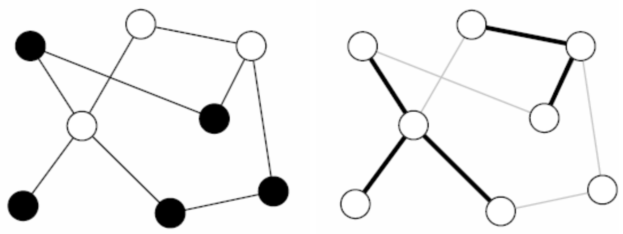
SPREADING PROCESS ON NETWORKS

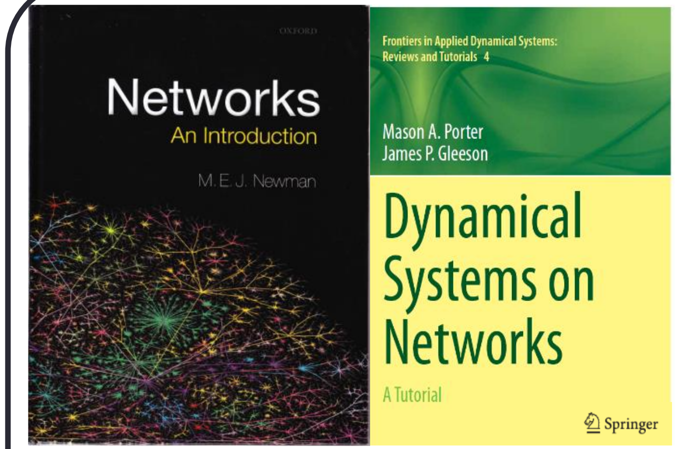
Lots of different types!

Dynamics on networks

- Site Percolation
- Bond Percolation



site percolation   bond percolation



See Chapter 3 of Porter & Gleeson, *Dynamical Systems on Networks: A Tutorial*, Springer (2016)

Percolation on complex networks: Theory and application

Ming Li [a], Run-Ran Liu [b], Linyuan Lü [c,b,d,*], Mao-Bin Hu [a], Shuqi Xu [c], Yi-Cheng Zhang [e]

Catastrophic cascade of failures in interdependent networks

Sergey V. Buldyrev, Roni Parshani, Gerald Paul, H. Eugene Stanley & Shlomo Havlin

Network reliability analysis based on percolation theory

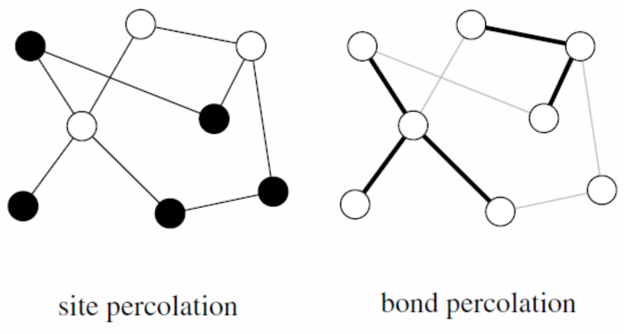
Daqing Li [a,*], Qiong Zhang [a], Enrico Zio [b,c], Shlomo Havlin [d], Rui Kang [a]

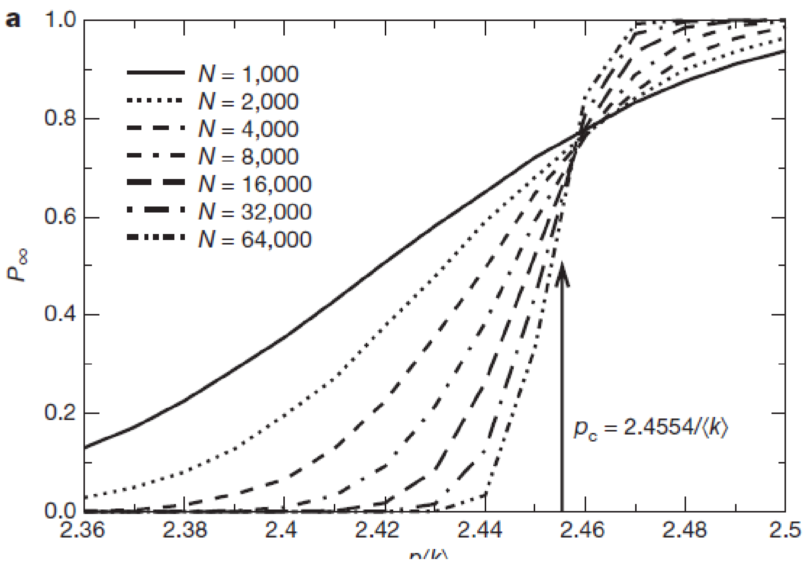
Assessing police topological efficiency in a major sting operation on the dark web

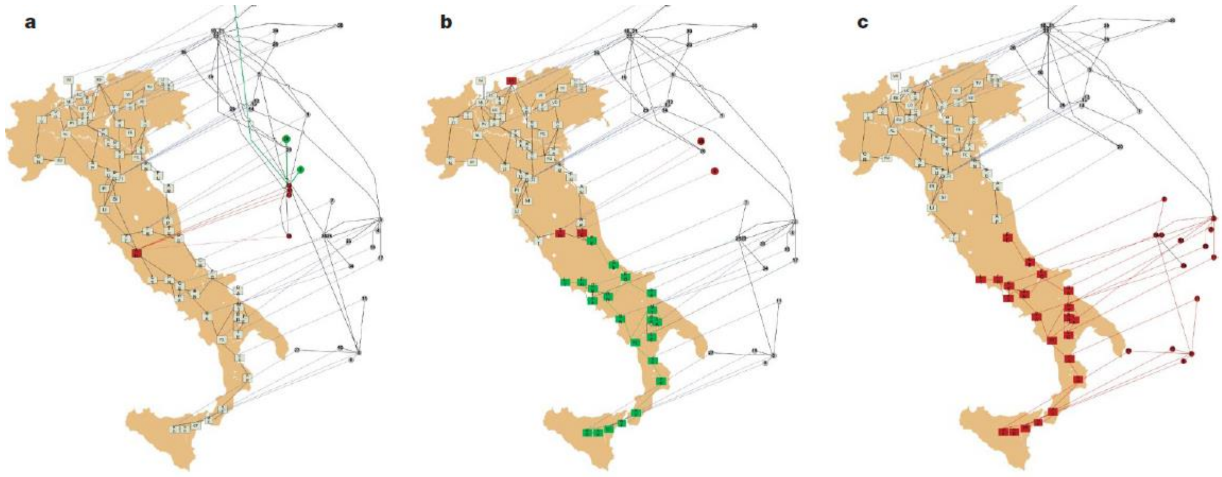
Bruno Requião da Cunha, Pádraig MacCarron, Jean Fernando Passold, Luiz Walmocyr dos Santos Jr., Kleber A. Oliveira & James P. Gleeson

