

Report Template coursework assignment A - 2019

CS4125 Seminar Research Methodology for Data Science

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1 Part 1 - Design and set-up of true experiment

1.1 Motivation

Students, researchers, and most people beyond these buckets believe that caffeine improves their productivity. Many even insist on having their morning coffee to do their work each day. We design an experiment to empirically evaluate the effects of caffeine as well as other intake trends on one’s mental acuity. Perhaps caffeine truly improves one’s performance in their studies or work. However, we may also investigate the existence of a placebo effect on one’s work ethic. We also investigate more complex effects of caffeine with its relationship on mental acuity, such as the influence of the caffeine-induced “crash”, and caffeine tolerance, and possible relationships between caffeine effectiveness and amount of sleep.

1.2 Theory

Academics have performed a host of analyses on the effects of caffeine on cognitive performance. For example, a study by Nehlig at UDS found that caffeine changes memory performance in nuanced ways, and it likely does not change the aggregation of long-term memory (Nehlig 2010). The study concluded that “caffeine cannot be considered a ‘pure’ cognitive enhancer”, although it may indirectly influence, and possibly enhance, one’s cognitive performance.(Nehlig 2010). Another study by Pasman et al. found that, when taking cognition tests, scores of subjects did not improve, but the tests were completed “approximately 10% faster” (Pasman et al. 2017). The findings of these studies suggest that, while caffeine may not directly improve cognitive performance, indirect factors may still lead users to enjoy increased efficiency when doing their work.

1.3 Research Question

The question which we investigate with our true experiment is the following: Does caffeine intake improve a student’s testing ability?

1.4 Participants

For convenience and consistency, the experiment will use students from TUDelft as participants. A sample from this population would likely generalize to broader student populations for the effects of caffeine intake. We can also find various levels of caffeine intake habits and regular sleep amounts. Lastly, because the students belong to the same university, we can expect that, with a lower variance in mental acuity, a smaller sample size could suffice for testing of statistical significance. One thing to consider is that, because TUDelft is a linguistically diverse university, we can make no assumptions about the language backgrounds of the students. As such, it is important to test subjects with means independent of reading ability, domain experience, etc. Administering of caffeine shall be transparent and consensual. Caffeine will be administered in commercially available forms and otherwise ordinary forms, with the possibility of caffeine-free doses.

1.5 Conceptual Model

Dependent Variable: IQ Test Score

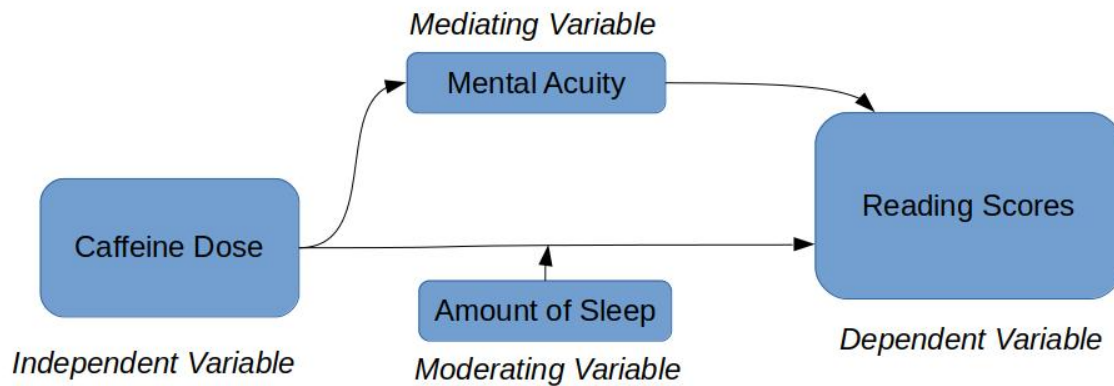
We would like to see if caffeine intake improves one's test taking abilities. In particular, the subject will be given an IQ test, which can be used as an unbiased approximation of the mental acuity of a subject. An IQ test would be the most fair form of judging a difference in test performance, independent of the background of the subject.

Independent Variable: Caffeine Administration (real or placebo)

Mediating Variables: Mental Acuity

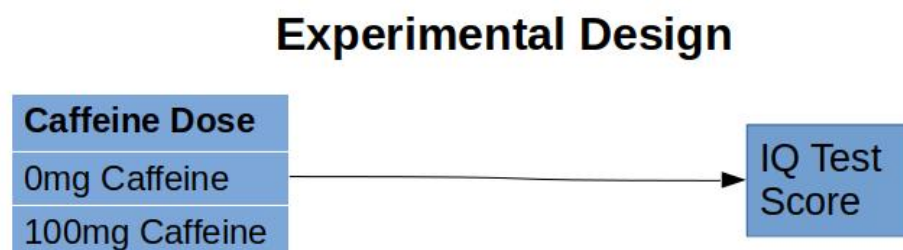
Mental acuity is perhaps the variable that we're more interested in, but there's no real way to measure this.

Moderating Variable: Amount of Sleep



1.6 Experimental Design

Our experiment is designed to watch test scores as related to caffeine ingestion. There will be two randomly assigned treatments, where subjects receive coffee with either 0mg of caffeine or 100mg of caffeine.



1.7 Experimental Procedure

For the collection of data from test subjects:

1. Randomly sample a test subject from a place of study, and preemptively, randomly assign treatment to the subject
 - Treatment is either a 100mg dose of caffeine or a 0mg dose of caffeine, both served through a cup of coffee
 - We would need to find people who have not had any caffeine yet on the given day
2. Collect some preliminary information about the subject, including estimated sleep amount
3. administer a waiting period for the subject to go about other activities and allow the caffeine to kick in
 - Duration of waiting period will be influenced by academic literature about caffeine
4. After the waiting period, administer the IQ test
 - The test should be realistically impossible to complete in the given amount of time to avoid statistically meaningless score distributions

1.8 Experiments and Suggested Statistical Analyses

To test the effects of caffeine intake on mental acuity, we would use a two-sample t-test.

2 Part 2 - Generalized linear models

2.1 Question 1: Twitter Sentiment Analysis (Between groups - single factor)

Set up libraries...

```
if (F) {  
  install.packages("base64enc", dependencies=T)  
  install.packages("twitterR", dependencies=T)  
  install.packages("plyr", dependencies=T)
```

```

install.packages("stringr", dependencies=T)
install.packages("container", dependencies=T)
}

library(container)

##
## Attaching package: 'container'
## The following object is masked from 'package:base':
##
##      remove
library(base64enc)
library(twitterR)
library(plyr)

##
## Attaching package: 'plyr'
## The following object is masked from 'package:twitterR':
##
##      id
## The following objects are masked from 'package:container':
##
##      count, empty
library(stringr)
library(car)

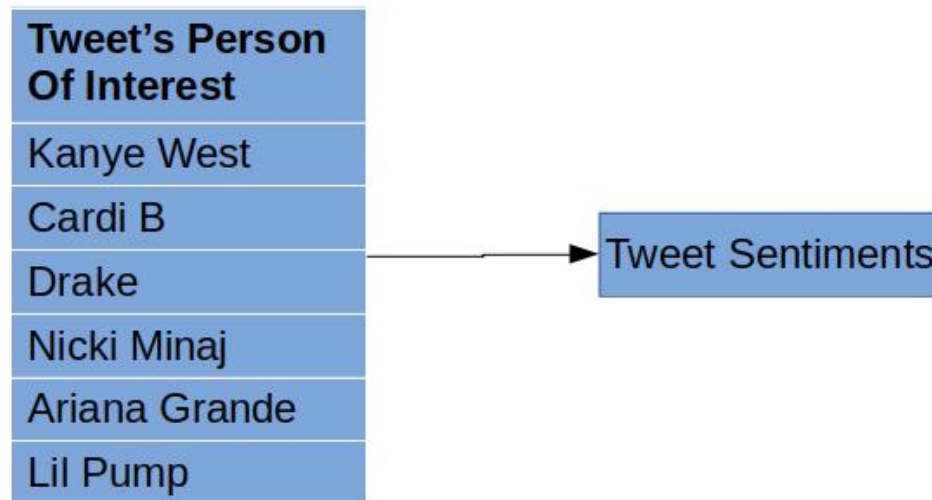
## Loading required package: carData

```

2.1.1 Conceptual Model

Our tweet analysis seeks to answer the following question: Is there a difference in the sentiment of the tweets related to the different celebrities? This question is explored in the following conceptual model.

Conceptual Model



2.1.2 Collecting Preliminary Data

Collect positive and negative words and related functions, set up Twitter oauth...

```
# load local textfiles listing key positive and negative words
#taken from https://github.com/mjhea0/twitter-sentiment-analysis
positive_words = scan('./data/positive-words.txt', what = 'character', comment.char=';') #read the posi
negative_words = scan('./data/negative-words.txt', what = 'character', comment.char=';') #read the nega
source("sentiment3.R")

# set up twitter session
source("twitter_keys.R") # imports consumer_key, consumer_secret, access_token, and access_secret
setup_twitter_oauth(consumer_key=consumer_key,
                    consumer_secret=consumer_secret,
                    access_token=access_token,
                    access_secret=access_secret)
```

```
## [1] "Using direct authentication"
```

Collect tweets about celebrities, calculate sentiment scores...

```

tweetsFilename = 'data/tweets.csv'
n_tweets = 1e3
celebHandles = Dict$new(c("Kanye West" = "@KanyeWest",
                          "Drake" = "@Drake",
                          "Ariana Grande" = "@ArianaGrande",
                          "Cardi B" = "@IAmCardiB",
                          "Lil Pump" = "@LilPump",
                          "Nicki Minaj" = "@NickiMinaj")
                        )
if (file.exists(tweetsFilename)) {
  tweets = read.csv(tweetsFilename, header=T)
} else {
  tweets = NULL
  for (name in celebHandles$keys()) {
    handle = celebHandles[name]
    retTweets = searchTwitter(handle, n=n_tweets, lang="en", resultType="recent")
    out = data.frame("text"=laply(retTweets, function(t)t$text()))
    out$name = name
    out$handle = handle
    tweets = rbind(tweets, out)
  }
  write.csv(tweets, file='data/tweets.csv', row.names=T)
}

tweets$score = score.sentiment(tweets$text, positive_words, negative_words)[[1]]

```

2.1.3 Tweet Sentiment Analysis

Is there a difference in the sentiment of the tweets related to the different celebrities? We inspect this question using context independent sentiment analysis of tweets about them. The data was collected as follows: 1. We used the Twitter api to collect the 1000 most recent tweets which contain the twitter tag of the celebrity of interest 2. Punctuation and links were removed from the tweets 3. Upon these “parsed” tweets, we counted the number of “positive” and “negative” words present in the tweet, as provided in lists for the assignment, and the tweets were given a sentiment score as a difference of the positive and negative word counts

Tweets about the following American music artists were collected: - Kanye West - Drake - Ariana Grande - Cardi B - Lil Pump - Nicki Minaj

2.1.4 Statistical Considerations:

Making no assumptions, we would expect that all tweets about our celebrities have similar sentiment scores. That is, the sentiment scores found in tweets about celebrities have come from the same, broad distribution of sentiment scores in tweets about all celebrities. The alternative hypothesis would be that tweet sentiment scores come from different distributions when we sample tweets about the various celebrities.

