MATH H113: Honors Introduction to Abstract Algebra

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- http://math.berkeley.edu/~vojta/H113.html/

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- Introduction
- Overview of the course
- Sets etc.
- Functions

For Friday: read sect. 0.1, 0.2 (esp. prop. 1 on p. 2 (optional bonus, find the mistake in prop. 1, hint: it includes $\emptyset$).

Grading

Homework, due weekly in class	30%
Midterm 1, Wed. 2016-02-17	15%
Midterm 2, Wed. 2016-04-06	20%
Final, Mon. 2016-05-12	35%

Exams will be cumulative

Book

Dummit & Foote Abstract Algebra (3rd Edition)

Intorduction to Material

You should know: $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ and how to add, subtract, multiply and divide them (where applicable). If you just look at any of these sets with just addition and subtraction (no multiplication, division, limits, calculus etc.) they are *groups*. Other examples of groups ($n \times n$ with addition and subtraction), invertible matrices with multiplication.

If we look at any of these sets with just addition, subtraction and multiplication (but not division, inequalities, limits, etc.) these are examples of *rings*. Other examples include $n \times n$ matrices, polynomials

If we look at $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ with $+, -, \times, \div$, these are examples of fields.

Start with the actual course: For sets, you should know union ($\bigcup$), intersection ($\bigcap$), $\subseteq$, $\subsetneq$, $\in$ (I don't use $\subset$, because it is ambiguous as to whether or not it is proper).
$\{a \in A | \text{some condition}\}$
Cartesian product: $A \times B$, $\{(a, b).a \in A, b \in B\}$ ((a, b) is an ordered pair) We can also have multiple products:
$A \times B \times C$
Infinity products:
$\prod\limits_{n=1}^{\infty} A_n$ or $\prod\limits_{z \in \mathbb{C}} A_z$ etc.
Finite products:
$\prod\limits_{i=1}^{n} A_i$

Since we generally don't work with limits, we won't work with $\sum\limits_{n=1}^{\infty} a_n$, $\prod\limits_{n=1}^{\infty}$, $\sum\limits_{x \in S} a_x$, or $\prod\limits_{x \in S} a_x$
Exception: if all but finitely many of the a_ns are 0, then some $\sum\limits_{n=1}^{\infty} a_n$ or $\sum\limits_{x \in S} a_x$ are really finite sums, so we may refer to those.
Likewise: Infinite products if *all but finitely many* ("almost all") of the a_i are 1, then $\prod\limits_{n=1}^{\infty} a_n$ or $\prod\limits_{x \in S} a_x$ are really finite.

In the other extreme: What are $\sum\limits_{i=1}^{n} a_i$ if $n = 0$ or $\sum\limits_{x \in S}$ with $S = \emptyset$?
0 (the identity of addition)
With the forumlas for combining sums: e.g.
$\sum\limits_{i=1}^{n} a_i + \sum\limits_{j=1}^{m} a_{n+j} = \sum\limits_{i=1}^{n+m} a_i$ (works if $m = 0$ or $n = 0$)
Similarly: $\prod_{i=1}^{n} a_i = 1$ if $n = 0$, $\prod\limits_{i \in S} a_i = 1$ if $S = \emptyset$.

Infinite union: $\bigcup\limits_{i=1}^{\infty} A_i$ or $\bigcup\limits_{i \in S} A_i$
and infinite intersection: $\bigcap\limits_{i=1}^{\infty} A_i$ or $\bigcap\limits_{i \in S} A_i$

What is $\bigcup\limits_{i=1}^{n} A_i$ when $n = 0$? $\emptyset$
Similarly $\bigcap\limits_{i=1}^{\infty}$ when $n = 0$? **undefined**, but usually you should say the whole set from which A_is are drawn.
$\bigcap\limits_{i=1}^{n} A_i = (\bigcup\limits_{i=1}^{n} A_i^{\mathrm{c}})^{\mathrm{c}} = \emptyset^{\mathrm{c}}$

Empty product: $\prod_{i \in S} A_i = \{\text{one element}\}$
Works with formulas like:
$\prod_{i \in S} \times \prod_{i \in T} \quad \prod_{i \in S \bigcup T} A_i$ if $S \bigcap T \neq \emptyset$ (the two are not equal, but naturally bijective).

Notation

$\mathbb{Z}^+ = \mathbb{Z}_{>0} = \{n \in \mathbb{Z} : n > 0\} = \{1, 2, 3, \ldots\}$
$\mathbb{Q}^+ = \mathbb{Q}_{>0} = \{n \in \mathbb{Q} : n > 0\}$
$\mathbb{R}^+ = \mathbb{R}_{>0} = \{n \in \mathbb{R} : n > 0\}$
$\mathbb{N} = \mathbb{Z}_{\geq 0} = \{n \in \mathbb{Z} : n \geq 0\} = \{0, 1, 2, \ldots\}$

Functions

You should know what a function $f : A \to B$ means (A is the domain, B is the codomain) (also $A \xrightarrow{f} B$) a subset Γ of $A \times B$ such that $\forall a \in A. \exists! b \in B$ for which $(a, b) \in \Gamma$

The image of the domain B $\{b \in B | \exists a \in A. b = f(a)\} = f(A) \subseteq B$
If $A' \subseteq A$ then $f(A') = \{f(a) : a \in A'\} =$ image of A'

If $B' \subset eqB$, then the preimage of B' is $f^{-1}(B') = \{a \in A : f(a) \in B'\}$
If $b \in B$ the *fiber over* b is $f^{-1}(b) = f^{-1}(\{b\}) = \{a \in A : f(a) = b\}$
not to be confused with $f^{-1}(b) = g(b)$ if f has an inverse function $f^{-1} = g$. We know from the context which it is.