Modeling of the Stably Stratified Atmospheric Boundary Layer

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ABSTRACT

The similarity which exists between the stably stratified atmospheric boundary layer and a wind tunnel boundary layer developing over a cold plate is illustrated. It is shown that mean flow and turbulence characteristics in the near wall region of the stratified boundary layer are well described by Monin and Obukhov's similarity theory, and that this theory provides a good basis for modeling in the laboratory of similar characteristics of the atmospheric surface layer. Various forms of stability parameters are shown to be universally related.

1. Introduction

It has long been known that certain dynamic characteristics of the airflow in the atmospheric boundary layer can be studied with advantage by scaling the atmospheric layer in a wind tunnel. Batchelor (1953) has given some encouragement to the view that the invariance of a single parameter, based on the stratification characteristics such as Richardson number, might suffice to guarantee dynamic similarity of stratified boundary layers in model and field. On a similar premise, Monin and Obukhov (1954) provided a theory according to which all profiles of mean and turbulent velocity characteristics are assumed to be universal functions scaled by a stability length. This theory, which will be outlined further, has provided the framework for most analyses of atmospheric boundary layer data. If it can be shown that the same universal shape functions are valid for the laboratory data also, then the credibility of the similarity theory is greatly enhanced, and at the same time wind tunnel modeling of the atmospheric boundary layer is shown to be possible. For this reason, the thermally stratified boundary layer has been studied in the U. S. Army meteorological wind tunnel at Colorado State University.

The present study is distinguished from two earlier ones, by Plate and Lin (1966) and Chuang and Cermak (1966), by being based on a more complete set of data obtained by Arya (1968) with a more sophisticated and accurate measuring procedure. These data will be discussed in the light of Monin and Obukhov's similarity theory, and compared with similar measurements in the atmospheric surface layer. Since only that portion of the boundary layer in which the shear stress is approximately constant has been studied extensively in the atmosphere, the discussion of the laboratory data is restricted to data obtained in the lowest 15% of the stratified layer. In the large test section of Colorado State University's meteorological wind tunnel, boundary layer thicknesses of about 70 cm were obtained so that the thickness of the region of interest is approximately 11 cm.

2. The similarity theory

The similarity theory of Monin and Obukhov is based on the assumptions that the flow is plane-homogeneous and that vertical fluxes are constant. Furthermore, the pertinent independent variables are assumed to be the height z, the density ρ_0, the wall shear stress τ_0, the wall heat flux H_0, and the stability parameter g/T_a. The roughness height z_0 and molecular properties μ and k are neglected because consideration is limited to the region $z \gg z_0 > 0$. From the above variables the following velocity, temperature and length scales are obtained:

The shear velocity u_*:

$$u_* = (\tau_0/\rho_0)^{\frac{1}{2}} \tag{1}$$

The friction temperature T_*:

$$T_* = -H_0/(\rho_0 C_p \kappa u_*) \tag{2}$$

The stability length L:

$$L = -u_*^3 \bigg/ \left[\kappa \frac{g}{T_a} \frac{H_0}{\rho_0 C_p}\right] \tag{3}$$

The quantities u_*, T_* and L are assumed to specify the dynamics of the flow field. In the above, κ is von Kármán's constant, C_p the specific heat of air at constant pressure, and T_a the absolute average temperature of the layer under consideration. All the mean flow and turbulent quantities when nondimensionalized by a proper combination of u_*, T_* and L must be universal functions of the stability ratio z/L. Thus, for the mean velocity and temperature distribution the theory predicts

$$S=\frac{\kappa z}{u_*}\frac{\partial U}{\partial z}=\phi\left(\frac{z}{L}\right), \tag{4}$$

$$R=\frac{z}{T_*}\frac{\partial T}{\partial z}=\phi_T\left(\frac{z}{L}\right), \tag{5}$$

which on integration give

$$U(z)-U(z_{\text{ref}})=\frac{u_*}{\kappa}\left[f\left(\frac{z}{L}\right)-f\left(\frac{z_{\text{ref}}}{L}\right)\right], \tag{6}$$

$$T(z)-T(z_{\text{ref}})=T_*\left[f_T\left(\frac{z}{L}\right)-f_T\left(\frac{z_{\text{ref}}}{L}\right)\right]. \tag{7}$$

In Eqs. (6) and (7) z_{ref} is a reference height which may be taken as some appropriate fraction of L. The so-called log-linear law has been suggested as a first approximation of Eqs. (6) and (7); i.e.,

$$U(z)-U(z_{\text{ref}})=\frac{u_*}{\kappa}\left[\ln\left(\frac{z}{z_{\text{ref}}}\right)+\beta\left(\frac{z-z_{\text{ref}}}{L}\right)\right], \tag{8}$$

$$T(z)-T(z_{\text{ref}})=T_*\left[\ln\left(\frac{z}{z_{\text{ref}}}\right)+\beta_T\left(\frac{z-z_{\text{ref}}}{L}\right)\right], \tag{9}$$

in which β and β_T are empirical constants which may depend on the range of z/L (see Taylor, 1960; Monin and Yaglom, 1965). In this form, the equations are free from the effect of the surface roughness z_0, which would correspond to the case where $z_{\text{ref}}=z_0$, $U_{\text{ref}}=0$. By writing the equation for a difference, the problem of deciding whether z_0 is a function of stability is avoided.

