

Report Template coursework assignment A - 2019

CS4125 Seminar Research Methodology for Data Science

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1 Part 1 - Design and set-up of true experiment

1.1 Motivation

Students, researchers, and most people beyond these buckets believe that caffeine improves their productivity. Many even insist on having their morning coffee to do their work each day. We design an experiment to empirically evaluate the effects of caffeine as well as other intake trends on one’s mental acuity. Perhaps caffeine truly improves one’s performance in their studies or work. However, we may also investigate the existence of a placebo effect on one’s work ethic. We also investigate more complex effects of caffeine with its relationship on mental acuity, such as the influence of the caffeine-induced “crash”, and caffeine tolerance, and possible relationships between caffeine effectiveness and amount of sleep.

1.2 Theory

Academics have performed a host of analyses on the effects of caffeine on cognitive performance. For example, a study by Nehlig at UDS found that caffeine changes memory performance in nuanced ways, and it likely does not change the aggregation of long-term memory (???). The study concluded that “caffeine cannot be considered a ‘pure’ cognitive enhancer”, although it may indirectly influence, and possibly enhance, one’s cognitive performance.(???). Another study by Pasman et al. found that, when taking cognition tests, scores of subjects did not improve, but the tests were completed “approximately 10% faster” (Pasma et al. 2017). The findings of these studies suggest that, while caffeine may not directly improve cognitive performance, indirect factors may still lead users to enjoy increased efficiency when doing their work.

1.3 Research Questions

Put succinctly, here are some of the questions that we investigate in this experiment: - Does caffeine intake change one’s mental acuity? - Does the “caffeine crash”, or the perceived exhaustion after wearing off of caffeine, change one’s mental acuity? - Does there exist a placebo affect in caffeine intake and one’s mental acuity? - Does caffeine tolerance change the effectiveness of a single dose of caffeine? - Does one’s recent amount of sleep change the effectiveness of caffeine at changing one’s mental acuity?

1.4 Participants

For convenience and consistency, the experiment will use students from TUDelft as participants. A sample from this population would likely generalize to broader student populations for the effects of caffeine intake. We can also find various levels of caffeine intake habits and regular sleep amounts. Lastly, because the students belong to the same university, we can expect that, with a lower variance in mental acuity, a smaller sample size could suffice for testing of statistical significance. One thing to consider is that, because TUDelft is a linguistically diverse university, we can make no assumptions about the language backgrounds of the students. As such, it is important to test subjects with means independent of reading ability, domain experience, etc. Administering of caffeine shall be transparent and consensual. Caffeine will be administered in commercially available forms and otherwise ordinary forms, with the possibility of caffeine-free doses.

1.5 Conceptual Model

Dependent Variable: reading test score Mental acuity is difficult to measure, so we will approximate it by administering a brief intelligence examination. In particular, the subject will be given an IQ test, which can be used as an unbiased approximation of the mental acuity of a subject.

Independent Variables:

- * Amount of Administered Caffeine (in mg)
- * Duration Since Caffeine Dose

Mediating Variables:

- * Caffeine Tolerance (as approximated by amount of caffeine per day)
- * Average Sleep Amount (in Hours)

Moderating Variables:

- * Perceived Intake of Caffeine (Placebo or No Placebo)

1.6 Experimental Design

The experiment explores the change in mental acuity as described by various factors around caffeine intake. There are many variables which may complicate and influence the change in mental acuity, such as caffeine tolerance and duration since dose, and these variables will be tested in experiments where other variables are controlled.

1.7 Experimental Procedure

For the collection of data from test subjects:

1. Randomly sample a test subject from a place of study, and preemptively, randomly assign treatment to the subject
 - * treatment would be in the form of a dose of caffeine, a placebo dose of caffeine, or no dose of caffeine
 - * We would need to find people who have not had any caffeine yet on the given day
2. Collect some preliminary information about the subject, including estimated sleep amount and estimated caffeine intake per day
3. administer a waiting period for the subject to go about other activities and allow the caffeine to kick in and/or wear off
 - * Duration of waiting period will be influenced by academic literature about caffeine
4. After the waiting period, administer the IQ test
 - * The test should be realistically impossible to complete in the given amount of time to avoid statistically meaningless score distributions

1.8 Experiments and Suggested Statistical Analyses

In this section, we describe the types of independent variables that we will observe and what statistical tests would allow for hypothesis testing:

1. Effect of Caffeine Intake on Mental Acuity -> two-sample t-statistic
2. Placebo Effect of Caffeine Intake -> two-sample t-statistic
2. Effectiveness of Caffeine Intake Over Time -> Regression Analysis
3. Influence of Caffeine Tolerance on Caffeine Effectiveness -> Regression Analysis
3. Influence of Sleep on Caffeine Effectiveness -> Multiple Regression Analysis

2 Part 2 - Generalized linear models

2.1 Question 1: Twitter Sentiment Analysis (Between groups - single factor)

Set up libraries...

```
##
## Attaching package: 'container'

## The following object is masked from 'package:base':
##
##      remove

##
## Attaching package: 'plyr'

## The following object is masked from 'package:twitterR':
##
##      id

## The following objects are masked from 'package:container':
##
##      count, empty

## Loading required package: carData
```

2.1.1 Collecting Preliminary Data

Collect positive and negative words and related functions, set up twitter oauth...

```
## [1] "Using direct authentication"
```

Collect tweets about celebrities, calculate sentiment scores...

2.1.2 Tweet Sentiment Analysis

Is there a difference in the sentiment of the tweets related to the different celebrities? We inspect this question using context independent sentiment analysis of tweets about them. The data was collected as follows: 1. We used the twitter api to collect the 1000 most recent tweets which contain the twitter tag of the celebrity of interest 2. Punctuation and links were removed from the tweets 3. Upon these “parsed” tweets, we counted the number of “positive” and “negative” words present in the tweet, as provided in lists for the assignment, and the tweets were given a sentiment score as a difference of the positive and negative word counts

Tweets about the following American music artists were collected: - Kanye West - Drake - Ariana Grande - Cardi B - Lil Pump - Nicki Minaj

2.1.3 Statistical Considerations:

Making no assumptions, we would expect that all tweets about our celebrities have similar sentiment scores. That is, the sentiment scores found in tweets about celebrities have come from the same, broad distribution of sentiment scores in tweets about all celebrities. The alternative hypothesis would be that tweet sentiment scores come from different distributions when we sample tweets about the various celebrities.

2.1.4 Assessing Homogeneity of Variance

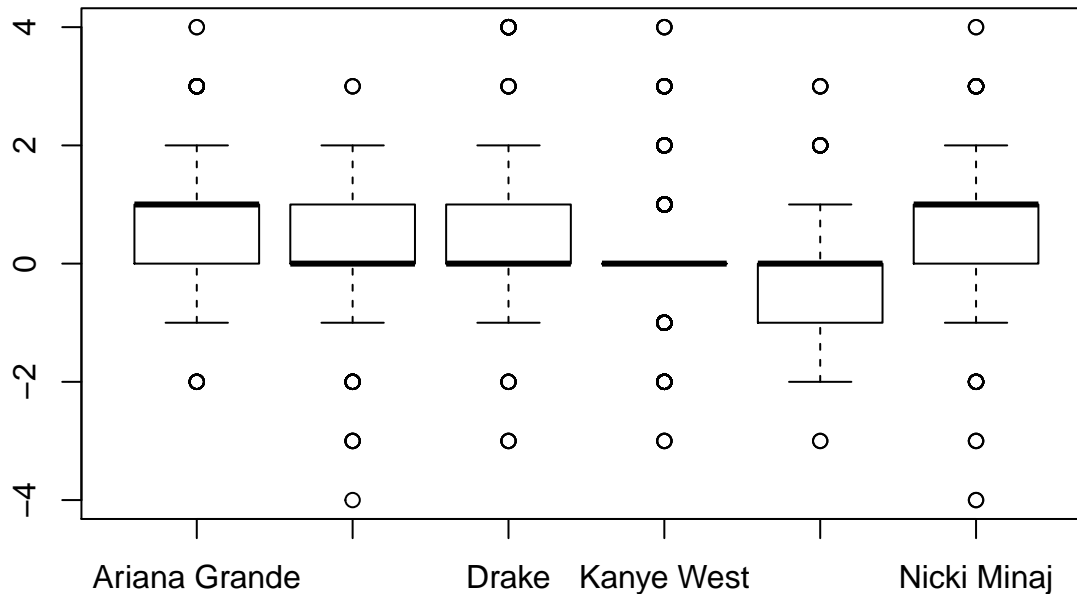
```
leveneTest(tweets$score, group=tweets$name)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##           Df F value    Pr(>F)
## group      5  3.0336 0.009731 **
##           5994
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We use the Levene Test to assess homogeneity of variance. At an alpha value of .05, our p-value is less than our alpha value, so we reject the assumption that the tweet sentiment distributions from the various celebrities have homogeneous variances.

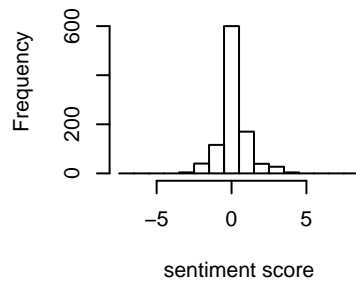
2.1.5 Graphical Examination of Means and Variations of Tweet Sentiment by Celebrity

