An improved metric for the analysis of swarms

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***Abstract*—This paper examines the distance metric as a mechanism to measure the internal movement of agents and introduces a new *magnitude based metric*. Both metrics are based upon the resultant internal movement of a swarm identified by analysing the changes in the inter-agent interactions. The two metrics differ in their approach to identifying the changes. The distance metric uses variations in the inter-agent spaces and the new metric uses the resultant magnitudes from an agent's *inter-agent vector magnitudes* that are calculated from an agent's fields.**

Both metrics allow a comparison of the effects of different swarming algorithms on a swarm's structure to idenfify the 'level' of change in a swarms internal structure which is an indication of the amount of unnecessary movement an algorithm creates. The magnitude based metric also provides additional information by allowing the current 'state' of a swarm to be identified i.e if is it expanding or a cohesive member of a swarm.

***Index Terms*—Swarming, Swarm Dynamics, Mobile Sensor Networks, Algorithms.**

I. Introduction

This paper presents a swarm model and then applies the two metrics to this model to present a comparison of the two metrics and also to use the new metric to highlight a swarms 'state'.

II. Swarm modelling

Currently, much swarm research uses field effects as the method of modelling inter-agent interactions [1], [2], [3], [5], [6], [7], [8], [9], [10], [11]. The models usually use two field effects to implement the swarming characteristic. These effects are *cohesion*, to draw agents closer, and *repulsion* to prevent agents colliding. Fields are the ranges around an agent that determine the effect other agents have upon its movement (Figure 1). It is usual for the cohesion field to have a radius C_b which is larger than the repulsion radius R_b. When an agent b' moves into the *cohesion field* of an agent b then b' is said to be a neighbour of b and is subject to cohesion. When an agent b' moves into the repulsion field of b then b has a tendency to move away from b', i.e. to be repulsed. When an agent b moves too close to an obstacle, i.e. within the obstacle repulsion range O_b, it has a tendency to move away from the obstacle.

III. Inter-agent vector magnitude effect on internal movement

Figure 2 shows the cohesion and repulsion vector contributions to $v_c(b)$ and $v_r(b)$ due to neighbour b', as given in (1, 2). Notice that the vectors are along the line of separation bb'.

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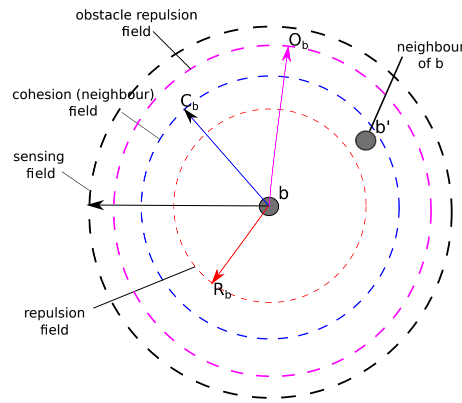

Fig. 1. Agent field effects

Equation (1) identifies the resultant cohesion effect of all the neighbours of b. $nbr(.)$ returns a set of all the agents in the swarm that are a neighbour of b. A neighbour is any agent that fails within the cohesion field.

$$v_c(b) = \frac{\sum\limits_{b' \in nbr(b)} bb'}{|nbr(b)|} \tag{1}$$

Equation (2) identifies the resultant repulsion effect of all the neighbours of b where $rep(.)$ returns a set of all agents that are within the repulsion field.

$$v_r(b) = -\frac{1}{|R(b)|}\left(\sum_{b' \in rep(b)} \left(1 - \frac{|bb'|}{R_b}\right) bb'\right) \tag{2}$$

Using the cohesion and repulsion vectors generated by the relationship of b to its neighbour a resultant vector can be calculated. This vector creates an agent characteristic that can be used as a metric. Summing the vectors creates a resultant vector with a magnitude that affects the agent. Summing the vectors also provides an indication of the direction an agent will move based on the relationship. This is the *inter-agent vector*.