2.1.5 Assessing Homogeneity of Variance

```

leveneTest(tweets$score, group=tweets$name)

## Levene's Test for Homogeneity of Variance (center = median)
##           Df F value    Pr(>F)

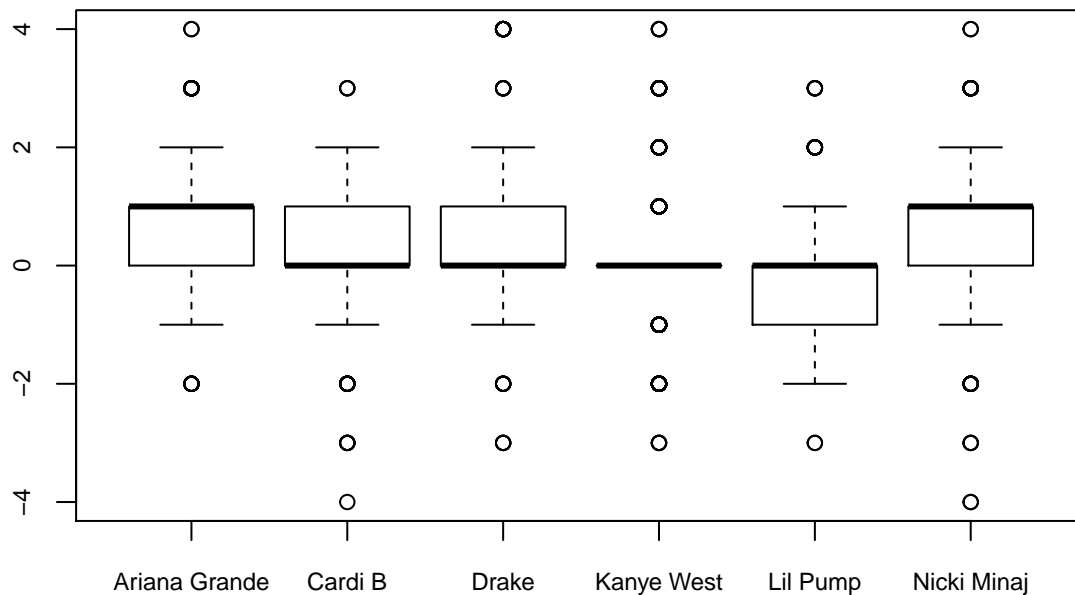
```

```
## group      5  3.0336 0.009731 **
##          5994
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We use the Levene Test to assess homogeneity of variance. At an alpha value of .05, our p-value is less than our alpha value, so we reject the assumption that the tweet sentiment distributions from the various celebrities have homogeneous variances.

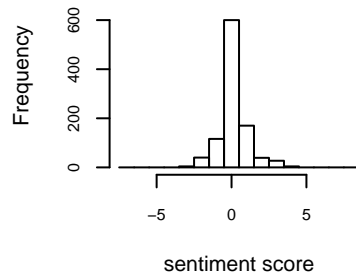
2.1.6 Graphical Examination of Means and Variations of Tweet Sentiment by Celebrity

```
par(cex.axis=.75)
boxplot(score ~ name, data=tweets)
```

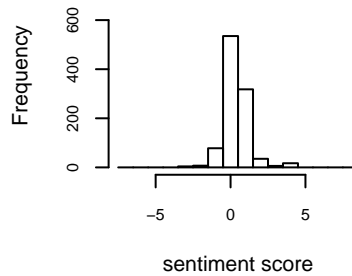


```
par(mfrow=c(2,3))
for (name in unique(tweets$name)) {
  hist(tweets[tweets$name == name,]$score, breaks = -8:8 + .5,
    main=paste("Tweet Sentiments for", name), xlab="sentiment score", ylim=c(0,750))
}
```

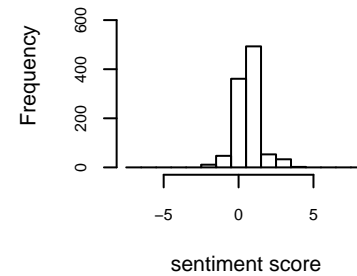

Tweet Sentiments for Kanye We



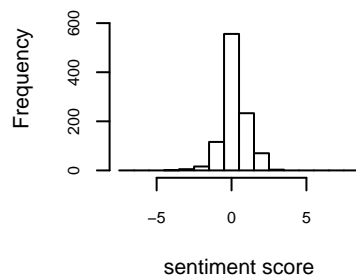
Tweet Sentiments for Drake



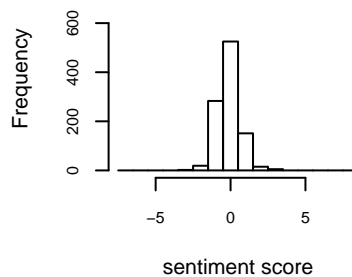
Tweet Sentiments for Ariana Gra



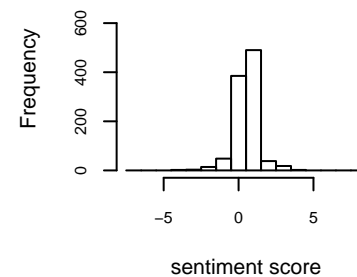
Tweet Sentiments for Cardi B



Tweet Sentiments for Lil Pump



Tweet Sentiments for Nicki Min



Above, we have produced visualizations of the sampled distributions of tweet sentiments about various American celebrity artists. They have different medians, levels of spread, and numbers of outliers. In particular:

- All celebrities were found to have a neutral sentiment score for the mode for their sampled tweet scores for Ariana Grande and Lil Pump, who have a mode tweet sentiment of +1
- Most celebrities have an interquartile range of 1, while Drake's tweet scores have an IQR of 2, and Kanye's tweet scores have an IQR of 0
- The boxplots show that a lot of celebrities have many outliers in their tweet score distributions, as defined by the boxplot function
- Distributions tend to center around neutral tweets, but some celebrities receive far less neutral tweets than others:
- Kanye West appeared to receive about ~650 neutral tweets
- Ariana Grande received only about ~250 neutral tweets, and most of her tweets were at a score of +1

2.1.7 Tweet Knowledge as Used to Describe Tweet Sentiment

```
tweets_lm0 <- lm(score ~ 1, data = tweets,
na.action = na.exclude)
tweets_lm1 <- lm(score ~ name, data = tweets,
na.action = na.exclude)
anova(tweets_lm0, tweets_lm1)
```

```
## Analysis of Variance Table
##
## Model 1: score ~ 1
## Model 2: score ~ name
##   Res.Df  RSS Df Sum of Sq    F    Pr(>F)
## 1     5999 4891
## 2     5994 4498   5    393.02 104.75 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Using the ANOVA function in R, we find a p-value which is less than 0.05. With that, we find that the distributions of sentiments of tweets about various artists come from different distributions.

2.1.8 Assessing Model Quality Using Tweet Knowledge as a Predictor

```
pairwise.t.test(tweets$score, tweets$name, paired=FALSE, p.adjust.method="bonferroni")

##
## Pairwise comparisons using t tests with pooled SD
##
## data: tweets$score and tweets$name
##
##           Ariana Grande Cardi B Drake   Kanye West Lil Pump
## Cardi B    < 2e-16      -      -      -           -
## Drake      9.1e-11      0.00096 -      -           -
## Kanye West < 2e-16      0.66175 2.9e-08 -           -
## Lil Pump   < 2e-16      < 2e-16 < 2e-16 7.6e-11    -
## Nicki Minaj 0.12731      2.7e-15 0.00031 < 2e-16    < 2e-16
##
## P value adjustment method: bonferroni
```

The Bonferroni Correction analysis shows that the tweet distributions for most pairs of celebrities differ from each other. However, for some pairs, it appears that we fail to reject that the tweets come from different distributions. In this example, assuming an alpha value of 0.05, we see that we fail to reject the given null hypothesis for the following pairs of celebrities: - Drake and Kanye West - Nicki Minaj and Cardi B - Nicki Minaj and Kanye West

2.1.9 Small Section for Scientific Publication

The modern music industry lends itself to creating artists into celebrities, celebrating them as personalities that receive a lot of attention outside of their music. Recent performances, works, and presentation in other media create seemingly distinct attitudes towards these artists. Public discourse about these artists can be found on online platforms, and so we ask: Is there a difference in the sentiment of the tweets related to the different celebrities?

For our analysis, we collect the 1000 most recent tweets which tag a small set of artists, listed as the following:

- Drake
- Kanye West
- Cardi B
- Nicki Minaj
- Ariana Grande
- Lil Pump

This selection of artists make music at intersecting areas of hip hop and pop music, but they have very different styles of music, online presences, and types of fans. One should also note that all tweets were also pulled from the Twitter API at the same time, and the artists will have had varying levels of activity and perhaps different types of attention surrounding them at the time of the tweet collection.

To gauge the sentiments of the tweets, we produce a “sentiment score” for each tweet using the following method: We use a collection of positively and negatively connotated words. A tweet initially has a sentiment score of zero, and the score is incremented and decremented for the presence of each positively and negatively

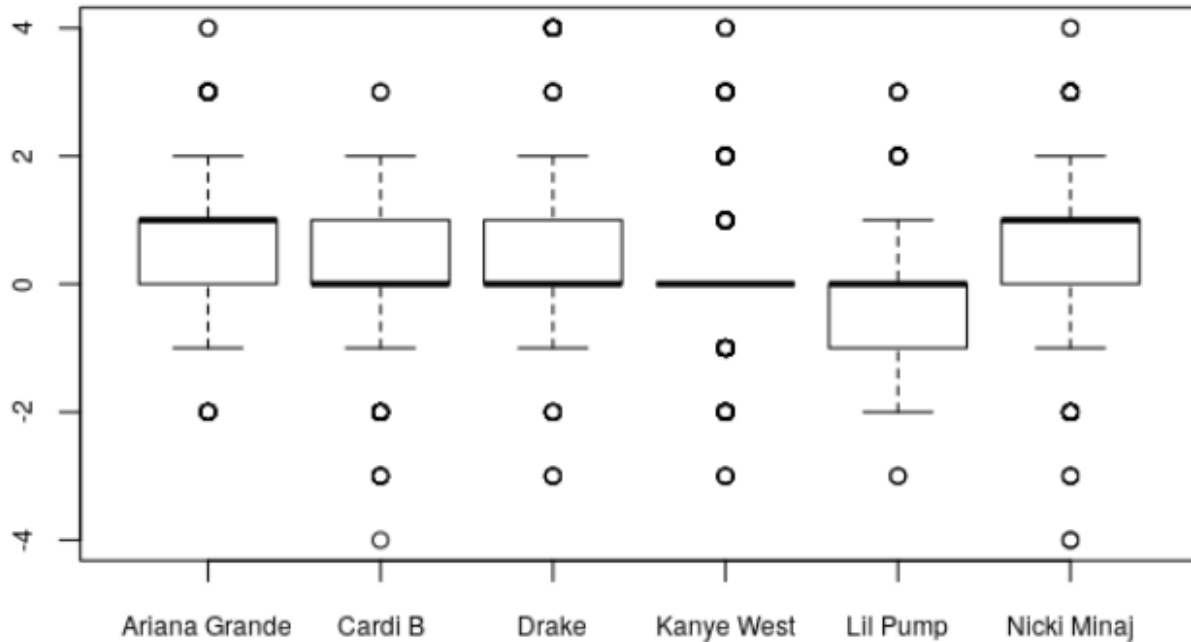


Figure 1: Tweet Score Distributions

connotated word, respectively. This lets us produce a dataset of the sentiment scores of 1000 tweets from each of these six artists.

This process gives us a sampling of the distribution of the sentiment of tweets related to given artists. To answer the posed question relating to homogeneity of tweet sentiments, we compare the distributions and check for statistically significant differences between these distributions.

First, we use the Levene Test to assess homogeneity of variance across tweet sentiment distributions. The test reports a p-value of 0.0097, so there is significant difference in the variances of these sentiment distributions. In another test, we create linear models to fit the sentiment score of a tweet to the person which it pertains to, as well as a linear model which does not use any predictors. An analysis of variance for the two linear models produces a p-value close to zero (at a computed $2.2e-16$), showing that the tweet sentiment distributions are significantly different per relevant celebrity.

Finally, our boxplot above helps to visually show the unique types of tweet sentiment distributions across celebrities. According to the whiskers and outlier points of the boxplots, no two tweet distributions are exactly the same. Most tweet sentiments, across celebrities, are often neutral or positive, but Kanye West's tweet sentiments are neutral especially often, and Lil Pump's tweets are negative far more often than for other artists. In general, the sentiments towards artists, as expressed in tweets, can vary a lot depending on who they pertain to.

2.2 Question 2 - Website visits (between groups - Two factors)

Set up libraries, load data...

```
if (F) {
  install.packages("gmodels", dependencies=T)
}
```

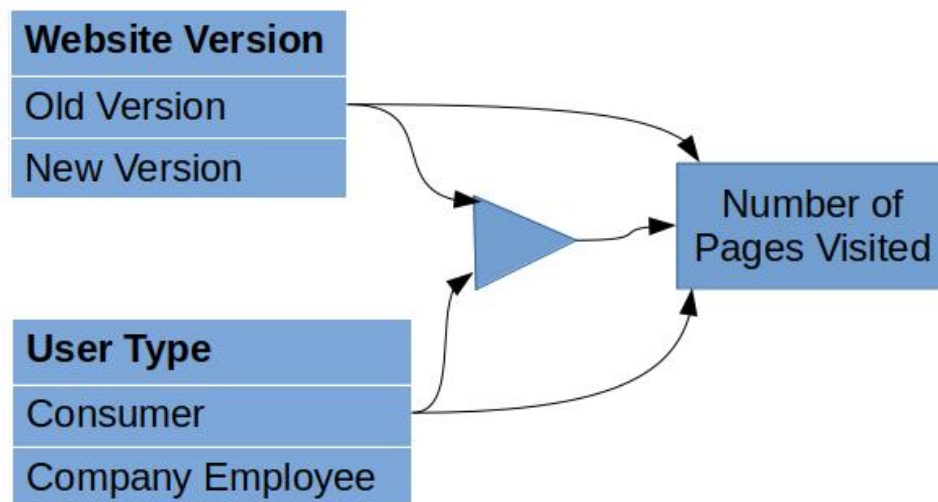
```
library(gmodels)

visits = read.csv("data/webvisit1.csv", header = TRUE)
visits$version[visits$version==0] = "Old"
visits$version[visits$version==1] = "New"
visits$portal[visits$portal==0] = "Consumer"
visits$portal[visits$portal==1] = "Company"
```

2.2.1 Conceptual model

We are tasked with analyzing the results of an A-B study of a webserver as administered in two different versions to two different groups. Also, notice that we are using the webvisit dataset 1. Through this analysis, we investigate linear modeling between groups of two different factors:

Conceptual Model



Independent Variables:

- Version of webserver (Old or New)
- Type of User (0=consumer, 1=company)
- All combinations of the previously listed independent variables

Dependent Variables:

- Number of pages visited

In this analysis, we inspect whether the independent variables, individually and/or in combination, effected the number of pages visited:

