Electrodynamics of a Moving Gaussian Source

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Abstract Based on closed-form analytical solutions to Maxwell's equations for a moving Gaussian source, the dynamics of electromagnetic fields are investigated via large amounts of precise numeric calculations and some analytical deductions. Propagations of electromagnetic fields exhibit behaviors of water waves and sound waves. Formulas for field propagation speeds are derived for moving sources. For a source moving below c/2, electromagnetic fields exhibit a behavior of "pseudo inertia" which is between inertial and non-inertia. Speed c acts as a "speed barrier" for a source. A source may break the speed barrier with a limited amount of energy.

Contents

Introduction

In [1], I give formulas for group speeds of electric field and magnetic field generated by a stationary Gaussian source. In [2], I give closed form analytical solutions to Maxwell's equations in response to moving sources. Based on formulas given by [2], we may investigate characteristics of the electromagnetic fields, analytically and numerically.

In section “Propagation of electromagnetic fields”, fields are calculated and displayed to show propagations affected by moving of the source, using formulas given in [2].

In section “Propagation speeds”, formulas of propagation speeds for the magnetic field and electric field are derived. Propagation speeds are calculated and displayed for sources moving at various speeds, including lower and higher than the standard light speed, and very near the standard light speed. The calculations show that in the instantaneous periods, the speeds are similar. The consequence is that at the field source, the fields look inertial; away from the field source, the inertial behavior fades out; in the stable period, the fields behave non-inertial, and the propagation speeds of the fields become the standard light speed minus the source moving speed, as Michelson-Morley experiment expected. This “pseudo inertia” behavior partially contributes to the result of the Michelson-Morley experiment being smaller than expected.

In section “Lux-barrier”, analytical solution to Maxwell’s equations for a Gaussian source moving at the standard light speed is given and proved. Fields are calculated using various source moving speeds, including lower and higher than the standard light speed. These calculations tell us that the maximum field magnitude changes with the source moving speed. Analytical formulas tell us that at the standard light speed, the maximum field magnitude is infinity. Away from the standard light speed, the maximum field magnitude is not infinity. The further away from the standard light speed, the smaller the maximum field magnitude. Thus, the standard light speed acts as a barrier for increasing the source moving speed. Given acceleration of the source moving, we may calculate the extra electromagnetic energy needed for breaking through the speed barrier.

First, let me quote the results from [2].

1D Maxwell’s equations along the x-axis are

$\frac{\partial H_y(x,\theta)}{\partial\theta}=\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x}$	(1d.1)
$\frac{\partial E_z(x,\theta)}{\partial\theta}=\eta\frac{\partial H_y(x,\theta)}{\partial x}-\eta J_z(x,\theta)$	(1d.2)

Where

$$\theta = ct$$

$$c=\frac{1}{\sqrt{\varepsilon\mu}}$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}}$$

The source is

$J_z(x,\theta)=e^{-a(x-v\theta)^2}$	(1.1)

where v is the moving speed of the source. v is a constant.

The initial values are

$H_y(x,0)=0$	(1.2)
$E_z(x,0)=0$	(1.3)

The solution is

$\xi=\sqrt{a}\theta$	(1.4)
$\varrho=2\sqrt{a}x$	(1.5)
$H_y(x,\theta)=\frac{1}{\sqrt{a}}\frac{e^{-ax^2}}{1-v^2}(esinh(\xi,\varrho)_1-esinh(v\xi,\varrho)_1+v\ eicoshi(\xi,\varrho)-eicoshi(v\xi,\varrho))$	(1.6)
$E_z(x,\theta)=\frac{1}{\sqrt{a}}\frac{\eta e^{-ax^2}}{1-v^2}(-eicoshi(\xi,\varrho)+v\ eicoshi(v\xi,\varrho)-v\ esinh(\xi,\varrho)_1+v\ esinh(v\xi,\varrho)_1)$	(1.7)

where $esinh(\xi,\varrho)_1$ and $eicoshi(\xi,\varrho)$ are hyper-exponential functions, see [3].

Propagation of electromagnetic fields

Effects of source moving

I’ll make drawings using (1.6) and (1.7) to give us a feeling of how the electromagnetic field propagates in space over time.

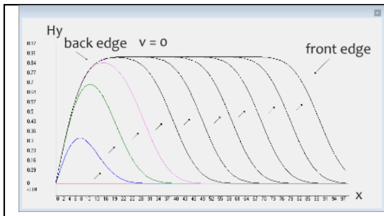


Fig.1. propagation of Hy, stationary source

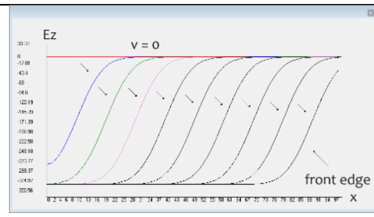


Fig.2. propagation of Ez, stationary source

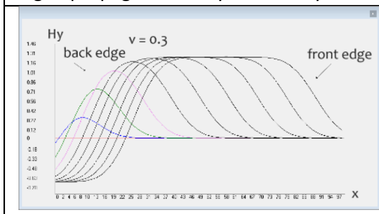


Fig.3. propagation of Hy, moving source

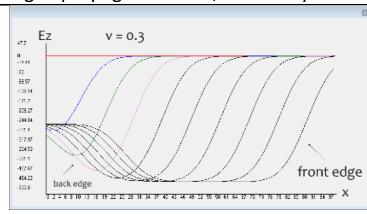


Fig.4. propagation of Ez, moving source

Each curve is a snapshot of the field at one time. The next curve on the right side of each curve shows the propagation of the field during the time interval between the two curves. The propagations are visually demonstrated by the moving of the front edges; I use small arrows to mark the directions of the moving of the front edges.

Comparing Fig.1 with Fig.3, the moving of the source alters the moving of the back edge.

Fig.2 shows that for a stationary source, the electric field does not have a moving back edge.

Fig.4 shows that the moving of the source generates a moving back edge for the electric field.

The same time interval is used in these drawings, so, the closer the two curves the slower the speed of the propagation. The dense curves of the back edges show a slow moving of the source. The sparse curves of the front edges show a fast propagation of the fields. The moving speed of the source is 0.3c. The drawings show that the interval of the dense curves is about one third of the interval of the sparse curves.

Water wave like field propagations

Calculations of the solutions to the Maxwell's equations show that the moving of the field source acts like a force pushing the fields forwards; since the speed of the front edge is not affected by this pushing, the effect is that the fields are squeezed and the fields go up, just like pushing the water and the water rise up. Thus, the field propagations look like water wave propagations, as shown in the following figures.

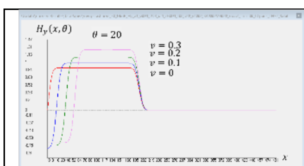


Fig.5. $H_y(x,\theta)$ $\theta = 20$

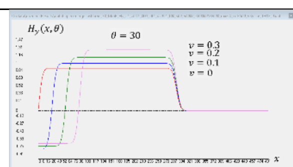


Fig.6. $H_y(x,\theta)$ $\theta = 30$

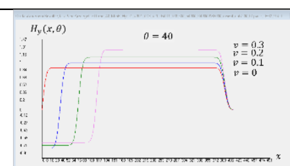


Fig.7. $H_y(x,\theta)$ $\theta = 40$

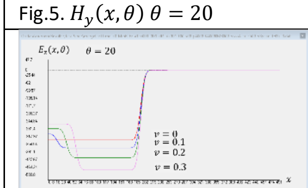


Fig.8. $E_z(x,\theta)$ $\theta = 20$

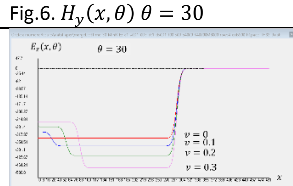


Fig.9. $E_z(x,\theta)$ $\theta = 30$

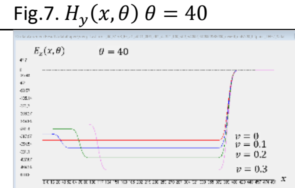


Fig.10. $E_z(x,\theta)$ $\theta = 40$

In these figures, we see following characteristics of the electromagnetic fields.

