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BACHELOR THESIS

Thesis Title

Author:
David Gauch

Supervisor:
Prof. Dr. Philippe Cudré-Mauroux
Giuseppe Cuccu

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eXascale Infolab
Department of Informatics

Boulevard de Pérolles 90 • 1700 Fribourg • Switzerland
phone +41 (26) 300 84 65 • diuf-secr@unifr.ch • www3.unifr.ch/inf

Abstract

David Gauch

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Neural networks have become a powerful tool for machine learning, particularly with the large amount of data now available. Deep learning has proven to be an effective method for many applications in various domains, but neural networks have also been used in the field of reinforcement learning to solve non-continuous control tasks. This approach is counterintuitive, since neural networks are continuous.The hypothesis is that discontinuous models such as binary trees would have an advantage in addressing these tasks. To test this hypothesis, a new function has been introduced that allows the tree to grow dynamically when tasks are too complex to be solved by its current structure. This is an important step towards architecture search for binary trees, and has been tested on different benchmarks with promising results. The results indicate that a growing binary tree could be an efficient model for solving many control tasks. Overall, the study provides insights into the use of discontinuous models for reinforcement learning, and the potential benefits of using dynamic structures such as growing binary trees for efficient and effective learning.

Keywords: reinforcement learning, black-box optimization, binary trees, architecture search

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Chapter 1

Introduction

Machine learning, particularly deep learning using neural networks, has greatly benefitted from the availability of large amounts of data. Neural networks have demonstrated significant potential in solving a wide range of tasks. By examining the potential of binary trees, this thesis aims to provide insights into alternative models that could complement or even surpass the performance of traditional neural networks in certain reinforcement learning scenarios.

1.1 Neural Networks

Neural networks are a type of machine learning that takes inspiration from the structure and function of neurons in the human brain. The model is composed of layers of interconnected artificial neurons, which process and transmit information. In neural networks, the input data is first passed through the first layer, and then each subsequent layer receives the output from the previous layer as input.

Neural networks learn by adjusting the connections, or weights, between neurons based on the input data and the desired output. The output of a single neuron is calculated by combining the inputs from the previous layer with the corresponding weights, and adding a bias term if applicable. This weighted sum is then passed through an activation function, which transforms the sum into the output of the neuron. Common activation functions include the sigmoid function, hyperbolic tangent (tanh), and rectified linear unit (ReLU). The choice of activation function depends on the specific task and the architecture of the network.

A neural network can be represented as a computational graph that combines a series of simple functions to produce complex and high-dimensional representations, which capture the underlying function that enables the network to make accurate predictions. This ability to learn multiple levels of abstraction through function composition is one of the key strengths of neural networks and sets them apart from traditional linear models. Figure 1.1 illustrate a human-readable way to represent neural networks.

The most widely used optimization algorithm for the learning process is gradient descent, which helps find the optimal weights by minimizing the error between the network's predictions and the actual output. Backpropagation is a powerful tool for efficiently calculating the gradient of the loss function with respect to the weights. This is done through a forward pass, where the predicted outputs and intermediate node values are determined, followed by a backward pass, where the gradient of the loss function with respect to each weight is calculated using the chain rule of calculus. The gradient is then used to update the weights through gradient descent until the weights converge to values that minimize the loss function. For backpropagation to be effective, the activation functions of the artificial neurons must be continuous and easily differentiable (Goodfellow, Bengio, and Courville, 2016).

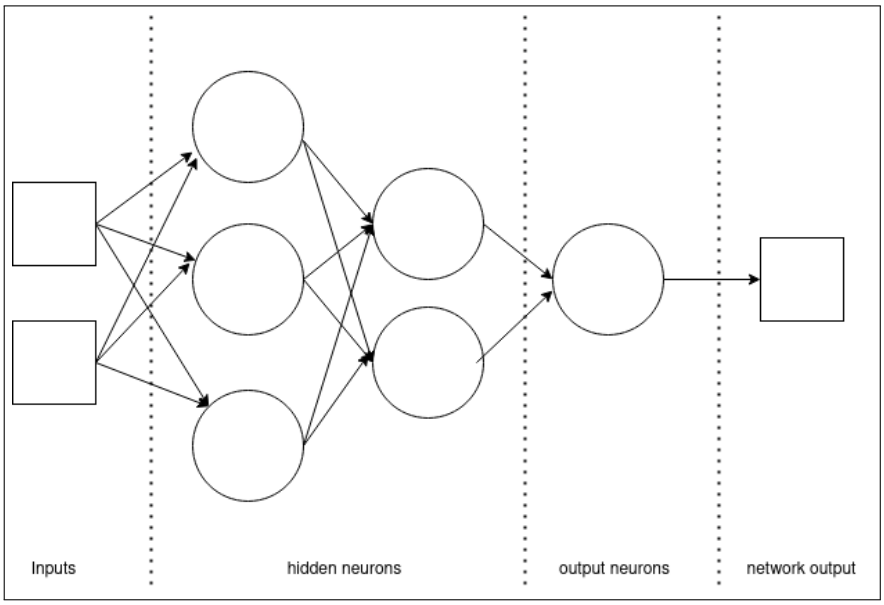


FIGURE 1.1: **Neural network** process incoming input data through a structure consisting of two hidden layers, one with three neurons and one with two, and an output layer with a single neuron. The output produced by the network is determined by passing it through an activation function, which represents the final output of the network.

The structure of a neural network, including the number of layers and number of neurons per layer, significantly impacts its ability to solve tasks. A larger number of layers allows for the approximation of more complex functions, but can also result in overfitting, where the model performs well on training data but poorly on new data. The field dedicated to finding optimal structures is referred to as neural architecture search.

1.2 Binary trees

Trees are a type of data structure that are commonly used in computer science and mathematics. They consist of nodes, which are connected by edges. Trees are special because they are a type of graph that is undirected, connected, and acyclic, meaning that nodes are connected by edges, but there are no loops or cycles in the graph.

In a tree, each node is either a parent or a child. The top node, with no parent, is called the root, while the bottom nodes with no children are called leaves. The distance from the root node determines the level of the node, with the root at level 0 and its children at level 1, and so on. Nodes on the same level are called siblings.

Binary trees are a specific type of tree in which every node, except for leaves, has at most two children, which are called the left and right nodes. Binary trees are easy to understand and visualize, making them useful for a variety of applications. An example of a binary tree is shown in Figure 1.2.

In addition to binary trees, there are other types of trees that are commonly used in computer science and data structures. One example is the "n-ary" tree, which allows nodes to have any number of children. Another example is the "balanced" tree, which is designed to keep the tree's height as small as possible, while still allowing for efficient searches and insertions. Balanced trees come in several different

varieties, such as red-black trees and B-trees, each with their own specific balancing algorithms and performance characteristics (Goodrich, n.d.).

For this project, a specific type of binary tree will be used, in which all nodes except the leaves have exactly two children, rather than at most two children like in classical binary trees.

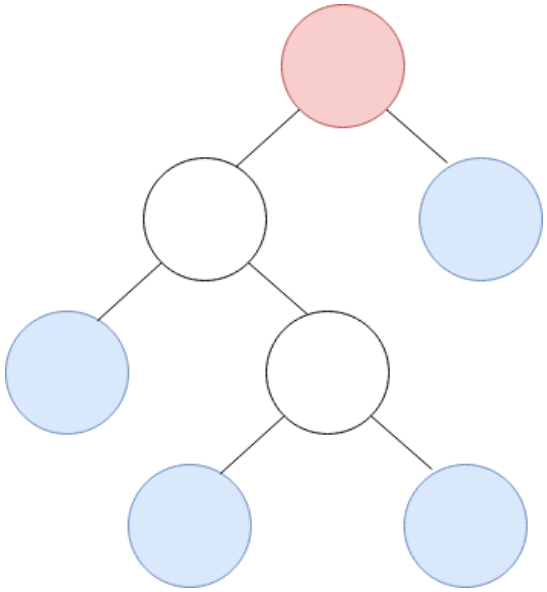


FIGURE 1.2: **Binary tree** where the blue nodes illustrate leafs and the red node is the root of the tree

Binary trees offer several advantages over other data structures. One of the main advantages is their ability to efficiently search, insert, and delete elements. Another advantage is their simplicity and interpretability, as the structure of a binary tree is easy to understand and visualize. This makes it easier to understand how decisions are made and identify errors in the model. In the context of reinforcement learning, binary trees can provide better approximation of discontinuities by allowing for different actions depending on the chosen path. This will be one of the motivations of the project discussed later.

1.3 Architecture search

Neural networks have seen significant advancements in recent years, due in large part to the availability of vast amounts of data. Deep neural networks have demonstrated their ability to solve a wide range of problems, such as image recognition and translation. However, finding the optimal number of layers and nodes, referred to as the architecture, for a neural network to efficiently solve a problem is still a challenging task. One approach is to try different architectures and evaluate their performance, which can be time and effort consuming when dealing with deep, complex neural networks as the structure needs to be designed by hand. Another approach is to use neural architecture search (NAS), which automates the process of finding an appropriate architecture for a specific task. Researchers continue to investigate effective methods for architecture search, with evolutionary algorithms, reinforcement learning, gradient-based optimization, or a combination of these techniques, being commonly used to explore the space of possible architectures(Elsken, Metzen, and Hutter, n.d.).

Until now, neural architecture search algorithms tend to be slow and expensive due to the need to train a large number of candidate networks to inform the search process. The paper by Mellor et al., n.d. showed that a possible solution to speed up this process would be to perform neural architecture search without any network training. The authors implemented a search algorithm called "NASWOT," which only makes observations on the initial untrained networks in the scope of convolutional networks.

The concept of automatically searching for structures that improve model performance without manual intervention can also be applied to binary tree models. A complex reinforcement learning task would usually require a larger binary tree, as this would increase the search space for policies and, thereby, improve the probability of finding good solutions. To achieve this, it is important to adapt the size of the tree according to the complexity of the task. By dynamically incrementing the size of the tree, various strategies can be implemented. The first decision is where to add or remove new nodes in the tree and how many to add. For example, nodes could be added from left to right until the level is full and then proceed to the next layer, or they could be added to a randomly selected leaf, as is the case in this project. Another decision is when to change the size of the tree dynamically. If the tree structure changes too rapidly, the algorithm may not have enough time to search for solutions in the actual search space, whereas waiting too long could result in the algorithm getting stuck in a local optimum and wasting time. Therefore, it is important to strike a balance and change the size of the tree at the right moment. Additionally, it is important to decide which functions the newly created nodes should have. Should they be the same as their parent nodes, or should they be different? Finally, another surprising consideration is whether the newly added nodes should be leaves, which is not the case in this project, as you will see later.

All of these decisions and more must be made and possibly compared when deciding which node-adding strategy to implement. It is important that, even if the tree structure changes, the mathematical equation of the tree remains invariant, which can be a challenging aspect to implement and can also limit the strategies that can be used.

1.4 Motivation and Hypothesis for Discontinuous Models in Reinforcement Learning

In this project, it is observed that current models are typically designed for continuous control, but many control tasks are not. Continuous control refers to tasks where the observation and action space are continuous and the control actions can take on any value within a continuous range of values. Examples of such tasks include robotic arm control and autonomous vehicle navigation. On the other hand, discontinuous control problems have discrete action spaces, which means that only a limited number of actions can be executed. Examples of such tasks include playing chess or running through a maze (Sutton and Barto, 2018).