The similarity theory further predicts that the Richardson number Ri given to

$$\text{Ri}=\frac{g}{T}\frac{(\partial T/\partial z)}{(\partial U/\partial z)^2}=F_1\left(\frac{z}{L}\right), \tag{10}$$

as well as the flux Richardson number

$$\text{Rf}=\frac{k_H}{k_M}\text{Ri}=\frac{g}{T}\frac{\overline{w't'}}{\overline{u'w'}(\partial U/\partial z)}=F_2\left(\frac{z}{L}\right), \tag{11}$$

are universal functions of z/L. Consequently, it follows also that

$$\frac{k_H}{k_M}=F_3\left(\frac{z}{L}\right). \tag{12}$$

Recently, Monin and Yaglom (1965) have extended the theory to include many other statistical characteristics of wind and temperature fields, e.g., second- or higher-order moments, spectral functions, etc.

Universal functions, $\phi(z/L)$, $f_i(z/L)$, $F_i(z/L)$, etc., cannot be predicted by the similarity theory alone. It has not been possible to determine the form of the functions from other theoretical considerations. Only their behaviour in the asymptotic sense of $z/L\rightarrow\pm\infty$ has been predicted in some cases. Therefore, the theory must necessarily be supplemented by experimental work. Much of the atmospheric data has generally been shown to support the similarity theory [for a complete review, see Monin and Yaglom (1965)]. But, large scatter in data partly due to difficulties of measurements and partly to variable conditions in the atmosphere itself, make it almost impossible to check some finer points of the theory and bring out the precise form of the universal functions. It is felt, after creating the proper conditions of the theoretical model in the wind tunnel, that controlled laboratory measurements can be used for this purpose.

3. Experimental arrangement

Experiments were performed in the U. S. Army meteorological wind tunnel (Plate and Cermak, 1963) at Colorado State University. The zero pressure gradient boundary layer develops along the floor of the 25 m long test section with $1.8\times1.8\ \text{m}^2$ cross section. Over the last 12.5 m of the test section, the floor consists of an aluminum plate. There are arrangements for heating or cooling of the aluminum floor as well as of the air for obtaining different stability conditions. During the present study, the temperature varied from ~4C at the floor to ~50C in the ambient air. Boundary layers for ambient air velocities of about 9, 6 and 3 m sec^{-1} were studied covering a range of stabilities from near neutral to moderately stable. Measurements were made at a distance of 24 m from the leading edge, where a boundary layer thickness of about 70 cm was obtained.

The mean velocities were measured with a 3.2 mm diameter standard pitot-static tube and mean temperature with a copper-constantan thermocouple. An elaborate hot wire technique (Arya and Plate, 1968; Arya, 1968) was employed to measure turbulent intensities and fluxes free of error due to the presence of large temperature fluctuations.

4. Results

The basic assumptions underlying the similarity theory are those of plane-homogeneity and constant fluxes. Plane-homogeneity in the sense that velocity and temperature profiles do not change noticeably with distance (in the direction of flow) has been approximately realized in our experiments, where the boundary layer was allowed to develop over a long distance (24 m). The thickness of the constant flux layer was very small. However, it has been suggested by Monin and Yaglom (1965) that the distribution of mean flow

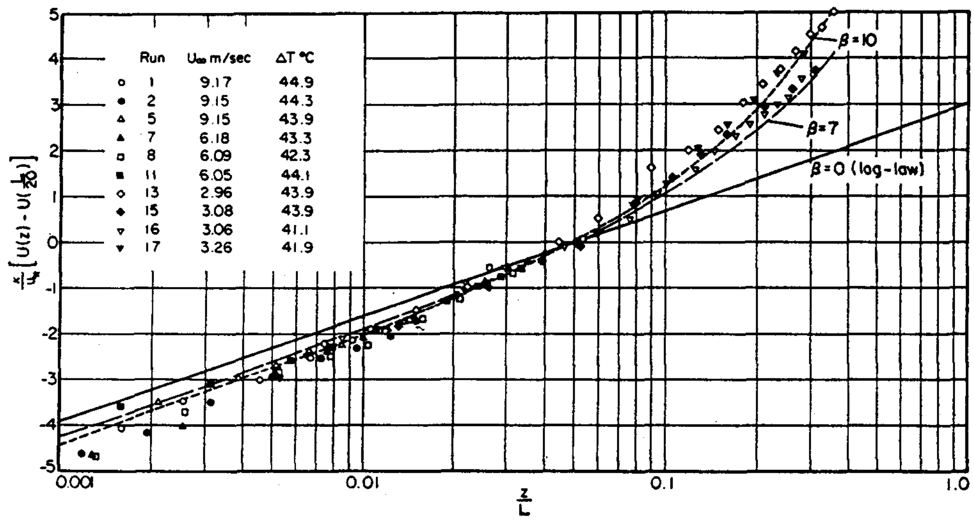

FIG. 1. Mean velocity distribution in relation to log-linear law.

fields and integral turbulent characteristics may not be very sensitive to even significant changes in fluxes. For example, it is known that experimental data on velocity profiles in pipe and neutral boundary layer flows follow a logarithmic law over a wide region, notwithstanding changes in momentum flux, even though the logarithmic law should hold only in constant momentum flux layers. For this reason, data from a layer of $0.01 \leq z/\delta \leq 0.15$ has been considered even though fluxes varied up to 50% in this layer.

