# NOTES AND CORRESPONDENCE

## Relationship between the Monin–Obukhov Stability Parameter and the Bulk Richardson Number at Sea under Unstable Conditions, Derived From a Turbulence-Closure Model

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ABSTRACT

Relationships between the Monin–Obukhov stability parameter and the bulk Richardson number are useful for explicit calculation of fluxes through the air–sea interface. The most straightforward method assumes simple proportionality and is supported well by experimental evidence in a recent work by Grachev and Fairall. On the other hand, the reference iterative method is regarded as more accurate. To recognize possible differences, calculated values of the proportionality factor as a function of wind speed and air–sea virtual potential temperature difference are shown. The calculation is based on commonly used sea roughness specifications and the Mellor–Yamada turbulence-closure model, which has been shown in previous studies to reproduce the three-sublayer structure of the atmospheric surface layer under convective conditions and to predict mean flow profiles that are consistent with empirical data. The results show that the proportionality factor varies with the wind speed and virtual potential temperature vertical differences and that this variability has a nonmonotonic character when wind speeds are smaller than 5 m s$^{-1}$.

### 1. Introduction

Calculation of the vertical fluxes of momentum, heat, and water vapor in the atmospheric surface layer, either in numerical models or other studies, is usually handled with algorithms based on the Monin–Obukhov theory. In a general situation, a set of nonlinear equations (e.g., Nickerson and Smiley 1975; Berkowicz and Prahm 1982) is solved using an iterative technique. Explicit approximate methods (e.g., Launiainen 1995; De Bruin et al. 2000) are also used. These methods are usually based on relationships between the Monin–Obukhov stability parameter $z/L$ and the bulk Richardson number $\mathrm{Ri}_B$ and are designed to work in most common situations.

The Monin–Obukhov theory generalizes the logarithmic wall law to thermally stratified flows in the surface layer. Over the sea, the stratification effects are usually less important than over the land. However, in some situations they are dominant. During winter outbreaks in coastal regions, cold air is advected over much warmer coastal waters and a large temperature difference is maintained despite the strong wind. Also, frequent cases of weak stratification contribute considerably to climatic budgets. Over the ‘‘warm pool’’ of the equatorial western Pacific Ocean, weak winds accompany convection over the sea. Both situations have significant importance: air–sea energy exchange in the outbreaks plays an important role in forecasting winter coastal storms, and the convective regime in the warm pool is crucial for climate and interannual climate variability studies. The estimation of the surface layer transfer in convective, weak wind conditions enjoyed some popularity in recent years, with two approaches forming: 1) the concept of a three-sublayer structure of the surface layer and 2) the wind gustiness correction, accounting for the large convective eddies penetrating into the surface layer. For a broader discussion of this matter, the reader is referred to Łobocki (2001a) and to the literature referenced therein. Note that these two approaches are not mutually exclusive and are sometimes combined (e.g., Grachev et al. 1997).

Although a majority of the aforementioned algorithms are based on empirical universal functions of the Monin–Obukhov theory, bulk relationships can be also derived from turbulence-closure models (Łobocki 1993, 2001a,b). It is interesting that the solution of the popular Mellor–Yamada level-2 model (Mellor and Yamada 1974) is capable of predicting the three-sublayer structure in strongly

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unstable conditions (Łobocki 2001a). In the former papers (Łobocki 1993, 2001b), explicit algorithms based on the model solutions were proposed that assume fixed aerodynamic roughness of the surface. The purpose of this note is to present the results of such an approach applied to the atmospheric surface layer over the sea.

When the wind speed $u$ and the virtual potential temperature $\theta_v = \theta(1 + 0.61q)$ (where $\theta$ is the potential temperature and $q$ is the specific humidity) are given at two heights $z_1$ and $z_2$, the subject relationship can be derived from the Monin–Obukhov theory (e.g., Berkowicz and Prahm 1982; Łobocki 1993):

$$\frac{z_2 - z_1}{L} = \frac{\frac{g}{T_{v0}}(z_2 - z_1)[\theta_v(z_2) - \theta_v(z_1)]}{[u(z_2) - u_1(z_1)]^2} \frac{\left[\ln\left(\frac{z_2}{z_1}\right) + \Psi_u\left(\frac{z_2}{L}\right) - \Psi_u\left(\frac{z_1}{L}\right)\right]^2}{\varphi_{\theta 0}\left[\ln\left(\frac{z_2}{z_1}\right) + \Psi_\theta\left(\frac{z_2}{L}\right) - \Psi_\theta\left(\frac{z_1}{L}\right)\right]}. \quad (1)$$

