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TRIVIAL PURSUIT OF THE HIGGS PARTICLE

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ABSTRACT:

The highly accurate Kadanoff lower-bound renormalization group for spin systems is generalized to models with local gauge symmetry. As an example it is applied to the Z_2 gauge-Higgs theory. The two critical exponents of the model are respectively predicted exactly and with 0.1% accuracy by a simple analytic calculation. The application of the technology to more complicated gauge groups in arbitrary spacetime dimension is described.

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It is becoming evident that triviality plays an essential role in models of the weak interaction (see [1] and these proceedings). In particular, if a continuum limit of a Higgs lattice gauge theory exists, it may even be possible to predict the Higgs mass [2,3].

One simple argument which suggests that the Higgs mass is predictable [1,2,3] is based upon the precepts of the renormalization group. If a continuum limit of a Higgs lattice gauge theory exists, it must correspond to a fixed point of the renormalization group transformation. The number of relevant directions at this fixed point equals the number of independent renormalized parameters in the theory. Yet universality suggests that the quartic coupling is irrelevant at this fixed point. Thus at least one parameter of the theory (e.g., the Higgs mass) might be predictable from the others.

Partially in order to explore this possibility, a Monte Carlo renormalization group analysis of the fixed-length SU(2) x U(1) standard model was performed [3]. This calculation was made using a specific "maximal truncation" approximation. Several fixed points do, in fact, appear in this scheme. Some of these fixed points possess "marginal" directions, suggesting that (in addition to the possibility of a calculable quartic coupling, as implied by universality) _a priori_ bounds on other renormalized couplings might be set. Before a definitive statement can be made, however, an accurate real-space renormalization group technique for lattice Higgs systems is needed. One candidate technique [4], a generalized lower-bound renormalization group, is discussed here.

The Kadanoff lower-bound renormalization group (LBRG) predicts [5] critical exponents of spin systems with a precision not achievable by any other simple RG method (see [6] for a review). The LBRG gives $d\nu=2-\alpha$ to 0.1% accuracy in the $d=2$ and 3 Ising models, and to 1% accuracy in four dimensions. For ϕ^4 field theory it yields a one-dimensional integral equation whose solution give ν correctly to first order in the ε-expansion. It is also a direct approximation to an infinite system, so true finite-size effects are absent.

This amazing precision, coupled with the present level of interest in the renormalization-group structure of gauge-Higgs systems (see, e.g., [1]), provides motivation to generalize this simple technique to systems with local gauge symmetry. The generalization presented here is applicable in arbitrary spacetime dimension, and reduces to the Kadanoff method for spin systems in the limit where the gauge coupling vanishes.

The crux of the LBRG is the observation that [since $\langle \exp(\Delta S)\rangle \geq \exp(\langle \Delta S\rangle)$] the addition to the action of an operator with vanishing expectation value <u>lowers</u> the free energy f. Interaction-moving operations satisfy this criterion by translational invariance. Variational parameters are introduced and are (easily) optimized at a fixed point to give a best lower bound for f. Critical exponents then follow directly from the recursion relations.

The application of the original LBRG to gauge theories is complicated by the fact that its block lattice points lie in the centers of hypercubes, while the gauge links {U} are only defined along its edges. It is therefore first necessary to use a prefacing transformation to map the original "plaquette" system to a "subsumed" model which allows parallel transport to the center of a hypercube. The blocking then yields a plaquette model with larger lattice spacing, ready for the next iteration.

An example facilitates explanation. Consider a single plaquette in a two-dimensional Z_2 gauge theory with vertices labelled (1,2,3,4) and action $S_{G,1234}=-\beta U_{12}U_{23}U_{34}U_{14}$ where $U_{ji}=U_{ij}=\pm 1$. Place a point c in the center of the plaquette. The corresponding subsumed model action is

$$\tilde{S}_{G,1234} = -\tilde{\beta}(V_{1c}U_{12}V_{2c} + V_{2c}U_{23}V_{3c} + V_{3c}U_{34}V_{4c} + V_{1c}U_{14}V_{4c})-C_{\beta} \tag{1}$$

