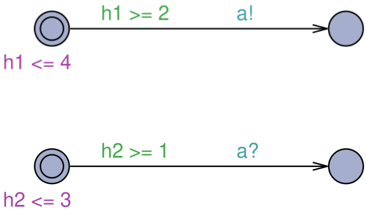
Embedded Systems Specification and Design
Model-based Design and Verification

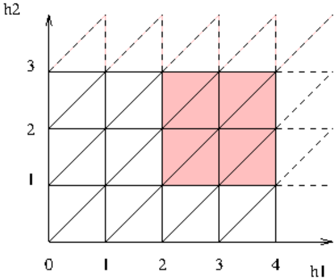
David Kendall

State space of timed automaton is infinite ?!!

- Introduction of real-valued clock variables to FSM makes its state space infinite
- How can we exhaustively explore infinite state space?
  - We can't
- How can we exhaustively explore state space of timed automaton?
  - Find an equivalent finite representation of its state space
- This is the key idea that makes model-checking of timed automata possible
- Finite representations
  - Region graph
  - Zone (simulation) graph

Zones and their representation





$$
\begin{array}{cccc}
M & h_0 & h_1 & h_2 \\
h_0 & (0, \leq) & (-2, \leq) & (-1, \leq) \\
h_1 & (4, \leq) & (0, \leq) & (\infty, <) \\
h_2 & (3, \leq) & (\infty, <) & (0, \leq)
\end{array}
$$

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\begin{array}{cccc}
M' & h_0 & h_1 & h_2 \\
h_0 & (0, \leq) & (-2, \leq) & (-1, \leq) \\
h_1 & (4, \leq) & (0, \leq) & (3, \leq) \\
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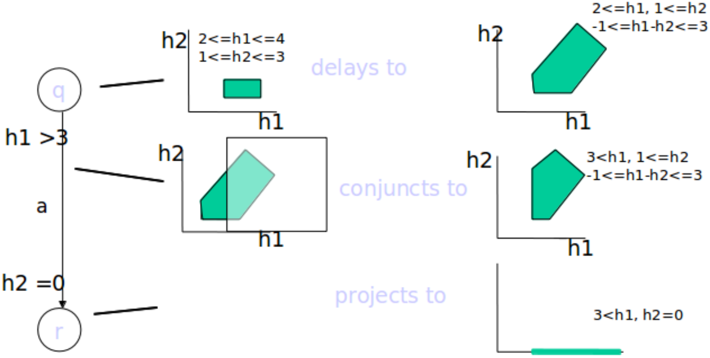
DIFFERENCE BOUND MATRICES

Operations on zones



Thus (q,2<=h1<=4,1<=h2<=3) =a => (r, 3<h1, h2=0)

Symbolic states and transitions

Symbolic state

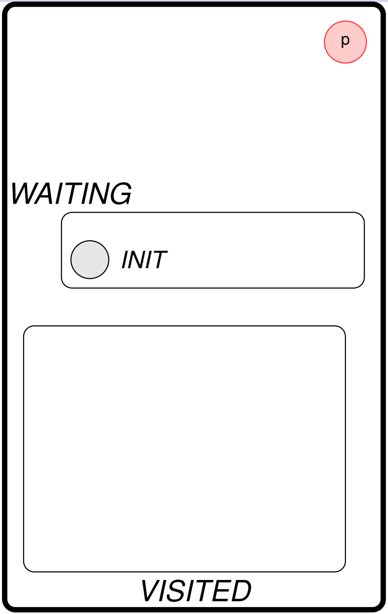
A symbolic state, $s = (\ell, \zeta)$, consists of:

- *a location vector*, ℓ, and
- *a zone*, ζ

Symbolic transition

If the edges of a network of timed automata justify a transition from location vector ℓ to location vector ℓ' and the delays to, conjuncts and projects operations transform the zone ζ to ζ', then for $s = (\ell, \zeta)$ and $s' = (\ell', \zeta')$, we say there is a *symbolic transition* from s to s' and write $s \Rightarrow s'$

Forward reachability



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$WAITING \leftarrow \{INIT\}$
$VISITED \leftarrow \emptyset$
while $WAITING \neq \emptyset$ **do**
    choose some $s = (\ell, \zeta)$ in $WAITING$
    if s satisfies p **then**
        return true
    else
        $NEW \leftarrow \{s' \mid s \Rightarrow s' \wedge s' \notin VISITED\}$
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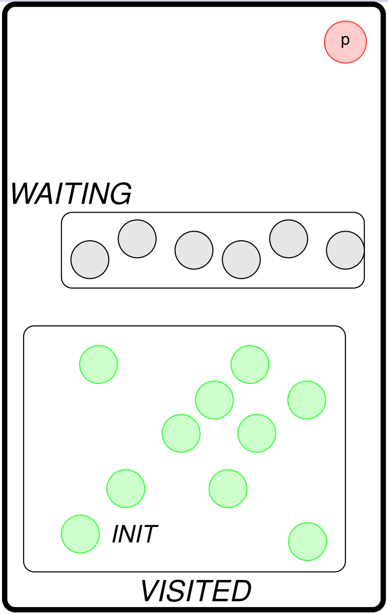

