Dingle's crusade and the twin's paradox

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Abstract Herbert Dingle is a rare figure in history who believes the paradox in the theory of special relativity is real and the theory should be discarded, while most others believe the paradox is an illusion caused by misunderstanding of the theory. For several decades he wrote many papers and books, trying to convince the others, in vain. I surveyed a debate participated by prominent scientists published in Nature between the 50s to 70s. One is neutral, one supports Dingle, and all others are against him by denying the existence of the paradox. I then surveyed 4 recent papers from 2003 to 2022, each of which provides its way to dismiss the paradox. The need of those recent papers is proof that Dingle did not lose his battle even though he was vastly outnumbered. Dingle used the Lorentz transformation to form a question and requested the science community to answer his question. He only got one unsatisfactory answer. I worked on his question and revealed a hidden nonreciprocal of the Lorentz transformation, thus answered Dingle's question and confirmed his claim that Lorentz is correct and Einstein is not. A paradox cannot be resolved but its debate can. Once a juxtaposition of views by Lorentz and Einstein's theories is made, there is no room for debate.

Keyword: Special Relativity, Paradox, Dingle, Lorentz transformation, principle of relativity

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Introduction

On June 8, 1973, Nature and Prof. Ziman apologized to Prof. Dingle:

> *there was no basis for attacking Professor Dingle's personal integrity and accordingly both Nature and Professor Ziman offer their sincere apologies to Professor Dingle" [1].*

Before the above apologies, on March 16 1973, H. L. Armstrong criticized the ways Dingle was treated:

> *When Dingle … and his opponents seem to agree on little except the fact that they are opponents,…*
> *In the first place, Dingle was once recognized to be an authority in this matter. If he has now come to different conclusions, either there are good reasons for these second thoughts, or else they are to be put down as a foible of old age. But the writing is certainly not that of a senile man.*
> *In the secod place, it would appear that none of his critics has faced Dingle's points that (a) he was discussing physics, not mathematics; and (b) that all of the alleged experimental verifications involve circular arguments in their interpretation.*
> *In the third place, it is should turn out that there is some truth in Dingle's views, the way in which they seem to have been brushed aside will not be likely to make science stand any higher in the public esteem. [2]*

The unfairness to Dingle continued after the apologies. On July 6 1973 J. H. Fremlin twisted Armstrong's support to Dingle:

> *Armstrong has suggested that if Dingle's work did not deserve serious thought and discussion it might have been better to ignore it completely.*
> *…his next two or three letters were indeed neglected. [3]*

Professor Herbert Dingle was once the president of the Royal Astronomical Society. But now he was in a bitter battle and vastly outnumbered. Prof. Ziman said that Dingle was one to one thousand outnumbered:

> *The fact that he, one man in a thousand, thinks differently is scarcely a major flaw in the scientific consensus. [4]*

The paradox and Dingle's question

The paradox is also referred to as the space-traveler paradox, the clock paradox, etc.

In response to McCrea's criticism [5], Dingle said:

> *Prof. McCrea wanders widely from the point… [6].*

So, what is the point of the debate on the presumed paradox?

The paradox was spotted when Einstein published his theory which became a rival to Lorentz's theory. So, the point of the debate should be that what in Einstein's theory, which is not in Lorentz's theory, causes the presumed paradox?

Dingle pointed out that the paradox was caused by the use of the principle of relativity:

> *…whereas the error is in the pre-mathematical ideas. The fundamental principle of relativity, in its application to the case of two bodies… [7].*

The principle of relativity is to say that the world is always symmetric. The justification of this symmetric view is provided by the first paragraph of [8]. I have pin-pointed errors in that paragraph. That is, Einstein's justification of a symmetric world does not hold. But let's focus on the debate.

