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CAN ELEMENTARY SCALAR PARTICLES EXIST?

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A B S T R A C T

Strong evidence supports the notion that pure ϕ^4 field theory is a non-interacting, "trivial" theory. Should such a situation persist when gauge fields are present, the idea that elementary scalars can break symmetries spontaneously is called into question. Here several examples [such as the O(N) gauge model] are explored, and it is shown how triviality can be avoided, and what phenomenological consequences may arise. A general method (the Monte Carlo renormalization group) for ascertaining whether a theory is trivial or not is presented and applied to the SU(2)-Higgs model. No evidence for a non-trivial continuum theory is found.

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1. - PROLEGOMENA

The concept of particle mass generation by spontaneous symmetry breaking[1) has become one of the most useful ideas in field theory. Typically this symmetry breaking is produced by the introduction of a scalar field. The new field supposedly interacts with itself in such a fashion (via a ϕ^4 coupling) as to generate a ground state which lacks a symmetry of the Lagrangian. At the classical level, gauge particles propagating in the asymmetric vacuum may then appear to be massive, while the underlying theory remains renormalizable. It is generally assumed that no significant changes occur in this picture due to quantum effects such as renormalization.

Such an assumption may not be correct, however. A large body of analytical[2)] and numerical[3),4)] evidence has been presented which indicates that pure ϕ^4 field theory is "trivial", i.e., non-interacting in the infinite cut-off limit in four space-time dimensions. This triviality seems to persist for all values of the bare coupling constant, and presumably therefore precludes the existence of spontaneous symmetry breaking in pure ϕ^4 field theory.

Clearly it is necessary to understand the phenomenon of triviality in detail. Such a non-trivial theory is a necessary precursor for spontaneous symmetry breaking by elementary scalar particles. Also, if pure ϕ^4 field theory is trivial but becomes non-trivial by the addition of gauge fields [as with, e.g., the O(N) gauge model in the large N limit], phenomenological constraints on the Higgs mass may result, as is discussed below. In particular, it is quite possible that the Higgs mass in the standard model is a calculable quantity.

In order to have a non-trivial theory, an ultra-violet-stable fixed point must occur in the (Gell Mann-Low) beta function of the theory. Using perturbative methods, it is typically only possible to observe such a stable fixed point if it occurs at zero coupling, that is, if the theory is asymptotically free. This is far from the most general possibility, however, and a non-perturbative method for examining the question of triviality or non-triviality of a theory (such as the standard model of the weak interaction) must be found.

The Monte Carlo renormalization group (or MCRG) is a natural candidate for such an analysis. By dealing directly with the scaling properties of the various fields and operators under consideration, it allows an economical (albeit numerical) formulation of the problem. Additionally, by restricting the analysis to a few carefully chosen operators, the MCRG allows accurate and systematically improvable approximations. As is shown below, the combination of numerical methods with judicious approximation can lead to techniques of some utility in the non-perturbative study of quantum field theory.

As an example of the efficacy of the MCRG, we use it to examine the case of SU(2) with a fundamental representation scalar field, the "SU(2)-Higgs model". The flow diagram and critical exponents are found, and their implications for the theory are discussed.

2. - BACKGROUND

It is appropriate at this juncture to recapitulate the motivation for approaching the question of triviality in this fashion. A simple example of one way that a non-trivial theory can be produced from a trivial scalar theory is provided by the O(N) gauge model.

Consider first the pure O(N) model without gauge interactions, defined by an interaction Lagrange density

$$\mathcal{L}_I = -\frac{\lambda}{8}\left(\varphi^2\right)^2 \tag{1a}$$

The running coupling obeys an equation

$$\frac{d\lambda(t)}{dt} = \frac{N\lambda^2(t)}{16\pi^2} \tag{1b}$$

in the large N limit. [This result follows easily from the work of Ref. 5) and has been rederived in many textbooks.] Here, $t = \ln(Q/\mu)$ where Q is the running momentum scale and μ is the renormalization point. Equation (1b) has the solution

$$\lambda(t) = \frac{\lambda_R}{1-\left(\frac{N}{16\pi^2}\right)\lambda_R t} \tag{2}$$

