1D Solution to Maxwell’s Equations in Response to a Moving Source

Author: David Wei Ge (gexiaobao.usa@gmail.com)

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Abstract A generic solution to Maxwell’s equations in response to a constant speed moving source is deduced and verified. The solution is for 1D fields in open space. Two examples are given. One example is a time-invariant Gaussian source. The other example is a trigonometric source with Gaussian decay. The results are used to examine the Michelson-Morley experiment. Electromagnetic fields are calculated using the solutions to Maxwell’s equations in response to a cosine source. The calculation results show significant differences between applying coordinate transformation to a stationary source solution and to a moving source solution, leading to a question whether non-null results of Michelson-Morley experiment should be expected.

Contents

Introduction

A generic moving source solution to Maxwell’s equations is deduced from the generic solution for any sources (see [1]). To ensure the correctness of the deduction, the solution obtained is further verified with Maxwell’s equations.

The generic moving source solution is applied to two examples to get specific solutions. One example is a time-invariant Gaussian source, the other example is a trigonometric source with Gaussian decay.

The results are used to examine the Michelson-Morley experiment.

Because the solutions to Maxwell's equations were not available, people made the following assumptions.

1. The stationary source solution in the rest frame must be a function of constant speed. see [3]
2. The moving source solution could be obtained by applying coordinate transformation to the stationary source solution in the rest frame. See section 6 of [3].

The second assumption is wrong because the moving source solution should be obtained by applying coordinate transformation to the moving source solution in the rest frame.

Now we know the solutions to Maxwell's equations. We may check to see whether the consequences of the wrong assumption are significant.

Electromagnetic fields are calculated using the solutions to Maxwell's equations in response to a cosine source. The calculation results show significant differences between applying coordinate transformation to a stationary source solution and to a moving source solution, leading to a question whether non-null results of Michelson-Morley experiment should be expected.

1D Generic Solution

1D Maxwell's equations along the x-axis are

$\frac{\partial H_y(x,\theta)}{\partial\theta}=\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x}$	(1d.1)
$\frac{\partial E_z(x,\theta)}{\partial\theta}=\eta\frac{\partial H_y(x,\theta)}{\partial x}-\eta J_z(x,\theta)$	(1d.2)

Where

$$\theta=ct$$

$$c=\frac{1}{\sqrt{\varepsilon\mu}}$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}}$$

The solution to (1d.1) and (1d.2), assuming 0-initial values, is (see [1])

$H_y(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\frac{\partial^{2n+1}J_z(x,0)}{\partial\theta^{2(n-m)}\partial x^{2m+1}}-\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\frac{\partial^{2n+2}J_z(x,0)}{\partial\theta^{2(n-m)+1}\partial x^{2m+1}}$	(1d.3)
$E_z(x,\theta)=-\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\frac{\partial^{2n+1}J_z(x,0)}{\partial\theta^{2(n-m)+1}\partial x^{2m}}-\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{\partial^{2n}J_z(x,0)}{\partial\theta^{2(n-m)}\partial x^{2m}}$	(1d.4)

Solution for constant speed moving source

Suppose the source is moving in a constant speed *v*:

$$J_z(x-v\theta,\theta)$$

Let

$\varsigma=x-v\theta$	(1)
$J_z(x-v\theta,\theta)=\Gamma_z(\varsigma,\theta)$	(2)

We have

$\frac{\partial^n J_z(x-v\theta,\theta)}{\partial x^n}=\frac{\partial^n\Gamma_z(\varsigma,\theta)}{\partial\varsigma^n}$	(3)
$\frac{\partial^n J_z(x-v\theta,\theta)}{\partial\theta^n}=\left(\frac{\partial}{\partial\theta}-v\frac{\partial}{\partial\varsigma}\right)^n\Gamma_z(\varsigma,\theta)$	(4)

Or

$\frac{\partial^n J_z(x-v\theta,\theta)}{\partial\theta^n}=\sum_{k=0}^{n}\binom{n}{k}(-1)^k v^k\frac{\partial^n\Gamma_z(\varsigma,\theta)}{\partial\varsigma^k\partial\theta^{n-k}}$	(5)

Substitute (5) into (1d.3) and (1d.4), we have the generic solution for a moving source:

$$H_y(x,\theta) = -\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)-k}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)+1-k}}$$	(6)
$$E_z(x,\theta) = -\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}}$$	(7)

Verifications of the moving source solution

Moving source solution: Electromagnetic fields given by (6) and (7) satisfy Maxwell's equations (1d.1) and (1d.2) for a moving source $J_z(x - v\theta, \theta)$.

Proof

By (7) and (3), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = -\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)+1-k}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)-k}}$$	(8)

By (6), we have

$$\frac{\partial H_y(x,\theta)}{\partial\theta} = -\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)-k}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)+1-k}}$$	(9)

Compare (8) and (9), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = \frac{\partial H_y(x,\theta)}{\partial\theta}$$

Thus, (6) and (7) satisfy (1d.1).

By (6) and (3), we have

$$\frac{\partial H_y(x,\theta)}{\partial x} = -\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+2}\partial\theta^{2(n-m)-k}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+3}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+2}\partial\theta^{2(n-m)+1-k}}$$

$$= -\sum_{n=1}^{\infty}\frac{\theta^{2n}}{(2n)!}\sum_{m=1}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}} - \sum_{n=1}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=1}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}}$$

$$= -\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k \frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}} - \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k \frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}}$$

$$+\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}\sum_{k=0}^{2n}\binom{2n}{k}(-v)^k\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^k\partial\theta^{2n-k}}+\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{k=0}^{2n+1}\binom{2n+1}{k}(-v)^k\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^k\partial\theta^{2n+1-k}}$$

Substitute (5) into above, we have

$\frac{\partial H_y(x,\theta)}{\partial x}=-\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}}-\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}}+J_z(x-v\theta,\theta)$	(10)

By (7), we have

$\frac{1}{\eta}E_z(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)+1}\binom{2(n-m)+1}{k}(-1)^k v^k\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}}-\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}\sum_{m=0}^{n}\sum_{k=0}^{2(n-m)}\binom{2(n-m)}{k}(-1)^k v^k\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}}$	(11)

Compare (10) and (11), we have

$$\frac{1}{\eta}E_z(x,\theta)=\frac{\partial H_y(x,\theta)}{\partial x}-J_z(x-v\theta,\theta)$$

Thus, (6) and (7) satisfy (1d.2).