The consequence of applying the principle of relativity is that the ether used by Lorentz cannot be used. The last sentence of section 6 of Einstein's 1905 paper says:

> *questions as to the "seat" of electrodynamic electromotive forces (unipolar machines) now have no point. [8]*

The word "seat" refers to ether. Lorentz also used "seat" to refer to ether. On other occasions Einstein may admit that ether exists. It does not matter whether Einstein admits the existence of ether or not. What matters is that the principle of relativity does not allow meaningful existence of ether, as Dingle points out:

> *Lorentz's thoery is imposible without an ether; Einstein's (because of its relativity postulate) is impossible with one. [9]*

Let me use math language to show Dingle's above statements.

Einstein's theory [8]: Rest coordinates: $(x, y, z, t); x, y, z, t \in R$ Moving coordinates: $(\xi, \eta, \zeta, \tau);\ \xi, \eta, \zeta, \tau\ \in R$	(E.1)
Lorentz's theory [10]: Rest ether: $(x, y, z, t); x, y, z, t \in R$ Dragged ether: $(\xi, \eta, \zeta, \tau);\ \xi, \eta, \zeta \in \{adjacent\ to\ the\ Earth\}$	(E.2)

Lorentz's ether-dragging is an explanation to a phenomenon, but Einstein's abstract movement of coordinates is an imagination which is hard to verify.

For those readers who are not familiar with electromagnetic field, let me use an airplane and sound waves to explain Lorentz's use of dragging-ether for waves:

$$(x, y, z, t): air\ on\ the\ surface\ of\ the\ Earth$$
$$(\xi, \eta, \zeta, \tau): cabin\ of\ the\ airplane$$
$$\xi, \eta, \zeta \in \{cabin\ of\ the\ airplane\}$$

For Lorentz's theory, length-contraction only occurs for {adjacent to the Earth}, or the airplane in the above example. For Einstein's theory, it is a coordinate-contraction applied to the whole universe.

Einstein's principle of relativity cannot be just for electrodynamics, it has to work for kinematics. See "I. KINEMATICAL PART" in [8].

Dingle says:

> *The essence of the theory, however, lies in its kinematical requirements, and if these are falsified the whole theory collapses.[11]*

What does it mean by "*its kinematical requirements*"? In our above example, from wave to kinematics is to treat the air-filled airplane as a solid rod. (E.1) becomes (E.3), and (E.2) becomes (E.4).

Einstein's theory for the moving rod: Rest coordinates: $(x, y, z, t); -\infty < x, y, z, t < +\infty$ Moving coordinates: $(\xi, \eta, \zeta, \tau);\ -\infty < \xi, \eta, \zeta, \tau < +\infty$ $rod: 0 \le \xi \le l, 0 \le \eta \le 0, 0 \le \zeta \le 0$ $l: rod\ length$	(E.3)
Lorentz's theory for the moving rod: Rest ether: $(x, y, z, t); -\infty < x, y, z, t < +\infty$ Moving rod: $(\xi, \eta, \zeta, \tau);\ 0 \le \xi \le l, 0 \le \eta \le 0, 0 \le \zeta \le 0$	(E.4)

Use a space-traveler, or a moving clock, to replace the rod, by setting the rod length to 0. We have

Einstein's symmetric theory for space-traveler: $(x, y, z, t);\ -\infty < x, y, z, t < +\infty$ $(\xi, \eta, \zeta, \tau);\ -\infty < \xi, \eta, \zeta, \tau < +\infty$ $clock\ A\ at\ x = 0$ $clock\ B\ at\ \xi = 0$	(E.5)
Lorentz's asymmetric theory for space-traveler: $(x, y, z, t);\ -\infty < x, y, z, t < +\infty$ $(\xi, \eta, \zeta, \tau);\ 0 \le \xi \le 0, 0 \le \eta \le 0, 0 \le \zeta \le 0$ $clock\ A\ at\ x = 0$	(E.6)

$clock\ B\ at\ \xi = 0$	

(E.5) and (E.6) exhibit the fundamental difference between Lorentz and Einstein's theories treating the space-traveler. Now, let's see how the difference affects the explanations of Lorentz transformation.

