Attempered renormalization-group scheme for the SU(2)-Higgs model

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The fact that most renormalization-group blocking schemes include each site link in many block links can generate spurious interactions in the block system. This shortcoming can lead to inconsistent flow diagrams in truncated calculations. A general method for avoiding this problem is formulated and applied to a Monte Carlo renormalization-group study of the SU(2)-Higgs model in four dimensions with scale factor $\sqrt{2}$.

I. PROLEGOMENA

In a recent publication by two of the present authors,[1] the results of a Monte Carlo renormalization-group study of the SU(2)-Higgs model were presented (see also Ref. 2). A major problem noted in this work was the fact that in most traditional blocking schemes, each site link (i.e., each link in the original system) is included in many block links. Although such transformations are valid within the framework of the renormalization group, in the context of a truncated scheme additional spurious interactions can be generated which produce a flow diagram inconsistent with known results for the theory (specifically, a separatrix at $\beta=0$ was generated[1]). In the previous work this problem was avoided by using a blocking transformation with a large scale factor ($b=3$). Additionally, the requirement that each site link appears in *one* block meant that a large fraction of site links appeared in *no* block link. Although the resulting flow diagram was reasonable, better schemes are certainly advisable when critical exponents are required.

One scheme which ameliorates this problem is presented here. It appears to be applicable to virtually all blocking schemes, but to illustrate its potential we have used it in conjunction with a $\sqrt{2}$ blocking transformation.[3] The resulting blocking scheme is so economical in its integration over degrees of freedom that a sensible flow diagram can be produced by blocking from a 2^4 lattice.

II. PROCEDURE

The action for the SU(2)-Higgs model in the fixed-length limit is defined by[1,2]

$$\begin{aligned} S &= \beta S_G + 2\kappa S_L \ , \\ S_G &= \sum_{\text{plaq}} (1-\tfrac{1}{2}\mathrm{Tr}U_{\text{plaq}}) \ , \\ S_L &= \sum_{n\mu} (1-\tfrac{1}{2}\mathrm{Tr}U_{n\mu}) \ , \end{aligned} \tag{1}$$

where the $\{U_{n\mu}\}$ are members of the fundamental representation of SU(2), and U_{plaq} represents the usual ordered product of four $U_{n\mu}$. Here, n represents a lattice site and μ a direction. We work in a gauge in which all of the scalar fields consist of doublets with a unit upper component and vanishing lower component. As explained in Ref. 2, this gauge choice can be maintained in the block system as well.

The unit vectors of the $\sqrt{2}$ block lattice are given by[3]

$$\begin{aligned} e_0' &= (1,0,1,0) = e_0 + e_2 \ , \\ e_1' &= (0,1,0,1) = e_1 + e_3 \ , \\ e_2' &= (1,0,-1,0) = e_0 - e_2 \ , \\ e_3' &= (0,1,0,-1) = e_1 - e_3 \ . \end{aligned} \tag{2}$$

Upon this lattice the blocking transformations for the scalar and gauge (link) fields are defined. The block scalar fields are chosen as the parallel transport of the sum of the four site scalar fields in the block, normalized so that the block scalar field is also of unit magnitude.

The definition of the block links is more complex, but is simplified for SU(2) by the fact that the sum of the elements of the group differs from another element by only an overall normalization factor. The most direct approach is therefore to define the block links as a normalized weighted average of site link paths connecting block points. Thus, for example, the simplest choice for calculating a block link going from the origin to the point $e_0'=(1,0,1,0)$ is to take the normalized sum of the two "elbow" paths connecting this point to the origin, one going through the point $e_2=(0,0,1,0)$ and the other through the point $e_0=(1,0,0,0)$. Additional "house" paths such as the path going via the route $(0,0,0,0)\rightarrow(0,1,0,0)\rightarrow(1,1,0,0)\rightarrow(1,1,1,0)\rightarrow(1,0,1,0)$ can also be included (with a different weighting factor in general). The nomenclature "elbow" and "house" is inspired by the appearance of the paths in line drawings.

Given the block links defined by this procedure, the algorithm defined in Ref. 1 can be used to determine the block couplings. A "maximal" truncation scheme,[1] in

which the block action is assumed to be of the same functional form as the site action, is employed. Basically, a set of loop equation identities is used to predict the block couplings in terms of measured correlation functions.

This naive blocking procedure has the flaw that spurious interactions are generated because each site link is included in the construction of several block links. (For example, in the above blocking scheme, the link U_{00} connecting the origin to the point e_0 is included in the calculation of the elbow block links connecting the origin to sites e'_0 and e'_2.) When this naive procedure is combined with the above scheme for extracting block couplings, an artificial "phase transition" at $\beta=0$ in the SU(2)-Higgs model is predicted by the renormalization flow structure.[1] One solution to this problem is to block using only paths which do *not* contain common links. The difficulties here include the fact that many site links are then not directly included in the block links, so the averaging is not performed well ("decimation" transformations typically induce long-range interactions).

