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SUPERCONDUCTING SURPRISES: The Novel Structure of the Intermediate State in Superconductors and Higgs Theories*

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Abstract

If a large square plate of type I superconductor is placed in a perpendicular magnetic field, the field will penetrate it and generate a pattern of superconducting and normal domains. The problem of interest here is to predict this pattern. Landau presented a solution in 1937 which consists of a periodic series of stripes (either vertical or horizontal) and thus breaks a discrete rotational invariance. Unfortunately this textbook solution is generally inconsistent with the Ginzburg-Landau equations. If these latter equations are used as a starting point then asymmetric elongated structures are predicted (at least near the critical field), again consistent with the breakdown of rotational invariance. Implications for Higgs theories are discussed.

1. Prolegomena

The problem of the magnetic intermediate state in superconductors is both extremely fascinating and extraordinarily difficult. It involves the application of concepts like highly degenerate ground states (reminiscent of spin glasses) as well as complex patterns of multiple length scales (reminiscent of fractals and chaos) and arcane mathematical structures most familiar perhaps to the string theorist. Yet for all the apparent complexity of its solution, the problem itself can be stated with deceptive ease: Consider a block of type I superconducting material placed in a weak magnetic field. Because of the Meissner effect the magnetic field is expelled. As the field strength is increased the magnetic field will penetrate the superconductor. The problem is to predict the pattern of penetration.

2. Background

The simplest paradigm of the intermediate state is a problem originally addressed by Landau [1]. Landau's problem involved a large square plate in a perpendicular magnetic field. He predicted that the magnetic field would penetrate the plate in a periodic series of stripes.

If this prediction were correct, the implications would be somewhat far-reaching, for evidently a discrete

rotational symmetry is broken (square plate → vertical stripes or horizontal stripes). Since Higgs theories are based upon the tenets of superconductivity as well, it would then be fair to ask if rotational symmetries are spontaneously broken in Higgs theories.

Subsequent experimental work revealed that the true situation was often far more complex, involving structures on many (if not all) length scales [2]. Curiously enough, the patterns often include elongated structures, suggesting that Landau's picture might be at least qualitatively correct. If the magnetic field is applied at a slightly <u>oblique</u> angle, the domain patterns found [3] also seem to agree qualitatively with Landau's model.

Before we are too critical of the early attempt of Landau, it is well to remember that at the time (1937) of his analysis, superconductivity was even less well understood than it is today. Most significantly, Landau's analysis predated the Ginzburg-Landau equations (1950) [4], and Abrikosov's germinal work (1957) [5] which first distinguished between type I and type II superconductors. Landau's work was <u>au fond</u> an exercise in classical electromagnetism, augmented with boundary conditions obtained via a phenomenological study of the Meissner effect.

3. Landau's Problem Revisited

A more careful analysis of the problem follows the Ginzburg-Landau approach. Assume that the free energy density $f(\underset{\sim}{x})$ of the system is well approximated by an expansion

$$f(\underset{\sim}{x}) = f_n + \alpha\,|\psi(\underset{\sim}{x})|^2 + \frac{1}{2}\beta\,|\psi(\underset{\sim}{x})|^4 + \frac{\underset{\sim}{H}^2}{4\pi} + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\underset{\sim}{\partial} - \frac{e^*}{c}\underset{\sim}{A}\right)\psi(\underset{\sim}{x})\right|^2 \tag{1}$$

where f_n is the free energy of the normal material, $\psi(\underset{\sim}{x})$ is the order parameter, $\underset{\sim}{H}$ is the magnetic field, and the rest are phenomenological parameters. Rescale so as to measure

$$\psi(\underset{\sim}{x}) \text{ in units } |\psi_\infty| \equiv \left(\frac{-\alpha}{2\beta}\right)^{\frac{1}{2}} \quad (\alpha<0) \tag{2a}$$

$$\underset{\sim}{x} \text{ in units } \xi \equiv \frac{\hbar}{\sqrt{-2m^*\alpha}} \tag{2b}$$

$$f(\underset{\sim}{x}) - f_n \text{ in units } \alpha|\psi_\infty|^2 \tag{2c}$$

$$\underset{\sim}{A}(\underset{\sim}{x}) \text{ in units } \Phi_0/2\pi\,\xi \tag{2d}$$

where $\Phi_0 \equiv 2\pi\hbar\, c/e^*$ is the elementary flux quantum. Also define

$$\lambda_\infty^2 \equiv \frac{m^* c^2}{4\pi|\psi_\infty|^2\, e^{*2}} \tag{3a}$$

$$\kappa^2 \equiv \lambda_\infty^2/\xi^2 \tag{3b}$$

whence

$$f(\underset{\sim}{x}) - f_n = \frac{1}{2}\left(|\psi(\underset{\sim}{x})|^2 - 1\right)^2 + |\underset{\sim}{D}\,\psi(\underset{\sim}{x})|^2 + \kappa^2\,\underset{\sim}{H}^2 \tag{4}$$