- The vertical gaps near x=0 show speed differences of the field source.
- The vertical gaps at the front edges show the speed differences of the field propagations. The gaps are very narrow, showing that the moving of source has very little effect on the speeds of the field propagations.
- The horizontal gaps show the effects of the source speeds on the field magnitudes. The faster the source moves, the higher the field magnitudes, as if the field is squeezed and the field goes up. This behavior shows that the electromagnetic field cannot be compressed, just like water.
- The horizontal gaps grow with the speed of the source. Later we will see that this behavior leads to a speed barrier.

Edge movements

From Fig.1 to Fig.4 we can see that the back edges occur at the times when the magnitudes of the fields start to increase; the front edges occur at the times when the magnitudes of the fields start to drop. That is, the edges are at the time when the fields gain or lose their accelerations in space. Let's show field derivatives with respect to space.

Using identities in [3], we have

$\frac{\partial H_y(x,\theta)}{\partial x} = \frac{1}{1-v^2}\left(-(\cosh(\varrho\xi) + v\ \sinh(\varrho\xi))e^{-\left(\frac{1}{4}\varrho^2+\xi^2\right)} + e^{-\left(v\xi-\frac{1}{2}\varrho\right)^2}\right)$	(2.1)
$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = \frac{1}{1-v^2}\left((\sinh(\varrho\xi) +\ v\ \cosh(\varrho\xi))e^{-\left(\frac{1}{4}\varrho^2+\xi^2\right)} - ve^{-\left(v\xi-\frac{1}{2}\varrho\right)^2}\right)$	(2.2)

Let's make drawings using (2.1) and (2.2).

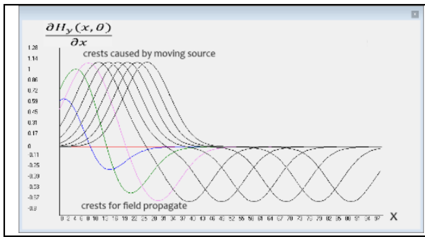


Fig.11. propagation of $\partial H_y(x,\theta)/\partial x$

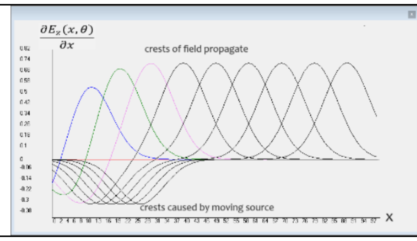


Fig.12. propagation of $\partial E_z(x,\theta)/\partial x$

The crests in these figures show the space and the time where/when the front edges and back edges begin.

So, the moving speed of the edges are the moving speeds of crests for field derivatives with respect to space:

$$\frac{\partial^2 F(x,t)}{\partial x^2} = 0$$

Fig.11 tells us that for the magnetic fields the maximum values are for the moving of the source, the minimum values are for the propagation of the field.

$back\ edge: \frac{\partial^2 H_y(x,\theta)}{\partial x^2} = 0; \frac{\partial^3 H_y(x,\theta)}{\partial x^3} < 0$	(2.3)
$front\ edge: \frac{\partial^2 H_y(x,\theta)}{\partial x^2} = 0; \frac{\partial^3 H_y(x,\theta)}{\partial x^3} > 0$	(2.4)

Fig.12 tells us that for the electric fields the minimum values are for the moving of the source, the maximum values are for the propagation of the field.

$back\ edge: \frac{\partial^2 E_z(x,\theta)}{\partial x^2} = 0; \frac{\partial^3 E_z(x,\theta)}{\partial x^3} > 0$	(2.5)
$front\ edge: \frac{\partial^2 E_z(x,\theta)}{\partial x^2} = 0; \frac{\partial^3 E_z(x,\theta)}{\partial x^3} < 0$	(2.6)

Propagation Speeds

Observation from the moving frame

In this section, the fields are observed from the moving frame. I'll use Galilean transformation to do the calculations.

Denote the moving frame by

$$(x_m, \theta_m)$$

Denote the states by

$$H_{my}(x_m, \theta_m)$$

$$E_{mz}(x_m, \theta_m)$$

Galilean transformation is given by

$\theta_m = \theta$	(3.1)
$x_m = x - v\theta$	(3.2)

$H_{my}(x_m,\theta_m) = H_y(x,\theta)$	(3.3)
$E_{mz}(x_m,\theta_m) = E_z(x,\theta)$	(3.4)

Apply the above transformation to (1.6) and (1.7), we have

$\xi_m = \sqrt{a}\theta_m$	(3.5)
$\varrho_m = 2\sqrt{a}(x_m + v\theta_m)$	(3.6)
$H_{my}(x_m,\theta_m) = \frac{1}{1-v^2}\frac{e^{-a(x_m+v\theta_m)^2}}{\sqrt{a}}(esinh(\xi_m,\varrho_m)_1 - esinh(v\xi_m,\varrho_m)_1 + v\ eicoshi(\xi_m,\varrho_m) - eicoshi(v\xi_m,\varrho_m))$	(3.7)
$E_{mz}(x_m,\theta_m) = \eta\frac{1}{1-v^2}\frac{e^{-a(x_m+v\theta_m)^2}}{\sqrt{a}}(-eicoshi(\xi_m,\varrho_m) + v\ eicoshi(v\xi_m,\varrho_m) - v\ esinh(\xi_m,\varrho_m)_1 + v\ esinh(v\xi_m,\varrho_m)_1)$	(3.8)

By making drawings for formulas (3.7) and (3.8) we can see propagations of electromagnetic fields in response to a moving source, observed in the moving frame.

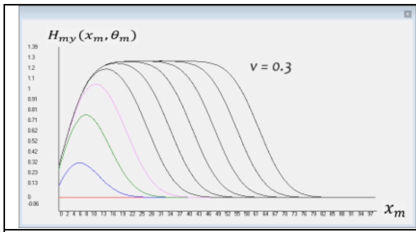
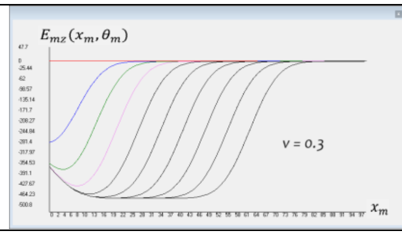
	

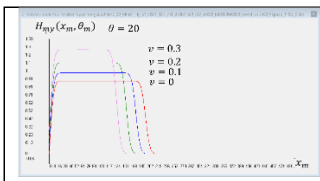
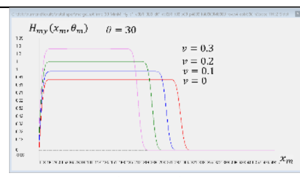
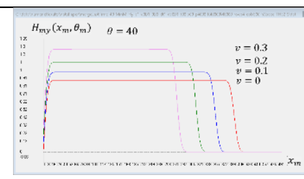
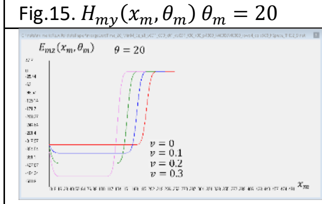
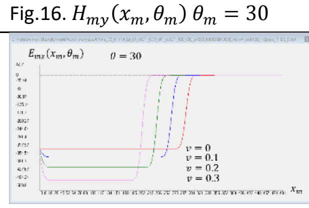
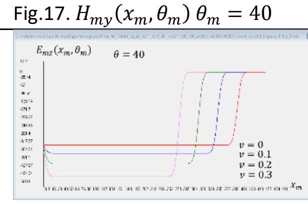
Fig.13. drawing of (3.7)	Fig.14. drawing of (3.8)

Comparing Fig.7 with Fig.1, we can see the effects of moving of the source on the magnetic field. Comparing Fig.8 and Fig.2, we can see the effects of moving of the source on the electric field.