These difficulties can, however, be obviated by using an attempered blocking scheme. ("Attempered" is here used in the sense "modify by blending.") The correlation functions defining the block couplings are calculated as averages over two (or even more, in general) blocked lattices, each of which, taken separately, includes each site link in only one block link. Thus, for example, if this procedure is applied to the above naive blocking scheme, the block links $\{U'=U_{\rm I}\}$ are calculated using one set of paths (including for U'_{00} the elbow path passing through e_2 as well as appropriately weighted house paths). The correlation functions defining the block couplings are then calculated. Before the site lattice is updated (i.e., using the same configuration), the complementary block links $\{U'=U_{\rm II}\}$ are calculated (e.g., including for U'_{00} the path passing through e_0 and the remaining weighted house links), and the correlation functions are evaluated for *this* set of block configurations. The two sets of correlation functions are then averaged. This procedure is equivalent to defining a projection operator[1] for the gauge fields:

$$P[\{U'\},\{U\}]=\tfrac{1}{2}\left[\prod_{\{U'\}}[\delta(U'-U_{\rm I})]+\prod_{\{U'\}}[\delta(U'-U_{\rm II})]\right]. \tag{3}$$

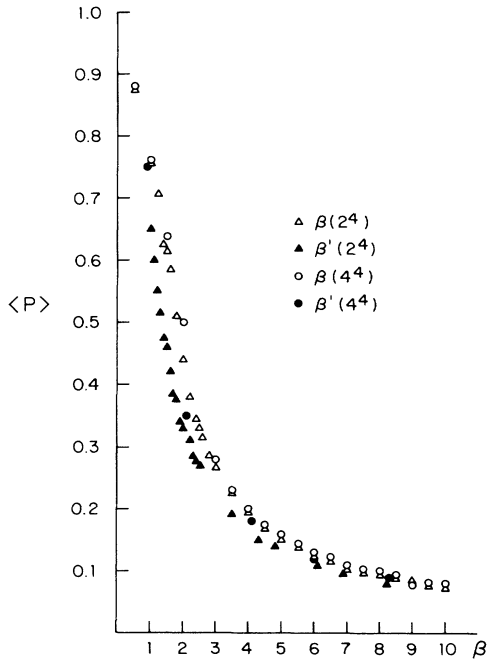

FIG. 2. Plot of average plaquette vs β and average block plaquette vs β' for various lattice sizes L. In all cases 500 measurement iterations were preceded by 50 equilibration steps.

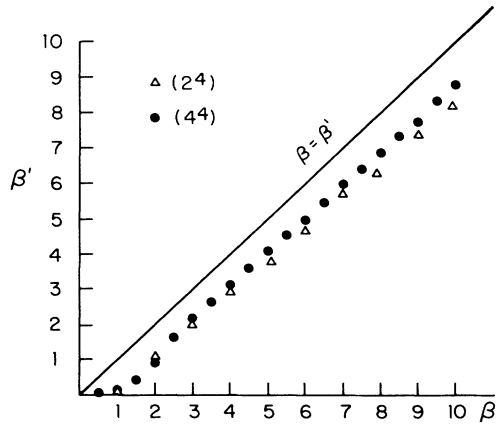

FIG. 1. Plot of β' vs β, obtained on a 4^4 lattice. Results are measured over 500 iterations following 50 equilibration steps.

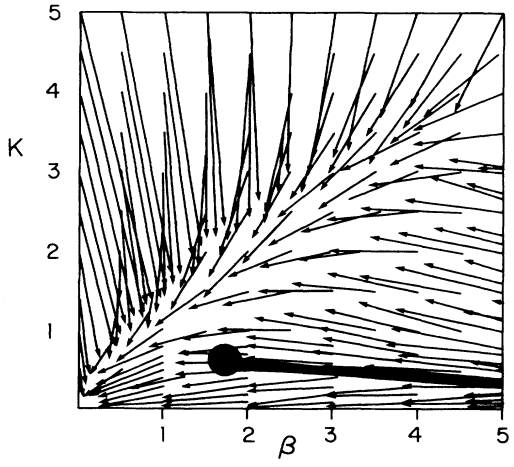

FIG. 3. Actual flows determined from a 2^4 lattice. Points are generated over 500 measurement iterations following 50 equilibration steps.

III. RESULTS AND CONCLUSIONS

The relative weight of the "house" links is adjusted (to 0.10) in order to give the asymptotic freedom (marginal) critical exponent

$$\left.\frac{\partial\beta'}{\partial\beta}\right|_{\beta\to\infty}=1 \tag{4}$$

and the plot of β' vs β is given in Fig. 1. In Fig. 2 we present results for the block plaquette as a function β', and compare it to the site plaquette plotted versus β (all at $\kappa=0$). The average plaquette is defined on a L^4 lattice as

$$P=\frac{\langle S_G\rangle}{6L^4}\ . \tag{5}$$

The complete flow diagram (determined from a 2^4 lattice) is plotted as Fig. 3. No qualitative difference is seen on a 4^4 lattice. From the flow diagram, a fixed point at $(\beta,\kappa)=(\infty,0.3)$ can be inferred. As suggested in Ref. 1, a marginal fixed point may exist near $(\beta,\kappa)=(1.6,0.6)$, but we were unable to confirm this point.

Our results suggest the following conclusions. First, the aforementioned difficulties which occur when a site link appears in many block links can be overcome with this method (no separatrix appears in the flow diagram at β equal to zero), which seems applicable to blocking transformations with *arbitrary* scale factor. Moreover, all site links are included in the calculation. Second, blocking with a $\sqrt{2}$ scale factor produces a reasonable picture of the flow diagram (even on a 2^4 lattice) when combined with our maximal truncation scheme.

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