where $\underset{\sim}{D} = i\,\underset{\sim}{\partial} - \underset{\sim}{A}$ is the covariant derivative. The corresponding Euler equations of motion are

$$\underset{\sim}{D}^2\,\psi - \psi + |\psi|^2\,\psi = 0 \tag{5a}$$

$$-\kappa^2\,[\partial^2\underset{\sim}{A} - \underset{\sim}{\partial}\,(\underset{\sim}{\partial}.\underset{\sim}{A})] = \mathrm{Re}\,[\psi^*\,\underset{\sim}{D}\,\psi] \tag{5b}$$

Note the appearance of the parameter κ. The value of κ determines [5] whether the superconductor is type I ($\kappa < 1/\sqrt{2}$) or type II ($\kappa > 1/\sqrt{2}$). Here we are primarily concerned with type I superconductors.

Consider Landau's problem of a flat square plate of type I superconductor in the xy plane with a magnetic field incident in the $\hat{z}$ direction. Deep inside the plate and away from the edges the magnetic field $\underset{\sim}{H}$ has only a $\hat{z}$ component. If the thickness of the plate is much greater than ξ, $\underset{\sim}{H}$ will be independent of $\hat{z}$. Thus

$$\underset{\sim}{H} = H(x,y)\,\hat{e}_z \tag{6}$$

The symmetries of the problem then allow us to choose

$$\underset{\sim}{A} = A(x,y)\ \hat{e}_y \tag{7}$$

If we consider the case where $H_o = |\underset{\sim}{H}|$ equals approximately H_c, the critical magnetic field, the order parameter $\psi(\underset{\sim}{x})$ will be small enough that we can linearize the Euler equations. The result is

$$(\underset{\sim}{D}_o^2 - 1)\ \psi(\underset{\sim}{x}) = 0 \tag{8a}$$

where

$$\underset{\sim}{D}_o = \frac{1}{i}\ \underset{\sim}{\partial} - H_o x\ \hat{e}_y \tag{8b}$$

After separating variables we find

$$\psi(\underset{\sim}{x}) = e^{iky}\ r(x) \tag{9a}$$

$$\{\partial_x{}^2 + [1 - H_o{}^2\ (x - \frac{k}{H_o})^2]\}\ r(x) = 0 \tag{9b}$$

Equation (9b) is much like the Schrodinger equation for the one dimensional harmonic oscillator, with an extremely important exception--the energy E equals one! Thus the normalization condition on $\psi(\underset{\sim}{x})$ requires that (for n a positive integer)

$$(2n+1)\ H_o = 1 \tag{10}$$

and thus $H_o \le H_c = 1$. (We should not worry about the seeming quantization of the magnetic field for, after all, this linearized analysis is only valid for $H_o \sim H_c = 1$).

Thus the solution to the linearized Ginzburg-Landau equations for $H_o = H_c = 1$ is simply

$$\psi(x,y) = \sum_k e^{iky - \frac{1}{2}(x-k)^2}\ A(k) \tag{11}$$

where the function A(k) is arbitrary in this limit. Recall that Landau predicted that $|\psi(x,y)|^2$ is independent of y and periodic in x (or vice versa). It is easy to see that the requirement that $|\psi|^2$ is independent of y demands that

$$A(k) \sim \delta\ (k-k_o) \tag{12a}$$

so that

$$|\psi|^2 \sim e^{-\ (x-k_o)^2} \tag{12b}$$

which is not periodic in x. Thus in the strictest sense Landau's solution is (oddly enough) inconsistent with the Ginzburg-Landau equations. (This inconsistency also persists in the full nonlinear case).

Are there any choices of A(k) which lead to a space-filling solution like Landau's? The answer is yes. Following Abrikosov's solution for type II superconductors [5], choose

$$A_p(k) = \sum_{n=-\infty}^{\infty} C_n\ \delta(k-nk_o) \tag{13a}$$

with

$$C_{n+p} = C_n \tag{13b}$$

for some integer p. Then

$$\psi_p(x,y) = \sum_{n=-\infty}^{\infty} e^{ink_o y - \frac{1}{2}(x-nk_o)^2} C_n \tag{14}$$

It is an immediate consequence that

$$\psi_p(x,y + \frac{2\pi}{k_o}) = \psi_p(x,y) \tag{15a}$$

$$\psi_p(x + pk_o, y) = e^{ipk_o y} \psi_p(x,y) \tag{15b}$$

and so $|\psi|^2$ is periodic (and ψ quasi periodic) on a square $(\Delta x, \Delta y) = (pk_o, 2\pi/k_o)$. The $\psi_p(x,y)$ can in fact be written in terms of Jacobi theta functions, e.g.,

$$\psi_1(x,y) = i\, c_o\, e^{-\frac{1}{2}x^2}\, \theta_3\left[ik_o(x+iy),\, e^{-\frac{1}{2}k_o^2}\right] \tag{16}$$