Neural networks, which are commonly used for modeling continuous control tasks, require continuous functions in backpropagation. The backpropagation algorithm adjusts the weights of the network by calculating the gradient of the loss function with respect to the weights. This is done by propagating the error backwards through the network, hence the name "backpropagation". The goal is to minimize the error of the network, which is repeated for many iterations until the model converges to a set of weights that minimize the error.

An example of a non-continuous control problem is the swing-up cartpole task. This task involves a pendulum fixed to a cart with a fixed joint above and a loose joint below, which is swung up in the first step and then stabilized in the second step (see Figure1.3). When attempting to approximate the function that models this task, it becomes apparent that the function will have a discontinuity. Due to the two distinct tasks involved, the agent must be able to recognize when the first task is complete and the second one begins. In the real world, there are many control problems that are not continuous.

The hypothesis of this project is that discontinuous models would have an advantage in addressing these tasks. Binary trees are used to approximate discontinuities by using a hyperplane to partition the observation space, and depending on the chosen subtree (by going to the left or right child), a different policy will be used. To test this hypothesis, multiple individuals can be evaluated in the environment (by running them through the fitness function) and their performance analyzed, along with other metrics such as the number of individuals that successfully solve the task. Hyperparameters also play an important role in improving the performance of individuals in the environment.

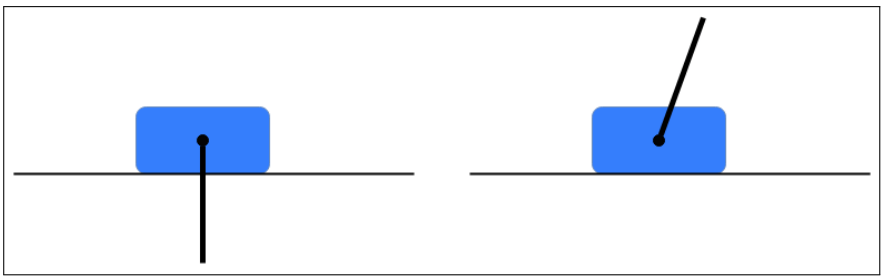


FIGURE 1.3: **Cart-pole swing up problem** On the left side, the initial state of the problem is depicted where the pole is pointing downwards and needs to be swung up by moving the blue cart on the horizontal line. On the right side, the cart has successfully managed to swing the pole up and now needs to stabilize it vertically.

1.5 Contribution

This thesis builds on prior work titled "Alternative Models for Direct Policy Search in Reinforcement Learning Control Problems" (Masanti, n.d.), which proposes using binary trees as an alternative to neural networks for reinforcement learning tasks. The main contributions of this work are:

- Refactoring the initial code provided to make it work and separating the modules into a more organized project structure.
- Implementing CMA-ES as an optimizer for more complex problems.
- Writing a setup to run experiments from scratch and using two environments with configuration files to make the project more customizable and easier to understand.
- Introducing a novel function that dynamically increases the size of the binary tree, which is a crucial step towards enabling architecture search for binary trees and has the potential to significantly improve their performance in solving reinforcement learning tasks. However, only one possible tree-growing

strategy will be tested, and no comparison of different structures will be made. The goal is solely to increase the tree size until the given task can be solved.

The contributions of this work aim to provide an alternative to neural networks for reinforcement learning tasks. Binary trees could potentially offer this alternative and this work aims to give a first insight into architecture search for binary trees, which could further improve their performance.

Chapter 2

Reinforcement learning basics

2.1 Reinforcement learning and continuous control

2.1.1 Reinforcement learning paradigm

A learning paradigm is a formal description of a framework that enables the learning process by defining the sources of information to learn from, establishing criteria for assessing the effectiveness of a learning solution, and identifying the available resources that can enhance the learning process. Essentially, a learning paradigm provides a formal description of the underlying principles and assumptions that guide how we learn and improve our knowledge and skills.

The paradigm involves the interaction between two main components: an agent and an environment. The environment represents the "world" in which the agent operates and provides information about its current state. The agent, on the other hand, is responsible for taking actions within the environment. It acts on the environment by performing available actions and controls, perceives and assesses the changes occurring in the environment via the senses at its disposal, and engages in a feedback loop that informs and modifies its future behavior. Each interaction between the agent and the environment follows a sequence: the agent receives observations based on the current state of the environment, selects an action to take, and transmits that action to the environment. In response, the environment updates its internal state and provides feedback to the agent in the form of updated observations and a reward signal. The reward signal indicates the success or suitability of the agent's action in completing a task, while the updated observations provide information about the new state of the environment. The reward function, also called the fitness, is the only signal required for this learning method to improve and estimate how good a solution is. Figure 2.1 depicts a single timestep of interaction between the agent and the environment.

Reinforcement learning is different from other machine learning approaches such as supervised learning, which uses labeled data to predict outputs for unseen data, and unsupervised learning, which seeks to find patterns in unlabeled data. Reinforcement learning is both a problem that can be addressed with specific solution methods and a field of study that examines the problem and its potential solutions. Unlike supervised learning, there are no labeled data available in reinforcement learning, so the agent learns by maximizing its performance based on the reward signal (Sutton and Barto, 1998; Sutton and Barto, 2018)

2.1.2 Continuous control

A control problem involves a dynamic system described by state variables, and the goal is to determine a strategy that leads the system to its desired target state. The

agent, which takes actions and is part of the environment, sends actions to determine the future behavior of the system. In continuous control problems, the system is observable at all times, and there is continuous interaction between the agent and the environment. Continuous actions are actions that can take on a continuous range of values, as opposed to discrete actions that can only take on a limited set of values.

An example of continuous control is the stabilization mode available in most drones nowadays that enable them to keep a stable position. In order to maintain stability, the drone's control system continuously adjusts the speed of the four rotors to keep the drone level and hovering in place. This is an example of continuous control because the drone's orientation can take on a continuous range of values, and the control system must make small adjustments to the rotor speeds to maintain stability. The drone must therefore continuously adjust inputs in the form of external factors like the wind or the gravity and change accordingly the rotor speed to remain stable.

To simplify the analysis of dynamic systems, time is often discretized into time-steps. This approach provides a way to break down the system's behavior into manageable intervals that can be analyzed and optimized. However, in practice, the accuracy of the analysis is often limited by the control frequency, which is the rate at which the system's inputs can be adjusted. If the control frequency is sufficiently high, the discretization of time becomes less critical, as the system's behavior can be accurately captured by the rate at which the inputs are adjusted. Therefore, the use of time-steps provides a useful tool for simplifying the analysis of dynamic systems, but the accuracy of the analysis ultimately depends on the system's control frequency (Franklin, Powell, and Emami-Naeini, 2014).

Both continuous control and reinforcement learning aim to design systems with richly structured perception, perform planning and control that adapt effectively to environmental changes, and exploit safeguards in the face of new scenarios (Recht, 2018). Continuous actions are particularly important in continuous control and reinforcement learning as they allow for greater precision and flexibility in controlling the behavior of the system.

2.2 Classical reinforcement learning

Some challenges cannot be solved through traditional problem-solving methods or supervised learning algorithms, and reinforcement learning is necessary to tackle them effectively. The Classical Reinforcement Learning framework is a comprehensive system designed with the sole purpose of enabling optimal interactions between agents and their environment. The framework is founded on the principle of trial-and-error learning, where the agent learns through experience by interacting with the environment, and receiving feedback in the form of rewards or penalties. The framework is structured to optimize the agent's behavior, allowing it to learn the best actions to take in any given situation. This broad framework is applicable across a range of contexts, and it has been successfully employed in various fields such as robotics, game-playing, and autonomous vehicles, among others.

In reinforcement learning, the policy is a key element of the framework that determines the actions an agent should take in different states of the environment. The policy is represented by a mapping from states to actions, and it can be either deterministic or stochastic. Deterministic policies specify a single action to take in each state, while stochastic policies specify probabilities for different actions to occur. The reward an agent receives depends on the chosen policy, and the sequence of states

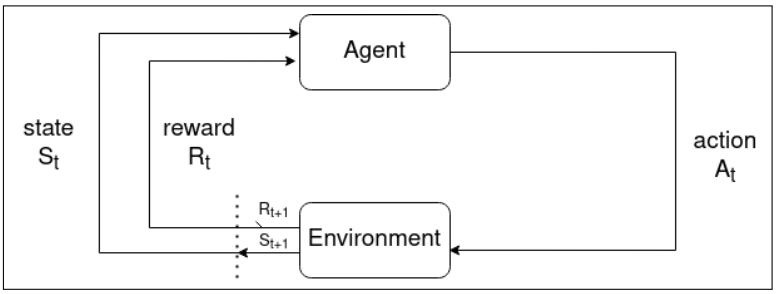


FIGURE 2.1: **Main interaction of the agent and the environment in reinforcement learning.** At the beginning (timestep t) the agent gets the observation S_t and the reward R_t from the environment. The agent performs then action A_t and sends it to the environment. The environment changes its state and returns a new observation S_{t+1} and a new reward R_{t+1}.

reached by the agent is called a Markov chain. In the Classical Reinforcement Learning framework, all reinformcement learning algorithms describe the problem as a Markov chain, which captures the essential properties of the agent's environment. This mathematical concept models systems that change over time in a way that depends only on the current state and not on the history of past states(Sutton and Barto, 1998; Sutton and Barto, 2018). The algorithms then analyze the interactions between the agent and its environment, focusing on observations, actions, and rewards. The rewards are often modeled to simplify the policy, which is the strategy that the agent uses to decide on its actions based on the current state of the environment. The modeling of rewards can be done using various techniques, including value function or q-function, among others.

The value function is a key concept in reinforcement learning that allows us to evaluate the effectiveness of different policies.$V_\pi(s)$ defines the expected total reward that an agent can expect to receive by following the policy π, starting from state s. One way to compute the value function is using the Bellman equation (2.1),

$$V_\pi(s) = R(s, \pi(s)) + \gamma \sum_{s' \in S} P_{s,s'}^{\pi(s)} V_\pi(s') \tag{2.1}$$

which expresses the value of a state in terms of the values of its successors(Barron and Ishii, 1989). $R(s, \pi(s))$ describes the reward obtained by doing the action chosen by $\pi(s)$ in state s.γ enables to give more or less importance to the rewards that occur later. $P_{s,s'}^{\pi(s)}$ describes the probability of reaching s', after executing the action chosen by $\pi(s)$ in the state s and $V_\pi(s')$ stands for the future reward collected by the agent following policy π starting from the state s'. It is also important to note, the Bellman equation does not have a closed-form solution, which makes it challenging to compute the value function in practice. The value function can be used to define an optimal policy, which is the policy that is expected to maximize the reward over time. Another way to analyze policies is using the Q-function.$Q_\pi(s, a)$ is defined as the expected total reward acquired by the agent following policy π starting from state s and taking action a. The Q-function can be related to the value function through the equation $V_\pi(s) = Q_\pi(s, \pi(s))$. A common method for finding the optimal Q-function is Q-learning, which is an iterative process that updates the Q-function based on experience (Watkins and Dayan, 1992).

Reinforcement learning is well-suited for autonomous systems that learn to achieve a desired outcome through trial and error. However, this paradigm presents a unique

challenge that is not encountered in supervised or unsupervised learning: balancing exploitation and exploration. Exploitation refers to the process of repeating actions that have resulted in positive rewards in the past, in order to maximize the cumulative reward. On the other hand, exploration involves trying new actions in order to potentially discover higher rewards and avoid getting stuck in a local optimum. Finding the right balance between these two approaches is crucial for the success of the learning process. There are a lot of strategies for this including $\epsilon - greedy$ selection and $Q - learning$, but still a lot of research is done to find more effective solutions (Coggan, n.d.).

