Dynamical Simulation of High School Social Interaction

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Abstract

Social interaction between high school students is studied, with the aim of informing future decisions by school administrators. This paper presents a computerized simulation, which predicts patterns in behaviors of an interacting student population. In addition, a survey is conducted with a sample of Grade 9 students at Appleby College, ON, Canada; the resulting quantitative description of the students' social interaction is, in turn, used to show a qualitative fitting between the simulation and real-life behaviors. The final simulation, whose parameters are tuned to the survey data, is shown to be a sufficient approximation of real-life behaviors except in some identified special cases. Extrapolated results from the simulation yields suggestions for school administration strategies: that conventional strategies often fail to increase social cohesion, and that increased diversity always increases inclusivity. Further research, with larger sample sizes and controlled experiments, is needed for verifying the validity of such extrapolations. Code for this project can be found at https://github.com/DavidGao48/Dynamical-Simulation-of-High-School-Social-Interaction.

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1 Introduction

Interaction between students has become increasingly important in the theory of education in North America. In the 1980's, Educational theorists and psychologists referred to peer-to-peer education as the "untapped potential" and the "neglected variable" [9, 18]. Since then, peer interaction has become crucial in the implementation of education [22, 25, 8].

A necessary consequence of this shift towards peer-to-peer learning is that learning becomes affected by the nature of adolescent interaction, which is complex and at times troubling. The spontaneous formation of primary groups (i.e. cliques, friend groups) is both hugely impactful in students' lives and completely out of administrative control [23]. This formation process, though omnipresent and indispensable, can also be marred by collective biases, and can consequently create unwanted divisions within a school [2, 6, 7, 12, 16, 17, 20]. Seeking a better understanding of this clique-formation process has therefore become a must for educational institutions, especially high schools.

Appleby College, an independent high school in Oakville, Ontario, is highly aware of this task. The leadership of the school has noticed that cliques at Appleby are often mildly separated across five factors: 1) gender, 2) ethnicity, 3) residency (living at school or commuting to school), 4) grade-of-entrance, and 5) native languages. Many actions have been taken by the leadership team, attempting to heal these divisions, and these actions have all focused on *conscious decisions* of students. In one experiment, students were asked to sit at assigned seats (deliberately mixed across the five factors) during dinner, rather than being allowed to sit freely. These efforts to remove the division across the five factors, however, have shown little to no effect, which demonstrates how little understanding there is on this problem.

The premise of this paper is that meaningful discussion about the clique-formation process can only begin once a quantitative model of this process is developed. In section 2, through an analysis of existing literature, it is hypothesized that the best such model is a computerized dynamical simulation, in which dots, representing students, make stochastic decisions at every point in time, and these decisions are dependent upon factors such as ethnicity and gender. The purpose of this paper is to create this dynamical simulation (based on the programming language Python), and to fit it to the behavior of Grade 9 students at Appleby College.

Though computerized models of social interaction are rare, there is an established method that couples sociology and mathematics. This field is known as Social Network Analysis. Analysts have developed algorithms to visualize existing networks of friendship, economic coalition, co-authorship, among other relationships. These algorithms have found application in education, econometrics, and informatics [21, 33, 13, 15]. The existing algorithms are mostly descriptive, not predictive (they help visualize existing data, rather than create new patterns); and deterministic, not stochastic (they rearrange whatever the user/programmer inputs, rather than generate unexpected results). We borrow techniques from these algorithms, but a new, predictive and stochastic algorithm is developed in section 3, along with an empirical study to validate this new algorithm. The results of this algorithm and empirical study are analyzed in section 4, and the most important social implications are found under the "Implications" section. [1]

2 Literature Review

Since the twentieth century, three groups of researchers have become interested in the phenomenon of primary group formation. The first were the sociometrists, who studied the dynamics of interpersonal relations (what causes cliques to form?) and their application to psychotherapy. The second were the econometrists who looked for a set of formulas that yield the time-evolution of interaction patterns (with sufficient information, can I predict who will be friends with whom a year from now?). The third were the social psychologists who, since the late twentieth century, have been seeking to trace problems of cohesion in modern society back to the inner dynamics of group formation (can I explain and predict political issues by understanding the interactions between large populations?). Each perspective holds valu-

[1] Throughout this paper, there are some descriptions which are necessarily technical. We have given simplified versions in the main body of this paper. The more mathematically rigorous and demanding explanations are in Appendix D.

able insights, but communication between the three has been limited, and a model that takes all three perspectives into account has yet to be developed.

2.1 The Sociometric Perspective

The term *sociometry* was first coined by the psychiatrist Jacob L. Moreno in 1934 [29]. Moreno envisioned a form of group psychotherapy based on understanding the dynamics of interpersonal relations. A survey of the journal *Sociometry* reveals that much of this discipline revolves around the *sociometric test* [29, 30, 36], an empirical method designed to reveal the fundamental structures within a society by disclosing the affinities, attractions and repulsions, operating between persons and persons [11]. The test involves a questionnaire, in which participants are asked to "evaluate" each other. For example, school-children might be asked to list the classmates that they would like to sit next to in class [34].

The results of these tests are plotted using the sociogram, a graph where individuals are represented by nodes, and friendships represented as edges that connect the nodes (see Diagram 1). For Moreno and his students, the sociogram allows for an intuitive view of social structures. More recently, with the use of computer algorithms, the sociogram can also yield quantitative symmetries and weight distributions within a society [27, 32, 31].

In 1974, Bruce D. Layton (State University of New York) and his colleague Chester A. Insko used the sociometric test to find a significant relation

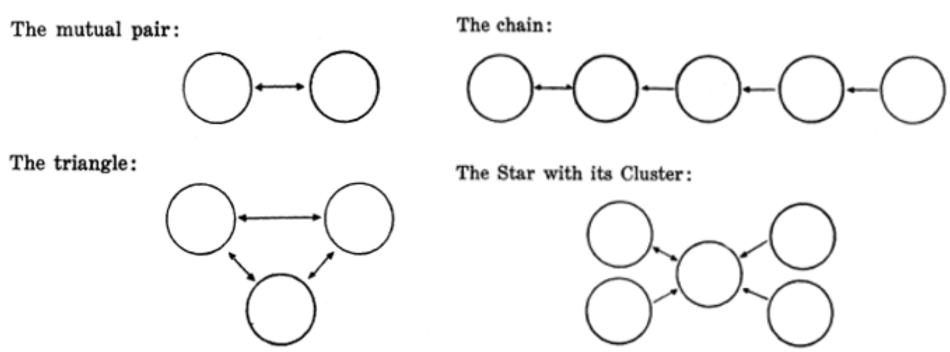

Diagram 1: Sample Sociograms, retrieved from [34]

between similarity and anticipated interaction [24]. Anticipated interaction increases as similarity increases, regardless of the rational reward-cost prediction. The authors named this phenomenon “Similarity-Attraction,” and described multiple explanations for it. An analysis of social dynamics cannot be complete without considering the capability of the Similarity-Attraction Effect to override reward-cost analysis, even in business and economic settings.