In the above, $L = (-\overline{u'w'})^{3/2}T_{v0}/\kappa g\overline{w'\theta_v'}$ is the Monin–Obukhov length, $\overline{u'w'}$ is the vertical kinematic momentum flux, $T_{v0}$ is the reference virtual temperature, $\kappa$ is the von Kármán constant, $g$ is the acceleration due to gravity, $\overline{w'\theta_v'}$ is the vertical kinematic virtual potential temperature flux, $\varphi_{\theta 0}$ is the (limit) value of the Monin–Obukhov dimensionless temperature gradient $\varphi_\theta$ at the neutral equilibrium, and $\Psi_u$ and $\Psi_\theta$ are the Monin–Obukhov diabatic corrections to the neutral (logarithmic) profiles for wind speed and temperature, respectively. For further explanations and details, readers are referred to Łobocki (1993). The first term on the right-hand side of (1) may be recognized as the bulk Richardson number $\mathrm{Ri}_B$.

In (1), it is assumed that eddy transfer diffusivities for scalar quantities (heat, water vapor) have identical values at a given level, or, equivalently, that the dimensionless (Monin–Obukhov scaled) temperature and humidity vertical gradients are the same. This simplification will be used throughout the paper. Furthermore, other possible differences between heat and water vapor transport mechanisms (e.g., values of roughness parameters for heat and water vapor) will also be neglected.

When $z_1$ is small (e.g., when surface roughness is used), the simplest way to approximate the relationship (1) is to take a simple proportionality following Deardorff (1968), that is,

$$z/L = C\mathrm{Ri}_B, \quad (2)$$

where $z$ is the measurement height, $\mathrm{Ri}_B = gz\Delta\theta_v/T_{v0}U^2$ is the bulk Richardson number, $U$ is the wind speed, $\Delta\theta_v$ is the virtual potential temperature difference between the measurement level and the sea surface (hereinafter referred to as air–sea virtual potential temperature difference), and $C$ is a dimensionless proportionality factor. Often, this simplification is adequate at sea, where logarithmic terms in (1) dominate over stability corrections.

The simplicity of (2) is tempting; however, the variability of $C$ cannot be neglected when stability effects are important. Using values of bulk transfer coefficients adopted from the literature, Hsu (1989) produced a table containing $C$ values for different wind speeds and air–sea virtual potential temperature differences. In unstable conditions, these values varied from 4.2 to 12.83 within the 1–25 m s$^{-1}$ of the 10-m wind speed and 1–20 K of the virtual potential temperature ranges. Launiainen (1995) derived an approximate formula with the $C$ factor depending on $z/z_0$ and $z/z_{0T}$ ratios ($z_0$ and $z_{0T}$ are the aerodynamic and thermal roughnesses, respectively); for the sea surface, he recommended a constant value of 11 for oceanic conditions. De Bruin et al. (2000) obtained $C$ as dependent on the bulk Richardson number; however, they did not specifically address the marine conditions. Extensive empirical material from the San Clemente Island Ocean Probing Experiment (SCOPE) and Tropical Ocean and Global Atmosphere (TOGA) Coupled Ocean–Atmosphere Response Experiment (COARE) has been used recently by Grachev and Fairall (1997) to derive relationships between $z/L$ and $\mathrm{Ri}_B$, with special attention paid to the light wind, unstable conditions. For these conditions, they also assumed proportionality (2); however, the wind gustiness effects were considered, involving the surface layer scales and the bulk Richardson number. Consideration of these effects under typical TOGA COARE conditions (assuming a fixed typical value of the boundary layer height) led to the formula:

$$z/L = C\mathrm{Ri}_B(1 + \mathrm{Ri}_B/\mathrm{Ri}_{BS})^{-1}, \quad (3)$$

where $\mathrm{Ri}_{BS}$ is a ''saturation'' bulk Richardson number that is dependent on the boundary layer height but is treated as a fixed value by Grachev and Fairall. The proposed correction was shown to improve the data fit at the extremely unstable conditions.