Fig.3 and Fig.4 show fields observed in the rest frame. The fields have a moving back edge showing the moving of the source.

Fig.13 and Fig.14 show fields observed in the moving frame. We can see that the moving back edges in Fig.3 and Fig.4 disappear in Fig.13 and Fig.14, as if the source were not moving. It demonstrates an "inertial" behavior of the electromagnetic fields.

Let's examine the effects of the moving of the source on the propagations of the fields.

 Fig.15. $H_{my}(x_m,\theta_m)\ \theta_m = 20$	 Fig.16. $H_{my}(x_m,\theta_m)\ \theta_m = 30$	 Fig.17. $H_{my}(x_m,\theta_m)\ \theta_m = 40$
 Fig.18. $E_{mz}(x_m,\theta_m)\ \theta_m = 20$	 Fig.19. $E_{mz}(x_m,\theta_m)\ \theta_m = 30$	 Fig.20. $E_{mz}(x_m,\theta_m)\ \theta_m = 40$

From these figures, we can see the following characteristics.

- Verical gaps do not appear at the front edges, demonstrating an "inertial" behavior.
- Vertical gaps only appear at the front edges. The gaps show the moving of the source.
- Field propagation speeds are slower for faster moving sources.

To derive speed formulas, we need derivatives with respect to space. By identities in [3], we have the following formulas.

$b_n = x_m - (1-v)\theta_m$	(4.1)
$b_p = x_m + (1+v)\theta_m$	(4.2)
$\frac{\partial H_{my}(x_m,\theta_m)}{\partial x_m} = -\frac{1}{2(1-v^2)}\left((1+v)e^{-ab_n^2} + (1-v)e^{-ab_p^2} - 2e^{-ax_m^2}\right)$	(4.3)
$\frac{\partial E_{mz}(x_m,\theta_m)}{\partial x_m} = \frac{\eta}{2(1-v^2)}\left((1+v)e^{-ab_n^2} - (1-v)e^{-ab_p^2} - 2ve^{-ax_m^2}\right)$	(4.4)

$\frac{(1-v^2)}{a}\frac{\partial^2 H_{my}(x_m,\theta_m)}{\partial x_m^2} = (1+v)b_n e^{-ab_n^2} + (1-v)b_p e^{-ab_p^2} - 2x_m e^{-ax_m^2}$	(4.5)
$\frac{1-v^2}{a\eta}\frac{\partial^2 E_{mz}(x_m,\theta_m)}{\partial x_m^2} = -(1+v)b_n e^{-ab_n^2} + (1-v)b_p e^{-ab_p^2} + 2vx_m e^{-ax_m^2}$	(4.6)

Propagation speed of the magnetic field

Let

$$\frac{\partial^2 H_{my}(x_m,\theta_m)}{\partial x_m^2} = 0$$

We have

$(1+v)b_n e^{-ab_n^2} + (1-v)b_p e^{-ab_p^2} - 2x_m e^{-ax_m^2} = 0$	(4.7)

Take derivative with respect to θ_m on both sides of (4.7), we have the speed formula for the propagation of the magnetic field:

$a_n = 1 - 2ab_n^2$	(4.8)
$a_p = 1 - 2ab_p^2$	(4.9)
$\frac{dx_m}{d\theta_m} = \frac{(1-v^2)(a_n e^{-ab_n^2} - a_p e^{-ab_p^2})}{(1+v)a_n e^{-ab_n^2} + (1-v)a_p e^{-ab_p^2} - (1-2ax_m^2)2e^{-ax_m^2}}$	(4.10)

Let's make drawings of (4.7) and (4.10) to see what they look like.

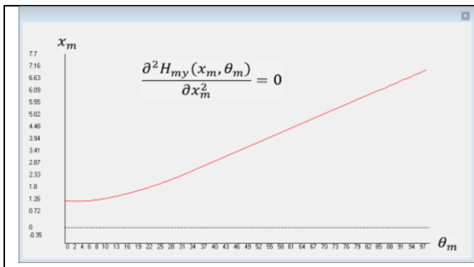


Fig.21. Time-space of the front edge

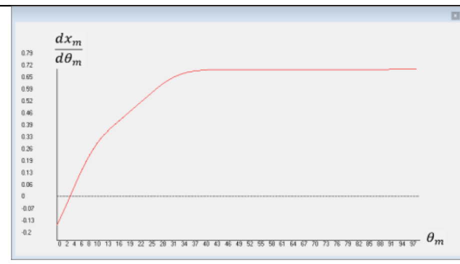


Fig.22. propagation speed of the magnetic field

Fig.22 tells us a few things:

1. There is a dynamic period where the propagation speed rises to a stable speed, just like in the case of a stationary source.
2. There is a stable period where the propagation speed looks constant, just like in the case of a stationary source.
3. Unlike the case for a stationary source, for the moving source, the speed starts from a negative value, not from 0.
4. Unlike the case for a stationary source, for the moving source, the stable speed is not the standard light speed c. It is c – v. The stable speed is lower than the standard light speed.

Propagation speed of the electric field

Let

$$\frac{\partial^2 E_{mz}(x_m,\theta_m)}{\partial x_m^2} = 0$$

We have

$(1+v)b_n e^{-ab_n^2} - (1-v)b_p e^{-ab_p^2} - 2vx_m e^{-ax_m^2} = 0$	(4.11)

Take derivative with respect to θ_m on both sides of (4.11), we have the speed formula for the propagation of the electric field

$\frac{dx_m}{d\theta_m} = \frac{(1-v^2)(a_n e^{-ab_n^2} + a_p e^{-ab_p^2})}{(1+v)a_n e^{-ab_n^2} - (1-v)a_p e^{-ab_p^2} - v(1-2ax_m^2)2e^{-ax_m^2}}$	(4.12)

Let's make drawings of (4.11) and (4.12) to see what they look like.

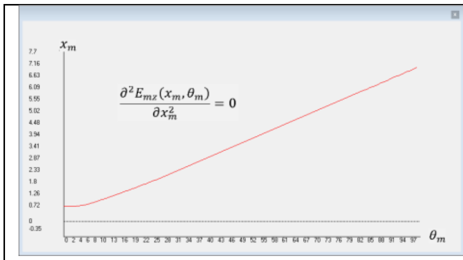


Fig.23. Time-space of the front edge

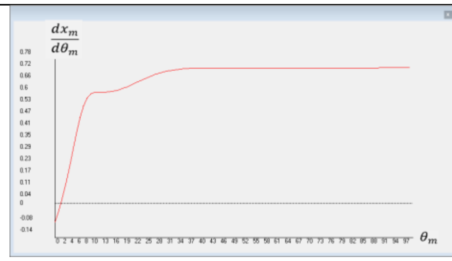


Fig.24. propagation speed of the electric field

Compared with the calculations for the magnetic field, the characteristics of the speed of the electric field are the same.

Effects of source speeds on field propagation speeds

Let's change the speed of the source to see how it affects the propagation speed of the electromagnetic field.