The Lorentz transformation is given below: ([8], [11], [12])

$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	(E.7)
$\tau = \beta\left(t - \frac{v}{c^2}x\right)$	(E.8)
$\xi = \beta(x - vt)$	(E.9)
$\eta = y$ $\zeta = z$	(E.10)
$t = \beta\left(\tau + \frac{v}{c^2}\xi\right)$	(E.11)
$x = \beta(\xi + v\tau)$	(E.12)

Let's see how Dingle shows the paradox. I quote Dingle's words below:

> *Here is the passage from Einstein's paper:* … $x = vt$
>
> $$\tau = t\sqrt{1-\frac{v^2}{c^2}}$$
>
> *Whence it follows that the time marked by the clock (viewed in the stationary system) is slow …*
>
> *And here is the parallel passage, leading to the opposite conclusion:* … $\xi = -v\tau$
>
> $$t = \tau\sqrt{1-\frac{v^2}{c^2}}$$
>
> *Whence it follows that the time marked by the clock (viewed in the stationary system) is fast …*
>
> …
>
> *To avoid this outcome it must be explained …* ***why*** *… the former result must be accepted as true while the latter must be rejected as false.[11]*

Let me answer Dingle's question. The answer to Dingle's question is in his use of

$$\xi = -v\tau$$

By Einstein's theory (E.5), the $\xi = -v\tau$ is valid and thus the paradox occurs as Dingle described.

By Lorentz's theory (E.6), the $\xi = -v\tau$ is invalid and thus "*the latter must be rejected as false*". The paradox does not occur.

By revealing a hidden nonreciprocal of Lorentz transformation, I answered Dingle's question and dismissed the space-traveler paradox: Lorentz is correct and Einstein is not, just as Dingle said:

> *the disproof of Einstein's theory … leaves Lorentz's intact* [9].

The debate

Dingle raised the above question and said "*it must be explained*". It seems I am the only one who responds to Dingle's request, works on his question and gives him an answer, after so many years. No one else is bothered to work on Dingle's question and find an answer. At that time only one addressed Dingle's question:

> *The correspondent in question – the only one, in my view, who has really faced the problem – admits that no such reason can be found, and concludes that the conception of an objective rate-ratio of two clocks is impermissible. I cannot agree, … [13]*

Dingle got a debate instead of an answer.

The debate was like a friendly contest among Dingle's opponents to see who could propose more possible refutations to Dingle. McCrea totally ignores Dingle's question and guesses what could be the possible errors Dingle made:

> *If Dingle obtains two different answers it must be because (a) … or (b) … or (c) … [14]*

Ziman lists reasons to dismiss Dingle from (a) to (h). [4]

I carefully reviewed the arguments of Dingle's opponents and found none of them was substantial.

S. F. Singer [15], W. Cochran [16], Farley, F. J. M., Bailey, J., and Picasso, E. [17], W. Whippman [18], and W. M. McCrea [5], [14], [19] all misunderstand Dingle. They all think that symmetry is what Dingle wants. They do not know that symmetry is the target of Dingle's criticisms. S. F. Singer says:

> *A basic problem, as I see it, is whether there exists symmetry between observer A who stays on the Earth and observer B who travels out and then returns. This symmetry does not exist (contrary to Prof. Dingle's statement) since B experiences forces while A does not, so that there is no a priori reason to expect them to be of the same age when they meet again.[15]*

Based on such a misunderstanding, they all think that it is Dingle who insists on keeping the principle of relativity.

McCrea, being a leading opponent of Dingle, directly drops the principle of relativity when dismissing the paradox:

> *Dingle is wrong…*

This distinction between the two observers is an absolute distinction. That it is so is shown by the simple fact that one of the men has to use an engine and the other not. This common-sense distinction cannot, of course, be removed by any 'principle of relativity'. Even if the traveler chooses to regard himself as at rest and the Earth as departing and returning, it is still he and not the man on the Earth who has to use the engine. Therefore, no mere appeal to a principle of relativity enables us to say whether or not the two men agree as to the duration of the journey. [5]

Ziman drags Einstein into the debate:

The clock paradox, and its resolution, was discussed in detail by Einstein himself, [4]

Ziman does not give a reference for his statement. According to Peter Pesic (2003 [20]), Einstein used two reasons to dismiss the paradox: acceleration and simultaneity. These two reasons are within the scope covered in this survey. Many believe Einstein dismissed the paradox in his 1905 paper by the following words:

From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $\frac{1}{2}tv^2/c^2$ (up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B.