QED.

Solution for time-invariant moving source

If the source is time-invariant when moving speed is 0 then the source can be written as

$J_z(x-v\theta,\theta)=J_z(x-v\theta)$	(12)
$\Gamma_z(\varsigma,\theta)=\Gamma_z(\varsigma)$	(13)

For such a source, we have

$\frac{\partial\Gamma_z(\varsigma,\theta)}{\partial\theta}=0$	(14)

Substitute (14) into (6) and (7), the last summations disappear, and we have

$$H_y(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=2(n-m)}^{2(n-m)}\binom{2(n-m)}{2(n-m)}(-1)^{2(n-m)}v^{2(n-m)}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)-k}}$$
$$-\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\sum_{k=2(n-m)+1}^{2(n-m)+1}\binom{2(n-m)+1}{2(n-m)+1}(-1)^{2(n-m)+1}v^{2(n-m)+1}\frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)+1-k}}$$

$$H_y(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}v^{2(n-m)}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}}+\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}v^{2(n-m)+1}\frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+2}}$$

$$H_y(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}}\sum_{m=0}^{n}v^{2(n-m)}+v\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+2}}\sum_{m=0}^{n}v^{2(n-m)}$$

$$\sum_{m=0}^{n}v^{2(n-m)}=\sum_{m=0}^{n}v^{2m}=\sum_{m=0}^{n}(v^2)^m=\frac{1-(v^2)^{n+1}}{1-v^2}$$

$$H_y(x,\theta)=-\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}}\frac{1-(v^2)^{n+1}}{1-v^2}+v\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+2}}\frac{1-(v^2)^{n+1}}{1-v^2}$$

$$E_z(x,\theta) = -\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{k=2(n-m)+1}^{2(n-m)+1}\binom{2(n-m)+1}{2(n-m)+1}(-1)^{2(n-m)+1}v^{2(n-m)+1}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+2(n-m)+1}\partial\theta^{0}}$$

$$-\eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{k=2(n-m)}^{2(n-m)}\binom{2(n-m)}{2(n-m)}(-1)^{2(n-m)}v^{2(n-m)}\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+2(n-m)}\partial\theta^{2(n-m)-k}}$$

$$E_z(x,\theta) = \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}v^{2(n-m)+1}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}v^{2(n-m)}\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n}}$$

$$E_z(x,\theta) = \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}}\sum_{m=0}^{n}v^{2(n-m)+1} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n}}\sum_{m=0}^{n}v^{2(n-m)}$$

$$E_z(x,\theta) = \eta v\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}}\frac{1-(v^2)^{n+1}}{1-v^2} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n}}\frac{1-(v^2)^{n+1}}{1-v^2}$$

$$E_z(x,\theta) = \eta v\sum_{n=0}^{\infty}\frac{\theta^{2n+2}}{(2n+2)!}\frac{1-(v^2)^{n+1}}{1-v^2}\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n+1}} - \eta\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\frac{1-(v^2)^{n+1}}{1-v^2}\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2n}}$$

We have the solution for a time-invariant moving source:

$H_y(x,\theta) = -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+3}(1-(v^2)^{n+1})}{(2n+3)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}}$	(15)
$E_z(x,\theta) = \frac{\eta v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}} - \frac{\eta}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-(v^2)^{n+1})}{(2n+1)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}}$	(16)
$v \neq 1$	

For $v = 1$ the above solution is also valid by taking a limit $v \to 1$; the limit exists.

$H_y(x,\theta) = \frac{\theta}{2}\left(J_z(x-\theta) - J_z(x)\right) - \frac{1}{2}\sum_{n=0}^{\infty}\frac{\theta^{2n+3}}{(2n+3)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}}$	(17)
$E_z(x,\theta) = -\frac{\eta}{2}\theta J_z(x-\theta) - \frac{\eta}{2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}}$	(18)
$v = 1$	

Verification of the solution for time-invariant moving source

Solution for a time-invariant moving source: For a time invariant source, $J_z(x)$, if the source moves in a constant speed v, $J_z(x - v\theta)$, then solution to Maxwell's equations (1d.1) and (1d.2) is (15) and (16).

Proof.

By (15), we have

$\frac{\partial H_y(x,\theta)}{\partial\theta} = -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-(v^2)^{n+1})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}}$	(17)

By (16), we have

$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}} - \frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-(v^2)^{n+1})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}}$	(18)

Compare (17) and (18), we have

$$\frac{\partial H_y(x,\theta)}{\partial\theta} = \frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x}$$

Thus, (15) and (16) satisfy (1d.1).

By (12), we have

$$\frac{\partial^n J_z(x-v\theta)}{\partial\theta^n} = \left(-v\frac{\partial}{\partial x}\right)^n J_z(x-v\theta) \quad (19)$$

By (19), we have

$$J_z(x-v\theta) = \sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}v^{2n}\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) - \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}v^{2n+1}\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x) \quad (20)$$