Even at the very beginning, Moreno recognized that the advantage of sociometry lies in the understanding of human relations proceeding forward from their state at emergence, instead of reasoning backward from their final state [30]. This paper presents a computer algorithm that is as strong as those used in Social Network Analysis, but proceeds forward from an initial state, as Moreno envisioned.

2.2 The Mathematical-Econometric Perspective

Relevant mathematical models can be loosely categorized into two types. The first type (call it the *General-Actions approach*) views small decisions such as 'where to sit at lunch' as unit actions. It assumes that these small decisions lead to the formation of social networks in the long run. The second type (call it the *Network-as-Action approach*) views the formation of a clique as a single unit action. It bypasses the details of the General-Actions approach, and directly considers each network as a whole.

The General-Action approach is analogous to a classical economic approach: the more utility an action provides, the more likely it is for someone to choose that action. A common way to quantify this concept is the Softmax function, which calculates the probability of an action to be proportional to the exponential of the utility:

$$Prob_a(x) = \frac{\exp \varepsilon U_a(x)}{\sum_{y \in Y} \exp \varepsilon U_a(y)} \tag{1}$$

where x is a stand-in variable for any action that may be chosen; U_a is the perceived utility of the action x; the sum on the denominator is taken over Y (the set of all possible actions); and ε is an error term [2]. This model

[2] See Appendix D for further explanation.

is commonly used as a robust alternative to the Nash Mixed Equilibrium [1, 10, 28].

What makes social networks more complex is the fact that the utility of each action depends, in turn, on the action chosen by all other individuals in the network. Examples of this phenomenon include group conformity (students conforming to the same fashion style) and reverse conformity (at a checkout, it is better to choose the line with the fewest people). To capture all of these phenomena in general terms, the following formula was developed by McFadden [28], Brock and Durlauf [5]:

$$U_a(x) = u_a(x) + S_a(x, \mu_a^e(\mathbf{x})) \tag{2}$$

where u_a is the inherent utility of action x (for example, the warmth of a jacket), and S_a is the social utility of action x (for example, the trendiness of a jacket), which is dependent on the expected behavior of other agents μ_a^e.

What renders this formula computationally tractable is the linear-in-means model, postulated by Charles F. Manski in 1993 [26]. Under this model, equation 2 is reduced to a linear equation:

$$U_a(x) = \alpha + \frac{\beta \sum_{j \in N_a} x_j}{|N_a|} + \gamma \mathbf{C_a} + \frac{\delta \sum_{j \in N_a} \mathbf{C_j}}{|N_a|} + \varepsilon_a \tag{3}$$

where agent a's perceived utility of action x_a is assumed to be linear with respect to the choices of all other agents (x_j), the characteristic of agent a ($\mathbf{C_a}$), and the characteristic of all other agents ($\mathbf{C_j}$). This model was furthered in 2009 by Yann Bramoullé and his Colleagues (Aix-Marseille University), who solidified this model by demonstrating that it was possible to identify the constants $\alpha, \beta, \gamma, \delta$ empirically.

The piece that completes the puzzle in the General-Action Model is the Lattice Model, first used by Lawrence E. Blume, economist at Cornell University. This is the idea that medium-scale social interactions can be aequately described on an integer lattice (a grid, though not necessarily two-dimensional). Blume pictures individuals as moving dots on the lattice. In Section 3, this iea is further developed so that movements on the lattice represent all individual social choices. This model suggests the possibility to compute the development process of a sociogram-like structure.

In contrast to the General-Action approach, the Network-as-Action approach considers the formation of a clique as a single unit action. Cliques are objects that interact with its members in two ways:

1. Members can produce utility (for example, crops or money) and place it in the clique's sharing pool;

2. Utility in the sharing pool gets distribute to members, not necessarily uniformly.

This approach was formally established in 1996 by Matthew O. Jackson an Asher Wolinsky [15].

Using this picture, the formation of a clique becomes a special kind of optimization problem. A clique is called "strongly efficient" if the total utility in the sharing pool is more than the total utility that would have been produced by any smaller clique:

$$v(g) \geq v(g')\forall g' \subset g; \tag{4}$$

while a clique is called "pairwise stable" if none of its members has an incentive to break a relationship, or equivalently, no plausible breakage of relationship will provide any members with extra utility:

$$\begin{cases} U_i^Y(g,v) \geq U_i^Y(g-ij,v) \& U_j^Y(g,v) \geq U_j^Y(g-ij,v)\forall ij \in g \\ U_i^Y(g,v) < U_i^Y(g+ij,v) \implies U_j^Y(g,v) < U_j^Y(g+ij,v)\forall ij \in g \end{cases} \tag{5}$$

where the g's are possible clique configurations; the v's represent the total utility produced in the sharing pool, and the U's represent the utility distributed to an individual members.

The important insight given by Jackson and Wolinsky is that equations 4 and 5 are not necessarily satisfied simultaneously. This result has since been cited in thousands of works, including Sanjeev Goyal's 2007 textbook *Connections: An Introduction to the Economics of Networks* [13]. This idea is useful in computerizing a model of student interaction, because pairwise stability can be thought of as the long-term tendency of a social structure left to evolve by itself, while strong efficiency can be thought of as the ultimate goal of educational intervention, although the concept of "utility" is more ambiguous when applied to student-student relationships.

2.3 The Social Psychological Perspective

Sociologically, a secondary tie is defined as a sustained interaction for an external goal (for example, the interaction between two violinists in a chamber music group); while a primary tie is an interaction without such a goal [19]. This section focuses on primary ties, since that is the form of interaction that most fundamentally affects students' lives [23].

The idea that biases dominate clique formation has only recently entered public attention. However, in as early as 1978, Ted Huston and George Levinge, professors of psychology at University of Massachusetts, published a meta-analysis of Interpersonal Attraction Research. According to the analysis, attraction, and therefore bias, can exist both pre-encounter and post-encounter. Pre-encounter attraction has been examined with respect to three independent variables: Physical Attractiveness, Impression of Behavior, and Attitude Similarity. Post-encounter attraction has been examined in lab settings with respect to verbal and non-verbal interactions. Interestingly, though Attitude Similarity has been confirmed to have a positive effect on attraction [6, 7, 35], other similarities (ethnicity, for example) have rarely been studied as an influence on attraction.