Fig.25 and Fig.26 show propagation speeds of electromagnetic fields. The speeds of the source are

$$v = 10\%c, 20\%c, 30\%c, 40\%c, 50\%c, 60\%c$$

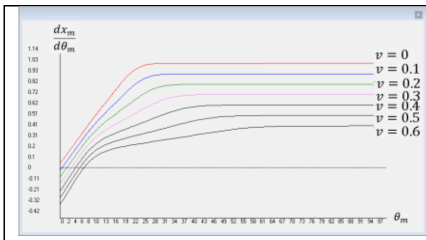


Fig.25. propagation speeds of the magnetic fields

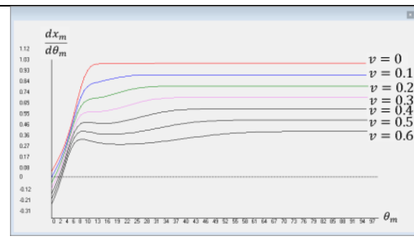


Fig.26. propagation speeds of the electric fields

Let v_e be the speed of the earth:

$$v_e = 0.006c$$

Fig.27 and Fig.28 show propagation speeds of electromagnetic fields. The speeds of the source are

$$v = v_e, 2v_e, 3v_e, 4v_e, 5v_e, 6v_e$$

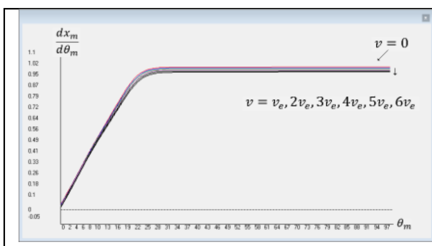


Fig.27. propagation speeds of the magnetic fields

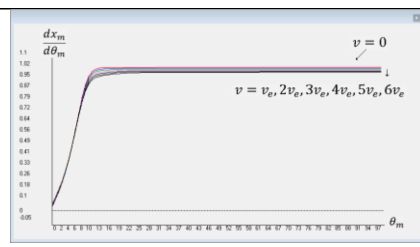


Fig.28. propagation speeds of the electric fields

If the field source moves faster then *c* then the propagation speeds of the fields will be negative. Let's make some calculations to investigate such situations.

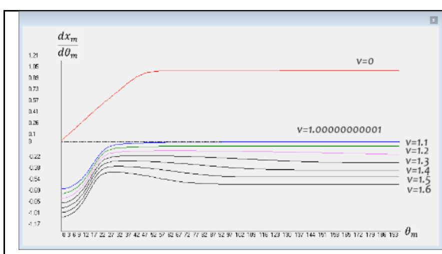


Fig.29. propagation speeds of the magnetic fields

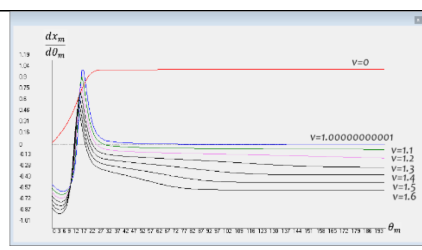


Fig.30. propagation speeds of the electric fields

Fig.29 and Fig.30 show propagation speeds of electromagnetic fields for the source moving faster than *c*.

Fig. 25 to Fig.30 show propagation speeds of the magnetic fields and electric fields corresponding to source moving speeds from 0 to 1.6, that is, from the stationary source to the source moving at 1.6 times the standard light speed. We can see the following characteristics.

1. The lengths of the dynamic period are almost the same. [1] tells us the length of the dynamic period depends on the Gaussian factor. Now we know that the speed of the source does not affect it.
2. In the stable period, the field propagation speed is $c - v$, that is, the standard light speed minus the source moving speed.
3. In the dynamic period, for different source moving speeds, the differences of the field propagation speeds are smaller than in the stable period. Thus, the electromagnetic fields exhibit some sort of inertial behavior in the dynamic period. Let's call it "pseudo inertia". For a true inertia the speed difference should be 0, independent of frame moving speed.

Comparing fields in rest and moving frames

Field differences

Denote

$\Delta_H(x,\theta) = H_y(x,\theta) - H_{my}(x_m,\theta_m)$	(4.13)
$\Delta_E(x,\theta) = E_z(x,\theta) - E_{mz}(x_m,\theta_m)$	(4.14)

Physically, it is almost impossible to measure the above differences. Michelson and Morley used mirrors to reflect lights to approach the above comparisons and used an ingenious design of detecting phase differences instead of magnitude differences.

Reflecting light alters the solutions to Maxwell's equations. It is quite difficult to truly measure (4.13) and (4.14). Here, we are doing mathematical calculations, not doing physical experiments. We do not need to deal with the problem Michelson and Morley had to solve.

Let's calculate (4.13) and (4.14) to see what they look like.

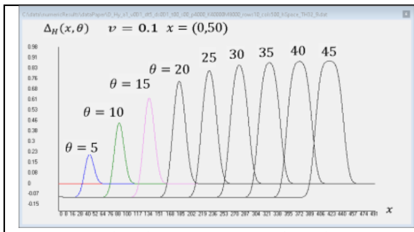


Fig.31. $\Delta_H(x,\theta), v = 0.1\ x = (0{,}50)$

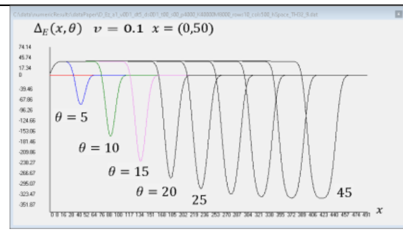


Fig.32. $\Delta_E(x,\theta), v = 0.1\ x = (0{,}50)$

Fig.31 and Fig.32 tell us the following characteristics of the field differences between the rest frame and the moving frame.

1. The field difference is a pulse signal traveling in space over time.
2. The magnitude of the pulse grows from 0 to a maximum value over the space.

The above 2 characteristics match the "pseudo inertia" behavior exhibited in the previous section when we investigate the field propagation speeds affected by a moving source.

Let's investigate the signal magnitudes. The signal magnitude can be defined as the maximum value at a space point over the time.

$\Delta_{maxH}(x) = \max\limits_{\theta>0} \lvert\Delta_H(x,\theta)\rvert$	(4.15)
$\Delta_{maxE}(x) = \max\limits_{\theta>0} \lvert\Delta_E(x,\theta)\rvert$	(4.16)

By drawing (4.15) and (4.16), we can see the signal magnitude distributions in the space.

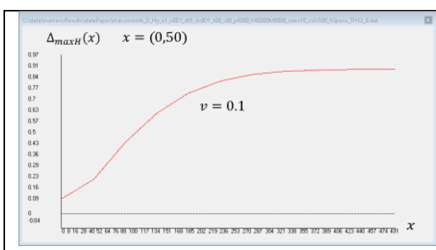


Fig.33. $\Delta_{maxH}(x)\ v = 0.1\ x = (0{,}50)$

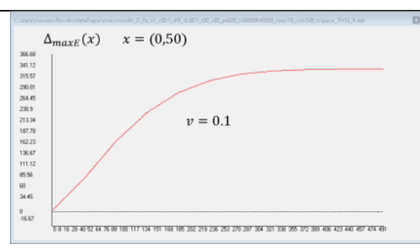


Fig.34. $\Delta_{maxE}(x)\ v = 0.1\ x = (0{,}50)$

Fig.33 and Fig.34 show the growth of the signals over space.

For Fig.31 and Fig.32, $v = 0.1\ x = (0{,}50)$, the time interval is 5: $\Delta_\theta = 5$.

Let's calculate using different source speeds and with a smaller time interval: $\Delta_\theta = 1$.