Dynamics on networks

- Site Percolation
- Bond Percolation







Dynamics on networks

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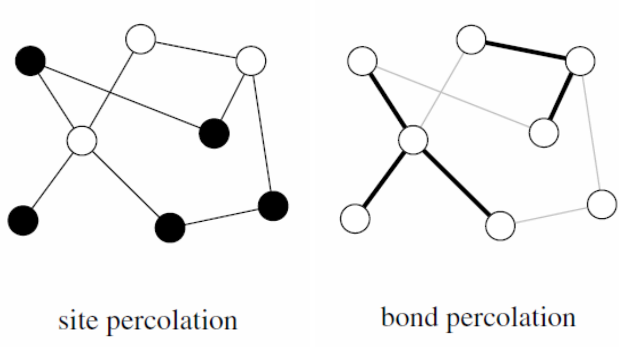
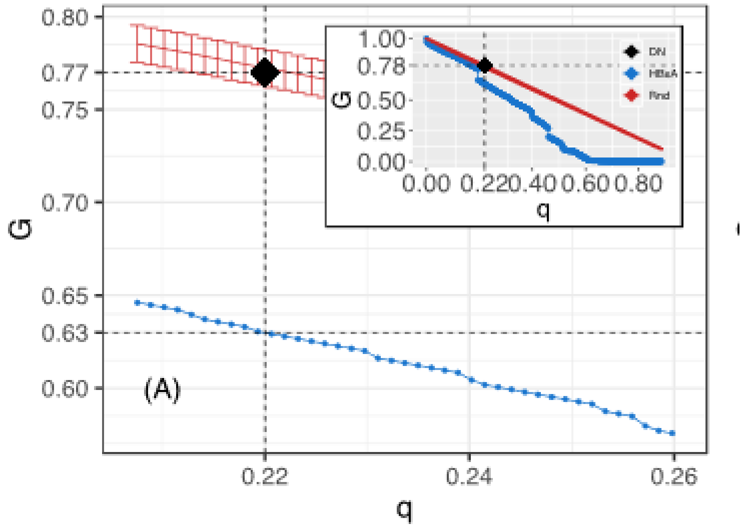
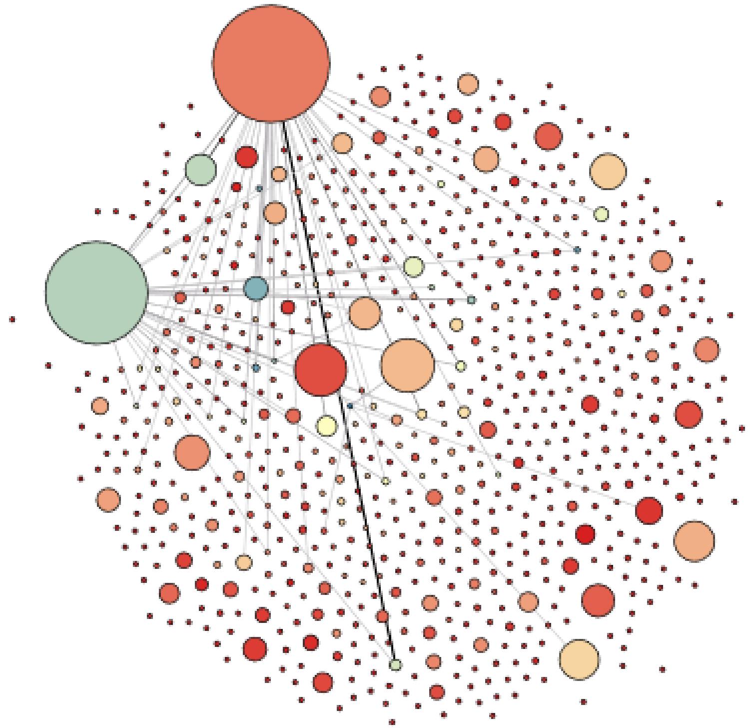
- Bond Percolation







Assessing police topological efficiency in a major sting operation on the dark web

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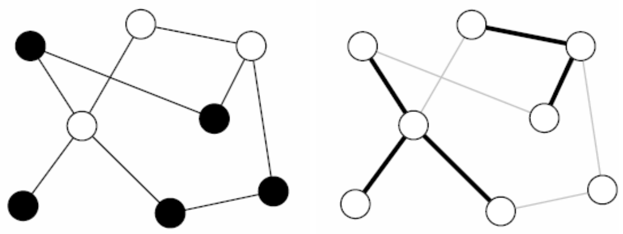
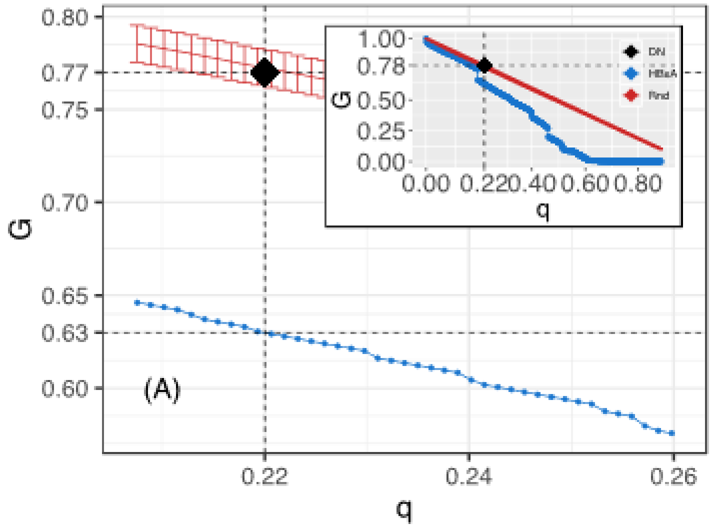
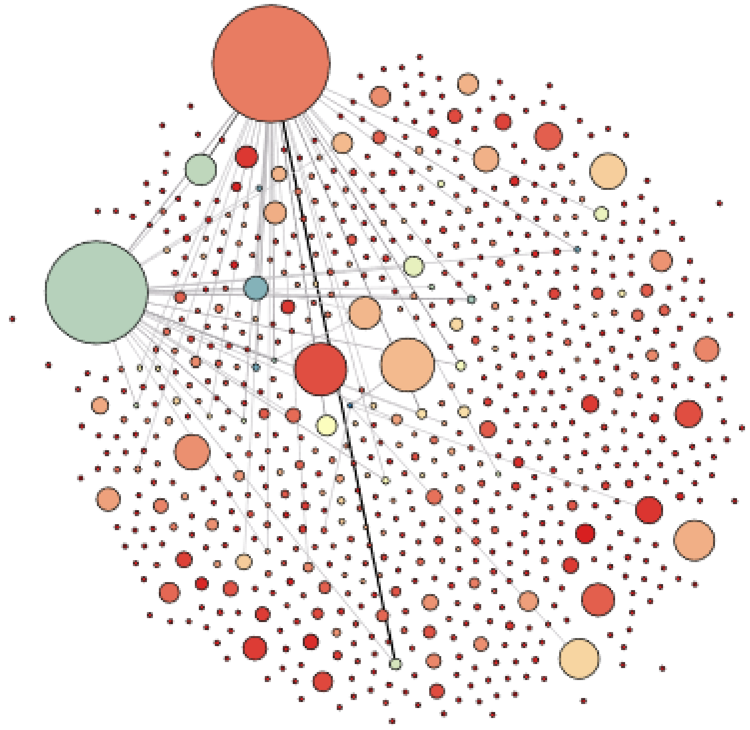


site percolation bond percolation





Assessing police topological efficiency in a major sting operation on the dark web

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Dynamics on networks

Threshold models of social contagion

- Each node i has a threshold R_i that is drawn from a distribution -> this does not change in time.
- Node states at any time:
  - 0 (inactive, not adopted, not infected, etc.) or 1 (active, adopted, infected, etc.)
  - At time $t = 0$ some (small) fraction ρ_0 of the nodes are in the active state

- Update rule: if inactive, node i compares its fraction m_i/k_i of active neighbours
  - m_i is the number of active neighbours
  - k_i is the node's degree
  - becomes active if $\frac{m_i}{k_i} \geq R_i$.

Threshold Models of Collective Behavior

Mark Granovetter

A simple model of global cascades on random networks

Duncan J. Watts*

Complex Contagions and the Weakness of Long Ties1

Damon Centola, and Michael Macy

Dynamics on networks

Threshold models of social contagion

- If active, node i does not change its state.
- Sometimes called complex contagion models
  - in contrast to the simple contagion of disease-spread models
  - because threshold models generally require a node to have more than one infected neighbour in order to become infected itself
- More on simple vs complex contagion later

Threshold Models of Collective Behavior

Mark Granovetter

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Dynamics on networks

Other discrete-state models:

- Voter model
- Axelrod model of opinion dynamics

Some continuous-state models:

- Bounded-confidence opinion dynamics models
- Metapopulation models
- Oscillator synchronisation

Related topics:

- Temporal networks
- Adaptive networks: dynamics *on* and *of* networks

MODELS OF SIMPLE CONTAGIONS, BRIEFLY

Dynamics on networks

SEIR type models for biological contagion

- States: *exposed, susceptible* (healthy), *infected, or recovered*
- Event are governed by 'hazard rate'; probability per unit time

