Solution to the Portal Problem

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Abstract

We present a solution to the classical portal physics problem, a thought experiment involving the interaction of an object with interconnected spatial portals. We model portals as cuts in a 2D spacetime manifold and develop a mathematical framework to describe the object's trajectory. Using this formalism, we demonstrate that the object remains stationary in its initial rest frame while being instantaneously transported in space. This result highlights the importance of considering global topological features in spacetime physics and has implications for our understanding of spacetime connectivity and causality.

1 Introduction

The classical portal physics problem is shown in figure 1. [1]

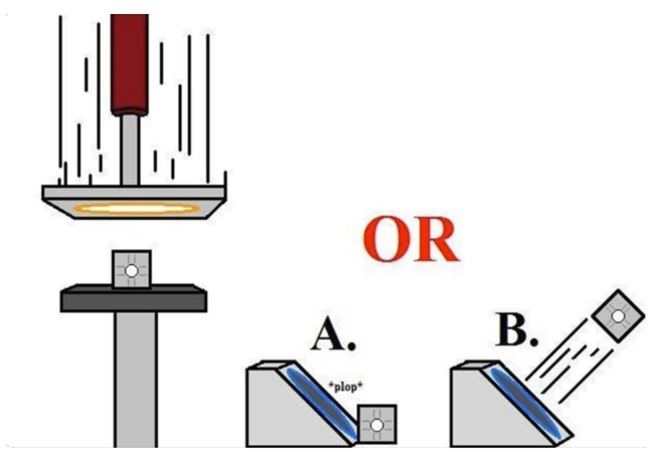

Figure 1: The classical portal physics problem.

We model portals as cuts in the spacetime manifold.

[1] The origins of this problem are unknown, lurking deep within the folkloric memescape of the internet.

2 Spacetime Setup

For simplicity, we assume a 2-dimensional smooth spacetime manifold, equipped with a flat metric. The spacetime may be Euclidean or Lorentzian. The manifold will be almost $\mathcal{R}^2$, giving us something close to Minkowksi or Euclidean space. The only deviation is the presence of the portal (see below), represented by smooth embeddings of line segments into M, with the additional structure that points on the `IN` portal are identified with corresponding points on the `OUT` portal.

We work **entirely** in the initial rest frame of the block, which we will assign co-ordinates (x_B, t) at each time t prior to the collision at co-ordinates (x_B, t_c).

The `IN` portal sits at a line segment in spacetime that connects the points (x_1, t_1) and (x_2, t_2). This indicates that the portal is moving in the x direction as time progresses from t_1 to t_2. The collision occurs at (x_B, t_c) which intersects this line section.

The `OUT` portal is stationary in the rest frame of the block, and sits at a vertical line segment connecting the points (x_3, t_1) and (x_3, t_2).

Figure 2 shows the spacetime setup with the worldlines of the portals and block prior to the collision.

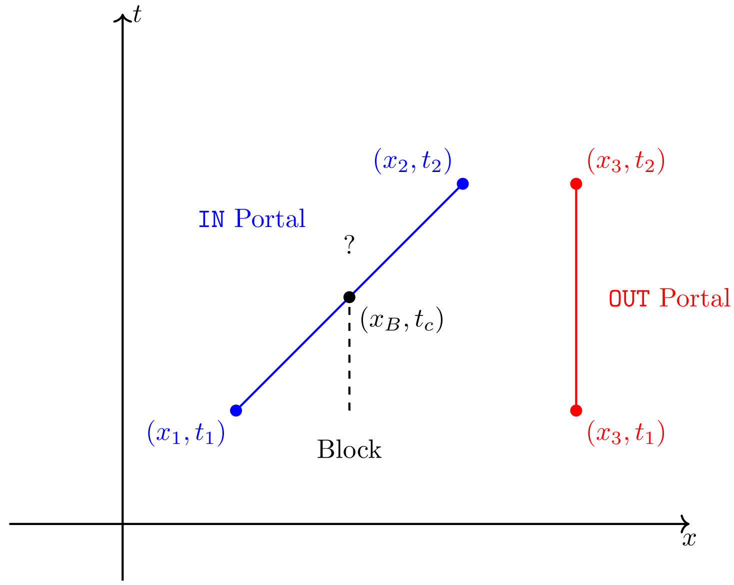

Figure 2: Worldlines of the portals and block prior to the collision.

3 Portal Behavior

The portal defines a mapping $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ that describes how a point (x, t) on the worldline of an object is transformed when it interacts with the portal (see [] for similar examples).

$$\Phi : (x, t) \mapsto \begin{cases} (x_3, t) & \text{if } (x, t) \in \texttt{IN} \text{ Portal}, \\ (x, t) & \text{otherwise.} \end{cases}$$

In other words, the portal "tapes" trajectories that intersect the `IN` portal at (x, t) in the rest frame of the block to the `OUT` portal , preserving the time coordinate t and shifting the spatial coordinate x to x_3.

4 Block-Portal Interaction

4.1 Intersection of Block and `IN` Portal Worldines

For the block with a worldline (x_B, t) to interact with the `IN` portal:

1. **Intersection Condition**: The object's worldline must intersect the `IN` portal at some point,
$$(x_B, t_c) \in \texttt{IN} \text{ Portal},$$
where t_c is the time of collision.