Educational psychologists Kyongboon Kwon and Michele Lease (Georgia University) noted in 2007 that children's social and emotional adjustments depend on the social environment, and that "attempts to understand the complexity of children's social lives are incomplete without the consideration of the social network in which children are embedded" [23]. Specifically, networks influence children's social and emotional adjustment through Group Homophily, a phenomenon where individuals in a clique tend to be more similar to each other than individuals outside the clique. This phenomenon has been observed in many later studies [20, 23, 35]. Group Homophily is dynamical by nature - it is a process where clique patterns may cause itself to develop recursively. Kwon and Lease could not, however, identify the effect of multiple *ethnicities* in Group Homophily, because 97% of their sample was either "white" or "black" [23]. This points again to the necessity of new data on this topic.

One popular explanation for the cause of Group Homophily starts by stating that most people's "knowledge" about sets of individuals (be it ethnic, gender, behavioral, or otherwise) come from media, books, and educational institutions [16]. These bits of information, often over-simplified, for a *micro-bias* which remains hidden in the individual's subconscious [2]. The micro-bias, in turn, exerts an influence on snap judgments that the individual makes about others [12]. In effect, these micro-biases become a new component in decision functions (equation 2), thus influencing the probability of friendship formations, which finally affects macroscopic clique structures in the long run.

2.4 Gap Analysis

Each of the three perspectives hold valuable insights. The sociometric perspective gives the most solid methodology for empirical studies of clique formation. Coupled with computerized Social Network Analysis tools, it gives powerful descriptive data, but is unable to produce predictive models. The econometric perspective gives the best predictive quantitative models, but the models also become easily intractable, and have not yet been computerized. The social psychological perspective most directly addresses the phenomenon of Group Homophily with tractable data, but needs more quantitative evidences. The three perspectives are complimentary, yet communication between them has been minimal. The current study proposes to use a sociometric methodology to inform a computerized dynamical model, which can motivate action research based on social psychological theories.

3 Methodology

3.1 Stochastic Simulation

Python and *tkinter* [3] are used to create a sociogram-like simulation[4] where an arbitrary amount of "students" are represented by dots on a $\mathbb{Z}^n$ (integer n-dimensions) lattice. The simulation is designed to effectively combine SNA

[3]Programming languages.

[4]The code for this project can be found at: https://github.com/DavidGao48/Dynamical-Simulation-of-High-School-Social-Interaction.

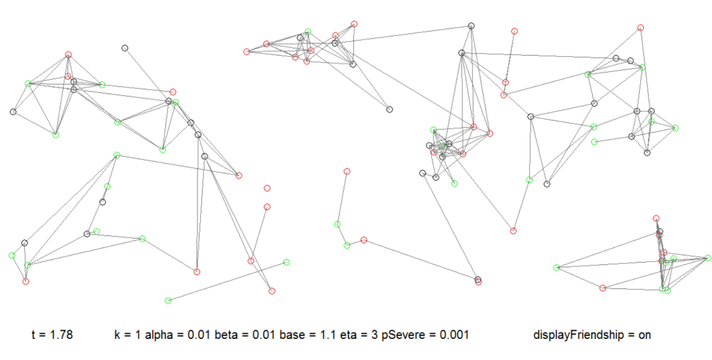

Diagram 2: Sample Frame in Stochastic Simulation (two dimensions)

techniques with predictive models (as discussed in Section 2.2) to build a realistic rendering of social interaction.

The dots are colored to denote one of the intrinsic differences between students (ethnicity, gender, residency, grade of entry, or language; experimenter's choice on each run of the simulation). Given any two dots of the same color, the corresponding two entities are said to be of the same type.

The dots move freely in the lattice. At each frame, each dot chooses a direction of movement $\dot{\mathbf{x}}$ (essentially, a choice of up or down, and left or right). Mathematically, the direction of movement is the time-derivative of the dot's position $\mathbf{x}$. Each dot chooses $\dot{\mathbf{x}}$ using a pseudo-random number generator[5], and a direction is more likely to be chosen if moving in that direction gives the student perceive utility. In other words, equation 1 (see Section 2.2) becomes:

$$Prob(\dot{\mathbf{x}}) = \frac{\exp \varepsilon U(\dot{\mathbf{x}})}{\sum_{\dot{\mathbf{y}}} \exp \varepsilon U(\dot{\mathbf{y}})}. \tag{6}$$

The utility function U is defined following the model of McFadden, Brock and Durlauf [28, 5], so equation 2 becomes:

$$U_a(\dot{\mathbf{x}}) = u_a(\dot{\mathbf{x}}) + S_a(\mathbf{x}_1, \ldots, \mathbf{x}_N, \dot{\mathbf{x}}_1, \ldots, \dot{\mathbf{x}}_N) + \epsilon_a \tag{7}$$

[5]A pseudo-random generator is an algorithm that spits out numbers that are "almost" random - they are not completely random, but are unpredictable enough to be seen as random.

where S_a is the perceived social utility of the direction $\dot{\mathbf{x}}$ - which is dependent on every other dot's position (the list $\mathbf{x}_1, \ldots, \mathbf{x}_N$) and every other dot's direction (the list $\dot{\mathbf{x}}_1, \ldots, \dot{\mathbf{x}}_N$). If, at any point in time, two students become less than or equal to a unit distance apart, they become "friends" (shown as a grey line connecting the two students). The most important parameters of the algorithm are k, α, and β. The amount by which students prefer to approach someone they are already "friends" with is denoted by k. The extent to which students prefer to socialize with someone of their *type* is denoted by α. The amount by which students dislike socializing with someone of a different *type* is denoted by β. Using these parameters, equation 3 becomes:

$$U_a(\dot{\mathbf{x}}) = \epsilon_a + \sum_{j \neq a} (k\phi_j + \alpha\tau_j + \beta(1 - \tau_j))\langle \mathbf{x}_j - \mathbf{x}_a, \dot{\mathbf{x}} \rangle \tag{8}$$

where the sum is taken over all other surrounding dots; ϕ_i is a Boolean representing whether the j^{th} surrounding dot is a friend with the a^{th} dot; ϵ is a random deviation variable, and the angular brackets take the inner product of the two vectors inside. The parameters k, α, β are hand-tuned[6] and these tunings are verified using a sociometric test (see Section 3.2)[7].

The resulting simulation, as a whole, takes into account all models as discussed in Section 2.2. Dots, as per the initial objective, should predict the behavior of students. The simulation is capable of updating at a rate of 5-10 frames per second, and display results purely visually. The final visual result, according to the initial objective, should display cliques as clusters with dense lines of connection within and sparse connections with other cliques. Results are displayed and discussed in Sections 4 and 5.

3.2 Sociometric Test

After receiving approval from Appleby College's Research Ethics Board, a pool of 60 anonymous volunteers (out of the population of 150 grade 9 students of Appleby College, Oakville, ON, Canada) was enlisted. This pool

[6]This means that the parameters don't find their optimal values automatically, rather, a programmer adjusts them manually.