for the running coupling "constant" $\lambda(t)$. Note that $\lambda(t=0) = \lambda_R$ defines the renormalized coupling λ_R. The running coupling $\lambda(t)$ becomes infinite at a momentum scale $t = 16\pi^2/(\lambda_R N)$, which is finite if λ_R is non-zero. Thus, the O(N) model in this large N limit is inconsistent unless λ_R vanishes, i.e., unless it is a non-interacting field theory. The O(N) model in the large N limit is therefore trivial. The existence of this inconsistency (or "Landau ghost") in the interacting theory is essentially a non-perturbative phenomenon, for Eq. (2) holds for all λ in this large N limit. This O(N) case can be contrasted with ordinary quantum electrodynamics, where the Landau ghost which appears in low orders of perturbation theory might still be exorcized by a non-perturbative analysis.

Interestingly enough, the addition of an O(N) gauge field to this theory rescues it from triviality. For small λ and g^2 the relevant equations for the running couplings can be written[6] as

$$\frac{dg}{dt} = -b_0 g^3 \frac{1}{32\pi^2} \tag{3}$$

where b_0 is a positive constant (as the gauge coupling is asymptotically free), and

$$\frac{d\lambda}{dt} = \frac{N}{16\pi^2}\left[\lambda^2 - 3\lambda g^2 + \frac{3}{4} g^4\right] \tag{4}$$

(where only the leading terms for large N are retained). Equation (4) can be rewritten in terms of a new variable

$$\zeta \equiv \lambda/g^2 \tag{5a}$$

to wit:

$$\begin{aligned}\beta_\zeta \equiv \frac{1}{g^2}\frac{d\zeta}{dt} &= \frac{N}{16\pi^2}\left[\zeta^2 + \left(\frac{b_0 - 3N}{N}\right)\zeta + \frac{3}{4}\right] \\ &\equiv \frac{N}{16\pi^2}(\zeta - \zeta_+)(\zeta - \zeta_-) \\ &\qquad \zeta_+ > \zeta_-\end{aligned} \tag{5b}$$

By the addition of enough fermion representations and by letting N increase without bound, the ratio b_0/N can be made as small as is desired (while still remaining positive). The two roots $\zeta_\pm$ then approach $\frac{1}{2}(3\pm\sqrt{6}) \approx (2.72, 0.28)$. What is quite remarkable is that these two roots (though irrational) in this limit are real and positive. It is this fact which ensures the existence of a non-trivial theory. Even more remarkably, there are physical ("phenomenological") consequences of this result, which are explored below.

The non-trivial O(N) gauge theory arises as follows. First, note that when t equals zero, $\zeta(t)$ equals

$$\zeta_R \equiv \lambda_R / g_R^2 \tag{6}$$

The function β_ζ is plotted in Fig. 1a, and the corresponding flow diagram in Fig. 1b (the root ζ_+ is taken as the larger of the two roots). If ζ_R is less than ζ_+, then as t increases $\zeta(t)$ flows toward the fixed limit ζ_-. Since $g(t)$ is asymptotically free, this means that $\lambda(t)$ is asymptotically free as well, and both couplings avoid the Landau ghost which haunted the pure O(N) model. If ζ_R equals ζ_+, $\zeta(t)$ will remain fixed at this value, and again both couplings $\lambda(t)$ and $g(t)$ will be asymptotically free and thus avoid the Landau ghost.

If, however, ζ_R is greater than ζ_+, the situation is vastly different. As t increases the running coupling $\zeta(t)$ continues to increase until $\lambda(t)$ is much larger than $g(t)$. At this point the coupling $g(t)$ becomes irrelevant, and $\lambda(t)$ finds itself ensnared once again by the Landau ghost. Thus, a consistent theory satisfies the bound

$$\zeta_R \leq \zeta_+ \tag{7a}$$

The quantity ζ_R also has physical import, for, to lowest order in the loop expansion

$$\left(\frac{m_H}{m_W}\right)^2 = \zeta_R \leq \frac{3+\sqrt{6}}{2} = \zeta_+ \tag{7b}$$

Here, m_H (the "Higgs" mass) is the renormalized mass of the scalar field, and m_W (the "W" mass) is the mass of the N-1 massive gauge boson. Thus, at least within the context of this artificial O(N) gauge model, the need for the existence of a non-trivial theory generates constraints (an upper bound on the mass of the "Higgs" boson). A more speculative extension of these ideas has been considered elsewhere[7)]. Similar ideas have also resulted in an upper bound on the mass of the Higgs boson in the standard model which is as low as 125 GeV[8)].

