Rules of Inference, Propositional Logic

Keith Burgess-Jackson

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Implication (One-Way) Rules ("$\therefore$" =df. "therefore")			
Name	**Abbreviation**	**Rule**	**Comments**
Modus Ponens	MP	$p \supset q$ p $\therefore q$	Pithy statement: "Anything materially implied by a truth is true."
Modus Tollens	MT	$p \supset q$ $\sim q$ $\therefore \sim p$	Pithy statement: "Anything that materially implies a falsehood is false."
Hypothetical Syllogism	HS	$p \supset q$ $q \supset r$ $\therefore p \supset r$	Pithy statement: "Material implication is transitive."
Disjunctive Syllogism	DS	$p \vee q$ $\sim p$ $\therefore q$	Pithy statement: "If at least one of two propositions is true, and one of them is false, then the other is true."
Constructive Dilemma	CD	$(p \supset q) \bullet (r \supset s)$ $p \vee r$ $\therefore q \vee s$	Pithy statement: "Both of these conditionals are true; at least one of their antecedents is true; therefore, at least one of their consequents is true."
Simplification	Simp	$p \bullet q$ $\therefore p$	Pithy statement: "Both propositions are true; therefore, each proposition is true."
Conjunction	Conj	p q $\therefore p \bullet q$	Pithy statement: "Each proposition is true; therefore, both propositions are true."
Addition	Add	p $\therefore p \vee q$	Pithy statement: "Any proposition may be disjoined to any proposition."
Replacement (Two-Way) Rules ("::" =df. "is logically equivalent to")			
De Morgan's Theorems	DM	$\sim (p \bullet q) :: \sim p \vee \sim q$ $\sim (p \vee q) :: \sim p \bullet \sim q$	Pithy statement: "The negation of the {conjunction/disjunction} of two propositions is logically equivalent to the {disjunction/conjunction} of the negations of the two propositions."
Commutation	Com	$p \vee q :: q \vee p$ $p \bullet q :: q \bullet p$	

Association	Assoc	$p \vee (q \vee r) :: (p \vee q) \vee r$ $p \bullet (q \bullet r) :: (p \bullet q) \bullet r$	
Distribution	Dist	$p \bullet (q \vee r) :: (p \bullet q) \vee (p \bullet r)$ $p \vee (q \bullet r) :: (p \vee q) \bullet (p \vee r)$	
Double Negation	DN	$p :: \sim\sim p$	Pithy statement: "Tildes can be added or subtracted in pairs."
Transposition	Trans	$p \supset q :: \sim q \supset \sim p$	Note the similarity between this rule and contraposition (from categorical logic): "All S are P" :: "All nonP are nonS."
Material Implication	MI	$p \supset q :: \sim p \vee q$	To say that p materially implies q is to say that either p is false or q is true.
Material Equivalence	ME	$p \equiv q :: (p \supset q) \bullet (q \supset p)$ $p \equiv q :: (p \bullet q) \vee (\sim p \bullet \sim q)$	"Biconditional" means "two conditionals." To say that p is materially equivalent to q is to say two things: first, that p and q materially imply each other; and second, that either both are true or both are false.
Exportation	Exp	$(p \bullet q) \supset r :: p \supset (q \supset r)$	Example: "If it's an even-numbered year and it's November, then there are Congressional elections" is logically equivalent to "If it's an even-numbered year, then, if it's November, then there are Congressional elections."
Tautology	Taut	$p :: p \vee p$ $p :: p \bullet p$	Be careful: p is *not* logically equivalent to $p \supset p$ or $p \equiv p$. To see this, suppose p is false; then $p \supset p$ and $p \equiv p$ are *true*.

There are 72 inferences represented on this chart. Each implication rule, of which there are eight, represents one inference, for a total of eight. Each logical equivalence, of which there are 16 (count the double colons), represents four inferences, for a total of 64. Eight plus 64 = 72. To see how one double colon generates four inferences, consider Double Negation: "p :: ~~p" generates (1) p; therefore, ~~p; (2) ~~p; therefore, p; (3) ~p; therefore, ~~~p; and (4) ~~~p; therefore, ~p.