[7]There are additional parameters which are necessary for program debugging and monitoring purposes; these are given in Appendix B

of volunteers received a survey each month, over a period of three months. The surveys varied each month, but all surveys contained the following five "chapters":

1. The first chapter asked the participant to log in. To ensure anonymity, each volunteer created a code-sentence (an arbitrary string of English characters) on their first day of participation. The volunteer used this code-sentence to log in each month.

2. The second chapter asked for basic information [8] about the participant, including: native language, gender (self-identified), ethnicity (self-identified), residency (boarding or commuting), and the Grade of Entrance (the year in which the participant joined Appleby College - 7, 8, or 9).

3. The third chapter ("Most Recent Interaction") asked the participant to recall the grade 9 student with whom they most recently ha a conversation (either online or in real life). The participant inputs basic information about this most recent interaction.

4. The fourth chapter ("Roommate Selection") asked the participant to think of who they would like to room with if they were to live at school residence[9]. The participant inputs basic information about their roommate selection.

5. The fifth chapter ("Most Frequent Interaction") asked the participant to recall the grade 9 student with whom they most frequently conversed in the past month. The participant inputs basic information about this most frequent interaction.

Note that the third and fifth chapters do not differentiate between primary and secondary ties, while the fourth looks specifically for primary ties.

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[8] From here on the term "basic information" will refer to this specific set of information

[9] Some participants are already living in residence. In that case, they were asked to imagine selecting a roommate for the next school year.

In each chapter some irrelevant distraction questions are mixed in to blur the purpose of the study, so as to minimize Social Desirability Bias[10].

Finally, results from the second chapter is compared with those from the third, fourth, and fifth. A statistical test[11] is performed on these results to yield the data presented in Section 4.2. This data is used to guide the tuning (see section 3.1) of the simulation's parameters. The participants' view of the survey is presented in Appendix A.

[10] Social Desirability Bias is a common error in psychological surveys, where participants over-report desirable traits or under-report undesirable traits.

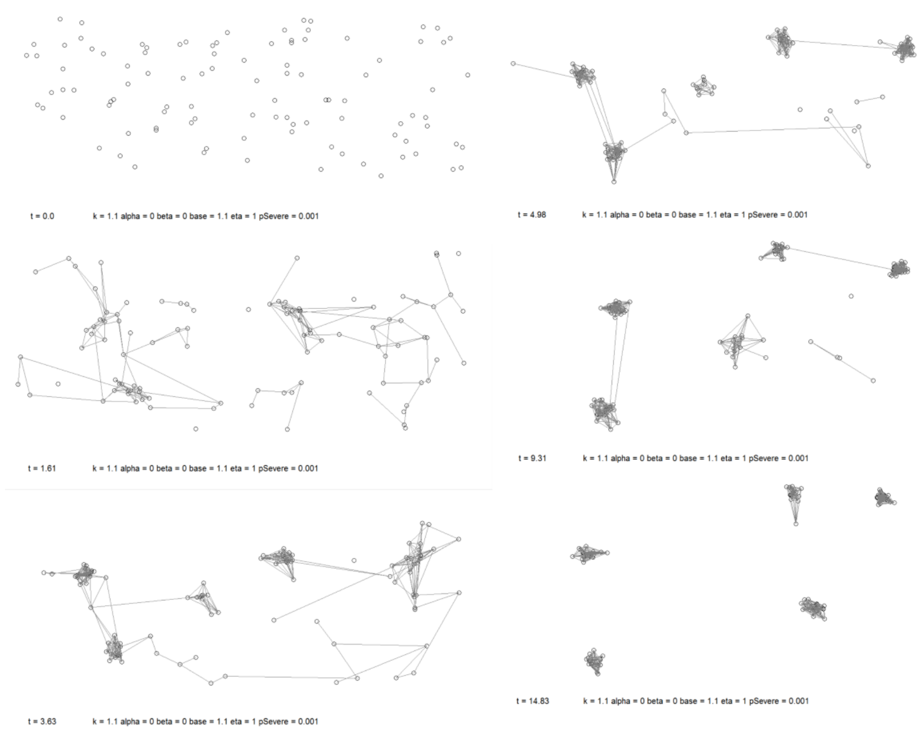
[11] See Appendix C for a detailed description of significance tests.

4 Results

4.1 Simulation Results

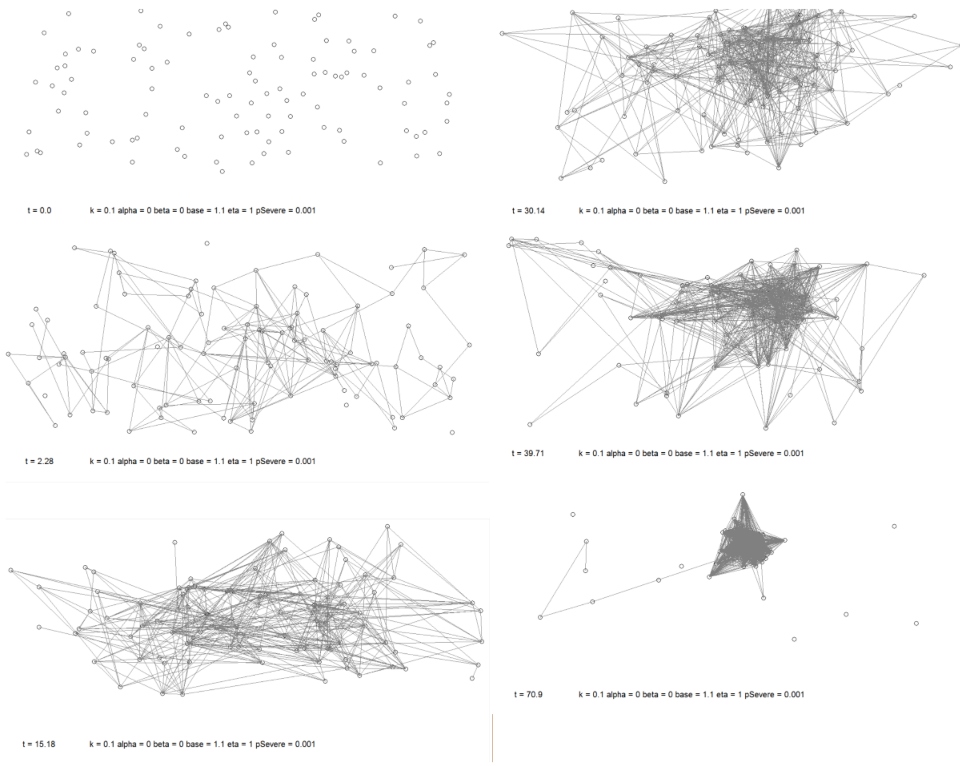
All of our simulations used a virtual population size of 100. The sample simulation results are called “sequences,” and they should be read vertically from the top-left corner.

Sequence 1 shows a social space that is homogeneous (all students in it are of the same type). The only bias built into the studens is their preference to interact with those they are already friends with. Note the rapidity of clique formation.



