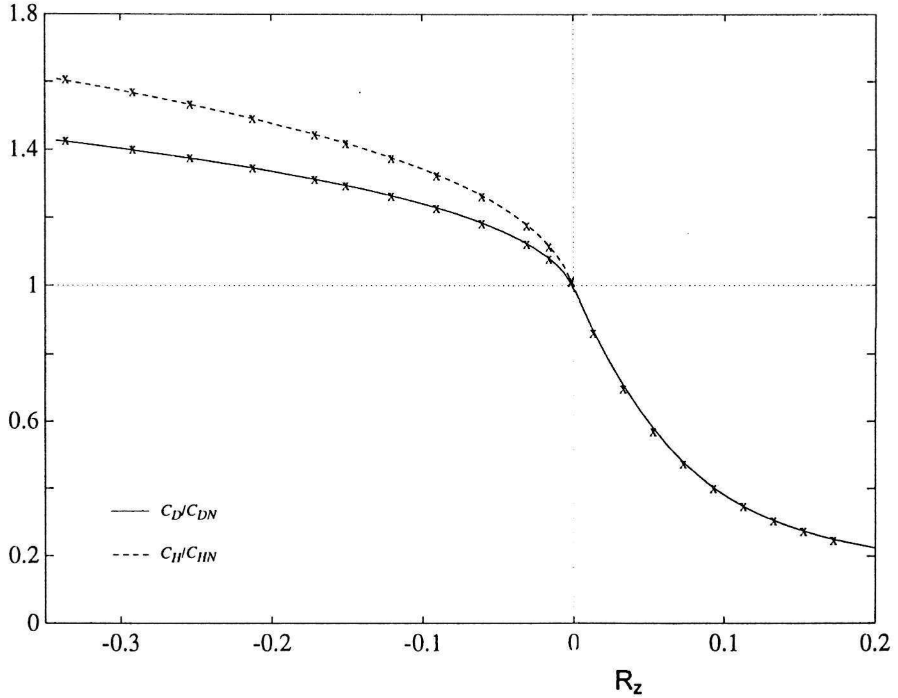

Fig. 4. Drag coefficient and bulk heat transfer coefficient (referred to 10 m) for diabatic cases. The continuous and dashed lines depict the C_D/C_{DN} and C_H/C_{HN} dependences based on iteration, while the crosses are those calculated according to (A5)–(A7) using the ζ from Equations (9) and (11). The roughness lengths used, $z_0 = 10^{-4}$ m and $z_0/z_T = 1$, correspond to the neutral case ($C_{DN} = C_{HN} = 1.2 \times 10^{-3}$).

for the unstable region from (11). As can be seen from the figures, the transfer coefficients calculated using the approximate ζ yield both for the drag coefficient and for the heat transfer coefficient values almost identical to those given by the iteration procedure. Accordingly, calculations using these bulk transfer coefficients yield fluxes almost identical to the iterated ones.

In some cases (e.g. in modelling), the procedure of $R_z \to \zeta(R_z) \to \psi_{M,H}(\zeta) \to C_{D,H}(\zeta)$ may be too long and cumbersome to do and just a short parameterization as $C_{D,H}(R_z)$ might be prefered. Then, for z_0 and z_T in question, results such as in Figures 4 and 5 can easily be expressed as a series expansion, to meet an accuracy of practical needs. For the unstable region, even a more simple expression may be defined. If we first roughly assume, as in Section 2.4, that $\psi_M \propto r_2(-\zeta)^{r_3}$ and $\psi_H \propto r_1(-\zeta)^{r_3}$, and we found ζ to be linearly dependent on R_z. Then, the bulk transfer coefficients can easily be shown to be approximated as