where each of the $V_{ic}= \pm 1$ is an element of Z_2 which runs between points c and i=1 to 4. Since $\mathrm{Tr}_{\{V\}}[\exp(-\tilde{S})] = \exp(-S)$ it follows that $(\tilde{\beta})'' = \beta$ and $C_\beta = -2(\tilde{\beta})'-\beta-\ln 16$, where $x'' \equiv (x')'$ and $\tanh x' \equiv \tanh^2 x$. In higher spacetime dimensions, the point c is placed in the center of the hypercube. The prefacing transformation can be performed analytically for discrete groups. For larger groups more interactions [e.g., $(VUV)^2$, $(VUV)(VUV)$

etc.] must be included. Note that the prefacing is <u>underconstrained</u> -- many subsumed models correspond to one plaquette model.

The machinery of the generalized LBRG can be illustrated using a Z_2 gauge theory in d=2 dimensions. Figure 1 defines the notation. The original lattice fields (e.g., U_{67}) run between the original site points, denoted by numerals 1-16. The new fields of the subsumed model (e.g., V_{6e}) go between the original site points and new points labelled by letters a through i. Block points are denoted by crosses, and block fields (e.g., U'_{ag}) connect these. Thus (a,c,i,g) is the boundary of a typical block of side length b=2, which is the RG scaling factor.

A gauge-invariant projection operator P can be defined for the subsumed model. The projection operator determines the renormalized action $S_{G,R}$ via $\exp(-S_{G,R}) = \mathrm{Tr}_{\{U,V\}} P \exp(-\tilde{S}_G)$. P is a product over all block links {U'} of terms like

$$P_{ag} = \exp[p_1 U'_{ag}(V_{6a}V_{6d}V_{10d}V_{10g} + V_{5a}V_{5d}V_{9d}V_{9g}) - N_{ag}] \quad (2)$$

where p_1 is a variational parameter. The requirement that the free energy remain invariant under the RG transformation implies that the trace of P_{ag} over U'_{ag} is unity, and thus

$$N_{ag} = (p_1)' \, (V_{6a}V_{6d}V_{5d}V_{5a})(V_{10d}V_{10g}V_{9g}V_{9d}) + C_1$$

with $C_1 \equiv (p_1)' + \ln 2$.

So far the RG transformation is <u>exact</u> (and intractable). The LB approximation is to move all interactions from the VUV terms and from the normalization terms (e.g. N_{ag}) in the product $P \exp(-\tilde{S}_G)$ into the shaded region (b,f,h,d), and equally to its counterparts in other blocks. The result for the contribution from block (a,c,i,g) is

$$
\begin{aligned}
&[P \exp(-\tilde{S}_G)]_{LB,acig} = \\
&\exp\{2\tilde{\beta}[U_{6,7}(V_{6b}V_{7b} + V_{6e}V_{7e}) \\
+\quad &U_{7,11}(V_{7f}V_{11f} + V_{7e}V_{11e}) \\
+\quad &U_{10,11}(V_{10e}V_{11e} + V_{10h}V_{11h}) \\
+\quad &U_{6,10}(V_{6e}V_{10e} + V_{6d}V_{10d})] \quad - \quad 4C_\beta \\
+\quad &p_1[U'_{ag}(V_{6a}V_{6d}V_{10d}V_{10g}) + U'_{gi}(V_{10g}V_{10h}V_{11h}V_{11i}) \\
&+ U'_{ci}(V_{7c}V_{7f}V_{11f}V_{11i}) + U'_{ac}(V_{6a}V_{6b}V_{7b}V_{7c})] \\
-\quad &p'_1[(V_{6b}V_{7b}V_{6e}V_{7e})(V_{10e}V_{11e}V_{10h}V_{11h}) \\
&+ (V_{6d}V_{6e}V_{10d}V_{10e})(V_{7e}V_{7f}V_{11e}V_{11f})] \\
&- 2C_1\}
\end{aligned}
\tag{3}
$$

