A Closed Form Analytical Solution to Maxwell's Equations in Response to a Time Invariant Gaussian Source

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Abstract A closed form analytical solution to Maxwell's equations in response to a time-invariant Gaussian source is deduced. A new family of functions appears in the solution. Functions in the new family are formed by inseparable or separable hyperbolic-exponential elements. A naming convention for the new functions is proposed. Characteristics of the new functions are studied. Graphics of the new functions and the electromagnetic fields are presented.

Contents

Introduction

We cannot expect that all electromagnetic fields can be described by known math functions, due to complexity of the dynamics of electromagnetic fields. The generic solution [1] to Maxwell's equations leads us to explore such unknown kingdoms.

In response to a time-invariant Gaussian field source, the generic solution [1] to Maxwell's equations gives us a specific analytical solution. The solution shows that the dynamics of the electromagnetic fields are inseparable mixtures of hyperbolic and exponential elements.

The mixing of hyperbolic and exponential elements were discussed in suspension-flow and Anosov flow ([2],[3],[4]). I am not familiar with the work in those areas. It is not obvious for me to see a math link between the functions that appear in the frame flows and the functions that appear in dynamics of electromagnetic fields. I am assuming the dynamics of electromagnetic fields are new functions not studied previously. The math links between the two areas can be an interesting research topic.

There are many ways of mixing hyperbolic and exponential elements to form new functions; let's call the new functions hyper-exponential functions. A naming convention is proposed for identifying hyper-exponential functions. Some characteristics of the hyper-exponential functions are studied. Graphic drawings of some hyper-exponential functions are presented.

1D fields are used to discover the hyper-exponential functions. The hyper-exponential functions also appear in 3D fields.

Field source and solution

Curl part of Maxwell's equations are shown below [5].

$\frac{\partial H(x,y,z,t)}{\partial t} = -\frac{1}{\mu}\nabla \times E(x,y,z,t)$	(1.1)
$\frac{\partial E(x,y,z,t)}{\partial t} = \frac{1}{\varepsilon}\nabla \times H(x,y,z,t) - \frac{1}{\varepsilon}J(x,y,z,t)$	(1.2)
$E,H,J \in R^3$ $x,y,z,t,\varepsilon,\mu \in R$ $J(x,y,z,t) = field\ source$	

To simplify notations, a time scale of *ct* is use. Maxwell's equations become

$c = \frac{1}{\sqrt{\varepsilon\mu}}$	(1.3)
$\eta = \sqrt{\frac{\mu}{\varepsilon}}$	(1.4)
$\theta = ct$	(1.5)
$\frac{\partial H(x,y,z,\theta)}{\partial \theta} = -\frac{1}{\eta}\nabla \times E(x,y,z,\theta)$	(1.6)
$\frac{\partial E(x,y,z,\theta)}{\partial \theta} = \eta\nabla \times H - \eta J(x,y,z,\theta)$	(1.7)

For (1.6) and (1.7) to be compatible with (1.1) and (1.2), I should have used $E\left(x,y,z,\frac{1}{c}\theta\right)$, $H\left(x,y,z,\frac{1}{c}\theta\right)$, and $J\left(x,y,z,\frac{1}{c}\theta\right)$. Since (1.1) and (1.2) are not used in this paper, I'll let (1.6) and (1.7) use the simpler notations.

Assume the initial values are zero. The solution to Maxwell's equations (1.6) and (1.7) is given below (see [1]).

$H(x,y,z,\theta) = \sum_{n=0}^{\infty}\frac{\theta^{2(n+1)}}{\left(2(n+1)\right)!}\sum_{m=0}^{n}(-1)^m\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(x,y,z,0)}{\partial\theta^{2(n-m)}} + \sum_{n=1}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=1}^{n}(-1)^{m+1}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(x,y,z,0)}{\partial\theta^{2(n-m)+1}}$	(1.8)
$E(x,y,z,\theta) = \eta\sum_{n=0}^{\infty}\frac{\theta^{2(n+1)}}{\left(2(n+1)\right)!}\sum_{m=0}^{n}(-1)^{m+1}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(x,y,z,0)}{\partial\theta^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}(-1)^{m+1}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(x,y,z,0)}{\partial\theta^{2(n-m)}}$	(1.9)

The above contents are taken from [1].