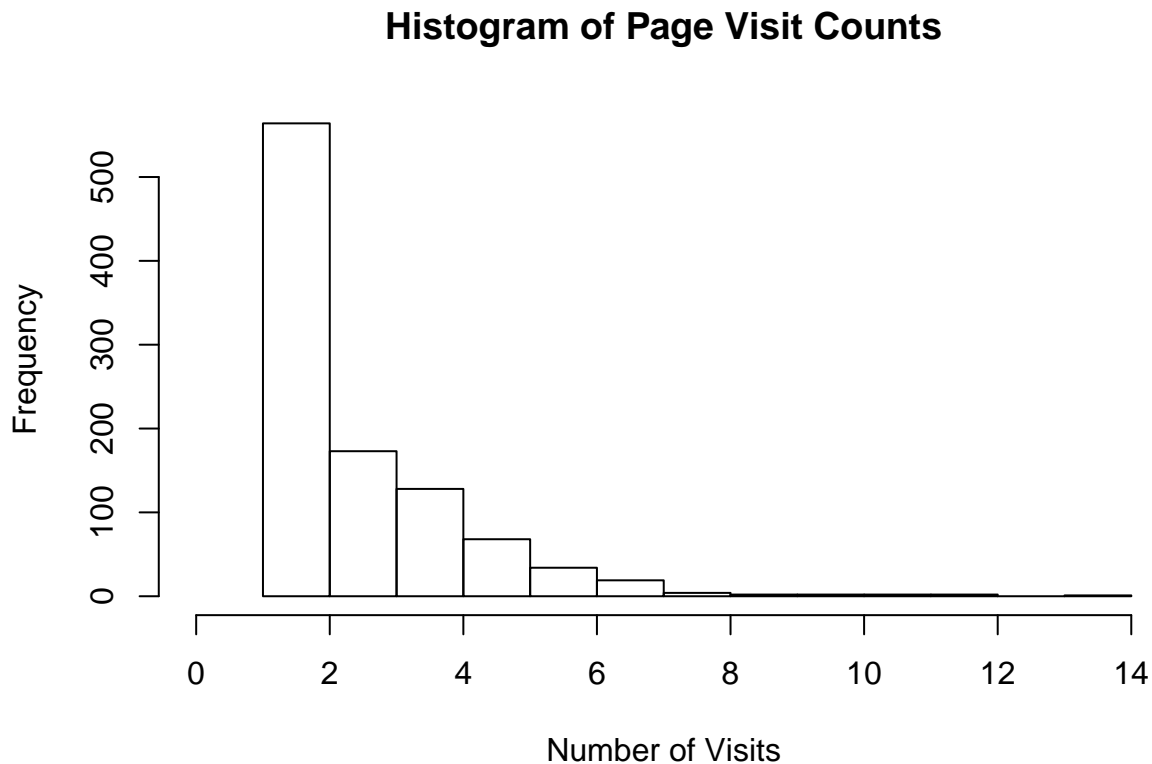
Null hypothesis: There is no observed difference in the number of pages visited based on either the versions, the portals, or a combination thereof used. The observed difference in the sample is based on a sampling error and there is no observed difference in the entire population.

Alternative hypothesis: The observed difference in the sample is a real effect plus some change variation.

2.2.2 Visual inspection

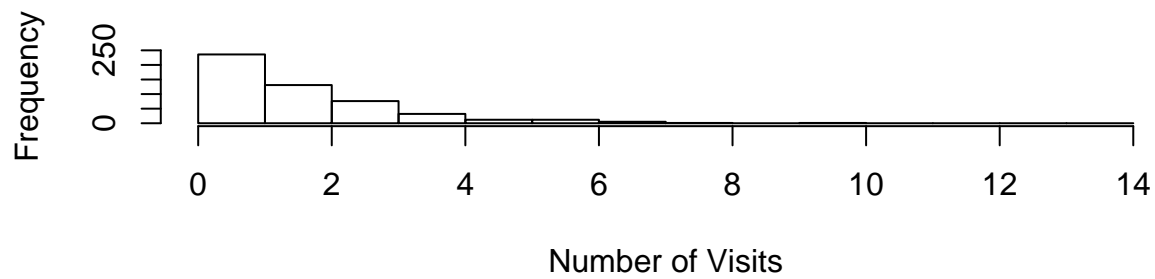
```
xlim = c(0, max(visits$pages))
ylim = c(0, 250)
breaks=max(visits$pages)

# histogram of all page visits
hist(visits$pages, xlab="Number of Visits", main="Histogram of Page Visit Counts", xlim=xlim)
```

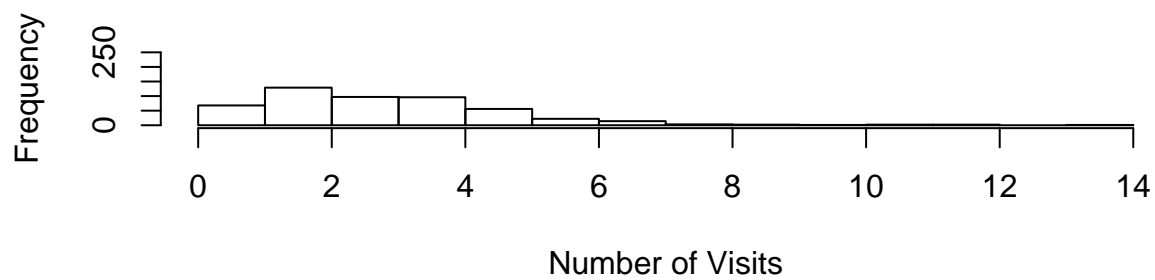


```
# histogram of page visits by different explanator variables:
# by design
par(mfrow=c(2,1))
for (v in unique(visits$version)) {
  hist(visits[which(visits$version == v),]$pages,
       xlab="Number of Visits", main=paste("Visits for version =", v),
       xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
}
```

Visits for version = Old

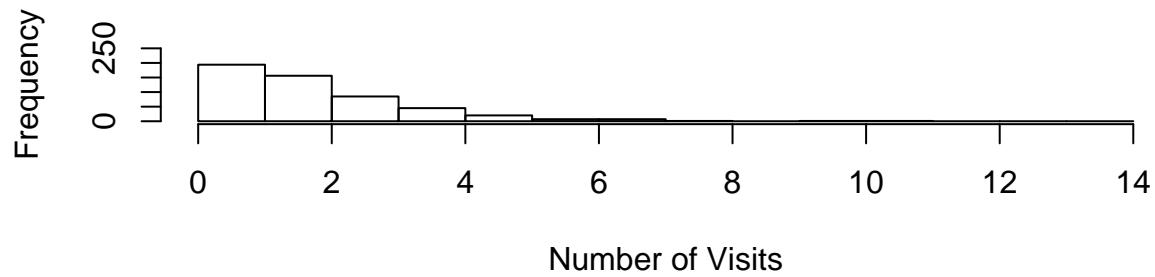


Visits for version = New

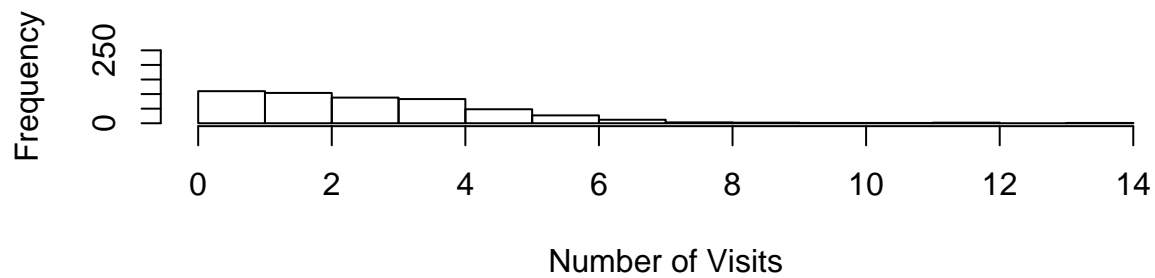


```
# by user type
par(mfrow=c(2,1))
for (p in unique(visits$portal)) {
  hist(visits[which(visits$portal == p),]$pages,
       xlab="Number of Visits", main=paste("Visits for portal =", p),
       xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
}
```

Visits for portal = Company

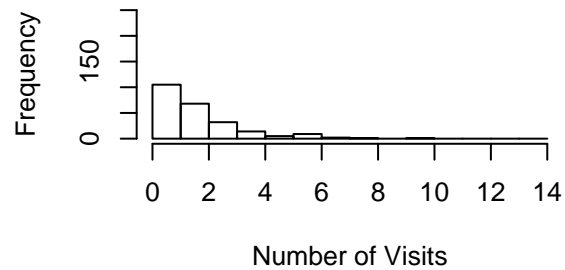
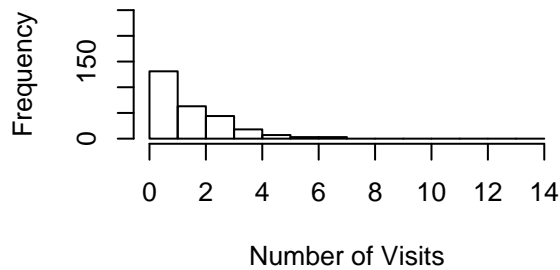


Visits for portal = Consumer

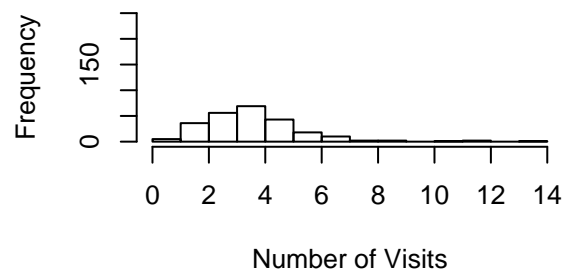
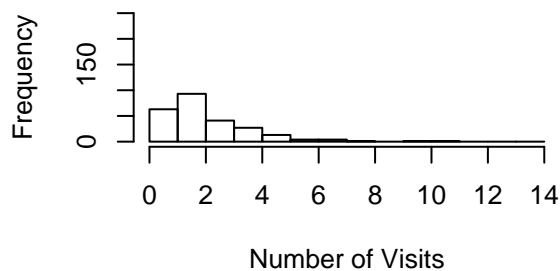


```
# by all combinations:
par(mfrow=c(2,2))
for (v in unique(visits$version)) {
  for (p in unique(visits$portal)) {
    hist(visits[which(visits$version == v & visits$portal == p),]$pages,
          xlab="Number of Visits",
          main=paste("Visits for version =", v, ", portal =", p),
          xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
  }
}
```

Visits for version = Old , portal = Compæ Visits for version = Old , portal = Consur



Visits for version = New , portal = Compæ Visits for version = New , portal = Consur



Upon visual inspection, it appears that the portal type doesn't change the distribution of page visit counts. However, in both cases, it appears that the version of the website causes the mean page visit count to shift to the right, and the page visit count distributions no longer seem as right skewed.

2.2.3 Normality check

Statistically test if variable page visits deviates from normal distribution

```
shapiro.test(visits$pages)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  visits$pages
## W = 0.83076, p-value < 2.2e-16
```

A simple Shapiro-Wilk normality test reveals, with a p-value of 2.2e-16, it is unlikely that the true distribution of page visit counts across all scenarios are sampled from normal distributions.

