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In Search of Landau's Ghost*

by

David J.E. Callaway
Department of Physics
The Rockefeller University
1230 York Avenue
New York, NY 10021-6399

ABSTRACT:

The concept that field-theoretic parameters such as the standard model Higgs mass might be predictable or bounded by self-consistency arguments is discussed within the framework of the real-space renormalization group. Results from the first such calculation for the SU(2)xU(1) standard model are reviewed. A new variational method for renormalization group calculations is presented.

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1. Prolegomena: Ghostbusting for Nonspecialists

Exorcism has become a prime industry for field theorists. In this context, "exorcism" means the elimination of a worrisome inconsistency in field theories known as a Feldman-Landau "ghost". This sort of ghost can be found in the momentum-dependent running coupling constant, corresponding to the effective charge of a particle as measured by a probe of a given mementum. If a Landau ghost is present, the running coupling becomes infinite at a certain finite probe momentum. This is a symptom of an inconsistency which also leads to such phenomena as tachyons (see [1] for a review).

The input needed for the computation of the running coupling constants $\{g_i(t)\}$ is the corresponding set of beta functions $\{\beta_i\}$:

$$\frac{dgi}{dt} = \beta_i \{g\}$$

where $t = \frac{1}{2} \ln (Q^2/\mu^2)$, Q is the probe momentum, μ sets the scale at which the renormalized couplings $\{g_{R,i}\}$ are defined:

$$g_{R,i} \equiv g_i \ (t=0)$$

Landau ghosts can be eliminated from the theory if there is a simultaneous zero (a "fixed point") of the beta functions which is approached by the $g_R(t)$ as t increases. In the language of the renormalization group, the $g_{R,i} \equiv g_i\ (t=0)$ must therefore lie within a domain of attraction of a fixed point. This domain of attraction can be as small as a single point. For example, in one-and two-component $\lambda\emptyset^4$ field theory it is known that the only fixed point occurs when λ_R is equal to zero. Discarding the highly unlikely possibility that the theory is asymptotically free, the domain of attraction of this fixed point is limited to the point itself. Thus λ_R is predicted to be zero, and the theory to be "trivial" or noninteracting. More generally [1], coupling constants and mass ratios can be predicted by such consistency conditions.

Obviously the idea that parameters such as the Higgs mass might be predictable from renormalization group consistency requirements is a highly prorocative one. The major difficulty in putting it into practice stems from the fact that nonperturbative techniques are needed for the analysis. This becomes clear from a perusal of results for the beta functions of theories like the standard model of the weak interaction.

Typically only the lowest-order terms in a perturbative expansion of these quantities are known. Analyses based upon perturbation theory possess the serious flaw that the only fixed points which can be analyzed are those near the origin (like in asymptotically free theories, which possess an attractive zero of the beta functions at the origin). Thus theories which are not asymptotically free will automatically appear to have Landau ghosts in general. This is especially problematic in models of the weak interaction, for it is difficult to construct theories which are both asymptotically free in all couplings and also exhibit complete spontaneous symmetry breaking (see [1] for a review).

2. The Real-Space Renormalization Group: An Exercise in Exorcism

Thus the developments of nonperturbative renormalization group techniques is essential for a proper understanding of models of the weak interaction. The real-space renormalization group (RSRG) is one particularly promising nonperturbative technique. (It should be understood that "renormalization group" refers here to the modern formalism of Wilson rather than to the historical architecture employed above). The RSRG involves separating the degrees of freedom of a system into blocks, then partially integrating over these to obtain a new system with a larger lattice spacing. At each step, the system can be parameterized by a set of coupling constants. As the degrees of freedom are systematically integrated out, the sequence of sets of couplings defines a renormalization group "flow" or trajectory as the lattice spacing increases.