Consider 1D fields using the x-axis. Maxwell's equations (1.6) and (1.7) become

$\frac{\partial H_y(x,\theta)}{\partial\theta} = \frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x}$	(1d.1)
$\frac{\partial E_z(x,\theta)}{\partial\theta} = \eta\frac{\partial H_y(x,\theta)}{\partial x} - \eta J_z(x,\theta)$	(1d.2)

The solution (1.8) and (1.9) becomes

$H_y(x,\theta) = -\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\frac{\partial^{2n+1}J_z(x,0)}{\partial\theta^{2(n-m)}\partial x^{2m+1}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\frac{\partial^{2n+2}J_z(x,0)}{\partial\theta^{2(n-m)+1}\partial x^{2m+1}}$	(1d.3)
$E_z(x,\theta) = -\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\frac{\partial^{2n+1}J_z(x,0)}{\partial\theta^{2(n-m)+1}\partial x^{2m}} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{\partial^{2n}J_z(x,0)}{\partial\theta^{2(n-m)}\partial x^{2m}}$	(1d.4)

Consider a time-invariant Gaussian source:

$J_z(x,\theta) = e^{-ax^2}; a > 0$	(1d.5)

Because

$$\frac{\partial^k J_z(x,\theta)}{\partial\theta^k} = 0; k > 0$$

The solution (1d.3) and (1d.4) become

$H_y(x,\theta) = -\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{\partial^{2n+1}J_z(x,0)}{\partial x^{2n+1}}$	(1d.6)
$E_z(x,\theta) = -\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\frac{\partial^{2n}J_z(x,0)}{\partial x^{2n}}$	(1d.7)

Apply (1d.5) to (1d.6) and (1d.7), and we have the solution for this time-invariant Gaussian source:

$$H_y(x,\theta) = \frac{e^{-ax^2}}{\sqrt{a}} \sum_{k=0}^{\infty} \frac{(2ax\theta)^{2k+1}}{(2k+1)!} \sum_{m=0}^{\infty} \frac{(-1)^m (\sqrt{a}\theta)^{2m+1}}{m!\,(2m+2k+2)}$$	(1d.8)
$$E_z(x,\theta) = \eta \frac{e^{-ax^2}}{\sqrt{a}} \sum_{k=0}^{\infty} \frac{(2ax\theta)^{2k}}{(2k)!} \sum_{m=0}^{\infty} \frac{(-1)^{m+1} (\sqrt{a}\theta)^{2m+1}}{m!\,(2m+2k+1)}$$	(1d.9)

Functions of Hyperbolic-Exponential Mixtures

Notice that

$$\sum_{k=0}^{\infty} \frac{\alpha^{2k+1}}{(2k+1)!} = \sinh\alpha\,; \sum_{k=0}^{\infty} \frac{\alpha^{2k}}{(2k)!} = \cosh\alpha$$

$$\sum_{m=0}^{\infty} \frac{(-1)^m \alpha^{2m}}{m!} = e^{-\alpha^2}; \sum_{m=0}^{\infty} \frac{(-1)^m \alpha^{2m+1}}{m!\,(2m+1)} = \int e^{-\alpha^2} d\alpha$$

We can see that the fields of (1d.8) and (1d.9) are formed by hyperbolic and exponential elements; the hyperbolic and exponential elements are inseparable. Also, time and space are inseparable in the hyperbolic elements.

Such inseparable mixtures of hyperbolic and exponential elements are new types of math functions not studied in literature, to my knowledge. Therefore, we need new names for these new types of functions.

I notice that the hyperbolic elements of sinh are related to following coefficients. I'll call it "odd-binomial coefficients".

$$p_{h,n,k} = \frac{(k+h)!\,n!\,(2(n+h)+1)!}{(n-k)!\,(n+h)!\,k!\,(2(k+h)+1)!}$$ $$h \ge 0; 0 \le k \le n$$	(f.1)

The hyperbolic elements of cosh are related to following coefficients. I'll call it "even-binomial coefficients".

$$q_{h,n,k} = \frac{(k+h+1)!\,(n+1)!\,(2(n+h))!}{(n-k)!\,(n+1+h)!\,(k+1)!\,(2(k+h))!}$$ $$h \ge 0; 0 \le k \le n$$	(f.2)

Collectively, we may call $p_{h,n,k}$ and $q_{h,n,k}$ "double-binomial coefficients".