Equation (3) identifies the *inter-agent vector* for agent b with respect to its neighbours.

$$v(b) = v_c(b) + v_r(b) \tag{3}$$

If the system is applying a weighted model then equation 3 becomes:

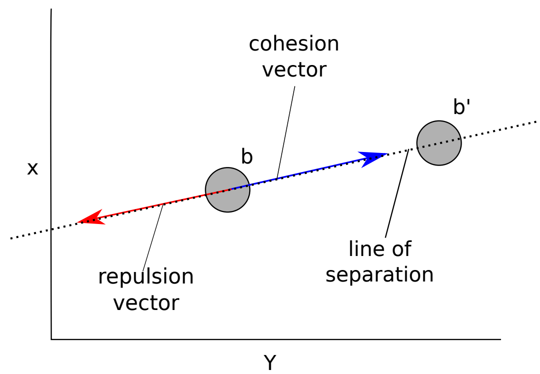

Fig. 2. Vectors on line of separation

Weight Component	Setting	Description
Sample Rate	100	ms - Unit sampling interval
k_c	5	weight adjuster for cohesion bias
k_r	15	weight adjuster for repulsion bias
k_d	0	weight adjuster for directional bias 0 for static baseline 100 from directional
Repulsion Boundary	70	units
Neighbour Distance	80	units
Speed	20	units/s

TABLE I
SWARM MODEL PARAMETERS

$$v(b) = k_c v_c(b) + k_r v_r(b) \tag{4}$$

where k_c and k_r are weightings to change the effect of each vector as shown in Table I.

IV. SWARM MOVEMENT ANALYSIS

The repulsion and cohesion vectors are generated for an agent through the interaction of their field effects (Figure 1). There are a limited number of interactions that can occur; These are illustrated in Figures 3, 4, 5 and 6.

The example data extracts (Tables II, III, IV and V) show simulation results that are produced by the parameters listed in Table I. The simulation consists of 200 agents over a 20 second period. The cohesion of an agent pair is shown as $k_c v_c$ and the repulsion as $k_r v_r$.

Figure 3 shows two agents within each other's cohesion fields but sufficiently distant to be outside of the repulsion fields. In this case $|k_c v_c| > 0$ and $|k_r v_r| = 0$: the result is the agent's resultant magnitudes cause the agents to move towards each other. Table II shows the repulsion magnitude with a value of 0. The only influence on the agent pairs are cohesive vectors.

Figure 4 shows two agents close together with repulsion dominating cohesion such that $|k_c v_c| < |k_r v_r|$. The resultant vector direct the agents away from each other.

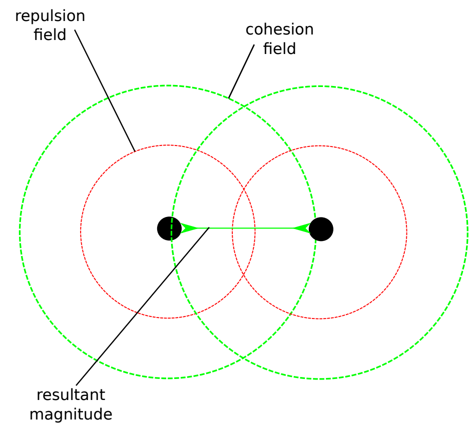

Fig. 3. Internal movement cohesion (no repulsion)

Log	Id	N.Id	Distance	Cohesion	Repulsion
0	1	3	70.50359	352.51799	0
0	1	100	71.78005	358.90027	0
0	1	151	78.33995	391.69979	0
0	2	99	72.04066	360.20334	0

TABLE II
DATA EXTRACT ($|k_r v_r| = 0$)

Table III shows the repulsion magnitude with a value greater than cohesion.

Figure 5 shows two agents close together but with cohesion vector magnitudes greater than the repulsion magnitudes $|k_c v_c| > |k_r v_r|$. The resultant vector draws the agents together. The magnitude of the resultant cohesion vector reduces due to the cancelling effect of the repulsion vector. Table IV shows a data extract with the cohesion magnitude greater than repulsion.