By (15) and (20), we have

$$\frac{\partial H_y(x,\theta)}{\partial x} = -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-v^{2n+2})}{(2n+2)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+3}(1-v^{2n+2})}{(2n+3)!}\frac{\partial^{2n+3}J_z(x)}{\partial x^{2n+3}}$$

$$-\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}v^{2n}\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) + \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}v^{2n+1}\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x) + \sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}v^{2n}\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) - \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}v^{2n+1}\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)$$

$$= -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+2}(1-v^{2n+2})}{(2n+2)!}\frac{\partial^{2n+2}J_z(x)}{\partial x^{2n+2}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+3}(1-v^{2n+2})}{(2n+3)!}\frac{\partial^{2n+3}J_z(x)}{\partial x^{2n+3}}$$

$$-\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)$$

$$+\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}v^{2n}\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) - \sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}v^{2n+1}\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)$$

$$= -\frac{1}{1-v^2}\sum_{n=1}^{\infty}\frac{\theta^{2n}(1-v^{2n})}{(2n)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}} + \frac{v}{1-v^2}\sum_{n=1}^{\infty}\frac{\theta^{2n+1}(1-v^{2n})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}}$$

$$-\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)$$

$$+J_z(x-v\theta)$$

$$= -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}(1-v^{2n})}{(2n)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-v^{2n})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}}$$

$$-\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}}{(2n)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n}}{\partial x^{2n}}J_z(x) + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}}{(2n+1)!}(v^{2n}-v^{2n+2})\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)$$

$$+J_z(x-v\theta)$$

$$= -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}(1-v^{2n}+v^{2n}-v^{2n+2})}{(2n)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-v^{2n}+v^{2n}-v^{2n+2})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}}$$

$$+J_z(x-v\theta)$$

Thus, we have

$$\frac{\partial H_y(x,\theta)}{\partial x} = -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}(1-v^{2n+2})}{(2n)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-v^{2n+2})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}} + J_z(x-v\theta) \quad (21)$$

By (16), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = -\frac{1}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n}(1-(v^2)^{n+1})}{(2n)!}\frac{\partial^{2n}J_z(x)}{\partial x^{2n}} + \frac{v}{1-v^2}\sum_{n=0}^{\infty}\frac{\theta^{2n+1}(1-(v^2)^{n+1})}{(2n+1)!}\frac{\partial^{2n+1}J_z(x)}{\partial x^{2n+1}} \quad (22)$$

Comparing (21) and (22), we have

$$\frac{1}{\eta}\frac{\partial E_z(x,\theta)}{\partial x} = \frac{\partial H_y(x,\theta)}{\partial x} - J_z(x-v\theta)$$

Thus, (15) and (16) satisfy (1d.1) and (1d.2).

QED.

Example 1 – Time-invariant Gaussian source

The source is

$J_z(x-v\theta,\theta)=J_z(x-v\theta)=e^{-a(x-v\theta)^2}$	(e1.1)
$\Gamma_z(\varsigma,\theta)=e^{-a\varsigma^2}$	(e1.2)

This is a time-invariat moving source. The solution is given by (15) and (16).

From (e1.1), we have

$\frac{\partial^{2n}}{\partial x^{2n}}J_z(x)=e^{-ax^2}(-1)^n a^n\sum_{k=0}^{n}(-1)^k\left(2\sqrt{a}x\right)^{2k}q_{0,n,k}$	(e1.3)
$\frac{\partial^{2n+1}}{\partial x^{2n+1}}J_z(x)=e^{-ax^2}2ax(-1)^{n+1}a^n\sum_{k=0}^{n}(-1)^k\left(2\sqrt{a}x\right)^{2k}p_{0,n,k}$	(e1.4)

Where

$p_{h,n,k}=\frac{(k+h)!\,n!\,(2(n+h)+1)!}{(n-k)!\,(n+h)!\,k!\,(2(k+h)+1)!}$ $h\geq 0;0\leq k\leq n$	(e1.5)
$q_{h,n,k}=\frac{(k+h+1)!\,(n+1)!\,(2(n+h))!}{(n-k)!\,(n+h+1)!\,(k+1)!\,(2(k+h))!}$ $h\geq 0;0\leq k\leq n$	(e1.6)

Substitute (e1.3) and (e1.4) into (15) and (16), we have

$\xi=\sqrt{a}\theta$	(e1.7)
$\varrho=2\sqrt{a}x$	(e1.8)
$H_y(x,\theta)=\frac{1}{\sqrt{a}}\frac{e^{-ax^2}}{1-v^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\sum_{k=0}^{n}(-1)^k\varrho^{2k+1}p_{0,n,k}$ $+\frac{1}{\sqrt{a}}\frac{ve^{-ax^2}}{1-v^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+1}(1-(v^2)^n)}{(2n+1)!}\sum_{k=0}^{n}(-1)^k\varrho^{2k}q_{0,n,k}$	(e1.9)
$E_z(x,\theta)=-\frac{1}{\sqrt{a}}\frac{\eta ve^{-ax^2}}{1-v^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}(1-(v^2)^{n+1})}{(2n+2)!}\sum_{k=0}^{n}(-1)^k\varrho^{2k+1}p_{0,n,k}$ $-\frac{1}{\sqrt{a}}\frac{\eta e^{-ax^2}}{1-v^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+1}(1-(v^2)^{n+1})}{(2n+1)!}\sum_{k=0}^{n}(-1)^k\varrho^{2k}q_{0,n,k}$	(e1.10)

Use the hyper-exponential functions (see [2]), (e1.9) and (e1.10) can be written as

$H_y(x,\theta)=\frac{1}{1-v^2}\frac{e^{-ax^2}}{\sqrt{a}}(esinh(\xi,\varrho)_1-esinh(v\xi,\varrho)_1+v\ eicoshi(\xi,\varrho)-eicoshi(v\xi,\varrho))$	(e1.11)
$E_z(x,\theta)=\frac{\eta}{1-v^2}\frac{e^{-ax^2}}{\sqrt{a}}(-eicoshi(\xi,\varrho)+v\ eicoshi(v\xi,\varrho)-v\ esinh(\xi,\varrho)_1+v\ esinh(v\xi,\varrho)_1)$	(e1.12)

Example 2 - Trigonometric source with Gaussian decay

The source is

$J_z(x-v\theta,\theta)=e^{-a(x-v\theta)^2+i\omega\theta}$	(e2.1)
$a>0;\ \omega>0$	

We have

$\varsigma=x-v\theta$	(e2.2)
$J_z(x-v\theta,\theta)=\Gamma_z(\varsigma,\theta)=e^{-a\varsigma^2+i\omega\theta}$	(e2.3)

