Couresework 3

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Problem 1: Probability of Event A

Let A be the event that the first child is a boy or that the last two children are girls. We have two scenarios:

- **First child is a boy**: The probability of the first child being a boy is $P(\text{boy}) = \frac{1}{2}$.
- **Last two children are girls**: The probability of the last two children being girls is $P(\text{girls}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Since these scenarios are mutually exclusive, we can add their probabilities:

$$P(A) = P(\text{boy}) + P(\text{girls}) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Problem 2: Conditional Probability

Let F represent the event of having fair hair, and H represent the event of having freckles. We want to find $P(H|F)$:

$$P(H|F) = \frac{P(F|H) \cdot P(H)}{P(F)} = \frac{0.4 \cdot 0.06}{0.2} = 0.12$$

Problem 3: Independence of Events

Let's define the events:

- $E1$: Scores have the same parity (both odd or both even).
- $E2$: Scores differ by 4.

To check for independence, we compare the joint probability with the product of individual probabilities:

$$P(E1 \cap E2) \stackrel{?}{=} P(E1) \cdot P(E2)$$

$$P(E1) = \frac{9}{36} + \frac{9}{36} = \frac{1}{2} \quad \text{(probability of same parity)}$$

$$P(E2) = \frac{1}{9} \quad \text{(probability of score difference by 4)}$$

$$P(E1 \cap E2) = \frac{1}{9}$$

Since $P(E1 \cap E2) \neq P(E1) \cdot P(E2)$, the events $E1$ and $E2$ are not **independent**.