It is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide. [8]

Luis Dias Ferreira (2021 [21]) believes the "*peculiar consequence*" referred to by Einstein is the paradox everyone is talking about. Actually it is not. What Einstein refers to is the phenomenon of "time-dilation" Lorentz conjectured. Einstein was introducing an unresolvable contradiction into his theory: a paradox. This paradox was spotted 6 years later by Paul Langevin. Ferreira says:

Translated in a more populare version, formulated by Paul Langevin in 1911, these statements gave rise to the "twin's paradox" [21].

The above understanding of Langevin is incorrect. Einstein's previous statements do not give rise to the "twin's paradox". Paul Langevin did not translate Einstein's statements. Paul Langevin discovered the paradox.

Looking at Einstein's statements, we see an asymmetric narrative, only A to B is described, not B to A. There is not a contradiction in those statements to raise a paradox. It conforms to Lorentz's theory: who is moving who's time is slower.

To understand Langevin's discovery, we need to go back to the very first sentence of Einstein's 1905 paper, where Einstein sets the goal for his theory:

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. [8]

The rest of his paper is to develop a theory of symmetry, represented by the principle of relativity, to explain the asymmetric time-dilation discovered by Lorentz and others.

Use symmetry to explain asymmetry, that is the paradox Langevin discovered, not from Einstein's asymmetric statements, but from the principle of relativity. Einstein did not think he had an obligation to follow the symmetric rule of the world he laid out, as we see him kept using Lorentz's asymmetric theory to explain time-dilation:

Thence we conclude that a balance-clock at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions. [8]

The time-dilation of an "equator clock" can only be explained by Lorentz's asymmetric theory, not by Einstein's principle of relativity, there is not a symmetric relative movement in this "equator clock" example.

McCrea, Einstein and others are correct: dropping the principle of relativity is the correct way to dismiss the paradox, but it is at an unaffordable cost of returning to Lorentz's theory and losing the presumed value of Einstein's theory. Therefore, many other debaters do not use a strategy of dropping the principle of relativity explicitly as McCrea does, but mix the two theories implicitly. Dingle protested the strategy of switching between Lorentz and Einstein's theories back and forth in the debate:

"the special theory of relativity" became an incompatible mixture of Lorentz's and Einstein's – a fact that preserved the theory from disproof, since any attack on the relativity aspect could be met by an appeal to Lorentz's non-relativistic ideas, and criticisms of those could be disposed of by a reversion to relativity. [9]

To keep the principle of relativity alive they accept that a symmetry of relative movement exists in the rest clock and the moving clock. But, they claim that asymmetries are inevitable in the process of making the two clocks meet. Various schemes are proposed. About these schemes, H. L. Armstrong comments:

his opponents seem to agree on little except the fact that they are opponents[2]

Dingle is also overwhelmed by his opponents' various answers:

It would be profitless to deal separately with the latest "answers" to my question; their diversity tells its own tale, and the writers may see their misjudgements corrected in my book. I have through the years put the question in various forms to meet successive obscurations, but in vain. Though wholly understandable to any ordinary layman it is at once submerged in such mathematical and metaphysical mysticism that normal readers either conclude that its simplicity must be deceptive and I deluded, or express to me their astonishment at the denseness of the "authorities". [22]

Prof. Ziman gives the readers what Dingle called *the denseness of the "authorities"*:

…Pauli's admirable treatise on Relativity Theory (originally published in 1921) which explains the position very clearly, and Introducing Relativity by W. G. V. Rosser, published in 1967, which devotes about 50 pages to this very topic, with particular reference to Dingle's arguments. … Time and the Space Traveller, by L. Marder (1971), which is entirely concerned with the theoretical analysis and experimental confirmation of relativistic time phenomena. Books such as these have been favourably reviewed, recommended as textbooks, and made the basis for undergraduate lecture in every university in the country. [4]

