

Symbol	Description	First Slide
$X$	Typical random variable of interest	3
$f$	Density of $X$ ; <i>unknown</i> ; $f \in L_2(\mathbb{R})$	3
$\varepsilon$	Typical error random variable, $X \perp \varepsilon$	3
$g$	Density of $\varepsilon$ ; <i>known</i> ; even; a.e. non-zero Fourier trans.	3
$Y$	$Y = X + \varepsilon$ ; typical observed random variable	3
$h$	$h = g * f$ ; Density of $Y$ ; <i>unknown</i> ; to be estimated	3
$\tilde{f}, \tilde{g}, \tilde{h}$	Twiddle always denotes Fourier transform $\tilde{f}(\omega) = \int e^{-i\omega x} f(x) dx$	6
$\ \cdot\ $	Standard $L_2$ norm: $\ f\ ^2 = \int  f ^2$	6
$h_n$	Density estimate of $h$ from $Y_1, \dots, Y_n$ i.i.d. as $Y$ (any $L_2(\mathbb{R})$ -consistent method)	10
$T$	Operator $L_2(\mathbb{R}) \rightarrow L_2(\mathbb{R})$ mapping $v \mapsto g * v$	10
$\ g * v - h_n\ ^2 + \alpha \ v^{(2)}\ ^2$	“Tikhonov functional” considered as a map $v \mapsto \ g * v - h_n\ ^2 + \alpha \ v^{(2)}\ ^2$	15
$f_n^\alpha$	<b>Our estimate of <math>f</math></b> ; the $v$ minimizing above Tikhonov functional	15
$\tilde{\varphi}_\alpha(\omega)$	$\tilde{\varphi}_\alpha(\omega) = \frac{\tilde{g}(\omega)}{\tilde{g}(\omega)^2 + \alpha \omega^4}$ ; satisfies $\tilde{f}_n^\alpha = \tilde{\varphi}_\alpha \tilde{h}_n$ .	17
$f^\alpha$	$f^\alpha = \arg \min_v \ g * v - h\ ^2 + \alpha \ v^{(2)}\ ^2$ ; i.e. minimizer of Tikhonov functional for <i>exact</i> density $h$	24
$\delta_n^2$	$\delta_n^2 = \mathbb{E} \ h_n - h\ ^2$ ; the MISE of estimator $h_n$ of $h$	26
$W(\cdot)$	Lambert $W$ function; $t = W(x)$ solves $te^t = x$ ; $W(x) \sim \log(x)$ as $x \rightarrow \infty$	30
$\mathcal{C}$	$\mathcal{C} = \{v \in L_2(\mathbb{R}), v \text{ is a pdf}\}$ ; set of square-integrable pdfs	31
$P_A$	$P_A u = \arg \min_{v \in A} \ v - u\ $ ; metric projection of $u$ to closed, convex $A$	31
$\hat{f}_n^\alpha$	$f_n^\alpha$ projected to some set of pdfs	31
$\mathcal{C}_t$	$\mathcal{C}_a = \{v \in \mathcal{C}, v(x) = 0 \ \forall x \notin [-t, t]\}$ ; set of square-integrable pdfs with support in $[-t, t]$	32
$\mathcal{S}$	$\mathcal{S} = \mathcal{S}(\{t_i\})$ ; Space of cubic splines with knots $\{t_i\}$	36
$B_i$	B-spline basis functions for $\mathcal{S}$	37
$\mathbf{M}, \mathbf{P}, \mathbf{G}$	Inner-product matrices for $g * B_i$ , $B_i^{(2)}$ , and $B_i$ , respectively	38