Figure 6 shows two agents close together with $|k_c v_c| = |k_r v_r|$ the resultant vector is a *null vector* and the agents have no influence upon each other due to the magnitude of the resultant vector being zero. Table V is an extract of data from the simulator. The data shows near equilibrium but due to the dynamic nature of a swarm system no agents meet the condition fully.

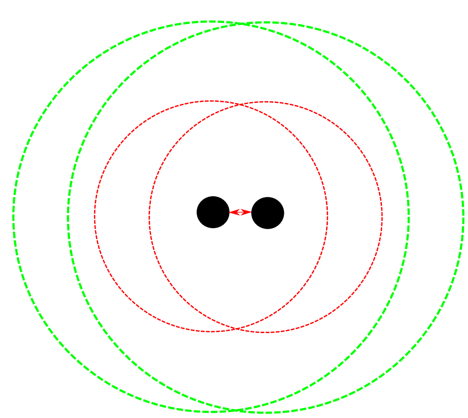

Fig. 4. Internal movement repulsion

Log	Id	N.Id	Distance	Cohesion	Repulsion
0	1	2	28.32522	141.62612	1544.86014
0	1	6	41.48517	207.42586	721.71736
0	1	7	35.26412	176.32064	1034.27101
0	1	8	43.54503	217.72518	637.90759

TABLE III
DATA EXTRACT ($|k_c v_c| < |k_r v_r|$)

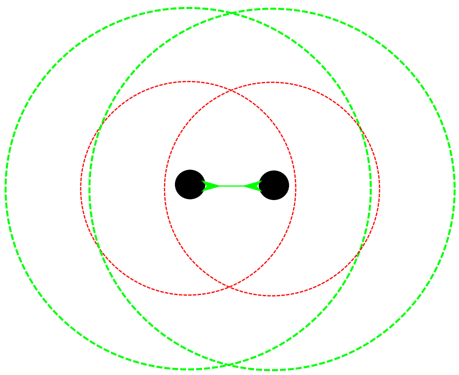

Fig. 5. Internal movement cohesion

Log	Id	N.Id	Distance	Cohesion	Repulsion
0	1	5	64.17214	320.86072	95.35676
0	1	9	63.88049	319.40248	100.58590
0	1	95	65.61522	328.07613	70.16681
0	1	152	63.10700	315.5350	114.68844

TABLE IV
DATA EXTRACT ($|k_c v_c| > |k_r v_r|$)

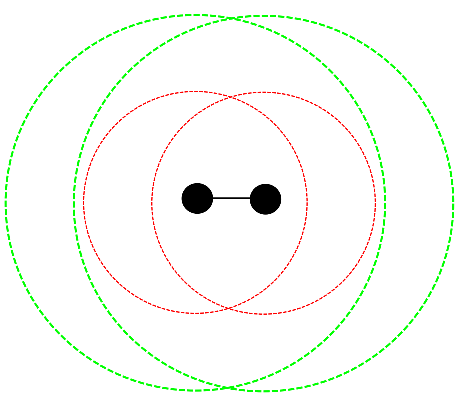

Fig. 6. Internal movement equilibrium

Log	Id	N.Id	Distance	Cohesion	Repulsion
7	76	91	55.39031	276.95156	276.94684
24	75	6	55.39032	276.95160	276.94661
32	72	38	55.39002603773678	276.95013	276.95370
35	63	64	55.39022	276.95113	276.94887

TABLE V
DATA EXTRACT ($|k_c v_c| \approx |k_r v_r|$)