It should be pointed out that it is not necessary for a theory to be asymptotically free in order to avoid a Landau ghost. It is only necessary to have a fixed point (i.e., a zero of the beta function) which is attractive as t increases. If this fixed point is located at the origin, then the theory is asymptotically free. Thus, asymptotic freedom is a sufficient, but unnecessary, condition for non-triviality.

In fact, it seems to be true[6)] that a theory cannot be both asymptotically free and still exhibit complete spontaneous symmetry breaking. For example, in the O(N) gauge model, N must be greater than or equal to 6 in order for both the gauge and quartic couplings to be asymptotically free. However, for one vector representation scalar, the symmetry is only broken from O(N) to O(N-1). Thus, only for N equal to two is the symmetry broken completely, and for, e.g., O(6) there remain ten massless non-Abelian gauge bosons corresponding to the generators of the remaining non-Abelian O(5) subgroup. This seems to be[6] the general pattern -- either there are a large number of massless gauge bosons, or else at least one coupling in the theory is not asymptotically free.

A class of exceptions to this general rule does exist; these are theories[9] which obey "eigenvalue" conditions, where every renormalized coupling in the theory is uniquely determined. Such theories could be correct, but they do exhibit "fine tuning" on the grandest scale possible.

Accordingly, in general it is necessary to consider the question of triviality in theories where at least one of the couplings is not asymptotically free. A new non-perturbative method must be employed to deal with this question; we propose (and have used) the MCRG for this purpose. Possible phenomenological results in general include upper bounds on Higgs masses [as in the case of the O(N) gauge model] or even calculable Higgs masses (as in the case of the eigenvalue theories[9]). We determine the flow diagram, fixed points and critical exponents of the SU(2)-fundamental representation Higgs model, and discuss the results in light of the above remarks.

3. - DESCRIPTION OF THE PROCEDURE

The Monte Carlo renormalization group[10,11,4] is a combination of the ideas of Monte Carlo simulation[12] with those of the real space renormalization group[13]. The system under consideration is divided up into "blocks" or collections of site variables and a smaller set of block variables is defined by averaging in some fashion over these site variables. The new block system is governed by a block renormalized action (defined by couplings $\{K'\}$) in the same fashion as the original site action (defined by couplings $\{K\}$) governed the site system. By studying the sequence of couplings $\{K\} \to \{K'\} \to \{K''\} \to \ldots$ as repeated blockings are performed, the critical properties of the system can be determined. For a lattice gauge theory, these critical properties determine the continuum limit of the theory[14].

These ideas can be formulated in a more precise mathematical fashion as follows. Consider a system of site variables $\{\phi\}$ governed by a site action $S\{\phi\}$. A block renormalized action $S'\{\phi\}$ is determined from the site action by use of the projection operator $P[\{\phi'\},\{\phi\}]$:

$$\exp[-S'\{\varphi'\}] = \mathrm{Tr}_{\{\varphi\}} P[\{\varphi'\},\{\varphi\}] \exp[-S\{\varphi\}] \tag{8}$$

where the trace (or functional integral) is only over the site variables $\{\phi\}$. The requirement that the renormalization group transformation preserve the partition function (vacuum-to-vacuum transition amplitude) of the system imposes the constraint

$$\mathrm{Tr}_{\{\varphi'\}} P[\{\varphi'\},\{\varphi\}] = 1 \tag{9}$$

The projection operator is otherwise entirely arbitrary. The generalization of Eqs. (8) and (9) to a case where both spin and gauge fields are present is obvious.

Only the fixed-length scalar SU(2)-Higgs model is considered here. The reasoning behind this simplification is as follows. First, in our initial study[4] of pure ϕ^4 field theory, the action flowed into the fixed-length theory (in that case the Ising model) exponentially fast in the number of blocking steps (specifically, the measured quartic coupling increased at this rate). This behaviour is expected to persist in the SU(2)-Higgs model. Indeed, recent calculations[15] support the hypothesis that the quartic coupling is irrelevant (in some renormalization scheme).