## 2. $\mathrm{Ri}_B$ and $z/L$ relationships

The surface-layer flux-profile relationships employed here are adopted from Łobocki (2001a) [his (79)–(80)]:

$$U = -\overline{u'w'}\left[\frac{I_{ud}(z_0, |L|, z)}{u_*} + \frac{I_{uc}(z_0, L, z)}{\overline{w'\theta_v'}g/T_{0v}}\right] \quad \text{and} \tag{4}$$

$$\Delta\theta_v = -\overline{w'\theta_v'}\left[\frac{I_{\theta d}(z_{0T}, |L|, z)}{u_*} + \frac{I_{\theta c}(z_{0T}, |L|, z)}{\overline{w'\theta_v'}g/T_{0v}}\right], \tag{5}$$

where $u_* = |\overline{u'w'}|^{1/2}$ is the friction velocity and $I_{ud}(z_0, |L|, z)$, $I_{uc}(z_0, |L|, z)$, $I_{\theta d}(z_{0T}, |L|, z)$, and $I_{\theta c}(z_{0T}, |L|, z)$, are integrals representing the lower (dynamic and transitional sublayer) and the upper (transitional and convective) parts of the wind speed and virtual potential temperature profiles, respectively. Depending on stability, either one of the two sublayers, or both of them, may be present. For example, when $z_{0T} < |L| < z$, $I_{\theta c} = 0$ and the virtual potential temperature profile is given by

$$I_{\theta d}(z_{0T}, |L|, z) = \frac{1}{\kappa\varphi_{0\theta}}\left[\ln\frac{z}{z_{0T}} + \Psi_\theta\left(\frac{z}{L}\right) - \Psi_\theta\left(\frac{z_{0T}}{L}\right)\right]. \tag{6}$$

A similar expression holds true for $I_{ud}$. Using solutions of a simplified second-order turbulence closure model (the Mellor–Yamada level-2 model), Łobocki (2001a) obtained approximated expressions allowing straightforward calculation of the integrals $I_{Xx}$ (here, $X = u, \theta$, $x = c, d$), by (6) and

$$\Psi_u(\zeta) = 0.6783\zeta^3 + 1.8673\zeta^2 + 2.5408\zeta, \tag{7}$$

$$\Psi_\theta(\zeta) = 0.3889\zeta^3 + 1.2488\zeta^2 + 2.0358\zeta, \tag{8}$$

and

$$I_{Xc}(z_1, |L|, z_2) = I'_{Xc}(z_2, z_2/L) - I'_{Xc}(z_1, z_1/L),$$
$$I'_{Xc}(z, \zeta) = \alpha_{Xc}z^{-1/3}\frac{1 - (-\zeta)^{-1/3}}{(-\zeta)^{-1/3}}, \tag{9}$$

where $\alpha_{uc} = 3.1$ and $\alpha_{\theta c} = 2.393$ are numerical constants determined by the model constants.

Within the empirical approach, Fairall et al. (1996) used empirical functions for the lower and upper parts of profiles and applied a blending procedure to achieve dimensionless gradients consistent with the three-sublayer concept, expressed as functions of the Monin–Obukhov parameter $z/L$. In (4)–(5), however, two different scalings are applied concurrently; the local convective scaling is directly obtained when the ''dynamic'' integrals in brackets on the right-hand side are zero (or when the free-convection profile extends down to the surface).

Equations (4)–(9), together with the definition of the Monin–Obukhov length, the bulk Richardson number, and the expressions determining the dynamic and thermal roughnesses, form a set of equations from which the desired relationship can be derived. The standard treatment adopted here is to use the Charnock formula, modified to include smooth flow effects as by Smith (1988):

$$z_0 = 0.018\frac{u_*^2}{g} + 0.11\frac{\nu}{u_*} \tag{10}$$

and to compute the thermal roughness parameter as in Liu et al. (1979) by

$$\frac{z_{0T}u_*}{\nu} = a\left(\frac{z_0u_*}{\nu}\right)^b, \tag{11}$$

where $\nu$ is the kinematic viscosity of the air and $a$ and $b$ are numerical constants that are specified for several intervals of $z_0u_*/\nu$ and tabulated in Liu et al. (1979).