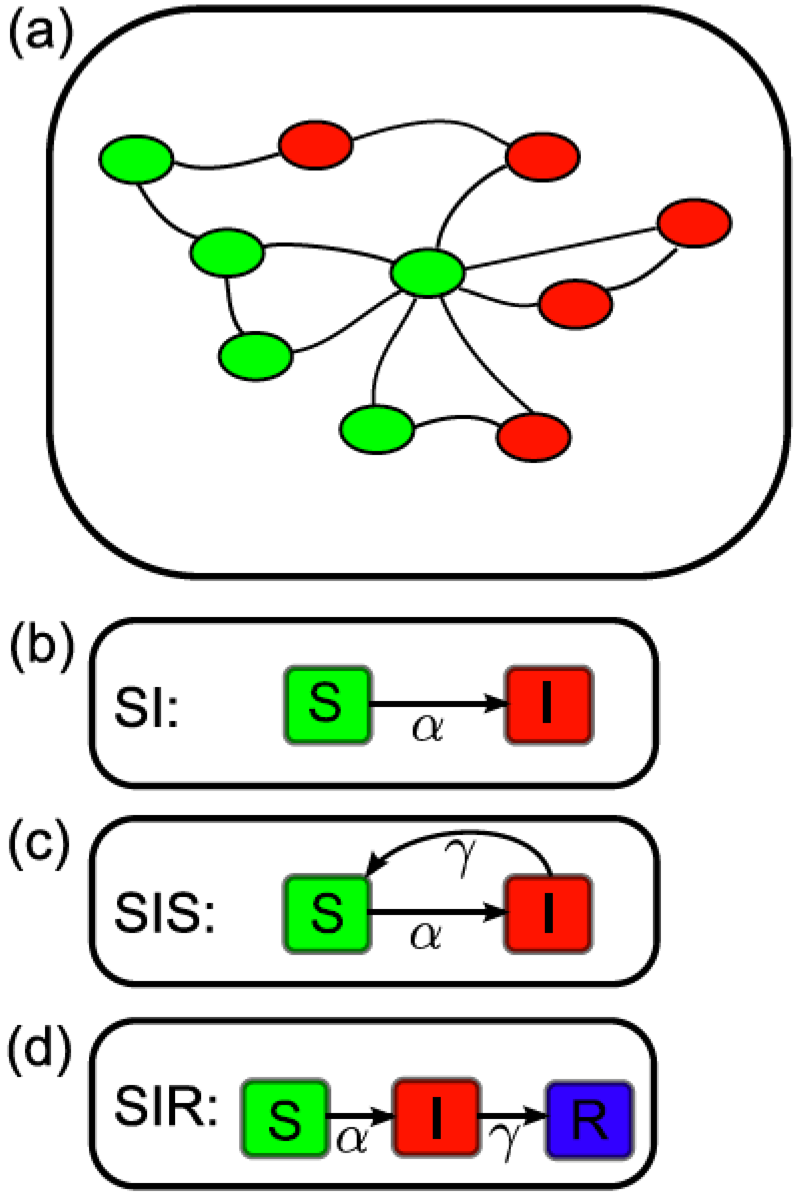
RESEARCH-ARTICLE

Epidemiological modeling of news and rumors on Twitter

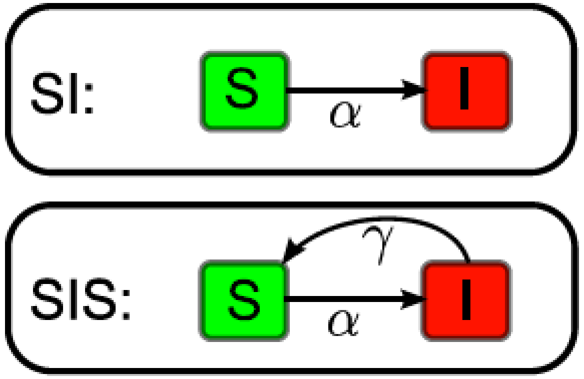
Authors: Fang Jin, Edward Dougherty, Parang Saraf, Yang Cao, Naren Ramakrishnan

Authors Info & Affiliations

- But how do we account for the network?



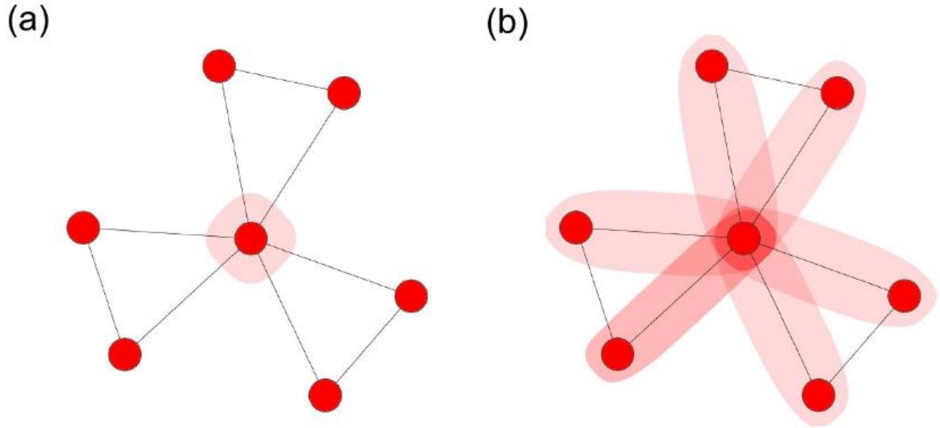
Dynamics on networks



Different levels of approximation are made

- a) Heterogeneous Mean-Field (MF)
- MF approximation assume states neighbouring nodes are statistically independent

Epidemic Spreading in Scale-Free Networks
Romualdo Pastor-Satorras and Alessandro Vespignani
Phys. Rev. Lett. **86**, 3200 – Published 2 April 2001



- b) Pair approximation and closure conditions

Epidemic processes in complex networks
Romualdo Pastor-Satorras, Claudio Castellano, Piet Van Mieghem, and Alessandro Vespignani
Rev. Mod. Phys. **87**, 925 – Published 31 August 2015