Reinforcement learning has been effective in solving a range of tasks, ranging from simple games to complex real-world problems in fields such as robotics, games and autonomous driving. However, it has also encountered challenges in these real-world applications (Zhu et al., 2020). Nevertheless, the paradigm is sufficient for addressing the desired tasks in this thesis.

In conclusion, reinforcement learning offers a powerful tool for training agents to make decisions in dynamic environments and optimize for a given reward signal. It can effectively solve a range of problems while also presenting the unique challenge of balancing exploitation and exploration.

2.3 Direct Policy Search

Direct policy search is a powerful technique for solving reinforcement learning problems that does not rely on value function approximation. Instead, it optimizes the policy directly, making it well-suited for problems with high-dimensional and continuous action spaces. In contrast to value-based methods that use a value function to estimate the quality of actions, direct policy search learns the optimal policy by searching directly in the space of policies.

Suppose you want to teach an agent how to navigate through a complex maze. The agent can move in four directions: up, down, left, and right. The maze has many obstacles, dead ends, and hidden paths, so it is not easy to find the shortest path to the goal.

Using direct policy search, the agent would start by taking random actions and evaluating how well they work. It might try going up, then down, then left, and so on until it reaches the goal. Over time, it would learn which actions work best in different parts of the maze and develop a policy for navigating through the maze efficiently.

One of the key advantages of direct policy search is its ability to handle complex and continuous control tasks, such as robotics and autonomous systems, which are typically difficult to solve using value-based methods. By separating the network into two components, one for learning intermediate representations of the input and another for learning the policy, smaller networks can be used to learn the policy, improving performance and reducing computation time(Cuccu, Togelius, and Cudre-Mauroux, 2019). This separation enables the use of smaller networks dedicated to policy learning.

Direct policy search can be implemented using various techniques, including gradient-based optimization methods and evolutionary algorithms. However, the use of direct policy search requires a learning algorithm that uses the reinforcement learning paradigm, without any framework set up. The choice of method depends on the specific problem and constraints and may involve trade-offs between computational efficiency and solution quality.

2.4 Neuroevolution

Neuroevolution is a technique that utilizes black-box optimization, such as evolutionary algorithms, to determine the parameters of a neural network. For instance, imagine we have a neural network with two input nodes, one hidden layer with two nodes, and one output node. In order to train this network using neuroevolution, we first define a fitness function that evaluates the network's performance. We then represent the network's weights as a list of values, which we can mutate and select to generate new individuals. These individuals will be used to create a population that will be evaluated using the fitness function.

In reinforcement learning, neuroevolution can be used for direct policy search, which eliminates the need for supervised learning. For example, let's say we want to train an agent to play a game where it must navigate a maze to reach a goal. Using neuroevolution, we can define the reward function such that the agent receives a positive reward for reaching the goal and a negative reward for hitting a wall. We can then use this reward function to evaluate the agent's performance and use an evolutionary algorithm to generate new agents until we find one that performs well.

However, there are limitations to consider when using neuroevolution. Firstly, these algorithms can be computationally expensive and may not be as efficient as gradient-based algorithms. Additionally, since the performance of randomized algorithms depends on random events such as mutations, their performance can vary significantly across runs, and there are no guarantees. Furthermore, neuroevolution algorithms only use the cumulative reward at the end of an episode and miss the correspondence between individual actions and per-step rewards.

For this project, a similar concept is applied but using binary trees instead of neural networks. The proposed name for this method is "Treeevolution".

2.5 Black-Box Optmization

In mathematics, optimization refers to the process of finding the maximum or minimum value of an objective function. Neural networks, for example, try to find the best weights for approximating an underlying function using techniques such as backpropagation and gradient descent. However, these techniques require knowledge of the derivative of the function, which may not always be available or may be too complex to compute (Schaul, n.d.). Black-box optimization is a method that does not rely on any assumptions about the function or its properties, and can be used to optimize any function approximator. It is based on a feedback score similar to reinforcement learning, and the parameter set is improved based on this score (Anderson, 1995).

For example, suppose you have a neural network that classifies images, and you want to optimize the weights of the network to improve its accuracy. You can use black box optimization methods to try different combinations of weights and observe the resulting classification accuracy. By iteratively adjusting the weights and observing the resulting accuracy, you can gradually converge to a set of weights that yield the highest accuracy.

Black-box optimization methods are generally less efficient than traditional techniques such as gradient descent because they do not take advantage of information about the structure of the function being optimized. This means they must explore

a larger space of possible solutions, which can be time-consuming. However, black-box optimization methods can be effective in situations where the function being optimized is highly complex or has a large number of variables, and traditional methods may not be applicable. They are also flexible and can be applied to a wide range of problems without requiring any knowledge of the function being optimized.

In optimization problems, techniques typically converge to a single optimum, which is referred to as unimodality. On the other hand, multimodality refers to the presence of multiple distinct optima in the objective function, which is more common in real-world applications. Solving multimodal problems requires exploration in addition to the exploitation used in single-optimum problems. For example, gradient descent only uses exploitation and can only find another local optimum through exploration by restarting with a different initialization.

Black-box optimization methods, which do not depend heavily on knowledge of the function, can be well-suited for handling multimodal problems because they can explore a larger space of possible solutions. However, one challenge in multimodal landscapes is avoiding getting stuck in a local optimum before reaching the global optimum. A solution to this challenge is to generate multiple viable parametrizations, each exploring a different area in the optimization space. This technique gives a better understanding of the landscape and provides direction for where the most improvement can be obtained. An example of a method for generating parametrizations with improving scores is evolutionary algorithms.

2.5.1 Random weight guessing

The simplest version of an optimizer is to randomly select the set of weights, also known as random initialization, and keep always the best performing individuals. Algorithm 1 shows this procedure. This procedure is called random weight guessing. By randomly initializing the weights, the network is able to explore a wide range of possible solutions, increasing the chances of finding a good global minimum. Even with its simple implementation, random weight guessing has shown some great results in Classic Control benchmarks from the OpenAI Gym (Oller, Glasmachers, and Cuccu, 2020). It's important to note that the initialization of the weights can have a significant impact on the performance of the neural network. Thus, the chosen range for the randomly selected weights will have an impact on the result. Choosing a suitable range for the randomly selected weights is important to ensure that the network can learn useful features and avoid getting stuck in a poor local minimum during the training process. However, it should be noted that random weight guessing will have some limitations with complex problems due to the large search space, because of the large number of possible weight combinations, which can make it difficult for the algorithm to find the global optimum.

Algorithm 1 `random weight guessing` algorithm

```
best_ind ← None          ▷ No best performing individual at the beginning
best_fit ← −inf          ▷ fitness of best performing individual
while stopping criterion not reached do
    generate a population randomly
    evaluate fitness of each individual
    if fitness of individual > best_fit then
        best_ind ← individual
        best_fit ← fitnessofindividual
```

2.5.2 Evolution strategies

Evolution strategies (ES) is a class of evolutionary algorithms that is specialized for optimization of continuous variables. Inspired by natural evolution, ES is a black-box optimization algorithm that uses a process of mutation and selection to search for good solutions to a given problem.

In the main loop of the algorithm, new individuals are created by mutating the parent individuals of the current generation. An individual in the context of ES refers to a specific set of parameters being optimized by the algorithm. A population is a group of individuals being considered by the algorithm at a given time, and a generation refers to one iteration of the main loop. The fitness of an individual is a measure of its performance or quality, based on the feedback score provided by the algorithm.

The main loop of the ES algorithm consists of creating new individuals from the parent individuals of the current generation, evaluating their fitness, and selecting the best-performing individuals to be the parent individuals for the next generation. This process continues until a sufficient solution is found, as determined by a stopping criterion.

To solve a simple example using ES, consider the function $f(x) = x^2$. The goal is to find the value of x that minimizes the function. ES would start by creating a population of random individuals (i.e., values of x). In each generation, the individuals would be mutated and evaluated based on their fitness (i.e., the value of $f(x)$). The best-performing individuals would be selected as parents for the next generation. This process would continue until a satisfactory solution is found. Because ES explores multiple individuals and paths of the optimization space, it has the potential to find the global minimum value of the function, even in the presence of local minima.

Algorithms differ in the number of offsprings created per generation, the number of selected individuals for the next generation, and how the mutation process is performed. Other than gradient-descent based methods, ES generates multiple individuals and by that explores different areas or paths of the optimization space independently, which can be beneficial for avoiding local optima and solving real-world problems that may require sophisticated exploration mechanisms. It is important to note that the efficiency of Evolution Strategies highly depends on factors like the population size, or the mutation and selection methods used. To maximize their performance, experimenting on these factors with different configuration settings might be useful(Salimans et al., 2017).

2.5.3 Covariance Matrix Adapation Evolution Strategy

CMA-ES (Covariance Matrix Adaptation Evolution Strategy) is a stochastic optimization algorithm that is used to optimize complex non-linear functions. It is a derivative-free optimization method that is particularly well-suited for high-dimensional problems. The algorithm works by maintaining a distribution of candidate solutions (i.e. a population of possible solutions) and adapts the distribution based on the performance of the solutions. It is based on the evolution strategy algorithm and uses a covariance matrix to adapt the distribution. The algorithm iteratively updates the distribution until it converges to a solution that is close to the global optimum (Akimoto et al., 2012).

The algorithm has several hyperparameters that can be adjusted to optimize its performance and highly influence its efficiency. Some of the most important ones

include: population size, step size (represented by "sigma" parameter), number of generations, number of parents (represented by "mu" parameter), and so on. The "mu" parameter represents the number of solutions (or parents) that are selected from the population to generate the next generation of solutions and it determines the balance between exploration and exploitation in the search process. The "sigma" parameter represents the step size of the search, it controls the scale of the search and determines how far the algorithm moves away from the current best solution in each generation, it also adjusts the standard deviation of the multivariate normal distribution that guides the search. CMA-ES is a robust optimization algorithm that is widely used. Figure 2.2 illustrates the evolution of the search distribution for CMA-ES on a simple quadratic function, which is a minimization task. The function is defined as $(x-5)^2+(y-5)^2$. The background of the plot indicates good solutions with dark red colors and less good solutions with lighter colors. The red cross represents the optimal solution at coordinates $(5,5)$. The CMA-ES algorithm starts with a standard deviation of one and an initial population initialized with zeros. In the early stages of the optimization process, the search distribution will grow, leading to a high level of exploration. The search distribution is represented by a yellow ellipse, where the center of the ellipse is the mean of the current solutions and the width and height of the ellipse are twice the standard deviation of the search distribution. As the optimization progresses, the search distribution becomes more focused on finding the best individuals and converging towards the global optimum.

2.6 Benchmarks for reinforcement leaning control problems

A benchmark environment in the context of reinforcement learning is a standard and well-defined scenario that serves as a reference point for evaluating and comparing the performance of different reinforcement learning algorithms. These environments usually provide a clear definition of the state space, action space, reward structure, and other problem specifications. Furthermore, having a standard benchmark environment promotes the reproducibility of research and facilitates the sharing of results among the RL community. It also allows for the development of a set of best practices for tackling specific types of reinforcement learning problems, as well as identifying new areas of research and potential improvements to existing algorithms. By utilizing a common benchmark environment, researchers and practitioners can build upon each other's work, accelerating the pace of innovation and discovery in the field of reinforcement learning (Sutton and Barto, 2018). A commun benchmark environment is OpenAI Gym, which will be used in this project.