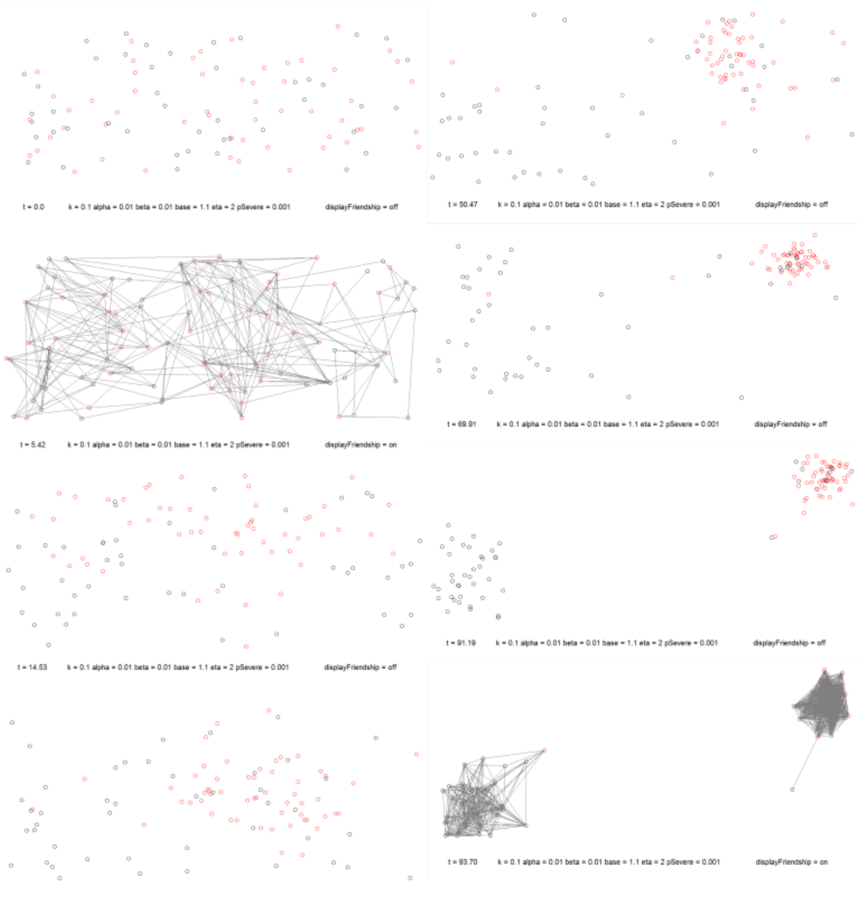
Sequence 1: Homogenous, k=1.1

Sequence 2 shows a homogeneous social space where, unlike in Sequence 1, students are more willing to interact with others to whom they do not have a connection yet. A single, large, high conformity clique forms over a long period of time.



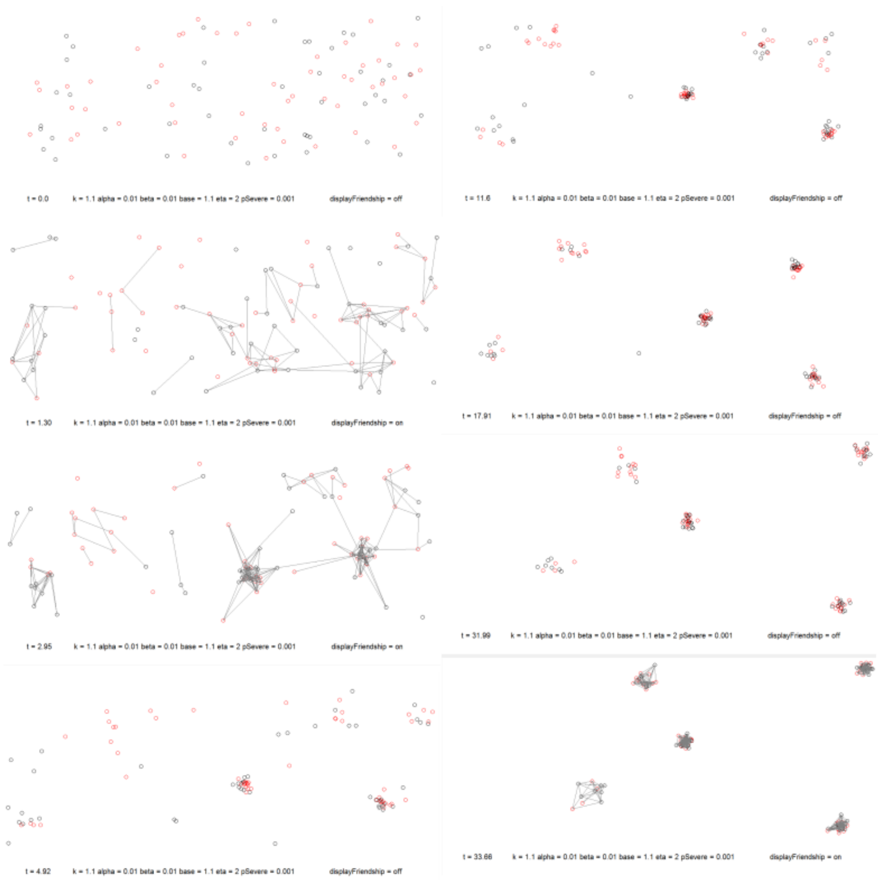
Sequence 2: Homogenous, k=0.1

Sequence 3 shows a two-type social space (two different types of students are denoted by the two colors, red and black). Even though character-based bias is very small ($\alpha = \beta = 0.01$), two large cliques with significantly unbalanced color distributions are formed. The friendship lines are hidden in frames 3-7 for a clearer view of the color distribution.



Sequence 3: Two-type, k=0.1, $\alpha = \beta = 0.01$

Sequence 4 shows a two-type social space with the same settings as Sequence 3, except students interact more strongly with existing friends (k is increased). Cliques now become smaller, but more heterogeneous (that is, more inclusive).