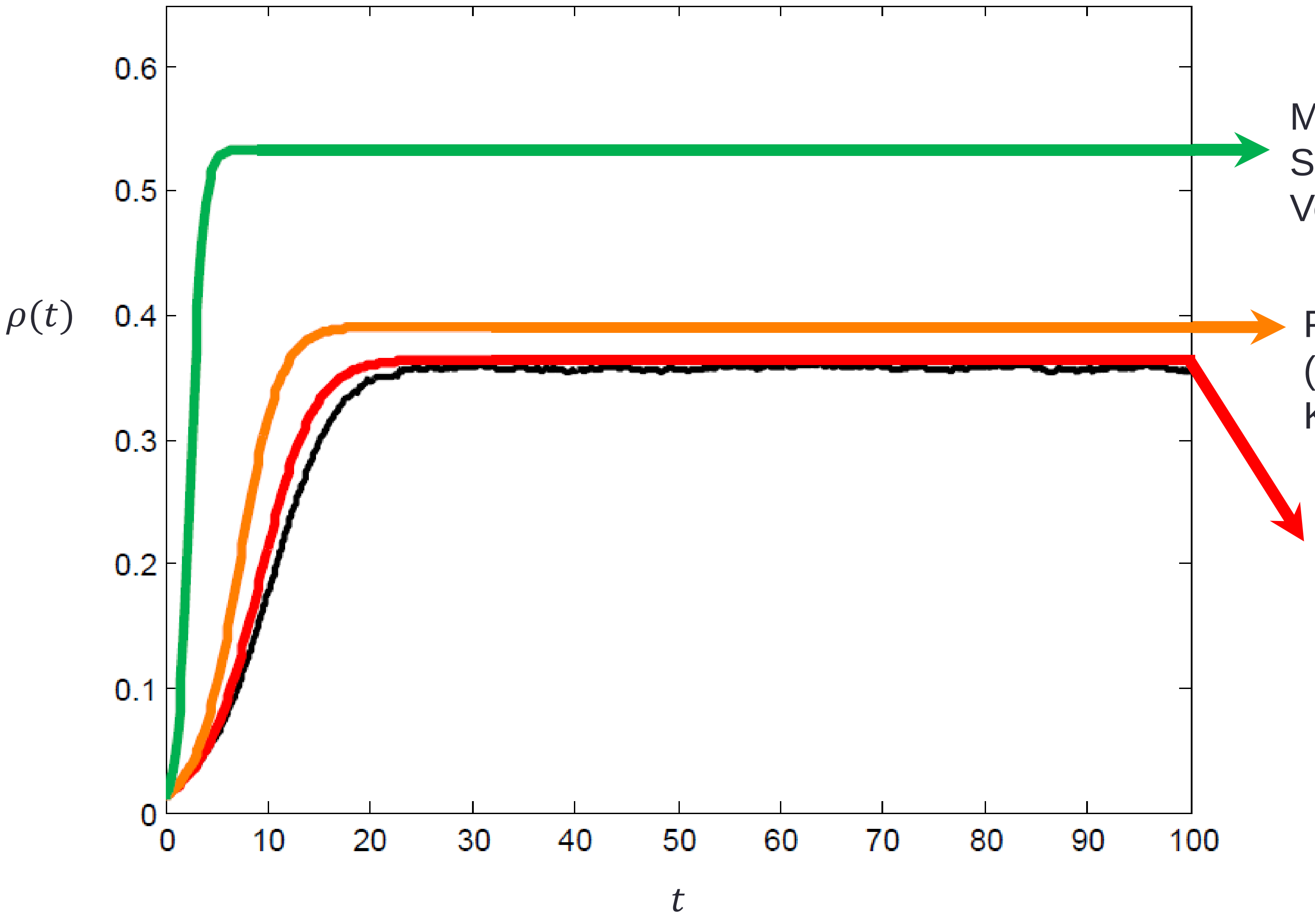
- Further…

Binary-State Dynamics on Complex Networks: Pair Approximation and Beyond

James P. Gleeson*

MACSI, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland
(Received 8 October 2012; published 29 April 2013)

Dynamics on networks



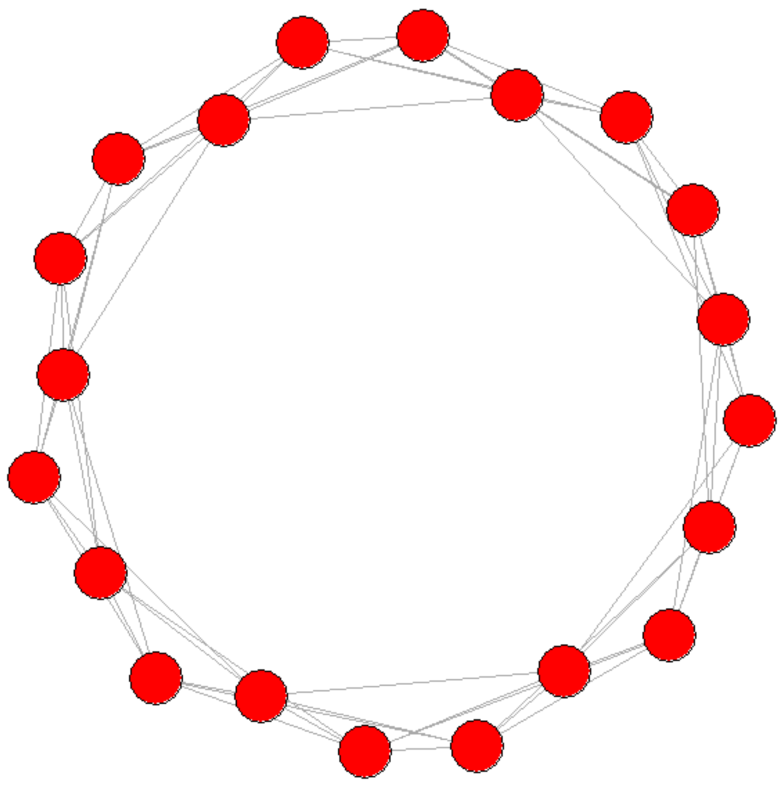
MF theory of Pastor-Satorras and Vespignani (2001)

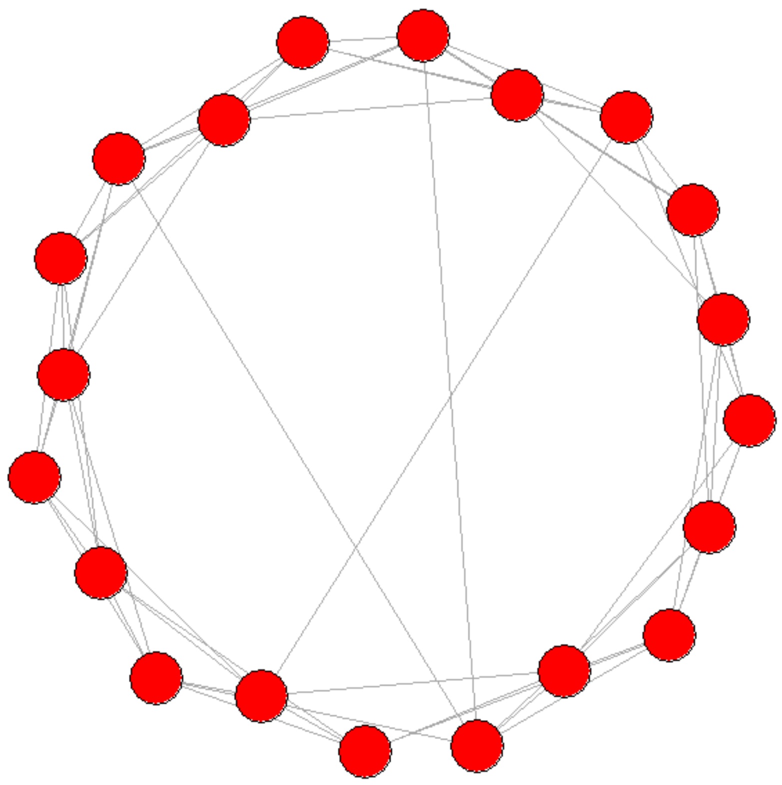
PA: Levin and Durrett (1996); Eames and Keeling (2002)

AME: Marceau et al. (2010), Lindquist et al. (2011)

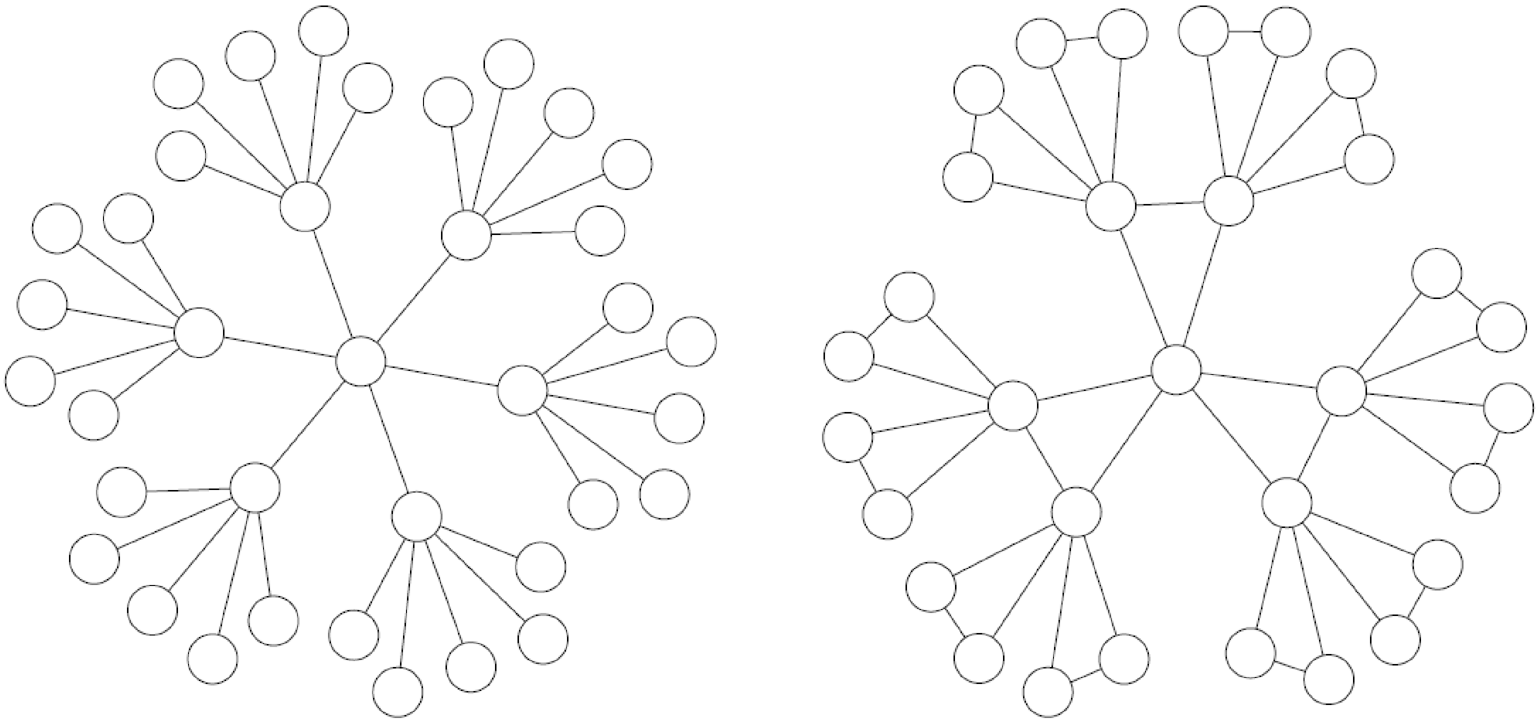
TYPES OF CONTAGIONS

Which network would a contagion spread faster on?