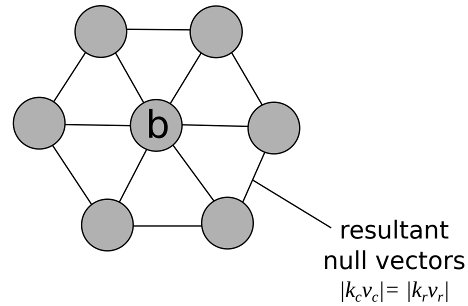

Fig. 7. Equilibrium with null vectors

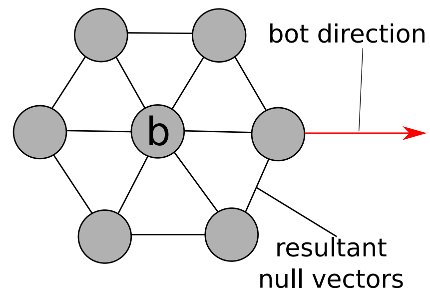

Fig. 8. (t)

V. INTERNAL MOVEMENT AND THE NULL VECTOR

When the two vectors (cohesion and repulsion) have magnitudes that are equal and opposite they produce a null vector. This indicates that two agents are optimally spaced for a given set of conditions. Although the agents are at an optimum position it does not mean the swarm is optimally distributed. If a swarm is in a confined space it is possible for an optimum position to be created where the vector magnitude is positive and there is a compression effect. This phenomenon is used in the identification of the emergent behaviour of *area flooding*.

If we consider the equilibrium state (Figure 6) the resultant vector of b is $(0,0)$. A null vector cannot be normalised to produce a directional vector ($\hat{v} = \frac{v}{|v|}$ if $v \neq 0$; 0 if $v = 0$) The effect of the resultant magnitude being a null vector is that the agent will remain stationary. If all agent pairs are in this condition the swarm will stop moving (Figure 7).

Due to the independent nature of the agents this situation is very rare. The residual motion that persists in a swarm is the background 'noise' or 'jitter' that an algorithm creates.

If a swarm is goal-based the addition of a *directional vector* will prevent all agents simultaneously producing null vectors (Figure 9, Equation 5). Where $k_d v_d(b)$ is the calculated directional vector to be applied to an agent.

$$v(b) = k_c v_c(b) + k_r v_r(b) + k_d v_d(b) \tag{5}$$

VI. RESIDUAL INTERNAL MOVEMENT (JITTER)

Due to the dynamic nature of a swarm maintaining optimum internal movement as in (Figure 7) a stationary swarm is highly unlikely. The agent pairs will fluctuate between the

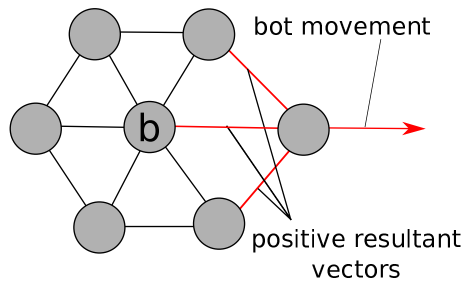

Fig. 9. (t + 1)

3 states (Figures 4, 5 and 6). This alternation between the states is *jitter*. The degree to which this variation occurs can be measured using either the change in distance between the agent pairs, or the change in the resultant magnitude between the agent pairs. Jitter is motion that is produced to maintain the structure of a swarm. A coordination algorithm that produces minimal jitter is generally more desirable. Jitter (the fluctuation in and between the states) is an indication of the efficiency of an algorithm and an integral component of a swarms measurable behaviour.

VII. Magnitude based metric

Magnitude based internal movement (*agent resultant magnitude*) is measured by identifying the balance between the repulsion and cohesion between agents. 'Jitter' in the case of the *agent resultant magnitude* metric is measured as the variance of the magnitude lengths created by the agents. The identification of this variance produces the clarifying part of the *agent resultant magnitude* metric. The *agent resultant magnitude* is identified by Gazi and Passino [10] and Barnes et al [4] as a 'resultant characteristic' of a swarm. There are two ways of using the cohesion and repulsion in identifying a resultant vector. The two vectors can be added as absolute values to give an overall 'size' to the magnitude that is affecting each relationship. Alternatively the resultant magnitude can be the sum of the actual magnitudes. The repulsion vector has a negative magnitude and the cohesion vector has a positive magnitude. In this paper the magnitude analysis will be based on summing the two actual vectors to determine the result of the inter-agent interaction, although both mechanisms could be used for a comparitive analysis. This paper will refer to the resultant magnitude as the 'agent resultant magnitude' of the relationship. The 'state' of a swarm is therefore the effect the environmental constraints and algorithms have upon the agent resultant magnitude. the 'agent resultant magnitude' can be considered part of the 'quality' measure for a swarm's performance.