2.2.4 Model analysis

```
pages_model = glm(pages~version+portal+version:portal,
                  data=visits, na.action=na.exclude)
summary(pages_model)
```

```
##
## Call:
## glm(formula = pages ~ version + portal + version:portal, data = visits,
##      na.action = na.exclude)
```



```
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -3.0694  -1.0694  -0.1266   0.8734   9.9306
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      2.5242     0.0981  25.730 < 2e-16 ***
## versionOld       -0.5353     0.1360  -3.936 8.85e-05 ***
## portalConsumer     1.5452     0.1392  11.104 < 2e-16 ***
## versionOld:portalConsumer -1.4075     0.1957  -7.191 1.26e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 2.386778)
##
##      Null deviance: 3054.7  on 998  degrees of freedom
## Residual deviance: 2374.8  on 995  degrees of freedom
## AIC: 3710.1
##
## Number of Fisher Scoring iterations: 2
```

Because our data does not follow a normal distribution, we use a generalized linear model to assess the ability of the website version and visitor type, as well as an interaction effect between said conditions, to predict page visit counts. According to our models, we recognize the following statistically significant trends:

- with $p = 8.85e-5$, we expect the old version of the website to receive .53 fewer visits on average
- with $p = 2e-16$, we expect a consumer to visit the webpage 1.54 times more on average
- with $p = 1.26e-12$, we expect consumers using the old version of the portal to visit the page 1.4 times less on average

2.2.5 Simple effect analysis

```
visits$interaction = interaction(visits$portal, visits$version)
allPortalsVersion0 = c(1,-1,0,0)
allPortalsVersion1 = c(0,0,1,-1)
SimpleEff = cbind(allPortalsVersion0, allPortalsVersion1)
contrasts(visits$interaction) = SimpleEff
simpleEffectModel = aov(pages ~ interaction, data=visits, na.action=na.exclude)
summary.lm(simpleEffectModel)
```

```
##
## Call:
## aov(formula = pages ~ interaction, data = visits, na.action = na.exclude)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.0694  -1.0694  -0.1266   0.8734   9.9306
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      2.67725     0.04893  54.714 <2e-16 ***
## interactionallPortalsVersion0 -0.77260     0.06958 -11.104 <2e-16 ***
## interactionallPortalsVersion1 -0.06887     0.06882  -1.001   0.317
```

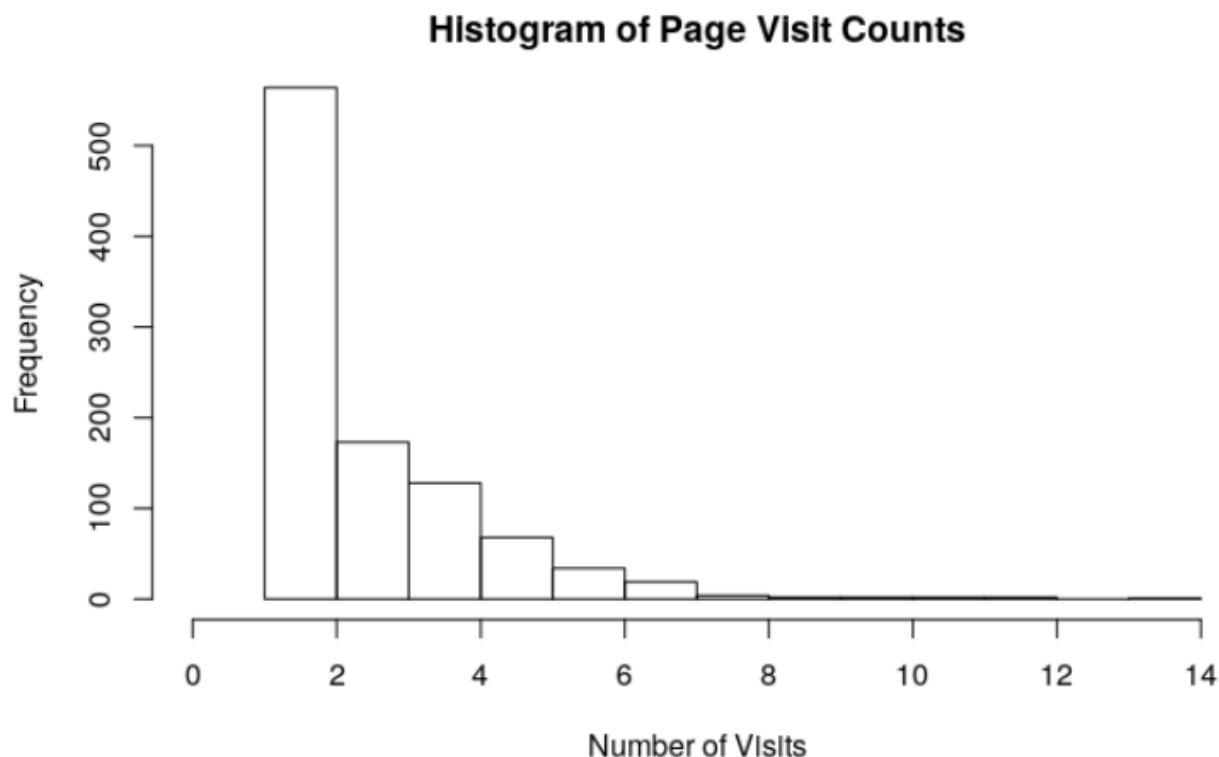


Figure 2: Distribution of Page Visits

```
## interaction                -1.23908    0.09786 -12.661    <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.545 on 995 degrees of freedom
## Multiple R-squared:  0.2226, Adjusted R-squared:  0.2202
## F-statistic: 94.94 on 3 and 995 DF,  p-value: < 2.2e-16
```

Our analysis shows that, indeed, there is an interaction effect, but only in some cases:

- For version 0, the type of user doesn't change the page visit count. The test specifically finds a p-value of 0.317, so we don't have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.
- For version 1, the type of user indeed changes the page visit count. The test specifically finds a p-value of $2e-16$, so we have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.

2.2.6 Report section for a scientific publication

We are tasked with analyzing the results of an A-B study of a webserver as administered in two different versions to two different groups. In particular, we inspect an old version and a new version of the website, collecting how many visits the website receives for each of the versions. We collect these observations for two distinct audiences: consumers and company employees. We investigate the distributions, with some plots shown below.

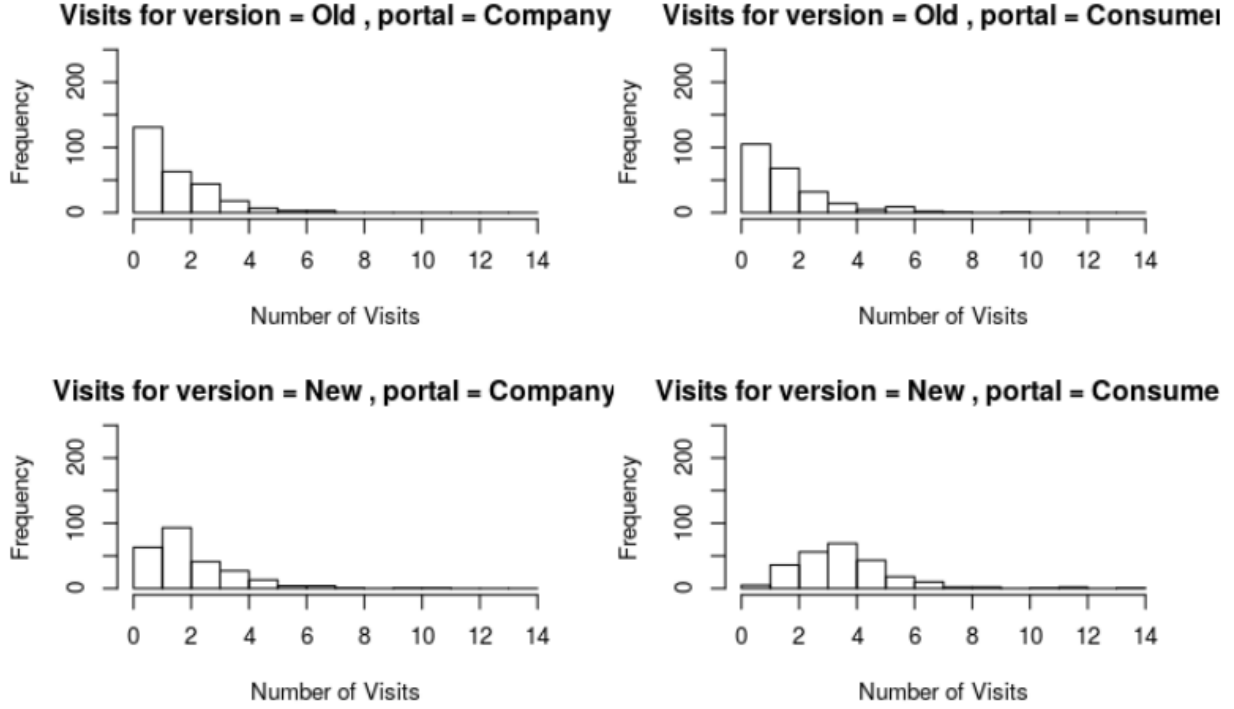


Figure 3: Distributions of page visits for all combinations of consumers and website versions.

Visually, the distributions of page visits don't appear to be normal in most distributions. Especially across page visits of all conditions, the distribution of page visits does not look normal. With a Shapiro Test, reporting a p-value extremely close to zero (at $p=2.2e-16$), we confirm that the distribution of all page visits across all visitors and versions is far from a normal distribution.

Because our data does not follow a normal distribution, we use a generalized linear model to assess the ability of the website version and visitor type, as well as an interaction effect between said conditions, to predict page visit counts. According to our models, we recognize the following statistically significant trends:

- with $p = 8.85e-5$, we expect the old version of the website to receive .53 fewer visits on average
- with $p = 2e-16$, we expect a consumer to visit the webpage 1.54 times more on average
- with $p = 1.26e-12$, we expect consumers using the old version of the portal to visit the page 1.4 times less on average

We then perform an analysis of the extent of the interaction effect with an analysis of variance. Our analysis shows that, indeed, there is an interaction effect, but only in some cases:

- For version 0, the type of user doesn't change the page visit count. The test specifically finds a p-value of 0.317, so we don't have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.
- For version 1, the type of user indeed changes the page visit count. The test specifically finds a p-value of $2e-16$, so we have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.

In general, we find that the newer version of our website aggregates at least as many page visits, if not more, from a user, across all user types. All else equal, the provider of this website should use the newer version of their page in all cases.

2.3 Question 3 - Linear regression analysis

Set up libraries, load data...

```
if (F) {  
  install.packages("ggpubr", dependencies=T)  
  install.packages("ggExtra", dependencies=T)  
  install.packages("ppcor", dependencies=T)  
  install.packages("mctest", dependencies=T)  
}  
  
library(ggpubr)  
  
## Loading required package: ggplot2  
## Loading required package: magrittr  
##  
## Attaching package: 'magrittr'  
## The following object is masked from 'package:container':  
##  
##      add  
##  
## Attaching package: 'ggpubr'  
## The following object is masked from 'package:plyr':  
##  
##      mutate  
## The following object is masked from 'package:container':  
##  
##      rotate  
library(ggExtra)  
library(car)  
library(mctest)  
library(ppcor)  
  
## Loading required package: MASS  
airfare <- read.csv(file="data/airfare.csv", header=T)
```

2.3.1 Conceptual model

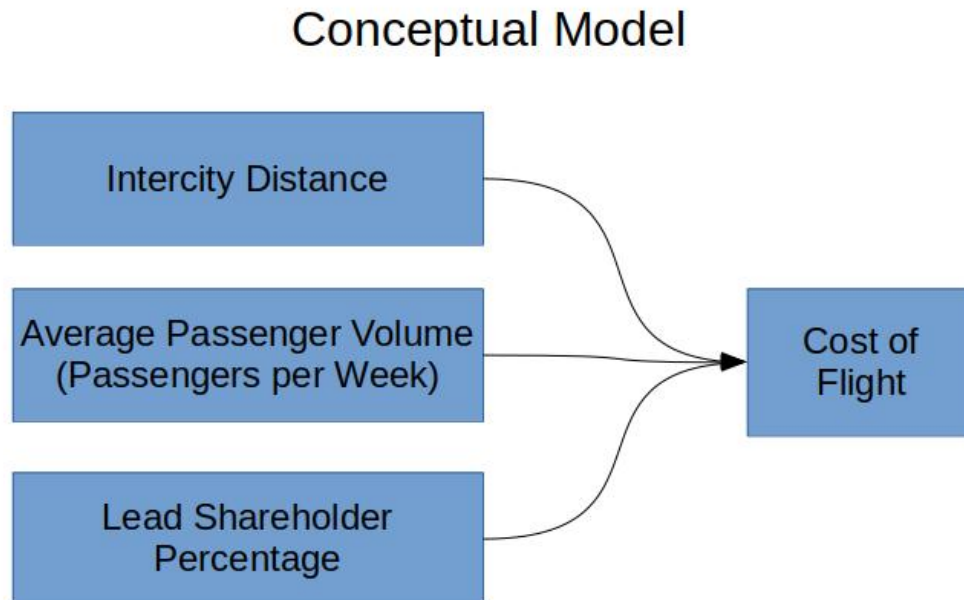
For a self-guided linear regression analysis, we investigate a dataset which records airfares from cities to cities. We would like to see if the chosen independent variables can be used to predict the price of a ticket from one city to another. For the analysis:

Dependent Variable: Average Fare

The average price of the ticket to get from City1 to City2

Independent Variables:

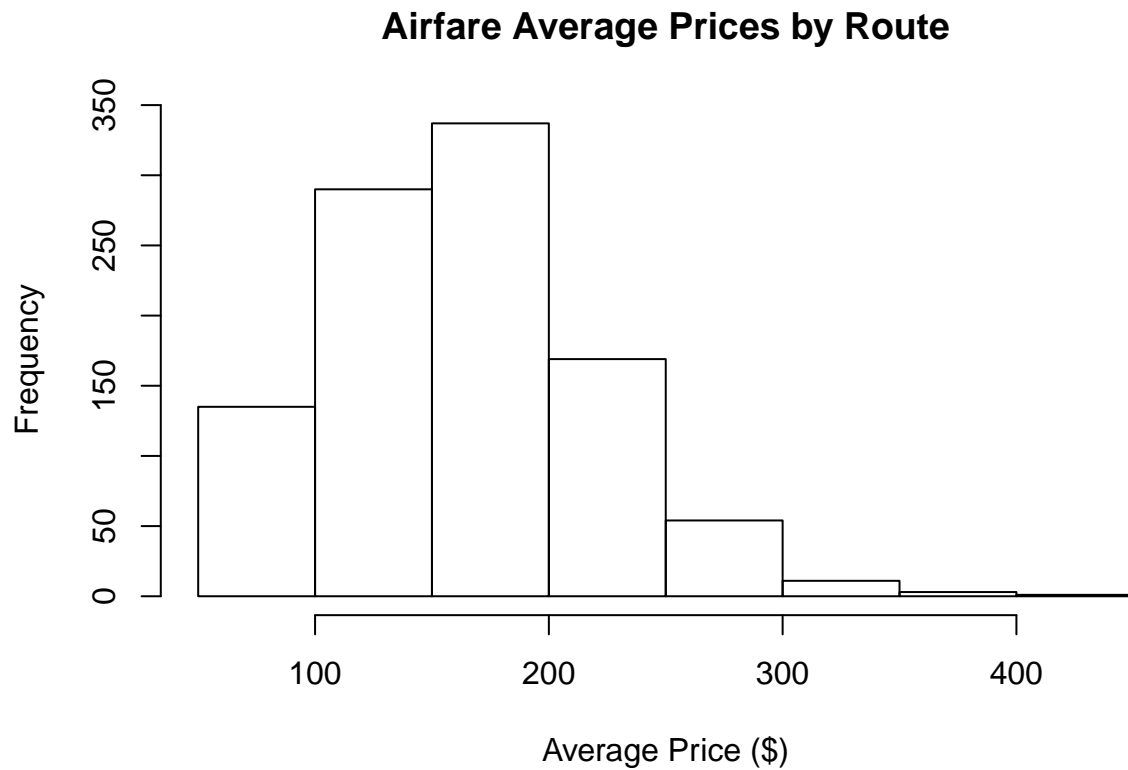
- Distance - the distance between City1 and City2
- Average Weekly Passengers - the average number of passengers that fly from City1 to City2 per week
- Lead Share Percentage - the Percentage of the flights from City1 to City2 which are served through the leading airline of the route