Which network would a contagion spread faster on?



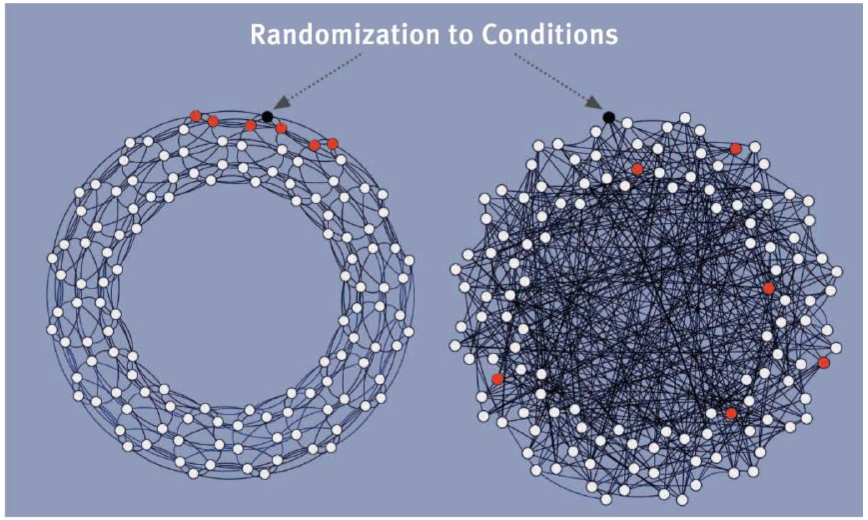
(a) (b)

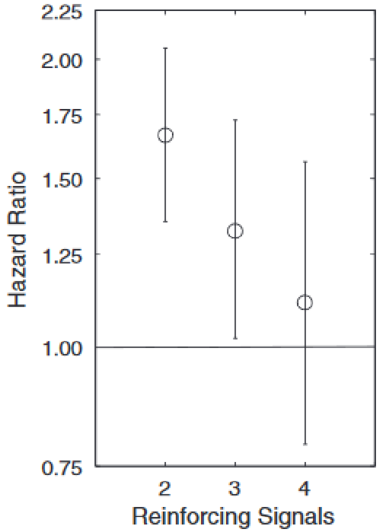
Type of contagion models

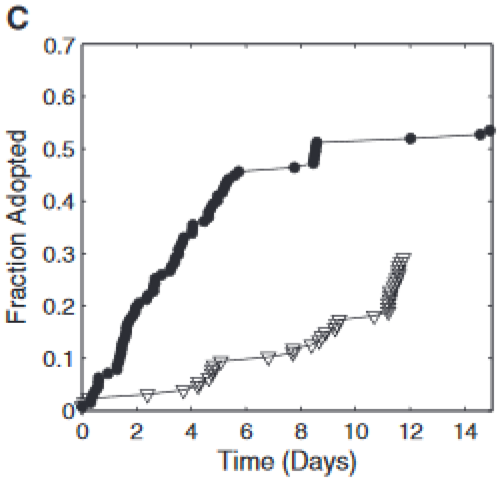
The Spread of Behavior in an Online Social Network Experiment

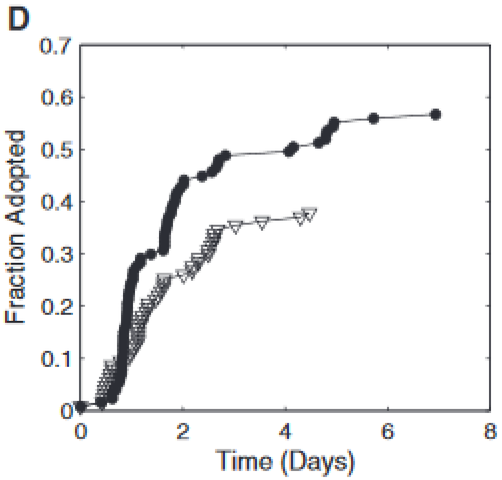
Damon Centola
+ See all authors and affiliations

- Two types that are of interest in this setting
  - Simple contagion
  - Complex contagion
- How do they spread differently?
  - Simple contagion - faster on a network with low clustering
  - Complex contagion - faster on a network with high clustering









Complex contagion and clustering

- Found both in spread of behaviours and topics

RESEARCH-ARTICLE

Differences in the mechanics of information diffusion across topics: idioms, political hashtags, and complex contagion on twitter

Authors: Daniel M. Romero, Brendan Meeder, Jon Kleinberg Authors Info & Affiliations

Complex Contagions: A Decade in Review

Douglas Guilbeault, Joshua Becker and Damon Centola*

Complex contagion and clustering

- Clustering has been incorporated in to continuous time dynamics (your SIR type models)

Propagation dynamics on networks featuring complex topologies

Laurent Hébert-Dufresne, Pierre-André Noël, Vincent Marceau, Antoine Allard, and Louis J. Dubé
Phys. Rev. E **82**, 036115 – Published 27 September 2010

- And complex contagion

Mathematical modeling of complex contagion on clustered networks

David J. P. O'Sullivan*, Gary J. O'Keeffe, Peter G. Fennell and James P. Gleeson

Mathematics Applications Consortium for Science and Industry, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland

- But would be great to have a model for it using branching process to model cascade spread

Complex contagion and clustering

- Cascade dynamics for simple contagion: ICM
  - Simulation based

Maximizing the Spread of Influence through a Social Network

David Kempe[*]
Dept. of Computer Science
Cornell University, Ithaca NY
kempe@cs.cornell.edu

Jon Kleinberg[†]
Dept. of Computer Science
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Éva Tardos[‡]
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Cornell University, Ithaca NY
eva@cs.cornell.edu

  - Branching processes

Branching process descriptions of information cascades on Twitter

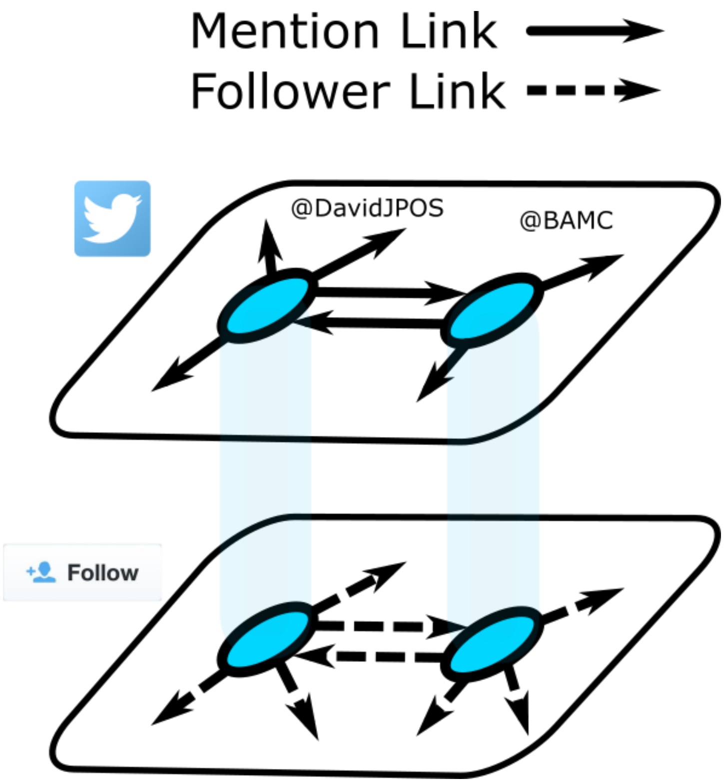
James P Gleeson, Tomokatsu Onaga, Peter Fennell, James Cotter, Raymond Burke, David J P O'Sullivan