Continuum limits of a lattice theory are approached by taking the lattice spacing to zero, and thus the dimensionless correlation length to infinity. These limits then are found on critical surfaces where ξ is infinite. Under a RSRG transformation with scale factor b, $\xi \to \xi/b$ and so points on the critical surface are mapped <u>into</u> the critical surface. Fixed points under this transformation define the nature of the continuum limits taken within their domain of attraction. Specifically, the number of relevant directions at a RSRG fixed point (the number of axes along which the flow is repulsive) equals the number of independent renormalized parameters of all continuum theories defined on this portion of the critical surface. <u>Marginal</u> directions (those in which the flow goes <u>through</u> the fixed point) give rise to <u>bounds</u> on coupling constants. The derivatives of the block couplings with respect to the original couplings can also be used to find the anomalous dimensions of various operators. If these are nonzero, the theory is likely nontrivial.

The economy of the RSRG is now evident. Instead of studying all of the uncountably infinite ways of approaching each of the points on a critical surface, only a handful of fixed points must be analyzed. These will determine if the theory is nontrivial, and if a parameter (like the Higgs mass) is predictable or bounded from consistency requirements.

It was for these reasons that Roberto Petronzio and I recently performed [5] the first Monte Carlo renormalization group study of the (bosonic) SU(2)xU(1) standard model of the weak interaction. Our analysis used the simplest possible realization of the renormalization group. A comprehensive blocking transformation was employed. The resulting block action was approximated by one of the original (Wilson) form, but with effective couplings determined by Schwinger-Dyson identities. Several fixed points appeared in the resultant flow diagram, which implied (and was thus consistent with) many known results. One very interesting finding was that no fixed point appeared at finite SU(2) gauge coupling with as many relevant directions as there are renormalized parameters in the theory. Thus the possibility of a priori constraints on the coupling constants and/or mass ratios in the theory appears to be a realistic one.

Nevertheless, our optimism must be tempered with realism. Although corrections to our results can be obtained systematically to arbitrary accuracy, the analysis of errors in any RSRG calculation is not yet an exact science. A philosophy for choosing the optimal blocking transformation is needed. One such philosophy for gauge theories [4] is based upon the Kadanoff lower-bound renormalization group for spin systems [2]. This approach is now described.

3. Generalizing the Lower - Bound Renormalization Group to Gauge Theories

The Kadanoff lower-bound renormalization group (LBRG) predicts [2] critical exponents of spin systems with a precision not achieved by any other simple RG method (see [3] for a review). The LBRG gives $d\nu=2-\alpha$ to 0.1% accuracy in the d=2 and 3 Ising models, and to 1% accuracy in four dimensions. For ϕ^4 field theory it yields a one-dimensional integral equation whose solution give ν correctly to first order in the ε-expansion. It is also a direct approximation to an infinite system, so true finite-size effects are absent.

This amazing precision, coupled with the present level of interest in the renormalization-group structure of gauge-Higgs systems (see, e.g., [1]), provides motivation to generalize this simple technique to systems with local gauge symmetry. The generalization presented here is applicable to all gauge groups in arbitrary spacetime dimension, and reduces to the Kadanoff method for spin systems in the limit where the gauge coupling vanishes.

The crux of the LBRG is the observation that [since $\langle\exp(\Delta S)\rangle \geq \exp(\langle\Delta S\rangle)$] the addition to the action of an operator with vanishing expectation value lowers the free energy f. Interaction-moving operations satisfy this criterion by translational invariance. Variational parameters are introduced and are (easily) optimized at a fixed point to give a best lower bound for f. Critical exponents then follow directly from the recursion relations.

The application of the original LBRG to gauge theories is complicated by the fact that its block lattice points lie in the centers of hypercubes, while the gauge links {U} are only defined along its edges. It is therefore first necessary to use a prefacing transformation to map the original "plaquette" system a "subsumed" model which allows parallel transport to the center of a hypercube. The blocking then yields a plaquette model with larger lattice spacing, ready for the next iteration.