```
boxplot(score ~ name, data=tweets)
```

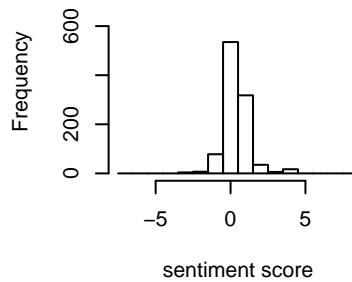


```
par(mfrow=c(2,3))
for (name in unique(tweets$name)) {
  hist(tweets[tweets$name == name,]$score, breaks = -8:8 + .5,
    main=paste("Tweet Sentiments for", name), xlab="sentiment score", ylim=c(0,750))
}
```

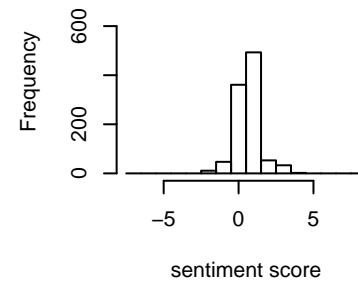
Tweet Sentiments for Kanye We



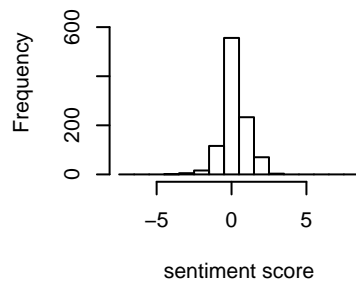
Tweet Sentiments for Drake



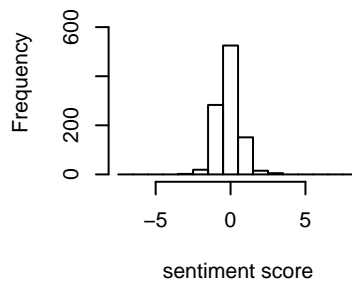
Tweet Sentiments for Ariana Gra



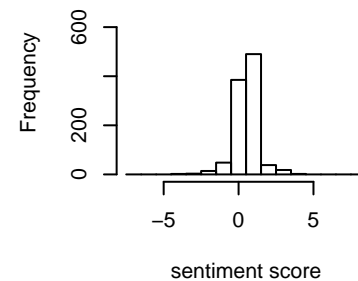
Tweet Sentiments for Cardi B



Tweet Sentiments for Lil Pump



Tweet Sentiments for Nicki Min



Above, we have produced visualizations of the sampled distributions of tweet sentiments about various American celebrity artists. They have different medians, levels of spread, and numbers of outliers. In particular:

- All celebrities were found to have a neutral sentiment score for the mode for their sampled tweet scores for Ariana Grande and Lil Pump, who have a mode tweet sentiment of +1
- Most celebrities have an interquartile range of 1, while Drake's tweet scores have an IQR of 2, and Kanye's tweet scores have an IQR of 0
- The boxplots show that a lot of celebrities have many outliers in their tweet score distributions, as defined by the boxplot function
- Distributions tend to center around neutral tweets, but some celebrities receive far less neutral tweets than others:
- Kanye West appeared to receive about ~650 neutral tweets
- Ariana Grande received only about ~250 neutral tweets, and most of her tweets were at a score of +1

2.1.6 Tweet Knowledge as Used to Describe Tweet Sentiment

```
tweets_lm0 <- lm(score ~ 1, data = tweets,
na.action = na.exclude)
tweets_lm1 <- lm(score ~ name, data = tweets,
na.action = na.exclude)
anova(tweets_lm0, tweets_lm1)
```

```
## Analysis of Variance Table
##
## Model 1: score ~ 1
## Model 2: score ~ name
##   Res.Df  RSS Df Sum of Sq    F    Pr(>F)
## 1     5999 4891
## 2     5994 4498   5    393.02 104.75 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Using the ANOVA function in R, we find a p-value which is less than 0.05. With that, we reject the hypothesis that the sentiment score of a tweet cannot be predicted by knowing which celebrity the tweet pertains to.

2.1.7 Assessing Model Quality Using Tweet Knowledge as a Predictor

```
pairwise.t.test(tweets$score, tweets$name, paired=FALSE, p.adjust.method="bonferroni")

##
## Pairwise comparisons using t tests with pooled SD
##
## data: tweets$score and tweets$name
##
##           Ariana Grande Cardi B Drake   Kanye West Lil Pump
## Cardi B    < 2e-16      -      -      -           -
## Drake      9.1e-11      0.00096 -      -           -
## Kanye West < 2e-16      0.66175 2.9e-08 -           -
## Lil Pump   < 2e-16      < 2e-16 < 2e-16 7.6e-11    -
## Nicki Minaj 0.12731      2.7e-15 0.00031 < 2e-16    < 2e-16
##
## P value adjustment method: bonferroni
```

The Bonferroni Correction analysis shows that the tweet distributions for most pairs of celebrities differ from each other. However, for some pairs, it appears that we fail to reject that the tweets come from different distributions. In this example, assuming an alpha value of 0.05, we see that we fail to reject the given null hypothesis for the following pairs of celebrities: - Drake and Kanye West - Nicki Minaj and Cardi B - Nicki Minaj and Kanye West

2.1.8 Findings

Through statistical significance tests and inspection of visualizations, we believe that the sentiment score distribution from tweets pertaining to various American celebrity artists do not come from the same distribution. The sampled distributions of sentiment scores about artists have different variances and median sentiment scores. However, some pairs of celebrities appear to come from similar distributions.

2.2 Question 2 - Website visits (between groups - Two factors)

Set up libraries, load data...

2.2.1 Conceptual model

We are tasked with analyzing the results of an A-B study of a webserver as administered in two different versions to two different groups. Also, notice that we are using the webvisit dataset 1. Through this analysis, we investigate linear modeling between groups of two different factors:

Independent Variables:

- * Version of webserver (0 or 1)
- * Type of User (0=consumer, 1=company)
- * All combinations of the previously listed independent variables

Dependent Variables:

- * Number of pages visited

In this analysis, we inspect whether the independent variables, individually and/or in combination, effected the number of pages visited:

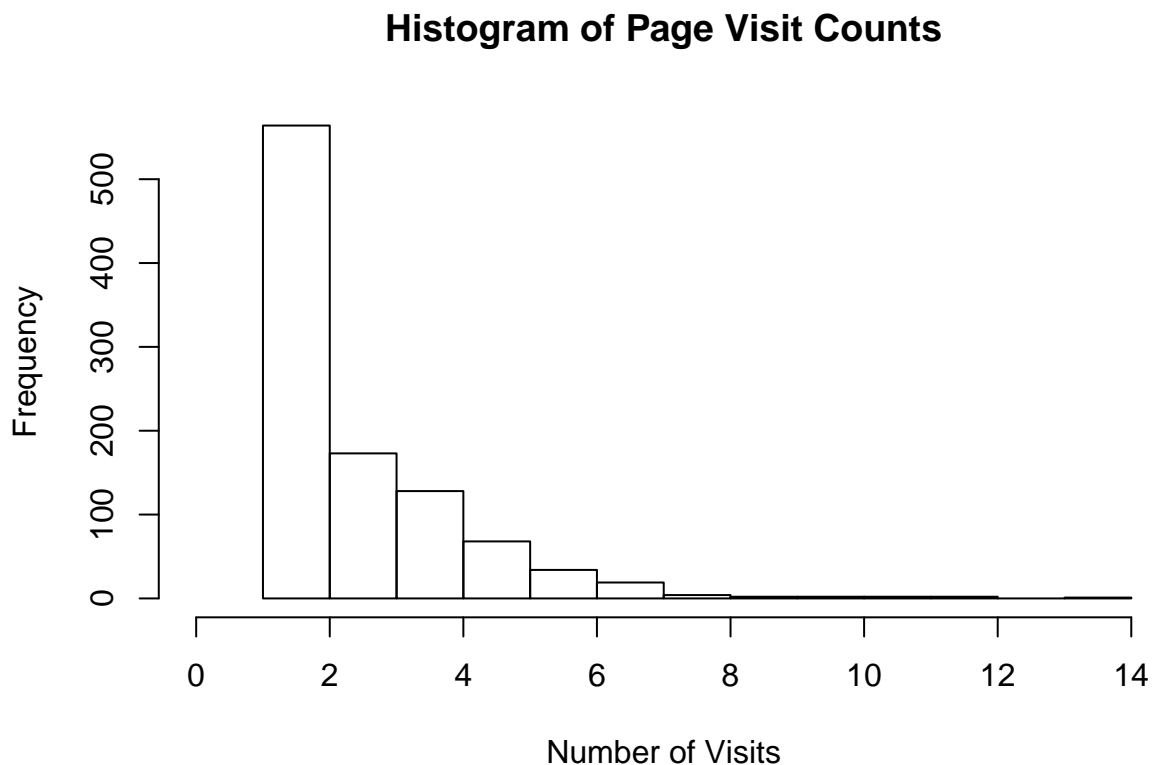
Null hypothesis: There is no observed difference in the number of pages visited based on either the versions, the portals, or a combination thereof used. The observed difference in the sample is based on a sampling error and there is no observed difference in the entire population.

Alternative hypothesis: The observed difference in the sample is a real effect plus some change variation.

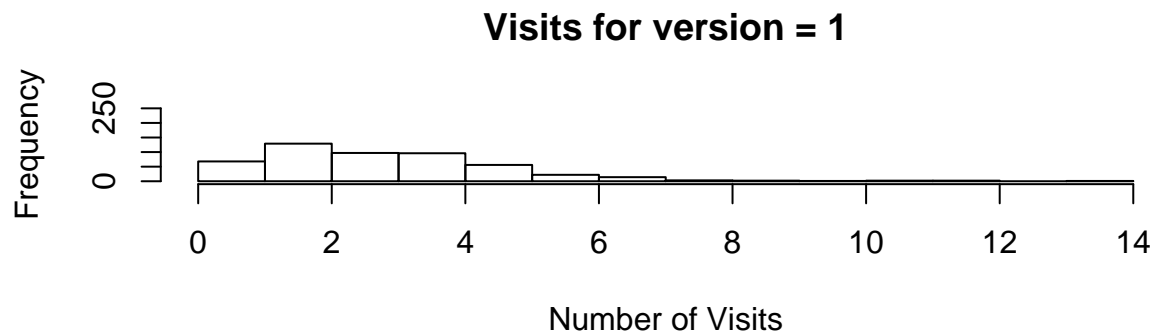
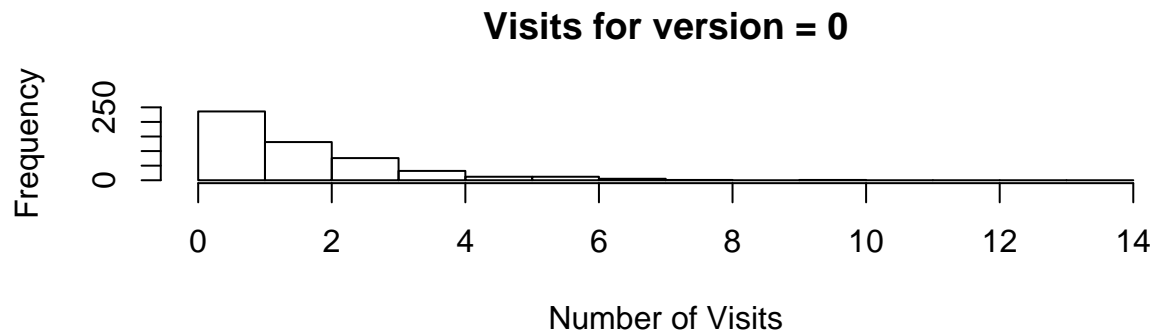
2.2.2 Visual inspection

```
xlim = c(0, max(visits$pages))
ylim = c(0, 250)
breaks=max(visits$pages)

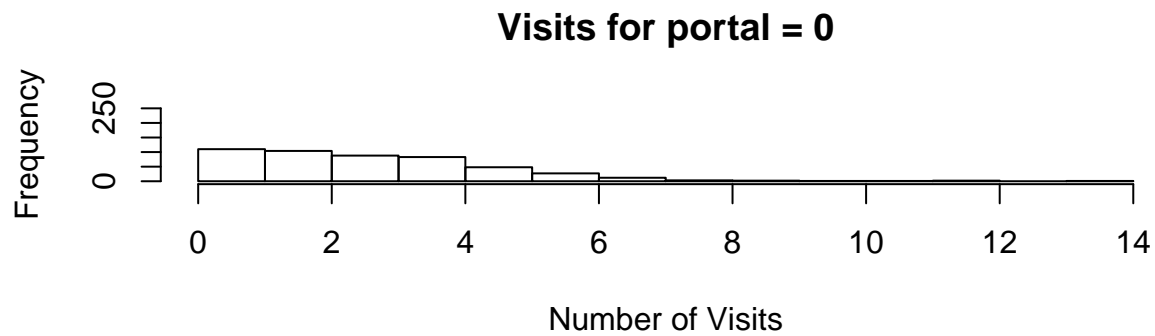
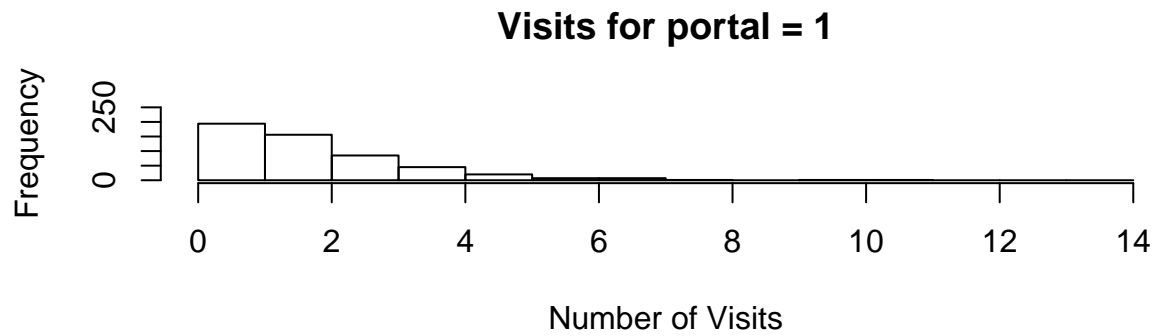
# histogram of all page visits
hist(visits$pages, xlab="Number of Visits", main="Histogram of Page Visit Counts", xlim=xlim)
```



```
# histogram of page visits by different explanator variables:
# by design
par(mfrow=c(2,1))
for (v in unique(visits$version)) {
  hist(visits[which(visits$version == v),]$pages,
       xlab="Number of Visits", main=paste("Visits for version =", v),
       xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
}
```

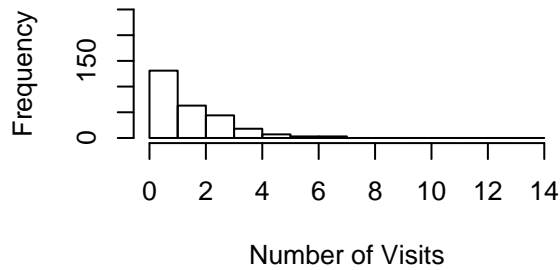



```
# by user type
par(mfrow=c(2,1))
for (p in unique(visits$portal)) {
  hist(visits[which(visits$portal == p),]$pages,
       xlab="Number of Visits", main=paste("Visits for portal =", p),
       xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
}
```