Measured mean velocity and temperature profiles are shown in non-dimensional form in Figs. 1 and 2. The reference height was chosen as $z_{ref}=0.05L$. The friction temperature T_* and friction velocity u_* required for calculating L were found by extrapolating from measured fluxes the values for the wall. These values were found in good agreement with those determined from profiles slopes by assuming the lower part of the profiles represented by the logarithmic law of the wall. The values of u_* obtained by these methods are compared in Table 1. Also listed are values of u_* which are obtained by calculating the wall shear stress from the boundary layer parameters, i.e., the momentum thickness θ and the displacement thickness δ^* using Ludwieg and Tillmann's (1950) formula

$$\tau_0 = 0.246 \rho U_\infty^2 \left(\frac{U_\infty \theta}{\nu}\right)^{-0.268} 10^{-0.678 H_f}, \qquad (13)$$

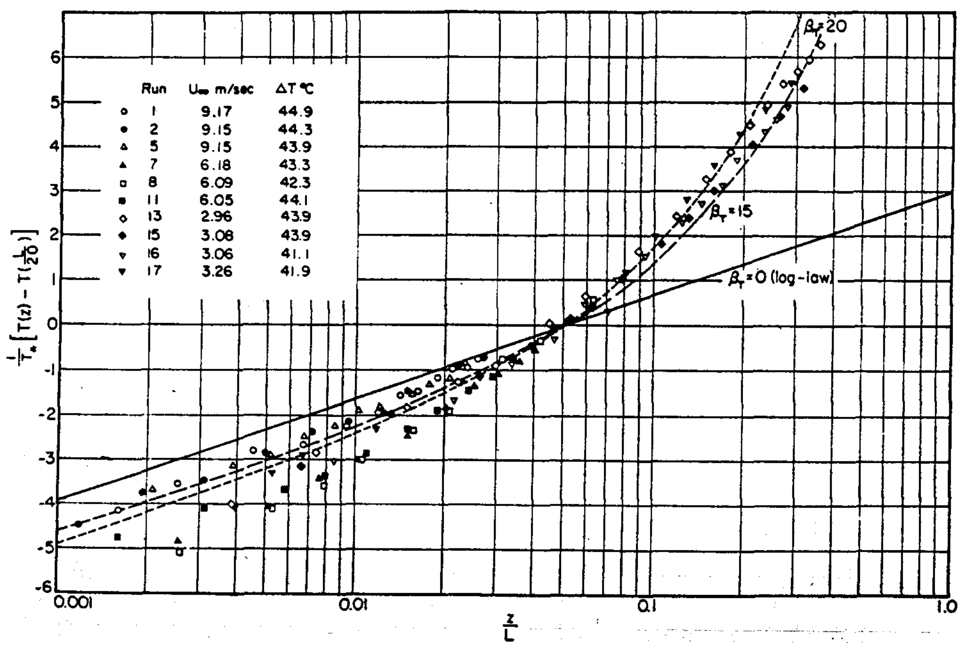

FIG. 2. Mean temperature distribution in relation to log-linear law.

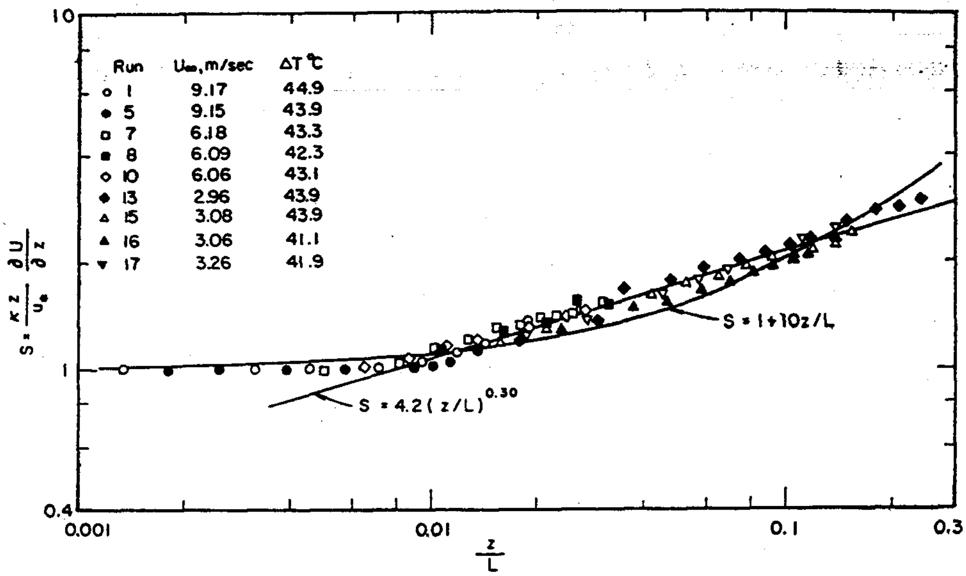

FIG. 3. Dimensionless wind shear coefficient as a function of stability parameter.

where U_∞ is the velocity outside of the boundary layer, θ the momentum thickness, and H_f the form factor. This equation, which is used extensively in boundary layer work, had been used in the study by Plate and Lin (1966). It gives a value of u_* which is substantially too high for large stabilities, as compared with the values found by the other two methods.