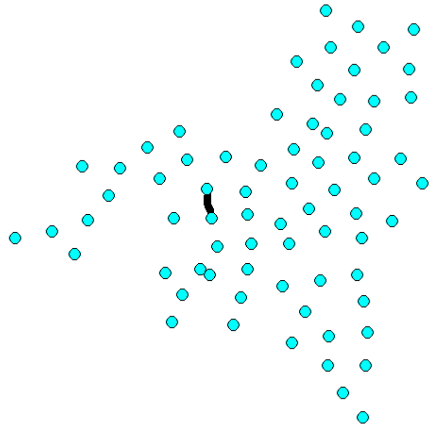
Journal of Complex Networks, Volume 8, Issue 6, 1 December 2020, cnab002,
https://doi.org/10.1093/comnet/cnab002
Published: 20 March 2021 **Article history** ▾

CASCADES

Cascade

- How might we construct cascades to compare a simple model to?
- Irish marriage referendum dataset was extensive
  - Allowed us to generate retweet cascades
  - See how information spreads
    - Across the network
  - How the cascades were created
  - Summarized
  - Comparison to synthetic cascades -> ICM





Cascade

- Dataset was extensive
  - Allowed us to generate retweet cascades

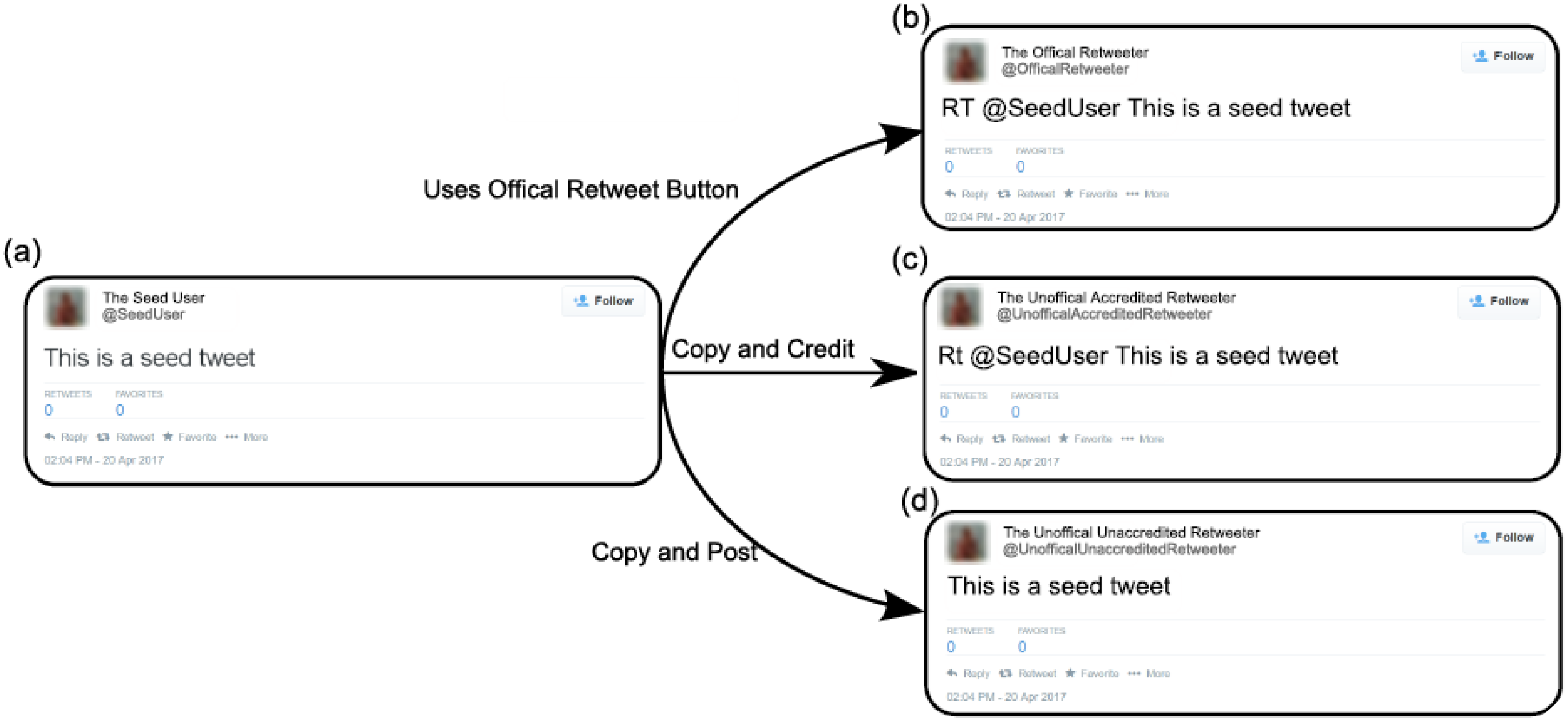
The Structural Virality of Online Diffusion

Sharad Goel, Ashton Anderson
Stanford University, Stanford, California, 94305 {scgoel@stanford.edu, ashton@cs.stanford.edu}
Jake Hofman, Duncan J. Watts
Microsoft Research, New York, New York 10016 {jmh@microsoft.com, duncan@microsoft.com}

- **Corpus identification:** We take the full dataset of tweets and identify the ones that were part of a retweet cascade, either as *seed* content (the original post) or as a retweet (the rebroadcasted post).
- **Parent attribution:** For each retweet and user who sent the retweet, we ascribe a parent: the user who most likely introduced the user to the retweeting content that was retweeted.