$$C_D/C_{DN} = 1 + l_1(-\zeta)^{r_3}, \tag{12}$$

$$C_E/C_{EN} = 1 + l_2(-\zeta)^{r_3}, \tag{13}$$

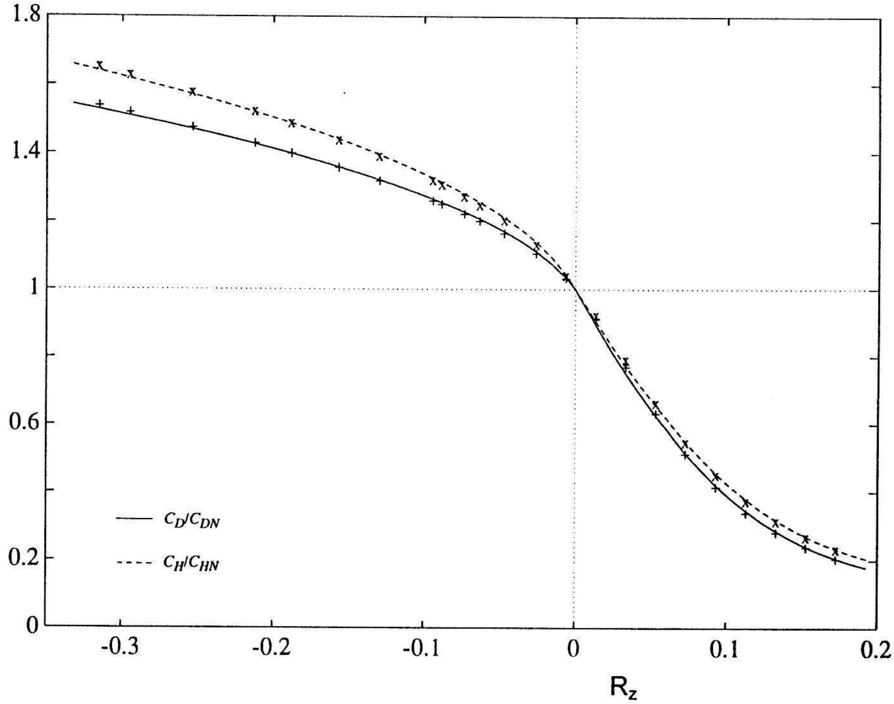

Fig. 5. As Figure 4 but for $z_0 = 10^{-2}$ m and $z_0/z_T = 7.3$ (corresponding to $C_{DN} = 3.4 \times 10^{-3}$ and $C_{HN} = 2.7 \times 10^{-3}$).

where l_1 and l_2 are coefficients. An example is given in Figure 6, for a case of $z_0 = 10^{-4}$ m and $z_0/z_T = 1$ for which $l_1 = 0.73$ and $l_2 = 1.05$ and $r_3 \simeq 0.5$. The approximation may be seen to follow the iterated results well, for the case of small z_0 and $z_0 = z_T$.

For the stable region, results based on the simple form (5) are given in Figure 6 as well. For small R_z they follow the iterated results but for $R_z > 0.5$ those bulk transfer coefficients diminish much more drastically. The reason is the difference between the universal functions of (A8) and (A9) discussed in Section 2.3 above.

3.2. Gradient Method Transfer Coefficients

In the gradient method of flux calculations or modelling, the gradient-form transfer coefficients are calculable e.g. according to Equation (A4), using first u_* from (A5) and in the ψ-functions the ζ values estimated above, to avoid any iteration. Specifically, for near-neutral and stable conditions $-0.2 < \zeta < 1$, assuming the log-linear region to be valid, we get the following simple expressions

$$u_* = V_z k(\ln z/z_0)^{-1}(1 - bR_z) = C_{DN}^{1/2} V_z(1 - bR_z) \tag{14}$$

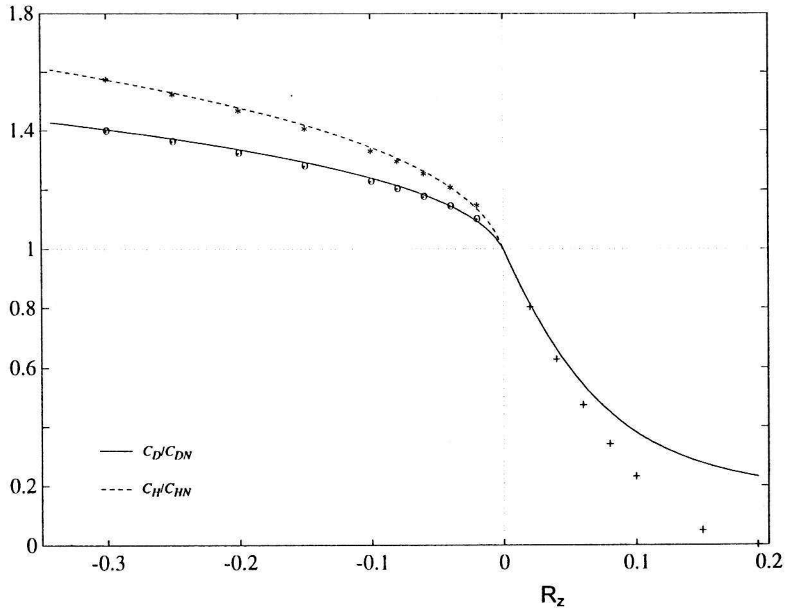

Fig. 6. Continuous and dashed lines depict the C_D/C_{DN} and C_E/C_{EN} based on iteration, conditions as in Figure 4. ⊙ and * denote approximations from Equations (12) and (13). Values for the stable region are from Equation (5) denoted as +.