The experimental profiles shown in Figs. 1 and 2 are seen to agree well with the shape predicted by the similarity theory. In particular, the data of Fig. 1 agree with the log-linear law [Eq. (8)] with $\beta=10$. This value lies at the upper end of the range of β values between 7 (McVehil, 1964; Gurvich, 1965) and 10 (Zilitinkevich and Chalikov, 1968), reported in the meteorological literature.

TABLE 1. Friction velocities in wind tunnel at various wind speeds.

	u_* (m sec^{-1})		
U_∞ (m sec^{-1})	From flux measurements	From mean velocity profiles	Calculated from Ludwieg and Tillmann's (1950) formula
9.15	0.305	0.295	0.294
6.10	0.178	0.171	0.200
3.26	0.075	...	0.103

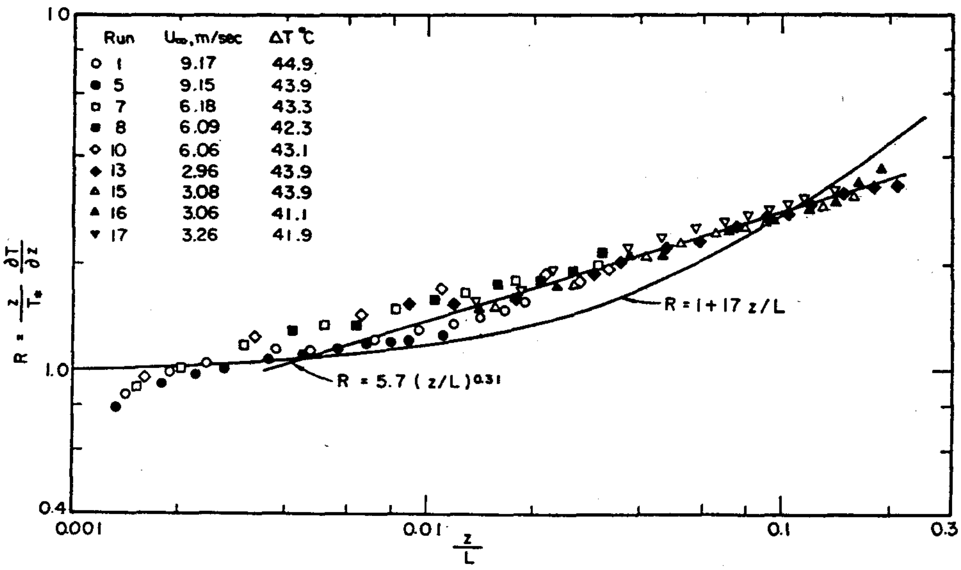

FIG. 4. Dimensionless heat flux coefficient as a function of stability parameter.

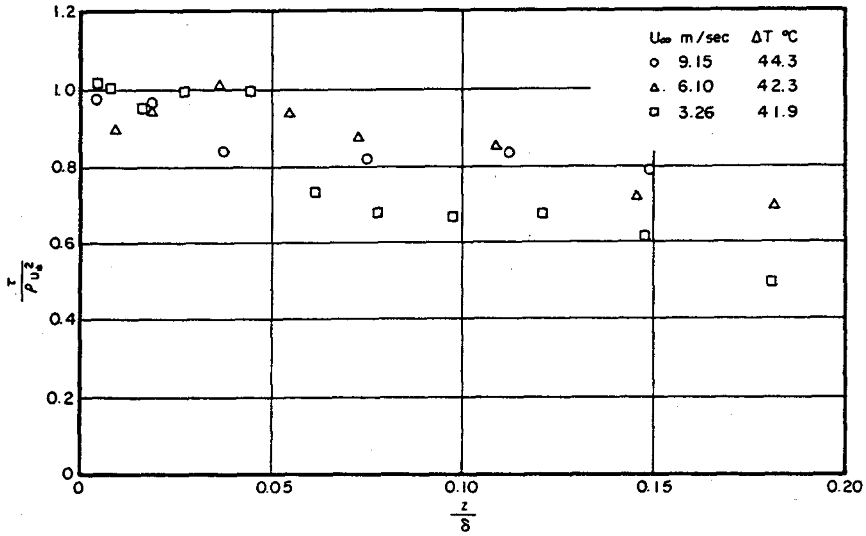

FIG. 5. Variation of shear stress with height.

For β_T a value of about 17 has been observed, agreeing with expectations. In stable conditions β_T is expected to be larger than β because the ratio of exchange coefficients, which is approximately given by

$$\frac{k_H}{k_M}=\frac{1+\beta(z/L)}{1+\beta_T(z/L)}, \tag{14}$$

decreases with increase in stability.

The difference between the values of β observed in the laboratory and in the field can be attributed in part to differences in the range of z/L used, and to errors in u_* and T_*. The latter became apparent in the study of Plate and Lin (1966). They used the shear velocity value based on the Ludwieg-Tillmann equation, and found a value of $\beta=7$, in agreement with McVehil (1964). But corrections for the shear velocity according to the results reported here led to a value which was closer to $\beta=10$.