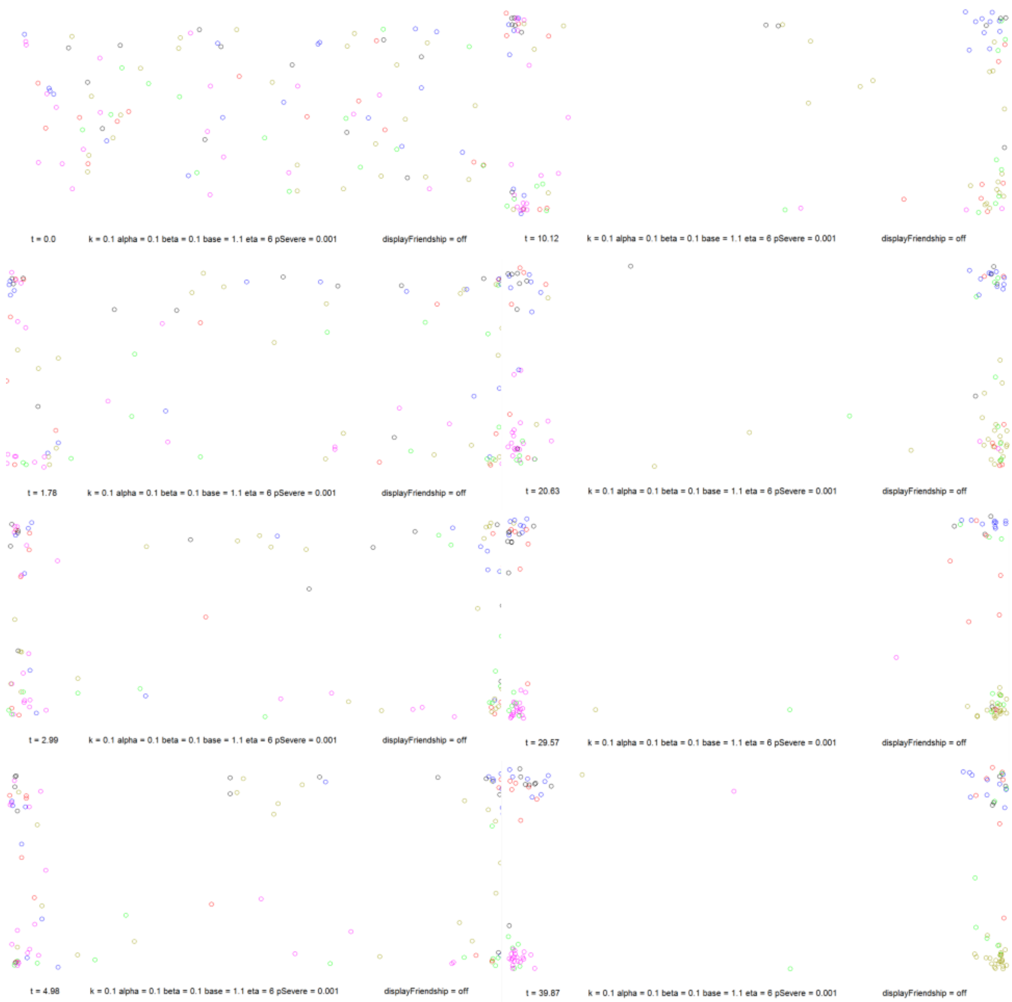


Sequence 4: Two-type, k=1.1, $\alpha = \beta = 0.01$

Sequence 5 shows a three-type social space. The three types initially separate into three cliques, but two of these cliques integrate.

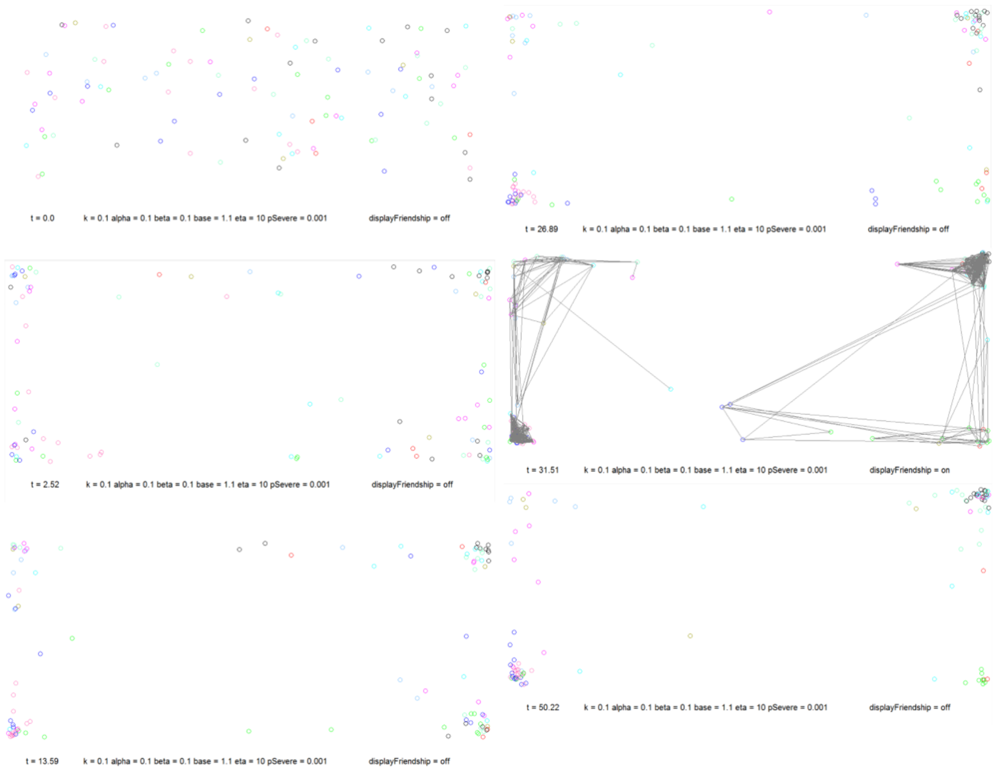
Sequence 5: Three-type, k=0.1, $\alpha = \beta = 0.1$

Sequence 6 shows a six-type social space. Cliques are relatively inclusive - they contain a mix of different types, but there is still a tendency for some cliques to exclude specific types (for example, blue dots are excluded from the bottom cliques).



Sequence 6: Six-type, k=0.1, $\alpha = \beta = 0.1$

Sequence 7 shows a ten-type social space. Multiple cliques form; each clique is very inclusive - they all contain a mix of different types.

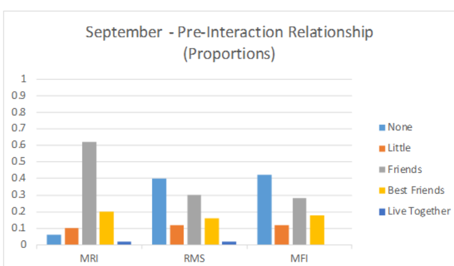


Sequence 7: Ten-type, k=0.1, $\alpha = \beta = 0.1$

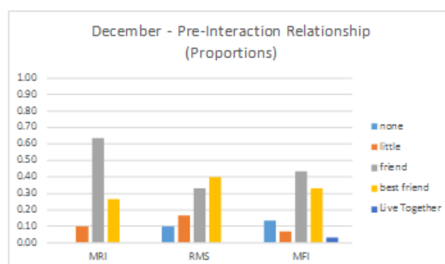
4.2 Sociometric Test Results

All sociometric test results are displayed in "graphs". Each graph contains three clusters and each cluster contains four bars. The clusters are arranged by time: the first cluster is the data collected in September; the second cluster October; and the third December. Within each cluster, the four bars are arranged by the "chapter" of survey (see Section 3.2: "MRI" for "Most Recent Interaction"; "RMS" for "Roommate Selection"; "MFI" for "Most Frequent Interaction").