We get

$$f(x)=e^{-ax^2}$$

$$\frac{\partial^n\Gamma_z(\varsigma,0)}{\partial\theta^n}=(i\omega)^n f(x)$$

$$\frac{\partial^n \Gamma_z(\varsigma,0)}{\partial \varsigma^n} = \left.\frac{\partial^n J_z(x-v\theta,\theta)}{\partial x^n}\right|_{\theta=0} = \frac{d^n f(x)}{dx^n}$$

Thus

$\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)-k}} = \frac{\partial^{2m+k+1}}{\partial\varsigma^{2m+k+1}}\frac{\partial^{2(n-m)-k}\Gamma_z(\varsigma,0)}{\partial\theta^{2(n-m)-k}} = (i\omega)^{2(n-m)-k}\frac{\partial^{2m+k+1}}{\partial x^{2m+k+1}}f(x)$	(e2.4)
$\frac{\partial^{2n+2}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k+1}\partial\theta^{2(n-m)+1-k}} = \frac{\partial^{2m+k+1}}{\partial\varsigma^{2m+k+1}}\frac{\partial^{2(n-m)+1-k}\Gamma_z(\varsigma,0)}{\partial\theta^{2(n-m)+1-k}} = (i\omega)^{2(n-m)+1-k}\frac{\partial^{2m+k+1}}{\partial x^{2m+k+1}}f(x)$	(e2.5)
$\frac{\partial^{2n+1}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)+1-k}} = \frac{\partial^{2m+k}}{\partial\varsigma^{2m+k}}\frac{\partial^{2(n-m)+1-k}\Gamma_z(\varsigma,0)}{\partial\theta^{2(n-m)+1-k}} = (i\omega)^{2(n-m)+1-k}\frac{\partial^{2m+k}}{\partial x^{2m+k}}f(x)$	(e2.6)
$\frac{\partial^{2n}\Gamma_z(\varsigma,0)}{\partial\varsigma^{2m+k}\partial\theta^{2(n-m)-k}} = \frac{\partial^{2m+k}}{\partial\varsigma^{2m+k}}\frac{\partial^{2(n-m)-k}\Gamma_z(\varsigma,0)}{\partial\theta^{2(n-m)-k}} = (i\omega)^{2(n-m)-k}\frac{\partial^{2m+k}}{\partial\varsigma^{2m+k}}f(x)$	(e2.7)

Substitute the above into (6) and (7), and separate sine and cosine sources, we get the moving source solutions.

Sine source solution

$J_z(x-v\theta,\theta) = e^{-a(x-v\theta)^2}\sin(\omega\theta)$	(e2.14)
$\varrho = 2\sqrt{a}x$	(e2.15)
$\xi = \omega\theta$	(e2.16)
$\varphi = \frac{\sqrt{a}}{\omega}$	(e2.17)
$\begin{aligned} H_y(x,\theta,v) &= \frac{e^{-ax^2}}{\omega}\sum_{n=1}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n-1}\varphi^{2m+1}\sum_{h=0}^{n-m-1}\binom{2n-2m}{2h+1}(v\varphi)^{2h+1}\sum_{k=0}^{m+h+1}(-1)^k\varrho^{2k}q_{0,m+h+1,k} \\ &+\frac{e^{-ax^2}}{\sqrt{a}}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\varphi^{2m+2}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h}(v\varphi)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k} \end{aligned}$	(e2.18)
$\begin{aligned} E_z(x,\theta,v) &= -\eta\frac{e^{-ax^2}}{\omega}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\varphi^{2m}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h}(v\varphi)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k}q_{0,m+h,k} \\ &+\eta\frac{e^{-ax^2}}{\sqrt{a}}\sum_{n=1}^{\infty}\frac{(-1)^n\xi^{2n+1}}{(2n+1)!}\sum_{m=0}^{n-1}\varphi^{2m+1}\sum_{h=0}^{n-m}\binom{2n-2m}{2h+1}(v\varphi)^{2h+1}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k} \end{aligned}$	(e2.19)

Cosine source solution

$J_z(x-v\theta,\theta) = e^{-a(x-v\theta)^2}\cos(\omega\theta)$	(e2.8)
$\varrho = 2\sqrt{a}x$	(e2.9)
$\xi = \omega\theta$	(e2.10)
$\varphi = \frac{\sqrt{a}}{\omega}$	(e2.11)
$\begin{aligned} H_y(x,\theta,v) &= \frac{e^{-ax^2}}{\sqrt{a}}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\varphi^{2m+2}\sum_{h=0}^{n-m}\binom{2n-2m}{2h}(v\varphi)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k} \\ &-\frac{e^{-ax^2}}{\omega}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\varphi^{2m+1}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h+1}(v\varphi)^{2h+1}\sum_{k=0}^{m+h+1}(-1)^k\varrho^{2k}q_{0,m+h+1,k} \end{aligned}$	(e2.12)
$\begin{aligned} E_z(x,\theta,v) &= -\eta\frac{e^{-ax^2}}{\sqrt{a}}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\varphi^{2m+1}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h+1}(v\varphi)^{2h+1}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k} \\ &-\eta\frac{e^{-ax^2}}{\omega}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\varphi^{2m}\sum_{h=0}^{n-m}\binom{2n-2m}{2h}(v\varphi)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k}q_{0,m+h,k} \end{aligned}$	(e2.13)

Michelson-Morley experiment

Once the analytical solutions to Maxwell's equations are available, it is possible to examine Michelson-Morley experiment analytically.

The Michelson-Morley experiment involves two beams of light, and uses mirrors to make the two beams meet. Keep in mind that the two beams should be treated as being independent even though they can be generated by one light source.

Assumptions made in the history

We saw some math analysis of the experiment treating the beams as bullets travelling at a constant speed. Because it is to detect phase differences of the two beams, the experiment result should be produced by treating the beams as electromagnetic waves which are solutions to Maxwell's equations.

Since the solutions to Maxwell's equations were not available, the experiment expectations were made based on inaccurate/wrong guesses. Let's examine those guesses first.