Following the conventions of Ref. 15), the action can be written as

$$S = -K \sum_{x} \sum_{\mu>0} \mathrm{Tr}\,[\alpha^{+}_{x+\mu}\, U(x,\mu)\, \alpha_x] + \beta \sum_{\square} (1-\tfrac{1}{2}\mathrm{Tr}\, U_{\square}) \tag{10}$$

where $U(x,\mu)$ and α_x are fundamental representation SU(2) matrices. The symbol $U_\square$ represents the usual ordered product about an elementary plaquette, and the integration measure is

$$\prod_{x} [d^3\alpha_x \prod_{\mu>0} d^3U(x,\mu)] \tag{11}$$

where dg^3 represents the invariant Haar measure[16] in SU(2). Essentially by a gauge choice Eq. (10) can be rotated into a form in which all of the $\{\alpha_x\}$ are equal to unity. Then it is easy to generalize the SU(2) heat-bath algorithm of Creutz[16)] to the SU(2)-Higgs model in order rapidly to generate site lattice configurations.

Next it remains to specify the blocking transformation. The projection operators for both the gauge and scalar fields are taken to be simple products of delta functions,

$$P[\{\varphi'\},\{\varphi\}] = \prod_i \delta[\varphi'_i - g_i\{\varphi\}] \tag{12}$$

where $\{\phi'\}$ and $\{\phi\}$ represent generically the gauge and the scalar fields in the case of each projection operator.

The system is divided into blocks with two sites on a side. Block lattice points are located at the site nearest the origin (see Fig. 2 for a two-dimensional projection). The block scalar fields α'_i are located at these block prints, and are defined to be the average of the site scalar field at that point and the parallel transport of each of the four site fields located one unit away in the block:

$$\alpha'_i = \left(\alpha_i + \sum_{\mu=1}^{4} U(x_i,\mu)\,\alpha_{i+\mu}\right) N \tag{13}$$

where N is chosen so that α'_i has unit determinant. The block scalar field α'_i has the gauge transformation properties of the site scalar field α_i located at the same point.

The block links are defined in a fashion which is a generalization of the approach used in Ref. 17). Each block link is the sum of the nineteen paths of the form shown in Fig. 3, normalized to unit determinant,

$$\tilde{U}(x,\mu) = \frac{\sum_{path} U_{path}}{\det\left(\sum_{path} U_{path}\right)} \tag{14}$$

It is convenient to gauge away the scalar degrees of freedom in the block system as well. This can be effected by a gauge rotation

$$U'(x_i,\mu) = W^{+}_{i+\mu}\,\tilde{U}(x_i,\mu)\,W_i \tag{15}$$

where

$$W_i = M\left[1 + \sum_{\mu=1}^{4} U(x_i,\mu)\right] \tag{16}$$

and M insures that the $\{W\}$ have unit determinant. The $\{U'\}$ are taken as the "final" block variables of the system. The block action can then be written entirely in terms of the $\{U'\}$.

In order to complete the machinery for the computation, a method of extracting block renormalized couplings must be constructed. We have presented a general method for this extraction[11),4)]; however, the present problem allows an expedient simplification. If the blocking transformation averages the site fields sufficiently completely, the block action will be well described by a form like the site action, Eq. (10)

$$S' = -K' \sum_x \sum_{\mu>0} \mathrm{Tr}\left[U'(x,\mu)\right] + \beta' \sum_{\Box} \left(1 - \tfrac{1}{2}\mathrm{Tr}\, U'_{\Box}\right) \tag{17}$$

where a gauge in which the $\{\alpha'\}$ equal unity is chosen, using Eqs. (15) and (16).

Each element U in the fundamental representation of SU(2) can be written

$$U = a_0 + i\,\vec{a}\cdot\vec{\sigma} \tag{18}$$

where $a \equiv (a_0, a_1, a_2, a_3) = (a_0, \underset{\sim}{a})$ is a four-component vector and the $\underset{\sim}{\sigma}$ are the familiar 2×2 Pauli spin matrices. The Haar measure in this notation is given by[16)]:

$$dU = \frac{\delta(a^2-1)}{2\pi^2}\, d^4a = \frac{1}{4\pi^2}\sqrt{1-a_0^2}\, da_0\, d\cos\theta\, d\varphi \tag{19}$$

This notation and measure is equally valid for the site or blocked system.