The effect of the range of z/L is attributed to the fact that the velocity distribution law is only approximately represented by the log-linear law. This is evident if the functions $\phi(z/L)$ and $\phi_T(z/L)$ are considered directly, experimentally determined values of which are shown in Figs. 3 and 4. Some departure from the linear behavior is noticed. In fact, over most of the range z/L of the data, S and R are better represented empirically by the power law relations

$$S=A(z/L)^p, \tag{15}$$

$$R=B(z/L)^q, \tag{16}$$

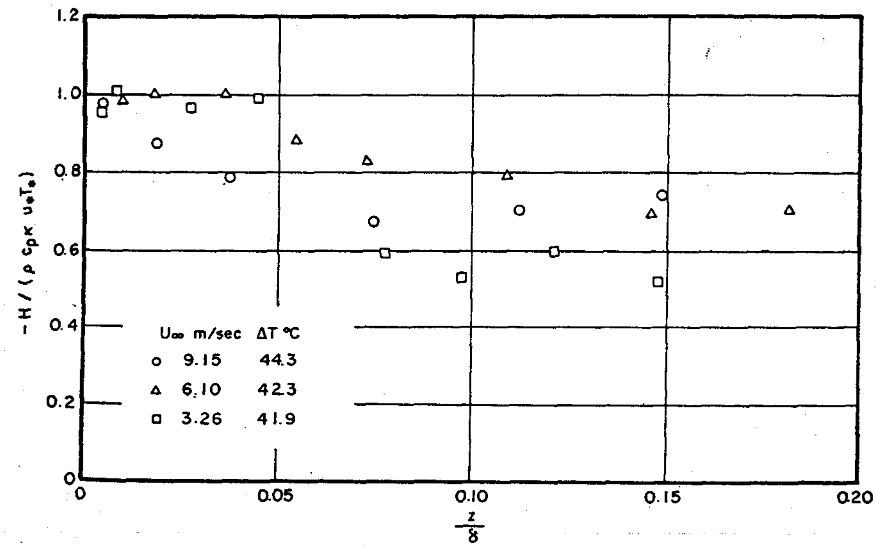

FIG. 6. Variation of heat flux with height.

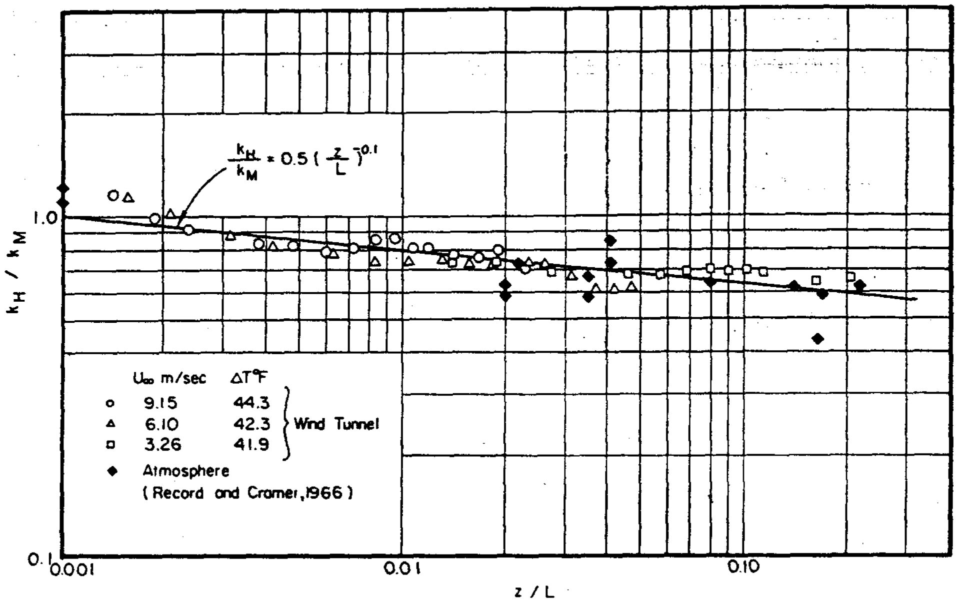

FIG. 7. Ratio of exchange coefficients as a function of stability.

leading also to power law forms of the velocity and temperature distributions. Over the ranges shown, p and q are approximately equal to 0.3. Over other ranges, one would expect to find different values of p and q. Power laws, in a somewhat different form, have been used before (Priestley, 1959), and it has been argued that in the limit, as $z/L \rightarrow 0$, the exponent should reduce to zero, thus leading to the logarithmic law. This factor makes power law representations of the temperature and velocity distributions of limited usefulness.

The measured turbulent fluxes of heat and momentum are shown in Figs. 5 and 6. The molecular terms $\mu(\partial U/\partial z)$ and $k(\partial T/\partial z)$ were added to the hot wire measured fluxes $-\rho\overline{u'w'}$ and $-\rho c_p\overline{w't'}$ to obtain total fluxes. Although the fluxes are far from constant in the near-wall layer, the deviations of the local from the mean over the layer rarely exceeds 20%, thus providing reason for assuming constant fluxes as a first approximation.