and

$$K_{M,H,E} = \frac{k^2 V_z z(1 - bR_z)^2}{\ln z/z_0(1 + bR_z(\ln z/z_0 - 1))} = \frac{kC_{DN}^{1/2} V_z z(1 - bR_z)^2}{1 + bR_z(kC_{DN}^{-1/2} - 1))}, \tag{15}$$

which will serve as good approximations for the rather smooth surfaces of $\ln z_0 \simeq \ln z_T$ i.e. with the Reynolds analogy valid, such as over water and snow surfaces in light and moderate winds.

4. Summary and Conclusions

Formulae for practical determination of the Obukhov stability parameter ζ using simple aerodynamic observations were studied semianalytically using the connection between ζ and the bulk Richardson number R_z given by the flux-profile relationships. Conditions having roughness lengths of z_0 of 10^{-5} to 10^{-1} m and sublayer ratios z_0/z_T of 0.5 to 7.3 were considered. To find the numerical corrections and verification, the "correct" ζ was determined by iteration. For near-neutral and moderately stable stratification Equation (6), originally based on the log-linear universal profile assumption, was found to give reasonable estimates, while for cases satisfying the Reynolds analogy, the stability parameter and bulk transfer coefficients reduce to the very simple expressions of (4) and (5). The form of the series expansion (9), based on the universal functions of Holtslag and De Bruin (1988), gives similar results but seems to extend to stronger stability and is therefore essential. For the unstable region, good ζ values for practical calculations are obtainable from the linear form (11).

Large and Pond (1982) suggested for sea conditions simple linear formulae of the form $\zeta \approx 7.14R$ for $R_z > 0$ and $\zeta \approx 10.20R_z$ for $R_z < 0$. Donelan's (1982) proposal was $\zeta = 6.0R_z$ for the stable region and $\zeta = 7.6R_z$ for the unstable region. Andreas and Murphy (1986) suggested for sea ice leads and polynyas $\zeta \approx 8.0R_z$ for $-10 < \zeta < 2$. In the light of experience gained in this study, a linear formula is especially tenable for the unstable region, and the result by Large and Pond (1982) is very comparable with Equation (11) for (z_0, z_T) in oceanic conditions, whereas the forms of Donelan (1982) and Andreas and Murphy (1986) might represent a somewhat "rougher" surface. For the stable region, the forms listed yield smaller estimates than either (6) or (9), and smaller than the iteration procedure using recognised universal functions; generally, a linear form for ζ is supposed to be valid in stable conditions for only a rather narrow region of stability. None of the literature formulae listed expresses the z_0/z_T ratio explicitly, but actually they are found for conditions above water or ice of $\ln z_0 \sim \ln z_T$.

Finally, when applied for the various universal functions, the results given by the present formulae yield transfer coefficients and fluxes (Figures 4 and 5) which are almost identical to the iterated ones, especially for $z_0 < 10^{-2}$ m. For specific practical procedures those can be simplified as a set of R_z-power expansion or, as simple approximations are (12) and (13).

An algorithm (NDP-FORTRAN) for calculation of turbulent fluxes using the stability relations described above is available from the author on request.

Acknowledgments

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Appendix

Flux-profile relations are based on the Monin–Obukhov similarity theory (see Garratt, 1992):

1. Turbulent fluxes (a-forms) of momentum (τ), sensible heat (H) and water vapour (E) expressed in the gradient (b-) and bulk (c-) forms read

$$\tau = -\rho\overline{u'w'} \simeq \rho K_M \frac{\partial V}{\partial z} = \rho u_*^2 = \rho C_D V_z^2, \tag{A1}$$

$$H = \rho c_p \overline{\Theta' w'} \simeq -\rho c_p K_H \frac{\partial \Theta}{\partial z} = \rho c_p C_H (\Theta_s - \Theta_z) V_z \tag{A2}$$

