Embedded Systems Specification and Design
Model-based Design and Verification

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Real-time Systems

Development of embedded systems is challenging due to:

- concurrency
- communication
- real-time
- resource constraints

So far, we have considered concurrency and communication.

What about real-time?

How to include explicit reference to time in models and specifications?

If a message is sent, it is eventually delivered – OK

If a message is sent, it is delivered within 45ms – ????

The essence of a formal method

Informally...

The system meets its specification

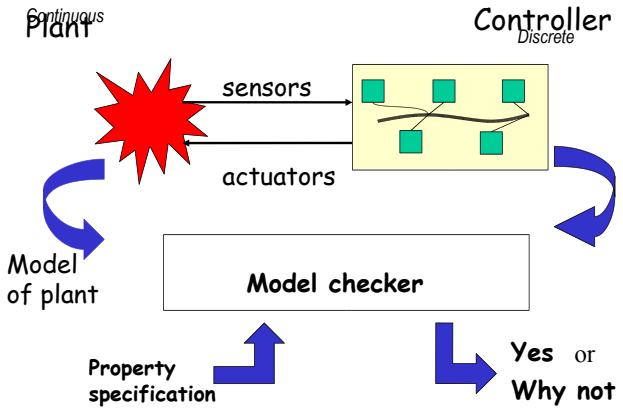
Formally...

$$\mathrm{Sys} \models \mathrm{Spec}$$

A formal language has well-defined

- syntax
- semantics
- rules for reasoning about the relationship between expressions

Model checking for embedded systems



Model-checking allows systems engineers to make practical use of formal methods

Model-checking provides the method for reasoning about the relationship between a model and a specification

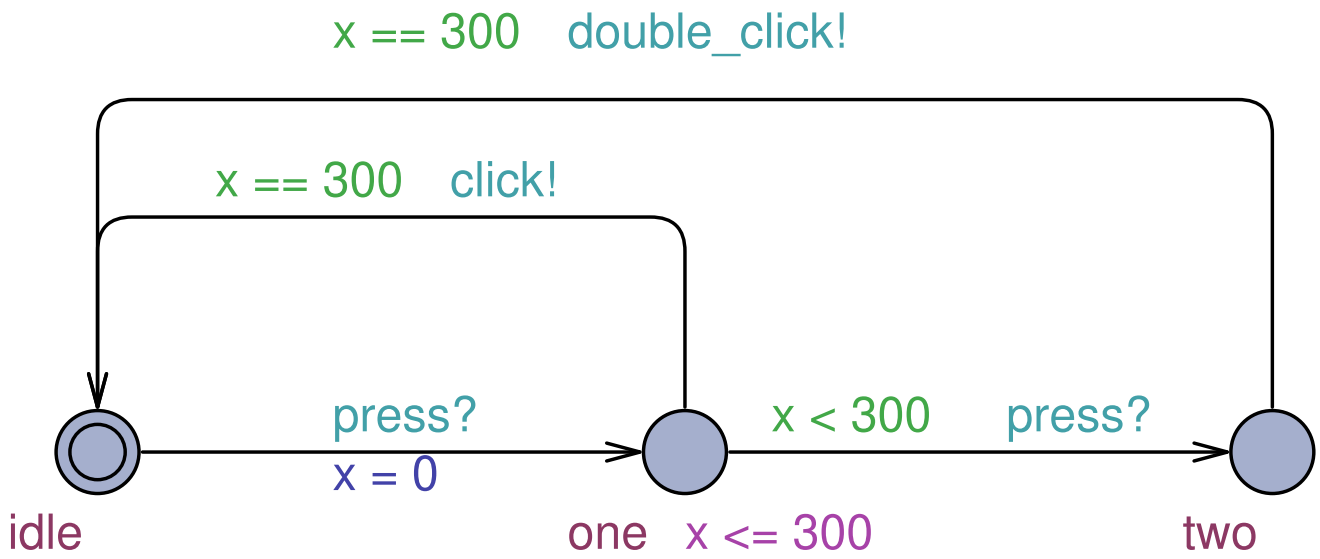
For real-time, we need

- a language for models that includes explicit references to quantitative time - timed automata
- a language for specifications that includes explicit references to quantitative time - timed temporal logic

Timed Automaton

- Timed automaton – finite-state machine extended with clock variables.
- Each clock variable evaluates to a real number.
- All clocks progress synchronously (and keep perfect time!)
  - This is an *abstraction*
- Clocks can be compared against lower and upper bounds. Comparisons can be strict or non-strict
- An invariant condition is associated with each location.
- Each edge has a guard, an action label and a set of clocks to reset.
- A system is modelled as a network of timed automata in parallel.