The ratio k_H/k_M of the exchange coefficients has been calculated from measured fluxes and is show in Fig. 7 as a function of z/L. It is seen to decrease with increase in stability as expected from theoretical and physical considerations (Ellison, 1957; Stewart, 1959). An empirical power law relation

$$\frac{k_H}{k_M} = 0.5\left(\frac{z}{L}\right)^{-0.1}, \tag{17}$$

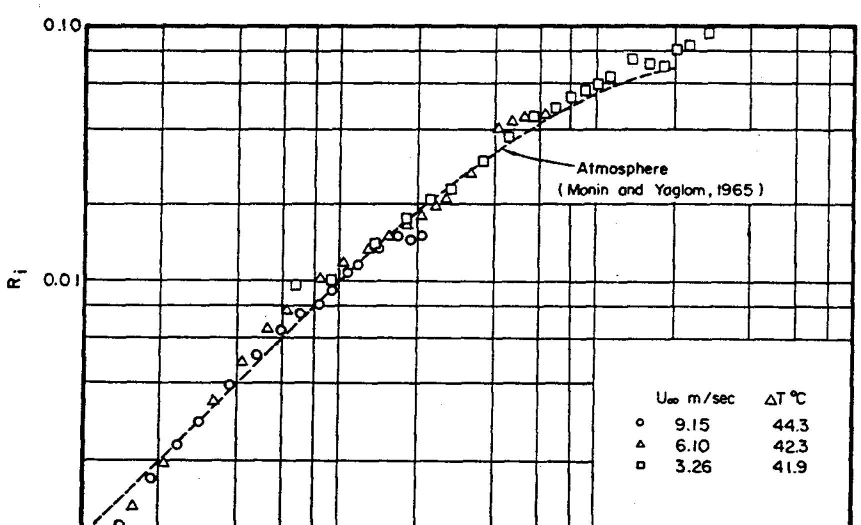

FIG. 8. Ri as a function of z/L.

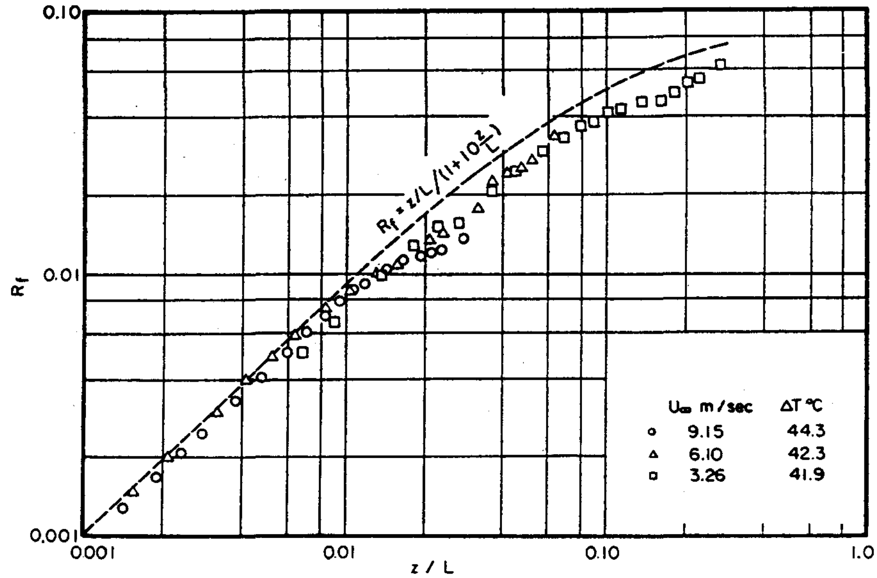

FIG. 9. Rf as a function of z/L.

is seen to fit our data better than Eq. (14) with $\beta_T=17$ and $\beta=10$. For comparison, some measurements in the atmospheric surface layer by Record and Cramer (1966) are also shown in Fig. 7. They scatter in a wide band about the mean curve through our experimental data.

According to Monin and Obukhov's similarity theory, Ri and Rf must be universal functions of z/L. Figs. 8 and 9 show that unique functions of z/L exist for Ri and Rf in our experiment. Whether these functions are really universal can be seen only after comparing data from different sources. No such data are available from other laboratory measurements. However, Monin and Yaglom (1965) have reported a curve Ri vs z/L attributed to Gurvich (1965) which is represented in Fig. 8 for comparison. It agrees remarkably well with our wind tunnel results. There is thus no doubt that Ri and z/L are uniquely related by a universal function. Thus, our results confirm a very important result of the similarity theory, which would allow one to use conveniently measured Ri in place of z/L in all similarity representations.

Normalized turbulent intensities $(\overline{u'^2})^{\frac{1}{2}}/u_*$, etc., and heat transfer and shear stress correlation coefficients $\gamma_{u'w'}$ and $\gamma_{w't'}$ both as functions of Ri are shown in Figs. 10 and 11. Some tendency for these quantities to decrease with increase in stability is seen. The wind tunnel results are compared in Figs. 12 ,13 and 14 with some data from the surface layer of the atmosphere reported by Mordukhovich and Tsvang (1966). Separate symbols have been used for data points at different heights to bring out any height dependency besides that implicit in Ri. The following observations can be made