The interaction-moving separates the original system into blocks within which the summations over $\{U,V\}$ can be performed independently. The renormalized couplings β_R and C_R are found from

$$
\begin{aligned}
\exp\,[\beta_R(U'_{ag}U'_{gi}U'_{ci}U'_{ac}) + C_R] &= [\exp(-S_{G,R})]_{acig} \\
&= \mathrm{Tr}_{\{U,V\}}[P \exp(-\tilde{S}_G)]_{LB,acig}
\end{aligned}
\tag{4}
$$

The variational parameter p_1 is determined from the extremum condition

$$
0 = \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial K_\alpha}\;\frac{\partial K_\alpha}{\partial p_i}
\tag{5}
$$

where $\alpha = 1,\ldots, n$; $K_{R,n} \equiv C_R$. This can be solved easily at a fixed point [1], for there $\partial f/\partial K_\alpha \equiv e_\alpha$ is a left eigenvector of the matrix $D_{\alpha\beta} \equiv \partial K_{R,\alpha}/\partial K_\beta$ with eigenvalue b^d ($= 2^2$). In fact, Eq. (5) is then a determinant, since e_α is proportional to cof $(\Delta_{\alpha n})$, $\Delta_{\beta\alpha} \equiv D_{\alpha\beta} - b^d \delta_{\alpha\beta}$. The result at the infinite β fixed point is $\beta_R = (\frac{1}{2}(2\tilde{\beta})")"$, $(p_1)' = \frac{1}{2}(2\tilde{\beta})"$, giving the exact limit $\partial\beta_R/\partial\beta \to 1 = b^y$. Thus $y=0$ (the fixed point is <u>marginal</u>). No fixed points exist at finite β. Exact blocking yields $\beta_R = ((\tilde{\beta})")" = \beta"$ by comparison.

Scalar "Higgs" fields are easily included in the formalism. Consider for simplicity fixed-length Z_2 fields $\sigma_n = \pm 1$ (variable-length scalars are treated in [1]). The Higgs terms in the original action S as well as in the renormalized action S_R can be written entirely in terms of gauge-invariant objects like $h_{6,7} \equiv \sigma_6\, U_{6,7}\, \sigma_7$ and $h'_{ac} \equiv \sigma'_a\, U'_{ac} \sigma'_c$ respectively. For scaling factor b=2 in two dimensions nothing larger than a plaquette can be included. Thus the allowable Higgs terms for plaquette (6,7,10,11) are:

$$
\begin{aligned}
O_{Plaq} &\equiv h_{6,7}\, h_{7,11}\, h_{10,11} h_{6,10} \\
O_2 &\equiv \tfrac{1}{2}\,(h_{6,7} + h_{7,11} + h_{10,11} + h_{6,10}) \\
O_3 &\equiv \tfrac{1}{2}\, h_{6,7}(h_{7,11} + h_{6,10})\,(1 + O_{Plaq}) \\
O_4 &\equiv 2\, O_{Plaq}\, O_2 \\
O_5 &\equiv \tfrac{1}{2}\, h_{6,7}\, h_{10,11}\,(1 + O_{Plaq})
\end{aligned}
\tag{6}
$$

and their contribution to the action is $-\sum_{i=2}^{5} K_i O_i$. The Higgs contribution to the full action is the sum over all plaquettes of this quantity (note that $\beta \equiv K_1$ and likewise $\beta_R \equiv K_{R,1}$; $C_R \equiv K_{R,6}$ etc.). The projection operator Q for the scalars is a product over all block fields of terms like this for σ'_a:

$$Q_a = \exp[p_2 \sigma_a' \Sigma - \frac{1}{2} L_2(\Sigma^2 - 4)$$

$$- M_2 \sigma_1 \sigma_2 \sigma_5 \sigma_6 \, V_{1a} V_{2a} V_{5a} V_{6a} - C_2]$$

$$\Sigma \equiv V_{1a}\sigma_1 + V_{2a}\sigma_2 + V_{5a}\sigma_5 + V_{6a}\sigma_6 \qquad (7)$$

where the normalization $\mathrm{Tr}_{\sigma'} Q = 1$ implies that

$4L_2 = (2p_2)'$, $2M_2 = (2L_2)'$, $-C_2 = -2L_2 + M_2 - \ln 2$.