If the *agent resultant magnitude* is a negative value (absolute values would prevent this analysis) the swarm's bias is to expand. This is seen in the chaotic stage of a swarm. If the *agent resultant magnitude* is positive then the swarm is exhibiting a tendency to contract and this indicates the swarm is a cohesive entity. This could also be described as the swarm being 'sticky' as the agents bias is to 'pull' towards each other.

The *agent resultant magnitude* on its own does not give a complete measure of a swarm's internal state. There needs to be a qualifying component to the metric that identifies the degree of deviation in the resultant magnitude, this is the *jitter*. The smaller the degree of deviation the more uniform the structure of the swarm. These two components identify the degree to which a swarm has progressed towards a stable state.

The *agent resultant magnitude* provides a view of the swarm's state through the balance between the repulsive and the cohesive vectors that are being applied to each agent. The variance component identifies the degree to which the swarm has stabilised. The ideal status for inter-agent interactions would be for the agents to have a resultant vector (*agent resultant magnitude*) of zero or above. This would indicate that the agents are distributed such that they are at their distribution limit (outer most range of the cohesion field) or at a level that causes the agents to 'pull' together. The ideal degree of deviation is zero as this indicates an even distribution of agents. Therefore for a fuller indication of a swarms *state* both measurements need to be combined. The deviation from the mean clarifying the internal movement and the *agent resultant magnitude* providing an indication of the 'compression' that a swarm is logically experiencing (cohesiveness). These two aspects of a swarm's features are not considered by Gazi and Passino [10] or Barnes et al [4] as a means of quantifying the structure of a swarm in terms of stability.

VIII. Distance based metric

The distance based metric considers the effect of the resultant vectors upon a swarm in terms of how the agents are physically distributed: i.e. only the inter-agent distances and the deviation from the mean of the agents (jitter) are considered. As with the *agent resultant magnitude* metric the variations are important to determine the agent distribution. The standard deviation from the mean allows the internal 'characteristic' of the measure to be realised. If the standard deviation is zero then all the agents are evenly spaced. The distance metric does not take into consideration the vector magnitudes between the agents as discussed above. The metric therefore is unable to identify the potential state of the swarm in terms of its cohesive or repulsive state.

Navarro and Fernando describe a mean distance error metric that is based on the variations in distances between inter-agent spaces [12]. This is the same as the standard deviation of the distance based internal movement metric as described is here.

IX. Magnitude based internal movement model

Using the formulae for the calculation of cohesion (Equation 1, page 1) and repulsion (Equation 2, page 1) for every agent and its neighbours it is possible to calculate an *agent resultant magnitude* value (sum of agent resultant magnitudes). This value represents the overall potential of an agent. This magnitude when normalised produces a component of the *movement-destination vector* for a swarm. If the agent resultant magnitude is zero (null vector) then the agent will not move. $P(b)$ is the *inter-agent resultant magnitude vector* for agent b defined by:

$$P(b) = k_c v_c(b) + k_r v_r(b) \tag{6}$$

Although it is possible for agent b to have a resultant vector of null there could still be a variation in the constituent components. The variation calculation (standard deviation) is shown in X. 7 is the mean of the *agent resultant magnitudes* for an agent and its neighbours where $|nbr(b)|$ is the number of neighbours.

$$\mu_p(b) = \frac{P(b)}{|nbr(b)|} \tag{7}$$

To identify the swarm based *agent resultant magnitude* 7 must be extended to iterate over all the agents in the swarm. 8 shows $\mu_p(S)$ as the swarm based magnitude where the swarm iteration is shown as $\sum_{b \in S}$ and $\sum_{b \in S} |nbr(b)|$ calculates the total number of inter-agent relationships.