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An example facilitates explanation. Consider (Fig. 1) a single plaquette in a two-dimensional Z_2 gauge theory with vertices labelled (1,2,3,4) and action $S_{G,1234}=-\beta U_{12}U_{23}U_{34}U_{14}$ where $U_{ji}=U_{ij}=\pm 1$. Place a point c in the center of the plaquette. The corresponding subsumed model action is

$$\tilde{S}_{G,1234} = -\tilde{\beta}(V_{1c}U_{12}V_{2c} + V_{2c}U_{23}V_{3c} + V_{3c}U_{34}V_{4c} + V_{1c}U_{14}V_{4c})-C_{\beta} \tag{1}$$

where each of the $V_{ic} = \pm 1$ is an element of Z_2 which runs between points c and i=1 to 4. Since $\mathrm{Tr}_{\{V\}}[\exp(-\tilde{S})] = \exp(-S)$ it follows that $(\tilde{\beta})'' = \beta$ and $C_{\beta} = -2(\tilde{\beta})'-\beta-\ln 16$, where $x'' \equiv (x')'$ and $\tanh x' \equiv \tanh^2 x$. In higher spacetime dimensions, the point c is placed in the center of a hypercube. The prefacing transformation can be performed analytically for discrete groups. For larger groups more interactions [e.g., $(VUV)^2$, $(VUV)(VUV)$ etc.] must be included. Note that the prefacing is <u>underconstrained</u> -- many subsumed models correspond to one plaquette model.

The machinery of the generalized LBRG can be illustrated using a Z_2 gauge theory in d=2 dimensions. Figure 2 defines the notation. The original lattice fields (e.g., U_{67}) run between the original site points, denoted by numerals 1-16. The new fields of the subsumed model (e.g., V_{6e}) go between the original site points and new points labelled by letters a through i. Block points are denoted by crosses, and block fields (e.g., U'_{ag}) connect these. Thus (a,c,i,g) is the boundary of a typical block of side length b=2, which is the RG scaling factor.

A gauge-invariant projection operator P can be defined for the subsumed model. The projection operator determines the renormalized action $S_{G,R}$ via $\exp(-S_{G,R}) = \mathrm{Tr}_{\{U,V\}} P \exp(-\tilde{S}_G)$. Here P is a taken to be a product over all block links {U'} of terms like

$$P_{ag} = \exp[p_1 U'_{ag}(V_{6a}V_{6d}V_{10d}V_{10g} + V_{5a}V_{5d}V_{9d}V_{9g}) - N_{ag}] \quad (2)$$

where p_1 is a variational parameter. The requirement that the free energy remain invariant under the RG transformation implies that the trace of P_{ag} over U'_{ag} is unity, and thus

$$N_{ag} = (p_1)' \ (V_{6a}V_{6d}V_{5d}V_{5a})(V_{10d}V_{10g}V_{9g}V_{9d}) + C_1$$

with $C_1 \equiv (p_1)' + \ln 2$.

So far the RG transformation is <u>exact</u> (and intractable). The LB approximation consists of moving all interactions from the VUV terms and from the normalization terms (e.g. N_{ag}) in the product $P \exp(-\tilde{S}_G)$ into the shaded region (b,f,h,d), and equally to its counterparts in other blocks. The result for the contribution from block (a,c,i,g) is

$$
\begin{aligned}
& [P \exp(-\tilde{S}_G)]_{LB,acig} = \\
& \exp\{2\tilde{\beta}[U_{6,7}(V_{6b}V_{7b} + V_{6e}V_{7e}) \\
+ \quad & U_{7,11}(V_{7f}V_{11f} + V_{7e}V_{11e}) \\
+ \quad & U_{10,11}(V_{10e}V_{11e} + V_{10h}V_{11h}) \\
+ \quad & U_{6,10}(V_{6e}V_{10e} + V_{6d}V_{10d})] \quad - \quad 4C_\beta \\
+ \quad & p_1[U'_{ag}(V_{6a}V_{6d}V_{10d}V_{10g}) + U'_{gi}(V_{10g}V_{10h}V_{11h}V_{11i}) \\
& + U'_{ci}(V_{7c}V_{7f}V_{11f}V_{11i}) + U'_{ac}(V_{6a}V_{6b}V_{7b}V_{7c})] \\
- \quad & p'_1[(V_{6b}V_{7b}V_{6e}V_{7e})(V_{10e}V_{11e}V_{10h}V_{11h}) \\
& + (V_{6d}V_{6e}V_{10d}V_{10e})(V_{7e}V_{7f}V_{11e}V_{11f})] \\
& - 2C_1\}
\end{aligned}
\tag{3}
$$