The LB approximation of moving all interactions equally into the region (b,f,h,d) and its counterparts in other blocks is made as before. The LB contribution to $\exp(-S_R)$ from block (a,c,i,g) is the sum over all contained {U,V} of

$$[PQ \exp(-\tilde{S})]_{LB,acig} =$$

$$\exp\{4(2K_2O_2 + K_3O_3 + K_4O_4 + K_5O_5)$$

$$- \frac{1}{2} L_2[(\sigma_6 V_{6e} + \sigma_7 V_{7e} + \sigma_{10} V_{10e} + \sigma_{11} V_{11e})^2 - 4]$$

$$- M_2(\sigma_6\sigma_7\sigma_{10}\sigma_{11} \, V_{6e}V_{7e}V_{10e}V_{11e}) - C_2$$

$$+ p_2(\sigma_a' V_{6a}\sigma_6 + \sigma_c' V_{7c}\sigma_7 + \sigma_i' V_{11i}\sigma_{11} + \sigma_g' V_{10g}\sigma_{10})\}$$

$$\times [P \exp(-\tilde{S}_G)]_{LB,\, acig} \qquad (8)$$

The six block couplings $\{K_R\}$ are evaluated analytically in terms of the $\{K\}$, and the p_i are determined by Eq. (5). The familiar [5] Ising fixed point is recovered at p_2=0.76 and infinite β and p_1, yielding $d\nu$=2-α = 1.998 to 0.1% accuracy. No distinct new fixed points appear at finite β [β always decreases when Eqs. (5) are applied].

Thus it is seen that by the use of a prefacing transformation the accurate Kadanoff LBRG can in fact be applied to systems with local gauge symmetry. In the case of the two-dimensional Z_2-Higgs theory, the two critical exponents are respectively determined exactly and to 0.1% accuracy by a simple analytical calculation. The method is applicable to more complicated gauge groups in higher spacetime dimension, though numerical techniques [7] may then be needed. Often however an invariant subspace exists which can vastly simplify the calculation (for instance, the space of plaquette products on opposite sides of the cube for a Z_2 gauge theory in three dimensions). Moreover in all cases the generalization does reduce to the original technology [5] for spin systems in the limit of vanishing gauge coupling. Thus the good results for spin systems are automatically recovered as a special case.

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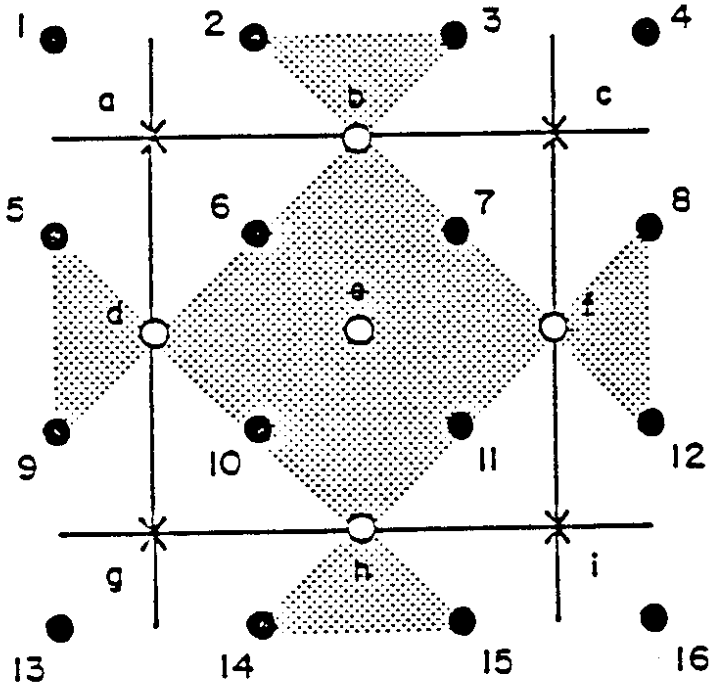

Fig. 1

Definition of notation