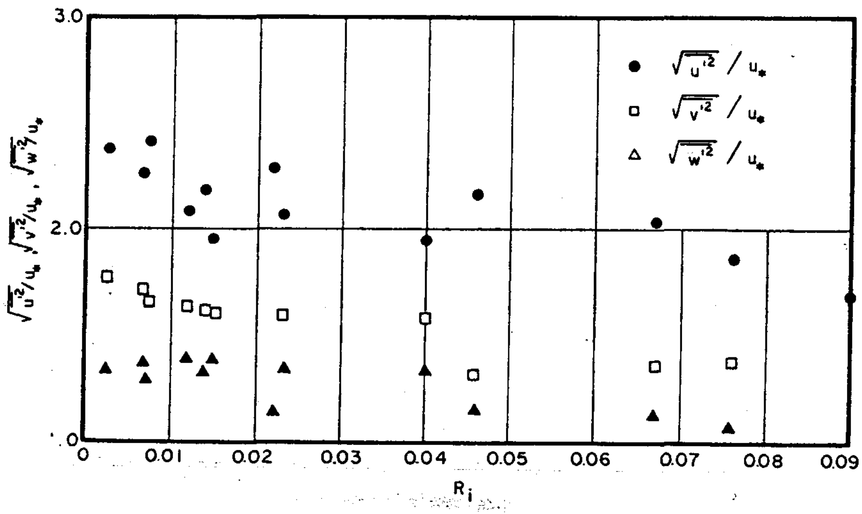

FIG. 10. Variation of $(\overline{u'^2})^{\frac{1}{2}}/u_*$, $(\overline{v'^2})^{\frac{1}{2}}/u_*$ and $(\overline{w'^2})^{\frac{1}{2}}/u_*$ with Ri.

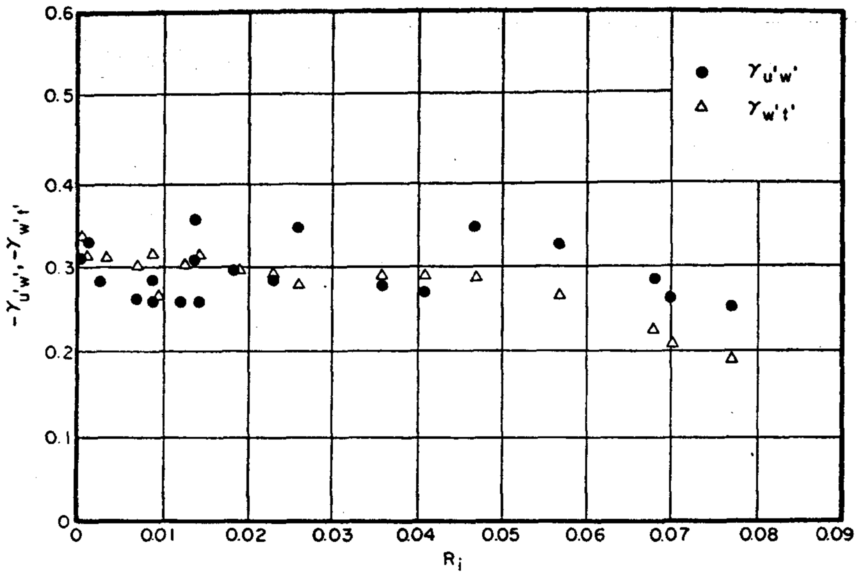

FIG. 11. Variation of correlation coefficients $\gamma_{u'w'}$ and $\gamma_{w't'}$ with Ri.

from this comparison: 1) wind tunnel and atmospheric surface layer data are in good agreement; 2) the latter show too much of scatter to give any precise form of the universal functions, if they exist; and 3), in contradiction with the prediction of the similarity theory, the values of $(\overline{u'^2})^{\frac{1}{2}}/u_*$ and $(\overline{t'^2})^{\frac{1}{2}}/T_*$, both in the wind tunnel and the atmosphere, depend markedly on height. A similar height dependency was observed by Cramer (1967), among others. Calder (1966) reexamined the theoretical basis of the similarity theory and indicated that variances of the horizontal velocity components are not uniquely determined by the same set of scaling parameters that determine the mean velocity and temperature profiles. He also suggests the possibility of a breakdown of the similarity theory for certain of the other characteristics. The height dependence of the experimental data of $(\overline{t'^2})^{\frac{1}{2}}/T_*$ indicates that this quantity may be one of them.

Available field data on the trend of $(\overline{t'^2})^{\frac{1}{2}}/T_*$ are somewhat contradictory. For example, Monin's (1962) results show $(\overline{t'^2})^{\frac{1}{2}}/T_*$ decreasing with increase in stability, while Cramer's data as well as the present wind tunnel data indicate the reverse trend. This serious difference needs to be resolved by more careful field measurements of temperature fluctuations together with those of fluxes.

The theoretical problem of defining a suitable length scale for eliminating the height dependency has at

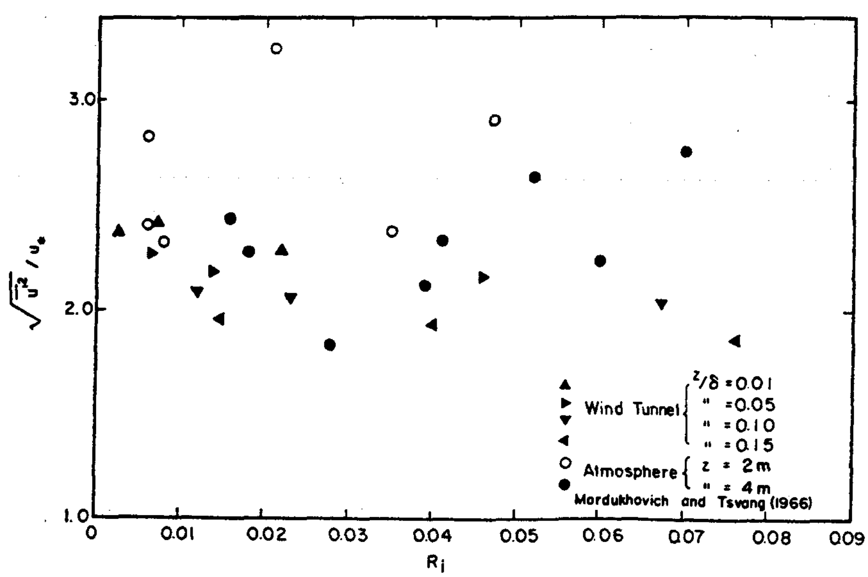

FIG. 12. Comparison of $(\overline{u'^2})^{\frac{1}{2}}/u_*$ with atmospheric data.

