Claim: If $G \cong H$, then $\overline{G} \cong \overline{H}$, where $\overline{G}$ and $\overline{H}$ are the complements of G and H respectively.

Proof: Let $G = (V_G, E_G)$ and $H = (V_H, E_H)$ be simple graphs. Assume $G \cong H$, meaning there exists an isomorphism $f : V_G \to V_H$ such that for any two vertices $u, v \in V_G$, u and v are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H.

Now, let's define a function $g : V_G \to V_H$ such that $g(u) = f(u)$ for all $u \in V_G$. It's clear that g is also a bijection.

We will show that g is an isomorphism between the complements $\overline{G}$ and $\overline{H}$.

Vertex Correspondence: Since g is a bijection, it establishes a one-to-one correspondence between the vertices of $\overline{G}$ and $\overline{H}$.

Edge Correspondence: Let (u, v) be an edge in $\overline{G}$, then (u, v) is not an edge in G. Since $G \cong H$, $f(u)$ and $f(v)$ are not adjacent in H. Therefore, $(f(u), f(v))$ is an edge in $\overline{H}$. This implies that $(g(u), g(v)) = (f(u), f(v))$ is an edge in $\overline{H}$. Conversely, if (u, v) is not an edge in $\overline{G}$, then $(f(u), f(v))$ is not an edge in $\overline{H}$.

Isomorphism: Since g preserves adjacency, it is an isomorphism between $\overline{G}$ and $\overline{H}$.

Therefore, if $G \cong H$, then $\overline{G} \cong \overline{H}$.

Q2

(a).To find the total number of edges, we can use the formula for the number of edges in a complete graph:

Number of edges in

$$K_n = \frac{n(n-1)}{2}$$

(b). In the case of Kn, it is bipartite if and only if n is even. When n is odd, Kn cannot be bipartite because it would require an odd number of vertices in each partition, which is not possible. Therefore, Kn is bipartite when n is **even**.