2.3.2 Visual inspection

Graphical analysis of the distribution of the dependent variable, e.g. histogram, density plot

```
hist(airfare$averageFare, main="Airfare Average Prices by Route", xlab="Average Price ($)")
```

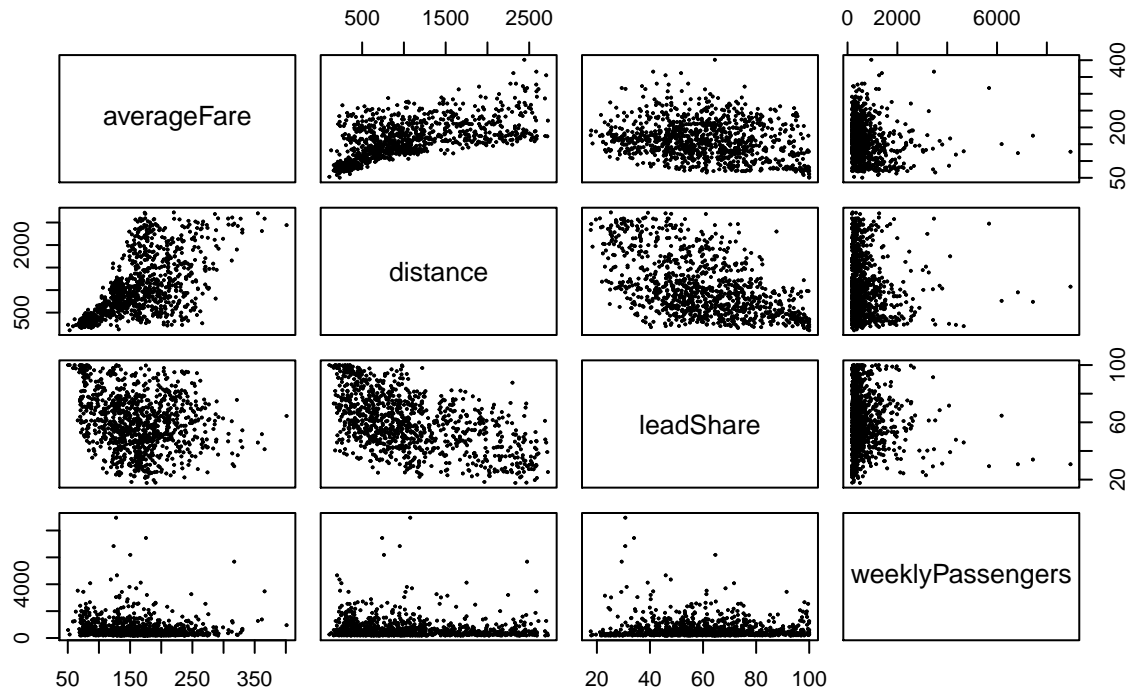


The price of airfares appears normally distributed, centered somewhere around 175 units. The prices are right skewed, although this is expected because the price has a lower bound of 0.

2.3.3 Scatter plot

```
# Basic Scatterplot Matrix
pairs(~averageFare+distance+leadShare+weeklyPassengers,data=airfare,
      main="Airfare Scatterplot Matrix", pch=10, cex=.2)
```

Airfare Scatterplot Matrix



We produce a scatterplot matrix for our independent and dependent variables. Some of the scatterplots which stand out:

- Average fare seems to increase with distance, which is quite intuitive.
- There doesn't appear to be a strong relationship between the percentage of flights owned by the leading airline and the average fare price.
- It's very hard to see a relationship, visually, between the amount of weekly passengers for a route and the price of the flight. However, the routes which have extremely high weekly passenger counts seem to have lower prices. This trend is visually supported by very few data points, though.

2.3.4 Linear regression

Conduct a multiple linear regression (including confidence intervals, and beta-values)

```
fare_model0 = lm(averageFare ~ 1, data=airfare, na.action=na.exclude)
confint(fare_model0)
```

```
##                2.5 %    97.5 %
## (Intercept) 159.9397 166.8111
```

```
coef(fare_model0)
```

```
## (Intercept)
##      163.3754
```

```
fare_model1 = lm(averageFare ~ distance, data=airfare, na.action=na.exclude)
confint(fare_model1)
```

```
##                2.5 %    97.5 %
## (Intercept) 104.59931149 115.30817862
## distance      0.04621403   0.05487025
```

```
coef(fare_model1)
```

```
## (Intercept)      distance
## 109.95374505    0.05054214
```

```
anova(fare_model0, fare_model1)
```

```
## Analysis of Variance Table
##
## Model 1: averageFare ~ 1
## Model 2: averageFare ~ distance
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      999 3062270
## 2      998 2006500   1    1055770 525.12 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the distance of the flight. The test reports a p-value of 2.2e-16, so indeed we reject that the average fare price is independent of the distance between the cities. We also report the confidence interval and the weights of the independent variables above.

```
fare_model2 = lm(averageFare ~ distance + weeklyPassengers,
                 data=airfare, na.action=na.exclude)
confint(fare_model2)
```

```
##              2.5 %      97.5 %
## (Intercept)  108.094615921 120.196476496
## distance      0.045639171   0.054299165
## weeklyPassengers -0.008967769 -0.001700977
```

```
coef(fare_model2)
```

```
##      (Intercept)      distance weeklyPassengers
##    114.145546209      0.049969168      -0.005334373
```

```
anova(fare_model1, fare_model2)
```

```
## Analysis of Variance Table
##
## Model 1: averageFare ~ distance
## Model 2: averageFare ~ distance + weeklyPassengers
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      998 2006500
## 2      997 1989934   1      16567  8.3003 0.004049 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the number of weekly passengers of the route. The test reports a p-value .004049, so indeed we reject that the average fare price is independent of the number of weekly passengers. We also report the confidence interval and the weights of the independent variables above.

```
fare_model3 = lm(averageFare ~ distance + weeklyPassengers + leadShare,
                 data=airfare, na.action=na.exclude)
confint(fare_model3)
```

```
##              2.5 %      97.5 %
## (Intercept)  77.041006409 106.566226301
## distance      0.049439643   0.059686992
```



```
## weeklyPassengers -0.008156413 -0.000855535
## leadShare          0.111695523  0.451401752
```

```
coef(fare_model3)
```

```
##      (Intercept)      distance weeklyPassengers      leadShare
##      91.803616355      0.054563317      -0.004505974      0.281548638
```

```
anova(fare_model2, fare_model3)
```

```
## Analysis of Variance Table
##
## Model 1: averageFare ~ distance + weeklyPassengers
## Model 2: averageFare ~ distance + weeklyPassengers + leadShare
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      997 1989934
## 2      996 1969017   1      20917 10.581 0.001181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the percentage of flights which are provided by the lead airline. The test reports a p-value .001181, so indeed we reject that the average fare price is independent of the percentage of flights which are controlled by the leading airline of the route. We also report the confidence interval and the weights of the independent variables above.

2.3.5 Examine assumption

```
X = airfare[c('distance', 'weeklyPassengers', 'leadShare')]
Y = airfare['averageFare']
imcdiag(x=X, y=Y)
```

```
##
## Call:
## imcdiag(x = X, y = Y)
##
##
## All Individual Multicollinearity Diagnostics Result
##
##              VIF    TOL      Wi      Fi Leamer  CVIF Klein
## distance      1.4252 0.7016 211.9735 424.3723 0.8376 1.5679    0
## weeklyPassengers 1.0274 0.9733  13.6824  27.3922 0.9866 1.1303    0
## leadShare      1.4201 0.7042 209.4375 419.2952 0.8391 1.5623    0
##
## 1 --> COLLINEARITY is detected by the test
## 0 --> COLLINEARITY is not detected by the test
##
## * all coefficients have significant t-ratios
##
## R-square of y on all x: 0.357
##
## * use method argument to check which regressors may be the reason of collinearity
## =====
```

```
pcor(X, method='pearson')
```

```
## $estimate
```

```
##           distance weeklyPassengers  leadShare
## distance      1.0000000      -0.1491478 -0.5409332
## weeklyPassengers -0.1491478        1.0000000 -0.1369035
## leadShare      -0.5409332      -0.1369035  1.0000000
##
## $p.value
##           distance weeklyPassengers  leadShare
## distance      0.000000e+00      2.193641e-06 5.292396e-77
## weeklyPassengers 2.193641e-06      0.000000e+00 1.410984e-05
## leadShare      5.292396e-77      1.410984e-05 0.000000e+00
##
## $statistic
##           distance weeklyPassengers  leadShare
## distance      0.00000      -4.762660 -20.307730
## weeklyPassengers -4.76266        0.000000 -4.363857
## leadShare      -20.30773      -4.363857  0.000000
##
## $n
## [1] 1000
##
## $gp
## [1] 1
##
## $method
## [1] "pearson"
```

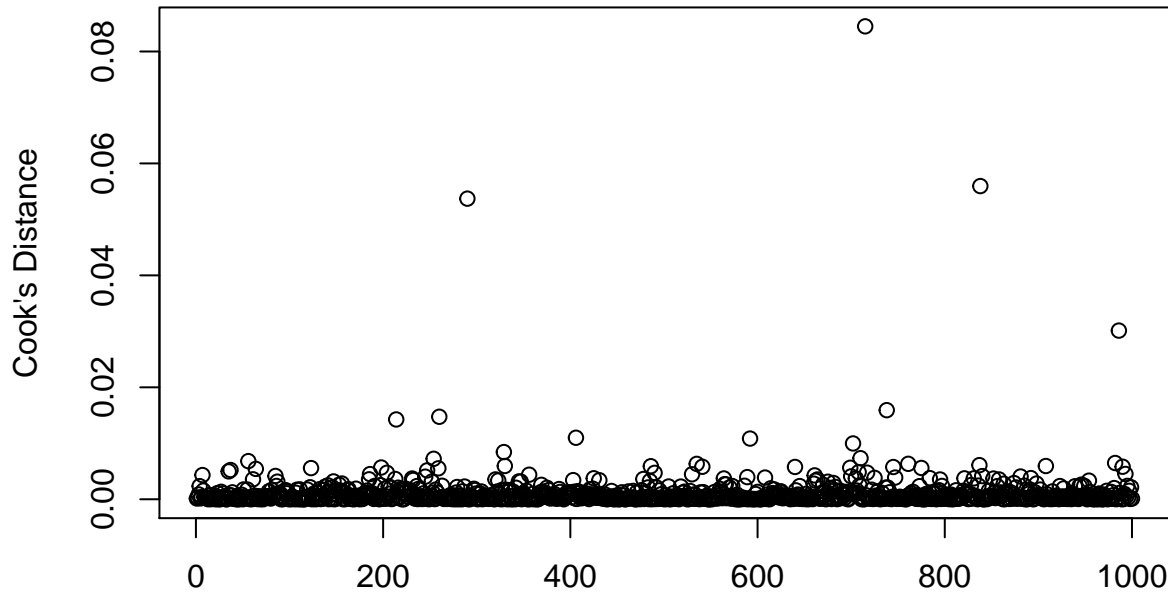
The output of the partial correlation coefficients analysis shows that, for all combinations, the testing of independence of all pairs of independent variables produces p-values close to zero. That is, all pairs of independent variables are found to have some amount of correlation.

2.3.6 Impact analysis of individual cases

Examine effect of single cases on the predicted values (e.g. DFBeta, Cook's distance)