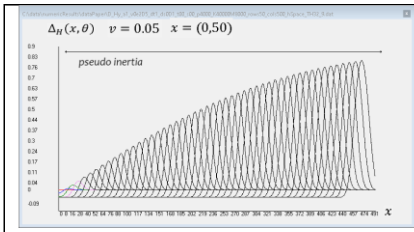


Fig.35. $\Delta_H(x,\theta), v = 0.05$

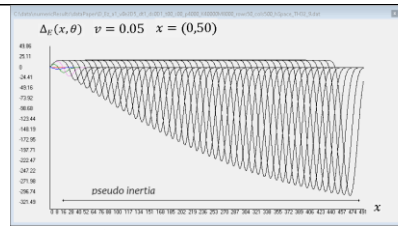


Fig.36. $\Delta_E(x,\theta), v = 0.05$

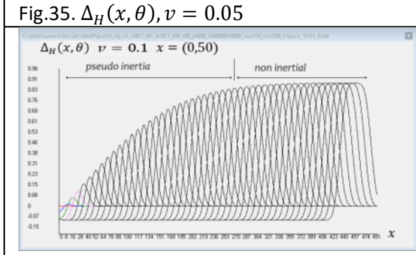


Fig.37. $\Delta_H(x,\theta), v = 0.1$

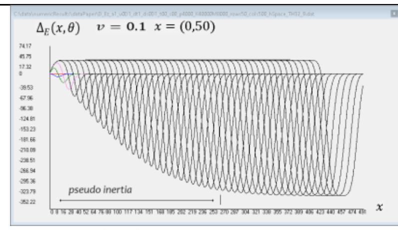


Fig.38. $\Delta_E(x,\theta), v = 0.1$

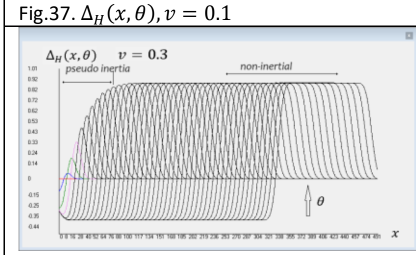


Fig.39. $\Delta_H(x,\theta), v = 0.3$

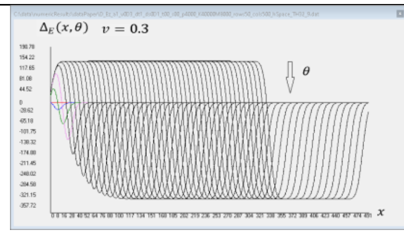


Fig.40. $\Delta_E(x,\theta), v = 0.3$

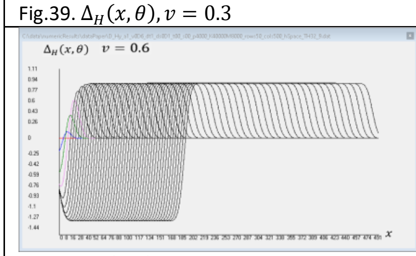


Fig.41. $\Delta_H(x,\theta), v = 0.6$

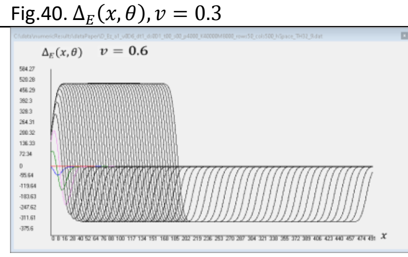


Fig.42. $\Delta_E(x,\theta), v = 0.6$

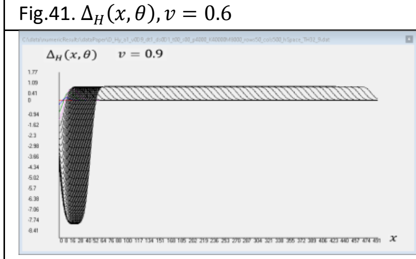


Fig.43. $\Delta_H(x,\theta), v = 0.9$

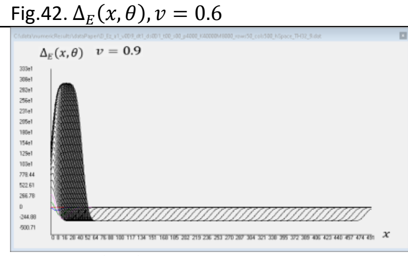


Fig.44. $\Delta_E(x,\theta), v = 0.9$

The above figures show that the source moving speed affects the magnitudes of the field differences. Let's investigate this effect.

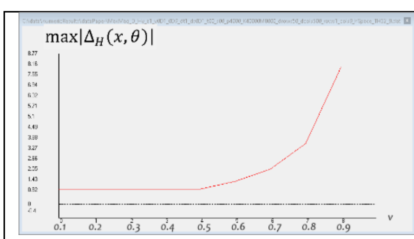


Fig.45. $\max|\Delta_H(x,\theta)|\,(v)$

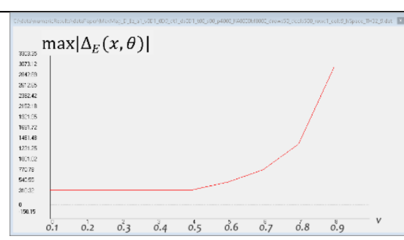


Fig.46. $\max|\Delta_E(x,\theta)|\,(v)$

The above figures show a surprising discovery that c/2 is a special speed. To verify this discovery, I made calculations near c/2, as shown below.

v	$\max\|\Delta_H(x,\theta)\|\,(v)$	$\max\|\Delta_E(x,\theta)\|\,(v)$
0.488	0.88622692545275794	333.86854742407877
0.489	0.88622692545275794	333.86854742407877
0.5	0.88622692545275794	333.86854742407877

0.501	0.88977893717801959	335.20669791475643
0.502	0.89334521401061151	336.55022250379022

The above calculations verified the discovery.

From these figures and calculations we see following characteristics:

- Speed c/2 separates two distinct speed ranges. Below c/2, the max magnitudes are constants. Above c/2, the max magnitudes start growing to infinity. See Fig.45 and Fig.46
- Below c/2, field propagations exhibit a “pseudo inertia” behavior. The length of the “pseudo inertia” depends on the speed of the source. The faster the source moves the shorter the “pseudo inertia” period. See Fig.35 to Fig.40
- Above c/2, field propagations exhibit a “speed barrier” behavior. See Fig.41 to Fig.44.

The above characteristics are further investigated in the next two subsections.

Let’s investigate the magnitudes of $\Delta_H(x,\theta)$ and $\Delta_E(x,\theta)$ for sources below c/2 first.

Magnitudes of field differences of slow moving sources

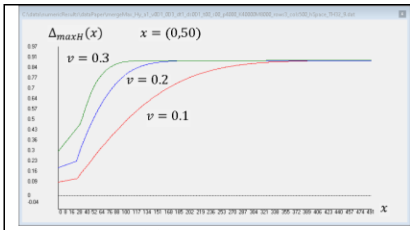


Fig.47. $\Delta_{maxH}(x)$ $v = 0.1, 0.2, 0.3$

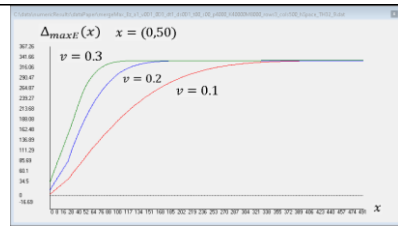


Fig.48. $\Delta_{maxE}(x)$ $v = 0.1, 0.2, 0.3$

From these figures, we can see:

- For different source moving speeds, the magnitudes of the field differences reach the same value at far enough spaces
- The slower the source moves the longer space it needs to reach the final value.
- The final value for the electric field is 333.86854742407877
- The final value for the magnetic field is 0.88622692545275794

From calculations, we have following conclusions.