Once the value of the Monin–Obukhov length is found, fluxes can be explicitly calculated from (4)–(5). Therefore, the key to the solution process is calculating $z/L$ from $\mathrm{Ri}_B$. However, because of the dependence of the roughness on friction velocity in (10), $z/L$ depends not only on $\mathrm{Ri}_B$, but also on an additional parameter. Therefore, $C$ is a function of two arguments that can be chosen with some freedom. Figure 1a displays $C$ as a function of the wind speed $U$ and the air–sea virtual potential temperature difference $\Delta\theta_v$, as obtained by solving the system (4)–(11) (10-m measurement height was used in these calculations). We restrict the discussion to the range of light and moderate winds, thereby excluding the effects of the presence of spray, which begin at around 10 m s$^{-1}$, and to a moderate air–sea virtual potential temperature difference to exclude possible effects of fog formation. For wind speeds larger than 5 m s$^{-1}$, the computed value of $C$ decreases with the growing wind speed at a rate of 0.5–1 per 1 m s$^{-1}$ and of 0.05 per 1 K. This behavior corresponds well with the Hsu (1989) table, in which $C = 8.02$ for $U = 10$ m s$^{-1}$ and $\Delta\theta_v = -10$ K, $C = 8.23$ for $U = 10$ m s$^{-1}$ and $\Delta\theta_v = -5$ K, and $C = 10.72$ for $U = 5$ m s$^{-1}$ and $\Delta\theta_v = -5$ K. In the same region, the Grachev–Fairall parameterization produces much smaller variations of $C$ with growing wind speed (Fig. 1b), however, the dependence on the air–sea virtual potential temperature difference has the opposite character. At around 3–4 m s$^{-1}$, a maximum in the dependence of $C$ on wind speed appears in the results, with $C$ values varying from around 11 to 14; this feature is absent in Fig. 1b. For lower wind speeds, the $C$ factor diminishes with decreasing wind speed, which is qualitatively in agreement with the Grachev–Fairall (1997) results; however, this decrease is less intense. This is understandable, recalling that no wind gustiness correction is used here. Introducing this correction into the model would lead to further decrease of $C$ at low wind speeds because it would be divided by $(1 + \mathrm{Ri}_B/\mathrm{Ri}_{Bc})$.

Figure 2 displays the $C$ dependence on $U$ and $\Delta\theta_v$ as inferred from the SCOPE and TOGA COARE experiments [see Grachev and Fairall (1997) for a relevant description]. Here, the $U$–$\Delta\theta_v$ domain was divided into regularly spaced averaging bins (1 m s$^{-1}$ by 0.5 K), where bin-averaged $C$ values were computed from individual measurements falling into the bin. The dataset was preedited to exclude measurements when 1) the wind direction was outside the $\pm 90°$ range relative to the ship course, 2) the rain rate was over 0.5 mm h$^{-1}$,