The interaction-moving separates the original system into blocks within which the summations over {U,V} can be performed independently. The renormalized couplings β_R and C_R are found from

$$
\begin{aligned}
\exp[\beta_R(U'_{ag}U'_{gi}U'_{ci}U'_{ac}) + C_R] &= [\exp(-S_{G,R})]_{acig} \\
&= \mathrm{Tr}_{\{U,V\}}[P \exp(-\tilde{S}_G)]_{LB,acig}
\end{aligned}
\tag{4}
$$

The summations over the {U} and {V} are then performed. The block renormalized couplings β_R and C_R are found from the partial partition functions Z_+ and Z_-:

$$e^{2\beta_R} = \frac{Z_+}{Z_-} \tag{5a}$$

$$e^{2C_R} = Z_+ Z_- \tag{5b}$$

where

$$Z_+ = 2\, e^{(2\tilde{\beta})'} (A^2 + B^2) \tag{6a}$$

$$Z_- = 4\, e^{(2\tilde{\beta})'} (AB) \tag{6b}$$

and

$$A \equiv e^{2p_1'} + \gamma\, e^{-2p_1'} \tag{7a}$$

$$B \equiv 1 + \gamma \tag{7b}$$

$$\gamma \equiv \cosh\, [2.(2\tilde{\beta})'] \tag{7c}$$

$$= \exp\, [2.(2\tilde{\beta})'']$$

so that

$$\tanh \beta_R = \tanh^2 [p_1' - (2\tilde{\beta})''] \tanh^2 p_1' \tag{7d}$$

The variational parameter p_1 is determined from the extremum condition

$$0 = \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial K_\alpha} \frac{\partial K_\alpha}{\partial p_i} \tag{8}$$

where $\alpha = 1,..,n$; $K_{R,n} \equiv C_R$. This can be solved easily at a fixed point [2], for there $\partial f/\partial K_\alpha \equiv e_\alpha$ is a left eigenvector of the matrix $D_{\alpha\beta} \equiv \partial K_{R,\alpha}/\partial K_\beta$ with eigenvalue b^d $(= 2^2)$. In fact, Eq. (8) is then a determinant, since e_α is proportional to $\text{cof}\,(\Delta_{\alpha n})$, $\Delta_{\beta\alpha} \equiv D_{\alpha\beta} - b^d \delta_{\alpha\beta}$.

For a system with two coupling constants (β and C), the extremum condition at a fixed point is

$$\frac{\partial C_R}{\partial \beta}\ \frac{\partial \beta_R}{\partial p_1} + \frac{\partial C_R}{\partial p_1}\left[\frac{\partial C_R}{\partial C} - \frac{\partial \beta_R}{\partial \beta}\right] = 0 \tag{9a}$$

where here

$$\frac{\partial C_R}{\partial C} \equiv b^d = 4 \tag{9b}$$

The solution of Eqs. (9) is

$$p_1' = \frac{1}{4}\,\ell n\ \gamma \tag{10a}$$

$$= \frac{1}{2}(2\tilde{\beta})'' \tag{10b}$$

When Eqs. (10) are substituted into Eq. (7d), the result is the RG flow equation

$$\beta_R = \left[\frac{1}{2}\ (2\tilde{\beta})''\right]'' \tag{11}$$

which can be compared (cf. Fig. 3) with the exact result $\beta_R = \left[(\tilde{\beta})''\right]'' = \beta''$. [Note however that the extremum equation Eq. (9a) is strictly valid only at a fixed point].

The critical exponent for the fixed point at infinite β can be easily calculated,

$$\lim_{\beta \to \infty} \frac{\partial \beta_R}{\partial \beta} = 1 = b^y \tag{12}$$

The fixed point is therefore predicted to be <u>marginal</u> (y=0) as per the known result. The critical exponent for the pure Z_2 gauge theory is thus given <u>exactly</u> by the LBRG.