We can determine the finite set of parameters $\{C_n, k_o\}$ by expanding the Ginzburg-Landau free energy in powers of $(H_c-H)/H_c$ about the linear regime and minimizing. This can be accomplished by inserting a fictitious parameter ϵ in the GL equations,

$$\underset{\sim}{D}^2 \psi - \psi + \epsilon |\psi|^2 \psi = 0 \tag{17a}$$

$$-\kappa^2 \left[\partial^2 \underset{\sim}{A} - \underset{\sim}{\partial} (\underset{\sim}{\partial} . \underset{\sim}{A})\right] = \epsilon \,\mathrm{Re}\left[\psi^* \underset{\sim}{D} \psi\right] \tag{17b}$$

$$\underset{\sim}{A} = \underset{\sim}{A}_o + \epsilon \underset{\sim}{A}_1 + \epsilon^2 \underset{\sim}{A}_2 + \ldots \tag{17c}$$

$$\psi = \psi_o + \epsilon \psi_1 + \epsilon^2 \psi_2 \ldots \tag{17d}$$

Compute the first correction (which is of order ϵ^2) to the free energy and set ϵ to one. The result is (see also [5,6]):

$$\kappa^{-2} (f - f_n) = \frac{1}{2} + B^2 - \frac{(B-1)^2}{1-(2\kappa^2 - 1)\beta} + \ldots \tag{18}$$

where B is the flux per unit area and

$$\beta = \frac{\overline{|\psi|^4}}{(\overline{|\psi|^2})^2} \tag{19}$$

is explicitly independent of B and the normalization of ψ. Here the bar denotes a spatial average over x and y.

Recall that Abrikosov's analysis concerned type II superconductors (where $\kappa > 1/\sqrt{2}$). When κ exceeds $1/\sqrt{2}$ the free energy Eq. (18) is minimized by making β as small as possible. This leads to [5] the standard prediction of a triangular-lattice array of vortices for a type II superconductor. Here we go <u>beyond</u> Abrikosov and look at $\kappa < 1/\sqrt{2}$, for which the minimum free energy occurs when

$$\beta \,\tilde{<}\, \frac{1}{1-2\kappa^2} \tag{20}$$

Note therefore that if $\kappa \,\tilde{<}\, 1/\sqrt{2}$, β will be quite large.

For the case p=1, β is given by

$$\beta\ (k_o) = \sum_{m=-\infty}^{\infty} e^{-\frac{1}{2} k_o^2 m^2} \sum_{n=-\infty}^{\infty} e^{-\frac{1}{2} (\frac{2\pi}{k_o})^2 n^2} \qquad (21)$$

[whence it is obvious that $\beta(k_o) = \beta\ (\frac{2\pi}{k_o})$]. Away from its minimum value $\beta\ (k_o = \sqrt{2\pi}) \simeq 1.18$, $\beta(k_o)$ is well approximated by

$$\beta(k_o) = \frac{\sqrt{2\pi}}{k_o} + \frac{k_o}{\sqrt{2\pi}} \qquad (22)$$

Since

$$|\psi(x,y)|^2 = |\psi(x, y + \frac{2\pi}{k_o})|^2 \qquad (23a)$$

$$= |\psi(x+k_o, y)|^2 \qquad (23b)$$

we see that $|\psi|^2$ is $x \leftrightarrow y$ symmetric only when $k_o^2 = 2\pi$ (where $\beta = 1.18$). Note however that the extremum condition Eq. (20) implies that β is much greater than this value for $\kappa^2 \sim \frac{1}{2}$. Thus elongated structures are preferred. Since the free energy associated with $\beta\ (k_0)$ is degenerate with that for $\beta(2\pi/k_0)$, a domain pattern of bands meeting at right angles is to be expected. Moreover this high degree of degeneracy assures us that the domain patterns so generated will be largely unpredictable and irreversible as parameters like the temperature and magnetic field are cycled. Note also that this degeneracy should disappear when an oblique magnetic field is applied, and the long axes

of the elongated patterns will (by the Meissner effect) align themselves with the component of the magnetic field parallel to the plate.

Since the Ginzburg-Landau model of superconductivity is actually a Higgs theory, it would be no surprise if the phenomena described here (spontaneous breakdown of discrete rotational symmetries, fractal domain structures, etc.) appeared in Higgs models in general. It is therefore to be expected that numerical analyses of lattice Higgs models will prove to be of interest in understanding these nonlinear effects.

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