XJCO1511: Introduction to Discrete Mathematics
2023/24
Coursework 4
Released: Monday, April 22, 2024, at 8 a.m.
Due: Friday, May 10, by 5 p.m.

1. [4 marks] Let G and H be simple graphs. Prove that if $G \cong H$ then $\overline{G} \cong \overline{H}$.

2. A simple graph in which each pair of distinct vertices are adjacent is called a *complete graph*. We denote by K_n the complete graph on n vertices.
A simple bipartite graph with bipartition (X, Y) such that every vertex of X is adjacent to every vertex of Y is called a *complete bipartite graph*. If $|X| = m$ and $|Y| = n$, we denote this graph with $K_{m,n}$.

   (a) [1 marks] How many edges does K_n have?

   (b) [1 marks] For which values of n is K_n bipartite?

   (c) [1 marks] How many edges does $K_{m,n}$ have?

   (d) [1 marks] For which values of m and n is $K_{m,n}$ Eulerian?

3. [4 marks] For a non-negative integer r, a simple graph is *r-regular* if all of its vertices have degree r. Prove that if a bipartite graph $G = (V, E)$, with bipartition (X, Y), is r-regular, for some positive integer r, then $|X| = |Y|$.