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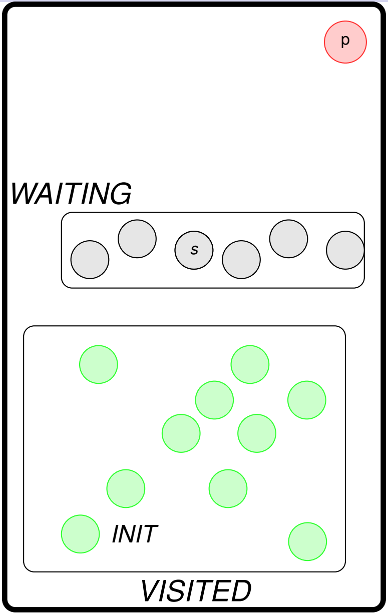

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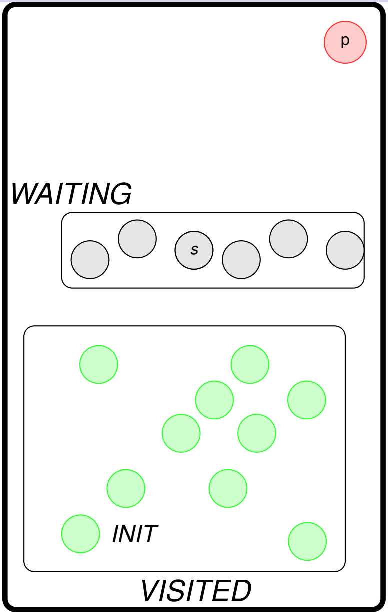

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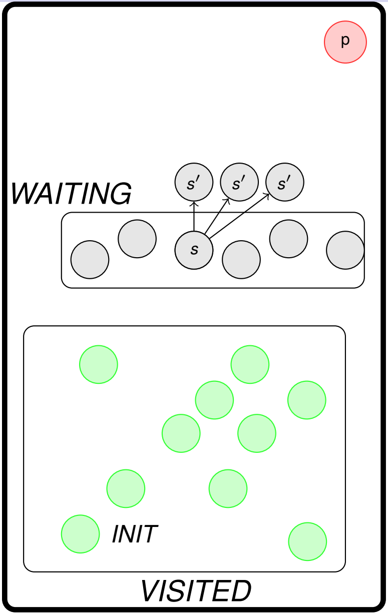

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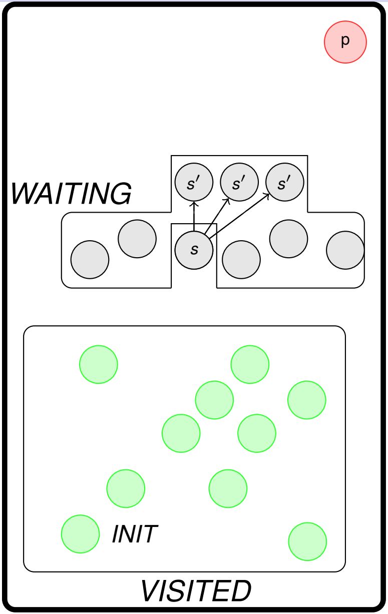

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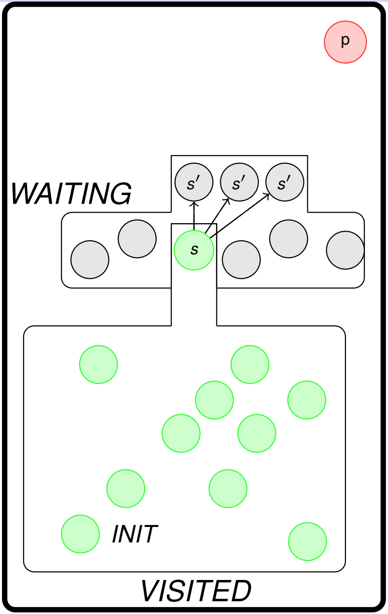