We may use following 8 functions to form a function family of hyperbolic-exponential mixtures, and call them hyper-exponential functions.

$$esinhi(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)}}{(2(n+m))!} \sum_{k=0}^{n} (-1)^k k^d p_{h,n,k} \varrho^{2(k+s)}$$	(f.3)
$$esinh(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)}}{(2(n+m))!} \sum_{k=0}^{n} (-1)^k k^d p_{h,n,k} \varrho^{2(k+s)+1}$$	(f.4)
$$eisinhi(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)+1}}{(2(n+m)+1)!} \sum_{k=0}^{n} (-1)^k k^d p_{h,n,k} \varrho^{2(k+s)}$$	(f.5)
$$eisinh(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)+1}}{(2(n+m)+1)!} \sum_{k=0}^{n} (-1)^k k^d p_{h,n,k} \varrho^{2(k+s)+1}$$	(f.6)
$$ecoshi(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)}}{(2(n+m))!} \sum_{k=0}^{n} (-1)^k k^d q_{h,n,k} \varrho^{2(k+s)}$$	(f.7)
$$ecosh(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)}}{(2(n+m))!} \sum_{k=0}^{n} (-1)^k k^d q_{h,n,k} \varrho^{2(k+s)+1}$$	(f.8)
$$eicoshi(\xi,\varrho)_{m,s,h,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)+1}}{(2(n+m)+1)!} \sum_{k=0}^{n} (-1)^k k^d q_{h,n,k} \varrho^{2(k+s)}$$	(f.9)
$$eicosh(\xi,\varrho)_{m,h,s,d} = \sum_{n=0}^{\infty} \frac{(-1)^n \xi^{2(n+m)+1}}{(2(n+m)+1)!} \sum_{k=0}^{n} (-1)^k k^d q_{h,n,k} \varrho^{2(k+s)+1}$$	(f.10)
$m \ge 0; s \ge 0; h \ge 0; d \ge 0$	

The above functions adopt the following naming convention.

$$e[i]\{\sinh|\cosh\}[i](\xi,\varrho)_{m,s,h,d}$$

- i – for integration

- e – for exponential
  - $e: power\ of\ \xi\ is\ even$
  - $ei: power\ of\ \xi\ is\ odd$
- sinh - coefficient $p_{h,n,k}$ is used
  - $sinh: power\ of\ \varrho\ is\ odd$
  - $sinhi: power\ of\ \varrho\ is\ even$
- cosh - coefficient $q_{h,n,k}$ is used
  - $cosh: power\ of\ \varrho\ is\ odd$
  - $coshi: power\ of\ \varrho\ is\ even$

4 integers are used to describe how the hyperbolic-exponential elements are mixed.

- m – for time attribute
- s – for space attribute
- h – for hyperbolic attribute
- d – for differential attribute

A short version of the naming for $d = 0$ can be used,

$e[i]\{\sinh\|\cosh\}[i](\xi,\varrho)_{m,s,h} = e[i]\{\sinh\|\cosh\}[i](\xi,\varrho)_{m,s,h,0}$	(f.11)

A shorter version of the naming for $d = 0, s = 0, h = 0$ can be used,

$e[i]\{\sinh\|\cosh\}[i](\xi,\varrho)_{m} = e[i]\{\sinh\|\cosh\}[i](\xi,\varrho)_{m,0,0,0}$	(f.12)

The shortest version of the naming for $d = 0, s = 0, h = 0, m = 0$ can be used

$e[i]\{\sinh\|\cosh\}[i](\xi,\varrho) = e[i]\{\sinh\|\cosh\}[i](\xi,\varrho)_{0,0,0,0}$	(f.13)

Identities of the Hyperbolic-Exponential Mixtures

A few identities of the hyperbolic-exponential mixtures are listed below. A comprehensive table of identities should be listed in a reference book.