To describe the assumptions made in the history, let's use a math function to represent a beam of light:

$$f(x, \theta, v)$$

Where v is the source moving speed, v is a constant. (x, θ) is the rest frame.

There are two guesses in the history (or say, currently).

Guess 1 is made for beam 1. It is made by assuming that the stationary source solution to Maxwell's equations is a function of constant speed,

$f(x, \theta, 0) = f_1(x - \theta)$	(mm.1)

such as

$f_1(x - \theta) = \cos\big(\omega(x - \theta)\big)$	(mm.2)

Guess 2 is made for beam 2. It is made by assuming that the moving source solution to Maxwell's equations is produced by applying coordinate transformation to the stationary source solution.

People (including Einstein, see section 6 of [3]) had to make such a guess because they did not know what was the moving source solution to Maxwell's equations; something like (mm.1) and (mm.2) were all they could start with and rely on.

Michelson and Morley made Guess 2 using Galilean coordinate transformation, and got beam 2 as

$f_2(x, \theta, v) = \cos\big(\omega(x + v\theta - \theta)\big)$	(mm.3)

The experiment was designed to detect the phase difference between $f_1(x - \theta)$ and $f_2(x, \theta, v)$; the expected result was $v\theta$.

Measuring $v\theta$ is difficult. Michelson and Morley made an ingenious design of changing the speed v by rotating the experiment device. Thus, a difference of "phase by speed" is generated.

$f_2(x, \theta, v_1) = \cos\big(\omega(x + v_1\theta - \theta)\big)$ $f_2(x, \theta, v_2) = \cos\big(\omega(x + v_2\theta - \theta)\big)$ $\underline{phase\ differece\ = (v_1 - v_2)\theta}$	(mm.4)

Now we do not need to measure $v\theta$. The expectation of the experiment is that when the speeds v_1 and v_2 change, $(v_1 - v_2)\theta$ also changes, which can be observed via an interference pattern change.

The experiment failed to detect changes of $(v_1 - v_2)\theta$. It is called a null-result of the Michelson-Morley experiment.

To explain this so-called null-result, Lorentz proposed to use a different coordinate transformation in Guess 2, his calculations treat the light beams as bullets, not as waves. Using it to explain the disappearance of wave phase difference is questionable because there is not a phase in a bullet. Let me explain it in more detail.

Lorentz coordinate transformation is

$\gamma = \dfrac{1}{\sqrt{1 - v^2}}$ $x = \gamma x' + v\gamma\theta'$ $\theta = v\gamma x' + \gamma\theta'$	(mm.5)

We have

$f_2'(x', \theta', v) = \cos\big(\omega\gamma(1 - v)(x' - \theta')\big)$	(mm.6)

The speeds of $f_1(x - \theta)$ and $f_2'(x', \theta', v)$ are the same, as Lorentz wanted. But the frequencies are different, generating a problem of comparing phases.

On top of the Lorentz coordinate transformation, Einstein added a new state transformation to produce a so-called Lorentz Covariance of Maxwell's equations, forming a theory known as the special theory of relativity.

Michelson, Morley, Lorentz and Einstein all used the above two guesses as the foundation for their work. Let's examine these two guesses.

From the analytical solutions to Maxwell's equations, we know Guess 1 is wrong. For details, see [4].

Let's examine Guess 2.

Suppose we have a solution to Maxwell's equations for a moving source in the rest frame:

$$H_y(x,\theta,v)$$

$$E_z(x,\theta,v)$$

Then, beam 1 is

Beam 1: $H_y(x,\theta,0)$ $E_z(x,\theta,0)$	(mm.7)

Guess 2 is

Beam 2 by Guess 2 and Galilean coordinate transformation: $H_y(x+v\theta,\theta,0)$ $E_z(x+v\theta,\theta,0)$	(mm.8)

We can see that Guess 2 does not produce Beam 2 in Michelson-Morley experiment; Guess 2 is to observe Beam 1 from the moving frame.

Beam 2 in Michelson-Morley experiment is an independent beam; it should be applying a coordinate transformation to a moving source solution, not to a stationary source solution which is Beam 1.

Correct Beam 2 via Galilean coordinate transformation applied to the moving source solution: $H_y(x+v\theta,\theta,v)$ $E_z(x+v\theta,\theta,v)$	(mm.9)

We now see that Guess 2 is also wrong.

Using the wrong Guess 2, there is only one beam of light, beam 1. Beam 2 disappears. But there are two observers; one observer is at the rest frame, another observer is at the moving frame.

For the Michelson-Morley experiment, there are two beams of light and one observer.

What is the consequence of making the wrong guess (Guess 2)? Is there a problem using (mm.8), instead of (mm.9), to get the expected result of the Michelson-Morley experiment?

To answer the questions, I made some calculations using (mm.7), (mm.8) and (mm.9).

From the calculations, we can see that using (mm.8) to replace (mm.9) makes significant differences.

I am presenting my calculations below.

Calculation formulas

To make the calculations easier, the following field source is used:

$$J_z(x-v\theta,\theta)=\sqrt{a}e^{-a(x-v\theta)^2}\cos\left(\sqrt{a}\theta\right)$$

The solution become

$\varrho=2\sqrt{a}x$	(e2.9)
$\xi=\sqrt{a}\theta$	(e2.10a)
$\omega=\sqrt{a}$	(e2.11a)
$H_y(x,\theta,v)=e^{-ax^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m}{2h}(v)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k}$ $-e^{-ax^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h+1}(v)^{2h+1}\sum_{k=0}^{m+h+1}(-1)^k\varrho^{2k}q_{0,m+h+1,k}$	(e2.12a)
$E_z(x,\theta,v)=-\eta e^{-ax^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h+1}(v)^{2h+1}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k}$ $-\eta e^{-ax^2}\sum_{n=0}^{\infty}\frac{(-1)^n\xi^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m}{2h}(v)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k}q_{0,m+h,k}$	(e2.13a)