Then, Prof. Ziman gives explanations for dismissing the paradox by acceleration, among his other possible ways of resolutions:

(c) To make this comparison, the clocks must be accelerated (or decelerated) … (d) … To distinguish the slower working clock … one must use the theory of general relativity, which takes account of accelerations. … [4]

G. F. R. Ellis gives several ideas about using acceleration and gravity to dismiss the paradox, saying his reasons may not as fatal to Dingle as Ziman's, but it still contributes to dismiss Dingle in a way:

In fact, while special relativity does not deal adequately with gravity, it does quite adequately cope with accelerated motion. In special relativity, just as in general relativity, the answer to Professor Dingle's "question" is: the fastest working clock between any two events is one that travels between them by

free fall. Any other clock traveling between these events necessarily experiences inertial forces, which a physicist moving with the clock might interpret as being due to a (uniform) gravitational field; a physicist moving with the "fastest" clock would experience no such forces (he would be an "inertial observer").

This completely answers Professor Dingle's "question". It leaves unsettled the further question as to what it is that prescribes this particular structure for space-time. In special relativity, this structure is simply taken as given a priori; while this may not be thought to be a completely satisfctory answer (and general relativity gives a better one), it is certainly at least a logically consistent answer. [23]

J. H. Fremlin sent his calculations of using acceleration to Dingle. Fremlin said Dingle was impatient with him:

I continued a desultory private correspondence for some time. I stopped when he answered a particularly detailed analysis which I gave, …, showing where I believed his error to lie, by saying simply that it was not his business to find the error in my analysis. …

His basic error has been to suppose … and at no time therefore has he discussed quantitatively the results of this acceleration. [3]

Dingle does not think acceleration can be used to dismiss the paradox:

…accelerations…The only escape is by supposing, as some do, that the initial acceleration causes a change of rate which persists when the motion becomes uniform. This is arbitrary, and to justify McCrea's formula the change would astonishingly have to be exactly equal to the Lorentz value…it would stutify modern astronomy,… [24]

Some discussions are irrelevant to the paradox.

- Special relativity is mathematically correct and thus must be true. McCrea [5], [14], Max Born [9], Ziman [4], J. H. Fremlin [3].
  Dingle [6], [9], [24] pointed out that math correctness is irrelevant.
- Special relativity is verified experimentally and thus should not be challenged. McCrea [5], W. Cochran [16], F. J. M. Farley, J. Bailey, and E. Picasso [17], John Ziman [4].
  L. Essen [25], H. L. Armstrong [2] and Dingle [9], [11] pointed out that the experiments cannot be used to verify special relativity.
- You doubt Special Relativity because you do not understand it.
  W. Stewart Brown provided a deduction of Lorentz transformation to help:
  Recent discussion about the special theory of relativity suggests the need for clarification, which this note is intended to provide.[12]
  J. L. Synge advice the readers to abandon Newtonian physics in order to understand Einstein's theory:
  …(a) the concepts used in the special theory of relativity as ordinarily understood, and (b) the concept of clocks that run regularly, as understood by Professor Dingle. … Because (b), …is equivalent to Newton's concept of absolute time, and because relativeistic physics appears to me to represent nature more closely than Newtonian physics does, I cast my vote for the abandonment of (b) and the retention of (a).[26]
- Anderson Nettleship uses biological time to explain the asymmetry in space-traveler [27]. He recognizes that Special Relativity is a theory of observation. But it seems Mr. Nettleship does not notice that the time-dilations shown in the experiments are mechanical.

Dismiss the paradox by rules

G. E. Stedman [28] points out that a third inertial frame or clock, clock C, is needed to form the events. Stedman touches the essence of the paradox: the formulation of the paradox must be symmetric.

R. Jacob [29] perfectly formulates the paradox with a third clock, clock C, as Stedman does. But like McCrea, Ziman and all Dingle's opponents, he does not distinguish the differences between Lorentz and Einstein's theories.