```
plot(cooks.distance(fare_model3), main="Airfare Cook's Distances", ylab = "Cook's Distance", xlab="")
```

Airfare Cook's Distances



The plotting of cook's distance shows that all DFBeta are far less than one. Of the 1000 airfare data points, only a handful of points have values which seem to deviate from the rest, but we do not believe that these values would not reveal any sort of influence of single cases on predicted values.

2.3.7 Report section for a scientific publication

we investigate a dataset which records airfares from cities to cities. We would like to see if the chosen independent variables can be used to predict the price of a ticket from one city to another. In particular, we observe the flight fares of flights and inspect their relationships with the distance traveled in the flight, the number of passengers that take the route each week, and the percentage of flights which are provided by the airline which serves the given route the most.

We plot the relative frequencies of airfare prices in our dataset. The prices seem somewhat normal, although the distribution is, of course, right tailed because the prices approach a lower bound of \$0.

We produce a scatterplot matrix to help visualize the potential relationships between the potential predictors of airfare. At a glance, it appears that airfare increases as the intercity distance is increases, as one would expect, although the airfares vary a lot. When observing the relationship between the airfare with respect to the percentage of route share, it appears that the airfare has a weak relationship with the lead share percentage, but perhaps there is a slight negative correlation between these measures. Finally, routes with a higher throughput of passengers per week appear to weakly decrease in price of airfare as the throughput increases.

To numerically assess the relationships between these potential predictors of airfare, we produce linear models and report the conclusions below:

Fare Predictors	2.5% Confidence		97.5% Confidence	
	Slope	Fare Slope	Slope	p-value
distance	0.04621403	0.05054214	0.05487025	2.2e0-16
distance, lead share	-0.0896	-0.00533	-0.0017	.004049

Fare Predictors	2.5% Confidence Slope	Fare Slope	97.5% Confidence Slope	p-value
distance, lead share, weekly passengers	.1116	0.2815	0.045	.001181

As showed above, the various measurements related to routes have statistically significant linear relationships with airfares. As we saw in our visualizations, the cost of the flight increases with distance, and the cost decreases with the percentage of flights provided by the leading airline. However, An intuition we attempted to derive from the plots is inconsistent with the results of our models: the airfare of a route actually increases with the number of passengers that take the route each week. The error in visualization is reasonable because of how dense the plot is.

A partial correlation coefficient analysis shows that, for all combinations, the testing of independence of all pairs of independent variables produce p-values close to zero. That is, all pairs of independent variables are found to have some amount of correlation. In particular, the distance of the flight and the lead share percentage are quite strongly negatively correlated, with an r value of -0.54. The number of weekly passengers is negatively correlated with both the lead share perentage and the distance of the flight, although quite weakly, at r values of -0.137 and -0.149, respectively.

Finally, we inspect how extraneous points might influence the reliability of our linear regression models. Our dataset contains exactly 1000 points, and a visual inspection shows that only about five points have Cook's distances which are especially larger than the rest. The highest Cook's distance we observe is also just around 0.1. With that, we don't believe that the small number of deviators heavily influence what airfares our model might predict.

As we have explored, we can predict the cost of airfare for a given flight route by using some characteristics of the route: airfares tend to decrease with the number of passengers which use the route each week; airfares also decrease as the percentage of flights provided by some leading airline increase; finally, and maybe obviously, the airfare of a route tends to increase as the distance that the route covers increases.

2.4 Question 4 - Logistic regression analysis

Set up libraries, collect data...

```
if (F) {
  install.packages("caret")
  install.packages('questionr')
  install.packages('pscl')
}

library(questionr)
library(caret)

## Loading required package: lattice

library(gmodels)
library(ggpubr)
library(ggExtra)

shf <- read.csv("data/logisticDataStatureHandFoot.csv")
```

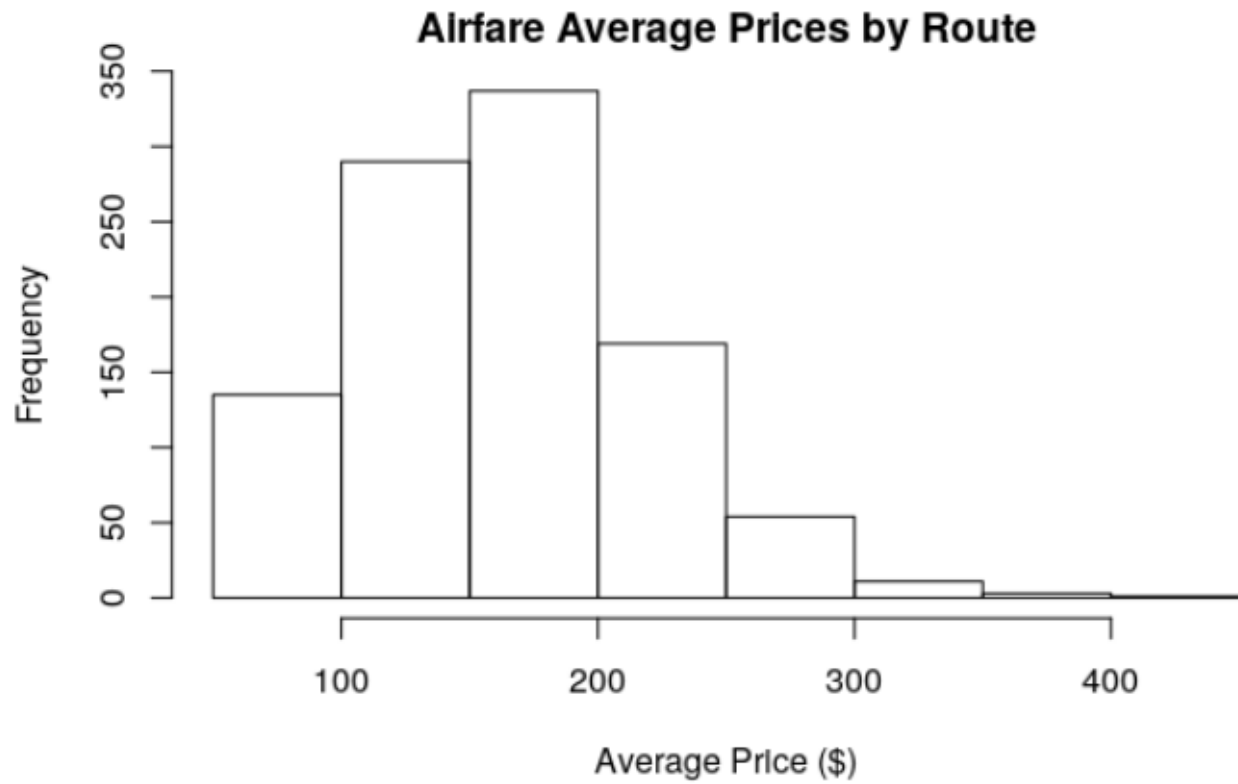


Figure 4: Airfare Histogram.

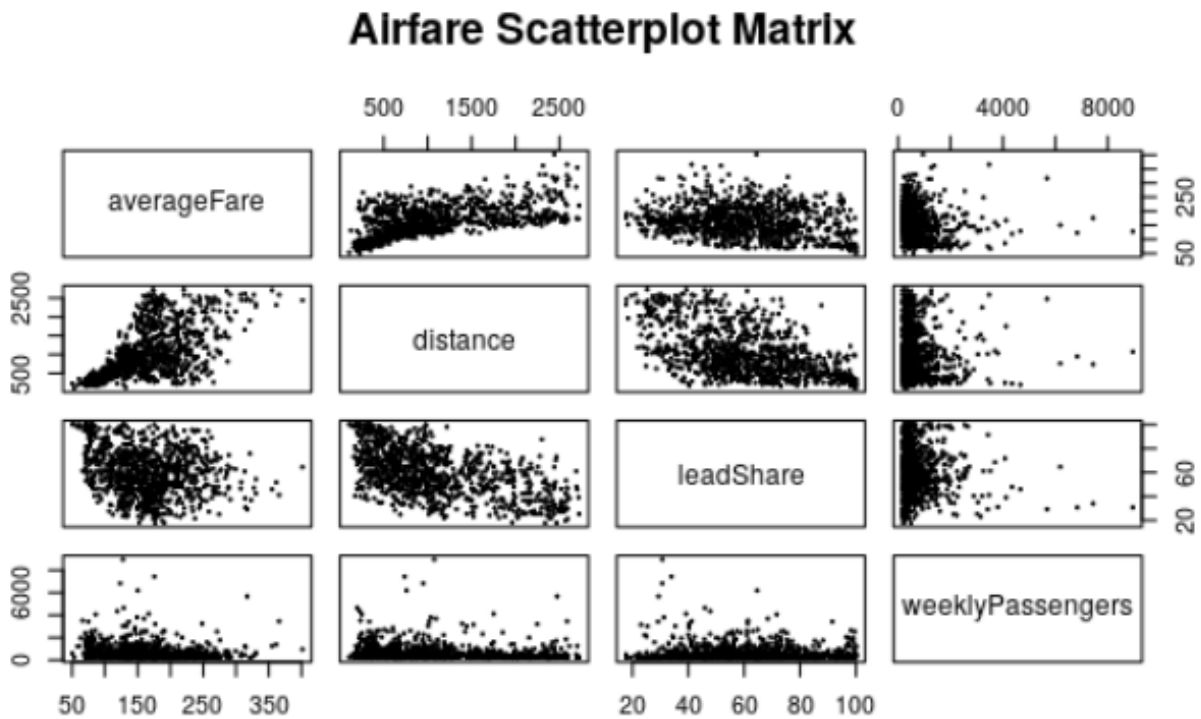


Figure 5: Scatterplots visualizing the relationships between the collected metrics related to airfare.

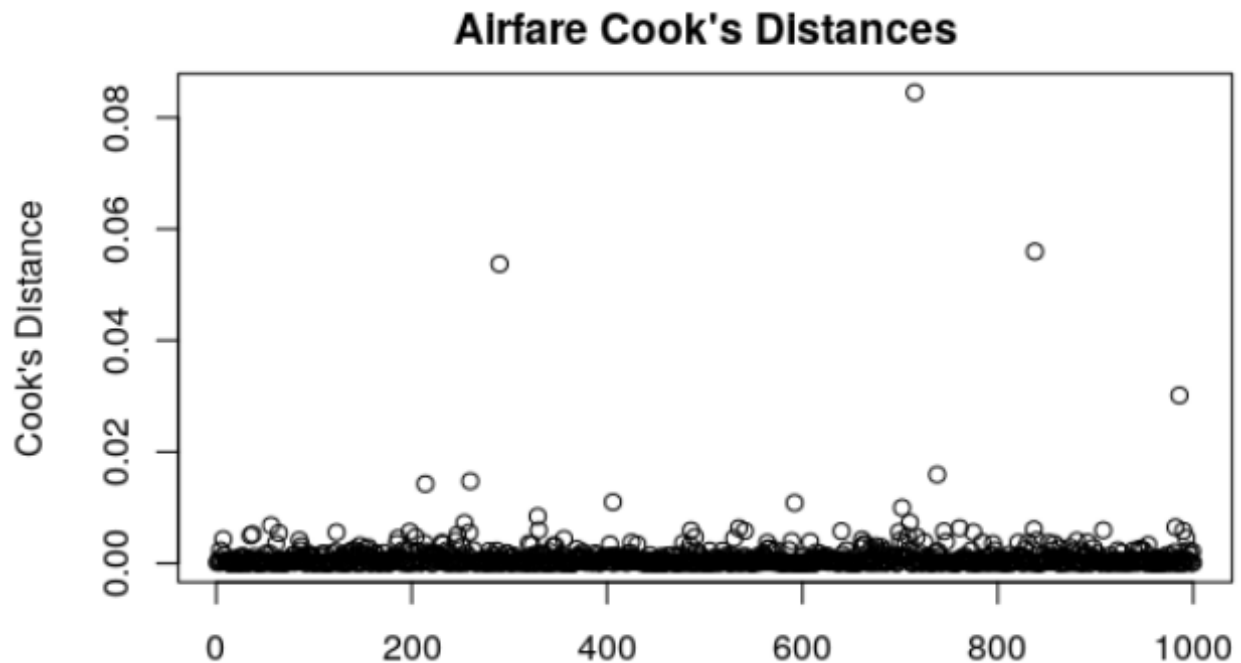


Figure 6: Cook's Distances as measures from our model built upon all regressors. Only handful of points appear to deviate from the rest.

2.4.1 Conceptual model

In this logistic regression analysis, we consider some size measurements of subjects and look for a relationship between these measurements and the sex of the subject. Of course, we assume that the lengths of the hands and feet of our subjects as well as their sexes are independent of those observations in other subjects.

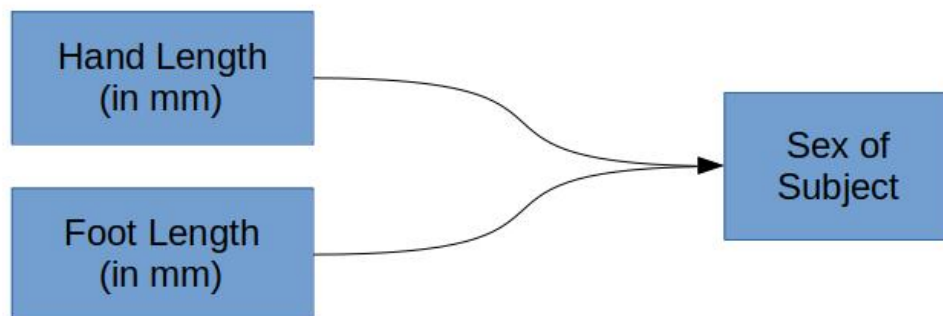
Dichotomous Dependent Variable: sex

Note: the experiment collected and recorded this variable as a "gender". We shall call this variable "sex" because we believe that this is what the experimenters were actually observing.

Independent Variables:

- Hand Length (in mm)
- Foot Length (in mm)

Conceptual Model

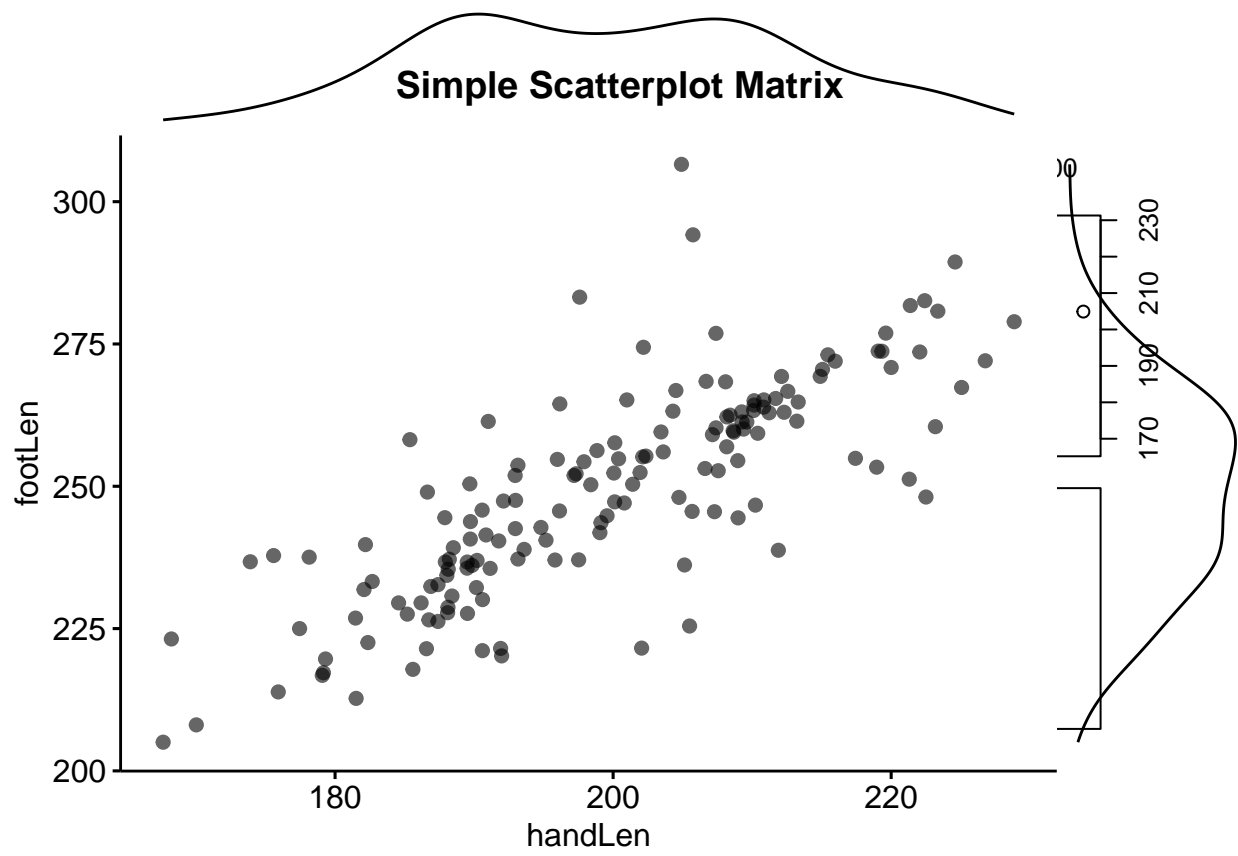


The Null Hypothesis would suggest that the independent variables, which are the lengths of hands and feet, do not have statistically significant relationships with the sex of the subject. The Alternative Hypothesis, then, would be that there is a statistically significant relationship between the lengths of hands and feet and the sex of the subject.

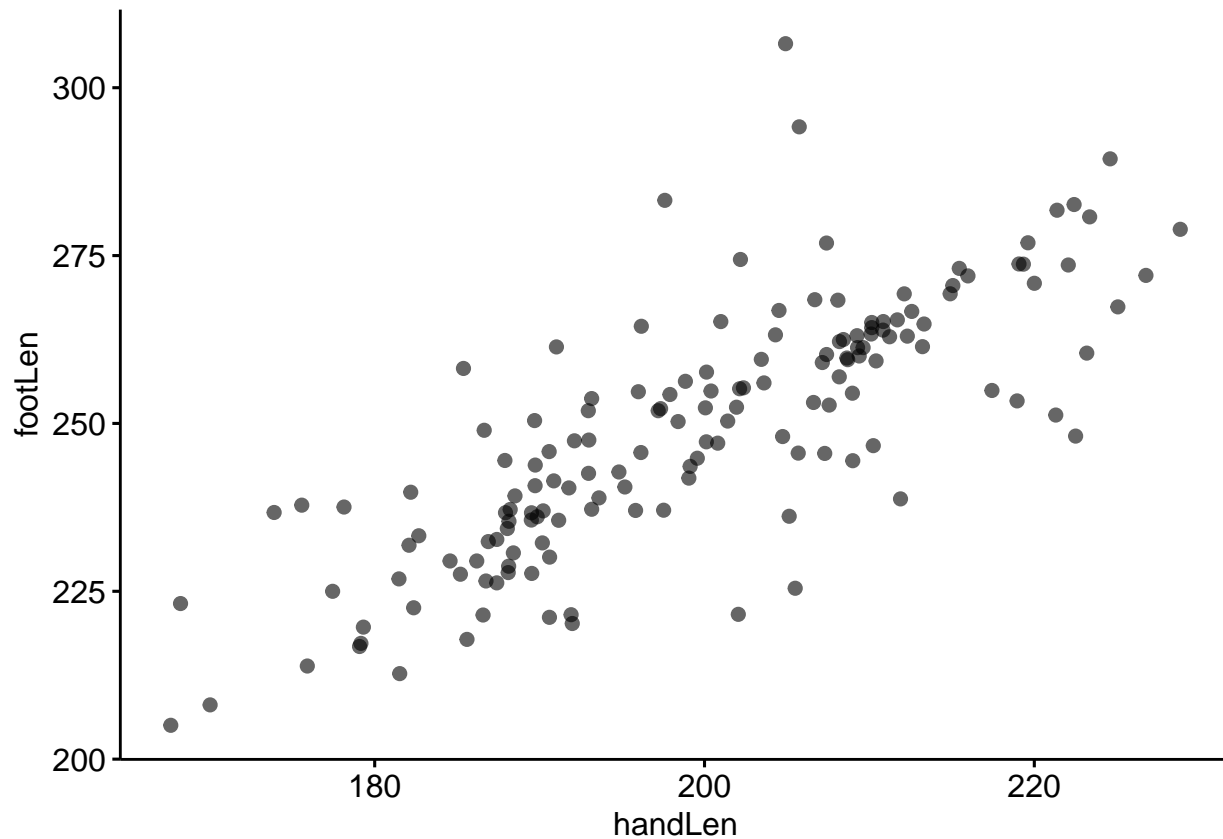
2.4.2 Visualization of Data