$\lim_{x\to\infty} \Delta_{maxH}(x) = \max\|H_y(x,\theta)\| \approx 0.88622692545275794$	(4.17)
$\lim_{x\to\infty} \Delta_{maxE}(x) = \max\|E_z(x,\theta)\| \approx 333.86854742407877$	(4.18)

The importance of the above conclusions is that it is independent of the source moving speeds.

Effects of “pseudo inertia” at the speed of the earth

From Fig.35 and Fig. 36 we can see that when the source is moving at 5% of the standard light speed, $v = 0.05$, the length of the “pseudo inertia” period is longer than 50 meters.

Because the earth is moving at about 0.6% of the standard light speed, we expect that the length of the “pseudo inertia” period for the earth speed is much longer than 50 meters. Let’s make calculations using speeds near the earth speed.

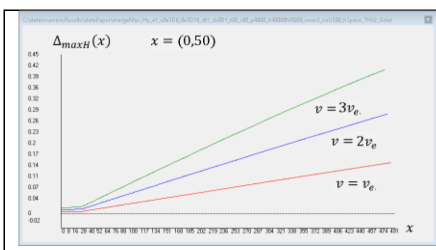


Fig.49. $\Delta_{maxH}(x)$ $v = v_e, 2v_e, 3v_e$

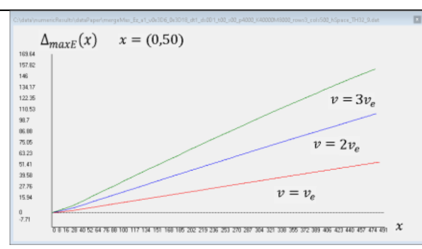


Fig.50. $\Delta_{maxE}(x)$ $v = v_e, 2v_e, 3v_e$

We may use regression lines to estimate the length of the “pseudo inertia” period.

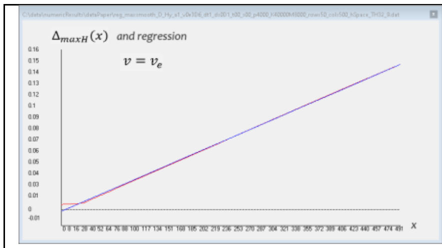


Fig.51. $\Delta_{maxH}(x)$ *and regression*, $v = v_e$

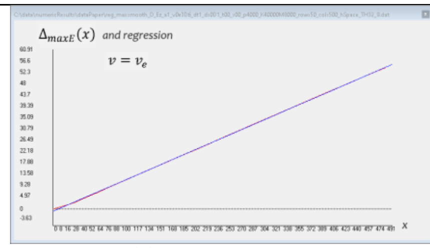


Fig.52. $\Delta_{maxE}(x)$ *and regression*, $v = v_e$

These figures show that the regression lines match $\Delta_{maxH}(x)$ and $\Delta_{maxE}(x)$ very well. The regression lines are produced by least-square method, in the following formulas.

$Regression_H(x) = 0.000296511 \cdot x - 0.00151389$	(4.19)
$Regression_E(x) = 0.112462 \cdot x - 0.824744$	(4.20)

From (4.17), we have

$$Regression_H(x_p) = 0.8862269 \rightarrow x_p \approx 2994$$

From (4.18), we have

$$Regression_E(x_p) = 333.8685474 \rightarrow x_p \approx 2976$$

The two regression lines give almost the same estimation of the length of the "pseudo inertia" period: about 3 km.

$$x_p \approx 3000\ meters$$

Let's define an accumulated effect of magnetic/electric field differenc as

$$S_H(x) = \left| \int_0^x Regression_H(\sigma) d\sigma \right|$$

and

$$S_E(x) = \left| \int_0^x Regression_E(\sigma) d\sigma \right|$$

The lengths of the light paths of various Michelson-Morley style experiments are less than 40 meters. From the above calculations, we have the following data.

x	$\frac{\Delta_{maxH}(x)}{\max\lvert H_y(x,\theta)\rvert}$	$\frac{\Delta_{maxE}(x)}{\max\lvert E_z(x,\theta)\rvert}$	$\frac{S_H(x)}{S_H(x_p)}$	$\frac{S_E(x)}{S_E(x_p)}$
10	0.16%	0.09%	$2 \cdot 10^{-5}\%$	0.0024%
30	0.83%	0.76%	0.0066%	0.0005%
50	1.5%	1.4%	0.022%	0.01%

Physically, there are lots of factors affecting the results of Michelson-Morley experiments, such as the use of mirrors, different light sources, etc. Mathematically, the above calculations tell us that the behavior of the "pseudo inertia" is a factor which cannot be ignored.

The "pseudo inertia" behavior of the electromagnetic fields also tells us that

- The very small values of the results of Michelson-Morley experiments might not be interpreted as null-results
- Maxwell's equations were discovered via physical experiments on the earth, and thus, naturally used the earth as its coordinate system. In a small space range, the inertia behavior dominates and thus there is very little difference whether Maxwell's equations are using the absolute rest frame as its coordinate system or using the earth as its coordinate system.

High speed movement of the source

When the source is moving at high speed, unique patterns appear in the propagations of the fields. Let's make calculations and investigate the effects.

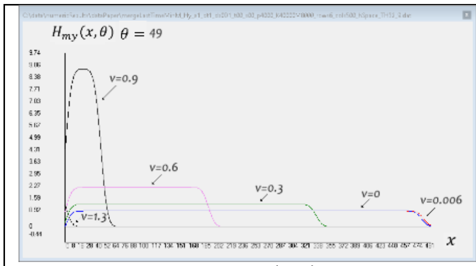


Fig.53. source speeds vs $H_{my}(x,\theta)$

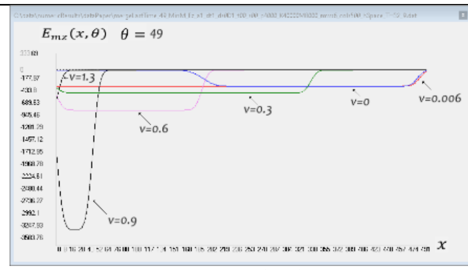


Fig.54. source speeds vs $E_{mz}(x,\theta)$

These figures show field magnitudes at time 49 for different source moving speeds *v*. *v*=0.006 is near the earth speed. *v*=0 is the stationary source. We can see the following characteristics.

1. Increase the source speed from 0 to 0.9, the magnitudes of the fields grow
2. For *v*=1.3 the field magnitudes become smaller than for *v*=0.9, showing that when the source speed is faster than the standard light speed, the field magnitudes shrink.

These characteristics tell us that the standard light speed acts as a speed barrier. The closer to this barrier, the more electromagnetic energy is required. Away from this barrier, either lower or faster than this barrier, less energy is required. That means, once the source breaks this barrier, less energy is needed. Let's call this speed barrier "lux-barrier".

The discovery of the lux-barrier calls for more investigations. The next section presents some investigations.

Lux-barrier

I'll investigate the lux-barrier from the rest frame. Such phenomena would be more likely observed in a particle accelerator.

For now I limit my investigations to

$$v \geq 0$$

Observing from the rest frame and borrowing concepts from sound waves, we may classify the source moving speeds into 3 ranges:

1. Sub-luxic speed: $v < 1$
2. Super-luxic speed: $v > 1$
3. Lux-barrier speed: $v = 1$

The solution to Maxwell's equations given by (1.6) and (1.7) is invalid when the source is moving at speed of *c*, that is, when $v = 1$. Let's find out the solution for $v = 1$.