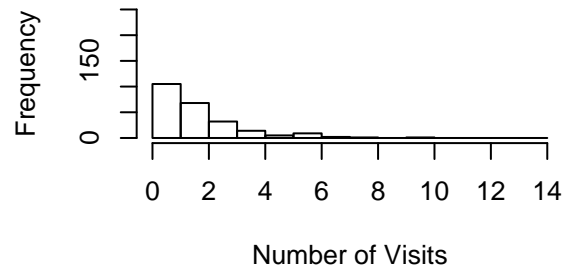


```
# by all combinations:
par(mfrow=c(2,2))
for (v in unique(visits$version)) {
  for (p in unique(visits$portal)) {
    hist(visits[which(visits$version == v & visits$portal == p),]$pages,
          xlab="Number of Visits",
          main=paste("Visits for version =", v, ", portal =", p),
          xlim=xlim, ylim=ylim, breaks=seq(0,breaks,1))
  }
}
```

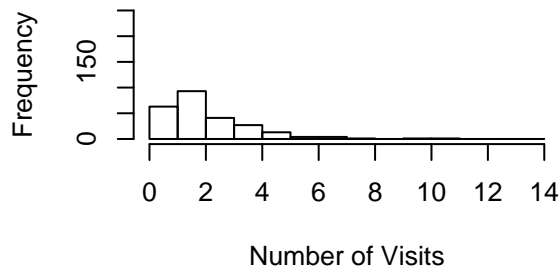
Visits for version = 0 , portal = 1



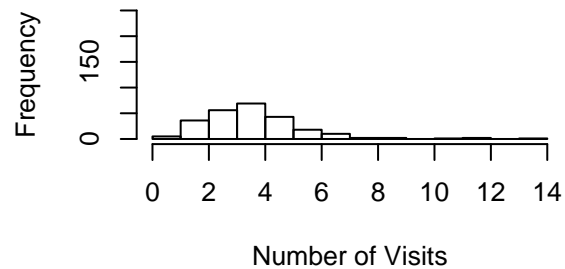
Visits for version = 0 , portal = 0



Visits for version = 1 , portal = 1



Visits for version = 1 , portal = 0



Upon visual inspection, it appears that the portal type doesn't change the distribution of page visit counts. However, in both cases, it appears that the version of the website causes the mean page visit count to shift to the right, and the page visit count distributions no longer seem as right skewed.

2.2.3 Normality check

Statistically test if variable page visits deviates from normal distribution

```
shapiro.test(visits$pages)
```

```
##  
## Shapiro-Wilk normality test  
##  
## data: visits$pages  
## W = 0.83076, p-value < 2.2e-16
```

A simple Shapiro-Wilk normality test reveals, with a p-value of 2.2e-16, it is unlikely that the true distribution of page visit counts across all scenarios can be described by a linear model.

2.2.4 Model analysis

We construct linear models for the number of page visits as described by some factors:

- model0 - page visit count without predicting variables
- model1 - page visit count as linearly described by website version
- model2 - page visit count as linearly described by user type
- model3 - page visit count as linearly described by website version and user type, independently

- model3 - page visit count as linearly described by website version and user type, with interaction effects

```
pages_model0 = lm(pages~1, data=visits,
  na.action=na.exclude)
pages_model1 = lm(pages~version, data=visits,
  na.action=na.exclude)
pages_model2 = lm(pages~portal, data=visits,
  na.action=na.exclude)
pages_model3 = lm(pages~version+portal, data=visits,
  na.action=na.exclude)
pages_model4 = lm(pages~version+portal+version:portal,
  data=visits, na.action=na.exclude)
```

```
anova(pages_model0, pages_model1)
```

```
## Analysis of Variance Table
##
## Model 1: pages ~ 1
## Model 2: pages ~ version
##   Res.Df    RSS Df Sum of Sq   F    Pr(>F)
## 1      998 3054.7
## 2      997 2671.5   1    383.16 143 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model 1 attempts to predict the page visit count using page version, and an ANOVA test finds, with p-value = 2.2e-16, that page visit count is not independent of the page version.

```
anova(pages_model0, pages_model2)
```

```
## Analysis of Variance Table
##
## Model 1: pages ~ 1
## Model 2: pages ~ portal
##   Res.Df    RSS Df Sum of Sq   F    Pr(>F)
## 1      998 3054.7
## 2      997 2866.5   1    188.14 65.435 1.734e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model 2 attempts to predict the page visit count using user type, and an ANOVA test finds, with p-value = 1.734e-15, that page visit count is not independent of the user type.

```
anova(pages_model3, pages_model4)
```

```
## Analysis of Variance Table
##
## Model 1: pages ~ version + portal
## Model 2: pages ~ version + portal + version:portal
##   Res.Df    RSS Df Sum of Sq   F    Pr(>F)
## 1      996 2498.3
## 2      995 2374.8   1    123.42 51.709 1.264e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Because model1 and model2, which use page version and user type as predictors, can be used to explain trends in page visits, we would expect that a model using both of these predictors would work as well. This is how we constructed model3. Now we use an ANOVA test to see whether including interaction effects would

further help predict the number of page visits, as we construct in model4. Through this ANOVA test with a p-value of 1.264e-12, we see that interaction effects can be further used to explain the number of page visits seen.

2.2.5 Simple effect analysis

```
visits$interaction = interaction(visits$portal, visits$version)
allPortalsVersion0 = c(1,-1,0,0)
allPortalsVersion1 = c(0,0,1,-1)
SimpleEff = cbind(allPortalsVersion0, allPortalsVersion1)
contrasts(visits$interaction) = SimpleEff
simpleEffectModel = aov(pages~interaction, data=visits, na.action=na.exclude)
summary.lm(simpleEffectModel)
```

```
##
## Call:
## aov(formula = pages ~ interaction, data = visits, na.action = na.exclude)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
##	-3.0694	-1.0694	-0.1266	0.8734	9.9306

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
## (Intercept)	2.67725	0.04893	54.714	<2e-16 ***
## interactionallPortalsVersion0	0.06887	0.06882	1.001	0.317
## interactionallPortalsVersion1	0.77260	0.06958	11.104	<2e-16 ***
## interaction	1.23908	0.09786	12.661	<2e-16 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.545 on 995 degrees of freedom
## Multiple R-squared:  0.2226, Adjusted R-squared:  0.2202
## F-statistic: 94.94 on 3 and 995 DF, p-value: < 2.2e-16
```

Our analysis shows that, indeed, there is an interaction effect, but only in some cases:

- For version 0, the type of user doesn't change the page visit count. The test specifically finds a p-value of 0.317, so we don't have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.
- For version 1, the type of user indeed changes the page visit count. The test specifically finds a p-value of 2e-16, so we have reason to believe that there is a statistically significant difference in page visit counts for the consumer vs business users when using this version.

2.2.6 Report section for a scientific publication

We analyzed the number of page visits for a given website. In particular, we inspected the effects of the type of user and the version of the website presented to the type of user. In general, we find that the number of page visits is indeed dependent on the type of user visiting the page and the version of the page. We also found that some conditions of the experiment seem to have interaction effects. In particular, we find that for users that are using portal version 0, there is a statistically significant difference in the trends of page visit counts for the business users and the consumer users.

2.3 Question 3 - Linear regression analysis

Set up libraries, load data...