Cascade Construction

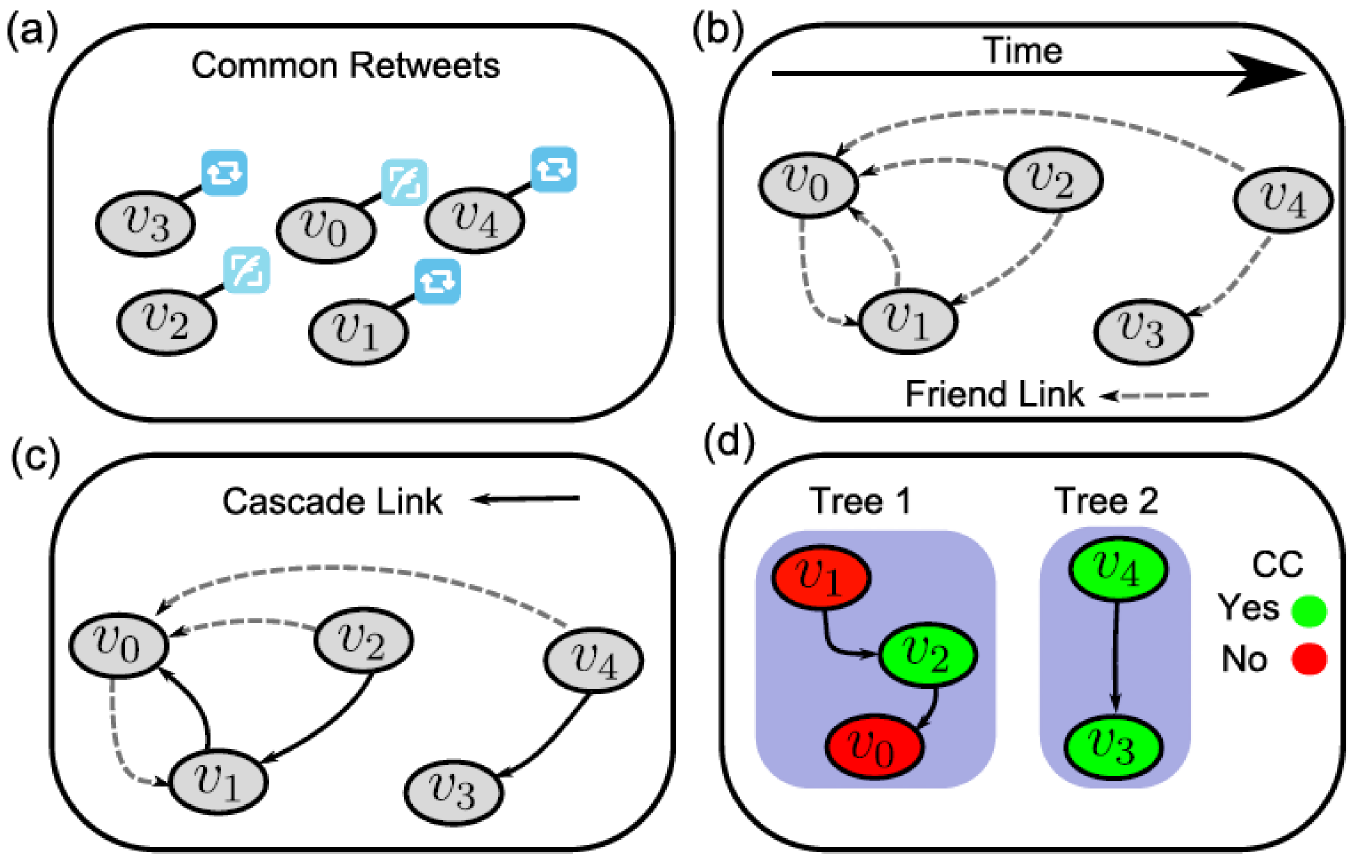
- Corpus identification



- Goel et al., Management Science, 62, 180 (2015)

Cascade Construction

- Patent attribution for tweets with common text



- Goel et al., Management Science, 62, 180 (2015)

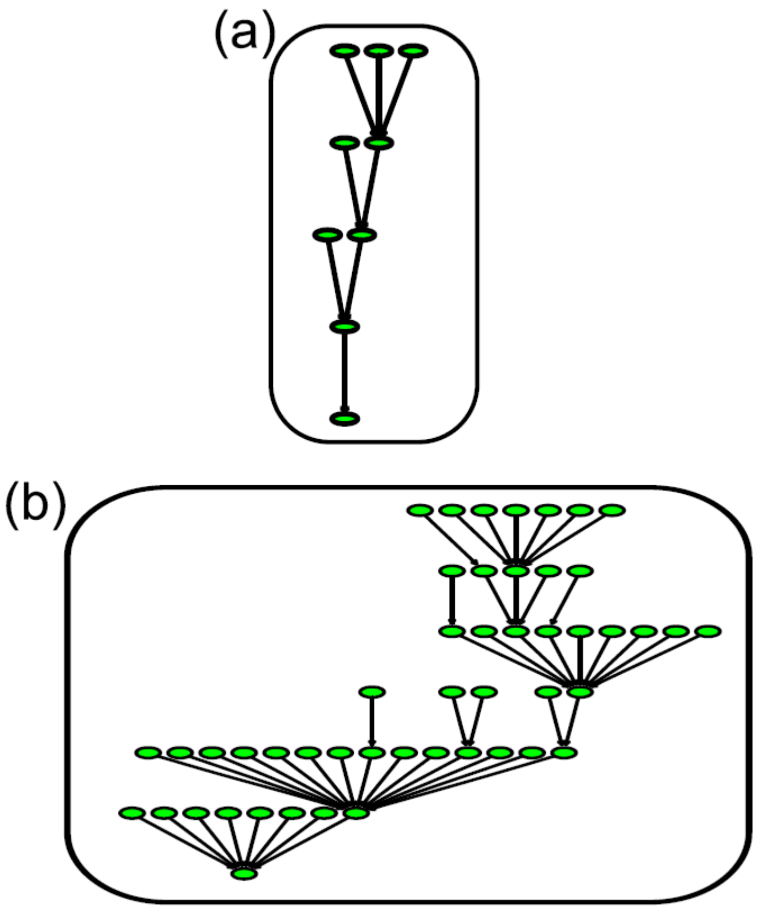
How to summarise the spread?

- How can we best summarise the structure or size of a cascade?

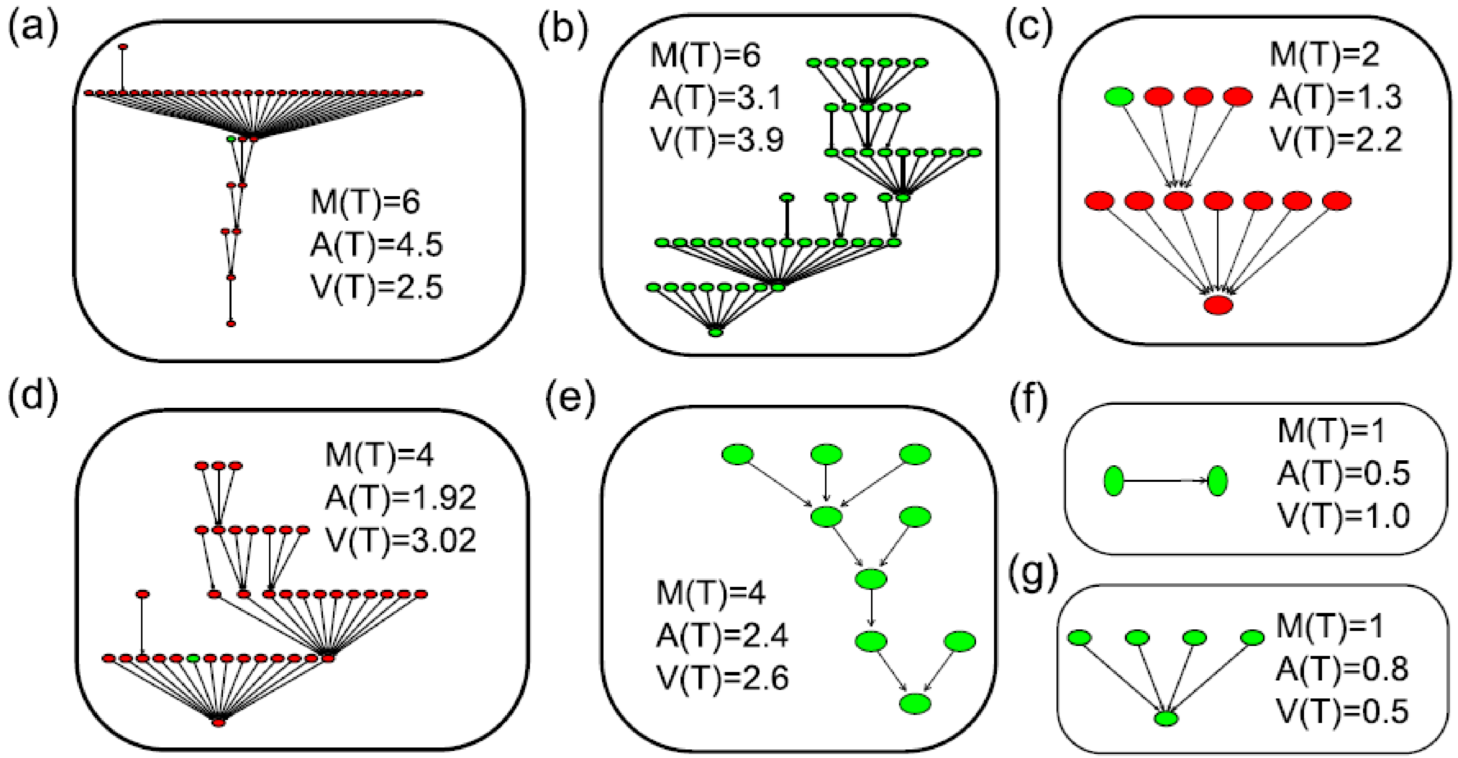
$$M(T) = \max_{l} d_{lj}$$

$$A(T) = \frac{1}{n}\sum_{l}^{n} d_{lj}$$

$$V(T) = \begin{cases} \frac{1}{n(n-1)} \sum_{l=1}^{n} \sum_{j=1}^{n} d_{lj} & \text{if } n > 0, \\ 0 & \text{if } n = 0 \end{cases}$$



How to summarise the spread?