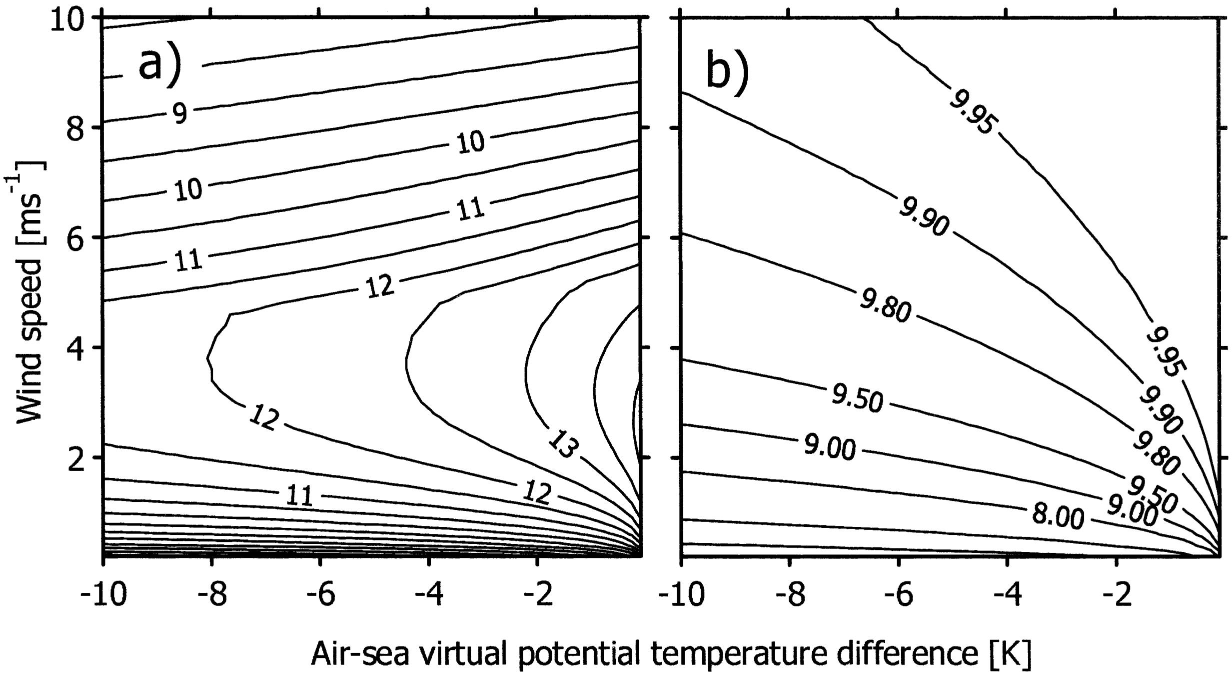


FIG. 1. Proportionality coefficient $C$ in (2) as a function of the 10-m wind speed $U$ and the virtual potential temperature difference $\Delta\theta_v$ between the 10-m height and sea surface as calculated (a) from the equation set (4)–(11) described in this paper and (b) by the Grachev–Fairall method (3). Note that contour lines in (b) also correspond to fixed values of the bulk Richardson number.

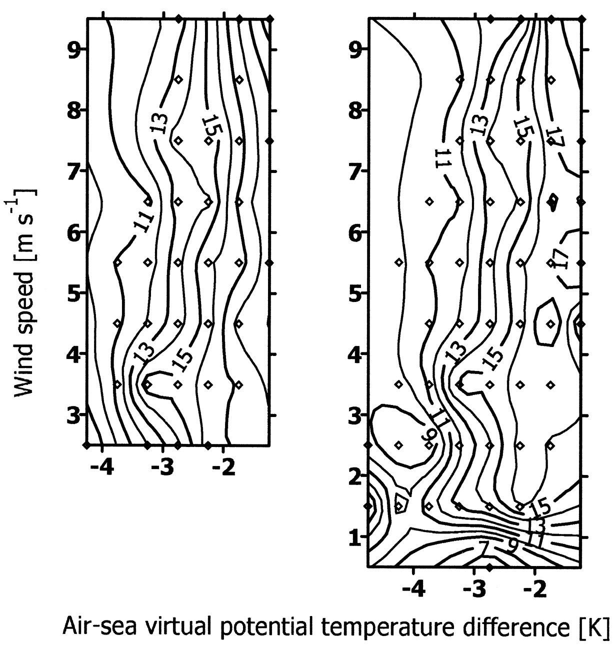


FIG. 2. Proportionality coefficient $C$ in (2) as calculated from the SCOPE and the TOGA COARE data (see description in text), obtained using bin-averaged values for which the 95% confidence interval bounds for the mean were within (left) 20% of the mean or (right) a factor of 3. Open diamonds mark locations of the bins (center points) from which qualified data were used.

3) the data were flagged for the ship plume or maneuver contamination, or 4) the mean wind vector tilt exceeded 10°. Next, individual $C$ values were obtained using fluxes measured by the inertial-dissipation method, tallied into averaging bins, and averaged. Averaging of ln $C$ was chosen (the geometric mean of $C$), to reflect better the nature of this parameter; also, the $C$ probability distribution is close to normal. Only a part of the domain of Fig. 1 was covered by the data. For every bin, 95% confidence intervals for the mean value were calculated. Because of the large scatter and data scarcity in some regions, many of the intervals were excessively wide. The left-hand panel in Fig. 2 was produced using means for which the confidence bounds were not farther away than 20% of the average; the remaining data were discarded. Hence, the plot was made with a limited number of points for which the mean appeared to be relatively precise. On the other pole, in the right-hand panel, 300% relative uncertainty was allowed; that is, the actual mean could have been 3 times greater or smaller than the estimate, assuming a 95% probability level. This allows some insight into the low-wind-regime area, yet these results are uncertain.