```
pairs(~handLen+footLen,data=shf,
      main="Simple Scatterplot Matrix")

p1Q4 <- ggscatter(shf, x = "handLen", y = "footLen",
                  palette = "jco",
                  size = 2, alpha = 0.6)
ggMarginal(p1Q4, type = "density")
```



```
plot(p1Q4)
```

2.4.3 Logistic Regression

```
shf$sex[shf$gender==1] <- 'male'
shf$sex[shf$gender==2] <- 'female'
shf$sex = factor(shf$sex)

model0 = glm(sex ~ 1, data = shf, family = binomial())
model1 = glm(sex ~ handLen, data = shf, family = binomial())
model2 = glm(sex ~ handLen + footLen, data = shf, family = binomial())

anova(model0, model1, model2, test="Chisq")

## Analysis of Deviance Table
##
## Model 1: sex ~ 1
## Model 2: sex ~ handLen
## Model 3: sex ~ handLen + footLen
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1         154      214.714
## 2         153      105.843  1  108.872 < 2.2e-16 ***
## 3         152       75.671  1   30.172 3.954e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

#pander(anova(model0,model1,model2,test = "Chisq" ),
#        caption = "Model comparison of binominal variable of sex")
```

We use of ANOVA for the comparison of our models:

- From random classification of subjects, we find that adding in Hand Length as a predictor improves our linear model. Statistically, the chance that Hand Length as an indicator improves our model without it being a truly good indicator is $2.2\text{e-}16$: we find that hand length has a relationship with the sex of a subject.
- Similarly, we find that additionally adding Foot Length as a predictor improves our model. Statistically, the chance that Foot Length as an indicator improves our previous model, with just Hand Length as an indicator, without it being a truly good indicator is $3.954\text{e-}8$: we find that foot length has a relationship with the sex of a subject.

2.4.4 Visualization of Results

```
sexProbs = predict.glm(model2, shf, type="response")
sexPreds = sapply(sexProbs, function(x) if (x>.5) 'male' else 'female')
sexPreds = factor(sexPreds)
```

```
odds.ratio(model2)
```

```
## Waiting for profiling to be done...
```

```
##              OR      2.5 % 97.5 %      p
## (Intercept) 2.0070e-27 1.3563e-37 0.0000 2.107e-09 ***
## handLen      1.1157e+00 1.0375e+00 1.2116 0.005186 **
## footLen      1.1731e+00 1.0975e+00 1.2737 2.180e-05 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
pscl::pR2(model2)
```

```
##          llh      llhNull          G2      McFadden      r2ML
## -37.8354149 -107.3571538  139.0434779    0.6475743    0.5922311
##          r2CU
##    0.7899157
```

```
dnn = c('predicted', 'observed')
sexTable = table(sexPreds, shf$sex, dnn=dnn)
sexConfusionMatrix = confusionMatrix(sexTable)
sexConfusionMatrix
```

```
## Confusion Matrix and Statistics
```

```
##
```

```
##          observed
```

```
## predicted female male
```

```
##   female      66      7
```

```
##   male        9     73
```

```
##
```

```
##              Accuracy : 0.8968
```

```
##              95% CI : (0.8378, 0.9398)
```

```
##   No Information Rate : 0.5161
```

```
##   P-Value [Acc > NIR] : <2e-16
```

```
##
```

```
##              Kappa : 0.7932
```

```
##
```

```
##   McNemar's Test P-Value : 0.8026
```

```
##
```

```
##           Sensitivity : 0.8800
##           Specificity : 0.9125
##           Pos Pred Value : 0.9041
##           Neg Pred Value : 0.8902
##           Prevalence : 0.4839
##           Detection Rate : 0.4258
##           Detection Prevalence : 0.4710
##           Balanced Accuracy : 0.8962
##
##           'Positive' Class : female
##
```

2.4.5 Report section for a scientific publication

Using logistic regression, we use some body size measurements of subjects and look for their relationships with the sex of the given subject. In particular, we use the lengths of a subject's hand and foot as a predictor, and of course, we assume that the lengths of the hands and feet of our subjects as well as their sexes are independent of those observations in other subjects.

We build a logistic regression model. With an ANOVA test, we find that:

- addition of hand length to the predictor free model creates a stronger model, with ANOVA reporting a p-value of 2.2e-16
- addition of the foot length to the previous model, using hand length, creates an even stronger model, ANOVA reporting a p-value of 3.954e-8

To further evaluate our model, we calculate the following:

- the pseudo-r-squared associated with our model is .7899
- the odds-ratio related to hand length is 1.116, with a 95% confidence interval of [1.03, 1.21]
- the odds-ratio related to foot length is 1.117, with a 95% confidence interval of [1.01, 1.27]

With that, we find both predictors to build a strong logistic regression model for predicting the sex of a subject. We can quickly visualize the performance of our logistic regression model with a confusion matrix, shown model. The model has an accuracy of .8968, which is quite good when only using lengths of body parts.

		Observed	
		Female	Male
Predicted	Female	66	7
	Male	9	73

In conclusion, We find that, when predicting the sex of an individual, hand length and foot length can each be used to attempt to predict the sex of the individual. Using these measurements as indicators are both statistically significantly better than attempting to predict the sex of a person by random chance. Our model considering both of these factors achieves a training error of ~89.7%.

3 Part 3 - Multilevel model

Collecting data and setting up libraries...

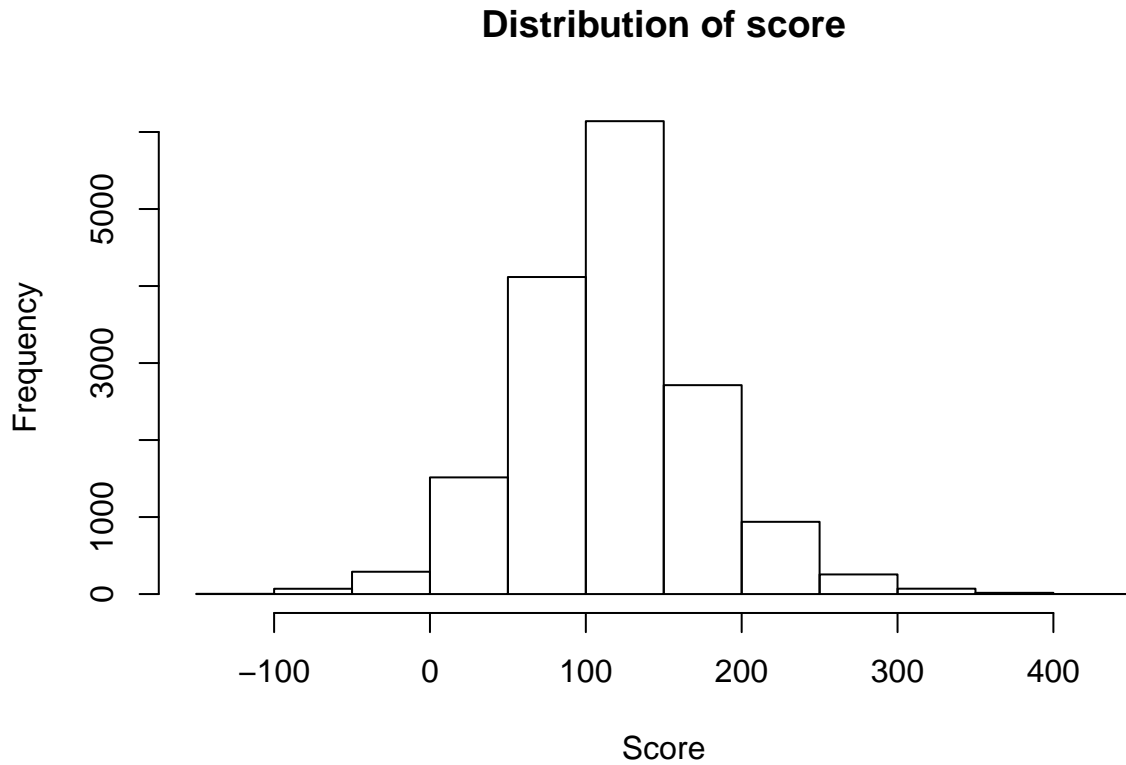
```
library(ggplot2)
library(hexbin)
```