```
## Loading required package: ggplot2
## Loading required package: magrittr
##
## Attaching package: 'magrittr'
## The following object is masked from 'package:container':
##
##      add
##
## Attaching package: 'ggpubr'
## The following object is masked from 'package:plyr':
##
##      mutate
## The following object is masked from 'package:container':
##
##      rotate
## Loading required package: MASS
```

2.3.1 Conceptual model

For a self-guided linear regression analysis, we investigate a dataset which records airfares from cities to cities. We would like to see if the chosen independent variables can be used to predict the price of a ticket from one city to another. For the analysis:

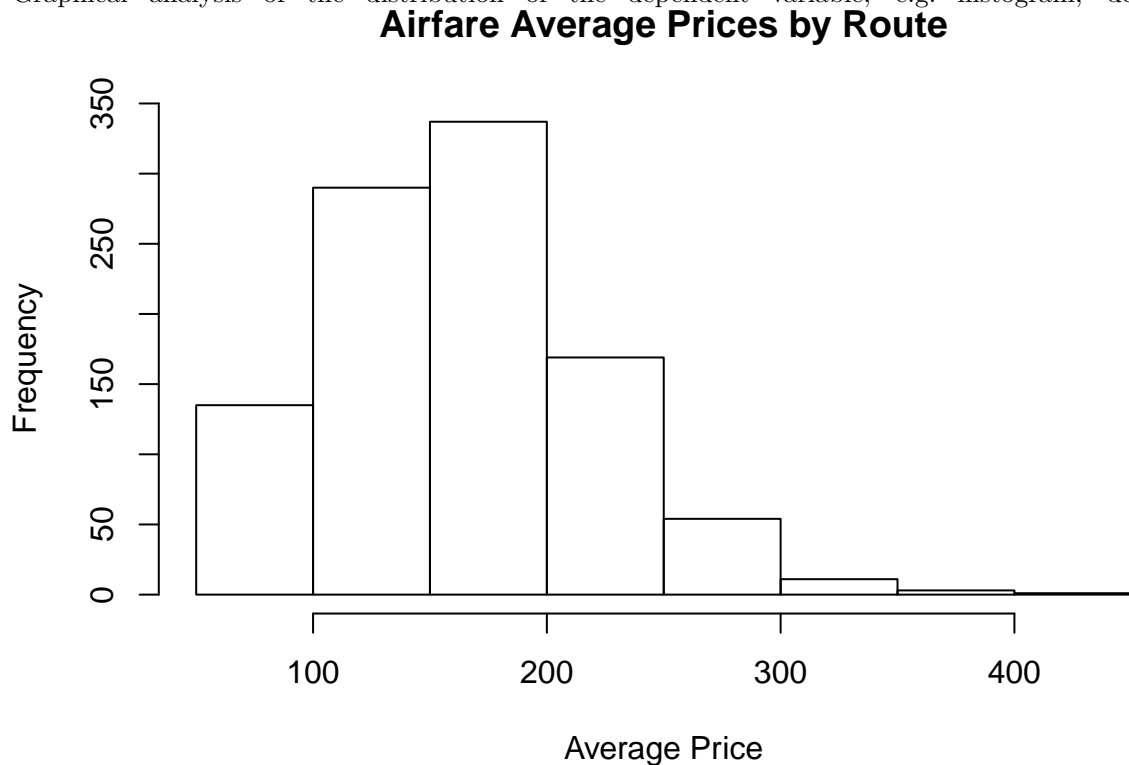
Dependent Variable: Average Fare The average price of the ticket to get from City1 to City2

Independent Variables:

- * * Distance - the distance between City1 and City2
- * Average Weekly Passengers - the average number of passengers that fly from City1 to City2 per week
- * Lead Share - the Percentage of the flights from City1 to City2 which are served through the leading airline of the route

2.3.2 Visual inspection

Graphical analysis of the distribution of the dependent variable, e.g. histogram, density plot

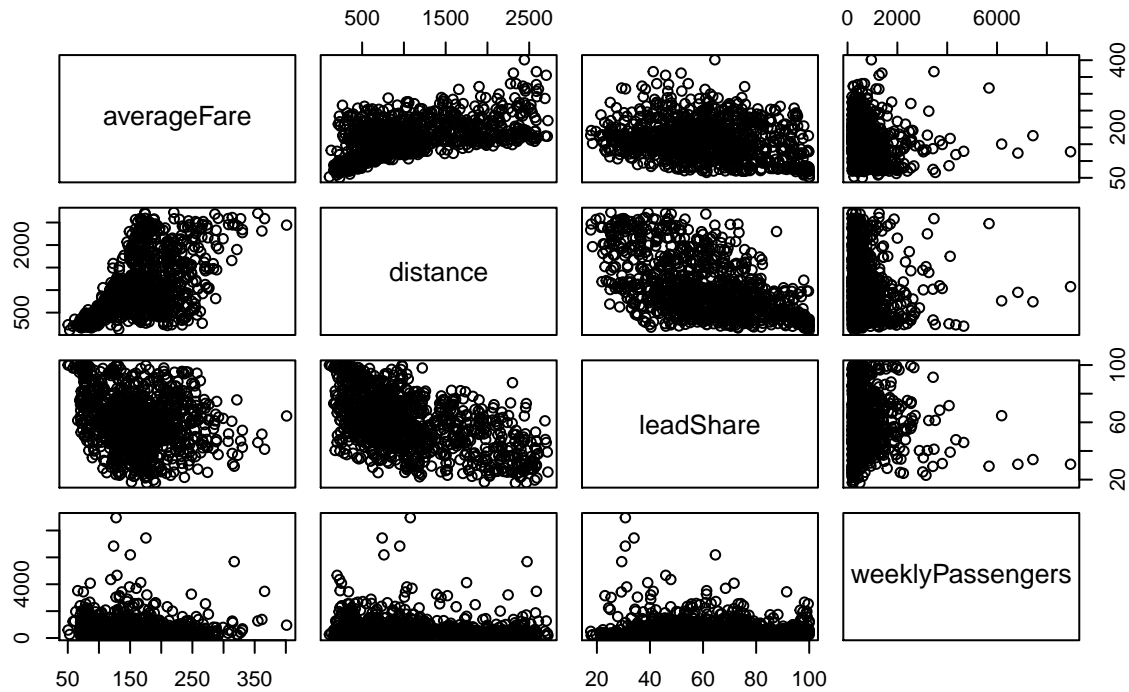


The price of airfares appears normally distributed, centered somewhere around 175 units. The prices are right skewed, although this is expected because the price has a lower bound of 0.

2.3.3 Scatter plot

```
# Basic Scatterplot Matrix
pairs(~averageFare+distance+leadShare+weeklyPassengers,data=airfare,
      main="Simple Scatterplot Matrix")
```

Simple Scatterplot Matrix



We produce a scatterplot matrix for our independent and dependent variables. Some of the scatterplots which stand out:

- Average fare seems to increase with distance, which is quite intuitive.
- There doesn't appear to be a strong relationship between the percentage of flights owned by the leading airline and the average fare price.
- It's very hard to see a relationship, visually, between the amount of weekly passengers for a route and the price of the flight. However, the routes which have extremely high weekly passenger counts seem to have lower prices. This trend is visually supported by very few data points, though.

2.3.4 Linear regression

Conduct a multiple linear regression (including confidence intervals, and beta-values)

```
fare_model0 = lm(averageFare ~ 1, data=airfare, na.action=na.exclude)
confint(fare_model0)
```

```
##                2.5 %    97.5 %
## (Intercept) 159.9397 166.8111
```

```
coef(fare_model0)
```

```
## (Intercept)
##      163.3754
```

```
fare_model1 = lm(averageFare ~ distance, data=airfare, na.action=na.exclude)
confint(fare_model1)
```

```
##                2.5 %    97.5 %
## (Intercept) 104.59931149 115.30817862
## distance      0.04621403   0.05487025
```



```
coef(fare_model1)
```

```
## (Intercept)      distance
## 109.95374505    0.05054214
```

```
anova(fare_model0, fare_model1)
```

```
## Analysis of Variance Table
##
## Model 1: averageFare ~ 1
## Model 2: averageFare ~ distance
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      999 3062270
## 2      998 2006500   1   1055770 525.12 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the distance of the flight. The test reports a p-value of 2.2e-16, so indeed we reject that the average fare price is independent of the distance between the cities. We also report the confidence interval and the weights of the independent variables above.

```
fare_model2 = lm(averageFare ~ distance + weeklyPassengers,
                 data=airfare, na.action=na.exclude)
confint(fare_model2)
```

```
##              2.5 %      97.5 %
## (Intercept)  108.094615921 120.196476496
## distance      0.045639171   0.054299165
## weeklyPassengers -0.008967769 -0.001700977
```

```
coef(fare_model2)
```

```
##      (Intercept)      distance weeklyPassengers
##    114.145546209      0.049969168      -0.005334373
```

```
anova(fare_model1, fare_model2)
```

```
## Analysis of Variance Table
##
## Model 1: averageFare ~ distance
## Model 2: averageFare ~ distance + weeklyPassengers
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      998 2006500
## 2      997 1989934   1     16567  8.3003 0.004049 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the number of weekly passengers of the route. The test reports a p-value .004049, so indeed we reject that the average fare price is independent of the number of weekly passengers. We also report the confidence interval and the weights of the independent variables above.