Consider the dynamics of a single fluctuating block link, denoted by a'. For a given system of block links a' can be thought of as fluctuating in a given "local field". If the block action is assumed to be of the form Eq. (17), then, in the notation of Eq. (18) the part of the action involving a' is

$$S'(a') = (\beta' b_0 + 2K')a'_0 + \beta' |\underset{\sim}{b}'||\underset{\sim}{a}'| \cos\theta' \quad . \tag{20}$$

Here, $b' = (b_0, \underset{\sim}{b})$ is a four-component "local field" [proportional to an element of SU(2)] which equals the sum of the six incomplete plaquettes (products of three links) with which a' interacts. With the measure Eq. (19) and using integration by parts it follows that for fixed b'

$$(2K' + \beta' b_0') = \beta' |\underset{\sim}{b}'| \left\langle \frac{a_0'}{\sqrt{1-a_0'^2}} \cos\theta' \right\rangle + \left\langle \frac{a_0'}{1-a_0'^2} \right\rangle \equiv f(b_0', |\underset{\sim}{b}'|) \tag{20a}$$

$$\beta' |\underset{\sim}{b}'| = \frac{\left\langle \frac{a_0'}{1-a_0'^2} \right\rangle \langle a_0' \rangle - \left\langle \frac{a_0'^2}{1-a_0'^2} \right\rangle + 1}{\left\langle \frac{a_0'^2 \cos\theta'}{\sqrt{1-a_0'^2}} \right\rangle - \langle a_0' \rangle \left\langle \frac{a_0'}{\sqrt{1-a_0'^2}} \cos\theta' \right\rangle} \equiv g(b_0', |\underset{\sim}{b}'|) \tag{20b}$$

Instead of fixing b', it is more practical for this problem to bin together the functions $f(b_0', |\underset{\sim}{b}'|)$ and $g(b_0', |\underset{\sim}{b}'|)$ for all the block links and minimize the (least squares) function

$$\chi^2 = \sum_{b_0', |\underset{\sim}{b}'|} \left\{ \left[(2K' + \beta' b_0') - f(b_0', |\underset{\sim}{b}'|) \right]^2 + \left[\beta' |\underset{\sim}{b}'| - g(b_0', |\underset{\sim}{b}'|) \right]^2 \right\} \cdot \rho(b_0', |\underset{\sim}{b}'|) \tag{21}$$

in order to find the best values of K' and β'. Here, ρ is a weight factor (the number of points in the bin).

By following the above procedure it is possible to extract a pair of constants, K' and β', which describe (to a certain level of accuracy) the block action. If the chosen blocking transformation is a very good one, this description will be highly accurate. The main test of this accuracy of reproduction is, of course, whether the flow diagram and critical exponents so produced agree with known results in previously explored regions of the flow diagram.

4. - RESULTS

All calculations were performed using the above blocking scheme on a 6^4 lattice. Twenty-five bins in b_0 (ranging from -6 to 6) and $|\underset{\sim}{b}|$ (ranging from 0 to 6) were used. Correlations were measured over 2,500 iterations, following an equilibration of 250 iterations. In the least-squares fit, the weight factor equalled the number of points in the bin. Bins with less than 1,250 points were excluded. Each point took roughly 5,600 seconds of time on a CDC 875.

It is important to recall that the procedure employed here (truncation of the renormalization group to two couplings) is an approximate procedure based upon the assumption that the blocking transformation averages well the site fields. Accordingly, the first point to verify is that the flow diagram thus generated agrees with known results in appropriate limits.

The first known result is the fixed point when the couplings (K,β) equal $(0,\infty)$. This fixed point should have a critical exponent associated with asymptotic freedom, that is, in this approximation scheme

$$(0,\infty) \longrightarrow \frac{\partial\beta'}{\partial\beta} \cong 1 \tag{22}$$

Therefore, a plot of β' versus β at $K = 0$ should exhibit the following properties: when β is zero, β' should also be zero, for this model is then mapped into itself. The renormalized coupling β' should always be less than β (for the origin is a point of attraction) but as β increases β' should approach the line $\beta' = \beta$ and should be asymptotic to it <u>with the same slope</u>. These features are in evidence in Fig. 4.