$\varrho\ eicoshi(\xi,\varrho) = eicosh(\xi,\varrho)$	(i.1)
$\frac{\partial}{\partial\varrho} eicoshi(\xi,\varrho) = \frac{1}{2}\big(eicosh(\xi,\varrho) - eisinh(\xi,\varrho)\big)$	(i.2)
$\frac{\partial}{\partial\xi} esinh(\xi,\varrho)_1 = eisinh(\xi,\varrho)$	(i.3)
$\varrho\ eisinh(\xi,\varrho)_{m,s,h,d} = eisinhi(\xi,\varrho)_{m,s+1,h,d}$	(i.4)
$\frac{\partial}{\partial\varrho} esinh(\xi,\varrho)_{m,s,h} = 2esinhi(\xi,\varrho)_{m,s,h,1} + (2s+1)esinhi(\xi,\varrho)_{m,s,h}$	(i.5)
$esinhi(\xi,\varrho)_{1,1,0} - 4esinhi(\xi,\varrho)_{1,0,0,1} = esinhi(\xi,\varrho)_{1,1,1}$	(i.6)
$eisinhi(\xi,\varrho)_{1,1,1} - 2eisinhi(\xi,\varrho)_1 + \xi = eicoshi(\xi,\varrho)$	(i.7)
$\frac{\partial}{\partial\xi} eisinhi(\xi,\varrho)_{m,s,h,d} = esinhi(\xi,\varrho)_{m,s,h,d}$	(i.8)
$eisinh(\xi,\varrho) = e^{-\xi^2}\sinh(\varrho\xi)$	(i.9)
$ecoshi(\xi,\varrho) = e^{-\xi^2}\cosh(\varrho\xi)$	(i.10)
$ecosh(\xi,\varrho) = \varrho e^{-\xi^2}\cosh(\varrho\xi)$	(i.11)
$eisinhi(\xi,\varrho) = \frac{1}{\varrho} e^{-\xi^2}\sinh(\varrho\xi)$	(i.12)
$1 + esinhi(\xi,\varrho)_{1,1,1} - 2\ esinhi(\xi,\varrho)_1 = ecoshi(\xi,\varrho)$	(i.13)
$\frac{\partial}{\partial\xi} eicoshi(\xi,\varrho)_{m,s,h,d} = ecoshi(\xi,\varrho)_{m,s,h,d}$	(i.14)
$\varrho\ esinh(\xi,\varrho)_1 = esinhi(\xi,\varrho)_{1,1,0}$	(i.15)
$\frac{\partial\ eisinh(\xi,\varrho)}{\partial\xi} = \varrho\ esinhi(\xi,\varrho)$	(i.16)
$\frac{\partial\ eisinh(\xi,\varrho)}{\partial\varrho} = \xi\ ecoshi(\xi,\varrho)$	(i.17)
$e^{-\xi^2}\cosh(\varrho\xi) + 2\ esinhi(\xi,\varrho)_1 - esinhi(\xi,\varrho)_{1,1,1} = 1$	(i.18)

(i.9) to (i.12) show that for some functions in the hyper-exponential family, their hyperbolic and exponential elements are separable.

Closed-Form Analytical Solution

Using the new hyper-exponential functions, the solution (1d.8) and (1d.9) can be written in closed-form.

$\xi = \sqrt{a}\theta$	(2.1)
$\varrho = 2\sqrt{a}x$	(2.2)
$H_y(x,\theta) = \frac{1}{\sqrt{a}}e^{-ax^2}esinh(\xi,\varrho)_1$	(2.3)
$E_z(x,\theta) = -\eta\frac{1}{\sqrt{a}}e^{-ax^2}eicoshi(\xi,\varrho)$	(2.4)

We may verify the above closed-form analytical solution using the function identities.