Scalar "Higgs" fields are easily included in the formalism. Consider for simplicity fixed-length Z_2 fields $\sigma_n = \pm 1$ (variable-length scalars are treated in [2]). The Higgs terms in the original action S as well as in the renormalized action S_R can be written entirely in terms of gauge-invariant objects like $h_{6,7} \equiv \sigma_6 U_{6,7} \sigma_7$ and $h'_{ac} \equiv \sigma'_a U'_{ac} \sigma'_c$ respectively. For scaling factor b=2 in two dimensions nothing larger than a plaquette can be included. Thus the allowable Higgs terms for plaquette (6,7,10,11) are:

$$O_{Plaq} \equiv h_{6,7}\, h_{7,11}\, h_{10,11} h_{6,10}$$

$$O_2 \equiv \tfrac{1}{2}\,(h_{6,7} + h_{7,11} + h_{10,11} + h_{6,10})$$

$$O_3 \equiv \tfrac{1}{2}\, h_{6,7}(h_{7,11} + h_{6,10})\,(1 + O_{Plaq})$$

$$O_4 \equiv 2\, O_{Plaq}\, O_2$$

$$O_5 \equiv \tfrac{1}{2}\, h_{6,7}\, h_{10,11}\,(1 + O_{Plaq}) \tag{13}$$

and their contribution to the action is $-\sum_{i=2}^{5} K_i O_i$. The Higgs contribution to the full action is the sum over all plaquettes of this quantity (note that $\beta \equiv K_1$ and likewise $\beta_R \equiv K_{R,1}$; $C_R \equiv K_{R,6}$ etc.). The projection operator Q for the scalars is a product over all block fields of terms like this for σ'_a:

$$Q_a = \exp[p_2 \sigma_a' \Sigma - \frac{1}{2} L_2 (\Sigma^2 - 4)$$

$$- M_2 \sigma_1 \sigma_2 \sigma_5 \sigma_6 \; V_{1a} V_{2a} V_{5a} V_{6a} - C_2]$$

$$\Sigma \equiv V_{1a}\sigma_1 + V_{2a}\sigma_2 + V_{5a}\sigma_5 + V_{6a}\sigma_6 \qquad (14)$$

where the normalization $Tr_{\sigma'} Q = 1$ implies that

$4L_2 = (2p_2)'$, $2M_2 = (2L_2)'$, $-C_2 = -2L_2 + M_2 - \ln 2$.

The LB approximation of moving all interactions equally into the region (b,f,h,d) and its counterparts in other blocks is made as before. The LB contribution to $\exp(-S_R)$ from block (a,c,i,g) is the sum over all contained {U,V} of

$$[PQ \exp(-\tilde{S})]_{LB,acig} =$$

$$\exp \{4(2K_2O_2 + K_3O_3 + K_4O_4 + K_5O_5)$$

$$- \frac{1}{2} L_2[(\sigma_6 V_{6e} + \sigma_7 V_{7e} + \sigma_{10} V_{10e} + \sigma_{11} V_{11e})^2 - 4]$$

$$- M_2(\sigma_6\sigma_7\sigma_{10}\sigma_{11} \; V_{6e}V_{7e}V_{10e}V_{11e}) - C_2$$

$$+ p_2(\sigma_a' V_{6a}\sigma_6 + \sigma_c' V_{7c}\sigma_7 + \sigma_i' V_{11i}\sigma_{11} + \sigma_g' V_{10g}\sigma_{10})\}$$

$$\times [P \exp(-\tilde{S}_G)]_{LB,\, acig} \qquad (15)$$

The six block couplings $\{K_R\}$ are evaluated analytically in terms of the {K}, and the p_1 are determined by Eq. (8). The familiar [2] Ising fixed point is recovered at p_2=0.76 and infinite β and p_1, yielding $d\nu=2-\alpha$ = 1.998 to 0.1% accuracy. No distinct new fixed points appear at finite β [β always decreases when Eqs. (8) are applied].

Thus it is seen that by the use of a prefacing transformation the accurate Kadanoff LBRG can in fact be applied to systems with local gauge symmetry. The method can be applied to gauge-Higgs systems with arbitrary gauge group in any spacetime dimension, though in general numerical techniques [6] may be needed. It should be noted, however, that invariant subspaces of coupling constants often exist and can vastly simplify the calculation [2]. The (marginal) critical exponent for two-dimensional Z_2 gauge theory was predicted exactly, and good qualitative agreement with the flow equation was found. When Z_2 scalars were also included, the good results (0.1% accuracy) for the Ising limit were recovered as expected. No unexpected additional spurious fixed points appeared, and again a good flow diagram resulted.