Timed Automaton Example 1

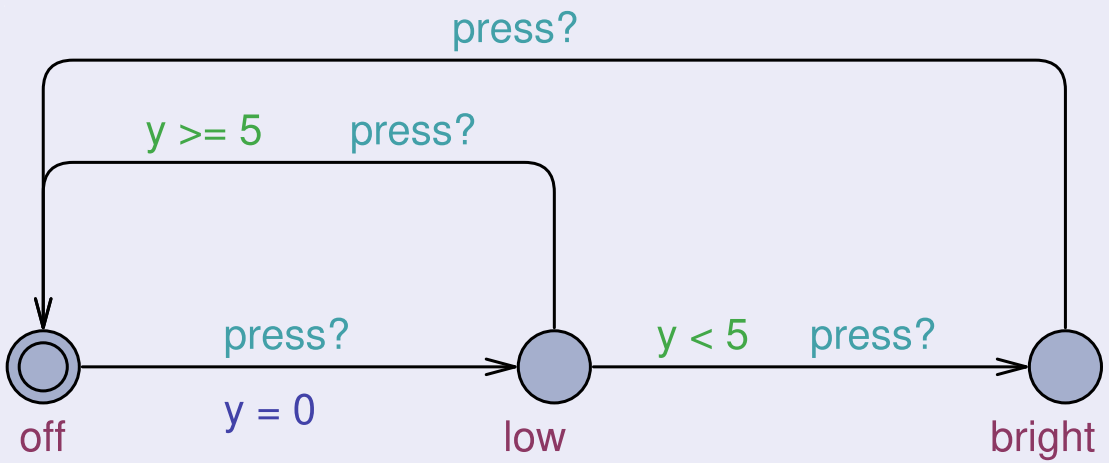


The example shows

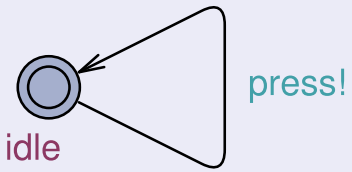
- Locations
- Edges
- Clock Guards and Invariants
- Synchronisation Labels
- Clock Resets

Timed Automaton Example 2

Lamp



User

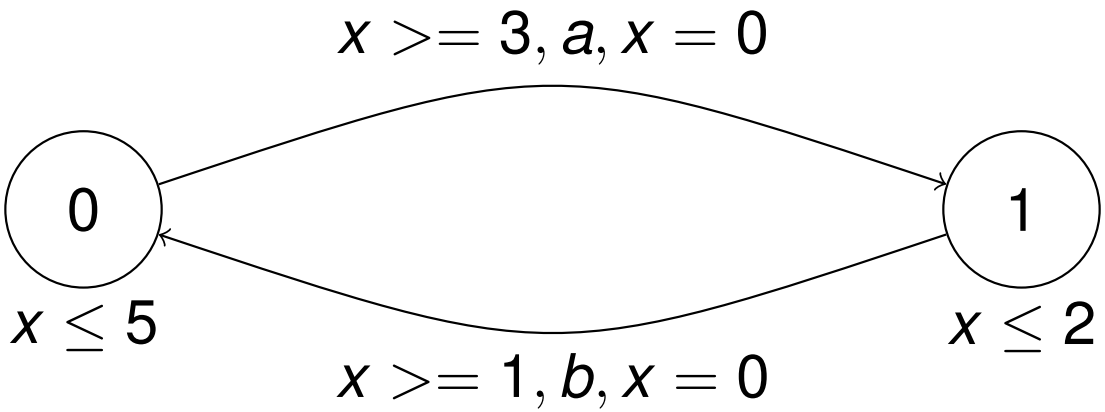


We need a User to interact with the Lamp

Anatomy of a timed automaton

Exercise:

- Name the parts...
- Location, Edge, Label, Guard, Reset, Invariant



Some notation for clocks

Assume C is a set of clock variables

Clock Guards and Invariants

- $B(C)$ is the set of conjunctions over simple conditions of the form $x \bowtie k$ or $x - y \bowtie k$, where $x, y \in C$, $k \in \mathit{Int}$ and $\bowtie$ is one of $<, \leq, =, \geq, >$

Clock Valuations

- A clock valuation v gives the current value of each clock in C, i.e. $v(x)$ gives the value of clock x
- $v \in [C \to \mathbb{R}_{\geq 0}]$ (can be written as $v \in \mathbb{R}_{\geq 0}^{C}$)

Clock Resets

- If $r \subseteq C$ then $v[r]$ is a clock valuation in which all the clocks in r have the value 0, and all other clocks have the same value as given by v, i.e. $\forall\, x \in r : v[r](x) = 0$ and $\forall\, x \in C \setminus r : v[r](x) = v(x)$

Some notation for clocks (ctd)

Time passes

- $v + d$ is the clock valuation in which all clocks have their value in v increased by d time units
- formally $v + d$ maps each clock $x \in C$ to the value $v(x) + d$

Constraint satisfaction

- Assume $g \in B(C)$ is some constraint
- $v \models g$ is the notation that means g is true for clock valuation v.