```
fare_model3 = lm(averageFare ~ distance + weeklyPassengers + leadShare,
                 data=airfare, na.action=na.exclude)
confint(fare_model3)
```

```
##              2.5 %      97.5 %
## (Intercept)  77.041006409 106.566226301
## distance      0.049439643   0.059686992
```

```
## weeklyPassengers -0.008156413 -0.000855535
## leadShare      0.111695523  0.451401752

coef(fare_model3)

##      (Intercept)      distance weeklyPassengers      leadShare
##      91.803616355      0.054563317      -0.004505974      0.281548638

anova(fare_model2, fare_model3)

## Analysis of Variance Table
##
## Model 1: averageFare ~ distance + weeklyPassengers
## Model 2: averageFare ~ distance + weeklyPassengers + leadShare
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      997 1989934
## 2      996 1969017   1      20917 10.581 0.001181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our ANOVA test finds that the average price of the fare is dependent on the percentage of flights which are provided by the lead airline. The test reports a p-value .001181, so indeed we reject that the average fare price is independent of the percentage of flights which are controlled by the leading airline of the route. We also report the confidence interval and the weights of the independent variables above.

2.3.5 Examine assumption

```
X = airfare[c('distance', 'weeklyPassengers', 'leadShare')]
Y = airfare['averageFare']
imcdiag(x=X, y=Y)

##
## Call:
## imcdiag(x = X, y = Y)
##
##
## All Individual Multicollinearity Diagnostics Result
##
##              VIF    TOL      Wi      Fi Leamer  CVIF Klein
## distance      1.4252 0.7016 211.9735 424.3723 0.8376 1.5679    0
## weeklyPassengers 1.0274 0.9733  13.6824  27.3922 0.9866 1.1303    0
## leadShare      1.4201 0.7042 209.4375 419.2952 0.8391 1.5623    0
##
## 1 --> COLLINEARITY is detected by the test
## 0 --> COLLINEARITY is not detected by the test
##
## * all coefficients have significant t-ratios
##
## R-square of y on all x: 0.357
##
## * use method argument to check which regressors may be the reason of collinearity
## =====
pcor(X, method='pearson')

## $estimate
```

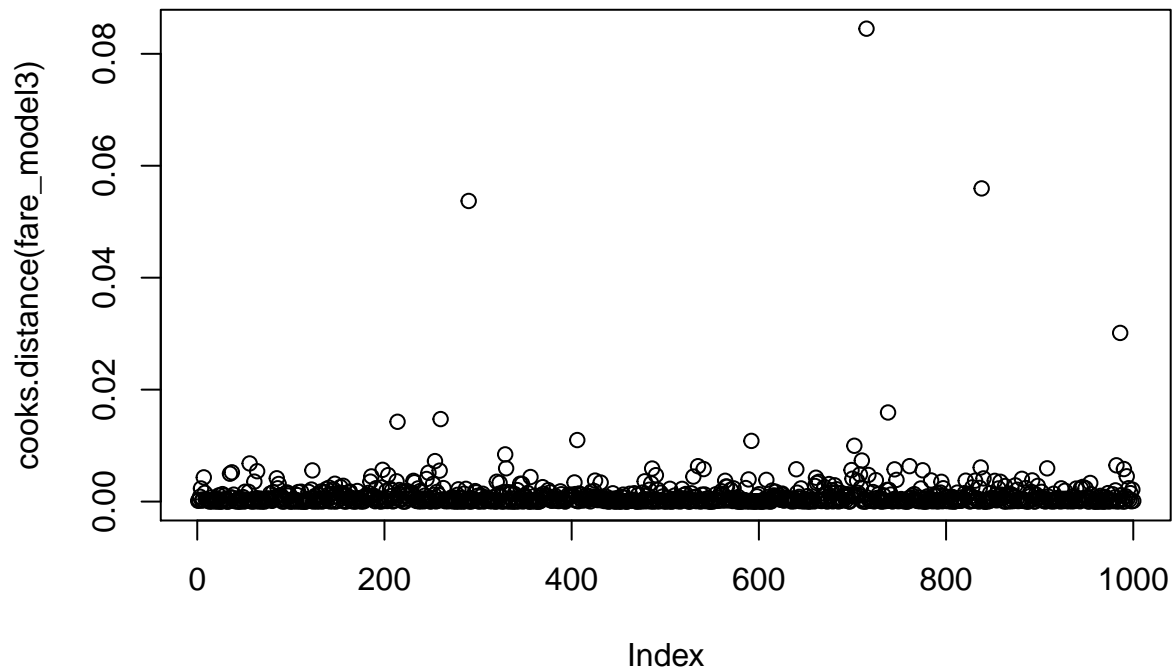
```
##           distance weeklyPassengers  leadShare
## distance      1.0000000      -0.1491478 -0.5409332
## weeklyPassengers -0.1491478        1.0000000 -0.1369035
## leadShare      -0.5409332      -0.1369035  1.0000000
##
## $p.value
##           distance weeklyPassengers  leadShare
## distance      0.000000e+00      2.193641e-06 5.292396e-77
## weeklyPassengers 2.193641e-06      0.000000e+00 1.410984e-05
## leadShare      5.292396e-77      1.410984e-05 0.000000e+00
##
## $statistic
##           distance weeklyPassengers  leadShare
## distance      0.00000      -4.762660 -20.307730
## weeklyPassengers -4.76266        0.000000 -4.363857
## leadShare      -20.30773      -4.363857  0.000000
##
## $n
## [1] 1000
##
## $gp
## [1] 1
##
## $method
## [1] "pearson"
```

The output of the partial correlation coefficients analysis shows that, for all combinations, the testing of independence of all pairs of independent variables produces p-values close to zero. That is, all pairs of independent variables are found to have some amount of correlation.

2.3.6 Impact analysis of individual cases

Examine effect of single cases on the predicted values (e.g. DFBeta, Cook's distance)

```
plot(cooks.distance(fare_model3))
```



The plotting of cook's distance shows that all DFBeta are far less than one. Of the 1000 airfare data points, only a handful of points have values which seem to deviate from the rest, but we do not believe that these values would not reveal any sort of influence of single cases on predicted values.

2.3.7 Report section for a scientific publication

We performed a multiple regression on the average airfare of a route from one city to another. We found that the average airfare is dependent on the distance between cities, the amount of flights on the route owned by a dominant airline, and the average number of weekly passengers of the given route. We also found that each of these independent variables exhibit high degrees of correlation.

2.4 Question 4 - Logistic regression analysis

Set up libraries, collect data...

2.4.1 Conceptual model

In this logistic regression analysis, we consider some size measurements of subjects and look for a relationship between these measurements and the sex of the subject. Of course, we assume that the lengths of the hands and feet of our subjects as well as their sexes are independent of those observations in other subjects.

Dichotomous Dependent Variable: sex Note: the experiment collected and recorded this variable as a "gender". We shall call this variable "sex" because we believe that this is what the experimenters were actually observing.