2.6.1 OpenAI Gym

Open Ai Gym is a toolkit that provides a variety of environments for developing and comparing reinforcement learning algorithms. One of its main advantages is that is uses the same interface for every task which enables an easy comparison and reproduction of results. It offers a range of environments for training agents, including classical control problems, Atari games, and physics simulations which vary in difficulty. OpenAI Gym offers tools for evaluating and visualizing the performance of the algorithms such as pre-built plotters and metrics. All of this gives big advantages for the research community in the field of reinforcement learning (*OpenAI Gym Beta* 2016). There are many different categories of environments available. The less complex environments are the classical control problems and can usualy be solved

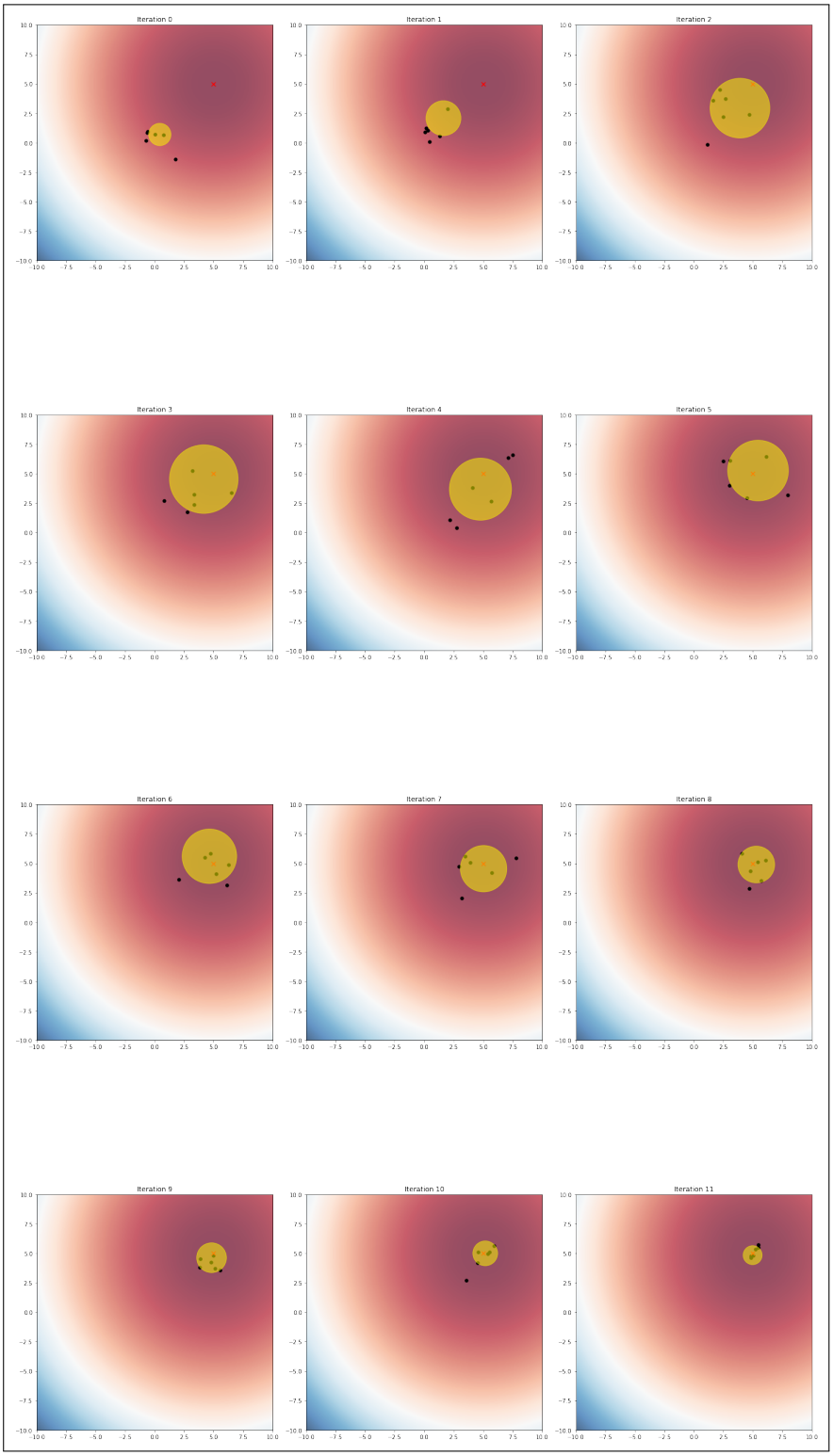


FIGURE 2.2: **Optimization of a 2D problem with CMA-ES** Illustration of a population reaching the global optimum in twelve generations. The background displays the fitness landscape, with red colors indicating higher scores. The red cross indicates the optimal score. The population is represented by black dots, and the yellow ellipse represents the search distribution.

rapidely. The environments used in the context of this project are of the category of Box2D environments. Those problems are harder to solve and are highly configurable. Another category are Atari games, which are a collection of classic video games from the 1980s that were released for the Atari 2600 console. These games are relatively simple by modern standards, but they are still challenging for machine learning algorithms because they require the agent to learn to make decisions in a complex and dynamic environment.

Some of the Atari games included in OpenAI Gym are Pong, Breakout, Space Invaders, and Pac-Man. These games have become popular benchmarks for reinforcement learning algorithms because they are simple enough to be used as a starting point for research, but complex enough to pose a challenge(Brockman et al., 2016). Figure 2.3 illustrates some of the Atari games available in OpenAI Gym.

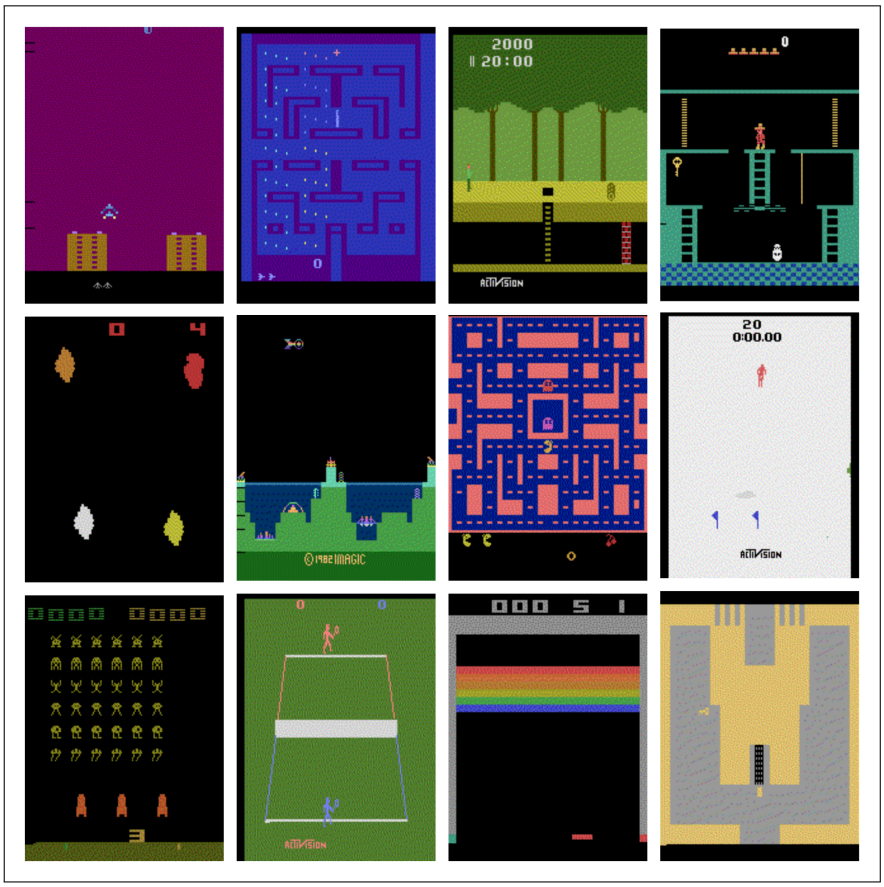


FIGURE 2.3: **Some Atari games of OpenAI Gym** Illustration of a subset of the Atari environments available in OpenAI Gym. The represented environments from left to right and from the top to the bottom are: Air Raid, Alien, Pitfall, Montezuma Revenge, Asteroids, Atlantis, Ms Pacman, Skiing, Space Invaders, Breakout and Adventure

To start working with the toolkit, the first step is to generate an instance of a specified envrionment. This can be done with the predefined function $gym.make()$ to which we pass the name of the environment we want to generate as parameter. The environment can then be stored as a variable and can be reset to its initial state with the $reset()$ function which is typically done at the beginning of an episode and gives out the observations of the current state and some extra information. The observation is often used to get an action from a model which is then passed as argument to the predefined function $step()$. This function returns the next state, the

reward obtained, a boolean indicating wheter the episode is over and some extra information too. These are just some basic functions that enable to start developing and evaluating reinforcement learning algorithms with the help of OpenAI Gym.

Chapter 3

Method

This chapter presents the core contribution of the project, focusing on the binary tree model with a particular emphasis on two key functions. The efficiency of the model was tested on two OpenAI Gym environments, which will be discussed. Additionally, some basic reinforcement learning components and their adaptations for this project will be presented. The code for this chapter is written in Python and can be found on Github[1].

3.1 Model

Models are tools used to represent a range of functions, and the number of functions a model can represent depends on the specific model used. However, every model can only approximate a finite number of functions.

This project employs binary trees as a unique method to approximate control policies using the reinforcement learning approach. Compared to traditional neural network models, binary trees offer potential benefits, including improved performance and interpretability. Additionally, binary trees do not require back-propagation or continuous functions, which makes them a more efficient and effective solution.

The binary tree model is being used to solve continuous control problems in this project. It takes in information about the current state of the environment and outputs the appropriate action for the agent to take. The goal is to use the binary tree to approximate the best action for the agent to take given the current state, making the model an effective tool to approximate control policies.

3.1.1 `Node` module

A node in a binary tree is composed of a pointer that points to its parent node, a function from a defined function class, and an assigned weight that adjusts the importance of the decision or computation made at that node. Additionally, the node has two pointers - one to its left child and one to its right child - that are used to traverse the tree and make decisions based on the input data and the functions applied at each node. The basic functioning of this module was already implemented at the beginning of the project. An important addition that needs to be made is to have a pointer to the parent, which was not necessary before having a method to dynamically increase the tree size. Each node in the tree needs to know its parent node in order to transmit the updated number of weights and nodes once new nodes are added to the tree, as you will see in 3.1.4. Figure 3.1 illustrates a binary tree with a single node to show the different elements that compose a node in this model and the pointers that are assigned to it.

[1] *https://github.com/DavidGauc/btree_model/*

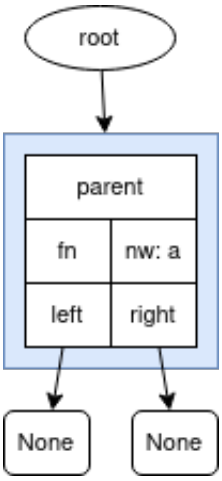


FIGURE 3.1: **Components of a single-node tree** Representation of a binary tree with a single node. The *root* pointer points to the node, and the *parent* pointer points to nothing as it is a single-node tree. The main components of the node are a function fn that is implemented in the function module, an amount of weights nw it contains, and two pointers to its left and right child ($left$, $right$) which point to None in this case.

3.1.2 `Functions` module

Each node in the binary tree contains a function that is used to make decisions or perform computations based on the input data. In the project, three function types were implemented: constant, linear, and perceptron.