Zero valuation

- $\mathbf{0}_C$ is a clock valuation in which all clocks in C have the value 0

Exercise

Assume $C = \{x, y, z\}$, $v = [x \mapsto 1, y \mapsto 2, z \mapsto 3]$ and $r = \{x, z\}$.

- Write out the following valuations:
  1. $v + 3$
  2. $v[r]$
  3. $v[r] + 10$
  4. $(v + 10)[r]$
- Give clock valuations v such that
  1. $v \models x \leq 5 \wedge z \geq 2$
  2. $v \models y \leq 5 \wedge z \geq 2$

Timed Automaton Definition

A timed automaton is a tuple (L, ℓ_0, C, A, E, I), where

- L is a finite set of locations,
- $\ell_0 \in L$ is the initial location,
- C is a finite set of clocks,
- A is a finite set of actions, co-actions and the internal τ action,
- $E \subseteq L \times B(C) \times A \times 2^C \times L$ is a set of edges between locations with a guard, an action and a set of clocks to be reset
  - we often write $\ell \xrightarrow{g,a,r} \ell'$ for $(\ell, g, a, r, \ell') \in E$
- $I \in [L \to B(C)]$ assigns invariants to locations

Timed Transition System (TTS)

A timed transition system is a tuple $(S, s_0, \mathcal{L}, \longrightarrow)$ where

- S is the set of states
- s_0 is the initial state
- $\mathcal{L} = \mathbb{R}_{\geq 0} \cup A$ is the set of labels (assuming A is some set of discrete actions disjoint from $\mathbb{R}$)
- $\longrightarrow \subseteq S \times \mathcal{L} \times S$ is the transition relation, e.g. $s \xrightarrow{3.5} s'$, $s \xrightarrow{a} s'$.

Timed Automaton Semantics

The semantics of a timed automaton $A = (L, \ell_0, C, A, E, I)$ is given by a timed transition system, constructed as follows:

- The set S of states is the set of all possible combinations of locations and clock valuations (ℓ, v) where $v \models I(\ell)$, i.e. the invariant is satisfied
- The initial state s_0 is given by $(\ell_0, \mathbf{0}_C)$, i.e. the initial location with the zero clock valuation
- The set $\mathcal{L}$ of labels is $\mathbb{R}_{\geq 0} \cup A$
- The transition relation $\longrightarrow$ is given by the rules TA.1 and TA.2, following

Timed Automaton Semantics (ctd)

Definition (TA.1 - Action Transitions)

$(\ell, v) \xrightarrow{a} (\ell', v')$ iff $\exists\, \ell \xrightarrow{g,a,r} \ell' \in E : v \models g \wedge v' = v[r] \wedge v' \models I(\ell')$

Definition (TA.2 - Time Transitions)

$(\ell, v) \xrightarrow{d} (\ell, v + d)$ iff $d \in \mathbb{R}_{\geq 0}$ and $\forall\, d' : 0 \leq d' \leq d \Rightarrow v + d' \in I(\ell)$.

Timed Automaton Behaviour

A behaviour (run, execution, trace) of a timed automaton is an alternating sequence of time transitions and action transitions of its timed transition system, starting in the initial state

- Time passes, action occurs, time passes, action occurs, time passes, action occurs, ...
- e.g. $(idle, 0) \xrightarrow{150} (idle, 150) \xrightarrow{press?} (one, 0) \xrightarrow{250} (one, 250) \xrightarrow{press?} (two, 250) \xrightarrow{\cdots}$

Actions are instantaneous (take no time)

Time-consuming activities are modelled by distinct start and end actions, e.g. `start_transmission`, `end_transmission`

Exercise

Consider just the Lamp automaton from an earlier slide

1. Write down the components of the tuple for the Lamp automaton
2. Use the rules TA.1 and TA.2 to derive a (finite prefix of a) possible behaviour of the Lamp automaton

Properties of Time

- Deterministic whenever $s \xrightarrow{d} s'$ and $s \xrightarrow{d} s''$ then $s' = s''$, i.e. it's not possible to reach different states simply by passage of time
- Dense for any two time points $d1 < d2$, there exists a third point d such that $d1 < d < d2$
- Non-Zeno there is no bound on the progress of time, i.e. for any real value c, the time can progress beyond c.

Deadlock and Time-deadlock

- A state s in the TTS of a TA is a deadlocked state if there is no time delay d and action a such that $s \xrightarrow{d} s'' \xrightarrow{a} s'$
- A state s in the TTS of a TA is a time-deadlocked state if every execution from s is a Zeno execution. A Zeno execution is an infinite trace of a system in which the progress of time is bounded by some upper limit $c \in \mathbb{R}_{\geq 0}$
- Give examples of TA with deadlocked and time-deadlocked states.

Uppaal

- www.uppaal.org
- Integrated environment for modelling, simulation and verification of real-time systems
- Developed by universities of Uppsala and Aalborg
- Systems modelled as collection of extended TA
- Applications: real-time controllers, communication protocols etc
- Gearbox controller: Mecel AB and Uppsala