```
library(lattice)
library(nlme)

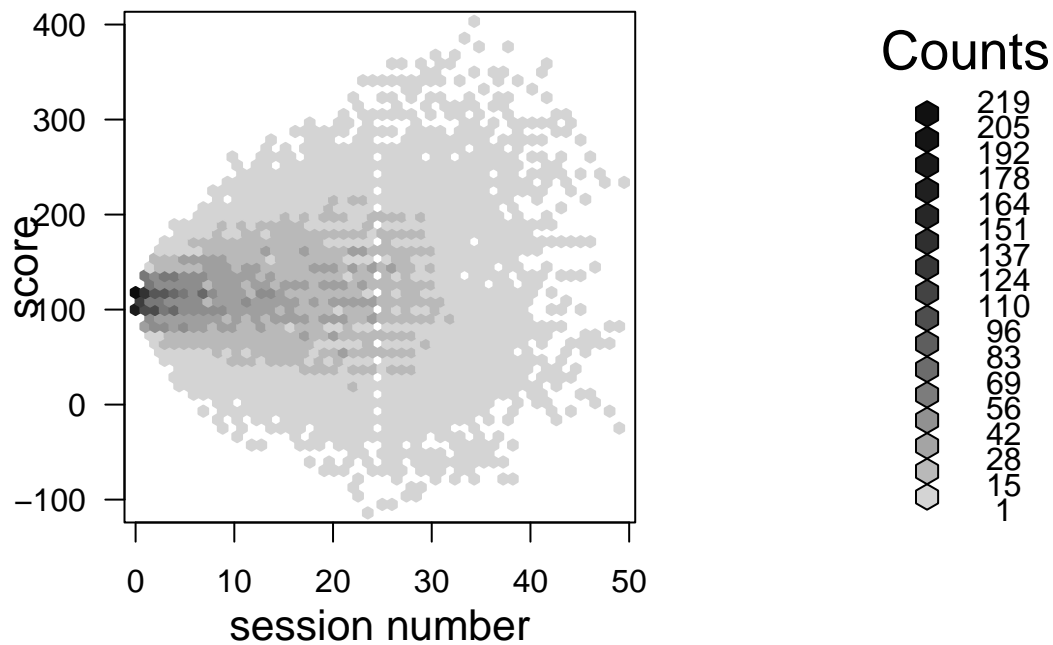
learningData<-read.csv("./data/set1.csv", header = TRUE)
```

3.1 Visual inspection

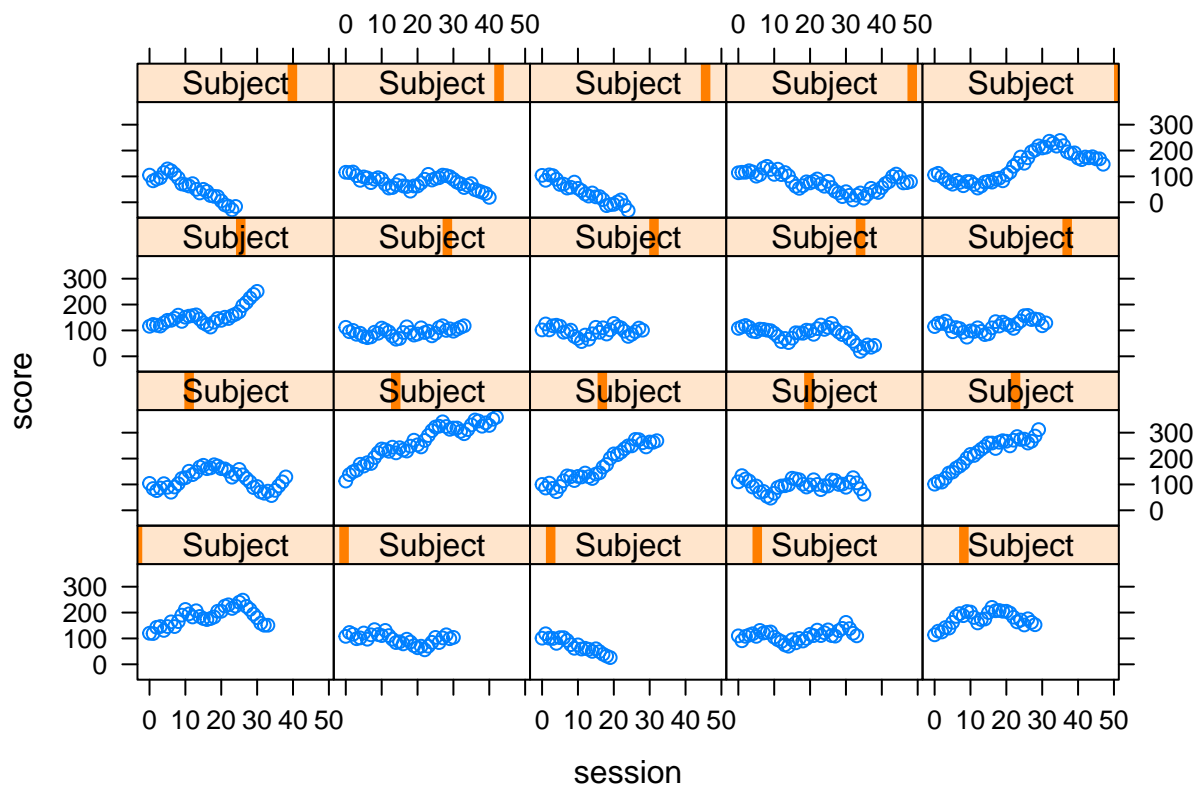
```
hist(learningData$score, xlab="Score", main="Distribution of score")
```



```
plot(hexbin(learningData$score ~ learningData$session, xbins=50, xlab="session number", ylab="score"))
```



```
xyplot(score~session | Subject, data=learningData[learningData$Subject %in% seq(1,191,10),])
```



Through a visual inspection:

- scores alone seem normally distributed
- When plotting a scatterplot of scores against session number, it appears that the score of a testee still centers around ~100 as session number increases, although one might see that the center increases slightly with session number. Also, the spread of scores increases as a function of session number.
- We sample some ~20 subjects from our data and plot their scores as a function of the session after which

they were tested. We see quite inconsistent trends, where some subjects score similarly before and after sessions, some subjects have increasing scores over time, and some subjects even have decreasing scores over time.

3.2 Multilevel analysis with scientific findings

Is there significant variance between the participants in their score?

```
randomInterceptOnly <- lme(score ~ 1, data = learningData,
                           random = ~1|Subject, method = "ML")
summary(randomInterceptOnly)
```

```
## Linear mixed-effects model fit by maximum likelihood
## Data: learningData
##      AIC      BIC    logLik
## 162710.9 162734 -81352.45
##
## Random effects:
## Formula: ~1 | Subject
##      (Intercept) Residual
## StdDev:      46.52747 35.25763
##
## Fixed effects: score ~ 1
##              Value Std.Error   DF  t-value p-value
## (Intercept) 116.8139  2.097897 15627 55.68142      0
##
## Standardized Within-Group Residuals:
##      Min           Q1           Med           Q3           Max
## -4.22644590 -0.61530909  0.01016836  0.62959973  4.10477262
##
## Number of Observations: 16128
## Number of Groups: 501
```

```
intervals(randomInterceptOnly, 0.95)
```

```
## Approximate 95% confidence intervals
##
## Fixed effects:
##           lower      est.    upper
## (Intercept) 112.7019 116.8139 120.9259
## attr("label")
## [1] "Fixed effects:"
##
## Random Effects:
## Level: Subject
##           lower      est.    upper
## sd((Intercept)) 43.68637 46.52747 49.55334
##
## Within-group standard error:
##           lower      est.    upper
## 34.86891 35.25763 35.65067
```

We find that there is very high variance between the scores of each subject on a given session. With a p-value of 0, we find approximately no chance that, if the distributions of scores as a function of lesson number came from the same distribution, we would find a collection of this sort of data. That is, We reject the null

hypothesis that the scores of a subject as a function of the number of lessons he had received are not from the same distribution. Subjects' scores respond differently to receiving lessons.

Does session have an impact on people score?

```
randomInterceptSession <- lme(score ~ session,
                              data = learningData, random = ~1|Subject, method = "ML")
summary(randomInterceptSession)
```

```
## Linear mixed-effects model fit by maximum likelihood
## Data: learningData
##      AIC      BIC    logLik
## 162545.2 162575.9 -81268.58
##
## Random effects:
## Formula: ~1 | Subject
##      (Intercept) Residual
## StdDev:      46.5146 35.06933
##
## Fixed effects: score ~ session
##              Value Std.Error   DF  t-value p-value
## (Intercept) 111.0676  2.143371 15626 51.81911      0
## session      0.3682  0.028356 15626 12.98493      0
## Correlation:
##      (Intr)
## session -0.206
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.120041920 -0.613554431  0.009847298  0.627208531  3.952634923
##
## Number of Observations: 16128
## Number of Groups: 501
```

```
anova(randomInterceptOnly, randomInterceptSession)
```

```
##           Model df      AIC      BIC    logLik    Test
## randomInterceptOnly      1 3 162710.9 162734.0 -81352.45
## randomInterceptSession    2 4 162545.2 162575.9 -81268.58 1 vs 2
##           L.Ratio p-value
## randomInterceptOnly
## randomInterceptSession 167.7298 <.0001
```

With a p-value of $<.001$, we find that there is a correlation between the number of sessions attended by a subject and the change in their test score. In particular, we see that a subject's score increases by ~ 0.368 points per training session that they attend.

3.2.1 Report section for a scientific publication

We inspect a dataset recording a longitudinal analysis which, for a group of subjects, records the scores they achieve after each training session for some undescribed task. We're interested in seeing if the scores on the exercise systematically vary with respect to the number of training sessions completed.

First, we would like to analyze, as individuals are trained at the task more over time, whether they consistently change their post-training scores. We assess this with a Linear Mixed-Effect Model, and with a p-value of

$<.001$, we find that there is a correlation between the number of sessions attended by a subject and the change in their test score. In particular, we see that a subject's score increases by ~ 0.368 points per training session that they attend.

We also inspect if the scores of individuals vary significantly after a given number of training sessions. Again, we use a Linear Mixed-Effect Model and find a high variance between the scores of each subject on a given session. With a p-value of 0, we find that the subjects have significant variance in their scores.

With this analysis, we believe that the task being trained, whatever the task happens to be, does indeed improve the subjects at the task being tested upon. However, the test is scored at a maximum of 300, and subjects only improved their scores by about .37 points per session, which sounds like a very small improvement.

Nehlig, A. 2010. "Is Caffeine a Cognitive Enhancer?" Journal Article. <https://www.ncbi.nlm.nih.gov/pubmed/20182035>.

Pasman, Wilrike J, Ruud Boessen, Yoni Donner, Nard Clabbers, and Andre Boorsma. 2017. "Effect of Caffeine on Attention and Alertness Measured in a Home-Setting, Using Web-Based Cognition Tests." Journal Article. <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5608989/>.