2. **Mapping Application**: If the intersection occurs, the portal applies the mapping:
$$(x_B(t_c), t_c) \mapsto (x_3, t_c).$$
This indicates that at time t_c, the object is instantly transported to the spatial position x_3, while the time coordinate remains unchanged.

4.2 Expression of the Mapped Trajectory

After interaction with the portal, the object's trajectory can be described by:

$$\Phi : (x, t) \mapsto \begin{cases} (x_3, t) & \text{if } (x, t) \in \texttt{IN} \text{ Portal}, \\ (x, t) & \text{otherwise.} \end{cases} \tag{1}$$

Remaining in the initial rest frame of the block, the block remains stationary, as there is no change to the trajectory of the block. The complete trajectory is shown in figure 3

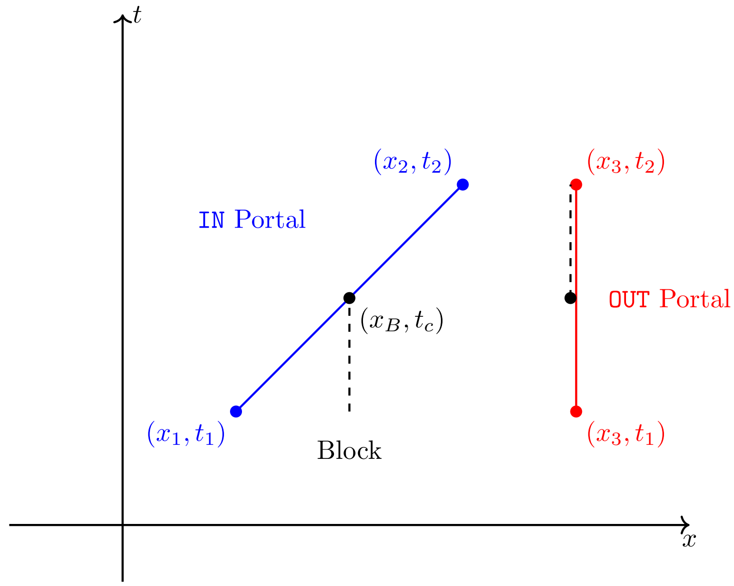

Figure 3: Worldlines of the portals and the block before and after the collision. The block transitions from its initial trajectory at x_B to the `OUT` portal, reappearing at x_3.

5 Conclusion

The object retains its time coordinate and stationary status, while its spatial coordinate is instantaneously shifted by the portal from x_B to x_3. Crucially, the block's worldline is continuous in the time coordinate but discontinuous in the spatial coordinate at the moment of portal interaction. This preserves the block's state of motion (stationary in its initial rest frame) while allowing for the instantaneous change in position. Thus solution **A** is correct.

The key insight is that portals are not intrinsic elements of the system's dynamics but rather topological features of the spacetime manifold. By altering the topology, portals provide pathways that modify the connectivity of spacetime, enabling instantaneous spatial transitions while preserving temporal continuity. By conceptualizing portals as topological constructs, we can better understand their role in shaping the global structure of spacetime and their impact on the possible trajectories of objects within that spacetime. This perspective allows for the separation of global structural changes from local dynamical laws.

Future work may extend this formalism to higher-dimensional spacetimes and investigate the interplay between topological portals and various physical phenomena, potentially uncovering novel insights into the nature of spacetime and connectivity. This important problem deserves further research, not least due to its myriad connections to other well known problems in physics. For instance, in general relativity, wormholes serve as bridges connecting distant regions of spacetime without directly affecting the local gravitational dynamics governed by the Einstein field equations [2, 5]. Similarly, in condensed matter physics, topological defects such as vortices or dislocations alter the global properties of materials without fundamentally changing their local physical behavior [3]. Another example can be found in the realm of quantum field theory, where non-trivial topological configurations give rise to phenomena like anyons, which exhibit exotic statistics due to the underlying topological structure [4].

The solution presented here preserves the block's state of motion, which is consistent with conservation of energy and momentum in the block's initial rest frame. However, this raises questions about how these conservation laws apply across different regions of a topologically non-trivial spacetime. Furthermore, whilst no causality was violated here more complex portal configurations can lead to ostensibly pathological behaviors such as closed timelike curves.

References

[1] John Byron Manchak. *Global Spacetime Structure*. Cambridge University Press, 2013.

[2] M. S. Morris, K. S. Thorne, and U. Yurtsever. "Wormholes, Time Machines, and the Weak Energy Condition." *Physical Review Letters*, vol. 61, no. 13, 1988, pp. 1446-1449.

[3] T. W. B. Kibble. "Topology of Cosmic Domains and Strings." *Journal of Physics A: Mathematical and General*, vol. 9, no. 8, 1976, pp. 1387-1398.

[4] Frank Wilczek. "Quantum Mechanics of Fractional-Spin Particles." *Physical Review Letters*, vol. 49, no. 14, 1982, pp. 957-959.

[5] Matt Visser. *Lorentzian Wormholes: From Einstein to Hawking*. AIP Press, 1995.