One thing agreed upon by both sides is that they all said that their treatments of paradox can be understood by an ordinary layman and thus demanded the other side to accept immediately, despite the fact that prominent scientists debated on it in Nature for 35 years.
I treat the space-traveler paradox by (E.5) and (E.6), which are deduced involving concepts of electrodynamics/ether, wave/kinematics, and reciprocal/nonreciprocal of Lorentz transformation. I am not sure such deductions can be fully understood by laymen.

On the other hand, I agree that the paradox can be understood and treated by laymen, if unnecessary factors are stripped. The basic phenomenon is simple: the time of a moving clock runs slower than a resting clock.
Let's set two rules, layman-rules, for the debate, just like setting rules for a fair game.

1. The formulation of the paradox must be symmetric.
   The vivid narrative of the "space traveler" breaks the rule and should not be used. To get the vividness unnecessary asymmetries are introduced. Stedman and Jacob's symmetric formulations are good.
2. The juxtaposition of conclusions drawn by both Lorentz and Einstein's theories must be made.
   Lorentz's theory: the movement is absolute. If I run to you then I am moving and you are not. If you run to me then you are moving and I am not. If we both run then we are moving and the Earth is not.
   Einstein's theory: there is not an absolute movement. If I run to you, I take it as you move to me, and you take it as I move to you.

The first rule avoids giving a layman overwhelming physics and mathematics. The second rule makes it easy for a layman to tell when the referee unfairly gives the score to Einstein while the true winner is Lorentz, Dingle's opponents are such unfair referees.

Let's apply the layman-rules to Jacob's symmetric formulation:

1. Clock C at the center, clock A at the left side of clock C in a distance, clock B at the right side of clock C in the same distance.
2. Clock A moves to right and clock B moves to left
3. Clocks A and B meet at the center

4. Draw conclusions by the two theories:
   Lorentz's theory: the times of clock A and clock B are the same and lag behind the time of clock C.
   Einstein's theory: the time of clock A is ahead of the time of clock B because clock B is moving, and the time of clock B is ahead of the time of clock A because clock A is moving, this is a paradox. In a similar way, Clock A is faster than clock C and clock C is faster than clock A, this is another paradox. Clock B is faster than clock C and clock C is faster than clock B, this is yet another paradox.

The above treatment of the paradox can be understood by laymen.

Apart from the acceleration, the relativistic simultaneity is another major reason for dismissing the paradox (W. H. McCrea 1967 [14], W. Stewart Brown 1968 [12]). Recently it has become a more popular reason over the acceleration, Peter Pesic (2003 [20]), G. Alencar (2022, [30]). When a unique treatment is discovered to dismiss the paradox, what really works is not the newly discovered treatment, but the inevitable asymmetry in the process.

If only symmetric processes are used then only symmetric outcomes will be produced, no matter what treatments are used, i.e., acceleration, relativistic simultaneity, or some new treatments invented by future generations of physicists if they will still be taught as students have been. Possible symmetric outcomes include:

1. clock A is slower than clock B and clock B is slower than clock A
2. clock A is faster than clock B and clock B is faster than clock A
3. clock A and clock B are the same

The first 2 symmetric outcomes are paradoxes.
The 3rd symmetric outcome is what Luis Dias Ferreira (2021 [21]) produces:

> *The final conclusion here is that, despite the reality of time dilation, it is not true that the "traveling twin" returns younger than the one who "stayed at home": they meet again at exactly the same age. [21]*

Ferreira believes that he digs out a new treasure from Special Relativity by overcoming the difficulties of common sense:

> *…we discover that Special Relativity is yet a wonderful box full of surprises … our difficulties arise from approaches based, after all, on 'common sense'. [21]*

By Ferreira's deduction, the time-dilation is undetectable. This is inconsistent with the experiment results and presumably predicted by Special Relativity, and Einstein believes that the traveling twin is younger. Therefore, Ferreira actually refutes Einstein and Special Relativity.