In the limit of large β, the model reduces to an SU(2) spin system with a symmetry-breaking second order phase transition[18),19)] at $K_c = 0.355\pm0.010$. If, as is expected, the SU(2) ϕ^4 scalar theory is trivial, then this phase transition will be associated with a critical exponent $y = \nu^{-1} = 2$. For one relevant renormalization group operator (associated with K), y is defined by

$$y = \left.\frac{\ln\left(\partial k'/\partial k\right)}{\ln b}\right|_{k=k^*} \tag{23}$$

where in Eq. (23), b is the scale factor of the transformation (here b equals 2). Thus, it is to be expected that a plot of K' versus K should cross the line $K' = K$ at $K^* \cong 0.355$ with a slope $\partial K'/\partial K$ equal to four if the fixed point is Gaussian (trivial). This expected slope and crossing are shown as a dashed line in Fig. 5; the data points follow these well. A least squares fit to the last six points yields $K^* \cong 0.37$, $\partial K'/\partial K = 4.14$, or $y = 2.05$, $\nu = 1/y = 0.49$. The last can be compared with the expected value $\nu = \frac{1}{2}$ for a Gaussian (trivial) theory.

It should be noted that this critical exponent is, in fact, a new result. Although it is expected that multicomponent scalar ϕ^4 theories should be trivial, the evidence[2)-4)] for triviality so far exists only for single component theories and the large N limit discussed above. Our data also support the notion[18),19)] that this phase transition at infinite β is second order rather than first order. Had the transition been first order, the renormalization group formalism would still have worked[20)] but would have predicted a value $y = 4$ (or $\nu = 0.25$) and thus $\partial K'/\partial K = 8$.

Finally, it remains to present the full flow diagram of the theory. This is given in Fig. 6. Note that this entire flow diagram is controlled by two fixed points - the one previously mentioned at $(K,\beta) = (0.355,\infty)$ and the trivial fixed point at the origin (0,0). The latter fixed point occurs at values of the coupling constants for which the theory is non-interacting (like an ideal gas or an Ising system at infinite temperature). All couplings are also irrelevant here, so there is evidently no non-trivial continuum limit in the fundamental SU(2)-Higgs model. Other workers have also failed to find indications of such a limit[15),19),21),22)].

A possible objection which might be raised against the above analysis is the following. Suppose that at the endpoint $\approx(0.6,1.6)$ of the separatrix [connecting this point to $(0.355,\infty)$] a marginal fixed point existed. Then it could be argued that our analysis might have missed this point, for the renormalization flow could continue in the same direction and go "through" the fixed point. Even if such a point existed however (and we believe it does not), it should not yield a non-trivial continuum limit. The reason is that in taking a continuum limit one follows the flow lines backwards, and so one must follow this point to the region of small quartic coupling. However, in this region this imagined fixed point moves off[19),21)] to $\beta \to -\infty$, a region unlikely to have physical import. Thus, in any event the SU(2)-fundamental Higgs model appears to be trivial.

5. - SUMMARY AND OUTLOOK

We have discussed the general physical meaning of triviality, and have shown that it is far from obvious that the triviality of a ϕ^4 theory implies that a unified gauge-Higgs system is also trivial. In particular, we showed how the O(N) model in the large N limit (which is trivial) can be turned into an interacting theory by the inclusion of gauge fields in the problem.

It was then pointed out that in order to study the question of triviality in realistic models a non-perturbative method of analysis must be developed. We proposed the Monte Carlo renormalization group as an effective and efficient tool for this purpose, and applied it to the SU(2)-(fundamental representation Higgs) model. As predicted by perturbative arguments[7], no non-trivial continuum limit was found. Accordingly, we are continuing the analysis in the full SU(2)×U(1) theory. Possible phenomenological consequences (described above) include the possibility that the Higgs mass can be bounded or even calculated directly.

After the completion of this work a preprint was received containing similar ideas[23].

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FIGURE CAPTIONS

Fig. 1a Plot of the beta function β_ζ versus ζ.

Fig. 1b Flow diagram corresponding to Fig. 1a.