$$\mu_p(S) = \frac{\sum\limits_{b \in S} P(b)}{\sum\limits_{b \in S} |nbr(b)|} \tag{8}$$

X. Variance in agent resultant magnitude metric

The mechanism just described provides an overall indication of the internal movement based on inter agent vectors that produce the *agent resultant magnitude*. This model however is not sufficient to give an indication of the swarm 'state' as an overall metric. To improve the metric clarification is required in terms of the deviation from the *agent resultant magnitude* norm. The variation in the metric is the standard deviation of the entire swarm from the mean of the inter-agent potential magnitudes (Equation 7).

The standard deviation is calculated as Equation 9 where $\sigma_p(S)$ is the standard deviation at a time t and $\mu_p(S)$ is the mean at the same point in time. $\sum_{b \in S} \sum_{b' \in nbr(b)}$ iterates over every agent in the swarm and its neighbours and $\sum_{b \in S} |nbr(b)|$ calculates the total number of inter-agent relationships.

$$\sigma_p(S) = \sqrt{\frac{\sum\limits_{b \in S} \sum\limits_{b' \in nbr(b')} \Big(P(b') - \mu_p(S)\Big)^2}{\sum\limits_{b \in S} |nbr(b))|}} \tag{9}$$

The metric for the internal movement is a set of numbers, the mean and standard deviation of the swarms internal *agent resultant magnitude* derived from each agent and its neighbour interactions (14). The pair $\mu_p(S)$, $\sigma_p(S)$ may be written informally as:

$$\psi_p = \mu_p(S) \pm \sigma_p(S) \tag{10}$$

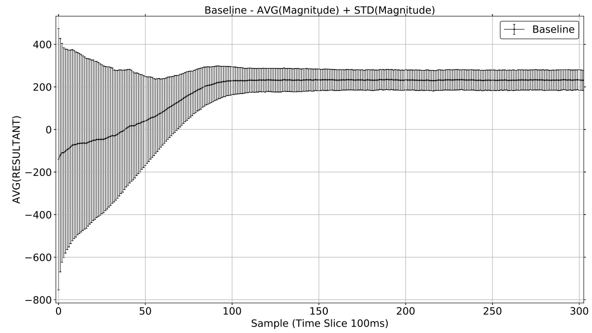

Fig. 10. Baseline internal movement - magnitude

XI. Distance metric

The distance based internal movement is measured by identifying the mean length of the vectors between an agent and its neighbours. As with the *agent resultant magnitude* a coordination algorithm produces 'jitter' which is the variations from the mean. In the case of the distance based metric the jitter is identified by the changes in the distances rather than the changes in vector magnitude (*agent resultant magnitude*). The distance metric is the mean and the standard deviation 'jitter' of the inter-agent distances.

XII. Calculating distance based internal movement

The relative position vector generated for an agent b to its neighbour b', bb', is shown in (1). The magnitude of that vector gives the distance between two agents. For an individual agent the average magnitude $\mu_d(b)$ is calculated as 11 where b is the agent and $|nbr(b)|$ is the number of neighbours.

$$\mu_d(b) = \frac{\sum\limits_{b' \in nbr(b)} |bb'|}{|nbr(b)|} \tag{11}$$

Equation 11 identifies the mean distance for an individual agent. The mean distance for a swarm is calculated by 12. All the inter-agent interactions must be included for the swarm (S). $\sum_{b \in S} |nbr(b)|$ calculates how many inter-agent relationships exist in the swarm and $\sum_{b' \in nbr(b)} |bb'|$ calculates the total distance between each agent and its neighbours. $\sum_{b \in S}$ iterates over all the agents in the swarm (S).

$$\mu_d(S) = \frac{\sum\limits_{b \in S} \sum\limits_{b' \in nbr(b)} |bb'|}{\sum\limits_{b \in S} |nbr(b)|} \tag{12}$$