In 2005, Claus Lämmerzahl published a 32-pages paper providing his insights into Special Relativity and Lorentz Invariant. In his paper he briefly talks about the twin paradox and offers his resolution. He first rejects the idea of acceleration:

> *Here we emphasize thet contrary to what is often stated, it is not necessary to have an acceleration of one of the twins or of the moving clock. This is important since sometimes the effect is said to be caused by the acceleration. Since accelerations are beyond the Poincare transformation, this would mean that the effect is not due to the LT and, thus, no genuine SR effect. [31]: page 83*

He then proposes an asymmetric formulation by a rest clock and a moving clock. He uses a third clock to carry the time information of the moving clock back to the rest clock.
Dingle points out that in such a case the Lorentz transformations do not commute [7]. The relativistic synchronization of the third clock and the rest clock must be re-examined. Let's not go into such math details and assume Lämmerzahl's scheme works: the third clock can carry the message back.
Let's apply the layman-rules to the formulation of Lämmerzahl: modify his formulation to be symmetric and then make a juxtaposition of Lorentz and Einstein's conclusions.

1. Clock A, clock B and clock R all at the same place
2. Clock A starts moving right, clock B starts moving left.
3. At a predetermined time (how? It does not matter), Clock C moves right from clock B, carrying clock B's information, say, the local time of clock B; clock D moves to left from clock A, carrying clock A's information, say, the local time of clock A.
   Due to symmetry, we cannot use any excuses to say clock C and clock D carry different information, without arbitrarily introducing asymmetries.
4. Clock C and clock D meet at clock R
5. Draw conclusions by the two theories:
   By **Lorentz's theory**: clock C and clock D give the same time which lags behind the time of clock R.
   By **Einstein's theory**: clock C lags behind clock D and clock D lags behind clock C, this is a paradox.

Once a symmetric narrative is used, how physics and mathematics work is irrelevant.

Dismiss the paradox by logics

Suppose Lämmerzahl and Jacob's examples are actually performed then it is possible to see the results predicted by Lorentz, but there is no way to show something consistent with Einstein's theory because they are physically impossible.
An answer to a paradox cannot be verified or disproved by experiment, as Dingle says:

> *…what I have advanced is not a theory which, in the traditional scientific manner, can be left to be justified or condemned by experiemnt. [9]*

Let me use the famous Barber's Paradox to explain what Dingle means:

> Once upon a time there was a barber who shaves all those, and those only, who do not shave themselves.
> Question: does the barber shave himself?

If someone gives us an answer, say, the barber shaves himself, or the barber does not shave himself, then we cannot use "experiments" to verify or disprove his answer. We can only use logic to disprove his answer. That is, use pencil and paper to disprove the answer, as Dingle did and mocked by his opponents: we are using lots of experiments to prove Special Relativity, and you are using pencil and paper to disprove it. To dismiss the paradox we must change the narrative: replace "Once upon a time there was a barber …" with "There is not a barber …".

The paradox in Special Relativity must also be dismissed in exactly the same way.

> Once upon a time there was a theory which can explain all asymmetric phenomena by a reason of symmetry.
> Asymmetric phenomenon: the time of a moving clock runs slower than that of a rest clock.
> Symmetric explanation: the time-dilation is caused by the relative movement.
> Question: For two clocks making a relative movement, which clock's time is slower?

Experiments verified that the time of moving clock runs slower, but the experiments cannot answer the question and dismiss the paradox. To dismiss the paradox, we must change the narrative to "There is not a theory ….". That is exactly what has been done time and time again. From Einstein (1905 [8]) to G. Alencar (2022, [30]), in more than 100 years, the paradox has indeed been dismissed over and over again, all by dropping the principle of relativity, implicitly, except McCrea who dropped it explicitly.

Conclusion

I did not mention many other papers Dingle published, especially his book [32]. Even for the debate, I did not mention two other references he published in Nature [33], [34]. What I presented already proved the following conclusion.

The paradox cannot be dismissed while keeping the principle of relativity.
Misunderstanding of the paradox is due to an ignorance that Lorentz used his transformation nonreciprocal and Einstein used it reciprocally.

The paradox is simply a logical error, not a demonstration of mysterious powers which, proudly claimed by supporters, cannot be understood by common sense. Do not waste your talent trying to harness the ghostly powers.

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