The above solution can be calculated by

$H_y[\varrho,\xi,v,N]\approx e^{-\frac{1}{4}\varrho^2}\sum_{n=0}^{N}\frac{(-1)^n\xi^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m}{2h}(v)^{2h}\sum_{k=0}^{m+h}(-1)^k\varrho^{2k+1}p_{0,m+h,k}$ $-e^{-\frac{1}{4}\varrho^2}\sum_{n=0}^{N}\frac{(-1)^n\xi^{2n+3}}{(2n+3)!}\sum_{m=0}^{n}\sum_{h=0}^{n-m}\binom{2n-2m+1}{2h+1}(v)^{2h+1}\sum_{k=0}^{m+h+1}(-1)^k\varrho^{2k}q_{0,m+h+1,k}$	(e2.12b)

$$E_z[\varrho,\xi,v,N] \approx -\eta e^{-\frac{1}{4}\varrho^2} \sum_{n=0}^{N} \frac{(-1)^n \xi^{2n+2}}{(2n+2)!} \sum_{m=0}^{n} \sum_{h=0}^{n-m} \binom{2n-2m+1}{2h+1} (v)^{2h+1} \sum_{k=0}^{m+h} (-1)^k \varrho^{2k+1} p_{0,m+h,k}$$
$$-\eta e^{-\frac{1}{4}\varrho^2} \sum_{n=0}^{N} \frac{(-1)^n \xi^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \sum_{h=0}^{n-m} \binom{2n-2m}{2h} (v)^{2h} \sum_{k=0}^{m+h} (-1)^k \varrho^{2k} q_{0,m+h,k} \quad \text{(e2.13b)}$$

The above formulas are independent of field source frequency, showing that for this kind of source, the characteristics of the electromagnetic fields are independent of the source frequency, that is, for a blue light or for a red light, the behaviors are the same.

For large ϱ and ξ, large N is needed for accuracy. The calculations of (e2.12b) and (e2.13b) with large N can be challenging for a personal computer.

I assembled a desktop computer with all the parts from Amazon, using an Intel 24 core CPU (Core i-9 14900K) and a Nvidia 3036 core GPU (GeForce 4096 TI, 16 G memory). I spent a lot of effort coding CUDA on the GPU. But I could not get good HPC performance from the GPU. On reading some reports from other people, including a report from University of Tsukuba ([5]), I gave up my efforts to try to use GeForce GPU. Even though [5] is a report from 10 years ago, its results match what I got with my newer GPU and CPU. I ended up relying on Intel's CPU to make the calculations.

Boost multiprecision library [6] was used to make the calculations. I used a decimal precision of 5000 digits.

My C++ source code is uploaded to Github for anyone who wants to examine my calculations.

Propagation of electromagnetic field

Let's use field snapshots at two times to see how the electromagnetic field propagates in space over time.

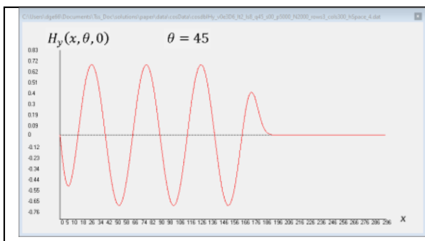

Fig.1. magnetic field at $\theta = 45$

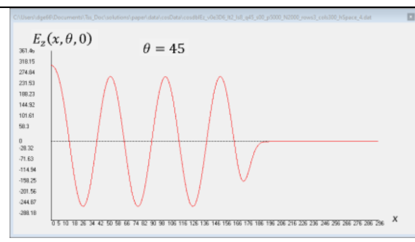

Fig.2. electric field at $\theta = 45$

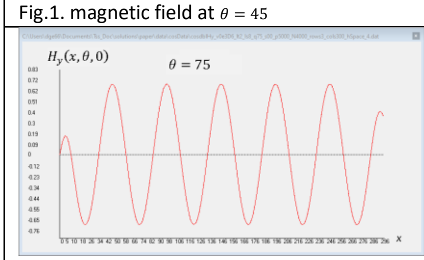

Fig.3. magnetic field at $\theta = 75$

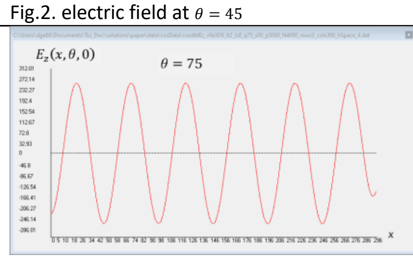

Fig. 4. electric field at $\theta = 75$

Fig.1 shows a snapshot of the magnetic field at time $\theta = 45$.
Fig.3 shows a snapshot of the magnetic field at time $\theta = 75$.

Fig.2 shows a snapshot of the electric field at time $\theta = 45$.
Fig.4 shows a snapshot of the electric field at time $\theta = 75$.

These figures show the beam of light propagating in space over time.

Phase differences

Let the source move at the earth speed:

$$\mathrm{v} = 0.006$$

Magnetic field:

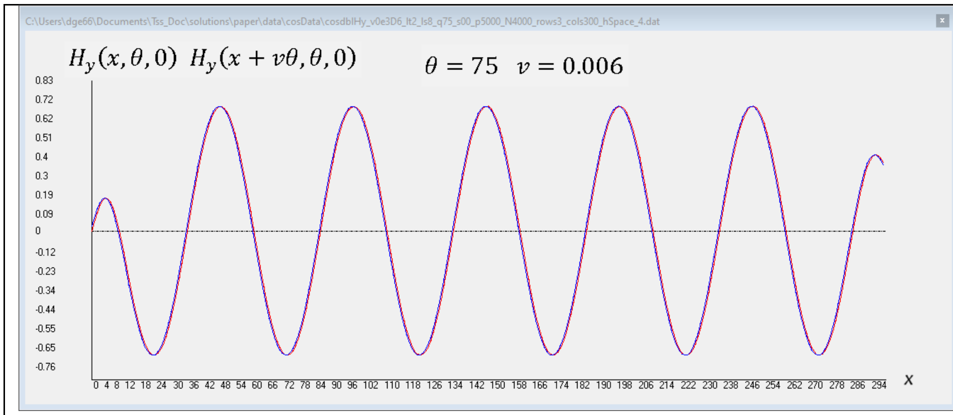

Fig.5. magnetic fields at $\theta = 75$, use Guess 2: comparing (mm.7) and (mm.8)