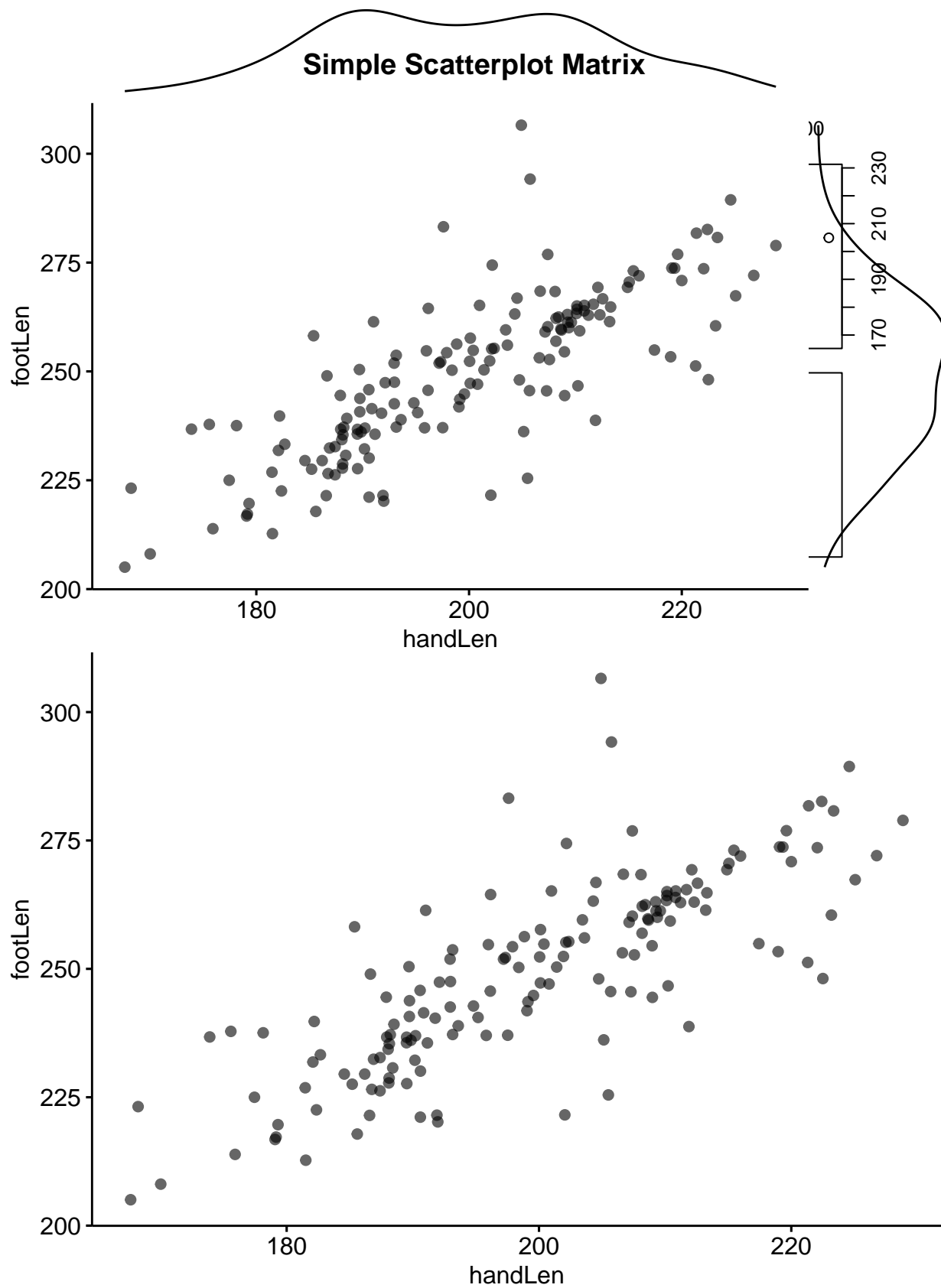
Independent Variables:

- * Hand Length (in mm)
- * Foot Length (in mm)

The Null Hypothesis would suggest that the independent variables, each individually and combinations of them, do not have statistically significant trends such that we can reliably predict the sex of a new subject given their feature size measurements. The Alternative Hypothesis, then, would be to say that we can reliably

attempt to predict the sex of a subject given their feature size measurements because we find different sexes to exhibit different trends in these measurements.

2.4.2 Visualization of Data



2.4.3 Logistic Regression

```
## Analysis of Deviance Table
##
## Model 1: sex ~ 1
## Model 2: sex ~ handLen
## Model 3: sex ~ handLen + footLen
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      154    214.714
## 2      153    105.843  1  108.872 < 2.2e-16 ***
## 3      152     75.671  1   30.172 3.954e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We use of ANOVA for the comparison of our models:

- From random classification of subjects, we find that adding in Hand Length as a predictor improves our linear model. Statistically, the chance that Hand Length as an indicator improves our model without it being a truly good indicator is $2.2e-16$: we reject that Hand Length is independent of the sex of a subject.
- Similarly, we find that additionally adding Foot Length as a predictor improves our model. Statistically, the chance that Foot Length as an indicator improves our previous model, with just Hand Length as an indicator, without it being a truly good indicator is $3.954e-8$: We reject that Foot Length is independent of the sex of the subject.

2.4.4 Visualization of Results

```
## [1] "training accuracy: 0.896774193548387"
```

2.4.5 Report section for a scientific publication

We find that, when predicting the sex of an individual, Hand Length and Foot Length can each be used to attempt to predict the sex of the individual. Using these measurements as indicators are both statistically significantly better than attempting to predict the sex of a person by random chance. Our model considering both of these factors achieves a training error of $\sim 89.7\%$.

3 Part 3 - Multilevel model

Collecting data and setting up libraries...

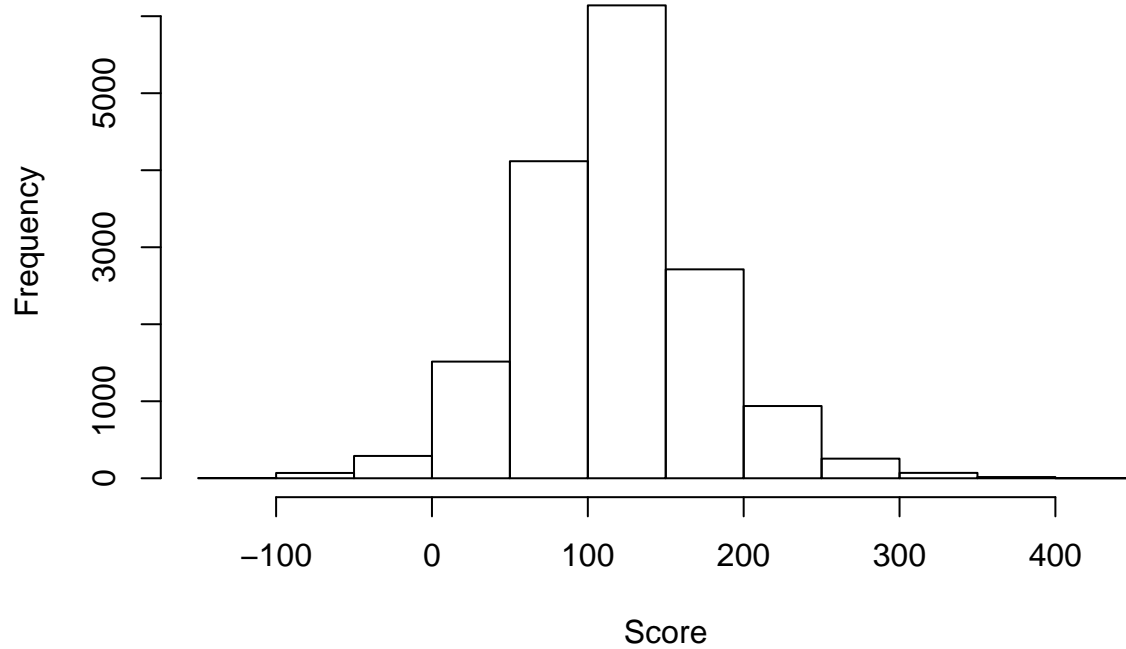
```
library(ggplot2)
library(hexbin)
library(lattice)
library(nlme)

learningData<-read.csv("./data/set1.csv", header = TRUE)
```

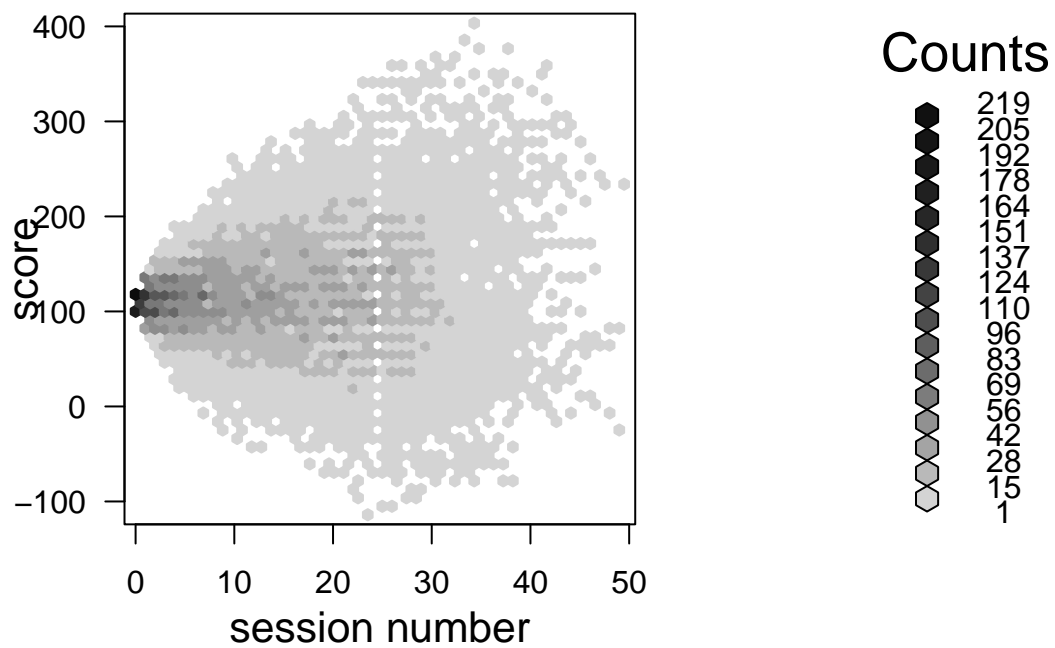
3.1 Visual inspection