The constant function returns the weights as output regardless of the input values, while the linear function returns the dot product of the weights and the observations it received as input. The perceptron uses the sigmoid activation function

$$\frac{1}{(1+\exp(-x))} \tag{3.1}$$

to transform the dot product between the weights and observations (denoted as x) into a scalar value that can be used to perform computations.

Each instance of the function class contains the number of inputs and outputs, the weights used by the function, and the number of times the function has been activated. The weights can be learned or fixed, and the input and output values can take a specific range.

The module also includes a way to set the required amount of weights needed for each function and return the unused weights. Additionally, it increases a counter each time a function is activated in the tree. This allows us to see which nodes and functions were activated the most during the process, which is important for analysis, especially when determining which tree structure is most convenient for solving different problems.

An important add-on that was implemented in this project is the ability to copy a function. This feature will be used in the node insertion strategy presented later.

For non-leaf nodes in the tree, it is convenient to use linear functions because their output will be a scalar value that is useful for traversing the tree. The details of how the functions, weights, and pointers are used to make decisions and perform computations in the tree can be found in the activate function in 3.1.3.

3.1.3 `BTree` module

This module implements the binary tree model. A binary tree is composed of interconnected nodes, each of which holds a function. The illustration in Figure 3.2 shows a binary tree with a root node and two child nodes. A binary tree is a struc-

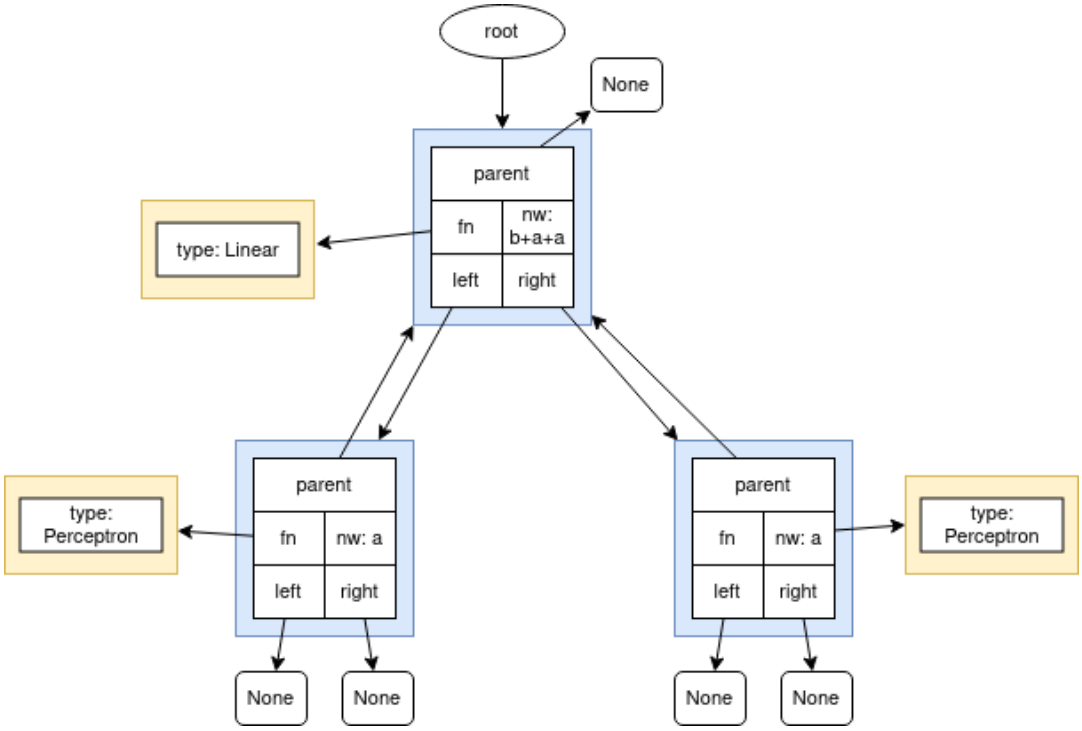


FIGURE 3.2: **Components of a tree with three nodes** Representation of a tree with a root node that contains a linear function and two child nodes with perceptrons. Each node displays its pointer to the corresponding function instance (represented by yellow blocks) and the connections to each other. The number of weights nw of the parent node is incremented by the number of weights of its children nodes.

ture made up of linked nodes that contain functions that are used to make decisions or perform computations based on input data. Each node in the tree has a function, which can be one of the types implemented in the project: constant function, linear function, or perceptron. It also has two child nodes that can be either leaf nodes or internal nodes.

The module enables the retrieval of basic information about the binary tree, such as its current structure, the number of weights or nodes it has, and the activations that occurred during an experiment. The main contribution of this project is enabling the tree to grow dynamically with a specific node insertion strategy, which will be presented in 3.1.4.

3.1.4 model functioning

The process of using the tree to make decisions or perform computations is referred to as activation. This procedure enables the use of a classical data structure like the binary tree for solving reinforcement learning problems. The model uses the reinforcement learning paradigm by getting observations as inputs and outputting actions for the agent to take through the activation of the binary tree. The resulting

activation of the model is a good policy approximator which makes use of the advantages discussed in the introduction. An important role is taken by the function that determines how to traverse the tree in such a way that the observations result in convenient actions for the agent to take by getting information about the current state of the environment. The function implemented for this model is represented as pseudocode in Algorithm 2. The `activate_function` starts at the root of the tree and navigates through the links between nodes based on the output of the current node's function. The functions that are not leaf nodes use a linear function in the case of this project. This means the function takes the observations as inputs and gives out a scalar. If the value of the scalar is positive, the left child node is chosen, else the right child node is chosen. By repeating this, the function will eventually get to the bottom of the tree. When reaching a leaf node, the function will return the output obtained by passing the observations as input to the function implemented in that particular node. In the case of this project, leaf nodes will either use a constant function or perceptrons. The outputs will need to be interpreted differently depending on whether the action space is discrete or continuous, as you will see in 3.4. But basically, this will tell the agent which actions it should perform.

The construction of the tree involves determining the decision points in the tree, which are selected based on the input data and the problem to be solved. The tree can be trained and updated by adjusting its functions, weights, and links between nodes.

Compared to other models like neural networks and decision trees, binary trees have both advantages and limitations. One advantage is interpretability, as the decision points and functions used in the tree can be explained. However, the efficiency of the tree is sensitive to its size, which can be a limitation. This can be mitigated through pruning techniques or other methods that optimize the tree structure.

Algorithm 2 `activate` function

```
1: function ACTIVATE(obs)
2:     node ← self.root                          ▷ starting from the root
3:     while True do
4:         if node is a leaf then return node.fn(obs)     ▷ output of node's function
5:         else if node.fn(obs) >= 0 then
6:             node ← node.left                  ▷ go to the left child node
7:         else
8:             node ← node.right                 ▷ go to the right child node
```

The difficulty of tasks can vary, making it necessary to have the ability to adjust the size of the tree accordingly. For instance, simpler problems like the Cartpole in OpenAI Gym can be solved with a smaller tree, while more complex problems demand a larger tree. Increasing the size of the tree enables more complex decision-making and can enhance the model's performance. However, it also increases the risk of overfitting.

In this project, a function has been implemented to dynamically increase the size of the tree as the complexity of the problem increases. This process of finding an optimal structure is referred to as architecture search. The implemented function demonstrates only one of many possible techniques for growing the tree structure. This technique is a simple approach and will add only a minimal amount of complexity while maintaining the model's invariance. It is important to note that the strategy is restricted in the sense that it can only add nodes and not remove them, the functions of the added nodes are taken from the previous tree, and the nodes

are added by randomly selecting the place where they will be added. All of these factors can be changed in order to create new strategies that will further develop architecture search for binary trees.

The initial idea was to add one new node at a time, but it was quickly abandoned because it would not maintain the invariance of the model. For example, imagine an initial tree with five nodes, where the output of the linear function in the root decides to go to the left node and from there on, the linear function of that node decides to go to its right node. If we add one new node as a left child of that particular node, the model of the node would have changed because the relative position of the node before the addition is not maintained. This is a problem in our case where we want to add the minimal amount of new complexity while keeping the model invariant.

Therefore, another solution was to add two new nodes directly as child nodes, but this also presented difficulties in maintaining the same model within the new, larger tree structure. In this case, the node that has the same relative position to its parent as the node had before the insertion to its parent would need to take over the function and properties of the old leaf in order to maintain this invariance. That's when the idea of having one of the newly inserted nodes as the parent of the leaf node where the nodes will be inserted, and the other new node as its sibling emerged. In this way, the leaf node will remain a leaf while keeping the whole model invariant.

Another decision that had to be made was whether to maintain the activations upon growing the tree. Initially, the idea was to reset the activation counter when adding the nodes. However, with the new implementation, it was more accurate to take over the current activations for the node that was set as the new parent and to set the counter to zero for the node put as the sibling. This approach helped maintain the model invariant.

Finally, the number of weights and nodes needed to be updated throughout the tree when inserting the new nodes. This issue surfaced once the tree implementation was completed, and it did not work correctly because the nodes did not get the correct amount of weights for their functions to perform. To resolve this, a pointer for each node of the tree was added to its parent node. This allowed traversing the tree from bottom to top while updating the number of weights and nodes during the traversal.

The actual strategy for randomly selecting the place to add two new nodes proceeds as follows:

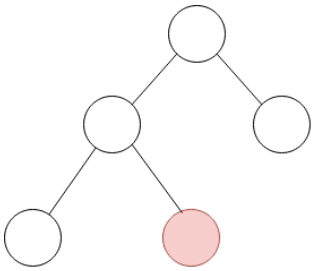
- It begins by traversing the tree randomly until it reaches a leaf node. The pseudo code shown in Algorithm 3 illustrates the implementation of the *pick_random_leaf* function which does this job
- It verifies if the selected leaf is the root of the tree or not
- If it is, a new parent node is created
- If not, the leaf's parent node is duplicated to create a new parent node
- The new parent node is then set as the parent of the current leaf node and its duplicate.

The new nodes are added to the tree in such a way that the current leaf node's relative position to its previous parent node remains unchanged.For more details about the implementation the pseudo code of the function is shown in Algorithm 4.

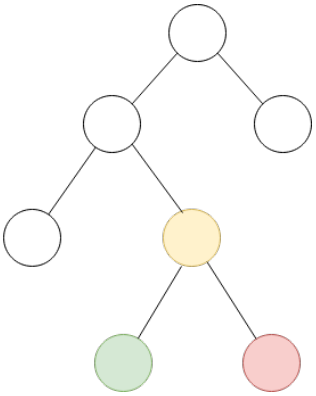
Algorithm 3 `pick_random_leaf` function

```
function PICK_RANDOM_LEAF
    current ← self.root                          ▷ starting from the root
    last_direction ← None
    while current is not the leaf do
        if random >= 0.5 then                    ▷ random is between 0 and 1
            current ← current.left               ▷ go to the left child node
            last_direction ←' left'              ▷ store relative position of current node to its
parent
        else
            current ← current.right              ▷ go to the right child node
            last_direction ←' right'
    return (current, last_direction)
```

It is important to note that the function does not add two child nodes directly to the leaf node, but instead adds one node as the parent of the leaf node and the other as its sibling. As shown in Figure 3.3, this is an example of adding two nodes to a binary tree. Figure 3.3a shows a binary tree with five nodes before using the node adding strategy. Figure 3.3b then shows the binary tree incremented by two new node with their position in the tree. After the addition of these new nodes, all the links in the tree must be updated, and information regarding the number of weights and the number of descendants must be refreshed. To achieve this, the function employs a process of propagating the information from the current leaf node to the root of the tree.