The most prominent feature of Fig. 2 is that $C$ diminishes with growing buoyant instability over most of the domain. This feature is only partially reflected in model results and is even of the opposite character in the Grachev–Fairall simplified computational method. Also, $C$ values are underrated by both methods. A weakly discernible ridge at $U = 3.5$ m s$^{-1}$ may be seen as

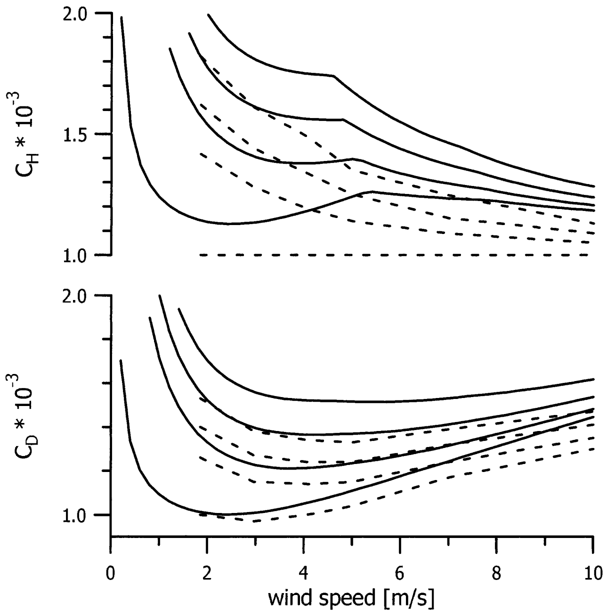


FIG. 3. Bulk transfer coefficients (bottom) $C_D$ and (top) $C_H$, as a function of the 10-m wind speed, for four different values of the air–sea virtual potential temperature difference: −10, −5, −2, and 0 K (from the uppermost to the lowest curves, respectively). The solid line is for the current model, and the dashed line is Smith (1988).

corresponding to the one in model prediction. In the ''uncertain'' part of the domain, below 2 m $s^{-1}$ (right-hand panel), a branching of this ridge is likely (supported by the data from five bins). The sharp decrease with ceasing wind in the region below 1 m $s^{-1}$ corresponds to the smooth flow regime represented by (10) in the model.

Bulk transfer coefficients $C_D = -\overline{u'w'}/U^2$ and $C_H = -\overline{w'\theta'_v}/(U\Delta\theta_v)$, calculated from the model, are presented in Fig. 3 as a function of wind speed at 10-m height for different values of the air–sea virtual potential temperature difference, in comparison with Smith (1988). Both of the two coefficients are slightly (typically by 10%) higher than Smith's; the difference at low wind speeds is due to Liu et al.'s (1979) thermal roughness parameterization. Also, the thermal instability influence on the bulk transfer coefficients is stronger in the current model.

## 3. Conclusions

The proportionality factor in the relationship between $z/L$ and $\mathrm{Ri}_B$ was investigated using approximate solutions of a turbulence-closure model, compatible with the empirical Monin–Obukhov universal functions and the three-sublayer structure of the atmospheric surface layer in convective conditions. The aerodynamic and thermal roughness of the sea is parameterized using the Charnock formula as modified by Smith (1988) and Liu et al. (1979). For wind speeds ranging from 5 to 10 m $s^{-1}$, the dependence of this factor on wind speed and air–sea virtual potential temperature difference agrees qualitatively and quantitatively with Hsu's (1989) table. For weak winds, there is a decreasing tendency of the proportionality factor with diminishing wind speed, which seems qualitatively similar to the gustiness effects in Grachev and Fairall (1997). Experimental data are still too sparse and too scattered to allow for a precise and credible fitting/verification in the two-dimensional parameter space. The model results reflect part of the observed variability and show some improvement over estimations made with the simpler form (3).

Some problems related to the model should be mentioned and possibly addressed in further studies. The thermal roughness parameter may not necessarily have the same meaning and value under purely free convection as it does in other cases, when the dynamic sublayer is present. Also, the model predictions differ from some studies, in which the free convection heat flux over smooth surfaces is predicted to be proportional to the 4/3 power of temperature difference (such as Kondo and Ishida 1997). The smooth flow conditions, represented by the second term on the right-hand side of (10), invite singularity when $u_* = 0$, which is circumvented when the gustiness concept is used.

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