

# Ampliació d'Algorísmia

## Exercicis d'Examen

Alumnes d'AA 2021 - Q2

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### Parameterization

**Problem 5.** Maximizing support

Consider an undirected graph  $G = (V, E)$ . We say that a set  $S \subseteq V$  supports a vertex  $u \in V$  (or  $u$  is supported by  $S$ ) if  $u$  and all its neighbors in  $G$  belong to  $S$ .

The Max Restricted Support problem is defined as follows: Given an undirected graph  $G = (V, E)$  and an integer  $s$ ,  $1 \leq s \leq |V|$ , find a subset  $S \subseteq V$  with  $|S| = s$  such that the number of vertices supported by  $S$  is maximum over all sets with  $s$  vertices.

- a) Show that the bounded version of this optimization problem under the natural parameterization or when the parameter is the value  $s$  belongs to XP.

*Solution.*

Let  $S$  be a supporting set and  $T$  be a supported set, where  $|S| = s$  and  $|T| = t$  are their respective cardinalities. There exist  $\mathcal{O}(n^k)$  such sets of size  $k$ .

To find the supported set  $T$  from a supporting set  $S$ , we loop through all its vertices, and check the edges of each vertex. If and only if all the edges are within  $S$  the vertex belongs to  $T$ . Checking up to  $n$  edges for  $s$  vertices costs  $\mathcal{O}(n \times s)$ .

Similarly, to find the supporting set  $S$  from a supported set  $T$ , we loop through all its vertices, adding them and all the vertices they are connected to to  $S$ . It takes  $\mathcal{O}(n \times t)$  checks to note which vertices of  $V$  belong to  $S$ , then  $n$  time to construct it.

If we parameterize either problem, with parameter  $k$ , it takes  $\mathcal{O}(n^k \times n \times k)$  time to check all the possible sets to find the best solution restricted by the parameter. As such, the parameterized problem belongs to XP.  $\square$

- b) Provide a FPT algorithm for the optimization problem parameterized by the treewidth of  $G$ .

*Solution.*

[YOUR FPT ALGORITHM HERE]