By (2.2) and (2.4), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = \varrho e^{-ax^2}eicoshi(\xi,\varrho) - e^{-ax^2}2\frac{\partial}{\partial\varrho}eicoshi(\xi,\varrho)$$

Substitute (i.1) and (i.2) into the right side, we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = e^{-ax^2}eisinh(\xi,\varrho)$$

By (i.9), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = e^{-ax^2-\xi^2}\sinh(\varrho\xi)$$

By (2.1) and (2.3), we have

$$\frac{\partial}{\partial\theta}H_y(x,\theta) = e^{-ax^2}\frac{\partial}{\partial\xi}esinh(\xi,\varrho)_1$$

Substitue (i.3) into the right side, we have

$$\frac{\partial}{\partial\theta}H_y(x,\theta) = e^{-ax^2}eisinh(\xi,\varrho)$$

By (i.9), we have

$$\frac{\partial}{\partial\theta}H_y(x,\theta) = e^{-ax^2-\xi^2}\sinh(\varrho\xi)$$

It is thus verified that the solution (2.3) and (2.4) satisfy Maxwell's equation (1d.1).

By (2.2) and (2.3), we have

$$\frac{\partial}{\partial x}H_y(x,\theta) = -e^{-ax^2}\varrho\, esinh(\xi,\varrho)_1 + e^{-ax^2}2\frac{\partial}{\partial\varrho}esinh(\xi,\varrho)_1$$

Substitute (i.4) and (i.5) into the right side, we have

$$\frac{\partial H_y}{\partial x} = e^{-ax^2}\left(4esinhi(\xi,\varrho)_{1,0,0,1} + 2esinhi(\xi,\varrho)_1 - esinhi(\xi,\varrho)_{1,1,0}\right)$$

Substitute (i.6) into the right side, we have

$$\frac{\partial H_y}{\partial x} = e^{-ax^2}\left(-esinhi(\xi,\varrho)_{1,1,1} + 2esinhi(\xi,\varrho)_1\right)$$

By (2.1) and (2.4), we have

$$\frac{\partial}{\partial\theta}E_z(x,\theta) = -\eta e^{-ax^2}\frac{\partial}{\partial\xi}eicoshi(\xi,\varrho)$$

Substitute (i.7) into the right side, we have

$$\frac{\partial}{\partial\theta}E_z(x,\theta) = -\eta e^{-ax^2}\frac{\partial}{\partial\xi}\left(\xi + eisinhi(\xi,\varrho)_{1,1,1} - 2eisinhi(\xi,\varrho)_1\right)$$

Substitute (i.8) into the right side, we have

$$\frac{\partial}{\partial\theta}E_z(x,\theta) = -\eta e^{-ax^2}\left(1 + esinhi(\xi,\varrho)_{1,1,1} - 2esinhi(\xi,\varrho)_1\right)$$

Substitute the result of $\frac{\partial H_y}{\partial x}$ we have reached previously into the right side, we have

$$\frac{\partial}{\partial\theta}E_z(x,\theta) = \eta\frac{\partial H_y}{\partial x} - \eta e^{-ax^2}$$

Substitute (1d.5) into the right side, we have

$$\frac{\partial}{\partial\theta}E_z(x,\theta) = \eta\frac{\partial H_y}{\partial x} - \eta J_z(x,\theta)$$

It is thus verified that the solution (2.3) and (2.4) satisfy Maxwell's equation (1d.2).

Characteristics of Hyperbolic-Exponential Mixtures

Characteristics of hyperbolic-exponential mixtures should be listed in a reference book. Below, I'll list a few of them.

$esinh(0,\varrho)_{m,s,h,d} = 0; m > 0$	(c.1)
$esinh(\xi,0)_{m,s,h,d} = 0$	(c.2)
$eicoshi(0,\varrho) = 0$	(c.3)
$eicoshi(\xi,0) = \int_0^\xi e^{-\tau^2}d\tau$	(c.4)

By (c.1) and (c.2), we have

$$H_y(x,0) = 0$$

$$H_y(0,\theta) = 0$$

By (c.3) and (c.4), we have

$$E_z(x,0) = 0$$

$$E_z(0,\theta) = -\eta\frac{1}{\sqrt{a}}\int_0^{\sqrt{a}\theta} e^{-\tau^2}d\tau$$

We can see that at the origin space point, E_z is an error function (erf, [6]) and its magnitude is inversely proportional to $\sqrt{a}$.