Solution at the lux-barrier

Solution at the lux-barrier. At the lux-barrier speed, the solution becomes

$v = 1$ $J(x,\theta) = e^{-a(x-\theta)^2}$	(5.1)
$H_y(x,\theta) = \frac{1}{2}\left(\theta e^{-a(x-\theta)^2} - \frac{1}{\sqrt{a}} e^{-ax^2} eicoshi(\xi,\varrho)\right)$	(5.2)
$E_z(x,\theta) = -\frac{\eta}{2}\left(\theta\, e^{-a(x-\theta)^2} + \frac{1}{\sqrt{a}} e^{-ax^2} eicoshi(\xi,\varrho)\right)$	(5.3)

Proof:

Because

$$eicoshi(0,\varrho) = 0$$

We have

$$H_y(x,0) = 0$$

$$E_z(x,0) = 0$$

Initial values (1.2) and (1.3) are satisfied.

By identities (i.2) and (i.1) in [3], we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x}=\frac{1}{2}\left(2\left(\frac{\varrho}{2}-\xi\right)\xi e^{-a(x-\theta)^2}+e^{-ax^2}e^{-\xi^2}\sinh(\varrho\xi)\right) \tag{5.4}$$

By identities (i.14) and (i.10) in [3], we have

$$\frac{\partial H_y(x,\theta)}{\partial\theta}=\frac{1}{2}(2\left(\frac{\varrho}{2}-\xi\right)\xi e^{-a(x-\theta)^2}+e^{-ax^2}e^{-\xi^2}\sinh(\varrho\xi)) \tag{5.5}$$

(5.4) and (5.5) show (1d.1) holds.

By (i.2), (i.9) and (i.1) in [3], we have

$$\frac{\partial}{\partial x}H_y(x,\theta)=-a(x-\theta)\theta e^{-a(x-\theta)^2}+\frac{1}{2}e^{-ax^2}e^{-\xi^2}\sinh(\varrho\xi) \tag{5.6}$$

By (i.14), (i.10) and (i.2) in [3], we have

$$\frac{1}{\eta}\frac{\partial}{\partial\theta}E_z(x,\theta)=-\frac{1}{2}e^{-a(x-\theta)^2}-a\theta(x-\theta)e^{-a(x-\theta)^2}-\frac{1}{2}e^{-ax^2}e^{-\xi^2}\cosh(\varrho\xi) \tag{5.7}$$

From (5.6) and (5.1), we have

$$\frac{\partial H_y(x,\theta)}{\partial x}-J_z(x,\theta)=-\frac{1}{2}e^{-a(x-\theta)^2}-a\theta(x-\theta)e^{-a(x-\theta)^2}-\frac{1}{2}e^{-ax^2}e^{-\xi^2}\cosh(\varrho\xi) \tag{5.8}$$

(5.7) and (5.8) show (1d.2) holds.

QED

From the solution at the lux-barrier, we have the following conclusion.

Lux-barrier magnitude. The source moving at the lux-barrier generates electromagnetic fields growing from 0 to infinity.

Proof.

Since for the stationary source, we have

$$E_{zs}(x,\theta)=-\frac{\eta}{\sqrt{a}}e^{-ax^2}eicoshi(\xi,\varrho)$$

(5.2) and (5.3) can be written as

$$H_y(x,\theta)=\frac{1}{2}\theta e^{-a(x-\theta)^2}+\frac{1}{2\eta}E_{zs}(x,\theta) \tag{5.2a}$$

$$E_z(x,\theta)=-\frac{\eta}{2}\theta\, e^{-a(x-\theta)^2}+\frac{1}{2}E_{zs}(x,\theta) \tag{5.3a}$$

We have

$$H_y(\theta,\theta)=\frac{1}{2}\theta+\frac{1}{2\eta}E_{zs}(\theta,\theta) \tag{5.2b}$$

$$E_z(\theta,\theta)=-\frac{\eta}{2}\theta+\frac{1}{2}E_{zs}(\theta,\theta) \tag{5.3b}$$

Because

$$|E_{zs}(x,\theta)|<\infty$$

We have

$$\lim_{\theta\to\infty}H_y(\theta,\theta)=\infty \tag{5.2c}$$

$$\lim_{\theta\to\infty}E_z(\theta,\theta)=-\infty \tag{5.3c}$$

QED

The above conclusion verifies analytically the phenomenon of the lux-barrier discovered in the last section via calculations of the electromagnetic fields.

Field propagation speeds for sources at lux-barrier

Propagation speed of the magnetic field

By (5.6), we have

$$\frac{\partial^2}{\partial x^2}H_y(x,\theta)=\frac{1}{2}a\left(\left(4a(x-\theta)^2\theta-(x+\theta)\right)e^{-a(x-\theta)^2}+(x+\theta)e^{-a(x+\theta)^2}\right) \tag{5.9}$$

Let

$$\frac{\partial^2}{\partial x^2}H_y(x,\theta)=0$$

We have

$$4a(x-\theta)^2\theta-x-\theta+(x+\theta)e^{-4ax\theta}=0 \tag{5.10}$$

Take derivative with respect to space, we have the propagation speed of the magnetic field:

$$\frac{dx}{d\theta}=\frac{8a(x-\theta)\theta+1-4a(x-\theta)^2-(1-4ax(x+\theta))e^{-4ax\theta}}{8a(x-\theta)\theta-1+(1-4a\theta(x+\theta))e^{-4ax\theta}} \tag{5.11}$$

Let’s calculate (5.10) and (5.11) and make drawings of them.

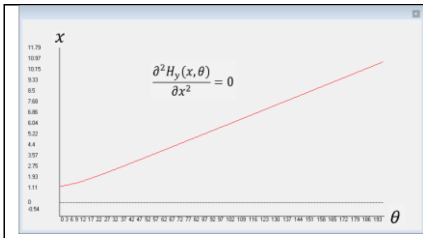


Fig.55. propagation of the magnetic field

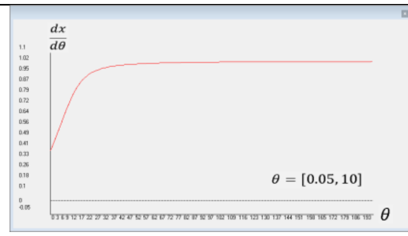


Fig.56. propagation speed of the magnetic field

It looks the field speed behaves the same as for sources not at lux-barrier speed.

Propagation speed of the electric field

From (5.4), we have

$$\frac{2}{\eta a}\frac{\partial^2 E_z(x,\theta)}{\partial x^2}=2\theta e^{-a(x-\theta)^2}-(1+4a(x-\theta)\theta)(x-\theta)e^{-a(x-\theta)^2}+(x+\theta)e^{-a(x+\theta)^2} \tag{5.12}$$

Let

$$\frac{\partial^2 E_z(x,\theta)}{\partial x^2}=0$$

We have

$$3\theta-x-4a(x-\theta)^2\theta+(x+\theta)e^{-4ax\theta}=0 \tag{5.13}$$

Take derivative with respect to space, we have the propagation speed of the magnetic field:

$$\frac{dx}{d\theta}=\frac{3+8a(x-\theta)\theta-4a(x-\theta)^2+(1-4ax(x+\theta))e^{-4ax\theta}}{1+8a(x-\theta)\theta-(1-4a\theta(x+\theta))e^{-4ax\theta}} \tag{5.14}$$

Let’s calculate (5.13) and (5.14) and make drawings of them.