It is a pleasure to thank R. Petronzio for informative discussions.

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Figure captions

Figure 1 — Schematic diagram of prefacing transformation

Figure 2 — Definition of notation for LBRG

Figure 3 — Plot of LBRG recursion relation (solid line) and exact recursion relation (dashed line) for a pure Z_2 gauge theory in two dimensions.

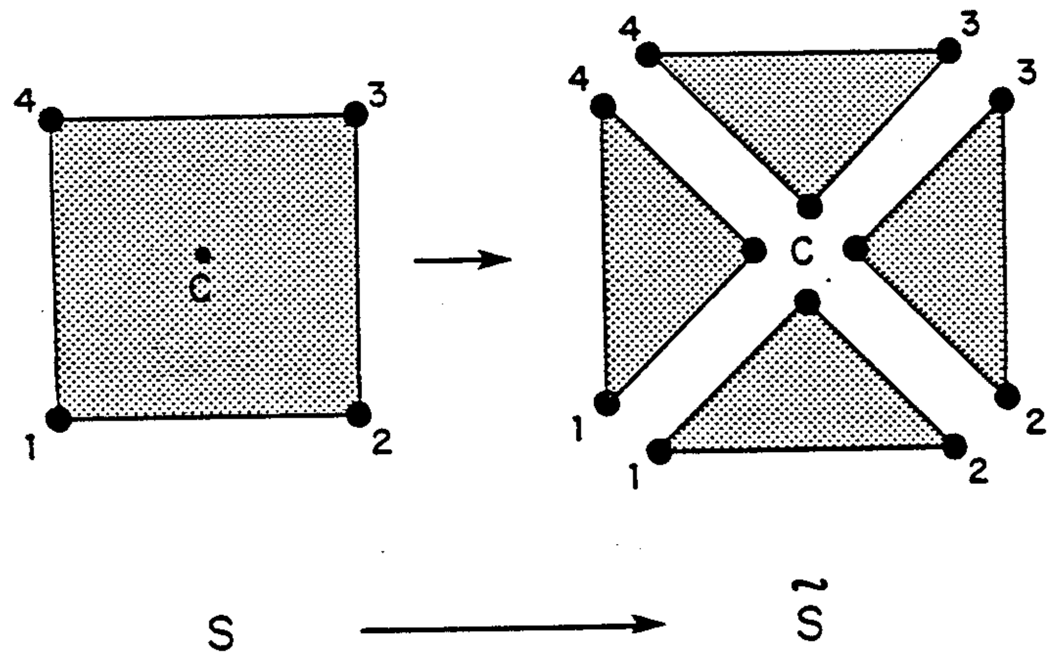

Fig. 1

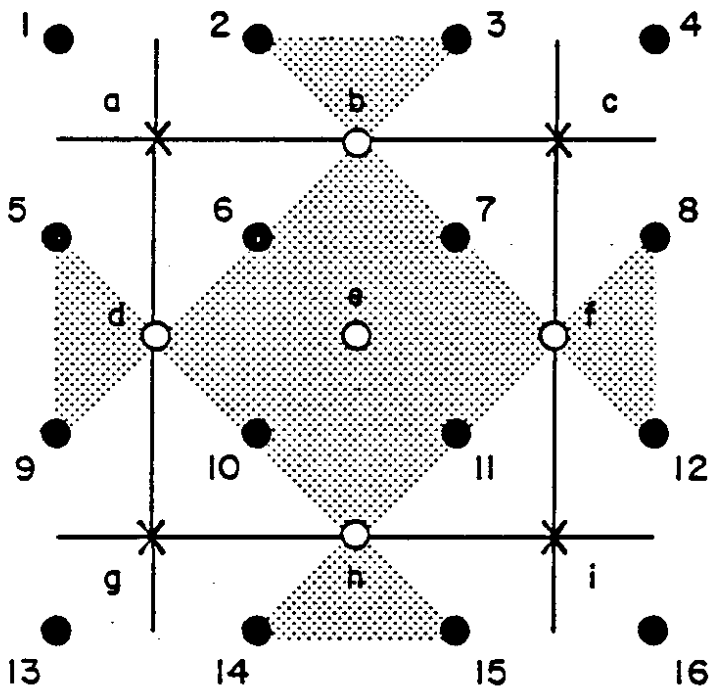

Fig. 2

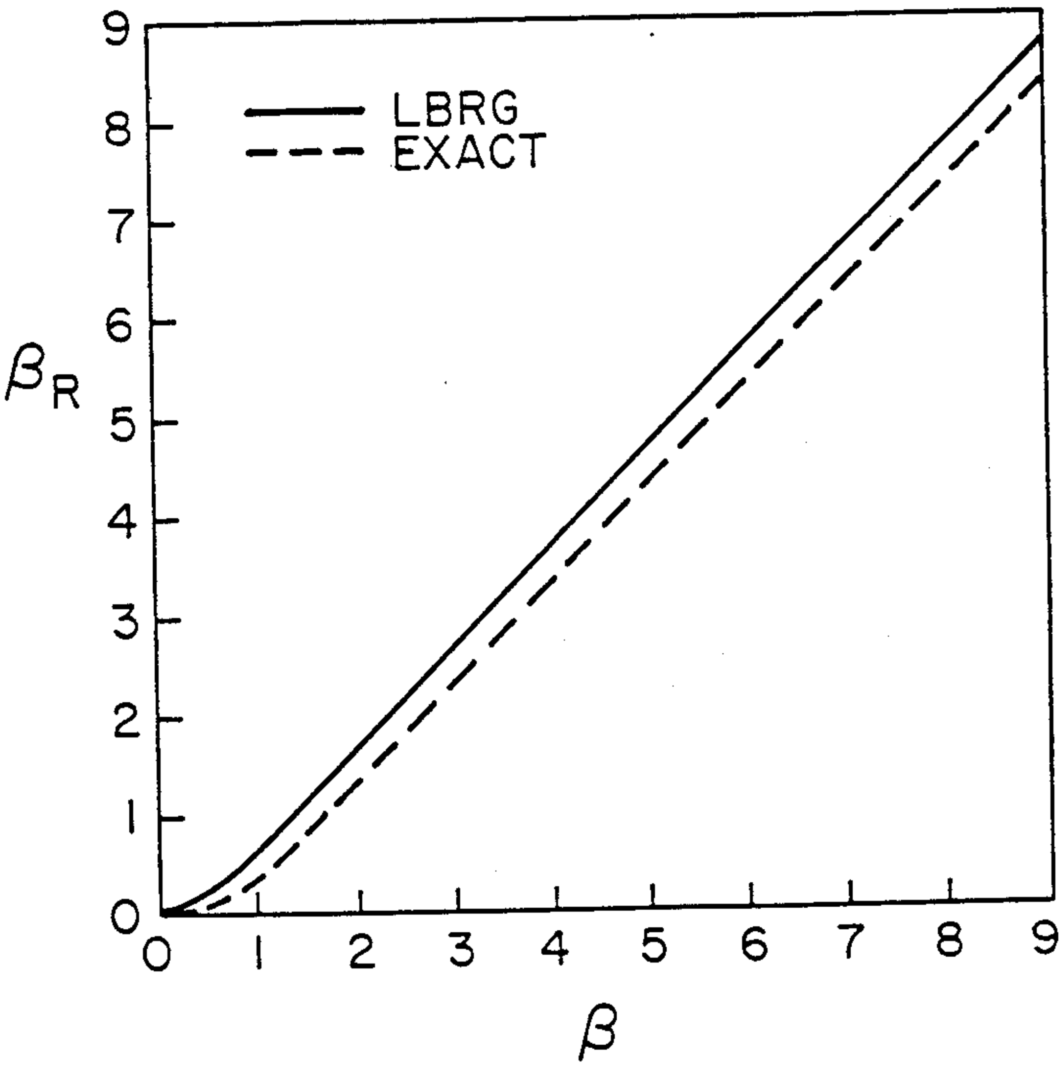

Fig. 3