For all significance tests[12], the significance level is marked according to a legend on the right, where α_{sig} is the significance level. The observed proportions are denoted by "pˆ", followed by the "chapter" name. All null-hypotheses are marked by diagonally lined bars to the left of each cluster, and denoted by "p0". For example, in Graph 1, lined bars represent the proportion of gender similarity expected in interaction, if gender bias had no effect whatsoever on the probability of interactions. Each test was sent to the same pool of 60 volunteers, and rates of response varied from 50% to 83%.

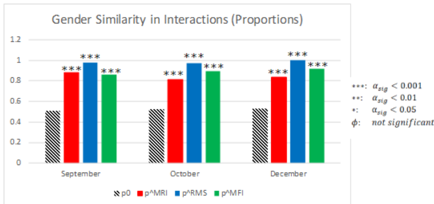


(a) Graph 1

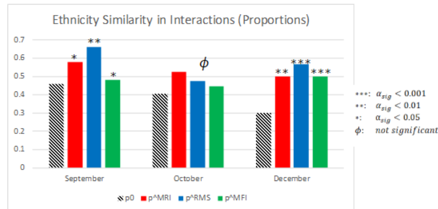


(b) Graph 2

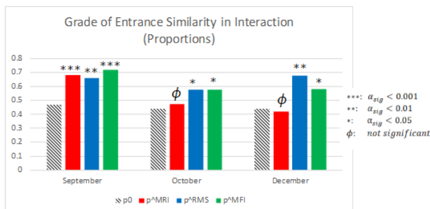
[12] Again, see appendix C for a detailed description of significance tests used in this section.



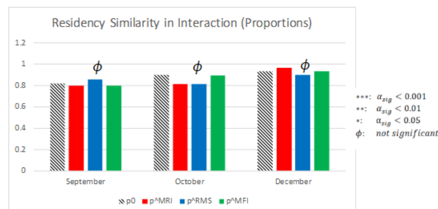
(c) Graph 3



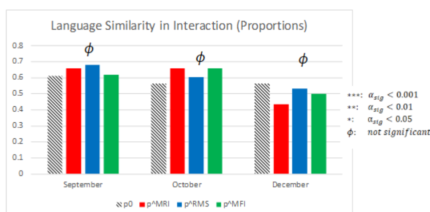
(d) Graph 4



(e) Graph 5



(f) Graph 6



(g) Graph 7

5 Discussion

In this section, the behavior illustrated by the simulation sequences is compared to the sociometric test results. All references to "Sequences" are directed to pages 13-19; and "Graphs" are directed to pages 20-21.

The core features to be discussed are: cohesion - how densely connected individuals are; variability - how free individuals are to move around the social space; diversity - the number of different types of individuals present in the social space; and inclusivity - the existence of diversity within each clique.

Sequences 1 and 2 illustrate the dilemma between cohesion and variability. Sequence 2 shows a social structure that is very cohesive, but lacks variability; Sequence 1 shows the opposite. It is an inherent property of the simulation that a social structure too cohesive loses variability, and vice versa. It is crucial to analyze all other simulation sequences with the first two sequences as baselines.

In all sequences, a shrinking process is observed in cliques over time. If there is a consistent preference to interact with someone that is already a friend, then over time it is less and less likely for students to interact with someone that is not yet a friend, even if the initial preference k is extremely small. This result is validated by Graphs 1 and 2: in the beginning of the school year (Graph 1) it was more likely for two students who didn't know each other to interact (because most students didn't know each other to begin with); however, three months later (Graph 2), it was almost certain that if one finds a pair of students talking to each other, they already knew each other before that conversation. This agrees with Kwon and Lease's interpretation of Group Homophily [23].

Sequence 3 suggests that, whenever there is an overwhelming binary categorization of individuals (such as an incorrect distinction between local and international students), a deep social division is inevitable; and even if the bias based on this categorization (α and β) is negligibly small, the long-term effect is not negligible. This is validated by Graph 3, where gender similarity is found to be the strongest influencing factor upon interaction[13], scoring the

[13] All participants identified themselves as ether male or female, though an "other"

highest possible significance value (three stars) consistently.

There is another much-discussed binary categorization at Appleby College: the distinction between students that live at school (boarders) and those that commute to school every morning (commuters). At Appleby, there has been a perceived division between boarders and commuters. However, Graph 6 shows that the boarder-commuter categorization has no statistically significant effect on clique formation. Not only does this result defy the school's expectation, it also contradicts the predictions of Sequence 3 (see previous paragraph). Equally surprisingly, in Graph 7, no significant Group Homophily was found with respect to language similarity. Given this data, it may be advisable for the school's leadership to shift their focus from the idea of boarder-commuter dichotomy and of language barriers, towards other Group Homophily effects that show more statistical significance.

Graphs 4 and 5 illustrate two contrasting over-time patterns: over time, ethnicity similarity has an increasing strength of influence on clique formation, while grade-of-entrance has a decreasing strength of influence (note that the strength of influence is the significance level).

It is no surprise that ethnicity similarity's strength of influence increases over time. Under Jackson and Wolinsky's model [15], a clique's pairwise stability should uniformly increase over time. Thus, if ethnicity similarity increases pairwise stability, ethnicity's strength of influence should increase over time. Since, in September, ethnicity similarity seemed to have a positive effect on friendship formation (most notably, on Roommate Selection), the fact that this effect generally grows stronger over three months fits the theoretical prediction. The simulation accounts for this because it implements the equations of McFadden, Brock and Durlauf [28, 5] (see Section 3.1). In Sequences 3 and 5, once there is a small, initial separation between different types of dots, this separation only grows larger over time.

Graph 3, on the other hand, shows grade-of-entrance's decreasing strength of influence over time. In September, around 70% of student-to-student conversations happened between students who entered Appleby in the same grade. However, by December this effect had mostly disappeared - except

option was available. Note also that the RMS Chapter should be disregarded, since all roommates at Appleby are of the same gender.

for in roommate selection. It may be explained by the fact that roommate selection is the criteria that focuses most on primary ties, and that secondary ties behave differently, lying outside the focus of this paper. However, this drastic decrease in significance value over time demands a more comprehensive explanation.

All of this can be summarized as follows: This paper has presented a dynamical simulation, which adequately describes social interactions between students, except for when student-student behaviors do not follow general models for social interaction due to the special circumstances of a school environment.

Limitations

The sociometric test is limited by its sample size, and in fact, by the population of Appleby College. It was made clear at the start of the project that the study will focus only on grade 9 students at the school. There are, in total, 120 grade-9 students at Appleby. It would be incorrect to sample more than 60 out of the 120 students, since the sampling distribution would be too far from a Normal Distribution. Therefore, exactly 60 were selected. Over the three months of the study, it was inevitable that interest faded, so each month the number of responses decreased from 83% to 50%.