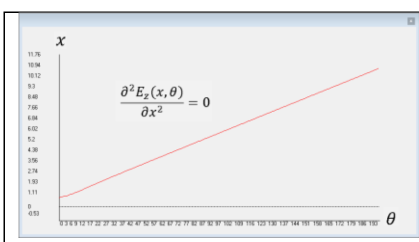


Fig.57. propagation of the electric field

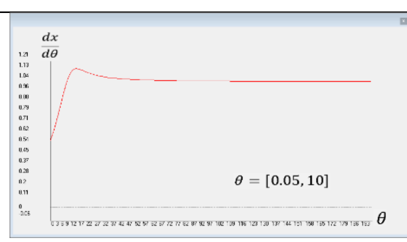


Fig.58. propagation speed of the electric field

It looks the field speed behaves the same as for sources not at lux-barrier speed.

Break the lux-barrier

The solution (1.6) and (1.7) is derived by assuming the speed of the source is a constant. If the speed is not constant then we need to re-derive the solutions to Maxwell’s equations. I am not going to do it here. I’ll make some comments on the energy involved for breaking the lux-barrier.

- At the lux-barrier, the magnitudes of the fields grow to infinity, but the fields need time to grow. If the source’s speed breaks the lux-barrier in a limited time then the energy involved is limited. In another word, breaking of the lux-barrier is possible.

- The faster the source breaks the lux-barrier, the less electromagnetic energy involved. But that means larger acceleration and thus larger mechanical energy needed to accelerate the source.

Field propagation patterns

One way to look for super-luxic charged particles might be looking for the patterns of the electromagnetic fields they generate. Different kinds of sources will produce different patterns. Here I'll present a few patterns for Gaussian sources.

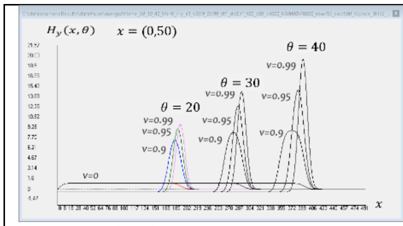


Fig. 59. $H_y(x,\theta)$ $v=0, 0.9, 0.95, 0.99$

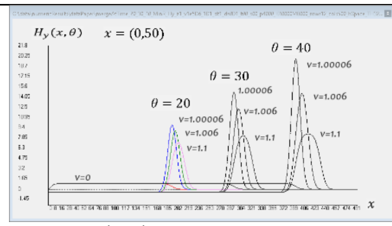


Fig.60 . $H_y(x,\theta)$ $v=0, 1.00006, 1.006, 1.1$

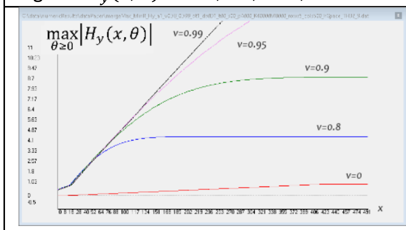


Fig. 61. $\max_{\theta\geq 0}|H_y(x,\theta)|$ $v=0, 0.8, 0.9, 0.95, 0.99$

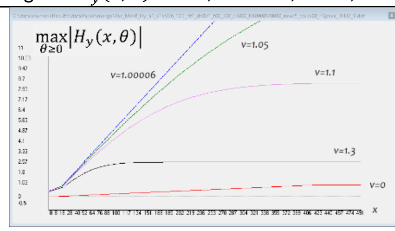


Fig.62. $\max_{\theta\geq 0}|H_y(x,\theta)|$ $v=0, 1.00006, 1{,}05, 1.1, 1.3$

Fig.59 shows magnetic field propagations for sub-luxic sources.

Fig.61 shows magnetic field magnitudes for sub-luxic sources.

Fig.60 shows magnetic field propagation for super-luxic sources.

Fig.62 shows magnetic field magnitudes for super-luxic sources.

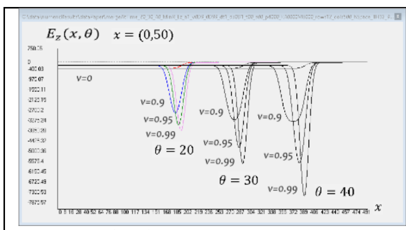


Fig. 63. $E_z(x,\theta)$ $v=0.9, 0.95, 0.99$

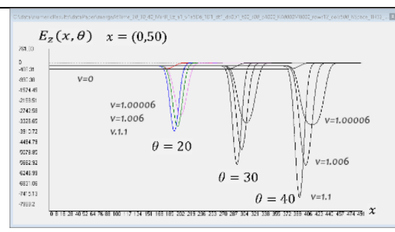


Fig. 64. $E_z(x,\theta)$ $v=1.00006, 1.006, 1.1$

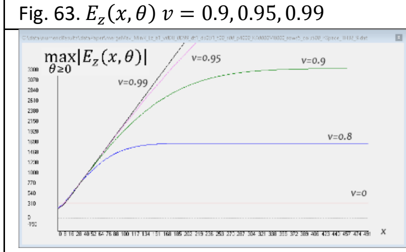


Fig.65 . $\max_{\theta\geq 0}|E_z(x,\theta)|$ $v=0, 0.8, 0.9, 0.95, 0.99$

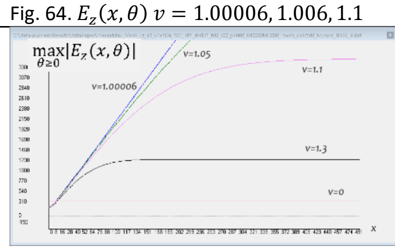


Fig.66. $\max_{\theta\geq 0}|E_z(x,\theta)|$ $v=0, 1.00006, 1{,}05, 1.1, 1.3$

Fig.63 shows electric field propagations for sub-luxic sources.

Fig.65 shows electric field magnitudes for sub-luxic sources.

Fig.64 shows electric field propagations for super-luxic sources.

Fig.66 shows electric field magnitudes for super-luxic sources.

Summary

- The electromagnetic fields exhibit sound wave-like and water wave-like behavior.
- Like water, the electromagnetic fields cannot be compressed.
- Like sound, a speed barrier exists.
- The speed barrier for a field source is the standard light speed c.
- For sources below c or above c, field propagations exhibit distinct patterns.
- For sources below c/2 and above c/2, field propagations exhibit distinct behaviors.
- For a fast moving source, it is possible to break the speed barrier with a limited amount of energy.
- For a slow moving source, the electromagnetic fields exhibit a “pseudo inertia” behavior. At the source the fields are inertial; away from the source the inertial behavior fades out In a limited space range.
- The length of the “pseudo inertia” depends on the speed of the source. The faster of the source the shorter of the range
- At the speed of the earth, the length of the “pseudo inertia” is longer than 3000 meters.
- At the speed of the earth, in a space range less than 2 meters, the inertia behavior dominates. Thus, there is not much difference whether Maxwell’s equations are using the earth as the coordinate system or using the absolute rest frame as the coordinate system.

All the above characteristics of the electromagnetic fields are discovered from a large amount of numerical data calculated from the analytical formulas. We need to study the formulas and figure out the reasons and theoretical rules for these characteristics, and gain insights into the nature of the electromagnetic fields.

References

[1] David Wei Ge, Speeds of Electromagnetic Fields of a Gaussian Source, September 2023, DOI: 10.13140/RG.2.2.17243.99362, http://dx.doi.org/10.13140/RG.2.2.17243.99362

[2] David Wei Ge, Speeds of Electromagnetic Fields of a Gaussian Source, September 2023, DOI: 10.13140/RG.2.2.17243.99362, http://dx.doi.org/10.13140/RG.2.2.17243.99362

[3] David Wei Ge, A Closed Form Analytical Solution to Maxwell's Equations in Response to a Time Invariant Gaussian Source, August 2023, DOI: 10.13140/RG.2.2.15985.70245, http://dx.doi.org/10.13140/RG.2.2.15985.70245