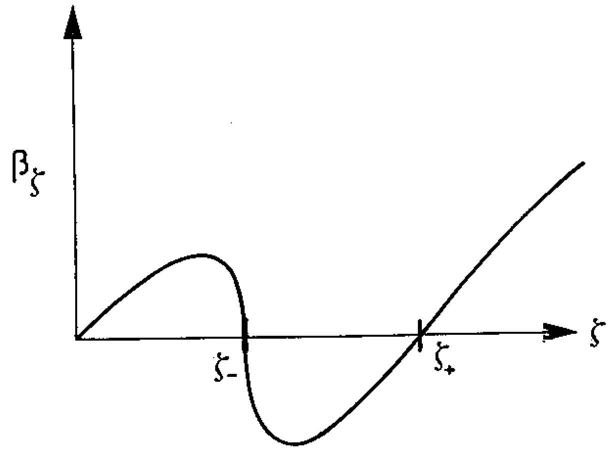
Fig. 2 Two-dimensional projection of the blocking scheme. Heavy dots indicate block lattice points. Dashed lines depict block boundaries.

Fig. 3 Examples of the nineteen paths whose normalized sum forms a block link.

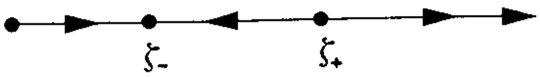
Fig. 4 Plot of β' versus β for $K = 0$. The line $\beta' = \beta$ is also shown.

Fig. 5 Plot of K' versus K for $\beta = 4$. The line $K' = K$ is plotted as a solid line. A line of slope 4 passing through the critical point $K^* = 0.355$ is plotted as a dashed line.

Fig. 6 Flow diagram of the SU(2)-Higgs model in the (K,β) plane.



(1a)



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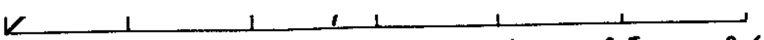