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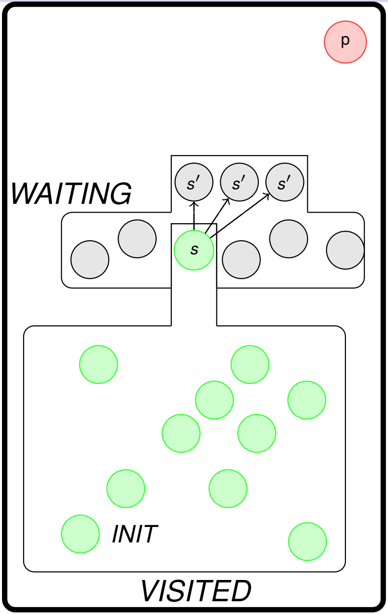

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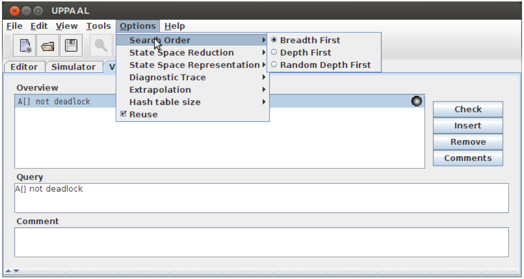
Search order



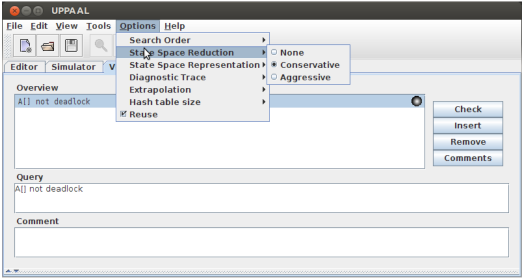
Breadth First:

- Search state space in breadth-first order
- Good for complete search and shortest/fastest trace

Depth First:

- Search state space in depth-first order
- Better than breadth-first if counter-example expected

State space reduction



- Inclusion abstraction
- Active-clock reduction
- ...

Inclusion abstraction

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    else
        $NEW \leftarrow \{s' \mid s \Rightarrow s' \wedge \nexists \zeta' \supseteq \zeta : (\ell, \zeta') \in VISITED\}$
        $WAITING \leftarrow WAITING \cup NEW \setminus \{s\}$
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    end if
end while
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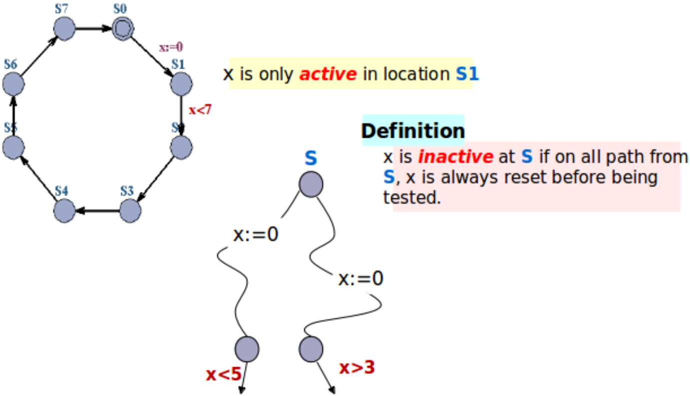
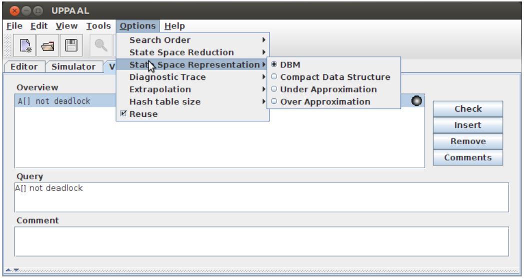
Active clock reduction



- Inactive clocks need not be stored
- ⇒ smaller DBMs
- ⇒ smaller state space

State space representation



Under-approximation

- Uses bit-state hashing
- Does not guarantee to explore all states
- So, if verifier claims that some state is unreachable, it may be wrong
- Useful mainly for discovering bugs, not for proving their absence

Over-approximation

- Uses convex hull abstraction
- May explore too many states
- So, if verifier claims that some state is reachable, it may be wrong
- Can prove absence of bugs but when it fails you need to ask if it's a genuine failure or a failure caused by the abstraction

State space explosion summary

- Infinite number of states can be represented finitely by using symbolic representation, e.g. *zones*
- State space explosion
  - State space may be finite but often still too large to handle with current computing power
- Attacks on state space explosion
  - Store fewer states
  - Store smaller states

And finally...

Much recent research has pushed model-based development using timed automata into new areas

- Priced timed automata
  - Reason about resource usage, e.g. energy
- Timed games
  - Controller synthesis
- Probabilistic timed automata
  - Reason about probabilistic systems, e.g. Bluetooth
- Statistical model-checking
  - Don't explore full state space, but use statistical techniques to calculate confidence in results based on partial exploration