```
hist(learningData$score, xlab="Score", main="Distribution of score")
```

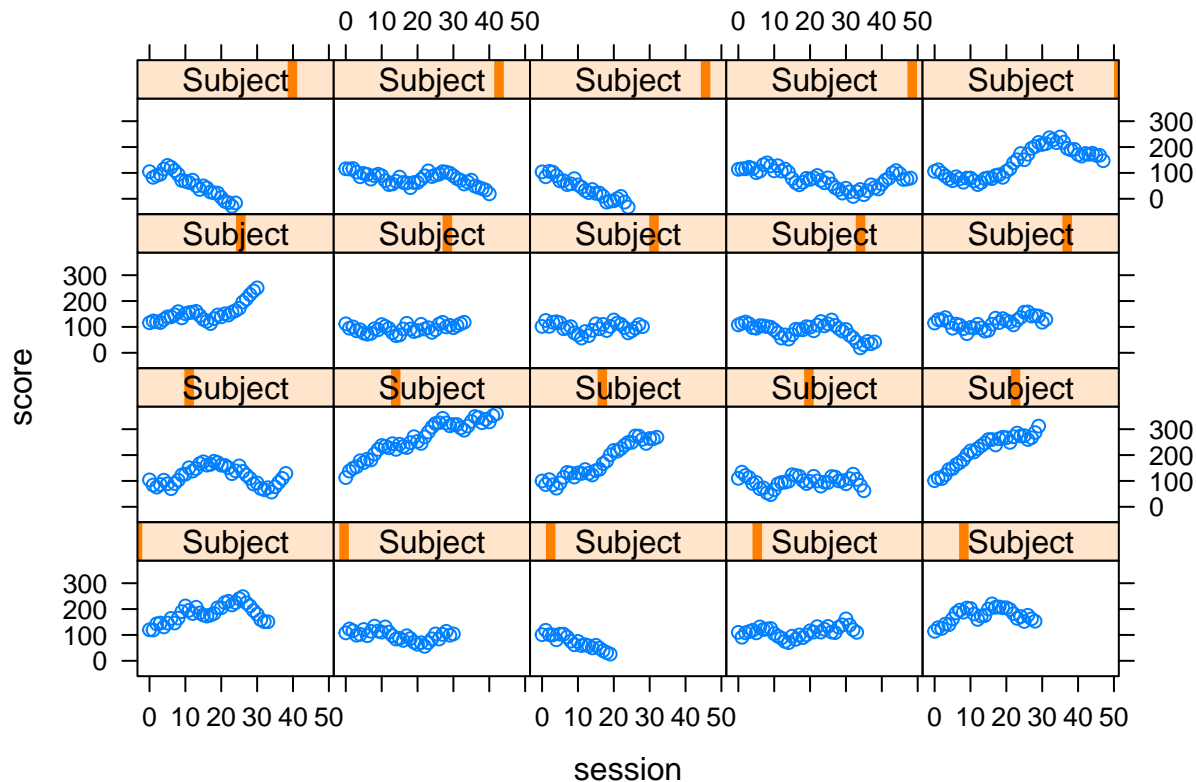

Distribution of score



```
plot(hexbin(learningData$score ~ learningData$session, xbins=50, xlab="session number", ylab="score"))
```



```
xyplot(score~session | Subject, data=learningData[learningData$Subject %in% seq(1,191,10),])
```



Through a visual inspection:

- scores alone seem normally distributed
- When plotting a scatterplot of scores against session number, it appears that the score of a testee still centers around ~100 as session number increases, although one might see that the center increases slightly with session number. Also, the spread of scores increases as a function of session number.
- We sample some ~20 subjects from our data and plot their scores as a function of the session after which they were tested. We see quite inconsistent trends, where some subjects score similarly before and after sessions, some subjects have increasing scores over time, and some subjects even have decreasing scores over time.

3.2 Multilevel analysis with scientific findings

Is there significant variance between the participants in their score?

```
randomInterceptOnly <- lme(score ~ 1, data = learningData,
                             random = ~1|Subject, method = "ML")
summary(randomInterceptOnly)
```

```
## Linear mixed-effects model fit by maximum likelihood
## Data: learningData
##      AIC      BIC    logLik
## 162710.9 162734 -81352.45
##
## Random effects:
## Formula: ~1 | Subject
##      (Intercept) Residual
## StdDev:      46.52747 35.25763
##
```

```
## Fixed effects: score ~ 1
##           Value Std.Error   DF  t-value p-value
## (Intercept) 116.8139  2.097897 15627 55.68142      0
##
## Standardized Within-Group Residuals:
##      Min           Q1           Med           Q3           Max
## -4.22644590 -0.61530909  0.01016836  0.62959973  4.10477262
##
## Number of Observations: 16128
## Number of Groups: 501
```

```
intervals(randomInterceptOnly, 0.95)
```

```
## Approximate 95% confidence intervals
##
## Fixed effects:
##           lower      est.      upper
## (Intercept) 112.7019 116.8139 120.9259
## attr(,"label")
## [1] "Fixed effects:"
##
## Random Effects:
## Level: Subject
##           lower      est.      upper
## sd((Intercept)) 43.68637 46.52747 49.55334
##
## Within-group standard error:
##           lower      est.      upper
## 34.86891 35.25763 35.65067
```

We find that there is very high variance between the scores of each subject on a given session. With a p-value of 0, we find approximately no chance that, if the distributions of scores as a function of lesson number came from the same distribution, we would find a collection of this sort of data. That is, We reject the null hypothesis that the scores of a subject as a function of the number of lessons he had recieved are not from the same distribution. Subjects' scores respond differently to recieving lessons.

Does session have an impact on people score?

```
randomInterceptSession <- lme(score ~ session,
                              data = learningData, random = ~1|Subject, method = "ML")
summary(randomInterceptSession)
```

```
## Linear mixed-effects model fit by maximum likelihood
## Data: learningData
##           AIC           BIC       logLik
## 162545.2 162575.9 -81268.58
##
## Random effects:
## Formula: ~1 | Subject
##           (Intercept) Residual
## StdDev:      46.5146 35.06933
##
## Fixed effects: score ~ session
##           Value Std.Error   DF  t-value p-value
## (Intercept) 111.0676  2.143371 15626 51.81911      0
## session      0.3682  0.028356 15626 12.98493      0
```

```
## Correlation:
##      (Intr)
## session -0.206
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.120041920 -0.613554431  0.009847298  0.627208531  3.952634923
##
## Number of Observations: 16128
## Number of Groups: 501
anova(randomInterceptOnly,randomInterceptSession)

##              Model df      AIC      BIC    logLik    Test
## randomInterceptOnly      1  3 162710.9 162734.0 -81352.45
## randomInterceptSession    2  4 162545.2 162575.9 -81268.58 1 vs 2
##              L.Ratio p-value
## randomInterceptOnly
## randomInterceptSession 167.7298  <.0001
```

We perform an analysis to observe the linear regressions of scores on the number of sessions from a subject across all subjects. We create linear models with and without use of session as an explanatory variable. We then compare these linear models, and with a p-value of $<.001$, we find that it cannot be the case that scores of a subject are independent of the number of sessions they attend. That is, constructing a linear model for the score of a subject is changed when we consider the number of sessions they attend.

Pasman, Wilrike J, Ruud Boessen, Yoni Donner, Nard Clabbers, and Andre Boorsma. 2017. "Effect of Caffeine on Attention and Alertness Measured in a Home-Setting, Using Web-Based Cognition Tests." Journal Article. <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5608989/>.