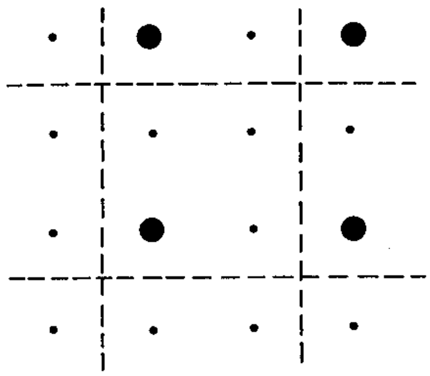

Fig. 2

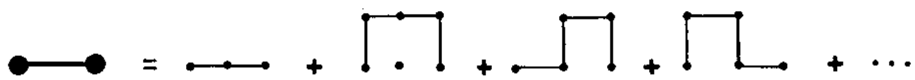

Fig. 3

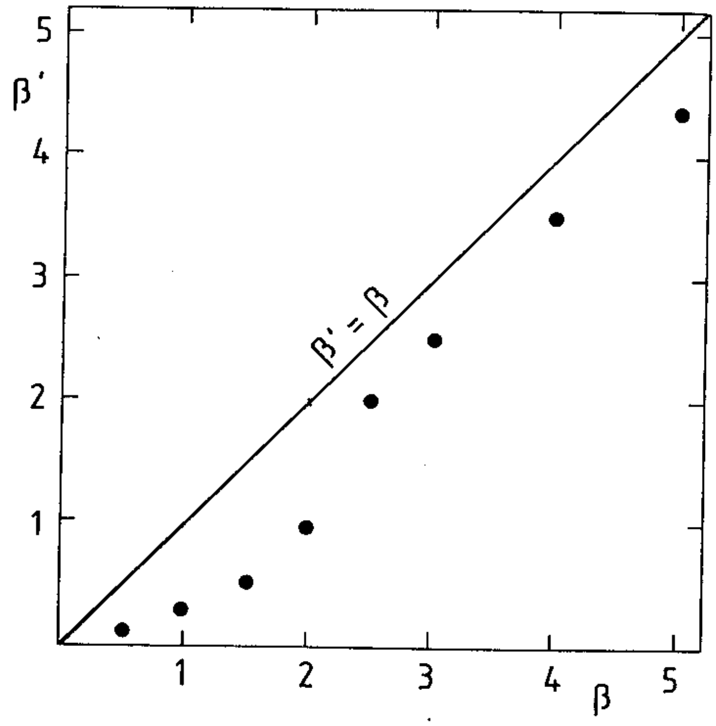

Fig. 4

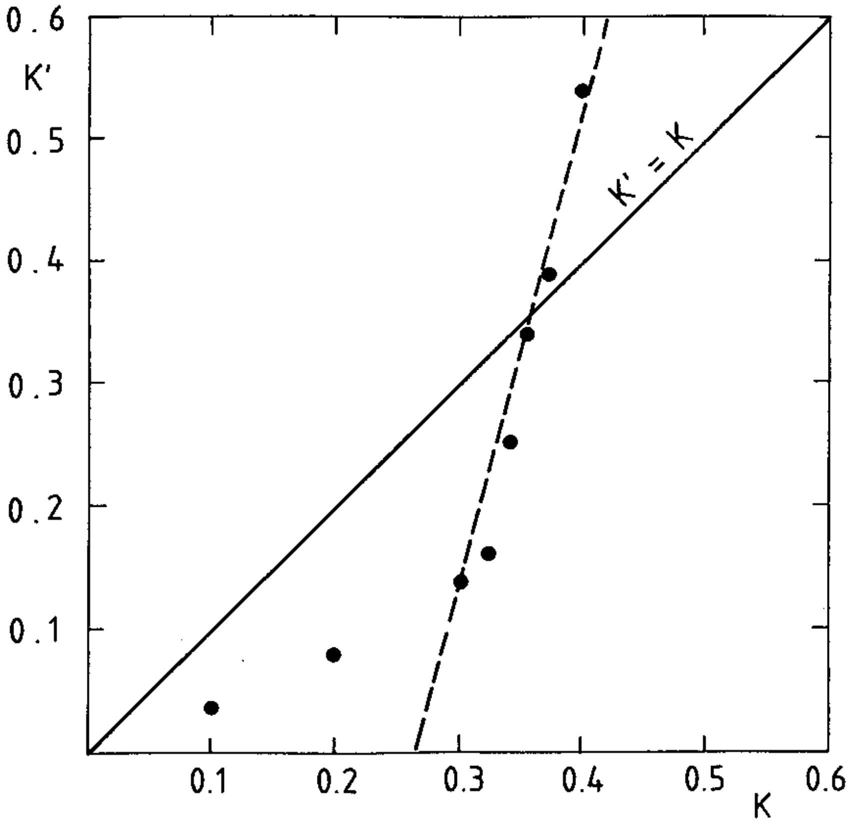

Fig. 5

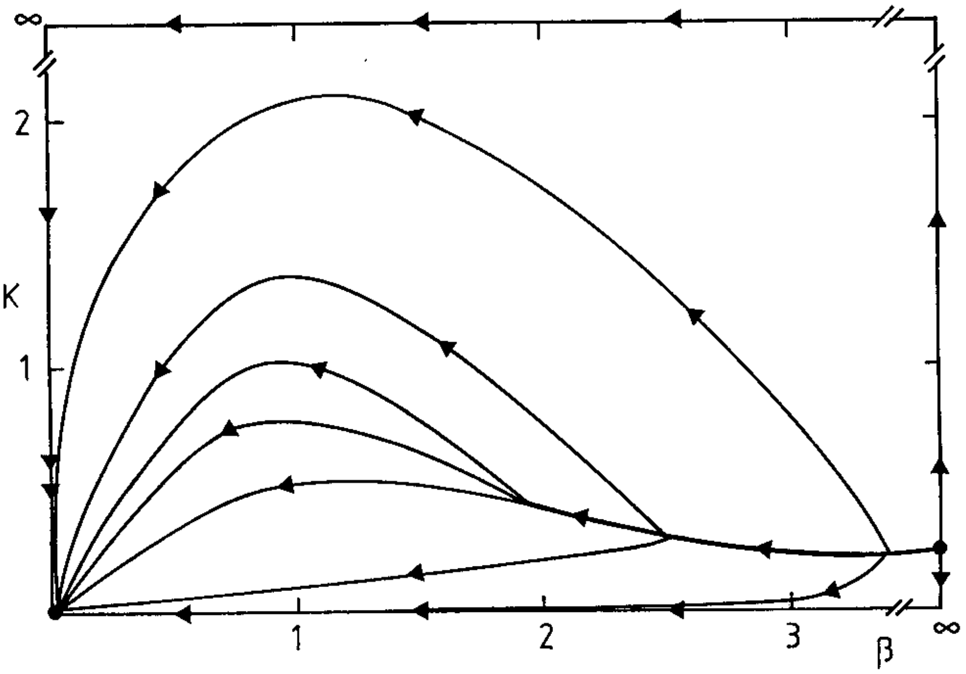

Fig. 6