What is a *Thing*?

David Jaz Myers

Johns Hopkins University

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Galileo's Argument

Suppose that heavier things fell faster than lighter ones. Then, if we tied a light stone to a heavy stone, it would slow the heavy stone down because it falls slower. But the whole thing is heavier than its parts, so it should speed up. This is a contradiction, so we know that things fall at the same speed regardless of their weight.

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What about tying the stones together makes them *part of the same thing*?

Basic Questions

▶ What is a *thing*?

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- How do things come to be, and cease?
- ► How can we set up a system to make or maintain the things we want, and end the things we don't?

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▶ Given a model of some system, what *things* are in this model?

Idea: If you pull on part of a thing, the rest will come with.

Idea: If you constrain part of a thing, the rest is constrained as well.

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The question "Is this a thing?" will be answered in terms of:

► The relationship between constraints on the parts and constraints on the whole.

The Two Noodles Thought Experiment

[noodle waving]

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To answer this, we need

- A notion of "system" (or "model"),
- A notion of "part",
- A notion of "constraint",
- ▶ An understanding of how the constraints of some part of the system constrain other parts.

Formalizing Our Question

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So, we should model a system by its type of behaviors!

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Whatever they are, they form a category \mathcal{B} ! (The morphisms will be functions sending behaviors to behaviors.)

But we want to reason about behaviors using *logic*, so we need the category $\mathcal B$ of behavior types to be a *topos*.

The Briefest Introduction to Toposes

A topos is a category where you can do logic.

Definition

A topos is a category that has

- a terminal object and pullbacks,
- ▶ an internal hom $(-)^X$ (right adjoint to $X \times -$).
- a subobject classifier Prop.

The Briefest Introduction to Toposes

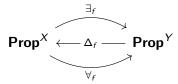
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Given $f: X \to Y$, we get an adjoint triple:



What is a Part?

- ▶ If B_S is the type of possible behaviors of our system S, and P is a part of S,
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- ▶ and every behavior $p: B_P$ arises in this way (since P is considered as part of S, not on its own).

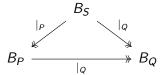
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Definition

If B_S is the behavior type of some system S, a part P of S is an epimorphism $|_P: B_S \twoheadrightarrow B_P$.

A part P contains Q (written $P \ge Q$) if there is an epi $|_Q : B_P \twoheadrightarrow B_Q$ so that



Compatibility and the Lattice of Parts

Definition

Behaviors $p: B_P$ and $q: B_Q$ of parts P and Q are compatible if there is a behavior s of the whole system which restricts to both of them:

$$\mathfrak{c}(p,q) :\equiv \exists s : B_S. p = s|_P \wedge s|_Q = q.$$

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▶ The union $B_{P \cup Q}$ of parts P and Q has behaviors given by compatible pairs of behaviors from P and from Q:

$$B_{P\cup Q}:\equiv\{(p,q):B_P\times B_Q\mid \mathfrak{c}(p,q)\}.$$

▶ The intersection $B_{P \cap Q}$ of parts P and Q has behaviors which are either behaviors from P or from Q, but considered equal if they are compatible:

$$B_{P\cap Q}:\equiv \frac{B_P+B_Q}{\mathfrak{c}}.$$

Parts as Equivalence Relations

Given a part $B_S woheadrightarrow B_P$, we can consider the equivalence relation on behaviors of S

$$s \sim_P s' \iff s|_Q = s'|_Q$$

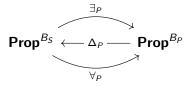
that is, $s \sim_P s'$ if they involve the same behavior of Q, if "Q sees them to be the same".

Constraints

We will equate a *constraint* ϕ on the behaviors of a part P with predicate "satisfies ϕ " on B_P . That is, $\phi: B_P \to \mathbf{Prop}$.

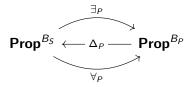
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A quick calculation gives:

$$\Delta_P \circ \exists_P \phi(s) = \exists s'. \, s \sim_P s' \land \phi(s')$$

$$\Delta_P \circ \forall_P \phi(s) = \forall s'. \ s \sim_P s' \Rightarrow \phi(s')$$

Induced Constraints

Definition

A constraint ϕ on a part P induces two interesting constraints on a part Q.

• "Is compatible with ϕ ": $\Diamond_Q^P :\equiv \exists_Q \circ \Delta_P$

$$\Diamond_Q^P \phi(q) :\equiv \exists s : B_S. \, s|_Q = q \wedge \phi(s|_P).$$

• "Ensures ϕ ": $\square_Q^P := \forall_Q \circ \Delta_P$

$$\Box_Q^P \phi(q) :\equiv \forall s : B_S. \, s|_Q = q \Rightarrow \phi(s|_P).$$



Properties of Induced Constraints

Claim

- If $\phi \Rightarrow \psi$, then $\lozenge_Q^P \phi \Rightarrow \lozenge_Q^P \psi$ and $\square_Q^P \phi \Rightarrow \square_Q^P \psi$
- $ho \ \Diamond_P^P = \Box_P^P = \mathrm{id}$
- $ightharpoonup \Box_Q^P \dashv \Diamond_P^Q$

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Measuring with Numbers

Suppose we have a notion of size $\#B_P : \mathbb{R}$ for each behavior type we are considering (and their subtypes)

We can then define the *constraint ratio* for $\phi: B_P \to \mathbf{Prop}$

$$constr(\phi, P) :\equiv \frac{\#B_P - \#\{\phi\}}{\#B_P}$$

as a measure of "how constrained P is by ϕ ".

Then the *constraint rate* for $\phi: B_P \to \mathbf{Prop}$ and part Q

$$\mathsf{R}(\phi, Q) := rac{\mathsf{constr}(\lozenge_Q^P \phi, Q)}{\mathsf{constr}(\phi, P)}$$

as a measure of "how constrained Q is by ϕ , relative to how constraining ϕ is".



Examples

[graph time]