(A) Illustration of the initial binary tree before the addition of the new nodes. The red node represents the randomly selected leaf node, from which the node addition process will start.



(B) Representation of the tree aftesbr adding two nodes using the `add_node` function. The yellow node is the newly created parent node and the green node is the sibling of the previously existing red nodesb. It is important to note that the red node retains its relative position to the parent node

FIGURE 3.3: Addition of two nodes in a binary tree with the `add_node` function

The function for expanding the size of the tree adapts to the complexity of the problem by growing based on a stagnation threshold, which is determined by a lack of improvement in the score for a certain number of steps. A larger tree has a greater search space, which enables it to handle more complex problems. However,

Algorithm 4 `add_node` function

function ADD_NODE
current, *last_direction* = *pick_random_leaf*() ▷ go to a leaf (*current*) randomly
if *current* is the root **then**
create *new_parent* node with a linear function
take over the number of activations from *current* to *new_parent*
last_direction ←′ *left*′ ▷ doesn't matter if its the root
`self.root` ← *new_parent* ▷ *new_parent* is the root
else
copy *current*'s parent into *new_parent* node
take over the number of activations from *current*'s parent to *new_parent*
create *copy_node* ▷ copy of *current*
if *last_direction* is 'right' **then** ▷ maitain relative position to parent node
new_parent.right ← *current* ▷ set *current* as right child of *new_parent*
new_parent.left ← *copy_node*
else if *last_direction* is 'left' **then**
new_parent.left ← *current*
new_parent.right ← *copy_node*
else
raise error
Set *new_parent*'s parent to *current*'s parent ▷ fix parent links
Set *current*'s parent to *new_parent*
Set *copy_node*'s parent to *new_parent*
if *new_parent* is not the root **then**
if *last_direction* is 'right' **then**
set *new_parentasrightchildofitsparentnode*
else if *last_direction* is 'left' **then**
set *new_parentasleftchildofitsparentnode*
else
raise error
if *new_parent* is not the root **then**
parent_iter ← *parentofnew_parent* ▷ starting point for progating up
while True **do**
number of wheights of *parent_iter* is incremented by wheights of newly created nodes
number of nodes of *parent_iter* is incremented by nodes of newly created nodes
if *parent_iter* is root **then**
break
else
Set *parent_iter* to its parent ▷ Go one node upwards
Add to the number of weights of *new_parent* the number of weights of its two children nodes
Add to the number of nodes of *new_parent* the number of nodes of its two children nodes

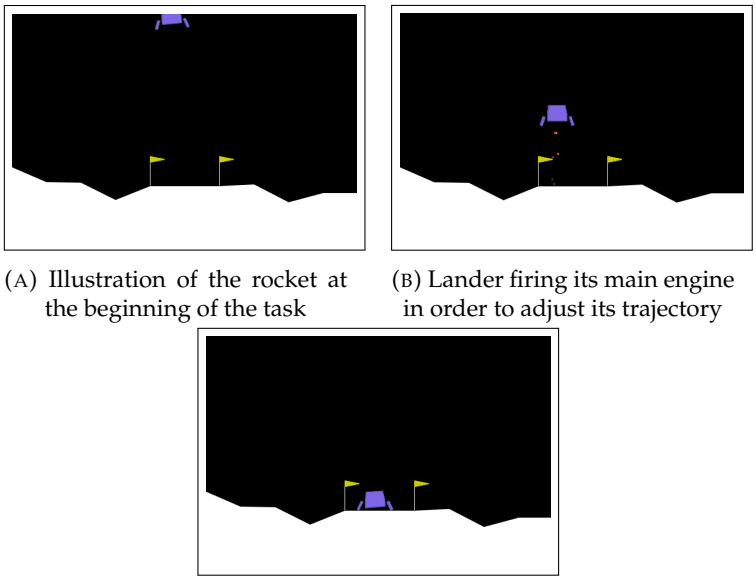
it also increases computational time, making it crucial to strike a balance between expanding the tree too rapidly or too slowly.

3.2 Environments

For this project, two environments from the Box2D category of OpenAI Gym were utilized[2]. These environments are more complex than the "Classical Control" problems and offer greater configurability. Box2D is a 2D physics engine designed for games that enables objects to move in a realistic manner, enhancing game interactivity(*Box2D: Overview* n.d.).

3.2.1 Lunar Lander

The Lunar Lander environment simulates a scenario where a rocket must land between two flags on the surface of the Moon. The rocket has three engines that can either be fired at full speed or turned off. This environment is available in both a continuous and a discrete version. In this project, the discrete version was utilized. For the continuous version to work the output needs to be normalized in order to give out values in the range of -1 and 1. The figure shown in Figure 3.4 depicts the various states that the rocket can be in during the landing process.



(A) Illustration of the rocket at the beginning of the task

(B) Lander firing its main engine in order to adjust its trajectory

(C) Successful landing between the two flags

FIGURE 3.4: Different states of the Lunar Lander environmment

Action space

The environment has four available actions: do nothing, fire the left engine, fire the right engine, or fire the main engine pointing downwards. The strength at which the engines fire cannot be adjusted and is fixed, resulting in a discrete action space with a dimension of 4. In practice the action with the biggest value obtained through the model is chosen and a number from 0 to 3 is sent to the environment.

[2]https://www.gymlibrary.dev/environments/box2d/

Observation space

The observation space for the Lunar Lander contains eight values. Two of them are booleans that indicate whether the corresponding leg of the lander is touching the Moon's surface or not, while all the other values are continuous.

TABLE 3.1: Observation values for the Lunar lander

	Min	Max
coordinates of the lander in x	-1.5	1.5
coordinates of the lander in y	-1.5	1.5
linear velocity in x	-5.0	5.0
linear velocity in y	-5.0	5.0
angle	-3.14	3.14
angular velocity	-5.0	5.0
left leg touching ground	0	1
right leg touching ground	0	1

Rewards

For the agent in the Lunar Lander environment, it receives a reward for successfully landing on the landing pad starting from the top of the screen. The reward points in the default implementation are calculated as follow:

- $-100 \times \sqrt{state[0] \times state[0] + state[1] \times state[1]}$: This calculates a penalty for the horizontal position and velocity of the lander, where $state[0]$ and $state[1]$ are the normalized horizontal position and velocity, respectively. The closer the lander is to the center of the viewport, the closer the value of $state[0]$ will be to 0, and the less penalty it will incur. The penalty is scaled by -100 to make it a significant factor in the reward.

- $-100 \times \sqrt{state[2] \times state[2] + state[3] \times state[3]}$: This calculates a penalty for the vertical position and velocity of the lander, where $state[2]$ and $state[3]$ are the normalized vertical position and velocity, respectively. The calculation is similar to the one for the horizontal position.

- $-100 \times |state[4]|$: This calculates a penalty for the angle of the lander, where $state[4]$ is the angle of the lander with respect to the vertical axis. The more the lander is tilted, the higher the penalty.

- $10 \times state[6]$: This adds a bonus for having the first leg of the lander in contact with the ground, where $state[6]$ is equal to 1 if the first leg is in contact, and 0 otherwise. The bonus is scaled by 10 to make it a relatively small factor in the reward.

- $10 \times state[7]$: This adds a bonus for having the second leg in contact with the ground. The calculation is similar to the one for the first leg.

- $-0.30 \times main\_engine$: This calculates a penalty for each frame the main engine is firing. In the case of a discrete action space $main\_engine$ is either 1 or 0.

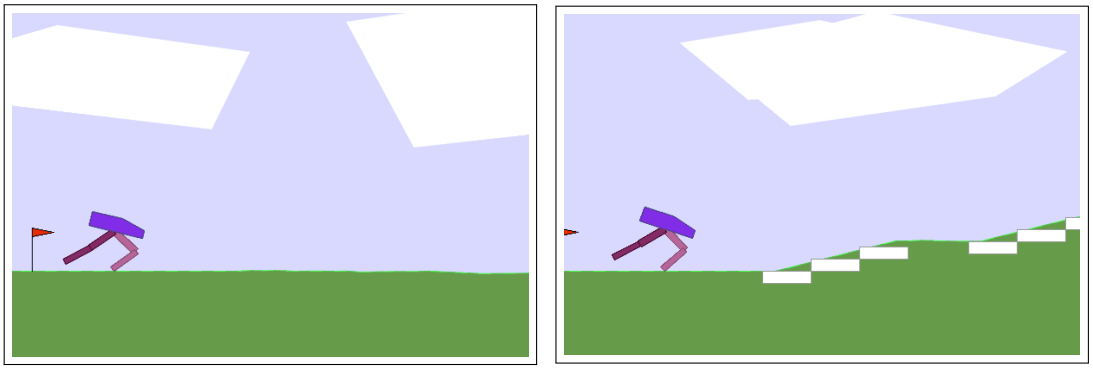
- $-0.03 \times side\_engine$: This calculates a penalty for each frame the a side engine is firing. In the case of a discrete action space $side\_engine$ is either 1 or 0.

An additional reward of -100 or +100 points for crashing or landing safely respectively is obtained at the end of the episode. The final reward is the sum of all of these terms. The design of the reward encourages the agent to land the lander safely on the landing pad with minimum velocity, at a suitable angle, and with both legs in contact with the ground.

The task is considered solved when the agent accumulates a total reward of 200 points or higher. It's worth noting that the exact rewards given for each action, state or event are not fixed and can be adjusted to fine-tune the agent's behavior, based on the specific implementation of the environment.

3.2.2 Bipedal Walker

This environment simulates a two-legged robot attempting to walk as far as possible on uneven terrain. There are two versions available: a "normal" version (Figure 3.5a) and a more challenging "hardcore" version (Figure 3.5b) which includes obstacles. The robot is composed of a hull and two legs, each with two joints, one connecting to the hull and the other allowing the leg to bend. Figure 3.5 illustrates the robot in action on both versions.



(A) Bipedal walker in the "normal" environment version

(B) Bipedal walker in the "hardcore" environment version

FIGURE 3.5: Bipedal walker performing on both environment versions

Action space

The actions of the bipedal walker are continuous, with four actions available, corresponding to the motor speed values of each joint. The values range from -1 to 1 and determine the movement and stability of the robot. It's possible to adjust the range of values in different implementations of the environment.

Observation space

The observation space for the bipedal walker has a dimension of 24 and consists of continuous values, as well as a few boolean values that indicate whether the legs are in contact with the ground or not. The observation space includes information such as the angle, angular velocity, linear velocity, and position of the torso of the robot. It's worth noting that the exact position of the robot is not explicitly stated in the observation space, but it can be derived from other observations, such as the linear and angular velocities of the joints.