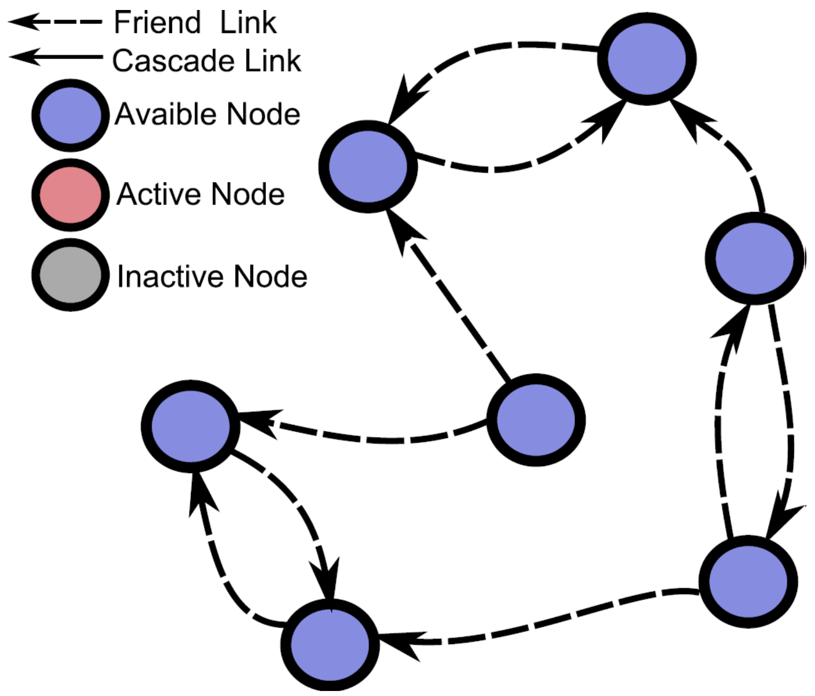
$$M(T) = \max_l d_{lj}$$

$$A(T) = \frac{1}{n} \sum_l^n d_{lj}$$

$$V(T) = \begin{cases} \frac{1}{n(n-1)} \sum_{l=1}^n \sum_{j=1}^n d_{lj} & \text{if } n > 0, \\ 0 & \text{if } n = 0 \end{cases}$$

The Cascades Models

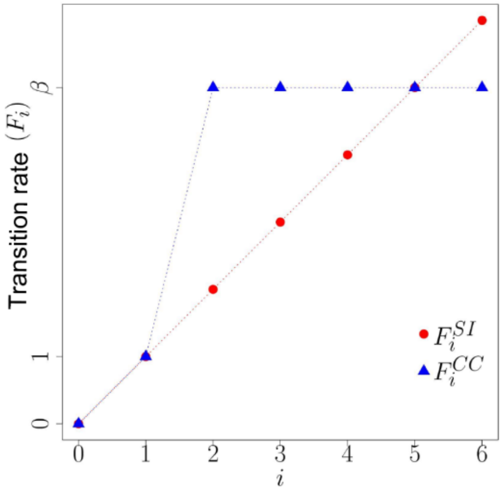
- We have
  - The empirical cascade & networks
  - Some cascade summary measures
    - (Focus on V(T))
- Need synthetic cascades
  - ICM & LAM
- Both
  - Popular models
  - Simple mechanisms
  - Capture global features
  - Fast to simulate
  - Simple contagion

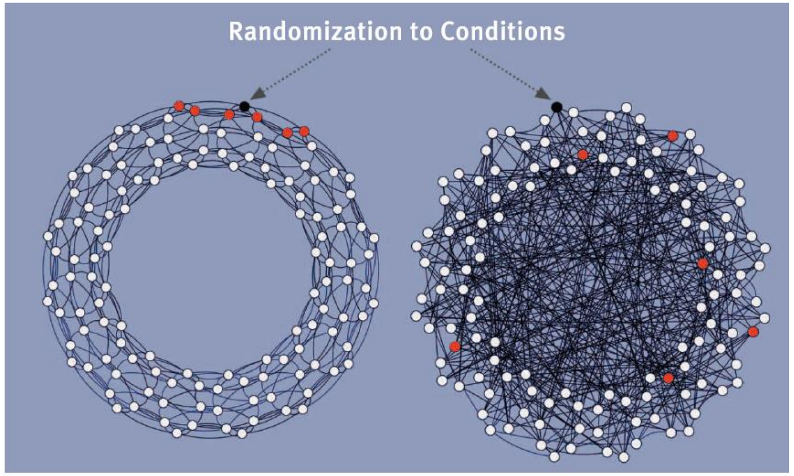


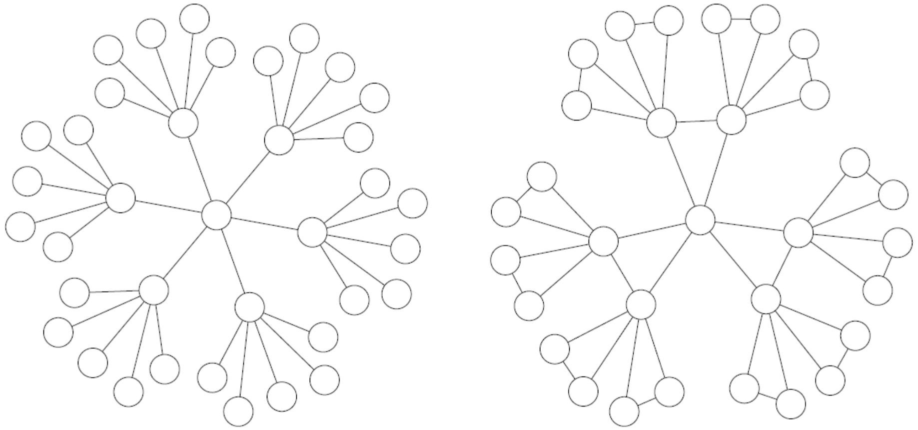
Finally, lets play with some cascades

- Two types that are of interest in this setting
  - Simple contagion
  - Complex contagion

- How do they spread differently?
  - Simple contagion
    - Faster on a network with high clustering or low?



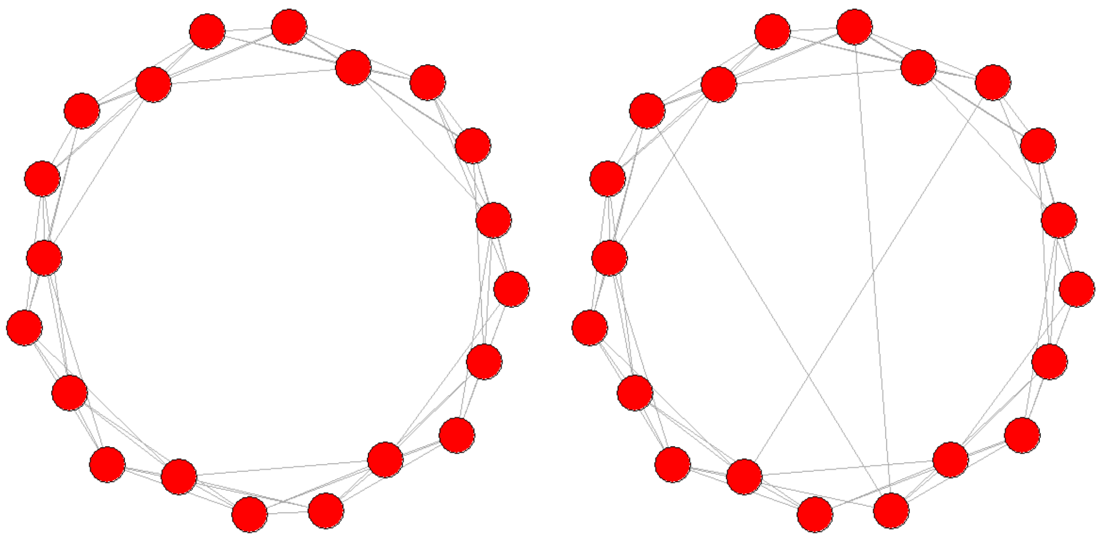




(a) (b)

Again, another little break with R

- 7_cascade_simulation.R
- Recreate the cascades results from with the cascades fitted
- Use the ICM model to see how simple contagion spreads on network with various amounts of clustering



SUMMING UP