$$\underset{\text{(a)}}{E = \rho \overline{q' w'}} \simeq \underset{\text{(b)}}{-\rho K_E \frac{\partial q}{\partial z}} = \underset{\text{(c)}}{\rho C_E (q_s - q_z) V_z}, \tag{A3}$$

where V_z denotes wind speed at height z, u_* friction velocity, Θ potential temperature, q specific humidity, and ρ is air density and c_p specific heat capacity of air. $\Theta_s - \Theta_z$ and $q_s - q_z$ are the differences in potential temperature and specific humidity between the surface and the atmosphere, respectively. The gradient form transfer coefficients are defined as

$$K_M = \frac{k u_* z}{\phi_M(z/L)}, \quad K_H = \frac{k u_* z}{\phi_H(z/L)}, \quad K_E = \frac{k u_* z}{\phi_E(z/L)} \tag{A4}$$

and the bulk transfer coefficients as

$$C_D = (u_*/V_z)^2 = k^2(\ln z/z_0 - \psi_M(z/L))^{-2}, \tag{A5}$$

$$C_H = \alpha_{HN} k^2 (\ln z/z_0 - \psi_M(z/L))^{-1} (\ln z/z_T - \psi_H(z/L))^{-1}, \tag{A6}$$

$$C_E = \alpha_{EN} k^2 (\ln z/z_0 - \psi_M(z/L))^{-1} (\ln z/z_q - \psi_E(z/L))^{-1}, \tag{A7}$$

where z_0, z_T and z_q are the roughness lengths for velocity, temperature and water vapour, respectively. ϕ_M, ϕ_H and ϕ_E are the gradient form- and ψ_M, ψ_H and ψ_E are the integrated form-universal functions. α_{HN} and α_{EN} denote the neutral ratios of the eddy diffusivities of sensible heat and water vapour to that of momentum.

In the universal functions, the argument z/L is defined with respect to the Obukhov length as Equation (1) in the text and, using (A1) to (A3) and (A5) to (A7), z/L can be expressed in the form of Equation (2) studied in the paper.

2. For the universal functions, the forms used are as follows:

a) For near-neutral and stable region $\zeta > 0$, the log-linear form of

$$\begin{aligned} &\phi_M \simeq \phi_H \simeq \phi_E = 1 + b\zeta \quad \text{and} \\ &\psi_M = \psi_H = \psi_E = -b\zeta \end{aligned} \tag{A8}$$

where $b \sim 5.2$ (Webb, 1970).

The extension to stronger stability (cf. the discussion in Section 2.3) uses a form suggested by Holtslag and De Bruin (1988), i.e.

$$\begin{aligned} &\phi_M = \phi_{H,E} = 1 + a_1\zeta + (1 + c_1 - d_1\zeta)\zeta b_1 \exp(-d_1\zeta) \quad \text{and} \\ &\psi_M \simeq \psi_{H,E} = -b_1 c_1/d_1 - a_1\zeta - b_1(\zeta - c_1/d_1)\exp(-d_1\zeta) \end{aligned} \tag{A9}$$

where $a_1 = 0.7$, $b_1 = 0.75$, $c_1 = 5$, $d_1 = 0.35$.

The ϕ-form above was not given originally by Holtslag and De Bruin (1988) but was obtained as a derivative from $\phi = 1 - \zeta\ \mathrm{d}\psi/\mathrm{d}\zeta$. From (A9), the form approaches that of (A8) with $b = 5.2$, for small values of ζ.

b) For the unstable region the Businger *et al.* (1971) and Dyer (1974) type forms of

$$\begin{aligned} \phi_M &= (1-\gamma_1\zeta)^{-1/4} \quad \text{and} \\ \psi_M &= 2\ln\left(\frac{1+\phi_M^{-1}}{2}\right) + \ln\left(\frac{1+\phi_M^{-2}}{2}\right) - 2\overline{\mathrm{arc}}\tan\phi_M^{-1} + \pi/2 \end{aligned} \tag{A10}$$

and

$$\begin{aligned} \phi_{H,E} &= (1-\gamma_2\zeta)^{-1/2} \quad \text{and} \\ \psi_{H,E} &= 2\ln\left(\frac{1+\phi_{HE}^{-1}}{2}\right) \end{aligned} \tag{A11}$$

were used the constants being $\gamma_1 \simeq \gamma_2 \simeq 15$ to 16. Results by Högström (1988) suggest slightly different values of $\gamma_1 = 19.3$ and $\gamma_2 = 12$.

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