TABLE 3.2: Observation values for the Bipedal Walker

	Min	Max
hull angle speed	-3.14	3.14
angular velocity	-5.0	5.0
horizontal speed	-5.0	5.0
vertical speed	-5.0	5.0
position of joints	-3.14	3.14
joints angular speed	-5.0	5.0
left leg contact with ground	0	1
right leg contact with ground	0	1
10 lidar rangefinder measurements	-1.0	1.0

Rewards

A reward is given to the robot when it is able to move forward without falling. Falling is defined as the hull touching the ground (horizontal position less than 0) and it is penalized by -100 points. If the bipedal walker reaches the end of the environment, it accumulates 300 points. The episode is also terminated if the horizontal position of the walker is greater than the length of the terrain. The default calculations for the reward are following:

- $130 \times pos[0]/SCALE$: This encourages the agent to move forward. $pos[0]$ is the normalized horizontal position of the walker and $SCALE$ is a normalization factor that enables to receive 300 points on completion of the task.
- $-5.0 \times |state[0]|$: This calculates a penalty for deviating from keeping the head straight. $state[0]$ is the normalized angular velocity of the walker's head. The more the walker's head deviates from being straight, the higher the penalty.
- $-0.00035 \times MOTORS\_TORQUE \times np.clip(|a|), 0, 1)$: This calculates a penalty for the use of motor torque by the agent the calculation is done for each motor of the walker. $MOTORS\_TORQUE$ is a constant that represents the maximum torque that a motor can apply. The larger the torque applied by a motor, the larger the penalty. The use of $np.clip$ ensures that the torque used is clipped to the range $[0, 1]$.

The "normal" version is considered solved when 300 points are earned within 1600 time steps. For the "hardcore" version, the same amount of points has to be earned within 2000 time steps. The goal is to achieve the highest possible reward while avoiding falling and moving as efficiently as possible. Like for the Lunar Landar, the values can be adjusted to fine-tune the agent's behaviour.

3.3 Control Loop and Performance Evaluation

To evaluate the efficiency of a model in solving tasks in a given environment, a bridge is needed between the model and the environment. This interface extracts actions from the model, which are then performed by the agent and evaluated in relation to its performance in solving the given problem. It is a crucial component in reinforcement learning algorithms, providing the mechanism for feedback and evaluation of the agent's performance, allowing it to learn and optimize its behavior.

The `fit` function, at the core of this interface, implements the control loop, evaluating the quality of the agent's actions based on the rewards provided by the environment. The action is extracted from the model. A distinction is also made between environments with a discrete action space and those with a continuous action space, as shown in Figure 3.4. Once the agent performs the action provided by the model, it receives feedback on the execution of its action. This feedback contains information about the new state of the environment, including the agent itself, as well as a reward that determines if the action was suitable for solving the task. If the problem is solved, the loop finishes.

The efficiency of the agent is calculated by summing the rewards received over the course of a single episode, which is the output of the `fit` function. This score is used to guide the learning process and update the agent's parameters, helping it to continually improve its performance. The pseudo-code in Algorithm 5 illustrates the basic elements of the `fit` function.

Algorithm 5 `fit` function

```
function FIT(ind)
    reset the environment and get the observations
    set the weights of ind in the model
    score ← 0
    done ← False                                   ▷ False is a boolean
    for number of step nsteps do
        action ← env_model_interface.get_action(model,obs)
        get new state of environment after executing action step
        increment score with the obtained reward from the action step
        if done then
            break
    return score                                   ▷ in our case −score as we use CMA-ES
```

3.4 Adjusting Action Selection for Discrete and Continuous Control Tasks

When we're trying to control things, like robots or machines, we need to make decisions about what actions to take. These actions can be either specific things to do or more continuous movements. Environments with either a discrete or continuous action space, as well as discrete or continuous observation spaces, need to be distinguished.

OpenAI Gym helps us by telling us whether we're dealing with specific actions or continuous movements. If we're dealing with specific actions, we just use the number of actions in the program. If we're dealing with continuous movements, we need to rescale our actions to make sure they fit within the allowed movements. We create a special program for this, called a lambda function. This lambda function should take as input an action that needs to be between zero and one (e.g., logistic function) and rescale that action into the boundaries allowed for the function.

Next, a function is created to extract the action from the environment and readapt the output to be consistent with the type of action space. This function is used by the control loop to retrieve the action from the model that the agent should execute. The function takes the model and the current observation of the environment as inputs. It first obtains the action through activation of the binary tree. If the environment

is determined to be continuous by the above procedure, the action is rescaled and returned as the output. This means that all outputs of the binary tree are used and they correspond to the signal to send to each control, which requires regression to approximate an action based on the given outputs. If the environment is discrete, the function selects the action with the highest activation by examining the leaf reached by the activation of the binary tree. The procedure is outlined in pseudocode in Algorithm 6.

Algorithm 6 `get_action` function

```
function GET_ACTION(model, obs)
    action ← model.activate(obs)          ▷ action obtained from model activation
    if environment type is continuous then
        rescale the action within the action boundaries
    else if environment type is discrete then
        pick action with highest activation
    else
        raise Error
    return action
```

3.5 Challenges

- The main difficulty was encountered in implementing the `add_node` function. The goal is to generate a model that is functionally equivalent to the previous best performing model but still capable of improvement. This is achieved by adjusting more weights. If improvement is not achieved, the `add_node` function can be called again until the critical complexity is reached and better scores are obtained. When adding new nodes, it's important to maintain the direction chosen during activation and the number of times each node's function was activated before the node addition. This means that if a node's function chose to go to the left child node during activation, the new larger tree should also go the same way, and the activation should lead to the same leaf but within a larger tree. The code memorizes the last direction chosen when randomly going through the tree, and when adding new nodes, the randomly selected leaf keeps its relative position to its new parent nodes.
- Another challenge was ensuring that each node always knew the number of weights it and its descendants had. This information is crucial for the functions within the nodes. When adding new nodes to a tree, the number of weights for all ancestor nodes of the newly added nodes is no longer correct, as they must now include the number of weights of those newly added nodes. The same holds true for the number of nodes, which also needs to be updated. The number of nodes for a given node corresponds to the number of descendants of that node, including itself. In the implementation of the `add_node` function, this information was propagated upward through the tree starting from the parent node of the one of the two newly inserted nodes (the one that is set as parent of the leaf which was chosen randomly). The number of weights and nodes of the newly inserted nodes was added to the current node iteratively. Finally, the number of weights and nodes of the new parent node, which was not included in the loop, was also increased by the number of weights of its two children.

Chapter 4

Experiments

In this chapter, the objective is to address the research questions of this study through a series of experiments. The experiments performed in this study involve solving the "Lunar Lander" and "Bipedal Walker" control tasks from the OpenAI Gym library using a binary tree model. The results obtained from these experiments are analyzed and discussed in terms of the efficiency of the binary tree model in solving these control tasks.

4.1 Scientific experiments

A basic scientific experiment involves several steps that ultimately lead to a conclusion based on the observations made. The first step is to observe a phenomenon and formulate a hypothesis about how or why it works. Next, an experiment is designed and executed to validate or disprove the hypothesis. A crucial step is to then analyze and interpret the data obtained from the experiment and, finally, draw a conclusion. It is important to note that in computer science, the process can be more complex because experimentation often involves creating something that did not exist previously. However, the same scientific methods must still be applied to study and understand the newly created system.

This experiment aims to see if the implemented node insertion strategy gives better results in solving control tasks.

1. The first thing the experiment should show is if the environment can be solved with binary trees as an alternative model to neural networks.

2. In the second stage, the experiment should determine if the actual node insertion method gives a significant advantage.

4.2 Experimental Design and Implementation

The initial structure of the tree was set to a single-node tree, with either a perceptron or constant function. The `add_node` function was implemented to allow the tree to have linear functions in its decision-making nodes (all except the leaves) and either perceptrons or constants at its leaves when the tree has more than one node. The experiment was optimized using random weight guessing and CMA-ES. Although random weight guessing showed promising results, especially in the early stages of solving the environments, the analysis will focus on CMA-ES as it showed more potential for these problems. A target score was set for each environment, and the experiment would stop once this target score was reached (the target score has to be negative for CMA-ES). The `add_node` function was called when a stagnation threshold was reached, which grew proportionally to the tree size to allow for rapid

exploration of the optimal solution in smaller trees, and slower exploration as the tree grew and the search space became larger. As CMA-ES works with a covariance matrix, its size was adjusted when nodes were added to the tree, with a new matrix created and the best-performing individual reset. This allowed us to see how the individual with the best fitness evolved with changes to the tree structure. The control loop searched for individuals that performed well with the current tree size, and if the threshold was reached, it would grow the tree and continue until the target score, indicating that the environment was solved, was reached. The values for specific tasks can be adjusted in configuration files.

4.2.1 Visualization

To evaluate the performance of the binary tree in solving the environments, two plots were used. First, a score over generations line plot was used to indicate the current best performing individual. The scores were negated to have positive scores on the plot, as CMA-ES requires negative results. This plot shows if the tree with increased size tends to have more individuals reaching high scores.

The second plot is a log-scale histogram of the mean scores. For this, the mean score of each population is calculated. This plot shows if the model overall tends to have more individuals achieving high scores or not.

4.3 Results

The experiments for the lunar lander were all run on an Acer Spin SP513-52N with an Intel Core i7-8550U CPU and 7.7 GB RAM, as they were solved rapidly. No other programs were run at the same time during the execution of the experiments. The bipedal walker experiments were run on a remote server of the University of Fribourg as the time to run them was longer. The server has 256 processors of the brand AMD with each having 64 CPU cores and 251 GB RAM. Both plots described in 4.2.1 were used to analyze the performance of the models solving the two environments.

4.3.1 Lunar Lander

The Lunar Lander was solved rapidly using the binary tree implementation, with a target score of 270 (-270 with CMA-ES) chosen to indicate a smooth landing between the two flags in the environment. The initial tree structure, a single node tree with a perceptron function, was found to be too simple to solve the task, although it could solve the task in some cases. The use of one linear function as the root and two perceptrons as child nodes was found to be effective in reaching the target score when using CMA-ES as the optimizer. However, this was not the case with constant functions for the leaves, where a larger structure was required to achieve the same result. The number of steps, which indicates the maximum number of steps the agent can execute per episode, was set to 300.

For the CMA-ES optimizer, the initial standard deviation (also called *sigma*) was set to 0.3, and the starting point (also called *mu*) is an array with a size equal to the number of weights in the binary tree, with their values chosen randomly between zero and one. The optimum is suggested to lie within $mu \pm 3 * sigma$, according to the documentation [1]. Furthermore, an option was set to determine the maximum

[1] https://cma-es.github.io/apidocs-pycma/cma.evolution_strategy.CMAEvolutionStrategy.html

number of iterations done by the optimizer. This value allows the tree to grow whenever the maximum number of iterations is reached. As explained before, the number of iterations should be small for a small tree structure with few weights and longer for a larger tree. This was achieved by setting the maximum number of iterations to be the multiplication of the number of weights of the current tree and a scalar. The scalar for this experiment was set to two and is called the *stag_step*.

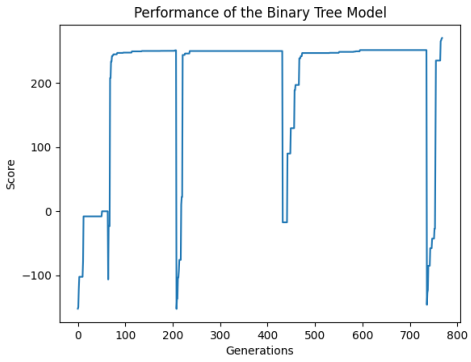
Figure 4.1a shows the evolution of the best-performing individual over the generations in a line plot. The large score reductions in the plot indicate the use of the `add_node` function, which increases the size of the binary tree. With the increasing tree size, both the best-performing individual and the covariance matrix of CMA-ES are reset. This means that the best score obtained by an individual is reset to minus infinity, and once a population passes through the experiment, the score of its best-performing individual is overtaken as the best score. This method enables us to see if the new structure of the tree reaches high scores quicker than the preceding one. The plot shows that the initial tree structure, a single-node tree with a perceptron function, achieves low scores. However, by adding two nodes to the tree, which means having a linear function as the root and two child nodes with perceptrons, scores over 200 points are reached. Also, the steepness of the slope that shows the rapidity at which those scores were reached for tree sizes with more than one node is similar. This shows that further increases (after having a tree with three nodes) in the size of the binary tree do not significantly increase the scores and the execution time to reach high scores. The figure also shows that the single-node tree was searched for optimal solutions over a few generations, and that the number of generations searching for solutions increases with the size of the tree. This can be seen by the distance between the depressions of the graph. The bars in the graph that result from these separations represent the evolution of the best scores obtained with a certain tree structure.