XIII. Variance in distance metric

The mechanism above provides an overall indication of the distribution of the agents. This model, as with the agent resultant magnitude model, is not sufficient to give an indication of the internal distribution of the agents. The addition of the standard deviation from the norm clarifies the distribution within

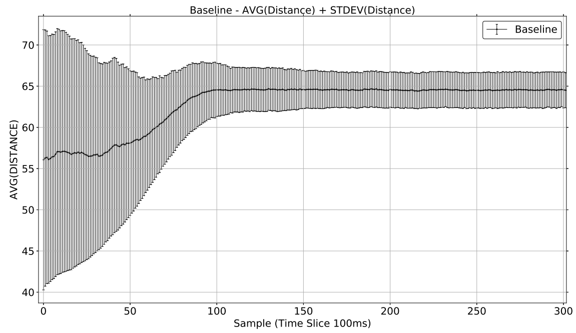

Fig. 11. Baseline internal movement - distance

the swarm as shown in 13. $(|bb'| - \mu(S))^2$ is the square of the difference in a distance to the mean and $\sum_{b\in S} \sum_{b'\in nbr(b)}$ calculates the number of inter-agent interactions.

$$\sigma_d(S) = \sqrt{\frac{\sum_{b\in S} \sum_{b'\in nbr(b)} \left(|bb'| - \mu_d(S)\right)^2}{\sum_{b\in S} |nbr(b)|}} \tag{13}$$

The distance metric for the internal distribution of the agents is the pair consisting of $\mu_d(S)$, $\sigma_d(S)$ the mean and the standard deviation of the swarms internal resultant distances from every agent in the swarm. This can be written informally as:

$$\psi_d = \mu_d(S) \pm \sigma_d(S) \tag{14}$$

XIV. Conclusion - metric comparison

The two metrics appear to be similar in terms of the measurement of the structure of a swarm. The main difference is in how these two metrics can be used when examining the state of the swarm.

Both metrics identify the state of a swarm with respect to variations in the dispersement of the agents from an average distribution.

The main difference in the metrics is that the distance metric is based upon the physical *distribution* of the agents and the magnitude based metric is based upon the logical *interaction* of the agents.

The distance based metric provides and analysis of the actual distribution of the agents at a point in time and allows the agitation of the swarm to be assessed without considering the possible distribution of agents that the field effects *could* produce.

The *agent resultant magnitude* metric provides a view of the interaction magnitude. This provides an indication of the swarms potential movement. This is independent of the physical distribution. The lack of dependence on the physical distribution allows the metric to be used in heterogeneous field effect swarms 12 where the physical distribution may vary.

Combining the two metrics allows a deeper evaluation of a swarm to be made. Consider the following: the repulsion field

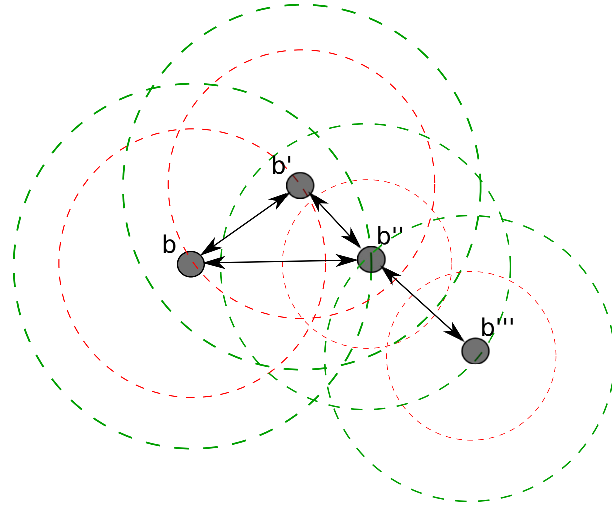

Fig. 12. Complex swarm interactions

is increased but the internal distances do not change as a result the *agent resultant magnitude* rises: This indicates 'something' is confining the swarm's distribution. This analysis could be used in identifying effective swarm distribution for the coverage of a sensor array as discussed by Ramaithitima et al. [13]

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