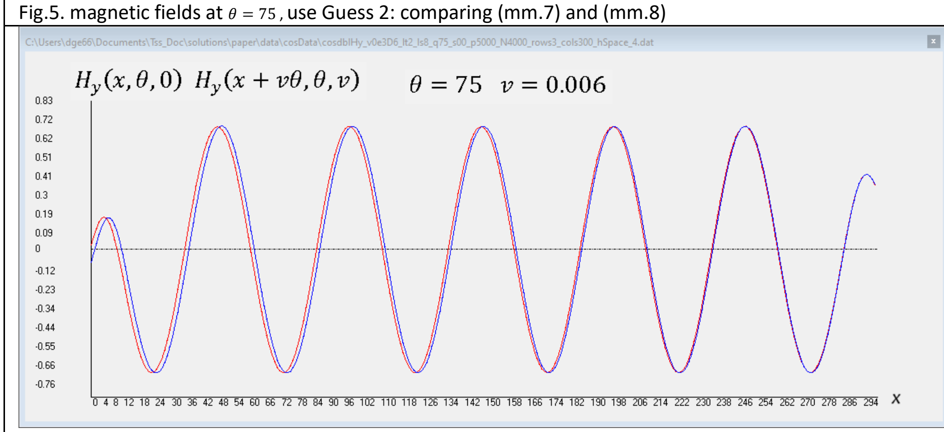

Fig.6. magnetic fields at $\theta = 75$, use moving source solution: comparing (mm.7) and (mm.9)

Electric field:

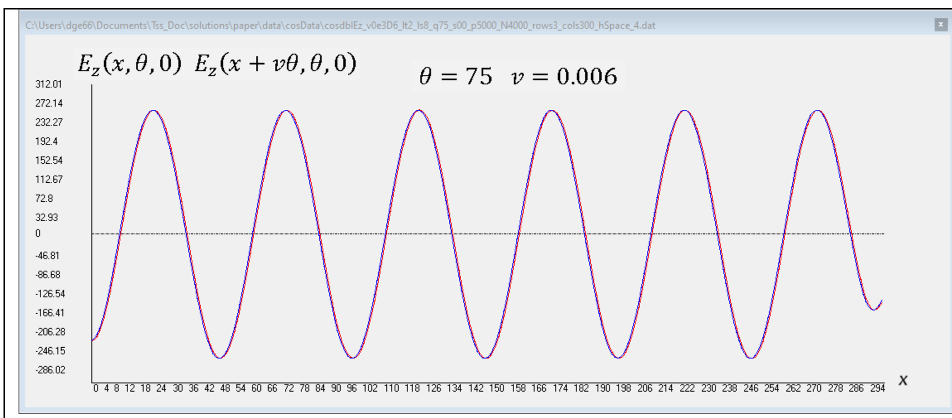

Fig.7. electric fields at $\theta = 75$, use Guess 2: comparing (mm.7) and (mm.8)

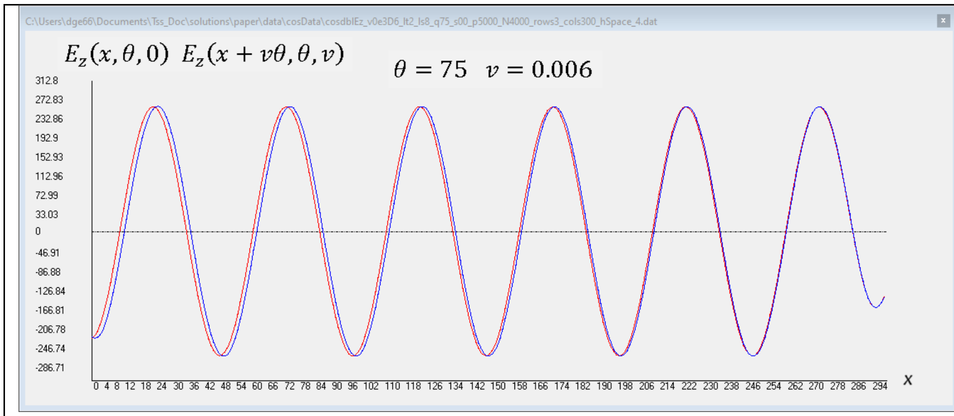

Fig.8. electric fields at $\theta = 75$, use moving source solution: comparing (mm.7) and (mm.9)

Fig.5 and Fig.7 show consistent phase differences. This is the expectation of Guess 2. The phase difference is supposed to be $v\theta$. Because θ is small in these figures due to the limitations of my computing power, the phase differences in Fig.5 and Fig.7 are small.

Fig.6 and Fig. 8 show non-consistent phase differences for the moving source solution. For these few cycles, we see both large phase differences and very small phase differences like “null-results”.

These figures show significant differences between applying coordinate transformation to the solutions of the stationary source and the moving source.

Effects of source speeds

Use the following two source speeds to generate electromagnetics fields:

$$v_1 = 0.006$$

$$v_2 = 0.003$$

Magnetic field:

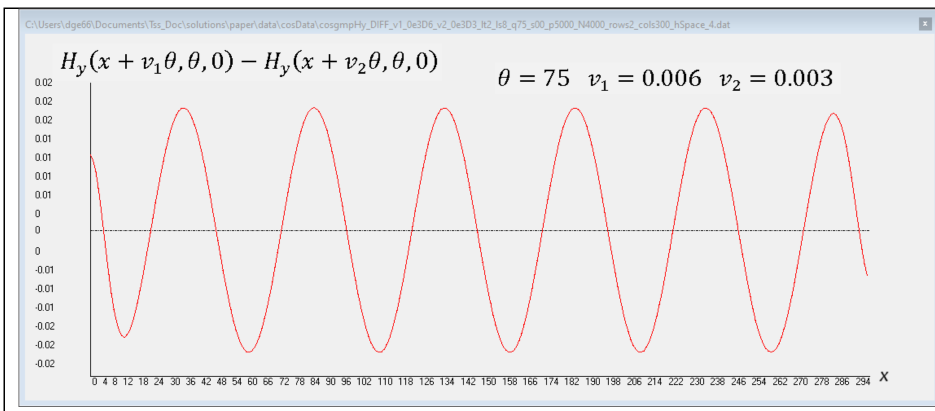

Fig.9. magnetic field difference at $\theta = 75$, use Guess 2

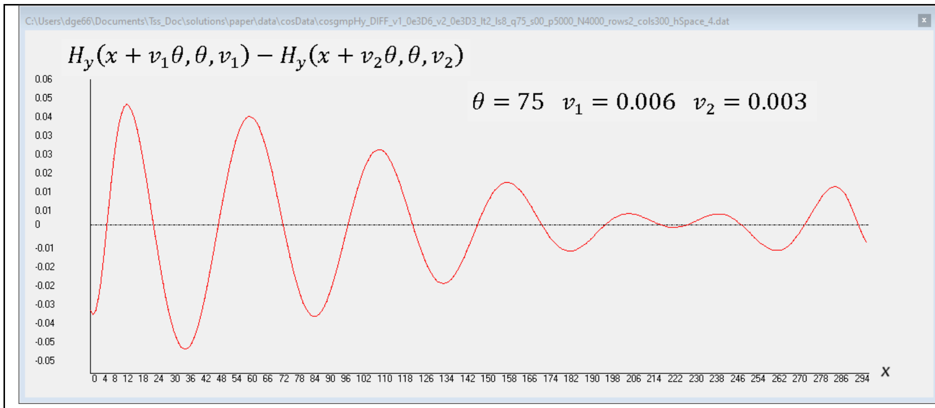

Fig.10. magnetic field difference at $\theta = 75$, use moving source solutions

Electric field:

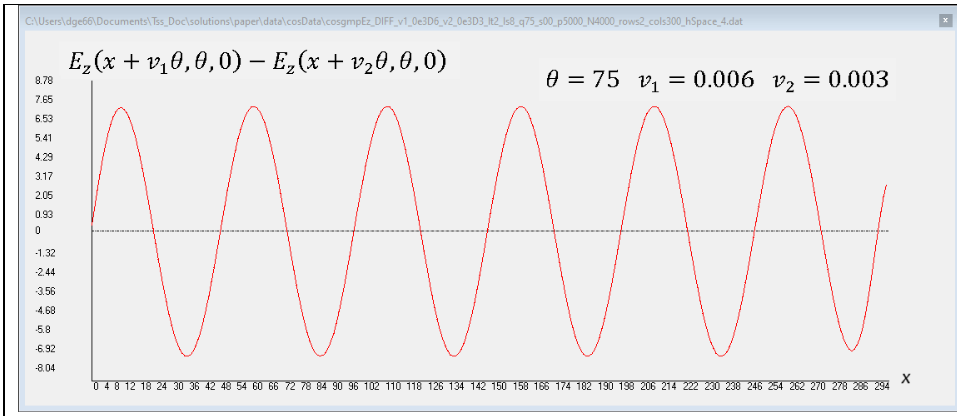

Fig.11. electric field difference at $\theta = 75$, use Guess 2

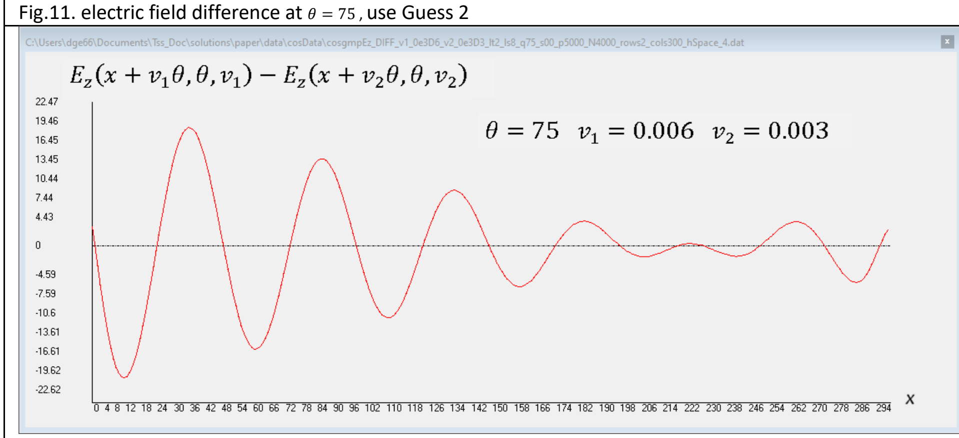

Fig.12. electric field difference at $\theta = 75$, use moving source solutions

Fig. 9 and Fig.11 show the electromagnetic field produced by applying coordinate transformation to the solution to Maxwell's equations of the stationary source.

Fig. 10 and Fig.12 show the electromagnetic field produced by applying coordinate transformation to the solution to Maxwell's equations of the moving source.

These figures show significant differences between applying coordinate transformation to the solutions of the stationary source and the moving source.

Small $H_y(x+v_1\theta,\theta,v_1)-H_y(x+v_2\theta,\theta,v_2)$ and $E_z(x+v_1\theta,\theta,v_1)-E_z(x+v_2\theta,\theta,v_2)$ represent "null-result". We do see that these values are getting small over the space. My computer is not powerful enough to calculate for longer time and space.

Summary

Generic analytical solutions to Maxwell's equations for moving sources are obtained and proved for 1D fields. The same approach can be used to get solutions for 3D fields.

Electromagnetic fields are calculated using the solutions to Maxwell's equations in response to a cosine source. The calculation results show significant differences between applying coordinate transformation to a stationary source solution and to a moving source solution, leading to a question whether non-null results of Michelson-Morley experiment should be expected.

Accurate expectations of the Michelson-Morley experiment can be obtained by calculating the solutions to Maxwell's equations. But the computing power required is far beyond a desktop computer can offer.

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