Figure 4.1b shows a log-scaled histogram of the mean scores achieved. The y-axis shows the mean scores, and the x-axis shows the number of populations that achieved these scores in log-scale. The mean is calculated over 15 individuals representing one population. The plot shows that relatively few populations achieve low scores in this environment. Most populations have a score of about 0 points, although populations with a high mean remain high. The graph shows the mean of the populations over all tree sizes.

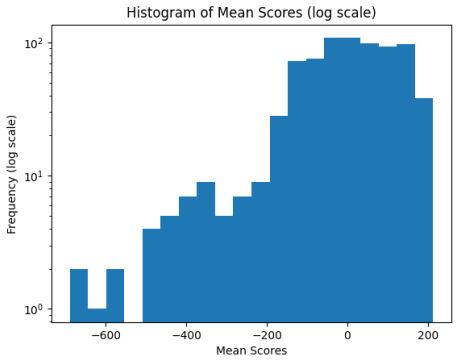
4.3.2 Bipedal walker

The Bipedal Walker was not solved with the current implementation, as 300 (-300 for CMA-ES) points were not obtained throughout the experiment. However, a larger tree seemed to find more individuals with high fitness. In this experiment, the initial structure showed better results when starting with a constant as the single node's function rather than with a perceptron. The maximum number of steps was set to 1600. For the CMA-ES optimizer, the same values were chosen for the parameters as for the Lunar Lander experiment. The *stag_step* variable, explained in 4.3.1, was set to 0.2 in this case.

To assess the impact of tree size on performance, the experiment was run for 45 minutes. Figure 4.2a shows the line plot of scores over generations. Similar to the Lunar Lander, large reductions in the plot indicate an addition of nodes to the tree. With small trees, the individuals perform poorly (less than zero points). From the fifth bar onwards (which corresponds to a binary tree with nine nodes, as we start with one node and always add two nodes with the `add_node` function), the score of



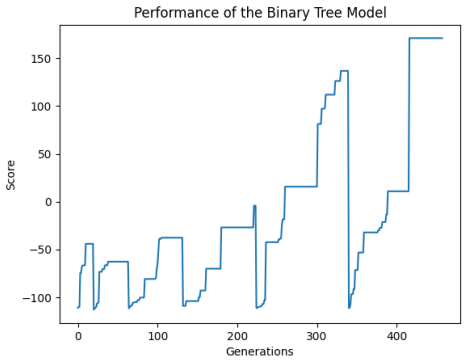
(A) **Score over generations lineplot of the lunar lander environment** Evolutions of the best performing individual in the lunar lander environment with an increasing binary tree size over the generations



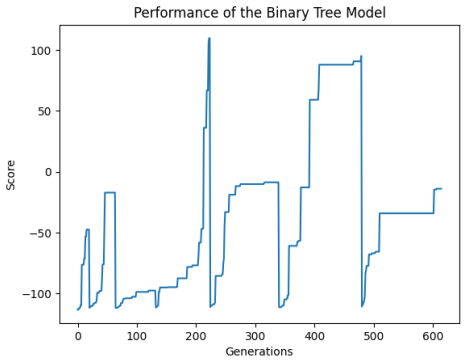
(B) **log-scale histogram of the mean scores obtained in the lunar lander environment** The scores where obtained with a growing binary tree until one individual obtained a score of at least 270 points

FIGURE 4.1: Plots of the lunar lander experiment

the best-performing individuals with the model increases rapidly, achieving scores over 100 points. However, it is important to note that this is not always the case. Some experiments achieve lower scores even with big tree structures. For example, when the experiment was run for one hour, the results in Figure 4.2b showed different results. The fourth and sixth bars indicate very high scores (seven and eleven nodes), but the fifth and seventh bars show that the individuals had scores of less than zero points (nine and thirteen nodes). This shows that an increasing size of the tree does not necessarily mean that individuals with high scores will be generated. In the case of this example, the highest scores were obtained with a tree structure of seven nodes.



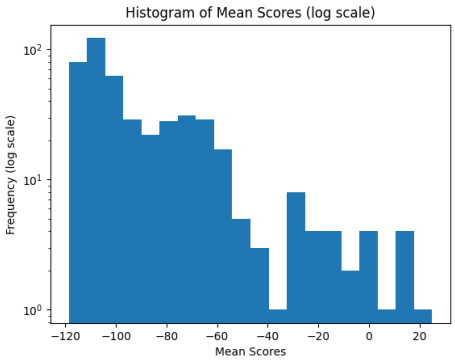
(A) **Score over generations lineplot of the bipedal walker** Evolutions of the best performing individual in the bipedal walker environment run for 45 minutes with an increasing binary tree size over the generations



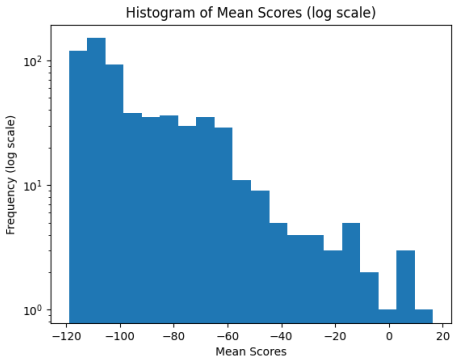
(B) **Score over generations lineplot of the bipedal walker** Evolutions of the best performing individual in the bipedal walker environment run for one hour with an increasing binary tree size over the generations

The log-scaled histogram in Figure 4.3a provides the mean scores of the populations, as calculated over 15 individuals. The experiment was run for 45 minutes. The histogram shows that, given the complexity of the environment, a large proportion of populations had low mean scores. Most of the means were below zero points and the amount decreased as the mean scores increased. There were no populations with a mean score over 20 points after 45 minutes of the experiment, which indicates that

the task is still far from being solved (the target score is 300 points). The results of the same histogram after running the experiment for one hour (Figure 4.3b) showed similar results.



(A) **log-scale histogram of the mean scores obtained in the bipedal walker environment run for 45 minutes** The scores where obtained with a growing binary tree for one hour even if the environment was not solved



(B) **log-scale histogram of the mean scores obtained in the bipedal walker environment run for 1 hour** The scores where obtained with a growing binary tree for one hour even if the environment was not solved

It is important to note that all of these experiments were run using the default reward functions, without fine-tuning. In the case of the more complex Bipedal Walker environment, this led to suboptimal results. The walker often became stuck in local optima, which prevented it from exploring better ways of moving. For example, the walker often remained balanced on its two legs without falling, as shown in Figure 4.4. This is because it does not receive a large penalty for remaining in that state, whereas taking a step forward, which could lead to learning a more efficient way to walk, would result in a fall and a penalty.

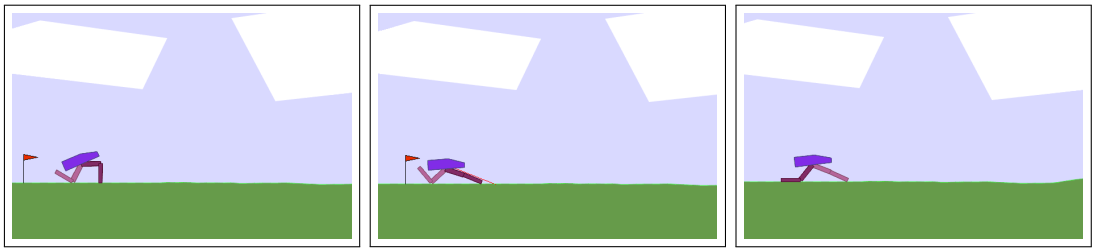


FIGURE 4.4: Different states of the Bipedal walker environment where the walker got stuck in local optima

Chapter 5

Conclusion

5.1 Conclusion

In this work, we improved the binary tree model as an alternative to neural networks in solving reinforcement learning problems by adding a function that enables trees to grow dynamically depending on the complexity of the task to be solved. Following conclusions can be made:

- The first goal of the project was to make the model work, which was successfully achieved as the project enabled testing the model on two different OpenAI Gym environments.
- Using CMA-ES instead of the previously used random weight guessing showed some interesting results, as it improved the model's ability to solve tasks. However, updating the covariance matrix of the optimizer each time the tree grows in size was a challenge, as the shape of the matrix would also need to be increased. To address this, we created a new covariance matrix whenever the tree size changes. A more accurate solution to this problem could further improve the model's efficiency in solving complex tasks.
- The code of this project was refactored to enable easy introduction of new environments by creating a new configuration file, and to allow for rapid changes of each component of the model, as most functions perform a single task.
- The newly implemented function that enables the model to increase the size of the tree by randomly selecting the place to add new nodes was also successfully implemented. During experiments, the tree grew in size if it got stuck for a while without obtaining better scores, and the activations of the tree during the task were also printed. This information could be beneficial in implementing new strategies, as it shows which paths of the tree are more likely to be activated for a certain task.

The model, together with the node insertion strategy, was able to solve the `Lunar Lander` environment with discrete actions rapidly. However, for the `Bipedal Walker` environment, which has a continuous action space, it had more difficulties than expected, often getting stuck with relatively low scores. Despite this, the simplicity of the `add_node` function makes it a comprehensive approach and a first step towards architecture search for binary trees.

Overall, the binary tree model shows some interesting advantages in theory and is able to solve some relatively simple problems from OpenAI Gym with a simple implementation. However, with the current node insertion strategy and without fitness shaping, it was not yet capable of solving the `Bipedal Walker`environment

of OpenAI Gym. Therefore, it would be interesting to continue working on binary trees and architecture search combined with it, to see how they perform in the future as an alternative to neural networks.

5.2 Future Work

The continuation of this work includes multiple directions. Some ideas for future work are listed here:

- One approach could be to introduce fitness shaping to determine if the `Bipedal Walker` task can be solved with the current implementation of the binary tree.
- After that, it would be interesting to modify the covariance matrix of the CMA-ES optimizer instead of recreating it from scratch each time the tree grows in size. Recreating the matrix each time results in losing all the learning from previous runs, which is suboptimal. Instead, it would be better to keep the invariant matrix values, modify the changing ones, and add the new ones that did not exist before.
- It would also be valuable to investigate the model's performance on other environments to evaluate its robustness in solving reinforcement learning problems and gain insights into its capabilities and limitations.
- Furthermore, exploring other methods for architecture search using binary trees would be a crucial aspect. The current project's scope is limited to randomly selecting the place to insert new nodes, adding new nodes when a linear threshold based on the tree's size is reached, and setting the functions of the nodes equally for all nodes of the tree (only distinguishing between leaf and non-leaf nodes), among other aspects. All of these aspects could be modified and tested to improve architecture search for binary trees.
- Once the model demonstrates robust capabilities in solving the tasks, it would be interesting to compare the performance of the binary tree model to that of traditional neural networks on various tasks, providing a better understanding of the potential advantages and disadvantages of using binary trees as an alternative.

Finally, binary trees remain one possibility for an alternative to neural networks. However, seeking other models that address the current limitations of neural networks is still state-of-the-art and will likely be the focus of future research.

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