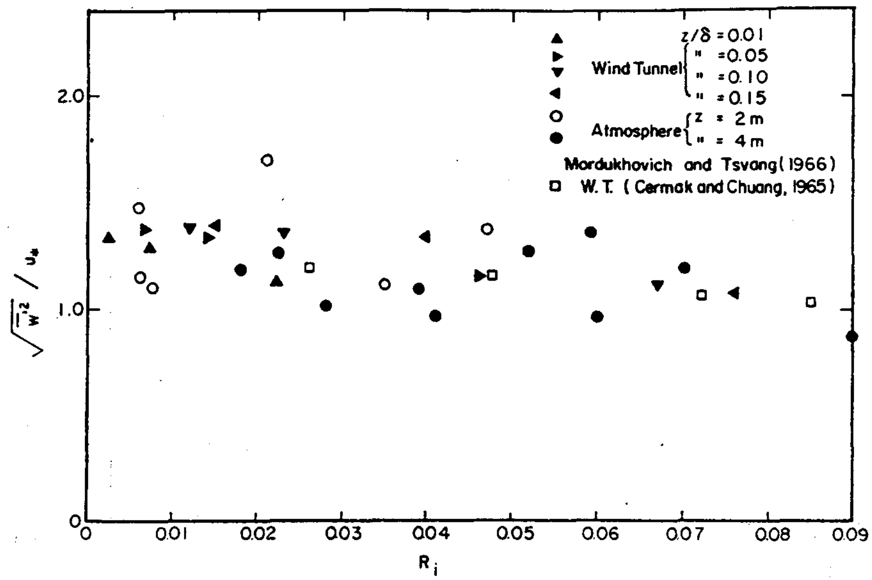

FIG. 13. Comparison of $(\overline{w'^2})^{\frac{1}{2}}/u_*$ for atmospheric data.

present not been solved. But for the purpose of modeling the atmosphere it is encouraging to note that field data and laboratory data scatter about the same mean curve, thus making it likely that the scaling of the stability effect by the length L has also scaled the unknown additional length scale.

5. Conclusions

The experimental data presented in this paper show conclusively what had been suspected before on the basis of laboratory experiments by Plate and Lin (1966) and Chuang and Cermak (1966); namely, that the surface layer of the stably stratified atmosphere can be modeled in the wind tunnel. The modeling must be done in the lower 15–20% of the boundary layer. Modeling requires that for identical profile shapes to exist in the laboratory and in the field the ratio of the height z to the Monin and Obukhov length L must be the same in field and model; i.e., a z/L identity suffices to model both the mean velocity and the mean temperature distribution, when the velocity is nondimensionalized by the shear velocity, and the temperature profile by the shear temperature. As far as the turbulent

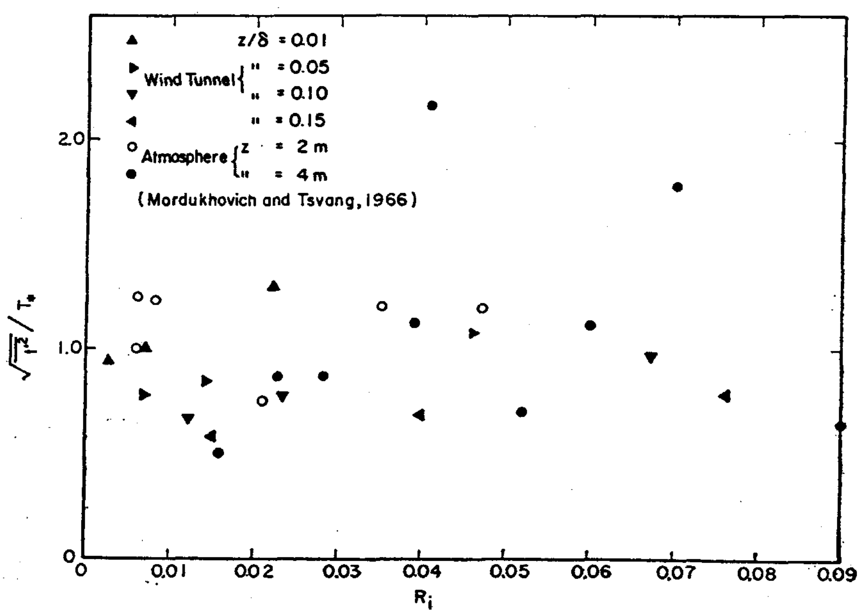

FIG. 14. Comparison of $(\overline{t'^2})^{\frac{1}{2}}/T_*$ for atmospheric data.

energy and fluxes are concerned, there appears to exist an additional length scale associated with the height dependency. This scale is apparently scaled also by the dynamic scaling requirements for the mean profiles.

The experimental results are generally in excellent agreement with field data if scaled according to the similarity theory of Monin and Obukhov, thus providing both a proof for the validity of the theory and a justification for modeling the stably stratified atmospheric boundary layer in a wind tunnel.

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