The significance level of a data set is proportional to $\sqrt{N}$, where N is the sample size. As a result, if a significance level decreases over time (say, in graph 3, as discussed previously), it is impossible to determine whether this is simply due to a decreased sample size, or due to an actual decrease in the strength of influence of the independent variables. Similarly, when expected significance is not found, it may simply be that a larger sample (hence a larger population) is needed.

Implications

Once the validity of the dynamical simulation has been established, school administrators should take into account its implications. The first implication is the error in believing that, to make a society more cohesive and inclusive, members of the society need to be more willing to interact with someone they are not yet friends with (corresponding to a small k value). This belief is true in a homogeneous social setting, where there are no distinguishing features between individuals that cause similarity bias, as demonstrated by Sequence 1 and 2. It is incorrect to extrapolate this belief to real-world settings, where there is always some, albeit small, similarity bias [2]. As Sequences 3 and 5 show, when there exists a small similarity bias (α and β), a decrease in k can only lead to *more* Group Homophily, and hence less inclusiveness.

The second implication is that, in order to make a school's social interaction more inclusive, so that cliques are not segregated with respect to types, there are only two options: to make the school completely homogeneous (as in Sequence 1), or to completely embrace diversity (as in Sequence 7). By focusing on the progression from Sequence 3 to Sequences 5, 6, and 7, it is evident that inclusivity gradually increases as diversity increases. When all else is equal, a two-type social space is the lease inclusive, while a ten-type social space is almost perfectly inclusive. Of course, a homogeneous (one-type) social space is also perfectly inclusive, since there is nothing with respect to which Group Homophily might occur, but such a space is impossible in practice (as mentioned earlier). Therefore, the school administrator's best bet is to embrace the idea that diversity is inclusivity.

This second implication, however, extrapolates simulation results. To verify that there is a causal relationship between diversity and inclusivity, it is necessary to conduct controlled social experiments; sociometric tests would not suffice. Nevertheess, the possibility that there exists a causal relationship is high.

Future Research

Future research should focus on the following two agendas:

1. To devise a controlled experiment, which occurs in clinical settings rather than over long months of data collection, that is able to verify the causal relationship between diversity and inclusivity.

2. To verify that findings in this paper hold in larger populations.

6 Conclusion

Social interactions between grade 9 students at Appleby College were studied, and a dynamical simulation of such interactions was developed. It was found that whenever social interactions fit under existing theoretical models, they are reflected accurately by the dynamical simulation. However, it is unclear whether results from the simulation continue to hold in situations that are not clearly reflected by existing mathematical models. Implications of the simulation challenge many beliefs held at Appleby College, but some of these implications need further verification by a separate, short-term, controlled experiment. Overall, a quantitative model of social interaction has been successfully developed, and its results are promising: they have the power of making the statement that "diversity is inclusivity" a mathematical conjecture.

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7 Appendix

Appendix A

The following questions were sampled from the Most-Recent Interaction section of the third survey. Questions that are very similar to the following were used in every other section.

Appendix B

The adjustable parameters of the simulation code are as follows:

t	The count of time frames that have elapsed since the "start" button was pressed.
k	Friend-Attraction; the extent to which students prefer to interact with their friends, rather than to interact with someone unknown.
α	Similarity-Attraction; the extent to which students prefer to interact with someone of the same type.
β	Difference-Repulsion; the extent to which students dislike interacting with someone of a different type.
$base$	The base in the exponentiation of utility. It is simply e^{ε}.
η	The number of different types of students in the simulation (i.e. the number of different colors present in the social space).
$pSever$	The expected probability that, at a given frame, a pair of friends will decide to sever their friendship.
$displayFriendship$	The Boolean parameter that shows whether friendships are currently set as visible or invisible.

Appendix C

For each set of data analyzed in section 4.2, the significance level was calculated as follows:

The null hypothesis H_0 was that similarity in the specific characteristic (gender, ethnicity, etc.) had no impact on the probability of interaction, and the alternative hypothesis H_a was that similarity had a positive impact on the probability of interaction. The expected Probability of Similarity (PoS), given the null hypothesis, is calculated as

$$\hat{p} = \sum_c Prob(c)^2.$$

where $Prob(c)$ is the probability that, when a random student is selected from the population, he/she will be in the category c.

It was assumed that the sampling distributions of PoS followed a normal distribution:

$$p_{obs} \sim \mathcal{N}(\hat{p}, \sigma),$$

and hence the likelihood of the null hypothesis given the observed data was calculated as:

$$\alpha(p_{obs}) = \mathscr{L}(H_0|p_{obs}) = \frac{1}{\sqrt{2\pi}\sigma}\int_{p_{obs}}^{\infty} \exp(-\frac{(x-\hat{p})^2}{2\sigma^2})dx.$$

Appendix D

Additional Details on Equations

The variable ε in equation 1 represents the degree of rationality of the population being modelled. The probability of each action is normalized from the distribution $P'(x) = \exp \varepsilon U(x)$; while

$$\frac{\partial P'}{\partial U} = \varepsilon P'.$$

Thus, if ε is large, then the model reflects a population whose decisions are very much decided by considering the utility of each posible action. On the other hand, if $\varepsilon = 0$, then the model reflects a population whose decisions are not influence by rational considerations of utility at all.

For equation 2, let the interacting agents in a social space be numbered from 1 to N. Arrange the action taken by each individual - except for the individual a - into a vector:

$$\mathbf{x} = (x_1, x_2, \ldots, x_{a-1}, x_{a+1}, \ldots, x_N).$$

The actual value of $\mathbf{x}$ is unknown to agent a. However, agent a has a hypothesized probability distribution for $\mathbf{x}$. Call this the perceived probability distribution $\pi\mathbf{x}$. Then we define a's perceived expected action of the entire society as

$$\bar{\mathbf{x}}^{\mathbf{e}}_{\mathbf{a}} = \int_{\mathbb{R}^N} \mathbf{x}\pi(\mathbf{x})d^N\mathbf{x}.$$

Now, if we take the average of the components of the vector $\bar{\mathbf{x}}^{\mathbf{e}}_{\mathbf{a}}$, that gives us a's expected mean action of the entire society:

$$\mu^e_a = (N-1)^{-1}\mathbf{I}^{\mathbf{T}}\bar{\mathbf{x}}^{\mathbf{e}}_{\mathbf{a}}.$$

The model presented by Brock and Durlauf states that the utility of an action consists of three terms: the inherent utility of the action, the social utility, and some random noise; and that the social utility is a function

$$S_a(x_a, \mu^e_a) \in \mathbb{R},$$

which depends on both the action itself and a's expected mean action of society.