Drawings of Functions

For numerical applications, standard computer libraries should be developed to provide function values for the hyper-exponential functions.

For showing the electromagnetic field (2.3) and (2.4), we need function values of $esinh(\xi,\varrho)_1$ and $eicoshi(\xi,\varrho)$.

Fig.1 shows the function $esinh(\xi,\varrho)_1$. The horizontal axis is for ξ. Each curve is for one value of ϱ. It shows that the larger ϱ the larger value of the function, just like a sinh function. It shows that the larger ξ the larger value of the function, but the effect only lasts for a short period of time.

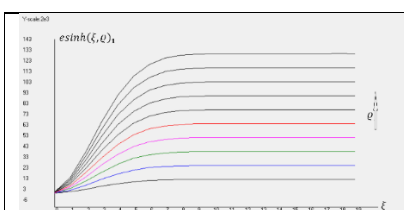

Fig. 1. function $esinh(\xi,\varrho)_1$

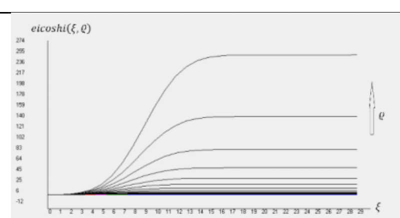

Fig.2. function $eicoshi(\xi,\varrho)$

Fig.2 shows function $eicoshi(\xi,\varrho)$. The horizontal axis is for ξ. Each curve is for one value of ϱ. Comparing with Fig.1., we can see that the two functions look similar.

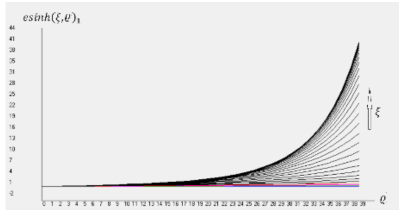

Fig.3. function $esinh(\xi,\varrho)_1$

Fig.3 shows the function $esinh(\xi,\varrho)_1$ from a different perspective. The horizontal axis is for ϱ. Each curve is for one value of ξ.

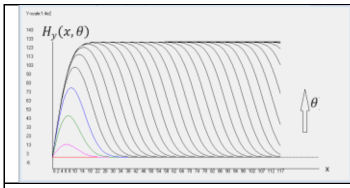

Fig.4. Magnetic field $H_y(x,\theta)$

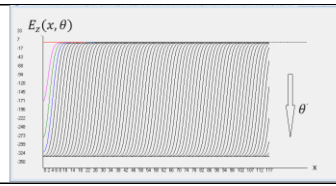

Fig.5. Electric field $E_z(x,\theta)$

Fig.4 shows magnetic field $H_y(x,\theta)$. The horizontal axis is for x. Each curve is for one value of time $\theta = ct$. From the figure, we can see following characteristics of the magnetic field.

$$\lim_{t\to\infty} H_y(x,ct) = a\ constant\ curve\ of\ x$$

Fig.5 shows electric field $E_z(x,\theta)$. The horizontal axis is for x. Each curve is for one value of time $\theta = ct$. From the figure, we can see following characteristics of the electric field.

$$\lim_{t\to\infty} E_z(x,ct) = a\ constant$$

Summary

This paper and [1] show a typical approach of getting closed-form analytical solutions to Maxwell's equations.

1. Apply the given initial values and field sources to the generic solution to Maxwell's equations to get specific solutions in summations of series.
2. A specific solution in summations of series may lead to known math functions, and thus, closed form analytical solution is obtained. Paper [1] shows 3 examples of this situation.
3. A specific solution in summations of series may lead to math functions not reported in literature. We need to study the new math functions. Then, such new functions become known math functions. Thus, a closed-form analytical solution is obtained. This paper shows an example of this situation.

Mixtures of hyperbolic-exponential elements form a new family of functions. One function in this family describes a type of dynamic magnetic field; one function in this family describes a type of dynamic electric field. The fields are invoked by a time-invariant Gaussian source.

This paper presents a few characteristics of hyper-exponential functions. Much research is needed to fully reveal the characteristics of this new family of functions. Standard computer libraries for these new functions should be developed for numeric applications.

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